Willingness to fight on: Environmental quality in dynamic contests

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Abstract

We show that the prevalence of prolonged contests in professional tennis drops sharply when the ambient environment deteriorates through heat or pollution. We develop a dynamic model of multi-battle competition to investigate how the disutility from a protracted competition shapes agents' willingness to fight on. Our theory predicts that a poor environment amplifies the momentum of a competitor's head start. We show how model primitives including preferences for improved working conditions (environmental amenities in our setting) can be inferred from battle-to-battle transition probabilities. We provide clean evidence that heat and pollution affect individuals' incentives to compete strategically. Model estimates show that in a contest between equally able rivals at the median prize of $15,100, the value of a head start is $130 to $370 higher in a degraded environment compared with a climate-controlled one.

Keywords: Dynamic contests, multi-battle competitions, strategic momentum, discouragement effect, revealed environmental preferences

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1 Introduction

Research of mostly theoretical nature considers dynamic competitions in which a winner is determined through a multi-battle contest.[1] These adversarial competitions require persistent effort from participants and are used widely in innovation, product advertising, job promotion, litigation and conflict, political campaigns, and sports (Corchon, 2007; Konrad, 2009). In a best-of-N contest, the agent who wins the majority of N battles is declared the winner and takes the prize. Agents' performance as they transition from one battle to the next is determined by relative ability and continuation values, which jointly determine their effort choices. Economic incentives in such dynamic multi-battle contests produce a result known as strategic momentum: When facing an equally able rival, the agent who wins battles early on and moves closer to overall victory will raise equilibrium effort relative to her opponent, increasing the likelihood that she continues winning.[2]

In this paper, we develop a dynamic contest model of complete information in which agents incur a fixed cost from engaging in each battle—say an opportunity cost or disutility from protracted competition that is invariant to effort and unobserved by the researcher. When this per-battle disutility grows, the importance of an early win is magnified. We show how the shift in momentum due to variation in the unobservable disutility (fixed cost) reveals its magnitude. The intuition is as follows. Consider two ex-ante symmetric agents competing in a best-of-three contest. There is a fixed cost per battle. To win the contest and collect the prize, an agent needs to win two battles. By design, one agent wins the first battle and faces win-loss continuation values in the second battle whose difference grows when a higher fixed cost makes the prospect of competing in a third battle (should she lose the second battle) less attractive. Similarly, in the second battle the agent who lost the first battle now faces win-loss continuation values whose difference narrows as the prospect of competing in a third battle and incurring another round of fixed cost (should she win the second battle) becomes less attractive. Thus, as per-battle disutility grows,

[1] See, for example, Harris and Vickers (1987), Snyder (1989), Konrad and Kovenock (2009), Gelder (2014), and Feng and Lu (2018).

[2] In an intriguing example, Klumpp and Polborn (2006) compare sequential to simultaneous multi-district elections to explain the "New Hampshire effect" in sequential US primaries, whereby gaining a head start can be crucial to the final win even when there are many districts to fight over.

the agent who starts winning raises effort relative to her opponent and the contest is (in expectation) resolved sooner. Our focus is in showing how the change in interim transition probabilities reveals the per-battle disutility and the value of a head start.

Our specific setting links contest theory to environmental valuation. The economic agents are tennis professionals competing in best-of-three outdoor contests (matches) in which the per-battle disutility shifts with environmental quality, namely heat and air pollution. We show how the increase in momentum as the ambient environment deteriorates identifies agents' preferences over environmental amenities. We provide evidence that heat and pollution affect individuals' willingness to compete strategically.

Empirical work is relatively scarce in the contest literature. Szymanski (2003) argues that testing contest theory is challenging, as ability is rarely observed or the stakes are low, with the exception of sporting contests. Boudreau et al. (2016) argue that data rarely meet the econometric demands of theoretical models. In a study of best-of-three contests in professional tennis—the same contest as in our application—Malueg and Yates (2010) provide observational evidence of strategic momentum. The player who wins two of three sequential battles (tennis sets) is the winner and takes the match prize. When equally skilled agents compete, theory predicts that the battle-1 winner is more likely than not to win battle 2.[3] Thus, the likelihood that the match requires a third battle for closure is less than 0.5. Absent the strategic behavior predicted by theory, 0.5 is the probability that a match completes in two battles, i.e., battle scores 2-0 or 0-2 with probability $0.5^2 + 0.5^2$. Malueg and Yates (2010) report that in a sample of 351 matches between equally able athletes, identified using betting-market odds, the proportion of three-battle matches is 0.36. This is statistically significantly lower than 0.5.[4]

To test and apply our amplified-momentum theory, we assemble a dataset on Women's Tennis Association (WTA) tournament matches played outdoors in Australia and China.

[3]Having won battle 1, a player is one step away from victory, advantaged by the near prospect of taking the prize after exerting costly effort in the current battle 2; in contrast, the battle-1 loser can possibly take the prize only after incurring effort cost over two more battles. To this model we add an unobserved fixed cost per battle and show how it is identified from environmental shocks and informative of preferences.

[4]As an alternative to strategic incentives, Malueg and Yates (2010) do not find evidence that the momentum observed in the field is driven by the "psychology" of a recent win. Specifically, in a smaller sample of 125 matches between equal-odds players that do reach a third battle, the player who won the most recent battle, the battle-2 winner, is as likely to lose as to win the third battle.

Plausibly, an agent's disutility from a prolonged competition is higher when it is uncomfortably hot or visibly polluted. The 75th percentile of the mean temperature distribution over Australian Open matches, played in Melbourne's summer, is 30 °C (86 °F). The 75th percentile of the mean PM2.5 distribution over China Open matches, played in Beijing's fall, is 133 μg/m^3, a level that is visible to the naked eye.[5] While such conditions are harsh, they are quite common in our settings and are not extreme or rare.

The raw data already suggest that environmental degradation increases momentum. Consider a subset of 591 matches in our sample in which agents are ex-ante relatively similar (as implied by pre-match betting odds). In this subsample, the proportion of three-battle matches is 0.41. Strikingly, in the 192 matches played in poor environments—defined here as either temperature over 29 °C or PM2.5 over 100 μg/m^3—the proportion of three-battle matches falls to 0.30, i.e., there is more momentum. By contrast, in the 399 matches in which both temperature and PM2.5 were below these cutoffs, the proportion of three-battle matches is 0.46. An equality test, $(0.30, N = 192)$ vs. $(0.46, N = 399)$, has a p-value of 0.0001. The data indicate that first battles are not shorter in a poor environment. This suggests that the underlying mechanism is not explained by idiosyncratic player sensitivity to heat or pollution, as such player asymmetry would shorten not only the match but also battle 1. Reduced-form evidence also suggests that when asymmetric players compete, a poor environment does not raise a weaker player's overall chance against a stronger opponent.

Using the entire range of opponent asymmetry, we estimate the dynamic contest model by maximum likelihood on a sample of 2200 to 2700 matches (depending on the availability of alternative ability measures). By one set of estimates, raising temperature from 27 to 37 °C reduces a contestant's expected payoff by \$670 when facing an equal-odds rival; raising PM2.5 from 150 to 250 μg/m^3 reduces a player's expected payoff by \$1800. How environmental shocks, working through a per-battle fixed cost, shift momentum is moderated by the size of the contest prize and may be asymmetric across players (e.g., a player about to retire, with potentially lower reputation gains from winning).

[5] PM2.5 is particulate matter of diameter up to 2.5 micron. The 24-hour US standard is 35 μg/m^3.

Intuitively, at each node of the dynamic contest, an agent weighs the benefit and cost of supplying effort, which includes the fixed cost of extending the competition. In our setting, how players differentially respond after winning vs. losing the first battle, in bad compared to good environments, provides a measure of the damage from exposure to degraded environments. More generally, the differential dynamic response when battle opportunity costs are high compared to low reveals the magnitude of these fixed costs.

The increase in momentum due to players' reduced willingness or discouragement to fight on extends beyond sport contests into other economic contexts. Internal promotions typically involve multiple battles among candidates. The lengthy procedure is designed to encourage them to contribute to their organization and display their capacities, talents, skills, and personalities vis-à-vis those judged desirable. The endurance race induces productive effort and enables the organization to identify which contestant to promote (DeVaro, 2006).[6] The discouragement effect, however, means that the multi-stage screening can be compromised when candidates' incentives to fight on are blunted, even giving up prematurely. Our study highlights that a deteriorating competitive environment amplifies momentum and, by the same token, can be mitigated in an environment that maintains the candidates' incentives as the competition extends. Universities, for example, can improve the competitive environment by offering research grants, teaching support, office facilities, and training and mentoring.

Other settings in which momentum induced by difficult working conditions may prevent participants from reaching their potential include multi-stage R&D and political competitions (Grossman and Shapiro, 1987; Klumpp and Polborn, 2006). A planner may offer research subsidies to encourage laggards to stay in an R&D race, to the benefit of society, as in the multi-stage search for a vaccine amid a pandemic (WHO, 2020). Campaign restrictions can hinder a political contest by increasing the discouragement effect, whereas technologies such as social media can reduce the fixed cost of campaigning on.

Our paper contributes to a sparse empirical literature on agents' incentives in contest

[6]Bognanno (2001) finds that 80% of high-level executives were promoted internally and that even before promotion a successful candidate earned more than peers. This suggests the presence of interim ranks and the potential relevance of discouraged laggards to a firm's performance (Goltsman and Mukherjee, 2011).

settings. For instance, Brown (2011) studies the superstar effect in competitions. Contest models in this literature are typically offered to guide interpretation of reduced-form (regression) analysis and not for direct estimation of economic primitives, as is our case. An exception is Ferrall and Smith Jr. (1999)'s test of momentum in team sports using a structural best-of-five contest model based on Lazear and Rosen (1981) and Rosen (1986). Genakos and Pagliero (2012), Huang et al. (2014), and Boudreau et al. (2016) estimate contest models with varying levels of structure and that differ in design and context, e.g., one-shot innovation tournaments. Related to our application, a mostly nonstructural literature in environmental economics quantifies the impact of heat or pollution on socioeconomic outcomes.[7] The tractable model we estimate can generally be applied to dynamic competitions in which shocks to fighting on—which encompass institutional, political, and operational factors—may lead competitors who fall behind to simply give up.

2 Data and institutions

Our data consist of outdoor WTA matches held in Australia and China, including environmental conditions during match time. Melbourne in January can be hot: Percentiles 50, 75, and 90 of the mean temperature distribution over the 2004-2016 Australian Open matches in our sample are 25, 30, and 34 °C. Beijing's air quality in the fall varies and can be poor: Percentiles 50, 75, and 90 of the mean PM2.5 distribution over China Open matches are 52, 133, and 220 μg/m^3. Data for the Beijing-based China Open start only in 2008, so we further compile data for five other outdoor WTA tournament series held annually in China. Figure 1 and Table 1 report the wide variation in environmental quality in our sample (yet freezing weather is not applicable).

We examine women's singles series, in which each match is a best-of-three contest between two players. In such matches, the first player to win two tennis sets (battles) over her rival, with one battle played immediately after the other on the same day, wins the match. Thus, battle transitions that yield a match win are either win-win (in two

[7] A selected list includes heat- or PM-induced mortality and defensive expenditures (Deschenes and Greenstone, 2011; Chen et al., 2013; Salvo, 2018; Ito and Zhang, 2019); and output, productivity, and labor supply (Dell et al., 2014; Hanna and Oliva, 2015; Archsmith et al., 2018; Somanathan et al., 2019).

sets), win-lose-win or lose-win-win (three-setters). The contest winner earns cash (sample median = \$15,092) and WTA ranking points and, except in the series' final round, plays another opponent on another day, typically a day or two later.[8]

To describe a series, consider the Australian Open, where the "main draw" features 128 qualifying athletes. The single winners of the 64 matches played by the 128 players in round 64 progress to round 32, and similarly for subsequent rounds. There are more matches in round 64 than in all six later rounds combined ($32 + 16 + 8 + 4 + 2 + 1 = 63$ matches). It helps to think of the "median" contest as a round-64 match, not the famed series final. Athletes who do not qualify for a series' initial round 64, based mainly on WTA ranking points, may do so in "qualifying" matches held shortly before round 64.

The Australian Open is one of the four "Grand Slam" tennis series attended by the world's top athletes. The China Open was upgraded in 2009 to "Premier Mandatory" status and also attracts top players. The five other WTA series played in China are less prestigious but attract similar professionals.[9] In the player-by-match distribution in our sample, the median player's world rank is 48 in Melbourne, 28 in Beijing, and 63 in other Chinese venues. Because main-draw matches in the China Open start only in round 32, its median player ranks even higher than in the Australian Open. Excluding round-64 matches in the Australia Open, the median player rank happens to equal that in the China Open, at 28. Figure 2 shows that players in Melbourne, Beijing, and other Chinese locations rank among the world's top 100 tennis athletes, including at the very top.

Match-level data. Our sample consists of main-draw WTA matches in the following years and locations: summer 2004-2016 in Melbourne; fall 2008-2016 in Beijing; fall 2011-2016 in Guangzhou; winter 2013-2016 in Shenzhen; and summer/fall 2014-2016 in Hong Kong, Tianjin and Wuhan. We cannot extend a given venue's matches to earlier years due to high-frequency environmental conditions not being available or because a given series

[8]In terms of sample, we do not consider doubles matches, which would require that we model strategies between players within a team. Men's singles matches in the Australian Open are best-of-five contests. Subsequent research can study momentum in the context of teams, longer contests, and gender differences. Future research accessing more granular data can model battles as tennis games or tennis points, both of which are subunits of a tennis set, but will need to contend with game serve advantage.

[9]Other series are the Guangzhou International Women's Open, Hong Kong Open, Shenzhen Open, Tianjin Open, and Wuhan Open.

was recently introduced. In total, we observe 1651 scheduled matches in Melbourne, 499 matches in Beijing, and 661 matches in other locations in China, of which 98%, 95%, and 94%, respectively, were completed. The remaining 2%, 5%, and 6% of matches at these respective locations were won either by "retirement," due to an injured player's withdrawal, or, less frequent still, by "walkover," when a player does not attend.

Matches follow a prescheduled order and start between 10 am to 7 pm local time. In addition to match status (e.g., completed), we observe player characteristics (name, WTA rank, ranking points), and the number of tennis games each player won in each battle. For example, player X wins two battles and loses one, with sequential scores of 6-4, 6-7, and 6-2 (in games won).[10] Rain other than a light shower typically delays play. Due to controversy over the Australian Open's heat policy,[11] playing in ambient heat has been common even in the few instances in which a retractable roof was available (the Rod Laver and Hisense arenas).

The mean length of battle 1, as measured by the number of tennis games played, is similar across locations: 9.2 in Melbourne, 9.4 in Beijing, and 9.3 elsewhere in China. Compared with battle 1, battle 2 tends to be shorter by about 0.2 game. Upon losing battle 1, the likelihood that the battle-1 loser wins the next two battles to win the match is less than 1 in 5, and similar across locations. Third battles tend to last 9.5 games.

Opponent symmetry vs. asymmetry. A scheduled match's opponents are typically confirmed a day or two in advance. We observe pre-match betting odds according to a leading prediction market (bet365). From these odds, we calculate implied match winning probabilities prior to the start of play. For example, the odds for players X and Y are 1.57 and 2.37, implying winning probabilities of 64% (i.e., 100%/1.57) and 42% (100%/2.37), with a winning probability difference between the two players of $64 - 42 = 22\%$ in absolute value.[12] Figure 3(a) shows the distribution of this winning probability difference over

[10]In the example, player Y wins battle 2 by a tie-breaker after both players are tied at 6 games each. A battle consists of at most 13 games, the last decided by a tie-breaker, except in third battles in Grand Slam matches in which a player must win 2 games more than her rival and at least 6 games, e.g., 8-6.

[11]A player may feel that heat works to her advantage. Even a referee's call for a short break between battles can be controversial. Our findings apply to hot, and not specifically extreme, weather. We subsequently show robustness of our main results when dropping 66 matches for which a retractable roof appears to have been closed to protect from heat or rain (Table A.4).

[12]Betting \$100 on player Y winning the match pays out \$237, netting \$137, should she win. Other online

matches in the sample. We are missing odds prior to 2008 (about 500 matches).

By incorporating available information, including expected location- and time-specific conditions that affect athletes differentially, the difference in players' winning probabilities captures the ex-ante asymmetry: for instance, compare the above match's winning probability difference of 22% with 72% in another match among asymmetric competitors. A player may have a history of performing better on a hard court or in hot weather than her opponent, and such expected conditions should be reflected in betting prices. Matches with a low winning probability difference are those in which the opponents' relative strength—comparative ability, fitness, or motivation—was deemed most symmetric.

As alternative measures of the relative strength of athletes paired in matches in our sample, Figure 3 also shows (panel b) the distribution of the absolute difference in opponents' WTA ranks, e.g., a difference of 30 for a match between players ranked 25 and 55; and (panel c) the distribution of the difference in opponents' WTA ranking points or z-scores, e.g., a difference of 2 standard deviations for a match between players with points 5 and 3 standard deviations above the mean in the worldwide population of WTA players.[13] These alternative measures are based on aggregates over time and integrated over an athlete's performance at venues with varying court surfaces and environments.

The pairwise correlation coefficient for (a) the winning probability difference against (b) the rank difference (in log) is 0.43, or (c) the z-score difference is 0.61. The left panels of Figure 4 show that as each asymmetry measure grows, the stronger player is indeed more likely to win the match. (We will later refer to the right panels.) For each measure, we group matches in 10 same-width bins, with the first bin starting at the minimum difference and the last bin ending at the 95th percentile of the difference distribution over asymmetric matches.[14] Matches in which player ranks differ by 100 or more tend to be

betting sites report similar odds to bet365. 64% and 42% exceed 100% due to bookmaker fees (about 6%).

[13]The time-adjusted world z-score is a player's unadjusted WTA ranking points minus the mean over all worldwide WTA players (on the match's date), divided by the standard deviation over all worldwide WTA players. The adjustment accounts for changes in the variance of WTA ranking points over time.

[14]Using the sample of matches without missing measures, we implement three separate regressions. In each case, we regress an indicator that the match was won by the stronger player (alternatively defined as that with the higher winning probability, the better rank, or the higher z-score) on the corresponding asymmetry measure, i.e., as the respective regressor, the winning probability difference, log rank difference, or z-score difference. The R^2 of the first regression, at 9%, is more than double that of the others.

those between weaker players, for which rank difference is a noisier measure of asymmetry.

Exogenously varying environments. We observe temperature and particle pollution, among other ambient weather conditions, mostly in the form of 1-hour readings (several sources, described in Table 1 notes). For Melbourne, we lack 1-hour PM2.5 prior to 2015, so we use the Victoria EPA's 1-hour Airborne Particle Index (API) instead. We verified that daily mean PM2.5, available once every 3 days, closely tracks the 1-hour API aggregated into daily means. Specifically, at each of two sites less than 9 km from the venue, the pairwise correlation coefficient between PM2.5 and API is 0.94 over 2004-2016. One API unit is associated with 15 $\mu g/m^3$ PM2.5, so a maximum of 5.1 units in the Australian Open match sample corresponds to 77 $\mu g/m^3$ PM2.5. For Beijing, PM2.5 was measured at the US Embassy, located 19 km from the venue in 2008 and 11 km thereafter. In north China, PM2.5 fluctuates substantially from one day to the next, due to exogenous shifts in atmospheric stagnation (He et al., 2019).

3 Reduced-form regressions

We now document that environmental quality affects contest duration in professional tennis. We later interpret the relationships in the data through the lens of a dynamic contest model, in which we explicitly characterize players' incentives and effort choices at each state, and quantify these agents' preferences for environmental amenities.

Amplified momentum. Spectators, advertisers, and organizers presumably derive value from a tighter and longer match. Consider the probability that a match ends in three battles rather than two. Without conditioning on player asymmetry, the probability that a match lasts three battles is about one-third. When opponents are more symmetric, the proportion of three-battle matches (three-setters) is higher. This proportion is 0.40 in the first quartile of the players' winning probability difference distribution.[15] Recall that absent dynamic behavior, the probability that a match between symmetric players lasts three battles would be 0.5.

[15]Similarly, the probability of ending in three battles is 0.36 and 0.35 in the first quartiles of the rank difference and z-score difference distributions. In contrast, the probability of a three-setter is 0.20 for matches in the fourth quartile of the winning probability difference distribution.

Table 2 reports our key finding: A degraded environment, whether heat or pollution, sharply reduces the probability that an ex-ante even match is played over three battles. Two players may be symmetric when starting a match, but a degraded environment induces more asymmetric outcomes, as it raises the advantage in battle 2 from having won battle 1. Start with very asymmetric pairings, in column 1. In the subsample of matches for which opponents' winning probabilities differ by at least 65% ($N = 566$), heat or pollution does not change the probability that a match lasts three battles (0.20 in this subsample). In contrast, in relatively symmetric pairings in columns 2 to 8, heat or pollution significantly lower the probability that a match lasts three battles. Columns 2 to 6 progressively shrink the subsample based on opponents' winning probabilities, from at most 30% apart ($N = 700$) to at most 10% apart ($N = 194$). The likelihood of a three-setter is flat at about 0.4, even as the definition of symmetry becomes more stringent (i.e., $\leq 30\%$ to $\leq 10\%$ difference). Compared to matches played in cooler and cleaner air, the probability of a three-setter is about (i) 18 percentage points lower when the temperature exceeds 29 °C, and this estimate is significant at the 1% level even as standard errors grow; and (ii) 10 percentage points lower when PM2.5 exceeds 100 $\mu g/m^3$, with loss of precision. Comparing columns 7 and 3, estimates grow in magnitude upon adding controls for series by round (e.g., Australian Open's round 64),[16] year, time-of-day, humidity, wind speed, and rain. (We do not use all of these shifters in our structural model, to keep it parsimonious, but could do so in principle.) Column 8 specifies finer PM2.5 bins, namely 100 to 200 $\mu g/m^3$ and over 200 $\mu g/m^3$. We write our first key result:

Fact 1 (Battle-2 transition probability): *In a contest between players with fairly even strengths, as measured by similar winning probabilities, the battle-1 loser is more likely to lose than to win in battle 2, and this momentum grows as the environment deteriorates.*

As in the subsequent structural analysis, column 9 pools the estimation sample over the entire range of opponent asymmetry ($N = 2130$). Here, we control for the winning probability difference and its interactions with heat and pollution indicators. Naturally,

[16] Winning a contest later in a series, and in more prestigious series, carries a higher cash (and continuation/reputation) prize, so we condition on series-round. Three-setters tend to be more prevalent in later vs. earlier rounds, and in more prestigious series, even among relatively symmetric pairings.

environmental quality impacts transitions less when a top 10 player faces a rival ranked 100-200. The patterns reported in Table 2 are robust to extending the sample to include qualifying matches (Tables A.1 and A.2) and to controlling for opponents' age difference.[17]

Table 2 regression results are readily seen in the combined match-environment data. Figure 5 partitions the set of completed matches along two dimensions: (i) into 10 bins for the winning probability difference, namely a zero-difference bin and nine same-width bins in increasing order of opponent asymmetry along the horizontal axis; and (ii) whether either temperature or PM2.5 exceeded a threshold (marked by red circles), against a "control" in which both environmental variables were below their respective cutoff (green squares). The plots show increased momentum whether we specify temperature cutoffs of 27 or 29 °C and PM2.5 cutoffs of 100 or 200 μg/m^3.

Table 3, columns 1 to 4, report that the battle-1 loser's gain in terms of tennis games won in battle 2 relative to battle 1 shrinks as the environment deteriorates. Because a battle is won with 6 or 7 games, a 0.4 to 1.1 reduction in relative games won is large in magnitude. This is another manifestation of momentum amplified by a poor environment.

Heterogeneity vs. dynamic incentives. The next section formalizes the within-match dynamics of player incentives. However, a natural question is whether the environment's effect on the likelihood that the battle-1 loser successfully fights back in battle 2 can be explained by an alternative hypothesis. Were one opponent, the battle-1 loser, more sensitive to the environment than her opponent, this might also explain why she already fell behind in battle 1. By this alternative hypothesis, momentum would be due to physiology (biology) rather than dynamic incentives.

Suppose one player were systematically more sensitive to the environment. Such public information would likely be incorporated into betting odds. For example, prediction markets would level the odds of a player with poor resistance to heat and who meets an otherwise less able opponent on a hot Melbourne afternoon. By Fact 1, we use betting odds to control for differences in player strength.

Now suppose that heterogeneous environmental sensitivity were instead idiosyncratic,

[17] Across matches, the median age difference is 3.6 years. In the player-by-match distribution, percentiles 25, 50, and 75 of player age are 22, 25, and 27 years (players are young and professional careers short).

and thus not incorporated into betting odds. If a publicly even player were privately weaker than her opponent when playing in a poor environment, we would observe not only a shorter match but also a shorter *first* battle, which the data show not to be the case, as Fact 2 summarizes next. Columns 5 to 8 of Table 3 show that the length of battle 1, in the tennis games played, is not lower in a low-quality environment. For example, in column 6's relatively symmetric subsample, battle 1 is longer by a statistically insignificant 0.1 game when the temperature exceeds 29 °C compared to matches played in cooler air. This small and insignificant difference is robust as we reduce the subsample to define symmetry more strictly (as we did in Table 2, columns 2 to 6).

Fact 2 (Battle-1 competitiveness): *In a contest between players with fairly even strengths, as measured by similar winning probabilities, the length of the first battle does not change significantly as the environment deteriorates.*

The evidence thus suggests that heterogeneous environmental sensitivity—whether public or private—is not the key driver of shorter contests. We subsequently show that momentum amplified by environmental degradation obtains in a smaller sample of matches with similar winning probabilities and with tight battle-1 outcomes, e.g., 7-6.

Environment and randomness. Table 4 suggests that match outcomes are not more random, or less predictable, in poor environments. As environmental quality declines, the less favored opponent—when betting odds indicate there is one—is about as likely to lose the match (columns 1 to 4) and as likely to lose battle 1 (columns 5 to 7) compared to matches in milder environments.[18] Column 9 indicates that the ex-ante winning probability of the match winner, with a sample mean of 0.67, is not significantly associated with environmental quality. Heat or pollution do not level the playing field, in the sense that "upsets" do not become significantly more likely under environmental stress (Figure 4, right panels). If anything, to judge by the sign of most point estimates, upsets become somewhat less likely as the environment deteriorates. We summarize this relationship in the data as follows:

[18]To the extent that betting markets use heat or pollution forecasts, prices may already incorporate any randomness that is driven by a poor environment. If we instead define the favorite as the opponent with the strictly better rank or higher z-score (aggregates over time), similar results obtain.

Fact 3 (Favorite still wins): *Upsets, defined as the less favored player winning the contest, or winning battle 1, do not become more likely as the environment deteriorates.*

Table A.3 considers tennis points, which are a subunit of a tennis game.[19] We similarly find that the favorite wins a share of tennis points in the match—and in battle 1—at least as large in a poor environment compared with a mild one.

We also compared the proportion of uneven first and second battle outcomes, defined as scores of 6-0 or 6-1 (tennis games), for matches that are played in varying environments. Taking cutoffs of 29 °C and 100 μg/m^3, uneven first battles are as prevalent with heat or pollution $(0.193, N = 802)$ as under cool and clean air $(0.196, N = 1880)$; the p-value of an equality test is 0.86. In contrast, uneven *second* battles are more prevalent in heat or pollution (0.241) than in a mild environment (0.211), with a p-value of 0.09. If we lower the temperature cutoff to 27 °C to include more matches played in poor environments, the prevalence of uneven first battles again does not vary with heat or pollution, yet the p-value of an equality test of uneven second battle prevalence is now 0.006, namely $(0.247, N = 1064)$ with heat or pollution vs. $(0.202, N = 1618)$ otherwise.

Withdrawals. Our analysis conditions on completed matches: 2722 of 2811 scheduled matches, for a 97% completion rate. Due to large penalties, no-shows and retiring from the court rarely happen in these high-profile contests; our sample includes 15 walkovers and 74 retirements. A player needs a medical reason for either. Here we briefly examine whether the environment affects this additional margin of labor supply. For Melbourne, we find that temperatures were on average similar for the 27 scheduled but not completed matches, compared to the 1624 completed matches in the sample: 26.2 against 26.1 °C, respectively. For Beijing, PM2.5 was on average 33% higher for the 23 non-completed matches compared to the 444 completed matches: 115 against 87 μg/m^3, respectively.[20] In all of 7 no-show cases in Beijing, the absentee had played in a preceding round, so she had traveled to Beijing and was possibly injured. We conclude that an adverse environment, and pollution in Beijing in particular, may reduce the likelihood that a scheduled match

[19] A tennis game can be won by winning at least 4 tennis points. For this analysis, we obtained total tennis points won by each player in a match and in battle 1.

[20] The p-value of a one-sided test of equality is 0.08. For other Chinese venues, average PM2.5 for the 37 non-completed matches and the 614 completed matches were similar; respectively, 53 and 50 μg/m^3.

is completed, but this effect is of limited significance.

We also checked that the distribution of players' WTA rank in Melbourne in hotter-than-usual annual events (2006, 2009, and 2014) was similar to that in less hot events (2004, 2011, and 2015).

4 Amplified momentum in a contest model

We develop a dynamic contest model that lends itself naturally to estimation. The model offers a lens through which to interpret the reduced-form results. We set up a best-of-three contest (a tennis match), and solve for the transition probabilities between battles (tennis sets as stages of the contest) as a function of economic primitives. These primitives include marginal effort cost and fixed cost parameters that, in the empirical model, shift with observables such as ability, heat, and pollution.

4.1 Model setup

Two possibly asymmetric players, labeled h and l, face marginal costs $c_h \geq c_l > 0$ that are constant over the range of effort x. The opponents engage in a contest in which up to three battles are held sequentially. In each battle, players simultaneously choose effort $x_i \geq 0$, $i \in \{l, h\}$, conditional on preceding battle outcomes. We model the contest technology following Tullock (1980), specifying the battle-transition probabilities as

$$p(x_i, x_j) = \frac{(x_i)^k}{(x_i)^k + (x_j)^k}, \quad i, j \in \{l, h\}, i \neq j, \tag{1}$$

for the case $\max\{x_i, x_j\} > 0$, where $p_i = p(x_i, x_j)$ is the probability that i wins a battle given her effort choice and that of her opponent, and $p(0, 0) = 0.5$ when both players choose zero effort. Technology parameter k, $0 < k \leq 1$, measures the degree of randomness in the winner selection process, reflecting the contest success function's discriminatory power. A smaller k corresponds to a noisier technology. In particular, as k approaches 0, relative effort does not matter and the winner is effectively a random draw.[21]

[21] The Tullock family of contest success functions is the most popular in the contest literature. Skaperdas (1996) and Fu and Lu (2012), respectively, provide axiomatization and micro foundations for it. Following

The player who wins at least two of the three battles wins the contest. The single prize for winning the contest is V_i (the subscript reflects a potentially asymmetric prize). Consistent with the data, we implicitly assume a sufficiently large penalty for no-shows such that players show up to play. We introduce a per-battle fixed cost, specifically an environmental disutility parameter δ (hereafter disutility), which scales with environmental degradation and captures the disutility to a player from added exposure to adverse environmental conditions. Much like the situation of spectators who do not exert player effort but still suffer disutility from heat or pollution (which may induce lower attendance at the arena), we assume δ enters player utility separately from effort cost; we formalize this below.[22] We further introduce an environmental effort cost factor $\lambda > 0$ by which a player's constant marginal effort cost, λc_i, also scales with environmental degradation.

The environment enters the model both as a direct utility shifter and by shifting marginal cost. Despite our model accommodating both channels, we learn that marginal cost shifts do not affect players differentially and do not drive transition probabilities. The environmental effort cost factor λ thus cannot be identified from empirical winning probabilities, which are at the heart of our design. In contrast, the first shifter, disutility δ, does shift transitions. To show that λ "cancels out," we keep it in the model. (λ can increase both in a poor environment and with the battle number, as players grow tired.) With richer cost specifications in Appendix B, we show that even when a poor environment makes the cost function more convex (less concave), it does not increase momentum.[23]

4.2 A one-shot game: The building block

We solve for the subgame perfect equilibrium by backward induction, so it is convenient to consider a single battle. Let $\overline{V}_i$ and $\underline{V}_i$ denote continuation value to player i if she,

Klumpp and Polborn (2006) and Malueg and Yates (2010), we formulate cost to depend on the effort chosen in each battle, not the cumulative effort, but we can allow the mapping from player ability to marginal cost to vary by battle. The structural approach allows us to pool the estimation sample over the entire range of asymmetry. We focus on pure-strategy subgame perfect Nash equilibria.

[22] Inspired by Gelder (2014), the environment entered an early version of the model by discounting continuation value from future battles exposed to adverse conditions, not through an additive fixed cost.

[23] We specify player i's effort cost in battle b as $C_i(x_i; \lambda, b) = a_b c_i\ [(x_i + 1)^{\lambda} - 1]$, $\lambda > k$. Cost is convex (or, less naturally, concave) in effort, e.g., $\lambda = 2$ (or $\lambda = 0.5$). Marginal cost is $a_b \lambda c_i (x_i + 1)^{\lambda - 1}$, noting that players growing tired from one battle to the next could be captured by $a_3 \geq a_2 \geq a_1 > 0$.

respectively, wins and loses the one-shot game, with win-loss prize spread $\Delta V_i := \overline{V}_i - \underline{V}_i$. Given her rival's action x_j, player i solves:

$$\arg\max_{x_i} \frac{(x_i)^k}{(x_i)^k + (x_j)^k}\overline{V}_i + \frac{(x_j)^k}{(x_i)^k + (x_j)^k}\underline{V}_i - \lambda c_i x_i - \delta = \frac{(x_i)^k}{(x_i)^k + (x_j)^k}\Delta V_i + \underline{V}_i - \lambda c_i x_i - \delta,$$

where the terms of the expression account for the expected benefit of effort, the effort cost, and the environmental disutility. At an interior solution, with $\Delta V_i > 0$, player i equates marginal effort benefit with marginal effort cost:

$$\frac{(x_i)^{k-1}(x_j)^k}{[(x_i)^k + (x_j)^k]^2} k\Delta V_i = \lambda c_i, \quad i \in \{l, h\}, j \neq i.$$

An isomorphic representation of the problem has player asymmetry entering the contest success function directly, $p_i = (\gamma_i x_i)^k / [(\gamma_i x_i)^k + (\gamma_j x_j)^k]$, with parameters $\gamma_i, \gamma_j > 0$ capturing players' skill, and homogeneous marginal cost c_i, c_j normalized to 1. In this representation, a unit of effort contributes more to a win, the more skilled the player.

Solving the system of best-response functions, and noting that $c_i x_i / \Delta V_i = c_j x_j / \Delta V_j$, the optimal bidding strategies as a function of prize spreads (and other parameters) are

$$x_i(\Delta V_i, \Delta V_j) = \frac{k\Delta V_i}{\lambda c_i} \frac{(\frac{c_i}{c_j}\frac{\Delta V_j}{\Delta V_i})^k}{[1 + (\frac{c_i}{c_j}\frac{\Delta V_j}{\Delta V_i})^k]^2}, \quad i, j \in \{l, h\}, i \neq j. \tag{2}$$

Multiplying both sides of (2) by λc_i, equilibrium effort cost $\lambda c_i x_i$ does not depend on the environmental factor λ, as effort x is inversely proportional to λ in equilibrium. For an equal prize spread, this effort cost is equal across players.

The reduced-form winning probabilities are

$$p_i = p(x_i(\Delta V_i, \Delta V_j), x_j(\Delta V_j, \Delta V_i)) = \frac{1}{1 + (\frac{c_i}{c_j}\frac{\Delta V_j}{\Delta V_i})^k}, \quad i, j \in \{l, h\}, i \neq j. \tag{3}$$

Facing equal incentives, a lower relative marginal cost makes a player's success more likely.

Figure 6 illustrates the best-response functions when players are symmetric, and when they are not (whether heterogeneity enters through marginal cost or through the contest prize). In both symmetric and asymmetric cases, as her rival's effort increases from zero, a player's optimal response is to increase her effort level. As the rival's effort increases beyond a threshold, the player responds by cutting back on effort.

4.3 Transition probabilities

Let binary variable χ_{bi} indicate the event in which battle b is won by player i, i.e., $\chi_{bi} = 1$ denotes her battle win and $\chi_{bi} = 0$ denotes her loss. We use $\Delta V_{bi} = \overline{V}_{bi} - \underline{V}_{bi}$ to denote player i's win-loss prize spread in battle b. Note this is in general history-dependent.

In a final battle 3, played only when each previous battle was won by a different player (i.e., $\chi_{2l} \neq \chi_{1l}$), the winner takes the contest prize and the loser earns 0. Setting win-loss spreads $\Delta V_{3l} = V_l$ and $\Delta V_{3h} = V_h$ in (3) yields battle-3 transition probabilities:

$$p_{3l} := \Pr\left(\chi_{3l} = 1 \,|\chi_{2l} \neq \chi_{1l}\right) = \frac{(c_h V_l)^k}{(c_l V_h)^k + (c_h V_l)^k}, \quad p_{3h} := \Pr\left(\chi_{3l} = 0 \,|\chi_{2l} \neq \chi_{1l}\right) = 1 - p_{3l}.$$

In battle 2, a player's winning probability is state-dependent. There are two states, depending on whether battle 1 was won or lost by the low-cost player. Starting with a history $\chi_{1l} = 1$ (battle 1 was won by player l), continuation values conditional on battle-2 outcomes are:

$$\text{if } \chi_{2l} = 1 \text{ (contest ends 2-0)} \begin{cases} \overline{V}_{2l|\chi_{1l}=1} = V_l, \\ \underline{V}_{2h|\chi_{1l}=1} = 0, \end{cases} \tag{4}$$

$$\text{if } \chi_{2l} = 0 \text{ (contest continues 1-1)} \begin{cases} \underline{V}_{2l|\chi_{1l}=1} = p_{3l} V_l - \lambda c_l x_l(V_l, V_h) - \delta, \\ \overline{V}_{2h|\chi_{1l}=1} = p_{3h} V_h - \lambda c_h x_h(V_h, V_l) - \delta. \end{cases} \tag{5}$$

Having won battle 1, player l's continuation values are (i) $\overline{V}_{2l|\chi_{1l}=1} = V_l$ from winning battle 2 and the contest, and (ii) $\underline{V}_{2l|\chi_{1l}=1} = p_{3l} V_l - \lambda c_l x_l(.) - \delta$ from losing battle 2, which means taking the contest to battle 3 and incurring further effort cost and environmental exposure. Player h, having lost battle 1, faces continuation values $\overline{V}_{2h|\chi_{1l}=1} \gtreqless 0$ and $\underline{V}_{2h|\chi_{1l}=1} = 0$ from, respectively, winning and losing battle 2. When player h's continuation value from taking the contest to battle 3 is negative ($\overline{V}_{2h|\chi_{1l}=1} < 0$), she can do better by exerting 0 effort and securing a continuation value of 0: In this situation, player l exerts an effort infinitesimally higher than 0 and wins the contest. Other than this corner solution, we obtain actions by plugging battle-2 win-loss incentives $\Delta V_{2i|\chi_{1l}=1} = \overline{V}_{2i|\chi_{1l}=1} - \underline{V}_{2i|\chi_{1l}=1}$ into one-shot effort (2). Plugging these actions into technology (1) yields conditional battle-2 transitions. Appendix A reports actions and transitions at all contest nodes.

Given the alternative history $\chi_{1l} = 0$ (battle 1 was lost by player l), continuation values conditional on battle-2 outcomes are expressed similarly to (4) and (5). For example, player l's continuation values are $\overline{V}_{2l|\chi_{1l}=0} = p_{3l}V_l - \lambda c_l x_l(V_l, V_h) - \delta$ from winning battle 2, triggering further effort and exposure, and $\underline{V}_{2l|\chi_{1l}=0} = 0$ otherwise.[24] Again, for an interior solution, using each player's win-loss spread $\Delta V_{2i|\chi_{1l}=0}$ in (2) yields optimal actions, and battle-2 transitions conditional on history $\chi_{1l} = 0$ follow from (1).

We pause to discuss how a degraded environment amplifies momentum in the model. With only one player winning battle 1, in battle 2 players' win-loss spreads are asymmetric even when marginal cost and the contest prize are equal across players, i.e., $c_i = c$ and $V_i = V$. To see this, we derive battle-2 incentives for this symmetric player case. Start by considering a possible battle 3. Using battle-3 win-loss spreads $\Delta V_{3i} = V$ in effort (2), battle-3 effort cost is $\lambda cx = kV/4$ (and invariant to the environmental factor, as higher λ induces lower effort); moreover, each player's winning probability is 0.5. Now move back to battle 2. The player who won battle 1 faces continuation values from battle 2 of V from winning and $V/2 - kV/4 - \delta$ from losing, with a difference of $(2 + k)V/4 + \delta$. By contrast, the player who lost battle 1 faces continuation values from battle 2 of $V/2 - kV/4 - \delta$ from winning and 0 from losing, with a difference of $(2 - k)V/4 - \delta$.

Two key points are immediate. First, even absent environmental shocks $\delta = 0$, the battle-1 winner enjoys a larger spread and chooses higher effort in battle 2 relative to the battle-1 loser (as $k > 0$). Such differential battle-2 incentives yield momentum. Second, as environmental disutility δ grows, the battle-1 winner's win-loss spread in battle 2 increases relative to that of her rival, amplifying momentum.

Back to the general asymmetric player case, we complete the derivation of transition probabilities, to be taken to the data. Battle-1 transitions are obtained from continuation

[24]Like $\overline{V}_{2h|\chi_{1l}=1}$ above, $\overline{V}_{2l|\chi_{1l}=0}$ can be negative, in which case player l exerts 0 effort in battle 2 to avoid a third battle. In contrast, a battle-1 winner's win-loss spread in battle 2 is always positive.

values conditional on battle-1 outcomes, plugging the associated win-loss spreads for each player in (2) and the resulting effort in technology (1). For example, player l's continuation values in the event she, respectively, wins and loses battle 1 are a function of battle-2 conditional transitions, continuation values, effort cost, and environmental disutility:[25]

$$\overline{V}_{1l} = \Pr(\chi_{2l} = 1|\chi_{1l} = 1)\overline{V}_{2l|\chi_{1l}=1} + \Pr(\chi_{2l} = 0|\chi_{1l} = 1)\underline{V}_{2l|\chi_{1l}=1} - \lambda c_l x_{2l|\chi_{1l}=1} - \delta, \quad (6)$$

$$\underline{V}_{1l} = \Pr(\chi_{2l} = 1|\chi_{1l} = 0)\overline{V}_{2l|\chi_{1l}=0} + \Pr(\chi_{2l} = 0|\chi_{1l} = 0)\underline{V}_{2l|\chi_{1l}=0} - \lambda c_l x_{2l|\chi_{1l}=0} - \delta. \quad (7)$$

For clarity, the first equation is player l's continuation value from winning battle 1 ($\chi_{1l} = 1$), leading her to a battle-2 win ($\chi_{2l} = 1$) or loss ($\chi_{2l} = 0$) with effort cost and disamenity.

4.4 Identification and illustration

Identification follows from the discussion above. Consistent with amplified momentum as documented in the descriptive analysis, environmental quality shifts transition probabilities via the per-battle environmental disutility parameter δ. As δ increases from 0, players are exposed to a more degraded environment.

As in the one-shot game, one can show that in each subgame a player's transition probabilities, effort cost, and expected payoff in equilibrium do not depend on the environmental effort cost factor λ. Environmental degradation shifts the marginal cost of effort λc_i, but this is offset by an inversely proportional response in effort $x_i(.)$. Because λ is not identified from the data, on implementation we set it to 1. Notice that environmental parameters λ and δ are similar in that, in principle, they function through their impact on total costs (effort cost and fixed cost).

Randomness k in the winner-selection technology is identified from variation in transition probabilities, holding constant both environmental disutility and players' relative strength c_l/c_h. We illustrate with symmetric contest prizes $V_i = V = 1$ (and subsequently consider asymmetric prizes, e.g., reputation win-loss differentiated through player age).

Figure 7 illustrates transition probabilities as a function of the per-battle environmental

[25]The example is for an interior solution. In contrast, if $\overline{V}_{2l|\chi_{1l}=0} < 0$, a battle-1 loss leads player l to exert 0 effort in battle 2, with a battle-1 continuation value from loss of $\underline{V}_{1l} = -\delta$.

disutility parameter δ when players are symmetric, and when there is moderate player asymmetry $c_h = 2c_l$.[26] As the environment deteriorates, momentum increases, with a rising probability that the battle-1 winner also wins battle 2. Heat or pollution reduces the probability that battle 2 is won by the player who lost battle 1, and thus the likelihood that the contest transitions to a third battle falls, which is consistent with the descriptive analysis. For comparison, in battles 1 and 3, each player has won an equal number of battles (zero or one): Facing the same win-loss spread, equally able players respond equally to the environment, and thus winning probabilities for these battles, of 0.5, do not depend on the environment.

Figure A.1 plots overall contest winning probabilities for two degrees of randomness, for $k = 1$ in the top panels and for $k = 0.7$ (more randomness) in the bottom panels. Among symmetric players (left panels), a contest winning probability of 0.5 does not depend on the environment as, intuitively, this affects both players equally. With asymmetric players (right panels), the difference in contest winning probabilities may widen slightly as δ grows. This suggests that the likelihood of an upset changes little—perhaps falls slightly—as the environment deteriorates, consistent with the descriptive analysis.[27]

5 Dynamic contest model estimates

5.1 Empirical implementation

For each match $n = 1, ..., N$, we observe alternative proxies for each opponent i's relative ability at the time of the match, a_{in}, based on betting odds, rank, or z-score, respectively, $odds_{in}$, $rank_{in}$, or $zscore_{in}$. We specify marginal effort cost parametrically:

$$c_{in} = f(a_{in}, \theta, \epsilon_{in}), \tag{8}$$

[26] Subsequent model estimates indicate that stronger degrees of asymmetry, such as $c_h = 5c_l$, are not uncommon in the data (Figure A.6). In such cases, a poor environment still increases momentum but the slope is flatter. Intuitively, even in the unlikely event that battle 1 is won by the weak player, her winning battle 2 is unlikely and environmental degradation can provide little momentum.

[27] Prediction markets may or may not capture this environment-induced change in overall winning odds. In any case, the widening in favor of the more able player appears small. Figure A.2 plots player l's contest winning probability for asymmetry up to $c_h = 5c_l$, as δ varies. Figure A.3 shows the probability that a contest lasts three battles, as opposed to two. Figure A.4 shows effort choices in battles 1 to 3.

where θ is a parameter to be estimated, and player-match shock ϵ_{in} is a mean-zero normally distributed error, i.i.d. over matches n and players i with variance σ_ϵ^2, and invariant within a match.[28] Relying on parameter k, which already captures randomness in mapping effort to success, our main empirical model sets $\sigma_\epsilon = 0$. Marginal cost is, alternatively, given by $c_{in} = (odds_{in})^\theta$, $c_{in} = (rank_{in})^\theta$, or $c_{in} = e^{-\theta(zscore_{in})}$. Note that z-scores are occasionally negative, for the few players below world average, which explains the variation in the forms of specification. The constraint $\theta > 0$, which may be imposed during estimation, implies that marginal cost increases in the odds or the rank and decreases in the z-score. We find that different measures of player strength yield similar results.

We observe mean temperature, T_n, and PM2.5, P_n, and make functional form assumptions relating ambient conditions to the per-battle environmental disutility parameter:

$$\delta_n = g(T_n, P_n, \delta_T, \delta_P), \tag{9}$$

where δ_T, δ_P are unrestricted parameters to be estimated, governing how disutility varies in the respective temperature and PM2.5 excess (°C and μg/m^3) relative to cutoffs $\underline{T}$ and $\underline{P}$:

$$\delta_n = \delta_0 + \delta_T \ln(1 + \max(T_n - \underline{T}, 0)) + \delta_P \ln(1 + \max((P_n - \underline{P})/100, 0)). \tag{10}$$

The temperature and pollution cutoffs and the intercept δ_0 are either specified or estimated. In our data, we rarely observe matches played under both heat and pollution, for example, no match was played at a temperature above 27 °C and PM2.5 above 150 μg/m^3 at the same time. Alternatively, we estimate models with dummy variables, such as

$$\delta_n = \delta_0 + \delta_T 1(T_n > \underline{T}) + \delta_P 1(P_n > \underline{P}). \tag{11}$$

The vector of parameters to be estimated is then, at a minimum, $\Psi = (k, \theta, \delta_T, \delta_P)$. We can also impose the model constraint that $0 < k \leq 1$. However, neither this constraint on the technology parameter nor the constraint on the parameter that governs how ability maps to marginal cost ($\theta > 0$) turn out to bind during optimization. Our main model

[28] While ϵ_{in} and ϵ_{jn} can be correlated, transition probabilities shift with opponents' relative strength.

takes the match's prize money as a proxy for V_{in}, but note that this ignores a winner's continuation value from playing the next round and prestige as the series final approaches.

5.2 Likelihood function

For a guess of parameters Ψ, we calculate the likelihood contribution for every match n in the sample as follows. Compute $c_{in} = f(a_{in}, \theta)$, $c_{jn} = f(a_{jn}, \theta)$, and $\delta_n = g(T_n, P_n, \delta_T, \delta_P)$ from (8) and (9), and use $c_{ln} = \min(c_{in}, c_{jn})$ and $c_{hn} = \max(c_{in}, c_{jn})$ to label the players (weakly) low-cost and high-cost. Denote possible outcomes by $\iota_n = \mathbf{ab}(\mathbf{c})$, where **a**, **b**, and (for three-setters) **c** label the players who won battles 1, 2, and 3. A match has 6 mutually exclusive outcomes, $\iota_n \in \mathcal{O} := \{\mathbf{ll}, \mathbf{lhl}, \mathbf{lhh}, \mathbf{hll}, \mathbf{hlh}, \mathbf{hh}\}$. For example, facing a relatively high-cost opponent ranked 40, a player ranked 20 wins battles 1 and 2, such that the match outcome is **ll**, i.e., the low-cost player wins the match in two sets.

Model-predicted probabilities follow from the transition probabilities derived earlier. For example, matches with outcomes **ll** and **lhl** contribute, respectively:

$$\begin{aligned}
\Pr(\mathbf{ll}\,|\Psi\,) &= \Pr\left(\chi_{1l}=1\right)\Pr\left(\chi_{2l}=1\,|\chi_{1l}=1\,\right), \\
\Pr(\mathbf{lhl}\,|\Psi\,) &= \Pr\left(\chi_{1l}=1\right)\Pr\left(\chi_{2l}=0\,|\chi_{1l}=1\,\right)\Pr\left(\chi_{3l}=1\,|\chi_{1l}\neq\chi_{2l}\,\right).
\end{aligned}$$

Appendix A reports all expressions. Write match n's likelihood contribution, conditional on parameters Ψ, as $\prod_{\iota_n\in\mathcal{O}}\left(\Pr(\iota_n\,|\Psi\,)\right)^{1(\iota_n)}$, where indicator $1(\iota_n)$ is 1 if outcome ι_n was observed in match n and 0 otherwise. The likelihood across all matches is

$$\prod_{n=1}^{N}\prod_{\iota_n\in\mathcal{O}}\left(\Pr(\iota_n\,|\Psi\,)\right)^{1(\iota_n)}. \tag{12}$$

Now we take logs, and our task is to find parameters Ψ that maximize the log likelihood. To test robustness, we restrict the sample to matches in which the prize money is not too high, to matches for which opponent asymmetry is low, and so on.

5.3 Results

Table 5 shows that estimates are robust across alternative ability measures. Across columns, the technology parameter k is stable at about 0.7, implying that effort yields a win with some degree of randomness. Alternative θ parameters imply that marginal cost increases in the betting odds (inverse winning probability) and the WTA rank, and decreases in the WTA z-score. In the isomorphic representation, in which asymmetry shifts technology, the degree to which a player converts effort into success decreases in the odds and the rank, and increases in the the z-score.

With regard to the key economic primitives, we obtain that the environmental disutility grows as ambient (i) temperature increases from $\underline{T} = 27$ °C and (ii) PM2.5 increases from $\underline{P} = 150$ μg/m^3 (in this table we fix the poor environment cutoffs and set the intercept $\delta_0 = 0$). Parameters δ_T, δ_P are estimated to be significant both statistically and economically. Across columns, the fixed cost per battle grows by $\hat{\delta}_T =$\$120 to \$150 as the temperature shifts from 27 to $27 + e - 1 \approx 28.7$ °C. The coefficient is multiplied by 1000 because the match's prize money—the proxy for the contest prize here—is expressed in thousands of dollars, per Table 1. Euler's number e follows from the natural logarithm in (10);[29] this logarithmic form implies that over the range of temperature variation, to a sample maximum of 44 °C, impacts grow at a diminishing rate. The concavity may stem from heat relief technologies being adopted as conditions deteriorate, like ice misters when players switch sides on the court after every odd tennis game.[30] Thus the disutility grows by twice as much, $2\hat{\delta}_T =$\$250 – 300, for a temperature shift from 27 to $27 + e^2 - 1 \approx 33.4$ °C, and by $3\hat{\delta}_T =$\$370 – 450 for a shift from 27 to 46 °C.

To read the coefficient on pollution, the per-battle disutility grows by $\hat{\delta}_P =$\$1100 to \$1400 for a large PM2.5 shift from 150 to $(1.5 + e - 1) \times 100 \approx 322$ μg/m^3. The disutility grows by half as much, $0.5\hat{\delta}_P =$\$570 – 700, for a still sizable PM2.5 shift from 150 to $(1.5 + e^{0.5} - 1) \times 100 \approx 215$ μg/m^3.

[29] That is, for a shift in the temperature covariate in (10) from $\ln(1+27-27) = 0$ to $\ln(1+27+e-1-27) = 1$.

[30] WSJ (2014) may help interpret: "Tennis is, of course, not a terrible sport to play in the heat. There are no helmets or protective pads. There are plenty of stops and starts. Players can rest in the shade between games with ice-filled towels. They can take bathroom breaks and call for medical timeouts."

Taking betting odds to measure ability, Table 6 further reports on robustness. Column 1 allows the technology parameter to vary by battle k_b, $b \in \{1, 2, 3\}$, with point estimates of 0.67, 0.67 and 0.67, indicating that the contest success function's discriminatory power is similar during the course of a match. Column 2 allows for battle-specific marginal cost parameters θ_b, $b \in \{1, 2, 3\}$, with point estimates of 0.87, 0.85, and 0.88, suggesting that the mapping from ability to marginal cost varies little within a match. The estimated environmental disamenities hardly change. In column 3, slopes δ_T, δ_P remain significant when the fixed cost intercept δ_0 is estimated, at \$56, and statistically insignificantly different than zero. This suggests that setting $\delta_0 = 0$ in our preferred specification (Table 5, column 1) is not too restrictive. In column 4, we estimate (rather than specify) the poor environment cutoffs subject to the constraints $25 \leq \underline{T} \leq 35$ °C and $100 \leq \underline{P} \leq 300$ μg/m^3. With estimated cutoffs of $\hat{\underline{T}} \approx 26.1$ °C and $\hat{\underline{P}} \approx 220$ μg/m^3, the heat slope is slightly lower and the pollution slope is higher. With technology and environment interacting in column 5, randomness falls (k increases) as the environment worsens, but not significantly so.

Model predictions. Figure 8 further interprets our preferred estimates, providing a welfare analysis that underscores one strength of structural work. The figure illustrates a match with prize money at the sample median, \$15,100 (we later illustrate for a lower-prize match). For a given environment, the vertical intercept in panel (a) shows the increase in likelihood that a player wins battle 2 after winning vs. losing battle 1 against an equally able player. Compared to a "threshold" environment, this momentum (likelihood increase) grows by 3 percentage points when the temperature shifts from 27 to 37 °C and by 8 percentage points when PM2.5 shifts from 150 to 250 μg/m^3.[31] The panels report on cases of increased asymmetry along the horizontal axis. In panel (b), the probability of a tighter three-battle contest against an equally able player falls by up to 4 percentage points in the illustrated poor environments compared with a threshold one—again, compare the vertical intercepts. In panel (c), environmental degradation slightly reduces the likelihood of an upset. Reassuringly, the stronger player's model-predicted winning probability rises linearly with the betting-market winning probability difference. The expected utility loss

[31] Momentum is calculated as $\Pr(\chi_{2l} = 1 \,|\chi_{1l} = 1) - \Pr(\chi_{2l} = 1 \,|\chi_{1l} = 0)$ (equivalent if we write h instead of l in the subscript, because one player's win is the other player's loss).

from heat and pollution at the outset of a contest is shown in panels (d) and (e) for strong and weak players, respectively. In a symmetric contest, raising temperature from 27 to 37 °C reduces a player's expected payoff, net of her effort cost, by \$670; raising PM2.5 from 150 to 250 μg/m^3 reduces a player's expected payoff by \$1800.

These model estimates inform the continuation values to a player from winning relative to losing the first battle (calculated from (6) and (7)) and, importantly, how this value difference—the value of a head start—grows as the environment worsens. At 27 °C and 150 μg/m^3, in a symmetric contest with median prize (\$15,100), this differential value from winning vs. losing battle 1 is \$9610 − \$1150 = \$8470. At 37 °C (and 150 μg/m^3), the differential value from winning battle 1 is \$130 higher (i.e., \$8600). At 250 μg/m^3 (and 27 °C), the differential value from winning battle 1 is \$370 higher (i.e., \$8840) than at 150 μg/m^3.

Figure 9 is similar to Figure 8 except that it illustrates a lower-prize match, at the 25th-percentile prize money of \$8700. Comparing panels (a) and (b) across the two figures, environmental degradation amplifies momentum and reduces three-setter occurrences to a greater degree when the contest prize is lower.[32]

Figure A.5 is similar to Figure 9 in illustrating a lower-prize match except that it takes WTA rank to measure ability. As stated earlier, environmental impacts are similar across alternative ability measures. Figure A.6 shows how alternative ability measures in the data map onto estimated marginal cost. Fitted marginal cost ranges from 1 to as high as 30, depending on the measure and parametric form; see the left panels. The right panels of Figure A.6 compare the low-cost player's winning probability advantage predicted by betting markets to that predicted by the empirical model.[33] When the ability measure is based on betting odds, model predictions line up fairly well with betting-market predictions. In fact, panel (b) suggests that the model predicts a somewhat better chance for the strong player in poor environments compared with betting-market predictions.

On the possibility of negative expected payoffs at the start of a match (e.g., Fig-

[32]Figure 9 shows that compared to a threshold environment, momentum increases by 5 percentage points when the temperature shifts from 27 to 37 °C and by 14 percentage points when PM2.5 shifts from 150 to 250 μg/m^3. The environmental impact is larger as we further reduce the prize money.

[33]The model's prediction for a player-l win advantage is computed as $\Pr(\mathbf{ll}\cup\mathbf{lhl}\cup\mathbf{hll})-\Pr(\mathbf{hh}\cup\mathbf{hlh}\cup\mathbf{lhh})$.

ure 9(e)), notice that harsh penalties for no-shows—such as cancelled sponsorship contracts or angry fans, to be added to these payoffs—would induce (weaker) players to show up to play. Our model specifies environmental disutility for every battle played. We assume that δ is incurred also for battle-2 corner solutions in which a player chooses minimal exertion yet has to remain in court in battle 2 after losing battle 1.[34]

Age heterogeneity. Table 7 illustrates how our framework is amenable to the analysis of player heterogeneity. We allow environmental disutility (10) to shift by age, specifically, the coefficient on the heat covariate $\ln(1+\max(T_n-\underline{T},0))$ is now $\delta_T+\delta_T^S$ if a player's age on match day exceeds the 75th percentile of the player-by-match distribution (27.2 years), and remains δ_T otherwise. Superscipt S denotes such senior (or seasoned) professionals. Similarly, the coefficient on the pollution covariate is $\delta_P+\delta_P^S$ if a player is in the upper quartile of the age distribution and δ_P otherwise. The top panel suggests that for senior players the disutility (i) from heat is 40-60% higher (e.g., 0.053+0.118 vs. 0.118, in column 1) and (ii) from pollution 10-30% higher relative to the rest of the players. While positive, the seniority differential for the pollution disamenity is not statistically significant.

Possible interpretations for such age heterogeneity are that players in the upper quartile of the age distribution (i) are not as young, e.g., the sample includes players aged 15 years, and (ii) they are wealthier—the oldest players are aged 44 and the data show that professionals who are active in this labor market over longer careers tend to be better ranked. The bottom panel of Table 7 goes one step further and scales the contest prize V_{in} by a factor ρ if a player's age is in the upper quartile, i.e., ρV_{in}, and V_{in} otherwise (as in Table 5, with the match prize money as a proxy). With parameter ρ estimated at 0.95-0.99 across ability measures, there is suggestive evidence that senior players slightly discount winning relative to younger athletes; for example, with a longer career ahead and thus reputation gains adding to the cash prize, a young battle-1 loser may still fight on.[35] Importantly, the complementarity between momentum and adverse environmental

[34] An alternative model can specify δ only for the third (identifying) battle. Through their battle-2 effort, players can influence the likelihood that battle 3 is avoided, without angering their fan base too much. Specifying δ as a fixed entry cost only in battle 3 would not change equilibrium behavior or estimated parameters relative to our model, but it would raise expected payoffs (because δ is incurred at most once).

[35] Consider a senior player i, with a short career left, facing a median rival j as driving a wedge $V_i < V_j$.

conditions is stable (compare $\hat{\delta}_T$ and $\hat{\delta}_P$ to that in Table 5), as is its interaction with player seniority (compare Table 7's top and bottom panels).

Sample composition. Table 8 analyzes sensitivity to the likelihood function and to sample composition. In column 1, we use only observed vs. predicted battle-2 transitions to pin down the model parameters, i.e., alternative criterion function (17) in the appendix instead of all transitions in (12). In principle, the win-loss spread V to a match winner should incorporate value from moving closer to the WTA series final and the associated fame and sponsorship benefits. Thus column 2 drops quarterfinal, semifinal, and final matches from the estimation sample. Estimates change little. Similarly, column 3 drops matches with prize money higher than the 75th percentile ($25,896), to control for possibly different behavior in such matches. Column 4 shows that estimates are also robust to dropping Australian Open matches (a Grand Slam series), in a sample comprising only Chinese venues. Column 5 restricts the sample to 700 matches in which the ex-ante winning probability difference is at most 30%, obtaining a smaller $\hat{k}$ (more randomness in this sample) and a larger $\hat{\delta}_T$ and $\hat{\delta}_P$ (and the estimated standard error on $\hat{\delta}_P$ is large).[36]

Dummy variables. Table A.5 considers a dummy-variable specification for environmental disutility, i.e., (11). Column 1 shows that the per-battle fixed cost increases by (i) \$190 at temperatures above 27 °C and (ii) \$670 at PM2.5 levels above 150 μg/m^3. Relative to column 1, column 2 additionally estimates the per-battle fixed cost intercept δ_0 at a statistically insignificant \$30; and column 3 additionally estimates poor environment cutoffs at $\underline{\hat{T}} \approx 26.9$ °C and $\underline{\hat{P}} \approx 210$ μg/m^3. Column 4 specifies finer environment bins. Estimates of heat stress are similar for $T_n > 29$ °C compared to $T_n \in (27, 29]$; estimates of PM2.5 stress grow for $P_n > 200$ μg/m^3 compared to $P_n \in (100, 200]$.

Subsequent matches in a series. The extent to which winning a match enhances player value through prestige and by attracting sponsors is unobservable. But on implementing the model one can increase the contest prize V_{in} to include, on top of the match's prize money, value from subsequent rounds within the WTA series. Such a model variant

[36] To capture "unforseen reasons, [by which] one player simply turned out to be better on match day" (Malueg and Yates, 2010), we restricted the column 5 sample to 320 matches with tight battle-1 outcomes comprising at least 10 tennis games (i.e., 6-4, 7-5, 7-6). We obtain even larger $\hat{\delta}_T$ and $\hat{\delta}_P$.

would plausibly assume that players are forward-looking and internalize the effect of current match effort on playing in a subsequent round. There are different ways to model a player's expectations regarding future matches' opponent abilities and environments. To simplify, and because our findings are unlikely to qualitatively change, the implementation that follows assumes that going forward a player expects to meet opponents of similar caliber (reasonable on average) in mild environments (the modal environment).

Using Table 5 estimates and for every series (venue-year), we begin by computing the expected payoff to participating in the final contest, for which there are no subsequent contests.[37] We add the final's expected payoff to the semifinal's prize money to obtain a new measure of the semifinal's contest prize, and use this measure to calculate the semifinal's expected effort cost and thus expected payoff. We backward induct and consider the perspective of a player choosing effort in the quarterfinal, for whom the contest prize is the quarterfinal's prize money plus continuation value from reaching the semifinal (which itself subsumes the chance of reaching the final). We proceed recursively to the series' first round of contests.

Equipped with a contest prize vector V that now includes a winner's continuation value from participating in a series' subsequent rounds, we supply V_{in}—along with ability a_{in}, environmental conditions T_n, P_n, and match outcomes ι_n—to criterion function (12). We obtain a new parameter vector. We iterate until the parameter vector converges.

In this "matches ahead" implementation of V, Table 9 reports that the fixed cost per battle increases (i) by $\hat{\delta}_T =$\$260 as temperature shifts from 27 to $27 + e - 1 \approx 28.7$ °C, and (ii) by $\hat{\delta}_P =$\$3490 for a large PM2.5 shift from 150 to $(1.5 + e - 1) \times 100 \approx 322$ μg/m^3. These estimates are about double what we obtained with match prize money as a proxy for V, which ignores continuation value (Table 5). The intuition, as we discuss next, is that the valuation of behavior underlying the amplified momentum in the data is larger when we consider that more is at stake than match day's cash reward alone (Figure A.7).

[37] This is the winning probability times the final's prize money less the expected cost. For a symmetric match played in a mild environment, the only parameter estimate this relies on is $\hat{k}$.

6 Discussion

We find that heat and pollution affects elite workers' willingness to compete over the successive battles that determine a contest winner. In our setting, a battle (tennis set) lasts on average just under one hour, played over more than 9 tennis games and 60 tennis points, with each tennis point consisting of a sequence of back and forth shots between players until a miss ends the rally. A quick battle can be over in 20 minutes, yet long battles can extend over 90 minutes, in particular third battles in Grand Slam matches. Adverse environmental shocks amplify the discouragement effect.

Given a good measure of the award enjoyed by the contest winner, we show that how players differentially respond after winning vs. losing the first battle, in bad vs. good environments, reveals their fixed entry cost into an optional third battle. We interpret this entry cost as informative of contestants' preferences over environmental amenities.

In our setting, we specify two alternative measures of the contest prize. The first measure of V considers the contest's prize money alone. In a contest between equally able athletes, raising temperature from 27 to 37 °C reduces a player's expected payoff, net of her effort cost, by \$670; raising PM2.5 from 150 to 250 μg/m^3 reduces a player's expected payoff by \$1800. The second measure of V adds, to a match's prize money, value from playing a subsequent match in the series. Because the second measure of V is larger, the amplified momentum we observe in the data is rationalized through a higher fixed cost.

One limitation of our structural estimates is that more comprehensive measures of V than those we implemented would include the impact of a match win on a player's state of fame (e.g., become a celebrity), sponsorship prospects (attract a major sports brand), and WTA ranking points (qualify as a seed player in a subsequent series). We leave the implementation of such continuation value, within a more general dynamic model than ours, to future research. Such work can assess how V interacts with players' heterogeneous ability to sustain wins all the way to a series final.[38]

Specific to our setting, we note that these young players are among the world's top

[38] As noted previously, betting odds, on which we base our preferred measure of opponents' relative ability, may partly capture differential motivation and fitness.

athletes, typically in top physical (and mental) form. They enjoy high earning potential over short and intense professional careers. They hire professional managers to advise them on competitive strategy. They invest substantial monetary and nonmonetary effort to stay healthy and fit. In analogy to preferences for environmental amenities estimated here, it is conceivable that to reduce fatigue these players are willing to pay thousands of dollars to fly in a business-class cabin, as they move from one WTA series to the next.

With Australian Open revenues nearing half a billion dollars per series, it is clear that the types of markets examined here create substantial welfare to the economic agents involved, including spectators, advertisers, suppliers, and organizers. Faced with a warming climate and growing popularity of venues subject to poor ambient air,[39] adapting contest design through widespread use of retractable roofs and air conditioning may result in a greater proportion of contests reaching three battles. Our model estimates show, for a median cash prize, that three-setters when played between equally able rivals in a degraded environment are as (un)likely as when asymmetric players—with pre-match winning probability differences of 30-50%—meet in a climate-controlled environment (e.g., Figure 8(b)). In principle, with sufficient data, reduced-form work can deliver such findings. Our structural estimates enable other counterfactual analyses. For the high-income players themselves, a demographic group for which there is limited empirical evidence, the individual disutility from heat and pollution exposure can run in the order of hundreds to thousands of dollars per hour. Such numbers can be interpreted as a statement about how difficult it is to compete at a high level as the environment deteriorates.

Future work with a continued emphasis in this specific setting can construct a player-level panel over many years and venues to examine individual heterogeneity in the distaste for, and adaptation to, adverse environmental shocks. For example, does heat disutility $\delta_{T,it}$ vary across home region, gender, skill level, and tenure of players i over time t?

Multi-battle contests are ubiquitous in the economy, with settings as diverse as labor, innovation, advertising, politics, foreign relations, and litigation. We believe that the willingness to win early, to avoid the fixed cost of participating in a protracted competition

[39]China in particular and the routinely polluted urban developing world in general (Marlier et al., 2016) offer a growing fan base. Poor air quality may also result from wildfires in the (warming) rich world.

and instead redeploy resources to outside opportunities, has broad appeal. For example, rising licensing costs for existing patents from other inventors may induce innovators who fall behind in a patent race to throw in the towel. A tightening high-skill labor market may raise the opportunity cost of a protracted internal promotion, inducing key managers who fall behind to reduce effort relative to peers who get ahead, compromising the screening procedure and the organization's overall effort supply.

Beyond professional sport and environmental valuation, our empirical framework can be used to study alternative theories of behavior[40] and individual heterogeneity in dynamic competitions, e.g., gender differences (Mago et al., 2013; Gauriot and Page, 2018; Gill and Prowse, 2014; Gonzalez-Diaz and Palacios-Huerta, 2016; Jetter and Walker, 2015; Cohen-Zada et al., 2017). In particular, models such as the one we estimate have the potential to test the relative strengths of economic incentives vs. psychological factors at explaining empirically observed momentum, in addition to uncovering economic primitives of interest, such as preferences, opportunity costs, and technological parameters.

References

Archsmith, J., Heyes, A., and Saberian, S. (2018). Air quality and error quantity: Pollution and performance in a high-skilled, quality-focused occupation. Journal of the Association of Environmental and Resource Economists, 5(4):827–863.

Bognanno, M. L. (2001). Corporate tournaments. Journal of Labor Economics, 19(2):290–315.

Boudreau, K. J., Lakhani, K. R., and Menietti, M. (2016). Performance responses to competition across skill levels in rank-order tournaments: Field evidence and implications for tournament design. RAND Journal of Economics, 47(1):140–165.

Brown, J. (2011). Quitters never win: The (adverse) incentive effects of competing with superstars. Journal of Political Economy, 119(5):982–1013.

Chen, Y., Ebenstein, A., Greenstone, M., and Li, H. (2013). Evidence on the impact of sustained exposure to air pollution on life expectancy from China's Huai River policy. Proceedings of the National Academy of Sciences, 110(32):12936–12941.

Cohen-Zada, D., Krumer, A., and Shtudiner, Z. (2017). Psychological momentum and gender. Journal of Economic Behavior & Organization, 135:66–81.

[40] For example, Table A.6 further allows for (and finds some evidence of) psychological state dependence.

Corchon, L. C. (2007). The theory of contests: A survey. Review of Economic Design, 11(2):69–100.

Dell, M., Jones, B. F., and Olken, B. A. (2014). What do we learn from the weather? The new climate-economy literature. Journal of Economic Literature, 52(3):740–798.

Deschenes, O. and Greenstone, M. (2011). Climate change, mortality, and adaptation: Evidence from annual fluctuations in weather in the US. American Economic Journal: Applied Economics, 3(4):152–185.

DeVaro, J. (2006). Internal promotion competitions in firms. RAND Journal of Economics, 37(3):521–542.

Feng, X. and Lu, J. (2018). How to split the pie: Optimal rewards in dynamic multi-battle competitions. Journal of Public Economics, 160:82–95.

Ferrall, C. and Smith Jr., A. A. (1999). A sequential game model of sports championship series: Theory and estimation. Review of Economics and Statistics, 81(4):704–719.

Fu, Q. and Lu, J. (2012). Micro foundations of multi-prize lottery contests: A perspective of noisy performance ranking. Social Choice and Welfare, 38(3):497–517.

Gauriot, R. and Page, L. (2018). Psychological momentum in contests: The case of scoring before half-time in football. Journal of Economic Behavior & Organization, 149:137–168.

Gelder, A. (2014). From Custer to Thermopylae: Last stand behavior in multi-stage contests. Games and Economic Behavior, 87:442–466.

Genakos, C. and Pagliero, M. (2012). Interim rank, risk taking, and performance in dynamic tournaments. Journal of Political Economy, 120(4):782–813.

Gill, D. and Prowse, V. (2014). Gender differences and dynamics in competition: The role of luck. Quantitative Economics, 5(2):351–376.

Goltsman, M. and Mukherjee, A. (2011). Interim performance feedback in multistage tournaments: The optimality of partial disclosure. Journal of Labor Economics, 29(2):229–265.

Gonzalez-Diaz, J. and Palacios-Huerta, I. (2016). Cognitive performance in competitive environments: Evidence from a natural experiment. Journal of Public Economics, 139:40–52.

Grossman, G. M. and Shapiro, C. (1987). Dynamic R&D competition. Economic Journal, 97(386):372–387.

Hanna, R. and Oliva, P. (2015). The effect of pollution on labor supply: Evidence from a natural experiment in Mexico City. Journal of Public Economics, 122:68–79.

Harris, C. and Vickers, J. (1987). Racing with uncertainty. Review of Economic Studies, 54(1):1–21.

He, J., Liu, H., and Salvo, A. (2019). Severe air pollution and labor productivity: Evidence from industrial towns in China. American Economic Journal: Applied Economics, 11(1):173–201.

Huang, Y., Vir Singh, P., and Srinivasan, K. (2014). Crowdsourcing new product ideas under consumer learning. Management Science, 60(9):2138–2159.

Iso-Ahola, S. E. and Mobily, K. (1980). "Psychological momentum": A phenomenon and an empirical (unobtrusive) validation of its influence in a competitive sport tournament. Psychological Reports, 46:391–401.

Ito, K. and Zhang, S. (2019). Willingness to pay for clean air: Evidence from air purifier markets in China. Journal of Political Economy, Forthcoming.

Jetter, M. and Walker, J. K. (2015). Game, set, and match: Do women and men perform differently in competitive situations? Journal of Economic Behavior & Organization, 119:96–108.

Klumpp, T. and Polborn, M. K. (2006). Primaries and the New Hampshire effect. Journal of Public Economics, 90:1073–1114.

Konrad, K. A. (2009). Strategy and Dynamics in Contests. Oxford University Press, New York.

Konrad, K. A. and Kovenock, D. (2009). Multi-battle contests. Games and Economic Behavior, 66(1):256–274.

Lazear, E. P. and Rosen, S. (1981). Rank-order tournaments as optimal labor contracts. Journal of Political Economy, 89(5):841–864.

Mago, S. D., Sheremeta, R. M., and Yates, A. (2013). Best-of-three contest experiments: Strategic versus psychological momentum. International Journal of Industrial Organization, 31(3):287–296.

Malueg, D. A. and Yates, A. J. (2010). Testing contest theory: Evidence from best-of-three tennis matches. Review of Economics and Statistics, 92(3):689–692.

Marlier, M. E., Jina, A. S., Kinney, P. L., and DeFries, R. S. (2016). Extreme air pollution in global megacities. Current Climate Change Reports, 2(1):15–27.

Rosen, S. (1986). Prizes and incentives in elimination tournaments. American Economic Review, 76(4):701–715.

Salvo, A. (2018). Electrical appliances moderate households' water demand response to heat. Nature Communications, 9(5408):1–14.

Skaperdas, S. (1996). Contest success functions. Economic Theory, 7(2):283–290.

Snyder, J. M. (1989). Election goals and the allocation of campaign resources. Econometrica, 57(3):637–660.

Somanathan, E., Somanathan, R., Sudarshan, A., and Tewari, M. (2019). The impact of temperature on productivity and labor supply: Evidence from Indian manufacturing. Journal of Political Economy, Forthcoming.

Szymanski, S. (2003). The economic design of sporting contests. Journal of Economic Literature, 41(4):1137–1187.

Tullock, G. (1980). Efficient rent seeking. In Buchanan, J., Tollison, R., and Tullock, G., editors, Toward a Theory of the Rent-Seeking Society, pages 97–112. Texas A&M University Press, College Station.

WHO (2020). DRAFT landscape of COVID-19 candidate vaccines. World Health Organization, September 9.

WSJ (2014). Some tennis players like it hot: Temperatures reached 108 degrees Fahrenheit at the Australian Open on Tuesday. The Wall Street Journal, January 14.

Table 1: Descriptive statistics for the Australian Open, the China Open, and other WTA contests in China

Variables	N	Mean	Std.Dev.	Min.	Max.
Australian Open: Melbourne in the summer					
Difference in winning probability between the two players (%)	1127	49.42	26.98	0	95.16
Difference in WTA rank between the two players	1641	69.47	97.31	0	1200
Difference in WTA ranking points between the two players	1641	1670.12	1919.22	1	12,847
Difference in age between the two players (years)	1651	4.39	3.43	0.00	26.45
Completed match (yes=1)	1651	0.98	0.13	0	1
Match's prize money (win-loss cash spread in $ $\times$ 1000)	1651	45.85	116.23	8.38	1400.18
Battle 1 length in tennis games (among completed matches)	1624	9.19	1.95	6	13
Battle 2 length in tennis games (among completed matches)	1624	9.05	1.95	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	1624	0.79	2.53	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	1624	0.31	0.46	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	1624	0.16	0.37	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	498	9.47	2.72	6	30
Total points played in the match (among completed matches)	1038	140.48	42.29	63	358
Hour match starts (among completed matches)	1651	14.01	3.02	7	23
Airborne Particle Index (mean during match)	1651	0.58	0.51	0.25	5.09
PM2.5 (μg/m^3, mean during match)	251	9.20	5.35	0.00	56.67
Temperature ($^\circ$C, mean during match)	1651	26.15	6.24	12.80	43.98
Relative humidity (%, mean during match)	1651	55.85	18.57	15.31	102.21
Wind speed (m/s, mean during match)	1651	3.84	1.51	0.24	8.93
Any rain, however light (yes=1, during match)	1628	0.17	0.38	0	1
China Open: Beijing in the fall					
Difference in winning probability between the two players (%)	493	42.31	24.33	0	92.16
Difference in WTA rank between the two players	497	39.23	52.55	0	526
Difference in WTA ranking points between the two players	497	2101.16	2054.34	5	11,145
Difference in age between the two players (years)	498	4.08	3.04	0.04	23.32
Completed match (yes=1)	499	0.95	0.22	0	1
Match's prize money (win-loss cash spread in $ $\times$ 1000)	499	40.42	71.99	4.02	554.82
Battle 1 length in tennis games (among completed matches)	474	9.40	2.00	6	13
Battle 2 length in tennis games (among completed matches)	474	9.03	1.89	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	474	0.68	2.59	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	474	0.32	0.47	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	474	0.16	0.37	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	153	9.55	2.01	6	13
Total points played in the match (among completed matches)	375	141.71	40.63	66	246

Table 1: Descriptive statistics for the Australian Open, the China Open, and other WTA contests in China

Variables	N	Mean	Std.Dev.	Min.	Max.
Hour match starts (among completed matches)	499	15.20	2.93	10	23
PM2.5 (μg/m^3, mean during match)	467	88.05	94.05	3.67	520.33
Temperature (°C, mean during match)	499	18.91	4.61	7.72	31.25
Relative humidity (%, mean during match)	499	46.12	18.98	14.59	97.80
Wind speed (m/s, mean during match)	499	2.40	1.11	0.38	7.96
Any rain, however light (yes=1, during match)	499	0.06	0.24	0	1
Five other WTA series in China: Guangzhou, Hong Kong, Shenzhen, Tianjin, Wuhan in the fall or winter					
Difference in winning probability between the two players (%)	656	38.79	23.26	0	92.16
Difference in WTA rank between the two players	661	68.14	84.10	1	1014
Difference in WTA ranking points between the two players	661	1140.73	1410.57	0	8065
Difference in age between the two players (years)	661	4.28	3.24	0.01	18.76
Completed match (yes=1)	661	0.94	0.23	0	1
Match's prize money (win-loss cash spread in $ × 1000)	661	10.21	21.48	1.28	235.89
Battle 1 length in tennis games (among completed matches)	624	9.32	1.93	6	13
Battle 2 length in tennis games (among completed matches)	624	9.23	1.98	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	624	0.78	2.47	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	624	0.30	0.46	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	624	0.17	0.37	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	190	9.34	1.91	6	13
Total points played in the match (among completed matches)	542	140.54	40.52	71	294
Hour match starts (among completed matches)	661	14.57	2.64	10	22
PM2.5 (μg/m^3, mean during match)	651	49.93	35.63	7.00	310.67
Temperature (°C, mean during match)	661	24.76	5.17	8.61	34.42
Relative humidity (%, mean during match)	661	61.16	17.48	15.26	98.19
Wind speed (m/s, mean during match)	661	2.97	1.63	0.14	10.46
Any rain, however light (yes=1, during match)	661	0.16	0.37	0	1

Notes: Singles match series include 2004-2016 for the Australian Open in Melbourne, 2008-2016 for the China Open in Beijing, and 2011-2016 for other Women's Tennis Association (WTA) series in China (Guangzhou from 2011, Shenzhen from 2013, and Hong Kong, Tianjin, and Wuhan from 2014). Match data including pre-match betting odds are from flashscore.com except for odds in 2008, which are from tennis-data.co.uk, and total points played in a match, which are from oncourt.info. Player age and match prize money are extracted from wtatennis.com. We adjust nominal US$ for inflation using the urban CPI published by the US BLS (base December 2016). Particle pollution is measured at air monitors near match venues by Victoria Environment Protection Authority (Melbourne, average across Alphington and Footscray sites); US Department of State (Beijing, Guangzhou); Hong Kong Environmental Protection Department (Hong Kong); and Chinese Ministry of Environmental Protection (other cities). Victoria EPA 1-hour PM2.5 are available only from January 2015, and we use the 1-hour Airborne Particle Index instead. US Department of State 1-hour PM2.5 are missing for Guangzhou in 2011, and we use Hong Kong EPD 1-hour PM2.5 instead (for this month only). Weather data are compiled by NASA (3-hour means), except for temperature and wind speed in Melbourne, which are available from Victoria EPA (1-hour means). Mean environmental conditions are taken over the three 1-hour periods that encompass the hour in which the match started and the two subsequent hours, e.g., 2:00 pm to 4:59 pm for a match that started at 2:23 pm.

Table 2: Environmental quality and momentum: Probability of transiting to a third battle

Dependent variable is 1 if match lasts three battles, 0 otherwise.	(1) High asymm.	(2)	(3)	(4) More symmetric opponents	(5)	(6)	(7)	(8)	(9) Entire
Win. prob. diff. (from betting):	$\geq 65\%$	$\leq 30\%$	$\leq 25\%$	$\leq 20\%$	$\leq 15\%$	$\leq 10\%$	$\leq 25\%$	$\leq 25\%$	range
Temperature > 29 °C (yes=1)	-0.004	-0.154***	-0.178***	-0.178***	-0.216***	-0.202***	-0.206***	-0.204***	-0.157***
	(0.042)	(0.037)	(0.041)	(0.046)	(0.055)	(0.068)	(0.052)	(0.053)	(0.044)
Temp. $> 29 \times$ win. prob. diff.									0.002***
									(0.001)
PM2.5 > 100 μg/m^3 (yes=1)	0.031	-0.101*	-0.134**	-0.121	-0.101	-0.036	-0.178**		-0.103*
	(0.074)	(0.059)	(0.065)	(0.076)	(0.085)	(0.104)	(0.081)		(0.062)
PM2.5 $> 100 \times$ win. prob. diff.									0.002*
									(0.001)
PM2.5 $\in (100, 200]$ μg/m^3 (yes=1)								-0.160*	
								(0.088)	
PM2.5 > 200 μg/m^3 (yes=1)								-0.226*	
								(0.134)	
Winning probability difference (%)									-0.004***
									(0.000)
Additional controls	No	No	No	No	No	No	Yes	Yes	Yes
Observations	566	700	591	497	312	194	583	583	2130
R-squared	0.000	0.019	0.025	0.024	0.034	0.029	0.113	0.114	0.054
Number of regressors	2	2	2	2	2	2	52	53	58
Mean of dependent variable	0.20	0.38	0.41	0.41	0.41	0.40	0.41	0.41	0.31

Notes: An observation is a completed match in a singles WTA tournament series held in Australia or China. The dependent variable is 1 if the match lasts three battles and 0 if the match lasts two battles. See Table 1 for sample venues and years. Column 1 restricts the estimation sample to matches for which the two opponents' absolute difference in winning probability according to pre-match betting odds is at least 65%. Columns 2 to 6 restrict the sample to matches for which this ex-ante difference does not exceed 30%, 25%, 20%, 15%, or 10%, respectively. Columns 7 and 8 repeat the estimation sample of column 3 (winning probability difference does not exceed 25%) with additional controls for series by round (indicators for each of the seven venues interacted with indicators for rounds 64, 32, 16, 8, 4, 2, and 1=series final), year, time-of-day (indicators for matches starting before 9 am, 9 am to 12 pm, 12 to 3 pm, 3 to 6 pm, 6 to 9 pm, and after 9 pm, local time), humidity, wind speed, and rain. Column 9 considers the entire range of opponent asymmetry in the sample, but interacts environmental factors with (and controls for) the winning probability difference. OLS estimates. Standard errors, clustered on day (effectively, day by venue), are in parentheses. ***Significantly different than zero at 1%, **at 5%, *at 10%.

Table 3: More evidence of state dependence and little evidence of heterogeneous sensitivity: Battle-1 loser's shift in performance in battle 2 (columns 1 to 4) & Battle-1 competitiveness (columns 5 to 8)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	Battle-1 loser's tennis games won in battle 2 relative to battle 1				Battle-1 length in tennis games			
Win. prob. diff. (from betting):	$\leq 30\%$	$\leq 25\%$	$\leq 25\%$	$\leq 25\%$	$\leq 30\%$	$\leq 25\%$	$\leq 25\%$	$\leq 25\%$
Temperature > 29 °C (yes=1)	-0.436**	-0.533**	-0.504*	-0.477	0.056	0.051	0.091	0.071
	(0.222)	(0.235)	(0.291)	(0.293)	(0.163)	(0.184)	(0.255)	(0.257)
PM2.5 > 100 μg/m^3 (yes=1)	-0.509*	-0.667**	-0.509		-0.050	0.100	-0.301	
	(0.298)	(0.328)	(0.407)		(0.245)	(0.262)	(0.306)	
PM2.5 $\in (100, 200]$ μg/m^3 (yes=1)				-0.284				-0.466
				(0.425)				(0.335)
PM2.5 > 200 μg/m^3 (yes=1)				-1.141*				0.164
				(0.620)				(0.435)
Additional controls	No	No	Yes	Yes	No	No	Yes	Yes
Observations	700	591	583	583	700	591	583	583
R-squared	0.007	0.011	0.116	0.118	0.000	0.000	0.093	0.095
Number of regressors	2	2	51	52	2	2	51	52
Mean of dependent variable	0.91	0.93	0.91	0.91	9.46	9.51	9.53	9.49

Notes: An observation is a completed match in a singles WTA tournament series held in Australia or China. In columns 1 to 4, the dependent variable is the battle-1 loser's change in tennis games won in the second battle relative to the first battle, e.g., -1 if the battle-1 loser wins 4 games in battle 1 and 3 games in battle 2. In columns 5 to 8, the dependent variable is the length of the first battle in tennis games, e.g., 10 if the battle-1 score is 6-4. The estimation sample is restricted to matches for which the two opponents' absolute difference in winning probability according to pre-match betting odds does not exceed 25% (30% for columns 1 and 5). See Table 2 for a description of additional controls in columns 3, 4, 7, and 8. OLS estimates. Standard errors, clustered on day, are in parentheses. ***Significant at 1%, **at 5%, *at 10%.

Table 4: Environmental quality does not significantly change the predictability of match outcomes (upsets)

Dependent variable:	Betting market's favorite player wins the match (yes=1)				Betting market's favorite player wins battle 1 (yes=1)				Win. prob. of match winner
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Temperature > 29 °C (yes=1)	0.039	0.062*	0.062*	0.069	0.013	0.037	0.040	0.024	0.004
	(0.028)	(0.034)	(0.034)	(0.057)	(0.028)	(0.038)	(0.038)	(0.061)	(0.012)
Temp. > 29 × win. prob. diff.				-0.000				0.000	
				(0.001)				(0.001)	
PM2.5 > 100 μg/m^3 (yes=1)	0.043	0.031		-0.009	0.022	-0.020		-0.041	-0.011
	(0.034)	(0.039)		(0.080)	(0.039)	(0.047)		(0.077)	(0.145)
PM2.5 > 100 × win. prob. diff.				0.001				0.001	
				(0.001)				(0.001)	
PM2.5 $\in (100, 200]$ μg/m^3 (yes=1)			0.032				0.014		
			(0.050)				(0.055)		
PM2.5 > 200 μg/m^3 (yes=1)			0.030				-0.086		
			(0.048)				(0.059)		
Winning probability diff. (%)	0.005***	0.005***	0.005***	0.005***	0.005***	0.005***	0.005***	0.005***	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
Additional controls	No	Yes	Yes	Yes	No	Yes	Yes	Yes	No
Observations	2113	2086	2086	2086	2113	2086	2086	2086	2157
R-squared	0.086	0.105	0.105	0.105	0.066	0.092	0.093	0.092	0.003
Number of regressors	3	56	57	58	3	56	57	58	2
Mean of dependent variable	0.73	0.73	0.73	0.73	0.69	0.69	0.69	0.69	0.67

Notes: An observation is a completed match in a singles WTA tournament series held in Australia or China over the entire range of opponent asymmetry. Columns 1 to 8 exclude matches for which prediction markets deem both opponents as having equal odds of winning, and thus there is no favorite player and the dependent variable is undefined. In columns 1 to 4, the dependent variable is 1 if the betting market's favorite player wins the match and 0 if there is a match upset. In columns 5 to 8, the dependent variable is 1 if the betting market's favorite player wins the first battle and 0 if there is a battle-1 upset. In column 9, the dependent variable is the (ex-ante) winning probability of the match winner (expressed as a proportion). See Table 2 for a description of additional controls in columns 2 to 4 and 6 to 8. OLS estimates. Standard errors, clustered on day, are in parentheses. ***Significant at 1%, **at 5%, *at 10%.

Table 5: Contest model estimates: Continuous measures of heat and pollution

Alternative measure of player strength:	Pre-match betting (1)	WTA rank (2)	WTA z-score (3)
k, randomness in winner selection	0.668	0.720	0.734
	(0.026)	(0.024)	(0.025)
θ, ability to marginal cost mapping	0.869	0.485	0.189
	(0.063)	(0.035)	(0.015)
Environmental disutility parameters:			
δ_T, coefficient on $\ln(1 + \max(T_n - \underline{T}, 0))$	0.123	0.136	0.149
	(0.026)	(0.025)	(0.032)
δ_P, coefficient on $\ln(1 + \max((P_n - \underline{P})/100, 0))$	1.136	1.402	1.336
	(0.070)	(0.068)	(0.072)
Number of parameters	4	4	4
Observations	2157	2670	2670
Median contest prize V (\$ $\times$ 1000)	18.300	15.092	15.092
Log likelihood	-2897	-3700	-3735

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament series held in Australia or China over the entire range of opponent asymmetry. A match's likelihood contribution is defined using all possible transitions. Marginal cost is alternatively modeled as $c_{in} = (odds_{in})^{\theta}$ in column 1, $c_{in} = (rank_{in})^{\theta}$ in column 2, and $c_{in} = e^{-\theta(zscore_{in})}$ in column 3 (and missing odds prior to 2008 explain column 1's smaller sample). The intercept to per-battle disutility δ_0 is set to \$0, and temperature and PM2.5 pollution cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. The contest prize V_{in} is proxied by the match's prize money; the table reports the median in the given estimation sample. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for $k = 1$ and $\theta = \delta_T = \delta_P = 0$. Estimates are robust to specifying Matlab's unconstrained fminsearch solver and other initial values. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

Table 6: Contest model estimates: Robustness checks

Robustness check:	Random. parameter by battle (1)	MC parameter by battle (2)	Estimate, not fix, δ_0 (3)	Estimate, not fix, $\underline{T}$ & $\underline{P}$ (4)	Random. shift w/ environ. (5)
Battle-1 randomness k_1, battle-invariant k as in main model, or random. intercept k_0	0.674	0.669	0.655	0.667	0.662
	(0.053)	(0.030)	(0.024)	(0.027)	(0.029)
Battle-2 randomness k_2	0.668				
	(0.055)				
Battle-3 randomness k_3	0.666				
	(0.053)				
Randomness coefficient k_T on heat covariate $\ln(1+\max(T_n-\underline{T},0))$					0.006
					(0.018)
Randomness coeff. k_P on pollution covariate $\ln(1+\max((P_n-\underline{P})/100,0))$					0.091
					(0.059)
Battle-1 ability-to-marginal-cost θ_1, or battle-invariant θ as in main model	0.865	0.874	0.888	0.871	0.871
	(0.069)	(0.106)	(0.055)	(0.066)	(0.067)
Battle-2 ability-to-marginal-cost θ_2		0.854			
		(0.111)			
Battle-3 ability-to-marginal-cost θ_3		0.875			
		(0.084)			
Environmental disutility parameters:					
δ_T, coefficient on the heat covariate $\ln(1+\max(T_n-\underline{T},0))$	0.125	0.122	0.090	0.112	0.121
	(0.030)	(0.029)	(0.035)	(0.021)	(0.030)
δ_P, coefficient on the pollution covariate $\ln(1+\max((P_n-\underline{P})/100,0))$	1.140	1.133	1.090	1.901	1.024
	(0.072)	(0.071)	(0.069)	(1.855)	(0.092)
δ_0, per-battle disutility intercept, or set to 0			0.056		
			(0.043)		
$\underline{T}$, temperature cutoff (°C), or set to 27				26.110	
				(0.147)	
$\underline{P}$, pollution cutoff (μg/m^3), or set to 150				220.000	
				(18.255)	
Number of parameters	6	6	5	6	6
Observations	2157	2157	2157	2157	2157
Log likelihood	-2897	-2897	-2897	-2897	-2897

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament series held in Australia or China. A match's likelihood contribution is defined using all possible transitions. Marginal cost is modeled as $c_{in} = (odds_{in})^{\theta}$. Column 1 allows the randomness parameter to vary by battle $b \in \{1, 2, 3\}$. Column 2 allows the mapping from player ability to marginal cost to vary by battle $b \in \{1, 2, 3\}$. The intercept to per-battle disutility δ_0 is set to \$0, except in column 3 where this intercept is estimated. Except in column 4, the poor environment cutoffs are set to 27 °C and 150 μg/m^3; column 4 estimates these cutoffs imposing the constraints $25 \leq \underline{T} \leq 35$ °C and $100 \leq \underline{P} \leq 300$ μg/m^3. Column 5 allows the winner-selection technology to shift with heat and pollution, specifying randomness as $k_n = k_0 + k_T \ln(1+\max(T_n-\underline{T},0)) + k_P \ln(1+\max((P_n-\underline{P})/100,0))$. The contest prize V_{in} is proxied by the match's prize money. Solver Knitro using the interior-point algorithm, with initial values for all parameters set to 0 except for $k = 1$ ($k_b = 1$ in column 1 and intercept $k_0 = 1$ in column 5) and for $\underline{T} = 25$ °C and $\underline{P} = 100$ μg/m^3 in column 4. Randomness parameters (including k_n $\forall n$ in column 5) are constrained to lie between 0 and 1. Marginal cost parameters are constrained to be positive. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

Table 7: Contest model estimates: Age heterogeneity (player age in the upper quartile)

Alternative measure of player strength:	Pre-match betting	WTA rank	WTA z-score
	(1)	(2)	(3)
k, randomness in winner selection	0.665	0.717	0.730
	(0.022)	(0.023)	(0.025)
θ, ability to marginal cost mapping	0.873	0.488	0.191
	(0.053)	(0.034)	(0.015)
Environmental disutility parameters (w/ heterog.):			
δ_T, coefficient on the heat covariate	0.118	0.133	0.143
$\ln(1 + \max(T_n - \underline{T}, 0))$	(0.025)	(0.025)	(0.030)
δ_T^S, coeff on: 1(age > 27.2 y) × heat covariate	0.053	0.066	0.081
(add to δ_T, for senior players only)	(0.038)	(0.039)	(0.045)
δ_P, coefficient on the pollution covariate	1.139	1.405	1.379
$\ln(1 + \max((P_n - \underline{P})/100, 0))$	(0.070)	(0.069)	(0.797)
δ_P^S, coeff on: 1(age > 27.2 y) × pollution covariate	0.264	0.455	0.110
(add to δ_P, for senior players only)	(1.402)	(2.253)	(2.687)
Number of parameters	6	6	6
Log likelihood	-2897	-3699	-3735
	(4)	(5)	(6)
k, randomness in winner selection	0.665	0.717	0.730
	(0.023)	(0.022)	(0.024)
θ, ability to marginal cost mapping	0.874	0.491	0.192
	(0.054)	(0.032)	(0.015)
Contest prize: Heterogeneity factor:			
ρ, parameter on the contest prize if 1(age > 27.2 y)	0.989	0.951	0.971
(otherwise equal to 1)	(0.034)	(0.034)	(0.034)
Environmental disutility parameters (w/ heterog.):			
δ_T, coefficient on the heat covariate	0.120	0.139	0.145
$\ln(1 + \max(T_n - \underline{T}, 0))$	(0.026)	(0.021)	(0.028)
δ_T^S, coeff on: 1(age > 27.2 y) × heat covariate	0.049	0.044	0.069
(add to δ_T, for senior players only)	(0.039)	(0.033)	(0.041)
δ_P, coefficient on the pollution covariate	1.146	1.444	1.362
$\ln(1 + \max((P_n - \underline{P})/100, 0))$	(0.070)	(0.072)	(0.071)
δ_P^S, coeff on: 1(age > 27.2 y) × pollution covariate	0.208	0.245	-0.026
(add to δ_P, for senior players only)	(1.221)	(2.375)	(2.557)
Number of parameters	7	7	7
Log likelihood	-2897	-3699	-3734
Observations	2157	2670	2670

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament series held in Australia or China. Compared with the specification for environmental disutility in Table 5, i.e., (10), here we add an indicator for a player's age greater than 27.2 years (the 75th percentile in the player-by-match distribution of age) interacted with each of the two environmental covariates. The intercept to per-battle disutility δ_0 is set to \$0, and temperature and PM2.5 pollution cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. In the top panel, the contest prize V_{in} is proxied by the match's prize money. In the bottom panel, V_{in} is the match's prize money scaled by a factor ρ if a player is aged over 27.2 years, and the prize money otherwise. A match's likelihood contribution is defined using all possible transitions. Marginal cost is alternatively modeled as $c_{in} = (odds_{in})^\theta$ in columns 1 & 4, $c_{in} = (rank_{in})^\theta$ in columns 2 & 5, and $c_{in} = e^{-\theta(zscore_{in})}$ in columns 3 & 6. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$, $\theta > 0$, and $0 < \rho < 2$, and initial values for $k = \rho = 1$ and $\theta = \delta_T = \delta_P^S = \delta_P = \delta_P^S = 0$. (Superscript S denotes a "senior" player.) Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

Table 8: Contest model estimates: Sample composition

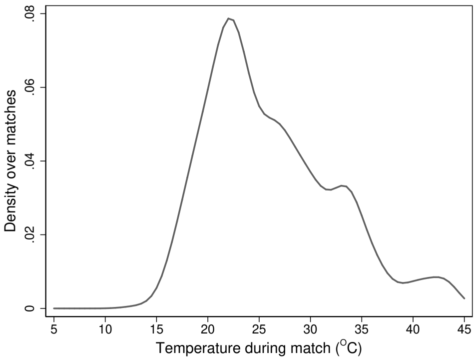
Estimation sample:	Only battle-2 transitions in match (1)	Drop quarterfinal to final matches (2)	Drop ≥ 75 percentile prize matches (3)	Only matches in China (4)	Restrict asymmetry to win. prob. diff. $\leq 30\%$ (5)
k, randomness in winner selection	0.669	0.662	0.675	0.686	0.588
	(0.029)	(0.027)	(0.029)	(0.032)	(0.163)
θ, ability to marginal cost mapping	0.854	0.874	0.830	0.836	0.966
	(0.077)	(0.065)	(0.067)	(0.075)	(0.334)
Environmental disutility parameters:					
δ_T, coeff. on $\ln(1+\max(T_n-\underline{T},0))$	0.123	0.125	0.125	0.122	0.294
	(0.029)	(0.026)	(0.027)	(0.025)	(0.099)
δ_P, coeff. on $\ln(1+\max((P_n-\underline{P})/100,0))$	2.049	1.142	1.137	1.120	1.490
	(0.187)	(0.069)	(0.071)	(0.074)	(1.427)
Number of parameters	4	4	4	4	4
Observations	2157	1912	1578	1048	700
Median contest prize V (\$ $\times$ 1000)	18.300	14.066	11.106	9.083	12.917
Log likelihood	-1204	-2556	-2128	-1446	-1121

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament held in Australia or China. Marginal cost is modeled as $c_{in} = (odds_{in})^{\theta}$. Compared with the estimation sample in Table 5(1): Column 1 defines a match's likelihood contribution using only battle-2 transitions. Column 2 drops round 4 (quarterfinal) to round 1 (series final) matches from the sample. Column 3 drops matches with prize money higher than the 75th percentile (\$25,896) from the sample. Column 4 drops Australian Open matches, with the estimation sample comprising only Chinese venues. Column 5 restricts the sample to matches in which the two opponents' absolute difference in winning probability is at most 30%. The intercept to per-battle disutility δ_0 is set to \$0, and the poor environment cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. The contest prize V_{in} is proxied by the match's prize money; the table reports the median in the given estimation sample. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for $k = 1$ and $\theta = \delta_T = \delta_P = 0$. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

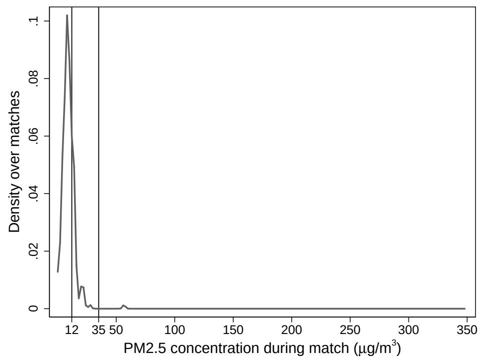
Table 9: Contest model estimates: Augmenting V_{in} with continuation value from subsequent matches in a series

	(1)
k, randomness in winner selection	0.667
	(0.027)
θ, ability to marginal cost mapping	0.870
	(0.065)
Environmental disutility parameters:	
δ_T, coefficient on $\ln(1 + \max(T_n - \underline{T}, 0))$	0.256
	(0.058)
δ_P, coefficient on $\ln(1 + \max((P_n - \underline{P})/100, 0))$	3.490
	(0.102)
Number of parameters	4
Observations	2157
Median contest prize V prize money & some continuation value (\$ $\times$ 1000)	32.857
Log likelihood	-2898

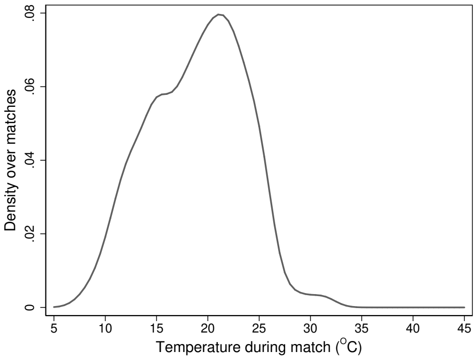
Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament series held in Australia or China. A match's likelihood contribution is defined using all possible transitions. Marginal cost is modeled as $c_{in} = (odds_{in})^{\theta}$. The intercept to per-battle disutility δ_0 is set to \$0, and temperature and PM2.5 pollution cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. The contest prize V_{in} is proxied by the match's prize money augmented with continuation value from remaining in the series and competing in at least one subsequent match, which the contest winner is entitled to (the loser is eliminated from the series). See the text on players' expectations of future conditions of play and the recursive procedure that yields continuation value at each round in a series for every vector of parameters Ψ. For every iteration on Ψ, we compute the associated augmented contest prize vector V and re-estimate the contest model. We iterate until Ψ converges: adopting the sup norm with a tolerance of .0001, this happens after 108 iterations, with Ψ changing little from the second iteration on after adding continuation value to prize money in V. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for $k = 1$ and $\theta = \delta_T = \delta_P = 0$ (at each iteration of estimation). Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.



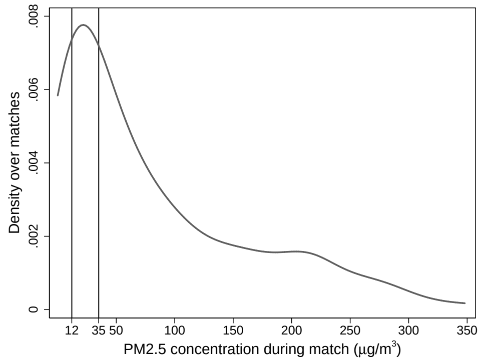
(a) Temperature, Melbourne matches



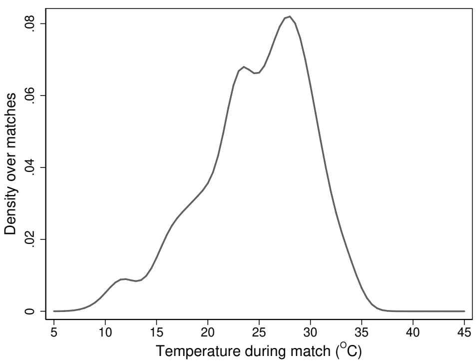
(b) PM2.5, Melbourne matches



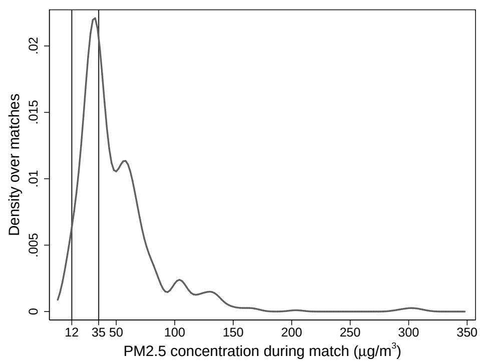
(c) Temperature, Beijing matches



(d) PM2.5, Beijing matches

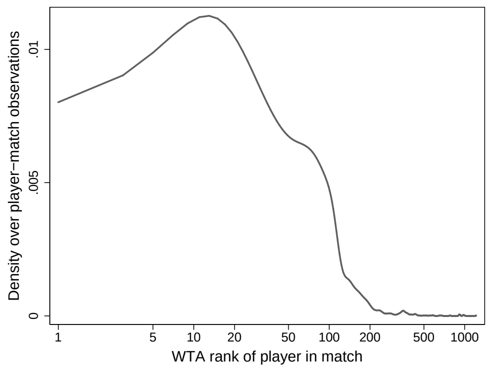


(e) Temperature, other Chinese matches

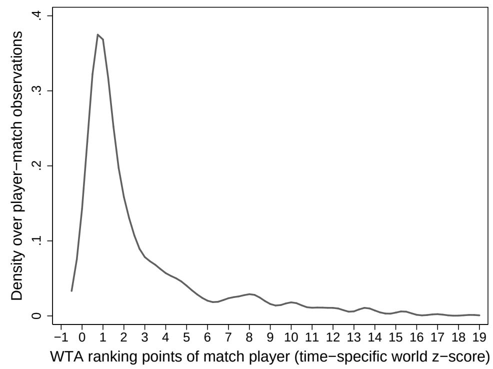


(f) PM2.5, other Chinese matches

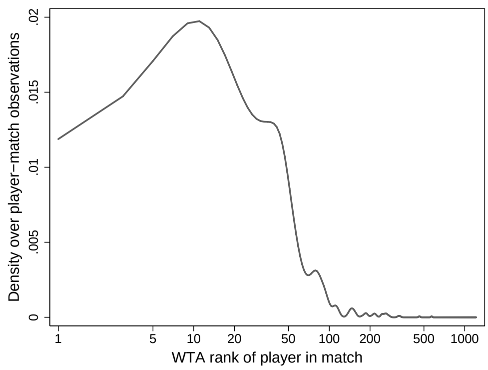
Figure 1: [**Data**] Distribution of ambient temperature and PM2.5 over matches: Australian Open (Melbourne, summer 2004-2016), China Open (Beijing, fall 2008-2016), and other WTA series in China (Guangzhou, Hong Kong, Shenzhen, Tianjin, Wuhan, 2011-2016). An observation is a match. We take mean environmental conditions, recorded at sites close to the venues, over the three 1-hour periods that encompass the hour in which the match started and the two subsequent hours, e.g., 2:00 pm to 4:59 pm for a match that started at 2:23 pm. In the right panels, the vertical lines mark the US annual and 24-hour primary PM2.5 standards; for readability, we do not show PM2.5 up to the sample maximum of 520 μg/m^3 (Table 1). Sources: Victoria EPA, US Department of State, Hong Kong EPD, Chinese Ministry of EP, NASA.



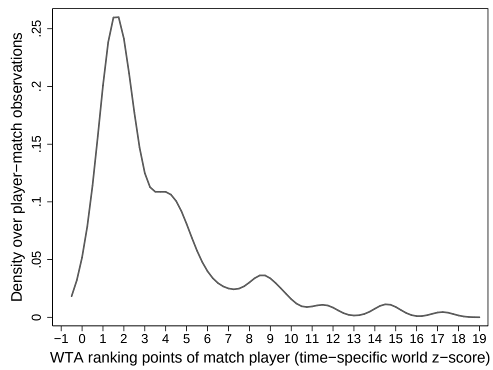
(a) WTA rank, Melbourne



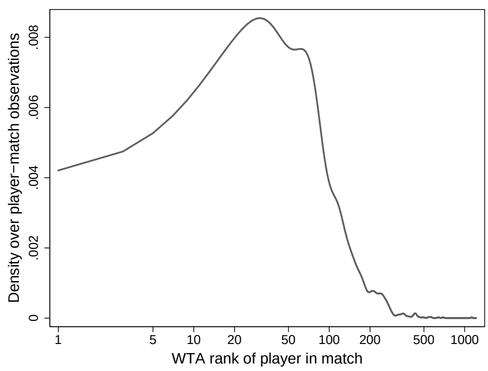
(b) WTA z-score, Melbourne



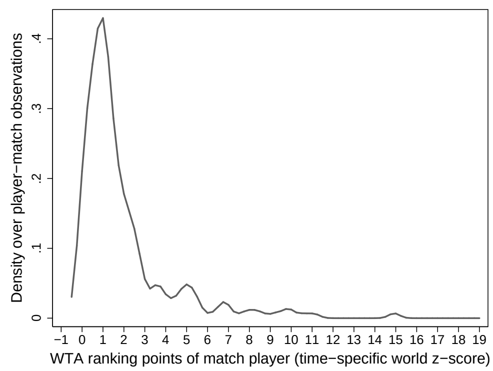
(c) WTA rank, Beijing



(d) WTA z-score, Beijing

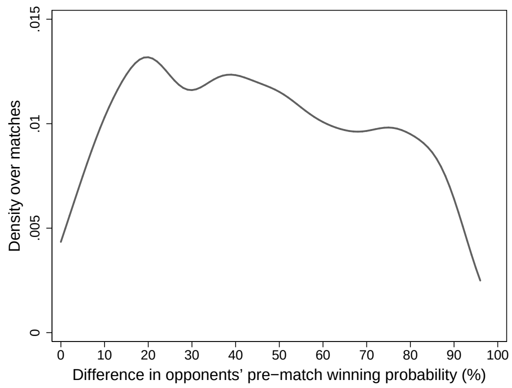


(e) WTA rank, other China

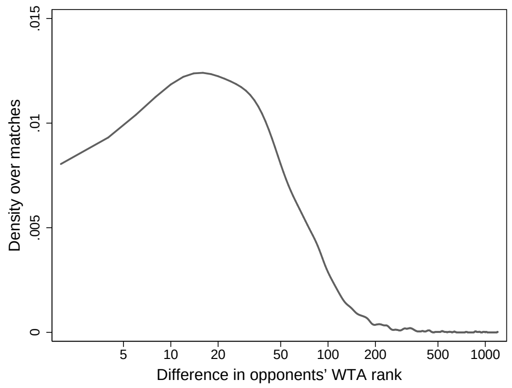


(f) WTA z-score, other China

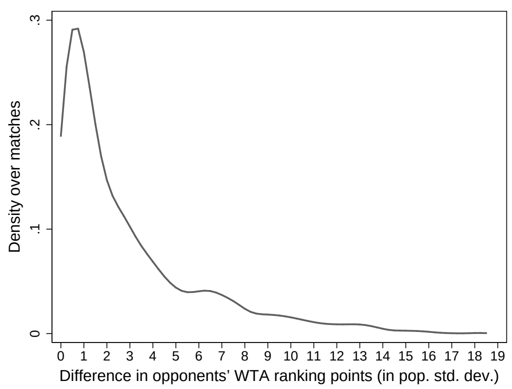
Figure 2: [**Data**] Distribution of WTA rank (log scale) and WTA z-score over player by match observations: Australian Open (Melbourne, summer 2004-2016), China Open (Beijing, fall 2008-2016), other WTA series in China (Guangzhou, Hong Kong, Shenzhen, Tianjin, Wuhan, 2011-2016). The top-ranked player worldwide has rank 1. Due to shifts in the worldwide distribution of points across players over time, for each player by match in the sample we compute a time-adjusted world z-score: From her observed points we subtract the mean points over all worldwide WTA players at the time the match was played, and divide by the standard deviation of points over all worldwide WTA players at the time. Most players are well above world average. Source: flashscore.com, tennis-data.co.uk.



(a) Winning probability difference (%)

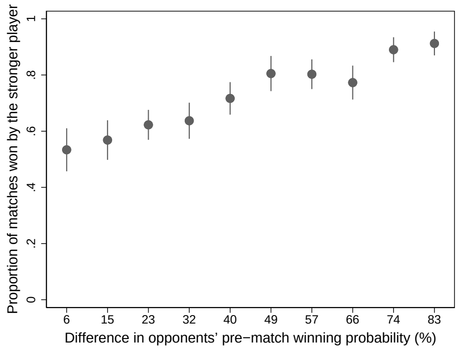


(b) WTA rank difference

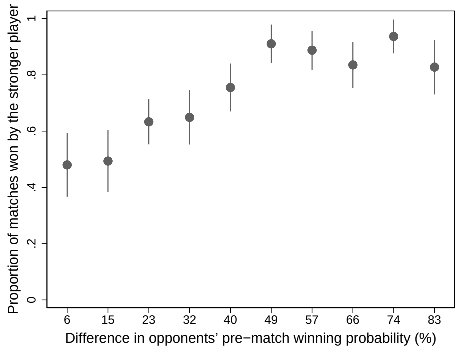


(c) WTA z-score difference

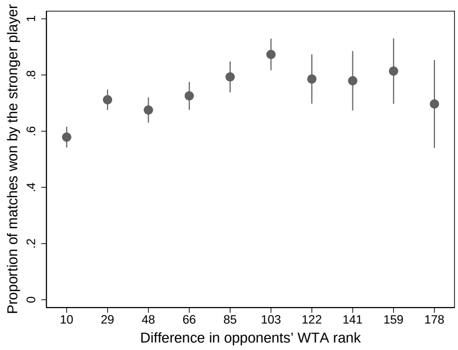
Figure 3: [**Data**] Distribution of paired opponents' relative strength over matches in our sample: Absolute difference in the two players' (a) winning probabilities implied from pre-match betting odds (%), (b) WTA rank (log scale), and (c) WTA z-scores. An observation is a match, comprising two opponents. Source: flashscore.com, tennis-data.co.uk.



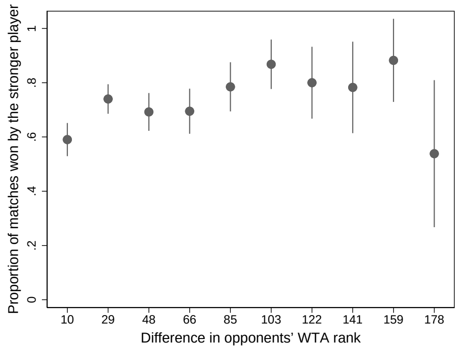
(a) Winning probability difference (%)



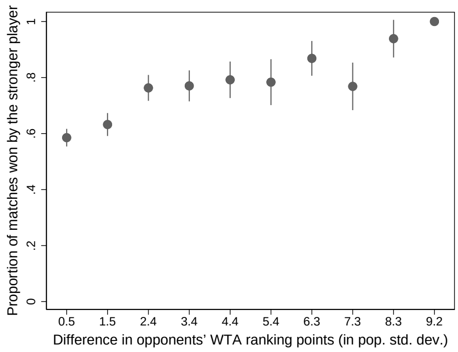
(b) Restrict to > 27 °C or > 100 μg/m^3



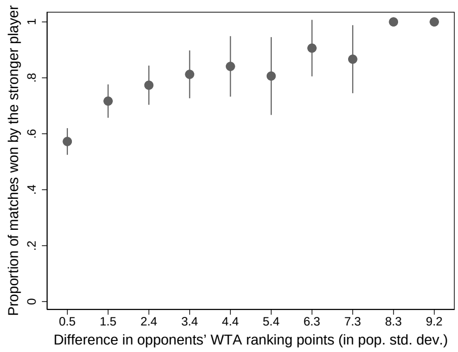
(c) WTA rank difference



(d) Restrict to > 27 °C or > 100 μg/m^3

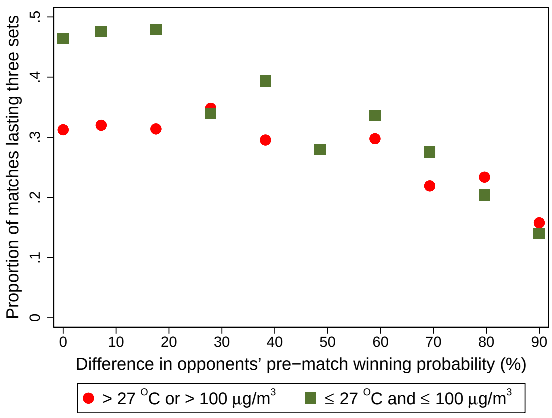


(e) WTA z-score difference

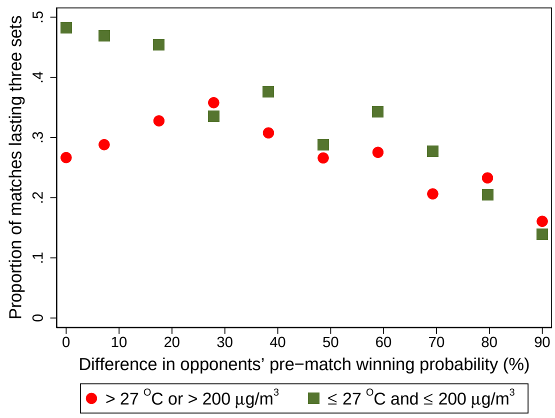


(f) Restrict to > 27 °C or > 100 μg/m^3

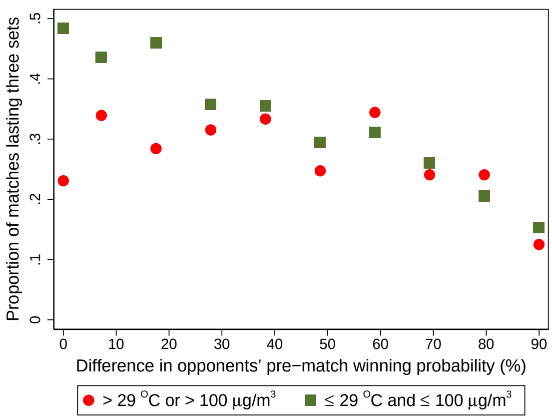
Figure 4: [**Data**] Proportion of matches won by the stronger player against the degree of opponent asymmetry, as measured alternatively by the absolute difference in the two players' pre-match winning probabilities, ranks, or z-scores. For each measure, we group matches in 10 bins of equal width, with the first bin starting at the minimum difference and the last bin ending at the 95th percentile of the distribution of the respective difference over asymmetric matches (otherwise the stronger player is undefined). Bins are labeled at the midpoint difference. The right panels consider only matches played in temperatures above 27 °C or PM2.5 above 100 μg/m^3, i.e., about two-fifths of the matches considered in the left panels.



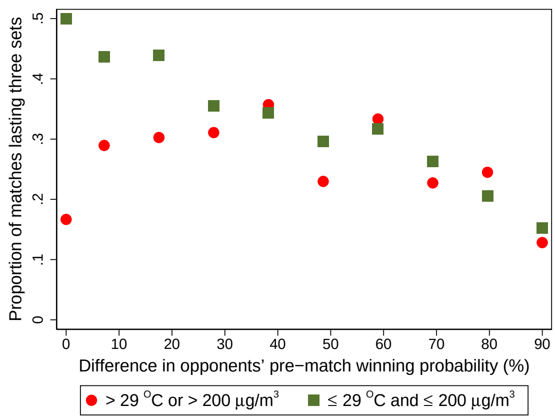
(a) Cutoffs 27 °C or 100 μg/m^3



(b) Cutoffs 27 °C or 200 μg/m^3



(c) Cutoffs 29 °C or 100 μg/m^3



(d) Cutoffs 29 °C or 200 μg/m^3

Figure 5: [**Data**] Environmental quality and momentum. Among fairly symmetric players, the proportion of matches lasting three sets (rather than two) drops sharply when either temperature or PM2.5 exceeds its respective cutoff, marked by the red circles, compared to when both conditions are milder, marked by the green squares. Across the panels, we vary the cutoffs: (a) 27 °C or 100 μg/m^3, (b) 27 °C or 200 μg/m^3, (c) 29 °C or 100 μg/m^3, and (d) 29 °C or 200 μg/m^3. We group matches in 10 bins, the first bin for matches with equal odds, and nine other bins of same width labeled at the midpoint difference, up to the maximum winning probability difference in the sample.

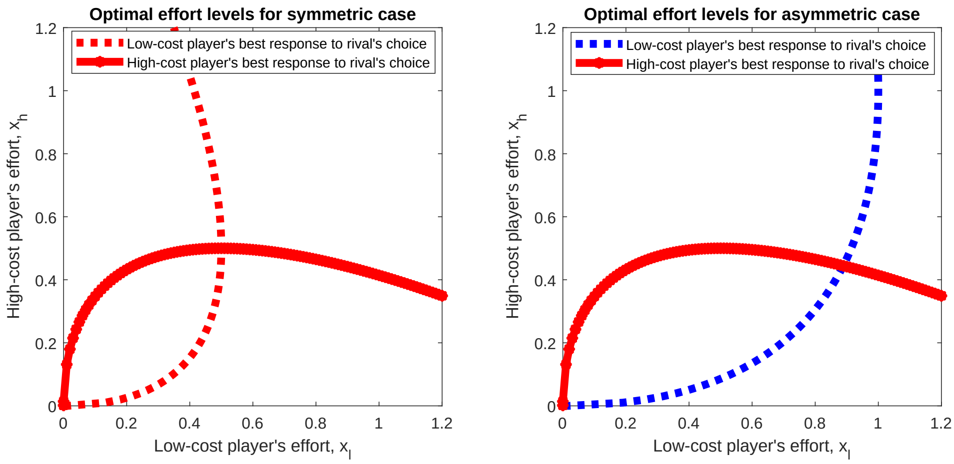


Figure 6: [**Model**] Best-response functions in a single battle: Two examples. Left panel: The symmetric player case, $c_h = c_l$ (homogeneous marginal cost) and $\Delta V_h = \Delta V_l = V$ (symmetric win-loss prize spread). Right panel: The asymmetric player case, $c_h = 2c_l$ (best-response functions are equivalent if we set $\Delta V_h = 0.5\Delta V_l$ instead). We illustrate with technology $k = 1$ (low randomness) and normalize $\lambda c_h = 0.5V$.

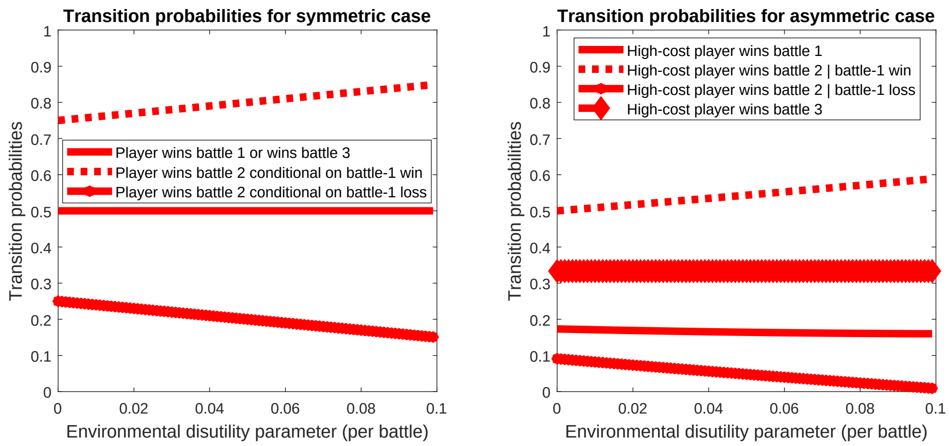
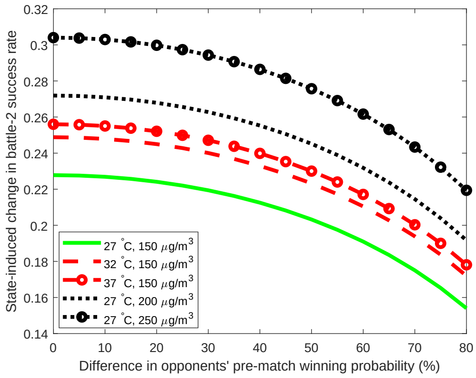
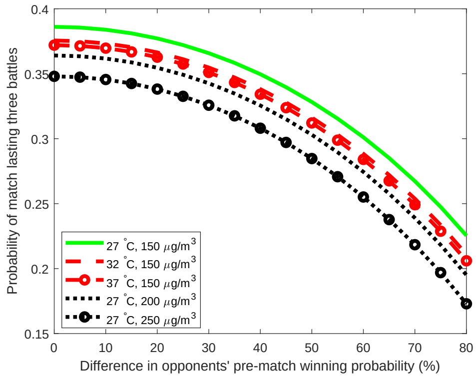


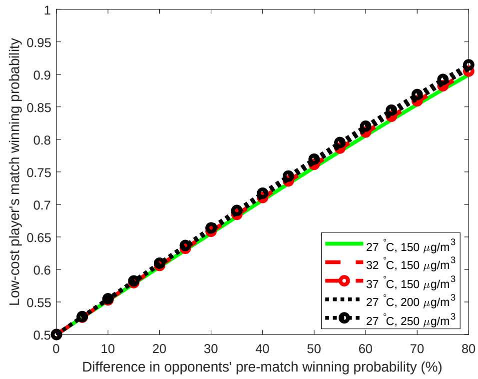
Figure 7: [**Model**] Battle transition probabilities, as a function of the per-battle environmental disutility parameter δ. In the model, a higher δ captures a more adverse environment, i.e., a stronger dose of heat or pollution. Left panel: Players are symmetric, $c_h = c_l$. Right panel: Players are asymmetric, $c_h = 2c_l$. We illustrate with technology $k = 1$ (low randomness), symmetric contest prizes $V_h = V_l = V = 1$, and $\lambda c_h = 0.5V$.



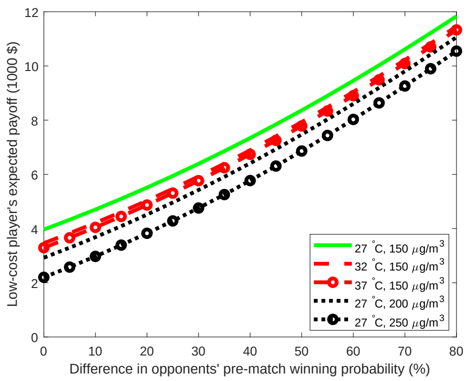
(a) Stronger player's battle-2 success



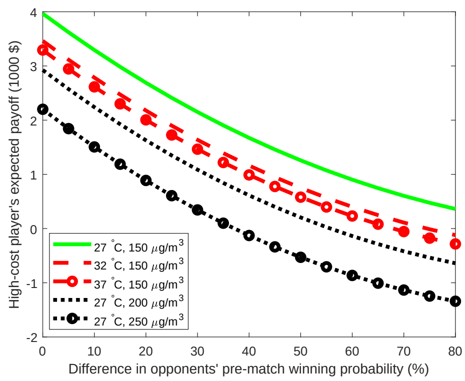
(b) Probability of match lasting 3 battles



(c) Match outcome predictability

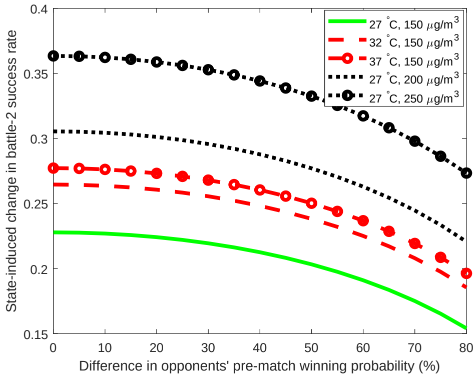


(d) Stronger player's expected payoff

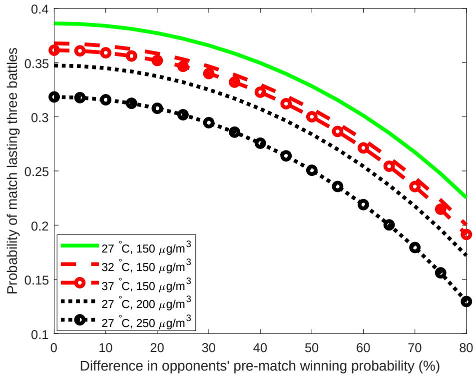


(e) Weaker player's expected payoff

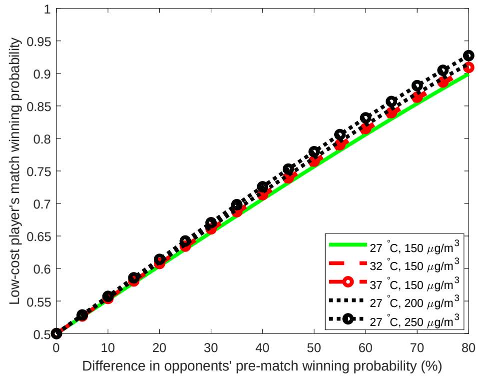
Figure 8: [**Model estimates: Odds-based ability measure, median prize**] Model predictions against the degree of opponent asymmetry evaluated at the median cash prize in the sample (\$15,092; $N = 2811$ matches). Model according to Table 5, column 1, with player strength based on betting odds, $c_{in} = (odds_{in})^{\theta}$. Footnote 31 defines the state-induced change in battle-2 success rate shown in panel (a).



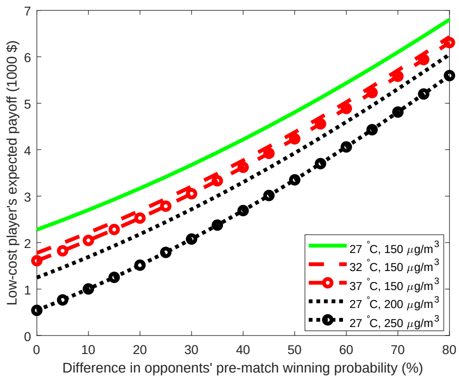
(a) Stronger player's battle-2 success



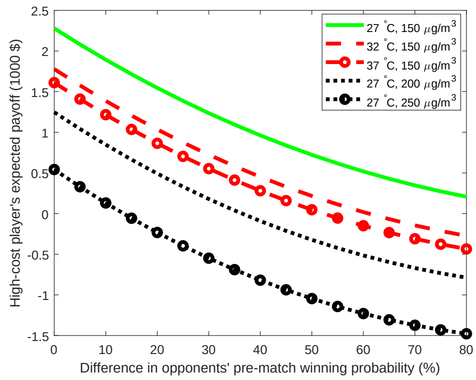
(b) Probability of match lasting 3 battles



(c) Match outcome predictability



(d) Stronger player's expected payoff



(e) Weaker player's expected payoff

Figure 9: [**Model estimates: Odds-based ability measure, 25th-percentile prize**] Model predictions against the degree of opponent asymmetry evaluated at the 25th-percentile cash prize in the sample (\$8672; $N = 2811$ matches). Model according to Table 5, column 1, with player strength based on betting odds, $c_{in} = (odds_{in})^{\theta}$.

A Dynamic contest model: Theory and estimation

A.1 Optimal player effort and transition probabilities

We proceed by backward induction. In battle 3, the winner takes the contest prize and the loser earns 0, so optimal bidding strategies are given by (2) with $\Delta V_{3i} = V_i - 0 = V_i$ and $\Delta V_{3j} = V_j$:

$$x_{3i} = x_i(V_i, V_j) = \frac{V_i}{\lambda c_i} \frac{k(\frac{c_i V_j}{c_j V_i})^k}{[1 + (\frac{c_i V_j}{c_j V_i})^k]^2}, \quad i, j \in \{l, h\}, i \neq j.$$

In battle 2, given a history $\chi_{1l} = 1$, players l and h choose effort levels:

$$\begin{aligned} x_{2l|\chi_{1l}=1} &= x_l(\overline{V}_{2l|\chi_{1l}=1} - \underline{V}_{2l|\chi_{1l}=1}, \overline{V}_{2h|\chi_{1l}=1} - \underline{V}_{2h|\chi_{1l}=1}), \\ x_{2h|\chi_{1l}=1} &= x_h(\overline{V}_{2h|\chi_{1l}=1} - \underline{V}_{2h|\chi_{1l}=1}, \overline{V}_{2l|\chi_{1l}=1} - \underline{V}_{2l|\chi_{1l}=1}), \end{aligned}$$

where win-loss spreads $\Delta V_{2i|\chi_{1l}=1} = \overline{V}_{2i|\chi_{1l}=1} - \underline{V}_{2i|\chi_{1l}=1}$ are calculated from (4) and (5), i.e., we plug these in (2) to obtain reduced-form effort choices. Transition probabilities follow from (1):

$$\begin{aligned} \Pr(\chi_{2l} = 1 \,|\chi_{1l} = 1) &= \frac{(x_{2l|\chi_{1l}=1})^k}{(x_{2l|\chi_{1l}=1})^k + (x_{2h|\chi_{1l}=1})^k}, \\ \Pr(\chi_{2l} = 0 \,|\chi_{1l} = 1) &= 1 - \Pr(\chi_{2l} = 1 \,|\chi_{1l} = 1). \end{aligned}$$

For simplicity, we consider the situation in which players exert positive effort in battle 2. The expressions can easily be extended to allow for the corner solution, i.e., when δ is too high and the battle-1 loser's continuation value from winning battle 2 is negative; thus she chooses to lose battle 2 by exerting 0 effort and the contest ends after two battles. (Both players still incur a battle-2 environmental disutility δ, as they are in court despite the minimal exertion.) Specifically, following player h's defeat in battle 1, a corner solution in battle 2 obtains when the continuation value from continuing play in a third battle is negative. That is, the expected benefit of effort in battle 3 would be less than the sum of effort cost and environmental disutility:

$$V_h \frac{1}{1 + (\frac{c_h V_l}{c_l V_h})^k} \leq k V_h \frac{(\frac{c_h V_l}{c_l V_h})^k}{[1 + (\frac{c_h V_l}{c_l V_h})^k]^2} + \delta,$$

or, equivalently:

$$\frac{\delta}{V_h} \geq \frac{1}{1 + (\frac{c_h V_l}{c_l V_h})^k} [1 - k \frac{(\frac{c_h V_l}{c_l V_h})^k}{1 + (\frac{c_h V_l}{c_l V_h})^k}].$$

Given an alternative history $\chi_{1l} = 0$, choices are:

$$
\begin{aligned}
x_{2l|\chi_{1l}=0} &= x_l(\overline{V}_{2l|\chi_{1l}=0} - \underline{V}_{2l|\chi_{1l}=0}, \overline{V}_{2h|\chi_{1l}=0} - \underline{V}_{2h|\chi_{1l}=0}), \\
x_{2h|\chi_{1l}=0} &= x_h(\overline{V}_{2h|\chi_{1l}=0} - \underline{V}_{2h|\chi_{1l}=0}, \overline{V}_{2l|\chi_{1l}=0} - \underline{V}_{2l|\chi_{1l}=0}),
\end{aligned}
$$

with continuation values conditional on battle-2 outcomes (and battle-1 history):

$$
\text{if } \chi_{2l} = 1 \text{ (contest continues 1-1)} \begin{cases} \overline{V}_{2l|\chi_{1l}=0} = p_{3l}V_l - \lambda c_l x_l(V_l, V_h) - \delta, \\ \underline{V}_{2h|\chi_{1l}=0} = p_{3h}V_h - \lambda c_h x_h(V_h, V_l) - \delta, \end{cases} \tag{13}
$$

$$
\text{if } \chi_{2l} = 0 \text{ (contest ends 0-2)} \begin{cases} \underline{V}_{2l|\chi_{1l}=0} = 0, \\ \overline{V}_{2h|\chi_{1l}=0} = V_h. \end{cases} \tag{14}
$$

Optimal player actions then follow from (2), yielding transition probabilities:

$$
\begin{aligned}
\Pr(\chi_{2l} = 1 \,|\chi_{1l} = 0) &= \frac{(x_{2l|\chi_{1l}=0})^k}{(x_{2l|\chi_{1l}=0})^k + (x_{2h|\chi_{1l}=0})^k}, \\
\Pr(\chi_{2l} = 0 \,|\chi_{1l} = 0) &= 1 - \Pr(\chi_{2l} = 1 \,|\chi_{1l} = 0).
\end{aligned}
$$

Again, these expressions can be extended to allow for the battle-2 corner solution in which, having lost battle 1, player l exerts 0 effort to avoid taking the contest to a third battle where her continuation value is negative.

In battle 1, there is no history to condition on. Both opponents' continuation values in the alternative events that player l, respectively, wins and loses battle 1 are given by:

if $\chi_{1l} = 1$ (contest continues 1-0), (15)

$$
\begin{cases}
\overline{V}_{1l} = \Pr(\chi_{2l} = 1|\chi_{1l} = 1)\overline{V}_{2l|\chi_{1l}=1} + \Pr(\chi_{2l} = 0|\chi_{1l} = 1)\underline{V}_{2l|\chi_{1l}=1} - \lambda c_l x_{2l|\chi_{1l}=1} - \delta, \\
\underline{V}_{1h} = \Pr(\chi_{2l} = 1|\chi_{1l} = 1)\underline{V}_{2h|\chi_{1l}=1} + \Pr(\chi_{2l} = 0|\chi_{1l} = 1)\overline{V}_{2h|\chi_{1l}=1} - \lambda c_h x_{2h|\chi_{1l}=1} - \delta,
\end{cases}
$$

if $\chi_{1l} = 0$ (contest continues 0-1), (16)

$$
\begin{cases}
\underline{V}_{1l} = \Pr(\chi_{2l} = 1|\chi_{1l} = 0)\overline{V}_{2l|\chi_{1l}=0} + \Pr(\chi_{2l} = 0|\chi_{1l} = 0)\underline{V}_{2l|\chi_{1l}=0} - \lambda c_l x_{2l|\chi_{1l}=0} - \delta, \\
\overline{V}_{1h} = \Pr(\chi_{2l} = 1|\chi_{1l} = 0)\underline{V}_{2h|\chi_{1l}=0} + \Pr(\chi_{2l} = 0|\chi_{1l} = 0)\overline{V}_{2h|\chi_{1l}=0} - \lambda c_h x_{2h|\chi_{1l}=0} - \delta.
\end{cases}
$$

Finally, plugging differences in continuation values in (2) yields battle-1 actions:

$$
x_{1i} = x_i(\overline{V}_{1i} - \underline{V}_{1i}, \overline{V}_{1j} - \underline{V}_{1j}), \quad i, j \in \{l, h\}, i \neq j.
$$

Transition probabilities are:

$$
\Pr(\chi_{1l} = 1) = \frac{(x_{1l})^k}{(x_{1l})^k + (x_{1h})^k}, \quad \Pr(\chi_{1l} = 0) = 1 - \Pr(\chi_{1l} = 1).
$$

Players' effort choices battle by battle, as a function of environmental degradation δ

and cost asymmetry, are illustrated in Figure A.4. For example, in the top-left panel for symmetric players (with $k = 0.7$), battle-1 effort rises with δ, but not steeply.

A.2 Likelihood contribution

A tennis match's likelihood contribution is computed from the battle-transition probabilities derived above, which are functions of optimal player effort. We list within-match outcome probabilities predicted by the model here:

$$
\begin{aligned}
\Pr(\mathbf{ll}\,|\Psi) &= \Pr(\chi_{1l}=1)\Pr(\chi_{2l}=1\,|\chi_{1l}=1)\\
\Pr(\mathbf{lhl}\,|\Psi) &= \Pr(\chi_{1l}=1)\Pr(\chi_{2l}=0\,|\chi_{1l}=1)\Pr(\chi_{3l}=1\,|\chi_{2l}\neq\chi_{1l})\\
\Pr(\mathbf{lhh}\,|\Psi) &= \Pr(\chi_{1l}=1)\Pr(\chi_{2l}=0\,|\chi_{1l}=1)\Pr(\chi_{3l}=0\,|\chi_{2l}\neq\chi_{1l})\\
\Pr(\mathbf{hll}\,|\Psi) &= \Pr(\chi_{1l}=0)\Pr(\chi_{2l}=1\,|\chi_{1l}=0)\Pr(\chi_{3l}=1\,|\chi_{2l}\neq\chi_{1l})\\
\Pr(\mathbf{hlh}\,|\Psi) &= \Pr(\chi_{1l}=0)\Pr(\chi_{2l}=1\,|\chi_{1l}=0)\Pr(\chi_{3l}=0\,|\chi_{2l}\neq\chi_{1l})\\
\Pr(\mathbf{hh}\,|\Psi) &= \Pr(\chi_{1l}=0)\Pr(\chi_{2l}=0\,|\chi_{1l}=0)
\end{aligned}
$$

As an alternative to criterion function (12), for which a match's likelihood contribution is defined using all possible transitions, the model can be estimated using only battle-2 outcomes, conditional on the battle-1 realization. Given a history $\chi_{1l} \in \{0, 1\}$, there are two possible outcomes. For example, given player l winning battle 1 ($\chi_{1l} = 1$) in match n, battle 2 may be won by either player $\iota_{2,n} \in \{\mathbf{l}, \mathbf{h}\}$ with conditional probabilities $\Pr(\chi_{2l} = 1\,|\chi_{1l} = 1)$ and $\Pr(\chi_{2l} = 0\,|\chi_{1l} = 1)$. The likelihood is

$$\prod_{n=1}^{N}\prod_{\iota_{2,n}\in\{\mathbf{l},\mathbf{h}\}}\left(\Pr(\chi_{2l}\,|\chi_{1l};\Psi)\right)^{1(\iota_{2,n})}, \tag{17}$$

whereby only model predictions for battle-2 transitions are taken to the data.

A.3 Further model variants

We implemented more general versions of the main model—with a match's prize money as a proxy for V_{in}—along two dimensions. We briefly outline these here.

Random coefficients. Instead of fixing the variance of the player-match level cost shocks $\sigma_\epsilon^2 = 0$—see (8)—we specified a mean-zero i.i.d. normal error entering additively into each player's marginal effort cost, $c_{in} = f(a_{in}, \theta) + \epsilon_{in}$. Now σ_ϵ is an additional parameter to be estimated. This allows for a serially correlated shock that is common over battles within a match, which randomness parameter k does not account for. For example, one of the otherwise equally able opponents suffers from a bad cold on match day, which can work to her rival's advantage over the course of the match. Estimation is by simulated likelihood.

Our findings are robust to this model variant. On introducing random coefficients over 1000 simulations (each with $2 \times N$ player-match ϵ_{in} draws), we obtained parameters $(\hat{k}, \hat{\theta}, \hat{\delta}_T, \hat{\delta}_P)$ that on average line up with the point estimates obtained in the main model. During optimization we constrain $\sigma_\epsilon < 0.2$ and verified that marginal cost is necessarily positive $\forall\ i, n$. The constraint turns out not to bind and the mean $\hat{\sigma}_\epsilon$ is 0.017 (compare to Figure A.6 showing estimated marginal cost). This suggests that serially correlated shocks are of low magnitude relative to estimated "base" marginal cost, $f(a_{in}, \theta)$.

Psychological momentum. We extended our model to allow for another form of state dependence studied in psychology and economics (Iso-Ahola and Mobily, 1980; Malueg and Yates, 2010). This is based on an abstract psychological impact on a player's current performance from recently winning a battle—abstract in that it is not formalized through economic incentives.[41] We introduced the psychological channel either (i) additively to marginal cost, by which a player i's marginal cost c_{inb} in battle $b \in \{2, 3\}$ of match n is $c_{in} + \eta$ if she won the preceding battle $b - 1$ and is c_{in} otherwise, or (ii) multiplicatively to marginal cost, by which marginal cost c_{inb} in battle $b \in \{2, 3\}$ of match n is ηc_{in} if player i won battle $b - 1$ and is c_{in} otherwise. During optimization we impose $\eta > -1$ in specification (i) and $\eta > 0$ in specification (ii) to ensure that state-dependent marginal cost is necessarily positive $\forall\ i, n, b$.

In battle 1, there is no immediate history and thus no momentum. Battle 2, in contrast, may have psychology complementing strategic momentum. Identification of psychology follows from battle-3 outcomes in the subset of three-battle matches as the state varies.

Table A.6 shows some evidence of a psychological effect for some specifications. A recent win can reduce a player's marginal cost. Taking WTA rank to measure ability, we obtain $\hat{\eta}$ = -0.31 < 0 in column 2's additive specification, and $\hat{\eta}$ = 0.85 < 1 in column 4's multiplicative specification (i.e., a 15% reduction in marginal cost). Taking betting odds to measure ability, we again obtain that a recent win lowers marginal cost ($\hat{\eta}$ = -0.07 < 0 in column 1 and $\hat{\eta}$ = 0.94 < 1 in column 3) but now the marginal cost reduction is statistically insignificant.

The marginal significance of psychological state dependence may partly be due to these agents' experience in handling loss/win. Importantly, our finding of strategic momentum amplified by environmental shocks is robust as we enrich the model to allow for psychological mechanisms.

B An alternative effort cost specification

The model we developed and estimate above specifies linear effort cost $\lambda c_i x_i$, where $\lambda > 0$ is a factor by which environmental conditions shift the slope. That is, marginal cost is constant in player i's effort x_i and increases in a poor environment. We briefly consider an alternative specification in which a poor environment makes the effort cost function more convex:

$$C_i(x_i; \lambda, b) = a_b c_i[(x_i + 1)^{\lambda} - 1],$$

[41] Malueg and Yates (2010) were unable to reject the null that a psychological effect was absent in a sample of 125 three-battle matches among equally able players. We repeat their statistical test on the 246 three-setters in our sample where opponents' ex-ante winning probabilities are at most 30% apart. We find that the battle-2 winner wins 55% of third battles; the p-value for a two-tailed test of H_0 = 0.5 is 0.13. Though we lack power, any psychological momentum does not appear to grow with environmental degradation. Among symmetric-player three-battle matches, the incidence of lose-win-win is 0.49 in a poor environment compared to 0.57 in a mild environment; the p-value of an equality test is 0.33.

where parameter $\lambda \geq 1$ scales with environmental degradation, and $b = 1, 2, 3$ indexes the battle number. Parameters $a_3 \geq a_2 \geq a_1 > 0$ reflect player i growing more tired as the contest progresses. Here, marginal cost $a_b \lambda c_i (x_i + 1)^{\lambda - 1}$ increases with (i) battle number b, (ii) the environmental factor λ, (iii) effort x_i, and (iv) individual-specific cost $c_i > 0$. We show that even when a poor environment makes the cost function more convex, it does not increase momentum, in contrast to the descriptive analysis. Here we shut the direct utility channel by setting the environmental disutility parameter $\delta = 0$.

Before proceeding, we briefly note that we later consider an arguably less natural variant in which the cost function can be concave in effort, i.e., we then require only $\lambda > k > 0$, allowing $\lambda < 1$.

Mirroring the analysis above, consider battle 3, reached in the event that each previous battle was won by a different player. For simplicity, assume a symmetric contest prize V. Facing a win-loss prize spread $\Delta V_i = V$ and given her rival's action x_j, player i solves:

$$\arg\max_{x_i} \frac{(x_i)^k}{(x_i)^k + (x_j)^k} V - a_3 c_i [(x_i + 1)^\lambda - 1].$$

(For simplicity, we omit the battle number from the effort subscript, i.e., here x_i denotes x_{3i}.) The first-order conditions (FOC) are:

$$\frac{(x_i)^{k-1}(x_j)^k}{[(x_i)^k + (x_j)^k]^2} kV = a_3 \lambda c_i (x_i + 1)^{\lambda - 1}, \quad i, j \in \{l, h\}, i \neq j. \tag{18}$$

Thus, in equilibrium, we have:

$$\frac{x_l}{x_h} (\frac{x_l + 1}{x_h + 1})^{\lambda - 1} = \frac{c_h}{c_l} \geq 1,$$

which means that the high-cost player h exerts lower effort than the low-cost player l. (Note that $\frac{x_l}{x_h} \geq 1$ means $\frac{x_l}{x_h} \geq \frac{x_l+1}{x_h+1} \geq 1$.) Moreover, players' effort levels x_l and x_h change in the same direction when the environment λ shifts, because:

$$x_l (x_l + 1)^{\lambda - 1} = \frac{c_h}{c_l} x_h (x_h + 1)^{\lambda - 1}.$$

We proceed considering the symmetric player case, $c_i = c$ and $C_i(.) = C(.)$. With $x_l = x_h = x$, we can write the FOC as:

$$\frac{kV}{4a_3 c} = \lambda x \left(x + 1\right)^{\lambda - 1}.$$

implying that effort x decreases when λ increases (the environment deteriorates).

Recall battle-2 incentives when players have symmetric costs. In battle 2, the player who won battle 1 faces continuation values of (a) V from winning battle 2 and ending the contest, vs. (b) $V/2$ minus the subsequent effort cost from having to play in a third battle. For her part, the player who lost battle 1 faces continuation values in battle 2 of (c) $V/2$ minus the effort cost that would follow a win that takes the contest to a third battle, vs.

(d) 0 from losing outright.

Thus, to generate strengthened momentum from a poor environment, a higher λ would need to induce increased battle-3 effort cost such that expected payoffs (b) and (c) from transiting to battle 3 fall for both of the respective players. We next show that in general this is *not* so. We show that degrading environmental conditions between $\lambda = 1$ (linear effort cost) and $\lambda = 2$ (convex effort cost) *lower* battle-3 equilibrium effort cost, increasing expected payoffs (b) and (c). This weakens the incentive for players to decide the contest in two battles. (Below we extend this analysis to concave effort cost.)

Let $\beta = \frac{kV}{2a_3c} > 0$. Battle-3 equilibrium effort levels when the environment deteriorates between $\lambda = 1$ and $\lambda = 2$ follow from the FOC:

$$x(\lambda = 1) = \frac{1}{2}\beta, \quad x(\lambda = 2) = \sqrt{\frac{1}{4}\beta + \frac{1}{4}} - \frac{1}{2}. \tag{19}$$

Battle-3 effort cost is lower in the degraded environment when

$$\begin{aligned} C(x(\lambda = 2)) &< C(x(\lambda = 1)) \\ \iff a_3c[(x(\lambda = 2) + 1)^2 - 1] &< a_3c[(x(\lambda = 1) + 1)^1 - 1] \\ \iff (x(\lambda = 2) + 1)^2 &< x(\lambda = 1) + 1 \\ \iff (\sqrt{\frac{1}{4}\beta + \frac{1}{4}} + \frac{1}{2})^2 &< \frac{1}{2}\beta + 1. \end{aligned}$$

After a little algebraic manipulation, this condition simplifies to $\beta^2 > 0$, a condition that holds.

In sum, battle-3 equilibrium effort and effort cost are lower in the degraded environment $\lambda = 2$ compared with $\lambda = 1$. Thus, in battle 2, both players' continuation values in the event that the contest transitions to a third battle are higher in the degraded environment compared with the milder one, weakening the incentive for the contest to be decided in two battles, in contrast to the data pattern we document.

It is for this reason that our chosen effort cost specification is linear $C_i(x_i; \lambda, b) = a_b\lambda c_i x_i$, whereby higher λ—or higher battle-specific marginal cost parameter a_3, for that matter—induce lower effort, but the same effort cost in battle 3. Therefore, the environment does not induce a change in battle-2 momentum through the effort cost channel. Importantly, we allow for a direct utility channel and thus do not constrain the per-battle environmental disutility parameter δ to be zero.

Cost function concave in effort. For generality, we relax the restriction $\lambda \geq 1$ and allow $\lambda < 1$ (while requiring $\lambda > k > 0$, which guarantees a pure-strategy equilibrium in each battle).

We can rewrite the battle-3 FOC (18) (with player-specific cost c_i and symmetric prize V) as

$$\frac{(x_i)^k(x_j)^k}{[(x_i)^k + (x_j)^k]^2}\left[\frac{1}{(x_i + 1)^\lambda}\left(1 + \frac{1}{x_i}\right)\right] = \frac{a_3\lambda}{kV}c_i, \quad i, j \in \{l, h\}, i \neq j.$$

Because both $\frac{1}{(x_i+1)^\lambda}$ and $1 + \frac{1}{x_i}$ are decreasing in x_i, this still means that $x_i > x_j$ if

$c_i < c_j$, i.e., the low-cost player exerts higher effort than the high-cost player.

In addition to the environments $\lambda = 1$ and $\lambda = 2$ in (19), we compute battle-3 equilibrium effort level in the environment $\lambda = \frac{1}{2}$:

$$x(\lambda = \frac{1}{2}) = \frac{1}{2}\beta^2 + \beta\sqrt{1 + \frac{1}{4}\beta^2}.$$

We can then compare the equilibrium effort cost when the environment deteriorates between $\lambda = \frac{1}{2}$ and $\lambda = 1$. Battle-3 effort cost is lower in the degraded environment when

$$\begin{aligned}
C(x(\lambda = 1)) \quad &< \quad C(x(\lambda = \frac{1}{2})) \\
\iff a_3c[(x(\lambda = 1) + 1)^1 - 1] \quad &< \quad a_3c[(x(\lambda = \frac{1}{2}) + 1)^{1/2} - 1] \\
\iff (x(\lambda = 1) + 1)^2 \quad &< \quad x(\lambda = \frac{1}{2}) + 1 \\
\iff \frac{1}{4}\beta^2 + \beta \quad &< \quad \frac{1}{2}\beta^2 + \beta\sqrt{1 + \frac{1}{4}\beta^2},
\end{aligned}$$

a condition which holds because $\frac{1}{4} < \frac{1}{2}$ and $1 < \sqrt{1 + \frac{1}{4}\beta^2}$.

Here, again, battle-3 equilibrium effort cost is lower in the degraded environment $\lambda = 1$ compared with $\lambda = \frac{1}{2}$.

Effort cost channel. Overall, whether environmental degradation makes the cost function (i) less concave, proxied by a shift from $\lambda = \frac{1}{2}$ to $\lambda = 1$, or (ii) more convex, proxied by a shift from $\lambda = 1$ to $\lambda = 2$, for both shifts a worse environment similarly induces lower effort costs in battle 3, i.e., $C(x(\lambda = \frac{1}{2})) > C(x(\lambda = 1)) > C(x(\lambda = 2))$. Thus, in battle 2, both players' continuation values in the event that the contest transitions to a third battle rise in the worse environment compared with the milder one. This weakens the incentive for players to decide the contest in two battles, in contrast to the data. We thus choose a linear effort cost and focus on a direct utility channel through a per-battle fixed cost δ. As we note in the text, the environmental parameters λ and δ are similar in that, in principle, they function through their impact on total costs (effort cost and fixed cost).

Table A.1: Descriptive statistics in an extended sample with qualifying matches, for which data are available

Variables	N	Mean	Std.Dev.	Min.	Max.
Australian Open: Melbourne in the summer					
Difference in winning probability between the two players (%)	1517	44.40	26.95	0	95.16
Difference in WTA rank between the two players	2303	72.16	107.09	0	1200
Difference in WTA ranking points between the two players	2303	1235.39	1759.90	0	12,847
Difference in age between the two players (years)	2323	4.44	3.49	0.00	26.45
Completed match (yes=1)	2323	0.98	0.12	0	1
Match's prize money (win-loss cash spread in \$ $\times$ 1000)	2323	34.11	99.71	2.24	1400.18
Battle 1 length in tennis games (among completed matches)	2288	9.20	1.93	6	13
Battle 2 length in tennis games (among completed matches)	2288	9.12	1.94	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	2288	0.86	2.49	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	2288	0.32	0.47	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	2288	0.17	0.37	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	727	9.54	2.81	6	30
Hour match starts (among completed matches)	2323	13.89	2.92	7	23
Airborne Particle Index (mean during match)	2323	0.55	0.44	0.25	5.09
PM2.5 (μg/m^3, mean during match)	419	7.01	5.35	0.00	56.67
Temperature (°C, mean during match)	2323	25.28	6.06	12.80	43.98
Relative humidity (%, mean during match)	2323	57.13	17.74	15.31	102.21
Wind speed (m/s, mean during match)	2323	3.41	1.11	0.68	10.23
Any rain, however light (yes=1, during match)	2274	0.16	0.37	0	1
China Open: Beijing in the fall					
Difference in winning probability between the two players (%)	610	40.27	24.31	0	92.16
Difference in WTA rank between the two players	663	49.82	79.57	0	790
Difference in WTA ranking points between the two players	663	1672.75	1942.43	1	11,145
Difference in age between the two players (years)	664	4.22	3.22	0.04	23.32
Completed match (yes=1)	666	0.95	0.21	0	1
Match's prize money (win-loss cash spread in \$ $\times$ 1000)	666	31.45	64.24	1.34	554.82
Battle 1 length in tennis games (among completed matches)	634	9.36	1.96	6	13
Battle 2 length in tennis games (among completed matches)	634	9.10	1.90	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	634	0.77	2.56	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	634	0.33	0.47	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	634	0.17	0.37	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	210	9.39	1.98	6	13
Hour match starts (among completed matches)	666	14.55	2.88	10	23
PM2.5 (μg/m^3, mean during match)	634	88.20	93.03	0.67	520.33

Table A.1: Descriptive statistics in an extended sample with qualifying matches, for which data are available

Variables	N	Mean	Std.Dev.	Min.	Max.
Temperature (°C, mean during match)	666	19.76	4.41	7.72	31.25
Relative humidity (%, mean during match)	666	43.66	18.70	14.59	97.80
Wind speed (m/s, mean during match)	666	2.50	1.28	0.23	7.96
Any rain, however light (yes=1, during match)	666	0.07	0.25	0	1
Five other WTA series in China: Guangzhou, Hong Kong, Shenzhen, Tianjin, Wuhan in the fall or winter					
Difference in winning probability between the two players (%)	978	39.16	23.67	0	92.34
Difference in WTA rank between the two players	994	110.87	157.06	1	1132
Difference in WTA ranking points between the two players	994	846.91	1229.11	0	8065
Difference in age between the two players (years)	1011	4.18	3.28	0	23.89
Completed match (yes=1)	1024	0.95	0.21	0	1
Match's prize money (win-loss cash spread in \$ $\times$ 1000)	1024	6.94	17.82	0.20	235.89
Battle 1 length in tennis games (among completed matches)	977	9.22	1.97	6	13
Battle 2 length in tennis games (among completed matches)	977	9.11	1.94	6	13
Battle-1 loser's games won in battle 2 versus battle 1 (completed)	977	0.79	2.44	-6	7
Match lasts 3 battles (yes = 1, among completed matches)	977	0.31	0.46	0	1
Battle-1 loser wins the match (yes=1, among completed matches)	977	0.17	0.38	0	1
Battle 3 length in tennis games (among matches lasting 3 battles)	300	9.30	1.93	6	13
Hour match starts (among completed matches)	1024	13.95	2.59	10	22
PM2.5 (μg/m^3, mean during match)	1014	44.64	31.05	3.00	310.67
Temperature (°C, mean during match)	1024	24.76	5.55	8.61	34.42
Relative humidity (%, mean during match)	1024	60.35	18.16	15.26	98.19
Wind speed (m/s, mean during match)	1024	3.02	1.79	0.14	10.46
Any rain, however light (yes=1, during match)	1024	0.16	0.36	0	1

Notes: This extended sample adds qualifying matches played with the same best-of-three format and held at the same venues shortly before main-draw matches for the WTA tournament series described in Table 1. Qualifying matches are missing for the Australian Open prior to 2009 and for the China Open prior to 2010. Pre-match betting odds for qualifying matches prior to 2012 are mostly missing.

Table A.2: Environmental quality and momentum, in an extended sample with qualifying matches, for which data are available

Dependent variable is 1 if match lasts three battles, 0 otherwise.	(1) High asymm.	(2)	(3)	(4)	(5) More symmetric opponents	(6)	(7)	(8)	(9) Entire
Win. prob. diff. (from betting):	$\geq 65\%$	$\leq 30\%$	$\leq 25\%$	$\leq 20\%$	$\leq 15\%$	$\leq 10\%$	$\leq 25\%$	$\leq 25\%$	range
Temperature > 29 °C (yes=1)	-0.013	-0.116***	-0.133***	-0.127***	-0.160***	-0.157***	-0.155***	-0.151***	-0.131***
	(0.039)	(0.031)	(0.033)	(0.039)	(0.045)	(0.054)	(0.045)	(0.046)	(0.039)
Temp. > 29 × win. prob. diff.									0.002**
									(0.001)
PM2.5 > 100 μg/m^3 (yes=1)	-0.007	-0.057	-0.094*	-0.086	-0.053	0.020	-0.113		-0.068
	(0.068)	(0.051)	(0.055)	(0.062)	(0.075)	(0.091)	(0.071)		(0.057)
PM2.5 > 100 × win. prob. diff.									0.002
									(0.001)
PM2.5 $\in (100, 200]$ μg/m^3 (yes=1)								-0.080	
								(0.077)	
PM2.5 > 200 μg/m^3 (yes=1)								-0.205**	
								(0.098)	
Winning probability difference (%)									-0.003***
									(0.000)
Additional controls	No	No	No	No	No	No	Yes	Yes	Yes
Observations	672	1,092	943	798	498	323	921	921	2,908
R-squared	0.000	0.010	0.013	0.012	0.019	0.019	0.090	0.091	0.051
Number of regressors	2	2	2	2	2	2	69	70	76
Mean of dependent variable	0.21	0.38	0.40	0.40	0.39	0.38	0.41	0.41	0.32

Notes: An observation is a completed match in a singles WTA tournament series held in Australia or China, including qualifying matches held prior to matches in the main draw. The dependent variable is 1 if the match lasts three battles and 0 if the match lasts two battles. Table A.1 describes the extended sample. Column 1 restricts the estimation sample to matches for which the two opponents' absolute difference in winning probability according to pre-match betting odds is at least 65%. Columns 2 to 6 restrict the sample to matches for which this ex-ante difference does not exceed 30%, 25%, 20%, 15%, or 10%, respectively. Columns 7 and 8 repeat the estimation sample of column 3 (winning probability difference does not exceed 25%) with additional controls for series by round (now including indicators for qualifying rounds 1, 2, and 3), year, time-of-day, humidity, wind speed, and rain (see Table 2 notes for further details). Column 9 considers the entire sample but interacts environmental factors with (and controls for) the winning probability difference. OLS estimates. Standard errors, clustered on day, are in parentheses. ***Significant at 1%, **at 5%, *at 10%.

Table A.3: Environmental quality does not significantly change the predictability of tennis point outcomes

Dependent variable:	Share of in-play points won in total by the betting market's favorite player			Share of in-play points won in battle 1 by the betting market's favorite player		
	(1)	(2)	(3)	(4)	(5)	(6)
Temperature > 29 °C (yes=1)	0.003	0.003	0.003	0.010*	0.009	0.009
	(0.004)	(0.006)	(0.006)	(0.006)	(0.010)	(0.010)
PM2.5 > 100 μg/m^3 (yes=1)	0.006	0.005		0.006	0.006	
	(0.006)	(0.007)		(0.009)	(0.010)	
PM2.5 $\in (100, 200]$ μg/m^3 (yes=1)			0.003			0.012
			(0.008)			(0.012)
PM2.5 > 200 μg/m^3 (yes=1)			0.011			-0.011
			(0.010)			(0.011)
Winning probability diff. (%)	0.001***	0.001***	0.001***	0.001***	0.001***	0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Additional controls	No	Yes	Yes	No	Yes	Yes
Observations	1,864	1,837	1,837	1,151	1,124	1,124
R-squared	0.137	0.153	0.153	0.079	0.100	0.101
Number of regressors	3	56	57	3	52	53
Mean of dependent variable	0.54	0.54	0.54	0.54	0.54	0.54

Notes: An observation is a completed match in a singles WTA tournament series held in Australia or China over the entire range of opponent asymmetry, for which we observe total tennis points played in the match (source: oncourt.info, 2018-2016) or the number of tennis points played in battle 1 (source: flashscore.com, 2012-2016). A tennis point is a subunit of a tennis game, e.g., a game can we won by winning at least 4 points. We exclude matches for which prediction markets deem both opponents as having equal odds of winning, and thus there is no favorite player and the dependent variable is undefined. In columns 1 to 3, the dependent variable is the share of in-play points won in total by the betting market's favorite player. In columns 4 to 6, the dependent variable is the share of in-play points won in battle 1 by the betting market's favorite player. See Table 2 for a description of additional controls in columns 2 to 3 and 5 to 6. OLS estimates. Standard errors, clustered on day, are in parentheses. ***Significant at 1%, **at 5%, *at 10%.

Table A.4: Contest model estimates: Dropping a few climate-controlled matches

Alternative measure of player strength:	Pre-match betting (1)	WTA rank (2)	WTA z-score (3)
k, randomness in winner selection	0.662	0.715	0.728
	(0.026)	(0.027)	(0.025)
θ, ability to marginal cost mapping	0.896	0.497	0.197
	(0.064)	(0.039)	(0.015)
Environmental disutility parameters:			
δ_T, coefficient on $\ln(1 + \max(T_n - \underline{T}, 0))$	0.122	0.135	0.145
	(0.026)	(0.026)	(0.033)
δ_P, coefficient on $\ln(1 + \max((P_n - \underline{P})/100, 0))$	1.137	1.261	1.342
	(0.069)	(0.210)	(0.071)
Number of parameters	4	4	4
Observations	2109	2605	2605
Log likelihood	-2823	-3609	-3643

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament series held in Australia or China. Compared with the estimation sample in Table 5, we drop 66 matches for which our review of press articles on LexisNexis indicated that a retractable roof, when available, was closed to protect from ambient environmental conditions, namely heat and rain. (In practice, missing values for some covariates such as betting odds imply that less than 66 matches are dropped.) A match's likelihood contribution is defined using all possible transitions. Marginal cost is alternatively modeled as $c_{in} = (odds_{in})^{\theta}$ in column 1, $c_{in} = (rank_{in})^{\theta}$ in column 2, and $c_{in} = e^{-\theta(zscore_{in})}$ in column 3. The intercept to per-battle disutility δ_0 is set to \$0, and temperature and PM2.5 pollution cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. The contest prize V_{in} is proxied by the match's prize money. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for $k = 1$ and $\theta = \delta_T = \delta_P = 0$. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

Table A.5: Contest model estimates: Dummy variables (bins) for heat and pollution

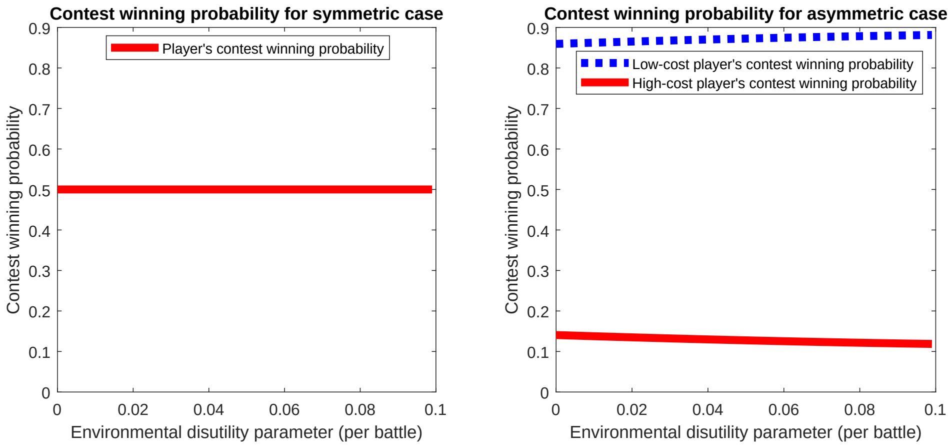
Robustness check:	Disutility intercept $\delta_0 = 0$ (1)	Estimate, not fix, δ_0 (2)	Estimate, not fix, $\underline{T}$ & $\underline{P}$ (3)	Two heat & PM2.5 bins (4)
k, randomness in winner selection	0.662	0.656	0.664	0.661
	(0.027)	(0.028)	(0.024)	(0.028)
θ, ability to marginal cost mapping	0.880	0.889	0.875	0.880
	(0.064)	(0.066)	(0.057)	(0.070)
Environmental disutility parameters:				
δ_T, coefficient on $T_n > \underline{T}$	0.188	0.163	0.185	
	(0.040)	(0.073)	(0.040)	
δ_P, coefficient on $P_n > \underline{P}$	0.667	0.642	1.093	
	(0.062)	(0.065)	(0.068)	
δ_0, per-battle disutility intercept (or set to 0)		0.029		
		(0.061)		
$\underline{T}$, temperature cutoff (°C) (or set to 27)			26.890	
			(0.052)	
$\underline{P}$, pollution cutoff (μg/m^3) (or set to 150)			209.060	
			(2.674)	
δ_{T1}, coefficient on $T_n \in (27, 29]$ °C				0.174
				(0.053)
δ_{T2}, coefficient on $T_n > 29$ °C				0.197
				(0.040)
δ_{P1}, coefficient on $P_n \in (100, 200]$ μg/m^3				0.019
				(0.148)
δ_{P2}, coefficient on $P_n > 200$ μg/m^3				1.095
				(0.068)
Number of parameters	4	5	6	6
Observations	2157	2157	2157	2157
Log likelihood	-2897	-2897	-2897	-2896

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament held in Australia or China. A match's likelihood contribution is defined using all possible transitions. Marginal cost is modeled as $c_{in} = (odds_{in})^{\theta}$. The intercept to per-battle disutility δ_0 is set to \$0, except in column 2 where this intercept is estimated. In columns 1 and 2, temperature and PM2.5 pollution cutoffs are set to 27 °C and 150 μg/m^3. Column 3 estimates these cutoffs imposing the constraints $25 \leq \underline{T} \leq 35$ °C and $100 \leq \underline{P} \leq 300$ μg/m^3. Column 4 specifies two heat bins (the lowest starting at 27 °C) and two PM2.5 bins (the lowest starting at 100 μg/m^3). The contest prize V_{in} is proxied by the match's prize money. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for all parameters set to 0 except for $k = 1$ and, in column 3, initial values for $\underline{T} = 25$ °C and $\underline{P} = 100$ μg/m^3. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

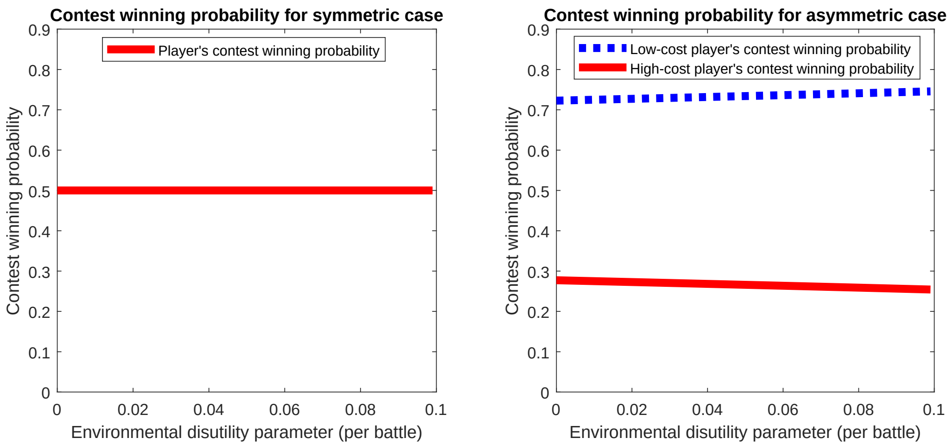
Table A.6: Contest model estimates: Allowing for psychological state dependence on preceding battle outcome

Psychology of preceding-battle won:	Additively enters player's MC this set if she won preceding set		Multiplicat. enters player's MC this set if she won preceding set	
Alternative measure of player strength:	Pre-match betting	WTA rank	Pre-match betting	WTA rank
	(1)	(2)	(3)	(4)
k, randomness in winner selection	0.655	0.703	0.647	0.668
	(0.029)	(0.025)	(0.029)	(0.028)
θ, ability to marginal cost mapping	0.886	0.491	0.912	0.550
	(0.066)	(0.037)	(0.070)	(0.043)
η, add to c_{in} in battle $b > 1$ if i won battle $b-1$ in match n	-0.067	-0.313		
	(0.088)	(0.138)		
η, multiply with c_{in} in battle $b > 1$ if i won battle $b-1$ in match n			0.939	0.847
			(0.061)	(0.050)
Environmental disutility parameters:				
δ_T, coeff. on $\ln(1+\max(T_n - \underline{T}, 0))$	0.127	0.145	0.130	0.158
	(0.027)	(0.025)	(0.028)	(0.028)
δ_P, coeff. on $\ln(1+\max((P_n - \underline{P})/100, 0))$	1.169	1.436	1.191	1.579
	(0.070)	(0.066)	(0.070)	(0.066)
Number of parameters	5	5	5	5
Observations	2157	2670	2157	2670
Log likelihood	-2897	-3699	-2897	-3698

Notes: Maximum likelihood estimates. An observation is a completed match in a singles WTA tournament held in Australia or China. A match's likelihood contribution is defined using all possible transitions. Marginal cost is modeled as $c_{in} = (odds_{in})^\theta$ in columns 1 & 3 and $c_{in} = (rank_{in})^\theta$ in columns 2 & 4. In columns 1 & 2, a player's marginal cost in battle $b \in \{2,3\}$ shifts additively by η if she won battle $b-1$, i.e., marginal cost c_{inb} is $c_{in} + \eta$ if i won match n's battle $b-1$ and c_{in} otherwise. In columns 3 & 4, a player's marginal cost in battle $b \in \{2,3\}$ is multiplied by η if she won battle $b-1$, i.e., marginal cost c_{inb} is ηc_{in} if i won match n's battle $b-1$ and c_{in} otherwise. The intercept to per-battle disutility δ_0 is set to \$0, and temperature and PM2.5 pollution cutoffs $\underline{T}$ and $\underline{P}$ are set to 27 °C and 150 μg/m^3. The contest prize V_{in} is proxied by the match's prize money. Solver Knitro using the interior-point algorithm, constraints $0 < k \leq 1$ and $\theta > 0$, and initial values for $k = 1$ and $\theta = \delta_T = \delta_P = 0$. We constrain $\eta > -1$ in columns 1 & 2 and $\eta > 0$ in columns 3 & 4 such that marginal cost is necessarily positive $\forall\ i, n, b$. The initial value for η is set to 0 in columns 1 and 2 and to 1 in columns 3 and 4. Standard errors, in parentheses, are obtained from the Hessian estimate of the asymptotic covariance matrix.

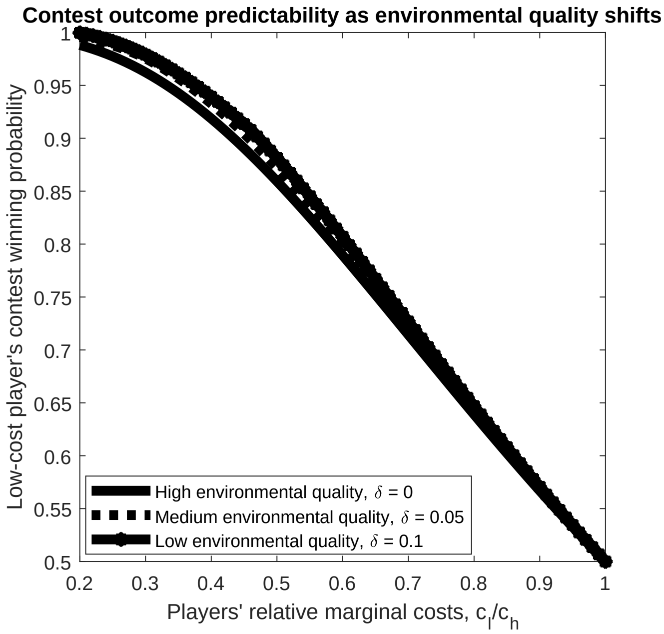


(a) Low randomness, $k = 1$

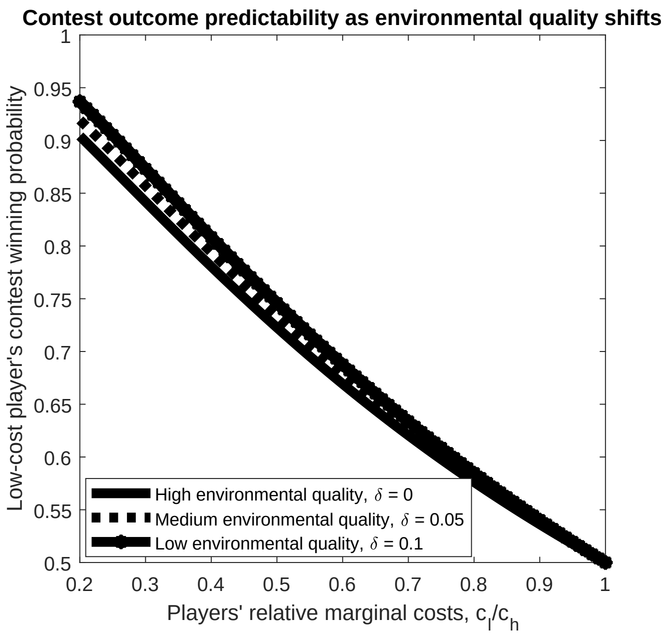


(b) More randomness, $k = 0.7$

Figure A.1: [**Model**] Contest winning probabilities, as a function of the per-battle environmental disutility parameter δ. In the model, a higher δ captures a more adverse environment, i.e., a stronger dose of heat or pollution. Left panels: Players are symmetric, $c_h = c_l$. Right panels: Players are asymmetric, $c_h = 2c_l$. Top panels (a) set technology $k = 1$ and bottom panels (b) set $k = 0.7$ (more randomness). We illustrate with symmetric contest prizes $V_h = V_l = V = 1$ and $\lambda c_h = 0.5V$.

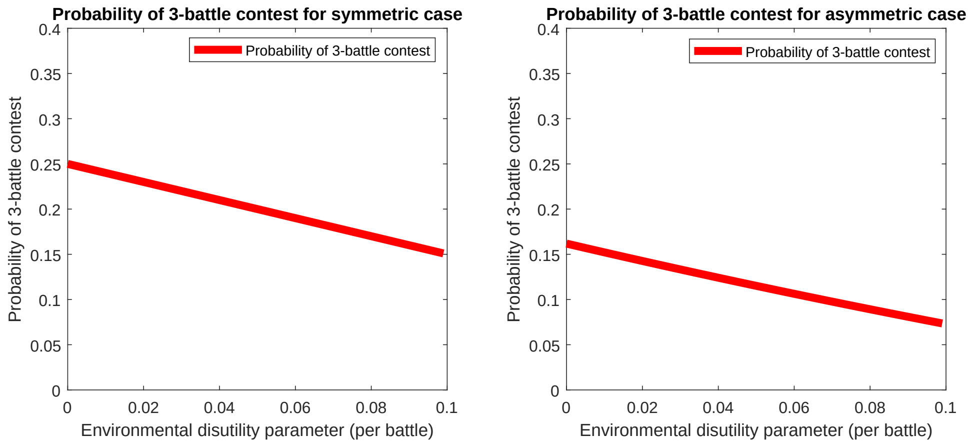


(a) Low randomness, $k = 1$

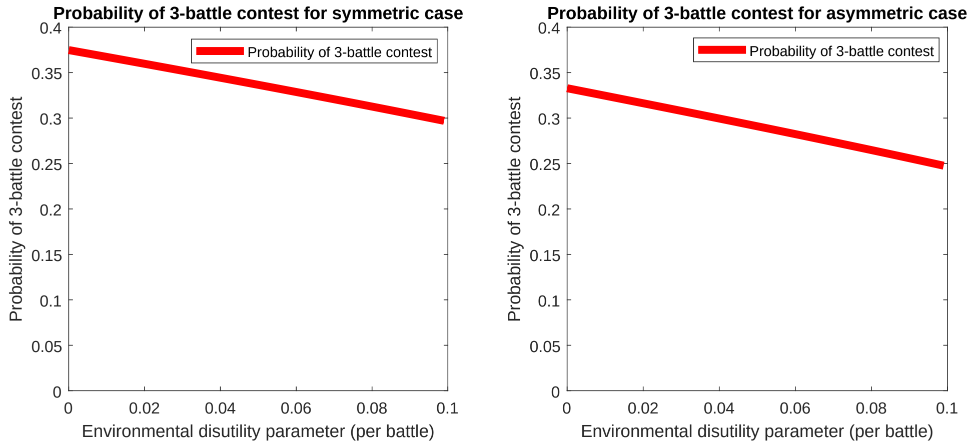


(b) More randomness, $k = 0.7$

Figure A.2: [**Model**] Predictability of the contest winner as a function of: opponent asymmetry c_l/c_h (along the horizontal axis within a panel); environmental disutility δ (different curves within a panel); and randomness in the contest technology k (across panels). The vertical axis plots the low-cost player's contest ex-ante winning probability. We illustrate with symmetric contest prizes $V_h = V_l = V = 1$ and $\lambda c_h = 0.5V$.

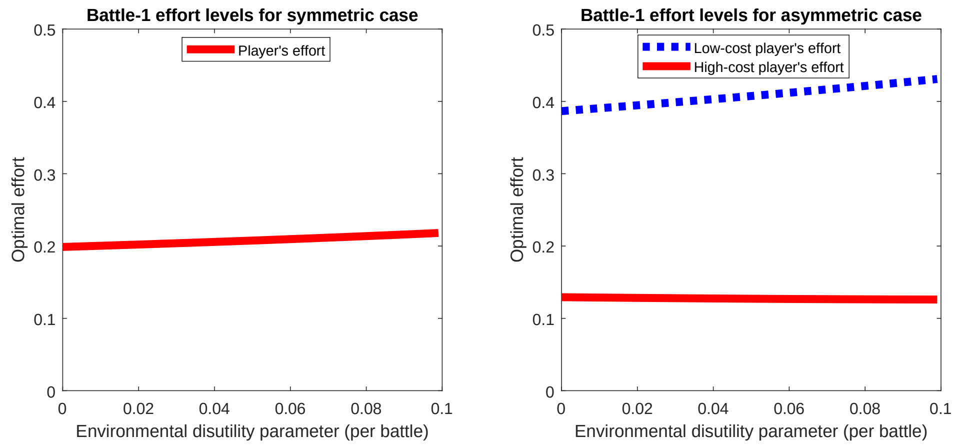


(a) Low randomness, $k = 1$

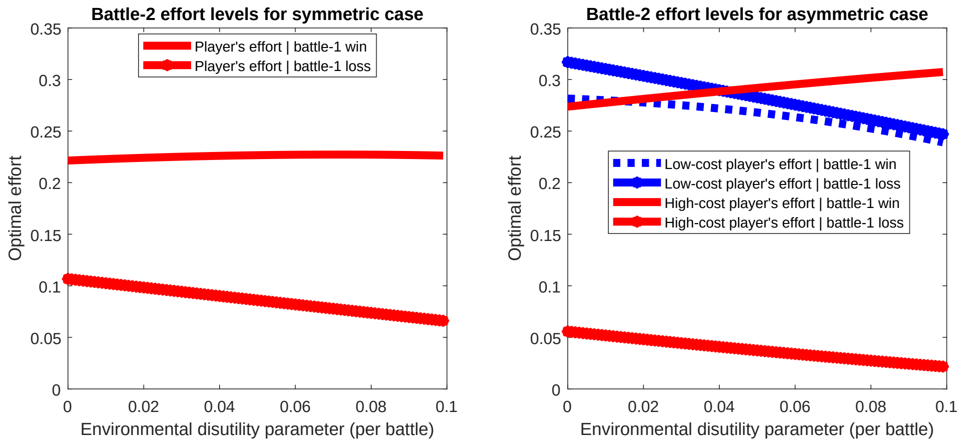


(b) More randomness, $k = 0.7$

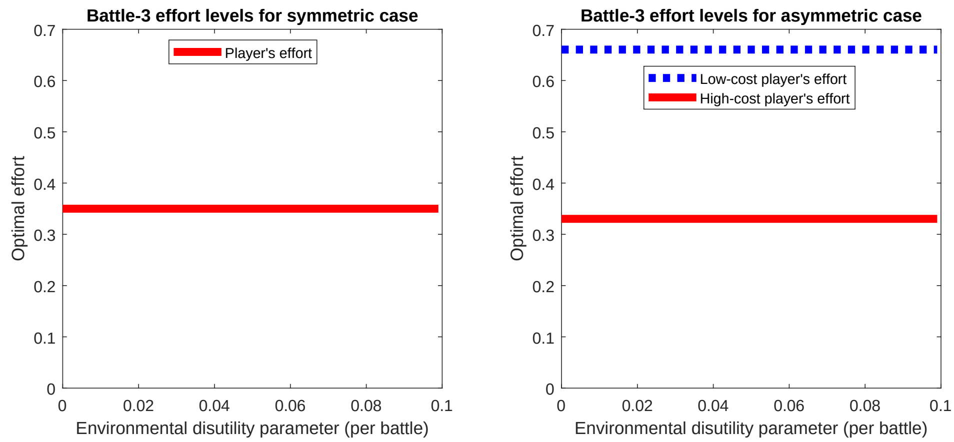
Figure A.3: [**Model**] Probability that the contest finishes in three battles, as a function of the per-battle environmental disutility parameter δ. Left panels: Players are symmetric, $c_h = c_l$. Right panels: Players are asymmetric, $c_h = 2c_l$. Top panels (a) set technology $k = 1$ and bottom panels (b) set $k = 0.7$ (more randomness). We illustrate with symmetric contest prizes $V_h = V_l = V = 1$, and $\lambda c_h = 0.5V$.



(a) Optimal effort in battle 1

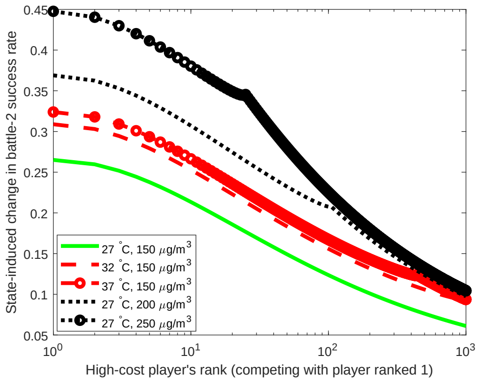


(b) Optimal effort in battle 2

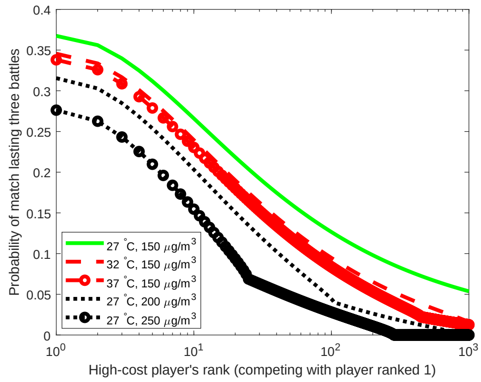


(c) Optimal effort in battle 3

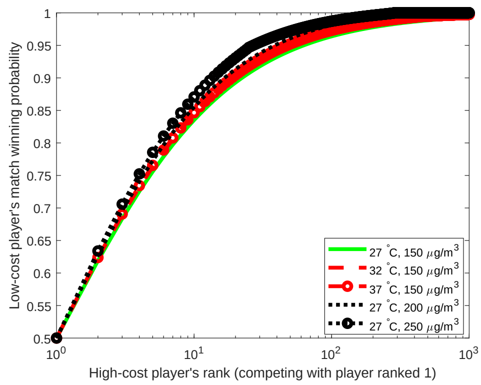
Figure A.4: [**Model**] Optimal effort choices in battles 1 (top panels), 2 (middle panels) and 3 (bottom panels), as a function of the per-battle environmental disutility parameter δ. Left panels: Players are symmetric, $c_h = c_l$. Right panels: Players are asymmetric, $c_h = 2c_l$. We illustrate with $k = 0.7$, symmetric contest prizes $V_h = V_l = V = 1$, and $\lambda c_h = 0.5V$.



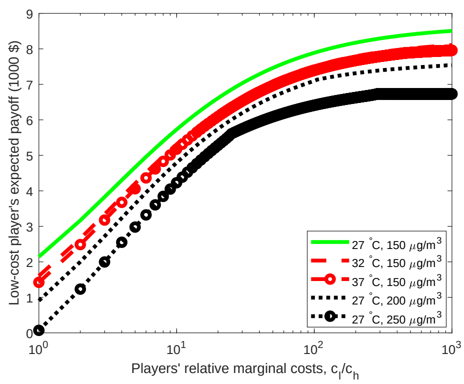
(a) Stronger player's battle-2 success



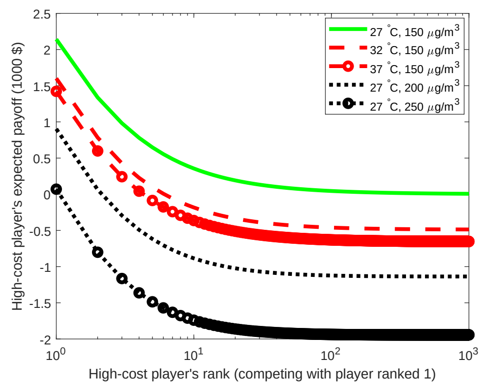
(b) Probability of match lasting 3 battles



(c) Match outcome predictability

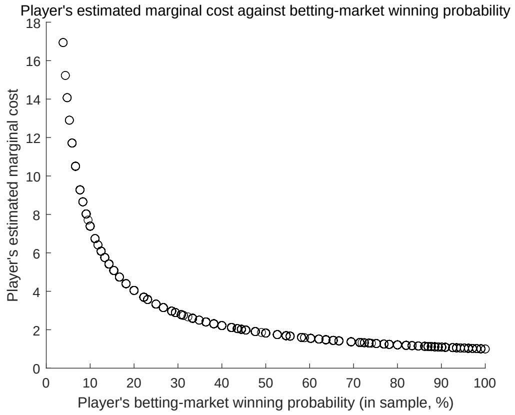


(d) Stronger player's expected payoff

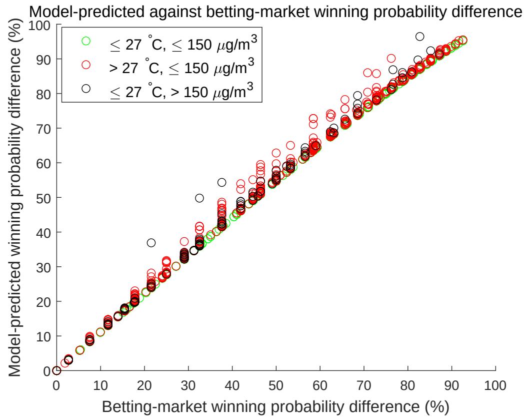


(e) Weaker player's expected payoff

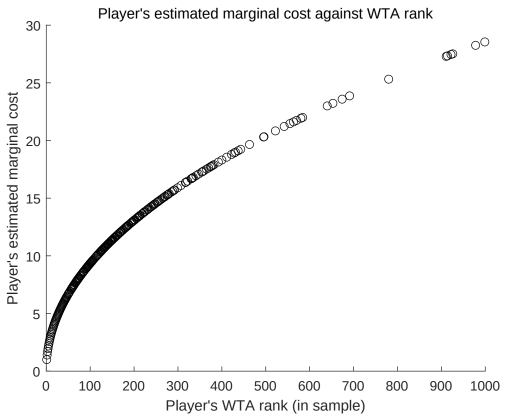
Figure A.5: [**Model estimates: Rank-based ability measure, 25th-percentile prize**] Model predictions against the degree of opponent asymmetry evaluated at the 25th-percentile cash prize in the sample ($8672; $N = 2811$ matches). Model according to Table 5, column 2, with player strength based on WTA rank, $c_{in} = (rank_{in})^{\theta}$.



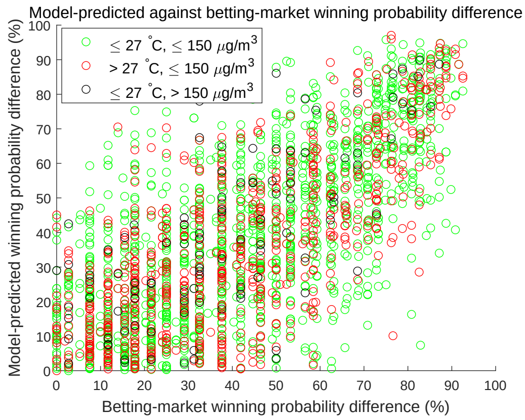
(a) $\hat{c}_{in}$ against (inverse) $odds_{in}$



(b) Pr(l wins) − Pr(h wins): Fitted vs. betting

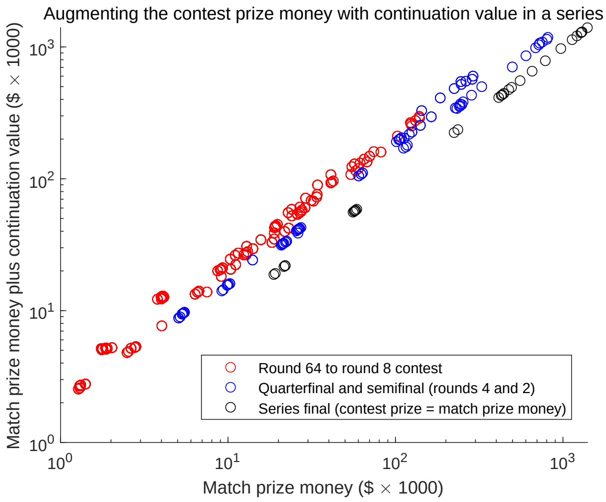


(c) $\hat{c}_{in}$ against $rank_{in}$

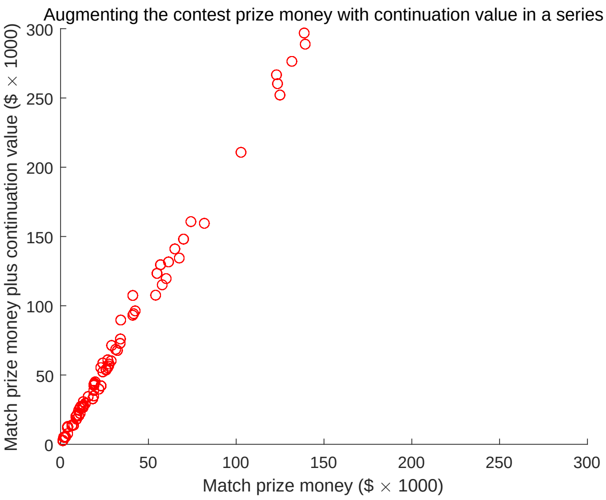


(d) Pr(l wins) − Pr(h wins): Fitted vs. betting

Figure A.6: [**Estimated model diagnostics**] Further diagnostics for estimated models reported in: (top panels) Table 5, column 1, with ability based on betting odds, $c_{in} = (odds_{in})^{\theta}$; and (bottom panels) Table 5, column 2, with ability based on WTA rank, $c_{in} = (rank_{in})^{\theta}$. Each circle in these scatters represents a match in the sample; the horizontal axes show data whereas the vertical axes show model predictions. The left panels show fitted marginal cost against the corresponding betting odds- or rank-based ability measure in the data. The right panels show the model-predicted against the betting-market winning probability difference across players; matches are color coded according to ambient conditions.



(a) Contests in all rounds of a WTA series (log scale)



(b) Contests in all rounds but quarterfinal, semifinal, and final

Figure A.7: [**Model estimates**] Augmenting the contest prize money with continuation value to a player from winning the match and remaining in the WTA series. Augmented contest prize V, defined as prize money plus continuation value per estimates in Table 9, against prize money. Each circle in a scatter represents a match. The top panel (in log scale) shows all contests in the estimation sample (with observed betting odds, $N = 2157$): the median V is estimated at \$32,857 and the median prize money is \$18,300 for this sample. Matches are color coded according to the round in the WTA series: final matches, shown in black, lie along the diagonal because there is no subsequent round of contests. For better visualization, the bottom panel (in linear scale) plots contests up to round 8 only. See the text on assumed player beliefs regarding the strength of the opponents and environmental quality they expect to encounter in future matches (in this implementation, the contest prize does not vary within series-year-round so many circles are overlaid).