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Asset Pricing and the Carbon Beta of Externalities

Abstract

Climate policy needs to set incentives for actors who face imperfect, distorted markets and large uncertainties about the costs and benefits of abatement. Investors price uncertain assets according to their expected return and risk (carbon beta). We study carbon pricing and financial incentives in a consumption-based asset pricing model distorted by technology spillover and time-inconsistency. We find that both distortions reduce the equilibrium asset return and delay investment in abatement. However, their effect on the carbon beta and risk premium of abatement can be decreasing (when innovation spillovers are not anticipated) or increasing (when climate policy is not credible). Efficiency can be restored by carbon pricing and financial incentives, implemented in our model by a regulator and by a long-term investment fund. The regulator commands carbon pricing and the fund provides subsidies to reduce technology costs or to boost investment returns. The investment subsidy creates a financial incentive that complements the carbon price. In this way the investment fund can support climate policy when the actions of the regulator fall short. These instruments must also consider the investment risk and the sequence of their implementation. The investment fund can then pave the way for carbon pricing in later periods by preventing a capital misallocation that would be too expensive to correct. Thus the investment fund improves the feasibility of ambitious carbon pricing.

JEL-Codes: Q540, D810, G120.

Keywords: carbon budget, CCAPM, policy instruments, external effect.

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1. Introduction

Carbon pricing is a key element of climate policy in many jurisdictions including important players such as China and the European Union. Part of its appeal is that putting a price on an externality is a simple yet powerful way to correct the associated market failure and align economic activity and investment behavior with social well-being. Yet the design of the optimal carbon pricing scheme faces many challenges. First, little is known about the future during which carbon pricing will take effect. To neglect unknowns in the design of climate policy would mean to ignore the careful consideration of uncertainties and risks in the investment decisions that climate policy aims to (re)direct. Second, focusing solely on the climate change externality would deny any further market distortions. Market-based climate policy via carbon pricing depends on well-functioning markets. Understanding how to address uncertainty and investment risk as well as additional market failures is therefore essential for carbon pricing.

Economic models have long been used to estimate optimal carbon prices. Early studies focused on climate change in a deterministic setting (Nordhaus, 1992). These "integrated assessment models" have become more complex by distinguishing world regions, energy technologies and multiple greenhouse gases (cf. Bertram et al., 2015). As a methodological backbone in assessment reports, these deterministic models are now an authoritative source of carbon pricing information (IPCC, 2018).

Several studies incorporate uncertainty in the economic decision-making within these models, initially limiting uncertainty to a few alternative "states of the world" (e.g. Giannousakis et al., 2021). More recently, fully stochastic models suggest there are substantial implications of incorporating uncertainties; those relating to climate change, for example, can result in high carbon prices (Cai et al., 2016). These models are solved numerically, hence disentangling how uncertainties specifically affect investment decisions is not always possible.

When investors are faced with more than one uncertain option (or asset) to invest in, the key to understanding their investment decision is the correlation with their expected returns. Investments with a high return in "bad" states of the world are preferable to investments that pay off in "good" states.[1] To see why this is, consider carbon pricing; the fundamental trade-off is between the resources consumed today and the investment in emissions abatement. Consider also, uncertainty about the severity of climate change impacts; investments in abatement become especially beneficial when climate change impacts are bad. In this state of the world, consumption would be low (reduced by climate change impacts) and would hence negatively correlate with the return

[1]Formally, this intuitive property follows from the curvatures of the utility function (decreasing marginal utility).

on abatement investment. In contrast, consider uncertainty over economic growth; investments in abatement become especially beneficial when total emissions are substantial, which coincides with high growth and prosperity. Consumption would be high and hence correlate positively with the return on abatement investment.

The consumption-based capital asset pricing model (CCAPM) provides a formal framework for these intuitions (Lucas, 1978). The correlation between asset returns and growth in consumption, as measured by the *beta* of the investment, is decisive for the valuation (pricing) of investments relative to the benchmark of a (hypothetical) risk-free investment. The *beta* determines the risk premium, i.e. by how much the rate of return of an asset exceeds (or falls short of) the risk free rate of return in equilibrium; put differently the risk premium is the extra return demanded (or foregone) by the investor in compensation for bearing the associated investment risk. For abatement, Dietz et al. (2018) estimate the *climate beta* (also: *carbon beta*) at $\phi \approx 1$.

Gollier (2021) builds a CCAPM to study the optimal carbon price compatible with a 2 degree target for the European Union. The model takes the perspective of a benevolent social planner and abstracts from further market distortions.

The list of market distortions that are potentially relevant for carbon pricing is long. Staub-Kaminski et al. (2014) offer a three-part classification of obstacles to climate policy: economic agents may respond irrationally to climate policy (in this study we focus on rational agents), regulators may fail in creating an effective regulatory framework, and additional market failures may distort pricing. A prime example of the latter occurs when the benefits of technological innovation cannot be fully appropriated, which drives a wedge between private returns and social returns (Jaffe et al., 2005). The regulator can address the market failure either by finding a second-best carbon pricing that takes the distortion into account, or (following the intuition of the "Tinbergen rule") by introducing supplementary policies that address the distortion directly (for example using a technology subsidy as in Kalkuhl et al., 2012).

In contrast, regulatory failure to create a credible carbon price may arise from an inability of the government to commit itself, leaving economic agents in doubt as to whether an announced policy will be implemented (Kalkuhl et al., 2020). Consequently, economic agents will not act in accordance with the carbon price and may strategically choose to hold back from investing (Gersbach and Glazer, 1999).

In the context of climate policy, market distortions have been shown to have substantial impact on its success, resulting in lock-in to inferior technology (Kalkuhl et al., 2012) or asset stranding (Kalkuhl et al., 2020). Analyses of carbon pricing in stochastic climate-economy models, however, have thus far abstracted from additional distortions.

In our paper, climate economics and asset pricing meet to improve our understanding of how additional distortions affect the pricing of risks, the associated risk premium demanded, and the optimal policy response to address the distortions. We study the asset pricing problem for emission abatement projects in a market economy where prices are distorted by a technology externality and the inability of the regulator to credibly commit to a carbon pricing policy.

We consider a decentralized economy that is populated by three agents: a firm-owning household, a regulatory authority and a long-term investment fund. The firm-owning household (henceforth simply household) is endowed with the (stochastic) economic product and chooses consumption to maximize (expected) welfare but must constrain total emissions to keep an emission permit budget. To this end, the household controls emissions by investing in emission abatement projects.

The regulatory authority is in charge of the carbon pricing policy, i.e. issuance of emission permits and their management. The resulting carbon price, however, may be distorted by additional market failures such as inappropriate technological learning, giving rise to a carbon price with a non-optimal growth rate. Furthermore, when the announced emission budget is not credible, such that investors (the household) expect an injection of additional permits in the second period, the announcement will fail to set the desired incentive to invest.

The investment fund is investigated as a potential remedy to overcome distortions. The fund anticipates household and government actions and may be designed to support abatement projects via a subsidy paid on project benefits or via an upfront technology subsidy.

We find that the distortions (technological externalities, and commitment problems) affect the asset return as well as the risk premium of abatement. But while the direct (*ceteris paribus*) effect on asset returns is a reduction in both cases, their direct effect on carbon betas and risk premiums is different: *ceteris paribus* the technological externality reduces the risk premium while non-credibility of the emission budget raises the risk premium. Regarding investment in abatement, the direct effects of the former on risk and return counteract each other, while they are reinforcing in the latter case. In equilibrium both distortions trigger an underinvestment in abatement in the first period relative to the socially optimal level.

We consider three policy options to address the distortions: modifying the carbon price via an intertemporal trading ratio for emission permits, an investment subsidy to boost the return of investment in abatement, and a technology subsidy to reduce the costs of the abatement technology. We show that all instruments can be used to restore the first-best allocation. Moreover, the investment subsidy can support a sub-optimally low (nonzero) carbon price such that their combined incentive to abate matches any carbon price.

When the regulator lacks a commitment device and thus the announced climate policy is not credible, a conflict of interest game emerges: since the household is assumed to be unaware of any benefits of climate policy, it prefers a lower emission permit budget than the regulator. Once the household invests according to this preference, it becomes very costly for the regulator to then enforce a tight permit budget. When the household anticipates that its preference will become a “self-fulfilling prophecy” they will act accordingly. This distortion, too, can be addressed by investment or technology subsidies that complement the sub-optimally low investment incentive of the non-credible climate policy announcement.

In summary, we highlight two main results:

First, we confirm a key insight of Gollier (2021) in a decentralized economy with distortions: that the risk premium on abatement is substantial, putting the socially optimal rate of return well above the risk-free rate. This rate is decisive for discounting returns and the timing of abatement activities. Hence ignoring risks has welfare costs and leads to a misallocation between consumption and investment projects. The primarily ethical debate on social discounting needs to be complemented by identifying and quantifying the macro-economic risks for investors. Otherwise, climate economics would focus on the quantitatively less important component of the social discount rate.

Second, the impact of market failures on the risk premium deserves more attention as private sector investors may mis-price risk in the distorted economy relative to the socially optimal level of the risk premium. Similarly, we quantify the welfare losses and risk premium of time-inconsistency when regulators cannot commit to their policies. We highlight the sequencing of policies in which a long-term fund paves the way to ambitious carbon pricing when the regulator might fail to implement a credible long-term carbon price path.

The paper is organized as follows. In section 2 we discuss the literature and provide a rationale for our research question. Section 3 presents the social planner model for the normative benchmark and the decentralized economy. In section 4 we explore policies. Sections 5 and 6 are dedicated to the calibration of the model and its numerical results. The final section offers the conclusion and outlook.

2. Literature

Mainstream models in climate economics have primarily relied on deterministic cost-benefit approaches (Nordhaus, 2017, 2018) or on cost-effectiveness analyses within Integrated Assessment Models (IAMs) often used in the IPCC assessment reports (IPCC, 2018). Both approaches share the use of a risk-free social discount rate. The social discount rate might then be used either as a normative benchmark (Stern et al., 2006) or to replicate observed market behavior (Nordhaus, 2007). Both approaches ignore the specific macro-economic risks arising from the uncertain behavior of the key elements making-up the climate-economic model. Yet, real-world investors demand a risk premium based on the covariance of uncertainties.

More recently, researchers have begun to investigate the risk premium of physical and transitional risks. One important strand of literature explores the physical risks of climate change including the damage to productive assets. A second strand of literature focuses on transitional risk reflecting costs arising from the transformation to a low carbon economy (Giglio et al., 2021). A further strand of literature traces the impact of government policies on the carbon beta or the risk premium.

Physical risks of climate change: The majority of studies on the role and implications of uncertainty in climate-economic models have so far focused on climate change impacts. A wide literature on this topic was developed using stochastic general equilibrium models with recursive preferences (a la Epstein and Zin, 1989, 1991). Within this modeling framework, studies have mostly focused on two sources of uncertainty, namely damage uncertainty and growth uncertainty. Several papers report on the effect of damage uncertainty and its implication for climate policy, for example Crost and Traeger (2014), Rudik (2020) and more recently Hambel et al. (2021), who find the social cost of carbon to be heavily driven by the damage specification assumptions. Other papers studied the impact of growth uncertainty (Jensen and Traeger, 2014; Cai et al., 2013; Cai and Lontzek, 2019). These stochastic equilibrium models with recursive preferences have the advantage of capturing how policy incorporates anticipated learning and the ability to calculate the optimal tax on carbon emissions. Most of these models treating uncertainty are, however, solved numerically, with little attention to analytical insights (but see Golosov et al. 2014, Van der Ploeg and van den Bremer 2018 and Hambel et al. 2021 for examples of closed-formed solutions of stochastic IAMs).

A more recent approach to studying the question of physical risks regard climate change as an asset pricing problem, where CO_2 in the atmosphere, or inverse abatement levels, are considered assets. Financial-economics models of decisions under risk can provide interesting insights into the implications of uncertainty in the calculation of the carbon price. Bansal et al. (2016) explores the impact of long-term risks, as related to expected growth and volatility of future economic prospects, and of climate change on the social cost of carbon and asset prices; Dietz et al. (2018) estimate the β-climate, defined as the elasticity of climate damages with respect to the increase in global mean temperature, where they obtained a positive and close to one climate-beta value for investment maturities of up to about one hundred years. This result is primarily due to the dominating positive effect of uncertainty over an emissions-neutral technological progress. Daniel et al. (2019) estimate the social cost of delay in implementing CO_2 prices in a dynamic asset pricing model with recursive preference.

Transitional risks: This strand of literature focuses on risk related to the transition to a carbon-free economy. Gollier (2021) discusses transition risk in an asset pricing framework, where he analyses the effects of uncertainties over abatement technologies and economic growth on the dynamics of efficient carbon prices, interest rates and risk-premiums. His results highlight the positive correlation between aggregate consumption and marginal abatement costs along the optimal abatement path, thus implying a positive carbon beta and an efficient growth rate of expected carbon prices greater than the risk-free rate. Lemoine (2021) derives an analytical model to portray the different channels through which uncertainty affects the social cost of carbon, and goes on to quantitatively estimate the impact of different sources of uncertainties on the marginal value of emission reductions.

Government policies, asset prices and multiple externalities: In addition to the above cited literature, exploring the effects of uncertainty over climate-economy interactions on asset prices, our paper also contributes to the work on the interaction between government actions and asset prices. Pastor and Veronesi (2012), Baker et al. (2016) and Kelly et al. (2016) are examples of the impact of policy uncertainty on the prices of assets that are exposed to different degrees of climate policy risk. Our paper looks more precisely into the implications of a regulator's lack of commitment in implementing a credible long-term carbon price trajectory on the beta of abatement investments and therefore their risk premium. On top of the above-mentioned areas of research, our modeling framework takes into consideration the role of multiple externalities explored in climate economics. In fact, it has been shown extensively in the recent literature that additional externalities have far reaching consequences for the design of policy instruments: R&D investments, learning-by-doing investments (Jaffe et al., 2005; Kalkuhl et al., 2012), interaction with the fiscal system (Goulder, 2013; Franks et al., 2015), and lack of commitment (Kalkuhl et al., 2020) all require well-designed policy packages for an efficient implementation.

Drawing on these strands of literature we build a decentralized market equilibrium version of the CCAPM model from Gollier (2021), where we include a technology externality as well as political economy constraints on carbon pricing in order to investigate their effect on the carbon beta.

3. The model

The focus of our research is to explore instruments that support an efficient implementation of climate policy in an economy with additional distortions and uncertainty. We consider (a) inefficient timing due to the ignoring of technological spillovers, and (b) lack of commitment power of the regulator. We first characterize the socially efficient solution from a social planner perspective. Our presentation closely follows the social planner model of Gollier (2021) but introduces technological learning. For the subsequent analysis of instruments, we introduce decentralized problems for all agents and solve for a market equilibrium of the decentralized economy.

3.1. Social planner benchmark

Assume a social planner who considers utility of consumption $u(C_t)$ in two periods $t = 0, 1$. Consumption C_t is the residual of income Y_t and abatement expenditures A_t. Income Y_t is an endowment and is associated with emissions $E_t = Q_t Y_t$ at an emission intensity of Q_t. The abatement level K_t reduces emissions in period t; furthermore, abatement K_0 in the first period has a spillover effect on future abatement cost, such that abatement at $t = 0$ affects abatement cost at $t = 1$: $A_1(K_0, K_1)$. For spillovers related to technology learning, K_0 reduces the cost of future abatement, i.e. ${}^{\partial A_1}/_{\partial K_0} < 0$. Carbon uptake in natural sinks, such as the ocean, remove carbon from the atmosphere at the emissions decay rate of δ. The planner's objective is to limit emissions to a carbon budget of T.

The exogenous carbon budget (rather than a Pigouvian carbon price that reflects the social cost of carbon) is calibrated to current EU policy in the numerical application of our model, which relies on a carbon budget consistent with carbon neutrality by 2050 in its *Green Deal*. Using a carbon budget approach avoids the uncertainties in quantifying climate damages.

The problem of the planner is thus:

$$\max_{K_0, K_1} u(C_0) + e^{-\rho}\mathbb{E}[u(C_1)] \tag{1}$$

$$\text{such that } C_0 = Y_0 - A_0(K_0) \tag{2}$$

$$C_1 = Y_1 - A_1(K_0, K_1) \tag{3}$$

$$T = (Q_0 Y_0 - K_0)e^{-\delta} + Q_1 Y_1 - K_1 \tag{4}$$

For an interior solution, the first order conditions of the planner yield the following asset pricing equation.

$$u'(C_0)A_0' = e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\underbrace{\frac{\partial A_1}{\partial K_0}}\right)\right] \tag{5}$$

Equation (5) is the asset pricing condition for investment in abatement. The form is analogous to Gollier (2021, equation (2)) but includes the underlined term for pricing the technological spillover. Technology learning with $\partial A_1/\partial K_0 < 0$ thus implies an increased social return on investment.

To derive the risk premium in a beta form representation, we view abated emissions in equation (5) as an asset with cost A'_0 and expected gross return $R^A_1 = \left(\partial A_1/\partial K_1 - e^{\delta}\partial A_1/\partial K_0\right)/A'_0$. We use the following Lemma to rewrite (5) using a beta-form representation of the risk premium.

Lemma 1. *Consider a representative agent with time-additive expected utility, with a subjective discount rate* ρ *and a constant relative risk aversion* ξ*, in a discrete-time setting with a risk-free asset traded each period. Assuming the relative growth rate of consumption* $g^c_\tau = c_\tau/c_t - 1$ *and gross return* $R_\tau = \frac{e^{-\delta\tau}A'_\tau}{A'_t} = e^{-\delta\tau}R^A_\tau$ *to be jointly lognormally distributed, then*

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[R^A_\tau\right]\right) = \delta + \frac{1}{\tau}\ln R^f + \frac{1}{\tau}\xi\sigma_{g^c}Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_t}\right]\sigma[\ln R^A_\tau]$$

and in beta-form

$$\mathbb{E}\left[\frac{A'_\tau}{A'_t}\right]^{\frac{1}{\tau}} = e^{\delta + r^f + \phi\eta}$$

with

$$\phi = \frac{Cov\left[r_\tau, \tilde{g}^c_\tau\right]}{Var\left[\tilde{g}^c_\tau\right]} \quad \text{and} \quad \eta = \frac{1}{\tau}\xi Var\left[\tilde{g}^c_\tau\right]$$

and r^f*,* r_τ*, and* $\tilde{g}^c_\tau$ *represent respectively* $\ln R^f$*,* $\ln R_\tau$*, and* $\ln\frac{c_\tau}{c_t}$*.* τ *is future time period* $t+1$*.*

The proof is provided in Appendix E. Based on Lemma 1, we have the expression for the two-period risk premium of abatement investments:

$$A'_0 = e^{-(r_f+\phi\eta)}\mathbb{E}\left[e^{-\delta}\frac{\partial A_1}{\partial K_1} - \frac{\partial A_1}{\partial K_0}\right] \tag{6}$$

That is, the growth rate of social marginal abatement costs should exceed the risk-free rate r_f by a risk-premium $\phi_t\eta_t$ where η_t and ϕ_t are the systematic risk premium and "carbon beta" respectively, as defined in Lemma 1.

3.2. The market economy

The market economy is populated by three agents: a representative, firm-owning household, a regulatory authority who imposes carbon pricing and a long-term investment fund whose subsidies tempt additional abatement. This section introduces the agents in turn.

In common with the social planner, the household faces the intertemporal decision of allocating its income between consumption and abatement at $t = 0, 1$ for a limited budget of emissions. However, the household only partially anticipates the technology externality (ψ) and does not anticipate emissions decay (δ). We will first assume that the announced permit budget T_0 of the regulator is credible and hence the regulators policy goal will be met. However due to externalities, the market economy will not achieve the efficient allocation of abatement for a given carbon budget as in the social planner economy. We therefore introduce several policy instruments to address the misallocation due to these externalities: the government can complement its budget of emission permits T_0 with an intertemporal trading ratio (γ) to control the growth rate of emissions. Additionally, we introduce a technology subsidy (κ) and a bonus on long-term abatement projects (σ).

Second, we relax the assumption of a credible announcement of T_0. The ensuing commitment problem of the regulator provides an additional distortion to the household decision. Since the subsidy (κ) and the bonus (σ) are delegated to an independent authority (the long-term investment fund), greater commitment is achieved.

Carbon pricing

The household is subject to regulation via an emission permit budget T_0 specified by the regulator. The regulator controls the growth rate of the resulting carbon price by discounting emission permits at an intertemporal trading ratio when banked. At $t = 1$ a banked permit covers e^{γ} emissions (instead of 1), i.e. the intertemporal trading ratio γ acts like an interest rate on saved permits.[2] For example, the regulator can incentivize early abatement using $\gamma < 0$. The necessary abatement at $t = 1$ can be expressed in terms of abatement at $t = 0$ and the emission permit budget:

$$T_0 = (Q_0 Y_0 - K_0) + e^{-\gamma}(Q_1 Y_1 - K_1) \tag{7}$$

Technology & investment subsidies

The long-term investment fund can play two roles by either reducing investment costs or enhancing the benefits of investment. Investment costs are reduced by paying a subsidy rate of κ on abatement expenditures $A_0(K_0)$, which helps to internalize the technology externality. Efficient management of the fund's resources suggests that the fund pays the technology subsidy only for

[2]*Intertemporal trading ratios* are due to Leiby and Rubin (2001). We assume that the regulator adjusts T_0 in anticipation of the allocation such that in the end emissions do not exceed the carbon budget T, i.e. for $\gamma \neq 0$ we have $T_0 = T + E_1(e^{-\gamma} - 1)$, where $E_1 = Q_1 Y_1 - A1(K0, K1)$ is the permit demand at t=1.

additional projects $\Delta_0 = K_0 - \bar{K}_0$ beyond a baseline $\bar{K}_0$ but this does not affect the incentive to invest.

$$\begin{aligned} C_0 &= Y_0 - A_0(\bar{K}_0) - \Big(A_0(K_0) - A_0(\bar{K}_0)\Big)(1-\kappa) + \Gamma_0 \\ &= Y_0 - A_0(K_0)(1-\kappa) + \kappa A_0(\bar{K}_0) + \Gamma_0 \end{aligned} \tag{8}$$

Second, the long-term investment fund can subsidize abatement investments by paying a bonus on the benefits of abatement projects. To this end, the fund offers a financial contract at $t = 0$ that commits the fund to paying a bonus (σ) at $t = 1$ on top of the expected return, which is the value of abated emissions in period $t = 1$. Specifically, for any additional emission reduction Δ_0 (monitored and verified as additional relative to the baseline $\bar{K}_0$ by the fund) the fund pays a bonus σp_1 on the corresponding period $t = 1$ permits $\Delta_1 = e^{\gamma}\Delta_0$ where p_1 is the permit price at $t = 1$.

$$\begin{aligned} C_1 &= Y_1 - A_1(K_0, K_1(\bar{K}_0 + \Delta_0)) + \underbrace{\sigma p_1 \Delta_1}_{\text{fund bonus}} - \underbrace{p_1 \omega}_{\text{new permits}} + \Gamma_1 \\ &= Y_1 - (1+\sigma)A_1(K_0, K_1(K_0)) + \sigma A_1(\bar{K}_0, \bar{K}_1) + \sigma p_1 \Delta_1 - p_1\omega + \Gamma_1 \end{aligned} \tag{9}$$

To establish the price p_1, we include the option to buy ω new permits in the permit market in the household's budget equation (9). If additional permits were sold at $t = 1$ the permit budget T_0 would be inflated by $e^{-\gamma}\omega$ permits (in terms of period 0 permits). The adjusted permit budget takes this into account. Together, (9-10) determine the demand for new permits.

$$T_0 + \underbrace{e^{-\gamma}\omega}_{\text{new permits}} = (Q_0 Y_0 - \bar{K}_0 - \Delta_0) + e^{-\gamma}(Q_1 Y_1 - K_1) \tag{10}$$

Note that unless the regulator relaxes the permit budget at $t = 1$ (which we consider in Section 4.2), the demand for new permits will meet an inelastic supply of $\omega = 0$ new permits by the regulator. In this case, the main benefit of introducing the permit market is to explicitly price emission permits at $t = 1$.

Equations (8) and (9) include lump-sum recycling of revenues Γ_t. Both subsidy payments for σ and κ will draw on the funds of the long-term investment fund. Similarly carbon pricing may generate revenues for the regulator if emission permits T_0 are not freely allocated but auctioned. We assume that the revenue side of these policies is budget neutral for the household, for example due to the regulator recycling any revenues from carbon pricing lump-sum to the households in the same period in which the revenues accrued, and likewise the fund is financed by the regulator via a lump-sum tax at $t = 0$ to cover the technology subsidy κ, and a lump-sum tax at $t = 1$ to cover the investment subsidy σ.

Market equilibrium

Together with the objective to maximize welfare, the household's problem hence becomes

$$\max_{\{K_0\}} u(C_0) + e^{-\rho} u(C_1) \tag{11}$$

$$\begin{aligned} \text{subject to (8), (9)} &: \text{ budget equations} \\ \text{(10)} &: \text{ emission permit budget} \\ \text{and given } (T_0, \gamma) &: \text{ the regulator's instruments} \\ (\sigma, \kappa) &: \text{ the fund's instruments} \end{aligned}$$

The first order condition of optimality for the household requires the permit price p_1 to reflect marginal abatement costs ($\mathbb{E}[u'(C_1)p_1] = \mathbb{E}[u'(C_1)A_1']$) and investment follows an asset pricing equation that balances (marginal) investment costs and their (discounted marginal) benefits (derived in Appendix B).

$$u'(C_0)\,(1-\kappa)\,A_0' = e^{-(\rho-\gamma)}\mathbb{E}\left[u'\,(C_1)\left((1+\sigma)\,p_1 - e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)\right] \tag{12}$$

The benefit of abatement K_0 is determined by the permit price p_1 (and technological learning). Equation (12) shows how carbon pricing and the investment subsidy σ are complementary in setting the incentive for investment K_0. Any shortfall in the expected carbon price p_1 can be corrected by an appropriately set investment bonus σ. The following proposition records this finding.

Proposition 1. *Carbon pricing and the subsidy of the investment fund are complementary for the incentive to invest in abatement. By offering $\sigma = p_1^*/p_1 - 1$ the fund can lift the investment incentive of any $p_1 > 0$ to the level of a desired carbon price p_1^*.*

If households cannot appropriate (or do not anticipate) any of the technology learning ${\partial A_1}/{\partial K_0}$ then they will act as if ${\partial A_1}/{\partial K_0} = 0$. We model partial appropriation of the return on innovation by introducing a scaling parameter ψ as a measure of the market failure into (12), where we substitute marginal abatement costs A_1' for p_1.

$$u'(C_0)(1-\kappa)A_0' = e^{-(\rho-\gamma)}\mathbb{E}\left[u'(C_1)\left((1+\sigma)A_1' - (1-\psi)e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)\right] \tag{13}$$

For $\psi = 1$ technological progress at time $t = 1$ from abatement at time $t = 0$ is a pure externality. That is, households do not take into consideration the feedback effect of abatement learning. For any $\psi < 1$, part of the externality is anticipated and thus internalized. For $\psi = 0$ there is no technology externality (we adopted this approach from Fischer and Newell 2008). Equation (13) shows how the household prices the abatement investment.

To see how risk takes effect in the asset pricing equation, we can re-express equation (13) using the risk free rate $r_f = \rho - \log\left(\mathbb{E}[u'(C_1)]/u'(C_0)\right)$.

$$\begin{aligned}(1-\kappa)A_0' =& e^{-(r_f-\gamma)}\mathbb{E}\left[(1+\sigma)A_1' - (1-\psi)e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right] \\ &+ e^{-\rho}\mathrm{Cov}\left(\frac{u'(C_1)}{u'(C_0)}, e^{\gamma}A_1'(1+\sigma) - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\end{aligned} \tag{14}$$

Equation (14) underlines the specific role of covariance in pricing abatement. It is precisely the covariance term in (14) which translates into the risk premium. The technology externality (ψ), as it appears in the covariance term, will therefore have an effect on the risk premium.

Similarly, the instruments γ and σ will have a direct effect on the risk premium in addition to their effect on the expected risk-free return (first term in the right-hand side of (14)). In contrast to κ, these two policies affect the balance of marginal abatement costs (A_1') and the technological spillover term, hence like ψ, the two affect the strength of the covariance.

When asset return and marginal utility of consumption are uncorrelated, causing the covariance term in (14) to vanish, so will the risk premium. Intuitively, while the asset return remains uncertain, it would have no systematic effect on the marginal utility of consumption.

We summarize these insights in a proposition.

Proposition 2. *When emissions decay (δ) or technology learning (ψ) are external to the household's decision problem, the asset pricing is distorted from the socially optimal rule in both the risk-free return and the risk premium. Of the three instruments, intertemporal trading ratio (γ) and investment subsidy (σ) have a direct effect on the risk premium.*

With the assumption of (log)normality as in Lemma 1 we can rewrite (14) in beta form:

$$
\begin{aligned}
(1-\kappa)\,A_0' =& e^{-(r_f+\tilde{\phi}\tilde{\eta})}\mathbb{E}\left[e^{\gamma}\left((1+\sigma)\,A_1' - (1-\psi)\left({}^{\partial A_1}/_{\partial K_0}\right)\right)\right] && (15)\\
\tilde{\phi} =& \frac{\mathrm{Cov}\left[\log\left(\left(A_1'(1+\sigma) - e^{-\gamma}(1-\psi)\frac{\partial A_1}{\partial K_0}\right)/A_0'\right), \log\left({}^{\mathbb{E}[u'(C_1)]}/_{u'(C_0)}\right)\right]}{\mathrm{Var}\left[\log\left({}^{\mathbb{E}[u'(C_1)]}/_{u'(C_0)}\right)\right]} && (16)\\
\tilde{\eta} =& \xi \mathrm{Var}\left[\log\left(\frac{\mathbb{E}[u'(C_1)]}{u'(C_0)}\right)\right] && (17)
\end{aligned}
$$

The asset pricing equations of the household (15) and social planner (6) show a very similar structure. In contrast to the household, however, the planner takes into consideration the decay rate δ, which is not part of the household's pricing equation since it is not anticipated, as mentioned earlier. Instead, the marginal abatement in period 1, in the household's derived pricing equation (15), is amplified by the intertemporal trading ratio γ. Where there is no technology learning, the intertemporal trading ratio is optimal when it is equal in absolute terms to the decay rate. When γ is greater than optimal, it implies an underinvestment in the initial period 0. Equation 16 reveals that not only the asset rate of return (expectation over the left-hand side in (15)) but also risk premium affect the carbon beta, as evident by the influence of ψ, γ and σ on $\tilde{\phi}$.

4. Policy analysis

Carbon pricing, by setting T_0, is efficient if there are no distortions. In the following, we explore how the market equilibrium is distorted in comparison to the social planner solution. We begin by focusing on the case where the emission permit budget of the regulator is credible, but later relax this assumption to consider the effect when the regulator has a commitment problem.

Perfect commitment to a fixed permit budget T_0 guarantees that the climate policy goal is not exceeded. But the intertemporal allocation of mitigation (K_0, K_1) may not be achieved when the carbon pricing signal is distorted by additional market failures. We discuss the failure to consider emissions decay (δ) and failure in innovation (technology externality ψ).

The case where the regulator cannot commit to T_0 is discussed as a sequential game with lack of commitment. When the regulator cannot commitment to the announced high-ambition climate policy, investors may lock the economy into a low-mitigation path, forcing the regulator to reconsider the ambition of the climate policy.

4.1. Intertemporal distortions

Parameters δ and ψ distort the asset pricing equation of the household compared to the social optimum.

To derive instruments σ and κ that address the distortions we equate the asset returns expressions of household and social planner from equations (5) and (13).

$$e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho+\gamma}\mathbb{E}\left[u'\left(C_1\right)\frac{1}{(1-\kappa)}\left((1+\sigma)A_1' - (1-\psi)\,e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)\right] \quad (18)$$

The expression simplifies to

$$(1-\kappa) = \frac{(1+\sigma)e^{\gamma}\mathbb{E}[u'\left(C_1\right)A_1'] + (1-\psi)\,\mathbb{E}[u'\left(C_1\right)(-\frac{\partial A_1}{\partial K_0})]}{e^{-\delta}\mathbb{E}[u'(C_1)A_1'] + \mathbb{E}[u'(C_1)(-\frac{\partial A_1}{\partial K_0})]} \quad (19)$$

4.1.1. Intertemporal trading ratio

Equation (19) contains the two distortions, $\delta > 0$ and $\psi > 0$. The intertemporal trading ratio γ can in market equilibrium easily play the role of emissions decay δ. Indeed for $\sigma = \kappa = 0$ we have

$$e^{\gamma} = e^{-\delta} + \frac{\psi\mathbb{E}\left[u'\left(C_1\right)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[u'\left(C_1\right)A_1'\right]} \quad (20)$$

Thus if there was no technology externality ($\psi = 0$) one could simply set $\gamma = -\delta$. But if $\psi > 0$ then an appropriate choice of the intertemporal trading ratio γ can simultaneously address this market failure.

4.1.2. Investment subsidy

The optimal subsidy on investment returns σ is additive in the two distortions which can be (simultaneously) addressed as follows:

$$\sigma = \left(e^{-(\delta+\gamma)} - 1\right) + \frac{\psi\mathbb{E}\left[u'\left(C_1\right)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{e^{\gamma}\mathbb{E}[u'\left(C_1\right)A_1']} \quad (21)$$

Any $\gamma \neq -\delta$ can be addressed by $(1 + \sigma) = \exp(-\delta - \gamma)$. The second term shows how σ addresses an externality in technological learning. Here, σ boosts the benefit on investment (the denominator) to make up for the share of learning ψ that cannot be appropriated (in the numerator). Rearranging (21) for $\gamma = -\delta$ shows this balance:

$$\sigma e^{\gamma}\mathbb{E}[u'\left(C_1\right)A_1'] = \psi\mathbb{E}[u'\left(C_1\right)(-\frac{\partial A_1}{\partial K_0})]$$

4.1.3. Technology subsidy

The optimal use of the technology subsidy κ is not as cleanly additive-separable in the two distortions as for σ above.

$$\kappa = \frac{(e^{-\delta} - e^{\gamma})\mathbb{E}[u'(C_1)A_1']}{\mathbb{E}[u'(C_1)(e^{-\delta}A_1' - \frac{\partial A_1}{\partial K_0})]} - \frac{e^{\gamma}\mathbb{E}[u'(C_1)(\sigma A_1' + \psi\frac{\partial A_1}{\partial K_0})]}{e^{-\delta}\mathbb{E}[u'(C_1)(A_1' - \frac{\partial A_1}{\partial K_0})]} \tag{22}$$

Note though how the first term vanishes for $\gamma = -\delta$. Then if $\psi > 0$ (and $\sigma = 0$), it is optimal to set κ as:

$$\kappa = \frac{\psi\mathbb{E}[u'(C_1)(-\frac{\partial A_1}{\partial K_0})]}{\mathbb{E}[u'(C_1)(A_1' - \frac{\partial A_1}{\partial K_0})]}$$

That is, κ is set to the share ψ of the learning effect that is not anticipated (as part of the overall benefit in the denominator).

Equation (22) also shows which combinations of (σ, κ) can be used.

The three policy instruments (γ, κ and σ) play, in essence, a similar role in attempting to correct the distortions arising from the externalities or factors of political economy. Yet, the timing of their implementation is different. κ as a technology subsidy is paid upfront to reduce abatement costs in period 0.

Hence there is no commitment problem for κ. Both γ and σ, however, are announced at $t = 0$ and implemented at $t = 1$ and thus susceptible to a commitment problem. For σ, the problem is less severe, as σ is a contractually agreed bonus and benefits from the commitment power of the legal system. Furthermore, independence from the regulator gives the investment fund more commitment power.

4.1.4. Instruments and the carbon beta

To further characterize how optimal instruments relate to the carbon beta, we divide the optimal asset pricing equations (6) and (15).

$$\frac{\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} = e^{(r_f+\tilde{\phi}\eta)-\gamma-(\delta+r_f+\phi\eta)} \tag{23}$$

The optimal κ is

$$\kappa = 1 - e^{(\phi - \tilde{\phi})\eta + (\gamma + \delta)} \frac{\mathbb{E}\left[(1+\sigma) A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} \tag{24}$$

The optimal σ is

$$\sigma = e^{(\tilde{\phi} - \phi)\eta - (\gamma + \delta)}(1-\kappa) \frac{\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{\mathbb{E}\left[A_1'\right]} - \frac{\mathbb{E}\left[A_1'\right] + \mathbb{E}\left[(1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[A_1'\right]} \tag{25}$$

The optimal instruments κ in (24) and σ in (25) are increasing in the spread between between the social carbon beta and the market carbon beta ($\tilde{\phi} - \phi$). Both optimal instruments, κ and σ, increase with $\tilde{\phi}$ as well as the spread. Hence when markets estimate a risk premium above the risk premium of the social planner, one of the subsidy instruments of the fund can be increased to compensate, and vice versa if the market's risk premium is lower.

In summary, the instruments of the investment fund can be used to affect asset returns and thus steer the economy towards the socially optimal equilibrium.

The role of the risk premium (carbon beta), however, remains unclear. In section 6 we turn to numerical simulations to shed light on the role of the carbon beta as the key determinant of the risk premium, and to estimate order-of-magnitudes of the distortionary effects on the intertemporal allocation. But first, we discuss the commitment problem of the regulator.

4.2. Climate policy commitment problem

A regulator who is not bound to carry out the announced climate policy may reconsider the emission permit budget at $t = 1$ and opt for a more lenient policy by issuing additional emission permits. The regulator may be prompted to reconsider the original policy if the investor does not invest as expected at the time the policy was announced. Gersbach and Glazer (1999) argue that investors may trigger the reconsideration by the regulator by strategically choosing not to invest. Below, we suggest a discrete game of regulator and investor that creates a similar incentive problem. When we use the numerical model to compute the payoff structure for plausible assumptions in section 6.2, we find that it is indeed rational for the investor to hold up on investment.

4.2.1. The commitment problem

To introduce the commitment problem into the model, we consider the case where the regulator and household consider two discrete possibilities for the ultimate emission budget. When the

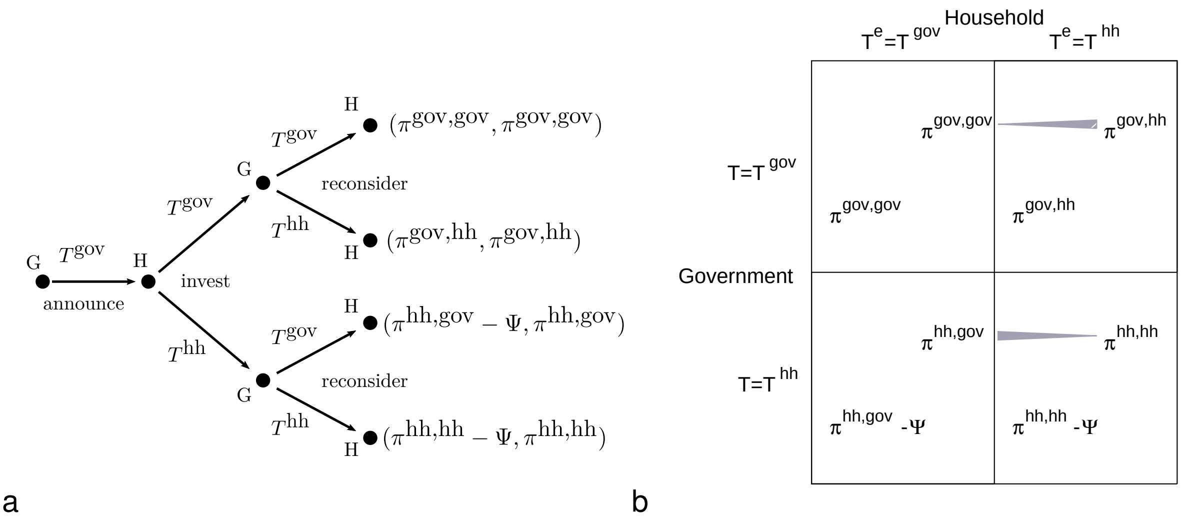

Figure 1: Commitment problem: The game structure is shown in extended form (panel a) where the government (G) announces the intended budget (T^{gov}), households (H) invest according to T^{gov} or T^{hh} (stage 1) before G decides (stage 2) to remain steadfast (T^{gov}) or tumble in their resolve (T^{hh}). Panel b summarizes the payoff structure.

regulator remains steadfast, no further emissions permits are issued, and the ultimate budget will be equal to the announced budget $T_0 = T^{gov}$. Otherwise, a regulator who topples in their resolve will issue additional emission permits ultimately imposing a high emission budget of $T_0 = T^{hh}$ with $T^{hh} < T^{gov}$.

We assume that when the regulator is able to commit to the emission budget $T_0 = T^{gov}$ at $t = 0$, the expectation of the household regarding the emission budget at $t = 1$ will be in line with the announcement, i.e. $T^e = T^{gov}$. When the announcement is not credible, the household will expect $T^e = T^{hh}$.

This setup gives rise to a simple two-stage game where the household can take the investment decision K_0 either based on $T^e \in \{T^{gov}, T^{hh}\}$ at $t = 0$, whereupon the regulator faces the decision at $t = 1$ to stick with the announced emissions budget $T = T^{gov}$ or issue more permits to a total of $T = T^{hh}$ (cf. Figure 1a).

The household realizes a payoff π^{ij} ($i, j \in \{gov, hh\}$) when acting in expectation of T^j while the regulator implements T^i. For the regulator, we assume a benevolent objective function, such that the regulator, too, maximizes household payoff π^{ij}. The regulator, though, incurs the cost of failing to meet the announced emission budget, which we capture in a penalty term Ψ. The penalty may include the anticipated climate change damages to the economy, loss of reputation and cost of non-compliance with international climate treaties (cf. Kalkuhl et al., 2020, for a similar approach). Obviously, the level of Ψ will be decisive for the preference of regulator and the structure of the game.

Figure 1b presents the payoffs in a 2×2 matrix. If the decision of the regulator is given, the household will always prefer to act in accordance with it, as this allows an efficient and hence welfare maximizing abatement choice (K_0, K_1). We can therefore conclude $\pi^{gov,gov} > \pi^{gov,hh}$ and $\pi^{hh,hh} > \pi^{hh,gov}$ from our model. We indicate this preference by arrows in Figure 1b.

If there were not costs to political failure (i.e. $\Psi = 0$) both players would share the same preferences and the game would have two Nash equilibria in symmetric strategies: (T^{gov}, T^{gov}) and (T^{hh}, T^{hh}). Our modeling assumptions further imply $\pi^{hh,hh} > \pi^{gov,gov}$ because the smaller budget implies higher abatement costs – and we have not included the benefits of avoided climate impacts in the model. Hence for $\Psi = 0$ the game has the structure of a no conflict coordination game (assurance game): there are two alternative Nash equilibria but there is no conflict as the players prefer the same equilibrium.

For higher penalty values (i.e. $\Psi > \pi^{hh,hh} - \pi^{gov,gov}$ 0) the more interesting case of a conflicting interests game emerges: the regulator will prefer the ambitious policy equilibrium (T^{gov}, T^{gov}) when the penalty exceeds the difference in payoff between the two equilibria. We summarize our analysis of the game structure in the following proposition.

Proposition 3. *The game between regulator and household defined in Figure 1 has two Nash equilibria in* (T^{gov}, T^{gov}) *and* (T^{hh}, T^{hh}). *Depending on the policy cost relaxing the permit budget* (Ψ)*, the structure is either an* assurance game *or a* conflicting interest coordination game*:*

1. Assurance game: *for* $\Psi = 0$ *both players prefer the low ambition equilibrium* (T^{hh}, T^{hh})*. The outcome of the sequential game is* (T^{hh}, T^{hh})*.*
2. Conflicting interest coordination: *if* $\Psi > \pi^{hh,hh} - \pi^{gov,gov}$ *the regulator prefers* (T^{gov}, T^{gov}) *in conflict with the household's preference. The outcome of the sequential game depends further on* Ψ*:*
   (a) *For* $\Psi \leq \pi^{hh,hh} - \pi^{gov,hh}$ *the outcome remains* (T^{hh}, T^{hh})*.*
   (b) *For* $\Psi > \pi^{hh,hh} - \pi^{gov,hh}$*, the outcome remains* (T^{gov}, T^{gov}) *as* T^{gov} *becomes a dominating strategy for the regulator.*

Proof. Stability of the Nash equilibria follows from the discussion above. In the sequential game, the household can commit by moving first and thus determine which equilibrium is played. The preference of the household is always for the low ambition equilibrium (T^{hh}, T^{hh}). However, when the regulator always, and despite its inefficiency, prefers the stringent policy (T^{gov}, T^{hh}) to the efficient but low ambition (T^{hh}, T^{hh}) (i.e. $\Psi > \pi^{hh} - \pi^{gh}$), T^{gov} is a dominant strategy for the regulator. In the sequential game, the household will anticipate this and also play T^{gov}. □

In the following, we are interested in the case of conflicting interest between regulator and household where the regulator's commitment problem introduces a deviation from the announced policy (case 2a). We therefore select $\Psi \in (\pi^{hh,hh} - \pi^{gov,hh}, \pi^{hh,hh} - \pi^{gov,hh})$.

Intuitively the distortion arises from the realization of the household that by delaying investment, as if they knew that the regulator would issue more permits up to T^{hh} at $t = 1$, the household creates a situation where the inefficiency of rushing through abatement at the last minute for T^{gov} at $t = 1$ is so costly that the regulator prefers to revise the policy and to issue additional permits.

4.2.2. Investment fund policies

We can see how the lack of commitment affects the household's incentive to invest from equation (12). The permit price p_1 therein is closely related to the shadow price of the emission permit budget equation (equation 10, see also λ in equation B.7), such that inflating the permit budget T_0 via additional permits ω at $t = 1$ implies a lower p_1.

As discussed before, the permit price determines the return on investment. The asset pricing equation (12) shows how the investment subsidy σ can counteract this effect by amplifying the remaining carbon price. If the investment fund has stronger commitment power than the regulator, it can overcome the commitment problem with an appropriate subsidy.

We will numerically investigate this in section 6.2.

5. Calibration

In most parameters choices, we follow Gollier (2021), which we summarize in Table 1. New parameters are introduced to the model in the extension to technology learning.

Abatement cost function with learning.

Continued use of a given technology builds experience which translates into an improved efficiency of the technology. A prominent approach that captures such technology learning *by doing* is to make its marginal cost dependent on the past cumulative investment (see Guo and Fan, 2017, for a recent example). Samadi (2018) reports technological *learning rates* for electricity generation, where a learning rate of lr indicates a decrease in costs for each doubling of cumulative installed capacity. The study includes estimates for future learning rates for renewable energy technologies, reporting 3-5% for wind turbines and 12-20% for solar photovoltaics.

We build on the abatement cost function of Gollier (2021) at $t = 1$ ($A_1(K_1) = \theta K_1 + \frac{1}{2} b K_1^2$). In line with Guo and Fan (2017), we include the learning dynamics in the non-linear term. Here, past

Parameter Descriptions	Notations	Values
annual rate of pure preference for the present	ρ	0.5%
parameter of relative risk aversion	ξ	3
annual probability of a macroeconomic catastrophe	p	1.7%
mean growth rate of production in a business-as-usual year	μ_{bau}	2%
volatility of the growth rate of production in a business-as-usual year	σ_{bau}	2%
mean growth rate of production in a catastrophic year	μ_{cat}	-35%
volatility of the growth rate of production in a catastrophic year	σ_{cat}	25%
production in the first 15-year period (in trillion US$)	Y_0	315
annual rate of natural decay of CO_2 in the atmosphere	δ	0.5%
carbon intensity of production in period 0 (in $GtCO_2e/GUS\$$)	Q_0	2.10×10^{-4}
carbon intensity of production in period 1 (in $GtCO_2e/GUS\$$)	Q_1	1.85×10^{-4}
expected carbon budget (in $GtCO_2e$)	μ_T	40
standard deviation of the carbon budget (in $GtCO_2e$)	σ_T	10
slope of the marginal abatement cost functions (in $GUS\$/GtCO_2e^2$)	b	1.67
slope of marginal abatement cost with learning (in $GUS\$/GtCO_2e^2$)	c_0	5.04
technology learning rate (in percent)	lr	20.0
marginal cost of abatement in the BAU, first period (in $GUS\$/GtCO_2e$)	a_0	23
expected future log marginal abatement cost in BAU	μ_θ	2.31
standard deviation of future log marginal abatement cost in BAU	σ_θ	1.21

Table 1: Benchmark calibration of the two-period model.

experience is simply given by first period investment K_0.

$$A_1(K_1, K_0) = \theta K_1 + \frac{1}{2} c_0 K_1^2 K_0^{-\alpha}$$

We explore an optimistic scenario with learning rate of $lr = 20\%$ at the upper end of the empirically observed learning rates (Samadi, 2018). The learning rate lr translates to the learning elasticity parameter as $lr = 1 - 2^{\alpha}$.

For consistency with the original calibration, we adjust $c_0 < b$ such that $A_1(K_1) \approx A_1(K_1, K_0)$, taking (K_0, K_1) from the equilibrium without technology learning.

6. Numerical results

In this section we consider two distortions that rationalize an underinvestment: technology learning as an externality to the household, and non-credibility of the emission budget such that households expect additional emission permits to be issued. For both distortions, we analyze their impact on asset pricing and risk premium and suggest investment fund policies to overcome the resulting inefficiencies. To address the distortions, we consider the timing of carbon pricing (via

scenario	K_0	$E[K_1]$	p_0	$E[p_1]$	r_f	ϕ	η	$r_f + \phi\eta$	welfare
$\psi = 0$	35.0	62.2	81.5	120.8	1.29	1.30	2.26	4.23	0.0
$\psi = 1$	32.5	64.5	77.3	127.0	1.26	1.02	2.26	3.57	-0.2
$T^e = T^{gov}$	35.0	62.2	81.5	120.8	1.29	1.30	2.26	4.23	0.0
$T^e = T^{hh}$	17.5	78.5	52.2	178.5	1.26	2.02	2.31	5.92	-15.8

Table 2: Overview of distortions. We report the effects of an externality in technology learning (parameter ψ) and the (equilibrium) effect of (wrongly) expecting a doubling of the carbon budget T by issuing additional permits in the second period. Welfare is given as the difference to undistorted equilibrium in *bps* of constant welfare-equivalent consumption levels.

the intertemporal trading ration γ of the regulator) and fund instruments consisting of subsidies on up-front capital costs and premium on investment returns.

Table 2 gives a preview of key insights: we summarize the impact of the two distortions on key variables of the model. Both distortions delay abatement, and hence produce a steeper carbon price path. In the distorted equilibria, the 'carbon beta' ϕ lies below the reference case. As a measure of the distortion generated when the regulator's announcement of the carbon budget is not credible, we show a scenario where the household invests in anticipation of a higher T but has to face the originally announced low T in the second period.

The welfare costs of the distortion are greater for the commitment problem (about 16 bps compared to just 0.2 bps). The effect of the technology externality on ϕ is also reduced but results in a smaller underinvestment, and the associated welfare costs are hence lower.

6.1. Technology externality

Figure 2 shows asset return and the required rate of return by investors for the economy extended by technological learning.

Asset returns were computed from the expected benefit in (13) over the investment costs (left-hand side), the required rate of return is given by $r_f + \phi\eta$ in the same equation. While (13) demands that the two expressions balance in equilibrium, Figure 2 shows the out of equilibrium values (except where corresponding lines intersect).

When technology learning is an externality ($\psi = 1$, square points in Figure 2a), the asset return is lower than with full appropriation ($\psi = 0$) because the social benefits of innovation are ignored by the household. All else being equal, this would shift the equilibrium to the left. That is, the household would underinvest in K_0.

However $\psi = 1$ has a second effect: it reduces the carbon beta and hence the risk premium (shifting the red line downwards). The carbon beta captures the covariance of the asset return with

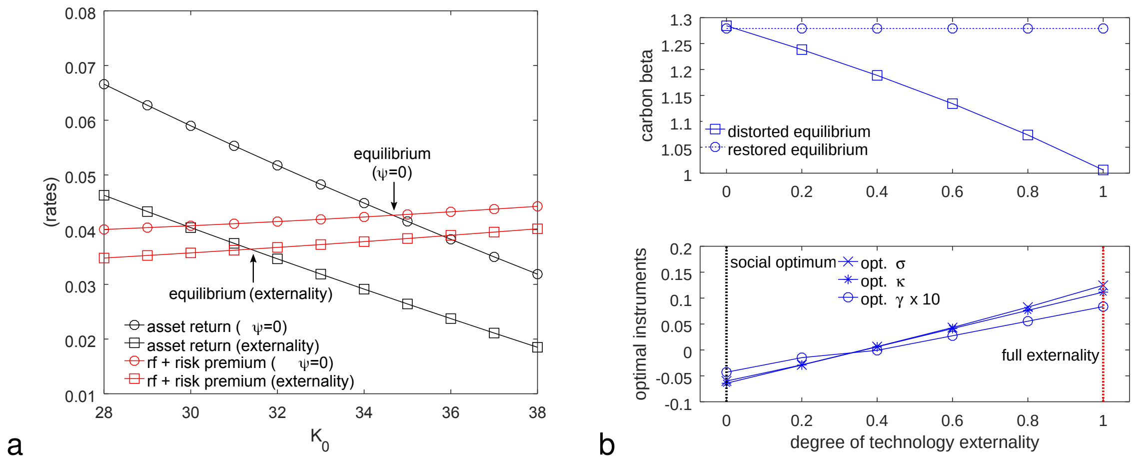

Figure 2: Technology externality. Panel a shows the actual asset return as a function of K_0 around the equilibrium (in black) as well as the return demanded by investors (in red). Intersection curves of the same bullet shape indicate an equilibrium. Panel b documents the implementation of optimal policies as a variation of ψ over the range $\psi \in [0.0, 1.0]$. At the top, we show the carbon beta in the distorted equilibrium and the equilibrium restored to the social optimum by the optimal instrument. At the bottom, we show the optimal instruments (σ, κ, γ).

consumption C_1. To see why the carbon beta decreases in ψ, note that according to (16) consumption covaries with the sum of A_1' and $-(\partial A_1/\partial K_0)$. The individual covariances of both summands with consumption are positive following the argument that high growth implies high consumption and simultaneously necessitates high investment K_0, which raises A_1' and lowers $\partial A_1/\partial K_0$. The covariance of the sum is the sum of the covariances, thus when $\psi = 1$ eliminates $\partial A_1/\partial K_0$ from the sum, the remaining covariance will be lower. Thus, the greater the extent to which technology learning is external (larger ψ), the lower the carbon beta from the perspective of the household.

In equilibrium, the reduced carbon beta implies a lower required rate of return which would raise investment K_0. The implications of ψ on the asset return and the required rate of return including the risk premium thus work in opposite directions.

In this calibration, the asset pricing effect is dampened by the risk premium effect but the former outweighs the latter: the overall effect of the technology externality is an underinvestment in K_0.

Policies

The regulator and the investment fund can use their instruments to restore the efficient equilibrium (cf. Figure 2b).

Additional numbers are shown in the appendix in Table G.5, which reports the financial variables for variations of the externality parameter ψ and both policy instruments σ and κ.

Both κ and σ ultimately cause an increase in the rate of return r_A. But the κ-policy works by reducing the asset price I_0 (net costs) while the σ-policy works by increasing asset benefit B_1.[3]

Climate policy uphill battle

With this endogenous change of the carbon beta, climate policy support becomes an uphill battle. Successful climate policy produces more abatement, for example via the investment premium σ of the fund, resulting in a higher (carbon) beta of abatement and thus the risk premium demanded for subsequent investment in abatement projects. Figure 3a illustrates this effect. The investment subsidy σ has the effect of increasing investment in the first period but decreases marginal effectiveness as it increases the carbon beta with higher abatement levels. The asset return is more greatly reduced when consumption is low and thus the covariance of asset return and consumption is strengthened. An investment that pays off in bad states of the world is worth more than an investment that pays off in good states. Paradoxically, successful climate policy leads to a higher risk premium for additional abatement; the more policy makers want to push green abatement technologies the higher the risk premium demanded by investors.

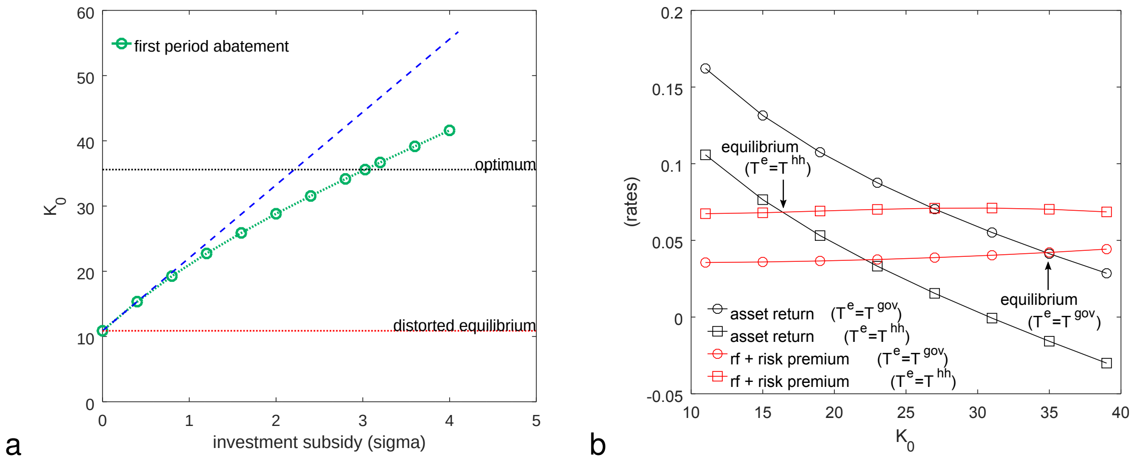

Figure 3: Panel a shows the effect of the fund subsidy on capital accumulation. The distorted equilibrium is the case of the non-credible climate policy from section 4.2. Panel b shows, for the case of commitment failure, the asset return as a function of K_0 around the equilibrium (in black) as well as the return demanded by investors (in red). Intersection curves of the same bullet shape indicate an equilibrium.

6.2. *Policy failure: non-credible climate policy*

In this section we numerically explore scenarios where the regulator is unable to commit to the low emission budget $T_0 = T^{gov}$ announced at $t = 0$.

[3]The asset price (benefit) moves in the same direction when the asset benefit (price) is affected by σ (κ) but in both cases this effect is smaller such that in total, the rate of return r_A increases.

Figure 3b visualizes the impact of shifting from a stringent budget of $T^{hh} = 40$ GtCO2e to a lenient budget of $T^{gov} = 80$ GtCO2e. The rate of return is substantially lower for the inflated budget, which reflects the deteriorated carbon price for this budget. This implies underinvestment in K_0. Relaxing the permit budget also affects the carbon beta. The risk premium is considerably higher for the larger budget mainly due to a more than two-fold increase of the carbon beta. Intuitively, for a lenient budget there are more likely to be excess permits in period $t = 1$. Economic output Y_1, which is tightly correlated with consumption C_1, determines emissions and hence whether the permit constraint is binding or non-binding. In the latter case, no more abatement is required and the carbon price is minimal, hence uncertainty about abatement costs and climate become less relevant. When uncertainty about economic growth dominates, the remaining variance in asset returns will be more closely related to it, resulting in a larger covariance which translates into a higher carbon beta. When the investor subsequently demands a higher risk premium, the effect on K_0 works in the same direction as the reduced rate of return, thus amplifying the underinvestment.

Non-credible regulator

Figure 4a shows the payoff matrix constant equivalent consumption levels of the household welfare. We normalize $\pi^{gov,gov}$ to zero to ease comparison. The payoff structure supports two outcomes as Nash equilibria when regulator and household decide simultaneously: $(T^{gov,gov}, T^{gov,gov})$ and (T^{hh}, T^{hh}). In such a *conflicting interest coordination game*, where either one of the two equilibria is preferred by one actor, actors can pick the game outcome if they can credibly commit to a strategy. In our case, the household moves first (deciding on investment K_0 at $t = 0$) and can therefore select (T^{hh}, T^{hh}).

Commitment by investment fund subsidies

The investment fund can support the regulator by subsidizing investment in abatement at $t = 0$ to raise investment K_0. As K_0 gets close to its optimal level for a low emission budget T^{gov}, the inefficiency of imposing T^{gov} even on a household that expected T^{hh} shrinks. Any $\sigma > 0$ thus reduces the incentive for the regulator to revise the announced policy of $T = T^{gov}$.[4]

Ideally, the fund subsidizes K_0 up to its optimal level for the T^{gov} budget. In this case, (T^{gov}, T^{hh}) is identical to (T^{gov}, T^{gov}), and these two become Nash equilibria instead of formerly (T^{hh}, T^{hh}). This game-theoretic setting provides compelling insights into the sequencing of climate policy; the investment fund can pave the path for carbon pricing in later periods. The inability of the regulator to commit to the announced carbon price in a credible way creates a situation where

[4]This is exacerbated by the increasing inefficiency of the (T^{hh}, T^{hh}) outcome: as σ increases, the choice of K_0 will be inefficiently high.

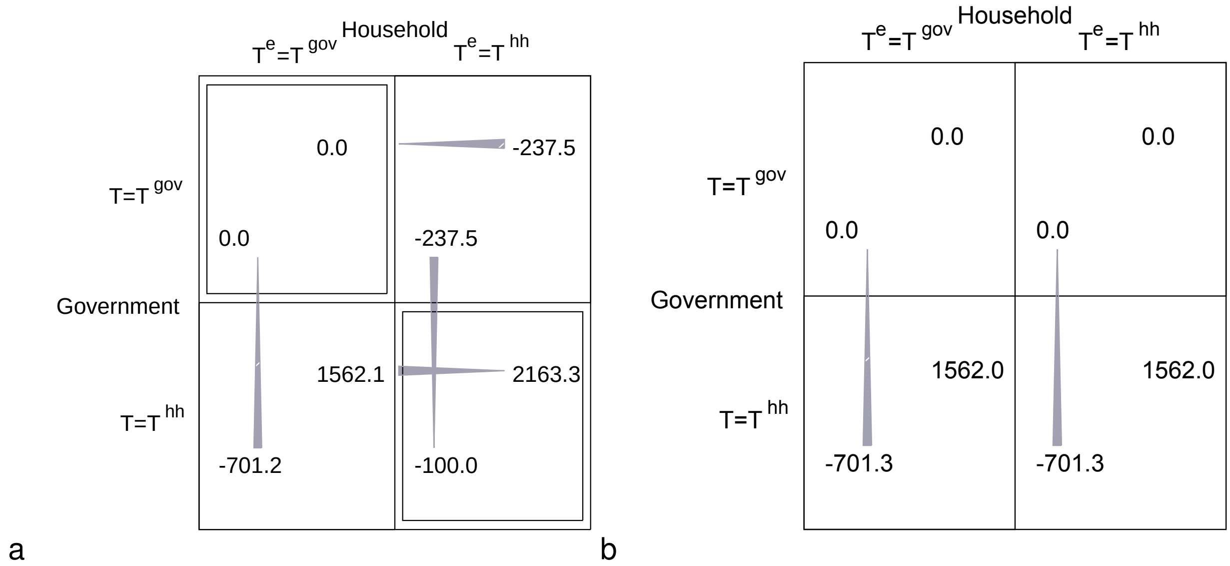

Figure 4: Payoff structure for the commitment game. Gray arrows indicate preferences, gray boxes indicate Nash equilibria without interference from the investment fund (panel a) and with σ policy by the fund (panel b). Payoffs are in constant-equivalent consumption levels (billion US$ for the 30 year time period).

households demand a higher risk premium and subsequently underinvest. When the investment fund pays subsidies, it incentivizes the household to increase abatement in earlier periods. This eliminates the conflict and reduces the risk premium to its socially optimal level. The intertemporal allocation of investment is not distorted. Additionally, due to the investment funds strategy, the social optimum is a Nash equilibrium. It should also be noted that carbon pricing needs to remain an essential part of the policy package, otherwise welfare costs will increase, and the budget constraint will be violated.

Proposition 4. *The sub-optimal outcome of the sequential game with conflicting interest coordination can be avoided by setting one of the subsidy instruments of the fund (σ or κ) to its welfare maximizing level.*

Table 3 presents additional numbers from the numerical simulations. Row 1 is the reference case with credible commitment, or (T^{gov}, T^{gov}). Rows 2-5 are (T^{hh}, T^{hh}) and (T^{gov}, T^{hh}), without and with optimal fund interventions, respectively. Rows 6-7 show analogous results for κ instead of σ. To compute rows 3, 5 and 7 we ran the model for $T = T^{hh}$ to calculate the first stage decision of K_0. Then, taking K_0 as given we ran the model for the second stage of K_1 for $T = T^{gov}$. The investment decision in these row was taken with the expectation of a rate of return $E[r_A]$ reported in the row above, and therefore we omit values that related to the investment decision.

Baseline with no credible carbon budget $T^{hh} = T^{gov}$															
T^{hh}	T^{gov}	σ	p_0	$E[p_1]$	I_0	$E[B_1]$	$E[r_A]$	K_0	$E[K_1]$	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
40	40	0.01	81.5	120.6	81.5	151.2	4.12	35.0	62.2	1.26	1.30	2.28	4.22	0.000	0.0
Noncredible carbon budget with $T^{hh} > T^{gov}$															
T^{hh}	T^{gov}	σ	p_0	$E[p_1]$	I_0	$E[B_1]$	$E[r_A]$	K_0	$E[K_1]$	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
100	100	0.01	44.1	74.9	44.1	91.7	4.89	12.6	24.3	1.34	2.92	2.27	7.97	-0.000	0.0
100	40	0.01	44.1	205.8	44.1	385.4		12.6	83.0					-0.294	110.1
100	100	3.14	81.5	33.5	81.5	132.7	3.25	35.0	7.8	1.46	1.57	2.29	5.06	-0.181	-79.7
100	40	3.14	81.5	120.6	81.5	580.4		35.0	62.2					-0.000	0.0
T^{hh}	T^{gov}	κ	p_0	$E[p_1]$	I_0	$E[B_1]$	$E[r_A]$	K_0	$E[K_1]$	r_f	ϕ	η	$r_f + \phi\eta$	ΔW	ΔA_1
100	100	0.01	43.5	75.0	43.5	90.6	4.90	12.6	24.3	1.34	2.92	2.27	7.97	0.000	0.1
100	40	0.01	43.5	205.9	43.5	380.7		12.6	83.0					-0.294	110.2
100	100	0.76	19.7	33.5	19.7	32.1	3.25	35.0	7.9	1.46	1.57	2.29	5.06	-0.181	-79.7
100	40	0.76	19.7	120.6	19.7	140.3		35.0	62.2					0.000	0.0

Table 3: Policy failures fixed by adjusted σ. T^{hh} denotes the carbon budget expected by the household. In the baseline, the household accepts the regulators announced carbon budget as credible, hence $T^{hh} = T^{gov}$. In the following, the household expects the regulator's budget to be exceeded ($T^{hh} > T^{gov}$) and chooses K_0 based on this expectation. We explore scenarios where the regulator topples in his policy ($T^{gov} = T^{hh}$) or remains steadfast ($T^{gov} < T^{hh}$). When the long-term investment fund acts ($\sigma > 0$), its subsidy aims to lift K_0 to its optimum level. The last columns show efficiency costs of inaction by the regulator. We show the differences in welfare (ΔW in percent balanced growth equivalent change) and abatement costs (ΔA_1 in percent change) relative to the optimal solution for the carbon budget that is *implemented*.

Rows 1 and 2 mirror the equilibria in Figure 1a for low and high T, respectively.[5] As discussed above, moving to a larger emission budget lowers the asset return while increasing the equilibrium risk premium, and investment in equilibrium K_1 is reduced.

In equilibrium, the asset return is higher for the larger budget (row 2 versus row 1) due to a higher growth rate of the carbon price – but at a much lower price level (cf. p_0, $\mathrm{E}[p_1]$). The carbon beta is more than double for the higher budget.

In the policy scenario (row 4) the investment subsidy σ is chosen such that K_0 (and hence p_0) match the reference case. Notice that the investment benefit B_1 includes the fund investment subsidy, such that the benefits (and subsequently the asset return) almost match the reference case benefits (row 1) despite a much lower carbon price at $t = 1$.

The two cost metrics ΔW and ΔA_1 in the last columns measure gains in welfare (in percent balanced growth equivalents) and abatement costs (percent), respectively, relative to the planner solution with the same carbon budget. Abatement costs are substantially higher in $t = 1$ where the household expects additional emission permits which are not supplied by the regulator (row 3). The subsequent welfare loss underlines the temptation for the regulator to revise the announced policy. The scenario with intervention by the investment fund (row 4) shows similar welfare losses; it distorts the economy unnecessarily because the fund policy is undercut by the regulator revising the budget. However the substantially reduced abatement costs show that the fund policy has prepared the economy for a more ambitious carbon budget.

Rows 6-7 show analogous computations for a technology subsidy (κ) instead of the investment subsidy (σ). The result is almost identical. We highlight two differences:

- The technology subsidy reduces investment costs rather than boosting their benefit, which is reflected in much lower I_0 and B_1. The resulting $r_A = \log(B_1/I_0)$ is the same.
- As discussed before, the difference between the technology subsidy and investment subsidy is in the time period over which they are paid. This lends an advantage to the technology subsidy which, by definition, has no commitment problem.

We summarize the results of the numerical analysis in Table 4,

which reports the changes for asset prices and risk premium caused by the policy instruments under *ceteris paribus* conditions and in a general equilibrium setting. In the first case, abatement investments are fixed; in the second-setting abatement investments are adjusted to the optimal level.

[5]Note though that we used a larger $T^{hh} = 100$ compared to $T^{hh} = 80$ in Figure 3b and Figure 4. This serves to (a) exacerbate the effects while (b) simultaneously illustrating their robustness.

All else being equal, the technology externality reduces the risk premium while non-credibility of the emission budget raises the risk premium.

	asset return		risk premium	
	ceteris paribus	equilibrium	ceteris paribus	equilibrium
technology externality (ψ)	⊖	⊖	⊖	⊖
non-credibility (T^{gov})	⊖	⊕	⊕	⊕

Table 4: Summary of distortion effects on asset return and risk premium. We distinguish the direct effect, keeping everything else the same (*ceteris paribus*), i.e. without adjustment of the investment decision K_0, and the entirety of equilibrium effects, i.e. after K_0 is adjusted to the new (distorted) equilibrium. We show reductions and increases relative to the undistorted equilibrium using ⊖ and ⊕.

7. Conclusions

This paper studies transition risk induced by climate policy in a CCAPM model, i.e. we focus primarily on risks to cash flows arising from a transition to a low-carbon economy induced by a carbon budget. The carbon budget captures physical risks of climate change (such as damages on productive assets) indirectly: these risks are not modeled explicitly but are reflected in the budget as they determine the choice of the carbon budget by the regulator. This modeling approach allows us to trace the carbon beta of abatement investments to climate policy instruments, e.g. carbon pricing, subsidies on up-front capital costs and investment premium. Additionally, we considered the risk premium induced by the lack of commitment of the regulator.

Studying climate policy through the lens of financial economics provides several crucial insights.

First, an investment that pays off in bad states of the world (with low economic growth) is worth more than an investment that pays off in good states (high economic growth). This basic truth from financial economics carries over to abatement investment and hence climate policy analysis. The covariance, and therefore the risk premium, are key for the design of climate policy instruments. Policy instruments change the investment pathway and therefore, the climate beta. We find that the effect of climate policy instruments on the risk premium of abatement investments may be neutral, increasing or decreasing depending on the nature of the distortion that is addressed by the instrument.

Second, financial market actors such as an investment fund can in principle address the distortions by setting financial incentives for green investment but need to take into account the distorted risk perception of investors. As we have shown, financial incentives for investors can complement a carbon pricing policy and cure its dynamic inefficiency or pave the path towards more ambitious carbon pricing. The policy failure experiments have also emphasized the importance of carbon pricing. Carbon prices which reflect a lack of commitment exhibit a substantial potential to cause an increase in the risk premium, which then acts as a brake on abatement and climate policy.

Third, the nascent literature on applying asset pricing theory to climate change mitigation has focused its analysis on how risk and uncertainty affect first-best mitigation policies and the associated social costs of carbon or carbon price trajectories which are consistent with the carbon budget. The avenue taken in this paper intends to connect the second-best analysis with this financial economics approach. We have shown that the risk premium is a fundamental endogenous variable determined by the regulator and agents on the financial markets, e.g. a long-term investment fund. The welfare losses of an distorted risk premium might be significant given the mis-allocation of capital.

This study takes a first step to discuss climate policy as an asset pricing problem in a second-best setting. While the simple framework illustrates the key role of correlated risks our analysis remains stylized in many aspects with room for improvements and extensions. Not all of the distortions that we considered are modeled endogenously, and integrating a micro-foundation for the distortion (as with the technology externality) could produce further insights. Of course, extensions could add additional distortions to the model. Short- and long-termism of investors or the introduction of incomplete markets into the model may be particularly interesting to shed light on the role of institutional investors.

While early assessments of optimal carbon pricing and second-best policy instruments relied on deterministic settings, more recent work has included uncertainties and asset pricing logic in the discussion of optimal carbon prices. Climate policy assessment can benefit enormously when climate economics is combined with financial economics. The large uncertainties that are ubiquitous in the assessment of climate policy can translate to substantial risk premiums in the calculus of investors, and it is important that policy instruments take this into account. Getting a better understanding of the level and structure of risk premiums will help avoid the mis-allocation of scarce resources. This is an important part of climate economics.

References

Baker, S. R., Bloom, N., and Davis, S. J. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, 131(4):1593–1636.

Bansal, R., Kiku, D., and Ochoa, M. (2016). Price of long-run temperature shifts in capital markets. *Capital Markets: Market Efficiency eJournal*.

Bertram, C., Luderer, G., Pietzcker, R. C., Schmid, E., Kriegler, E., and Edenhofer, O. (2015). Complementing carbon prices with technology policies to keep climate targets within reach. *Nature Climate Change*, 5(3):235–239.

Cai, Y., Judd, K. L., and Lontzek, T. S. (2013). The Social Cost of Stochastic and Irreversible Climate Change. *Working Paper 18704*, (January):1–38.

Cai, Y., Lenton, T. M., and Lontzek, T. S. (2016). Risk of multiple interacting tipping points should encourage rapid CO2 emission reduction. *Nature Climate Change*, 6(5):520–525.

Cai, Y. and Lontzek, T. S. (2019). The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734.

Cochrane, J. (2001). *Asset pricing*. Princeton Univ. Press, Princeton [u.a.].

Crost, B. and Traeger, C. (2014). Optimal CO2 mitigation under damage risk valuation. *nature climate change*, (4):631—-636.

Daniel, K. D., Litterman, R. B., and Wagner, G. (2019). Declining CO2 price paths. *Proceedings of the National Academy of Sciences*, 116(42):20886–20891.

Dietz, S., Gollier, C., and Kessler, L. (2018). The climate beta. *Journal of Environmental Economics and Management*, 87:258–274.

Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969.

Epstein, L. G. and Zin, S. E. (1991). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. *Journal of Political Economy*, 99(2):263–286.

Fischer, C. and Newell, R. G. (2008). Environmental and technology policies for climate mitigation. *Journal of Environmental Economics and Management*, 55(2):142–162.

Franks, M., Edenhofer, O., and Lessmann, K. (2015). Why Finance Ministers Favor Carbon Taxes, Even if They Do Not Take Climate Change into Account. *Environmental and Resource Economics*.

Gersbach, H. and Glazer, A. (1999). Markets and regulatory hold-up problems. *Journal of Environmental Economics and Management*, 37(2):151–164.

Giannousakis, A., Hilaire, J., Nemet, G. F., Luderer, G., Pietzcker, R. C., Rodrigues, R., Baumstark, L., and Kriegler, E. (2021). How uncertainty in technology costs and carbon dioxide removal availability affect climate mitigation pathways. *Energy*, 216:119253.

Giglio, S., Kelly, B., and Stroebel, J. (2021). Climate Finance. *Annual Review of Financial Economics*, 13. doi:10.1146/annurev-financial-102620-103311.

Gollier, C. (2021). The cost-efficiency carbon pricing puzzle. CEPR Discussion Paper DP15919. Available at SSRN: https://ssrn.com/abstract=3805342.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88.

Goulder, L. H. (2013). Climate change policy's interactions with the tax system. *Energy Economics*, 40:S3–S11.

Guo, J. X. and Fan, Y. (2017). Optimal abatement technology adoption based upon learning-by-doing with spillover effect. *Journal of Cleaner Production*, 143:539–548.

Hambel, C., Kraft, H., and Schwartz, E. (2021). Optimal carbon abatement in a stochastic equilibrium model with climate change. *European Economic Review*, 132:103642.

IPCC (2018). *Global Warming of 1.5°C – An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening* *the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty*. International Panel on Climate Change, Geneva, Switzerland.

Jaffe, A. B., Newell, R. G., and Stavins, R. N. (2005). A tale of two market failures: Technology and environmental policy. *Ecological Economics*, 54(2-3):164–174.

Jensen, S. and Traeger, C. P. (2014). Optimal climate change mitigation under long-term growth uncertainty: Stochastic integrated assessment and analytic findings. *European Economic Review*, 69:104–125.

Kalkuhl, M., Edenhofer, O., and Lessmann, K. (2012). Learning or lock-in: Optimal technology policies to support mitigation. *Resource and Energy Economics*, 34(1):1–23.

Kalkuhl, M., Steckel, J. C., and Edenhofer, O. (2020). All or nothing: Climate policy when assets can become stranded. *Journal of Environmental Economics and Management*, 100:102214.

Kelly, B., Pastor, L., and Veronesi, P. (2016). The price of political uncertainty: Theory and evidence from the option market. *The Journal of Finance*, 71(5):2417–2480.

Leiby, P. and Rubin, J. (2001). Intertemporal permit trading for the control of greenhouse gas emissions. *Environmental and Resource Economics*, 19(3):229–256.

Lemoine, D. (2021). The climate risk premium: how uncertainty affects the social cost of carbon. *Journal of the Association of Environmental and Resource Economists*, 8(1):27–57.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica: Journal of the Econometric Society*, pages 1429–1445.

Nordhaus, W. (2007). Critical assumptions in the stern review on climate change. *Science*, 317(5835):201–202.

Nordhaus, W. (2018). Projections and uncertainties about climate change in an era of minimal climate policies. *American Economic Journal: Economic Policy*, 10(3):333–60.

Nordhaus, W. D. (1992). An optimal transition path for controlling greenhouse gases. *Science*, 258(5086):1315–1319.

Nordhaus, W. D. (2017). Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523.

Pastor, L. and Veronesi, P. (2012). Uncertainty about government policy and stock prices. *The Journal of Finance*, 67(4):1219–1264.

Rudik, I. (2020). Optimal climate policy when damages are unknown. *American Economic Journal: Economic Policy*, 12(2):340–73.

Samadi, S. (2018). The experience curve theory and its application in the field of electricity generation technologies–a literature review. *Renewable and Sustainable Energy Reviews*, 82:2346–2364.

Staub-Kaminski, I., Zimmer, A., Jakob, M., and Marschinski, R. (2014). Climate policy in practice: a typology of obstacles and implications for integrated assessment modeling. *Climate Change Economics*, 5(01):1440004.

Stern, N. H., Peters, S., Bakhshi, V., Bowen, A., Cameron, C., Catovsky, S., Crane, D., Cruickshank, S., Dietz, S., Edmonson, N., et al. (2006). *Stern Review: The economics of climate change*, volume 30. Cambridge University Press Cambridge.

Van der Ploeg, R. and van den Bremer, T. (2018). The risk-adjusted carbon price. OxCarre Papers, CESifo Working Paper No. 7592. University of Oxford.

Appendix A. Asset pricing equation of the social planner

The Lagrangian reads

$$\mathcal{L} = u(Y_0 - A_0(K_0)) + e^{-\rho}\mathbb{E}[u(Y_1 - A_1(K_0, K_1))] \tag{A.1}$$

$$+ \lambda[(Q_0 Y_0 - K_0)e^{-\delta} + (Q_1 Y_1 - K_1) - T] \tag{A.2}$$

We consider first order conditions with respect to K_0 and K_1.

$$\frac{\partial \mathcal{L}}{\partial K_0} \stackrel{!}{=} 0 \Leftrightarrow u'(C_0)(-A'_0) + e^{-\rho}\mathbb{E}\left[u'(C_1)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] + \lambda[-e^{-\delta}] = 0 \tag{A.3}$$

$$\Leftrightarrow u'(C_0)(-A'_0) + e^{-\rho}\mathbb{E}\left[u'(C_1)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] = \lambda e^{-\delta} \tag{A.4}$$

$$\Leftrightarrow e^{\delta}u'(C_0)A'_0 + e^{-\rho+\delta}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_0}\right)\right] = -\lambda \tag{A.5}$$

$$\frac{\partial \mathcal{L}}{\partial K_1} \stackrel{!}{=} 0 \Leftrightarrow e^{-\rho}\mathbb{E}\left[u'(C_1)\left(-\frac{\partial A_1}{\partial K_1}\right)\right] + \lambda[-1] = 0 \tag{A.6}$$

$$\Leftrightarrow e^{-\rho}\mathbb{E}\left[u'(C_1)\frac{\partial A_1}{\partial K_1}\right] = -\lambda \tag{A.7}$$

Eliminate λ.

$$e^{\delta}u'(C_0)(A'_0) + e^{-\rho+\delta}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1}\right)\right] \tag{A.8}$$

$$u'(C_0)(A'_0) = e^{-\rho-\delta}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1}\right)\right] - e^{-\rho}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_0}\right)\right]$$

Appendix B. Asset pricing equation of the household

To derive its asset pricing equation, we restate the the problem of the household (11).

$$\mathcal{L} = u(Y_0 - A_0(K_0)) \tag{B.1}$$

$$+ e^{-\rho}\mathbb{E}\left[u(Y_1 - A_1(K_0, K_1) + \sigma p_1 e^{\gamma}(K_0 - \bar{K}_0) - p_1\omega)\right] \tag{B.2}$$

$$+ \lambda[T_0 - (Q_0 Y_0 - K_0) - e^{-\gamma}(Q_1 Y_1 - K_1 - \omega)] \tag{B.3}$$

For the optimal abatement choice K_1 at $t = 1$ we have

$$\frac{\partial \mathcal{L}}{\partial K_1} = 0 \quad \Leftrightarrow \quad 0 = e^{-\rho}\mathbb{E}[u_1'(-A_1')] + \lambda[e^{-\gamma}] \tag{B.4}$$

$$\lambda = e^{-\rho+\gamma}\mathbb{E}[u_1'A_1'] \tag{B.5}$$

For period $t = 1$, we first consider the choice of additional permits ω.

$$\frac{\partial \mathcal{L}}{\partial \omega} = 0 \quad \Leftrightarrow \quad 0 = e^{-\rho}\mathbb{E}[u_1' \cdot (-p_1)] + \lambda[-e^{-\gamma}] \tag{B.6}$$

$$\lambda = e^{-\rho+\gamma}\mathbb{E}[u_1' p_1] \tag{B.7}$$

Note that the shadow price λ of the permit budget equation is the expected permit price p_1 (when properly discounted and converted to utility units). By substituting in (B.5) we learn that the permit price p_1 reflects marginal abatement costs A_1':

$$e^{-\rho+\gamma}\mathbb{E}[u_1'A_1'] = e^{-\rho+\gamma}\mathbb{E}[u_1' p_1] \tag{B.8}$$

$$\mathbb{E}[u_1'A_1'] = \mathbb{E}[u_1' p_1] \tag{B.9}$$

Abatement K_0 at $t = 0$ follows from:

$$\frac{\partial \mathcal{L}}{\partial K_0} = 0 \quad \Leftrightarrow \quad 0 = u_0'(-(1-\kappa)A_0') + e^{-\rho}\mathbb{E}[u_1'(-\frac{\partial A_1}{\partial K_0} + \sigma p_1 e^{\gamma})] + \lambda \tag{B.10}$$

$$(1-\kappa)\,u_0'A_0' = e^{-\rho}\mathbb{E}[u_1'(-\frac{\partial A_1}{\partial K_0} + \sigma p_1 e^{\gamma})] + \lambda \tag{B.11}$$

$$= e^{-\rho}\mathbb{E}[u_1'(-\frac{\partial A_1}{\partial K_0} + \sigma p_1 e^{\gamma})] + \lambda \tag{B.12}$$

$$\tag{B.13}$$

Again, we substitute for λ.

$$(1-\kappa)\,u_0'A_0' = e^{-\rho}\mathbb{E}[u_1'(-\frac{\partial A_1}{\partial K_0} + \sigma p_1 e^{\gamma})] + e^{-\rho+\gamma}\mathbb{E}[u_1' p_1] \tag{B.14}$$

$$= e^{-\rho+\gamma}\mathbb{E}\left[u_1'\left((1+\sigma)p_1 - e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)\right] \tag{B.15}$$

Finally, we can use (B.9) to substitute the permit price with marginal abatement costs.

$$u_0' A_0' = e^{-\rho+\gamma}\mathbb{E}[u_1' \, (1-\kappa)^{-1} \left((1+\sigma)A_1' - e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)] \tag{B.16}$$

Appendix C. Optimal instruments

Based on (B.16) and the social optimum condition, we try to characterize optimal policies γ, σ, κ. We start by equating the discounted benefits of investment (once we move $(1-\kappa)$ to the RHS, the remaining costs are the same).

$$e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho+\gamma}\mathbb{E}\left[u'\,(C_1)\,\frac{1}{(1-\kappa)}\left((1+\sigma)A_1' - (1-\psi)\,e^{-\gamma}\frac{\partial A_1}{\partial K_0}\right)\right] \tag{C.1}$$

$$(1-\kappa) = \frac{(1+\sigma)e^{\gamma}\mathbb{E}[u'\,(C_1)\,A_1'] + (1-\psi)\,\mathbb{E}[u'\,(C_1)\,(-\frac{\partial A_1}{\partial K_0})]}{e^{-\delta}\mathbb{E}[u'(C_1)A_1'] + \mathbb{E}[u'(C_1)(-\frac{\partial A_1}{\partial K_0})]} \tag{C.2}$$

Appendix C.1. Investment subsidy sigma

There are two possible distortions, $\gamma \neq -\delta$ and $\psi > 0$. The optimal subsidy on investment returns σ is additive in the two distortions which can be (simultaneously) addressed as follows (equation (21) follows from (19) with some basic algebra):

$$\sigma = \left(e^{-(\delta+\gamma)} - 1\right) + \frac{\psi\mathbb{E}[u'\,(C_1)\,(-\frac{\partial A_1}{\partial K_0})]}{e^{\gamma}\mathbb{E}[u'\,(C_1)\,A_1']} \tag{C.3}$$

Any $\gamma \neq -\delta$ can be addressed by $(1+\sigma) = \exp(-\delta-\gamma)$. The second term shows how σ addresses an externality in technological learning. Here, σ boosts the benefit on investment (the denominator) to make up for the share of learning ψ that is not anticipated (in the numerator). Rearranging (21) for $\gamma = -\delta$ shows this balance:

$$\sigma e^{\gamma}\mathbb{E}[u'\,(C_1)\,A_1'] = \psi\mathbb{E}[u'\,(C_1)\,(-\frac{\partial A_1}{\partial K_0})]$$

Appendix C.2. Technology subsidy kappa

The optimal use of the technology subsidy κ is also additive in the two distortions. Note how the first term vanishes for $\gamma = -\delta$. In absence

$$\kappa = \frac{(e^{-\delta} - e^{\gamma})\mathbb{E}[u'(C_1)A_1']}{\mathbb{E}[u'(C_1)(e^{-\delta}A_1' - \frac{\partial A_1}{\partial K_0})]} - \frac{e^{\gamma}\mathbb{E}[u'\,(C_1)\,(\sigma A_1' + \psi\frac{\partial A_1}{\partial K_0})]}{e^{-\delta}\mathbb{E}[u'(C_1)(A_1' - \frac{\partial A_1}{\partial K_0})]} \tag{C.4}$$

If otherwise $\gamma = -\delta$, then the first parenthesis on the RHS vanishes. Then if $\psi > 0$ (and $\sigma = 0$), κ is set to the fraction ψ of the expected learning effect as a share of expected (true) benefits of the investment (with full anticipation). That is:

$$\kappa = \frac{\psi\mathbb{E}[u'(C_1)(-\frac{\partial A_1}{\partial K_0})]}{\mathbb{E}[u'(C_1)(A_1' - \frac{\partial A_1}{\partial K_0})]} \tag{C.5}$$

That is, κ is set to the share ψ of the learning effect that is not anticipated (as part of the overall benefit in the denominator).

Equation (22) also shows that combinations of (σ, κ) can be used.

Appendix D. Beta forms

Using Lemma 1, we can go from the asset pricing equation with marginal utilities

$$u_0'A_0' = e^{-\rho}\mathbb{E}[u_1'e^{\gamma}\,(1-\kappa)^{-1}\left((1+\sigma)A_1' - e^{-\gamma}\,(1-\psi)\,\frac{\partial A_1}{\partial K_0}\right)] \tag{D.1}$$

to the *beta form asset pricing* equation

$$(1-\kappa)A_0' = e^{-(r_f+\tilde{\phi}\eta)}\mathbb{E}\left[(1+\sigma)\,e^{\gamma}A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]$$

Take the social optimum asset pricing from the planner model:

$$\frac{\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{A_0'} = e^{\delta+r_f+\phi\eta} \tag{D.2}$$

$$\frac{\mathbb{E}\left[(1+\sigma)\,A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{(1-\kappa)A_0'} = e^{(r_f+\tilde{\phi}\eta)-\gamma} \tag{D.3}$$

$$\tag{D.4}$$

$$\frac{\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} = e^{(r_f+\tilde{\phi}\eta)-\gamma-(\delta+r_f+\phi\eta)} \tag{D.5}$$

$$\frac{\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} = e^{r_f+\tilde{\phi}\eta-\gamma-\delta-r_f-\phi\eta} = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)} \tag{D.6}$$

Isolate κ

$$\frac{\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)} \tag{D.7}$$

$$\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)}(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] \tag{D.8}$$

$$e^{(\phi-\tilde{\phi})\eta+(\gamma+\delta)}\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]/\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] = (1-\kappa) \tag{D.9}$$

$$\kappa = 1 - e^{(\phi-\tilde{\phi})\eta+(\gamma+\delta)}\frac{\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[A_1' - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]} \tag{D.10}$$

Isolate σ

$$\mathbb{E}\left[(1+\sigma)A_1' + (1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)}(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] \tag{D.11}$$

$$(1+\sigma)\mathbb{E}\left[A_1'\right] + \mathbb{E}\left[(1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)}(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] \tag{D.12}$$

$$\mathbb{E}\left[A_1'\right] + \sigma\mathbb{E}\left[A_1'\right] + \mathbb{E}\left[(1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right] = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)}(1-\kappa)\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right] \tag{D.13}$$

$$\sigma = e^{(\tilde{\phi}-\phi)\eta-(\gamma+\delta)}(1-\kappa)\frac{\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{\mathbb{E}\left[A_1'\right]} - \delta\frac{\mathbb{E}\left[A_1'\right] + \mathbb{E}\left[(1-\psi)\left(-\frac{\partial A_1}{\partial K_0}\right)\right]}{\mathbb{E}\left[A_1'\right]} \tag{D.14}$$

Appendix E. Proof of Lemma 1

Lemma 1. *Consider a representative agent with time-additive expected utility, with a subjective discount rate ρ and a constant relative risk aversion ξ, in a discrete-time setting with a risk-free* *asset traded each period. Assuming the relative growth rate of consumption* $g_\tau^c = c_\tau/c_0 - 1$ *and gross return* $R_\tau = \frac{e^{-\delta\tau}A'_\tau}{A'_0} = e^{-\delta\tau}R_\tau^A$ *to be jointly lognormally distributed, then*

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[R_\tau^A\right]\right) = \delta + \frac{1}{\tau}\ln R^f + \frac{1}{\tau}\xi\sigma_{g^c}Corr\left[\ln R_\tau^A, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau^A]$$

and in beta-form

$$\frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \delta + r^f + \phi\eta$$

or

$$\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]^{\frac{1}{\tau}} = e^{\delta + r^f + \phi\eta}$$

with

$$\phi = \frac{Cov\left[r_\tau, \tilde{g}_\tau^c\right]}{Var\left[\tilde{g}_\tau^c\right]} \qquad \text{(Appendix E.1)}$$

$$\eta = \frac{1}{\tau}\gamma Var\left[\tilde{g}_\tau^c\right] \qquad \text{(Appendix E.2)}$$

and r^f*,* r_τ*, and* $\tilde{g}_\tau^c$ *represent respectively* $\ln R^f$, $\ln R_\tau$, *and* $\ln\frac{c_\tau}{c_0}$.

Proof. Given the form of the utility function assumed, $u(c) = c^{1-\xi}/(1-\xi)$ then

$$\frac{u'(c_\tau)}{u'(c_0)} = \left(\frac{c_\tau}{c_0}\right)^{-\xi} = \exp\left\{-\xi\ln\left(\frac{c_\tau}{c_0}\right)\right\},$$

which is lognormally distributed, $\ln\left(\frac{c_\tau}{c_0}\right) \sim N\left(\bar{g}, \sigma_{g^c}^2\right)$

$$\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right] = \mathbb{E}\left[\exp\left\{-\xi\ln\left(\frac{c_\tau}{c_0}\right)\right\}\right] = \exp\left\{-\xi\bar{g} + \frac{1}{2}\xi^2\sigma_{gc}^2\right\}$$

and

$$\frac{\sigma\left[u'(c_\tau)/u'(c_0)\right]}{\mathbb{E}\left[u'(c_\tau)/u'(c_0)\right]} = \sqrt{e^{\xi^2\sigma_{gc}^2} - 1} \approx \xi\sigma_{gc}$$

The price of a risky asset can be expressed as[6]:

$$P_{i,\tau} = \mathbb{E}\left[m_\tau B_{i,\tau}\right]$$

[6]See Cochrane (2001) for more general asset pricing models

with $B_{i,\tau}$representing the payoff of the risky asset and m_τ the stochastic discount factor (also known as the state-price deflator) which is defined as $m_\tau \equiv \beta u'(c_\tau)/u'(c_0)$ with β the discount factor

$$\mathbb{E}[m_\tau B_{i,\tau}] = \mathbb{E}[m_\tau]\,\mathbb{E}[B_{i,\tau}] + \text{Cov}[B_{i,\tau}, m_\tau]$$

$$P_{i,\tau} = \mathbb{E}[m_\tau]\,\mathbb{E}[B_{i,\tau}] + \text{Cov}[B_{i,\tau}, m_\tau]$$

According to the asset pricing model, even though expected returns can vary across assets and time, expected discounted returns should be the same equal to 1. Then,

$$1 = \mathbb{E}[m_\tau]\,\mathbb{E}[R_{i,\tau}] + \text{Cov}[R_{i,\tau}, m_\tau]$$

$$\mathbb{E}[R_{i,\tau}] = \frac{1}{\mathbb{E}[m_\tau]} - \frac{1}{\mathbb{E}[m_\tau]}\text{Cov}[R_{i,\tau}, m_\tau]$$

$$\mathbb{E}[R_{i,\tau}] = R_t^f - \frac{\text{Cov}[R_{i,\tau}, u'(c_\tau)/u'(c_0)]}{\mathbb{E}[u'(c_\tau)/u'(c_0)]}$$

$$= R_t^f - \frac{\sigma[u'(c_\tau)/u'(c_0)]}{\mathbb{E}[u'(c_\tau)/u'(c_0)]}\sigma_t[R_{i,\tau}]\,Corr[R_{i,\tau}, u'(c_\tau)/u'(c_0)]$$

Hence,

$$\mathbb{E}[R_{i,\tau}] = R_t^f - \xi\sigma_g\sigma_t[R_{i,\tau}]\,Corr[R_{i,\tau}, u'(c_\tau)/u'(c_0)]$$

□

Instead of using marginal rate of substitution, the relation between *expected returns* and *relative consumption growth* can be approximated via $g_\tau^c = \frac{c_\tau}{c_0} - 1$. This is obtained by applying a first-order Taylor approximation of $u'(c_\tau)$ around c_0.

$$\frac{u'(c_\tau)}{u'(c_0)} \approx \frac{u'(c_0) + u''(c_0)(c_\tau - c_0)}{u'(c_0)} = 1 - \xi(c_0)g_\tau$$

where $\xi(c_0) = -c_0 u''(c_0)/u'(c_0)$ is the relative risk aversion of the individual evaluated at time 0 consumption level, and $g = c_\tau/c_0 - 1$ is the relative growth rate of consumption over the period. Replacing the above equation, we get

$$\mathbb{E}[R_\tau] - R^f \approx \xi(c_0)\text{Cov}[R_\tau, g_\tau^c] \qquad \text{(Appendix E.3)}$$

Assuming a time-additive utility with constant relative risk aversion parameter and the consumption growth to be lognormally distributed

$$\ln(1+g_\tau^c) \equiv \ln\left(\frac{c_\tau}{c_0}\right) \sim N(\bar{g}^c, \sigma_g^2) \qquad \text{(Appendix E.4)}$$

then

$$\mathbb{E}[R_\tau] - R^f = \sigma[R_\tau]\sqrt{e^{\xi^2\sigma_g^2} - 1}Corr\left[R_\tau, \left(\frac{c_\tau}{c_0}\right)^{-\xi}\right] \qquad \text{(Appendix E.5)}$$

$$\mathbb{E}[R_\tau] - R^f \approx \xi\sigma_{g^c}\sigma[R_\tau]Corr\left[R_\tau, \frac{c_\tau}{c_0}\right] \qquad \text{(Appendix E.6)}$$

One can make a stronger assumption, and assume that both gross rate of return R_i of the asset and the consumption growth g^c are jointly lognormally distributed. In that case,

$$\mathbb{E}[\ln(R_\tau)] - \ln R^f + \frac{1}{2}\text{Var}[\ln R_\tau] = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau] \qquad \text{(Appendix E.7)}$$

equivalently,

$$\ln(\mathbb{E}[R_\tau]) - \ln R^f = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma[\ln R_\tau] \qquad \text{(Appendix E.8)}$$

Following Gollier, P_0 is equivalent to A'_0 and $B_\tau = e^{-\delta\tau}A'_\tau$, and β the discount factor can be expressed as $\beta = e^{-\rho\tau}$. Hence,

$$R_\tau = \frac{B_\tau}{P_0} = \frac{e^{-\delta}A'_\tau}{A'_0} = e^{-\delta}R_\tau^A \qquad \text{(Appendix E.9)}$$

Then we can re-write equ. (43) as

$$\mathbb{E}\left[\ln\left(\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right)\right] - \ln R^f + \frac{1}{2}\text{Var}\left[\ln\left(\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right)\right] = \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln(R_\tau)\right]$$

$$\ln\left(\mathbb{E}\left[\frac{e^{-\delta\tau}A'_\tau}{A'_0}\right]\right) = \ln R^f + \xi\sigma_{g^c}Corr\left[\ln R_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln(R_\tau)\right]$$

$$-\delta\tau + \ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \ln\left(\frac{1}{e^{-\rho\tau}\mathbb{E}\left[u'(c_\tau)/u'(c_0)\right]}\right) + \xi\sigma_{g^c}Corr\left[\ln R_\tau^A, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R_\tau^A\right)\right]$$

$$\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right) = \delta\tau + \rho\tau - \ln\left(\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right]\right) + \xi\sigma_{g^c} Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R^A_\tau\right)\right]$$

$$\underbrace{\frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{A'_\tau}{A'_0}\right]\right)}_{g} = \delta + \underbrace{\rho - \frac{1}{\tau}\ln\left(\mathbb{E}\left[\frac{u'(c_\tau)}{u'(c_0)}\right]\right)}_{r^f} + \frac{1}{\tau}\xi\sigma_{g^c} Corr\left[\ln R^A_\tau, \ln\frac{c_\tau}{c_0}\right]\sigma\left[\ln\left(R^A_\tau\right)\right]$$

It can be expressed in "beta-form"

$$g = \delta + r^f + \phi\eta \qquad \text{(Appendix E.10)}$$

with,

$$\phi\left[r_\tau, \tilde{g}^c_\tau\right] = \frac{\text{Cov}\left[r_\tau, \tilde{g}^c_\tau\right]}{\text{Var}\left[\tilde{g}^c_\tau\right]}$$

$$\eta = \frac{1}{\tau}\xi\text{Var}\left[\tilde{g}^c_\tau\right]$$

Note:
A random variable X has log-normal distribution, if the random variable $Y = \ln X$ is normally distributed. Let μ be the mean of Y and σ^2 be the variance of Y so that $Y = \ln X \sim N(\mu, \sigma^2)$.
For $Y \sim N(\mu, \sigma^2)$ and $\xi \in \mathbb{R}$ we have

$$\mathbb{E}_t\left[e^{-\xi Y}\right] = \exp\left\{-\xi\mu + \frac{1}{2}\xi^2\sigma^2\right\}$$

$$var_t\left[e^{-\xi Y}\right] = \left(\mathbb{E}_t\left[e^{-\xi Y}\right]\right)^2\left[e^{-\xi^2\sigma^2} - 1\right]$$

Deriving expressions for optimal instruments

$$e^{-(\rho+\delta)}\mathbb{E}\left[u'(C_1)\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = e^{-\rho+\gamma}\mathbb{E}\left[u'(C_1)\frac{1+\sigma}{1-\kappa}A'^n_1\right]$$

$$\mathbb{E}\left[u'(C_1)e^{-\delta}\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)\right] = \mathbb{E}\left[u'(C_1)e^{\gamma}\frac{1+\sigma}{1-\kappa}A'^n_1\right] \qquad \text{(Appendix E.11)}$$

With $Z_1 := e^{-\delta}\left(\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right)$ and $Z_2 := e^{\gamma}\frac{1+\sigma}{1-\kappa}A'^n_1$ we have

$$\mathbb{E}\left[u'(C_1)Z_1\right] = \mathbb{E}\left[u'(C_1)Z_2\right] \quad \text{(Appendix E.12)}$$

$$\mathbb{E}\left[u'(C_1)\right]\mathbb{E}\left[Z_1\right] + \mathrm{Cov}\left[u'(C_1), Z_1\right] = \mathbb{E}\left[u'(C_1)\right]\mathbb{E}\left[Z_2\right] + \mathrm{Cov}\left[u'(C_1), Z_2\right] \quad \text{(Appendix E.13)}$$

$$\mathbb{E}\left[Z_1\right] = \mathbb{E}\left[Z_2\right] + \mathbb{E}\left[u'\left(C_1\right)\right]^{-1}\left(\mathrm{Cov}\left[u'\left(C_1\right), Z_2\right] - \mathrm{Cov}\left[u'\left(C_1\right), Z_1\right]\right)$$

$$e^{-\delta}\left(\mathbb{E}\left[A_1'\right] - \mathbb{E}\left[e^{\delta}\frac{\partial A_1}{\partial K_0}\right]\right) = e^{\gamma}\frac{1+\sigma}{1-\kappa}\mathbb{E}\left[A_1'^{n}\right] + \frac{1}{\mathbb{E}\left[u'\left(C_1\right)\right]}\left(\mathrm{Cov}\left[u'\left(C_1\right), Z_2\right] - \mathrm{Cov}\left[u'\left(C_1\right), Z_1\right]\right) \quad \text{(Appendix E.14)}$$

Appendix F. Beta forms of optimal instruments

Applying Lemma 1, we rewrite the asset pricing equation

$$u'(C_0)A_0' = e^{-\rho+\gamma}\mathbb{E}\left[u'(C_1)\frac{1+\sigma}{1-\kappa}\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)\right] \quad \text{(Appendix F.1)}$$

$$1 = \mathbb{E}\left[e^{-\rho}\frac{u'(C_1)}{u'(C_0)}e^{\gamma}\left(\frac{\left(A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}\right)}{A_0'}\frac{1+\sigma}{1-\kappa}\right)\right] \quad \text{(Appendix F.2)}$$

We define $A_1'^{n}$ such that

$$A_1'^{n} = A_1' - (1-\psi)\frac{\partial A_1}{\partial K_0}$$

Now we need that $g_c \equiv \frac{C_1}{C_0} - 1$ and R_1 with

$$R_1 = e^{\gamma}\frac{A_1'^{n}(1+\sigma)}{A_0'(1-\kappa)} \quad \text{(Appendix F.3)}$$

to be jointly lognormally distributed.

Then

$$\frac{\mathbb{E}\left[A_1'^{n}\frac{1+\sigma}{1-\kappa}\right]}{A_0'} = e^{-\gamma+r_f+\tilde{\phi}\eta} \quad \text{(Appendix F.4)}$$

Here $\tilde{\phi}$ reflects the covariance of the asset return (with partial spillover) and subject to the various instruments.

Remember from (6) that it is optimal to price abatement according to

$$\frac{\mathbb{E}\left[\frac{\partial A_1}{\partial K_1} - e^{\delta}\frac{\partial A_1}{\partial K_0}\right]}{A_0'} = e^{\delta + r_f + \phi\eta} \qquad \text{(Appendix F.5)}$$

With some algebraic manipulation equation (Appendix F.5) is transformed into an analogous form

$$\mathbb{E}\left[e^{\phi\eta - \tilde{\phi}\eta}\frac{A_1'^n}{A_0'}\frac{(1+\sigma)}{(1-\kappa)}e^{(\gamma+\delta)}\right] = e^{\delta + r_f + \phi\eta} \qquad \text{(Appendix F.6)}$$

We solve for optimal fund instruments by comparing expectations in (Appendix F.5) and (Appendix F.6).

Appendix G. Additional numerical results

(a) Externality (for $\psi = 1$ learning is fully external)

ψ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	83.32	146.70	3.77	1.28	1.10	2.30148	2.54	3.81
0.250	82.00	142.56	3.69	1.27	1.08	2.30111	2.50	3.76
0.500	80.56	138.00	3.59	1.26	1.07	2.30072	2.45	3.71
0.750	78.95	132.94	3.47	1.25	1.05	2.30031	2.41	3.65
1.000	77.10	127.26	3.34	1.23	1.03	2.29987	2.36	3.59

(b) Technology subsidy

κ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	77.17	127.12	3.33	1.22	1.02	2.30000	2.36	3.57
0.100	72.88	121.42	3.40	1.24	1.07	2.30079	2.46	3.70
0.111	72.39	120.79	3.41	1.25	1.08	2.30089	2.47	3.72
0.200	68.25	115.32	3.50	1.27	1.13	2.30188	2.59	3.87
0.300	63.30	108.61	3.60	1.31	1.20	2.30341	2.77	4.07
0.400	57.90	101.20	3.72	1.35	1.30	2.30552	2.99	4.34
0.500	51.97	92.82	3.87	1.40	1.42	2.30849	3.29	4.69

(c) Investment subsidy

σ	I_0	B_1	r_A	r_f	ϕ	η	$\phi\eta$	$r_f + \phi\eta$
0.000	77.17	127.12	3.33	1.22	1.02	2.30000	2.36	3.57
0.125	81.41	135.85	3.41	1.25	1.08	2.30089	2.47	3.72
0.200	83.80	140.89	3.46	1.26	1.11	2.30147	2.55	3.81
0.400	89.62	153.50	3.59	1.30	1.19	2.30315	2.74	4.04
0.600	94.86	165.04	3.69	1.34	1.27	2.30492	2.93	4.26
0.800	99.60	175.73	3.79	1.37	1.35	2.30671	3.11	4.48

Table G.5: Technology externality. *Note: Optimal policies are merged into the parameter variations (in-between the equidistant steps of the parameter variation).*