Let the Worst One Fail:
A Credible Solution to the Too-Big-To-Fail Conundrum

Thomas Philippon[*]   Olivier Wang[†]

March 2021

Abstract

We study time-consistent bank resolution mechanisms. When interventions are ex post efficient, a government cannot commit not to inject capital into the banking system. Contrary to common wisdom, however, we show that the government may still be able to implement the first best allocation because it can use the distribution of bailouts across multiple banks to provide ex ante incentives. We show that the efficient mechanism has the feature of a tournament. If each bank's net transfer from the government can only depend on its own performance, no credible mechanism can prevent maximal risk-taking by all banks. In stark contrast, using relative performance evaluation during the crisis can implement the first-best risk level while remaining credible. In particular, we analyze properties of credible tournament mechanisms that provide support to the best performing banks and resolve the worst performing ones. We extend our framework to allow for contagion and imperfect competition among banks. Our mechanism continues to perform well if banks are partially substitutable and if banks are heterogeneous in their size, interconnections, and thus systemic risk, as long as bailout funds can be targeted to particular banks.

---

[*]New York University Stern School of Business, CEPR and NBER, tphilipp@stern.nyu.edu
[†]New York University Stern School of Business, olivier.wang@nyu.edu

Governments often bail out large financial firms during financial crisis because they perceive that the economic costs of letting these firms fail exceed the fiscal costs of the bailouts themselves. This recurrent issue came to a head during the global financial crisis (GFC) of 2008-2009 because of the magnitude and scope of the bailouts. In the aftermath of the Great Recession, governments pledged to end the "too-big-to-fail" problem, and G20 Leaders endorsed the global implementation of a set of reforms for systemically important banks (SIBs). These financial stability reforms rely on three pillars: capital requirements (and other forms of loss absorbing capacity), enhanced supervision, and resolution regimes. The reforms have achieved significant progress along the first two dimensions. Capital requirements have roughly doubled and the supervision of large banks has become tighter (Financial Stability Board, 2021). These evolutions are somewhat uneven across jurisdictions, but regulators and market participants view banks as significantly safer than before the GFC.

The same cannot be said, however, of the third pillar: resolution regimes. Despite 10 years of efforts, there is still no consensus about the ability of governments to resolve large banks during times of economic stress. The root of the skepticism is that one cannot expect policy makers to let a majority of banks – or even a signficant number of large ones – fail at the same time. As a result, the argument goes, the expectation of bailouts will remain and will continue to distort funding costs and to feed moral hazard.

We argue that this skepticism is misplaced. More precisely, while we agree with the premise (letting several large banks fail is not a realistic option), we show that the pessimistic conclusion does not follow. The logic of the standard argument is flawed in two ways. Firstly, it assumes that if regulators cannot let a majority of banks fail then no bank can fail at all. Secondly, it assumes that private incentives depend only on the average level of the bailout. We show that both arguments are incorrect.

The main idea of our paper is to apply the logic of tournaments to the issue of too-big-to-fail in the context of imperfect resolution regimes. We assume that it impossible for governments to credibly commit not to intervene to support the financial sector as a whole during a crisis. However, this does not mean that the government has to support every bank in the same way. Time consistency might pin down the size of the bailout but it does not generally pin down its distribution, and the distribution of bailout funds (or taxes) matters for incentives.

We write a simple model where bailouts can be *ex post* efficient because of a neg-

ative externality on the real economy when the financial system is undercapitalized. Bailout anticipations affect the incentives of banks to engage in costly risk mitigation strategies *ex ante*. When we assume, as in the existing literature, that bailout funds are distributed in a symmetric way across banks, we obtain the standard moral hazard results: bailouts inefficiently increase risk taking as in Chari and Kehoe (2016), create strategic complementarities across banks' risk management choices as in Farhi and Tirole (2012), and the situation is worse the deeper the pockets of the government. This line of argument strongly calls for limiting the funds available for bailouts and tying the hands of regulators ex post to the extent possible.

To establish our first main result we use the systemic risk model of Acharya et al. (2016) where the negative externality on the real economy depends on the aggregate capital shortfall in the banking system. In this case the optimal bailout takes the form of a weakly increasing function $\mathcal{M}(K - R)$ where K is the aggregate capital requirement and R the aggregate return. With N banks, time consistency requires that the set of bailout payments satisfies $\sum_{i=1}^{N} m_i = \mathcal{M}(K - R)$ for any value of $R = \sum_{i=1}^{N} r_i$. This places no restrictions of the distribution of $\{m_i\}$ around its mean. In stark contrast to the conventional results, we then show that we can implement the first best equilibrium by conditioning government support on a relative performance mechanism such as a rank-order tournament, in banks performing above the median gets a higher m than banks performing below the median. The scheme is fully time consistent since it takes as given the overall size of the bailout. Punishing the banks that perform poorly while rewarding those who perform well works because, despite knowing that the median bank will be saved, each individual bank strives to make sure it does not end up in the lower half. This race to the top generate first-best ex ante incentives for all the banks.

The optimal contract requires punishment of bad banks. When we extend our model by adding limited liability constraints, we find that the common wisdom regarding deep pockets is overturned. We show that the set of implementable policies improves monotonically with fiscal slack. The more slack, the more incentives the government can provide, the less moral hazard, and with enough slack the first best is always implementable despite limited liability. When the limited liability constraint binds, our model offers a macro-prudential justification for increasing TLAC requirements and also for mandating clawback provisions in executive compensation contracts. The reason is that these contracts reduce the tightness of the constraint and therefore increase the

range of time consistent outcomes. For the same reason, we show that although the fire sales that occur during systemic crises must be met by larger bailouts, they also make it easier to provide ex-ante incentives. Fire sales hurt the outside options of weak banks relative to the transfers proposed by the regulator. Reducing bank leverage improves risk-taking incentives when fire sales discount are deep enough.

Our baseline framework assumes that banks are highly substitutable, in the sense that capital surpluses in one bank can compensate for capital shortfall in another. We show that this pure systemic risk model can be viewed as the optimal outcome of a process that allows the resolution authority to merge banks at a low cost. If healthy banks can absorb the assets and customers of any failing bank, then only the aggregate capital of the sector matters. If the social cost of mergers is too high, however, bailouts become more attractive, which spurs moral hazard.

We next study a model where banks are imperfect substitutes, for instance because of soft information, specialization across activities and locations, or market power. Lack of substitution worsens the time consistency problem as each individual bank knows it will be partly insured against its own poor returns to the extent that it would be costly for other banks to pick up the slack. We introduce the concept of ϵ-commitment to ensure continuity of the limit of mechanisms. A mechanism is ϵ-credible if welfare deviates by less than ϵ from its ex post optimum. We can then recast our first result in more general terms. We show that the 'size' of the set of implementable outcomes is proportional to $\epsilon\eta$ where η is the elasticity of susbtitution between capital surpluses located in different banks. The Acharya et al. (2016) loss function assume $\eta = \infty$ which is why the first best is always implementable. On the other hand, when η is small, the first best is not implementable in the usual (strong) time consistent fashion.

When banks are differentiated, however, the notion of renegotiation-proof contracts in Fudenberg and Tirole (1990) becomes quite appealing. If the government promises a set of transfers, banks can block a deviation that would leave them worse off. Under this weaker form of time consistency the government can choose among Pareto optimal allocations. The government cannot directly commit to punish weak banks but it can commit not to renege on its promised support to well performing banks. The core time-inconsistency problem is still present but our tournaments can once again implement the first-best level of safety, albeit at a higher cost (that is, larger bailouts) than in the case of perfect bank substitutability. Numerically, we find that the cost decreases rapidly towards the first best cost as banks become more substitutable.

Finally, we consider a different form of heterogeneity, arising from financial linkages between banks that generate comovement in returns. These linkages capture a variety of "contagion" forces, such as cross-exposures, fire sales, or domino effects, as studied in the financial networks literature. We show how contagion leads to a natural notion of systemic risk: banks are more systemic when their performance has a stronger effect on the rest of the system. In turn, more systemic banks should act more prudently, and so a resolution mechanism must strive to give them stronger incentives. ex post, however, the government may consider these "super-spreader" banks too interconnected to fail (Haldane, 2013). Our main finding is that the constraints that financial linkages impose on bank resolution depend crucially on how bailout funds attributed to one bank spill over to other banks.

If a form of "ring-fencing" applies to public funds and bailout money cannot flow throughout the system to benefit other banks indirectly, our tournament mechanism remains credible and efficient under minor amendments. A bank's rank in the tournament is determined by its ex post performance, as in the baseline model, but now weighted by its systemic risk. A subtle constraint appears if ring-fencing is not possible, and bailout money can instead spillover to other banks. A first intuition would be that these spillover effects can reduce costs ex post, as it is now possible to rescue some banks indirectly, working through the linkages. The countervailing and dominating force, however, is that spillovers actually worsen the credibility problem. It becomes optimal to target the most systemic bank, as this is a cheap way to save the whole system. But this makes the moral hazard problem unsolvable, because the most systemic bank will now be completely insured and thus maximize risk-taking, thereby endangering the whole system.

Related literature Bailouts are risky bets. Some succeed, some drag down the sovereign, as shown in Acharya et al. (2014). Our main contribution is to show how to use the classic rank-order tournament mechanisms Lazear and Rosen (1981) to overcome the pervasive time inconsistency problem that generates or worsens moral hazard in bank risk-taking (Farhi and Tirole 2012, Keister 2016, Chari and Kehoe 2016).

Our results differ from existing results in the literature in two important ways. First, the literature has concluded that bailouts cause moral hazard in the same setting. Farhi and Tirole 2012 restrict the set of contracts and tools available to the government.

Chari and Kehoe 2016 study an economy that is ex post efficient but the planner uses a welfare function that is utilitarian. So the planner distorts ex post a Pareto optimum. They therefore assume a extreme form of lack of commitment as their problem would be solved by a (Fudenberg and Tirole, 1990) mechanism. Second, the literature argues that the moral hazard problem is worst in countries with ample fiscal space: the narrative is that if banks expect the sovereign to be able to bail them out even in deep crises, they have no reason to self-insure. We find that fiscal capacity has the opposite effect, once richer mechanisms such as ours are used. Since a sovereign with larger fiscal capacity is able to transfer a larger amount to the banking sector as a whole, it also has more flexibility in the distribution of transfers across banks, which tends to relax incentive constraints and reduce moral hazard.

Our paper also relates to the strategic substitutabilities among banks during ex post fire sales, and the resulting ex ante incentives to build financial muscle, as in Perotti and Suarez (2002) and Acharya and Yorulmazer (2007a; 2007b). Instead of considering strategic substitutabilities driven by a competition for cheap assets, we show how a well-designed competition for government support can implement efficient ex ante safety. This approach relates to Kasa and Spiegel (2008), who show that using relative instead of absolute performance evaluation in bank closures can reduce costs. Unlike us, they do not consider how a tournament-like mechanism can implement the first best risk-taking. They also assume that regulators can fully commit, while our core insight is that tournaments mitigate the time-consistency problem.

1   A Model of Systemic Crises And Government Interventions

We present our baseline environment before defining the first-best allocation. The key feature of our model is that banks decide how much risk to take, anticipating government support policies in case of a systemic crisis that hits many banks at the same time.

1.1   Environment

We consider a two-period model with $N \geq 2$ banks and a "government", that should be viewed as combining fiscal and monetary authorities. At $t = 0$, the government

Table 1: Balance Sheet

Assets	a_i	Liabilities	
Gross Value	$r_i a_i$	Capital	e_i
		Deposits	d_i

announces a bailout rule mapping realized returns on banks' assets to government transfers, as described below. Each bank then chooses and safety investment $x_i \in [0,1]$. Uncertainty is resolved at time $t = 1$. Uncertainty consists of a shock s common to all banks as well as bank specific shocks. We define state $s = 0$ as the normal state and the states $s \neq 0$ as the crisis states. The probability of the normal state is $\mathbb{P}[s = 0] = p_0$. The crisis states are distributed on some compact set $\mathcal{S}$ so that $\int_{\mathcal{S}} p_s ds = 1 - p_0$.

Banks. At time 0 banks have assets a and deposits with face value d due at time 1. We denote by $r_{i,s}$ the gross asset return of bank i in state s at time 1 and by $m_{i,s}$ the cash injection from the government. Table 1 shows the balance sheet of bank i at time 1.

Capital denotes the sum of equity (tier 1) and other loss absorbing capacity such as junior unsecured bailinable bonds. We say that a bank is well capitalized when $e_i \geq \underline{e} a_i$, and its capital surplus is $e_i - \underline{e} a_i$. The notion of well capitalized is defined in the welfare function below. Banks maximize expected capital returns net of transfers $u_i = \mathbb{E}[\max\{0, e_{i,s}\}]$. The gross returns are given by

$$r_i^s = \begin{cases} f(x_i) + \xi_i & \text{with probability } p_0 \\ r_{i,s} \sim G(. \mid x_i, s) & \text{with probability } p_s \end{cases} \tag{1}$$

The shocks ξ_i are i.i.d. across banks and the returns $r_{i,s}$ are bounded. The expected return in the normal state f is decreasing, bounded, and concave over $[0,1]$ and attains a strict maximum at 0. The shock s is common to all banks. The cumulative distribution $G(x_i, s)$ of the return $r_{i,s}$ is ranked by stochastic dominance.[1]

Assumption 1. *$G(r \mid x_i, s)$ is decreasing and continuously differentiable in x for all r.*

---

[1]In section 6 we will allow the distribution of $r_{i,s}$ to depend on other banks' safety investments x_j as well.

The function f thus captures the risk/return tradeoff that banks face. Banks can improve their crisis return by increasing x, at the cost of lower returns $f(x)$ in normal times. The maximal risk banks can take, $x = 0$, leads to a highest expected return $f(0)$ in the good state but the worst exposure in crisis states.

Government. The government observes the aggregate state at time 1 as well as the banks' returns $r_{i,s}$. We will normalize the parameters of the model so that the normal state is indeed normal, i.e, no crisis and no bailout. The government's ex post value

$$V\left(\{e_i, a_i\}_{i=1..N}\right)$$

is concave and weakly increasing in e_i, decreasing in a_i, and strictly increasing in e_i when $e_i < \underline{e}a_i$. To simplify the notations we often write $V\{e_i\}$ since $\{e_i\}_{i=1..N}$ are the only random parts of the function, but the function itself also depends on a_i and the parameter $\underline{e}$. Finally, V is flat at its maximum when all banks are well capitalized: $V = \bar{V}$ when $e_i \geq \underline{e}a_i$ for all $i = 1..N$. This *defines* what we mean by a "well capitalized" banking system.

The government has the option to mitigate the consequences of financial distress by implementing transfers $\{m_{i,s}\}$. The total cost $M = \sum_i m_{i,s}$ is subject to a shadow cost of public transfers $\Gamma(M; \gamma)$ which is positive, weakly convex and strictly increasing for all $M > 0$. We index the cost of fund to γ which measures the inverse of fiscal slack. The function $\Gamma(M; \gamma)$ is increasing in γ and super-modular in (M, γ). Ex ante aggregate welfare is thus defined as

$$\mathbb{E}\left[R + V\{e_{i,s} + m_{i,s}\} - \Gamma(M_s; \gamma)\right]. \tag{2}$$

where $R = \sum_i a_i r_{i,s}$ is the random aggregate asset return.

Discussion of Assumptions We wish to focus our analysis on the issue of systemic risk. We therefore assume

Assumption 2. *Well calibrated TLAC*. $\frac{d_i}{a_i} \leq \min\{r_{i,s}\} < \frac{d_i}{a_i} + \underline{e}.$

Assumption A2 implies that deposits are safe but that banks can be undercapitalized. This choice captures the idea that regulations implemented after the GFC have made it *theoretically* possible to resolve a failing bank without taxpayers money.

8

TLAC requirements have been calibrated so that, using data from the worst financial crises on record, there would be enough capital, in virtually all cases, to resolve a bank in distress without involving small depositors and without using taxpayer money. We therefore assume that the limited liability of TLAC investors and equity holders does not bind *directly*. This does not mean that limited liability will not bind once we try to induce the right incentives, but it allows us to focus on the real issue with resolution regimes, namely that of systemic risk. Even if it is theoretically possible to cover the losses of one large banks with TLAC, it does not solve the issue of whether it is credible or even possible to resolve several large banks at the same time, which is what our paper is about.

The variable x captures the efforts of the bank to mitigate its systematic risk. It includes investment in liquid or safe assets with a low return as well as investments in monitoring and screening technologies and risk governance in general. We assume that x is not contractible. More precisely, we think of x as the residual discretion that bankers have once they have fulfilled their quantitative regulatory requirements, such as Tier 1 ratios, TLAC and LCR. The post-crisis policy response has focused on ensuring a minimum level x but these regulations are necessarily imperfect due to informational delays, signal jamming, off-balance sheet transactions, etc. Some private sector discretion always remains, so we normalize the regulatory level of safe investment to zero and view x as the residual investment in safety, above and beyond what can be enforced ex ante.

The variable m_i is the net transfer to bank i across all discretionary policies: the most obvious interpretation is that of direct equity injections, but we can also think of other implicit and explicit subsidies such as credit guarantees and loans at a reduced interest rate. Philippon and Skreta (2012) discuss these policies in the context of an adverse selection model, and Philippon and Schnabl (2013) in the context of a debt-overhang model. What matters in our model is the net subsidy component of these policies, i.e., the excess payment that the government makes compared to current market prices.

We assume in our baseline that returns are iid across banks, which means, among other things, that we ignore contagion. We extend the model to allow for contagion in Section 6.

Finally, our paper focuses on payoffs in the crisis state. In general, the planner might want to use information from the normal state to provide ex ante incentives.

In practice there are two reasons why this is not feasible. The empirical reason is that returns in normal states contain little information about returns in crisis states. For instance, Acharya et al. (2016) find that the cross-section of returns only begin to predict returns during the GFC after the end of 2006. Relative returns during the boom years contain no useable information for estimating performance during the crisis. We thus assume that $\mathbb{VAR}(\xi_i) \gg \mathbb{VAR}(\epsilon_i)$. The theoretical reason is that $f(x_i)$ is a decreasing function of x so an incentive scheme would have to punish a firm for good performance and these schemes are not robust to hidden trading as shown in Innes (1990) and Nachman and Noe (1994).

1.2 No Bailout

Consider first the allocations when bailouts are ruled out by assumption. Consider first the privately optimal solution. Under A2, maximizing e_i is equivalent to maximizing $r_{i,s}a_i$. Let $\tilde{x}$ be the privately optimal safe return of a bank anticipating $m = 0$ in all states:

$$\tilde{x}_i \equiv \arg \max_{0 \leq x_i \leq 1} p_0 a_i f(x_i) + (1 - p_0) a_i \mathbb{E}[r_{i,s} \mid x_i]. \tag{3}$$

By stochastic dominance the function $\mathbb{E}[r_{i,s} \mid x]$ is increasing in x and the concavity of f guarantees the existence of a unique solution. Since have assumed that the safety investment set is the same for all banks, $\tilde{x}_i = \tilde{x}$ is the same for all i.

Consider next the socially optimal allocation. Since f is concave it is optimal for the planner to set the same level of safety for all the banks. The return in the normal state is therefore $\sum_i f(x_i)$ and crisis returns is $\sum_i r_{i,s}$. To simplify the exposition, we assume the private sector is well capitalized in the good state: $f(x^*) > \underline{r}$. We can define the no-bailout optimal solution as

$$\mathbf{x}_0^* = \arg \max_{\mathbf{x}} \sum_i a_i \left(p_0 f(x_i) + (1 - p_0) \mathbb{E}[r_{i,s} \mid x_i]\right) + \mathbb{E}\left[V\left(\{e_{i,s}\}_i\right) \mid \mathbf{x}\right] \tag{4}$$

where $\mathbf{x}_0^* = \left(x_{1,0}^*, .., x_{N,0}^*\right)$ is the vector of safety investment by banks. The concavity of V guarantees the existence of a unique solution. We maintain throughout the paper the assumption that banks are well capitalized in the normal state. We also assume that the efficient safety investment without bailout is not stuck in the left corner.

Assumption 3. $0 < x_{i,0}^*$ and $f\left(x_{i,0}^*\right) > k_i$ for all i.

10

Note that, since V is an increasing function, we have $x_{i,0}^* \geq \tilde{x}$ for all i. The planner prefers higher safety investments than what banks would choose individually.

1.3   First-Best Allocation with Bailouts

Define $M \equiv \sum_i m_i$ as the state contingent aggregate bailout. Assumption A2 guarantees that $M = 0$ in the normal state since the option to bailout can only decrease the optimal level of ex ante safety (i.e., the solution of the full program is always such that $x^* \leq x_0^*$, therefore $f(x^*) > \underline{r}$ since f is decreasing). The program of the planner is therefore

$$(\mathbf{x}^*, \mathbf{m}^*) = \arg \max_{\mathbf{x}, \mathbf{m}} p_0 \sum_i a_i f(x_i) + (1 - p_0) \sum_i a_i \mathbb{E}[r_{i,s} \mid x_i]$$
$$+ \mathbb{E}\left[V\left(\{r_{i,s} a_i + m_{i,s} - d_i\}_i\right) - \Gamma(M; \gamma)\right]$$

We define the ex post optimal vector of bailouts as

$$\mathbf{m}^*(\mathbf{r}) \equiv \arg \max_{\{m_i\}_i} V\left(\{r_{i,s} a_i + m_{i,s} - d_i\}_i\right) - \Gamma(M; \gamma).$$

We assume that $\mathbf{m}^*(\underline{\mathbf{r}}) > 0$. A positive bailout in the worst state is typically part of the first best allocation. This is in line, for instance, with the theoretical results in Keister (2016) in the context of a Diamond and Dybvig (1983) model. More generally, it is not difficult to imagine that the government is more efficient than the private sector at providing some form of catastrophe insurance. In this case, it would be inefficient to force the private sector to fully self-insure completely against very unlikely but very costly crises. The issue is therefore not the existence of strictly positive bailout probability, but rather what the anticipation of a bailout does to private incentives for safety.

2   A Pure Systemic Risk Model

In this section we follow Acharya et al. (2016) and assume that the value function depends only on the *aggregate* capital surplus of the banking sector:

$$V(\{e_i, a_i\}) = V\left(\sum_i (e_i - \underline{e} a_i)\right) \tag{5}$$

11

where V is increasing and concave. For instance, the systemic expected shortfall in Acharya et al. (2016) uses the piecewise linear case $V = \min(0, E - \underline{e}A)$. The assumption behind this loss function is that the banking sector has specific expertise that is not easily replicated by non-bank actors, but that banks within the sector are good substitute for one another. With this loss function, the government does not care about the distribution of returns across banks, but only about the aggregate capital shortfall of the banking sector. In other words, we assume that the expertise that makes banks socially valuable, for instance their ability to lend to SMEs and households, is transferable across banks but not outside the banking system. If a bank fails, its outstanding assets and new lending can be picked up by other surviving banks. By definition, when the system is solvent, it is possible to transfer assets and liabilities to solvent banks. By contrast, when the banking system is insolvent, the planner cannot avoid a disruption that has real welfare costs because it is costly to transfer bank assets outside the banking sector, either to deep-pocket private investors or to the government itself, and it is difficult to raise bank equity quickly in a crisis. We relax these assumptions in Section 5, but view them as a good starting point to capture the deadweight loss from an undercapitalized banking system.

2.1 Ex Post Optimal Bailout

Define the *aggregate* return as $R \equiv \sum_i a_i r_{i,s}$ and the aggregate gross requirement as $K \equiv \sum_i (\underline{e}a_i + d_i)$. The ex post optimal bailout is then simply a function of the aggregate return. We define the maximized value function as

$$\mathcal{V}(R - K; \gamma) \equiv \max_{M \geq 0} V(R + M - K) - \Gamma(M; \gamma),$$

and the optimal bailout as

$$\mathcal{M}(K - R; \gamma) \equiv \arg \max_{M \geq 0} V(R + M - K) - \Gamma(M; \gamma). \tag{6}$$

We will use the following simple Lemma.

Lemma 1. *The solution $x^*(\theta, \kappa)$ to the problem $\max_x f(x - \theta) + g(k - x)$ where f and g are concave is increasing in θ and κ with slopes less than one, i.e., such that $x^* - \theta$ is decreasing in θ and $k - x^*$ is increasing in k.*

Proof. Appendix. □

12

Applying the Lemma to our problem we can characterize the optimal bailout.

Proposition 1. *The maximized value function $\mathcal{V}$ is increasing and concave in $R - K$, and decreasing in γ. The bailout $\mathcal{M}(K - R; \gamma)$ is increasing in $K - R$ and decreasing in γ. There exists a threshold $\mathcal{K}(\gamma) \in [0, K]$, decreasing in γ such, that $\mathcal{M} = 0$ for $R \geq \mathcal{K}(\gamma)$.*

Proof. First note that if $R > K$ the solution is obviously $M = 0$. We can therefore restrict our attention to $R < K$ and $M \geq 0$. Because V is concave the Lemma implies that $\mathcal{M}(R, K)$ is increasing in $K - R$ with slope less than one. The comparative statics with respect to γ come directly from the fact that $\Gamma(M; \gamma)$ is increasing and super-modular. The fact that $\mathcal{V}$ is concave comes from the fact that V is concave and the fact that $\mathcal{M}$ has a slope less than 1. The definition of $\mathcal{K}(\gamma)$ is the same as in the next example. $\square$

The value function $\mathcal{V}$ is concave and differentiable irrespective of the shape of V and Γ. The bailout function, on the other hand, may or may not be convex, and is usually not differentiable. For instance, when the sytemic externality is piecewise linear $V = \min(0, E - \underline{e}A)$ and the fiscal cost of fund is quadratic $\Gamma = \gamma M^2$, then the bailout is flat at $(2\gamma)^{-1}$ when the crisis is severe and then linearly decreasing (in R) to zero when the return is between $K - (2\gamma)^{-1}$ and K.

Example: Linear Cost of Fund Suppose that the cost of fund is linear

$$\Gamma(M) = \gamma |M|$$

The quasi-linear preferences of the planner imply that the ex post optimal bailout takes the simple form of a put option on the aggregate return R:

Lemma 2. *With linear cost of funds, the optimal aggregate bailout is $\mathcal{M} = \max\{0, \mathcal{K}(\gamma) - R\}$ where $\mathcal{K}(\gamma) \in [0, K]$ is decreasing.*

Proof. First note that if $R > K$ the solution is obviously $M = 0$. We can therefore restrict our attention to $R < K$ and $M \geq 0$. To exploit the quasi-linear preferences we change variable from M to $\hat{M} \equiv M + R - K$. We can rewrite the loss minimization problem (6) as

$$\max_{\hat{M} \geq R-K} V\left(\hat{M}\right) - \gamma\left(\hat{M} + K - R\right)$$

13

If $\hat{M} = R - K$ the solution is $M = 0$. If $\hat{M} > R - K$, then it solves

$$\hat{M}(\gamma) = \arg\max_{\hat{M}} \left\{ V\left(\hat{M}\right) - \gamma\hat{M} \right\}$$

which is negative and decreasing in γ. Since $M = \hat{M} + K - R$, we then get $M = \mathcal{K}(\gamma) - R$ with $\mathcal{K}(\gamma) = \hat{M}(\gamma) + K$. Putting the two cases together, we therefore get $M = \max\{0, \mathcal{K}(\gamma) - R\}$. $\qquad\square$

The planner has an aggregate target $\mathcal{K}(\gamma)$ which depends on the aggregate capital requirement K and the cost of public fund γ. If the private sector delivers the target by itself $(R > \mathcal{K})$, then the planner does not intervene. If the private sector falls short of the target $(R < \mathcal{K})$ then the planner replenishes aggregate capital up to the target to $\mathcal{M}(R) + R = \mathcal{K}$. The replenishment may not be complete $(\mathcal{K} < K)$ when public funds are costly and when V approaches its maximum smoothly from the left.

2.2   First Best

With the welfare function (5), the first best solution solves

$$\mathbf{x}^* = \arg\max_{\mathbf{x} \geq 0} p_0 \sum_i a_i f(x_i) + (1 - p_0) \sum_i a_i \mathbb{E}\left[r_{i,s} \mid x_i\right] + \mathbb{E}\left[\mathcal{V}\left(\sum_i a_i r_{i,s} - K\right) \mid \mathbf{x}\right]$$

The loss function is decreasing in R and increasing in γ which implies that

$$\tilde{x} \leq x_i^* \leq x_{i,0}^*.$$

The planner always wants more safety than the privately optimal choice under no bailout $\tilde{x}$, but requires less than in the optimal case without bailouts x_0^* because the option to bail out limits downside risks. Notice that optimal safety may depend on bank size because of the non-linear loss function. Let us define

$$\epsilon_{i,s} \equiv r_{i,s} - \mathbb{E}\left[r_{i,s} \mid x_i, s\right].$$

Let $G_\epsilon(. \mid x_i, s)$ be the distribution ϵ_i and let $\varepsilon \equiv \sum_i a_i \epsilon_i$ be the aggregate of bank level shocks.

Lemma 3. *Optimal safety does not depend on size when G_ϵ does not depend on x. Optimal safety is increasing in size if G_ϵ satisfies second order stochastic dominance in x.*

Proof. Suppose G_ϵ does not depend on x. Define $\bar{r}(x, s) = \mathbb{E}[r_{i,s} \mid x, s]$. We have

$$\mathbf{x}^* = \arg\max_{\mathbf{x} \geq 0} p_0 \sum_i f(x_i) + (1 - p_0) \int_s \sum_i \bar{r}(x_i, s) \, dP(s)$$

$$+ \frac{1}{a_i} \int_s dP(s) \int_\varepsilon \mathcal{V}\left(\sum_i a_i \bar{r}(x_i, s) + \varepsilon - K\right) d\bar{G}_\epsilon(\varepsilon)$$

where $\bar{G}_\epsilon(\varepsilon)$ is the convolution of the distributions G_ϵ. It does not depend on $\mathbf{x}$. Therefore

$$\frac{1}{a_i} \frac{\partial}{\partial x_i} \mathbb{E}[\mathcal{V}(R) \mid \mathbf{x}, s] = \bar{r}_x(x_i, s) \, \mathbb{E}[\mathcal{V}'(R) \mid \mathbf{x}, s]$$

and the optimal choice of x_i does not depend on the size of bank i. See Appendix for the second part of the proof. ☐

We get scale independence if return volatility does not depend on x. An example is $r_{i,s} = \alpha(x_i) + s + \epsilon_i$ where α is increasing. This implies $R = \sum_i a_i \alpha(x_i) + As + \varepsilon$ where ε is independent of $\mathbf{x}$. On the other hand there are realistic cases where x would affect the volatility of r. For instance $r_{i,s} = \alpha(x_i) + s + (1 - x_i)\epsilon_i$. In that case efficiency requires large banks to invest more in safety. We say that a crisis is systemic if it necessitates a bailout and moderate otherwise, i.e., when $R < \mathcal{K}$. We summarize our results in the following proposition.

Proposition 2. *The social optimum is characterized by $(\mathbf{x}^*, \mathcal{M}(K - R; \gamma))$. Safety investments $\mathbf{x}^*$ are increasing in γ, in A, and in the mean and variance of s, and decreasing in $\underline{e}$, and satisfy $(\tilde{x}, ..\tilde{x}) < \mathbf{x}^* < \mathbf{x}_0^*$.*

Propositions 1 and 2 put some discipline on the range of outcomes that are consistent with optimal regulations and interventions. There are no bailouts in moderate states. Once the capital shortfall is large enough, the planner finds it optimal to transfer bailout funds to banks. The shape of the bailout is then pinned down by fiscal capacity. When the fiscal cost is linear (e.g., the US), it is optimal to fully insure the banking system against further downside risk. When the fiscal cost is convex (e.g., Ireland, Greece, Cyprus), the bailout increases less than one for one with the losses.

In the first best, the government *mandates* the optimal safety vector $\mathbf{x}^*$, thus avoiding moral hazard. In the rest of the paper we study what happens when x is unobserved by the government. The model then includes the potential for a strong form of moral hazard. When $M^* > 0$ the aggregate return net of government transfer does not depend on x. Anticipating this, banks might discount the systemic states and increase their risk taking.

2.3 Moral Hazard under No Commitment and Symmetric Bailouts

We now assume that x cannot be observed and we impose a time-consistency, or "credibility", constraint. The government is restricted to rules $\{m_i\}$ that are ex post optimal, even off the equilibrium path. Therefore

$$\sum_i m_{i,s} = \mathcal{M}(K - R) \tag{7}$$

for all possible values of R where $\mathcal{M}(K - R)$ is defined in (6). We define a symmetric bailout as follows.

Definition 1. A bailout is symmetric if, for all $(i, j) \in [1 : N]^2$ and all $s \in \mathcal{S}$, we have $\frac{m_{i,s}}{a_i} = \frac{m_{j,s}}{a_j}$.

When all banks of ex ante identical a symmetric bailout is one where they all get the same amount of money. When banks' sizes vary, the definition simply allows proportionality with size. In a symmetric bailout we have $m_{i,s} = a_i \frac{\mathcal{M}(R)}{A}$ and $a_i r_{i,s} + m_{i,s} = a_i \left(r_{i,s} + \frac{\mathcal{M}(K-R)}{A} \right)$. The best response of bank i is therefore

$$\beta(\mathbf{x}_{-i}) = \arg\max_{x_i \geq 0} p_0 a_i f(x_i) + (1 - p_0) a_i \left(\mathbb{E}[r_{i,s} \mid x_i] + \Omega(x_i; \mathbf{x}_{-i}) \right) \tag{8}$$

where $\mathbf{x}_{-i}$ is the vector of safety investments by all banks except bank i, and Ω is defined as $\Omega(\mathbf{x}) \equiv \frac{1}{A} \mathbb{E}[\mathcal{M}(K - R) \mid \mathbf{x}]$, which we can write as

$$\Omega(\mathbf{x}) = \frac{1}{A} \int \mathcal{M}(K - R) \, d\Phi_N(R \mid \mathbf{x}). \tag{9}$$

The distribution Φ_N is the convolution of the underlying ones: $R \mid \mathbf{x} \sim \sum_{i=1}^N a_i r_{i,s} \mid \mathbf{x}$.

Lemma 4. $\Omega(\mathbf{x})$ *is continuous, decreasing, and satisfies the increasing difference condition in* $(x_i, \mathbf{x}_{-i})$ *for all* i.

Proof. We use the standard notations $R_{-i} = \sum_{j \neq i} a_j r_{j,s}$ and

$$\Phi_N(R \mid \mathbf{x}) = \mathbb{P}\left(\tilde{R} < R \mid \mathbf{x}\right)$$

$$= \int_s \mathbb{P}\left(\sum_{i=1}^N a_i r_{i,s} < R \mid \mathbf{x}, s\right) p_s ds$$

$$= \int_s \mathbb{P}\left(a_1 r_{1,s} < R - R_{-1} \mid \mathbf{x}, s\right) p_s ds$$

$$= \int_s \int_{R_{-1}} G\left(\frac{R - R_{-1}}{a_1} \mid x_1, s\right) d\Phi_{N-1}(R_{-1} \mid \mathbf{x}_{-1}, s) p_s ds$$

Since $G(. \mid x_i, s)$, is decreasing in x_i, so is $\Phi_N(R \mid \mathbf{x})$. Since $\mathcal{M}$ is decreasing in R, $\Omega(x_i; \mathbf{x}_{-i})$ in decreasing in x_i for any i. Since $G(. \mid x, s)$ is $\mathcal{C}^1$ in x we have

$$\frac{\partial \Phi_N(R \mid \mathbf{x})}{\partial x_i} = \int_s \int_{R_{-1}} \frac{\partial G\left(\frac{R-R_{-1}}{a_1} \mid x_i, s\right)}{\partial x_i} d\Phi_{N-1}(R_{-i} \mid \mathbf{x}_{-i}, s) p_s ds$$

is negative and increasing in $\mathbf{x}_{-i}$ since $\Phi_{N-1}(. \mid \mathbf{x}_{-i}, s)$ is decreasing in $\mathbf{x}_{-i}$. Therefore $\frac{\partial \Omega}{\partial x_i}$ is increasing in $\mathbf{x}_{-i}$. $\square$

Lemma 4 immediately implies that, for all possible value of $\mathbf{x}_{-i}$, the best response is bounded above by the private equilibrium: $\beta(x_{-i}) \leq \tilde{x}$. Our game takes place on compact sets with a finite number of players, continuous choices and continuous reward functions, therefore we know that at least one Nash equilibrium exists and any solution satisfies $\hat{x} \leq \tilde{x}$. We summarize our discussion in the following proposition.[2]

Proposition 3. *All equilibria with no commitment and symmetric bailouts have the following properties:*
(i) Lack of commitment creates strategic complementarities in risk taking: $\beta(x_{-i})$ *is increasing.*
(ii) Safety is too low $(\hat{x}_i < x_i^*)$ *and the probability of a systemic crisis is too high*

---

[2]Given risk-neutrality, it is without loss of generality to focus on pure strategies. Fudenberg and Tirole (1990) show that with risk-averse agents, it is possible to maintain some incentives once we allow for mixed strategies.

$\Phi_N \left(M_0 \mid \hat{\mathbf{x}} \right) > \Phi_N \left(M_0 \mid \mathbf{x}^* \right)$.

(iii) Moral hazard worsens when γ decreases.

(iv) If $\beta \left(\mathbf{0} \right) = 0$ a full unraveling equilibrium exists with minimum safety, maximum systemic risk, and maximum bailout $x_i = 0$ for all i.

Proof. (i) Because $\frac{\partial \Omega}{\partial x_i}$ is increasing in x_{-i}. (ii) Because Ω is decreasing. (iii) Because $\mathcal{M}$ is decreasing in γ. (iv) follows from the fact that f is maximized at $x = 0$. $\qquad\square$

Lack of government commitment creates strategic complementarities between banks: if all banks reduce their safety the probability of a bailout increases, which reduces the marginal incentives to hedge against systemic crises. Lack of government commitment can generate an extreme form of moral hazard where banks make no investment in safety. A marginal increase Δx_i reduces the bank's expected bailout.

Strategic Complementarities and Uniqueness   While strategic complementarities are a realistic feature, they can open the door to multiple equilibria if those complementarities are too strong. We can in principle deal with multiple equilibria: there is a set of equilibria, and each time we say that safety is increasing we mean it in the Strong Set Order sense of Topkis (1978) and Milgrom and Shannon (1994). Alternatively, we could allow the government to act as a coordination device and select the equilibrium with highest safety. These solutions are feasible but they create a large burden of notations without changing the economic insights. It is more convenient to have a unique equilibrium to state our main results in the next section. We therefore assume that Ω is not too convex or that f is concave enough.

Assumption 4. *The slope of the best response $\beta \left(\mathbf{x}_{-i} \right)$ is less than one.*

3   Credible Tournaments

The previous section has shown that when the government lacks commitment, standard contracts lead to moral hazard. In stark contrast, we now show that the government can use relative performance evaluation among multiple banks to solve the moral hazard problem and implement the first-best allocation in a time-consistent fashion. The reason is that the credibility constraint only affects the aggregate bailout $M = \sum_i m_{i,s}$, and leaves enough leeway to the government to structure the *distribution* of bailouts across banks. In particular, the government can use a relatively simple tournament

scheme that rewards banks according to their ranking while maintaining credibility. The time consistency constraint is

$$\sum_{i=1}^{N} m_{i,s} = \mathcal{M}(K - R)$$

for all possible realization of the aggregate return R. For simplicity we illustrate our main result in the case where banks are ex ante identical and consider size differences in the Appendix. Thus we assume $a_i = 1$ for all banks. Bank i chooses its safety investment as

$$\hat{x}_i = \arg \max_{x_i \geq 0} p_0 f(x_i) + (1 - p_0) \left(\mathbb{E}[r_{i,s} \mid x_i] + \mathbb{E}[m_{i,s}(\mathbf{r}) \mid \mathbf{x}] \right). \tag{10}$$

3.1 Bonus-Malus Implementation

Two Banks. We build intuition by considering the case of two banks. We define the tournament rule $\mathcal{T}$ with two banks as

$$m_i = \begin{cases} \frac{\mathcal{M}(K-R)}{2} + \Delta & r_{i,s} > r_{j,s} \\ \frac{\mathcal{M}(K-R)}{2} - \Delta & r_{i,s} < r_{j,s} \end{cases}$$

Note that $\mathbf{P}[r_{1,s} > r_{2,s}] = H_s(x_1, x_2)$ where is increasing in x_1 and decreasing in x_2. The best response function for bank 1 is therefore

$$\hat{x}_1 = \beta_1(\Delta, x_2) = \arg \max_{x_1} p_0 f(x_1) + (1 - p_0) \left(\mathbb{E}[r_{1,s} \mid x_1] + \Omega(x_1, x_2) \right) + 2\Delta \int_s H_s(x_1, x_2) p_s ds.$$

When $\Delta = 0$ this best response corresponds to the one discussed in Proposition 3. The crucial departure from perfect insurance and the ensuing moral hazard comes from Δ, which rewards the best bank and punishes the other one. We can then state our first main proposition.

Proposition 4. *With $N = 2$, there exists a unique Δ^* that implements the social optimum $(x^*, x^*, \mathcal{M}(K - R))$.*

Proof. The objective function is super-modular in (x_1, Δ) since H is increasing in x_1 therefore x_1 is increasing in Δ. Suppose that $x_2 = x^*$. Clearly $\hat{x}_1(0, x^*) < x^*$. On the other $\lim_{\Delta \to \infty} x_1(\Delta, x^*) = 1$. Since x_1 is continuous there is a unique Δ^* such that $x_1(\Delta^*, x^*) = x^*$. The same holds for x_2 by symmetry. $\qquad \square$

Note that Δ^* is unique in the class of mechanisms that we consider but there are other classes of mechanisms that can implement the first best. We know from Proposition 3, however, that all of them must use relative performance evaluations.

N **Banks.** It is straightforward to extend our results to N banks. In fact, it is easier since there are more degrees of freedom. A possible rule is

$$m_i = \frac{\mathcal{M}(K-R)}{N} + \Delta \times \mathcal{I}(r_i - \text{med}(\mathbf{r}))$$

where function $\mathcal{I}$ is such that $\mathcal{I}(y < 0) = -1$, $\mathcal{I}(0) = 1$, and $\mathcal{I}(y > 0) = 1$ and $\text{med}(\mathbf{r})$ is the median return. By definition of the median

$$\sum_i^N \mathcal{I}(r_i - \text{med}(\mathbf{r})) = 0$$

so $\sum_i^N m_i = \mathcal{M}(R)$ and the rule is credible. Denote $H_N^{\text{med}}(\theta)$ the probability that $r_i > \text{med}(\mathbf{r})$ when all other banks play the same $x_j = x_{-i}$ and bank i plays x_i. Then bank i solves

$$\hat{x}_i = \beta_i(\Delta, \mathbf{x}_{-i}) = \arg\max_\theta (1 - p_0) x_i + (1 - p_0)(\mathbb{E}[r_{i,s} \mid x_i] + \Omega(x_i, \mathbf{x}_{-i})) + 2\Delta \int_s H_{s,N}^{\text{med}}(x_i, \mathbf{x}_{-i}) p_s ds.$$

$H_{s,N}^{\text{med}}$ is increasing in x_i, and we give the expression in the Appendix. Following the same steps as for $N = 2$ we have:

Proposition 5. *For any number $N \geq 2$ of banks, there exists a unique Δ^* that implements the social optimum $(\mathbf{x}^*, \mathcal{M}(K - R))$.*

The simplicity of our "median" rule makes it attractive, but there are many other more complex rules that can achieve the same objective, even within the class of tournaments. For instance, different prizes could be attributed to banks according to their exact ranking in terms of returns, and not just whether they are above or below the median.

The implementation above might require large punishment in equilibrium. A bank with a bad draw needs to be punished to provide ex ante incentives. There are, however, practical limits on punishments. The first limit is that the planner might not be *able* to punish because of limited liability. The second limit is that the planner might not

be *willing* to punish because of imperfect substitutability between banks. We consider each one in turn.

3.2   Limited Liability

Let us now consider the case where government transfers and taxes are constrained by limited liability (LL). There are two ways to write limited liability. The strict form is $m_i \geq 0$ for all banks in all states, which simply rules out negative transfers. This constraint typically leaves equity holders with a surplus. A weaker form of limited liability is $a_i r_i + m_i \geq d_i$, which allows negative transfers of residual equity value, but not more. In section 3.3 we show how these two cases can be interpreted as polar cases of a richer model with fire sales and mark-to-market accounting in resolution. Our first result holds under strict (and therefore also weak) limited liability.

Proposition 6. *Even under strict limited liability $(m_i \geq 0)$, tournament incentives rule out moral hazard $(\hat{x} > \tilde{x})$ and implement the first best when the cost of fund is low, i.e., there exists $\hat{\gamma} > 0$ such that $\hat{x} = x^*$ for any $\gamma < \hat{\gamma}$.*

Proof. The proof has two steps. Let x^{max} be the maximum implementable level of safety. The first step is that the planner can always improve upon purely private incentives. From (10) it is clear that any bailout function with $m_i (r_i < \text{med}(\mathbf{r})) = 0$ and $m_i (r_i > \text{med}(\mathbf{r})) = 2\mathcal{M}/N$ satisfies $\hat{x}_i > \tilde{x}$. Therefore $x^{max} > \tilde{x}$. The second step is that when $\gamma \to 0$ the government can fully insure downside risk: $\lim_{\gamma \to 0} \mathcal{V} = \bar{V}$ and $\lim_{\gamma \to 0} x^* = \tilde{x}$. Therefore $\lim_{\gamma \to 0} x^* < x^{max}$. □

The limit result is easy to prove in the general case of the model with additive capital surpluses. Characterizing the second best allocation is a lot more complicated, however, so we use the following special case with binary outcomes. We assume that all banks are ex ante identical with size a. At time 1 banks are randomly allocated into two groups, L and H, with sizes N_L and N_H such that $N_L + N_H = N$. The probability that any particular bank ends up in group H is simply $h = N_H/N$. The returns of bank i are determined jointly by its risk management, its group, and the aggregate state:

$$r_i^s = \begin{cases} f(x_i) + \xi_i & \text{in the normal state} \\ s + x_i \mathbb{I}_{i \in H} & \text{in crisis state } s \end{cases}$$

The model thus works as follows. Banks make ex ante safety choices x_i. If a bank ends up in group L its return is s irrespective to x. If a bank is in group H, its safety choice matters ex post as its return is $s + x_i$. The key point is that there is no way for the planner to distinguish a bank in group L from a bank in group H who chose $x = 0$. The first best requires all banks to choose some $x^* > 0$ and we assume that the private equilibrium gives $\tilde{x} < x^*$. Let us consider the implementation of a symmetric equilibrium x. When all banks make the same choice the aggregate return does not depend on the random selection of the groups H and L:

$$R = A(s + hx)$$

In particular, the first best solves $x^* = \arg\max_{x \geq 0} p_0 A f(x) + (1 - p_0) A(\bar{s} + hx) + \mathbb{E}[\mathcal{V}(As + Ahx - K)]$. Let us now consider incentive constraints. If one bank deviates, the aggregate return depends both on s and on the group selection. Define $\tilde{X}_H = \sum_{i \in H} x_i$, the return is then

$$\tilde{R} = As + ah\tilde{X}_H.$$

If a bank deviates it will choose $x = 0$ so that it can hide among the legitimate banks of group L. Because banks' outcomes are binary, the bailout takes the simple form $m_L = a\mu_L$ for group $\tilde{L}$ (banks with low returns) and $m_H = a\mu_H$ to group $\tilde{H}$ (banks with high returns). The time consistency constraint is

$$\left(N - \tilde{N}_H\right) a\mu_L + \tilde{N}_H a\mu_H = \mathcal{M}\left(K - \tilde{R}\right)$$

Consider the incentive constraint of bank 1 given that all the other banks play x^*. If bank 1 chooses $x_1 = x^*$ its expected payoffs are

$$p_0 a f(x) + (1 - p_0) a \left(\bar{s} + hx + \mathbb{E}[(1 - h)\mu_L(s) + h\mu_H(s)]\right)$$

If bank 1 instead chooses $x_1 = 0$ its expected payoffs are

$$p_0 a f(0) + (1 - p_0) a \left(\bar{s} + \mathbb{E}[(1 - h)\mu_L(s) + h\mu_L(s, N_H - 1)]\right)$$

because ex post with probability $1 - h$ it belongs to group L and thus the fact that $x = 0$ does not matter. In this state nobody (except bank 1) is aware of the deviation, neither ex ante nor ex post and the payoff must be the same $\mu_L(s)$ as in equilibrium. With probability h it belongs to group H. In that case the planner learns that at

22

least one bank has deviated as the number of high types, $\tilde{N}_H = N_H - 1$, is not N_H as expected. The incentive constraint of bank 1 is therefore

$$(1 - p_0) h (x + \mathbb{E} [\mu_H (s) - \mu_L (s, N_H - 1)]) > p_0 (f (0) - f (x)) \qquad (11)$$

Minimizing $\mu_L (s, N_H - 1)$ is good for incentives. If the planner can lower the return μ_L sufficiently, it can implement the first best, i.e., satisfy (11) with $x = x^*$. With limited liability, the first best may not be implementable. The limited liability constraint depends only on the return of bank i, not on N_H. Therefore without loss of generality we can write $\mu_L (s)$ which is either 0 under strict limited liability, or $\mu_L = d/a - s$ under weak limited liability. Once we have minimized μ_L we find the maximum value μ_H^* using the time consistency constraint $h\mu_H^* = \mathcal{M}/A - (1 - h) \mu_L (s)$ or $h (\mu_H^* (s) - \mu_L (s)) = \mathcal{M}/A - \mu_L (s)$ and the IC constraint (11) becomes

$$(1 - p_0) (hx + \mathbb{E} [\mathcal{M} (s) /A - \mu_L (s)]) > p_0 (f (0) - f (x)) .$$

Proposition 7. *The highest implementable safety under limited liability is decreasing in the cost of public funds γ and decreasing in the size of the banking sector A. Incentives and ex ante leverage constraints d/a are substitutes under strict LL, but complement under weak LL.*

Proof. We know that $\mathcal{M} (s) /A$ is decreasing in γ so it is immediate that the IC improves when γ decreases. We also know that $\mathcal{M}/A$ is decreasing in A. Under strict LL we have $\mu_L = 0$ and know that $\mathcal{M}$ increases with the capital shortfall $K - R = \underline{e}A + \frac{d}{a}A - R$. $\mathcal{M}$ is therefore increasing in d/a and the IC tightens when ex ante leverage is lower. Under weak liability, on the other hand, we have $\mu_L = d/a - s$ and since the slope of $\mathcal{M}$ is less than one we have that $\mathcal{M} (s) /A - \mu_L (s)$ is decreasing in d/a. A lower leverage then loosens the IC constraint. $\qquad\square$

Proposition gives a striking result with respect to fiscal slack: a lower γ *increases* safety. This is exactly the opposite of the conventional wisdom based on symmetric contracts. Proposition gives a rationale for leverage limits and higher capital requirements. It also gives a macro-prudential reason for clawback provisions to reduce the binding limited liability constraint.

3.3 Fire Sales

We show that fire sales are useful for incentives. Suppose that the return r_i that we have described so far denotes the fundamental value that assets would recover to after the crisis. In the midst of the crisis, however, asset values can be temporarily lower, equal to $\chi(1 - r_i)$, where $\chi \in [0, 1)$ is a fire sale discount on assets. We treat χ as fixed to simplify, but our results would extend to a stochastic χ that is potentially correlated with returns, as would be the case, for instance, when endogenizing asset prices using "cash-in-the-market pricing".

Suppose that during the crisis, the regulator is constrained to net transfers m_i that cannot expropriate bank shareholders *at current market prices*. Thus shareholders have the choice between accepting resolution and obtaining a payoff $r_i + m_i - d$, with assets left at book value within the bank until the crisis is over, or liquidating assets at fire sale prices immediately.

The shareholder participation constraint is therefore $ar_i + m_i - d \geq \max\{0, (1 - \chi)ar_i - d\}$ or

$$
\begin{cases}
m_i + ar_i \geq d & \text{if } r_i \leq \frac{d}{(1-\chi)a} \\
m_i + \chi ar_i \geq 0 & \text{if } r_i \geq \frac{d}{(1-\chi)a}
\end{cases}
$$

For deep fire sale discounts $\chi \to 1$, the constraint converges to weak limited liability. For moderate discounts, the constraint writes $m_i + \chi ar_i \geq 0$, and strict limited liability corresponds to the case without fire sales $\chi = 0$. Just like weak LL is easier to satisfy than strict LL, a deeper fire sale discount χ allows the regulator to impose tougher punishments on weak banks during the crisis, and therefore relaxes the incentive constraint for all banks ex-ante.

We can now generalize our result on the effect of leverage on the incentive constraint

$$
(1 - p_0)(hx + \mathbb{E}[\mathcal{M}(s)/A - \mu_L(s)]) > p_0(f(0) - f(x)).
$$

We always have

$$
\frac{\partial \mathbb{E}[\max\{\frac{d}{a} - r, -\chi r\}]}{\partial(d/a)} = P\left(\frac{d}{(1-\chi)a} - hx\right)
$$

where $P(x) = \int_{s \leq x} p_s ds$. With a linear cost of funds, $\mathbb{E}[\mathcal{M}'(\underline{e}A + \frac{d}{a}A - R)] =$

24

$\mathbb{P}\left[s < \frac{\mathcal{K}(\gamma)}{A} - hx\right] = P\left(\frac{\mathcal{K}(\gamma)}{A} - hx\right)$ where $P(x) = \int_{s \leq x} p_s ds$. Thus, starting from leverage d/a, locally tightening the leverage constraint relaxes the incentive constraint if and only if the fire sale discount in case of crisis is deep enough:

$$\chi > \hat{\chi} = 1 - \frac{A}{\mathcal{K}(\gamma)}\frac{d}{a}$$

Proposition 8. *Suppose the cost of funds is linear and let $\hat{\chi} = 1 - \frac{A}{\mathcal{K}(\gamma)}\frac{d}{a}$. Incentives and ex ante leverage constraints d/a are complement if the fire sale discount χ is higher than $\hat{\chi}$, and substitutes otherwise.*

4 Mergers and Resolution Authority

We have used the benchmark loss function $V\left(\sum_i e_i - \underline{e}a_i\right)$ to establish our first main result with and without limited liability. In all these cases policy makers intervene using taxes and transfers.[3] These instruments are used extensively in practice but there is another tool that is used extensively and requires a modification of the baseline model: merger of weak banks with strong ones. In this section we are going to endogenize the distribution of assets and liabilities by giving the government a resolution authority.

Definition 2. Resolution authority is a technology with which, for any undercapitalized bank $e_i < \underline{e}a_i$ the government can write equity claims to 0 and transfer (some of) the assets and deposits to another bank or another set of banks.

We have already discussed the issue of strict versus weak LL so we focus here on the case of weak LL where regulators have the authority to wipe out shareholders of failing banks. It is straightforward to extend the results to the case of strict LL. To discuss mergers we need to specify a value function over different sets of existing banks. We consider the following value function

$$V\{e_i, a_i\} = V\left(\sum_{i=1}^{N} a_i v(y_i)\right), \tag{12}$$

---

[3]One should keep in mind that taxes can also be levied ex ante. Banks could all pay the same tax at time 0 and recoup their payments at time 1 based on the tournament rule described above. This is one example of a policy that alleviates the limited liability constraint, as discussed in Section 4.

where y_i is defined as the percentage surplus of bank i

$$y_i \equiv \frac{e_i}{a_i} - \underline{e}.$$

The functions V and v are increasing and (weakly) concave with $v\left(0\right) \geq 0$ and $V\left(0^+\right) = \bar{V}$. For instance, in our application below we use $v\left(y\right) = \min\left(0, y\right)$, while the benchmark model of Acharya et al. (2016) corresponds to $v\left(y\right) = y$. This value function has two key properties that make it appealing to study mergers and capital shortfalls. The first property is that it is neutral with respect to the combination of similarly capitalized banks. If $y_i = y_j = y$ then we get $a_i v\left(y_i\right) + a_j v\left(y_j\right) = \left(a_i + a_j\right) v\left(y\right)$ so nothing is gained or lost by combining the banks. This is clearly a desirable feature of any welfare function. The second key property is that the concavity of v around $y = 0$ captures the degree of substitution between capital shortfalls and surpluses: $v\left(y\right) = \min\left(0, y\right)$ implies zero substitution while $v\left(y\right) = y$ implies perfect substitution, with most realistic cases somewhere in between. For instance, in a fire sales model, distressed banks are forced to sell, while banks with surpluses take advantage of low prices, but they do not pick up the slack one for one. On the other hand, this value function does not capture two economic forces that may be important in some context: taste for variety and market power. We study these issues in Section 5.

4.1   An Aggregation Result

Let us now consider the case of mergers. We first define a merger allocation and its cost.

Definition 3. A merger allocation is a matrix α where $\alpha_{i,j} \in [0, a_i]$ are the assets from bank i transferred to bank j and $\sum_{j=1}^{N} \alpha_{i,j} = a_i$. The cost of the merger allocation is $\tau \sum_{i=1}^{N} \left(a_i - \alpha_{i,i}\right)$.

The idea here is simple. Mergers reallocate assets and the cost of transfer is τ per unit of assets. Consider for instance the sale $\alpha_{i,j}$ from bank i to bank j. The net welfare gain is then

$$V\left(.. + \left(a_i - \alpha_{i,j}\right) v\left(y_i\right) + \left(\alpha_{i,j} + a_j\right) v\left(\frac{\alpha_{i,j} y_i + a_j y_j}{\alpha_{i,j} + a_j}\right)\right) - V\left(.. + a_i v\left(y_i\right) + a_j v\left(y_j\right)\right) - \tau \alpha_{i,j}$$

$$(13)$$

Since v is concave and V increasing we know that the first difference is positive and the question is whether it is high enough to cover the cost $\tau\alpha_{i,j}$. Would the shareholders of bank j approve the merger? Under Assumption A2 of well callibrated TLAC the value of assets exceeds that of liabilities, so the merger increases shareholder value, but the merged bank might still be undercapitalized (if $\alpha_{i,j}y_i + a_jy_j < 0$), in which case the regulator might want to provide bailout funds to bank j. We return to these issues later. Let N_- and N_+ the sets of undercapitalized and well capitalized banks. When we study mergers the two key state variables are the mass of failed assets

$$A_- = \sum_{i \in N_-} a_i$$

and the aggregate surplus equity $E - \underline{e}A$. We have the following results.

Proposition 9. *Let V^{post} be the post-merger welfare value: $V^{post} = \max_\alpha V\left(\alpha, \{e_i, a_i\}\right) - \tau \sum_{i=1}^N (1 - \alpha_{i,i}) a_i$. We have $V^{post} \geq V\left(Av\left(\frac{E - \underline{e}A}{A}\right)\right) - \tau A_-$. As $\tau \to 0$ any value function of the type (12) converges to the value function $V\left(Av\left(\frac{E - \underline{e}A}{A}\right)\right)$.*

The proposition is useful because it shows that the value function in our benchmark case is without loss of generality when mergers are frictionless. In particular, all our previous results apply.

Corollary 1. *Tournament bailouts implement the first best as in Propositions 5 and 6 when mergers are frictionless. In particular, when $E > \underline{e}A$, frictionless mergers achieve the first best without bailouts.*

Tournament bailouts implement the first best as the government is not forced to bailout out the bad performing banks. It can instead merge them with good banks and reward the shareholders of relatively well performing banks with additional funds if necessary. To understand the proof of the results, let us start from a marginal change. Taking the derivative of (13) we get

$$\frac{dV}{d\alpha_{ij}} = \left(v\left(y_j\right) - v\left(y_i\right) - \left(y_j - y_i\right)v'\left(y_j\right)\right)V'\left(.\right) - \tau$$

This formula contains all the economics of mergers in our model.

Lemma 5. *If $\tau = 0$ a marginal asset transfer increases welfare if and only if $y_j > y_i$ and the most attractive first merger is the one between the two furthest banks $i = \arg\min y_t$ and $j = \arg\max y_t$.*

This then suggests the following algorithm starting at $k = 0$ with the initial allocation $\{a_i, e_i\}$. Define

$$i^{(k)} \equiv \arg \min_{a_t > 0, y_t < 0} y_t^{(k)}$$

$$J^{(k)} \equiv \arg \max_t y_t^{(k)}$$

$$j_2^{(k)} \equiv \arg \max_{t \notin J^{(k)}} y_t^{(k)}$$

In words, $i^{(k)}$ is the worst bank among the ones with positive assets and negative surplus (pick any one in case there is a tie), $J^{(k)}$ is the set of best banks, and $j_2^{(k)}$ the next best one. Let $y^c(i, j, \alpha)$ be the capital combination function

$$y^c(i, j, \alpha) = \frac{\alpha_{i,j} y_i + a_j y_j}{\alpha_{i,j} + a_j}$$

The algorithm is then

1. Compute $i^{(k)}, J^{(k)}, j_2^{(k)}$. If $i^{(k)} \in J^{(k)}$, stop. Otherwise proceed.

2. If $y^c\left(i^{(k)}, J^{(k)}, a_{i^{(k)}}^{(k)}\right) > y\left(j_2^{(k)}\right)$ then transfer uniformly all the assets from $i^{(k)}$ to the banks in $J^{(k)}$. Set $a_{i^{(k)}}^{(k+1)} = 0$ and $y\left(J^{(k)}\right) = y^c$. Repeat step 1.

3. Otherwise define α such that $y^c\left(i^{(k)}, J^{(k)}, \alpha\right) = y\left(j_2^{(k)}\right)$, transfer α, set $a_{i^{(k)}}^{(k+1)} = a_{i^{(k)}}^{(k)} - \alpha$ and $y\left(J^{(k)}\right) = y\left(j_2^{(k)}\right)$. Repeat step 1.

It is easy to check that this algorithm provides the welfare $V\left(Av\left(\frac{E - \underline{e}A}{A}\right)\right) - \tau A_-$. The set of failed bank N_- decreases until either $N_- = \emptyset$ or all the banks have the same negative capital surplus. When $E > \underline{e}A$ the algorithm stops when $A_-^{(k)} = 0$, having relocated all failed assets to healthy banks. Capital is not typically equalized across all banks, but since all remaining banks are well capitalized we get $\bar{V}$. When $E < \underline{e}A$ the algorithm does not stop until all the banks have the same capital ratio $\frac{E - \underline{e}A}{A}$. In both cases the algorithm never transfers more than A_-.

4.2 Second Best Allocations

When mergers are costly ($\tau > 0$) it is not in general optimal to complete the merging algorithm and the value function does not fully converge to the benchmark model of the

28

previous sections. Characterizing the second best allocation is challenging, especially when we also consider endogenous bailouts. So we consider the simple case

$$V = \bar{V} + v \sum_{i}^{N} a_i \min(0, y_i) \tag{14}$$

This is a conservative value function since it assumes no benefit from banks with capital surpluses $e_i > \underline{e}a_i$. In particular, this specification maximizes the time consistency problem since, without mergers, it is ex post optimal to bail out *only* the banks with negative surplus.

Ex-post Interventions: Mergers and Bailouts Define the surplus of good banks and the shortfall of undercapitalized banks, respectively, as

$$Y_+ = \sum_{j \in N_+} a_j y_j$$
$$Y_- = \sum_{j \in N_-} -a_j y_j$$

hence $Y = Y_+ - Y_-$.

With the value function (14) mergers are useful only if they tap into unused capital surplus. An immediate implications is that, if $\tau > 0$ and the value function is only weakly concave as in (14), then when $Y < 0$ the merger process will not lead to equalization of capital surpluses across banks. When $Y_+ > 0$ there is untapped capital surplus and the attractive merger target is the bank with the worst shortfall. Under Assumption A2 we know that $d_i/a_i - r_i \geq 0$ hence $y_i \geq -\underline{e}$, so we make the following assumption to ensure that mergers are potentially useful:

Assumption: The merger cost satisfies $\tau < \underline{e}v$.

We can now describe the second best allocation. Define for $y \leq 0$ the cumulative shortfall function

$$\mathcal{Y}(y) = -\sum_{y_i \leq y} a_i y_i \in [0, Y_-]$$

and the following two cutoffs y_γ and y_τ

$$y_\gamma = \inf_i y_i \text{ s.t. } \Gamma'(Y_- - \mathcal{Y}(y_i)) \leq v$$

$$y_\tau = \sup_i y_i \text{ s.t. } \begin{cases} y_i \leq -\tau/v \\ \mathcal{Y}(y_i) \leq Y_+ \end{cases}$$

Both y_γ and y_τ are negative. To interpret y_γ, note first that is never ex-post optimal to give more than $m_j = -a_j y_j$ to bank j. Thus if bailouts are the only option, the ex post efficient allocation that maximizes incentives gives a full bailout $m_j = -a_j y_j$ to banks starting from $y_j = 0$, until the marginal cost of further bailouts Γ' exceeds the marginal benefit v, which happens at y_γ. To interpret y_τ, note that if mergers are the only option, the ex post efficient allocation merges all the banks with y_i below y_τ to banks with a capital surplus, starting with the worst bank $y_i = \min y$. The two inequalities defining y_τ capture the fact that the merging process stops when either the marginal return to merging the next bank falls below the cost τ, or the entire capital surplus Y_+ has been exhausted.

When bailouts and mergers are both available, the ex post efficient allocation that maximizes incentives only bails out the least undercapitalized banks. When $y_\tau < y_\gamma$ the two policies are combined as follows:

- banks with $y_i \in [y_\gamma, 0]$ are fully bailed out $m_i = -a y_i$ thus the aggregate bailout is $-\sum_{y_\gamma \leq y_i \leq y} a_i y_i = Y_- - \mathcal{Y}(y_\gamma)$.

- banks with $y_i \leq y_\tau$ are merged with good banks;

- banks with $y_i \in (y_\tau, y_\gamma)$ are left untouched.

If the two regions overlap $(y_\gamma < y_\tau)$, then we can define $y^* \in (y_\gamma, y_\tau)$ such that

$$y^* = \sup_i y_i \text{ s.t. } -y_i \Gamma'(Y_- - \mathcal{Y}(y_i)) \leq \tau.$$

Banks with $y_i \geq y^*$ are fully bailed out and banks with $y_i \leq y^*$ are merged. This result is captured formally by the following Lemma:

Lemma 6. *Suppose the government has spent $M \geq 0$ in bailout funds with resulting sets of banks N_- and N_+. If $Y_+ = 0$, then no merger takes place and further bailouts of distressed banks happen if $v > \Gamma'(M)$. If $Y_+ > 0$, a merger takes place if there is an i such that $\tau < -y_i \min(v, \Gamma')$.*

Proof. Consider a bank with $y_i < 0$. If the government does nothing the value is $\mathcal{V}_0 = V_{-i} + vay_i - \Gamma(M)$. If the government bails out the bank by some small amount

30
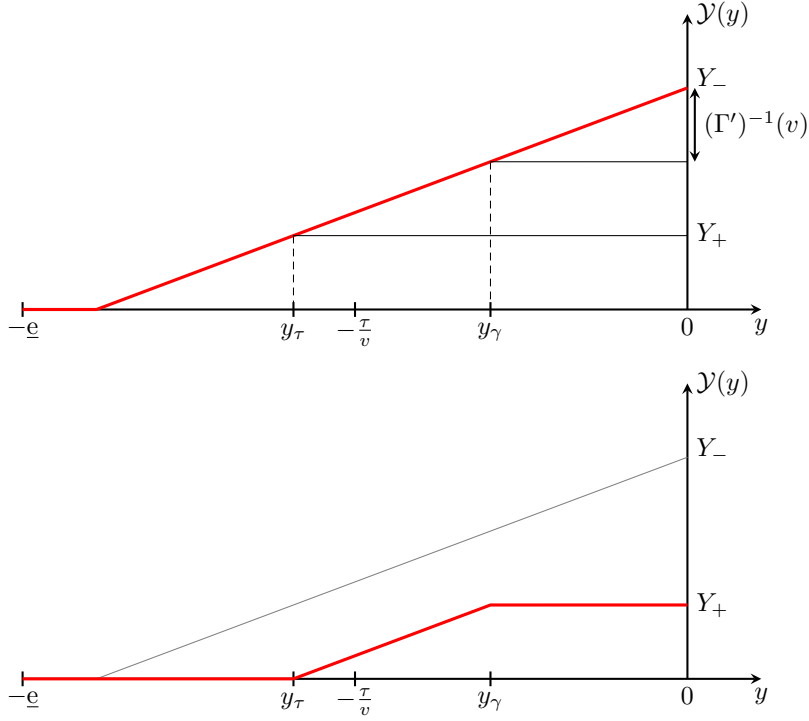
Figure 1: Cumulative shortfall function $\mathcal{Y}$ (in red) in the case $y_\tau < -\tau/v < y_\gamma$ before (top panel) and after (bottom panel) mergers and bailouts.

m the value becomes $\mathcal{V} = V_{-i} + v\,(ay_i + m) - \Gamma\,(M) - \Gamma'\,(M)\,m = \mathcal{V}_0 + (v - \Gamma'\,(M))\,m$ so a (partial) bailout improves welfare if and only if $v > \Gamma'\,(M)$. Consider a merger instead.

Consider next the sale of α of a_i. If $Y_+ = 0$ the acquiring bank has $y_j \leq 0$ so the value becomes $\mathcal{V} = V_{-i} + v\,(a_i - \alpha)\,y_i + v\alpha y_i - \tau\alpha - \Gamma\,(M) = \mathcal{V}_0 - \tau\alpha$. A simple merger reduces welfare. A merger cum recap would lead to $\mathcal{V}_0 + (v - \Gamma'\,(M))\,m - \tau\alpha$ which may be positive but is always worse than a simple bailout.

If $Y_+ > 0$ then a merger leads to $\mathcal{V} = V_{-i} + v\,(a_i - \alpha)\,y_i - \tau\alpha - \Gamma\,(M) = \mathcal{V}_0 - v\alpha y_i - \tau\alpha$ which is higher than $\mathcal{V}_0$ if $-vy_i > \tau$. Finally, a full merger of bank i $\alpha = a_i$ is better than a full bailout $m_i = -a_i y_i$ if $-y_i \Gamma'\,(M) > \tau$. $\qquad\qquad\square$

Ex-ante Incentives   We now show how mergers can act as a commitment device to reward good banks and punish weak ones and thus provide incentives, even when the loss function such as in (14) would call for fully bailing out weak banks absent the merger technology.

31

Suppose the cost of funds is quadratic $\Gamma(M) = \gamma\frac{M^2}{2}$. Then we can compute the thresholds as

$$y_\tau = -\frac{\tau}{v}, \quad y_\gamma = -\frac{v}{\gamma}, \quad y^* = -\sqrt{\frac{\tau}{\gamma}}.$$

Consider the case of 2 banks with the same shock structure as in section 3.2, with a single crisis state s to simplify. There are four possible events, depending on which banks end up in groups H and L. If both banks end up with a capital surplus, no policy intervention is needed. If both banks end up with capital shortfalls ($y_1 = y_2 = s - \frac{d}{a} - \underline{e} < 0$) then there is no merger as $Y_+ = 0$. We assume the cost of funds is low enough that the only time-consistent policy is to bailout both banks fully:

$$v \geq 2\gamma\left(\frac{d}{a} + \underline{e} - s\right) \tag{15}$$

Thus both banks obtain a payoff $\underline{e}$. Finally, the most interesting case occurs if only bank 1, say, ends up in group H. Then in an equilibrium with safety choices x, $Y_+ = y_1 = s + x - \frac{d}{a} - \underline{e}$ and $Y_- = -y_2 = -\left(s - \frac{d}{a} - \underline{e}\right)$. Suppose that even under the laissez-faire safety $\tilde{x}$, the strong bank has enough capital to absorb the distressed one:

$$\tilde{x} > 2\left(\frac{d}{a} + \underline{e} - s\right)$$

Thus whenever bank 1 succeeds, a full merger is feasible. (15) implies that if mergers are too costly, it is always optimal to fully bailout bank 2. If the cost of mergers τ is low enough, however,

$$y_2 \leq y^* \Leftrightarrow \tau \leq \tau^*(\gamma) = \gamma\left(\frac{d}{a} + \underline{e} - s\right)^2$$

then $y_2 \leq y^*$ and a merger is optimal. The merged bank's shareholders end up with 0, while the resulting equity of bank 1's shareholders is

$$r_1 + r_2 - 2\frac{d}{a} = 2s + x^* - 2\frac{d}{a}$$

Figure 2 illustrates how the thresholds y_τ, y_γ and y^* vary with the cost of mergers τ. A higher fiscal capacity (lower γ) makes bailouts more attractive and undermines the credibility of mergers, thus reducing τ^*.

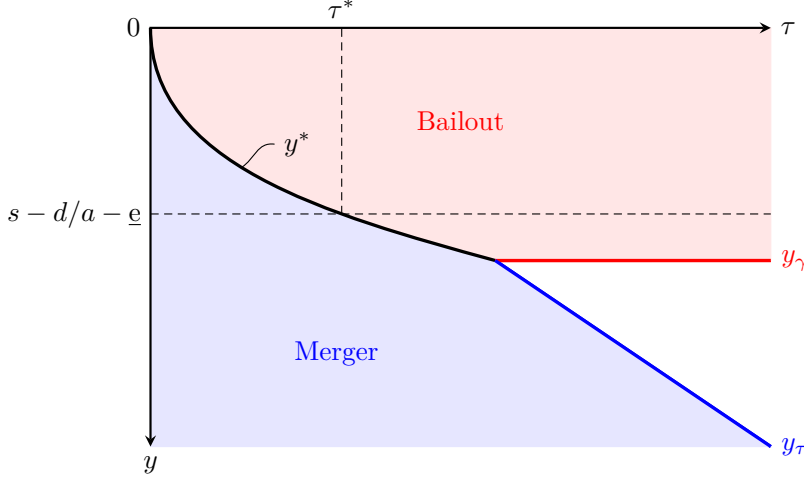We can now turn to ex-ante incentives. If $\tau > \tau^*$ and mergers are not used, then

Figure 2: Optimal ex-post policy as a function of the cost of mergers τ.

the equilibrium payoff of bank 1 under safety x is

$$p_0 f(x) + (1 - p_0) \left[h \left(x + s - \frac{d}{a} \right) + (1 - h) \underline{e} \right]$$

and the only equilibrium features $x_1 = x_2 = \hat{x}$ as in our general Proposition 3. Mergers are too costly and only full bailouts are credible, so we are back into the full moral hazard case with maximal risk-taking. Denote

$$\Delta_0 = p_0 \left[f(0) - f(x^*) \right] - (1 - p_0) \left[h \left(x^* + s - \frac{d}{a} \right) - \underline{e} \right] > 0$$

the difference between the payoff under $x = 0$ and the payoff under first-best safety x^*.

Contrast this with the case with mergers. If $\tau \le \tau^*$ then the equilibrium payoff under the first-best safety is

$$p_0 f(x^*) + (1 - p_0) \left[h^2 \left(x^* + s - \frac{d}{a} \right) + h(1 - h) \left(x^* + 2 \left(s - \frac{d}{a} \right) \right) + h(1 - h) \cdot 0 + (1 - h)^2 \underline{e} \right]$$

The first term in the bracket denotes the expected payoff if both banks succeed, and thus there is no government intervention. The second term denotes the expected payoff if bank 1 succeeds but bank 2 does not, hence bank 1 receives a surplus $s - \frac{d}{a}$ from the merger. The third term denotes the converse case in which the unsuccessful bank 1 is merged to the better bank 2. The last term captures the case in which both banks get fully bailed out.

33

If bank 1 instead chooses $x_1 = 0$ its expected payoffs are

$$p_0 f(0) + (1 - p_0)(1 - h)\,\underline{e}$$

Bank 1 only obtains a positive payoff $\underline{e}$ when bank 2 also fails hence both are bailed out, which happens with probability $1 - h$.

The incentive compatibility constraint holds if

$$\Delta_0 \le (1 - p_0)\left[\underline{e}(1 - h(1 - h)) + h(1 - h)\left(s - \frac{d}{a}\right)\right] \tag{16}$$

As usual, there exists $\hat{\gamma} > 0$ such that (15) and (16) hold for $\gamma < \hat{\gamma}$. Thus we find a striking discontinuity in the cost of mergers τ, that generalizes our limit aggregation result above:

Proposition 10. *Suppose that $\gamma < \hat{\gamma}$. Then for $\tau > \tau^*$ only the moral hazard safety level $\hat{x}$ (as defined in Proposition 3) is credibly implementable, while for $\tau \le \tau^*$ the first-best safety x^* is credibly implementable. Welfare decreases discontinuously at $\tau = \tau^*$.*

5 Horizontally Differentiated Banks

In Section 4 we show how mergers can be used to optimally combine capital shortfalls and surpluses under the assumption that the same banking activities can be performed under different ownership structures. This may not be a good assumption when banks are geographically specialized and rely on soft information, or when the regulators worry about excessive local concentration in deposit taking as emphasized by Drechsler et al. (2014). Suppose then that banks are imperfectly substitutable and the value function is

$$V\{e_i\} = V(\phi\{e_i\} - \phi\{\underline{e}\})$$

where

$$\phi\{e_i\} = \sum_{i=1}^{N} e_i^{\frac{\eta-1}{\eta}}$$

and $\eta > 1$ is the elasticity of substitution. This value function converges to the one in the pure systemic model (5) as $\eta \to \infty$. It also captures the fact that it becomes more costly to take away the positive equity $e_i = a_i r_i - d_i$ from bank i as it gets smaller.
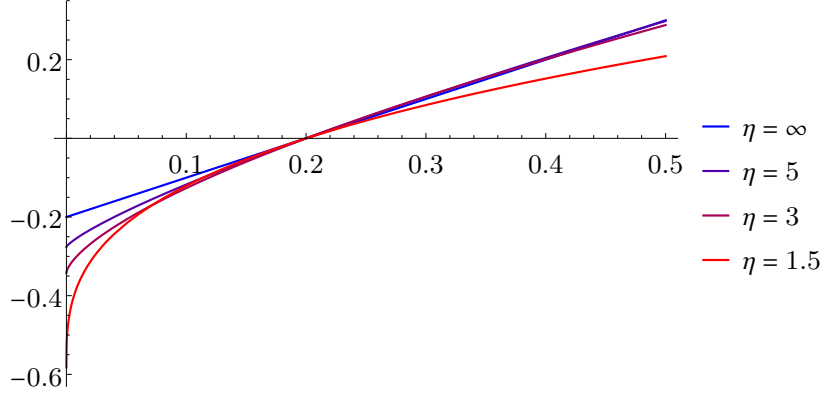
Figure 3: Function $e^{\frac{\eta-1}{\eta}} - \underline{e}^{\frac{\eta-1}{\eta}}$ for different values of η. Lower η makes the function more concave, wich increases incentives to use bailouts to offset large gaps $y = e - \underline{e} < 0$.

In this section we assume differentiability of f and V and a linear cost of funds $\Gamma(M) = \gamma M$ to simplify the exposition. Without commitment, perfect ex post efficiency requires equalizing the marginal returns to transfers m_i across banks i, that is for each i

$$\frac{\eta-1}{\eta} e_i^{\frac{-1}{\eta}} V'\{e_i\} = \gamma.$$

Thus the government will fully insure all banks by setting $e_i = e$ for some e, irrespectively of bank performance and $e > \underline{e}$ solves

$$\frac{\eta-1}{\eta} e^{\frac{-1}{\eta}} V'\{e\} = \gamma. \tag{17}$$

At first glance, it seems that imperfect substitutability brings back the extreme form of moral hazard that arose under individual contracts. Each bank knows that it will be perfectly insured by the government since other banks will not be able to step in and replace it in case of resolution. In particular, our previous tournament contracts are not credible in this context. This extreme result comes from the extreme assumption that the government does not want to deviate at all from the ex post optimum. Indeed, if banks are almost perfectly substitutable ($\eta \to \infty$), imperfect insurance should have negligible costs and the model's conclusions should approach those of the pure systemic risk model.

We now relax the assumption of complete lack of commitment in two ways; both allow to re-establish our main result. In the first relaxation, we introduce a small amount of commitment, by giving the planner the ability to deviate slightly from the

ex post optimum, by an amount at most $\epsilon > 0$. We call this notion ϵ-commitment. In the second, and independent, relaxation, we consider a less stringent notion of time-consistency: the government can only deviate from promises to achieve a Pareto-improvement relative to the ex ante contract. In other words, we consider renegotiation-proof contracts in the language of Fudenberg and Tirole (1990). This solution concept provides a weak form of commitment consistent with the political economy of bailouts.

5.1 ϵ-Commitment

Consider a mechanism that gives $m_i = e + d - r_i + \delta (r_i - \bar{r})$ to each bank, where e solves (17), so that $\sum_i (m_i + r_i) = M + R = Ne + D$. We will find a δ that is high enough to give incentives ex ante, while remaining low enough that the loss in ex post efficiency remains below some threshold ϵ. To second order,

$$\sum e_i^{\frac{\eta-1}{\eta}} = Ne^{\frac{\eta-1}{\eta}} \left(1 - \frac{\eta - 1}{\eta} \times \frac{1}{2\eta} \left(\frac{\delta}{e} \right)^2 \sigma_r^2 \right)$$

where σ_r is the standard deviation of returns. Setting $\delta > 0$ generates an additional loss relative to the ex post efficient allocation $V\{e_i\} - V\{e\}$, which to second-order writes

$$\Delta V = V'\{e\} \left[Ne^{\frac{\eta-1}{\eta}} - \sum e_i^{\frac{\eta-1}{\eta}} \right] = \frac{N}{2e\eta} \sigma_r^2 \gamma \delta^2.$$

Therefore ex post ϵ-efficiency allows to set any slope δ such that $\Delta V \leq \epsilon$ or

$$\delta \leq \bar{\delta} = \sqrt{\frac{2e}{N\gamma\sigma_r^2} \eta \epsilon}$$

We know that $\delta = \frac{N}{N-1} (1 + \gamma)$ can achieve the first-best hence a sufficient condition to implement the first-best is

$$\eta \epsilon > N \left(\frac{N}{N-1} \right)^2 \frac{(1 + \gamma)^2 \gamma \sigma_r^2}{2e}$$

For any non-zero ϵ, the first-best is ϵ-credibly implementable if there is enough substitutability η between banks. This shows how knife-edge the case of complete lack of commitment $\epsilon = 0$ is.

The right-hand side is increasing in γ and the variance of realized returns. More fiscal space (lower γ) leads to larger bailouts and thus lower welfare losses from ex-post equity dispersion, as banks are dispersed around a level closer to the unconstrained optimum that achieves $V'\{e\} = 0$. A contract with non-zero slope δ amplifies return differences arising from luck (in equilibrium), hence a lower variance of idiosyncratic risk makes stronger incentives δ less costly. Finally, the number of banks N only plays a role because we impose the ϵ bound on the total welfare loss V, and a larger numbers of banks N increases any welfare loss mechanically: if ϵ-efficiency applied to welfare per bank (i.e., $\Delta V \leq N\epsilon$) then $\bar{\delta}$ would be given by the same formula with $N = 1$.

5.2 Renegotiation-proof contracts

To simplify, consider the case of two banks. ex post, we assume the government can choose its preferred allocation subject to the constraint that each bank must be weakly better off than under the contractual allocation (C_1, C_2). Suppose without loss that ex post $r_1 > r_2$, then the government solves:

$$\max_{m_1, m_2} \quad r_1 + r_2 + V\left(\phi\{e_i\} - \phi\{\underline{e}\}\right) - \gamma M$$

$$\text{s.t. } e_1 = r_1 + m_1 - d_1 \geq \bar{e}_1 \qquad\qquad (\zeta_1)$$

$$e_2 = r_2 + m_2 - d_2 \geq \bar{e}_2 \qquad\qquad (\zeta_2)$$

Proposition 11. *There exists $\hat{\gamma}$ such that for $\gamma < \hat{\gamma}$ the tournament contract $(\bar{e}_1, \bar{e}_2)$ where $\bar{e}_1$ is the unique solution to*

$$\frac{\partial\phi}{\partial e_2}\left(\bar{e}_1, \bar{e}_1 - \frac{1+\gamma}{\alpha_1}\right) \times V'\left(\phi\left(\bar{e}_1, \bar{e}_1 - \frac{1+\gamma}{\alpha_1}\right) - \phi(\underline{e})\right) = \gamma \qquad (18)$$

and $\bar{e}_2 = \bar{e}_1 - \frac{1+\gamma}{\alpha_1}$ is renegotiation-proof and implements the first-best effort x^*.

In the limit perfect substitution $\eta \to \infty$, the renegotiation-proof tournament converges to our previous credible tournament. The renegotiation-proof "winner" payoff $\bar{e}_1$ (and therefore the payoff for the "loser" $\bar{e}_2 = \bar{e}_1 - \frac{1+\gamma}{\alpha_1}$) increases as η decreases. The reason is that when banks become less substitutable, it becomes less credible to punish the loser bank harshly. Ex post, the marginal benefit of bailing out the loser bank is higher when customers cannot easily switch to the winner bank. Thus incentives must be provided through a better "carrot" for the winner bank. Since the
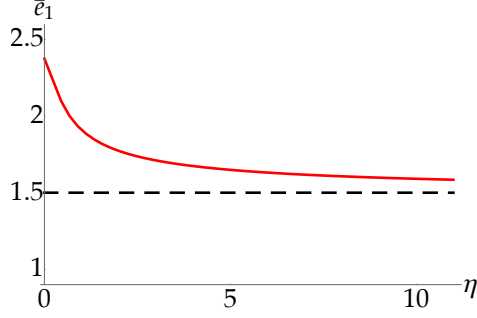
Figure 4: Renegotiation-proof prize $\bar{e}_1$ for the winner bank as a function of the elasticity of substitution η. Dashed line: $\bar{e}_1$ with perfectly substitutable banks. Parameters: $V(x) = -\frac{x^2}{2}$, $\gamma = 0.5$.

incentive condition pins down the difference in payoff between the two banks, the loser bank also ends up with a larger bailout. The expected cost of ex post interventions $\mathbf{E}[m_1 + m_2] = 2\bar{e}_1 - \frac{1+\gamma}{\alpha_1} - \mathbf{E}[r_1 + r_2]$ is thus higher when banks are less substitutable. The first-best expected cost of bailouts (assuming banks all chose x^*) would be instead $M_0 - \mathbf{E}[r_1 + r_2]$.

6 Financial Contagion

In this section we consider a different form of heterogeneity, arising from financial linkages between banks that generate comovement in returns. These linkages capture a variety of "contagion" forces, such as cross-exposures, fire sales, or domino effects, as studied in the financial networks literature (e.g., Caballero and Simsek 2013, Elliott et al. 2014, Acemoglu et al. 2015). The resulting return structure is significantly more complex than the one we have worked with so far: banks now have heterogeneous loadings on the aggregate risk factor η, and each bank is exposed to many other banks' idiosyncratic structural shocks ϵ_j.

We show how contagion leads to a natural notion of systemic risk: banks are more systemic when their performance has a stronger effect on the rest of the system. In turn, more systemic banks must act more prudently, and so a resolution mechanism must strive to give them stronger incentives. ex post, however, the government may consider these "super-spreader" banks too interconnected to fail (Haldane, 2013). Our main finding is that the constraints that financial linkages impose on bank resolution

depend crucially on how bailout funds attributed to one bank spill over to other banks.

If a form of "ring-fencing" applies to public funds and bailout money cannot flow throughout the system to benefit other banks indirectly, our tournament mechanism remains credible and efficient under minor amendments. A bank's rank in the tournament is determined by its ex post performance, as in the baseline model, but now weighted by its systemic risk.

A subtle constraint appears if ring-fencing is not possible, and bailout money can instead spillover to other banks. A first intuition would be that these spillover effects can reduce costs ex post, as it is now possible to rescue some banks indirectly, working through the linkages. The countervailing and dominating force, however, is that spillovers actually worsen the credibility problem. It becomes optimal to target the most systemic bank, as this is a cheap way to save the whole system. But this makes the moral hazard problem unsolvable, because the most systemic bank will now be completely insured and thus maximize risk-taking, thereby endangering the whole system.[4]

6.1   Restricted Bailouts

In this section, we focus on interconnectedness and simplify the other dimensions of the model, by assuming that all states s are systemic, and that f is differentiable. Suppose that conditional on a crisis, each bank i's return becomes a function of other banks j's returns through a linear relation

$$r_i = x_i + s + \epsilon_i + \sum_{j \neq i} \omega_{ij} r_j$$

We assume here that the interconnection between banks is based on pre-bailout returns r: at the ex post stage, bailouts do not spillover to other banks. The next subsection will consider the case in which bailout funds m_i cannot be "targeted" to bank i's shareholders, but also spill over to other banks j. In vector form, $\boldsymbol{r} = \boldsymbol{x} + \boldsymbol{s} + \boldsymbol{\epsilon} + \Omega \boldsymbol{r}$ with $\boldsymbol{\Omega} = \{\omega_{ij}\}$ where by convention $\omega_{ii} = 0$, which leads to

$$\boldsymbol{r} = \Lambda\left(\boldsymbol{x} + \boldsymbol{s} + \boldsymbol{\epsilon}\right) \tag{19}$$

---

[4]In the knife-edge case in which multiple banks are equally systemic, we can still use a tournament within them and thus restore incentives.

where $\boldsymbol{\Lambda} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$. Call Λ_{ij} the elements of Λ. The crisis value function in a contagion state becomes

$$V \left(\sum_i \lambda_i \left(x_i + s + \epsilon_i \right) + \sum_i m_i \right)$$

where $\lambda_i = \sum_j \Lambda_{ji}$ captures the *systemic risk* of bank i, that is how much other banks load on bank i's return, and thus how much bank i's return can affect the aggregate banking sector's shortfall through this form of financial contagion. Banks with higher weights λ_i are banks who have a high "network centrality": their returns have a relatively large impact on aggregate bank capital. The ex post optimality constraint remains unchanged: the total bailout has to satisfy

$$\sum_i m_i = M_0 - \sum_i r_i.$$

The only difference in the first-best allocation is that ex ante, more systemic banks should invest more in safety. In the differentiable f case, the first-best vector $\boldsymbol{x}^*$ now solves

$$f'(x_i^*) = - \left(\frac{1 - p_0}{p_0} \right) \lambda_i (1 + \gamma)$$

Our baseline symmetric model is nested by setting $\Omega = 0$ hence $\lambda_i = 1$ for all i. With heterogeneity, the first-best requires that higher λ_i banks must invest in higher safety x_i^*.

We show next that only slight modifications to our tournament mechanism are enough to accommodate in the presence of this fairly general form our financial contagion. Intuitively, under heterogeneous systemic risk, the ex post bailout distribution must incentivize more systemic banks to hedge more. This is achieved by promising such banks higher prizes upon winning the tournament, or raising the effect of safety on their probability of "wining the tournament". An asymmetric tournament contract can implement the first-best, by simply ranking banks ex post according to their systemic-weighted performance $\tilde{\lambda}_i r_i$ instead of their raw return r_i. For simplicity, consider the case of two banks:

Proposition 12. *Suppose $N = 2$. Then the following contract implements the first-best (x_1^*, x_2^*) credibly:*

- *If $\tilde{\lambda}_1 r_1 > \tilde{\lambda}_2 r_2$ then bank 1 obtains $m_1 = m_0 + \frac{1+\gamma}{2h} - r_1$ and bank 2 obtains*

$$m_2 = m_0 - \frac{1+\gamma}{2h} - r_2;$$

- If $\tilde{\lambda}_2 r_2 > \tilde{\lambda}_1 r_1$ then bank 1 obtains $m_1 = m_0 - \frac{1+\gamma}{2h} - r_1$ and bank 2 obtains $m_2 = m_0 + \frac{1+\gamma}{2h} - r_2$.

where $h = H'(\lambda_1 x_1^* - \lambda_2 x_2^*)$, H is the c.d.f. of $(\lambda_2 - \lambda_1)\eta + \lambda_2 \epsilon_2 - \lambda_1 \epsilon_1$, and

$$\tilde{\lambda}_1 = \lambda_1 + \Lambda_{21} + \det \Lambda - 1$$
$$\tilde{\lambda}_2 = \lambda_2 + \Lambda_{12} + \det \Lambda - 1$$

Proof. In the Appendix. □

Suppose, for instance, that bank 1 is systemic so $\omega_{21} = \omega \neq 0$ but bank 2 is not, $\omega_{12} = 0$. Then

$$\Lambda = \begin{pmatrix} 1 & 0 \\ w & 1 \end{pmatrix}$$

and

$$\lambda_1 = 1 + w, \quad \lambda_2 = 1$$
$$\tilde{\lambda}_1 = 1 + 2w, \quad \tilde{\lambda}_2 = 1$$

6.2 Contagious Bailouts

Finally, we consider the form of financial contagion that is hardest to overcome credibly. The regulator observes returns $\tilde{r}_i$ such that $\tilde{r} = \Lambda(x + s + \epsilon)$ as in the previous section, before deciding on a bailout policy. Suppose that bailout money m itself is also "contagious": it is now each bank j's post-bailout equity $r_j + m_j$, and not just r_j as in the previous section, that affects the value of other banks' assets r_i:

$$r_i = x_i + \sum_{j \neq i} \omega_{ij}(r_j + m_j) + s + \epsilon_i \tag{20}$$

Adding m_i on each side, we obtain in vector form $\boldsymbol{r} + \boldsymbol{m} = \boldsymbol{x} + \boldsymbol{m} + \Omega\left(\boldsymbol{r} + \boldsymbol{m}\right) + \boldsymbol{s} + \boldsymbol{\epsilon}$ which leads to

$$\boldsymbol{r} + \boldsymbol{m} = \Lambda\left(\boldsymbol{x} + \boldsymbol{m} + \boldsymbol{s} + \boldsymbol{\epsilon}\right)$$
$$= \tilde{\boldsymbol{r}} + \Lambda\boldsymbol{m}$$

The seemingly small difference relative to (19) turns out to be crucial in terms of policy implications. There is now an additional ex post asymmetry between banks: in the first-best allocation, not only should more systemic banks (those with a higher λ_i) invest more in liquidity x ex ante; but as we will show, it is also efficient to focus the ex post government intervention m on the most systemic bank. In the crisis state, the value function now writes

$$V\left(\sum_j \tilde{r}_j + \sum_i \lambda_i m_i\right)$$

The first-best vector of safety $\boldsymbol{x}^*$ is the same as in the previous section. ex post, however, since the shadow cost of public funds γ is the same for all banks i, a larger "bang for the buck" is obtained in terms of stabilizing the financial sector when the marginal dollar of public funds is allocated to the most systemic bank. Suppose that banks are strictly ranked according to their systemic risk, with bank 1 being the unique most systemic bank:

$$\lambda_1 > \lambda_2 \geq \cdots \geq \lambda_n$$

and banks cannot be taxed to fund other banks, so that $m_i \geq 0$ (otherwise the result would be strengthened further, as the planner would then redistribute from banks $i \geq 2$ to bank 1).

Lemma 7. *For any given realization of pre-bailout returns $\tilde{\boldsymbol{r}}$, the ex post optimal bailout policy is to give all the bailout $\mathcal{M}$ to bank 1:*

$$m_1 = \mathcal{M}$$
$$m_i = 0 \quad \forall i \geq 2$$

The total bailout is

$$\mathcal{M} = \frac{M_0}{\lambda_1} - \sum_{i=1}^N \tilde{r}_i$$

42

and decreases with λ_1.

For a given realization of returns, the loss is decreasing in λ_1: ex post, it is cheaper to inject funds through the most systemic bank, and the more systemic it is (the higher λ_1), the cheaper the total cost of intervening. However, this will backfire ex ante, as it becomes impossible to credibly punish bank 1 and reward other banks.

Proposition 13. *For any credible mechanism, the equilibrium features maximal risk-taking by bank 1, and the autarky risk-taking by other banks:*

$$x_1 = 0$$
$$x_i = \tilde{x}_i \quad \forall i \geq 2$$

and the equilibrium bailout is

$$\mathcal{M} = \frac{M_0}{\lambda_1} - \sum_{i=2}^{N} \lambda_i \tilde{x}_i - \sum_{i=1}^{N} \lambda_i (s + \epsilon_i)$$

The additional cost relative to the first-best is $\lambda_1 x_1^* + \sum_{i=2}^{N} \lambda_i (x_i^* - \tilde{x})$ which is increasing in λ_1.

The optimal bailout goes entirely to bank 1 and offsets one for one the idiosyncratic shock ϵ_1; but it also depends on the realization of all the idiosyncratic shocks $\{\epsilon_j\}_{j>1}$. When other banks do poorly, even if bank 1 has not suffered a negative idiosyncratic shock, the government still wants to inject equity into the system. It doesn't target directly the unlucky banks because it is cheaper to inject the money through the systemic bank 1.

The takeaway from this section is that financial contagion undermines credibility if and only if bailout funds can flow through the system and affect the performance of many banks besides the bank they are supposed to target. It is thus desirable to enforce a form of ring-fencing, where bailout money can be used to rescue specific institutions (in an asymmetric way, to provide incentives), but with some conditionality regarding its use. For instance, bailout funds should not be used primarily to repay debt to other banks (and it is always possible to bail out these downstream banks directly instead). Note that the economic force behind this finding is not the extent of moral hazard for the downstream banks, whose health is affected by systemic banks; it is instead that

43

heterogeneity in systemic risk undermines commitment power, as it is not credible not to bail out the most systemic institutions even when they perform poorly.

References

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015), "Systemic Risk and Stability in Financial Networks", *American Economic Review*, Vol. 105, pp. 564–608.

Acharya, Viral, Itamar Drechsler, and Philipp Schnabl (2014), "A Pyrrhic Victory? - Bank Bailouts and Sovereign Credit Risk", *Journal of Finance*, Vol. 69, pp. 2689–2739.

Acharya, Viral V, Lasse Heje Pedersen, Thomas Philippon, and Matthew Richardson (2016), "Measuring Systemic Risk", *The Review of Financial Studies*.

Acharya, Viral V. and Tanju Yorulmazer (2007a), "Too many to fail: An analysis of time-inconsistency in bank closure policies", *Journal of Financial Intermediation*, Vol. 16, pp. 1–31.

———— (2007b), "Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures", *The Review of Financial Studies*, Vol. 21, pp. 2705–2742.

Caballero, Ricardo J. and Alp Simsek (2013), "Fire Sales in a Model of Complexity", *The Journal of Finance*, Vol. 68, pp. 2549–2587.

Chari, V. V. and Patrick J. Kehoe (2016), "Bailouts, Time Inconsistency and Optimal Regulation: A Macroeconomic View", *American Economic Review*, Vol. 106, pp. 2458–2493.

Dávila, Eduardo and Ansgar Walther (2020), "Does size matter? Bailouts with large and small banks", *Journal of Financial Economics*, Vol. 136, pp. 1–22.

Diamond, Douglas W. and Philip H. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy*, Vol. 91, pp. 401–419.

Drechsler, Itamar, Philipp Schnabl, and Alexi Savov (2014), "The Deposits Channel of Monetary Policy The Deposits Channel of Monetary Policy", NYU.

Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson (2014), "Financial Networks and Contagion", *American Economic Review*, Vol. 104, pp. 3115–53.

Farhi, Emmanuel and Jean Tirole (2012), "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts", *American Economic Review*, Vol. 102, pp. 60–93.

Financial Stability Board (2021), "Evaluation of the effects of too-big-to-fail reforms".

Fudenberg, Drew and Jean Tirole (1990), "Moral Hazard and Renegotiation in Agency Contracts", *Econometrica*, Vol. 58, pp. 1279–1319.

Haldane, Andrew G (2013), *Rethinking the financial network*, pp. 243–278, Wiesbaden: Springer Fachmedien Wiesbaden.

Innes, Robert (1990), "Limited Liability and incentive contracting with ex ante action choices", *Journal of Economic Theory*, Vol. 52, pp. 45–67.

Kasa, Kenneth and Mark M. Spiegel (2008), "The Role of Relative Performance in Bank Closure Decisions", *FRBSF Economic Review*.

Keister, Todd (2016), "Bailouts and Financial Fragility", *Review of Economic Studies*, Vol. 83, pp. 704–736.

Lazear, Edward P. and Sherwin Rosen (1981), "Rank-Order Tournaments as Optimum Labor Contracts", *Journal of Political Economy*, Vol. 89, pp. 841–864.

Milgrom, Paul and Christina Shannon (1994), "Monotone Comparative Statics", *Econometrica*, Vol. 62, pp. 157–180.

Nachman, David and Thomas Noe (1994), "Optimal Design of Securities under Asymmetric Information", *Review of Financial Studies*, Vol. 7, pp. 1–44.

Perotti, Enrico C and Javier Suarez (2002), "Last bank standing: What do I gain if you fail?", *European Economic Review*, Vol. 46, pp. 1599–1622, Symposium on Finance.

Philippon, Thomas and Philipp Schnabl (2013), "Efficient Recapitalization", *Journal of Finance*, Vol. 68, pp. 1–42.

Philippon, Thomas and Vasiliki Skreta (2012), "Optimal Interventions in Markets with Adverse Selection", *American Economic Review*.

Topkis, Donald M. (1978), "Minimizing a Submodular Function on a Lattice", *Operations Research*, Vol. 26, pp. 305–321, URL: http://www.jstor.org/stable/169636.

Appendix

Heterogeneous Bank Size. In the general case with different bank sizes a_i, bank i chooses its safety investment to solve:

$$\hat{x}_i = \arg\max_{x_i \geq 0} p_0 f(x_i) + (1 - p_0) \left(\mathbb{E}\left[r_{i,s} \mid x_i \right] + \mathbb{E}\left[\frac{m_{i,s}(\mathbf{r})}{a_i} \mid \mathbf{x} \right] \right). \tag{21}$$

Suppose as in the first case of Lemma 3 that the first-best safety x^* does not depend on size. Importantly, due to the credibility constraint the reward Δ in the bonus-malus tournament cannot depend on size either: the gain of one bank is the loss of another. But if the tournament rule only compares raw returns to determine who wins and who loses, larger banks will in general choose a lower level of safety than smaller banks, because the potential prize Δ is smaller as a fraction of their assets.

We can solve this issue by considering the following handicapped tournament

$$m_i = \begin{cases} \frac{a_i}{A} \mathcal{M}(K - R) + \Delta & \lambda_i r_{i,s} > \lambda_j r_{j,s} \\ \frac{a_i}{A} \mathcal{M}(K - R) - \Delta & \lambda_i r_{i,s} < \lambda_j r_{j,s} \end{cases} \tag{22}$$

that compares weighted returns $\lambda_i r_i$ instead of raw returns to determine the bailout allocation. Given $\lambda = \frac{\lambda_1}{\lambda_2}$ the best response function for bank 1 is

$$\hat{x}_1 = \beta_1(\Delta, \lambda, x_2) = \arg\max_{x_1} p_0 f(x_1) + (1 - p_0)\left(\mathbb{E}\left[r_{1,s} \mid x_1 \right] + \Omega(x_1, x_2) \right) + 2\frac{\Delta}{a_1} \int_s \mathbf{P}\left[\lambda r_{1,s} > r_{2,s} | \mathbf{x} \right] p_s ds,$$

while the best response function for bank 2 is

$$\hat{x}_2 = \beta_2(\Delta, \lambda, x_1) = \arg\max_{x_2} p_0 f(x_2) + (1 - p_0)\left(\mathbb{E}\left[r_{2,s} \mid x_2 \right] + \Omega(x_1, x_2) \right) - 2\frac{\Delta}{a_2} \int_s \mathbf{P}\left[\lambda r_{1,s} > r_{2,s} | \mathbf{x} \right] p_s ds.$$

We thus look for a pair Δ, λ that implements the first-best:

$$x^* = \beta_1(\Delta, \lambda, x^*)$$
$$x^* = \beta_2(\Delta, \lambda, x^*)$$

To characterize when this is possible, we use a more specific example of returns:

$$r_i = x_i + s + \epsilon_i. \tag{23}$$

Then

$$\mathbf{P}\left[\lambda x_1 - x_2 > (1 - \lambda) s + \epsilon_2 - \lambda \epsilon_1\right] = H_s\left(\lambda x_1 - x_2; \lambda\right)$$

where $H_s\left(\cdot; \lambda\right)$ is the c.d.f. of $(1 - \lambda) s + \epsilon_2 - \lambda \epsilon_1$. The marginal incentives from the tournament for banks 1 and 2 are respectively

$$\frac{\partial}{\partial x_1}\left(2\frac{\Delta}{a_1}\int_s H_s\left(x_1, x_2; \lambda\right) p_s ds\right) = 2\Delta \frac{\lambda}{a_1}\int_s H_s'\left(\lambda x_1 - x_2; \lambda\right) p_s ds$$

$$\frac{\partial}{\partial x_2}\left(-2\frac{\Delta}{a_2}\int_s H_s\left(x_1, x_2; \lambda\right) p_s ds\right) = 2\frac{\Delta}{a_2}\int_s H_s'\left(\lambda x_1 - x_2; \lambda\right) p_s ds.$$

so as long as $\int_s H_s'\left(\lambda x_1 - x_2; \lambda\right) p_s ds > 0$ the two banks to choose the same x^* if $\lambda = \frac{a_1}{a_2}$.

Note that the condition $\int_s H_s'\left(\lambda x_1 - x_2; \lambda\right) p_s ds > 0$ imposes an upper bound on the relative size of the two banks. If a_1/a_2 is too large, then no λ can generate first-best incentives for the larger bank and we are back to the moral hazard unavoidable in a one-bank world, as shown by the following example:

Proposition 14. *Suppose that $N = 2$, $a_1 \geq a_2$, and returns follow (23) with ϵ_i distributed over $[0, \bar{\epsilon}]$. Then as long as*

$$\frac{a_1}{a_2} < 1 + \frac{\bar{\epsilon}}{x^* + \inf s}$$

the handicapped tournament (22) with $\lambda = \frac{a_1}{a_2}$ can implement the first best safety.

Proof. In the uniform case we can compute

$$H_s\left(\lambda x_1 - x_2; \lambda\right) = \int_0^{\bar{\epsilon}} G_\epsilon\left(\lambda \epsilon_1 + \lambda x_1 - x_2 - (1 - \lambda) s\right) d\epsilon_1$$

$$H_s'\left(\lambda x_1 - x_2; \lambda\right) = \int_0^{\bar{\epsilon}} g_\epsilon\left(\lambda \epsilon_1 + \lambda x_1 - x_2 - (1 - \lambda) s\right) d\epsilon_1$$

where G_ϵ and g_ϵ are the c.d.f. and p.d.f. of ϵ_1, respectively. Then for $x_1 = x_2 = x^*$

$$\lambda \epsilon_1 + \lambda x_1 - x_2 - (1 - \lambda) s \leq \bar{\epsilon} \Leftrightarrow \epsilon_1 \leq \frac{\bar{\epsilon} - (\lambda - 1)(x^* + s)}{\lambda}$$

Therefore

$$\int_s H_s'\left(\lambda x_1 - x_2; \lambda\right) p_s ds = \int_s \left(\int_0^{\bar{\epsilon}} g_\epsilon\left(\lambda \epsilon_1 + \lambda x_1 - x_2 - (1 - \lambda) s\right) d\epsilon_1\right) p_s ds$$

49

is positive if $\lambda < 1 + \frac{\bar{\epsilon}}{x^* + \inf s}$. $\qquad\square$

Proof of Proposition 11. We guess and verify that the ex post symmetric allocation $r_1 + m_1 = r_2 + m_2 = \frac{a}{2}$ is not renegotiation-proof, that is $\frac{a}{2} < C_1$.

Then it must be that the constraint $r_1 + m_1 \geq C_1$ binds, hence bank 1 gets C_1 and bank 2 gets y_2 such that

$$\frac{\partial \mathcal{Y}}{\partial y_2}(C_1, y_2) \times V'(\mathcal{Y}(C_1, y_2)) = \gamma$$

From the renegotiation-proofness principle, we can restrict attention to contracts with $C_2 = y_2$. As before, the first-best is implementable if C_1, C_2 satisfy (**??**):

$$\alpha_1 C_1 + \alpha_2 C_2 = 1 + \gamma$$

with $\alpha_2 = -\alpha_1$ hence $C_2 = C_1 - \frac{1+\gamma}{\alpha_1}$. We then look for a solution C_1 to

$$V'\left(\mathcal{Y}\left(C_1, C_1 - \frac{1+\gamma}{\alpha_1}\right)\right) = \frac{\gamma}{\frac{\partial \mathcal{Y}}{\partial y_2}\left(C_1, C_1 - \frac{1+\gamma}{\alpha_1}\right)}$$

As C_1 increases from 0 to ∞, the left-hand side decreases from $\lim_{y_2 \to 0} V'\left(\mathcal{Y}\left(\frac{1+\gamma}{\alpha_1}, y_2\right)\right)$ to 0 and the right-hand side increases from $\lim_{y_2 \to 0} \frac{\gamma}{\frac{\partial \mathcal{Y}}{\partial y_2}\left(\frac{1+\gamma}{\alpha_1}, y_2\right)}$ to γ. $\qquad\square$

Proof of Proposition 12. Suppose that bank i gets $m_i = \frac{M_0}{2} + \Delta - r_i$ and bank $j \neq i$ gets $m_j = \frac{M_0}{2} - \Delta - r_j$ if and only if $\tilde{\lambda}_i r_i > \tilde{\lambda}_j r_j$ where

$$\tilde{\lambda}_i = \lambda_i + \Lambda_{ji} + \det \Lambda - 1$$

Then $\tilde{\lambda}_1, \tilde{\lambda}_2$ solve the system

$$\tilde{\lambda}_1 \Lambda_{11} - \tilde{\lambda}_2 \Lambda_{21} = \lambda_1$$
$$\tilde{\lambda}_2 \Lambda_{22} - \tilde{\lambda}_1 \Lambda_{12} = \lambda_2$$

Therefore

$$\mathbf{P}\left[\tilde{\lambda}_1 r_1 > \tilde{\lambda}_2 r_2\right] = \mathbf{P}\left[\tilde{\lambda}_1\left[\Lambda_{11}\left(x_1 + s + \epsilon_1\right) + \Lambda_{12}\left(x_2 + s + \epsilon_2\right)\right] > \tilde{\lambda}_2\left[\Lambda_{22}\left(x_2 + s + \epsilon_2\right) + \Lambda_{21}\left(x_1 + s + \epsilon_1\right)\right]\right]$$

$$= \mathbf{P}\left[\left(\tilde{\lambda}_1\Lambda_{11} - \tilde{\lambda}_2\Lambda_{21}\right)\left(x_1 + s + \epsilon_1\right) > \left(\tilde{\lambda}_2\Lambda_{22} - \tilde{\lambda}_1\Lambda_{12}\right)\left(x_2 + s + \epsilon_2\right)\right]$$

$$= \mathbf{P}\left[\lambda_1\left(x_1 + s + \epsilon_1\right) > \lambda_2\left(x_2 + s + \epsilon_2\right)\right]$$

$$= \mathbf{P}\left[\lambda_1 x_1 - \lambda_2 x_2 > z\right]$$

where $z = \left(\lambda_2 - \lambda_1\right)s + \lambda_2\epsilon_2 - \lambda_1\epsilon_1$ has a conditional c.d.f. H. Therefore bank 1's optimal effort x_1 solves

$$\max_{x_1} \quad p_0 f\left(x_1\right) + \left(1 - p_0\right)\left\{H\left(\lambda_1 x_1 - \lambda_2 x_2\right) 2\Delta\right\}$$

leading to the first-order condition

$$f'\left(x_1\right) = \frac{-\left(1 - p_0\right)}{p_0}\lambda_1 H'\left(\lambda_1 x_1 - \lambda_2 x_2\right) 2\Delta.$$

Similarly, bank 2's optimal effort x_2 solves

$$\max_{x_2} \quad p_0 f\left(x_2\right) + \left(1 - p_0\right)\left[1 - H\left(\lambda_1 x_1 - \lambda_2 x_2\right)\right] 2\Delta$$

hence

$$f'\left(x_2\right) = \frac{-\left(1 - p_0\right)}{p_0}\lambda_2 H'\left(\lambda_1 x_1 - \lambda_2 x_2\right) 2\Delta.$$

Therefore, to implement effort levels $\left(x_1^*, x_2^*\right)$ that solve $f'\left(x_i^*\right) = \frac{-\left(1 - p_0\right)}{p_0}\lambda_i\left(1 + \gamma\right)$ we need

$$\Delta = \frac{1 + \gamma}{2H'\left(\lambda_1 x_1^* - \lambda_2 x_2^*\right)}$$

$\square$

A Illustrative example

In this section we consider a simple quadratic example that can be solved in closed form. There are two banks, with sizes $a_1 \geq a_2$. The value function is

$$V(R + M) = \min\left\{0, -\frac{v}{\beta}(K - R - M)^\beta\right\}, \quad \beta \geq 1$$

and the cost of funds is linear $\Gamma(M) = \gamma M$. There is only one systemic state, so we omit the s notation. Returns in the systemic state are linear in safety

$$r_i = x_i + \epsilon_i$$

with ϵ_i uniform between 0 and $\bar{\epsilon}$. The normal state return is

$$f(x_i) = -f\frac{x_i^2}{2}$$

Ex-post efficiency. The optimal bailout in the systemic state is

$$\mathcal{M}(K - R) = \max\left\{0, K - R - \left(\frac{\gamma}{v}\right)^{\beta-1}\right\}$$

Hence the optimized value is

$$\begin{aligned}
\mathcal{V}(R) &= V(R + \mathcal{M}) - \gamma\mathcal{M} \\
&= \begin{cases}
-\frac{v}{\beta}\left(\frac{\gamma}{v}\right)^{\beta(\beta-1)} - \gamma\left[K - R - \left(\frac{\gamma}{v}\right)^{\beta-1}\right] & \text{if } R \leq K - \left(\frac{\gamma}{v}\right)^{\beta-1} \\
\min\left\{-\frac{v}{\beta}(K - R)^\beta, 0\right\} & \text{otherwise}
\end{cases}
\end{aligned}$$

We assume that these returns are low enough that a bailout is always needed in the systemic state: $A(\sup x + 1) \leq K - \left(\frac{\gamma}{v}\right)^{\beta-1}$ hence

$$\mathcal{V}(R) = -\frac{v}{\beta}\left(\frac{\gamma}{v}\right)^{\frac{\beta}{\beta-1}} - \gamma\left[K - R - \left(\frac{\gamma}{v}\right)^{\beta-1}\right].$$

First Best. The first best safety x^* is the same for both banks and solves

$$x^* = \arg\max_x p_0 A f(x) + (1 - p_0)(Ax + \mathbb{E}[\mathcal{V}(R)|x])$$

$$= \arg\max_x p_0 A f(x) + A(1 - p_0)(1 + \gamma)x$$

hence

$$p_0 f'(x^*) = -(1 - p_0)(1 + \gamma)$$

$$x^* = \frac{q(1 + \gamma)}{f}$$

where $q = \frac{1-p_0}{p_0}$ is the odds ratio of a crisis. x^* is increasing in q and increasing in γ.

Moral hazard with symmetric bailouts. Suppose bailouts follow

$$m_i = \frac{a_i}{A}\mathcal{M}(K - R).$$

Then bank i solves

$$\hat{x}_i = \arg\max_x p_0 a_i f(x_i) + (1 - p_0)\left(a_i x_i + \frac{a_i}{A}\underbrace{\left[K - a_i x_i - a_j x_j - \left(\frac{\gamma}{v}\right)^{\beta-1}\right]}_{=\mathbb{E}[\mathcal{M}(K-R)|\mathbf{x}]}\right)$$

Thus

$$\hat{x}_i = \frac{q}{f}\left(1 - \frac{a_i}{A}\right) < x_i^*.$$

With symmetric bailouts, both banks take excessive risk, and the moral hazard problem is worse for the larger bank (high a_i/A). This is consistent with Dávila and Walther (2020)'s results on symmetric bailouts with small and large banks.

Tournament with bonus-malus. With symmetric banks $a_i = a$, the credible tournament described in section 3 with

$$\Delta = \frac{1}{2}\frac{a}{\epsilon}\left(\gamma + \frac{1}{2}\right)$$

implements the first best safety.

With asymmetric banks, under the condition

$$\frac{a_1}{a_2} \leq 1 + \frac{\bar{\epsilon}}{x^*} = 1 + \frac{\bar{\epsilon}f}{q(1+\gamma)}$$

then the handicapped tournament with

$$\Delta = \frac{1}{2} \frac{a_1}{\bar{\epsilon} - \left(\frac{a_1}{a_2} - 1\right)x^*} \left(\gamma + \frac{a_1}{A}\right)$$

implements the first best safety. Denoting $\lambda = \frac{a_1}{a_2}$ we can reexpress Δ as

$$\Delta = \frac{1}{2} \frac{\lambda}{1+\lambda} \frac{\left(\gamma + \frac{\lambda}{1+\lambda}\right)}{\bar{\epsilon} - (\lambda - 1)x^*}$$

which is increasing in λ. Thus we see that holding the aggregate size of the banking sector A fixed, more asymmetry requires a larger Δ.

Limited liability. As in the main text we consider a tournament rule that satisfies strong limited liability by transferring the total bailout $\mathcal{M}$ to bank 1 if $\lambda r_1 \geq r_2$ and to bank 2 otherwise. Bank 1 solves

$$\max_{x_1} p_0 a_1 f(x_1) + (1-p_0) \left(a_1 x_1 + \left[K - a_1 x_1 - a_2 x_2 - \left(\frac{\gamma}{v}\right)^{\beta-1}\right] \int_0^{\bar{\epsilon}} G_\epsilon (\lambda \epsilon_1 + \lambda x_1 - x_2) d\epsilon_1\right)$$

where G_ϵ is the c.d.f. of ϵ_1. With $a_1 = a_2 = a$ and $\lambda = 1$ the maximal implementable safety x^{max} satisfies

$$p_0 f'(x^{\mathrm{max}}) + (1-p_0) \left[\frac{1}{2} + \frac{K - A x^{\mathrm{max}} - \left(\frac{\gamma}{v}\right)^{\beta-1}}{a} \min\left\{1, \frac{1}{\bar{\epsilon}}\right\}\right] = 0$$

or

$$x^{\mathrm{max}} = \frac{q}{f + 2q \min\left\{1, \frac{1}{\bar{\epsilon}}\right\}} \left[\frac{1}{2} + \frac{K - \left(\frac{\gamma}{v}\right)^{\beta-1}}{a} \min\left\{1, \frac{1}{\bar{\epsilon}}\right\}\right]$$

which is indeed decreasing in γ (and above x^* for γ low enough) and in bank size a.