Voting Agendas and Preferences on Trees: Theory and Practice

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Abstract

We study how parliaments and committees select one out of several alternatives when options cannot be ordered along a "left-right" axis. Which voting agendas are used in practice, and how should they be designed? We assume preferences are single-peaked on a tree and study *convex* agendas where, at each stage in the voting process, the tree of remaining alternatives is divided into two subtrees that are subjected to a Yes-No vote. We show that strategic voting coincides with sincere, unsophisticated voting. Based on inference results and revealed preference arguments, we illustrate the empirical implications for two case studies.

1 Introduction

We study how parliaments and other committees vote to select one out of several alternatives in complex situations where not all available options can be ordered along a "left-right" axis.

For example, in a well-known abortion legislation case from the German Bundestag that we describe below, the main axis of conflict pitted the rights of women versus the rights of unborn life. The eight proposed bills contained provisions about deadlines that need to be respected for legal abortions, possible punishments for both women and doctors that perform illegal abortions,

the need for counseling, psycho-social indications, etc... Thus, certain pairs of alternatives were not comparable along the main axis. The German Bundestag used a particular, apparently well-designed agenda, and we offer here a theory and suitable inference tools that allow us to understand the rationale behind the voting procedure and the ensuing consequences for voting behavior and the induced outcome.

In another recent and dramatic case from the UK Parliament, the main conflict axis involved a "hard" vs. "soft" (or no-) Brexit. However, due to the complexity of the question and the many potential post-Brexit arrangements, some of the proposed bills were not easily comparable along this main conflict line. The employed voting agenda was rather unusual: Premier May's strategic calculations did not materialize, and she was subsequently forced to resign.

As in the two cases mentioned above, practically all democratic parliaments routinely use *sequential binary voting procedures* to select one of several alternatives (see the survey of Rasch [2000]). At each stage in a sequence of votes, and starting with the full set of alternatives, the set of remaining alternatives is divided into two strict subsets. Then, a binary Yes-No vote is taken on the two subsets. The subset that gains a majority of the votes advances to the next stage, while the other subset is discarded. This process is repeated until a single alternative remains and is formally elected. There is considerable variation concerning the precise divisions into two subsets that are put to vote at each stage. Well-known, stylized representatives include:

1) The amendment procedure (AP) is common in the Anglo-Saxon world. It works with a basic bill (proposed by the Government, say), amendments to that bills, amendments to amendments, etc. At each stage, two alternatives (the original bill and an amended version, say) are pitted against each other, and the winner advances to the next stage that has a similar structure.

2) The Successive Procedure (SP) is common in continental Europe and usually works with independent, fully-formed bills. At each stage, a single bill is voted upon (so to say, against the rest of the alternatives), and voting stops as soon as one alternative obtains a majority.

The *agenda* – defining which subsets of alternatives are considered at each voting stage – plays a crucial role in determining individual voting behavior and the identity of the elected alternative. How should agendas be designed? In previous work, we identified a special class of carefully constructed agendas ensuring that sincere voting at each stage constitutes a robust, dynamic equilibrium in any sequential binary voting procedure, as long as privately informed voters have single-peaked preferences on alternatives ordered on a *line*, e.g. when the underlying issue is one dimensional (see Kleiner and Moldovanu, [2017]). We also illustrated the use of such agendas in some (but

not all) parliaments, and gave examples of documented strategic behavior (so called "manipulations") in cases where the agenda was formed by different criteria.

We first extend our previous analysis to the much larger class of preferences that are single-peaked on an arbitrary *tree*. Trees represent ideological relations that go well beyond the one-dimensional "left-right" framework underlying single-peakedness on a line, but still avoid impossibility results that would result in fully-fledged multidimensional problems.[1] This class of preferences was introduced in an elegant paper by Demange [1982]. Demange showed that, although the induced majority dominance relation on alternatives is not necessarily transitive, every profile of single-peaked preferences on a tree admits a Condorcet winner.[2] This generalizes the classical insight, due to Black [1948], who showed that the peak of the median voter is a Condorcet winner for single-peaked preferences on a line.[3]

In this paper we introduce *convex* agendas on trees: at each stage in the sequential, binary voting process, the tree of remaining alternatives that has not yet been discarded is divided into two *subtrees* that are subjected to a binary Yes-No vote. Since subtrees are *connected* sets of alternatives, this roughly says that each of the two subsets of alternatives in each Yes-No vote is ideologically coherent (according to the logic induced by the original, underlying tree). Thus, it cannot be the case that "extreme left" and "extreme right" alternatives are grouped together in one subset, and a "moderate" compromise among those extremes only appears in the other subset.

Assume that preferences of incompletely informed agents are single-peaked with respect to an arbitrary tree, and that an arbitrary sequential, binary voting procedure with an arbitrary convex agenda is used. Our main theoretical result shows then that *sincere*, myopic voting is an ex-post perfect equilibrium (and hence that it does not depend on the agents' beliefs about each other), and that the Condorcet winner is elected in this equilibrium. This holds no matter what the voters' beliefs about other voters are, and what the information revealed during the voting process is. The ex-post nature of our dynamic equilibrium concept also embodies a notion of no-regret: even if agents were told ex-post what the actual preferences of others were, they would not want to revise their past voting behavior.[4] In this sense, our

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[1]We note that complex multidimensional voting problems are often divided into several simpler ones. See Poole [2005].

[2]In other words, cycles may occur, but they never occur at the top of the majority dominance relation.

[3]In that case the majority relation is acyclical.

[4]This should not be confused with the stronger notion of an equilibrium in dominant strategies. Such equilibria need not exist in our framework.

paper can also be seen as contributing to the *robust design* of dynamic voting procedures.

In order to conduct our empirical analysis, we first present inference results based the raw data that is available to the analyst (voting procedure, agenda, voting profiles). We formally derive those trees that would make the observed agenda convex and that maximize the number of observed Yes-No voting profiles that become single-peaked according to the derived tree. Convexity significantly restricts both the number of possible trees and the number of observed profiles that could be observed if our theory was correct. This specially tailored form of equilibrium *revealed-preference analysis* allows us, in principle, to pin down the underlying preference tree and to infer voters' preferences.

We conclude the paper by illustrating our theory's empirical implications for the above mentioned case studies from Germany and the UK. Both instances involved binary, sequential voting by more than 600 heterogeneous voters who dynamically selected one out of a relatively large number of alternatives. Party discipline (or the "whip") – whereby members of parliament have to vote according to a uniform party line – was either institutionally not imposed (Germany), or was not respected by many decisive voters (UK). As a consequence, in both cases the final outcome was highly uncertain. Therefore, both voting instances involved highly complex strategic situations, whose precise analysis seems, at least a priori, beyond the reach of standard theory. We also explain why in the Brexit case alternative explanations that drop convexity but take into account additional, external factors (i.e., the actual content of the alternatives and their political meaning) seem fruitful.

Finally, it is interesting to note that the analysis performed here resembles the one that would be necessary to infer valuations for auctioned objects (here preferences) from bids (here individual voting profiles) submitted at various prices in a dynamic auction procedure (here voting procedure and its agenda). If the dynamic auction procedure is suitably constructed (here our convexity assumption) and if the allowed class of valuations is suitably restricted (here our single-peakedness assumption), then the auction has a robust, ex-post perfect equilibrium[5]. Then, and only then, the beliefs of bidders about the valuations of others and the information released during the auction do not play a role. Otherwise, the observed bids mix valuations with beliefs and their dynamic updates in a much more complex way.

The paper is organized as follows: In the next subsection we review the

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[5]See, for example, Ausubel's [2004] generalization of the English auction for multiple goods.

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related literature. In Section 2, we recall some definitions and results about graphs that are trees. In Section 3, we introduce the social choice model, the sequential binary voting procedures and their agendas. In Section 4, we prove our main theoretical result that connects sincere and strategic voting for convex agendas. Section 5 contains the necessary inference results for the revealed preference analysis. In Section 6 we present two case studies, one each from the German and UK parliaments. Section 7 concludes.

Related literature

The study of strategic, sequential binary voting was pioneered by Farquharson [1969]. The literature has often assumed that agents are completely informed about the preferences of others (see, for example, the classic papers by Miller [1977], McKelvey and Niemi [1978] and Moulin [1979]). Under complete information, sophisticated voters can use backward induction: at each stage they foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. Under simple majority, a Condorcet winner is selected by sophisticated voters whenever it exists, independently of the particular structure of the binary voting tree, and independently of its agenda. If a Condorcet winner does not exist, then a member of the top Condorcet cycle is elected, and the agenda influences which particular element of the cycle prevails. The influence of agenda manipulations has been studied by Ordeshook and Schwartz [1987] and, more recently, by Barbera and Gerber [2017]. An observational equivalence between strategic voting and sincere voting was established by Austen-Smith [1987] for completely informed voters who use the amendment procedure with an endogenous agenda.

Chambers and Echenique's [2016] monograph contains a brief discussion of revealed preference analysis in social choice situations, while Poole and Rosenthal's [2000] seminal work offers a detailed analysis of roll-call voting in the US Congress. Heckman and Snyder [1997] revisit Poole and Rosenthal's estimation framework while emphasizing the fundamental lack of identification of preferences in multidimensional choice setting where both the voters' preferences and the policy alternatives' attributes are not observed. This is related to the difficulties we face here, even after imposing considerable more structure (see, for example, the Brexit case below). Kalandrakis [2015] assumes that policy alternatives are known vectors in Euclidean space, and that the analyst observes a series of binary choices made by a single individual. He characterizes acceptance/rejection records that are rationalizable via a concave utility function. Note that on a line, concave utility functions lead to single-peaked preferences. Closer to our own setting, Trick [1989] shows

that if there exists a tree that renders a profile of preferences single-peaked, then, under a very mild richness condition, such a tree is unique. Ballester and Haeringer [2011] provide necessary and sufficient conditions for a profile of preferences to be consistent with single-peakedness for some linear order on the alternatives. Both papers' conditions apply to preference profiles (unobservable) rather than to the observed, binary choices generated by a specific voting procedure with a specific agenda.

Several researchers have conducted empirical studies of voting behavior in the German parliament. Leininger [1993] and Pappi [1992] analyze the 1991 decision about the post-reunification location of the German capital. They assume sincere voting and attempt to reconstruct the legislators' preferences from the observed votes. They also conduct simulations with other, hypothetical voting procedures and compare the results. Pappenberger and Wahl [1995] look at the regulation of abortion in 1992, which we also analyze here, while Von Oertzen [2003] discusses several other cases from the Bundestag.

Ladha [1994] analyzes a large number of cases from the US Congress and focuses on instances where the agenda followed a natural left-right order on a line: he observes patterns of behavior with a monotonicity property naturally associated with sincere voting patterns.[6] In contrast, Riker [1958] and numerous followers have documented cases where strategic manipulation have probably occurred. In many of these cases, such as Riker's, it can be shown that the manipulation is induced by a non-convexity in the agenda.

Roughly speaking, the above mentioned empirical papers – and many other similar ones – try to infer preferences from observed behavior. An important difference from our paper is that they are all based on the premise that voting is sincere: there is no presumption of optimal individual behavior and no equilibrium analysis. Attempts to investigate when and why sincere voting might occur are often based on external explanation such as *home-style* a la Fenno [1978].

An early analysis of strategic, sequential, binary voting under incomplete information is offered by Ordeshook and Palfrey [1988]. These authors constructed relatively complex Bayes-Nash equilibria for *amendment* voting in a situation with three alternatives and with preference profiles that potentially lead to a Condorcet paradox. A Bayesian analysis crucially depends on the assumed agents' beliefs about others. In particular, a similar theoretical analysis of our real-life case studies (up to 8 distinct alternatives and more than 600 voters with heterogeneous preferences) does not seem to be feasible. Even if it were feasible, the analyst needs then to infer from the observed

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[6]The implications of Ladha's findings for strategic vs. sincere voting are also discussed by Groseclose and Miljo [2010].

voting data both the voters' beliefs and their preferences, and identification is much more complex.

Kleiner and Moldovanu [2017] showed that, under single-peaked, private-value preferences on a line, sincere voting constitutes an ex post perfect equilibrium in any sequential, binary voting procedure if the agenda is *convex*. Earlier special cases of this result who considered agendas that split the set of alternatives at each voting node into two disjoint sets can be found in Gershkov, Moldovanu and Shi [2017] and Jung [1989][7] (e.g., the amendment procedure is not covered by the earlier analysis). The Gershkov et al. analysis is devoted to the design of welfare maximizing procedures.[8] This requires the introduction of cardinal utilities: the selected social alternative (that maximizes average welfare under incentive constraints) coincides with the Condorcet winner (median welfare) only under rather special assumptions on the distribution and on the number of agents (this is also the theme of Pivato [2015]).

Kleiner and Moldovanu [2020] apply the above theory to explain both the emergence and rarity of *killer amendments* and illustrate it with a case study involving the Nazi party. Gershkov et al. [2019] consider single-peaked preferences on a line but assume that preferences are interdependent. In their model, not all alternatives are fixed ex-ante and the authors study the emergence and location of compromise alternatives (e.g., the location of a compromise deal in the Brexit case and the emergence of the composite flag of the Weimar Republic).

2   Graphs and Trees

We first briefly recall here several basic graph-theoretic definitions and a result that will be useful for our analysis below. While the concepts appear to be abstract, their utility will become apparent below.

Definition 1

1. A graph G *on a set of nodes* $\mathcal{A}$ *with typical elements* A, B, C, ... *is a set of unordered pairs of distinct elements of* $\mathcal{A}$, *called* edges.

2. A path P *of* G *is a sequence of distinct nodes* $A_1, ..., A_m$ *such that* (A_i, A_{i+1}) *is an edge for* $i = 1, 2, ..., m - 1$.

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[7]We wish to thank an anonymous referee and Tom Palfrey for pointing us Jung's paper.
[8]An extension to much more general environments is in Rachidi (2020).

3. *A graph is* connected *if, for any pair of nodes* A_i, A_j*, there is a path with initial node* A_i *and terminal node* A_j*.*

4. *A* cycle *(or circuit) is a path in which the initial node coincides with the terminal node.*

5. *The* degree *of a node* A_i*, denoted by* $d(A_i)$*, is the number of edges having* A_i *as element.*

Definition 2 *A tree* Ψ *is a connected graph that contains no cycles. A node A is a* leaf *of tree* Ψ *if it has degree 1; that is, there is exactly one edge of* Ψ *containing this node.*

In our application, nodes correspond to the social alternatives among which voters have to choose, and the edges in a graph correspond to ideological proximity relations among alternatives.

Theorem 1 (Berge, 1962) *Any one of the following equivalent properties characterizes trees:*

1. Ψ *contains no cycles and has* $k - 1$ *edges (where* k *is the number of nodes).*

2. Ψ *is connected and has* $k - 1$ *edges.*

3. Ψ *contains no cycles, and if a new edge is added, one, and only one, cycle is formed.*

4. Ψ *is connected but ceases to be so if any edge is deleted.*

5. *Any two nodes A and B in* Ψ *are linked by a unique path, denoted below by* P_{AB}*.*

In order to quantify the enumeration problem of finding a suitable ideological structure represented by a tree – but also to emphasize the richness of tree structures – we recall Cayley's famous formula (see Berge [1962]): the number of distinct trees with k nodes is k^{k-2}.

3 The Social Choice Model

We now apply the graph-theoretical structures to a social choice model. Suppose that there are $2n + 1$ voters who need to select one alternative out of a finite set $\mathcal{A}$ with $k \geq 2$ elements. The set of alternatives corresponds to the set of nodes of a graph, and this graph is assumed here to be a tree Ψ. Intuitively, two alternatives are directly connected by an edge if they are ideologically close, and are indirectly connected by a longer path if they are ideological more distant.

Each voter i is characterized by a preference relation $\succ_i$ on $\mathcal{A}$, and preferences are private: an agent only knows her own preference, but not others' preferences. Single-peakedness on trees requires that, on each isolated path (which can be seen as a line), the agent has a preferred alternative (the peak), and alternatives become worse from her point of view as one moves farther away (in terms of number of edges) from that peak. Formally, we have the following:

Definition 3

1. *An individual preference relation $\succ_i$ is an irreflexive, asymmetric, complete and transitive order on $\mathcal{A}$.*

2. *The preference $\succ_i$ is* single-peaked *on the path P_{AC} of Ψ if, for any node B that lies on this path, it is not the case that both $A \succ_i B$ and $C \succ_i B$ hold.*

3. *The preference $\succ_i$ is* single-peaked *on the tree Ψ if it is single-peaked on every path P of Ψ.[9]*

When a tree Ψ consists of a single path, we are in the classic case where alternatives can be ordered on a line, from "left" to "right". Single-peakedness on a tree with many distinct paths is thus a significant generalization of classic single-peakedness on a line, and many more preference profiles are potentially compatible with it. Nevertheless, a tree structure still restricts preferences in a way that allows for meaningful social choice, e.g., avoids standard impossibility results.

Definition 4 *Given a preference profile $\{\succ_i\}_{i=1}^{2n+1}$, a Condorcet winner is an alternative $CW \in \mathcal{A}$ such that $|\{i : CW \succ_i A\}| > |\{i : A \succ_i CW\}|$ for any $A \neq CW$.*

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[9]This is equivalent to the following: If A is the peak of $\succ_i$ and if B belongs to the path between A and C (i.e., it is nearer to A than C), then $B \succ_i C$.

The existence of a Condorcet winner for any profile of single-peaked preferences on a given tree was established by Demange [1982], a significant generalization of the classic result for lines due to Black [1948]. The existence of a Condorcet winner is naturally preserved for subsets of alternatives that preserve the tree structure:

Lemma 1 *Consider a tree Ψ and a subtree $\Psi' \subset \Psi$. If a preference relation $\succ_i$ is singled-peaked with respect to Ψ, then its natural restriction is single-peaked on Ψ'. In particular, there is a Condorcet winner among the alternatives in Ψ'.*

Proof. Take any two nodes (alternatives) in Ψ', A and C. Since Ψ' is a tree and hence connected, there exists a path P in Ψ' that goes from A to C. Since P is also a path in Ψ, the result follows by single-peakedness with respect to Ψ. The last part follows from Demange's result. ■

Finally, note that to define a game with incomplete information and to conduct a strategic analysis, it is usually necessary to also specify beliefs that agents hold about the other agents' preferences. Since our analysis will be *robust* – namely, independent of those beliefs and independent of other information that becomes available during the voting sequence – we need not specify beliefs here.

3.1 Voting Procedures and Their Agendas

At each stage of a sequential binary voting procedure the set of remaining alternatives (starting with the full set) is divided into two strict subsets who need not be disjoint. Each voter approves one of the two subsets. The subset that gains majority approval advances to the next stage, while the other subset is discarded. The process is repeated until a single alternative remains and is elected. More formally:

Definition 5   *1. A* binary tree *is a tree such that each node v has either 0 or 2 children, and exactly one node has no parent. Nodes without children are called* terminal.

    2. An agenda *assigns to each node of a binary tree a subset of $\mathcal{A}$ such that (i) a single alternative is assigned to every terminal node, (ii) every alternative in $\mathcal{A}$ is assigned to some terminal node, (iii) the set of alternatives assigned to a parent is the union of the sets of alternatives assigned to its children, and (iv) the set of alternatives assigned to each node is a proper subset of the alternatives assigned to its parent.*

3. *A* binary sequential voting procedure *is a finite binary tree together with an agenda.*

While binary sequential procedures can also be graphically described by means of binary trees – with the two branches protruding from each non-final node representing the Yes-No decision to be be made at that node (see, for example, Figures 2 and 5 in the Brexit case studied below) – such voting trees vary with the chosen procedure and should **not** be confounded with the distinct and fixed tree Ψ that governs the ideological proximity relations among alternatives.

The following important property connects the agenda of binary sequential voting procedures to the underlying structure of preferences:

Definition 6 *An agenda is* convex *with respect to a tree Ψ if, for each parent node in the binary voting tree, it divides the set of alternatives assigned to that node into two **subtrees** of Ψ that are assigned to the two children, respectively.*

The main ingredient in the above definition is the requirement that the division of alternatives at each voting stage is among two distinct, not necessarily disjoint, subsets that are ideologically connected. By the previous lemma, the restricted preferences continue to be single-peaked on each subtree, and thus, each binary division in a convex agenda is ideologically coherent: Consider two alternatives A and B that belong to one of the subtrees. Since the path P_{AB} connecting these alternatives is unique, all alternatives on the path P_{AB} must also belong to the same subtree. In other words, it cannot be the case that voters have to decide between, say, a subset of "centrist" alternatives on the one hand, and a subset containing only "extreme right" and "extreme left" alternatives on the other. In such a case, the unique path connecting the extreme nodes may need to go via one of the centrist nodes, violating the requirement that each subset is a connected subtree.

Convex agendas are necessarily *content based* rather than *procedural* (see Kleiner and Moldovanu [2017]). In other words, to construct such an agenda for a given tree, one needs to take into account the actual content of the proposed alternatives and also the logical/ideological connections between them. In contrast, purely procedural agendas follow predetermined rules that are independent of the content of alternatives. For example, the agenda may be defined in terms of some order in which alternatives were submitted to the relevant parliamentary committee or in terms of a formal denomination as main bill, amendment, amendment to amendment, etc.[10]

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[10]As Ladha [1994] documents for the US, in certain cases, procedural agendas may nevertheless be convex.

Example 1   *1. Consider the successive voting procedure on a set of alternatives $\mathcal{A}$. An agenda for this procedure is convex with respect to a tree Ψ if, at each stage, the alternative that is put to vote is a leaf of the subtree of remaining alternatives. If alternative C is considered at a particular stage, the binary division into two subtrees is $[C, \mathcal{A} \setminus \{C\}]$.*

2. Consider the amendment procedure on $\mathcal{A}$. An agenda for this procedure is convex with respect to a tree Ψ if, at each stage, both alternatives that are pitted against each other are leaves of the subtree of the remaining alternatives. If alternatives B and C are considered at a particular stage, the binary division in two subtrees is $[\mathcal{A} \setminus \{C\}, \mathcal{A} \setminus \{B\}]$.

Intuitively, both of the above agendas prescribe that more "extreme" alternatives should be put to a vote before more "moderate" ones. These agendas are indeed well-defined and convex based on the following lemma:

Lemma 2   *1. Any tree Ψ has at least two leaves.*

2. Let A be a leaf and denote by e the unique edge of Ψ that contains A. Then $\Psi \setminus \{e\}$ is a tree on $\mathcal{A} \setminus \{A\}$.

Proof. 1. *Let $P = \{A_1, ..., A_m\}$ be the longest path in Ψ. Then A_1 and A_m must be leaves. Alternatively, note that the sum of degrees in any graph equals twice the number of edges. By Theorem 1 we obtain:*

$$\sum d(A_i) = 2(k - 1) = 2k - 2$$

If there are fewer than two leaves we obtain:

$$\sum d(A_i) \geq 2(k - 1) + 1 = 2k - 1 > 2k - 2$$

which is a contradiction.

2. Consider any two nodes $B, C \in \mathcal{A} \setminus \{A\}$. Then the unique path P_{BC} that connects them in Ψ is also the unique path that connects them in $\Psi \setminus \{e\}$.
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4   Sincere and Strategic Voting on Trees

We now study strategic voting in binary sequential voting procedures. For each node v of the voting tree, let H_i^v denote the part of the history of play that is observable to player i at node v.[11] One common specification

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[11]And let $H^v \subset \times_i H_i^v$ be the set of consistent profiles of histories.

is that H_i^v consists of the aggregate number of Yes and No votes at each previous node, and i's own voting behavior at all previous nodes. Another possible specification is that H_i^v includes the individual voting behavior of every player at all previous nodes. None of our results below depend on the exact specification of H_i^v.

A strategy of player i associates to each non-terminal node of the binary voting tree, to each history leading to that node, and to each preference realization an action in the set {Yes, No}.[12]

Definition 7 *A strategy profile constitutes an* ex-post perfect equilibrium *if for every non-terminal node, and following any history, the agents play best responses for every realization of preferences.*[13]

Hence, a profile of voting strategies constitutes an ex-post perfect equilibrium if voters play best responses for each realization of preferences. Thus, no voter regrets her equilibrium strategy even after learning the preference realizations of all other voters. This is a particularly useful equilibrium notion for our empirical analysis because it does not depend on the (unobserved) beliefs voters entertain. We relate below strategic voting to the following concept of "unsophisticated" voting:

Definition 8 *A voting strategy for a binary sequential voting procedure is* sincere *if, at each stage in the voting sequence, it prescribes voting for the subset of alternatives that contains the most preferred alternative among all remaining ones. If that alternative is contained in both subsets that are put to vote at a certain stage, then a sincere voting strategy proceeds lexicographically (vote yes for the subset that contains the second-best alternative, and so on).*

Our notion of sincere voting takes an optimistic perspective and evaluates sets of alternatives based on a best-case analysis. It goes back to Farquharson [1969] and Miller [1977], and has been employed extensively in the literature (see Miller [2010] for a recent discussion). Our main theoretical result is as follows:

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[12]Formally, let P^Ψ denote the set of preferences that are single-peaked on the tree Ψ. A pure strategy for voter i is a mapping $\sigma_i : \bigcup_{v \in V'} H_i^v \times P^\Psi \to$ {Yes, No}.

[13]To formally define the equilibrium, let $A_{v,h^v}(\sigma, \succ)$ be the alternative that is selected if voters with preference profile $\succ$ use the strategy profile σ in the subgame starting at node v after observed history $h^v \in H^v$. The strategy profile σ is an ex-post perfect equilibrium if, for all i, non-terminal nodes v, histories $h^v \in H^v$, and for all $\succ \in \mathbf{P}^\Psi$, $A_{v,h^v}(\sigma, \succ) \succeq_i A_{v,h^v}(\sigma_i', \sigma_{-i}, \succ)$ holds for all strategies σ_i'.

Theorem 2 *Assume that preferences are single-peaked with respect to a tree Ψ and that a sequential binary procedure with a convex agenda is used. Then sincere voting is an ex-post perfect equilibrium, and the Condorcet winner is elected in this equilibrium.*

Proof. Assume first that all voters vote sincerely, and let CW be the Condorcet winner given the agents' preferences. We first show that CW must be elected under such a strategy profile.

Assume, by contradiction, that CW is not elected under sincere voting. Consider then the first stage in the voting process where the majority approved subtree is Ψ' such that $CW \notin \Psi'$. Then, there exist $m \geq n + 1$ agents whose preferred alternative among the remaining ones is in Ψ'. Denote those most preferred alternatives by $A_1, A_2, ..., A_m$, respectively (these need not be distinct). If $A_1 = A_2 = ... = A_m$ then there are $m \geq n + 1$ agents that prefer A_1 to CW, which is impossible by the definition of CW. Assume then without loss of generality that $A_1 \neq A_2$. Because Ψ' is a tree, there exists a unique path, $P_{A_1 A_2}$, which is entirely contained in Ψ' and which connects these two nodes. In particular, CW cannot be on this path since $CW \notin \Psi'$. Consider next the uniquely defined paths $P_{CW A_1}$ and $P_{CW A_2}$ in Ψ. Then there must exist an alternative, denoted by B, such that B belongs to $P_{CW A_1}$, $P_{CW A_2}$ and $P_{A_1 A_2}$. Otherwise, the concatenation of $P_{CW A_1}$, $P_{A_1 A_2}$ and $P_{CW A_2}$ contains a cycle, contradicting the assumption that Ψ is a tree.

By single-peakedness, we conclude that **all** agents whose most preferred alternative is **either** A_1 or A_2 prefer alternative B to CW. Arguing in the same manner for $A_3, ..., A_m$ shows that there must be an alternative in Ψ' that is preferred by $m \geq n + 1$ agents to CW, which is impossible. Thus, CW can never be eliminated, and will thus be elected under sincere voting.

We now argue that sincere voting is an ex-post perfect equilibrium. Fix an arbitrary preference profile and an arbitrary voter i. We show that given sincere behavior by all other voters, i has no profitable deviation from sincere voting. Consequently, sincere voting is an ex-post perfect equilibrium.

Observe first that sincere voting is a best response if only two alternatives remain. Consider a voting stage where the decision is between the two subtrees Ψ' and Ψ'' and assume that sincere voting is a best response in the subgame after this stage. Hence, if Ψ' gains a majority at this stage, it follows from the first part that the final outcome will be the Condorcet winner among the alternatives in Ψ', which we denote by C'. Similarly, if Ψ'' gains a majority, the final outcome will be the Condorcet winner among alternatives in Ψ'', denoted by C''.

To obtain a contradiction, suppose without loss of generality that i's peak is $A \in \Psi'$ but that he is strictly better off voting for Ψ''. Then, there must be

14

at least n other voters with peak in Ψ'' and it must hold that $C'' \succ_i C'$. Because $\Psi' \cup \Psi''$ is also a tree (that has been approved at the previous stage) there exists an alternative B that satisfies $B \in P_{AC'}$, $B \in P_{AC''}$ and $B \in P_{C'C''}$. Since $A, C' \in \Psi'$ and Ψ' is a tree, it must also hold that $B \in \Psi'$. Also, because alternative A is i's peak and because $B \in P_{AC''}$, single-peakedness implies $B \succeq_i C'' \succ_i C'$. Hence, $B \neq C'$.

We now consider two cases:

(1) Suppose that $B \notin \Psi''$. Since Ψ'' is a tree and $B \in P_{C'C''}$, $C' \notin \Psi''$. Also, for all $D \in \Psi''$, it must be the case that $B \in P_{C'D}$ (if not, then the concatenation of $P_{C'C''}$, $P_{C''D}$ and $P_{DC'}$ contains a cycle). By single-peakedness, every voter with a peak in Ψ'' prefers alternative B to C'. Since at least n other voters have a peak in Ψ'' and $B \succ_j C'$ for all such voters, we obtain a contradiction to the assumption that C' is the Condorcet winner among alternatives in Ψ'.

(2) Suppose that $B \in \Psi''$.[14] Since C'' is the Condorcet winner among the alternatives in Ψ'', if $C'' \neq B$ then at least $n + 1$ voters prefer C'' to B, and hence, by single-peakedness, they prefer B to C', contradicting the fact that C' is the Condorcet winner among alternatives in Ψ'. Hence, $C'' = B$ and $C'' \in \Psi'$. Since C' is the Condorcet winner in Ψ', at least $n + 1$ other voters prefer C' to C''; since C'' is the Condorcet winner among alternatives in Ψ'', $C' \notin \Psi''$. Since at least n other voters have a peak in Ψ'', there is a voter with peak in Ψ'' who prefers C' to C''. Denote his peak by D. Then, there is an alternative $E \neq C', C''$ such that $E \in P_{DC'}$, $E \in P_{DC''}$ and $E \in P_{C'C''}$ (otherwise the concatenation of $P_{DC'}, P_{DC''}$ and $P_{C'C''}$ contains a cycle). Since Ψ'' is a tree, $E \in \Psi''$. Since Ψ' is a tree and $C', C'' \in \Psi'$, we conclude $E \in \Psi'$. Therefore, $n + 1$ voters prefer C' to E and hence E to C'', contradicting the assumption that C'' is the Condorcet winner among alternatives in Ψ''. ∎

As is common in voting games, there are additional, trivial equilibria where, for example, voters coordinate to always vote Yes, no matter what their preferences are. In this case, no voter is pivotal and such strategies do form an ex-post perfect equilibrium. Call a strategy for voter i *responsive* if, for every non-terminal node v and for every history leading to this node, there is a preference realization such that i votes Yes at v, and another preference realization such that i votes No at v. We show in the Appendix that, for any convex binary sequential voting procedure, sincere voting is the unique ex-post perfect equilibrium in responsive strategies as long as each vote is between disjoint sets of alternatives (as, for example, in the successive

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[14]This case cannot occur in procedures where the binary decision is among disjoint subsets of alternatives as, e.g., in the successive procedure.

procedure). If the vote is among sets of alternatives that are not disjoint, the same alternative may be obtained following either branch of a given node. While in this case sincere voting need not be the unique equilibrium in responsive strategies, all ex-post perfect equilibria in responsive strategies will lead to the same outcome if the agenda is convex.

5 Inferring Trees from Voting Data

Recall that if the set of alternatives $\mathcal{A}$ has cardinality k, then there are k^{k-2} distinct trees on $\mathcal{A}$. Since in complex, real-life cases the tree structure is almost never explicit, an important criterion for assessing the power of the subsequent empirical analysis is: How arbitrary is the **analyst's** choice of a tree with respect to which preferences are potentially single-peaked? A first step towards answering this question was provided by Trick [1989]:

Proposition 1 (Trick [1989]) *Fix a profile of individual preferences such that each alternative in $\mathcal{A}$ is the peak of some voter. Then there exists at most one tree Ψ such that this profile of preferences is single peaked on Ψ.*

Proof. For the sake of completeness, we reproduce here the simple proof. Assume that the preferences are single-peaked on two distinct trees, Ψ and Ψ'. Then Ψ has an edge $e = (A, B)$ that is not contained in Ψ'. Consider then any node C on the path between A and B in Ψ' and its respective placement in Ψ. There must be such a node because, by assumption, $e = (A, B) \notin \Psi'$. There are two cases: either the path from A to C in Ψ contains B, or the path from B to C in Ψ contains A. In the first case, consider a voter i that has a peak on A. Then, we must have $A \succ_i B \succ_i C$, which implies that i's preferences cannot be single-peaked on Ψ'. The other case is similar, and this yields a contradiction. ∎

Trick's result uses as input the actual preferences of the voters. In particular, it is independent of the employed voting procedure and its agenda. But, in empirical applications we only observe sequences of "Yes" and "No" votes that are generated by a specific, known binary sequential procedure with a specific, known agenda. In general, such information alone is not sufficient to reconstruct the underlying preferences,[15] and therefore we cannot directly use Trick's result.

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[15]For example, if preferences are single-peaked on a line, an observed voting pattern may not be sufficient to completely rank alternatives below the peak. For this reason, most of the empirical literature focused on the identification of "ideal points" while making strong assumptions about the remaining profile, e.g. Euclidean preferences.

Even if we are given a specific voting procedure and observed voting profiles, it is not always possible to find a tree such that all voting profiles are consistent with sincere voting according to single-peaked preferences. To illustrate this in a simple example, consider an amendment procedure with three alternatives $\{A, B, C\}$ where the first vote is between A and B and the second between the winner of the first stage and C. Suppose that: A wins the first vote; some voters vote for A and then for C; others vote for B and then for A; still others vote against A in both stages. Assuming sincere voting, the first voting pattern implies $C \succ A \succ B$, the second $B \succ A \succ C$, and the last $B \succ A$ and $C \succ A$. Therefore, each alternative is the worst one for some voters and therefore, under singled-peaked preferences, must be a leaf of the underlying preference tree. This yields a contradiction since any such tree has only two leafs.

In contrast, we show below that assuming that the agenda was convex with respect to an underlying tree, allows us in some cases to pin down the tree from the observed voting data.

5.1 Inference under convexity: the successive procedure

In order to be able to derive a preference tree that encompasses all alternatives we need to assume that each proposal is put up for vote, i.e., the successive procedure does not stop early, and that each alternative receives at least one vote.[16] As we shall see, both assumptions were satisfied in the relevant German case discussed below.

Proposition 2 *Consider a successive voting procedure that does not stop until all alternatives are put to vote, and any set S of individual sequences of binary votes such that each alternative obtains at least one vote in its favor.[17] Then there exists at most one tree Ψ such that the successive procedure is convex with respect to Ψ, and such that all individual sequences in S are consistent with sincere voting - and hence equilbrium behavior - according to single-peaked preferences on Ψ.*

Proof. Suppose by contradiction that Ψ and Ψ' are two distinct trees with the desired properties. Then Ψ has an edge $e = (A, B)$ that is not contained in Ψ'. The unique path between A and B in Ψ' must contain an alternative $C \neq A, B$. Then, in tree Ψ either: (1) B lies on the path from A to C, or (2) A lies on the path from B to C.

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[16]If these assumptions are not satisfied, only partial inference is possible.

[17]This implies $|S| \geq k$.

Consider first case (1). Since the agenda is convex with respect to both trees Ψ and Ψ', alternative A must be put to a vote before B and C. By assumption, there is at least one voter i who votes for A: since voting is assumed to be sincere, A must be then i's peak among the remaining alternatives at that stage. If preferences are single-peaked with respect to Ψ, B must be i's peak after the elimination of A. Analogously, if preferences are single-peaked with respect to Ψ', voter i prefers C to B.

Assume first that, after A has been eliminated, B is put to vote before C. In order to be consistent with sincere voting according to preferences that are single-peaked on Ψ (Ψ'), i must vote for B (against B). Hence, i's observed voting profile cannot be consistent with both.

Assume next that C is put to vote before B. Then i must vote against C to be consistent with sincere voting according to preferences that are single-peaked on Ψ. But, because the agenda is convex, C must be a leaf when it is put to vote, and hence i will prefer C to any remaining alternative if preferences are single-peaked with respect to Ψ'. Therefore, i must vote for C in this case, and we again obtained a contradiction. The argument for case (2) is analogous. ∎

Our identification result implies that if preferences are single-peaked on some tree, if all voters play their equilibrium strategies, and if each alternative obtains at least one vote, then there is exactly one preference tree that explains all votes. Of course, in empirical applications with hundreds of legislators we do not expect every single individual sequence of votes to be explained by our model, and hence there might be no preference tree that explains all votes. In this case, we focus on the preference tree that maximizes the number of individual voting sequences that are consistent with sincere voting according to single-peaked preferences on this tree.

5.2 Inference under convexity: the amendment procedure

Unless the specific pairs of alternatives that are put to a vote against each other in an amendment procedure can be dynamically adjusted as a function of past results, convexity of static, fixed agendas is rather restrictive. Our next result — used below to analyze the Brexit case — shows that an a priori fixed agenda for an amendment procedure is convex if and only if the underlying tree governing single-peaked preferences is a *star* (i.e., a tree in which all alternatives except one are leafs): the alternative put to vote last (e.g., the status quo) must sit at the center of the star, and be directly connected by edges to all other alternatives. We note that fixed agendas are

common in parliaments that use procedural agenda formation rules.

Proposition 3 *Consider an amendment procedure with alternatives $A_1, A_2, ..A_k$ such that it is a-priori fixed that A_1 is put to vote against A_2, the winner against A_3, and so on, until the last alternative A_k. Such an agenda is convex with respect to a tree Ψ if and only if Ψ is a star with alternative A_k at its center.*

Proof. If Ψ is a star, then the given agenda is convex since each two alternatives that are put to vote against each other are leaves (note that A_k itself is a leaf of the remaining tree at the very last stage, when only one other alternative has survived until that point).

Conversely, assume that the agenda is convex with respect to a tree Ψ. Then alternatives A_k and A_{k-1} must be directly connected by an edge in Ψ. To see that, assume by contradiction that the unique path between A_k and A_{k-1} in Ψ contains another alternative A_j, $j \neq k, k-1$. Then, at the stage where A_j is put to vote, it cannot be a leaf of the tree of remaining alternatives since this tree still contains A_k and A_{k-1}, and since A_j lies on the path between A_{k-1} and A_k.

Consider next alternative A_{k-2} : we claim that it also must be directly connected by an edge to A_k. Again, assume by contradiction that this is not the case. Then the unique path between A_k and A_{k-2} contains another alternative A_i, $i \neq k, k-2$. Let $\widehat{i}$ be minimal with this property. If $\widehat{i} < k-2$, the argument follows exactly as above. The only other possibility is that $\widehat{i} = k-1$. Consider then the potential case where A_{k-2} wins against its opponent (i.e., one of the alternatives in the set $A_1, A_2, ..A_{k-3}$ that survived until that point). Then the remaining tree contains only A_{k-2}, A_{k-1}, A_k and, by assumption, A_{k-1} is between A_{k-2} and A_k. Thus the agenda pitting A_{k-2} against A_{k-1} cannot be convex. The argument for all other alternatives is analogous. ■

5.3 Inference under convexity: mixed procedures

It should be clear from the above results and their proofs that inference using convexity is highly dependent on the employed voting procedure. We next provide an illustration showing that, even under a convexity assumption, the preference tree is not always uniquely identified in mixed agendas that combine successive and amendment elements - this type of agendas was used in the Brexit applications discussed below.

Example 2 *Suppose that there are 4 alternatives A, B, C, D: the first vote is whether to implement A, and if A is eliminated, an amendment procedure will*

be used, with the first vote between B and C and the winner advancing against D. By our results above, this agenda is convex if the underlying preference tree contains the simple "star" subtree $B - D - C$ and if A is connected as a leaf to one of the other alternatives. This yields 3 possible trees. Suppose, for example, that we observe profiles where all voters who voted in favor of A also vote in favor of B: then A must be connected by an edge to either B or D. But, if C advances to the last stage, then the last vote cannot reveal any information pinning down where A was connected: all voters with initial peaks on A, B, D vote in favor of D independently of where A was connected (and all voters with peak on C vote in favor of C). Therefore, even if each alternative obtains at least one vote and each alternative is put to vote, there may be multiple trees such that the agenda is convex and such that voting is consistent with sincere voting according to single-peaked preferences.

6   Case Studies

We now apply our insights to two real cases from the German Bundestag and from the UK Parliament, respectively. The basic structure of the revealed preference analysis is as follows:

1. We review the general context of the vote and the specific content of the proposed alternatives.

2. We describe the employed binary, sequential voting procedure and its precise agenda.

3. We next present in concise form the real-life, observed voting profiles - these are sets of Yes-No sequences, one for each legislator - and the induced voting outcome. Voting profiles are available to us at the level of each individual legislator, and hence we can indeed draw conclusions at this level of behavior.

4. We formally infer a preference tree that makes the observed agenda convex, and that maximizes the number of observed voting profiles that are consistent with sincere voting (given single-peaked preferences on the presumed tree).

5. We discuss the results and the underlying assumptions. In particular, we sometimes offer alternative explanations of the observed data - these explanations are necessarily based on additional, external factors. In particular, a proposal's political content is taken then into account.

6.1 Abortion Law after the German Reunification

Prior to the 1992 reunification, abortions were strictly regulated in the Federal Republic of Germany, while the former Democratic Republic of Germany had a more liberal law. The reunification treaty required new, uniform legal foundations. After a long debate, 7 proposals were put up for vote in the Bundestag, covering a wide range of opinions and details. In ethical decisions it is customary to free members of the Bundestag from party discipline, and our assumption of incomplete information becomes then salient: support for various alternatives crosses party lines, and members of the same party vote in favor of different alternatives, introducing real uncertainty about the outcome.[18]

6.1.1 The proposed bills and the voting procedure

Following the Standing Orders of the Bundestag, voting proceeded according to the successive procedure. The agenda formation rule in those Standing Orders implicitly assume that the issue is one-dimensional and calls for voting on extreme alternatives first. The Elders' Council, headed by the Bundestag's president, suggested a very specific agenda.

We briefly describe here the proposed laws according to the order in which they were actually put up for vote, from **A** to **G**. The status quo is denoted by **H**.

A The Greens' proposal was very liberal and basically allowed any abortion.

B Similarly, the proposal by the Left party would allow any abortion, and there were only minor differences compared to proposal **A**.

C This proposal, coming from a subgroup of very conservative parliamentarians was extremely restrictive: it allowed an abortion only if the life of the mother was otherwise at stake.

D The Liberals proposed that abortions should be legal in the first 12 weeks of pregnancy, but only if the mother takes part in pregnancy counseling. Moreover, the proposal demanded punishment for women aborting after the first 12 weeks.

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[18]For example, in a recent case from 2018, Chancellor Merkel and a majority of legislators belonging to her governing party lost a landmark case that legalized gay marriage.

	Yes	No	Abstain	Total
A	17	632	6	655
B	17	633	3	653
C	104	492	57	653
D	74	575	4	653
E	236	402	16	654
F	272	369	16	657
G	**355**	**283**	**16**	**654**

Table 1: Aggregate vote outcomes

E The Social Democrats suggested instead that any abortion within the first 12 weeks should be legal, but without enforcing punishments for later abortions.

F The main proposal brought forward by conservatives and supported by the leaders of the ruling CDU/CSU, allowed abortions only under restrictive regulations: even early abortions would remain legal only under medical and/or psycho-social indications.[19] Both woman and treating doctor would be punished for an abortion after the first 12 weeks.

G This proposal was suggested by a group of legislators that crossed party lines: it was meant as a compromise between proposals **E** and **F**. An abortion within the first 12 weeks would not be punished. The woman needs to take part in a pregnancy counseling and the abortion must be performed by a doctor, but the ultimate decision stays with the woman.

H The status quo in the former Democratic Republic allowed an abortion in the first 12 weeks. In contrast, in the Federal Republic, an abortion required the presence of several "indications" that were not easy to fulfill.

Alternative **G**, the compromise among the main alternatives supported by the big parties, was elected at the final vote. Table 1 summarizes the voting results in the sequence of binary votes (see the Archives of Deutscher Bundestag [1992]):

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[19]This effectively handed the final decision to the doctor, who also had to explain the decision in writing.

6.1.2 Analysis

We assume throughout that the agenda was convex with respect to an underlying preference tree. This seems justified given the Bundestag's agenda formation rule to put more extreme alternatives up for vote first. If our theory is correct, we should be able to identify a preference tree such that most voting profiles are consistent with sincere voting according to single-peaked preferences on this tree.

For any given preference tree, only few voting profiles are consistent with sincere voting. Abstract for the moment from abstentions, and assume that voters can only vote Yes or No at each stage. This yields $2^7 = 128$ possible individual voting profiles. In the successive procedure with a convex agenda, each alternative is a leaf of the tree remaining at the time it is voted upon. Therefore sincere voting prescribes to vote Yes if the current proposal is the most preferred among the remaining alternatives, and No otherwise. Together, these features imply that the location of the peak completely determines the corresponding sincere voting strategy, i.e., this strategy is independent of how exactly alternatives are ranked below the peak. These considerations imply that, for each preference tree, out of the 128 possible voting profiles, only 8 are consistent with sincere voting according to strict and single-peaked preferences on this tree. This significant reduction in complexity well illustrates the empirical content of our proposed theory.

In reality, members of parliament choose not to cast a vote on a specific proposal, or to formally abstain. 658 voters participated in at least one vote, while 638 voters participated in all votes in the sequence, and we focus our analysis on the latter. More than 100 of these voters abstained at least once, which is why we include these voters in our analysis while treating an abstention as an expression of indifference. Table 2 summarizes all common voting profiles.

We first have to find the underlying preference tree. Since there are 8 alternatives, Cayley's formula implies that there are $8^6 = 262.140$ trees on which preferences could have been, at least theoretically, single-peaked. The observed agenda is convex "only" for $7! = 5040$ of these trees[20]. Since each alternative was actually voted upon and obtained at least one vote,

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[20]More generally, given a successive procedure with k alternatives, there are $(k - 1)!$ trees such that the agenda is convex with respect to this tree. Indeed, note that this claim trivially holds for $k = 2$ and suppose it holds if there are $k - 1$ alternatives. Consider a successive procedure with k alternatives and a preference tree for the $k - 1$ alternatives proposed last that makes the procedure from the second stage on convex. The complete procedure is then convex if and only if the first alternative is added as a leaf to any of the $k - 1$ alternatives that are considered later. It follows that there are $(k - 1)[(k - 2)!]$ trees that make the successive procedure convex.

Profile	Observations
NNNNYNY	203
NNNNNYN	122
NNYNNYN	85
NNNYNNY	71
NNANNYN	48
NNNNNNY	30
NNNNANY	13
NNYNNNN	9
NNYNNAN	8
YYNNYNY	5
YYNNYNA	5

Table 2: Vote profiles that are cast by at least 5 legislators.

Proposition 2 implies that all observed voting profiles could be consistent with sincere voting according to single-peaked preferences on **at most one** of these trees. While there is no tree that makes all voting profiles consistent, a vast majority of these are consistent with sincere voting and single-peaked preferences on either one the two trees shown in Figure 1 (601 and 610 out of 638 voting profiles, respectively). We show in the Appendix why no other preference tree explains more voting profiles.
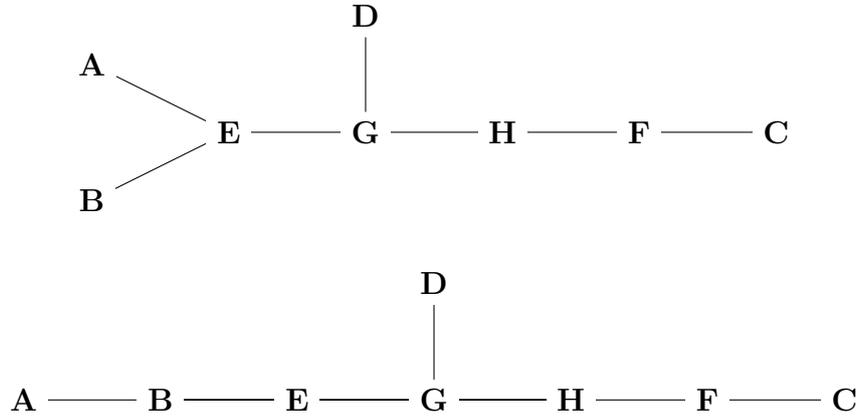

Figure 1: The two preference trees that maximize the number of voting profiles consistent with sincere voting and single-peaked preferences.

We now look for some external validity for the choice of preference tree by analyzing the content of the alternatives, and by using a preference survey. First, given the actual content of the proposed laws (e.g., their relative

Peak	A	B	C	D	E	F	G	H
Number (min)	17	0	102	72	204	130	30	1
Number (max)	23	4	158	77	224	188	55	7

Table 3: Number of legislators which prefer each alternative the most.

position on the main axis of conflict in this abortion case), both trees are reasonable. In particular, since there are only minor differences between proposals **A** and **B** and since the two trees differ only in the placement of **A** and **B**, content alone cannot be used to decide among the two trees.

Second, Pappenberger and Wahl conducted a post-voting preference survey. A majority of the reported preferences of more than 70 legislators were indeed single-peaked with respect to the first tree shown in Figure 1, Moreover, for no other tree (even non-convex ones) more profiles of reported preferences would be single-peaked.[21] Therefore, we will use this tree for our further analysis.

6.1.3 Inferred preferences

Since the chosen agenda was convex, our theoretical results predict that sincere voting constitutes a robust equilibrium. We can therefore infer the most preferred alternative of each legislator: for example, a legislator voting Yes at the first vote has a peak on **A**, a legislator who votes Yes for the first time at the second vote has a peak on **B**, and so on. Based on the record of voting profiles, the following table shows, for each alternative, how many legislators had a peak on that alternative.[22]

Although only a small minority of voters had a peak on the elected alternative **G**, it turns out that, under the above inferred possible distributions of peaks, this alternative was indeed the Condorcet winner. While some legislators criticized the voting procedure, our analysis implies that the Bundestag's president and the Elders' Council intuitively choose an agenda that made strategic voting unnecessary, and that ensured the election of the Condorcet winner - a compromise alternative that did not have much direct support. In other words, the employed agenda consistently extended the traditional "extremes first" doctrine from a line to a more complex tree that remained implicit in the process - no mean feat in this complex situation.

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[21]For example, note that only 7 preferences are single-peaked according to the linear order on alternatives that was suggested by Pappenberger and Wahl.

[22]Due to abstentions, we cannot precisely identify the peak of some legislators. We therefore display for each alternative a lower and an upper bound on the number of legislators that have a peak on this alternative.

6.2 The Brexit Voting Marathon

A voting marathon consisting of a sequence of eight binary votes was conducted by the British Parliament between March 12 and March 14, 2019. At stake was the shape and even the future of Brexit - UK's separation from the European Union - that was supposed to formally take place just two weeks later, on March 29, 2019.

The UK parliament has 649 members. Since Sinn Fein's 7 MPs do not take their seats, a majority of 322 was needed to pass legislation. The Tory (Conservative) government, supported by the North-Irish DUP had a very thin, theoretical majority of 324, but was facing many rebel members in favor of a hard Brexit. Thus, the outcome was highly uncertain.

The Parliament used relatively complicated sequential, binary agendas that mixed elements of the Amendment Procedure (AP) and the Successive Procedure (SP). This was necessary because some of the bills (such as May's negotiated deal) were complete pieces of legislation, while others were only partial amendments.

6.2.1 The First Voting Sequence

The first sequence of votes involved decisions about alternative courses of actions up to the official Brexit date on March 29, 2019. It consisted of 4 binary votes involving 5 alternatives:[23]

0 We denote by 0 a no-deal Brexit on March 29. Implicitly, this was the legal status quo unless further action was taken, and this was mentioned as such in May's motion 1.

1 May's deal with the EU.

2 May's no Brexit without a deal on March 29.

3 Malthouse: An alternative to May's deal (1) that would execute Brexit on March 29.[24]

4 Spelman: No Brexit without deal, ever (amendment to 2).

The agenda is illustrated in Figure 2. The first vote was on May's motion 1, according to SP: voting would have stopped in case of acceptance. But

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[23]Many other proposals were ultimately not put to a vote - agenda setting power lied with the former, powerful Speaker John Bercow.

[24]This was procedurally presented as an amendment to 2, but logically represented an altogether independent course of action.
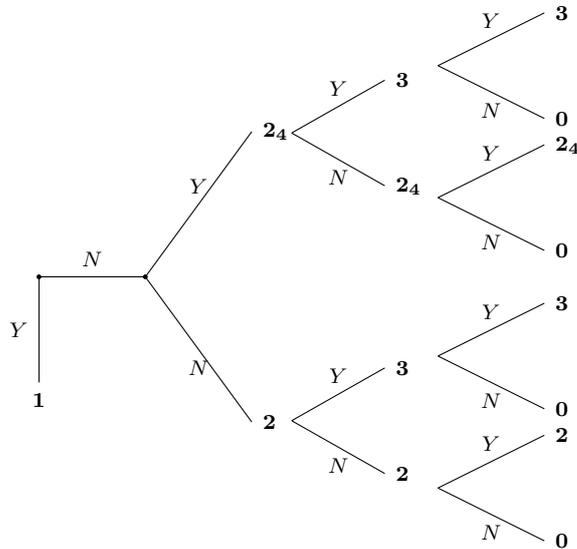
Figure 2: Voting procedure

motion 1 was defeated by 391 to 242 votes,[25] and a more traditional sequence according to AP followed. First, the Spelman amendment 4 narrowly passed. In other words, the original motion 2 was defeated against the amended version by 312 to 308 votes. Hence, motion 2 amended by 4, denoted here by 2_4, became the standing motion. Then, the Malthouse proposal 3 was defeated by 374 to 164 votes. Finally, the still standing motion 2_4 passed against the status quo 0 by 312 to 278 votes. To conclude, the agenda mixed a successive element (the vote on alternative 1) with an amendment element (the remaining stages). The observed voting profiles are summarized in the following table:

Inferring the tree based on convexity Assume first that preferences were single peaked on a tree, and that the employed agenda was convex with respect to that tree. Since the present agenda is analogous to the one of Example 2, the underlying tree cannot be pinned down even under a convexity assumption. Indeed, there are 4 trees for which the used agenda becomes convex: because of the amendment element (see Proposition 3) they must all contain a sub-tree that is a star with alternative 0 at the center, connected by edges to alternatives $2, 2_4$ and 3. In each of those

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[25]Note that this was tighter than the original defeat by 230 votes. An even tighter outcome was obtained at a later, third vote on the same issue. Thus, May's strategy, described below might have worked to some extent.

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Profile	Observations
NYNY	310
YNYN	94
YNAN	68
NNYN	65
YNNN	32
YANN	16
NNAN	11
AAAA	11
YAAN	7
YYNY	5
NNNN	5
Others	25

Table 4: Individual vote profiles for the first sequence of Brexit votes.

trees, alternative 1 - the only one that was voted according to SP - must be connected as a leaf to one of the remaining alternatives (see Proposition 2).

For the trees containing the edges $1 - 3$ or $1 - 2_4$, respectively, we find a relatively large number of observed profiles that is not consistent with single-peaked preferences on the respective tree:

1. In the tree with edge $1 - 3$, the profile $YNNN$ appearing 32 times is not consistent.

2. In the tree with edge $1 - 2_4$, the profile $YNYN$ appearing 94 times is not consistent.

In contrast, both for the star with center at 0 and for the tree containing the edge $1 - 2$, almost all observed profiles are consistent with sincere voting and single-peaked preferences on the respective tree.[26] The only inconsistent profile (among those explicitly listed in Table 1) is $YYNY$, which was cast only 5 times: it implies 1 being the peak and 2_4 being preferred to 0, which is not possible under single-peaked preferences. Thus, at least in principle, these two trees satisfy all our formal criteria of maximizing the number of consistent profiles.

An alternative explanation Recall that convex agendas are necessarily content-based: the meaning of the alternatives determines what is perceived as more extreme or more moderate, and hence what is put to vote first. But,

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[26]Here we interpret profiles that involve abstentions with a bias towards consistency.
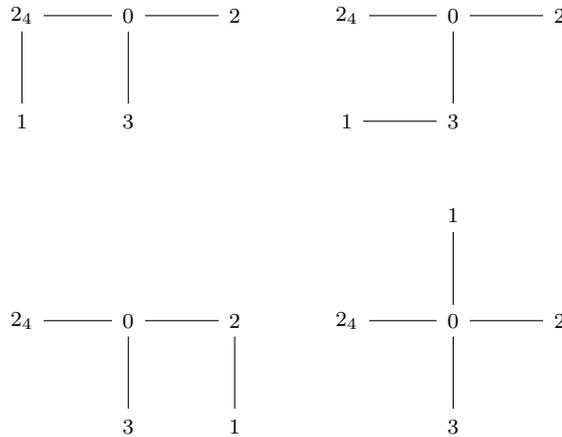
Figure 3: Preference trees that yield a convex agenda

the UK Parliament traditionally uses procedural agenda formation rules (e.g., status quo last, amendment to an amendment before the amendment itself, and so on). Thus, while sometimes agendas turn out to be convex, this is usually not by design.

Our inference exercise in the previous section did not take content into account, and this leads here to some unlikely inferred preference profiles. For example, for both trees identified above under the convexity assumption, any voter with peak on 2_4 (no Brexit without a deal, ever) should prefer alternative 0 (no deal Brexit) to alternative 2 (no Brexit without a deal on March 29). Similarly, if the underlying tree is the star, any voter with peak on May's deal 1 should prefer a no Brexit deal 0 to any other alternative. Or, if the tree is such that 1 is connected to 2, any agent with peak on 0 (no deal) should prefer alternative 2 (no Brexit without a deal on March 29) to May's deal 1.

Another main reason to question the convexity of the employed agenda is that at least one explicit deviation from convexity was actually part of prime minister May's strategy in order to get her deal through.[27] On January 15, 2019, prior to the voting marathon, May's negotiated Brexit deal with the EU has been rejected by a very large margin of 230 votes.[28] Nevertheless,

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[27]Note that, contrasting the German Bundestag, the UK Parliament has no rules for agenda formation that would generally lead to convexity.

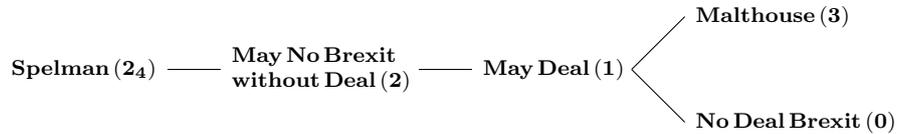[28]This was the largest defeat for a sitting government in history!

Figure 4: Preference tree underlying the first votes on Brexit

it was put to vote again, **before** the more "extreme" alternatives such as a no-deal Brexit or a new referendum (or, say, an arrangement whereby the UK remains in the EU common market and customs union) were formally discarded.[29] Here is what the Economist wrote about this strategy:[30]

> [...] Mrs May's plan is to hold yet another vote on her deal and to cudgel Brexiteers into supporting it by threatening them with a long extension that she says risks the cancellation of Brexit altogether. At the same time she will twist the arms of moderates by pointing out that a no-deal Brexit could still happen, because avoiding it depends on the agreement of the EU, which is losing patience. It is a desperate tactic from a prime minister who has lost her authority. It forces MPs to choose between options they find wretched when they are convinced that better alternatives are available. [...]

As the Economist explains, May's hope was that both Leavers and Remainers would finally unite behind her deal because each group perceived one of the remaining, extreme alternatives still on the table (and thus also a "lottery" among them) as catastrophic from their point of view.[31] Such an agenda, where a compromise is voted upon before the extremes, clearly violates convexity.

Given the above caveats, and using the alternatives' content, we therefore suggest that the five motions in this part of the voting marathon can be arranged on a tree as shown in Figure 4. To derive this tree we order the alternatives on a one-dimensional scale from soft (or no) Brexit to a hard Brexit, and we deviate from a simple linear order only when alternatives are not easily comparable along this axis.

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[29]The same strategy has been pursued by May' successor, Boris Johnson. It was repeatedly countered by a majority in Parliament who refused to vote for a deal while a no-deal Brexit was still an option (the Benn and Letwin amendments).

[30]"Whatever next?" Lead Article, *The Economist* March 16th 2019, page 11.

[31]Zeckhauser [1969] shows that introducing lotteries may destroy single-peakedness. Lotteries become relevant when the agenda is not convex because the anticipated outcome depends then on beliefs about others' preferences.

We assume below that preferences were single-peaked on this tree, and check whether the observed voting profiles are consistent with sincere voting. We also discuss the sincerity assumption in this non-convex case.

The following table summarizes the most frequently observed profiles and the single-peaked preference order on the tree that would generate each of the observed profiles given sincere voting, and given the agenda used.[32] We denote indifference between alternatives 1 and 2 by $1 \sim 2$, and the notation $1 \succ (2,3)$ summarizes that the preference could be either $1 \succ 2 \succ 3$ or $1 \succ 3 \succ 2$.

Profile	Observations	Implied single-peaked ranking
NYNY	310	$2_4 \succ 2 \succ 1 \succ (3,0)$
YNYN	94	$1 \succ (3,0,2) \succ 2_4$
YNAN	68	$1 \succ (2,0) \succ 3 \sim 2_4$
NNYN	65	$2 \succ 1 \succ (3,0) \succ 2_4, \quad 3 \succ 1 \succ (2,0) \succ 2_4,$
		$0 \succ 1 \succ (3,2) \succ 2_4$
YNNN	32	$1 \succ (0,2) \succ 2_4 \succ 3$
YANN	16	$1 \succ 0 \succ 2 \sim 2_4 \succ 3$
NNAN	11	$0 \succ 1 \succ 2 \succ 2_4 \sim 3, \quad 2 \succ 1 \succ 0 \succ 2_4 \sim 3$
AAAA	11	$1 \sim 2 \sim 2_4 \sim 3 \sim 0$
YAAN	7	$1 \succ 0 \succ 2 \sim 2_4 \sim 3$
YYNY	5	None
NNNN	5	$0 \succ 1 \succ 2 \succ 2_4 \succ 3, \quad 2 \succ 1 \succ 0 \succ 2_4 \succ 3$
Others	25	Diverse (including peaks on 2)

Table 5: Individual vote profiles for the first sequence of Brexit votes.

It follows from the above table that, with the exception of one profile that was observed just five times ($YYNY$) - the same inconsistent profile identified above - , all common profiles are indeed consistent with our assumption that voting was sincere according to single-peaked preferences on the constructed tree.[33] The selected alternative 2_4 was the Condorcet winner because it won the direct vote against alternative 2, the only other close contender.

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[32]We show all vote profiles that were cast by at least 5 voters.

[33]After alternative 1 was defeated by a large majority, the problematic profile YYNY is consistent with single-peaked preferences with a peak on 2_4. Out of the rare profiles that were used by 25 voters and that we didn't list, 14 voters cast profiles that are inconsistent with our assumption.
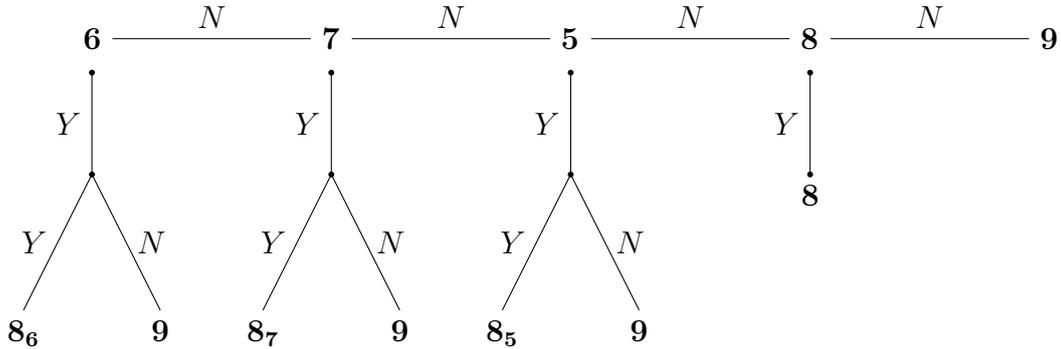
Figure 5: Voting procedure used for the second voting sequence

6.2.2 The Second Voting Sequence

The second sequence of votes can be seen as determining how to precisely continue the process, and how to implement the previous decision of not leaving the EU without a deal by March 29, 2019. The motions were:

5 Corbyn: extend Article 50^{34} + new Brexit approach (amendment to 8).

6 Wollaston: Hold a new referendum (amendment to 8).

7 Benn: Hold indicative votes (amendment to 8).[35]

8 May: Motion to delay the Brexit date.

9 We denote by 9 the status quo, a no-deal Brexit on March 29. Although Parliament has just excluded a no-deal Brexit "forever", without further legislative steps, including the approval of the EU, a Brexit on March 29 was still the legal default.[36]

The voting agenda is depicted in Figure 5. The agenda for this sequence was again a combination of SP and AP.

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[34]This was the legal step announcing the intention to leave the EU, including the deadline of March 29.

[35]The purpose was to find a deal that can be approved by a majority. For simplicity we ignore here the Powell amendment to this amendment, which would hold indicative votes while specifying a precise Brexit date of June 30.

[36]This has also been emphasized by the EU's leadership in the summit that followed the defeat of May's deal. The legal conundrum stemming from this status quo continued also after Brexit's delay and Johnson's premiership.
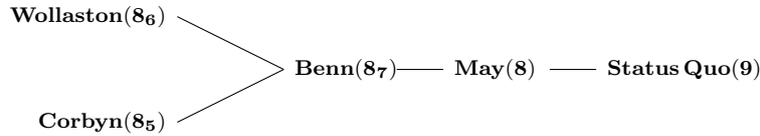
Figure 6: Preference tree underlying the second Brexit vote

May's basic motion 8 asked for a delay in the Brexit process, one that would give the parliament more time to approve a deal. The first vote was on amendment Wollaston 6 (new referendum). If accepted, the only other vote would be on May's motion 8 amended by 6, denoted by 8_6, pitted against the status quo. Wollaston was defeated by 85 to 334 votes. The second vote was on Benn's amendment 7. If accepted, the only other vote would be on motion 8_7 pitted against the status quo. Benn's amendment was narrowly defeated by 312 to 314 votes. The third vote was on Corbyn's amendment 5. If accepted, the only other vote would be motion 8_5 pitted against the status quo. Corbyn's amendment lost by 302 to 318 votes. Finally, as none of the amendments was successful, the un-amended motion 8 was pitted against the status quo, and passed by 413 to 202 votes.

Inferring the tree based on convexity   As above, we first assume that the agenda was convex with respect to an underlying tree. Analogously to a pure amendment procedure, the fact that the status-quo 9 could be potentially pitted against any other alternative forces the tree to be a star with alternative 9 at its center (see Proposition 3).

Then, a large number of observed profiles is inconsistent with single-peakedness on this star. For example, a voter with profile $YYYY$ (83 times) cannot have a peak on alternative 8 since she prefers all amendments to it. But any such agent would prefer alternative 9 at the last vote, which is incompatible with the Yes vote at the last step.

An alternative explanation   Taking now into account the alternatives' content, we assume that preferences for the second sequence were single-peaked on the tree shown in Figure 6. Then, only ten voting profiles are inconsistent with sincere voting according to single-peaked preferences on this tree given the employed agenda.

Table 6 summarizes all common profiles and the single-peaked preference orders that would generate each of these observed profiles given sincere voting and given the agenda. All common profiles are indeed consistent with our

Profile	Observed Number	Implied single-peaked preference relation
AYYY	202	$8_6 \sim 8_7 \succ 8_5 \succ 8 \succ 9$
NNNN	200	$9 \succ 8 \succ 8_7 \succ (8_5, 8_6)$
NNNY	103	Any with peak on 8
YYYY	83	$8_6 \succ 8_7 \succ 8_5 \succ 8 \succ 9$
AAAA	14	$8_6 \sim 8_7 \sim 8_5 \sim 8 \sim 9$
NYYY	10	$8_7 \succ (8_6, 8_5) \succ 8 \succ 9, \ 8_7 \succ 8_5 \succ 8 \succ (8_6, 9)$
NNNA	8	$9 \sim 8 \succ 8_7 \succ (8_5, 8_6)$
NYNY	6	$8_7 \succ 8 \succ (8_5, 8_6, 9), \ \ 8_7 \succ 8_6 \succ 8 \succ (8_5, 9)$
Others	23	Diverse

Table 6: Individual vote profiles for the second sequence of Brexit votes.

assumptions,[37] but the identification of the Condorcet winner is here more complex: either alternative 8_7 (Benn) or alternative 8 (May) could have been it. Alternative 8 very narrowly won against 8_7 by 314 to 312 votes, suggesting at first sight that 8 was the Condorcet winner. But, note that at that point in the voting sequence, alternative 8_5 (Corbyn) was still in play. For a voter with a peak on 8_5, sincere voting prescribes a vote **against** 8_7 even though he/she prefers 8_7 to 8. Since we do not have direct information on how many voters had a peak on 8_5, it is not completely clear which alternative was the Condorcet winner. On the other hand, the second vote in the sequence clearly pitted 8_7 vs. 8, so a home-style argument a la Fenno (see discussion below) might actually speak here **against** sincere voting and thus reinforce the view that alternative 8 (May) was the Condorcet winner. The identification difficulty described above is typical of non-convex agendas.

6.2.3 Why sincere voting?

We have argued that the employed agendas in the Brexit case were not convex, partly by tradition and partly by design. Thus, sincere voting need not constitute a strategic equilibrium. Nevertheless, we have shown that sincere voting based on single-peaked preferences on a tree yields precise predictions that agree well with the data. Why would legislators vote here sincerely?

An important force behind sincere, straightforward voting is the need to explain behavior and to make it transparent to constituents (see Fenno [1978]).[38] We observe a high correlation between MP's hawkish voting be-

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[37]Among the rare profiles cast by 23 voters, only 5 voters behaved inconsistently with our assumption.

[38]But recall that in non-convex procedures sincere voting might not always be the sim-

havior on Brexit and the percentage in favor of Leave in their constituency at the 2016 Referendum. Thus, an MP from a strong Leave constituency may find it difficult, if not impossible, to opportunistically vote Yes on a soft-Brexit alternative even if it yields some strategic gain.

This disciplining effect seems to be particularly relevant in the UK, where each member of parliament is individually elected (first past the post) in relatively small constituencies of about 70-80000 people each.[39] This should be contrasted with Germany, where a majority of legislators are elected on state-wide party lists (proportional representation), and are therefore not directly accountable to a local community. Moreover, even the directly elected legislators represent much larger, and possibly more diverse, constituencies of about 250000 people each. Thus, if sincere, transparent voting is a desideratum, a carefully designed agenda that induces it seems relatively more important in Germany than in the UK.

7 Conclusion

Even if sincere voting is being enforced via motives and institutions that lie outside the immediate scope of this paper, we strongly believe that having content-based agenda formation rules inducing convexity ensure a much smoother process both at the agenda setting stage and at the voting stage, and we recommend their use. A steady use of well-designed, convex agendas - that do not serve special interests and that tend to elect the Condorcet winner - establishes sincere voting as the *modus operandi* for members of parliaments, and frees them from the need to strategically asses each instance anew. As we saw above, Premier May's strategy of using a non-convex agenda, specially designed to create uncertainty, has badly backfired, and she lost her job. In fact, many examples of "deviations from sincere voting - so called "strategic manipulations" - can be traced back to a lack of convexity in the employed agenda. An early, very interesting paper analyzing such a case is Riker [1958].

We conclude by noting that our general method of inquiry can be extended to obtain a more robust inference of preferences even for non-convex agendas. Rather than assuming sincere voting one could compute equilibrium strategies and use these to infer preferences. However, as explained above, equilibrium computation is very complex, and inferences are then particularly sensitive to the exact (non-observable) beliefs held by voters.

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plest behavior to explain one's constituents!

[39]For example, Prime Minister Boris Johnson was elected to the relevant Parliament by gathering just 29000 votes in his constituency!

For future work, we propose instead to determine the strategies that survive the iterated elimination of weakly dominated strategies and to base inference on these strategies. Such an inference can yield bounds on the number of voters with each possible preference profile.

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Appendix

Uniqueness of Equilibrium in Sincere Strategies

Proposition 4 *For any convex binary sequential voting procedure, sincere voting is the unique ex-post perfect equilibrium in responsive strategies if each vote is between disjoint sets of alternatives.*

Proof. We show that, in every ex-post perfect equilibrium in responsive strategies, voting at a node v must be sincere if it is sincere at all following nodes. The result will follow by induction since sincere voting is the unique equilibrium in responsive strategies if only two alternatives remain,

Fix an ex-post perfect equilibrium in responsive strategies σ and a non-terminal node v where the vote is among two disjoint subtrees Ψ' and Ψ''. Any voter who prefers any alternative in Ψ' over any alternative in Ψ'' will vote for Ψ' in any ex-post perfect equilibrium in responsive strategies and, conversely, any voter who prefers any alternative in Ψ'' to any alternative in Ψ' will vote for Ψ''. In tree Ψ, there is exactly one edge that connects an alternative in Ψ' to an alternative in Ψ''. We denote these alternatives by A and B, respectively. Note also that any path connecting an alternative in Ψ' to an alternative Ψ'' must contain both A and B. Fix now a preference profile for all voters except i such that n voters have a peak at A and prefer any alternative in Ψ' to any alternative in Ψ'', and n voters have a peak at B and prefer any alternative in Ψ'' to any alternative in Ψ'. It follows that these voters vote for Ψ' and for Ψ'', respectively, and hence that voter i is pivotal at v. Voting for Ψ' will lead under the sincere continuation to the adoption of A, while voting for Ψ'' will lead to the adoption of B. If i's peak (among remaining alternatives) is in Ψ', she will prefer A to B (this is because the path from the peak to B must contain A) and it is a unique best response to vote for Ψ'. Analogous arguments apply if i's peak is in Ψ'' and we conclude that σ must prescribe a sincere vote for i at v. ∎

Identification of Preference Tree for the Vote on Abortion Law

The two trees shown in Figure 1 yield that, out of the 638 voters, 601 and 610 voters, respectively, casted ballots that are consistent with sincere voting. We now argue that there can be no other tree that both yields a convex agenda and explains a higher number of votes:

1. Because the last vote is between **G** and **H**, there must be an edge **G-H** for the agenda to be convex.

2. Since **F** is proposed at the second-to-last vote, it must be a leaf in the corresponding subtree containing alternatives **F,G** and **H**. This yields two possibilities: **F-G-H** or **G-H-F**. The vote profile $NNNNNYN$ was cast by 122 voters, which under sincere voting implies that these voter prefer **F** to **H** to **G**. Therefore, no tree that contains the subtree **F-G-H** can explain more votes.

3. Now we add alternative **E** as a leaf to the tree **G-H-F**. There are 203 voters casting vote profile $NNNNYNY$, which implies that they have a peak at **E** and prefer **G** to **H** and **G** to **F**. For this profile to be consistent, there must be an edge **E-G**. We conclude that any tree maximizing the number of consistent votes must contain the subtree **E-G-H-F**.

4. Now we add alternative **D** as a leaf to the tree **E-G-H-F**. There are 71 voters who voted $NNNYNNY$, which implies that **D** is their most preferred alternative, and **G** is their second-most preferred alternative. If a tree does not contain the edge **D-G**, these preferences will not be single-peaked, and this tree will, therefore, not maximize the number of consistent votes.

5. Now we add alternative **C** as a leaf to the tree consisting of the line **E-G-H-F** plus the edge **G-D**. There were 85 voters casting vote profile $NNYNNYN$, implying that **C** is their most-preferred alternative and **F** their second-most preferred alternative. Therefore, any tree maximizing the number of explained votes must contain the edge **F-C**.

6. It remains to place **A** and **B**. All voters that voted for **A** (or formally abstained in the first vote) have as their second-most preferred alternative **B** or **E**. Moreover, some voters are indifferent between **A** and **B** (or **A** and **E**, or **A** and **B** and **E**). This implies that the two trees shown in Figure 1 maximize the number of votes that are consistent.