Marriage Market and Labor Market Sorting

*Paula Calvo† Ilse Lindenlaub‡ Ana Reynoso§

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Abstract

We build a novel equilibrium model in which households' labor supply choices form the link between sorting on the marriage and sorting on the labor market. We first show that the nature of home production – whether partners' hours are complements or substitutes – shapes marriage market sorting, labor market sorting and labor supply choices in equilibrium. We then explore how sorting patterns in each market amplify or mitigate the gender gap in wages, as well as income inequality between and within households. To this end, we estimate our model on German data, and find that spouses' home hours are *complements*. We investigate to what extent complementarity in home hours drives sorting and inequality. We find that the home production complementarity – by strengthening positive marriage sorting and reducing the gender gap in hours and labor sorting – puts significant downward pressure on the gender wage gap and within household income inequality, but it fuels between household inequality. Our estimated model sheds new light on the sources of inequality in today's Germany, on the evolution of inequality over time, and on spatial inequality differences between East and West Germany.

Keywords. Sorting, Marriage Market, Labor Market, Hours, Household Income Inequality, Gender Wage Gap.

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†Yale University, **Address:** Department of Economics, Yale University, 28 Hillhouse Avenue, New Haven, CT 06520, US. **Email:** `paula.calvo@yale.edu`

‡Yale University, **Address:** Department of Economics, Yale University, 28 Hillhouse Avenue, New Haven, CT 06520, US. **Email:** `ilse.lindenlaub@yale.edu`

§University of Michigan, **Address:** Department of Economics, University of Michigan, 611 Tappan Av., Ann Arbor, MI 48109, US. **Email:** `areynoso@umich.edu`

1 Introduction

Positive assortative matching is a defining feature of both the labor market and the marriage market and has important implications. On the marriage market, the matching of partners with similar education impacts both within and between household income inequality. At the same time, positive sorting in the labor market between workers and firms or jobs reinforces wage inequality across skills. However, even though inequality in economic outcomes results from agents interacting in *both* the marriage and the labor market, the interplay between sorting on both markets has not yet been investigated.

This paper shows that there is a natural link between sorting in both markets and highlights the implications of this interaction for inequality. We build a new equilibrium model with rich heterogeneity and sorting on both markets to explore how sorting patterns in each market contribute to the gender gap in wages, as well as income inequality between and within households. We then investigate the key primitives behind sorting and inequality, where we assess the role of spousal complementarities in home production, women's relative productivity in home production, and women's relative productivity in the labor market. Our estimated model sheds new light on the causes of inequality in today's Germany, on the evolution of inequality over time and inequality differences across space.

We are motivated by three sets of facts, based on the 2010-2016 German Socioeconomic Panel (henceforth, GSOEP), that show a striking relationship between marriage and labor market choices. First, as is well documented in the literature, there is positive sorting in both markets: on spouses' education in the marriage market and between workers' education and jobs' skill requirements in the labor market. Importantly, men are better matched to jobs in the labor market, in the sense that for each given level of education they are matched to more demanding jobs. Second, men and women who are more strongly sorted in the marriage market (i.e. those whose education is more similar to their partner's education) are also more strongly sorted in the labor market (i.e. they tend to have the 'right' education level for the jobs they perform). Third, households' labor supply choices, and particularly the split between hours worked in the labor market vs. hours spent in home production, are an important link between the two markets: We not only show that time allocation choices relate to marriage market sorting (spouses with more similar education have a higher correlation of their hours worked; and similarly for hours in home production). But also labor supply choices have an impact on labor market sorting (controlling for hours worked eliminates most of the gender gap in sorting on the labor market).

We aim to explain these facts with a novel equilibrium model, in which households' endogenous labor supply choices form the link between the marriage and the labor market. The goal of this model is to disentangle – both qualitatively and quantitatively – the role of each market, and the corresponding sorting patterns, in shaping inequality. The model is static, and individuals differ in skills and face three decisions. First, in the marriage market, men and women choose whether and whom to marry. Second, there is a (collective) household problem, where each household formed in the marriage stage decides how much time to allocate to labor market vs. home production, which in turn determines the consumption of a private and public good. Last, in the labor market, individuals match with firms/jobs of different productivity, which determines their wages.

The crucial twist of our model is that for matching on the labor market, not only the individuals' skills matter but also the endogenous amount of hours worked. The optimal job match in the labor market depends on workers' *effective* skills, which are an increasing function of both skills and hours worked. Thus, hours worked increase individuals' productivity.[1] Since the time allocation choice of a couple depends on the characteristics of both partners (and thus on marriage sorting) and at the same time impacts the allocation of individuals to jobs (and thus labor market sorting), marriage market sorting affects labor market sorting. At the same time, when making their marital and household labor supply choices, individuals internalize that an increase in labor hours improves job quality and wages. Therefore, labor market outcomes and sorting also affect marriage market sorting. This interrelation between the two markets is the novel feature of our model but also makes the problem much more complex.

We focus on a tractable transferable utility (TU) representation of our model and characterize two benchmark equilibria depending on the model's primitives. Both feature positive assortative matching between workers and jobs in the labor market driven by productive complementarities. However, the two equilibria differ in household and marriage outcomes depending on properties of the home production function. On the one hand, if home production exhibits complementarity in partners' time inputs, a *monotone equilibrium* arises, characterized by positive sorting in the marriage market and labor hours that are increasing in both own and partner's skills. This equilibrium thus reflects a 'progressive' economy with a high frequency of two-earner households and where spouses are similar in terms of skills and their split between work and home production. The complementarity in home hours is therefore a force towards positive marriage sorting as well as balanced labor supply, labor market sorting and pay across gender. This leads to a narrow gender wage gap, low within-household income inequality, but high inequality between households. On the other hand, if partners' time inputs are substitutable in home production, a *non-monotone* equilibrium arises, featuring negative assortative matching in the marriage market and labor hours that are increasing in own but *decreasing* in partner's skill. This equilibrium reflects a 'traditional' economy with a high degree of household specialization and disparity in partners' skills – features that widen the gender wage gap and within-household income inequality but put downward pressure on between-household inequality.

The key insight from our model is that marriage and labor market sorting are linked in an intuitive way by households' labor supply choices. The nature of this link depends on whether spouses' hours in home production are complementary or substitutable, a feature that needs to be investigated empirically.

To do so, we first develop a quantitative version of our model that is amenable to estimation. We do so by imposing minimal changes to our baseline model in order to preserve its parsimony and key

[1]We base this assumption on our own evidence of a positive relationship between hours worked in the labor market and *hourly* wages in the GSOEP, both in Figure 4 and Table 9, column (3); and also on empirical evidence from the literature arguing that more hours worked lead to higher productivity, especially if it is costly to hand off clients, patients or customers to the next worker on the shift, for instance due to increased coordination costs. It has been shown that hours worked are of special importance in occupations in business and finance as well as legal occupations, which have high requirements for particular hours or considerable client contact (e.g. Goldin (2014)).

mechanism. We introduce marriage taste shocks to allow for mismatch in the marriage market; further, we allow for labor supply shocks to generate variation in hours choices even within the same couple-type. Last, we add a random productivity component to workers' skills to account for imperfect matching in the labor market. We show that this model is identified. Importantly, after parameterizing our model, we allow for two asymmetries between men and women that will be disciplined by the data: gender differences in home productivity and in labor market productivity (where the latter could also be interpreted as discrimination). Our objective is to use the estimated model to shed new light on the sources of gender wage and household income inequality in three contexts:

First, we focus on MODERN GERMANY – our benchmark estimation that illustrates the model mechanism and quantifies the role of sorting for inequality. We estimate the model on GSOEP data from 2010 to 2016 combined with data on job characteristics. Our model matches key features of the marriage market equilibrium (such as the degree of marital sorting and the correlation of home hours within couples) and the labor market equilibrium (such as moments of the wage distributions). To further validate the model, we show that it also reproduces the three stylized facts outlined above, even though they are not targeted in the estimation, as well as un-targeted moments of household and gender inequality.

In order to showcase our model's mechanism, we conduct comparative statics of the gender wage gap and within/between household wage inequality with respect to three parameters that significantly impact inequality: (i) women's relative productivity in the labor market, (ii) women's relative productivity in home production, (iii) the complementarity of partners' home production time. Our insights are the following: First, eliminating asymmetries in productivity across gender (whether at home or at work) naturally reduces the gender wage gap. But, interestingly, this is not the only way to reduce gender disparities: an increase in complementarity of partners' home production hours has qualitatively similar effects. Second, a decline in the gender wage gap tends to go hand in hand with a decline in the gender gaps of labor hours and labor market sorting, and with an increase in marriage market sorting. Third, while the effect of these parameter changes on overall household income inequality depends on the specific exercise, in all cases, the gender wage gap moves hand in hand with *within*-household inequality but in opposite direction as *between*-household inequality.

We next investigate the role of labor market and marriage market sorting for inequality more closely. To do so, we run counterfactuals where we impose either random labor market matching or random marriage market matching. We find that both labor market and marriage market sorting fuel overall income inequality and between-household inequality. But sorting on the marriage market tends to have the opposite effect as sorting on the labor market when it comes to *gender* disparities: While the presence of labor market sorting significantly increases the gender wage gap, marriage market sorting pushes it down. Similarly, labor market sorting pushes up within-household inequality while marriage market sorting tends to decrease this type of inequality. Our results show that sorting on both markets has a significant quantitative effect on inequality with labor market sorting cementing the advantage of men in a world where women work less in the labor market and more at home. In turn, marriage market sorting generates more balanced labor market outcomes (hours, sorting, and pay) across gender.

Second, having well understood our model mechanism and role of sorting, we focus on GERMANY OVER TIME. The last 25 years have shown a large decline in gender disparities (gender wage gap and within-household income inequality) and increases in overall household income inequality and its between-component. We ask whether and how our model can rationalize these trends. To this end, we also estimate our model on data from the beginning of the 1990s and focus on West Germany to not confound the impacts of technological change in the labor market and home production with the effects of the German reunification. Comparing the model estimates over time reveals significant changes in home production with today's Germany being characterized by increased male productivity at home and stronger complementarity in spouses' home hours, indicating a switch towards a more 'progressive' economy. Changes in home production technology account for around 22% of the observed decline in the gender gap and nearly for the entire drop in within-household inequality. In contrast, changes in labor market technology fueled inequality across the board, significantly pushing up household income inequality and its between-component, and preventing gender inequality from falling even further.

In the third context, we analyze spatial inequality differences through the lens of our model where we compare EAST AND WEST GERMANY today. Even after almost 30 years of the German reunification, significant differences in inequality, marriage market and labor market outcomes persist between the two regions. Inequality is significantly lower in the East both among households and across gender. We ask whether and how our model can rationalize these differences. Estimating the model on data from East Germany and comparing the outcomes to the West reveals large differences in home production technology (the East has stronger home production complementarities and higher TFP) and in labor market technology (the Eastern technology is less skill-biased and has a lower TFP). Both types of technological differences account for lower inequality in the East, with home production differences playing a particularly important role for gender disparities.

2 The Literature

Our main contribution is to develop a framework featuring both marriage *and* labor equilibrium that allows us to qualitatively and quantitatively understand how endogenous choices in both markets shape gender and household inequality. As such, this paper relates to four strands of literature: the literature on gender gaps in the labor market highlighting the importance of the gap in labor supply; sorting on the marriage market; sorting on the labor market; and the interaction between labor and marriage markets.

GENDER GAPS IN LABOR SUPPLY AND PAY. A growing literature studies the link between the gender gap in labor supply and the gender gap in pay. The standard channel works through earnings, where family and fertility choices have a permanent effect on the gender earnings gap (Dias, Joyce, and Parodi, 2018; Angelov, Johansson, and Lindahl, 2016; Kleven, Landais, and Søgaard, 2019). Because the wage rate is kept fixed in these papers, any gender gap in pay can only be attributed to *earnings* not to *hourly wages* (which is what we focus on). In assuming that hours worked affect workers' productivity in the market, we follow more closely the literature documenting significant labor market returns to hours (Goldin, 2014; Cortés and Pan, 2019; Gicheva, 2013). Other work links gender pay gaps to gender differences in preferences for work flexibility (Mas and Pallais, 2017 and Bertrand, Goldin, and Katz, 2010) and to sorting into occupations that require different hourly inputs (Erosa, Fuster, Kambourov, and Rogerson, 2017). Finally, there is work highlighting the importance of information frictions for gender pay gaps (without considering the marriage market): If employers believe that women have less market attachment relative to men, they get paid less (Albanesi and Olivetti, 2009, Gayle and Golan, 2011).

Our paper builds on this work in that we also propose the gender gap in hours as a key factor behind the gender pay gap. However, in contrast to both the purely empirical and the structural papers we cited, our work takes into account an endogenous marriage market which shapes labor supply choices.

Marriage Market Sorting. A growing literature measures marriage sorting in the data and finds evidence of positive assortative matching in education in the marriage market over the last decades in different contexts and increases in sorting over time (Eika, Mogstad, and Zafar, 2019; Greenwood, Guner, Kocharkov, and Santos, 2016b; Greenwood, Guner, and Vandenbroucke, 2017; Browning, Chiappori, and Weiss, 2011). We confirm these findings on positive marriage sorting on education for Germany.

Another approach has been to study marriage market sorting using structural models. Several papers have investigated how pre-marital investments in education interact with marriage patterns in a static framework (Chiappori, Iyigun, and Weiss, 2009 and Fernández, Guner, and Knowles, 2005) or a life cycle context (Chiappori, Dias, and Meghir, 2018) and how post-marital investments in a partner's career interact with family formation and dissolution (Reynoso, 2019). Further, there is structural work analyzing how exogenous changes in wages, education and family values (Goussé, Jacquemet, and Robin, 2017a), exogenous wage inequality shifts (Goussé, Jacquemet, and Robin, 2017b), the adoption of unilateral divorce (Voena, 2015; Reynoso, 2019), or different tax systems (Gayle and Shephard, 2019) affect household behavior and marriage sorting.[2]

Like in these papers, marriage market sorting is an important margin also in our model. While education is exogenous in our setting, we could think of the choice of how many hours to work as an 'investment' in individuals' effective skills. But this investment happens *post-marriage* market and *pre-labor* market, and therefore is impacted by marriage sorting while impacting labor market sorting, so the timing is different than in existing work. Crucially none of these papers endogenizes the labor market or features labor market sorting, which is the key addition of our work.[3]

Labor Market Sorting. A large body of literature investigates sorting on the labor market, documenting positive assortative matching between workers and firms (e.g. Card, Heining, and Kline,

[2] In a household model with *exogenous* marriage sorting (and exogenous labor market), Lise and Seitz (2011) analyze the effect of an increase in marriage sorting on between and within household consumption inequality.

[3] The exception is Fernández, Guner, and Knowles (2005) who endogenize the wages of their two worker types, low and high skill, but as in the other papers, their model does not feature any labor market sorting.

2013; Hagedorn, Law, and Manovskii, 2017; Bagger and Lentz (2018); Bonhomme, Lamadon, and Manresa, 2019); or workers and jobs (e.g. Lindenlaub, 2017; Lise and Postel-Vinay, Forthcoming; Lindenlaub and Postel-Vinay, 2020) without taking the marriage market into account. In this strand, our paper is perhaps closest to Pilossoph and Wee (2019b) who consider spousal joint search on the labor market to explain the marital premium on the labor market, but taking marriage market sorting as given. Our contribution is to explore how the forces that determine who marries whom and spouses' home production shape labor market sorting, participation and wages.

Interplay between Marriage and Labor Market. Our work is most closely related to a nascent literature investigating the interplay between marriage and labor markets. These papers have focused on the effects of spouses' joint labor search (Pilossoph and Wee, 2019a and Flabbi, Flinn, and Salazar-Saenz, 2020), changes in wage structure and home technology (Greenwood, Guner, Kocharkov, and Santos, 2016b), and changes in the skill premium (Fernández, Guner, and Knowles, 2005) on marital sorting and household inequality keeping the labor market in *partial* equilibrium.

To the best of our knowledge, this is the first paper that features both the marriage market and the labor market in *equilibrium* with market clearing and price determination in both markets. Further, jointly considering marriage and labor market *sorting* is novel and so is our mechanism of how the two markets and sorting margins are linked (i.e. through endogenous labor supply).

3 Descriptive Evidence

3.1 Data

We use two different data sources: the German Socioeconomic Panel (GSOEP) and the Employment Survey of 2012 (to which we refer as the BIBB Survey).

GSOEP: The GSOEP is a household survey conducted by the German Institute of Economic Research (in German: DIW Berlin) starting in 1984. In 1990, it was extended to include states from the former German Democratic Republic. The core study of the GSOEP surveys about 25,000 individuals living in 15,000 households each year. All individuals aged 16 and older respond to the individual questionnaire, while the head of household additionally answers the household questionnaire. This survey is suitable for our project not only because of its longitudinal nature, but also because the collected information is very rich: it includes comprehensive information on demographics (such as marital status, education level, nationality, and family background), labor market variables (including actual and contractual hours worked, wages and occupation), and home production (such as detailed time use variables). Important for our study, it contains the same information for the head of household and his/her spouse or partner. Additionally, it is possible to link all this information to marital and birth histories, which allows us to identify whether there are children in the household, and their ages. In our baseline sample, we pool observations from the period 2010-2016.

BIBB: The BIBB is our second source of data, collected in 2012 by the German Federal Institute of

Vocational Training (in German: BIBB) and the German Federal Institute for Occupational Safety and Health (in German: BAuA). This survey collects information on 20,000 individuals and is representative of the employed population working at least 10 hours per week. The BIBB survey contains information on the characteristics of employees (such as job satisfaction, tenure in their jobs, etc.), and the relationship between education and employment. Most important for us, it contains information about the characteristics of occupations, reported by each individual. In particular, respondents are asked about the job tasks performed and the knowledge required to perform them. We use this information to construct a measure of '*task complexity*' of each occupation, as a proxy for job productivity. We merge this measure back into the GSOEP, using four digits occupational codes.[4]

3.2 Empirical Evidence

We present several pieces of evidence related to sorting in the marriage market, sorting in the labor market, and the interaction between both markets. We then highlight how the allocation of hours between the labor market and home production is related to sorting patterns on each market. A description of the sample restrictions and the main variables can be found in Appendix D.1.

Marriage Market Sorting. We find evidence of positive assortative matching (PAM) in education in the German marriage market in line with the existing evidence (Eika, Mogstad, and Zafar (2019) for US and Germany and Greenwood, Guner, Kocharkov, and Santos (2016b), Greenwood, Guner, and Vandenbroucke (2017) for the US). Table 1 reports the matching frequencies by education in our data for the period 2010-2016, suggesting that almost 60% of individuals marry someone of the same level of education, with a correlation between the education level of spouses equal to 0.45.[5] The correlation of spouses' partners is our main measure of marriage market sorting.[6] Marriage market sorting also increased over time. For the period 1990-1996 the correlation between education of partners was 0.3931.

Table 1: Marriage Matching Frequencies by Education (2010-2016)

	Low Education Men	Medium Education Men	High Education Men
Low Education Women	**0.10**	0.05	0.02
Medium Education Women	0.13	**0.32**	0.11
High Education Women	0.03	0.07	**0.16**

Notes: Low education includes high school and vocational education with less than 11 years of schooling. Medium Education is vocational education with more than 11 years of schooling. High Education is defined as college and more.

Labor Market Sorting. We also document positive assortative matching in the labor market.

[4]For confidentiality reasons, the BIBB Public Use Files do not contain information of the 4-digit kldb 1992 classification of occupations of each individual (it contains 3-digit levels). However, we were able to obtain summary measures of occupational task content at the 4-digit level, kldb 1992, without reference to individual identifiers.

[5]We obtain a higher correlation (0.56) when we compute it based on years of education instead of education levels.

[6]As discussed in Chiappori, Dias, and Meghir (2020), the correlation is one of the measures that complies with two desirable properties – a 'monotonicity' condition and whether it captures the case of 'perfectly assortative matching' – that a measure of sorting (and sorting changes) should have. Eika, Mogstad, and Zafar (2019) propose an alternative measure of marriage sorting based on the frequency of couples' education relative to random matching. This measure equals 1.62 in Germany: individuals are 60% more likely to marry someone of the same education, relative to random matching.

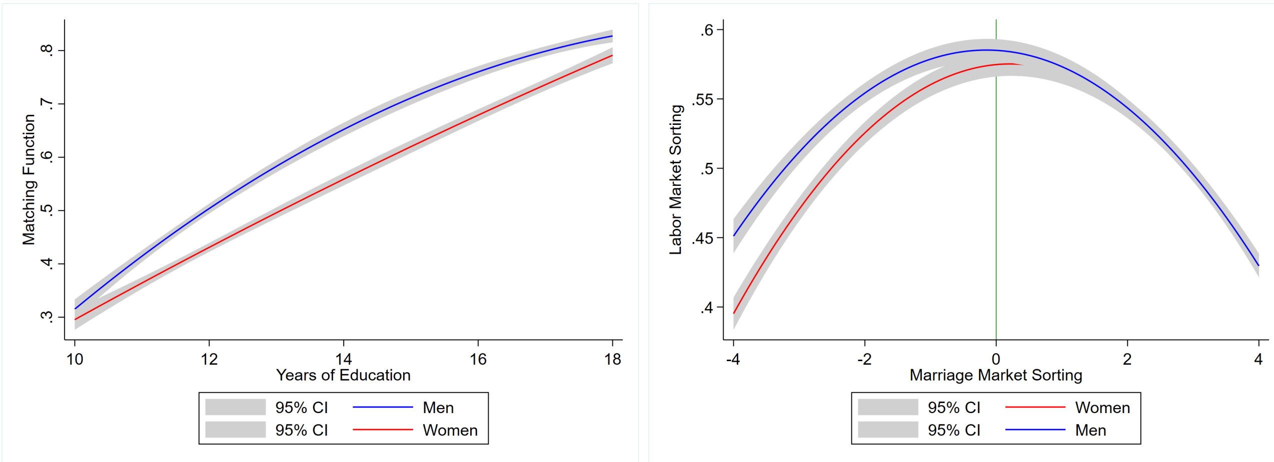

Figure 1:a. Labor Market Matching Function; b. Relation between Labor and Marriage Market Sorting

We do not have firm identifiers in the GSOEP, so we measure labor sorting based on the relationship between worker and *job* attributes, where a job is defined by the occupation of the individual. The match-relevant attribute of workers in the labor market is 'years of education'. In turn, for jobs we use information on the task requirement of each occupation to construct a measure of its *task complexity*. The correlation between years of education of workers and task complexity of jobs is 0.6024, indicating positive assortative matching between workers and jobs on the labor market.

Figure 1(a) plots the fitted matching function (job attributes as a function of worker characteristics) by gender, conditional on employment. Both men and women are positively sorted in the labor market, indicated by a positive slope of the matching function. However, men are 'better' matched than women: for every given education level, men are on average matched to better jobs than women, and the matching function is steeper for men for most education levels.[7] This pattern is also reflected in the correlation of worker and job attributes by gender, which is 0.62 for men and 0.60 for women.[8]

Labor Market Sorting and Marriage Market Sorting. Next we assess the relationship between labor market and marriage market sorting. To do so graphically, we measure marriage market sorting by the difference between the years of education of the individual under consideration and the years of education of his/her partner, with 'zero' indicating the maximum amount of sorting. We measure labor market sorting as before as the correlation between years of education (worker characteristic) and the task complexity of the occupation (job characteristic). We then plot the relation between labor market and marriage market sorting by gender in Figure 1(b), where the green vertical line indicates

[7]This pattern is robust to using education levels instead of years of education as the worker's attribute.

[8]Differences in labor market sorting across gender are larger (correlation 0.5465 for men vs. 0.4319 for women) if we do not condition on participation and we treat unmatched individuals as virtually matched to a job with attribute zero. This suggests that one dimension through which women are worse matched in the labor market is that they are less likely than men to participate. An alternative way of seeing the differential labor market sorting of men and women is to plot the share of male workers by task complexity. This shows that men are predominantly employed in more demanding jobs even when we condition on education (figures available upon request).

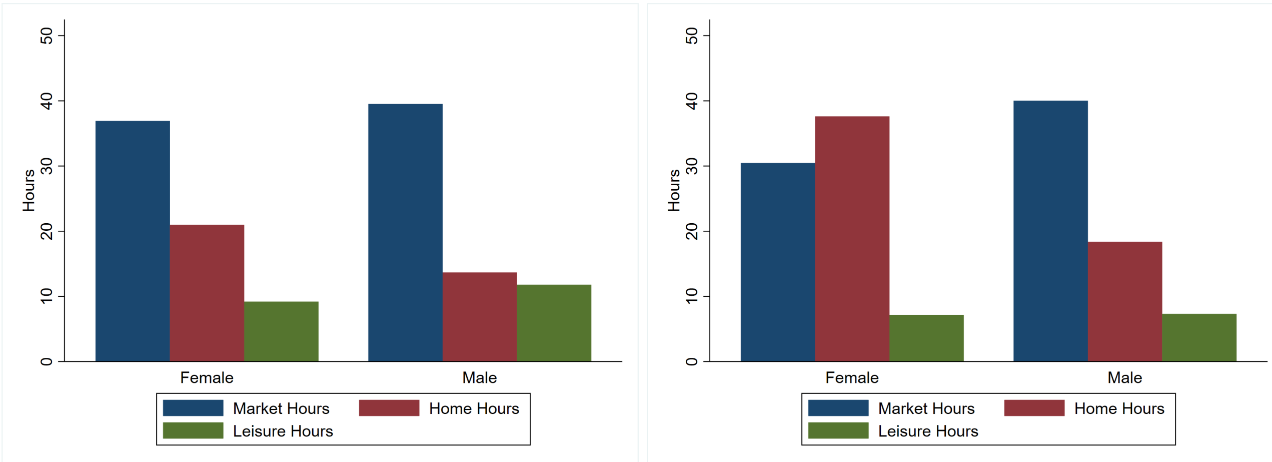

Figure 2: Time allocation for a. Singles and b. Couples

maximum marriage market sorting. The striking feature of this figure is that labor market sorting is maximized when marriage market sorting is maximized, both for men and for women.

THE ROLE OF HOURS. We now provide evidence on a salient link between the two markets: hours worked on the labor market vs. hours spent in home production. First, we document that the time allocation choice is 'impacted' by the partnership status as well as marriage market sorting. Second, we document that at the same time, the time allocation choice 'impacts' labor market sorting.[9]

First, as is well documented in the literature (Gayle and Shephard, 2019; Goussé, Jacquemet, and Robin, 2017b), an individual's time allocation between the activities 'work', 'home production' and 'leisure' is related to their partnership status. While among singles (left panel in Figure 2) gender differences in time allocation across activities are minor, these gender differences become much more pronounced for couples (right panel in Figure 2). Indeed, in couples, women spend about 10 hours less per week working on the labor market but about 10 hours more in home production compared to their male partners. Neither for couples nor for singles are there significant differences in leisure time across gender. This justifies that we abstract from leisure in our model below.

We also document the relationship between choice of hours and marriage market sorting. Figure 3 focuses on market and home production hours of partners and how they relate to their marriage sorting. The left panel plots the correlation between home production hours within couples against our summary measure of marriage market sorting (difference in partners' years of education); the right panel has the same structure only focussing on the correlation of labor hours within couples on the y-axis. Interestingly, both home production hours (left panel of Figure 3) and labor hours (right panel) are more complementary among those partners that are well sorted on education (whose difference in education is zero), as indicated by the inverse U-shape of the hours' correlation function. Note that the pattern for home production is even more pronounced than for labor hours, with a strong positive correlation of

[9] We use contractual hours as our measure of hours worked in the labor market. Our facts are robust, and in many cases the effects are even more sizable, if we consider actual market hours instead (defined as contractual hours plus overtime).

home hours among partners with the same education compared to partners whose education differs.[10]

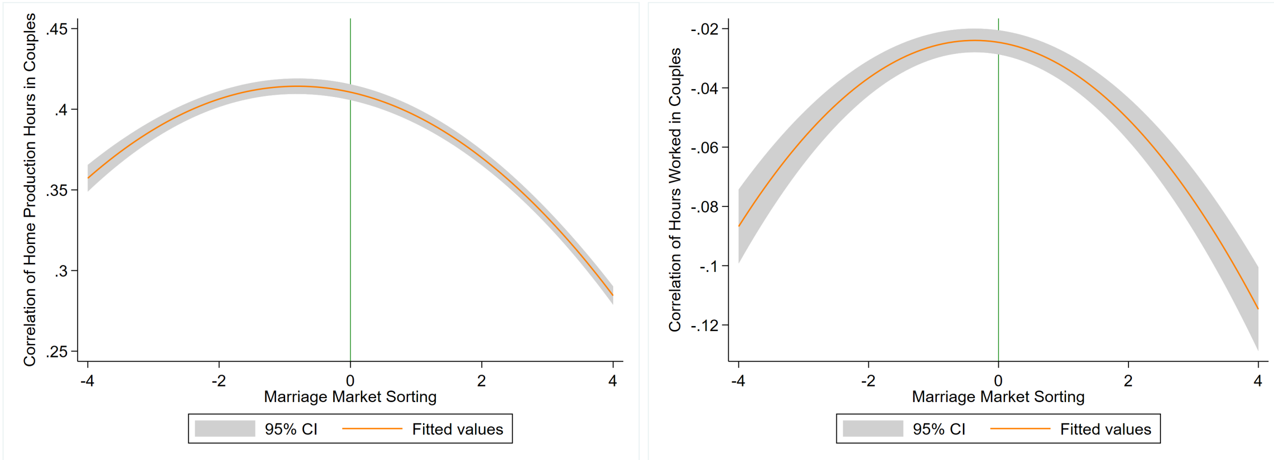

Figure 3: Time Allocation and Marriage Sorting

We further explore what drives this complementarity in home production hours between spouses, especially since most literature in family economics emphasizes household specialization. To do so, we first look at different components of home production. We find that there are less complementarities in terms of housework but particularly strong complementarities in terms of childcare (see Figure 17a in Appendix A). In line with childcare driving these complementarities, we find that inverse U-shaped complementarities in home production are only present for couples with children (see Figure 17b in Appendix A).

One concern is that the correlations between marital sorting and complementarities in hours displayed in Figure 3 are based on marriage market sorting bins that pool individuals from different education groups (e.g. the value zero on the x-axis includes couples where both partners have low education and also couples where both have high education). Our regression results in Tables 6 and 7 in Appendix A show that perfect sorting in the marriage market is associated with stronger complementarities in hours even if we control for the education of partners in a couple.

The second point we want to make is that the time split between labor market and home production also relates to labor market outcomes: We first show that in Germany there is a large hourly wage penalty for working part time, suggesting that hours are a productive input in the labor market. This is in line with, for instance, evidence by Goldin (2014) for the U.S. Figure 4a shows a sizable part time penalty, especially for women. The left panel shows the wage penalties of various groups *relative* to men working at least full-time (blue bar that equals one).[11] While full-time women have a wage penalty of 14 percentage points relative to full-time men, when they work part time the wage penalty increases to 20 percentage points.[12] Figure 4b shows that while few men work less than full time (less than 10%), nearly 50% of women do so, and are thus particularly affected by the documented wage penalties.

[10] In the left panel of Figure 3 we condition on both partners participating in the labor market, but this pattern holds if we do not condition on labor market participation.

[11] Full-time is defined as at least 36 contractual hours of work per week.

[12] The bars in Figure 4a are the estimates of a regression of the logarithm of hourly wages on an indicator variable for

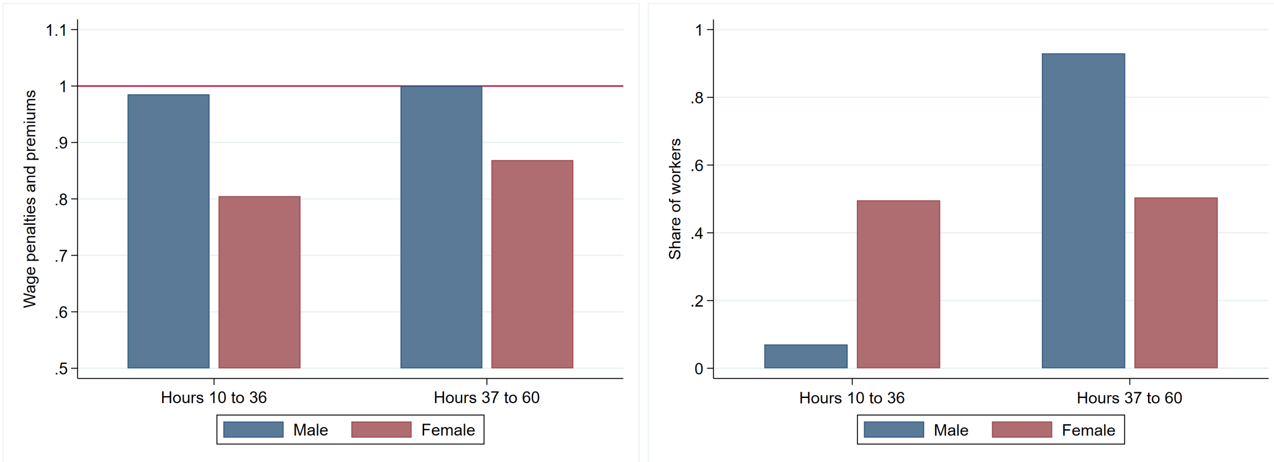

Figure 4: a. Part-Time Wage Penalties; b. Share of Workers in Full/Part Time Work

Moreover, we highlight that the number of hours worked are associated with *sorting* on the labor market. Indeed, when accounting for differences in hours worked across gender, the discrepancy in their matching functions from Figure 1 shrinks considerably. This is documented in Figure 5, where the solid lines are the matching functions by gender and identical to those in Figure 1. In turn, the dashed lines plot the residualized matching function for both genders, after partialling out hours worked. Nevertheless, we see that even when controlling for the number of hours worked, some differences in labor market sorting across gender persist, which must be accounted by other factors.

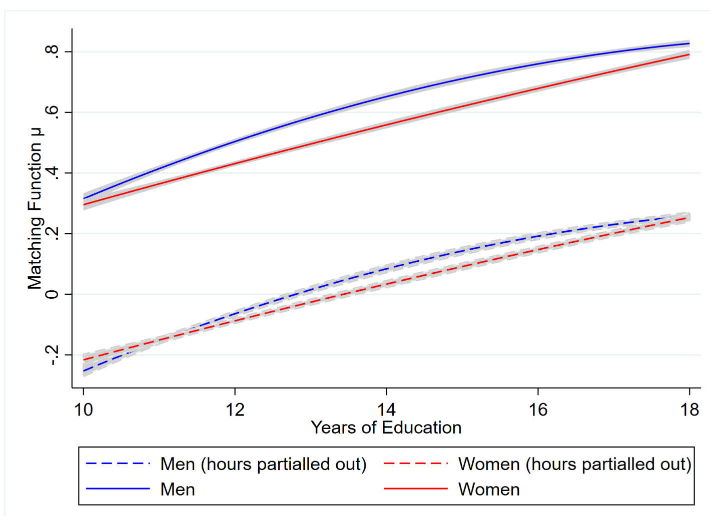

Figure 5: Time Allocation and Labor Market Sorting

In sum, we highlight three sets of facts. First, in both labor and marriage market, there is PAM. But on the labor market women are more mismatched than men, indicated by a lower matching function. Second, there is a strong relation between labor market and marriage market sorting with labor market

part time work, an indicator variable for being female and the interactions between both. We also include controls for age, marital status, whether the individual was born in Germany and his/her education level.

sorting (for both men and women) being maximized when marriage market sorting is. Third, the split between hours worked in the labor market vs. hours spent in home production is a potentially important link between the two markets. We not only show that time allocation choices depend on marriage market sorting but also they themselves have an impact on labor market sorting (where 'impact' really means 'is associated' here since we have not shown causality). Motivated by these facts we now build a model with endogenous labor *and* marriage market. We also use these facts to justify several assumptions and to guide our modeling choices regarding the link between labor and marriage markets, where we focus on the 'hours' margin. Finally, we come back to these facts when validating our estimated model.

4 The Model

We start with an overview: Men and women first decide whom to marry in a competitive market based on their education/skills. Each matched household then optimally chooses private consumption and the time allocation between home production and labor market work, which also pins down the public good consumption. Finally, individuals match with heterogenous firms in a competitive labor market.

4.1 Environment

There are two types of agents, individuals and firms. There is a measure one of firms. Firms are characterized by productivity $y \in \mathcal{Y}$ ($\mathcal{Y}$ is a closed and bounded interval), distributed according to a continuously differentiable cdf G, with positive density g.

Among the individuals, there is an equal measure of men (denoted by subscript m) and women (denoted by subscript f). The overall measure of men and women is one. Both men and women have exogenously given skills: Denote women's skills by $x_f \in \mathcal{X}_f = [0, \overline{x}_f]$, where x_f is distributed with a continuously differentiable cdf N_f, with positive density n_f. Analogously, men have skills $x_m \in \mathcal{X}_m = [0, \overline{x}_m]$, distributed according to the continuously differentiable cdf N_m with positive density n_m.

In the marriage market, men and women match on skills, so the relevant distributions for marriage matching are N_m and N_f. In the labor market, however, what matters for output is not only skills but also hours worked, which will be chosen optimally in each couple. Each individual is endowed with one unit of time that can be allocated to paid work in the labor market, denoted by $h_i, i \in \{f, m\}$, or non-paid work at home towards the production of a public good, $1 - h_i$ (based on Figure 2, which shows no difference in leisure across gender, we abstract from it). Note that $h_i = 0$ captures non-participation. By increasing hours worked in the labor market, each individual 'invests' in his/her *effective skill* $\tilde{x} := e(h, x)$, $\tilde{x} \in \tilde{\mathcal{X}}$, with endogenous cdf $\tilde{N}(X) := \mathbb{P}[\tilde{x} \leq X] = \frac{1}{2}\mathbb{P}[\tilde{x}_f \leq X] + \frac{1}{2}\mathbb{P}[\tilde{x}_m \leq X]$. We assume that e is twice differentiable, strictly increasing in each argument, (endogenous) hours worked and (exogenous) skill type, supermodular, and $e_h(\cdot, x)$ is strictly positive and finite for all x.[13] Thus, putting more labor hours is *as if* the worker is more skilled. The effective skill or index $\tilde{x}$ is the output-

[13] We use subscripts of functions to denote derivatives throughout.

relevant worker characteristic on the labor market. We base this assumption on evidence that more hours worked lead to higher productivity, especially if costs of handing off clients or customers to the next worker on the shift are high for instance due to coordination costs (see Goldin, 2014; Cortés and Pan, 2019, Gicheva, 2013 and our own evidence on the part-time wage penalty). This assumption that not only skills but also hours worked matter for labor market matching means that multiple attributes are matching-relevant even if the actual assignment is simplified and based on the index $\tilde{x}$.

Denote by $z(\tilde{x}, y)$ the output generated by an individual of type $\tilde{x}$ matched to a firm of type y. We assume that production function z is twice differentiable, increasing in each argument, and $z_{\tilde{x}}(\cdot, y)$ is strictly positive and finite for all y. Individuals and firms split the output they generate into wages and profits, where workers use their wages to finance the private consumption good $c_i, i \in \{f, m\}$.

The public good production function is given by p, which takes as inputs each couple's hours at home, so that $p(1-h_m, 1-h_f)$ is the public good produced by a couple spending $(1-h_m, 1-h_f)$ in home production (recall hours at home equal the hours *not* spent in the labor market). We assume that p is twice differentiable with $p_1 > 0, p_2 > 0, p_{11} < 0$ and $p_{22} < 0$ and the Inada conditions $\lim_{h_f \to 0} p_2(1-h_m, 1-h_f) = 0$ and $\lim_{h_f \to 1} p_2(1-h_m, 1-h_f) = \infty$ and similarly for p_1.

Denote the utility function of an individual by u, where $u(c_i, p)$ is the utility from consuming private good c_i and public good p. We assume that u is twice differentiable with $u_1 > 0, u_2 > 0, u_{11} \leq 0, u_{22} \leq 0$, and we further restrict the class of utility functions below.

The model is static and agents make three decisions. In the *marriage market stage*, men and women choose their partner to maximize their value of being married. The outcome is a marriage market matching function, matching each woman x_f to some man x_m (or single-hood), and a market clearing price. In the second stage, the *household decision problem*, each matched couple chooses private consumption and allocates hours to the various activities – work in the labor market and at home – under anticipation of the labor market outcomes (matching function and wage function). This stage renders both private consumption and public consumption (and thus the time allocation), pinning down individuals' effective types. In the third stage, the *labor market stage*, agents take marriage market and household choices as given and match with firms based on their effective skills so that their wage income is maximized (or equivalently, each firm chooses an effective worker type to maximize her profits). This problem pins down a labor market matching function and a market-clearing wage function.

Both matching markets, the labor and marriage market, are competitive (full information and no search frictions) and there is no risk. The two markets and sorting choices therein are linked through the labor supply choice, which can be interpreted as a pre-labor market and post-marriage market continuous investment in 'effective' skills. This link is the key element of our model.

4.2 Decisions

In terms of exposition, we will proceed backwards, so we describe the decision stages in reverse order.

Labor Market. Taking marriage market choices (in particular, the partner choice and the associated hours choices) as given, firms choose the *effective* worker type that maximizes their profits:

$$\max_{\tilde{x}} z(\tilde{x}, y) - w(\tilde{x}) \tag{1}$$

where $w : \tilde{\mathcal{X}} \to \mathbb{R}_+$ is the endogenous wage function taken as given. Market clearing pins down the labor market matching function $\mu : \tilde{\mathcal{X}} \to \mathcal{Y}$, mapping workers' effective skills to firm types. And if $\mathcal{X}$ is an interval (as it will be the case below) then the first order condition, which gives a differential equation in w, pins down the wage function as

$$w(\tilde{x}) = w_0 + \int_0^{\tilde{x}} z_{\tilde{x}}(t, \mu(t))dt, \tag{2}$$

where w_0 is the constant of integration. It is important to note that $w(\tilde{x})$ is the wage of worker $\tilde{x}$ *per unit of time* (recall our time endowment is normalized to one unit and we will give a specific interpretation to what that time unit is when going to the data below), *not* the worker's earnings.

The optimal matching μ, which features PAM if $z_{\tilde{x}y} > 0$, will match up the exogenous firm distribution G and the *endogenous* worker distribution of effective types, $\tilde{N}$, in a measure-preserving way. Importantly, the labor market matching function μ depends on the hours choice (through $\tilde{N}$), which in turn will depend on the marriage partner. Thus, sorting on the two markets is connected.

Household Problem. Each couple (x_f, x_m), taking the partner choice from the marriage market stage as given and anticipating the wage function and labor market matching function (w, μ), solves the following *cooperative household problem*. One partner (here wlog the man) maximizes his utility subject to the household budget constraint and a constraint that ensures a certain level of utility for the wife, by choosing the couple's private consumption and the hours allocation:

$$\begin{aligned} \max_{c_m, c_f, h_m, h_f} \quad & u(c_m, p(1-h_m, 1-h_f)) \\ s.t. \quad & c_m + c_f - w(\tilde{x}_m) - w(\tilde{x}_f) = 0 \\ & u(c_f, p(1-h_m, 1-h_f)) \geq \overline{v}, \end{aligned} \tag{3}$$

where at this stage $\overline{v}$ is taken as a parameter by each household (it will be a function of female skills and endogenously determined in the next stage, the marriage market stage). When solved for all feasible $\overline{v}$, problem (3) traces out the household's *pareto-frontier*. The solution to this problem yields the hours functions $h_i : \mathcal{X}_m \times \mathcal{X}_f \times \overline{v} \to [0, 1]$ and consumption functions $c_i : \mathcal{X}_m \times \mathcal{X}_f \times \overline{v} \to \mathbb{R}_+$. That is, for each partner in any matched couple (x_m, x_f) and for a given utility split $\overline{v}$, the problem pins down private consumption $c_m(x_m, x_f, \overline{v})$ and $c_f(x_m, x_f, \overline{v})$ and labor hours $h_m(x_m, x_f, \overline{v})$ and $h_f(x_m, x_f, \overline{v})$ (and therefore home production $p(1-h_m, 1-h_f)$). Because the household problem is set up in a cooperative way, these allocations are pareto-efficient for any given wage function.

Marriage Market. Anticipating for each *potential* couple the solution to the household problem

(h_f, h_m, c_f, c_m) (and implicitly the labor market outcomes (μ, w)), the value of marriage of husband x_m marrying wife x_f is given by the value of household problem (3) and thus:

$$\Phi(x_m, x_f, v(x_f)) := u(c_m(x_m, x_f, v(x_f)), p(1 - h_m(x_m, x_f, v(x_f)), 1 - h_f(x_m, x_f, v(x_f)))),$$

where we now make explicit that v, the marriage market clearing price, is an *endogenous* function of x_f and pinned down in the equilibrium of the marriage market. The marriage market problem for a man is then to choose the optimal female partner by maximizing this value:

$$\max_{x_f} \Phi(x_m, x_f, v(x_f)). \tag{4}$$

The FOC of this problem (together with marriage market clearing) determines the marriage market matching function $\eta : \mathcal{X}_f \to \mathcal{X}_m$, matching each woman x_f to man $\eta(x_f)$ and a transfer function $v : \mathcal{X}_f \to \mathbb{R}_+$, where $v(x_f)$ is the payoff of wife x_f. In particular, the FOC of (4) gives a differential equation in v, which can be solved given the marriage matching function η. In turn, the marriage matching function depends on the complementarities between men's and women's skills in Φ. Particularly tractable matching patterns result if the differential version of the Legros and Newman (2007) condition for positive sorting is satisfied. There is PAM on the marriage market in partners' skills if and only if

$$\Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}.$$

The marriage market matching function is then determined by market clearing, mapping N_f to N_m in a measure-preserving and – in the case of PAM – increasing way. Note that in principle, individuals can decide to remain single, which – given that there is an equal mass of men and women – will not happen in our baseline model if the value of marriage Φ is positive for all potential couples.

Figure 6 summarizes these decision stages and the endogenous variables they pin down.

4.3 Equilibrium

We now formally define equilibrium.

Definition 1 (Equilibrium)**.** *An equilibrium is given by a tuple $(\eta, v, h_m, h_f, c_f, c_m, \tilde{N}, \mu, w)$ such that*

1. *given (η, v) and (h_m, h_f), the tuple of labor market matching and wage function (μ, w) is a competitive equilibrium of the labor market;*
2. *given (η, v) and (μ, w), the hours and consumption functions (h_f, h_m, c_f, c_m) solve the household problem, pinning down the distribution of effective types $\tilde{N}$;*
3. *given (μ, w) and (h_m, h_f, c_f, c_m), the tuple of marriage market matching and transfer function (η, v) is a competitive equilibrium of the marriage market.*

We next define a *monotone* equilibrium, which will be our main benchmark below.

Figure 6: The Decision Stages of Individual $i \in \{f, m\}$ of Skill Type x_i

Stage:	*Marriage*	*Household*		*Labor*
Allocations:				
(married)	x_i *matches with* x_j	c_f, c_m, h_m, h_f	$\longrightarrow p$ and $\tilde{x}_i$	$\tilde{x}_i$ *matches with* y
(single)	*or single*	c_i, h_i		
Resources:				
(married)		$w_f(\tilde{x}_f) + w_m(\tilde{x}_m)$		
(single)		$w_i(\tilde{x}_i)$		

Definition 2 (Monotone Equilibrium)**.** *An equilibrium is monotone if it satisfies Definition 1 and:*

1. *labor market matching* μ *satisfies PAM,* $\mu(\tilde{x}) = G^{-1}(\tilde{N}(\tilde{x}))$*;*
2. *labor hours* h_i *are increasing in own type* x_i *and partner's type* x_j*,* $i \neq j, i, j \in \{f, m\}$*, as well as in transfer* v*;*
3. *marriage market matching* η *satisfies PAM,* $\eta(x_f) = N_m^{-1}(N_f(x_f))$*, and* v *is increasing in* x_f*.*

Thus, in a monotone equilibrium, there are three additional requirements relating to the three different stages of this model: (1.) matching on the labor market is PAM; (2.) hours worked in the labor market are increasing in own and in the partner's type; (3.) matching on the marriage market is PAM and the transfer to the wife is increasing in her type. Under (2.) and (3.), we obtain that a woman's effective type as a function of x_f, $\gamma_f(x_f) := e(x_f, h_f(x_f, \eta(x_f), v(x_f)))$, is strictly increasing in x_f since then $d\gamma_f/dx_f = e_{x_f} + e_{h_f}[\partial h_f/\partial x_f + (\partial h_f/\partial x_m)(\partial \eta(x_f)/\partial x_f) + (\partial h_f/\partial v)(\partial v/\partial x_f)] > 0$, implying that γ_f can be inverted (and similarly for γ_m). As a result, in a monotone equilibrium, the cdf of effective types has a closed form, where the probability that an effective type $\tilde{x} \leq X$ is given by:[14]

$$\tilde{N}(X) = \frac{1}{2} N_f(\gamma_f^{-1}(X)) + \frac{1}{2} N_m(\gamma_m^{-1}(X)). \tag{5}$$

This discussion highlights an important point: The equilibrium hours function, h_f, not only depends on her own skill type x_f but also on marriage market outcomes: the skill type of her partner, $\eta(x_f)$, as well as the transfer guaranteed to her in the marriage $v(x_f)$; and similarly for men's hours function

[14] To see this, observe that

$$\tilde{N}(X) = \frac{1}{2}\mathbb{P}[\gamma_f(x_f) \leq X] + \frac{1}{2}\mathbb{P}[\gamma_m(x_m) \leq X] = \frac{1}{2}\mathbb{P}[x_f \leq \gamma_f^{-1}(X)] + \frac{1}{2}\mathbb{P}[x_m \leq \gamma_m^{-1}(X)] = \frac{1}{2}N_f(\gamma_f^{-1}(X)) + \frac{1}{2}N_m(\gamma_m^{-1}(X)).$$

h_m. As a result, labor supply choices form the link between the marriage market (they are determined by the household and depend on who is matched to whom on the marriage market η) and the labor market (they affect the effective skills $\tilde{N}$ and thus labor market matching μ and wages w).

This interdependence of marriage and labor market sorting is the crucial feature of our model. But it also makes the problem challenging from a theoretical point of view since we seek the simultaneous equilibrium of two intertwined matching markets, which are related through the time allocation choice. Equilibrium existence requires to show that there is a fixed point in the space of continuous and bounded functions (the hours functions). Even aside from existence, the analysis of equilibrium properties and comparative statics is difficult in light of the equilibrium links; and it is further complicated by the possible feature of *imperfectly* transferable utility (ITU), where the hours functions depend on the transfer v (indicating that the overall split between public and private consumption depends on how private consumption is shared between partners).

To gain tractability and intuition into the main mechanisms of the model, we focus on a certain class of models (quasi-linear ones) that yields the *transferable utility* (TU) property.

4.4 Transferable Utility Representation

Under which conditions on primitives will this model with ITU be TU representable? We discuss the two stages where ITU will generally be present: the household problem and the marriage matching problem.

Household Stage. In the collective approach laid out above, the household maximization problem generally features ITU. This implies that the aggregate (household) demand for private and public consumption depend on distribution factors (e.g. the income distribution) and are determined simultaneously: The aggregate split into public and private consumption *cannot* be solved for independently of how private consumption is shared. In turn, if the household problem has the TU property, then the household agrees to maximize the sum of utilities (since in this case the pareto weight on each partner's utility or, equivalently, the Lagrange multiplier of the problem equals one) and aggregate demand for private consumption c and public consumption p can be determined independently of the couple's sharing rule for private consumption and thus independently of $\overline{v}$. As a consequence, the hours functions (h_f, h_m) are independent of $\bar{v}$. In this case, the household acts as a single decision-maker so the collective model collapses to the unitary model.

Marriage Market Stage. Generally, our model features ITU at the marriage market stage as well. In particular, the value of the marriage problem $\Phi(x_m, x_f, v(x_f))$ may not be additively separable in v. However, if the TU property ensues, and – with some abuse of notation – we obtain $\Phi(x_m, x_f) - v(x_f)$. As a result, supermodularity of the value of marriage in (x_f, x_m) is independent of properties of v (meaning the matching problem can be solved by maximizing the total value of marriage, independently of how this value is shared which is captured by v).[15]

[15] Note that depending on the precise utility function, we could also have $\Phi(x_f, x_m) - m(v(x_f))$ with some function m. The important point is that v is separable from the remaining part that depends on x_m.

The TU representation obtains if the utility function falls into a known class, the *Gorman form* (see Browning, Chiappori, and Weiss (2011) for a detailed discussion).[16] We give two examples of utility functions that can be represented as the Gorman form: (i) The utility function is linear in money/consumption (standard TU case), e.g., the quasi-linear utility $u(c,p) = c + k(p)$, for some function k. (ii) The utility is linear in money after a monotone transformation. Examples of this second case include $u(c,p) = m(c)k(p)$ or $u(c,p) = log(cp)$ or $u(c,p) = F(c+p)$ with F strictly increasing. See Appendix B.2 for the details of the TU representation of our model for these examples.

5 Analysis

5.1 The Quasi-Linear Class

Our model becomes considerably more tractable if it features the TU property. In this case, we can derive analytical properties of the model, which provide insights into the model's mechanism (see Appendix B.1 for further analysis of the general model). This is why we consider the quasi-linear class as our baseline. Specifically, we assume that the utility function is given by[17]

$$u(c_i, p) = c_i + p. \tag{6}$$

5.2 Monotone Equilibrium

Our first objective is to derive conditions under which any stable equilibrium is *monotone* in the sense of Definition 2. The monotone equilibrium will be our benchmark. We call an equilibrium stable if it is robust to small perturbations (see Appendix B.3.1 for a precise definition). We then show in that Appendix that if there exists an equilibrium there is at least one stable one.

Labor Market Stage. As is well-known, if technology z is supermodular, then the worker-firm assignment in the labor market will satisfy PAM, that is matching function μ is increasing. Workers with higher effective skills match up with more productive firms, where $\mu(\tilde{x}) = G^{-1}(\tilde{N}(\tilde{x}))$ is the firm matched to worker $\tilde{x}$. We reiterate that $\tilde{N}$ is endogenous. In turn, the equilibrium wage function is given by (2).

Household Stage. With quasi-linear utility (6), the household problem (3) takes the form:

$$\max_{h_m, h_f} w(\tilde{x}_m) + w(\tilde{x}_f) - \overline{v} + 2p(1-h_m, 1-h_f), \tag{7}$$

[16] More generally, with more than one private consumption good, the Gorman form of i's utility is given by $u^i(p, c_1, ..., c_n) = z^i(c_2, ..., c_n) + k(p)c_1$, which is *linear in at least one private consumption good*, here c_1, with common coefficient $k(p)$, meaning that the marginal utility w.r.t. c_1 is equalized across partners. Thus, utility can be costlessly transferred between partners at a constant rate using c_1. This finding on the TU representation under the Gorman form mimics a known result from consumer theory that aggregate demand is independent of the income distribution if and only if indirect utility can be represented by functions of the Gorman polar form.

[17] For the utility function, we choose the simplest functional form in the Gorman class to reduce notation that obscures the main mechanism and intuition. Generalizations to other utilities that comply with the Gorman form are possible, e.g. we can assume that $u = F(c+p)$ where F is strictly increasing.

where we substituted both the household's budget constraint and the wife's constraint to receive at least utility $\overline{v}$ into the objective. Clearly, as a consequence of TU, the overall split between public and private consumption (and thus the time allocation choice) can be made independently of how utility is shared, captured by $\overline{v}$. As a consequence, the hours functions (h_f, h_m) (and thus the public good) will only depend on types (x_f, x_m) but no longer on $\overline{v}$.

The FOCs for h_f and h_m are given by

$$w_{\tilde{x}_f} e_{h_f} - 2p_2 = 0 \tag{8}$$

$$w_{\tilde{x}_m} e_{h_m} - 2p_1 = 0, \tag{9}$$

where p_k indicates the partial derivative of p with respect to its $k's$ argument, $k \in \{1,2\}$. In any interior solution for the hours choices of partners, each of these FOCs equalizes the marginal benefit of an additional hour in the labor market, captured by the wage gain, with its marginal cost stemming from a reduction in home production that affects both partners (hence the multiplication by 2).

FOCs (8) and (9) give rise to two 'best-response' functions, one of wife's to husbands labor hours and one of husband's to wife's hours. We will focus on equilibria that are stable, i.e. where the husband's best-response function crosses the wife's best-response function from below in the (h_m, h_f)-space.

To characterize under which conditions an equilibrium is monotone, we need to pursue equilibrium comparative statics of the hours functions with respect to skill types, derived from the system of FOCs, evaluated at the equilibrium marriage market matching function η:

$$\frac{\partial h_f}{\partial x_f} = \frac{-\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 u}{\partial h_m^2} + \eta'(x_f)\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} p_{12}}{|H|} \tag{10}$$

$$\frac{\partial h_m}{\partial x_f} = \frac{-\eta'(x_f)\frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} + 2\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} p_{12}}{|H|} \tag{11}$$

$$\frac{\partial h_m}{\partial x_m} = \frac{-\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}\frac{\partial^2 u}{\partial h_f^2} + (\eta^{-1}(x_m))'\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} p_{12}}{|H|} \tag{12}$$

$$\frac{\partial h_f}{\partial x_m} = \frac{-(\eta^{-1}(x_m))'\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 u}{\partial h_m^2} + 2\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m} p_{12}}{|H|}. \tag{13}$$

The denominator in these expressions is given by $|H| = \frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 u}{\partial h_m^2} - 4p_{12}$ (where the notation indicates this is the determinant of the Hessian of the household problem). By our definition of stability (Definition 3 in the Appendix B.3.1), $|H|$ is positive and moreover $\partial^2 u/\partial h_f^2 \leq 0$ and $\partial^2 u/\partial h_m^2 \leq 0$. Further, we show in Lemma 1 (Appendix B.3.1) that if an equilibrium exists, then there is at least one stable one.

In any stable equilibrium, (10)-(13) are positive if home hours are complementary (p is supermodular), wages are supermodular in hours and skills, $\partial^2 w(\tilde{x}_f)/\partial h_f \partial x_f > 0$ and $\partial^2 w(\tilde{x}_m)/\partial h_m \partial x_m > 0$, and if marriage market matching is PAM, $\eta' > 0$. We will specify these conditions in terms of primitives below and give an interpretation. If (10)-(13) are positive, then the distribution of effective types, $\tilde{N}$, can be pinned down in closed form and is given by (5).

Marriage Market Problem. Given the equilibrium hours functions (h_f, h_m), we obtain the value of marriage Φ as the value of the household problem (7):

$$\Phi(x_m, x_f, v(x_f)) = w(x_m h_m(x_m, x_f)) + w(x_f h_f(x_f, x_m)) - v(x_f) + 2p(1 - h_m(x_m, x_f), 1 - h_f(x_m, x_f)). \quad (14)$$

Complementarities among partners' types in Φ determine marriage market matching patterns. Under TU, $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f = \Phi_{x_m x_f}$. If $\Phi_{x_m x_f} \geq 0$, then marriage matching is PAM, $\eta' > 0$.[18]

For consistency, we again adopt the male partner's perspective. Maximizing (14) with respect to x_f, while taking into account that $h_m(x_m, x_f)$ and $h_f(x_m, x_f)$ are already optimized so that they do not respond to further changes in x_f (by the Envelope Theorem), yields:

$$\Phi_{x_f} = 0 \qquad \Leftrightarrow \qquad w_{\tilde{x}_f} e_{x_f} - v_{x_f} = 0 \quad (15)$$

The transfer to the female partner, v, reflects the marginal impact of her type on her wage: When a man chooses a woman, he trades off the marginal benefits of choosing a higher type (which equals the marginal impact on her wage, $\partial w/\partial x_f = w_{\tilde{x}_f} e_{x_f}$) with the marginal costs (which equals the marginal increase in transfer to her, v_{x_f}). The higher the marginal wage return from a more productive female type, the larger is the increase in her compensation within the marriage. The reason why transfer v does not depend on the woman type's contribution to the public good p is that types only indirectly affect the public good production through the hours choice (and since hours were already optimized, the change in x_f has no impact on home production by the Envelope Theorem).

Then the cross-partial of Φ can be computed from (15) as:

$$\begin{aligned} \Phi_{x_f x_m} &= \frac{\partial^2 w}{\partial x_f \partial x_m} \\ &= \frac{\partial^2 w}{\partial x_f \partial h_f} \frac{\partial \widehat{h}_f}{\partial x_m} \end{aligned} \quad (16)$$

which is positive if wages are complementary in partners' skill types or, zooming in, when wages are supermodular in type and hours and when female labor hours are increasing in her partner's type, $\partial \widehat{h}_f/\partial x_m > 0$. Note that here, the comparative static of female hours with respect to male type is computed for *any* potential couple (x_f, x_m), not just the ones that form in equilibrium $(x_f, \eta(x_f))$ (we still need to determine η at this stage), and we use the *hat*-notation in (16) to make this distinction

[18] Under ITU, $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f \geq 0$ is equivalent to $\Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}$ (Legros and Newman, 2007). To see this, the FOC of problem (4) is given by $\Phi_{x_f} + \Phi_v v_{x_f} = 0$, while $\partial^2\Phi(x_m, x_f, v(x_f))/\partial x_m \partial x_f = \Phi_{x_m x_f} + \Phi_{x_m v} v_{x_f}$. Plugging the FOC into the latter condition gives the known condition for PAM in ITU problems $\partial^2\Phi(x_m, x_f, v(x_f)/\partial x_m \partial x_f \geq 0 \Leftrightarrow \Phi_{x_m x_f} \geq \frac{\Phi_{x_f}}{\Phi_v} \Phi_{x_m v}$. In the quasi-linear class, this becomes $\Phi_{x_m x_f} \geq 0$ since $\Phi_{x_m v} = 0$.

from the equilibrium comparative static (13) clear. In turn, wages are supermodular in type and hours if they are convex in effective types since $\partial^2 w/\partial x_f \partial h_f = w_{\tilde{x}_f\tilde{x}_f} e_{h_f} e_{x_f} + w_{\tilde{x}_f} e_{x_f h_f}$, meaning that the marginal wage return to skill increases when putting in more labor hours. The sorting conditions are intuitive: There is PAM in the marriage market, so that x_f is matched to $\eta(x_f) = N_m^{-1}(N_f(x_f))$, if labor hours of spouses are complementary in the sense that an individual's labor hours are increasing in partner's type, and if at the same time working more hours boosts the marginal wage return to skill.

Using the formula for $\partial \widehat{h}_f/\partial x_m$ (see Appendix), we can re-express $\Phi_{x_f x_m}$ in a more symmetric way, which highlights the importance of the home production function also at the marriage stage:

$$\Phi_{x_f x_m} = 2p_{12} \frac{\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f} \frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}}{|H|}. \tag{17}$$

Complementarity in home production (supermodular p) along with wages that are supermodular in skill and hours induce $\Phi_{x_f x_m} > 0$ and thus PAM in the marriage market.

We can now state our result on monotone equilibrium, expressing the discussed conditions in all three stages of the model in terms of primitives.

Proposition 1 (Monotone Equilibrium)**.** *If p is strictly supermodular and z is weakly convex in effective types $\tilde{x}$ and supermodular, then any stable equilibrium is monotone.*

The proof is in Appendix B.3.3. The first requirement, strict supermodularity of p, ensures that hours in home production (and thus also labor hours) of any two partners are complements. The second requirement, convexity of z in effective types, ensures that wages are convex in effective types, translating into the property that wages are supermodular in skill and hours, $\partial^2 w(\tilde{x}_i)/\partial h_i \partial x_i > 0, i \in \{f, m\}$. It is clear from (10)-(13) and (17) that any stable equilibrium that satisfies these two conditions will feature PAM on the marriage market and labor hours that are increasing in own and partner's type. Together, with the assumption that z is supermodular (which was imposed throughout), inducing PAM in the labor market, all three requirements for monotone equilibrium are satisfied, see Definition 2.

We provide details on stability of equilibrium in Appendix B.3.1, where we show that if an equilibrium exists there is at least one stable one.

5.3 Properties of Monotone Equilibrium

We now give more intuition for the properties of monotone equilibrium in Proposition 1. We also qualitatively link these properties to our stylized facts, before going to our quantitative analysis below.

Comparative Statics in Cross-Section of Couples. We first compute comparative statics in the cross-section of couples, taking equilibrium wage and matching functions as given. That is, how do outcomes change if we move up the type distribution of men or women? For concreteness, we compare two women with $x'_f > x_f$ and analyze the difference in their marriage sorting, household allocation of consumption and hours worked, and labor sorting, as well as the differences in their partners' outcomes.

Corollary 1 (Comparative Statics in the Cross-Section)**.** *Assume a monotone equilibrium. Compare two women with $x'_f > x_f$. Then:*

1. *Marriage Market: woman x'_f has a 'better' partner, $\eta(x'_f) > \eta(x_f)$ (PAM);*
2. *Household Allocation: Both partners in couple $(x'_f, \eta(x'_f))$ work more labor hours and less at home compared to couple $(x_f, \eta(x_f))$ and have thus less public consumption; in turn, both partners $(x'_f, \eta(x'_f))$ have more private consumption and also a higher utility than $(x_f, \eta(x_f))$;*
3. *Labor Market: Both partners $(x'_f, \eta(x'_f))$ have higher effective types and therefore better firm matches compared to $(x_f, \eta(x_f))$ (PAM in $(y, \tilde{x}_i)$ and also in (y, x_i)); and they have higher wages.*

Despite its stylized-ness, the monotone equilibrium of our model can account for several salient features of the data in a qualitative way.

First, there is PAM in the marriage market (resembling Table 1). At the heart of why couples match positively is the supermodular home production function, inducing supermodularity of marriage value Φ in skill types. Based on (17), increasing, say, the female skill x_f increases her wage and thus the marriage value $\Phi(x_f, x_m)$. Additionally augmenting her partner's skill x_m increases the marriage value even further if this change boosts her marginal wage return. Since there is no direct effect of male type on female wage, this effect must stem from a change in female labor hours. An increase in the partner's skill increases his labor hours if wages are complementary in skill and hours. This increase in his labor hours implies a decrease in his home hours. Because home hours are complementary among spouses if $p_{12} > 0$, she also reduces her home hours and increases her labor hours. This has a positive effect on Φ_{x_f} if *her* wage is complementary in her skill and labor hours, which is true if z is convex.

Second, labor hours are complementary within couples, meaning that labor hours are increasing in own skill and partner's skills: Increasing, say, the female type, not only makes her own labor hours go up but also induces her partner to work more. As a result, partners' hours co-move. There are two reasons behind this result. First, for a *given male partner type* x_m (for exogenous marriage matching), an increase in the female skill increases her labor hours if z is convex. But this reduces her home hours, inducing her partner to also work less at home and more in the market due to $p_{12} > 0$. As a result, *both* partners increase their labor hours as the female skill improves. Second, this complementarity gets an extra kick under endogenous marriage market sorting: If an increase in her skill x_f leads to a better partner x_m – which is the case under PAM in the marriage market – then this better partner puts more labor hours and less home hours (given wages are supermodular in (x_m, h_m) or given that z is convex). And since $p_{12} > 0$, the wife adjusts in the same direction (less home hours and more labor hours), reinforcing the co-movement of hours within a couple. Thus, PAM on the marriage market fuels the complementarity of hours within couples – a feature we saw in the data (Figure 3).

Third, more skilled couples have higher effective types since they work more, and therefore obtain a more productive labor market match. This implies there is PAM not only between firm productivity and effective types $(\tilde{x}, y)$ – which happens here by assumption of supermodular z – but also between firm productivity and raw education (x, y), another feature that is in line with the data (Figure 1(a)).

Relation between Labor Market and Marriage Market Sorting. The unique feature of our model is the link between labor and marriage market equilibrium and, in particular, labor and marriage sorting. This link becomes most transparent when highlighting how the labor market matching function depends on the marriage market matching function and vice versa.

Start with μ. Consider the total derivative $d\mu(\tilde{x}_i)/dx_i$, which – when positive – indicates PAM on the labor market in (x, y) (where we consider the matching between firms' y and *skills*, not effective skills):

$$\frac{d\mu(\tilde{x}_i)}{dx_i} = \mu' \left(e_x + e_h \left(\frac{\partial h_i}{\partial x_i} + \frac{\partial h_i}{\partial x_j} \eta' \right) \right) \tag{18}$$

Equation (18) illustrates how labor market sorting $d\mu(\tilde{x}_i)/dx_i$ depends on marriage market sorting η'. When marriage market matching is PAM, $\eta' > 0$, then $d\mu(\tilde{x}_i)/dx_i > 0$ (given that hours of spouses are complementary $\partial h_i/\partial x_j > 0$): the matching between (x, y) is given by an increasing function. The intuition is straightforward. PAM on the marriage market induces individuals with higher x_i to have a better partner x_j and therefore to work more hours, which translates into a higher effective type $\tilde{x}_i$ and thus a better labor market match $\mu(\tilde{x}_i)$ compared to when marriage market sorting is not PAM.

In a stylized way, this property of the monotone equilibrium is related to our empirical fact that labor market sorting is 'maximized' for positively sorted couples (Figure 1(b)).

Next consider η, highlighting that the link between marriage and labor market sorting also goes in the other direction: the cross partial $\Phi_{x_f x_m}$ (which determines the sign of marriage sorting) depends on labor market matching μ through the wage function. In (17), we can express the complementarities of wages in (h_f, x_f) as $\partial^2 w/\partial h_f \partial x_f = w_{\tilde{x}_f \tilde{x}_f} e_{h_f} e_{x_f} + w_{\tilde{x}_f} e_{x_f h_f} = (z_{\tilde{x}_f \tilde{x}_f} + z_{\tilde{x}_f y} \mu') e_{h_f} e_{x_f} + z_{\tilde{x}_f} e_{x_f h_f}$. Positive labor market sorting $\mu' > 0$ amplifies the convexity of the wage function in effective skills, strengthening the complementarity of wages in skills and hours and thus the complementarity of wage in partners' types (x_f, x_m), which results in $\Phi_{x_f, x_m} > 0$. Thus, labor market sorting affects marriage market sorting.

Labor Market Sorting across Gender. An interesting feature of our model is that it can generate a gender gap in labor market sorting: If the home production function is such that women spend relatively more time at home (e.g. if they are relatively more productive at home), then men will be 'better' matched on the labor market compared to women of the same skill. Thus, our competitive model can generate a gender gap in sorting and wages even in the absence of discrimination or differential frictions.

To see this, consider labor market sorting in terms of firm productivity and skills (y, x_i), and how it varies across gender $i \in \{f, m\}$. Consider a man and a woman with $x_f = x_m$. We say that x_m is 'better sorted' than x_f if $\mu(e(h_m, x_m)) > \mu(e(h_f, x_f))$. For each man and woman of equal skills, $x_f = x_m$, men x_m is better sorted if he works more hours on the labor market, $h_m(x_m, \eta^{-1}(x_m)) > h_f(\eta(x_f), x_f)$, which helps rationalizing our finding in the data on the differential sorting of men and women in the labor market (Figure 1(a)). Further controlling for hours worked, $h_m(x_m, \eta^{-1}(x_m)) = h_f(\eta(x_f), x_f)$, closes the sorting gap in the model and considerably shrinks it in the data (Figure 5).

5.4 Non-Monotone Equilibrium

The monotone equilibrium captures – albeit in a stylized way – several salient features of today's data. Some features of the monotone equilibrium, in particular the complementarity of spouses' hours, may be in contrast to the traditional and more standard view of the household, which relies on specialization. Historically, it is plausible that a different equilibrium was in place, in which partners' hours in home production were substitutable and where positive sorting on the marriage market was less pronounced, giving rise to specialization of household members. We capture this different regime in a stylized way by an equilibrium that – with some abuse – we call *non-monotone equilibrium* and we highlight the role played by properties of the home production function. We define a non-monotone equilibrium as the monotone one with two differences. First, there is NAM in the marriage market. And second, labor hours are *decreasing* in partner's type.

Proposition 2 (Non-Monotone Equilibrium)**.** *If p is strictly submodular and z weakly convex in effective types $\tilde{x}$ and supermodular, then any stable equilibrium is non-monotone.*

The proof is in Appendix B.3.5. This result highlights the key role of home production complementarities/substitutabilities in shaping equilibrium. Making hours at home substitutable, $p_{12} < 0$, gives rise to an equilibrium that relies on 'specialization', where a better partner puts more labor hours while own labor hours go down. At the same time, the partner spends less time in home production while own home production increases. This specialization within the household is clearly a force towards NAM in the marriage market, which indeed materializes. The reason is that increasing the partner's type pushes own labor hours *down*, hurting own labor market prospects especially for skilled individuals. Skilled individuals then prefer to match with less skilled partners.

The only feature that both equilibria have in common is PAM on the labor market not only in $(y, \tilde{x})$ but, importantly, also in (y, x). This follows from the positive slope of the labor matching function (18).[19]

Thus, complementarity vs. substitutability of home hours shapes equilibrium. In particular, $p_{12} \lesseqgtr 0$ determines whether marriage partners match positively and whether their hours – both at home and at work – are complementary. The monotone equilibrium captures 'progressive' times while the non-monotone one reflects a 'traditional' division of labor. To our knowledge, this mechanism in which home production complementarities are the key determinant of both marriage and labor market outcomes is new in the literature. We now investigate this property of home production and mechanism quantitatively.

6 Quantitative Model

The advantage of our parsimonious model is that we obtain analytical properties that clarify the model mechanism. However, in order to evaluate its quantitative importance and run counterfactuals, we

[19] Further note that the distribution of effective types $\tilde{N}(X) = \frac{1}{2}N_f(\gamma_f^{-1}(X)) + \frac{1}{2}N_m(\gamma_m^{-1}(X))$ will be pinned down just like in the monotone equilibrium since own labor hours are still increasing in own type so that γ_i is still monotone.

need to augment the model so that it can match the data. We do so by implementing some minimal departures from our baseline model in order to preserve its mechanism and underlying intuition.

6.1 Set-Up and Decisions

Our objective is to build a quantitative version of our baseline model that can match key facts of the data while minimally departing from the baseline model. To this end, we augment the model in each of the three stages, marriage market, household decision stage, and labor market, by including shocks that allow us to account for important features of the data: imperfect sorting and non-participation on both marriage and labor markets as well as heterogeneity in hours choices across couples of the same type.

We make three changes: First, in order to reflect the empirical heterogeneity in matching within education types in both the marriage and the labor market, we augment individuals' education/skill x by an idiosyncratic i.i.d. productivity component ν. We assume that individuals are characterized by *discrete* human capital $s := k(x, \nu) \in \mathcal{S}$ (where $\mathcal{S}$ is a discrete set), distributed according to cdf N_s. In the labor market, the match relevant attribute of a worker is her effective human capital $\tilde{s} := e(h, s)$ (instead of $\tilde{x}$), whose distribution we denote by $\tilde{N}_s$. So the firm's problem becomes

$$\max_{\tilde{s}} \quad z(\tilde{s}, y) - w(\tilde{s}).$$

Second, we account for heterogeneity in labor supply within couple and single types (note that in the marriage market individuals now choose partners based on their human capital s, instead of x in the baseline model) by introducing idiosyncratic labor supply shocks. We denote by $\delta_m^{h_m}$ and $\delta_f^{h_f}$ the idiosyncratic preference of man m and woman f for hours alternative $h_i, i \in \{f, m\}$. In this quantitative version of our model, hours are *discrete* elements of the choice set $\mathcal{H}$, $h_i \in \mathcal{H}$. Each decision-maker (single or couple) draws a vector of labor supply shocks, one for each discrete alternative h_i. These shocks realize after marriage.

In the household decision stage, partners now maximize utility *plus* labor supply shock, so the cooperative household problem reads:

$$\begin{aligned} &\max_{c_m, c_f, h_m, h_f} \quad u(c_m, p^C(1-h_m, 1-h_f)) + \delta_m^{h_m} \\ s.t. \quad & c_m + c_f - w(\tilde{s}_m) - w(\tilde{s}_f) = 0 \\ & u(c_f, p^C) + \delta_f^{h_f} \geq \bar{v}. \end{aligned} \tag{19}$$

where we introduce the notation p^C for the home production technology of couples.

Similarly, the consumption-time allocation problem of singles is given by

$$\max_{c_i, h_i} \quad u(c_i, p^U(1-h_i)) + \delta_i^{h_i} \tag{20}$$

where we denote by p^U the home production function of singles (Unmarried). This change (introduction of δ^h) will help us account for heterogeneity of hours *within* (s_f, s_m)-type couples and *within* s_i-type singles and for non-participation in the labor market (even if u turns out to be concave in the estimation).

Third, to accommodate the fact that marriage market matching on *human capital* s may not be perfectly assortative, we introduce an idiosyncratic taste shock for partners' s-types. We denote by β_m^s and β_f^s the idiosyncratic taste of man m and woman f for a partner with human capital $s \in \{\mathcal{S} \cup \emptyset\}$ where $s = \emptyset$ indicates the choice of remaining single. Each individual draws a vector of taste shocks, one for each discrete alternative s. Individuals in the marriage market here value potential partners not only for their impact on the economic joint surplus (as before) but also for their impact on the non-economic surplus (which depends on preference shocks β_f^s or β_m^s). The marriage problem of a man with s_m reads:

$$\max_s \quad \Phi(s, s_m, v(s)) + \beta_m^s$$

where the choice of marrying a woman with human capital type $s = s_f$ needs to be weighed against the choice of remaining single $s = \emptyset$ $(\Phi(\emptyset, s_m, v(\emptyset))$ denotes the economic value of remaining single).

Similar to the baseline model under quasi-linear utility, Φ is the value of the household problem, which captures the *economic* value from marriage. Different from the baseline model, due to the introduction of labor supply shocks that have not yet realized at the time of marriage, Φ is the *expected* economic surplus from marriage. The expectation is taken over the different hours alternatives of the couple whose choice probabilities are pinned down at the household stage. See Appendix C for details. This change (introduction of β^s) helps us account for imperfect marriage market sorting between human capital types (s_f, s_m) and for non-participation/single-hood. Contrary to the baseline model, marriage market matching is no longer pure (due to both the discreteness of the match attribute s and the idiosyncratic shocks β^s), which is why $\eta : \{\mathcal{S} \cup \emptyset\}^2 \to [0, 1]$ here denotes the matching *distribution* (as opposed to the matching function).

6.2 Functional Forms

To solve the model numerically and to estimate it, we assume the following functional forms.[20]

The production function on the labor market is given by

$$z(\tilde{s}, y) = A_z \tilde{s}^{\gamma_1} y^{\gamma_2} + K$$

where A_z is a TFP term, (γ_1, γ_2) are the curvature parameters reflecting the elasticity of output with respect to skill and firm productivity, and K is a constant.

[20] The functional forms for the production function and home production function below will not comply with all the assumptions we made in the analytical model to show that whenever an equilibrium exists there is at least one stable one. However, this is not an issue as the conditions we derived there were *sufficient*, not necessary. And in the quantitative model, we never ran into issues related to existence or stability.

For couples, the public good production function is assumed to be CES

$$p^C(1-h_m, 1-h_f) = A_p\Big[\theta\big(1-h_f\big)^{\rho} + (1-\theta)\big(1-h_m\big)^{\rho}\Big]^{\frac{1}{\rho}}$$

where A_p is the TFP in home production, θ is the relative productivity of a woman, and ρ is the complementarity parameter that determines the elasticity of substitution, $\sigma := 1/(1-\rho)$ (where $\sigma <$ $(>)1$ indicates that spouses' home hours are strategic complements (substitutes)). In turn, we assume that home production for single women and men are respectively given by

$$\begin{aligned} p^U(1-h_f) &= A_p\theta\big(1-h_f\big) \\ p^U(1-h_m) &= A_p(1-\theta)\big(1-h_m\big). \end{aligned}$$

The utility function of individual i is given by

$$u(c_i, p) = c_i + p$$

where $p \in \{p^C, p^U\}$ for spouses and singles and where we assume that both men and women have the same preferences. We adjust the private consumption of singles by the McClemens factor (singles consume .61 of their wage income relative to couples (Anyaegbu, 2010)).

Human capital as a function of skill and productivity shock is given by

$$s \propto x + \nu$$

where we assume that s is proportional to the sum of observed skill and unobserved productivity.

We allow workers' effective human capital to depend on gender:

$$\begin{aligned} \tilde{s}_f &= \frac{1-exp(-\psi s_f h_f)}{\psi} \qquad (21) \\ \tilde{s}_m &= s_m h_m \end{aligned}$$

where $\tilde{s}_f \to \tilde{s}_m$ as $\psi \to 0$. Effective skills of women are decreasing in ψ, which allows for a labor market penalty for women that could reflect gender discrimination or productivity differences.

In turn, we measure job productivity y directly in the data, so there is no need for parameterization.

Finally, both marriage taste shocks and labor supply shocks follow extreme-value type-I distributions:

$$\begin{aligned} \beta^s &\sim \text{Type I}(\bar{\beta}^t, \sigma_\beta^t) \qquad \text{for } t \in \{C, U\} \text{ and } s \in \{\mathcal{S} \cup \emptyset\} \\ \delta^{h^t} &\sim \text{Type I}(\bar{\delta}, \sigma_\delta) \qquad \text{for } t \in \{C, U\} \text{ and } h^t \in \mathcal{H} \end{aligned}$$

where we allow for different preference shock distributions for marriage partners and singles. We normalize the location parameter of both labor supply and marriage preference shocks to zero, $\bar{\delta} = \bar{\beta}^C = 0$ and we denote by σ_β^t and σ_δ the scale parameters of marriage/single and labor supply shocks respectively, where we normalize $\sigma_\beta^U = 1$. The type index t indicates *couples* or *singles* where we specify:

$$\delta^{h^t} = \begin{cases} \delta^{h_i}, i \in \{f, m\} & \text{if } t = U \\ \delta^{h_f} + \delta^{h_m} & \text{if } t = C. \end{cases}$$

We assume that when making hours choices, a decision-making unit draws *a single* labor supply shock, δ^{h^t}, that is extreme value distributed. In the case of singles, the decision-making unit is just one person and hence, as is standard, this agent draws a shock for each hours alternative. In the case of a household however, the decision-making unit is the couple. Therefore, a household draws a single shock for each *joint* time allocation of the spouses (equivalently, the sum of the spouses' shocks is assumed to be extreme-value distributed). We make this adjustment to the standard setting, where *each* individual agent draws an extreme-value shock when making a discrete choice, in order to obtain tractable choice probabilities that help with computation and identification of the model.[21]

6.3 Model Solution

Appendix C describes the solution of the quantitative model in detail. Our numerical solution consists of solving a fixed point problem in the wage function w (or, equivalently, in the hours function (h_f, h_m)). For any given wage function, individuals make optimal marriage and household choices as well as labor market choices. Labor market choices then give rise to a new wage function that, in equilibrium, needs to coincide with the initially postulated wage function. We design and implement a search algorithm that iterates between the problem of households and firms, producing a new wage function at each round, and that halts when the wage function satisfies a strict convergence criterion. Our procedure ensures that at convergence, both the labor *and* the marriage market are in equilibrium and households act optimally.

An interesting challenge in our fixed point algorithm is that when partners determine whether a particular hours choice is optimal (which – as discussed – can be understood as an 'investment' in effective skills), they must compare the payoff of this investment with alternative investments.[22] But the competitive wage only determines the price for *equilibrium* investment. In order to obtain the off-equilibrium wages without significantly perturbing the equilibrium wages, we use a *tremble strategy*. Trembling is used in game theory to pin down off-equilibrium choices. Here, we apply this idea to our

[21] Our approach is closely related to the approach taken by Gayle and Shephard (2019) who use this logic of households drawing one shock for each of the spouses' hours combinations for their numerical solution. Here we make this assumption upfront. The reason why it proves useful is that under TU, the household problem then becomes:

$$\max_{c_m, c_f, h_m, h_f} u(c_m, p(1-h_f, 1-h_m)) + u(c_f, p(1-h_f, 1-h_m)) + \delta^{h_f} + \delta^{h_m} \tag{22}$$

where we obtain the standard conditional logit choice probabilities for *joint* hours allocations given that the *sum* of shocks $\delta^{h_f} + \delta^{h_m}$ is extreme-value distributed.

[22] This issue is similar to the one in Cole, Mailath, and Postlewaite (2001) who study bargaining in a matching problem with pre-match investment.

context. We postulate that a small fraction of agents are tremblers who make a mistake by choosing off-equilibrium hours. This ensures that also off-equilibrium choices will be priced and individuals can compare *all* investment choices when solving the household problem. While trembling is a well-known concept in game theory, we believe the application to matching markets with investment is new.

7 Estimation

The overarching objective of the estimation is: (1) to assess whether partners' home production time is complementary or substitutable; (2) to quantify the model mechanism that links marriage and labor market outcomes and how it shapes within- and between-household inequality as well as gender disparities in labor market outcomes across time and space; (3) to perform comparative statics with respect to parameters impacted by policy or gender norms as well as by technological change to understand the effectiveness of these changes in reducing inequality.

7.1 The Data

We again use data from the German SOEP combined with information from the dataset of occupational characteristics (BIBB). The challenge is to bridge our static model with the panel data which is intrinsically dynamic and contains life-cycle features. We propose the following compromise between data and model. For the estimation of worker unobserved heterogeneity (which, as we explain below, will be done outside of the model), we exploit the full panel structure in order to make use of techniques that control for unobserved time-invariant characteristics. In turn, for the structural estimation of the model we construct a dataset that features each individual only once while accounting for his/her 'typical' outcomes. To be able to assess the typical outcomes, we focus in our baseline analysis on a restricted time period (2010-2016) so that each individual is captured in only one life-cycle stage. We consider each individual as one observation and generate 'summary measures' of this life-cycle stage for each individual. We focus on observations that are not too different in age (25-50). We then define for each individual the 'typical' occupation (based on a combination of tenure and job ladder features), 'typical' labor hours and 'typical' wage in that occupation, and 'typical' home hours while holding that occupation, as well as the 'typical' marital status. In line with our model, we only consider those individuals who are either married/cohabiting or have never been married and are thus single. We drop divorced and widowed people because they likely behave differently than the singles in our model. Our final sample contains 5,153 observations, 50% of which are men. See Appendix D.2 for the details on the sample construction.

7.2 Identification

We need to identify 11 parameters and two distributions. We group the parameters into 5 groups and discuss the identification group-wise. We have parameters pertaining to the home production function (θ, ρ, A_p), the production function $(\gamma_1, \gamma_2, A_z, K)$, labor supply shock and marriage preference shock distributions $(\sigma_\delta, \sigma^C_\beta, \bar{\beta}^U)$ and a wedge (ψ), which could be interpreted as a productivity wedge (reducing women's productivity relative to men) or a discrimination wedge (firms/jobs discriminate against women). Finally, we have the distributions of workers' human capital and jobs' productivity (N_s, G). We provide formal identification arguments in Appendix E.1 and summarize the logic here. Note that our estimation will be parametric. Nevertheless, we consider it useful to lay out non- and semi-parametric arguments in order to understand the source of variation in the data that pins down our parameter estimates. We will also clarify which parametric restrictions (mainly pertaining to the shock distributions) are important.

The home production function, and thus (θ, ρ, A_p), is identified from households' choice probabilities of the different home hours combinations for husband and wife of different s-types. The formal identification uses the assumption that the labor supply shock for different hours combinations of husband and wife follows a type-I extreme-value distribution. The intuition behind the argument is that the home production function dictates home hours choices of a couple.

The production function, and thus the tuple $(\gamma_1, \gamma_2, A_z, K)$, is identified from wage data. In our competitive environment, there is a tight link between wages and the marginal product (and thus technology), which allows us to do so. The curvature and TFP parameters, $(\gamma_1, \gamma_2, A_z)$, can be identified from the first derivative of the wage function (the marginal product), following arguments from the literature on the identification of hedonic models (Ekeland, Heckman, and Nesheim, 2004). In turn, the constant in the production function (K) can be identified from the minimum observed hourly wage.

The triple $(\sigma_\delta, \sigma^C_\beta, \bar{\beta}^U)$ associated with our three shock distributions is identified as follows (note that in each distribution we make one normalization choice). In the absence of any labor supply shocks, any two couples of the same type (s_f, s_m) would choose the same combination of hours. Hence, the variation in hours choices *by couple type* pins down the scale parameter of the labor supply shock distribution σ_δ. Similarly, in the absence of any preference shocks for marriage partners $(\sigma^C_\beta = 0)$, the model would produce perfect assortative matching on the marriage market with $corr(s_f, s_m) = 1$. The extent of marriage market sorting and mismatch (i.e. how partner choices vary with types) identifies the scale parameter of preference shocks for partners, σ^C_β. Note that the standard result in the literature that the scale parameter is not identified separately from the utility associated with the discrete choices (e.g. Keane, Todd, and Wolpin (2011)) does not apply in our context. The reason is that we are able to identify utility in a prior step from household labor supply choices. Importantly, we do not exploit variation in partner choices to identify the utility and therefore, this variation can be used to identify the scale of the marriage shock distribution. And the location of the preference shock for being single, $\bar{\beta}^U$, is identified from the fractions of singles across types, or equivalently, by the choice probabilities of remaining single by different types. The extreme-value assumption of the shock distributions is important for obtaining tractable choice probabilities that we use in our formal identification argument.

The productivity or discrimination wedge of women, ψ, is identified by the hourly gender wage gap *conditional on hours and s-type*. If there was no wedge, $\psi = 0$, women and men with the same hours-human capital bundle should receive the exact same wage. A gap can only be rationalized by $\psi \neq 1$.

Finally, the worker and job heterogeneity will be identified directly from the data. We use the empirical distributions of workers' human capital and occupations' productivity for (N_s, G).

We now summarize our identification results.

Proposition 3 (Identification)**.** *The production function and women's productivity wedge are identified. Under the assumption that labor supply shocks follow a type-I extreme value distribution, the home production function and the scale of labor shock are identified. If marriage shocks also follow a type-I extreme value distribution and if in addition Assumption D1 (Appendix) holds, then the parameters of the marriage shock distribution are identified.*

Our identification result informs the moments we choose to pin down our parameters. The first set of moments (5) relates to the division of labor and to the complementarity of hours within households (correlation of home production hours and human capital type, fraction of home production done by wife, correlation of spouses' home production hours, fraction of couples with both spouses working full time) and is supposed to identify the home production function. The second set of moments (4) relates to the hourly wage distribution (mean, variance, 90-10 and 90-50 inequality) and is meant to identify the production function. The third set of moments (2) concerns the marriage market (correlation of spouses' human capital types and fraction of single men) and is used to identify the marriage shock parameters. The fourth set of moments (9) relates to the hours variation across couples of given human capital (variance of wife's home hours for a given couple type, where we select 9 couple types), which identifies the scale of the labor supply shock. The last set of moments (2) relates to the gender wage gap conditional on the same human capital-work hours of men and women and identifies the female labor wedge. In total we have 22 moments, see Table 14 in Appendix E.1 for the details.

7.3 Two-Step Estimation

We propose a two-step estimation procedure. The first step estimates worker and job heterogeneity as well as the constant in the production function *outside* of the model. In a second step, given the worker and job distributions, we estimate the structural parameters of the primitives *within* the model.

7.3.1 First Step: Estimation Outside the Model

The first step of our estimation concerns the estimation of worker types (x, ν) and job types y. Except for x (education), these types are in principal unobserved to us. Moreover, even if we observe the educational group of a worker, it has no natural unit of measure in terms of productivity. Here we develop a strategy to estimate these types outside of the structural estimation. We then input the estimated type distributions into the model for the second step of the estimation.

Estimation of Worker Types. Let $ed \in \{hs, voc, c\}$ be the education level of a person (standing for *h*igh *s*chool or less, *voc*ational training and *c*ollege) and ν be his/her ability. Based on our theory, we specify an empirical model for hourly wages, where hourly wages are a function of effective types (which in turn are a function of education, ability and hours worked). That is, we assume the empirical log hourly wage of individual i at time t is given by

$$\ln w_{it} = \nu_i + \sum_{ed \in \{voc, c\}} \alpha^{ed} x_{it}^{ed} + \beta_1 h_{it} + \beta_2 h_{it}^2 + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{23}$$

where x_{it}^{ed} are indicators for the education group of an individual (meant to capture x in our model). This indicator is equal to one if individual i belongs to education group ed at time t. Coefficient α^{ed} identifies the 'value' of education ed in terms of log wage units where positive returns to education will be indicated by $0 < \alpha^{voc} < \alpha^c$. While these coefficients indicate the average return to education for all individuals in a certain category, ν_i is a person fixed effect meant to capture unobserved time-invariant ability, with model counterpart ν. So ν_i is a person specific deviation from the mean wage of his/her education group. In turn, h_{it} denote the hours worked in the market in a typical week (capturing the time 'investment' in labor productivity in our model). Finally, Z_{it} are additional time-varying controls for the individual (e.g. labor force experience and household size), κ_s and ρ_t are state and time fixed effects, and ϵ_{it} is a mean-zero error term.[23]

We thus make use of the dynamic features of the data (panel) to estimate individual *time-invariant* heterogeneity ν. For computational tractability, we divide individuals into two groups depending on their ν (above and below the median). We compute the level of the low and the high ability, ν_L and ν_H, based on the average fixed effect in their group. Hence, individuals belong to one of six (three education types times two ability types) human capital bins. Given our estimates, we are able to order individuals by their human capital $\widehat{s}_i = \widehat{\alpha}^{ed} x_i^{ed} + \widehat{\nu}_i$ (where we use an individual's 'typical' education group), giving us a global ranking of worker types. We use the empirical cdf over $\widehat{s}_i$ as our estimate for workers' human capital distribution N_s.

There are three challenges in implementing (23): First, hours worked are endogenous, so we need an instrument. In our model, there is a systematic relationship between the hours worked of an individual and the hours worked by his/her partner, so we use the partner's hours and squared hours as well as an indicator for whether the partner is present as instruments for own hours. Identification comes from changes in spousal labor hours over time. The exclusion assumption is that conditional on the individual fixed effect and education, partner's hours are exogenous in the wage regression and that partner's labor hours impact own wage *only* through own labor hours, which is satisfied in our model.

Second, we only observe wages for those who work and labor market participation is not random. To account for selection, we apply a Heckman selection correction. To do so, we need an instrument that affects participation but is excluded from wage regression (23). Since the variation in participation in our sample is mainly coming from females, we use the 'progressiveness' in a narrowly defined demographic

[23] We do not include occupation fixed-effect since in our model, conditional on $\tilde{s}$ (which we control for here by controlling for (x, ν, h)), the wage should not depend on any occupational characteristic in our competitive equilibrium. But even doing so – which we have done for robustness – does not significantly change the impact of the x's or ν's on the hourly wage.

cell. We proxy progressiveness by the fraction of women employed in each cell and use it as an IV at the selection stage. Third, even when we control for selection, using (23) we can only estimate types for those individuals who are employed for at least two periods in our panel. We therefore impute the fixed effects for those who we never observe participating using the *multiple imputation* method.

We provide the details on the sample as well as on the IV, selection and imputation in Appendix E.2. The estimation results of (23) are in Table 9 and details on the skill distribution are in Table 10.

Estimation of Occupation Types. The empirical counterpart of our model's firms are occupations (we do not observe firms in the GSOEP). We here describe how we measure the occupational types y in the data. Our objective is to obtain a one-dimensional ranking of occupations in terms of their task complexity, as a proxy for their productivity. Ideally, we want to use information that does not heavily rely on wages since wages are a function of the match, not just of the occupational type.

Our main dataset is the BIBB (comparable to the O*NET in the US), giving extensive information on task use in each occupation, were we focus on 16 tasks measured on a comparable scale (e.g. prevalence of problem solving, difficult decisions, responsibility for others etc.). We measure the occupations' types in two steps. First, we use a Lasso wage regression to select the important/pay-off relevant tasks. In a second step, we run a principal component analysis (PCA) to reduce the task dimensions further to a single one, where we select the first principal component as our one-dimensional occupation characteristic. Since this measure has only an ordinal interpretation, we define the occupational type by its rank in the distribution of task complexity. To this end, we transform the types coming out of the PCA ($\widehat{y}$) into their percentiles $y = \widehat{G}(\widehat{y})$, which are uniformly distributed. We use y as our measure for occupational types that we input in the model, noting that our production function is flexible enough to capture the true output as a function of non-transformed types $\widehat{y}$.[24]

It is important to note that we use the wage regression only to select the relevant tasks but we do not use the estimated coefficients. See Appendix E.3 for the details of this approach and for the alternative approaches that we pursued for robustness and which have led to similar results.

Estimation of Constant in Production Function. We assume that the constant in the production function is not shared between workers and firms but accrues to the worker in form of a minimum wage (the wage of someone with the lowest human capital who will be matched to the lowest productive occupation $y = 0$). This way, we obtain $K = 8$.

7.3.2 Second Step: Internal Estimation

There are ten remaining parameters of the model, $\Lambda \equiv (\theta, \rho, A_p, \gamma_1, \gamma_2, A_z, \sigma_\delta, \sigma_\beta^C, \bar{\beta}^U, \psi)$, that are not directly identified from the data in Step 1. They are disciplined by the moments described above, which are chosen based on our formal identification argument discussed in Section 7.2. To estimate those parameters, we apply the method of simulated moments (McFadden, 1989; Pakes and Pollard,

[24]This argument is standard in the literature, see for instance, Hagedorn, Law, and Manovskii (2017).

1989). For any vector Λ of structural parameters the model produces the 22 moments outlined above, $mom_{sim}(\Lambda)$, that will also be computed in the data, mom_{data}. We then use a global search algorithm to find the values of parameters that minimize the distance between simulated and observed moments. Formally, the vector $\hat{\Lambda}$ of ten estimates solves

$$\hat{\Lambda} = \arg\min_{\Lambda} \quad [mom_{sim}(\Lambda) - mom_{data}]'\mathcal{V}[mom_{sim}(\Lambda) - mom_{data}]$$

where $\mathcal{V}$ is specified as the inverse of the diagonal of the covariance matrix of the data.

7.4 Results and Fit

We report the parameters that we fixed outside of the structural estimation in Table 2. It contains our value for the minimum hourly wage of 8 Euros (K), and the three normalizations pertaining to parameters $(\bar{\delta}, \bar{\beta}^C, \sigma^U_\beta)$ of the shock distributions.

Table 2: Exogenously Fixed Parameters

Parameter		**Value**
Hourly Minimum Wage	K	8.00
Labor Supply Shock (location)	$\bar{\delta}$	0.00
Preference Shock for Partners (location)	$\bar{\beta}^C$	0.00
Preference Shock for being Single (scale)	σ^U_β	1.00

The estimated parameters are in Table 3. The home production function indicates that women are significantly more productive at home than men ($\theta = 0.91$). The (large) empirical share of wife's home production calls for this relatively high productivity at home. Importantly, our estimates indicate that spouses' hours at home (and therefore also in the labor market) are complements with $\rho = -0.53$, pushing the model towards the *monotone equilibrium* of the baseline model. The moment indicating a positive correlation of spouses' home hours informed this complementarity. In terms of labor market production function, our estimates indicate that it is concave in both the workers' effective skill as well as the jobs' productivity ($\gamma_1 < 1, \gamma_2 < 1$). Labor market TFP A_z is estimated to be higher than home production TFP A_p. The empirical gender wage gap *conditional* on hours and human capital calls for a female productivity/discrimination wedge, which we estimate to be $\psi = 0.93$, corresponding to an average productivity gap of -19.21%, across all types and hours.[25] Finally, regarding the marriage preference and labor supply shocks, our estimates indicate that the mean of the single shock is relatively high, $\bar{\beta}^U = 13.19$, needed to match the fraction of singles. In turn, the data calls for higher scale of labor supply shocks (to induce similar couples to choose different hours combinations) than of marriage preference shocks (which generates mismatch of married couples away from the perfectly assortative

[25] If we look at the average productivity across hours for a given s-type, the productivity gap implied by our ψ estimate ranges from -5.1% for the lowest s-type to -25.5% for the highest s-type. In line with the data, this implies higher gender wage gaps for workers with more human capital.

allocation in s-types), $\sigma_\delta > \sigma_\beta^C$. We also report the standard errors of the estimates.[26] The last column of the table reports the set of three moments that have the biggest impact on each parameter in estimation (Andrews, Gentzkow, and Shapiro, 2017 and Gayle and Shephard, 2019).[27] Our sensitivity analysis is in line with our identification arguments. For example, the fraction of female home hours m_1 is an important moment disciplining the female relative productivity in the household, θ. As another example, the female productivity wedge, ψ, is most related to the full time gender wage gaps m_{13}.

Table 3: Estimated Parameters

Parameter		Estimate	s.e.	Moments
Female Relative Productivity in Home Production	θ	0.91	0.02	m_1, m_3, m_{14-22}
Complementarity Parameter in Home Production	ρ	-0.53	0.20	m_1, m_3, m_{14-22}
Home Production TFP	A_p	23.68	0.48	m_1, m_5, m_{14-22}
Elasticity of Output w.r.t. $\tilde{s}$	γ_1	0.70	0.08	m_1, m_5, m_{14-22}
Elasticity of Output w.r.t. y	γ_2	0.17	0.12	m_1, m_5, m_{14-22}
Production Function TFP	A_z	35.92	2.77	m_1, m_4, m_{14-22}
Female Productivity Wedge	ψ	0.93	0.15	m_2, m_{13}, m_{14-22}
Preference Shock for being Single (location)	$\bar{\beta}^S$	13.19	0.16	m_5, m_{13}, m_{14-22}
Labor Supply Shock (scale)	σ_δ	7.09	0.32	m_1, m_3, m_{14-22}
Preference Shock for Partners (scale)	σ_β^C	0.19	0.03	m_4, m_5, m_{14-22}

Notes: s.e. stands for Standard Errors. *Moments* reports the three most important moments/groups of moments in estimation, based on our sensitivity analysis (details in footnote 27). $m_1, ..., m_{22}$ denote the 22 moments targeted in estimation, see Table 15 in Appendix E.4 for details.

Figure 7 summarizes the fit between model and data moments, where we plot all 22 moments (red dots indicate the level of these moments in the model) as well as their blue confidence interval of the corresponding data moment (computed from a bootstrap sample). We re-scaled some moments ($m_6 - m_9$) to be able to plot them all in the same graph. Table 15 in Appendix E.4 reports the fit in detail and indicates the moments corresponding to numbers 1-22. Our model achieves a good fit with the data, with nearly all model moments lying in the confidence interval of their data moments.

7.5 Model Validation

Our model achieves a good fit with the data along the moments we target. How does it perform along other dimensions of the data? In this section, we investigate the model fit regarding un-targeted moments. Our identification strategy mostly relied on targeting *average* moments instead of moments by detailed heterogeneity. We now assess whether our model can reproduce the rich patterns of marriage

[26]The covariance matrix of the estimator is computed as the sandwich matrix $Var = \left[D_m'\mathcal{V}D_m\right]^{-1}D_m'\mathcal{V}C\mathcal{V}'D_m\left[D_m'\mathcal{V}D_m\right]^{-1}$, where D_m is the 10×22 matrix of the partial derivative of moment conditions with respect to each parameter at $\Lambda = \hat{\Lambda}$ and C is the covariance matrix of the data moments.

[27]We compute the sensitivity of each parameter to moments as $|Sensitivity| = |-\left[D_m'\mathcal{V}D_m\right]^{-1}D_m'\mathcal{V}|$, defined by Andrews, Gentzkow, and Shapiro (2017), see previous footnote for notation. We report the three moments with the highest sensitivity in Table 3.

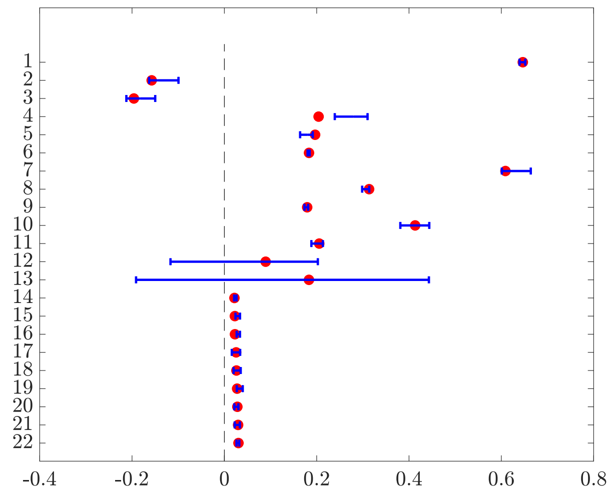

Figure 7: Model Fit: Model Moments (red) With Data Confidence Intervals (blue)

and labor market sorting and the link between them, as well as the role of hours worked in marriage and labor market sorting that we documented in Section 3.2.

Marriage Market Sorting. Below we display the matching frequency of marriages by three education types (low, medium and high) in the data and in the model. The main panel of Table 4 indicates the frequencies of different types of couples while the bottom row indicates the frequencies of single men by education and the right column gives the frequencies of female singles by education. Data frequencies are in parentheses. Note that we only targeted the *average* correlation of couples' *human capital* types (i.e. s-types), as s is the relevant matching characteristic on the marriage market in our model. We did not target marital matching on *education*, x, especially not the detailed matching frequencies. Nevertheless, the model is able to capture the most important features of educational sorting in the data: A considerable fraction of couples matches along the diagonal, while the off-diagonal cells indicate that mixed couples (especially high-low couples) are rare – a sign of positive assortative matching in education. Furthermore, our model captures that lower educated men and women (low plus medium) are more likely to be single compared to the highly educated.

Table 4: Marriage Matching Frequencies - Model and (Data)

	Low Educ Men	**Medium Educ Men**	**High Educ Men**	**Single Women**
Low Educ Women	0.0426 (0.0747)	0.0623 (0.0449)	0.0197 (0.0126)	0.0426 (0.0365)
Medium Educ Women	0.0656 (0.0860)	0.1574 (0.2159)	0.1115 (0.0695)	0.1148 (0.0747)
High Educ Women	0.0131 (0.0149)	0.0918 (0.0485)	0.0951 (0.0986)	0.0131 (0.0562)
Single Men	0.1016 (0.0527)	0.0689 (0.0714)	0.0000 (0.0430)	

Notes: *Low Educ* includes high school and vocational education with less than 11 years of schooling. *Medium Educ* is defined as vocational education with more than 11 years of schooling. *High Educ* is defined as college and more. Data frequencies are shown in parenthesis.

Labor Market Market Sorting. Despite the fact that we do not target labor market sorting in our estimation, our model matches the data reasonably well. We report in Figure 8 the labor market matching function for men (blue) and women (red), which is job productivity y as a function of individuals' human capital s, both in the data (left) and model (right). Our model captures that labor market sorting is PAM and that men are better matched for any given level of human capital. The model also captures that this differential sorting in the data is stronger at the top of the distribution, but it over-predicts the gender gap in sorting.

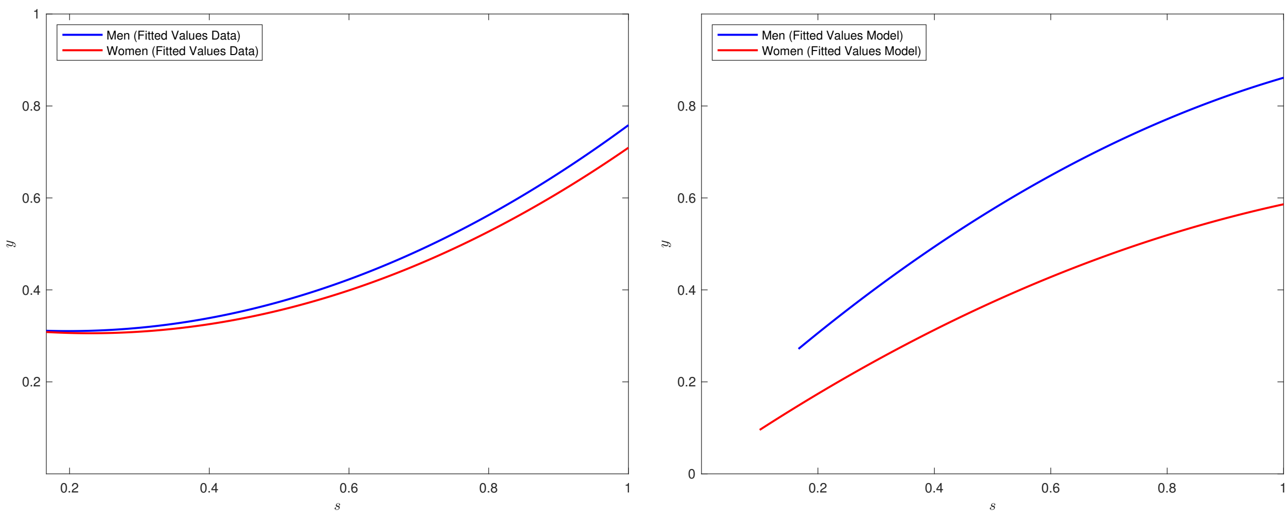

Figure 8: Labor Market Matching Function by Gender: a. Data and b. Model

Relationship between Labor Market Sorting and Marriage Market Sorting. We documented in Section 3.2 a striking link between labor market and marriage market sorting in the data, where labor market sorting is maximized when marriage market sorting is. Figure 9, which compares data and model, indicates that our model reproduces this pattern. Note that consistent with our quantitative model (and in contrast to Section 3.2), we here proxy marriage market sorting by spouses' differences in *human capital* s-types (as opposed to differences in education), also in the data. Similarly, labor market sorting is measured by the correlation of workers' human capital s and job complexity y.

Hours as the Link between Marriage Market Sorting and Labor Market Sorting. The key feature of our model is that marriage and labor markets are linked in equilibrium, namely through the household's time allocation choice. Here we show that the model replicates salient features of the data according to which hours are associated with both marriage and labor market outcomes. Figure 10 shows that both in data (left) and model (right), the correlation of spouses' home production hours is maximized when marriage market sorting is maximized (i.e. when partners' human capital is equalized $s_f = s_m$, indicated by the vertical line at 'zero'). This is a natural prediction of our model: Spouses of similar human capital can better align their hours and act on the hours complementarity in home production relative to couples whose human capital differences are large, which leads to more specialization in the household.

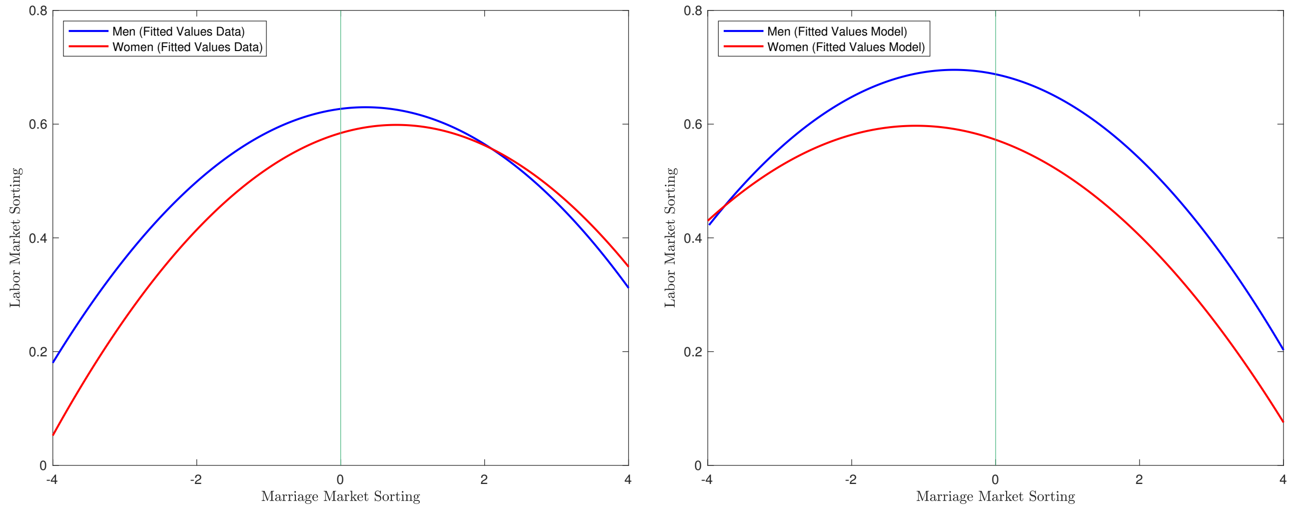

Figure 9: Labor Market Sorting and Marriage Market Sorting: a. Data and b. Model

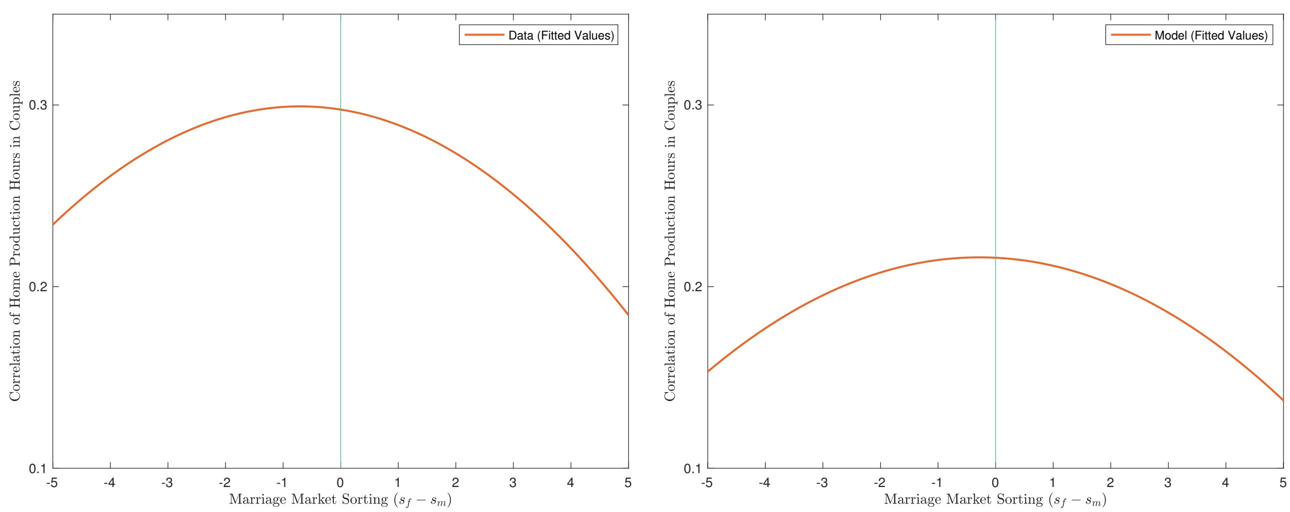

Figure 10: Spouses' Home Production Hours and Marriage Market Sorting: a. Data and b. Model

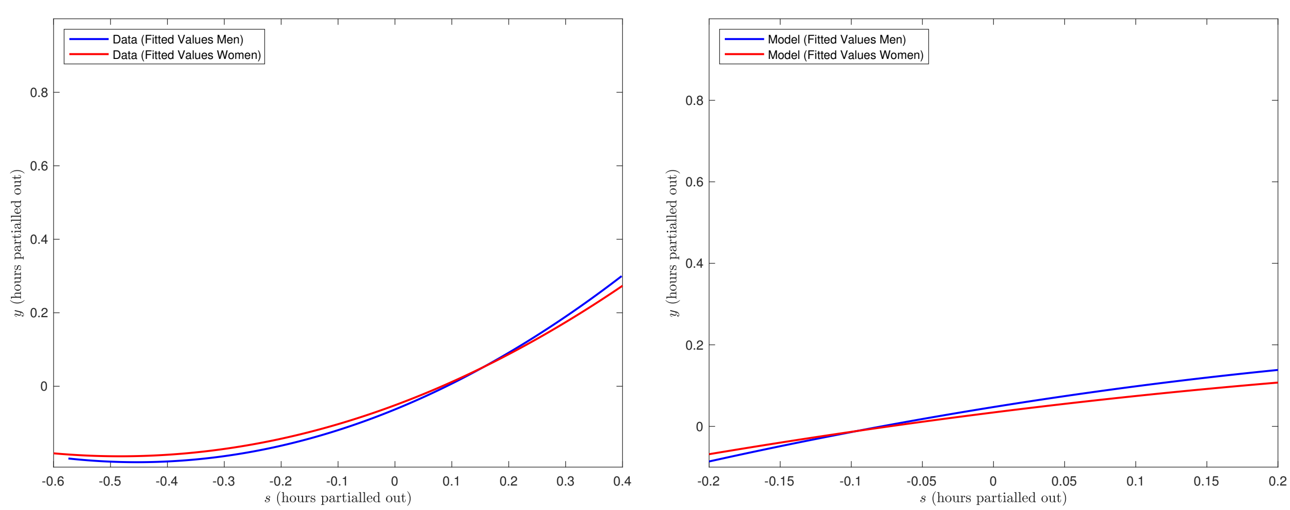

Figure 11: Labor Market Matching Function by Gender (Hours Partialled Out): a. Data and b. Model

Finally, households' time allocation choices are also related to labor market sorting. When re-plotting the labor market matching function from Figure 8 but *controlling* for hours worked, then the difference in sorting across gender nearly vanishes both in the data (left) and in the model (right) (Figure 11). This is indicated by similar labor market matching functions for both genders relative to Figure 8 where the sorting gap was more pronounced. Only for top human capital levels, the gender sorting gap favoring men persists.

8 Inequality Through the Lens of our Model

This section contains our main quantitative exercises. We use our model to shed new light on the sources of gender disparities in the labor market and household income inequality. Our analysis focusses on three different contexts: Today's Germany (Section 8.1), Germany across time (Section 8.2) and Germany across space (Section 8.3).

8.1 Inequality in Germany Today

We first focus on today's Germany, 2010-2016. We analyze the gender wage gap and income inequality within and between households through the lens of our model. We start with investigating the performance of our model in reproducing observed inequality. We then analyze comparative statics with respect to the model's key determinants of inequality. Finally, in order to highlight how the interplay between marriage and labor market affects inequality, we conduct counterfactual exercises where we shut down either labor market sorting or marriage market sorting and replace them by random matching. Understanding how sorting in the two markets mitigates or amplifies inequality will help us better understand the following exercises investigating inequality differences in Germany across time and space.

8.1.1 Inequality in Data and Model

To assess the extent of inequality in data and model, we mainly focus on two measures: the gender wage gap and household income variance, including its decomposition into between and within household component.[28] These statistics are reported in Table 5. First, regarding the inequality within households, our model captures the gap between men and women well: It predicts that the share of female wages in overall household wage income is 36% (in the data it is 35%). Importantly, while our model underestimates the level of the income variance (around 80 in the model versus 110 in the data), we capture the split of within- and between-household inequality accurately (55-45 split in the model vs. 53-47 in the data).[29] Last, our model produces a sizable gender wage gap (31.6%), overestimating the observed gap (19.31%).

[28] Our measure of the gender wage gap includes all individuals in the sample, single and in couples, conditional on participation in the labor market. In turn, both the female's share in household income and the total income variance and its decomposition are computed based on the sample of couples. All couples are included, independent of participation.

[29] The within-component is measured by the variance of wages within a couple, averaged across all couples. The between-component is measured as the variance of the average income of each couple.

Table 5: Gender and Household Inequality

	Model	**Data**
Share of Female Wage in Overall HH Wage Income	0.3604	0.3451
Total Income Variance (Couples)	80.5675	110.1422
Within Household Income Variance	44.4943	58.3308
... share in total variance	0.5523	0.5296
Between Household Income Variance	36.0732	51.8115
... share in total variance	0.4477	0.4704
Gender Wage Gap	0.3207	0.1931

Our model is thus able to reproduce key features of observed inequality that were not targeted in estimation. This out-of-estimation-sample performance makes our model an adequate tool through which we can investigate the main drivers of inequality, and understand the sources of heterogeneity in Germany over time and across space. We turn to these tasks next.

8.1.2 Comparative Statics

We begin with the comparative statics of our model, in order to highlight the key forces and mechanism behind inequality. The gender wage gap in our model is driven by (endogenous) gender differences in hours worked, and (exogenous) differences in human capital and labor productivity. In turn, differences in hours worked across gender are mostly impacted by the relative productivity of women at home, θ, the home production complementarity, ρ, and the labor market productivity wedge ψ. Clearly, if both $\theta = 0.5$ (men and women are equally productive at home) and $\psi = 0$ (men and women are equally productive in the labor market/women are not discriminated against in the labor market), this would eliminate the gender wage gap entirely. But given that $\theta \neq 0.5, \psi \neq 0$, the level of complementarities in home hours, ρ, is a third key determinant of inequality in our model. We are interested in the comparative statics effects of these parameters on the gender wage gap, and intra and inter-household income inequality.

One can easily think of policies and technological changes that impact these parameters. Anti-discrimination policies (such as gender quota or equal pay policies) can affect ψ. Universal childcare, parental leave policies (such as "daddy months") and tax reforms, might affect ψ and θ. As the time women are expected to put into home production decreases, they work more, providing incentives for men to spend more time in home production, which may reduce gender differences in productivity of home hours. Further, changes in home production technologies that make it easier to perform housework chores in which women traditionally specialized (Greenwood et al. (2016a), Greenwood (2019)), could affect θ and ρ. For instance, if partners' time is inherently more complementary in childcare relative to other home production activities (as we show in Appendix A), the outsourcing of and reduction of time dedicated to non-childcare activities shift the composition of home production towards more complementary tasks, affecting ρ. Finally, an increase over time in the returns to investments in children (Lundberg et al. (2016)) could also have impacted ρ, as parental time becomes more complementary in order to increase the quality of parental investments (in which both parents provide high quality time).

The Effect of ρ. We begin with investigating a change in home production technology that increases the complementarities in spouses' home hours. Recall that our estimate $\rho = -0.53$ indicates that home hours are strategic complements. We are interested in the effects on inequality when ρ becomes even more negative, and in the underlying mechanism (hours, labor sorting and marriage sorting).

Figure 12a plots the effect of ρ on different inequality measures: household income variance on the left y-axis and the gender wage gap on the right y-axis. It shows that a decline in ρ (moving from the right to the left on the x-axis) *decreases* the gender gap significantly. Starting from our estimate $\rho = -0.53$ and decreasing this parameter to -2 decreases the gender wage gap by 16%. This is due to a direct and several indirect effects: There is a direct effect of stronger complementarities on households' time allocation choices with spouses' hours becoming more aligned. As a result, women increase their labor hours while men decrease their labor hours, leading to a smaller gap in labor hours of couples, Figure 12b. Because of the opposite movement of labor hours, female labor market sorting tends to improve while male labor market sorting worsens significantly, putting downward pressure on the gender gap in labor market sorting (Figure 12c) and thus also in wages. Last, because complementarity in home production (and thus also in labor hours) among partners increases, the desire for positive sorting on the marriage market is stronger, resulting in more positive assortative matching, Figure 12d. The increased marriage sorting further aligns the hours choices of couples, reducing the wage gap even more.

How does this change in home production complementarities affect overall inequality and the split of income within and between households? Figure 12a shows that overall income inequality *declines* with stronger complementarities. This decline is driven by the decrease in within-household inequality (mirroring the decline in the gender wage gap), which dominates the increase in between inequality that stems from an increase in marriage sorting.

This comparative statics exercise also highlights what features of the data call for $\rho < 0$, pushing the economy towards the monotone equilibrium: If it was the case that $\rho > 0$ (i.e. if hours were strategic substitutes), then marriage market sorting would be random, see Figure 12d where marriage sorting drops to zero as ρ becomes positive, and the correlation of spouses' home hours will tend towards zero (not plotted here) even though in the data it is significantly positive (equal to 0.28, m_4, Table 15).

The Effect of θ. Next we are interested in the effect of women's relative productivity at home on our inequality measures. Clearly, the gender wage gap is increasing in θ, Figure 19a in Appendix F.1. Indeed, eliminating the gap in home productivity would cut the gender wage gap by half. The mechanism is as follows: increasing female home productivity increases her home hours at the expense of fewer labor hours while the opposite is true for men. This pushes up the labor hours gap within couples and overall, Figure 19b. As a result, women's labor market sorting deteriorates, while men's improves, Figure 19c. The increased gender inequality in labor market and home production outcomes decreases the incentive for positive sorting on the marriage market: marriage sorting declines as θ increases, Figure 19d.

Interestingly, overall household income inequality increases as women become more productive at home, Figure 19a. Here, this is driven by an increase in within-household inequality (mimicking the evolution of the gender wage gap), which dominates the decline in between-household inequality driven by a drop in marriage market sorting.

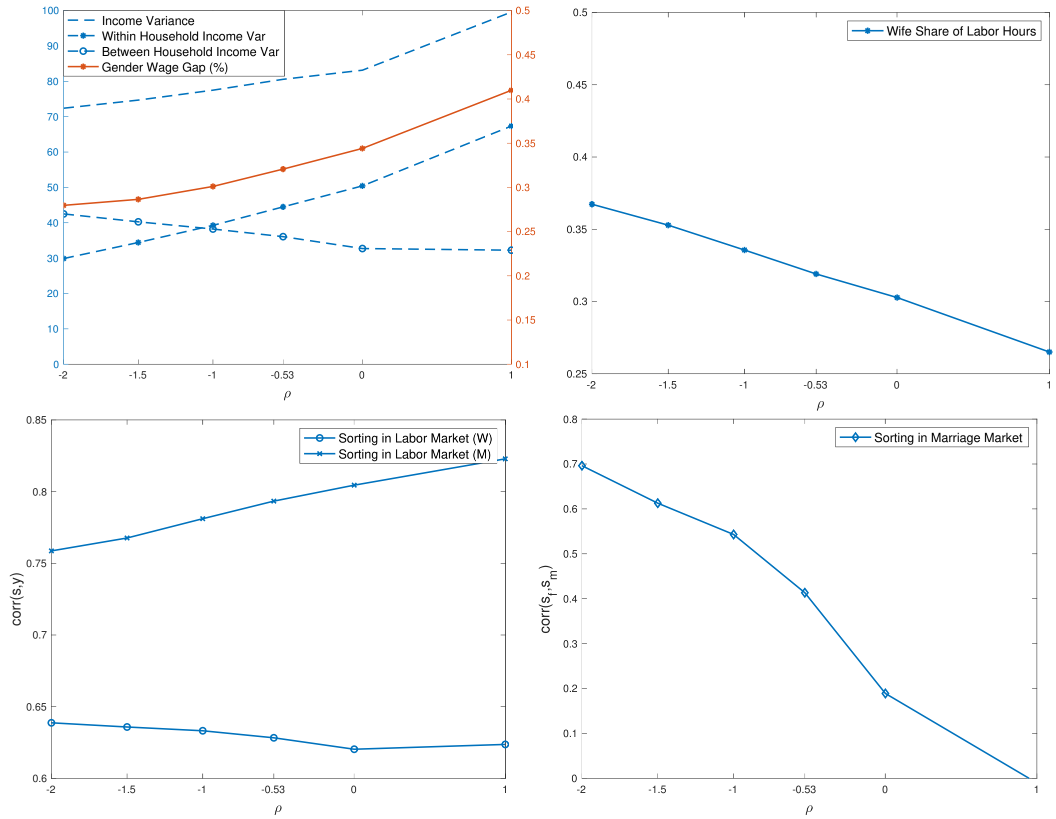

Figure 12: a. Inequality, b. Female Labor Hours , c. Labor Market Sorting, d. Marriage Market Sorting

The Effect of ψ. Last, we analyze the comparative statics of the female labor market wedge. Figure 20a in Appendix F.1 shows that increasing ψ from zero to 1.5 increases the gender wage gap by more than 50% (recall our estimate is $\psi = 0.93$, also indicated in the figure). Starting from our estimate and eliminating wedge ψ would reduce the gender gap by about one third. There is a direct effect of ψ on female productivity and thus wages but also several indirect effects: First, wife's labor hours decline in ψ relative to the husband's (and the female share of hours worked in a couple is inversely related to the gender wage gap since hours are a productive attribute, Figure 20b); second, this leads to a decline in female labor market sorting, Figure 20c, further fueling the gender wage gap; third, larger gender disparities on the labor market are associated with a decline in marriage market sorting, Figure 20d, since in a world where men and women are more unequal the motive for positive sorting weakens. The decline in marriage sorting reinforces the gap in hours between men and women and thus the gap in labor market sorting, further fueling the wage gap. Interestingly, the level of labor market sorting of men also declines, albeit to a lower extent than for women, Figure 20c, which is due to an equilibrium effect: As marriage market sorting declines, women are putting less hours in the labor market and more hours into home production. Since home (and thus market) hours are complementary among spouses, men also put less market hours, deteriorating their labor market sorting.

In Figure 20a, we contrast the effects of ψ on the wage gap with the effect on the variance of income, both within and across households. While an increase in ψ (a larger labor wedge between men and women) *decreases* the overall income variance because women's productivity effectively declines, there are interesting effects within and across households. Indeed, *within*-household inequality *increases*, moving in the same direction as the gender wage gap. In turn, *between*-household inequality *decreases*, mainly driven by the decline in marriage sorting.

Our take-away from these comparative statics exercises is as follows: First, eliminating asymmetries in productivity across gender (whether at home through $\theta \to 0.5$ or at work through $\psi \to 0$) reduces the gender wage gap. But interestingly this is not the only way to reduce gender disparities: an increase in home production complementarity (decrease in ρ, the key parameter of our model in shaping equilibrium) has qualitatively similar effects. Second, a decline in the gender gap tends to go hand in hand with (i) a decline in the labor hours gap and (ii) a decline in the labor market sorting gap and (iii) an increase in marriage market sorting. Third, while the effect of these parameters on overall income inequality depends on the specific exercise, in all cases, the gender gap positively co-moves with within-household inequality but negatively co-moves with between-household inequality.

8.1.3 Counterfactuals

We next investigate the role of labor market and marriage market sorting (and their interaction) for inequality more closely. To do so, we either shut down labor market sorting or marriage market sorting and replace them by random matching in the corresponding market (but we allow the agents to behave optimally regarding all other choices). We again plot each of the four inequality measures – gender wage gap, income variance, between and within household income variance – against our three key parameters impacting inequality (ρ, θ, ψ). We are interested in the counterfactual inequality measures both *at our baseline estimates* of (ρ, θ, ψ) and as we *vary these parameters*. We indicate the baseline estimate by a vertical black line. Solid lines correspond to our baseline model with endogenous labor and marriage sorting, while dashed lines correspond to counterfactuals where either labor or marriage sorting are shut down. We focus in the main text on how inequality (and counterfactuals) respond to home production complementarity ρ and note that the take-away for relative labor market productivity ψ and home productivity θ is similar, see Figures 21 and 22 in Appendix F.2.

We find that both labor market and marriage market sorting fuel *overall* inequality and *between* household inequality. This can be seen from Figures 13b and 13c, where the solid lines (income variance and between household income variance) lie above the counterfactual dashed curves that indicate random labor market matching (full dots) and random marriage market sorting (hollow dots). But sorting on the marriage market tends to have the opposite effect as sorting on the labor market when it comes to *gender* inequality: While labor market sorting significantly increases the gender gap, marriage market sorting puts downward pressure on the gender gap, see Figure 13a. This is indicated by the random labor market curve lying below the solid curve while the random marriage market curve lies above. Similarly, labor market sorting pushes up within-household inequality while marriage market sorting tends to decrease this measure of inequality, see Figure 13d.

Our results show that sorting on both markets has a significant quantitative effect on inequality with labor market sorting cementing the advantage of men in a world where women work less in the labor market. In turn, marriage market sorting generates more balanced labor market outcomes (regarding hours, sorting and pay) across gender. Furthermore, more sorting on either margin tends to increase overall and between household inequality.

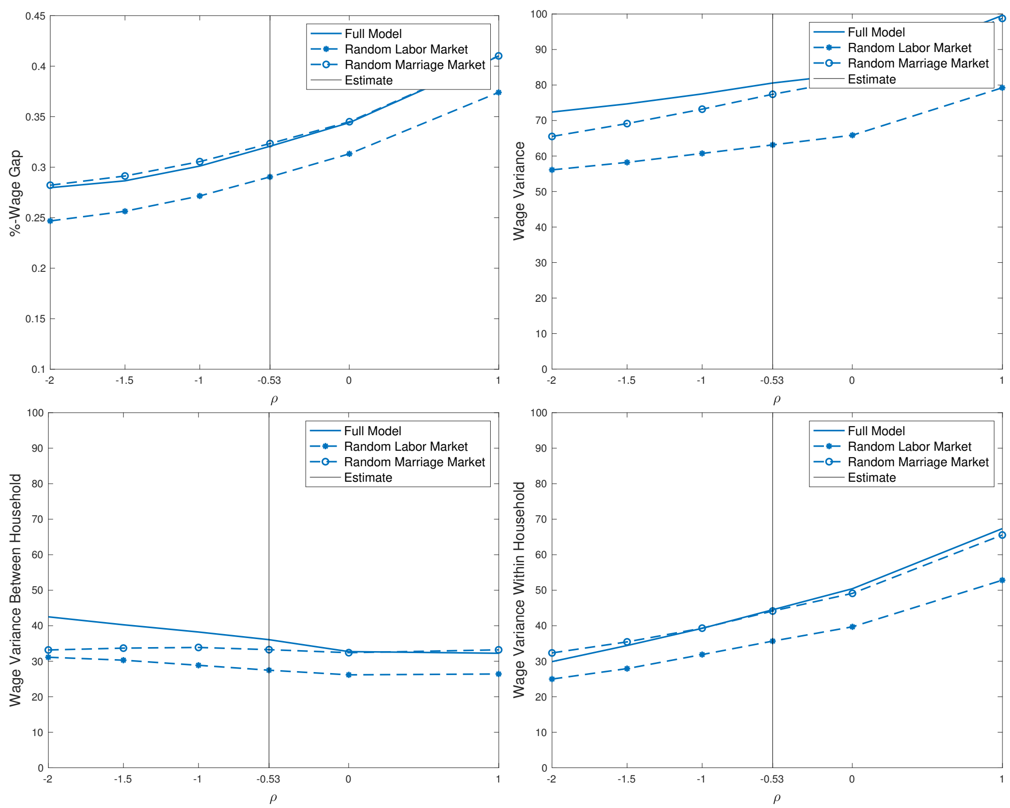

Figure 13: Wage Inequality Measures under Change in ρ: Full Model and Counterfactuals

8.2 Inequality in Germany Across Time

The German labor and marriage markets have significantly evolved over the past decades. We are interested in how this transformation affected inequality, both across gender and among households. We first use our model to investigate how its primitives have changed over time and how these changes affected inequality. We then document shifts in labor and marriage sorting and ask whether they amplified or mitigated inequality. The exercise compares *West* Germany today and before to not confound the impacts of technological change in the labor market and in home production with the impacts of the German reunification and convergence.

We estimate our model on the West sample in an earlier period, 1990-1996, and a recent period, 2010-2016.[30] We re-estimate all parameters of our model except those pertaining to the preference shocks, which we set to the level of our current period all-Germany benchmark (Section 7.4).[31] This is to tie our hands and force the model mechanism to explain the data, as opposed to giving changes in shock distributions a prominent role.[32] The model fit is in Table 16 in Appendix F.3, which also indicates that both labor and marriage market underwent statistically significant changes over time (column 4).

Here we highlight (mostly un-targeted) features of the data related to inequality shifts. The turquoise bars in Figure 14, left panel, indicate percentage differences of inequality today relative to the early period in the data. Household income variance is 13% higher today than 30 years ago, which masks diverging trends of within-household inequality (which declined by around 15%) and between-household inequality (which increased by more than 60%). In turn, the gender wage gap declined by 23% over this period. For comparison, the purple bars indicate the changes in inequality generated by the estimated model, suggesting that our model captures these features quite well even though they are untargeted.

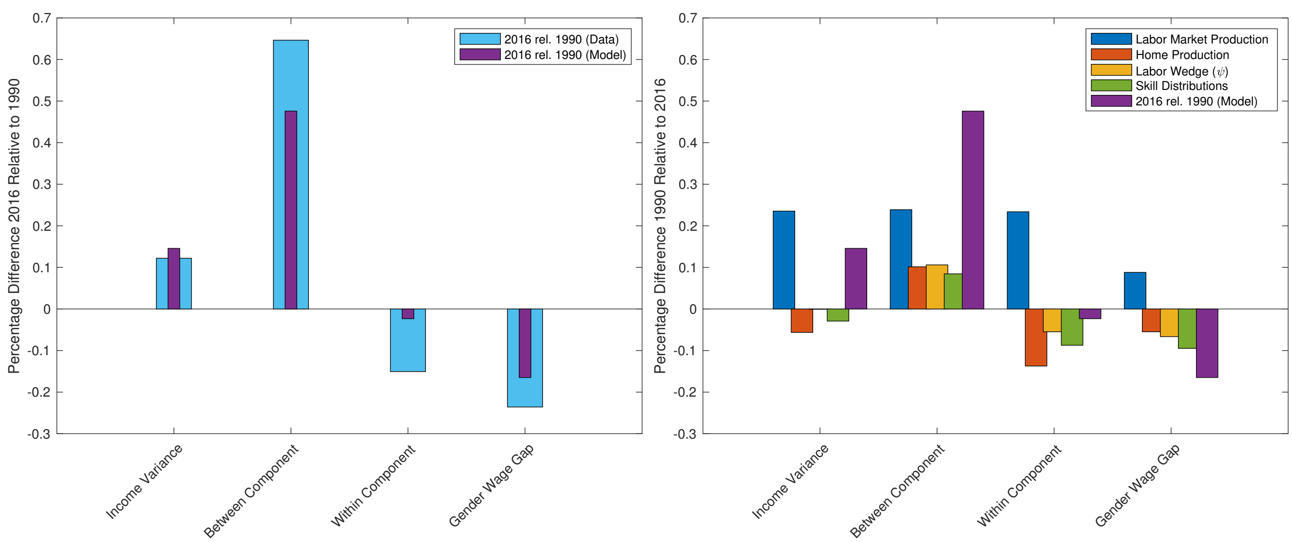

Figure 14: Inequality In Germany Over Time in Data and Model

We now zoom further into the model. We provide the parameter estimates for both periods and their standard errors, in Table 17, Appendix F.3. We emphasize significant changes in home production with today's West Germany being characterized by a lower θ (men became relatively more productive than in the past) and lower ρ (increased complementarity in spouses' home hours), as well as in the

[30]In the GSOEP, 1990 is the first year that features the time use variables used in our analysis of the later period.

[31]We target the same set of moments as in Table 14, except from moments M14 to M22, which were intended to identify the scale of labor supply shocks, which we do not re-estimate here.

[32]We had to make one exception to this rule: We free up the scale of the marriage market shock in the early period in order to give the model a chance to match the data.

labor productivity wedge ψ (which has declined over time). These changes indicate that Germany has become a more 'progressive' society and economy over the last three decades, with more gender equality at home and work. In turn, the labor market technology has become more convex in effective skills, resembling skill-biased technological change, and has a higher TFP than before.

We investigate the driving forces of inequality shifts over time, asking how much of the documented changes can be explained by changes in our model parameters. Figure 14, right panel, provides a detailed decomposition and re-displays, for comparison, the purple model bars (from the left panel) of shifts in the four outcomes when *all* parameters change from the early to the later level. The other bars give the percentage change in inequality outcomes relative to the early period (1990-1996) if one parameter group (where we consider changes in the labor market production function, blue, home production, orange, labor productivity wedge, yellow, and human capital distributions, green) changes *in isolation* while the others remain fixed at the early level.

We find that home production changes (orange bars) put significant downward pressure on inequality across gender. If only home production had changed over this period, within household inequality would have declined by 14% (which is essentially 100% of the observed shift) and the gender wage gap by more than 5% (accounting for 22% of the observed drop). In turn, home production changes put upward pressure on between household inequality (because they fostered more PAM on the marriage market), but this effect is dominated by the downward pressure on within inequality. The net effect of technological change in home production on overall household inequality is therefore negative. Splitting home production further into the contributions of our model's key parameters θ and ρ reveals that changes in complementarity parameter ρ were most important for the decline in gender inequality, see Figure 23 in Appendix F.3. Note that the decline in the labor market wedge ψ (yellow bars) had qualitatively and quantitatively similar effects on inequality as the advances in home production technology. Together, changes in home production and in the labor wedge account for most of the decline in gender inequality in the model, followed by changes in the human capital distribution (green bars). In contrast, changes in labor market technology (blue bars) fueled inequality across the board, significantly pushing up household income variance and its between-component, and preventing gender inequality from falling even further. Thus, technological change in home production and in the labor market have pushed inequality, and especially gender disparities, in opposite directions. According to our model, home production changes (along with the decline in the labor wedge ψ) were vital for more gender equality in the labor market and in households.

Our model delivers a clear mechanism for why the documented changes in home production and labor wedge have reduced gender inequality over time. Based on our comparative statics, Section 8.1.2, the estimated changes of both parameter groups have a direct effect pushing towards more equality, but also important indirect effects. In particular, both changes induced women to work more (leading to a decline in the gender gap of labor hours), which in turn made women sort relatively better on the labor market (reducing the gender gap in labor sorting). This in turn strengthened the desire for PAM in marriage, leading to an increase in marriage market sorting. Figure 15 displays these shifts in the model (purple bars) and shows that they match up well with the corresponding shifts in the data (turquoise bars). Our evidence and estimates suggest that Germany underwent significant changes over the last decades towards an equilibrium that strongly resembles the monotone equilibrium from our theory, with more co-movement of spouses' hours, labor market sorting and wages, as well as increased marriage sorting.

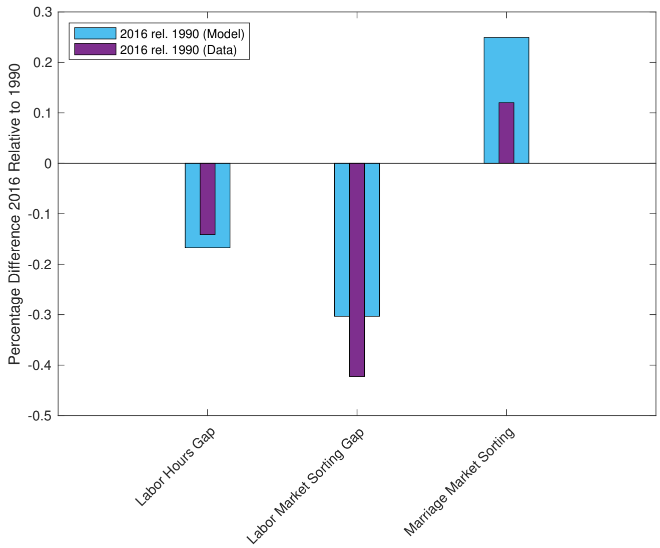

Figure 15: Mechanism Behind Inequality Changes Over Time

8.3 Inequality in Germany Across Space

Even after almost 30 years of the German reunification, significant differences in inequality, marriage market and labor market outcomes persist between East and West Germany. The turquoise bars in Figure 16, left panel, indicate percentage differences in inequality outcomes of East Germany relative to the West in the data. Inequality is lower in the East across the board: household income variance is around 35% lower, mainly driven by the within component which is almost 60% lower rather than by the between component, which is 10% lower. Further, the gender wage gap is almost 50% lower in the East.

We now investigate how the primitives of our model differ between East and West Germany today, and whether these differences can rationalize those persistent differences in inequality across regions. We estimate our model for East Germany on data from 2010-2016 and compare moments and parameters to our West sample from last section. We again tie our hands by keeping the preference parameters for both the East and West samples fixed to the same level of our all-Germany benchmark. Explaining differences between East and West Germany by preference heterogeneity would not generate much economic insight. The model fit is in Table 18 in Appendix F.4, along with a test of whether differences in moments across regions are statistically significant. Our model fits the targeted moments well. It underestimates the un-targeted inequality differences across space (purple bars, Figure 16), but not by a large margin.

We ask to what extent spatial differences in home production technology, labor market technology

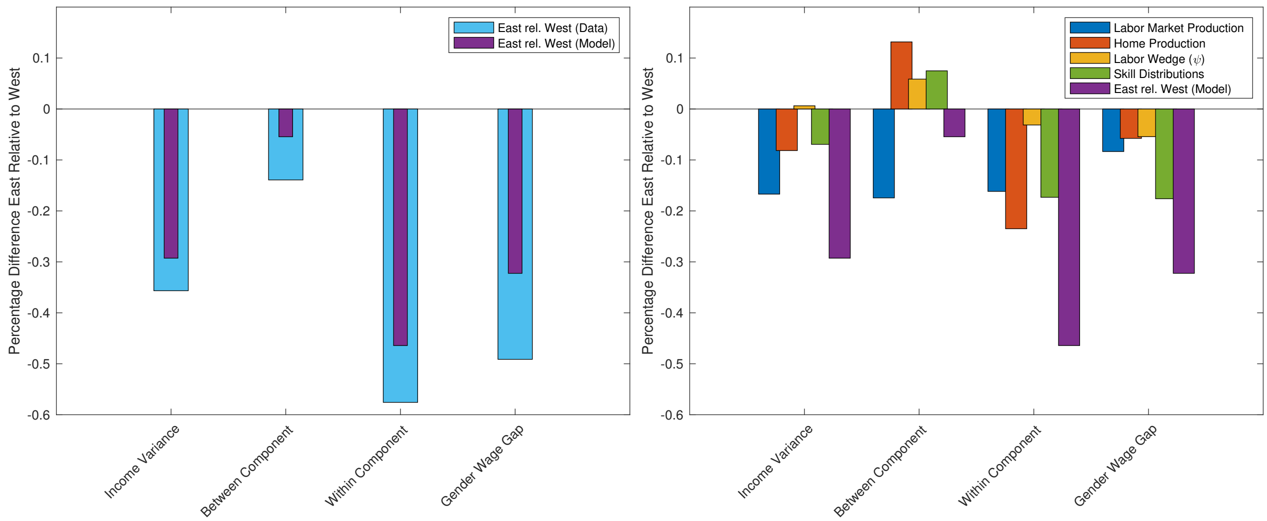

Figure 16: Inequality in East and West Germany in Data and Model

or labor wedge can account for the differences in inequality across regions. Table 19, Appendix F.4, displays the parameter estimates for East and West. We note significant differences in home production technology with the East having a more negative ρ (stronger home production complementarity) and a higher TFP. It also has a lower labor productivity wedge ψ. All this indicates that the East is more gender-equal in both home production and the labor market. In turn, the East is characterized by a labor market technology that is less convex in effective skills (less skill-biased) and has a lower TFP.

We follow the same approach as in the last section, and decompose inequality differences across space from Figure 16 (purple bars) into these primitives. The right panel gives the percentage change in inequality relative to the West benchmark if each of these parameters switches to its East value in isolation while the others remain fixed at the West level. It shows that both the Eastern German home production technology (orange) and labor market technology (blue) put significant downward pressure on gender gaps, measured by the wage gap or within household income inequality, and overall household income inequality. Only regarding between household inequality these two technological forces push in opposite directions: while the home production technology in the East fuels inequality (because stronger complementarities push towards more PAM in the marriage market), its labor market technology mitigates it compared to the West. Zooming further into our key parameters of home production, θ and ρ, reveals that complementarity parameter ρ is the main driver of inequality difference between East and West while differences in θ play a minor role (Figure 24, Appendix F.4). The lower labor wedge in the East (yellow) also pushes towards less gender inequality but the effect is quantitatively smaller.

In contrast to the last section where we analyzed inequality over time, here both technological forces (more 'progressive' home production and less skill-biased labor market production), pull on the same string, and put downward pressure on inequality in East Germany, both across gender and overall.

9 Conclusion

Employers value workers not only for their skills but also for their time input. In such a setting, if labor supply decisions are made at the household level (as opposed to the individual level) so that they depend on the characteristics of *both* spouses, then marriage market sorting affects labor market sorting. In turn, if individuals anticipate their hours choices as well as their sorting in the labor market and wages when deciding whom to marry, then labor market sorting affects marriage market sorting. The interaction of *both* the marriage and the labor market crucially impacts inequality across gender and within/between households. And policies affecting who marries whom (such as tax policies) or home production technologies (such as parental leave or universal childcare) can therefore mitigate or amplify inequality, calling for a better understanding of these spillovers across markets.

The interplay between labor market and marriage market and its effect on inequality is at the center of this paper. We build a novel equilibrium model in which households' labor supply choices form the natural link between sorting on the marriage and sorting on the labor market. We first show that in theory, the nature of home production – whether partners' hours are complements or substitutes – shapes marriage market sorting, labor market sorting and labor supply choices in equilibrium.

We then ask what is the nature of home production in the data. To this end, we estimate our model on data from today's Germany and find that spouses' home hours are strategic *complements*, pushing towards positive sorting in both markets and co-movement of labor hours of spouses. We then investigate the key drivers behind sorting and inequality based on primitives and find that the gender wage gap and within household income inequality would decrease if either gender productivity differences at home or in the labor market were reduced, but also if home production hours were even more complementary among partners. Further, sorting on both markets has a significant quantitative effect on inequality: Labor market sorting cements the advantage of men in a world where women work less in the labor market and more at home. In turn, marriage market sorting generates more balanced labor market outcomes – in hours, sorting, and pay – across gender.

Our main quantitative exercises ask whether and how our model can rationalize differences in inequality over time and across space. We find that differences in the home production regimes can account for a significant part of both the decline in gender inequality over time, as well as lower levels of inequality in East compared to West Germany.

References

S. Albanesi and C. Olivetti. Home production, market production and the gender wage gap: Incentives and expectations. *Review of Economic Dynamics*, 12(1):80 – 107, 2009.

I. Andrews, M. Gentzkow, and J. M. Shapiro. Measuring the sensitivity of parameter estimates to estimation moments. *The Quarterly Journal of Economics*, 132(4):1553–1592, 2017.

N. Angelov, P. Johansson, and E. Lindahl. Parenthood and the gender gap in pay. *Journal of Labor Economics*, 34(3):545–579, 2016.

G. Anyaegbu. Using the OECD equivalence scale in taxes and benefits analysis. *The Labour Gazette*, 4 (1):49–54, 2010.

J. Bagger and R. Lentz. An Empirical Model of Wage Dispersion with Sorting. *The Review of Economic Studies*, 86(1):153–190, 2018.

G. S. Becker. A theory of marriage: Part I. *Journal of Political Economy*, 81(4):813–846, 1973.

M. Bertrand, C. Goldin, and L. F. Katz. Dynamics of the gender gap for young professionals in the financial and corporate sectors. *American Economic Journal: Applied Economics*, 2(3):228–55, 2010.

S. Bonhomme, T. Lamadon, and E. Manresa. A distributional framework for matched employer employee data. *Econometrica*, 87(3):699–739, 2019.

J. N. Brown and H. S. Rosen. On the Estimation of Structural Hedonic Price Models. *Econometrica*, 50 (3):765–68, May 1982. URL https://ideas.repec.org/a/ecm/emetrp/v50y1982i3p765-68.html.

M. Browning, P. Chiappori, and Y. Weiss. Family economics. *Tel Aviv University, unpublished textbook manuscript*, 2011.

D. Card, J. Heining, and P. Kline. Workplace Heterogeneity and the Rise of West German Wage Inequality. *The Quarterly Journal of Economics*, 128(3):967–1015, 2013.

P. Chiappori, M. Iyigun, and Y. Weiss. Investment in schooling and the marriage market. *The American Economic Review*, 99(5):1689–1713, 2009.

P.-A. Chiappori, M. C. Dias, and C. Meghir. The marriage market, labor supply, and education choice. *Journal of Political Economy*, 126(S1):S26–S72, 2018.

P. A. Chiappori, M. C. Dias, and C. Meghir. Changes in assortative matching: Theory and evidence for the us and the uk. 2020.

H. L. Cole, G. J. Mailath, and A. Postlewaite. Efficient non-contractible investments in large economies. *Journal of Economic Theory*, 101(2):333 – 373, 2001.

P. Cortés and J. Pan. When time binds: Substitutes for household production, returns to working long hours, and the skilled gender wage gap. *Journal of Labor Economics*, 37(2):351–398, 2019.

M. C. Dias, R. Joyce, and F. Parodi. The gender pay gap in the uk: Children and experience in work. IFS Working Papers W18/02, 2018.

L. Eika, M. Mogstad, and B. Zafar. Educational assortative mating and household income inequality. *Journal of Political Economy*, 127(6):2795–2835, 2019.

I. Ekeland, J. J. Heckman, and L. Nesheim. Identification and estimation of hedonic models. *Journal of Political Economy*, 112(S1):S60–S109, February 2004. URL http://ideas.repec.org/a/ucp/jpolec/v112y2004is1ps60-s109.html.

A. Erosa, L. Fuster, G. Kambourov, and R. Rogerson. Hours, occupations, and gender differences in labor market outcomes. Working Paper 23636, National Bureau of Economic Research, 2017.

R. Fernández, N. Guner, and J. Knowles. Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality. *The Quarterly Journal of Economics*, 120(1):273–344, 2005.

L. Flabbi, C. J. Flinn, and M. Salazar-Saenz. Simultaneous search in the labor and marriage markets with endogenous schooling decisions. 2020.

G.-L. Gayle and L. Golan. Estimating a Dynamic Adverse-Selection Model: Labour-Force Experience and the Changing Gender Earnings Gap 1968–1997. *The Review of Economic Studies*, 79(1):227–267, 2011.

G.-L. Gayle and A. Shephard. Optimal taxation, marriage, home production, and family labor supply. *Econometrica*, 87(1):291–326, 2019.

D. Gicheva. Working long hours and early career outcomes in the high-end labor market. *Journal of Labor Economics*, 31(4):785–824, 2013.

C. Goldin. A grand gender convergence: Its last chapter. *American Economic Review*, 104(4):1091–1119, April 2014.

M. Goussé, N. Jacquemet, and J.-M. Robin. Household labour supply and the marriage market in the uk, 1991-2008. *Labour Economics*, 46:131 – 149, 2017a.

M. Goussé, N. Jacquemet, and J.-M. Robin. Marriage, labor supply, and home production. *Econometrica*, 85(6):1873–1919, 2017b.

J. Greenwood. *Evolving Households The Imprint of Technology on Life*. MIT Press, 2019.

J. Greenwood, N. Guner, G. Kocharkov, and C. Santos. Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation. *American Economic Journal: Macroeconomics*, 8(1):1–41, January 2016a. doi: 10.1257/mac.20130156. URL https://www.aeaweb.org/articles?id=10.1257/mac.20130156.

J. Greenwood, N. Guner, G. Kocharkov, and C. Santos. Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation. *American Economic Journal: Macroeconomics*, 8(1):1–41, 2016b.

J. Greenwood, N. Guner, and G. Vandenbroucke. Family economics writ large. *Journal of Economic Literature*, 55(4):1346–1434, December 2017.

M. Hagedorn, T. H. Law, and I. Manovskii. Identifying equilibrium models of labor market sorting. *Econometrica*, 85(1):29–65, 2017.

S. Heise and T. Porzio. Spatial wage gaps in frictional labor markets. 2019.

M. Keane, P. Todd, and K. I. Wolpin. The structural estimation of behavioral models: Discrete choice dynamic programming methods and applications. volume 4A, chapter 04, pages 331–461. Elsevier, 1 edition, 2011. URL https://EconPapers.repec.org/RePEc:eee:labchp:4-04.

H. Kleven, C. Landais, and J. E. Søgaard. Children and gender inequality: Evidence from denmark. *American Economic Journal: Applied Economics*, 11(4), 2019.

P. Legros and A. F. Newman. Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities. *Econometrica*, 75(4):1073–1102, 07 2007. URL http://ideas.repec.org/a/ecm/emetrp/v75y2007i4p1073-1102.html.

I. Lindenlaub. Sorting Multidimensional Types: Theory and Application. *The Review of Economic Studies*, 84(2):718–789, 2017.

I. Lindenlaub and F. Postel-Vinay. Multidimensional sorting under random search. 2020.

J. Lise and F. Postel-Vinay. Multidimensional skills, sorting, and human capital accumulation. *American Economic Review*, Forthcoming.

J. Lise and S. Seitz. Consumption Inequality and Intra-household Allocations. *The Review of Economic Studies*, 78(1):328–355, 01 2011. ISSN 0034-6527. doi: 10.1093/restud/rdq003. URL https://doi.org/10.1093/restud/rdq003.

S. Lundberg, R. Pollak, and J. Stearns. Family inequality: Diverging patterns in marriage, cohabitation, and childbearing. 2016.

A. Mas and A. Pallais. Valuing alternative work arrangements. *American Economic Review*, 107(12): 3722–59, 2017.

D. McFadden. A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, 57(5):995–1026, 1989.

A. Pakes and D. Pollard. Simulation and the asymptotics of optimization estimators. *Econometrica*, 57 (5):1027–1057, 1989.

L. Pilossoph and S. L. Wee. Assortative matching and income inequality: A structural approach. 2019a.

L. Pilossoph and S. L. Wee. Household search and the marital wage premium. 2019b.

A. Reynoso. The impact of divorce laws on the equilibrium in the marriage market. 2019.

S. Rosen. Hedonic prices and implicit markets: Product differentiation in pure competition. *Journal of Political Economy*, 82(1):34–55, Jan.-Feb. 1974. URL http://ideas.repec.org/a/ucp/jpolec/v82y1974i1p34-55.html.

L. S. Shapley and M. Shubik. The assignment game I: The core. *International Journal of Game Theory*, 1(1):111–130, 1971.

A. Voena. Yours, mine, and ours: Do divorce laws affect the intertemporal behavior of married couples? *American Economic Review*, 105(8):2295–2332, 2015.

Appendix

A Empirical Evidence

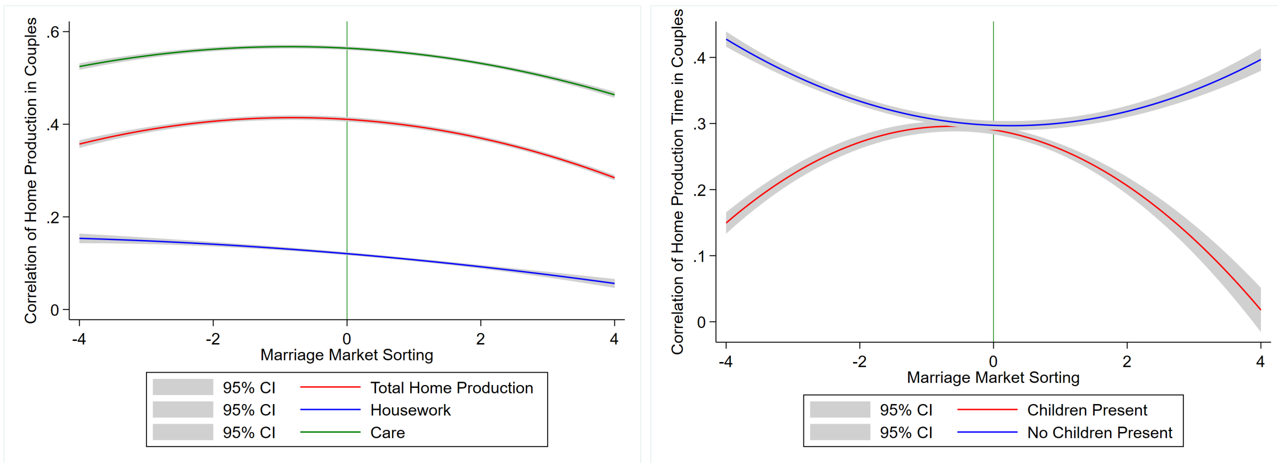

Figure 17: (a) Hours Correlation and Marriage Sorting by Home Production Components; (b) Hours Correlation and Marriage Sorting by Presence of Children

In Tables 6 and 7 we further explore the correlation between male and female hours, and how it is related to marriage market sorting. We run the following regressions at the couple level:

$$hh_i^f = \alpha_0 + \alpha_1 * hh_i^m + \alpha_2 * hh_i^m * PAM_i + \alpha_3 * yr\_educ_i^m + \alpha_4 * yr\_educ_i^f + \epsilon_i$$

where hh_i^f and hh_i^m are the home hours of the female and the male partner in couple i, respectively, and $hh_i^m * PAM_i$ captures the interaction between male hours at home and sorting in the marriage market. Our measure of sorting, PAM_i, takes value 1 when both partners in the couple have the same level of education, defined as in Section D.1). The variables $yr\_educ_i^f$ and $yr\_educ_i^m$ stand for the years of education of the female and male partner in couple i, respectively.

Results in Tables 6 and 7 suggest complementarities between home hours of the partners, as indicated by a positive and significant α_1. In many cases, the coefficient α_2 is also positive and significant, suggesting that complementarities are stronger when partners are better sorted in the marriage market, in line with our graphical results in Section 3.2. The coefficient of the interaction is larger when we condition on couples in which only the male partner works full time, and even more when children are present in the household. Also, in couples in which partners have higher education, hours at home are more complementary. In most cases, the coefficient associated to male years of education is positive, with women putting more hours at home if they marry a more educated spouse. The coefficient associated to own years of education is negative and significant, with more educated women putting less home hours.

Table 6: Home Hours Complementarities and Marriage Market Sorting

	(1) Female Hours	(2) Female Hours	(3) Female Hours	(4) Female Hours
Male Hours	0.581***	0.462***	0.484***	0.654***
	(0.028)	(0.020)	(0.028)	(0.041)
Male Hours * PAM	0.040	0.080***	0.104***	0.003
	(0.027)	(0.021)	(0.027)	(0.039)
Male Years of Education	0.190	0.050	-0.173	-0.214
	(0.121)	(0.108)	(0.136)	(0.133)
Female Years of Education	-0.666***	-0.806***	0.351**	-0.349***
	(0.127)	(0.114)	(0.147)	(0.132)
Observations	4,064	7,427	4,364	1,653
R-squared	0.155	0.110	0.097	0.336
Sample	Both Employed	All	Only Male FT	Both FT

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 7: Home Hours Complementarities and Marriage Market Sorting (different samples)

	(1) Female Hours	(2) Female Hours	(3) Female Hours	(4) Female Hours
Male Hours	0.202***	0.345***	0.284***	0.499***
	(0.021)	(0.040)	(0.029)	(0.047)
Male Hours * PAM	0.022	0.056	0.062**	0.170***
	(0.021)	(0.039)	(0.027)	(0.058)
Male Years of Education	0.081	-0.253***	-0.171	-0.402
	(0.120)	(0.098)	(0.135)	(0.318)
Female Years of Education	-0.272**	-1.267***	0.363**	-0.866***
	(0.128)	(0.103)	(0.145)	(0.241)
Observations	4,780	2,641	3,365	1,934
R-squared	0.029	0.159	0.049	0.107
Sample	Children	No Children	Children + Only Male FT	High Educ. Men

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

B Theory

B.1 General Model

We proceed backwards, and consider first the labor market problem, then the household problem and then the marriage market problem.

THE LABOR MARKET. Assume that z is supermodular. Then, the assignment will satisfy PAM in the labor market, that is the matching function $\mu : \tilde{\mathcal{X}} \to \mathcal{Y}$ is increasing and given by:

$$G(\mu(\tilde{x})) = \tilde{N}(\tilde{x})$$
$$\mu(\tilde{x}) = G^{-1}(\tilde{N}(\tilde{x}))$$

The FOC of the firm problem is given by

$$z_{\tilde{x}}(\tilde{x}, y) - w_{\tilde{x}}(\tilde{x}) = 0,$$

which holds at the equilibrium assignment, $z_{\tilde{x}}(\tilde{x}, \mu(\tilde{x})) - w_{\tilde{x}}(\tilde{x}) = 0$. Integrating on both sides, we obtain the equilibrium wage function as:

$$w(\tilde{x}) = w_0 + \int_0^{\tilde{x}} z_{\tilde{x}}(s, \mu(s))ds$$

From the assumptions on z, there is PAM in $(\tilde{x}, y)$, that is in workers' *effective* types and firms' productivities. But what does sorting in terms of workers' raw skill types depend on? Recall, in equilibrium, worker $\tilde{x}$ is matched to firm y:

$$y = \mu(\tilde{x}_i) = \mu(e(x_i, h_i(x_i, \eta(x_i), v(x_i))))$$

Even though a firm only cares about $\tilde{x}$, the worker can either be a man or a woman. They can have different skills x_f and x_m, yet be matched to the same firm y if their hours worked differ. Now to analyze sorting of men and women (and whether there is PAM in skills, i.e. in (x_f, y) and (x_m, y)), we should analyze how μ varies w.r.t. x_m and x_f separately. To analyze whether matching is monotone in women's underlying types x_f, we compute:

$$\begin{aligned}
\frac{\partial \mu}{\partial x_f} &= \frac{\partial \mu}{\partial e}\left(\frac{\partial e}{\partial x_f} + \frac{\partial e}{\partial h_f}\frac{dh_f}{dx_f}\right) \\
&= \frac{\partial \mu}{\partial e}\left(\frac{\partial e}{\partial x_f} + \frac{\partial e}{\partial h_f}\left(\frac{\partial h_f}{\partial x_f} + \frac{\partial h_f}{\partial \eta^{-1}(x_f)}\frac{\partial \eta^{-1}(x_f)}{\partial x_f} + \frac{\partial h_f}{\partial v}\frac{\partial v}{\partial x_f}\right)\right)
\end{aligned}$$

where the second effect is new to our framework, indicating that labor market sorting depends on (i) how hours worked vary with women's skills and (ii) how hours worked vary with her marriage partner's skill and (iii) how they vary with the marriage transfer. Similarly for men.

We can also see, that for a given x, the the firm match is increasing in hours worked:

$$\frac{\partial \mu(\tilde{x})}{\partial h} = \frac{\partial \mu(\tilde{x})}{\partial \tilde{x}} \frac{\partial e}{\partial h}$$

If $\frac{\partial e(h,x)}{\partial h} > 0$ which holds by assumption (higher hours worked imply a higher effective type), then $\frac{\partial \mu(\tilde{x})}{\partial \tilde{x}}$ implies $\frac{\partial \mu(\tilde{x})}{\partial h} > 0$ (PAM in the labor market in effective types implies that more productive firms will hire workers who put in more hours of work).

Similarly, for given hours worked, increasing skill x leads to a match with a better firm type.

The Household Problem. The problem of a couple with characteristics x_m and x_f can be written as the following cooperative household problem:

$$\max_{c_f, h_m, h_f} u(w(\tilde{x}_m) + w(\tilde{x}_f) - c_f, p(1-h_m, 1-h_f)) \tag{24}$$

$$s.t. \qquad u(c_f, p(1-h_m, 1-h_f)) \geq v$$

Let λ to be the Lagrange multiplier associated with the constraint. Men and women take as given the wage function in the labor market. The Lagrangian is then given by:

$$\mathcal{L}(c_f, h_m, h_f, \lambda) = u(w(\tilde{x}_m) + w(\tilde{x}_f) - c_f, p(1-h_m, 1-h_f)) + \lambda(u(c_f, p(1-h_m, 1-h_f)) - v)$$

The FOCs of this problem are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_f} = 0 \quad &\Leftrightarrow \quad -u^m_{c_m} + \lambda u^f_{c_f} &&= 0 && (25)\\
\frac{\partial \mathcal{L}}{\partial h_m} = 0 \quad &\Leftrightarrow \quad u_{c_m} \frac{\partial w(\tilde{x}_m)}{\partial h_m} &&= (u^m_p + \lambda u^f_p) \frac{\partial p(1-h_m, 1-h_f)}{\partial (1-h_m)} && (26)\\
\frac{\partial \mathcal{L}}{\partial h_f} = 0 \quad &\Leftrightarrow \quad u_{c_m} \frac{\partial w(\tilde{x}_f)}{\partial h_f} &&= (u^m_p + \lambda u^f_p) \frac{\partial p(1-h_m, 1-h_f)}{\partial (1-h_f)} &&\\
&\Leftrightarrow \quad u_{c_f} \frac{\partial w(\tilde{x}_f)}{\partial h_f} &&= \left(\frac{u^m_p}{\lambda} + u^f_p\right) \frac{\partial p(1-h_m, 1-h_f)}{\partial (1-h_f)} && (27)\\
\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad &\Leftrightarrow \quad & v &= u(c_f, p(1-h_m, 1-h_f) && (28)
\end{aligned}$$

FOCs (26) and (27) equate for each partner the marginal benefit from spending an extra hour in home production (rhs) with the marginal cost of doing so, which is the opportunity cost of not working that hour in the labor market and forgoing an increase in utility through increased private consumption (lhs). FOC (28) gives the female's constraint. In turn, FOC (25) equates the marginal utilities of husband and wife w.r.t. private consumption, taking into account how costly it is to transfer utility across partners which is captured by the Lagrange multiplier $\lambda = \partial u / \partial v$.

Equations (25)-(27) jointly imply that the efficient allocation of consumption and hours satisfies:

$$\frac{\frac{\partial p(1-h_m,1-h_f)}{\partial(1-h_f)}}{\frac{\partial w(\tilde{x}_f)}{\partial h_f}} = \frac{\frac{\partial p(1-h_m,1-h_f)}{\partial(1-h_m)}}{\frac{\partial w(\tilde{x}_m)}{\partial h_m}}$$

Along the Pareto frontier, the relative gain from putting one more hour into home production relative to its cost (forgone wages) is equalized across partners in a couple.

Conditions (25) to (28) implicitly define the consumption functions $c_m(x_m, x_f, v)$, the hours functions $h_m(x_m, x_f, v)$ and $h_f(x_f, x_m, v)$ for a given marriage partner and wife's compensation v, and the value of the problem

$$\Phi(x_m, x_f, v) = u(c_m(x_m, x_f, v(x_f)), p(1 - h_m(x_m, x_f, v(x_f)), 1 - h_f(x_m, x_f, v(x_f))))$$

is then maximized to solve for the marriage partner choice and the market clearing price v.

B.2 TU Representation: Examples

We give two examples of utility functions under which our problem features TU. First, consider a utility function that is multiplicatively separable in private and public consumption, $u(c,p) = m(c)k(p)$, where, as standard, m and k are assumed to be strictly increasing. Then, the wife's constraint in (3) reads $m(c)k(p) = \overline{v} \Leftrightarrow c = m^{-1}(\overline{v}/k(p))$, and the constrained household maximization problem (3) can be simplified as follows, when substituting in both the budget constraint and the constraint on the wife's utility:

$$\begin{aligned}
&\max_{h_m,h_f} m(w(\tilde{x}_m) + w(\tilde{x}_f) - m^{-1}(\overline{v}/k(p)))k(p) \\
\Leftrightarrow \quad &\max_{h_m,h_f} m([w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))] - m^{-1}(\overline{v})) \\
\Leftrightarrow \quad &\max_{h_m,h_f} [w(\tilde{x}_m) + w(\tilde{x}_f)][m^{-1}(k(p))]
\end{aligned}$$

These manipulations yield an objective function, which is linear in money, complying with the Gorman form. A second example is the log-utility function $u(c,p) = \log(c) + \log(p)$. We then obtain from (3):

$$\begin{aligned}
&\max_{h_m,h_f} \log\left(w(\tilde{x}_m) + w(\tilde{x}_f) - \frac{\exp(\overline{v})}{p}\right) + \log(p) \\
\Leftrightarrow \quad &\max_{h_m,h_f} \log\left((w(\tilde{x}_m) + w(\tilde{x}_f))p - \exp(\overline{v})\right) \\
\Leftrightarrow \quad &\max_{h_m,h_f} (w(\tilde{x}_m) + w(\tilde{x}_f))p
\end{aligned}$$

which is again linear in money and independent of $\overline{v}$. Here the arguments in the log are linear but they can be specified in a flexible way, e.g. CRRA utility for each component, as long as the overall utility function is log-additive in consumption of the private and the public good.

B.3 Monotone Equilibrium: Derivations and Proofs

B.3.1 Stability

We begin by analyzing stability of equilibrium. In particular, since our comparative statics apply to any stable equilibrium, where we refer to stability of the household problem. (In turn, stability in the marriage and labor market is trivially satisfied in the competitive equilibrium.) We therefore first define stability of the household problem.

Definition 3. *The equilibrium in the household stage, given by* (h_f, h_m)*, is stable for a given wage function* w *if*

$$w_{\tilde{x}_m\tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11} < 0 \tag{29}$$

$$w_{\tilde{x}_f\tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22} < 0 \tag{30}$$

$$\left(w_{\tilde{x}_m\tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11}\right)\left(w_{\tilde{x}_f\tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22}\right) - 4p_{12}p_{21} > 0. \tag{31}$$

We now explain what is behind this definition of stability. Stability is reflected in the properties of the spouses' 'best response' functions, which are implicitly given by the household's FOCs (8)-(9). The first FOC gives rise to a 'best response' function r_f for the woman's hours to man's hours, given by $h_f^* = r_f(h_m^*)$ for all $h_m^* \in [0, 1]$. The second FOC gives rise to a 'best response' function r_m for the man's hours to the woman's hours, given by $h_m^* = r_m(h_f^*)$ for all $h_f^* \in [0, 1]$. An equilibrium in the household stage is any solution (h_m^*, h_f^*) to (8)–(9), an intersection of the best response functions. It is stable if small perturbations in the hours choices induce the agents to converge back to equilibrium. This is true iff (29)–(31) hold:

Inequalities (29)–(30) require that, given the labor hour choice of the partner, own hours adjust properly if the marginal benefit from 'investing' (i.e. the marginal wage benefit from more labor hours) is higher/lower than the marginal cost (forgone home production), i.e., if higher then hours go up; if lower then hours go down.

In turn, inequality (31) requires that, at the crossing point of the two best response functions, $|\partial r_f/\partial h_m^*| < |\partial r_m^{-1}/\partial h_m^*|$. That is, the slope of r_f is smaller than the slope of the inverse of r_m when they cross in the (h_m^*, h_f^*)-space, or r_m crosses r_f from below (above) when the BR-functions are upward (downward) sloping at the crossing. To see that inequality (31) requires that, at the crossing of the two best response functions, $|\partial r_f/\partial h_m^*| \leq |\partial r_m^{-1}/\partial h_m^*|$, note that $|\partial r_m^{-1}/\partial h_m^*| > |\partial r_f/\partial h_m^*|$ is equivalent to

$$\begin{aligned}
\Leftrightarrow \quad & \left|-\frac{w_{\tilde{x}_f\tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22}}{2p_{12}}\right|\left|-\frac{w_{\tilde{x}_m\tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11}}{2p_{12}}\right| && > 1 \\
\Leftrightarrow \quad & \left|-\left(w_{\tilde{x}_f\tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22}\right)\right|\left|-\left(w_{\tilde{x}_m\tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11}\right)\right| && > (2p_{12})^2 \\
\Leftrightarrow \quad & \left(w_{\tilde{x}_f\tilde{x}_f} e_{h_f}^2 + w_{\tilde{x}_f} e_{h_f h_f} + 2p_{22}\right)\left(w_{\tilde{x}_m\tilde{x}_m} e_{h_m}^2 + w_{\tilde{x}_m} e_{h_m h_m} + 2p_{11}\right) && > (2p_{12})^2
\end{aligned}$$

where the last inequality follows from conditions (29)–(30), ensuring that in a stable equilibrium, both terms in brackets on the LHS are negative. The crossing of the best response functions described by (31) guarantees that small perturbations away from the equilibrium hours induce dynamics so that the resulting hours adjustments make the household converge back to the equilibrium hours.

With Definition 3 at hand, we can now show the following.

Lemma 1. *For any wage function w with the property that $\partial w/\partial h_i = w_{\tilde{x}_i} e_{h_i}$ is strictly positive and finite for all $h_i \in [0,1]$, a stable equilibrium of the household problem exists.*

Proof. We first show that, given $h_m^* \in [0,1]$, there exists a solution to (8), $w_{\tilde{x}_f} e_{h_f} = 2p_2$ that satisfies also (30). To see this, notice that, for any h_m^*, the LHS, $\partial w/\partial h_f = w_{\tilde{x}_f} e_{h_f}$, is always strictly positive and finite under the premise, while the RHS goes to infinity as h_f^* goes to one (i.e. when $1 - h_f$ goes to zero), and to zero as h_f^* goes to zero (and $1 - h_f$ to one) by the assumed Inada conditions on p. Hence, by the Intermediate Value Theorem, there is at least one solution to (8), and the first solution satisfies (29) (the RHS crosses the LHS from below when plotted against h_f). Denote this solution by $h_f^* = r_f(h_m^*)$ for each h_m^*, and notice that it is continuously differentiable in h_m^*.

Similarly, given $h_f^* \in [0,1]$, there exists a solution to (9) that satisfies also (29). We denote this solution by $h_m^* = r_m(h_f^*)$ for each h_f^*, which is continuously differentiable in h_f^*.

Next, we note that $r_f(0) > 0$ and $r_f(1) < 1$, which follows from the strictly positive and finite value of the LHS in (8), $w_{\tilde{x}_f} e_{h_f} = 2p_2$, and the boundary properties of p_2. Similarly, $r_m(0) > 0$ and $r_m(1) < 1$. Since r_m and r_f are continuous functions, there exists a pair (h_f^*, h_m^*) such that $h_f^* = r_f(h_m^*)$ and $h_m^* = r_m(h_f^*)$ and that also satisfies (31), i.e. $|\partial r_f/\partial h_m^*| \leq |\partial r_m^{-1}/\partial h_m^*|$.

Thus, a stable equilibrium of the household problem exists.[33] □

Lemma 1 shows that for *any* given wage function that satisfies the stated assumption on $\partial w/\partial h_i$, there exists at least one stable equilibrium. We now argue that, as a result, also for the *equilibrium* wage function, there exists at least one stable equilibrium. To show this, the equilibrium wage function needs to satisfy the property that $\partial w/\partial h_i$ is strictly positive and finite. It is given by (2), and so $\partial w/\partial h_i = z_{\tilde{x}_i}(\tilde{x}_i, \mu(\tilde{x}_i)) e_{\tilde{x}_i}$, which under the assumption of the model is strictly positive and finite.

Thus, whenever an equilibrium exists, we know that there exists at least one stable one. And any stable equilibrium satisfies the conditions of Definition 3, which we use to sign our comparative statics of the household problem.

[33]To see this graphically in the (h_m^*, h_f^*) space, notice that r_f starts above the inverse of r_h and ends below. Continuity implies there is a crossing between the two, and the first one is such that the inverse of r_m crosses the r_f from below. This implies that at the crossing point $|\partial r_f/\partial h_m^*||\partial r_m/\partial h_f^*| \leq 1$.

B.3.2 Spouses' Hours as Strategic Complements or Substitutes

Differentiating each FOC w.r.t. to (h_f, h_m), we obtain the slopes of the 'best response' functions for women (men) to men's (women's) hours, respectively:

$$\begin{aligned}
&0 = (w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f})dh_f + 2p_{12}dh_m + 2p_{22}dh_f \\
\Leftrightarrow \quad &\frac{dh_f}{dh_m} = -\frac{2p_{12}}{w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}} \qquad (32) \\
&0 = (w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m})dh_m + 2p_{12}dh_f + 2p_{11}dh_m \\
\Leftrightarrow \quad &\frac{dh_m}{dh_f} = -\frac{2p_{12}}{w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}} \qquad (33)
\end{aligned}$$

In any stable equilibrium (since the denominators of these expressions are negative), the best response functions are upward (downward) sloping, and thus hours are strategic complements (substitutes), if p is supermodular (submodular).

B.3.3 Proof of Proposition 1

We now check that under the specified conditions the equilibrium is monotone as specified in Definition 2.

1. PAM in the labor market in $(y, \tilde{x}_i)$ materializes due to $z_{\tilde{x}y} > 0$.

2. The properties of how own labor hours depend on own type and partner type follow from the equilibrium comparative statics that can be derived from the system of FOCs of the household problem. We differentiate system (8) - (9) w.r.t. x_f, taking as given the equilibrium wage function and the equilibrium marriage market matching function (meaning that hours do not only depend on own type but also on the partner's type along the equilibrium assignment η):

$$\begin{aligned}
A\frac{\partial h_f}{\partial x_f} + B\frac{\partial h_m}{\partial x_f} &= -(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f}) \\
C\frac{\partial h_f}{\partial x_f} + D\frac{\partial h_m}{\partial x_f} &= -\eta'(x_f)(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m})
\end{aligned}$$

where

$$\begin{aligned}
A &:= w_{\tilde{x}_f\tilde{x}_f}e^2_{h_f} + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22} \\
B &:= 2p_{12} \\
C &:= 2p_{12} \\
D &:= w_{\tilde{x}_m\tilde{x}_m}e^2_{h_m} + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}
\end{aligned}$$

Denote $|H| := AD - BC$, which is the determinant of the Hessian of the household problem and positive in any *stable* equilibrium, see stability condition (31) of Definition 3. Solving the system yields

$$\frac{\partial h_f}{\partial x_f} = \frac{-(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f})\left(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}^2 + w_{\tilde{x}_m}e_{h_mh_m} + 2p_{11}\right) + 2\eta'(x_f)((w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m}))p_{12}}{|H|}$$
$$= \frac{-\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f\partial x_f}\frac{\partial^2 u}{\partial h_m^2} + 2\eta'(x_f)\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m}p_{12}}{|H|}$$
$$\frac{\partial h_m}{\partial x_f} = \frac{-\eta'(x_f)\left(w_{\tilde{x}_f\tilde{x}_f}e_{h_f}^2 + w_{\tilde{x}_f}e_{h_fh_f} + 2p_{22}\right)(w_{\tilde{x}_m\tilde{x}_m}e_{h_m}e_{x_m} + w_{\tilde{x}_m}e_{x_mh_m}) + (w_{\tilde{x}_f\tilde{x}_f}e_{h_f}e_{x_f} + w_{\tilde{x}_f}e_{x_fh_f})2p_{12}}{|H|}$$
$$= \frac{-\eta'(x_f)\frac{\partial^2 u}{\partial h_f^2}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m} + 2\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f\partial x_f}p_{12}}{|H|}.$$

These expressions are equivalent to (10) and (11). They are positive in any stable equilibrium (where conditions (29)-(31) from Definition 3 are satisfied), meaning hours are increasing in own types and in partner's types, given that (i) hours are complementary in home production (p supermodular so that $p_{12} > 0$), (ii) matching on the marriage market is PAM ($\eta' > 0$), for which we will provide conditions below, and (iii) wages are convex in effective types (ensuring that $\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f\partial x_f} > 0$ and $\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m} > 0$), which can be ensured from primitives if z is weakly convex. To see this note that $\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f\partial x_f} = z_{\tilde{x}_f}e_{x_fh_f} + (z_{\tilde{x}_f\tilde{x}_f} + z_{\tilde{x}_fy}\mu'(\tilde{x}_f))e_{x_f}e_{h_f}$. Analogously, we can compute how hours respond to changes in male types (12) and (13) (omitted here for brevity).

3. In a stable equilibrium, PAM in the marriage market results if (as indicated in the text)

$$\Phi_{x_fx_m} = \frac{\partial^2 w}{\partial x_f\partial h_f}\frac{\partial \hat{h}_f}{\partial x_m}$$
$$= 2\frac{p_{12}\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f\partial x_f}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m}}{|H|} > 0$$

where we substituted in the first line the 'partial' equilibrium comparative static $\partial\hat{h}_f/\partial x_m$ (see below) to obtain to the second line. Thus, $\Phi_{x_fx_m} > 0$ if $z_{\tilde{x}\tilde{x}} \geq 0$ (which renders $\partial^2 w/\partial x_i\partial h_i > 0$, see part 2. above) and $p_{12} > 0$.

To show that $\partial\hat{h}_f(x_m, h_f)/\partial x_m > 0$ (i.e. hours are increasing in partner's type for *any* couple (x_f, x_m), not only along the equilibrium assignment, which is to be solved for in this step), we differentiate system (8) - (9) w.r.t. x_m for any given x_f:

$$\frac{\partial \hat{h}_m}{\partial x_m} = \frac{-\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m}\frac{\partial^2 u}{\partial h_f^2}}{|H|}$$
$$\frac{\partial \hat{h}_f}{\partial x_m} = \frac{2\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m\partial x_m}p_{12}}{|H|}$$

where we used the second expression to sign Φ_{x_f,x_m} above.

B.3.4 Proof of Corollary 1

The proof of Corollary 1 follows immediately from the properties of monotone equilibrium:

1. Follows from PAM on the marriage market.
2. That both $(x'_f, \eta(x'_f))$ work more labor hours and less at home compared to couple $(x_f, \eta(x_f))$ follows from the monotone equilibrium property that labor market hours are increasing in own and partner's types. That both partners of couple $(x'_f, \eta(x'_f))$ have more private consumption and less public consumption is due to the following: Woman x'_f has more private consumption than x_f since $c_f = v - p$ and $\partial v/\partial x_f = w_{\tilde{x}_f} e_{x_f} > 0$ and $dp/dx_f = -p_2(\partial h_f/\partial x_f + (\partial h_f/\partial x_m)\eta') - p_1(\partial h_m/\partial x_f + (\partial h_m/\partial x_m)\eta') < 0$. That the private consumption of partner $\eta(x'_f)$ is higher compared to the other couple's partner follows from $c_m = w_f + w_m - c_f$ and thus

$$\begin{aligned}\frac{dc_m}{dx_f} &= \frac{dw_f}{dx_f} + \frac{dw_m}{dx_f} - \frac{dc_f}{dx_f} \\ &= w_{\tilde{x}_f} e_{h_f}\left(\frac{\partial h_f}{\partial x_f} + \frac{\partial h_f}{\partial x_m}\eta'\right) + w_{\tilde{x}_f} e_{x_f} + w_{\tilde{x}_m} e_{h_m}\left(\frac{\partial h_m}{\partial x_f} + \frac{h_m}{\partial x_m}\eta'\right) + w_{\tilde{x}_m} e_{x_m}\eta' - \frac{\partial v}{\partial x_f} + \frac{dp}{dx_f} \\ &= (w_{\tilde{x}_f} e_{h_f} - p_1)\left(\frac{\partial h_f}{\partial x_f} + \frac{\partial h_f}{\partial x_m}\eta'\right) + (w_{\tilde{x}_m} e_{h_m} - p_2)\left(\frac{\partial h_m}{\partial x_f} + \frac{\partial h_m}{\partial x_m}\eta'\right) + w_{\tilde{x}_m} e_{x_m}\eta' > 0\end{aligned}$$

   where the positive sign follows from the FOCs of the household problem w.r.t. hours (the first two terms are positive if the FOCs hold). Finally, her utility increases in her type which follows from $\partial v/\partial x_f = w_{\tilde{x}_f} e_{x_f} > 0$. That her partner's utility increases in her type follows from $u = c_m + p$ and thus, $du/dx_f = dc_m/dx_f + dp/dx_f = w_{\tilde{x}_m} e_{x_m}\eta' > 0$ (where we applied the Envelope Theorem).
3. The first statement on effective types follows from 2. and the premise. The statement on PAM on the labor market follows from the comparison of effective types of the two couples and $z_{\tilde{x}y} > 0$. The last statement on wages follows since wages are increasing in effective types.

B.3.5 Proof of Proposition 2

We check that the properties of non-monotone equilibrium are satisfied under the specified conditions.

1. PAM in the labor market in $(y, \tilde{x}_i)$ materializes due to $z_{\tilde{x}y} > 0$.

2. The properties of how labor hours depend on own type and partner type follow straight from comparative statics expressions (10)–(13): $\partial h_i/\partial x_i > 0$ in a stable equilibrium if $p_{12} < 0$ and $\eta' < 0$ as well as $z_{\tilde{x}\tilde{x}} > 0$ (which renders $\partial^2 w/\partial x_i \partial h_i > 0$). Further, $\partial h_i/\partial x_j < 0$ under the same conditions. It remains to verify that $\eta' < 0$, see 3.

3. NAM in the marriage market results if

$$\Phi_{x_f x_m} = 2\frac{p_{12}\frac{\partial^2 w(\tilde{x}_f)}{\partial h_f \partial x_f}\frac{\partial^2 w(\tilde{x}_m)}{\partial h_m \partial x_m}}{|H|} < 0$$

Thus, in a stable eq., $\Phi_{x_f x_m} < 0$ if $z_{\tilde{x}\tilde{x}} > 0$ (which renders $\partial^2 w/\partial x_i \partial h_i > 0$ for $i \in \{f, m\}$) and $p_{12} < 0$.

C Solution of the Quantitative Model

The solution of our quantitative model consists of solving for a fixed point in the wage function (as a function of effective types) such that under this wage function, marital choices, household labor supply, and labor market sorting are all consistent. That is, we find the market-clearing wage function that induces households that form in the marriage market to optimally supply labor (pinning down their effective types) such that, when optimally sorting into firms on the labor market, this gives rise to that exact same wage function.

We first solve for the optimal matching in the marriage market and households' labor supply choices *given* a wage function. Given the induced labor supply decisions, individuals move to the labor market where they optimally match with firms. Sorting in the labor market endogenously determines a new wage function (again as a function of effective types) that supports this particular matching. Given this new wage function, new marriage and labor supply decisions are made that, in turn, again affect wages in the labor market. We iterate between the problem of households on the one hand and of workers and firms on the other until the wage function converges (until a fixed point in the wage function is found).

We next describe the solution in each decision stage, starting backwards from the labor market and then going to household and marriage problems. Finally, we outline the algorithm to find the fixed point.

C.1 Partial Equilibrium in the Labor Market ([lpe])

First, we show how we solve for the matching and wage functions in the labor market, (μ, w). Consider our *exogenous* distribution of firms, $y \sim G$, and *any given* distribution of effective types, $\tilde{s} \sim \tilde{N}_s$. Note that even though $\tilde{N}_s$ is an endogenous object in our model, from a partial equilibrium perspective where marital and household choices are taken as given, firms take the distribution $\tilde{N}_s$ as fixed.

To solve for the optimal matching between firms and workers note that the production function $z(\tilde{s}, y)$ is assumed to be supermodular. By the well known Becker-Shapley-Shubik result (Becker (1973) and Shapley and Shubik (1971)) the optimal matching in the labor market is positive assortative between y and $\tilde{s}$. Hence, a worker with effective skill $\tilde{s}$ matches to firm $\mu(\tilde{s})$, where

$$\mu(\tilde{s}) = G^{-1}[\tilde{N}_s(\tilde{s})].$$

Moreover, the wage function w is derived from the firms' first order conditions (2), evaluated at the optimal matching μ. In the quantitative model where G and $\tilde{N}_s$ are discrete, we approximate the integral in (2) numerically using trapezoidal integration.

The output from solving the equilibrium in the labor market given marital and household choices is the tuple (μ, w) as defined above.

C.2 Optimal Household Choices ([hh])

Second, we derive the solution of the household problem that yields spouses' optimal consumption, (c_f, c_m), their optimal labor supply (h_f, h_m), and the distribution of effective types $\tilde{N}_s$.

Individuals arrive at the household stage either as singles with human capital s_i or in a couple with human capital bundle (s_f, s_m). We denote the household human capital type by two-dimensional vector $\mathbf{s} = (s_f, s_m) \in \{\mathcal{S} \cup \emptyset\}^2$ where, for example, $(s_f, \emptyset)$ denotes the household of single woman of type s_f.

When solving their household problem agents *take as given* wage function w, the marriage market matching distribution η, and the marriage market clearing price v.

Given prices and marriage outcomes, couples solve problem (19) and singles solve problem (20). Replacing the budget constraint into the objective function and noting the transferable utility structure of the problem given the quasilinear utility function, the collective problem of couple (s_f, s_m) after labor supply preference shocks realize is given by:

$$\max_{h_m, h_f} w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^C(1 - h_m, 1 - h_f) + \delta_m^{h_m} + \delta_f^{h_f} \tag{34}$$

where $w(\tilde{s}_m)$ and $w(\tilde{s}_f)$ depend on hours through the effective human capital types (21).

Similarly, the problem of a single woman of type s_f after realization of her labor supply preference shock is

$$\max_{h_f} w(\tilde{s}_f) + p^U(1 - h_f) + \delta_f^{h_f} \tag{35}$$

and the problem of a single man s_m is given by

$$\max_{h_m} w(\tilde{s}_f) + p^U(1 - h_m) + \delta_m^{h_m}. \tag{36}$$

To derive aggregate labor supply and the distribution of effective types $\tilde{N}_s$, we need to introduce some notation.

We denote the *alternative* of hours that a decision maker chooses by $\mathbf{h} \in \{\mathcal{H} \cup \emptyset\}^2 := \{\{0, ..., 1\} \cup \emptyset\}^2$ (where $\emptyset$ indicates the hours of the non-existing partner when the individual is single). We denote by $\mathbf{h}^t$ the hours alternative chosen by a decision maker of type $t \in \{C, U\}$:

$$\mathbf{h}^t = \begin{cases} (h_i, \emptyset), i \in \{f, m\} & \text{if } t = U \\ (h_f, h_m) & \text{if } t = C. \end{cases}$$

where type $t = U$ indicates *single* and type $t = C$ indicates *couple*.

Also, we denote the economic utility associated with hours alternative $\mathbf{h}^t$ of household type $t \in \{C, U\}$ with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ by $\overline{u}_{\mathbf{s}}^t(\mathbf{h}^t)$, where

$$\overline{u}_{\mathbf{s}}^t(\mathbf{h}^t) = \begin{cases} w(\tilde{s}_i) + p^U(1 - h^t) & \text{if } t = U \\ w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^C(1 - h_m, 1 - h_f) & \text{if } t = C. \end{cases} \tag{37}$$

We obtain the optimal labor supply and private consumption (c_m, c_f, h_m, h_f) for each household by solving problems (34)-(36). Given our assumption that the labor supply shock distribution is Type-I extreme value, we then obtain the fraction of agents that optimally chooses each hours alternative. The probability that household type $t \in \{C, U\}$ with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ chooses hours alternative $\mathbf{h} \in \{\mathcal{H} \cup \emptyset\}^2$ is

$$\pi^t_{\mathbf{s}}(\mathbf{h}^t) \quad = \quad \frac{\exp(\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t)/\sigma_\delta)}{\sum_{\tilde{\mathbf{h}} \in \{\mathcal{H} \cup \emptyset\}^2} \exp(\overline{u}^t_{\mathbf{s}}(\tilde{\mathbf{h}}^t)/\sigma_\delta)} \tag{38}$$

Denoting the fraction of *households* who are type $\mathbf{s}$ by $\eta_{\mathbf{s}}$, the fraction of households who are of type $\mathbf{s}$ *and* choose hours alternative $\mathbf{h}$ is given by

$$\eta_{\mathbf{s}} \times \pi^t_{\mathbf{s}}(\mathbf{h}^t).$$

From this distribution of *household* labor supply we back out the distribution of *individual* labor supply. To do so, we compute the fraction of men and women of each *individual* human capital type, s_i in household $\mathbf{s}$, optimally choosing each *individual* hours alternative h_i associated with household labor supply $\mathbf{h}$. Given the distribution of individual labor supply, we can compute the distribution of effective human capital types, $\tilde{N}_s$. First, note that the support of the distribution is obtained by applying functional forms (21) for any combination of *individual* hours and skill types. Second, to each point in the support of $\tilde{s}(s, h)$ we attach the corresponding individual frequencies from the individual labor supply distribution backed out as explained above.

The output from solving the household problem given (w, μ, η) is the tuple $(h_f, h_m, c_f, c_m, \tilde{N}_s)$.

C.3 Partial Equilibrium in the Marriage Market ([mpe])

In the marriage stage, individuals draw idiosyncratic taste shocks for partners and single-hood, β^s_i, with $i \in \{f, m\}$ and $s \in \{\mathcal{S} \cup \emptyset\}$. At this stage, labor supply shocks are not yet realized. As a result, the *ex ante* economic value from marriage of type (s_f, s_m) is the expected value of (34); and the ex-ante economic value from female and male singlehood is the expected value of (35) and (36). In both cases, the expectation is taken over the distribution of δ-shocks. Denoting the utility transfer to a female spouse of type s_f by $v(s_f)$, the values of being married (economic plus non-economic) for a woman type s_f and a man type s_m in couple (s_f, s_m) are given by

$$\begin{aligned}
\Phi_f(s_m, s_f, v(s_f)) + \beta^{s_m}_f &:= v(s_f) + \beta^{s_m}_f \\
\Phi_m(s_m, s_f, v(s_f)) + \beta^{s_f}_m &:= \mathbb{E}_\delta \Big\{ \max_{h_m, h_f} w(\tilde{s}_m) + w(\tilde{s}_f) + 2p^C(1 - h_m, 1 - h_f) + \delta^{h_m}_m + \delta^{h_f}_f \Big\} + \beta^{s_f}_m - v(s_f) \\
&= \sigma_\delta \Big[\kappa + \ln \Big(\sum_{\mathbf{h}^C \in \mathcal{H}^2} \exp\{\overline{u}^C_{\mathbf{s}}(\mathbf{h}^C)/\sigma_\delta\} \Big) \Big] + \beta^{s_f}_m - v(s_f)
\end{aligned}$$

where $\kappa = 0.57722$ is the Euler constant, $\overline{u}$ is defined in (37) and $\mathbb{E}_\delta$ indicates the expectation is taken over the distribution of δ-shocks.

In turn, the value of being single for woman s_f is

$$\Phi_f(\emptyset, s_f) + \beta_f^{\emptyset} := \sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^U \in \mathcal{H}} \exp\{\overline{u}^U_{(s_f,\emptyset)}(\mathbf{h}^U)/\sigma_\delta\}\Big)\Big] + \beta_f^{\emptyset}$$

and for man s_m it is

$$\Phi_m(s_m, \emptyset) + \beta_m^{\emptyset} := \sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^U \in \mathcal{H}} \exp\{\overline{u}^U_{(\emptyset,s_m)}(\mathbf{h}^U)/\sigma_\delta\}\Big)\Big] + \beta_m^{\emptyset}$$

Every man s_m and every woman s_f choose the skill type of their partner or to remain single to maximize their value on the marriage market:

$$\max \quad \{\max_{s_f \in \mathcal{S}} \Phi_m(s_m, s_f, v(s_f)) + \beta_m^{s_f}, \Phi_m(s_m, \emptyset) + \beta_m^{\emptyset}\}$$
$$\max \quad \{\max_{s_m \in \mathcal{S}} \Phi_f(s_m, s_f, v(s_f)) + \beta_f^{s_m}, \Phi_f(\emptyset, s_f) + \beta_f^{\emptyset}\}$$

These problems thus also capture the marriage market participation margin.

In practice, using the transferable utility property of our model, we solve for the *optimal* marriage matching by maximizing the total sum of marital values across all individuals in the economy, using a linear program. We denote the matching distribution by η, which solves

$$\begin{aligned}
&\max_{\eta_{(s,s')} \in [0,1]} \sum_{(s,s') \in \{\mathcal{S} \cup \emptyset\}^2} \eta_{(s,s')} \times \big(\Phi(s,s') + \tilde{\beta}\big) \\
s.t. \quad &\sum_{s \in \mathcal{S}} \eta_s = 1/2 \\
&\sum_{s' \in \mathcal{S}} \eta_{s'} = 1/2
\end{aligned}$$

where $\eta_{(s,s')}$ denotes the mass of household type $(s,s') \in \{\mathcal{S} \cup \emptyset\}^2$ under matching η; η_s denotes the marginal distribution of η with respect to the first dimension; $\eta_{s'}$ denotes the marginal distribution of η with respect to the second dimension; $\Phi(s,s')$ denotes the economic value from marriage for the different types of households, $\Phi(s,s') = \{\Phi_m(s,s',v(s')) + \Phi_f(s,s',v(s')), \Phi_f(\emptyset,s'), \Phi_m(s,\emptyset)\}$; and $\tilde{\beta}$ denotes $\beta_f^{s_m} + \beta_m^{s_f}$ for couples and $\beta_i^{\emptyset}$ $(i = \{f,m\})$ for singles. Note that the restrictions of this linear program impose that the mass of women and men in all households (couples or singles) must be equal to the total mass of women and men in the economy (which is 1/2 for both sexes).

We obtain the equilibrium matching in the marriage market, η, by solving this linear program, taking prices and allocations in households and the labor market, (w, μ, h_f, h_m), as given.

C.4 General Equilibrium of the Model

Once we have derived the solution of each of the stages taking the output from the other stages as given, we solve for the *general equilibrium* of the model by searching for the prices, allocations, and assignments such that all markets are simultaneously in (partial) equilibrium. To preview, we start "backwards" from the output of the labor market stage with an arbitrary initial wage function indicating a wage offered to each effective type. In the household stage, each *potential* household takes those wages as given and makes their labor supply choices. These optimal labor supply choices (in each potential household) are then used by each individual in the marriage market to compute the value of single-hood and marriage with different partners, leading to marriage choices. The hours choices of *formed* households give rise to a distribution of *effective* types. With this endogenous distribution of effective types we go back to the labor market stage, where we match workers' effective types with firms' productivities optimally. This labor market matching gives rise to a new wage function supporting this allocation. With the new wage function at hand, we solve and update the household and marriage problems and iterate until we obtain a wage function from the labor market stage that when taken as given by households gives rise to an aggregate labor supply that, in turn, produces those same wages, i.e. until we have found the fixed point in the wage function.

C.4.1 Trembling Effective Types

A challenge in the search for the equilibrium is that each household type needs to face a wage for *any* hours choice in order to make its optimal labor supply choices. However, it may be the case that at a given iteration of our fixed point algorithm, the wage function is such that certain levels of hours are *not* chosen by some household types. Therefore, in the next iteration, agents would face a wage function that only maps *realized* effective types to a wage (i.e. a wage function 'with gaps in the support'), see subsection C.1. The problem then is that agents do not know the payoff from all potential hours choices when they try to make their optimal choice.

To fill in the gaps so that households of each type observe wages for *any* hours alternatives, we develop a trembling strategy. The trembling strategy consists of drawing a random sample of women and men and force them to supply a suboptimal amount of hours from the set of unchosen hours in each iteration. In practice, for each group of women with skill type s_f and each group of men with skill type s_m, we track their optimal choices for a given wage function and determine the hours that were *not* chosen with positive probability. We then draw a 1% random sample of women and men within each of those skill types and assign them uniformly to the unchosen hours. So we force maximally 1% of each skill type (the 'tremblers') to choose sub-optimal hours, or in other words, to tremble. Finally, we construct the distribution of effective types $\tilde{N}_s$ by taking into account both 'trembling' effective types and 'optimal' effective types.

C.4.2 Fixed Point Algorithm

To solve for the general equilibrium we denote by $\tilde{N}_s^*$ the distribution of *realized* effective types (based on *optimal* hours choices, not *trembling* hours choices). Similarly, we denote by w^* the wage as a function of *realized* effective types only, where recall that the *full support* wage function is denoted by w. The fixed point algorithm we designed to solve for the equilibrium is as follows:

0. Initiate a round-zero wage function for the full support of effective types, w^0.

At any round $r \geq 1$

1. Input w^{r-1} and solve [hh] and [mpe]. Update $\tilde{N}_s^{*r}$.
2. Input $\tilde{N}_s^{*r}$ and solve [lpe]. Update w^{*r}.
3. Update w^r:
   (a) We determine w^{*r} from step 2. above.
   (b) Simultaneously, we fill in the wage for effective types that did not realize at round r by solving step 2. for *trembling types*, yielding w^r.
4. Move to round $r+1$ by going back to step 1. above and continue iterating until the wage function converge, that is, $w^{r+1}(\tilde{s}) - w^r(\tilde{s}) < \epsilon$ for $\epsilon > 0$ and small, element-by-element (for each $\tilde{s}$).
5. (OUTPUT) Compute the general equilibrium as the tuple of outputs from [hh], [mpe], and [lpe] at the round where the wage function w^r converged.

D Sample Construction

D.1 Sample for Empirical Facts

In this Section, we describe the sample restrictions and the variables used to construct our empirical facts, presented in section 3.

D.1.1 Sample restrictions

To construct our sample, we pool observations from the period 2010-2016, for the original GSOEP samples and their refreshments. [34]

We apply the following demographic restrictions: we keep all individuals in private households, either single or in heterosexual couples (either married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, which could be either never married or the individual's first marriage.[35] We restrict our sample to individuals between 25 and 50 years old. We apply these restrictions at the individual level, but not at the couple level, which implies that in some cases, one of the spouses could be part of the sample, even if the other spouse is not.

Regarding the labor market, we exclude from our sample those individuals who are self-employed, those working in odd occupations (`kldb92`$\geq$ 9711) or employed individuals with missing data on the occupation code.

D.1.2 Variable description

We now describe the variables we use in the empirical analysis.

1. **Education Variables:** We classify individuals into three education levels: low education includes those with a high school degree or with a vocational degree, with less than 11 years of schooling. Medium education includes those individuals with a vocational degree and more than 11 years of schooling. High education includes those with a college degree or higher. Education levels are defined based on the ISCED 97 classification. Alternatively, we use years of education as our education measure, and we left-truncate this variable at 10 years of education.
2. **Marriage Market Sorting:** We define marriage market sorting as the difference between years of education of the partners in couples (own years of education minus partner's years of education). When the analysis is at the household level, we compute marriage market sorting as the difference between the years of education of the male partner and the years of education of the female partner, to be consistent across couples. We winsorize differences of years in education larger than 4 years in absolute value and we exclude from our sample couples in which the difference in years of education between the partners is larger than 6 years.

[34] We exclude from our analysis the migrants and refugees samples, the oversampling of low income individuals and single parents, and the oversampling of high income earners.

[35] Since cohabitants are defined as never married, they might be in a cohabiting relationship that is not the first one.

3. **Matching Function:** Our matching function is given by the task complexity of the occupation in which the individual is employed, defined at the 4 digits of the `kldb92` classification of occupations. For a detailed description of how this measure is constructed, refer to Appendix E.3.
4. **Labor Market Sorting:** We define labor market sorting as the correlation between the individual's years of education and their matching in the labor market, defined by the matching function.
5. **Hours:**
   (a) **Market hours**: We define market hours as the number of contractual hours an individual works in the labor market in a given week (excluding overtime). We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively.[36]
   (b) **Home Hours:** We measure home hours as the weekly time an individual allocates to the following activities: childcare, care for others, housework, running errand and repairs around the house, the car or garden work. Since home hours are measured by typical week day, we multiply these hours by 5. When home hours are not available, we use information on market hours to impute them, assuming a total of 70 hours available per week to allocate to home production and market work. We proceed in the same way when information on market hours is missing.
   (c) **Leisure Hours:** We measure leisure hours as the weekly hours allocated to hobbies and other leisure activities.

D.2 Estimation Sample

In this section we describe the sample restrictions and variable definitions of our estimation sample.

D.2.1 Sample restrictions

In order to construct our estimation sample, we use data from the period 2010-2016 of the original GSOEP panel and its refreshment (as discussed above). We apply the following restrictions:

1. **Age restrictions**: We restrict our attention to individuals between 25 and 50 years. In the case of couples, we keep in our sample those households in which at least one of the spouses is between 25 and 50 years old in some of the years in which they appear in the panel.
2. **Marital Status Restrictions**: We focus on individuals who are either single or in heterosexual couples (either married or cohabiting). We restrict our analysis to the first marital spell of the life of an individual, which could be either never married or the individual's first marriage. We drop observations corresponding to periods after the first marriage ended, or for which the end date of the first marriage cannot be identified. We keep in our sample individuals who are in cohabiting relationships (even if it is not the first one), or who cohabited in the past, as long as they never

[36] In our robustness exercises, we define actual hours as the number of hours an individual worked in her main occupation during the past week, independently of the hours set in the labor contract.

got married, or only got married once. We drop observations for which we can identify more than one spouse/partner during the sample period.

3. **Labor Market Restrictions:**
   (a) We exclude from our sample observations corresponding to individuals working in odd occupations (`kldb92`$\geq$ 9711) or employed but with missing data on the occupation code.
   (b) We exclude observations corresponding to employed individuals for whom information on hourly wages is missing.
4. **Additional restrictions:**
   (a) We exclude observations from individuals to whom we cannot assign a 'type', based on our estimation of individual's types described in Appendix E.2.
   (b) We exclude observations corresponding to individuals in couples but for whom information on the spouse/partner is not available in any of the years in which they appear in the sample.

D.2.2 Definition of typical occupation, hours, wages and marital status

In this section we explain the methodology to create summary variables for the individuals in the GSOEP panel, in line with the static nature of our model. We define the concepts of typical occupation, typical working hours, typical wages, and typical home hours. For each individual we also define the typical marital status and the corresponding variables for her/his partner.

Typical occupation: we apply the following rules to determine the typical occupation of an individual:

1. If the individual appears in the sample only once, we assign the occupation corresponding to that particular year.
2. If the individual appears more than once in the sample, but only reports one occupation, we assign to the individual the unique occupation that we observe over the sample period.
3. If an individual appears more than once in the sample, but he was out of the labor market or self- employed in at least one year, we check the tenure of 'not employed' or 'self-employment' (how many years he is not working or self-employed while he appears in the sample, even if not consecutive). If the individual was either not working or self-employed for strictly more than half of the time they appear in the sample, we consider 'not employed' or 'self-employment' as his typical occupation. If an individual was both 'not employed' and 'self-employed' during the time he appears in the sample, and exactly half of the time under each status, we consider 'not employed' as his typical occupation.
4. If we observe an individual more than once, and he only had one occupation other than 'not employed' or 'self-employment' during that period, we assign the individual to that occupation, except he was classified as 'not employed' or 'self-employed' based on the previous point.
5. If we observe an individual having more than one occupation during the period he appears on the sample, we follow these rules:

(a) We construct the difference in percentiles between the highest ranked occupation and the lowest ranked occupation the individual held during the sample period, where we rank occupations as described in section E.3.

(b) When the difference is larger than 0.1, we assign the individual to the highest ranked occupation he held.

(c) If this difference is lower than 0.1, we assign the individual to the occupation with the longest tenure.

(d) If the difference is lower than 0.1, but we observe the individual in more than one occupation with the same tenure, we assign the individual to the better ranked occupation (between those with the longest tenure).

(e) After applying all these rules, we exclude from our sample those individuals whose typical occupation is self-employment.

Typical market hours: For each individual, we define typical market hours as the mean of the contractual hours worked in the years he was working in the typical occupation, defined as above. We winsorize market hours to 10 and 60 hours, at the bottom and the top, respectively. For those individuals whose typical occupation is 'not employed', typical market hours takes value zero.

Typical wages: For each individual, we define typical wages as the mean of hourly wages earned in years the individual worked in the typical occupation.[37]

Typical home hours: For each individual, we define typical home hours (defined as in Appendix D.1) as the mean of hours the individual works at home while employed in the typical occupation. We impute missing home hours using data on labor market hours, as discussed in Appendix D.1.

Typical marital status: We define the typical marital status following these rules:

1. If the individual had only one marital status during the period we observe her in the sample, we consider that marital status as the typical one.
2. If the individual switched from being single to being married during the time she appears in the sample, we assign the marital status observed during the period she was employed in the typical occupation. If she was observed under both marital status while working in the typical occupation, we assign her to marriage.
3. We exclude from the sample those individuals that, even if assigned to marriage based on the previous rules, report more than one spouse during the time they appear in the sample. We also exclude their corresponding partners.

Corresponding variables for spouses: For every variable defined above (typical occupation, typical market hours, typical wages and typical home hours) we define the analogous variable for the individual's partner.

[37] Hourly wages are constructed based on inflation adjusted monthly earnings, divided by monthly contractual hours (constructed as weekly contractual hours times 4.3). Hourly wages are trimmed at the bottom and top 1% percentile. Data to deflate wages comes from the OECD: `https://data.oecd.org/price/inflation-cpi.htm`

Following the rules defined above, our *baseline sample* (all Germany, 2010-2016) has 5,153 individuals living in 3,094 households. Of these households, 2,059 are couples and 1,035 are single individuals (518 are single women and 517 are single men).

In turn, for our across-space exercise, we define the variable 'typical region' to classify individuals between East and West Germany. Following the criteria above, we assign them to the region in which they live while working in their typical occupation. We drop observations for which we cannot unequivocally assign individuals to only one region. For this exercise, we drop observations from Berlin following other papers in the literature, such as Heise and Porzio (2019). Our East-West sample spans 2010-2016 and contains 4,931 individuals living in 2,943 households. Of these households, 696 are in East Germany (24%) and 2,247 are in West Germany (74%).

Last, for our exercise that compares outcomes over time, we use data from the periods 1990-1996 and 2010-2016 and restrict the sample to those observations in West Germany. We follow the same criteria above to define the typical variables. Our *West sample for 1990-1996* consists of 2,381 individuals, living in 1,320 households, of which 1,061 are couples and 259 are singles. Our *West sample 2010-2016* contains 3,795 individuals living in 2,247 households, of which 1,548 are couples and 699 are singles.

E Estimation

E.1 Identification

Identification of the Worker and Job Distribution. We identify the distributions (G, N_s) directly from the data. We treat the distribution of occupational attributes G as observable. We identify the workers' human capital distribution N_s from workers' education and fixed effect in a panel wage regression. See Section 7.3 for the details on estimation.

Identification of the Remaining Parameters. For identification of the marriage market shock parameters we will make two further assumptions:

Assumption D1 (Identification)**.** .

1. *(Indifferent Type.) There exists some man $\hat{s}_m \in \mathcal{S}$ such that $\Phi(\hat{s}_m, s_f, v(s_f))/\sigma^C_\beta - \Phi(\hat{s}_m, \emptyset) = 0$ for some choice of female spouse $s_f \in \mathcal{S}$.*
2. *(Marriage Sorting.) There exists some male type $\hat{s}_m \in \mathcal{S}$ and choice of female spouse $s_f, s'_f \in \mathcal{S}$, $s_f \neq s'_f$, such that $\partial \left(\log \frac{\eta_{(s_f, \hat{s}_m)}}{\eta_{(s'_f, \hat{s}_m)}} \right) / \partial s_m \neq 0$.*

The first assumption says that there exists at least one male type and a choice of female partner, such that his economic value from marrying her (rescaled by σ^C_β) equals his economic value from being single (rescaled by σ^U_β). This assumption can be justified by the fact that there always is one long side of the marriage market (here w.l.g. men; if women were on the long side, then we would simply specify the argument in terms of women's choices), and thus there are men of some type(s) that are necessarily single. But then, with many types, there exists the 'marginal' type whose economic value of being married equals the economic value of being single.

The second assumption says that that there is marriage market sorting at least locally.

E.1.1 Proof of Proposition 3

Identification of the Production Function. We follow arguments on the estimation of hedonic models to show identification of the production function z. In principle, this argument is non-parametric, but in line with our parametric estimation, we focus here illustrate the parametric approach. We mainly follow Ekeland, Heckman, and Nesheim (2004), Section IV.D, and also make use of their discussion of the identification strategy proposed by Rosen (1974) and criticized by Brown and Rosen (1982). The identification is based on the firm's FOC and exploits the non-linearity of our matching model, which is an important source of identification just as in Ekeland, Heckman, and Nesheim (2004). Recall the firm's FOC in the model is given by:

$$w_{\tilde{s}}(\tilde{s}) = z_{\tilde{s}}(\tilde{s}, \mu(\tilde{s})) \tag{39}$$

This equation can be used to identify the parameters of interest. There are two steps:

1. Estimate the marginal return $w_{\tilde{s}}$ as the derivative of the kernel regression of w (observed) on $\tilde{s}$. Denote this estimate by $\widehat{w_{\tilde{s}}}$. We treat the derivative of the wage as observable. Also note that we only observe $\tilde{s}$ for men in the data (for women, there is – at this stage – an unknown productivity wedge ψ), and so for this argument we focus on the subsample of men.

2. Estimate FOC (39) after applying a log transformation and taking into account measurement error:

$$\log(\widehat{w}_{\tilde{s}}(\tilde{s})) = \log(z_{\tilde{s}}(\tilde{s}, \mu(\tilde{s}))) + \epsilon \tag{40}$$

   where, for concreteness, we assume the functional form for z, $z(\tilde{s}, y) = A_z \tilde{s}^{\gamma_1} y^{\gamma_2} + K$ (see main text), and where we treat $\tilde{s}$ and the matching μ as observed. Note that this functional form of z circumvents the identification problem of Rosen (1974), discussed in Brown and Rosen (1982) and Ekeland, Heckman, and Nesheim (2004), since the slope of the wage gradient in $\tilde{s}$ is *not* equal to the slope of the marginal product in $\tilde{s}$. We assume that ϵ is the measurement error of the marginal wage, with mean zero and uncorrelated with the rhs variables. Regression (40) identifies $(A_z, \gamma_1, \gamma_2)$.

In turn, the constant in the production function K is identified from the wage of the lowest productive type $s = 0$ (and thus $\tilde{s} = 0$), with $w(0) = \int_0^0 z_{\tilde{s}}(t, \mu(t))dt + K = K$.

Identification of the Female Productivity Wedge. We can identify ψ from the within sh- type (agents with the same s and same work hours h) wage gap across gender. Denote the gender wage gap within individuals of hours-human-capital type $sh = \overline{sh}$ in the data by $gap(\overline{sh})$, which we consider as observed for any $\overline{sh}$. Then, given the wage function and our assumption that effective skill types of women and men are given by $\tilde{s}_f = \dfrac{1 - exp(-\psi s_f h_f)}{\psi}$ and $\tilde{s}_m = s_m h_m$, the observed gender wage gap can be expressed as:

$$gap(\overline{sh}) = \frac{w(\overline{sh}) - w\left(\dfrac{1 - exp(-\psi\overline{sh})}{\psi}\right)}{w(\overline{sh})}$$

where in our model $w\left((1 - exp(-\psi\overline{sh}))/\psi\right)$ is the wage of a woman with $s_f h_f = \overline{sh}$ and $w(\overline{sh})$ is the wage of a man with $s_m h_m = \overline{sh}$; and where we made the dependence of the female wage on ψ explicit. We consider production function z and (marginal product $z_{\tilde{s}}$) as known at this stage (see previous step) and also, given that (G, N_s) were identified directly from the data and we observe which worker matches to which firm, we consider matching function μ as known.

Then for any observed $gap(\overline{sh}) \in [0, 1 - K/(w(\overline{sh})))$, the female wage is given by:

$$w\left(\frac{1 - exp(-\psi\overline{sh})}{\psi}\right) = \frac{w(\overline{sh})}{gap(\overline{sh}) + 1} \tag{41}$$

For a given (observed) μ, the RHS is independent of ψ, positive and takes values in $[w(\overline{sh})/(2 -$

$K/w(\overline{sh})), w(\overline{sh})]$ for any $gap(\overline{sh}) \in [0, 1 - K/(w(\overline{sh})))$. Note that the lowest possible value of the rhs is strictly larger than K, since $w(\overline{sh})/(2 - K/w(\overline{sh})) > K \Leftrightarrow (w(\overline{sh}) - K)^2 > 0$ for all $w(\overline{sh}) > K$.

In turn, the LHS is positive, finite and decreasing in ψ since

$$w\left(\frac{1 - exp(-\psi\overline{sh})}{\psi}\right) = \int_0^{(1-exp(-\psi\overline{sh}))/\psi} z_{\bar{s}}(t, \mu(t))dt + K.$$

It equals $w(\overline{sh})$ for $\psi \to 0$ and K for $\psi \to \infty$. Hence, by the Intermediate Value Theorem there exists a unique ψ for which (41) holds. Thus, ψ is identified from gender wage gaps of agents with the same hour-human-capital combination.

Identification of the Scale of the Labor Supply Shock. Recall that the choice set of singles differs from that of couples. In Appendix C, we introduced the notation where we denote the *alternative* of hours that a decision maker $t \in \{C, U\}$ chooses by $\mathbf{h}^t \in \{\mathcal{H} \cup \emptyset\}^2 := \{\{0, ..., 1\} \cup \emptyset\}^2$ with:

$$\mathbf{h}^t = \begin{cases} (h_i, \emptyset), i \in \{f, m\} & \text{if } t = U \\ (h_f, h_m) & \text{if } t = C. \end{cases}$$

where type $t = U$ indicates *single/unmarried* and type $t = C$ indicates *couple*.

Also, we denote the sum of economic utility and utility derived from preference shocks of decision-maker t with human capital type $\mathbf{s} \in \{\mathcal{S} \cup \emptyset\}^2$ by $\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t) + \delta^{\mathbf{h}^t}$, where

$$\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t) + \delta^{\mathbf{h}^t} = \begin{cases} u(c_i, p^U(1 - h_i)) + \delta^{h_i}, i \in \{f, m\} & \text{if } t = U \\ u(c_f, p^C(1 - h_f, 1 - h_m)) + u(c_m, p^C(1 - h_f, 1 - h_m)) + \delta^{h_f} + \delta^{h_m} & \text{if } t = C. \end{cases}$$

The probability that household type t with human capital $\mathbf{s}$ chooses hours alternative $\mathbf{h}$ is

$$\pi^t_{\mathbf{s}}(\mathbf{h}^t) = \frac{\exp(\overline{u}^t_{\mathbf{s}}(\mathbf{h}^t)/\sigma_\delta)}{\sum_{\tilde{\mathbf{h}} \in \{\mathcal{H} \cup \emptyset\}^2} \exp(\overline{u}^t_{\mathbf{s}}(\tilde{\mathbf{h}}^t)/\sigma_\delta)} \tag{42}$$

which follows from our assumption on the shock distribution (Type-I extreme value).

Let $\mathbf{h}^U = (0, \emptyset) := \mathbf{0}$ denote the hours for a single individual who puts all hours into home production and zero hours into market work. We consider alternative $\mathbf{h}^U = \mathbf{0}$ as our normalization choice and obtain for a single male:

$$
\begin{aligned}
\frac{\pi^U_{\mathbf{s}}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}}(\mathbf{0})} &= \frac{\exp(\overline{u}^U_{\mathbf{s}}(\mathbf{h}^U)/\sigma_\delta)}{\exp(\overline{u}^U_{\mathbf{s}}(\mathbf{0})/\sigma_\delta)} \\
\log\left(\frac{\pi^U_{\mathbf{s}}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}}(\mathbf{0})}\right) &= \frac{\overline{u}^U_{\mathbf{s}}(\mathbf{h}^U) - \overline{u}^U_{\mathbf{s}}(\mathbf{0})}{\sigma_\delta} \\
&= \frac{w(s_m h_m) - w(s_m 0) + p^U(1-h_m) - p^U(1-0)}{\sigma_\delta} \\
&= \frac{w(s_m h_m) + p^U(1-h_m) - p^U(1)}{\sigma_\delta}
\end{aligned} \tag{43}
$$

where the wage from not working is set to zero and where h_m is the male hours associated to this single household's hours choice, $\mathbf{h}^U = (h_m, \emptyset)$. Treating human capital type s_m as observed at this stage, the partial derivative of the LHS w.r.t. to s_m yields:

$$
\frac{\partial \log \frac{\pi^U_{\mathbf{s}}(\mathbf{h}^U)}{\pi^U_{\mathbf{s}}(\mathbf{0})}}{\partial s_m} = \frac{1}{\sigma_\delta}\frac{\partial w(s_m h_m)}{\partial s_m}.
$$

The LHS is observed in the data (how does the relative choice probability for hours alternative $\mathbf{h}^U \neq \mathbf{0}$ change in the population of male singles as one varies human capital types s_m), and on the RHS, $\frac{\partial w(s_m h_m)}{\partial s_m}$ is also observed (it is the effect of men's human capital on wages given the hours choice $\mathbf{h}^U \neq \mathbf{0}$). Thus, σ_δ is identified.

Identification of the Home Production Function. Let $\mathbf{h}^C = (1,1) := \mathbf{1}$ denote the vector of hours for couples in which both spouses put zero hours into home production and thus work full time in the labor market. Alternative $\mathbf{h}^C = \mathbf{1}$ is our normalization choice and we obtain the relative choice probabilities:

$$
\begin{aligned}
\frac{\pi^C_{\mathbf{s}}(\mathbf{h}^C)}{\pi^C_{\mathbf{s}}(\mathbf{1})} &= \frac{\exp(\overline{u}^C_{\mathbf{s}}(\mathbf{h}^C)/\sigma_\delta)}{\exp(\overline{u}^C_{\mathbf{s}}(\mathbf{1})/\sigma_\delta)} \\
\log\left(\frac{\pi^C_{\mathbf{s}}(\mathbf{h}^C)}{\pi^C_{\mathbf{s}}(\mathbf{1})}\right) &= \frac{\overline{u}^C_{\mathbf{s}}(\mathbf{h}^C) - \overline{u}^C_{\mathbf{s}}(\mathbf{1})}{\sigma_\delta} \\
&= \frac{w_f\left(\frac{1-exp(-\psi s_f h_f)}{\psi}\right) - w_f\left(\frac{1-exp(-\psi s_f)}{\psi}\right) + w_m(s_m h_m) - w_m(s_m) + 2p^C(1-h_m, 1-h_f)}{\sigma_\delta}
\end{aligned} \tag{44}
$$

where we used that $2p^C(0,0) = 0$ by assumption in our quantitative model. Note that the LHS of (44) (choice probabilities) is observed, and on the RHS, wages of men and women with types (s_f, s_m) conditional on hours are also observed in the data, and σ_δ is known at this stage. Thus, home production function p^C is non-parametrically identified since we can specify (44) for all hours alternatives $\mathbf{h}$ chosen in the data. Note that we can identify p from a couple C of any type $\mathbf{s} = (s_f, s_m)$.

By a similar argument the home production function of singles, p^U, is identified.

Identification of the Scale of the Marriage Taste Shock (Couples). Let $\eta_{(s_f,s_m)}$ be the probability that a man s_m chooses woman s_f on the marriage market. Under the assumption that the taste shock is extreme-value distributed (and following the same derivations as for the choice probabilities of hours), $\eta_{(s_f,s_m)}$ is given by:

$$\eta_{(s_f,s_m)} = \frac{\exp(\Phi(s_m, s_f, v(s_f)) - \bar{\beta}^C)/\sigma_\beta^C)}{\sum_{s'_f} \exp(\Phi(s_m, s'_f, v(s'_f)) - \bar{\beta}^C)/\sigma_\beta^C)}$$

where, as before, we denote by $\Phi(s_m, s_f, v(s_f))$ the expected value of man s_m from being married to woman s_f and paying her the transfer $v(s_f)$. This value is given by:

$$\begin{aligned}\Phi(s_m, s_f, v(s_f)) &:= \sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^C\in\mathcal{H}^2} \exp\Big\{\overline{u}_{\mathbf{s}}^C(\mathbf{h}^C)/\sigma_\delta\Big\}\Big)\Big] - v(s_f) \\ &= \sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^C\in\mathcal{H}^2} \exp\Big\{\frac{w(s_m h_m) + w\left(\frac{1-exp\{-\psi s_f h_f\}}{\psi}\right) + 2p^C(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big)\Big] - v(s_f)\end{aligned}$$

Using the ratio of probabilities of choosing two different women s_f and s'_f, we obtain:

$$\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(s'_f,s_m)}}\right) = \frac{\Phi(s_f, s_m, v(s_f)) - \Phi(s'_f, s_m, v(s_f))}{\sigma_\beta^C}$$

which, using the expression for $\Phi(s_m, s_f, v(s_f))$ from above, we can spell out as:

$$\begin{aligned}\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(s'_f,s_m)}}\right) = \frac{1}{\sigma_\beta^C}\Bigg(&\sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^C\in\mathcal{H}^2} \exp\Big\{\frac{w(s_m h_m) + w\left(\frac{1-exp\{-\psi s_f h_f\}}{\psi}\right) + 2p^C(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big)\Big] - \\ &- \sigma_\delta \Big[\kappa + \ln\Big(\sum_{\mathbf{h}^C\in\mathcal{H}^2} \exp\Big\{\frac{w(s_m h_m) + w\left(\frac{1-exp\{-\psi s'_f h_f\}}{\psi}\right) + 2p^C(1-h_m, 1-h_f)}{\sigma_\delta}\Big\}\Big)\Big] - v(s_f) + v(s'_f)\Bigg)\end{aligned}$$

To ease notation, let us denote the argument of the natural logarithms in the expression above by $\sum(s_f)$ for couple (s_f, s_m) and $\sum(s'_f)$ for couple (s'_f, s_m).

Taking the derivative w.r.t. s_m, we obtain:

$$\begin{aligned}
\frac{\partial\left(\log\frac{\eta_{(s_f,s_m)}}{\eta_{(s'_f,s_m)}}\right)}{\partial s_m} &= \frac{\sigma_\delta}{\sigma_\beta^C}\times\frac{\partial\left(\ln\sum(s_f)-\ln\sum(s'_f)\right)}{\partial s_m}\\
&= \frac{1}{\sigma_\beta^C}\times\left(\frac{1}{\sum(s_f)}\times\Big[\sum_{\mathbf{h}^C\in\mathcal{H}^2}\exp\{\overline{u}^C_{(s_f,s_m)}(\mathbf{h}^C)/\sigma_\delta\}\times\frac{\partial w(s_m h_m)}{s_m}\Big]-\right.\\
&\left.-\frac{1}{\sum(s'_f)}\times\Big[\sum_{\mathbf{h}^C\in\mathcal{H}^2}\exp\{\overline{u}^C_{(s'_f,s_m)}(\mathbf{h}^C)/\sigma_\delta\}\times\frac{\partial w(s_m h_m)}{s_m}\Big]\right)
\end{aligned}\tag{45}$$

The LHS is observed. In turn, all objects on the RHS are observed or identified at this stage (wages, home production, and the wage derivative w.r.t. type s_m, which, importantly, depends on hours) except for σ_β^C. If there exist types (s_f, s'_f, s_m) such that the LHS is not zero (as ensured by Assumption D1.2.), then for those types the expression in big parentheses on the RHS is not zero. Hence, σ_β^C is identified from (46).

Identification of the Scale of the Marriage Taste Shock (Couples). Let $\eta_{(s_f,s_m)}$ be the probability that a man s_m chooses woman s_f on the marriage market. Under the assumption that the taste shock is extreme-value distributed (and following the same derivations as for the choice probabilities of hours), $\eta_{(s_f,s_m)}$ is given by:

$$\eta_{(s_f,s_m)} = \frac{\exp(\Phi(s_m, s_f, v(s_f)) - \bar{\beta}^C)/\sigma_\beta^C)}{\sum_{s'_f}\exp(\Phi(s_m, s'_f, v(s'_f)) - \bar{\beta}^C)/\sigma_\beta^C)}$$

where, as before, we denote by $\Phi(s_m, s_f, v(s_f))$ the expected value of man s_m from being married to woman s_f and paying her the transfer $v(s_f)$. This value is given by:

$$\begin{aligned}
\Phi(s_m, s_f, v(s_f)) &:= \sum_{\mathbf{h}^C\in\mathcal{H}^2}\pi^C_{(s_f,s_m)}(\mathbf{h}^C)[w(s_m h_m)+w\left(\frac{1-exp(-\psi s_f h_f)}{\psi}\right)-v(s_f)+2p^C(1-h_m,1-h_f)]\\
&= \sum_{\mathbf{h}^C\in\mathcal{H}^2}\pi^C_{(s_f,s_m)}(\mathbf{h}^C)[w(s_m h_m)+w\left(\frac{1-exp(-\psi s_f h_f)}{\psi}\right)+2p^C(1-h_m,1-h_f)]-v(s_f)
\end{aligned}$$

Let $\eta_{(\emptyset,s_m)}$ be the probability that man s_m chooses single-hood. Then, using the ratio of probabilities of choosing woman s_f and being single, we obtain:

$$\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(\emptyset,s_m)}}\right) = \frac{\Phi(s_f, s_m, v(s_f))}{\sigma_\beta^C} - \frac{\Phi(\emptyset, s_m)}{\sigma_\beta^U} + \frac{\bar{\beta}^U}{\sigma_\beta^U} - \frac{\bar{\beta}^C}{\sigma_\beta^C}$$

We imposed the following normalizations of location and scale parameters: $\bar{\beta}^C = 0, \sigma_\beta^U = 1$. Hence:

$$\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(\emptyset,s_m)}}\right) = \frac{\Phi(s_f, s_m, v(s_f))}{\sigma_\beta^C} - \Phi(\emptyset, s_m) + \bar{\beta}^U$$

which, using the expression for $\Phi(s_m, s_f, v(s_f))$ from above, we can spell out as:

$$\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(\emptyset,s_m)}}\right) = \frac{1}{\sigma^C_\beta}\left(\sum_{\mathbf{h}^C\in\mathcal{H}^2}\pi^C_{(s_f,s_m)}(\mathbf{h}^C)[w(s_m h_m) + w\left(\frac{1-exp(-\psi s_f h_f)}{\psi}\right) + 2p^C(1-h_m, 1-h_f)] - v(s_f)\right)$$
$$-\sum_{\mathbf{h}^U\in\mathcal{H}}\pi^U_{(\emptyset,s_m)}(\mathbf{h}^U)[w(s_m h_m) + p^U(1-h_m)] + \bar{\beta}^U$$

Taking the derivative w.r.t. s_m, we obtain:

$$\frac{\partial\left(\log\frac{\eta_{(s_f,s_m)}}{\eta_{(\emptyset,s_m)}}\right)}{\partial s_m} = \frac{1}{\sigma^C_\beta}\sum_{\mathbf{h}^C\in\mathcal{H}^2}\frac{\partial\pi^C_{(s_f,s_m)}(\mathbf{h}^C)}{\partial s_m}[w(s_m h_m) + w\left(\frac{1-exp(-\psi s_f h_f)}{\psi}\right) + 2p^C(1-h_m, 1-h_f))]$$
$$+\frac{1}{\sigma^C_\beta}\sum_{\mathbf{h}^C\in\mathcal{H}^2}\pi^C_{(s_f,s_m)}(\mathbf{h}^C)\frac{\partial w(s_m h_m)}{\partial s_m} - \sum_{\mathbf{h}^U\in\mathcal{H}}\frac{\partial\pi^U_{(\emptyset,s_m)}(\mathbf{h}^U)}{\partial s_m}[w(s_m h_m) + p^U(1-h_m)] - \sum_{\mathbf{h}^U\in\mathcal{H}}\pi^U_{(\emptyset,s_m)}(\mathbf{h}^U)\frac{\partial w(s_m h_m)}{\partial s_m} \tag{46}$$

The LHS is observed. In turn, all objects on the RHS are observed or identified at this stage (wages, home production, the wage derivative w.r.t. type s_m and the derivative of choice probabilities of hours choices w.r.t. types s_m) except for σ^C_β. Further, by Assumption D1.2., there exists a couple (s_f, s_m), for which the sum of the first two terms on the RHS are not zero (implying that the LHS minus the sum of the last two terms on the RHS are not zero). Hence, σ^C_β is identified from (46).

Identification of the Location of the Marriage Taste Shock (Singles). We again use the probability of man s_m to choose woman s_f relative to choosing single-hood (see above):

$$\log\left(\frac{\eta_{(s_f,s_m)}}{\eta_{(\emptyset,s_m)}}\right) = \frac{\Phi(s_m, s_f, v(s_f))}{\sigma^C_\beta} - \Phi(s_m, \emptyset) + \bar{\beta}^U$$

Under Assumption D1.1., there exists at least one male type s_m and a female partner choice s_f such that s_m's economic value from marrying s_f (rescaled by σ^C_β, which is known at this stage) equals his economic value from being single, $\Phi(s_m, s_f, v(s_f))/\sigma^C_\beta - \Phi(s_m, \emptyset) = 0$. Therefore, the ratio of choice probabilities for choices $(s_f, \emptyset)$ by man s_m identifies $\bar{\beta}^U$. □

E.2 Estimation of Worker Types

Sample Selection. Our sample consists of individuals in the GSOEP from 1984-2016 who are between 20 and 60 years old and are either married/cohabiting or single. We exclude individual-year observations when the individual indicated self-employment and when he/she worked in an occupation that is not well defined (`kldb92`$\geq$ 9711). We also exclude observations with missing information on education or with missing (not zero) labor force experience. Our panel consists of around 200,000 person-year observations.

Key Variables. For *weekly hours*, we use reported contractual hours which, when positive, we winsorized by 10 hours from below and 60 hours from above to deal with outliers. For *labor force experience* we use the reported labor force experience, and – for men – we impute it by potential experience if this information is missing.[38] For *education*, we use three categories: In the group of *low education*, there are those whose highest degree is lower secondary, high school or vocational with weakly less than 11 years of education (around 35%). In the group of *medium education*, there are those with vocational degree and above 11 years of education (around 44%). In the group of *high education*, there are those with college degree or more (around 20%). For *occupation codes*, we use the variable `kldb92_current`, which consistently codes occupations across the entire panel. Our wage variable are *log hourly wages*, inflation-adjusted in terms of 2016 Euros. For the definition of 'demographical cells' in the selection stage below, we additionally use a variable that indicates whether *children below 9 years old* are in the household, *age bins* (≤ 25, > 25 and ≤ 40, > 40 and ≤ 50, > 50) and the *state* of residence.

Selection Equation. To account for selection into labor force participation in the wage regression, we first run a selection regression, where our excluded instrument is meant to capture the 'progressiveness of a demographic cell', which we proxy by the share of females working in a narrowly defined demographic cell. Our cells are defined by a combination of state, year, age and an indicator whether a child below 9 years is in the household.[39] When defining the value of this variable for a particular individual, we employ the 'leave-one-out' method and do not count the individual's labor force participation when computing this statistic. We further drop cells with less than five observations. We end up with 2,577 cells with more than 5 observations. Note that we experimented with additional cell characteristics (education and country of origin) but there is a trade-off between number of observations by cell and making the cells more specific to the demographic groups. Defining the cells by these additional variables led us to drop more than twice as many observations due to small cell size.

Our assumption on this IV is that the following exclusion restriction holds: 'progressiveness of a demographic cell' only affects wages through labor force participation but not in other ways.

We run the following probit selection regression:

$$emp_{it} = \alpha\, share_{j(i)t} + \sum_{ed \in \{voc,c\}} \alpha^{ed} x_{it}^{ed} + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{47}$$

where the dependent variable is an indicator of whether individual i is employed at time t, $share_{j(i)t}$ is the progressiveness measure in the demographic cell j of individual i at time t (given by the share of women working in the cell, see description of this IV above), x_{it} captures education indicators (indicator variables for medium and high education, so that low education is the reference group), Z_{it} is a vector

[38] For men, potential and actual experience are almost perfectly correlated, which is why this imputation should work well for them. For women, the correlation is much lower, which is why we do not impute here.

[39] German children make the transition from primary to high school/middle school at the age of 9, which implies longer school hours, at times also during the afternoon, making it easier for women to resume work. Our results are similar if we use children under 3 (when they enter kindergarten) or under 6 (when they enter primary school).

of demographic individual controls (linear and quadratic labor force experience in years, household size) and κ_s and ρ_t are state and year fixed effects and ϵ_{it} is a mean-zero error term. We cluster standard errors on the cell level.

The results are in Table 8, where we label our IV $share_{j(i)t}$ by *Share of Working Women in Cell*. There is a strong positive effect of the share of women working in the demographic cell on labor force participation of an individual in that cell (coefficient of 1.411 with standard error 0.0352).

Wage Regression. We further restrict the sample to those that are employed, work between 10 and 60 hours weekly (recall our treatment of outliers above), have non-missing hourly wage. Since we instrument hours worked and hours worked squared by (i) the hours worked by the partner, (ii) the hours worked by the partner squared and (iii) whether partner is present (where we set partner's hours to zero in both cases, if partner is present but not working and if partner is not present). We therefore drop observations whose partner reports to be employed but has zero reported labor hours and observations whose partner has missing employment and hours information. Since we include person fixed effects we also drop singleton observations (those who only show up in a single year of the panel). This leads to a sample of 114,000 person-year observations. Our regression specification is the following:

$$\ln w_{it} = \nu_i + \beta_0 IMR_{it} + \sum_{ed \in \{voc,c\}} \alpha^{ed} x_{it}^{ed} + \beta_1 h_{it} + \beta_2 h_{it}^2 + \beta_z' Z_{it} + \kappa_s + \rho_t + \epsilon_{it} \tag{48}$$

where ν_i is a person-fixed effect, IMR_{it} is the inverse mills ratio of individual i in year t from the selection probit regression, h_{it} are weekly hours worked, and x_{it}, Z_{it}, κ_s and ρ_t are as in the selection regression (47). Note that we could not include h_{it} into the selection equation since $h_{it} = 0$ versus $h_{it} > 0$ is a perfect predictor of employment. Nevertheless, based on our model, it is important to control for hours worked in the hourly wage regression. We again cluster standard errors on the cell level.

Table 9 contains the results. In column (1) an (2) we report the first stage regressions (for two variables to be instrumented: weekly hours and weekly hours squared) and column (3) contains the second stage regression. The three IV's for the hours worked variables (partner's hours, partner's hours squared and partner present) are not subject to the weak instrument problem according to the F-statistics. Moreover, the p-value of the Hansen's J statistic is 0.1629, suggesting that the over identifying restrictions are valid. Regarding the second stage, we note that the inverse mills ratio is positive and significant, indicating that individuals are positively selected into working and not controlling for selection here would have biased the coefficients upward. Moreover, we note that weekly hours worked have a strong positive effect on wages, justifying our model assumption that hours affect productivity and thus hourly wages.

Imputation. Based on these results, we are able to obtain x-types and ν-types (and thus s-types) for around 15,000 individuals, of which we are using a subset (see sample restrictions in Appendix D.2 for our structural estimation). We impute fixed effects of the remaining ones (around 13,000 individuals) based on the *multiple imputation* approach. Our auxiliary variables in this imputation are *education*, *gender* and *partner's hours*, which we chose based on the correlation between the variable to be imputed (ν_i) and the individual's characteristics available in the data set. We use a smaller subset of the individuals for whom we imputed the fixed effect for structural estimation (only those who comply with our final sample restrictions, Appendix D.2). In our final estimation sample, we have 5,153 unique individuals. For 25% of them we have imputed ν_i.

The resulting human capital distribution (s-types by education group) is displayed in Table 10.

We then use the raw sum of $s = \alpha^{ed}x^{ed} + \nu$ only to group agents into six bins and not as values of the human capital grid. Instead, we now normalize the support of the s-distribution such that grid points are equally spaced between zero and one.

Table 8: Selection Regression

	(1) Employed
Share of Working Women in Cell (share)	1.411***
	(0.0352)
Experience	0.0927***
	(0.00224)
Experience^2	-0.00165***
	(0.0000619)
Medium Educ	0.417***
	(0.00971)
High Educ	0.809***
	(0.0115)
HH Size	-0.0178***
	(0.00498)
Constant	-1.034***
	(0.0384)
Observations	197877
Pseudo R^2	0.140

Standard errors in parentheses
Controls that are included but not reported: state fixed effects, year fixed effects
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 9: Wage Regression

	(1) Weekly Hours Worked	(2) Weekly Hours Worked^2	(3) Log Hourly Wage
Partner's Weekly Hours Worked	-0.0374***	-2.726***	
	(0.00562)	(0.404)	
Partner's Weekly Hours Worked^2	0.000360**	0.0358***	
	(0.000150)	(0.0107)	
Partner Present	1.311***	73.90***	
	(0.130)	(7.825)	
Experience	0.210***	14.26***	0.0286***
	(0.0241)	(1.373)	(0.00149)
Experience^2	-0.000102	-0.0148	-0.000514***
	(0.000292)	(0.0177)	(0.0000179)
Medium Educ	-1.074***	-63.20***	0.0494***
	(0.121)	(7.453)	(0.00682)
High Educ	-0.226	-17.43	0.184***
	(0.296)	(17.05)	(0.0148)
HH Size	-0.987***	-55.23***	0.0188***
	(0.0373)	(2.220)	(0.00317)
Inverse Mills Ratio	-5.795***	-323.5***	0.144***
	(0.243)	(13.89)	(0.0218)
Weekly Hours Worked			0.0960***
			(0.0288)
Weekly Hours Worked^2			-0.00140***
			(0.000489)
Observations	113608	113608	113608
F			196.652
R^2			-0.142

Standard errors in parentheses

Controls that are included but not reported: individual fixed effects, state fixed effects, year fixed effects.

Columns (1) and (2) are the first stage regressions with weekly hours worked and weekly hours worked squared as dependent variables. Column (3) is the second stage regression. Weekly hours and weekly hours worked squared are instrumented in the second stage.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 10: Worker Distribution of s-Types by Education

educ/s-type	1/6	2/6	3/6	4/6	5/6	1	Total
1	751	0	477	0	0	0	1,228
2	0	1,199	0	0	1,359	0	2,558
3	0	0	0	216	0	1,151	1,367
Total	751	1,199	477	216	1,359	1,151	5,153

E.3 Estimation of Occupational Types

Sample Selection. Our main data source for measuring occupation types is the BIBB (see Section 3.1 for a detailed description). It contains data on task usage in 1,235 occupations defined by the 4-digit code `kldb92`, which we also use for our analysis in the GSOEP. This data is reported by individuals who work in these occupations. In order to reduce the problem of noisy reporting, we drop occupations in which the task information is based on less than 5 individuals. We are left with task data for 613 occupations. These are the most common occupations and we will base our structural estimation exercise on the 608 occupations that can be merged to the occupations held by individuals in our GSOEP sample.

Task Data. The BIBB contains data on how intensely different occupations use different types of tasks. These intensity measures are continuous and we normalized them to be between [0,1]. Table 11 gives an overview of the reported tasks.

	Task
1	Detailed Work
2	Same Cycle
3	New Tasks
4	Improve Process
5	Produce Items
6	Tasks not Learned
7	Simultaneous Tasks
8	Consequence of Mistakes
9	Reach Limits
10	Work Quickly
11	Problem Solving
12	Difficult Decisions
13	Close Gaps of Knowledge
14	Responsibility for Others
15	Negotiate
16	Communicate

Table 11: Tasks in the BIBB

Model Selection Stage. We merge the task data from the BIBB into occupations held by individuals in the GSOEP. As with the worker types, we here use the entire panel 1984-2016. We run a Lasso regression of log hourly wages on the task descriptors in Table 11 in order to systematically select the tasks that matter for the labor market. This procedure selects 13 tasks: all tasks in Table 11 except tasks 4, 9 and 12. The 13 selected tasks are important predictors of wages, which we will use going forward.

Principal Component Analysis. Because we want to reduce the occupational type to a single dimension, we collapse the information on the 13 selected tasks into a single measure using a standard dimension reduction technique (principal component analysis). We then select the first principal component, which – by far – captures the most variation of the underlying task variables in the sample of employed workers in the GSOEP: It captures around 43% of the underlying variation and – based on the loadings on the underlying task descriptors – our interpretation of this component is *task complexity* or *high skill requirement*. This interpretation is based on positive loadings on all task variables except *detailed work* and *same cycle*, which arguably are the only tasks that indicate routine work. All the other important tasks (as per the Lasso) indicate task complexity and have a positive loading on the first principal component. We report a scree-plot with eigenvalues of the different principal components and a plot with loadings of the first PC in Figure 18.

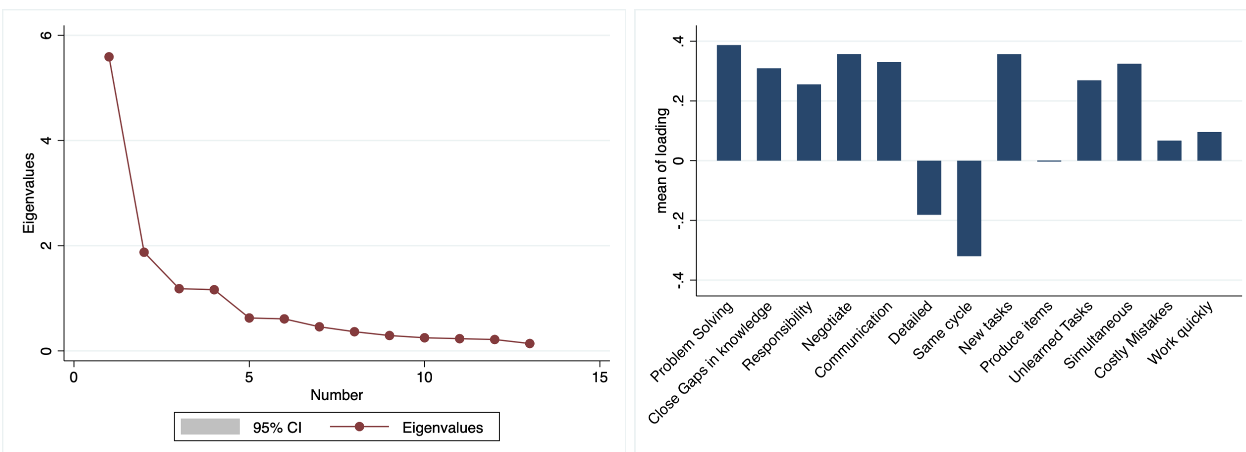

Figure 18: Principal Component Analysis

We then compute the mean of this measure by occupation and normalize its support between zero and one. We denote this measure by $\hat{y}$. Once matched to our main sample (see Appendix D.2), we compute our final measure of occupational types as the ranking of the occupation in the task complexity distribution, i.e. $y = \widehat{G}(\hat{y})$, which is uniformly distributed. Examples of occupations in the top 5% of the G include engineers and programmers. Examples of occupations in the lowest 5% include janitors and cleaners.

Alternative Approaches. We considered 2 alternative approaches for the estimation of occupational types. The first alternative is to not use wage data at all but instead, to directly reduce the multi-dimensional task data to a single measure by PCA. The second alternative is to rely more heavily on wages by first selecting important tasks via Lasso and then using the *predicted* wage based on these important tasks as our measure for occupation types. With all three approaches, we get very similar results. We show the summary statistics and correlations of all three measures in Tables 12 and 13.

Table 12: Summary Statistics of Alternative Measures

	mean	sd	min	max
y (baseline)	.573	.193	0	1
y(PCA)	.585	.188	0	1
y(Lasso)	.565	.196	0	1

Table 13: Correlation Across Alternative Measures

	y	y (PCA)	y (Lasso)
y (baseline)	1		
y (PCA)	0.983***	1	
y (Lasso)	0.946***	0.922***	1
Observations		608	

y is our baseline measure based on two steps: Lasso and PCA.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We also considered determining the occupational types through a fixed effect in the wage regression from above, which we used to recover worker types (equation (23)). This would have meant to run a two-way fixed effects regression. We chose our alternative approach that does not rely on occupational fixed effects for the following reasons: First, based on our model featuring a competitive labor market, the wage function does not depend on an occupational fixed effect/type when controlling for workers' effective types: all workers with the same effective type $\tilde{s}$ should be matched to the same occupation. Second, the two-way fixed effects approach is known to be problematic under limited mobility (limited mobility bias) and when one is interested in sorting (since the correlation between worker and firm/occupation fixed effects is often not accurately capturing sorting).

E.4 Internal Estimation

Our main estimation targets the 22 moments defined in Table 14. Below we provide details on how these moments are constructed both in the data and in the model.

Table 14: Moments

Moment Description	Definition
Fraction of Home Production by Wife (M1)	$\mathbb{E}\left[\frac{1-h_f}{(1-h_m)+(1-h_f)}\right]$
Correlation of Home Production Hours and Skill Type (M2-M3)	$corr(1-h_i, s_i), i \in \{f, m\}$
Correlation of Spouses' Home Hours (M4)	$corr(1-h_f, 1-h_m)$
Fraction of Couples with Both Spouses Working Full Time (M5)	$\frac{\#FullTimeCouples}{\#Couples}$
Mean Hourly Wage (M6)	$\mathbb{E}[w]$
Variance Hourly Wage (M7)	$Var[w]$
Upper Tail Wage Inequality (M8)	$\frac{w^{90}}{w^{50}}$
Overall Wage Inequality (M9)	$\frac{w^{90}}{w^{10}}$
Correlation between Spouses' Types (M10)	$corr(s_m, s_f)$
Fraction of Single Men (M11)	$\frac{\#SingleM}{\#M}$
Gender Wage Gap by Effective Type (M12-M13)	$\frac{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]-\mathbb{E}[w(s_i h_i)\|i=f,h_i=\hat{h},s_i=\hat{s}]}{\mathbb{E}[w(h_i s_i)\|i=m,h_i=\hat{h},s_i=\hat{s}]}$
Variance of Wife's Home Hours for given Couple Type (M14-M22)	$Var\left[\frac{1-h_f}{(1-h_m)+(1-h_f)}\middle\| s_f, \eta(s_f)\right]$ for all types s_f

For data moments related to home hours (M1 to M4), we define $1-h$ as the share of home hours in total time (defined as hours worked at home plus hours worked in the market), to make the data consistent with the model. To deal with the fact that different individuals report different number of total hours allocated to home production and labor market hours, we first constructed a common denominator for all individuals, given by the 95th percentile of the sum of home and market hours in the estimation sample (81.6 hours per week). Then, we use the hours allocated to home production by each individual to construct her share of home hours over their total time.[40] To construct the model

[40] We winsorize home hours to the value of the 95th percentile of total weekly hours.

counterpart of these moments, we define home hours as $1 - h_i$, where h_i are the hours worked in the labor market by individual i.

Data moments M1 and M4 are constructed based on the sample of individuals in couples, while moments M2 and M3 are based on the sample of all individuals (singles and in couples).

For data moment M5, we define the share of couples working full time as the share of total number of couples in which both spouses/partners work at least 37.5 hours per week in the labor market. Considering the 95th percentile of the total hours as individuals' overall time budget (as defined above), this corresponds to couples in which both partners work more than 46% of the available time. In the model, this moment is constructed as the share of couples in which both partners work more than 46% of the available time, corresponding to the 6th entry in the hours grid.

Wage moments (moments M6-M9) use data on hourly wages for all individuals in our estimation sample (singles and in couples), conditional on participation in the labor market. The model counterpart of these moments are constructed in the same way.

Moment M10 is constructed as the correlation of the spouses'/partners' s-types in the data and model. Moment M11 is the share of single men in the sample, both in the data and the model.

Moments M12 and M13 measure the gender wage gap by (s, h)-types. To construct these moments in the data, we we focus on two (s, h)-types: all individuals (single and in couples) of s-type 2 and s-type 5 that work full time in the labor market, where full time work is defined as described above. We construct the model counterpart of these two moments by computing the gender wage gap of individuals of s-type 2 and s-type 5 that work more than 46% of the time in the market, corresponding to the 6th entry in the hours grid.

Finally, moments M14 to M22 measure the variance of the share of female home hours in a couple within couple-type, where home hours are defined as described above. In our model there are 36 couple-types. However, to construct these moments (and reduce the number of moments), we group men and women into three types, both in the data and in the model.[41] This gives us 3×3 couple-types for which we construct the variance of the share of female home hours within couples (9 moments).

[41] Type 1 consists of s-types 1 and 2, Type 2 consists of s-types 3 and 4, and Type 3 consists of s-types 5 and 6.

Table 15: Targeted Moments

	Model	Data
1. Fraction of Home Production by Female Partner	0.6465	0.6500
2. Female Correlation of Home Production Hours and Skill Type	-0.1575	-0.1440
3. Male Correlation of Home Production Hours and Skill Type	-0.1960	-0.1820
4. Correlation of Spouses' Home Hours	0.2041	0.2800
5. Fraction of Couples with Both Working Full Time	0.1966	0.1800
6. Mean Hourly Wage	18.3346	18.2500
7. Variance Hourly Wage	60.8924	63.1600
8. Overall Wage Inequality (90th/10th ratio)	3.1355	3.0600
9. Upper Tail Wage Inequality (90th/50th ratio)	1.7941	1.7700
10. Correlation between Spouses' Types	0.4133	0.4133
11. Fraction of Single Men	0.2055	0.2000
12. Gender Wage Gap Full Time Workers (Human Capital Level 2)	0.0895	0.0800
13. Gender Wage Gap Full Time Workers (Human Capital Level 5)	0.1833	0.1600
14. Variance of Wife Home Hours in Couple Type 1	0.0218	0.0240
15. Variance of Wife Home Hours in Couple Type 2	0.0227	0.0290
16. Variance of Wife Home Hours in Couple Type 3	0.0229	0.0300
17. Variance of Wife Home Hours in Couple Type 4	0.0255	0.0250
18. Variance of Wife Home Hours in Couple Type 5	0.0262	0.0280
19. Variance of Wife Home Hours in Couple Type 6	0.0276	0.0340
20. Variance of Wife Home Hours in Couple Type 7	0.0282	0.0262
21. Variance of Wife Home Hours in Couple Type 8	0.0298	0.0280
22. Variance of Wife Home Hours in Couple Type 9	0.0306	0.0290

Notes: Moments are computed as discussed above (see Table 14 and the corresponding description).

F Quantitative Analysis

F.1 Comparative Statics

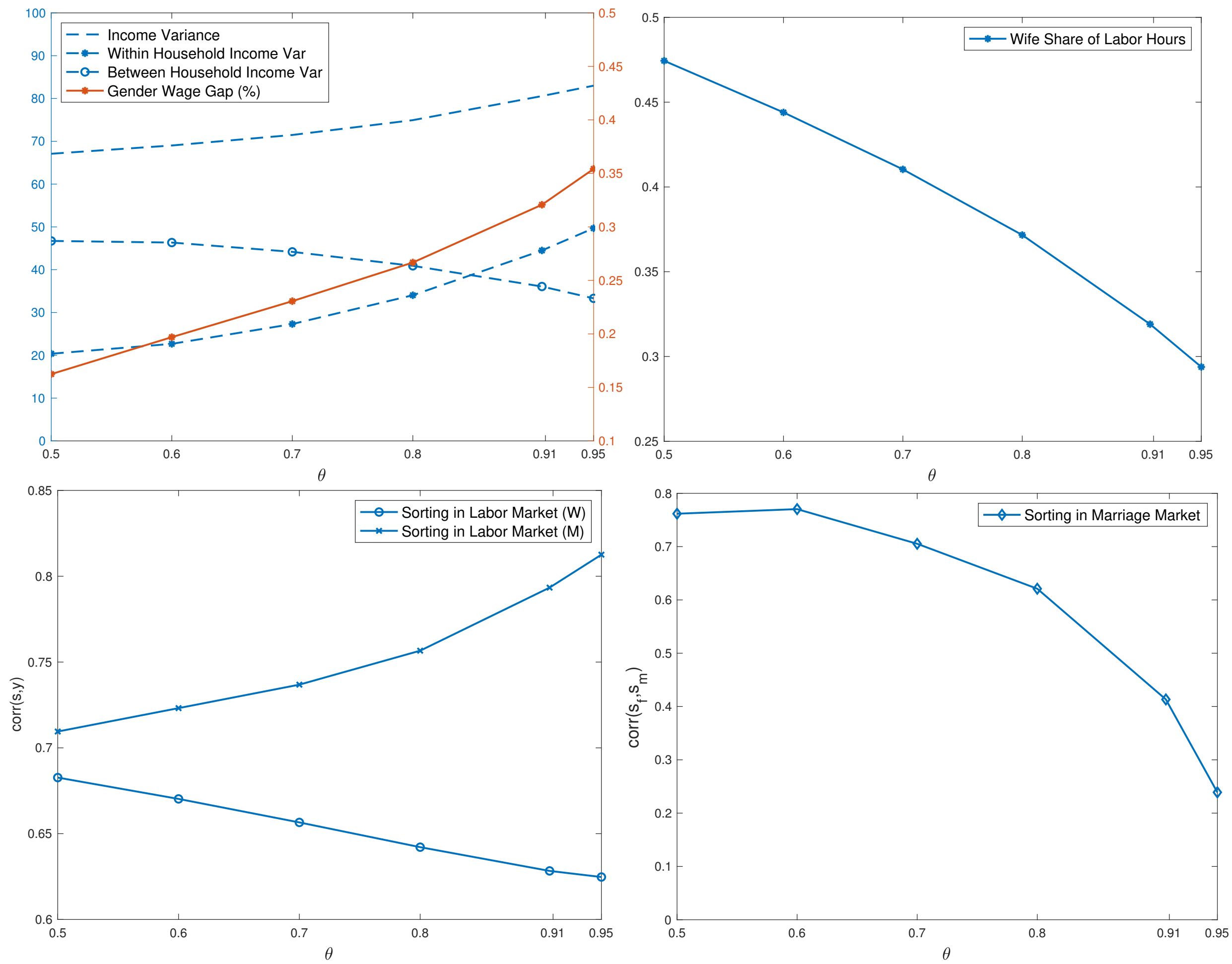

Figure 19: a. Inequality, b. Female Labor Hours , c. Labor Market Sorting, d. Marriage Market Sorting

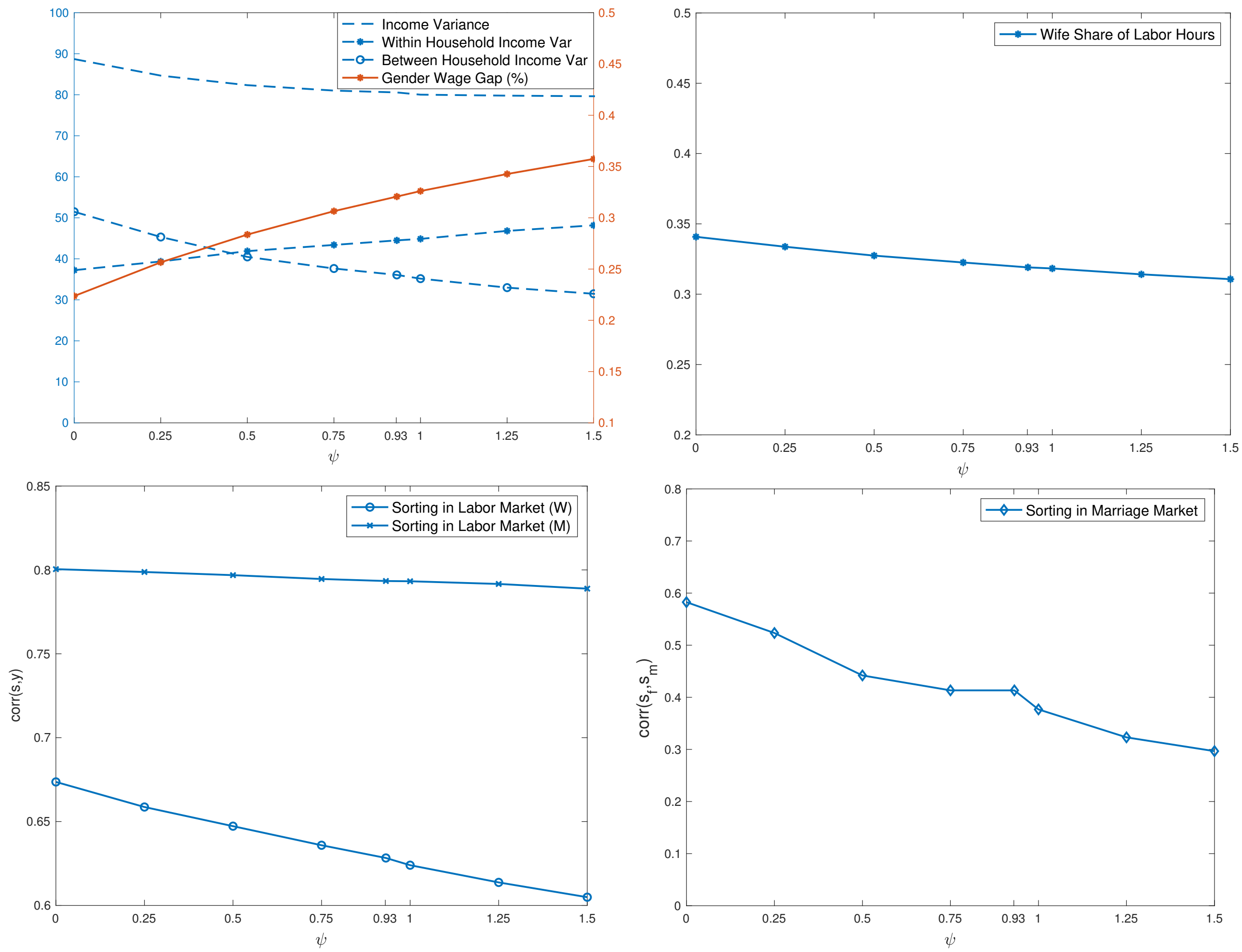

Figure 20: a. Inequality, b. Female Labor Hours , c. Labor Market Sorting, d. Marriage Market Sorting

F.2 Counterfactuals

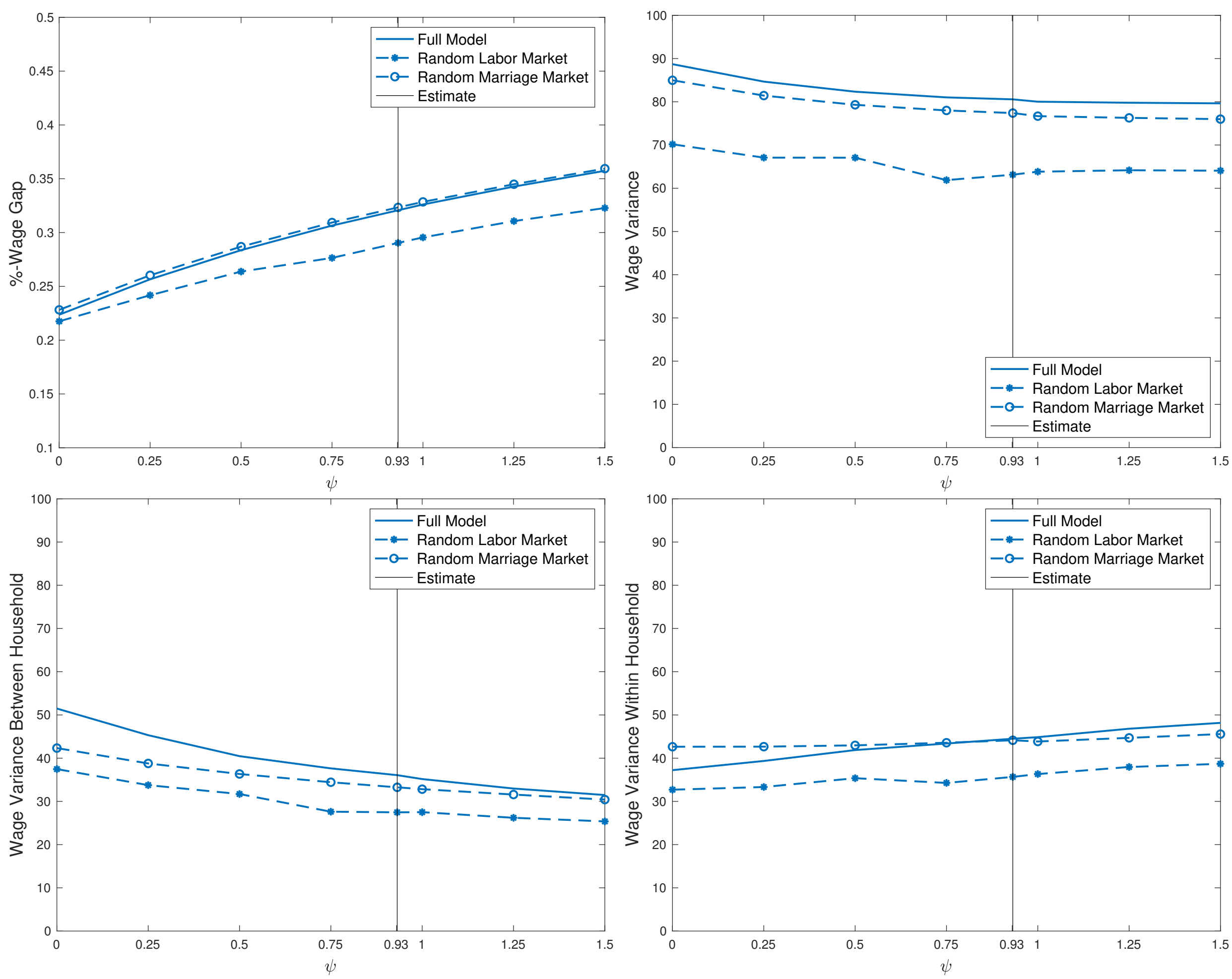

Figure 21: Wage Inequality Measures Under Change in ψ: Full Model and Counterfactuals

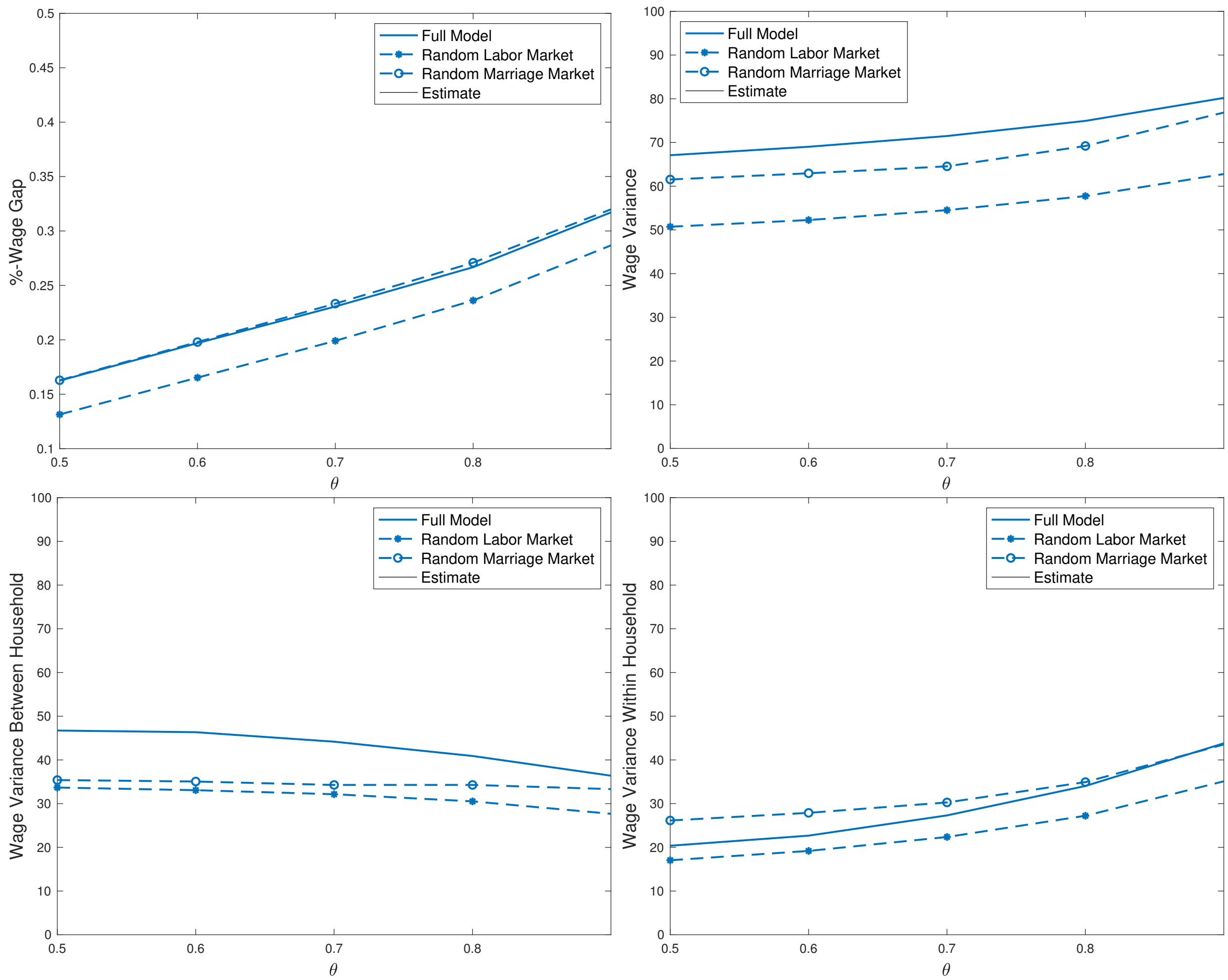

Figure 22: Wage Inequality Measures Under Change in θ: Full Model and Counterfactuals

F.3 Inequality Over Time

Table 16: Data and Model moments: Past and Current West Germany (1990-1996 vs 2010-2016)

	Past Period		**Current Period**		**Data Diff.**
	Model	Data	Model	Data	p-value
1. Fraction of Home Production by Wife	0.6705	0.6635	0.6520	0.6601	0.8293
2. Female Corr. of Home Production Hours and Skill Type	-0.1091	-0.1627	-0.1531	-0.1343	0.2576
3. Male Corr. of Home Production Hours and Skill Type	-0.1679	-0.0985	-0.2009	-0.1669	0.2453
4. Correlation of Spouses Home Hours	0.1049	0.2445	0.2003	0.3031	0.0261
5. Fraction of Couples with Both Working Full Time	0.1457	0.1769	0.1975	0.1486	0.0691
6. Mean Hourly Wage	18.4548	18.3032	19.1236	19.3400	0.0000
7. Variance Hourly Wage	51.3367	50.1047	60.7574	63.7997	0.0000
8. Overall Wage Inequality (90th/10th ratio)	3.0162	2.4644	3.0041	2.8988	0.0000
9. Upper Tail Wage Inequality (90th/50th ratio)	1.7587	1.6521	1.7665	1.7474	0.0552
10. Correlation between Spouses Types	0.3332	0.3316	0.3623	0.4143	0.0150
11. Fraction of Single Men	0.1107	0.1193	0.1700	0.1779	0.0000
12. Gender Wage Gap Full Time Workers ($\tilde{s}$ Type 2)	0.0993	0.0795	0.0882	0.1216	0.4333
13. Gender Wage Gap Full Time Workers ($\tilde{s}$ Type 5)	0.2153	0.2026	0.1922	0.1690	0.2665

Notes: Moments are computed as discussed above in Appendix E.4. The last column of the table reports the p-value of the hypothesis test of the differences between the data moments in the two samples being zero. We use standard T-test for differences in means (M1 and M6), standard test for differences in variances (M7) and standard tests for differences in proportions (M5 and M11). We use a Fisher transformation to construct test's statistic for the differences in correlations between samples (M2, M3, M4 and M10). We use a two-sample Wald test for differences in ratios across samples (M8, M9, M12 and M13).

Table 17: Estimated Parameters: Past and Current West Germany (1990-1996 vs 2010-2016)

		Past Period		**Current Period**	
		Estimate	s.e.	Estimate	s.e.
Female Relative Productivity in Home Production	θ	0.94	0.02	0.92	0.01
Complementarity Parameter in Home Production	ρ	-0.22	0.38	-0.67	0.10
Home Production TFP	A_p	22.50	0.64	23.17	0.32
Elasticity of Output w.r.t. $\tilde{s}$	γ_1	0.59	0.07	0.73	0.06
Elasticity of Output w.r.t. y	γ_2	0.21	0.12	0.20	0.10
Production Function TFP	A_z	33.53	2.89	35.38	2.11
Female Productivity Wedge	ψ	1.40	0.19	1.00	0.09
Preference Shock for Partners (scale)	σ_β^C	0.04	0.05	0.19	0.03

Notes: s.e. denotes standard errors. See Section 7.4 for a description of how these standard errors are computed. For the Current Period West Sample, we do not re-estimate σ_β^C, but we set this parameter (and corresponding standard error) to the estimate of our baseline all-Germany sample (see Table 3).

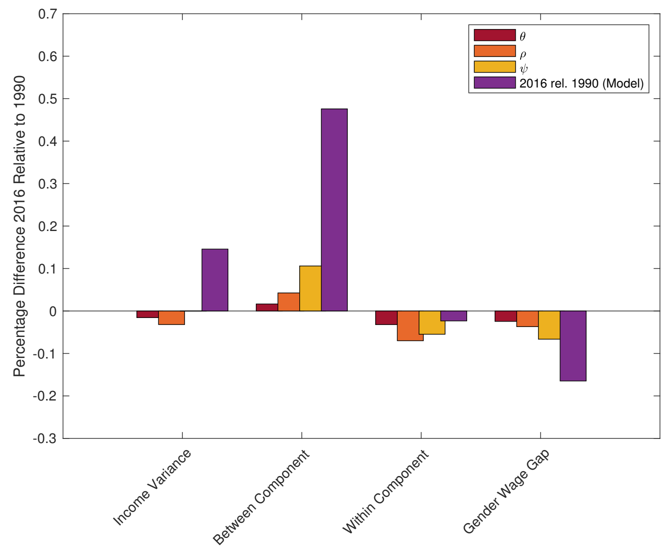

Figure 23: Inequality Changes Over Time: Detailed Decomposition

F.4 Inequality Across Space

Table 18: Data and Model moments: East and West Current Germany (2010-2016)

	East Germany		**West Germany**		**Data Diff.**
	Model	Data	Model	Data	p-value
1. Fraction of Home Production by Wife	0.6097	0.6024	0.6520	0.6601	0.0000
2. Female Corr. of Home Production Hours and Skill Type	-0.1308	-0.1415	-0.1531	-0.1343	0.8610
3. Male Corr. of Home Production Hours and Skill Type	-0.1254	-0.1621	-0.2009	-0.1669	0.5769
4. Correlation of Spouses Home Hours	0.2735	0.2458	0.2003	0.3031	0.1954
5. Fraction of Couples with Both Working Full Time	0.1530	0.2750	0.1975	0.1486	0.0000
6. Mean Hourly Wage	15.3328	14.8882	19.1236	19.3400	0.0000
7. Variance Hourly Wage	39.4078	47.9081	60.7574	63.7997	0.0000
8. Overall Wage Inequality (90th /10th ratio)	3.1282	2.9534	3.0041	2.8988	0.8754
9. Upper Tail Wage Inequality (90th /50th ratio)	1.8188	1.8454	1.7665	1.7474	0.3889
10. Correlation between Spouses Types	0.4128	0.3564	0.3623	0.4143	0.3599
11. Fraction of Single Men	0.2451	0.2374	0.1700	0.1779	0.0022
12. Gender Wage Gap Full Time Workers ($\tilde{s}$ Type 2)	0.0734	0.0870	0.0882	0.1216	0.2365
13. Gender Wage Gap Full Time Workers ($\tilde{s}$ Type 5)	0.1315	0.0747	0.1922	0.1690	0.2437

Notes: Moments are computed as discussed above in Appendix E.4. The last column of the table reports the p-value of the hypothesis test of the differences between the data moments in the two samples being zero. We use a standard t-test for differences in means (M1 and M6), standard test for differences in variances (M7) and standard tests for differences in proportions (M5 and M11). We use a Fisher transformation to construct test's statistic for the differences in correlations between samples (M2, M3, M4 and M10). We use a two-sample Wald test for differences in ratios across samples (M8, M9, M12 and M13).

Table 19: Estimated Parameters: East and West Current Germany (2010-2016)

		East Germany		**West Germany**	
		Estimate	s.e.	Estimate	s.e.
Female Relative Productivity in Home Production	θ	0.93	0.02	0.92	0.01
Complementarity Parameter in Home Production	ρ	-1.52	0.76	-0.67	0.10
Home Production TFP	A_p	25.37	0.50	23.17	0.32
Elasticity of Output w.r.t. $\tilde{s}$	γ_1	0.61	0.09	0.73	0.06
Elasticity of Output w.r.t. y	γ_2	0.17	0.15	0.20	0.10
Production Function TFP	A_z	33.36	3.07	35.38	2.11
Female Productivity Wedge	ψ	0.76	0.29	1.00	0.09

Notes: s.e. denotes standard errors. See Section 7.4 for a description of how these standard errors are computed.

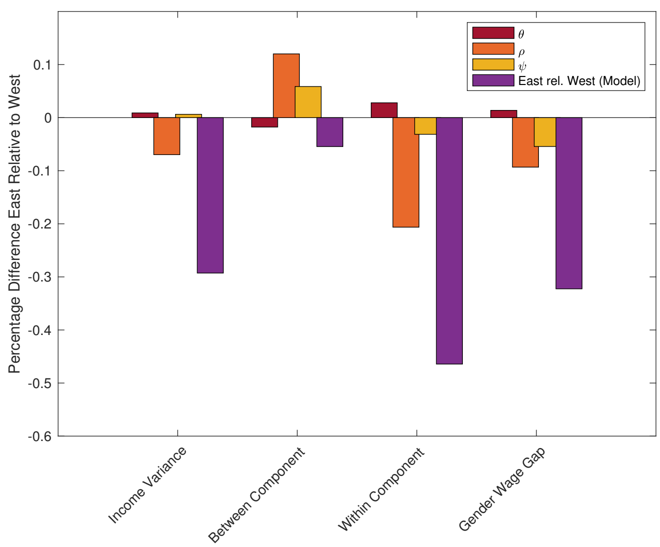

Figure 24: Inequality Differences between East and West Germany: Detailed Decomposition