Multihoming and oligopolistic platform competition*

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Abstract

We provide a general framework to analyze competition between two-sided platforms, in which buyers and sellers can multihome, and platforms compete on transaction fees charged on both sides. The framework allows buyers and sellers to have heterogenous benefits from using platforms for transactions, and additionally, buyers to have idiosyncratic preferences over using the different platforms. We study how key primitives such as the number of platforms, the fraction of buyers that find multihoming costly, the value of transactions for buyers and sellers, the degree of buyer heterogeneity, and the possibility of competition between sellers determine the level and structure of platform fees.

JEL classification: L11, L13, L4

Keywords: two-sided markets, ride-hailing, fee structure

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1 Introduction

A growing number of two-sided platforms intermediate transactions between buyers and sellers (or providers) of products and services. Ride-hailing platforms (Uber and Lyft), meal/grocery delivery platforms (Doordash, Grubhub, Postmates and UberEats), hotel booking platforms (Booking.com and Expedia), e-commerce marketplaces (Amazon, Lazada, Shopee, Taobao in South East Asia), and payment card platforms (AMEX, MasterCard and Visa), are among the best-known examples. Key features of the markets in which these platforms operate are: (i) the fees charged by platforms to each side are transaction based; (ii) sellers are free to join multiple competing platforms (a phenomenon known in the literature as "multihoming"), which they typically do, and (iii) buyers are free to join multiple competing platforms, and to decide which of these platforms to complete a transaction on, if any.

Our interest in studying these markets stems from the observation that these markets have matured with multiple platforms competing head-to-head, and with no sign of tipping to any one player. It has become increasingly easy for users on both sides to multihome.[1]

In our framework, users (buyers and sellers) have heterogenous valuations over transaction (or interaction) benefits and platforms charge users on each side per-transaction fees. All users can costless join multiple platforms. Platforms are differentiated from the buyers' perspective, but are identical from the sellers' perspective. This captures the fact that in many two-sided market settings, sellers view competing platforms as more or less homogenous, while buyers usually have idiosyncratic preferences for using particular platforms over others.

We focus on the equilibrium fees that emerge from platform competition. The preference buyers have towards using certain platforms means that even though all buyers and sellers are multihoming, each platform has some market power over sellers. This reflects that sellers may lose too much business if they try to divert buyers to transact through lower fee platforms by delisting from platforms that charge more. Our framework highlights that multihoming in participation does not automatically lead to particular market outcomes because one also has to take into account users' preferences for transacting on certain platforms and not others. If buyers have strong preferences towards using a particular platform, the results resemble a competitive bottleneck type outcome in which platform competition is focused on attracting buyers and exploiting sellers (Armstrong, 2006; Armstrong and Wright, 2007). Total fees are also high in this case. If, on the other hand, buyers have very little platform loyalty, it is easy for sellers to divert buyers to use the lowest-fee platform to make transactions without worrying about buyers dropping out. In this case platforms have little market power over sellers (so seller fees tend to be low relative to buyer fees) and total fees are competed down close to cost.

We then use our framework to explore what happens when we change various primitives in the model. Our first major result is on how increased platform competition (i.e. entry) affects

[1] Following advancements in tools that make it easier for buyers to compare the options across multiple platforms, there has been a substantial shift in the capability and willingness on the buyer-side to multihome on platforms. For example, in the ride-hailing market, advancements in mobile phone technology and fare-comparison "metasearch" aggregators such as Google Maps, BellHop and RideGuru, allow more riders to easily compare fares across different ride-hailing apps, resulting in more active multihoming by riders. Similar aggregators have also become quite widely used for hotel booking platforms (Kayak and Trivago) and are currently emerging for food delivery platforms (Foodboss and Mealme).

the platforms' equilibrium total fee and fee structure. Increased platform competition has two effects. First, it makes the platforms more substitutable, which raises within-market competitive pressure, thus decreasing the platforms' market power over buyers and sellers (since sellers will find it easier to divert buyers' transactions to cheaper platforms). Second, the existence of a new platform for transactions triggers a market expansion effect, which partially mitigates the lower markup to buyers. On balance, we find that increased platform competition lowers total fees, and for a fairly wide class of distribution functions for users' valuations, decreases fees to sellers and increases fees to buyers.

Our second main result is to study what happens if some buyers face a cost to multihome. This gives rise to a partial multihoming equilibrium. Starting from our baseline setting with two-sided multihoming, a decrease in the fraction of buyers multihoming decreases fees to buyers, and increases fees to sellers and total fees. Intuitively, having more buyers singlehome makes it harder for sellers to divert buyers' transactions, increasing the platforms' market power over sellers. As a corollary, this result implies that buyer multihoming reduces the tendency for high seller fees that typically arises in the "competitive bottleneck" case when buyers all singlehome (Armstrong, 2006; Armstrong and Wright, 2007; Belleflamme and Peitz, 2019).

Our third main result concerns changes in the distribution of buyer and seller preferences. Increasing buyer heterogeneity (when the buyer-side fee is positive) or increasing the value buyers put on transacting with sellers, increases buyer fees and total fees, while lowering seller fees as platforms gain more market power over buyers and, indirectly, less market power over sellers. Meanwhile, increasing the value sellers put on transacting or allowing for competition between sellers gives platforms more power over sellers, and has the opposite effects.

Finally, we study the interaction between platform competition and buyers' homing behavior. Even though increased platform competition always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether most of the buyers are singlehoming or multihoming. When most of the buyers multihome, increased platform competition induces platforms to compete more intensely for sellers. However, when most of the buyers singlehome, platforms have monopoly power over providing access to their buyers for the multihoming sellers, and increased platform competition induces platforms to compete more intensely for buyers rather than for sellers.

1.1 Relevant literature

The literature on two-sided markets starts with the seminal papers by Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006), which provide a basic foundation for studying pricing schemes by monopoly and duopoly platforms.[2] In developing and investigating a model of oligopolistic platform competition, our study relates closely to the recent

[2] Subsequent developments in the two-sided market literature extend the canonical two-sided framework in various directions. Among others, Weyl (2010) provides a more general model of a monopoly two-sided platform and examines the source of welfare distortions in platform pricing; Hagiu (2006) considers platform pricing and commitment issues when two sides of the market do not participate simultaneously; White and Weyl (2016) consider general nonlinear tariffs that are conditional on participation decisions of customers on all platforms; Jullien and Pavan (2019) consider platform pricing under dispersed information; Karle et al. (2020) explore how the phenomenon of platform market tipping relates to the presence of seller competition on platforms.

contribution by Tan and Zhou (2021) that presents a model of oligopolistic multi-sided platform competition rooted in the membership pricing model of Armstrong (2006). They provide important insights on the impact of platform entry and on the extent of excessive or insufficient platform entry. However, their framework does not consider heterogeneity in interaction benefits, transaction fees, or multihoming by users, which are the focus of our setting.

As mentioned in the Introduction, when buyers are loyal to particular platforms, our results resemble the classic "competitive bottleneck" result obtained by Armstrong (2006) and Armstrong and Wright (2007), and recently revisited by Belleflamme and Peitz (2019). These studies typically start with a configuration of singlehoming on both sides, and show that multihoming on one side leads to a competitive bottleneck, whereby platforms no longer need to compete for the multihoming side due to the monopoly power over providing exclusive access to each (singlehoming) user on the other side. Thus, in these studies, buyer-side multihoming would shift the fee structure in favor of sellers by shutting down competition on the buyer side. In contrast, we specify that buyers and sellers are always free to multihome in our baseline setting, and explore factors that affect buyer loyalty to platforms (including whether some buyers only singlehome or some buyers have strong preferences to only transact on particular platforms).

Closer to our setting, Bakos and Halaburda (2019) do consider two-sided multihoming, although also in Armstrong's framework. They compare two-sided multihoming with the benchmarks of two-sided singlehoming and competitive bottleneck, showing that two-sided multihoming eliminates the strategic interdependence between the two sides in platforms' pricing (so that platforms have no incentive to cross-subsidize across the two sides). A key ingredient for their result is that the market is fully covered on both sides. In our framework, in which the market is not fully covered on either side, strategic interdependence is restored.

At a more general level, our analysis on the impact of user multihoming behaviour and its interaction with platform entry relates to several recent papers in the media literature that investigate similar issues (Ambrus et al. 2016; Athey et al. 2018; Anderson et al. 2019; Anderson and Peitz, 2020). Ambrus et al. and Athey et al. show that multihoming by media consumers can either increase or decrease the equilibrium number of ads that platforms admit, depending on the correlation of consumers preference and the extent to which advertisements generate negative externalities on consumers. Anderson et al. consider a model of multihoming media consumption based on the Salop (1979) circular city model, deriving the interesting property of "incremental value pricing" whereby platform entry has no effect on consumers but harms advertisers. Anderson and Peitz (2020) use an aggregative game approach to analyze oligopolistic media market in a competitive bottleneck setup. Assuming that consumers dislike ads, they find that platform entry benefits consumers but tends to hurt advertisers. While these results bear some resemblance to ours, the market they analyze and the underlying economic reasoning behind their results is quite different.

2 Model setup

There is a set $\mathbf{N} = \{1, .., n\}$, of $n \geq 1$ platforms which compete for a continuum of buyers and a continuum of sellers, both of measure one. Buyers and sellers wish to "interact" or "transact" with each other to create economic value. If we consider any buyer/seller pair, then we can assume without loss of generality that each such pair corresponds to one potential transaction. Each transaction must occur through one of the platforms, and it can occur only if there exists at least one platform that both sides of the buyer-seller pair join.[3] Let $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$ denote the fees charged by platform i to buyers and sellers for each transaction facilitated.

□ **Sellers.** Each seller is indexed by a draw of per-transaction surplus, v, which is invariant across platforms. The net seller utility from each transaction through platform i is $v - p_i^s$, while the utility from not transacting is normalized to zero. Following Rochet and Tirole (2003), we assume that seller surpluses do not vary across platforms. Specifically, $v \in [\underline{v}, \bar{v}]$ (where $\underline{v} \geq -\infty$ and $\bar{v} \leq \infty$) is drawn i.i.d across sellers from cumulative distribution function (CDF) G with density function g, in which $1 - G$ is log-concave.

□ **Buyers.** Transaction decisions are endogenously initiated by buyers. For each potential transaction with a given seller v, if a buyer transacts with the seller and does so through platform $i \in \mathbf{N}$, the net utility is

$$b_0 + \epsilon_i - p_i^b.$$

Here, b_0 is a buyer-specific component that measures the intensity to which a buyer desires a transaction (gross value for transaction), and it is drawn i.i.d across buyers and transactions, but invariant across platforms.[4] Then, ϵ_i is a buyer-platform match component that measures buyer preference for transaction platforms, and it is drawn i.i.d across buyers and platforms. Utility provided by the outside option (of not transacting) is zero. For all $i \in \mathbf{N}$, let F be the common CDF for $\epsilon_i \in [\underline{\epsilon}, \bar{\epsilon}]$ (where $\underline{\epsilon} \geq -\infty$ and $\bar{\epsilon} \leq \infty$) with log-concave density f. To simplify the notation and avoid the need to carry a negative sign throughout our analysis, denote $\epsilon_0 \equiv -b_0$ and F_0 be the CDF for $\epsilon_0 \in [\underline{\epsilon}_0, \bar{\epsilon}_0]$ with log-concave density f_0. Finally, we assume that ϵ_0, $\epsilon_{i \in \mathbf{N}}$, and v are independently drawn.

□ **Platforms.** We allow transaction fees, $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$, to be negative (e.g negative buyer fees in the case of rewards in payment platforms, and negative seller fees in the case of ride-hailing apps payments to riders). Facilitating each transaction involves a marginal cost of c, which is assumed to be constant and symmetric across all platforms. We focus on the transactional aspect of platforms and abstract from any participation benefits (or costs) and fees. Therefore, platform i's profit is written as

$$\Pi_i\left(\mathbf{p}_i; \mathbf{p}_{-i}\right) = \left(p_i^b + p_i^s - c\right) Q_i\left(\mathbf{p}_i; \mathbf{p}_{-i}\right),$$

where $\mathbf{p}_{-i}$ is the fees set by all other platforms excluding i while Q_i is the total volume of transactions facilitated by platform i, which will be determined in Section 3.

□ **Participation multihoming.** Both buyers and sellers are always allowed to join multiple platforms. We assume that participation is costless. Note if both sides multihome, the choice of

[3]By reframing each transaction as "platform-intermediated transactions" and the outside option below as "direct transactions", the model easily allows for scenarios where buyers and sellers can interact directly, e.g., payment card platforms and cash-based direct transactions. See Rochet and Tirole (2003) and Bedre-Defolie and Calvano (2013) for such interpretations.

[4]Stochastic b_0 is not necessarily for our analysis but simplifies the special case of a logit demand function, which we use as a leading functional form example. Otherwise, our analysis allows for deterministic b_0 as a special case when F_0 is a degenerate distribution.

which of these platform to use for a transaction is a-priori indeterminate. Following Rochet and Tirole (2003) and consistent with each of our motivating examples, we assume that, whenever a seller is available on multiple platforms, the buyer is the one that chooses which platform to complete the transaction on.

☐ **Timing and equilibrium.** The timing of the game is summarized as follows:

1. All n platforms simultaneously set their transaction fees with platform i's being $\mathbf{p}_i = \left(p_i^b, p_i^s\right)$;
2. Given the platform fee profile $\mathbf{P} = (\mathbf{p}_1, ..., \mathbf{p}_n)$, sellers and buyers observe all fees and their realized draws v and $(\epsilon_1, ..., \epsilon_n)$ and simultaneously decide which platform(s) to join.[5]
3. For each potential transaction, buyers observe their realized b_0 and choose whether to transact, and if so, through which platform.

Note we are assuming buyers' preferences across platforms are fixed but whether they want to make a transaction or not varies with each potential transaction.[6] Our equilibrium concept is pure-strategy subgame perfect Nash equilibrium (SPNE), and we focus only on symmetric equilibria where all platforms set the same fees. As a tie-breaking rule, we assume that, whenever a user is indifferent between joining and not joining a platform, she breaks the tie in favor of joining.

☐ **Illustrative example.** As an illustration, we consider ride-hailing services.[7] The market has a large number of routes indexed by route v, and each route is occupied by a driver (seller). Consider a rider (buyer) who wants to travel on a given route v. If the driver on this route is unavailable on any of the ride-hailing platforms, the rider is unable to reach this driver and has to use alternative forms of transport (e.g., public transport) to travel on this route, obtaining utility normalized to zero. If the driver on this route is available on platform i, then the rider can choose between engaging with the driver or using alternative forms of transport, depending on whether $b_0 + \epsilon_i - p_i^b$ is greater than zero. Here, b_0 is the rider-specific value for the convenience of using a ride-hailing service, and ϵ_i represents idiosyncratic preference for ride-hailing platforms i. Multihoming riders compare the net utility of the ride across platforms (that have access to the driver), and then order through one of the platforms. Finally, given that v is an arbitrary index, we can reorder it such that v is proportional to the differences in utility of the driver between driving and idling, which is typically negative due to the effort and cost involved with driving (recall that v can be negative).

3 Equilibrium analysis

☐ **Choice of transaction medium**. Consider a buyer that has joined a set Θ^b of platforms who wishes to transact with a seller v that has joined a set Θ^v of platforms. The buyer can either

[5] Assuming that buyers do not observe the seller-side fees, which may be more realistic in some cases, does not change our analysis. See also the discussion in Section 5.1.

[6] We can alternatively assume that buyers observe $(\epsilon_1, ..., \epsilon_n)$ only at the point of making transaction decisions. This does not affect the analysis of the baseline model, except that the ex-post nature of the draws means that buyers would always strictly prefer joining all platforms.

[7] A similar illustration applies to other examples such as meal/grocery delivery platforms, hotel booking platforms, e-commerce marketplaces, and payment platforms.

perform the transaction through one of the of platforms (that the pair has joined in common) or opt for the outside option (not transacting). Thus, the buyer uses platform $i \in \Theta^b \cap \Theta^v$ if

$$b_0 + \epsilon_i - p_i^b > \max_{j \in \Theta^b \cap \Theta^v} \left\{ b_0 + \epsilon_j - p_j^b, 0 \right\}.$$

Given that $\epsilon_0 = -b_0$, the condition becomes

$$\epsilon_i - p_i^b \geq \max_{j \in \Theta^b \cap \Theta^v} \left\{ \epsilon_j - p_j^b, \epsilon_0 \right\}.$$

As will be seen later, we will focus on the participation equilibrium in which all buyers multihome on all platforms, i.e. $\Theta^b = \mathbf{N}$ for all buyers, so that $\Theta^b \cap \Theta^v = \Theta^v$. For each seller v, denote the mass of buyers who wish to transact with the seller and do so using platform $i \in \Theta^v$, as

$$B_i^{(\Theta^v)} \equiv \Pr \left(\epsilon_i - p_i^b \geq \max_{j \in \Theta^v} \left\{ \epsilon_j - p_j^b, \epsilon_0 \right\} \right) \text{ for any } \Theta^v \subseteq \mathbf{N}. \tag{1}$$

From (1), note that a seller, by selecting the platform(s) she wants to join, can restrict the set of platforms that buyers can choose from to make their transactions. It should be emphasized that (1) does not necessarily equal to the mass of buyers who have joined i given that the transaction choice is endogenous.

Denote the symmetric fee equilibrium under multihoming buyers as $\hat{\mathbf{p}} = (\hat{p}^b, \hat{p}^s)$. We consider a platform i which deviates from the equilibrium and sets $\mathbf{p}_i = (p_i^b, p_i^s) \neq \hat{\mathbf{p}}$. Whenever convenient, we use $\mathbf{N}_{-i} \equiv \mathbf{N} \setminus \{i\}$ to denote the set of all platforms excluding i.

☐ **Buyer participation.** It is straightforward to see that the assumptions of (i) buyers get to choose the final medium for transaction and (ii) zero joining cost, together, imply that it is a weakly dominant strategy for any given buyer to join all platforms, regardless of the fees set by the platforms.[8] Thus, given our tie-breaking rule, we focus on the participation equilibrium with all buyers multihome on all platforms.

☐ **Seller participation.** The profile of seller participation generally depends on how the seller fee set by platform i compares to other platforms. To derive the equilibrium fees, it suffices to focus on the participation profile after an upward deviation by platform i, that is, $p_i^s \geq \hat{p}^s$. We derive in detail the case of a downward deviation in Section A.1 of the Appendix.

Given $p_i^s \geq \hat{p}^s$ and that all platforms $j \neq i$ set the lowest seller fee $\hat{p}^s$, it is clear that a seller either joins no platforms, joins all platforms except i (i.e., N_{-i}), or joins all platforms including i (i.e., N). For a seller v, the net surplus from joining all platforms including i is

$$(v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})} + (v - p_i^s)\, B_i^{(\mathbf{N})}$$

as illustrated in Figure 1 below (with $i = 1$ and $\mathbf{N}_{-i} = \{2, 3\}$).

[8] More formally, consider a buyer who contemplates joining an additional platform i after already joining another platform j. The additional participation on platform i is weakly beneficial for two reasons. First, the buyer gains access to any sellers who are available on platform i but not available on platform j, and this additional access is strictly beneficial if $\epsilon_i - p_i^b > \epsilon_0$. Second, even if joining platform i does not provide any additional access, a buyer can switch his transaction over to platform i if it provides a higher utility than transacting through platform j, i.e. if $\epsilon_i - p_i^b \geq \epsilon_j - p_j^b$. We relax the assumption of zero joining cost in Section 5.1.

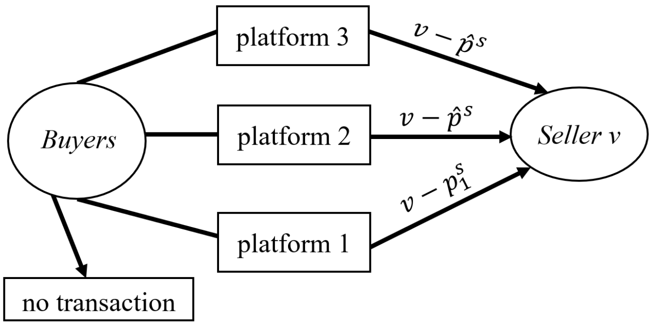

Figure 1: Buyer transaction pattern when seller v joins all platforms, including i.

If the seller quits platform i (in response to i's higher seller fee), it faces the following trade-off. First, it will divert some of the buyers who initially use platform i to switch to platforms $j \in \mathbf{N}_{-i}$. This raises the transactions on each of these platforms by

$$B_j^{(\mathbf{N}_{-i})} - B_j^{(\mathbf{N})} > 0$$

and allows the seller to enjoy the lower fee for each of these diverted transactions. Second, given that the market is not fully covered, some buyers will stop transacting (choosing the outside option) instead of transacting through other platforms. The seller will lose access to these buyers' transactions. To proceed, we define the following notation:

Definition 1 *For each given profile of buyer fees $(p_1^b, ..., p_n^b)$, buyers' loyalty to platform $i \in \mathbf{N}$ is defined as*[9]

$$\sigma_i \equiv 1 - \frac{\sum_{j \in \mathbf{N}_{-i}} (B_j^{(\mathbf{N}_{-i})} - B_j^{(\mathbf{N})})}{B_i^{(\mathbf{N})}} \in (0, 1). \tag{2}$$

Here σ_i measures buyer loyalty (in their transaction behavior) in the sense that it indicates the fraction of buyers who stop transacting when platform i ceases to be available for transactions with a given seller.[10] Note that σ_i is relevant even though buyers are multihoming on all platforms. Buyers can consider all options but may still have strong preferences towards using certain platforms to complete transactions, and be very reluctant to use others. Then, the change in seller v's net surplus from quiting the most expensive platform i can be written as

$$\underbrace{(p_i^s - \hat{p}^s)\,(1 - \sigma_i) B_i^{(\mathbf{N})}}_{\text{gain from diverting buyers to cheaper platforms}} - \underbrace{(v - p_i^s)\,\sigma_i B_i^{(\mathbf{N})}}_{\text{foregone transactions}}, \tag{3}$$

where the two components indicate the trade-off between saving on fees and attracting fewer transactions, as illustrated in Figure 2 below.

Solving for the indifference condition associated with (3) yields seller participation decisions:

[9] An alternative expression of (2) is $\sigma_i = 1 - \frac{\Pr\left(\epsilon_i - p_i^b \geq \max_{j \in \mathbf{N}_{-i}}\left\{\epsilon_j - p_j^b\right\} \geq \epsilon_0\right)}{\Pr\left(\epsilon_i - p_i^b \geq \max_{j \in \mathbf{N}_{-i}}\left\{\epsilon_j - p_j^b, \epsilon_0\right\}\right)}$.

[10] Our definition of loyalty is related to the concept of the aggregate diversion ratio (ADR) by Katz and Shapiro (2003), which measures the fraction of the total sales lost by a firm i (when its price rises by a small percentage amount) that are captured by all of the competing firms $j \neq i$. The inverse of buyer loyalty, $1 - \sigma_i$, is analogous to ADR in the sense that $1 - \sigma_i$ similarly measures the fraction of buyers who switch to the competing platforms. The key distinction is that in the definition of $1 - \sigma_i$ it is if buyers face an infinitely higher price to use platform

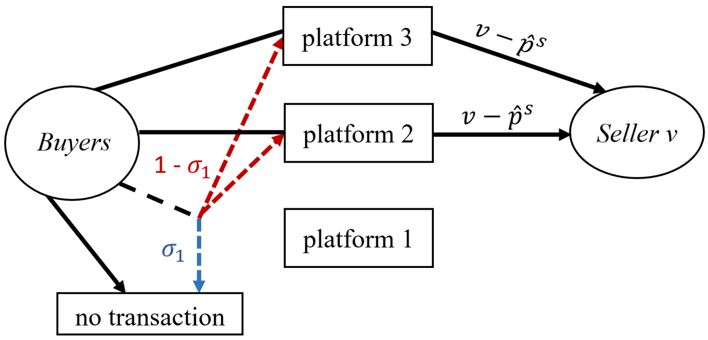

Figure 2: Buyer transaction pattern when seller v quits platform i, where $p_i^s \geq \hat{p}^s$.

Lemma 1 *Suppose $p_i^s \geq \hat{p}^s$. There exists a threshold*

$$\hat{v} = \frac{p_i^s - \hat{p}^s}{\sigma_i} + \hat{p}^s$$

such that a seller of type v joins no platform if $v < \hat{p}^s$, joins all platforms $j \neq i$ if $\hat{p}^s \leq v < \hat{v}$, and joins all platforms including i if $v \geq \hat{v}$.

Notice that the term σ_i plays a significant role in understanding how sellers react to changes in seller fees set by platforms. From (2), we can interpret the term σ_i as measuring the extent to which buyers *cannot be diverted in their transaction decisions.*

If σ_i is close to one, it means that platform i is not substitutable by other platforms $j \neq i$ for buyers who prefer i the most. Whenever platform i is not available for transactions with a particular seller, many existing buyers who have joined platform i will simply stop transacting with the seller (even though they have the option to transact with the seller through other platforms). In other words, it is hard for each seller to divert buyers to transact through the platform that the seller prefers. Thus, a large σ_i means that sellers are less likely to quit platform i following an increase in p_i^s, i.e.,

$$\frac{d\hat{v}}{dp_i^s} = \frac{1}{\sigma_i}$$

is small. If σ_i is close to zero instead, buyers on platform i are unlikely to stop transacting, so that it is easy for sellers to divert the buyers' choice of platform for completing transactions. In this case, sellers are more likely likely to quit platform i following an increase in p_i^s, i.e., $\frac{d\hat{v}}{dp_i^s}$ is large.

☐ **Volume of transactions.** Recall that each buyer-seller pair corresponds to one potential transaction. To derive the number of transactions Q_i facilitated by the deviating platform i, we count the number of buyers who use i to transact with each seller v, and sum this up over the

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i rather than the small percentage increase used to define ADR, reflecting that sellers delist from platform i in our case.

set of all sellers that are available on platform i. That is, for all $p_i^b \neq \hat{p}^b$ and $p_i^s \geq \hat{p}^s$, we have

$$\begin{aligned} Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)|_{p_i^s \geq \hat{p}^s} &= \int_{\{v | i \in \boldsymbol{\Theta}_v\}} B_i^{(\boldsymbol{\Theta}_v)} dG(v) \\ &= \left(1 - G\left(\hat{v}\right)\right) B_i^{(\mathbf{N})}, \end{aligned} \tag{4}$$

where the second equality is due to Lemma 1.

Our derivation of demand function (4) has focused on the case of an upward deviation $p_i^s \geq \hat{p}^s$, whereby platform i's increase in p_i^s trades off between fewer sellers participating (hence fewer transactions) and a higher fee. We relegate the case of $p_i^s < \hat{p}^s$ to Section A.1 of the Appendix and provide a sketch of the analysis here. When platform i sets $p_i^s < \hat{p}^s$, each seller faces a trade-off that is similar to, but the reverse of (3): by quitting the more expensive platforms $j \neq i$, the seller gains from diverting some buyers to use the cheaper platform i but loses transactions with buyers who switch to the outside option. Thus, in response to $p_i^s < \hat{p}^s$, some sellers join strictly less than n platforms (while still joining platform i). Specifically, there exists a sequence of cutoffs $\hat{v}_n \geq \hat{v}_{n-1} \geq ... \geq \hat{v}_1 = p_i^s$ such that a seller joins m platforms (including i) if and only if $v \in [\hat{v}_m, \hat{v}_{m+1})$, where $m \in \{1, 2..., n\}$. When buyers want to transact with sellers $v \in [\hat{v}_1, \hat{v}_n)$, they have fewer platforms to choose from, which implies that they are more likely to use platform i (if they transact at all). As such, platform i's decrease in p_i^s trades off between inducing sellers to multihome on fewer other platforms (which results in more transactions) and earning a lower fee.

Imposing specific distribution functions allows us to express the volume of transaction (when $p_i^s < \hat{p}^s$) in a simpler form:

□ **Example 1.** (Logit demand form) Suppose F and F_0 correspond to the Gumbel distribution with scale parameter μ so that (1) follows the standard logit form widely used in the industrial organization literature:

$$B_i^{(\Theta^v)} = \frac{\exp\left\{-p_i^b/\mu\right\}}{1 + \sum_{j \in \Theta^v} \exp\left\{-p_j^b/\mu\right\}}.$$

In Section A.1 of the Appendix, we show that $\hat{v}_m = (\hat{p}^s - p_i^s) \exp\left\{-p_i^b/\mu\right\} + \hat{p}^s$ is independent of m for all $m \geq 2$. As such, each seller either joins all platforms including i, joins only platform i, or joins no platforms, and so

$$Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)|_{p_i^s < \hat{p}^s} = \left(1 - G\left(\hat{v}_m\right)\right) B_i^{(\mathbf{N})} + \left(G\left(\hat{v}_m\right) - G(p_i^s)\right) B_i^{(\{i\})}.$$

3.1 Equilibrium fees

We now characterize the equilibrium in the first stage. In what follows, we assume that platform i's profit function

$$\Pi_i = \left(p_i^b + p_i^s - c\right) Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$$

is quasi-concave in (p_i^s, p_i^b).[11] In Section A.4 of the Appendix, we show that a sufficient condition for quasi-concavity is that (1) takes the standard logit form (as in Example 1) and that G is linear.[12]

For any arbitrarily given (symmetric) buyer fee p^b, we define the *buyer inverse semi-elasticity* as

$$X(p^b; n) \equiv \frac{B_i^{(\mathbf{N})}}{\partial B_i^{(\mathbf{N})}/\partial p_i^b}|_{p_i^b = p_j^b = p^b}, \tag{5}$$

which is a standard index that measures the competitive markup a firm can extract from buyers in a given equilibrium with n competing firms (see Perloff and Salop, 1985). Similarly, we define the buyer loyalty index as the symmetric counterpart of (2):

$$\sigma(p^b; n) \equiv \sigma_i|_{p_i^b = p_j^b = p^b} \in (0, 1) \tag{6}$$

which captures buyers' tendency to stop transacting when their most-preferred platform ceases to be available for transactions, i.e., how difficult it is for sellers to divert buyers' transactions across platforms. For an illustration:

☐ **Example.** In the case of logit demand form (Example 1), expressions (5) and (6) become

$$X(p^b; n) = \mu \left(\frac{1 + n \exp\{-p^b/\mu\}}{1 + (n-1)\exp\{-p^b/\mu\}} \right) \text{ and } \sigma(p^b; n) = \frac{1}{1 + (n-1)\exp\{-p^b/\mu\}}. \tag{7}$$

The standard first-order condition for optimal pricing leads to the following equilibrium.

Proposition 1 *A pure symmetric pricing equilibrium is characterized by all n platforms setting $\hat{\mathbf{p}} = (\hat{p}^b, \hat{p}^s)$ that solves*

$$\hat{p}^b + \hat{p}^s - c = X(\hat{p}^b; n) = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)} \sigma(\hat{p}^b; n). \tag{8}$$

Moreover, the solution $\hat{\mathbf{p}}$ to (8) is unique.

Condition (8) can be intuitively understood as the intersection of equilibrium conditions for the competition in the buyer-side and seller-side markets. To see this, we first denote $P^b(p^s)$ as a function defined implicitly by p^b that solves

$$p^b = \underbrace{c}_{\text{cost}} + \underbrace{X(p^b)}_{\text{market power over buyers}} - \underbrace{p^s}_{\text{cross-subsidy due to revenue from sellers}}; \tag{9}$$

[11] Our derivation below focuses on the upward deviation $p_i^s \geq \hat{p}^s$. In Section B of the Online Appendix, we verify that Q_i is always continuous, and that platforms cannot profitably deviate from the equilibrium in (8) by slightly decreasing p_i^s. The assumption of quasi-concavity rules out large deviations being profitable.

[12] Beyond the case of the standard logit demand, we numerically check that the profit function is indeed quasi-concave over a wide range of parameter values and distribution functions, suggesting that the quasi-concavity of the profit function may indeed hold quite generally.

and denote $P^s(p^b)$ as a function defined implicitly by p^s that solves

$$p^s = \underbrace{c}_{\text{cost}} + \underbrace{\frac{1-G(p^s)}{g(p^s)}\sigma(p^b;n)}_{\text{market power over sellers}} - \underbrace{p^b}_{\text{cross-subsidy due to revenue from buyers}} . \quad (10)$$

For each arbitrarily given (common) seller-side fee, $P^b(p^s)$ defined by (9) can be understood as a curve that maps out the "one-sided" equilibrium buyer fee. Likewise, $P^s(p^b)$ is a curve that maps out the "one-sided" equilibrium seller fee for each arbitrarily given buyer fee. Then, the equilibrium (8) is simply the unique intersection of the $P^s(p^b)$ and $P^b(p^s)$ curves, each representing the equilibrium condition on each side of the market.

This reinterpretation of the equilibrium provides an intuitive way to understand Proposition 1. Expression (9) represents the standard oligopoly pricing equilibrium (in setting the buyer fee) with competitive markup X, except that the price is adjusted downward (upward) because the platform is compensated by the positive (negative) seller fee collected from each transaction. Expression (10) is the sum of cost, adjusted by the buyer fee, plus the standard monopoly pricing markup $\frac{1-G(P^s)}{g(P^s)}$ that is discounted by the buyer loyalty index, $\sigma < 1$. The discount reflects that platform market power over sellers increases when it becomes harder for sellers to divert buyers to transact through different platforms. We discuss the implications of this equilibrium condition in the next subsection.

3.2 Discussion

□ **Comparison with pure membership models**. The pricing equations (9) and (10) in our pure transaction pricing model closely resemble those obtained in the pure membership pricing models of Armstrong (2006) and Tan and Zhou (2021) in the sense that there is a "*cross-subsidy adjustment*" on each side due to the two-sidedness of the market. However, there is a key conceptual difference in terms of how the subsidy adjustment arises in these two classes of models.

To see this, consider the determination of the seller fee (a similar logic applies for the buyer fee). In membership pricing models, the subsidy adjustment reflects the cross-group membership externality, whereby an increase in seller participation raises buyers' willingness to pay for platform membership. In cases where the cross-group externality is negative, e.g., if we replace "sellers" with "advertisers", then the subsidy adjustment would have a negative sign. However, in transaction pricing models, the cross-group externality is irrelevant in the determination of transaction fees. Transaction fees constitute an "insulating tariff" (to use Weyl's (2011) terminology), whereby buyers' per-transaction willingness to pay is independent of the mass of sellers. Instead, any subsidy adjustment in the transaction fees reflects the "*usage externality*" emphasized by Rochet and Tirole (2003; 2006) — an additional transaction caused by an increase in seller participation has a cost of c but generates an offsetting subsidy of p_B, so this subsidy should be taken into account in setting the price to sellers, and vice-versa. Moreover, the sign of the cross-subsidy adjustment is primarily determined by the nature of the value distributions of buyers and sellers.

□ **Significance of index σ.** In our model of usage externality, multihoming in participation does not automatically imply certain market outcomes because one has to take into account users' transaction behavior, which is summarized by σ. In our setup, all buyers are free to join all platforms and so they all fully multihome in terms of their participation. Yet, if $\sigma \to 1$, buyers' transaction pattern would exhibit a strong tendency of being loyal to a single platform, in the sense that they are likely to stop transacting whenever their most-preferred platform ceases to be available even though they have joined other alternative platforms. The resulting buyers' singlehoming-type behavior in transactions generate the familar "competitive bottleneck" outcome (Armstrong, 2006; Armstrong and Wright, 2007) despite the multihoming in participation. That is, buyers behave as if they only participate on a single platform. Platforms exert monopoly power over sellers (letting $\sigma \to 1$ in (10)) and compete intensely for buyers. Such a competitive bottleneck disappears in the opposite case of $\sigma \to 0$, whereby buyers' transaction pattern reflects a strong willingness to switch to other platforms if any particular platform is no longer available (this case is only possible if $n \geq 2$).

The index σ helps to make clear the nature of the platforms' market power over sellers, whereby the elasticity of seller participation is closely related to buyers' behavior (even if we ignore any cross-subsidization effect). In the extreme case where $\sigma \to 0$, each buyer necessarily purchases one product from each seller in equilibrium, and buyers are willing to do so through any of the n platforms. In this case, platforms have zero market power over sellers (recall that sellers view platforms as homogenous) because if one platform tries to charge more, sellers can always divert sales through one of the alternative (cheaper) platforms without losing any transactions. Thus, in our framework, each platform's market power over sellers stems primarily from the possibility that sellers may lose access to some buyers when they delist from the platform.

Finally, in the special case of $n = 2$ and the F_0 is a degenerate distribution at $\epsilon_0 = 0$ (i.e., each buyer obtains the same surplus from all potential transactions on the same platform), (8) recovers the duopoly equilibrium of Rochet and Tirole (2003, Proposition 3). Our result in (8) generalizes their pricing formula to oligopolistic platforms with transaction-specific buyer surpluses. A key difference is we relate the formula to the underlying distribution of buyers' and sellers' valuations over interaction benefits rather than expressing the formula in terms of reduced-form demand functions. Our approach offers two benefits. First, it enables sharp comparative static results with respect to platform competition, which we explore in Section 4. Second, it allows us to have a micro-founded understanding of the nature of the index σ, and the comparative statics of the factors which drive it, which we explore in Section 5.[13]

4 Impact of increased platform competition

In this section we explore how increased platform competition (i.e. entry) affects the platforms' equilibrium total fee and fee structure. We start by examining how an increase in n affects: (i)

[13] Rochet and Tirole (2003) label the term σ as the buyer "singlehoming index". We use buyer loyalty to highlight that σ is not necessarily tied to the homing behavior of buyers, although that is one factor we consider in Section 5.

the competition for buyers captured by (9), which depends on $X(p^b;n)$; and (ii) the competition for sellers captured by (10), which depends on $\sigma(p^b;n)$. Then, we combine these to get the overall effect of an increase in n.

We first state the following lemma, which we have proven in the proof of Proposition 1.

Lemma 2 *For each given p^b:*

- *The buyer inverse semi-elasticity $X(p^b;n)$ defined in (5) is decreasing in n.*
- *The buyer loyalty index $\sigma(p^b;n)$ defined in (6) is decreasing in n.*

□ **Intensified buyer-side competition.** The first part of Lemma 2 implies that the equilibrium buyer-side competitive markup, X, decreases with n. This reflects the standard intuition that platforms are more substitutable for buyers when buyers have more platforms to choose from.[14] Consequently, the buyer-side competition, as captured by the buyer-side curve $P^b(p^s)$ in (9), becomes more intense when n increases. We write $P^b(p^s;n)$ to make explicit this dependency on n. Graphically, when the number of platforms increases from n_1 to n_2, the buyer-side curve shifts downward from the solid line $P^b(p^s;n_1)$ to the dotted line $P^b(p^s;n_2)$ in Figure 3.

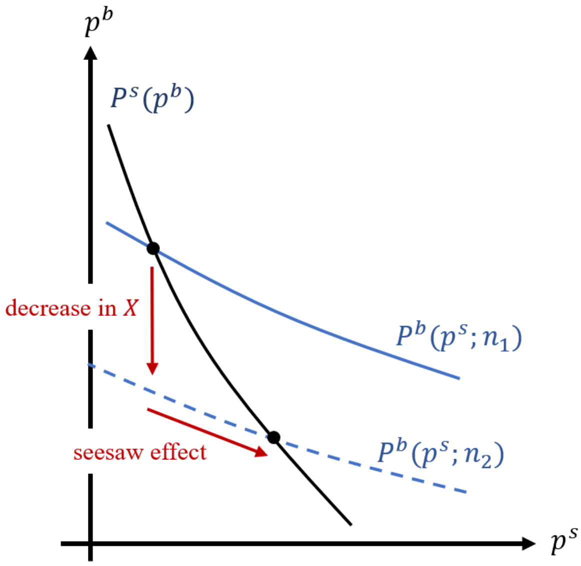

Figure 3: Reduced buyer markup X due to platform competition ($n_2 > n_1$)

All else equal, the shift in the buyer-side curve has two effects: (i) it decreases the equilibrium buyer fee *directly*; and (ii) it increases the equilibrium seller fee *indirectly* (through the movement along the $P^b(p^s)$ curve). The latter effect is the well-known *seesaw effect* in the two-sided market literature (Rochet and Tirole, 2006) — any change that makes it conducive to have a lower fee on one side will call for a higher fee on the opposite side (and vice-versa).

The economic intuition for the seesaw effect at work in our setting is as follow. When buyer-side competition becomes intensified (due to an increase in n), the buyer fee falls. A lower buyer

[14]This is an extension of the result by Zhou (2017) which considers the case where F_0 is a degenerate distribution at $\epsilon_0 = 0$.

fee implies a higher effective marginal cost of servicing sellers $c - p^b$, meaning that transactions generated from attracting seller participation become less valuable to the platforms and so the platforms compete less intensely for sellers. Therefore, the effect of higher n on the competitive markup *shifts the fee structure in favor of buyers*, in the sense that it induces a lower buyer fee and a higher seller fee.

☐ **Intensified seller-side competition.** The second part of Lemma 2 implies that sellers find it easier to divert buyers to transact through different platforms when n increases as the platforms become more substitutable. Sellers are more likely to quit platforms that charge high seller fees. Thus, the seller-side competition, as captured by the seller-side curve $P^s\left(p^b\right)$ in (10), becomes more intense when n increases. We write $P^s(p^b; n)$ to make explicit this dependency on n. Graphically, when the number of platforms increases from n_1 to n_2, the seller-side curve shifts downward from the solid line $P^s(p^b; n_1)$ to the dotted line $P^s(p^b; n_2)$, as shown in Figure 4.

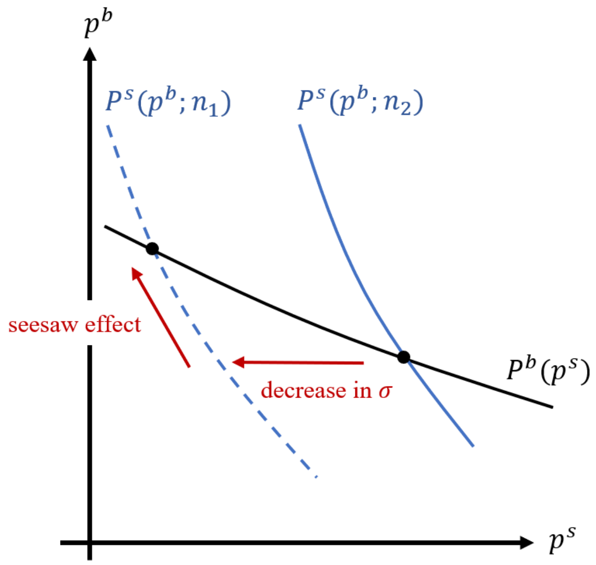

Figure 4: Reduced indivertible index σ due to platform competition ($n_2 > n_1$)

All else equal, the shift in the seller-side curve results in an immediate decrease in the seller fee, and an indirect increase in the equilibrium buyer fee through the movement along the $P^s\left(p^b\right)$ curve. The shift results in an immediate decrease in the equilibrium seller fee, and an indirect increase in the equilibrium buyer fee through the movement along the $P^s\left(p^b\right)$ curve (the seesaw effect). Consequently, we say that the effect of a higher n on buyer loyalty *shifts the fee structure in favor of sellers*, in the sense that it induces a lower seller fee and a higher buyer fee.

Combining the analyses, we conclude that an increase in n affects the equilibrium fees via two effects: *reduced buyer markup* (lower X) and *reduced (transaction) loyalty of buyers* (lower σ). An immediate impact of these two effects is a decrease in both the buyer-side and the seller-side markups that platforms earn in equilibrium, so that the equilibrium total fee unambiguously decreases with n (see Proposition 2 below).

However, these two effects shift the fee structure in opposite directions — the reduced buyer competitive markup intensifies the competition for buyers, while the decreased loyalty of buyers intensifies the competition for sellers. In the proof of the next proposition, we show that if density function f is weakly decreasing, then

$$\min_{p_b} \left\{ \left| \frac{\partial \sigma / \partial n}{\sigma} \right| - \left| \frac{\partial X / \partial n}{X} \right| \right\} > 0 \tag{11}$$

holds for all p^b. This means that the decrease in buyer loyalty index dominates the decrease in buyer competitive markup, which leads to the following formal result.[15]

Proposition 2 *(Increased platform competition) In the equilibrium characterized by Proposition 1, an increase in n (i.e. platform entry) decreases the total fee $\hat{p}^s + \hat{p}^b$. Furthermore, an increase in n decreases $\hat{p}^s$ if the density function f is weakly decreasing, and increases $\hat{p}^b$ if in addition the density function g is weakly decreasing*

A few remarks are in order. First, the key condition (11) says that buyer loyalty index σ is more elastic with respect to changes in n compared to the buyer-side competitive markup X. Whether this is true depends only on the preference distribution of the buyers, i.e., F and F_0. A sufficient condition for (11) is that the density function f is weakly decreasing, which is satisfied by some commonly used distributions such as the uniform distribution, the exponential distribution, the power law distribution, and the generalized Pareto distribution (for a certain range of parameter values). By way of comparison, linear demand (analogous to the uniform distribution) has been used in the related literature (e.g. Rochet and Tirole, 2003; Armstrong, 2006; Bakos and Halaburda, 2020).

Moreover, weakly decreasing density is certainly not a necessary condition. In Section A.4 of the Appendix, we show that (11) holds in our Example 1 whereby F and F_0 correspond to Gumbel distribution, which does not have monotone decreasing density. This suggests that (11) is indeed true quite generally.

Second, broadly speaking, an increase in n has two effects. First, it makes the platforms more substitutable, which raises the within-market competitive pressure, thus decreasing both X and σ. Second, it triggers a market expansion effect. This market expansion effect decreases the relative attractiveness of the buyers' outside option and so σ, thus reinforcing the first effect. In contrast, the market expansion effect increases X, thus partially mitigating the first effect on X. As a result, X tends to be less elastic towards changes in n than is σ, leading to property (11). This explains why under fairly general distributional assumptions, an increase in platform competition decreases the fee to sellers and the total fee, while increases the fee to buyers.

5 Determinants of buyer loyalty index

In this section, we explore how the market equilibrium outcome depends on (i) the cost of buyers multihoming; (ii) the value of transactions for buyers; (iii) buyer heterogeneity; and (iv) seller-side factors. We highlight that index σ plays a crucial role in understanding these comparative static exercises. To keep the exposition brief, we focus on presenting the main insights in this

[15] The comparative static below is applicable for all $n \geq 1$.

section and relegate further details and formal proofs of the propositions to Sections C and D of the Online Appendix.

5.1 Buyer participation and partial multihoming

The first determinant we consider is the fraction of buyers multihoming vs. singlehoming. Recall that in our benchmark setting, each buyer faces zero joining cost regardless of the number of platforms that he has already joined, and so all buyers multihome on all platforms in the equilibrium. To allow for some buyers to be singlehoming, we extend the model in Section 2 as follows.

Suppose that buyers obtain some stand-alone participation benefit (can be zero) from joining at least one platform and then incur a cost ψ (or a benefit if $\psi < 0$) for each additional platform joined. Buyers have heterogenous ψ, distributed according to some CDF F_ψ. Denote $1 - \lambda \equiv 1 - F_\psi(0)$ as the fraction of buyers with $\psi > 0$. Provided that these buyers expect each seller either multihomes on all platforms or joins no platform (which we we will show to be true in equilibrium), they do not expect to gain additional access to sellers by joining more than one platform. Hence, these buyers join at most one platform in the equilibrium, i.e., they singlehome. The remaining fraction $\lambda \equiv F_\psi(0)$ of buyers have $\psi \leq 0$ (i.e. there is some non-negative stand-alone benefit from joining additional platforms), so that these buyers multihome on all platforms in the equilibrium, as in the benchmark model. Notice that $\lambda = 1$ corresponds to our benchmark setting.

To keep the exposition as simple as possible for this application, we assume that singlehoming buyers observe only buyer fees and not seller fees.[16] These buyers hold passive beliefs (Hart and Tirole, 1990) on the unobserved seller fees, meaning they believe seller fees are equal to the equilibrium levels whenever they observe an off-equilibrium buyer fee. The alternative assumption of singlehoming buyers observing the seller side fees complicates our analysis but does not affect our main insights (see Section C.1 of the Online Appendix).

The derivation for this partial-multihoming model largely follows those in Section 3. We first note that the competition on the buyer side is independent of λ in the equilibrium. To see this, consider a singlehoming buyer's participation decision. The buyer will join the platform that yields the highest expected utility, taking into account the number of sellers on each of the platforms. Since the buyer does not observe seller fees and holds passive beliefs, he takes p_i^s as fixed at the equilibrium level $\hat{p}^s$, which is the same across all platforms. Given this, the singlehoming buyer expects the same set of sellers on each platform in the equilibrium and he will join only the platform that gives the highest per-transaction surplus. That is, he joins platform i if and only if $\epsilon_i - p_i^b \geq \max_{j \in \mathbf{N}}\{\epsilon_j - p_j^b\}$. After joining platform i, the buyer uses it for a transaction (with each seller) if $b_0 + \epsilon_i - p_i^b > 0$, or equivalently if $\epsilon_i - p_i^b \geq \epsilon_0$. Therefore,

[16] Recall in our benchmark setting with $\lambda = 1$, whether buyers observe seller fees or not does not affect the analysis. Consistent with our assumption here, Janssen and Shelegia (2015) note that vertical arrangements between sellers and platforms are typically confidential, and so are not observed by buyers. Hagiu and Halaburda (2014) and Belleflamme and Peitz (2019) have analyzed the implications of this informational assumption for pricing in two-sided markets.

the total mass of singlehoming buyers who use platform i for transactions is

$$\Pr\left(\epsilon_i - p_i^b \geq \max_{j\in\mathbf{N}}\left\{\epsilon_j - p_j^b, \epsilon_0\right\}\right),$$

which is exactly (1) whenever $\Theta^v = \mathbf{N}$.

However, the presence of some singlehoming buyers means sellers, whenever they quit one of the platforms, divert less buyers to other platforms for transactions, i.e., the transaction loyalty index increases when λ decreases. This allows platforms to exercise greater market power over sellers. Specifically, we can define the counterpart of (6) for this environment:

$$\sigma_\lambda(p^b; n) \equiv \lambda\sigma(p^b; n) + 1 - \lambda. \tag{12}$$

Notice that σ_λ is decreasing in λ. If $\lambda = 0$ then $\sigma_\lambda = 1$, i.e., buyers cannot be diverted to use other platforms for transactions because all of them join only one platform. If $\lambda = 1$ then $\sigma_\lambda < 1$ defined here corresponds to the benchmark definition in (6). Thus, σ_λ relates buyer transaction behavior with their participation homing behavior.

In this environment, a pure symmetric pricing equilibrium can be characterized by all platforms choosing $\hat{\mathbf{p}} = \left(\hat{p}^b, \hat{p}^s\right)$ that uniquely solves

$$\hat{p}_b + \hat{p}_s - c = X(\hat{p}^b; n) = \frac{1 - G\left(\hat{p}^s\right)}{g\left(\hat{p}^s\right)}\sigma_\lambda(p^b; n). \tag{13}$$

Given that an increase in λ always decreases σ_λ but does not affect X, the logic in Section 4 immediately implies the following result:

Proposition 3 *(Effect of buyer multihoming) In the equilibrium characterized by (13), a higher fraction of multihoming buyers (λ) increases $\hat{p}^b$, decreases $\hat{p}^s$, and decreases the total fee $\hat{p}^b + \hat{p}^s$.*

Proposition 3 is analogous to Proposition 5.3 of Rochet and Tirole (2003), but there are three important differences. First, their result focuses on competing associations (each that maximizes the volume of transactions) whereas our result considers proprietary platforms (that maximize profit). Second, our result does not rely on demand linearity and can accommodate an arbitrary number of platforms. Finally, their result is stated in terms of an exogenous increase in σ (the "singlehoming index" in their terminology) but does not clarify how does such an exogenous change relates to the homing behaviors of buyers.[17] Our approach provides a microfoundation that links such a change with the fraction of buyers singlehoming due to multihoming costs. This approach also has implications for the competitive bottleneck theory of Armstrong (2006) and Armstrong and Wright (2007) in that when more buyers multihome (an increase in λ), the competitive bottleneck initially faced by the seller side is reduced.

[17] In their primary example of an extended linear Hotelling model, such an exogenous increase in the buyer loyalty index corresponds to an increase in the marginal transportation cost of buyers for distances in the non-competitive hinterland of the rival platform while holding constant the transportation cost of all other segments of the Hotelling line.

5.2 Value of transactions and buyer heterogeneity

As noted earlier, one feature of index σ is that it responds differently to within-market competitive pressure and out-of-market competitive pressure. To formally develop this point, we extend the model in Section 2 by introducing two additional parameters: (i) β, which shifts the buyer utility from transacting relative to not transacting; (ii) $\gamma > 0$, which indicates the heterogeneity of buyer preferences. A buyer that uses platform i for a transaction receives

$$\beta + (b_0 + \epsilon_i)\gamma - p_i^b,$$

while the surplus from the outside option remains fixed at zero.[18] All other specifications remain the same.

This extension is mathematically equivalent to applying a linear transformation to the buyer fee charged by each platform. Specifically, after replacing each platform's buyer fee p_i^b with the "adjusted buyer fee" $\tilde{p}_i^b = \frac{p_i^b - \beta}{\gamma}$, the equilibrium characterization follows immediately from the benchmark model. The equilibrium condition is similar to (8):

$$\hat{p}^b + \hat{p}^s - c = \gamma X\left(\frac{\hat{p}^b - \beta}{\gamma}; n\right) = \frac{1 - G(\hat{p}^s)}{g(\hat{p}^s)}\sigma\left(\frac{\hat{p}^b - \beta}{\gamma}; n\right), \tag{14}$$

where $X(.; n)$ and $\sigma(.; n)$ are defined in (5) and (6). Before proceeding, it useful to state the following properties which have been established in the proof of Proposition 1:

Lemma 3 *For each given n:*

- *The buyer inverse semi-elasticity $X(.; n)$ defined in (5) is decreasing in its first argument.*
- *The buyer loyalty index $\sigma(.; n)$ defined in (6) is increasing in its first argument.*

The first part of Lemma 3 is a consequence of log-concavity of f. It reflects the standard intuition that an increase in the relative value of the outside option (i.e., a higher p^b) increases the competitive pressure faced by each platform, so that platforms earn a lower markup in the equilibrium. The second part of Lemma 3 states that when the outside option becomes more attractive for buyers, sellers find it harder to divert buyers as buyers become more likely to prefer not transacting.

□ **Higher value of transactions for buyers.** Given that an increase in β is equivalent to an decrease in p^b in Lemma 3, we have the following result:

Proposition 4 *(Value of transaction) In the equilibrium (14), an increase in the value of transactions for buyers (β) increases $\hat{p}^b$, decreases $\hat{p}^s$, and increases the total fee $\hat{p}^b + \hat{p}^s$.*

Proposition 4 implies that an increase in β which increases the value buyers put on transacting with sellers (relative to not transacting) makes competition for sellers more intense. This

[18] Notice that if $\gamma = 0$ then all buyers are homogenous. Our formulation follows the standard models of oligopolistic price competition (e.g., Anderson et al., 1992; and Anderson and Peitz, 2020), which typically impose a common scale parameter to all idiosyncratic components of consumer/buyer utility functions.

is intuitive. There are two relevant forces here. First, a higher β raises platforms' market power over buyers (they strongly desire transactions now) because transactions can only occur through platforms. Through the see-saw effect, this exerts a downward pressue on $\hat{p}^s$. The second force is more novel: when buyers highly value transacting with sellers, they are willing to use any platform for transaction (as opposed to not transacting). This makes it easier for each seller to, through her participation decision, divert buyers to use the platform that the seller desires. Consequently, platforms have weaker market power over the sellers, which exerts additional downward pressure on $\hat{p}^s$. In the extreme case where $\beta \to \infty$, the outside option becomes irrelevant and $\sigma \to 0$, implying that platforms would have no market power over sellers.

☐ **Increased buyer heterogeneity**. Without loss of generality, we assume $\beta = 0$ to focus on the effect of γ. Observe from (14) that the effect of an increase in γ critically depends on the sign of $\hat{p}^b$. Let us first focus on the case of $\hat{p}^b > 0$ (so the transaction is relatively expensive). An increase in γ raises the attractiveness of platform-mediated transactions (relative to the outside option) by dampening buyers' sensitivity towards the net cost of using platforms. Buyers are less likely to stop transacting, so that sellers find it easier to divert buyers' transactions, in the sense that index σ decreases. This weakens platforms' market power over sellers. As for the buyer side, γ has standard two effects: (i) increased differentiation between the platforms; and (ii) dampened buyer sensitivity towards the net cost of using platforms, which expands the market size. Both effects raise the market power platforms have over buyers. Combining the changes in market power over both sides of the market, we have:

Proposition 5 *(Effect of buyer heterogeneity) Suppose $\hat{p}^b > 0$. In the equilibrium (14), an increase in the extent of buyer heterogeneity (γ) increases $\hat{p}^b$, decreases $\hat{p}^s$, and increases the total fee $\hat{p}^b + \hat{p}^s$.*

The case of $\hat{p}^b \leq 0$ (so the transaction is subsidized) is more complicated.[19] In this case, an increase in γ decreases the attractiveness of platform-mediated transactions by dampening buyer sensitivity towards the net subsidy of using platforms $\hat{p}^b \leq 0$. This is in constrast to the previous case of $\hat{p}^b > 0$. Consequently, the index σ increases, strengthening platforms' market power over the sellers. As for the buyer side, the effect of γ is a priori unclear because it now has a market contraction effect, which offsets (and potentially dominates) the increase in market power from increased differentiation. Consequently, the overall effect of γ on the equilibrium fee is generally ambiguous in this case.

5.3 Seller-side factors

Motivated by the observation that our key index of interest, σ, depends only on buyers' behavior in our model, our analysis thus far has focused on buyer-side factors. Let us briefly discuss two key seller-side factors.

☐ **Higher value of transactions for sellers.** Suppose that the net seller utility from each transaction through platform i is $\alpha + v - p_i^s$, where α is a additive shifter parameter that

[19]See Anderson et al. (1992) for a similar discussion in the context of oligopolistic firms selling diffentiated products.

is invariant across platforms. In practice, an increase in α may correspond to sellers extracting more surplus from transactions or an industry-wide increase in the convenience benefit that platforms offer to the sellers. Obviously, a higher α allows platform to charge a higher $\hat{p}^s$ and, through the see-saw effect, exerts a downward pressure on $\hat{p}^b$. Formally, we prove in the appendix that the net effect is an increase in $\hat{p}^s$, a decrease in $\hat{p}^b$, and an increase in the total fee $\hat{p}^b + \hat{p}^s$. This result extends the insights of Proposition 5.1 of Rochet and Tirole (2003), which focuses on duopolistic competing associations (which maximize volume of transactions instead of profits).

□ **Competition between sellers**. In our analysis of seller participation, an implicit assumption is that potential transactions with each seller are irreplaceable. That is, once a seller v delists from platform i, buyers can only choose between transacting with the seller through other platforms or not transacting at all. More generally, one could imagine that v is an index for different "product categories", each occupied by multiple competing sellers. For each category v, once a seller delists from a platform, some of the buyers may switch to buy from competing sellers (within the same category), thus weakening the ability of each seller in diverting buyers through her participation decision. This suggests that more intense competition between sellers within each product category would increase index σ, so that we can alternatively interpret index σ as buyer loyalty to each platform relative to their loyalty to each seller. Nonetheless, a detailed exploration into this issue requires a microfounded model, which is outside the scope of the current paper.[20]

6 Interaction: platform competition and determinants of buyer loyalty

We are interested in the interaction between the determinants that drive the buyer loyalty index (Section 5) and the number of platforms that are competing (Section 4). Specifically, does the implication of increased platform competition in Proposition 2 change with the exogenous factors discussed in Section 5?

We first note that value of transactions (β) and buyer heterogeneity (γ) does not affect Proposition 2, which is perhaps not surprising given that any changes in β and γ are analogous to a shift in the buyer fee, as discussed in Section 5.2. In what follows, we focus on the interaction between n and the extent of buyer multihoming (λ), based on the model in Section 5.1.

Proposition 6 *(Increased platform competition) In the equilibrium with partial-multihoming buyers characterized by (13), an increase in n always decreases the total fee.*

1. *If the fraction of buyers multihoming goes to zero ($\lambda \to 0$), an increase in n increases $\hat{p}^s$ and decreases $\hat{p}^b$.*

[20] See, e.g., Guthrie and Wright (2007), Edelman and Wright (2015), and Wang and Wright (2020) for microfounded models of competing platforms in which sellers compete to attract buyers on and off the platforms.

2. *If the fraction of buyers multihoming goes to one ($\lambda \to 1$) and f and g are weakly decreasing, an increase in n decreases $\hat{p}^s$ and increases $\hat{p}^b$.*

In the case where (1) takes the standard logit form, we can obtain a stronger version of Proposition 6:

Corollary 1 *If F and F_0 correspond to the Gumbel distribution with scale parameter μ, then there exists a unique cutoff $\bar{\lambda} > 0$ such that:*

1. *If $\lambda < \bar{\lambda}$, an increase in n increases $\hat{p}^s$ and decreases $\hat{p}^b$.*

2. *If $\lambda \geq \bar{\lambda}$, an increase in n decreases $\hat{p}^s$, and increases $\hat{p}^b$ if in addition n is large enough.*

Proposition 6 highlights a novel finding of our paper: even though increased platform competition always reduces the total fee charged to the two sides, whether it shifts the fee structure in favor of buyers or sellers depends on whether most of the buyers are singlehoming or multihoming. A key step in our proof is showing that $\left|\frac{\partial \sigma_\lambda / \partial n}{\sigma_\lambda / n}\right|$ decreases when λ decreases. That is, when more buyers are singlehoming in participation, the loyalty index σ_λ becomes less responsive towards changes in n.

Intuitively, when most of the buyers multihome ($\lambda \to 1$), increased platform competition induces platforms to compete more intensely for sellers, as explained previously (following Proposition 2). However, when most of the buyers singlehome ($\lambda \to 0$), platforms have monopoly power over providing access to their buyers for the multihoming sellers. As such, increased platform competition induces platforms to compete more intensely for buyers rather than for sellers (the normal competitive bottleneck logic). We discuss the economic implication of this result for specific markets in the next two subsections.

6.1 Ride-hailing platforms

In the context of ride-hailing platforms, for each trip the riders (buyers) enjoy benefits while the drivers (sellers) incur efforts, so that $p^b > 0 > p^s$ in practice. Here, p^b is the fare set by the platforms, the negative value of p^s is the per-ride driver gross earning (or wage), and $p^b + p^s$ is the net commission that platforms earn from each ride. In this context, multihoming riders are those who compare and choose between multiple apps whenever they call for a ride, while singlehoming riders are those who do not do so. To facilitate exposition, we focus on the polar cases of all buyers singlehoming ($\lambda = 0$) and all buyers multihoming ($\lambda = 1$), while noting that the general qualitative insights remain the same for cases between these two extremes ($\lambda \in (0, 1)$). Figure 5 numerically illustrates this application, assuming that F and $F_0 \sim$ *Gumbel*.

☐ **Platform competition.** Buyer multihoming profoundly reverses the dynamics of platform competition. When riders are singlehoming, existing ride-hailing platforms respond to entry by cutting the fare to attract riders, and then reoptimize by offering less to drivers. However, when riders are multihoming, if the incumbent platforms naively continue to respond by cutting fares and driver wages, then some drivers will simply quit the lower-wage incumbents,

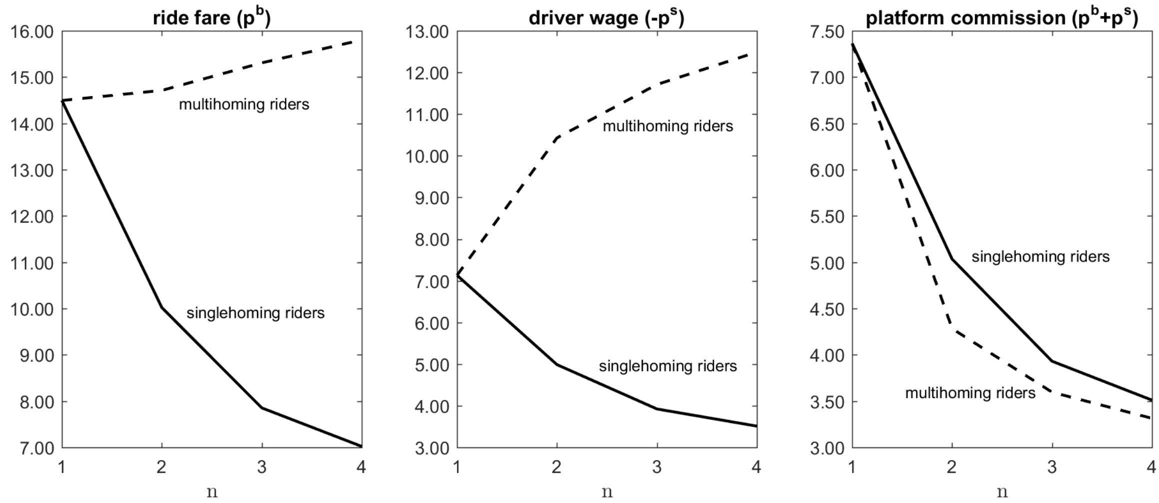

Figure 5: Ride-hailing market

knowing that they can still access a large portion of riders through other higher-wage platforms. Instead, our analysis suggests that the response in equilibrium would be the reverse: platforms increase wages to attract drivers, and then reoptimize the fare by charging more. The possibility of a fare increase following entry is in contrast to the conventional one-sided logic that high final product prices (in this case, rider fares) are caused by a lack of competition.[21]

□ **Platform merger and exit.** The industry of ride-hailing services has witnessed several high profile merger cases in recent years, including Didi-Uber in China (2016), Yandex-Uber in Russia (2017), Grab-Uber in South East Asia (2018), and Careem-Uber in Middle East (2019). Notably, each of these mergers has resulted in one of the platforms exiting the market entirely.[22] Based on analyzing what happens when n decreases by one, our analysis suggests that the effect of these mergers on the platform fee structure depends critically on the level of rider-multihoming. This provides an empirical implication: even in the absence of any cost-efficiency gain from the merger, it is possible for such a merger to result in lower fares for riders (if the extent of rider-multihoming is high) or higher earnings for drivers (if the extent of rider-multihoming is low). Regardless of the level of rider-multihoming, however, our model also predicts the total fee charged to the two sides will increase.

6.2 Payment card platforms

Payment card platforms typically offer card holders (buyers) a variety of card-usage benefits e.g. interest-free periods, cash rebates and loyalty rewards. Platforms then make money by charging transaction fees on merchants (sellers), so that $p^s > 0 > p^b$ in practice. Here, the negative value of p^b represents the various rewards on card transactions, p^s is the merchant fee (or interchange fee assuming the acquiring side is perfectly competitive), and $p^b + p^s$ is the profit margin earned by card issuers (or AMEX in the case of a proprietary card scheme). In this context, multihoming cardholders are those who have multiple cards to choose from at the

[21]See also Bryan and Gans (2019) for an investigation of how the multihoming behaviour of riders and drivers affects pricing (and welfare) then there are two competing ride-hailing platforms.

[22]Therefore, these merger cases are different from standard horizontal mergers involving differentiated products, where the merged entity would continue operating both of the original brands so as to maximize their joint profit.

point of transactions, while singlehoming cardholders are those who do not.

☐ **Platform competition and interchange fees.** Policymakers in some jurisdictions, including Australia, Europe, and United Kingdom, have claimed that payment card platforms set interchange fees too high. As summed up by Guthrie and Wright (2006), these authorities appear to view the lack of competition between platforms as a possible cause of high interchange fees. However, Proposition 6 suggests that this view by the authorities is true only when most of the cardholders are multihoming, whereby increasing inter-platform competition indeed helps to reduce the interchange fee paid by the merchant side to the cardholder side. Notably, the reverse view is true when the fraction of singlehoming cardholders is sufficiently large, whereby increasing inter-platform competition drives up the interchange fee instead, which seems to match the empirical evidence better (Rysman and Wright, 2015).[23]

7 Conclusion

This paper investigated two-sided market pricing by oligopolistic platforms when platforms set transaction fees on both user sides. We provided a framework which allows the underlying economic forces determining platform pricing to be anayzed. The following table summarizes the findings of our various comparative statics exercises. The second broad column describes the changes in platform market power over each side of the market, as defined in (9) and (10). The third broad column describes the changes in equilibrium fees in terms of fee structure and total fees.

	Platform market power		Equilibrium fees		
An increase in	on buyers	on sellers	buyers	sellers	total
extent of buyer multihoming (λ)	0	$-$	$+$	$-$	$-$
number of platforms (n) when λ is large	$-$	$-$	$+^*$	$-^*$	$-$
number of platforms (n) when λ is small	$-$	$-$	$-$	$+$	$-$
value of transactions for buyers (β)	$+$	$-$	$+$	$-$	$+$
buyer heterogeneity (γ) when $\hat{p}^b > 0$	$+$	$-$	$+$	$-$	$+$
buyer heterogeneity (γ) when $\hat{p}^b \leq 0$	*ambiguous*	$+$		*ambiguous*	
value of transactions for sellers (α)	0	$+$	$-$	$+$	$+$

Note: "0" = not changing; " + " = increase; " − " = decrease;

* = with additional conditions specified in Proposition 2.

There are two built-in asymmetries across the two sides in our model (i) buyers choose the platform to make their transaction on; (ii) sellers treat platforms as homogenous. The first one is natural, but the second one may not always apply. So one possible future direction is to extend the framework to study what happens if platforms are differentiated from the sellers' perspective as well. This would potentially generate richer equilibrium configurations where

[23]This provides another reason why increased platform competition can increase interchange fees separate from the existing explanation of Guthrie and Wright (2006) and Edelman and Wright (2015) which relies on the use of the no-surcharge rule (or price coherence more generally).

some sellers multihome on a subset of platforms while other sellers multihome on all platforms.

Another direction is to try to incorporate seller competition more explicitly into the current framework, which is particularly relevant when the sellers are merchants that compete on price. As we noted in Section 5, allowing for seller competition should increase the buyer loyalty index because it weakens the ability of each seller to divert buyers by delisting from a more expensive platform. Future research could formalize this result, and see whether increased seller competition indeed leads platforms to increase their fees to sellers and in total, while decreasing their fees to buyers.

A Appendix

A.1 Further details of demand derivation

In this appendix, we complete the demand derivation by considering the seller participation profile under a downward deviation $p_i^s < \hat{p}^s$. It is obvious that if a seller joins at least one platform, then the seller must also join platform i given that i charges the lowest seller fee. A seller will join i as long as $v \geq p_i^s$. However, the fact that all other platforms $j \neq i$ set $\hat{p}^s$ does not necessarily imply that the seller will join all these platforms together in a "block". This is because when $p_i^s < \hat{p}^s$, any additional platform that a seller joins will divert additional buyers away from the lowest-fee platform i to the newly joined platform. Therefore, the number of platforms a seller multihomes on will depend on v in general.

Consider a seller who chooses to join platform i together with $m-1$ other (symmetric) platforms. We denote this set of platforms as $\mathbf{N}_{i,m}$ (the seller joins m platforms in total, including i). Note that $\mathbf{N}_{i,1} = \{i\}$ and $\mathbf{N}_{i,n} = \mathbf{N}$ so m is bounded between 1 and n. The corresponding number of buyers who use i for transactions is

$$B_i^{(\mathbf{N}_{i,m})} = \Pr\left(\epsilon_i - p_i^b \geq \max_{j \in \mathbf{N}_{i,m}} \left\{\epsilon_j - p_j^b, \epsilon_0\right\}\right).$$

Clearly a higher m implies more buyers diverted from platform i since $B_i^{(\mathbf{N}_{i,m})}$ decreases with m. With a slight abuse of notation, let

$$B_0^{(\mathbf{N}_{i,m})} = \Pr\left(\epsilon_0 \geq \max_{j \in \mathbf{N}_{i,m}} \left\{\epsilon_j - p_j^b, \epsilon_i - p_i^b\right\}\right).$$

The following lemma states sellers' multihoming decision formally:

Lemma A.1 *Suppose $p_i^s < \hat{p}^s$. For $m = 2, ..., n$, define cutoffs*

$$\hat{v}_m \equiv \left(\hat{p}^s - p_i^s\right) \frac{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m+1})}}{B_0^{(\mathbf{N}_{i,m})} - B_0^{(\mathbf{N}_{i,m+1})}} + \hat{p}^s. \tag{15}$$

A type v seller joins no platform if $v \in [\underline{v}, p_i^s)$, joins only platform i if $v \in [p_i^s, \hat{v}_2)$, joins platform i together with $m-1$ randomly chosen symmetric platform(s) from $j \neq i$ if $v \in [\hat{v}_m, \hat{v}_{m+1})$, and joins all platforms if $v > \hat{v}_n$.

Proof. Consider a type v seller that has joined platform i and that is contemplating whether to join one of the platforms $j \neq i$ in addition. The utility of joining i alone (so $m = 1$) is $\left(v - p_i^s\right) B_i^{(\mathbf{N}_{i,0})}$, so this is superior than joining no platforms as long as $v \geq p_i^s$. Meanwhile the utility from joining another platform $j \neq i$ (so that $m = 2$) is $\left(v - p_i^s\right) B_i^{(\mathbf{N}_{i,1})} + \left(v - \hat{p}^s\right) B_j^{(\mathbf{N}_{i,1})}$. Comparing the two utilities yields the first cutoff

$$\hat{v}_2 \equiv \left(\hat{p}^s - p_i^s\right) \frac{B_i^{(\mathbf{N}_{i,0})} - B_i^{(\mathbf{N}_{i,1})}}{-B_i^{(\mathbf{N}_{i,0})} + B_i^{(\mathbf{N}_{i,1})} + B_j^{(\mathbf{N}_{i,1})}} + \hat{p}^s = \left(\hat{p}^s - p_i^s\right) \frac{B_i^{(\mathbf{N}_{i,0})} - B_i^{(\mathbf{N}_{i,1})}}{B_0^{(\mathbf{N}_{i,0})} - B_0^{(\mathbf{N}_{i,1})}} + \hat{p}^s.$$

Now suppose a seller has joined the set of platforms $\mathbf{N}_{i,m-1}$, i.e., platform i plus $m-2$ other platforms. Owing to the symmetry of all platforms $j \neq i$, the seller's utility can be written as $\left(v - p_i^s\right) B_i^{(\mathbf{N}_{i,m-1})} + (m-2)\left(v - \hat{p}^s\right) B_j^{(\mathbf{N}_{i,m-1})}$. The utility of joining one more platform — so that the seller joins platform i plus m other platforms, i.e. the set of platforms $\mathbf{N}_{i,m}$, is $\left(v - p_i^s\right) B_i^{(\mathbf{N}_{i,m})} + (m-1)\left(v - \hat{p}^s\right) B_j^{(\mathbf{N}_{i,m})}$. Comparing the two utilities yields cutoffs $\hat{v}_m$ (15) for all $m \leq n$. ■

Combining this with the case of upward deviation derived in the main text, the complete demand function faced by platform i is piece-wise defined by

$$Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) = \left\{ \begin{array}{cc} \sum_{m=1}^{n} \left[G\left(\hat{v}_{m+1}\right) - G\left(\hat{v}_m\right)\right] B_i^{(\mathbf{N}_{i,m})} & \text{if } p_i^s < \hat{p}^s \\ \left(1 - G\left(\hat{v}\right)\right) B_i^{(\mathbf{N})} & \text{if } p_i^s \geq \hat{p}^s \end{array} \right\}, \tag{16}$$

where we denote $\hat{v}_1 \equiv p_i^s$ and $\hat{v}_{n+1} \equiv \bar{v}$ (so that $G\left(\hat{v}_n\right) = 1$). Note that when $p_i^s < \hat{p}^s$, the volume takes into account sellers' heterogenous multihoming behavior. Figure 6 provides an illustration of function (16) assuming $n = 3$:

The left panel of Figure 6 depicts $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ when $p_i^s \geq \hat{p}^s$. In this case, only sellers with $v \geq \hat{v}$ join platform i, and the mass of buyers who use platform i to transact with each of these sellers is $B_i^{(\mathbf{N})}$, that is,

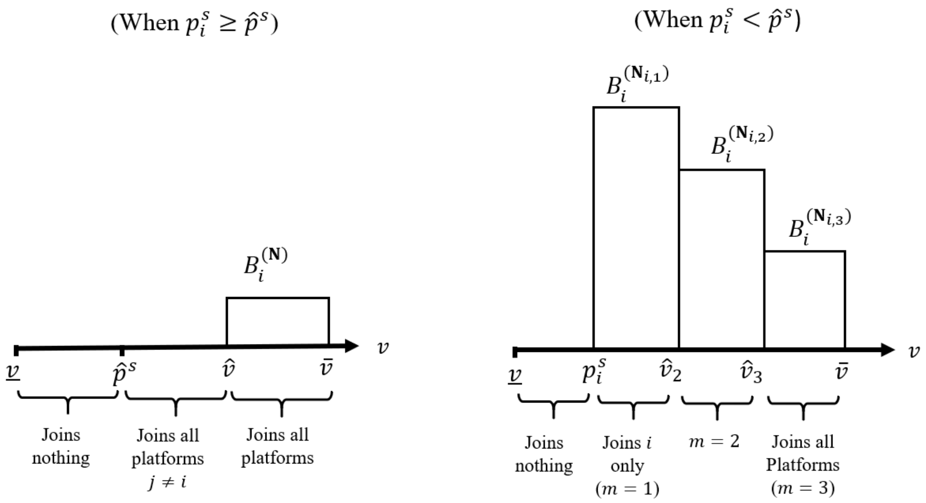

Figure 6: Seller multihoming and the associated transactions by buyers through i.

those who find i most attractive when all n platforms are available for transactions. The right panel of Figure 6 depicts the case of $p_i^s < \hat{p}^s$, where recall m denotes the number of platforms that a seller multihomes on in addition to platform i. Sellers with $v \in [p_i^s, \hat{v}_1)$ join platform i exclusively, so that buyers who transact with these sellers can only choose between transacting through i or transacting directly. The mass of buyers who use i to transact with these sellers is $B_i^{(\mathbf{N}_{i,1})}$, that is, those who find i more attractive than the outside option. Sellers with $v \in [\hat{v}_2, \hat{v}_3)$ join platform i and a randomly selected platform $j \neq i$, so that buyers who transact with these sellers can choose between transacting through i, j, or transacting directly. Notably, the mass of buyers who use i to transact with these sellers is $B_i^{(\mathbf{N}_{i,2})}$, which is smaller than $B_i^{(\mathbf{N}_{i,1})}$ due to the availability of an additional alternative platform for transactions.

If $B_i^{(\cdot)}$ satisfies the IIA property, it implies the ratio $B_i^{(\mathbf{N}_{i,m})}/B_0^{(\mathbf{N}_{i,m})}$ is independent of m. Denote the said constant ratio as χ. Through algebraic manipulations, we can simplify $\hat{v}_m$ in (15) as

$$\hat{v}_m = (\hat{p}^s - p_i^s)\,\chi + \hat{p}^s, \tag{17}$$

which is independent of m.

A.2 Proof of Proposition 1

We first state and prove the following two lemmas, which also prove Lemma 2 and Lemma 3 in the main text.

Lemma A.2 *Buyer-side inverse semi-elasticity $X(p;n)$ defined in (5) is decreasing in n and p.*

Proof. (**Lemma A.2**). Let $\epsilon_{(n)}$ denote the highest order statistic (out of n draws of ϵ), and denote

$$\bar{X}(\epsilon_0 + p; n) = \frac{\frac{1}{n}\left(1 - F(\epsilon_0 + p)^n\right)}{\int_{\epsilon_0+p}^{\bar{\epsilon}} [f(\epsilon)]\, dF(\epsilon)^{n-1} + f(\epsilon_0 + p)\, F(\epsilon_0 + p)^{n-1}}$$

as the buyer inverse semi-elasticity for given non-random outside option $\epsilon_0 + p$. Then, from definition (5) and exploiting the alternative expression of

$$\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon, \epsilon_0 + p^b\right\}\right) dF(\epsilon)^{n-1}\, dF_0(\epsilon_0) = \frac{1}{n} \int_{\underline{\epsilon}}^{\bar{\epsilon}-p} \left[1 - F(\epsilon_0 + p)^n\right] dF_0(\epsilon_0),$$

we can rewrite $X(p;n)$ as

$$\begin{aligned}\frac{1}{X(p;n)} &= \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[\frac{\int_{\epsilon_0+p}^{\bar{\epsilon}}[f(\epsilon)]\,dF(\epsilon)^{n-1}+f(\epsilon_0+p)\,F(\epsilon_0+p)^{n-1}}{\frac{1}{n}\left(1-F(\epsilon_0+p)^n\right)}\right]\left[\frac{\left(1-F(\epsilon_0+p)^n\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\left[1-F(\epsilon_0+p)^n\right]dF_0(\epsilon_0)}\right]dF_0(\epsilon_0)\\ &= \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[\frac{1}{\bar{X}(\epsilon_0+p;n)}\right]\left[\frac{\left(1-F(\epsilon_0+p)^n\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\left[1-F(\epsilon_0+p)^n\right]dF_0(\epsilon_0)}\right]dF_0(\epsilon_0).\end{aligned}$$

Define a new random variable $\tilde{\epsilon}_0\equiv\epsilon_0+p$ with support over $[\underline{\epsilon}_0+p,\bar{\epsilon}_0+p]$, and define the cdf of $\tilde{\epsilon}_0$ conditioned on it being smaller than $\epsilon_{(n)}$:

$$H(x;n,p)\equiv\Pr\left(\tilde{\epsilon}_0<x|\tilde{\epsilon}_0<\epsilon_{(n)}\right)=\frac{\int_{\underline{\epsilon}_0+p}^{x}\left(1-F(\tilde{\epsilon}_0)^n\right)f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}}\left(1-F(\tilde{\epsilon}_0)^n\right)f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}. \tag{18}$$

Then,

$$\frac{1}{X(p;n)}=\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[\frac{1}{\bar{X}(\tilde{\epsilon}_0;n)}\right]dH(\tilde{\epsilon}_0;n,p).$$

Lemma 4 of Zhou (2017) shows that $1/\bar{X}(\tilde{\epsilon}_0;n)$ is increasing in $\tilde{\epsilon}_0$ and n. Hence, to conclude that $\frac{1}{X(p;n)}$ is increasing in p and n, it remains to show that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ is increasing in n and p in the sense of first-order stochastic dominance (FOSD), i.e. $H(x;n,p)$ is decreasing in p and n at each given x.

Claim: *$\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ is FOSD increasing in p.* From the cdf function, the relevant derivative $\frac{\partial H(x;n,p)}{\partial p}$can be shown to be negative if

$$\frac{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0'(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}\geq\frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0'(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}. \tag{19}$$

Given $x\leq\bar{\epsilon}$, establishing (19) is equivalent to showing that the left-hand side of (19) is decreasing in x. If we define the distribution function

$$\begin{aligned}\tilde{H}(y;x) &= \Pr\left(\tilde{\epsilon}_0<y|\tilde{\epsilon}_0<\max\left\{\epsilon_{(n)},x\right\}\right)\\ &= \frac{\int_{\underline{\epsilon}_0+p}^{y}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}\quad\text{for } y\in[\underline{\epsilon}+p,x],\end{aligned}$$

then we can rewrite the left-hand side of (19) as $\int_{\underline{\epsilon}_0+p}^{x}\left[\frac{f_0'(y-p)}{f_0(y-p)}\right]d\tilde{H}(y;x)$. Log-concavity of f_0 implies that $\frac{f_0'(y-p)}{f_0(y-p)}$ is decreasing in y. Meanwhile it is easily verified from its definition, that $\tilde{H}(y;x)$ is FOSD increasing in x. Therefore, we conclude that the left-hand side of (19) is decreasing in x, so that inequality (19) indeed holds.

Claim: *$\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ is FOSD increasing in n.* From the cdf function, the relevant derivative $\frac{\partial H(x;n,p)}{\partial n}$ can be shown to be negative if

$$\frac{\int_{\underline{\epsilon}_0+p}^{x}\left[-\ln F(\tilde{\epsilon}_0)\,F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{x}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}\leq\frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[-\ln F(\tilde{\epsilon}_0)\,F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[1-F(\tilde{\epsilon}_0)^n\right]f_0(\tilde{\epsilon}_0-p)\,d\tilde{\epsilon}_0}, \tag{20}$$

so that $\frac{\partial H(x;n,p)}{\partial n}\leq 0$ if the left-hand side of (20) is increasing in x. Applying the same technique used in the previous claim, we can write the left-hand side of (20) as

$$\int_{\underline{\epsilon}_0+p}^{x}\left[\frac{-\ln F(y)\,F(y)^n}{1-F(y)^n}\right]d\tilde{H}(y;x).$$

Since $-\ln F(y)\geq 0$, we know that $\frac{-\ln F(y)F(y)^n}{1-F(y)^n}$ is increasing in y. This fact, together with the fact that $\tilde{H}(y;x)$ is FOSD increasing in x, implies that the left-hand side of (20) is increasing in x, and so the inequality in (20) indeed holds. ■

Lemma A.3 *The buyer loyalty index $\sigma(p;n)$ defined in (6) is strictly decreasing in n and increasing in p.*

Proof. (**Lemma A.3**). Rewrite $\sigma(p;n)$ as

$$\begin{aligned}\sigma(p;n) &= \frac{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[nF(\epsilon_0+p)^{n-1}(1-F(\epsilon_0+p))\right]dF_0(\epsilon_0)}{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\left[1-F(\epsilon_0+p)^n\right]dF_0(\epsilon_0)} \\ &= \Pr\left(\epsilon_{(n-1)}<\tilde{\epsilon}_0|\epsilon_{(n)}>\tilde{\epsilon}_0\right),\end{aligned}$$

where $\tilde{\epsilon}_0 \equiv \epsilon_0+p$. To show $\sigma(p;n)$ strictly increases with p, we write

$$\begin{aligned}\sigma(p;n) &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\Pr\left(\epsilon_{(n-1)}<y|\epsilon_{(n)}>y\right)\Pr\left(\tilde{\epsilon}_0=y|\tilde{\epsilon}_0<\epsilon_{(n)}\right)dy \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\Pr\left(\epsilon_{(n-1)}<y|\epsilon_{(n)}>y\right)dH(\tilde{\epsilon}_0;n,p),\end{aligned} \tag{21}$$

where $H(\tilde{\epsilon}_0;n,p)$ is the conditional distribution function defined in (18). We first observe that $\Pr\left(\epsilon_{(n-1)}<y|\epsilon_{(n)}>y\right)$ is strictly increasing in y:

$$\begin{aligned}\Pr\left(\epsilon_{(n-1)}<y|\epsilon_{(n)}>y\right) &= \frac{nF(y)^{n-1}(1-F(y))}{(1-F(y)^n)} \\ \frac{d}{dy}\Pr\left(\epsilon_{(n-1)}<y|\epsilon_{(n)}>y\right) &= \frac{1-F(y)-\frac{1}{n}(1-F(y)^n)}{(1-F(y)^n)^2}f(y)n^2>0.\end{aligned}$$

We also know from the proof of Lemma A.2 that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ associated with cdf H is FOSD increasing in p. This fact, together with the observation that the integrand of (21) is an increasing function, imply that $\sigma(p;n)$ is strictly increasing in p as required.

To show $\sigma(p;n)$ strictly decreases with n, we write

$$\sigma(p;n)=\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\Pr\left(\epsilon_{(n-1)}<\tilde{\epsilon}_0|\epsilon_{(n)}=y\right)\Pr\left(\epsilon_{(n)}=y|\epsilon_{(n)}>\tilde{\epsilon}_0\right)dy.$$

Then, we make the following two claims:

Claim: *For arbitrary constant $y\in[\underline{\epsilon},\bar{\epsilon}]$, $\Pr\left(\epsilon_{(n-1)}<\tilde{\epsilon}_0|\epsilon_{(n)}=y\right)$ is strictly decreasing in n and y.* By definition,

$$\begin{aligned}\Pr\left(\epsilon_{(n-1)}<\tilde{\epsilon}_0|\epsilon_{(n)}=y\right) &= \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}nF(\min\{\tilde{\epsilon}_0,y\})^{n-1}f(y)\,dF_0(\tilde{\epsilon}_0-p)}{nF(y)^{n-1}f(y)} \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left(\frac{F(\min\{\tilde{\epsilon}_0,y\})}{F(y)}\right)^{n-1}dF_0(\tilde{\epsilon}_0-p),\end{aligned}$$

which is clearly strictly decreasing in n and y.

Claim: *$\epsilon_{(n)}|_{\epsilon_{(n)}>\tilde{\epsilon}_0}$ is FOSD increasing in n.* By definition, the corresponding CDF is

$$\begin{aligned}\Pr\left(\epsilon_{(n)}<x|\epsilon_{(n)}>\tilde{\epsilon}_0\right) &= \frac{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[F(x)^n-F(\min\{\tilde{\epsilon}_0,x\})^n\right]dF_0(\tilde{\epsilon}_0-p)}{\int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[1-F(\min\{\tilde{\epsilon}_0,x\})^n\right]dF_0(\tilde{\epsilon}_0-p)} \\ &= \int_{\underline{\epsilon}_0+p}^{\bar{\epsilon}_0+p}\left[\frac{F(x)^n-F(\min\{\tilde{\epsilon}_0,x\})^n}{1-F(\min\{\tilde{\epsilon}_0,x\})^n}\right]dH(\tilde{\epsilon}_0;n,p),\end{aligned}$$

where $H(\tilde{\epsilon}_0;n,p)$ is the conditional distribution function defined in (18). We first observe that the integrand is decreasing in n: showing this is equivalent to showing $\frac{a^n-1}{b^n-1}$, $(1<a<b)$ is decreasing in n, which is easily verified from calculating $\frac{d}{dn}\left(\frac{a^n-1}{b^n-1}\right)\leq 0$. Likewise, the integrand is decreasing in $\tilde{\epsilon}_0$. These two observations, together with the proven result that the conditional random variable $\tilde{\epsilon}_0|_{\tilde{\epsilon}_0<\epsilon_{(n)}}$ associated with cdf H is FOSD increasing in n, implies $\Pr\left(\epsilon_{(n)}<x|\epsilon_{(n)}>\tilde{\epsilon}_0\right)$ is decreasing in n as required.

Using these two claims, we have for any $n' \geq n$,

$$\begin{aligned}
& \sigma(p;n) \\
> & \int_{\underline{\epsilon}_0}^{\tilde{\epsilon}_0} \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} = y\right) \Pr\left(\epsilon_{(n)} = y | \epsilon_{(n)} > \tilde{\epsilon}_0\right) dy \\
\geq & \int_{\underline{\epsilon}_0}^{\tilde{\epsilon}_0} \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} = y\right) \Pr\left(\epsilon_{(n')} = y | \epsilon_{(n')} > \tilde{\epsilon}_0\right) dy \\
= & \Pr\left(\epsilon_{(n'-1)} < \tilde{\epsilon}_0 | \epsilon_{(n')} > \tilde{\epsilon}_0\right) = \sigma\left(p;n'\right).
\end{aligned}$$

So $\sigma(p;n)$ is indeed strictly decreasing in n. ■

To prove Proposition 1, we first note that the demand derivatives, after imposing symmetry, are

$$\begin{aligned}
Q_i(\hat{\mathbf{p}};\hat{\mathbf{p}}) &= (1 - G(\hat{p}^s)) B_i^{(\mathbf{N})}|_{p_i^b = \hat{p}^b} \\
&= (1 - G(\hat{p}^s)) \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0) \\
\frac{dQ_i(\hat{\mathbf{p}};\hat{\mathbf{p}})}{dp_i^b} &= (1 - G(\hat{p}^s)) \frac{\partial B_i^{(\mathbf{N})}}{\partial p_i^b}|_{p_i^b = \hat{p}^b} \\
&= -(1 - G(\hat{p}^s)) \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0} \int_{\underline{\epsilon}}^{\bar{\epsilon}} f\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF(\epsilon)^{n-1} dF_0(\epsilon_0). \\
\frac{dQ_i(\hat{\mathbf{p}};\hat{\mathbf{p}})}{dp_i^s} &= -g(\hat{p}^s) \frac{d\hat{v}}{dp_i^s} B_i^{(\mathbf{N})}|_{p_i^b = \hat{p}^b} \\
&= \frac{-g(\hat{p}^s)}{\sigma(p^b;n)} B_i^{(\mathbf{N})}|_{p_i^b = \hat{p}^b}.
\end{aligned}$$

The standard first-order condition yields (8). To prove the existence and uniqueness of $\hat{p}^b$ and $\hat{p}^s$ defined in (8), denote $\hat{T} \equiv \hat{p}^b + \hat{p}^s$ and $M(\hat{p}^s) \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. From (8), for n fixed:

$$\hat{T} - c = X(\hat{p}^b) = M(\hat{p}^s)\,\sigma(\hat{p}^b). \tag{22}$$

Lemma A.2 and strict log-concavity of $1 - G$ implies that $X(.)$ and $M(.)$ are monotone decreasing functions. Define the generalized inverse functions as:

$$\begin{aligned}
X^{-1}(T) &= \inf\{p \in [\underline{\epsilon}, \bar{\epsilon}] : X(p) \geq T\} \\
M^{-1}(T) &= \inf\{p \in [\underline{v}, \bar{v}] : M(p) \geq T\},
\end{aligned}$$

so $X^{-1}(T)$ and $M^{-1}(T)$ are monotone decreasing functions. Therefore, we can express $\hat{p}^b = X^{-1}(\hat{T} - c)$ and $\hat{p}^s = M^{-1}\left(\frac{\hat{T}-c}{\sigma\left(X^{-1}(\hat{T}-c)\right)}\right)$, and rewrite $\hat{T} \equiv \hat{p}^b + \hat{p}^s$ as

$$\hat{T} = X^{-1}(\hat{T} - c) + M^{-1}\left(\frac{\hat{T} - c}{\sigma\left(X^{-1}(\hat{T} - c)\right)}\right), \tag{23}$$

where the right-hand side is monotone decreasing in $\hat{T}$ by the definitions of X^{-1} and M^{-1} and Lemma A.3. This implies that there exist a unique fixed point $\hat{T}$ that solves (23) and hence solves (22) by construction. Then, $\hat{p}^b$ and $\hat{p}^s$ can be uniquely determined from the one-to-one relations stated above.

A.3 Proof of Proposition 2

Denote $M(\hat{p}^s) \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$, and let the derivatives of X and σ with respect to the first argument be denoted X' and σ'. Total differentiation of (8), in matrix form, gives

$$\begin{bmatrix} 1 - X' & 1 \\ 1 - M\sigma' & 1 - \sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\hat{p}^b}{dn} \\ \frac{d\hat{p}^s}{dn} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial n} \\ M\frac{\partial \sigma}{\partial n} \end{bmatrix}. \tag{24}$$

Since $X' \leq 0$, $\frac{\partial M}{\partial \hat{p}^s} < 0$, and $\sigma' > 0$ (Lemma A.2 and Lemma A.3), the matrix in (24) has determinant

$$Det \equiv \underbrace{\left(1 - X'\right)\left(1 - \sigma \frac{\partial M}{\partial \hat{p}^s}\right)}_{>1} - 1 + M\sigma' > 0. \tag{25}$$

By Cramer's rule, and substituting for the equilibrium condition $M = \frac{X}{\sigma}$, we have

$$\frac{d\hat{p}^s}{dn} = \frac{1}{Det}\begin{vmatrix} 1 - X' & \frac{\partial X}{\partial n} \\ 1 - M\sigma' & M\frac{\partial \sigma}{\partial n} \end{vmatrix} = \frac{X}{Det}\left[\left(\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n}\right) + \left(\underbrace{\frac{\partial X}{\partial n}\sigma'}_{\leq 0} - \underbrace{\frac{\partial \sigma}{\partial n}X'}_{\geq 0}\right)\frac{1}{\sigma}\right]; \tag{26}$$

$$\frac{d\hat{p}^b}{dn} = \frac{1}{Det}\begin{vmatrix} \frac{\partial X}{\partial n} & 1 \\ M\frac{\partial \sigma}{\partial n} & 1 - \sigma\frac{\partial M}{\partial \hat{p}^s} \end{vmatrix} = \frac{X}{Det}\left[-\left(\frac{1}{\sigma}\frac{\partial \sigma}{\partial n} - \frac{1}{X}\frac{\partial X}{\partial n}\right) - \underbrace{\frac{\sigma}{X}\frac{\partial M}{\partial \hat{p}^s}\frac{\partial X}{\partial n}}_{\geq 0}\right]. \tag{27}$$

We know $\frac{\partial X}{\partial n} \leq 0$ and $\frac{\partial \sigma}{\partial n} \leq 0$ (Lemma A.2 and Lemma A.3), therefore

$$\frac{d\hat{p}^s}{dn} + \frac{d\hat{p}^b}{dn} = \frac{1}{Det}\left[M\left(\underbrace{\sigma'\frac{\partial X}{\partial n}}_{<0} - \underbrace{\frac{\partial \sigma}{\partial n}X'}_{>0}\right) - \underbrace{\sigma\frac{\partial M}{\partial \hat{p}^s}\frac{\partial X}{\partial n}}_{\geq 0}\right] \leq 0.$$

Denote

$$\Lambda \equiv \int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\int_0^{\bar{\epsilon}} f\left(\max\left\{\epsilon, \epsilon_0 + \hat{p}^b\right\}\right) dF\left(\epsilon\right)^{n-1} dF_0\left(\epsilon_0\right),$$

and let Λ' be its partial derivative wrt n. If f is decreasing, the fact that $F\left(\epsilon\right)^{n-1}$ is FOSD increasing in n implies $\Lambda' \leq 0$. Computing the relevant derivatives, we get

$$\begin{aligned} \frac{\partial \sigma/\partial n}{\sigma} - \frac{\partial X/\partial n}{X} &= \left(\frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right)F\left(\epsilon_0 + \hat{p}^b\right)^{n-1}\left(1 - F\left(\epsilon_0 + \hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[F\left(\epsilon_0 + \hat{p}^b\right)^{n-1}\left(1 - F\left(\epsilon_0 + \hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)} + \frac{\Lambda'}{\Lambda}\right) \\ &< 0, \end{aligned}$$

where the inequality is due to $\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right) < 0$ and $\Lambda' \leq 0$. Thus, $\frac{d\hat{p}^s}{dn} < 0$ from (26).

If g is decreasing, it implies that $\frac{\partial M}{\partial \hat{p}^s} \geq -1$ so (27) implies

$$\frac{d\hat{p}^b}{dn} \geq \frac{1}{Det}\left[\left((1+\sigma)\frac{1}{X}\frac{\partial X}{\partial n} - \frac{1}{\sigma}\frac{\partial \sigma}{\partial n}\right)\right]. \tag{28}$$

The right-hand side of (28) is strictly positive if and only if

$$\begin{aligned} 0 \quad > \quad & \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right)F\left(\epsilon_0 + \hat{p}^b\right)^{n-1}\left(1 - F\left(\epsilon_0 + \hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}-p}\left[F\left(\epsilon_0 + \hat{p}^b\right)^{n-1}\left(1 - F\left(\epsilon_0 + \hat{p}^b\right)\right)\right]dF_0\left(\epsilon_0\right)} \\ & + \sigma\left(\frac{1}{n} + \frac{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[\ln\left(F\left(\epsilon_0 + \hat{p}^b\right)\right)F\left(\epsilon_0 + \hat{p}^b\right)^n\right]dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left[1 - F\left(\epsilon_0 + \hat{p}^b\right)^n\right]dF_0\left(\epsilon_0\right)}\right) + \frac{\Lambda'}{\Lambda}(1+\sigma). \end{aligned}$$

We know $\Lambda' \leq 0$ if f is weakly decreasing. Meanwhile, applying L'Hopital rule twice shows that the first two components converges to zero when $\hat{p}^b \to (\bar{\epsilon} - \underline{\epsilon})$. Moreover, calculating the first derivative shows that the sum of the first two components to be increasing in $\hat{p}^b$, hence the sum is non-positive for all $\hat{p}^b \leq \bar{\epsilon} - \underline{\epsilon}$.

A.4 Results for logit buyer quasi-demand

In this section, we analyze in detail the special case when F and $F_0 \sim Gumbel(\mu)$ with a common scale parameter μ and a common location parameter normalized to 0 without loss of generality (allowing for a different location parameter for F_0 does not affect the analysis below). We first prove the quasi-concavity of the profit function when G is linear:[24]

Lemma A.4 *If F, $F_0 \sim Gumbel(\mu)$, and G is linear over $[\underline{v}, \bar{v}]$, then for all $\mathbf{p}_i = \left(p_i^s, p_i^b\right)$,*

$$Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right) = \left(1 - \frac{\hat{v} - \underline{v}}{\bar{v} - \underline{v}}\right) \times \frac{\exp\left\{-p_i^b/\mu\right\}}{1 + \exp\left\{-p_i^b/\mu\right\} + (n-1)\exp\left\{-\hat{p}^b/\mu\right\}},$$

where $\hat{v} = p_i^s + \left(p_i^s - \hat{p}^s\right)(n-1)\exp\left\{-\hat{p}^b/\mu\right\}$. Moreover, $Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$ is globally log-concave in $\mathbf{p}_i$.

Proof. We first consider $p_i^s \geq \hat{p}^s$. Substituting for the logit demand form and simplifying, Lemma 1 implies the result immediately. When $p_i^s < \hat{p}^s$, logit demand form and (17) implies

$$\hat{v}_m = \left(\hat{p}^s - p_i^s\right)\frac{B_i^{(\mathbf{N}_{i,m})}}{B_0^{(\mathbf{N}_{i,m})}} + \hat{p}^s = \left(\hat{p}^s - p_i^s\right)\exp\left\{-p_i^b/\mu\right\} + \hat{p}^s,$$

which is independent of m, so

$$\begin{aligned}
& Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)|_{p_i^s < \hat{p}^s} \\
= & \left(1 - G\left(\hat{v}_1\right) + \left(G\left(\hat{v}_1\right) - G\left(p_i^s\right)\right)\frac{B_i^{(\mathbf{N}_{i,1})}}{B_i^{(\mathbf{N}_{i,n})}}\right) B_i^{(\mathbf{N}_{i,n})} \\
= & \left(1 - G\left(\hat{v}_1\right) + \left(G\left(\hat{v}_1\right) - G\left(p_i^s\right)\right)\left(1 + \frac{(n-1)\exp\left\{-\hat{p}^b/\mu\right\}}{1 + \exp\left\{-p_i^b/\mu\right\}}\right)\right) B_i^{(\mathbf{N}_{i,n})} \\
= & \left(1 - G\left(p_i^s\right) + \left(\frac{G\left(\hat{v}_1\right) - G\left(p_i^s\right)}{\hat{v}_1 - p_i^s}\right)\left(\hat{p}^s - p_i^s\right)(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right) B_i^{(\mathbf{N})}
\end{aligned}$$

where the final line uses $\hat{v}_1 - p_i^s = (1 + \exp\left\{-p_i^b/\gamma\right\})(\hat{p}^s - p_i^s)$ and $B_i^{(\mathbf{N}_{i,n-1})} = B_i^{(\mathbf{N})}$. Linearity of G implies

$$\begin{aligned}
Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)|_{p_i^s < \hat{p}^s} &= \left(1 - \left(\frac{p_i^s - \underline{v}}{\bar{v} - \underline{v}}\right) + \left(\frac{1}{\bar{v} - \underline{v}}\right)\left(\hat{p}^s - p_i^s\right)(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right) B_i^{(\mathbf{N})} \\
&= \left(1 - \frac{\hat{v} - \underline{v}}{\bar{v} - \underline{v}}\right) B_i^{(\mathbf{N})}.
\end{aligned}$$

Finally, $Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$ is multiplicatively separable in p_i^s and p_i^b, whereby each multiplicative component is obviously log-concave in p_i^s and p_i^b respectively (a logit-demand form is necessarily log-concave). Given that log-concavity is preserved by multiplication, we conclude that $Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$ is log-concave in $\mathbf{p}_i = \left(p_i^s, p_i^b\right)$. ■

The following lemma is analogous to the second part of Proposition 2 in the main text. Note that the condition (29) below requires that seller quasi-demand $1 - G$ is not too log-concave, that is, ϵ_s is not too high relative to n. We first note that $\epsilon_s > 0$ if $1 - G$ is strictly log-concave, $\epsilon_s \geq 1$ if $1 - G$ is concave, and $\epsilon_s \leq 1$ if $1 - G$ is convex. The latter implies that (29) is immediately satisfied by all distributions with weakly decreasing densities (whereby $1 - G$ is convex).

Lemma A.5 *Suppose F, $F_0 \sim Gumbel(\mu)$. In the equilibrium characterized by Proposition 1, an increase in n always decreases seller fee $\hat{p}^s$, and increases buyer fee $\hat{p}^b$ if in addition*

$$4(n-1) > \epsilon_s(p) \text{ for all } p \in [\underline{v}, \bar{v}], \tag{29}$$

where $\epsilon_s(p) \equiv -\frac{d}{dp}\left(\frac{1-G(p)}{g(p)}\right)$ is the log-curvature index of seller quasi-demand.

[24]To determine the global quasi-concavity for other distribution functions, we rely on numerical calculations. We considered the case of F and $F_0 \sim Gumbel(\gamma)$ and $G \sim Normal(\mu_G, \sigma^2)$ for all combinations of the following parameter values: $n \in \{2, 3, 4\}$, $c \in \{0.1, 1\}$, $\mu \in \{0.5, 1, 2\}$, $\mu_G \in \{-4, 0, 3\}$, and $\sigma^2 = \{1, 2\}$. In all the cases considered, the quasi-concavity assumption was satisfied, suggesting it does not require very special conditions to hold. Details and codes of the numerical calculations are available from the authors upon request.

Proof. Following the proof of Proposition 2, it suffices to verify

$$\frac{1}{\sigma}\frac{\partial\sigma}{\partial n}-\frac{1}{X}\frac{\partial X}{\partial n}=\frac{-\exp\left\{-\hat{p}^b/\mu\right\}}{1+n\exp\left\{-\hat{p}^b/\mu\right\}}<0,$$

which follows from simple algebraic manipulations. Meanwhile, $\frac{\partial M}{\partial \hat{p}^s}=-\epsilon_s\left(p\right)$ by definition, so (27) implies

$$\frac{d\hat{p}^b}{dn}=\frac{1}{Det}\left[\frac{1}{X}\frac{\partial X}{\partial n}-\frac{1}{\sigma}\frac{\partial\sigma}{\partial n}+\epsilon_s\frac{\sigma}{X}\frac{\partial X}{\partial n}\right].$$

Substituting for the corresponding expressions,

$$\frac{d\hat{p}^b}{dn}=\frac{1}{Det}\left(\frac{\gamma\exp\left\{-\hat{p}^b/\mu\right\}^2}{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^3}\right)\left(\frac{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^2}{\exp\left\{-\hat{p}^b/\mu\right\}}-\epsilon_s\right).$$

Using the inequality of arithmetic and geometric means, we can bound $\frac{\left[1+(n-1)\exp\left\{-\hat{p}^b/\mu\right\}\right]^2}{\exp\left\{-\hat{p}^b/\mu\right\}}\geq 4\left(n-1\right)$, so that condition (29) implies the result. ■

References

Ambrus, A., E. Calvano, and M. Reisinger (2016). Either or both competition: A" two-sided" theory of advertising with overlapping viewerships. *American Economic Journal: Microeconomics 8*(3), 189–222.

Anderson, S. P., A. de Palma, and J. F. Thisse (1992). *Discrete choice theory of product differentiation.* MIT press.

Anderson, S. P., Ø. Foros, and H. J. Kind (2019). The importance of consumer multihoming (joint purchases) for market performance: Mergers and entry in media markets. *Journal of Economics & Management Strategy 28*(1), 125–137.

Anderson, S. P. and M. Peitz (2020). Media see-saws: Winners and losers in platform markets. *Journal of Economic Theory 186*, 104990.

Armstrong, M. (2006). Competition in two-sided markets. *The RAND Journal of Economics 37*(3), 668–691.

Armstrong, M. and J. Wright (2007). Two-sided markets, competitive bottlenecks and exclusive contracts. *Economic Theory 32*(2), 353–380.

Athey, S., E. Calvano, and J. S. Gans (2018). The impact of consumer multi-homing on advertising markets and media competition. *Management Science 64*(4), 1574–1590.

Bakos, Y. and H. Halaburda (2020). Platform competition with multihoming on both sides: Subsidize or not? *Management Science 66*(12), 5599–5607.

Bedre-Defolie, Ö. and E. Calvano (2013). Pricing payment cards. *American Economic Journal: Microeconomics 5*(3), 206–231.

Belleflamme, P. and M. Peitz (2019a). Platform competition: Who benefits from multihoming? *International Journal of Industrial Organization 64*, 1–26.

Belleflamme, P. and M. Peitz (2019b). Price disclosure by two-sided platforms. *International Journal of Industrial Organization 67*, 102529.

Bryan, K. A. and J. S. Gans (2019). A theory of multihoming in rideshare competition. *Journal of Economics & Management Strategy 28*(1), 89–96.

Caillaud, B. and B. Jullien (2003). Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics*, 309–328.

Edelman, B. and J. Wright (2015). Price coherence and excessive intermediation. *Quarterly Journal of Economics 130*(3), 1283–1328.

Guthrie, G. and J. Wright (2007). Competing payment schemes. *The Journal of Industrial Economics 55*(1), 37–67.

Hagiu, A. (2006). Pricing and commitment by two-sided platforms. *The RAND Journal of Economics 37*(3), 720–737.

Hagiu, A. and H. Hałaburda (2014). Information and two-sided platform profits. *International Journal of Industrial Organization 34*, 25–35.

Hart, O., J. Tirole, D. W. Carlton, and O. E. Williamson (1990). Vertical integration and market foreclosure. *Brookings papers on economic activity. Microeconomics 1990*, 205–286.

Janssen, M. and S. Shelegia (2015). Consumer search and double marginalization. *American Economic Review 105*(6), 1683–1710.

Jullien, B. and A. Pavan (2019). Information management and pricing in platform markets. *The Review of Economic Studies 86*(4), 1666–1703.

Karle, H., M. Peitz, and M. Reisinger (2020). Segmentation versus agglomeration: Competition between platforms with competitive sellers. *Journal of Political Economy 128*(6), 2329–2374.

Perloff, J. M. and S. C. Salop (1985). Equilibrium with product differentiation. *Review of Economic Studies 52*(1), 107–120.

Rochet, J.-C. and J. Tirole (2002). Cooperation among competitors: some economics of payment card associations. *RAND Journal of Economics 33*(4), 549–571.

Rochet, J.-C. and J. Tirole (2003). Platform competition in two-sided markets. *Journal of the European Economic Association 1*(4), 990–1029.

Rochet, J.-C. and J. Tirole (2006). Two-sided markets: a progress report. *The RAND Journal of Economics 37*(3), 645–667.

Rysman, M. and J. Wright (2015). The economics of payment cards. *Review of Network Economics 13*(3), 303–353.

Salop, S. C. (1979). Monopolistic competition with outside goods. *Bell Journal of Economics*, 141–156.

Tan, G. and J. Zhou (2021). The effects of competition and entry in multi-sided markets. *The Review of Economic Studies 88*(2), 1002–1030.

Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review 100*(4), 1642–72.

Wright, J. (2004a). One-sided logic in two-sided markets. *Review of Network Economics 3*(1).

Wright, J. (2004b). The determinants of optimal interchange fees in payment systems. *The Journal of Industrial Economics 52*(1), 1–26.

Zhou, J. (2017). Competitive bundling. *Econometrica 85*(1), 145–172.

Multihoming and oligopolistic platform competition: online appendix

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This online appendix contains proofs of omitted results and details from the main paper.

B Further properties of the baseline demand function

We examine the continuity and differentiability of the demand function $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ in (16).

Claim B.1 *For any $\hat{\mathbf{p}}$, $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ is continuous in p_i^b and p_i^s.*

Proof. Continuity with respect to p_i^b is obvious. To show continuity with respect to p_i^s, note from (15) that $\lim_{p_i^s \to \hat{p}^{s-}} \hat{v}_m = \hat{p}^s$ for $m = 2, ..., n$. Similarly, note from Lemma 1 that $\lim_{p_i^s \to \hat{p}^{s+}} \hat{v} = \hat{p}^s$. Thus,

$$\begin{aligned}
& \lim_{p_i^s \to \hat{p}^{s-}} Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\
= \; & \left[1 - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N})} + \sum_{m=0}^{n-2} \left[G\left(\hat{p}^s\right) - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N}_{i,m})} \\
= \; & \left[1 - G\left(\hat{p}^s\right)\right] B_i^{(\mathbf{N})} \\
= \; & \lim_{p_i^s \to \hat{p}^{s+}} Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right),
\end{aligned}$$

so $Q_i\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right)$ is continuous for all p_i^b and p_i^s, which includes $\left(p_i^b, p_i^s\right) = \hat{p}$. ■

Claim B.2 *For any $\hat{\mathbf{p}}$,*

$$\lim_{p_i^s \to \hat{p}^{s-}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) \geq \lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right).$$

Equality holds if in addition (i) $n = 2$, or (ii) F, $F_0 \sim$Gumbel(μ).

Proof. Consider first $p_i^s \geq \hat{p}^s$. Then the right-hand side derivative is

$$\begin{aligned}
\lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) &= \lim_{p_i^s \to \hat{p}^{s+}} -\frac{d\hat{v}}{dp_i^s} B_i^{(\mathbf{N})} g\left(p_i^s\right) \\
&= \lim_{p_i^s \to \hat{p}^{s+}} \frac{-B_i^{(\mathbf{N})}}{\sum_{j \in \mathbf{N}, j \neq i}\left(B_j^{(\mathbf{N})} - B_j^{(\mathbf{N}_{-i})}\right) + B_i^{(\mathbf{N})}} B_i^{(\mathbf{N})} g\left(p_i^s\right) \\
&= \frac{-B_i^{(\mathbf{N})}}{\sum_{j \in \mathbf{N}, j \neq i}\left(B_j^{(\mathbf{N})} - B_j^{(\mathbf{N}_{-i})}\right) + B_i^{(\mathbf{N})}} B_i^{(\mathbf{N})} g\left(\hat{p}^s\right).
\end{aligned}$$

Evaluating the above at $p_i^b = \hat{p}^b$, all platforms become symmetry so that functions $B_j^{(\Theta)}$ are the same for any set Θ and any given $j \in \Theta$. So, for simplicity we denote any such generic term as $B^{(\Theta)}$. Hence we have

$$\lim_{p_i^s \to \hat{p}^{s+}} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) = \frac{-B^{(\mathbf{N})} B^{(\mathbf{N})}}{nB^{(\mathbf{N})} - (n-1) B^{(\mathbf{N}_{-i})}} g\left(\hat{p}^s\right). \tag{B.1}$$

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When $p_i^s > \hat{p}^s$, the left hand side derivative is

$$\begin{aligned}
& \lim_{p_i^s \to \hat{p}^s -} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\
= & \lim_{p_i^s \to \hat{p}^s -} \sum_{m=0}^{n-1}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} g\left(\bar{v}_{m+1}\right) - \frac{d\hat{v}_m}{dp_i^s} g\left(\bar{v}_m\right)\right] B_i^{(\mathbf{N}_{i,m})} \\
= & \; g\left(\hat{p}^s\right)\left[\sum_{m=0}^{n-1}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} - \frac{d\hat{v}_m}{dp_i^s}\right] B^{(\mathbf{N}_{i,m})}\right],
\end{aligned} \tag{B.2}$$

where $\frac{d\hat{v}_n}{dp_i^s} = 0$ because $\hat{v}_n \equiv \bar{v}$, $\frac{d\hat{v}_0}{dp_i^s} = 1$ since $\hat{v}_0 \equiv p_i^s$, while

$$\begin{aligned}
\frac{d\hat{v}_m}{dp_i^s} &= \frac{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m-1})}}{B_i^{(\mathbf{N}_{i,m})} - B_i^{(\mathbf{N}_{i,m-1})} + mB_j^{(\mathbf{N}_{i,m})} - (m-1) B_j^{(\mathbf{N}_{i,m-1})}} \text{ for } m = 1, ..., n-1 \\
\frac{d\hat{v}_m}{dp_i^s}|_{p_i^b = \hat{p}^b} &= \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}},
\end{aligned}$$

in which $B^{(\cdot)}$ is as denoted earlier due to symmetry. Hence, evaluating at $p_i^b = \hat{p}^b$, ($B.2$) can be expanded

$$\begin{aligned}
& \frac{1}{g\left(\hat{p}^s\right)} \lim_{p_i^s \to \hat{p}^s -} \frac{dQ_i}{dp_i^s}\left(p_i^b, p_i^s; \hat{\mathbf{p}}\right) \\
= & -\frac{d\hat{v}_{n-1}}{dp_i^s} B^{(\mathbf{N})} + \sum_{m=1}^{n-2}\left[\frac{d\hat{v}_{m+1}}{dp_i^s} - \frac{d\hat{v}_m}{dp_i^s}\right] B^{(\mathbf{N}_{i,m})} + \left(\frac{d\hat{v}_1}{dp_i^s} - 1\right) B^{(\mathbf{N}_{i,0})} \\
= & \frac{-B^{(\mathbf{N}_{i,n-1})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\
& + \frac{B^{(\mathbf{N}_{i,n-2})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\
& + \sum_{m=1}^{n-2}\left[\frac{B^{(\mathbf{N}_{i,m+1})} - B^{(\mathbf{N}_{i,m})}}{(m+2) B^{(\mathbf{N}_{i,m+1})} - (m+1) B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}\right] B^{(\mathbf{N}_{i,m})} \\
& + \left(\frac{B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - 1\right) B^{(\mathbf{N}_{i,0})}.
\end{aligned} \tag{B.3}$$

By definition, proving differentiability at $\left(p_i^b, p_i^s\right) = \hat{p}$ requires us to show

$$\lim_{p_i^s \to \hat{p}^s -} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) = \lim_{p_i^s \to \hat{p}^s +} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right). \tag{B.4}$$

To prove this, we note that $\mathbf{N}_{i,n-1} = \mathbf{N}$ and that when all platforms are symmetry we have $\mathbf{N}_{i,n-2} = \mathbf{N}_{-i}$ (because both sets denote a set of $n-1$ symmetry platforms). Then, substituting for (B.1) we can rewrite (B.3) as

$$\lim_{p_i^s \to \hat{p}^s -} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) = \lim_{p_i^s \to \hat{p}^s +} \frac{dQ_i}{dp_i^s}\left(\hat{p}^b, p_i^s; \hat{\mathbf{p}}\right) + g\left(\hat{p}^s\right) \Phi\left(n\right),$$

where $\Phi\left(n\right)$ is defined as the last three lines of (B.3), i.e.

$$\begin{aligned}
\Phi\left(n\right) \equiv & \frac{B^{(\mathbf{N}_{i,n-2})} B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1) B^{(\mathbf{N}_{i,n-2})}} \\
& + \sum_{m=1}^{n-2}\left[\frac{B^{(\mathbf{N}_{i,m+1})} - B^{(\mathbf{N}_{i,m})}}{(m+2) B^{(\mathbf{N}_{i,m+1})} - (m+1) B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1) B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}\right] B^{(\mathbf{N}_{i,m})} \\
& + \left(\frac{B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - 1\right) B^{(\mathbf{N}_{i,0})}.
\end{aligned}$$

To conclude (B.4), it suffices to prove by induction that $\Phi\left(n\right) \geq 0$ for all $n \geq 2$. First, when $n = 2$ we have

$\mathbf{N}_{i,1} = \mathbf{N}$ so

$$\begin{aligned}\Phi(2) &= \frac{B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,1})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} + \left(\frac{B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - 1\right) B^{(\mathbf{N}_{i,0})} \\ &= \frac{2B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,1})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} - \left[\frac{B^{(\mathbf{N}_{i,0})}B^{(\mathbf{N}_{i,0})}}{2B^{(\mathbf{N}_{i,1})} - B^{(\mathbf{N}_{i,0})}} + B^{(\mathbf{N}_{i,0})}\right] = 0.\end{aligned}$$

Note that this also proves the first part of the claim, that is, the case of $n = 2$. By the inductive hypothesis, suppose $\Phi(n-1) \geq 0$. For $n \geq 3$, if we expand one more term from the summation in $\Phi(n)$ and rearrange terms we get

$$\begin{aligned}\Phi(n) &= \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N}_{i,n-1})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1)B^{(\mathbf{N}_{i,n-2})}} \\ &\quad + \left[\frac{B^{(\mathbf{N}_{i,n-1})} - B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1)B^{(\mathbf{N}_{i,n-2})}} - \frac{B^{(\mathbf{N}_{i,n-2})} - B^{(\mathbf{N}_{i,n-3})}}{(n-1)B^{(\mathbf{N}_{i,n-2})} - (n-2)B^{(\mathbf{N}_{i,n-3})}}\right] B^{(\mathbf{N}_{i,n-2})} \\ &\quad + \sum_{m=1}^{n-3}\left[\frac{B^{(\mathbf{N}_{i,m+1})} - B^{(\mathbf{N}_{i,m})}}{(m+2)B^{(\mathbf{N}_{i,m+1})} - (m+1)B^{(\mathbf{N}_{i,m})}} - \frac{B^{(\mathbf{N}_{i,m})} - B^{(\mathbf{N}_{i,m-1})}}{(m+1)B^{(\mathbf{N}_{i,m})} - mB^{(\mathbf{N}_{i,m-1})}}\right] B^{(\mathbf{N}_{i,m})} \\ &\quad + \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N})}}{nB^{(\mathbf{N})} - (n-1)B^{(\mathbf{N}_{i,n-2})}} \\ &= \frac{\left(2B^{(\mathbf{N}_{i,n-1})} - B^{(\mathbf{N}_{i,n-2})}\right)B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1)B^{(\mathbf{N}_{i,n-2})}} - \frac{B^{(\mathbf{N}_{i,n-2})}B^{(\mathbf{N}_{i,n-2})}}{(n-1)B^{(\mathbf{N}_{i,n-2})} - (n-2)B^{(\mathbf{N}_{i,n-3})}} + \Phi(n-1).\end{aligned} \tag{B.5}$$

By inductive hypothesis $\Phi(n-1) \geq 0$. Therefore, to prove $\Phi(n) \geq 0$, it remains to show

$$\frac{2B^{(\mathbf{N}_{i,n-1})} - B^{(\mathbf{N}_{i,n-2})}}{nB^{(\mathbf{N}_{i,n-1})} - (n-1)B^{(\mathbf{N}_{i,n-2})}} \geq \frac{B^{(\mathbf{N}_{i,n-2})}}{(n-1)B^{(\mathbf{N}_{i,n-2})} - (n-2)B^{(\mathbf{N}_{i,n-3})}}.$$

Rearranging the terms and cancelling out common coefficients, the inequality above is equivalent to

$$\begin{aligned}0 &\leq \frac{\left(B^{(\mathbf{N}_{i,n-2})} - B^{(\mathbf{N}_{i,n-1})}\right)}{B^{(\mathbf{N}_{i,n-1})}} - \frac{\left(B^{(\mathbf{N}_{i,n-3})} - B^{(\mathbf{N}_{i,n-2})}\right)}{B^{(\mathbf{N}_{i,n-3})}} \\ &\simeq \frac{\partial B^{(\mathbf{N}_{i,k})}}{\partial k}|_{k=n-1} - \frac{\partial B^{(\mathbf{N}_{i,k})}}{\partial k}|_{k=n-3}.\end{aligned} \tag{B.6}$$

We know $\frac{\partial B}{\partial k} \leq 0$, so we simply need to show that B is decreasing in k with a decreasing magnitude, i.e. B is convex in k. Recall that for $k \in \{0, ..., n-1\}$, we have

$$B^{(\mathbf{N}_{i,k})} = \int_{\underline{\epsilon}}^{\bar{\epsilon}-\hat{p}^b}\left[\frac{1 - F\left(\epsilon_0 + \hat{p}^b\right)^{k+1}}{k+1}\right] dF_0(\epsilon_0).$$

Convexity of B in k then follows from the observation that $\frac{1-F\left(\epsilon_0+\hat{p}^b\right)^{k+1}}{k+1}$ is convex in k, and that convexity is preserved by integration when the integrand is always positive over the entire region of integration. So, $\Phi(n) \geq 0$ for all $n \geq 2$ as required. Finally, in the special case of Gumbel distribution, IIA property of logit-demand form implies that the right-hand side of (B.6) equals zero, so that $\Phi(n) = 0$ for all $n \geq 2$. ■

C Buyer participation and partial multihoming

This section corresponds to Section 5.1 in the main text.

Demand derivation. We first derive the demand functions facing each platform. Consider a deviating platform i that charges $\left(p_i^b, p_i^s\right) \neq \left(\hat{p}^b, \hat{p}^s\right)$. Note that the decisions of buyers are shown in the main text and so are omitted here. Consider $p_i^s \geq \hat{p}^s$. For a seller with type v, we write her total surplus from joining all platforms $j \neq i$ as

$$\begin{aligned} & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \left((1-\lambda) B_j^{(\mathbf{N})} + \lambda B_j^{(\mathbf{N}_{-i})} \right) \\ = \; & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \tilde{B}_j^{(\mathbf{N}_{-i})}, \end{aligned} \tag{C.1}$$

where

$$\tilde{B}_j^{(\Theta)} \equiv (1-\lambda) B_j^{(\mathbf{N})} + \lambda B_j^{(\Theta)} \tag{C.2}$$

can be thought of as a "composite" buyer quasi-demand. Likewise, if the seller joins all platforms including i, then her total surplus is

$$\begin{aligned} & (v - \hat{p}^s) \left[(1-\lambda) \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})} + \lambda \sum_{j \in \mathbf{N}_{-i}} B_j^{(\mathbf{N})} \right] + (v - p_i^s) \left[(1-\lambda) B_i^{(\mathbf{N})} + \lambda B_i^{(\mathbf{N})} \right] \\ = \; & (v - \hat{p}^s) \sum_{j \in \mathbf{N}_{-i}} \tilde{B}_j^{(\mathbf{N})} + (v - p_i^s) \tilde{B}_i^{(\mathbf{N})}. \end{aligned} \tag{C.3}$$

Denote

$$\tilde{\sigma}_i \equiv 1 - \frac{\sum_{j \in \mathbf{N}_{-i}} (\tilde{B}_j^{(\mathbf{N})} - \tilde{B}_j^{(\mathbf{N}_{-i})})}{\tilde{B}_i^{(\mathbf{N})}}.$$

Comparing (C.1) and (C.3), we can pin down the threshold $\tilde{v}$ as in Lemma 1:

$$\tilde{v} = \frac{p_i^s}{\tilde{\sigma}_i} - \frac{1 - \tilde{\sigma}_i}{\tilde{\sigma}_i} \hat{p}^s.$$

Likewise, when $p_i^s < \hat{p}^s$, with similar calculations we can pin down thresholds $\tilde{v}_m$ for $m = 1, ..., n-1$ as in Lemma A.1:

$$\tilde{v}_m \equiv \frac{p_i^s \left[\tilde{B}_i^{(\mathbf{N}_{i,m})} - \tilde{B}_i^{(\mathbf{N}_{i,m-1})} \right] + \hat{p}^s \left[m \tilde{B}_j^{(\mathbf{N}_{i,m})} - (m-1) \tilde{B}_j^{(\mathbf{N}_{i,m-1})} \right]}{\tilde{B}_i^{(\mathbf{N}_{i,m})} - \tilde{B}_i^{(\mathbf{N}_{i,m-1})} + m \tilde{B}_j^{(\mathbf{N}_{i,m})} - (m-1) \tilde{B}_j^{(\mathbf{N}_{i,m-1})}}.$$

We can further define $\tilde{v}_n \equiv \bar{v}$ and $\tilde{v}_0 \equiv p_i^s$. Then the formal characterization of seller participation decisions is the same as Lemma A.1 after replacing the relevant thresholds with $\hat{v}_m$.

Equilibrium. To derive the equilibrium fees, it suffices to focus on the seller participation profile after an upward deviation by platform i, that is, $p_i^s \geq \hat{p}^s$. Note

$$\begin{aligned} \Pi_i &= \left(p_i^b + p_i^s - c \right) Q_i \\ &= \left(p_i^b + p_i^s - c \right) (1 - G(\tilde{v})) B_i^{(\mathbf{N})}, \end{aligned}$$

where we use $\tilde{B}_i^{(\mathbf{N})} = B_i^{(\mathbf{N})}$ given (C.2). We assume that Π_i is quasi-concave in $\left(p_i^b, p_i^s \right)$ so that the equilibrium can be characterized by the usual first-order condition. We numerically verified that Π_i is quasi-concave for $\lambda \in \{0.1, 0.5, 0.9\}$ over all the distributional and parameter configurations considered in the **baseline model. The** details and codes of the simulations are available from the authors upon request.

The demand derivatives, after imposing symmetry, can be calculated as follows:

$$\frac{dQ_i(\hat{\mathbf{p}}; \hat{\mathbf{p}})}{dp_i^b} = (1 - G(\hat{p}^s)) \frac{\partial B_i^{(\mathbf{N})}}{\partial p_i^b} |_{\mathbf{p} = \hat{\mathbf{p}}} = -\frac{1}{X} Q_i(\hat{\mathbf{p}}; \hat{\mathbf{p}})$$

and

$$\begin{aligned}\frac{dQ_i(\hat{\mathbf{p}};\hat{\mathbf{p}})}{dp_i^s} &= -\frac{d\tilde{v}}{dp_i^s}g(\hat{p}^s)B_i^{(\mathbf{N})}|_{\mathbf{p}=\hat{\mathbf{p}}}\\ &= -\frac{1}{\sigma_\lambda}\frac{g(\hat{p}^s)}{1-G(\hat{p}^s)}Q_i(\hat{\mathbf{p}};\hat{\mathbf{p}}).\end{aligned}$$

Then, the standard first-order conditions yield the equilibrium in (13).

Proof of Proposition 3. Denote $M \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. Applying total differentiation with respect to λ on (13) and writing in matrix form, we have

$$\begin{bmatrix} 1-X' & 1 \\ 1-M\sigma' & 1-\sigma_\lambda\frac{\partial M}{\partial \hat{p}^s}\end{bmatrix}\begin{bmatrix}\frac{d\hat{p}^b}{d\lambda}\\ \frac{d\hat{p}^s}{d\lambda}\end{bmatrix} = \begin{bmatrix}0\\ M\frac{\partial\sigma}{\partial\lambda}\end{bmatrix}.$$

Given that $\sigma_\lambda = \lambda\sigma + (1-\lambda)$ where $\sigma \in [0,1]$ is defined in (6), we know immediately from Lemma A.3 that $\frac{\partial\sigma_\lambda}{\partial\hat{p}^b} \geq 0$. Obviously, $\frac{\partial\sigma_\lambda}{\partial\lambda} \leq 0$. Then,

$$\overline{Det} \equiv \begin{vmatrix} 1-X' & 1 \\ 1-M\sigma' & 1-\sigma_\lambda\frac{\partial M}{\partial\hat{p}^s}\end{vmatrix} = \underbrace{(1-X')\left(1-\sigma_\lambda\frac{\partial M}{\partial\hat{p}^s}\right)}_{\geq 1} - 1 + M\underbrace{\frac{\partial\sigma_\lambda}{\partial\hat{p}^b}}_{>0} > 0. \tag{C.4}$$

By Cramer's rule,

$$\frac{d\hat{p}^b}{d\lambda} = \frac{1}{\overline{Det}}\begin{vmatrix}0 & 1\\ M\frac{\partial\sigma_\lambda}{\partial\lambda} & 1-\sigma_\lambda\frac{\partial M}{\partial\hat{p}^s}\end{vmatrix} > 0 \text{ and } \frac{d\hat{p}^s}{d\lambda} = \frac{1}{\overline{Det}}\begin{vmatrix}1-X' & 0\\ 1-M\sigma' & M\frac{\partial\sigma_\lambda}{\partial\lambda}\end{vmatrix} < 0$$

as required.

Proof of Proposition 6. Let $\overline{Det}$ be (C.4) and Det be (25). Applying total differentiation with respect to n on (13) and using the decomposition of $\sigma_\lambda = \lambda\sigma + (1-\lambda)$, we have

$$\frac{d\hat{p}^s}{dn} = \left[Det \times \left(\lambda\frac{d\hat{p}^s}{dn}|_{\lambda=1}\right) - (1-\lambda)\frac{\partial X}{\partial n}\right]\frac{1}{\overline{Det}},$$

where $\frac{d\hat{p}^s}{dn}|_{\lambda=1} < 0$ by Proposition 2 if f is decreasing. If $\lambda \to 1$, then $\frac{d\hat{p}^s}{dn} \to \frac{d\hat{p}^s}{dn}|_{\lambda=1} < 0$ by continuity. If $\lambda \to 0$, then $\frac{d\hat{p}^s}{dn} \to -\frac{\partial X}{\partial n}\frac{1}{\overline{Det}} \geq 0$.

Next

$$\frac{d\hat{p}^b}{dn} = \left[Det \times \left(\lambda\frac{d\hat{p}^b}{dn}|_{\lambda=1}\right) + \frac{\partial X}{\partial n}\left(1-\frac{\partial M}{\partial\hat{p}^s}\right)\right]\frac{1}{\overline{Det}},$$

where $\frac{d\hat{p}^b}{dn}|_{\lambda=1} > 0$ by Proposition 2 if f and g are decreasing. If $\lambda \to 1$, then $\frac{d\hat{p}^b}{dn} \to \frac{d\hat{p}^b}{dn}|_{\lambda=1} > 0$ by continuity. If $\lambda \to 0$ then $\frac{d\hat{p}^b}{dn} \to \frac{\partial X}{\partial n}\left(1-\frac{\partial M}{\partial\hat{p}^s}\right)\frac{1}{\overline{Det}} \leq 0$. Finally, the sum is

$$\frac{d\hat{p}^s}{dn} + \frac{d\hat{p}^b}{dn} = \lambda\left(\frac{d\hat{p}^b}{dn} + \frac{d\hat{p}^s}{dn}\right)|_{\lambda=1} + (1-\lambda)\frac{\partial X}{\partial n}\left(-\frac{\partial M}{\partial\hat{p}^s}\right)\frac{1}{\overline{Det}} \leq 0$$

by Proposition 2.

Proof of Remark 1. Continuing from the proof of Proposition 6, we can expand the difference as

$$\frac{d\hat{p}^b}{dn} - \frac{d\hat{p}^s}{dn} = \frac{1}{Det}\left[2\left(\frac{\partial X}{\partial n} - M\frac{\partial\sigma_\lambda}{\partial n}\right) - \sigma\frac{\partial M}{\partial\hat{p}^s}\frac{\partial X}{\partial n} - M\left(\sigma_\lambda'\frac{\partial X}{\partial n} - \frac{\partial\sigma_\lambda}{\partial n}X'\right)\right].$$

Substituting for the terms and simplifying, it can be shown that $\frac{d\hat{p}^b}{dn} - \frac{d\hat{p}^s}{dn} > 0$ if and only if

$$\left(\frac{\lambda}{1+(1-\lambda)(n-1)\exp\{-\hat{p}^b/\mu\}}\right)\frac{1+n\exp\{-\hat{p}^b/\mu\}}{\exp\{-\hat{p}^b/\mu\}}\left(2+\frac{\exp\{-\hat{p}^b/\mu\}}{1+(n-1)\exp\{-\hat{p}^b/\mu\}}\right)-2 > -\sigma_\lambda\frac{\partial M}{\partial \hat{p}^s}. \tag{C.5}$$

Denote LHS of (C.5) as $\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)$. We claim that there exists a unique threshold $\bar{\lambda}$ such that (C.5) — that is, $\frac{1}{\sigma}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right) > -\frac{\partial M}{\partial \hat{p}^s}$ — holds if and only if $\lambda > \bar{\lambda}$. We have

$$\frac{d\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)}{d\lambda} = \underbrace{\frac{\partial\psi}{\partial\lambda}}_{>0} + \underbrace{\frac{\partial\psi}{\partial\hat{p}^b}\frac{d\hat{p}^b}{d\lambda}}_{>0} > 0$$

and

$$\frac{d\hat{\sigma}}{d\lambda} = \underbrace{\frac{\partial\sigma_\lambda}{\partial\lambda}}_{>0} + \underbrace{\frac{\partial\sigma_\lambda}{\partial\hat{p}^b}\frac{d\hat{p}^b}{d\lambda}}_{>0} < 0,$$

so that $\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)$ is increasing in λ. Hence, $\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)$ is minimized at $\lambda = 0$ and maximized at $\lambda = 1$, in which

$$\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)|_{\lambda=0} = -2 < -\frac{\partial M}{\partial \hat{p}^s},$$

while it can be verified that

$$\begin{aligned}\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)|_{\lambda=1} &= \left(1+(n-1)\exp\{-\hat{p}^b/\gamma\}\right)\left[\frac{1+n\exp\{-\hat{p}^b/\mu\}}{\exp\{-\hat{p}^b/\mu\}}\left(2+\frac{\exp\{-\hat{p}^b/\mu\}}{1+(n-1)\exp\{-\hat{p}^b/\mu\}}\right)-2\right] \\ &> 4n-1 > 4(n-1),\end{aligned}$$

so that (29) implies that $\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right)|_{\lambda=1} > -\frac{\partial M}{\partial \hat{p}^s}$. Hence, by the intermediate value theorem, there exists a unique threshold $\bar{\lambda} \in [0,1]$ such that $\frac{1}{\sigma_\lambda}\eta\left(\lambda, n, \hat{p}^b(\lambda)\right) > -\frac{\partial M}{\partial \hat{p}^s}$ if and only if $\lambda > \bar{\lambda}$, as required.

C.1 Observable seller fee

We now extend our analysis in Section 5.1 to the case where buyers can observe the fees set on the seller side. We will focus on the polar cases where $\lambda \to 0$ and $\lambda \to 1$. We know that $\lambda \to 1$ corresponds to our baseline model. In what follows, we first derive the equilibrium for $\lambda \to 0$. Then, we focus on the case of logit buyer quasi-demand and show results that are analogous to Proposition 3 and Proposition 6.

Denote the symmetric fee equilibrium with $\lambda \to 0$ as $\tilde{p} = \left(\tilde{p}^b, \tilde{p}^s\right)$. Consider a deviating platform i that sets $p_i = \left(p_i^b, p_i^s\right) \neq \tilde{p}$, while all the remaining platforms continue to set $\tilde{p}$. We first note that the seller's participation decision is the same as in the baseline model, such that the number of sellers joining platform i is $1 - G\left(p_i^s\right)$. Then, a given buyer's total utility from participating and being able to transact with sellers through platform i is

$$\begin{aligned} U_i^b &= \max\left\{\epsilon_i - p_i^b, \epsilon_0\right\}\left(1 - G\left(p_i^s\right)\right) + \epsilon_0 G\left(p_i^s\right) \\ &= \max\left\{\epsilon_i - p_i^b - \epsilon_0, 0\right\}\left(1 - G\left(p_i^s\right)\right) + \epsilon_0. \end{aligned}$$

A buyer joins platform i if and only if $U_i^b \geq \max_{j\in\mathbf{N}} U_j^b$, and then uses it for a transaction (with each seller) if $\epsilon_i - p_i^b > \epsilon_0$. Therefore, the total mass of buyers using platform i for transactions (or buyer quasi-demand) is

$$\begin{aligned} B_i &\equiv \Pr\left(\left(\epsilon_i - p_i^b - \epsilon_0\right)\left(1 - G\left(p_i^s\right)\right) \geq \max_{j\in\mathbf{N}}\left\{\left(\epsilon_j - \tilde{p}^b - \epsilon_0\right)\left(1 - G\left(\tilde{p}^s\right)\right), 0\right\}\right) \\ &= \int_{\underline{\epsilon}}^{\bar{\epsilon}}\int_{\underline{\epsilon}}^{\bar{\epsilon}} 1 - F\left(\max\left\{\epsilon - \tilde{p}^b - \epsilon_0, 0\right\}\frac{1 - G(\tilde{p}^s)}{1 - G(p_i^s)} + p_i^b + \epsilon_0\right) dF^{n-1}(\epsilon) dF_0(\epsilon_0). \end{aligned}$$

Notice that when buyers observe the seller fee, their quasi-demand for platform i is decreasing in p_i^s because a higher seller fee decreases seller participation, making it less attractive to buyers.

The total demand is $Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right) = \left(1 - G\left(p_i^s\right)\right) B_i$. The usual demand derivative terms can be calculated as

$$-\frac{Q_i\left(\hat{\mathbf{p}}; \hat{\mathbf{p}}\right)}{dQ_i\left(\hat{\mathbf{p}}; \hat{\mathbf{p}}\right)/dp_i^b} = X(\tilde{p}^b; n)$$

and

$$\begin{aligned} -\frac{Q_i\left(\hat{\mathbf{p}}; \hat{\mathbf{p}}\right)}{dQ_i\left(\hat{\mathbf{p}}; \hat{\mathbf{p}}\right)/dp_i^s} &= \frac{1 - G(\tilde{p}^s) B_i}{g(\tilde{p}^s)\left(B_i + \int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{p^b+\epsilon_0}^{\bar{\epsilon}} f\left(\epsilon\right) dF^{n-1}\left(\epsilon\right) dF_0\left(\epsilon_0\right)\right)} \\ &= \frac{1 - G(\tilde{p}^s)}{g(\tilde{p}^s)}\delta(\tilde{p}^b; n), \end{aligned}$$

where we have defined, for any arbitrarily given (symmetric) equilibrium buyer fee p^b,

$$\delta(p^b; n) \equiv \left(1 + \frac{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{p^b+\epsilon_0}^{\bar{\epsilon}} f\left(\epsilon\right) dF^{n-1}\left(\epsilon\right) dF_0\left(\epsilon_0\right)}{\int_{\underline{\epsilon}_0}^{\bar{\epsilon}_0}\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left(1 - F\left(\max\left\{\epsilon, p^b + \epsilon_0\right\}\right)\right) dF^{n-1}(\epsilon) dF_0\left(\epsilon_0\right)}\right)^{-1}. \tag{C.6}$$

Here, $\delta(p^b; n)$ is an inverse measure of how changes in seller participation affect the total demand for platform i. The first component in (C.6) reflects that, each additional seller participating increases the number of transactions that can be made by inframarginal buyers who have participated on platform i. The second component in (C.6) reflects that, each additional seller participating also makes platform i more attractive to buyers, expanding the number of buyers that participate on platform i.

Provided that the profit function $\Pi_i = \left(p_i^b + p_i^s - c\right) Q_i\left(\mathbf{p}_i; \hat{\mathbf{p}}\right)$ is quasiconcave in $\mathbf{p}_i$, the usual first-order condition shows that a pure symmetric pricing equilibrium is characterized by all platforms setting $\tilde{\mathbf{p}} = (\tilde{p}^b, \tilde{p}^s)$ which solves

$$\tilde{p}^b + \tilde{p}^s - c = X(\tilde{p}^b; n) = \frac{1 - G(\tilde{p}^s)}{g(\tilde{p}^s)}\delta(\tilde{p}^b; n). \tag{C.7}$$

C.2 Implications of multihoming and platform competition

To derive further results, we focus on the special case in which F, $F_0 \sim$ Gumbel with scale parameter μ. In this case, it can be shown that

$$\delta(\tilde{p}^b; n) = \frac{1 + n\exp\left\{-\tilde{p}^b/\mu\right\}}{1 + \left(n + \frac{n-1}{\mu}\right)\exp\left\{-\tilde{p}^b/\mu\right\}},$$

which is decreasing in n and increasing in $\tilde{p}^b$. Meanwhile, the expressions for $X\left(p^b; n\right)$ and $\sigma\left(p^b; n\right)$ follow from (7) in Section A.4.

The following proposition corresponds to Proposition 3. We compare the equilibrium with $\lambda \to 1$ ($\hat{p}^b$ and $\hat{p}^s$) and the equilibrium with $\lambda \to 0$ ($\tilde{p}^b$ and $\tilde{p}^s$).

Proposition C.1 *(Effect of buyer multihoming) Suppose $\mu \geq 1$. A change from $\lambda \to 0$ to $\lambda \to 1$ decreases total fees ($\hat{p}^b + \hat{p}^s < \tilde{p}^b + \tilde{p}^s$), decreases the seller fee ($\hat{p}^s < \tilde{p}^s$), and increases the buyer fee*

($\hat{p}^b > \tilde{p}^b$).

Proof. First, it is straightforward to verify that $\mu \geq 1$ implies $n \exp\left\{-\tilde{p}^b/\mu\right\} \geq \frac{1}{\mu} - 1$, which implies

$$\sigma(\tilde{p}^b; n) = \frac{1}{1 + (n-1)\exp\left\{-\tilde{p}^b/\mu\right\}} < \frac{1 + n\exp\left\{-\tilde{p}^b/\mu\right\}}{1 + \left(n + \frac{n-1}{\mu}\right)\exp\left\{-\tilde{p}^b/\mu\right\}} = \delta(\tilde{p}^b; n).$$

From the respective equilibrium conditions, $\sigma(\tilde{p}^b; n) < \delta(\tilde{p}^b; n)$ immediately implies $\hat{p}^b + \hat{p}^s < \tilde{p}^b + \tilde{p}^s$. Next, define the function $P^s\left(p^b\right) = X(p^b; n) - p^b + c$, which is decreasing in p^b. Notice $\tilde{p}^s = P^s\left(\tilde{p}^b\right)$ and $\hat{p}^s = P^s\left(\hat{p}^b\right)$. Using this notation, $\tilde{p}^b$ is pinned down by

$$X(\tilde{p}^b; n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1 - G\left(P^s\left(\tilde{p}^b\right)\right)} - \delta(\tilde{p}^b; n) = 0.$$

We know $\sigma(\tilde{p}^b; n) < \delta(\tilde{p}^b; n)$, so

$$X(\tilde{p}^b; n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1 - G\left(P^s\left(\tilde{p}^b\right)\right)} - \sigma(\tilde{p}^b; n) > 0.$$

Notice the left-hand side of this expression is decreasing in p^b. To show $\tilde{p}^b < \hat{p}^b$, suppose by contradiction $\tilde{p}^b \geq \hat{p}^b$. Then it implies

$$X(\hat{p}^b; n)\frac{g\left(P^s\left(\hat{p}^b\right)\right)}{1 - G\left(P^s\left(\hat{p}^b\right)\right)} - \sigma(\hat{p}^b; n) \geq X(\tilde{p}^b; n)\frac{g\left(P^s\left(\tilde{p}^b\right)\right)}{1 - G\left(P^s\left(\tilde{p}^b\right)\right)} - \sigma(\tilde{p}^b; n) > 0,$$

contradicting the definition of $\hat{p}^b$. Therefore, we must have $\tilde{p}^b < \hat{p}^b$, which immediately implies $\hat{p}^s < \tilde{p}^s$. ■

The next proposition corresponds to the second part of Proposition 6 in the main text (recall that the first part of the proposition is exactly Proposition 2).

Proposition C.2 *(Increased platform competition) Suppose buyers observe seller fees. In the equilibrium characterized by (C.7), an increase in n (i.e. platform entry) decreases the total fee $\tilde{p}^s + \tilde{p}^b$. Furthermore, an increase in n decreases the buyer fee $\tilde{p}^b$ if $\exp\left\{-\tilde{p}^b/\mu\right\} > \frac{1}{\mu}$, and increases the seller fee $\tilde{p}^s$ if in addition $\exp\left\{-\tilde{p}^b/\mu\right\} > \Gamma$, where Γ is a threshold defined in (C.10) and Γ is decreasing in μ.*

Proof. Denote $M\left(\hat{p}^s\right) \equiv \frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. Total differentiation on the equilibrium (C.7), in matrix form, gives

$$\begin{bmatrix} 1 - X' & 1 \\ 1 - M\frac{\partial \delta}{\partial \tilde{p}^b} & 1 - \delta\frac{\partial M}{\partial \tilde{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\tilde{p}^b}{dn} \\ \frac{d\tilde{p}^s}{dn} \end{bmatrix} = \begin{bmatrix} \frac{\partial X}{\partial n} \\ M\frac{\partial \delta}{\partial n} \end{bmatrix}. \tag{C.8}$$

Since $X' \leq 0$, $\frac{\partial M}{\partial \tilde{p}^s} < 0$, so accordingly the matrix in (C.8) has determinant

$$Det \equiv \underbrace{(1 - X')\left(1 - \delta\frac{\partial M}{\partial \tilde{p}^s}\right)}_{>1} - 1 + M\underbrace{\frac{\partial \delta}{\partial \tilde{p}^b}}_{>0} > 0.$$

By Cramer's rule,

$$\frac{d\tilde{p}^s}{dn} = \frac{1}{Det}\begin{vmatrix} 1-X' & \frac{\partial X}{\partial n} \\ 1-M\frac{\partial \delta}{\partial \tilde{p}^b} & M\frac{\partial \delta}{\partial n}\end{vmatrix} = \frac{1}{Det}\left[\left(M\frac{\partial \delta}{\partial n}-\frac{\partial X}{\partial n}\right)+M\left(\underbrace{\frac{\partial \delta}{\partial \tilde{p}^b}\frac{\partial X}{\partial n}}_{<0}-\underbrace{\frac{\partial \delta}{\partial n}X'}_{\geq 0}\right)\right];$$

$$\frac{d\tilde{p}^b}{dn} = \frac{1}{Det}\begin{vmatrix} \frac{\partial X}{\partial n} & 1 \\ M\frac{\partial \sigma}{\partial n} & 1-\sigma\frac{\partial M}{\partial \tilde{p}^s}\end{vmatrix} = \frac{1}{Det}\left[\left(\frac{\partial X}{\partial n}-M\frac{\partial \delta}{\partial n}\right)-\underbrace{\delta\frac{\partial M}{\partial \tilde{p}^s}\frac{\partial X}{\partial n}}_{\geq 0}\right].$$

Clearly, $\frac{d\tilde{p}^b}{dn}+\frac{d\tilde{p}^s}{dn}<0$. Next, after appropriate substitutions, we can arrive at

$$\begin{aligned} & M\frac{\partial \delta}{\partial n}-\frac{\partial X}{\partial n} \\ = \; & \frac{\mu\exp\left\{-\tilde{p}^b/\mu\right\}}{1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}\left(\frac{\exp\left\{-\tilde{p}^b/\mu\right\}}{1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}-\frac{1+\exp\left\{-\tilde{p}^b/\mu\right\}}{\mu+(\mu n+n-1)\exp\left\{-\tilde{p}^b/\mu\right\}}\right), \end{aligned}$$

which is positive if and only if $\exp\left\{-\tilde{p}^b/\mu\right\}>\frac{1}{\mu}$. Therefore, $\frac{d\tilde{p}^b}{dn}<0$ if $\exp\left\{-\tilde{p}^b/\mu\right\}>\frac{1}{\mu}$ holds. Similarly, we can calculate

$$\begin{aligned} & M\left(\frac{\partial \delta}{\partial \tilde{p}^b}\frac{\partial X}{\partial n}-\frac{\partial \delta}{\partial n}\frac{\partial X}{\partial \tilde{p}^b}\right) \\ = \; & -\frac{\mu\exp\left\{-\tilde{p}^b/\mu\right\}^2\left[1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}+\exp\left\{-\tilde{p}^b/\mu\right\}\right]}{\left[1+(n-1)\exp\left\{-\tilde{p}^b/\mu\right\}\right]^3\left[\mu+(n+\mu n-1)\exp\left\{-\tilde{p}^b/\mu\right\}\right]}. \end{aligned}$$

Substituting these expressions and rearranging, we get $\frac{d\tilde{p}^s}{dn}>0$ if and only if

$$\frac{2}{\exp\left\{-\tilde{p}^b/\mu\right\}}+\frac{1}{n+1}\left(\frac{1}{\exp\left\{-\tilde{p}^b/\mu\right\}}+1\right)^2-\mu-(n-1)\left(\mu\exp\left\{-\tilde{p}^b/\mu\right\}-1\right)<0. \tag{C.9}$$

Notice that the left-hand side of (C.9) is decreasing in $\exp\left\{-\tilde{p}^b/\mu\right\}$. Therefore, an application of the intermediate value theorem implies that there exists a unique cutoff Γ, defined implicitly by

$$\frac{2}{\Gamma}+\frac{1}{n+1}\left(\frac{1}{\Gamma}+1\right)^2-\mu-(\mu\Gamma-1)(n-1)=0, \tag{C.10}$$

such that (C.9) holds if and only if $\exp\left\{-\tilde{p}^b/\mu\right\}>\Gamma$. Notice that Γ is decreasing in μ and n. ■

To the extent that $\exp\left\{-\tilde{p}^b/\mu\right\}$ is bounded below (e.g. when $\tilde{p}^b\leq 0$ so that $\exp\left\{-\tilde{p}^b/\mu\right\}\geq 1$), then the conditions in Proposition C.2 hold whenever μ is sufficiently large (i.e. platforms are sufficiently differentiated from buyers' perspective).

D Value of transactions and buyer heterogeneity

This section corresponds to Section 5.2 in the main text.

Proof of Proposition 4. By linearity, note that $\frac{\partial X}{\partial \beta}=-\frac{\partial X}{\partial \hat{p}^b}=-\frac{1}{\gamma}X'$ and $\frac{\partial \sigma}{\partial \beta}=-\frac{\partial \sigma}{\partial \hat{p}^b}=-\frac{1}{\gamma}\sigma'$, where X' and σ' are the derivatives of X and σ with respect to the first argument. Denote $M\left(\hat{p}^s\right)\equiv$

$\frac{1-G(\hat{p}^s)}{g(\hat{p}^s)}$. Total differentiation of the equilibrium (14), in matrix form, gives

$$\begin{bmatrix} 1-X' & 1 \\ 1-\frac{M}{\gamma}\sigma' & 1-\sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\hat{p}^b}{d\beta} \\ \frac{d\hat{p}^s}{d\beta} \end{bmatrix} = \begin{bmatrix} -X' \\ -\frac{M}{\gamma}\sigma' \end{bmatrix}. \tag{D.1}$$

The matrix in (D.1) has a strictly positive determinant

$$Det = -\left(1-X'\right)\sigma\frac{\partial M}{\partial \hat{p}^s} + \frac{M}{\gamma}\sigma' - X > 0,$$

where $\sigma' > 0$ and $X' \leq 0$ by Lemma 3 and $\frac{\partial M}{\partial \hat{p}^s} < 0$ by strict log-concavity of $1-G$. By Cramer's rule,

$$\frac{d\hat{p}^s}{d\beta} = \frac{-1}{Det}\left[\frac{M}{\gamma}\sigma' - X'\right] = -\left(1 + \frac{(1-X')}{Det}\sigma\frac{\partial M}{\partial \hat{p}^s}\right) \in (-1,0) \tag{D.2}$$

$$\frac{d\hat{p}^b}{d\beta} = \frac{1}{Det}\left[\left(\frac{M}{\gamma}\sigma' - X'\right) + \sigma\frac{\partial M}{\partial \hat{p}^s}X'\right] = 1 + \frac{1}{Det}\sigma\frac{\partial M}{\partial \hat{p}^s} \in (0,1)\,. \tag{D.3}$$

Moreover, $\frac{d\hat{p}^s}{d\beta} + \frac{d\hat{p}^b}{d\beta}$ has the same sign as $\sigma\frac{\partial M}{\partial \hat{p}^s}X' \leq 0$.

Proof of Proposition 5. We first note from (D.2) and (D.3) that $\hat{p}^b - \beta$ is strictly decreasing in β while $\hat{p}^s + \beta$ is strictly increasing in β. Denote

$$\begin{aligned} \bar{\beta}^b &= \sup\{\beta : \hat{p}^b - \beta \geq 0\} \\ \bar{\beta}^s &= \sup\{\beta : \hat{p}^s + \beta \leq 0\}. \end{aligned}$$

Assume $\beta \leq \min\{\bar{\beta}^b, \bar{\beta}^s\}$ so that $\hat{p}^b - \beta \geq 0$ and $\hat{p}^s + \beta \leq 0$ in what follows. From equilibrium (14),

$$\frac{\partial X}{\partial \gamma} = -\left(\frac{\hat{p}^b - \beta}{\gamma^2}\right)X' \geq 0 \quad \text{and} \quad \frac{\partial \sigma}{\partial \gamma} = -\left(\frac{\hat{p}^b - \beta}{\gamma^2}\right)\sigma' \leq 0.$$

Total differentiation of the equilibrium (14), in matrix form, gives

$$\begin{bmatrix} 1-X' & 1 \\ 1-\frac{M}{\gamma}\sigma' & 1-\sigma\frac{\partial M}{\partial \hat{p}^s} \end{bmatrix} \begin{bmatrix} \frac{d\hat{p}^b}{d\gamma} \\ \frac{d\hat{p}^s}{d\gamma} \end{bmatrix} = \begin{bmatrix} X - \left(\frac{\hat{p}^b-\beta}{\gamma^2}\right)X' \\ -\left(\frac{\hat{p}^b-\beta}{\gamma^2}\right)\sigma' M \end{bmatrix},$$

where the matrix on the lef hand side is the same as in (D.1). By Cramer's rule,

$$\frac{d\hat{p}^b}{dn} = \frac{1}{Det}\left[\left(X + \gamma\frac{\partial X}{\partial \gamma}\right) - M\frac{\partial \sigma}{\partial \gamma} - \sigma\frac{\partial M}{\partial \hat{p}^s}\left(X + \gamma\frac{\partial X}{\partial \gamma}\right)\right] \geq 0$$

and

$$\begin{aligned} \frac{d\hat{p}^s}{dn} &= \frac{1}{Det}\left[-\left(X + \gamma\frac{\partial X}{\partial \gamma}\right) + M\frac{\partial \sigma}{\partial \gamma} + M\sigma'\frac{X}{\gamma}\right] \\ &= \frac{1}{Det}\left[-\left(X + \gamma\frac{\partial X}{\partial \gamma}\right) + M\left(-\sigma'\frac{\hat{p}^b - \beta}{\gamma^2} + \sigma'\frac{X}{\gamma}\right)\right] \\ &= \frac{1}{Det}\left[-\left(X + \gamma\frac{\partial X}{\partial \gamma}\right) + \sigma'\frac{M}{\gamma^2}\left(\hat{p}^s + c + \beta\right)\right] \leq 0, \end{aligned}$$

where we used the equilibrium condition $\hat{p}^b + \hat{p}^s + c = \gamma X$ in the final equality. Finally,

$$\frac{d\hat{p}^b}{dn} + \frac{d\hat{p}^s}{dn} = \frac{1}{Det}\left[M\sigma'\frac{X}{\gamma} - \sigma\frac{\partial M}{\partial \hat{p}^s}\left(X + \gamma\frac{\partial X}{\partial \gamma}\right)\right] \geq 0.$$