ENDOGENOUS PRODUCTION NETWORKS AND NON-LINEAR MONETARY TRANSMISSION *

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Abstract

I develop a tractable sticky-price model, where input-output linkages are formed *endogenously* through firms' optimizing decisions. My model delivers cyclical properties of networks that are consistent with those I estimate using aggregate, sectoral and firm-level data, both unconditionally and conditional on identified real and monetary shocks. Crucially, I show that my model jointly rationalizes multiple empirically documented *non-linearities* associated with monetary transmission, which cannot be explained by workhorse models. First, the magnitude of real GDP's response to a monetary shock is procyclical in my model. This *cycle dependence* occurs because in expansions the level of productivity is high, encouraging firms to connect to more suppliers, which in turn facilitates stronger downstream propagation of price rigidity. Second, short-run non-neutrality of real GDP is higher following periods of loose monetary policy. The latter *path dependence* happens as under nominal rigidities, higher supply of money erodes the real prices charged by suppliers, encouraging more connections and hence a stronger contagion of stickiness to customer firms. Third, large monetary expansions make the production network denser, and hence have a disproportionally larger effect than small monetary expansions; on the other hand, large monetary contractions break the network and hence have a disproportionally smaller effect on GDP. The latter *size dependence* holds even under fully *time-dependent* pricing.

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1 Introduction

Intermediate inputs, and production networks that facilitate their trade, are an important factor in model economies' production structures. According to the Bureau of Economics Analysis, in the United States almost 50 per cent of all produced goods are then re-used as intermediate inputs. Though a key factor of production overall, the relative importance of intermediate is much less pronounced in recessions: as can be seen from panel (a) of Figure 1, the intermediate's share in US total output drops sharply in periods of economic slack. Moreover, adjustment on the extensive margin plays an important part in the above cyclicality: panel (b) uses data on input-output linkages across US publicly listed firms, and clearly shows that the average number of suppliers drops in recessions.

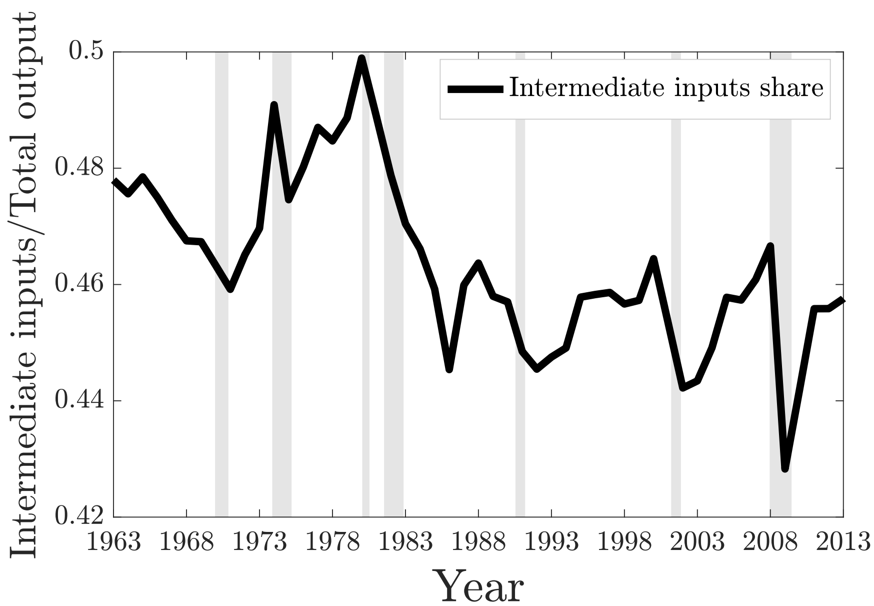
This paper develops a sticky-price New Keynesian model that allows input-output linkages across sectors to be determined endogenously by firms' optimizing decisions, and thus vary over the business cycle. Crucially, I show that not only can such model account for the observed variations in the importance of intermediate inputs, but also jointly rationalizes key empirically established non-linearities in *monetary transmission*, which cannot be explained by workhorse models. A key novel mechanism proposed in this paper is the relationship between the density of the network, which determines how important intermediate inputs are, and the strength of strategic complementarities in price setting created by the roundabout production structure that amplify the effect of monetary shocks. In particular, I show that the above novel mechanism can explain the dependence of the strength of monetary transmission on the *phase of the business cycle*, *past monetary policy* and the *size of the shock* without introducing state-dependent pricing and in a manner that is consistent with econometric evidence.

First, the effect of monetary policy shocks on real GDP exhibits *cycle dependence* in my model, as the degree of short-run non-neutrality is much stronger in expansionary states. This is consistent with the recent model-free econometric studies that have found the response of real GDP to monetary shocks to be extremely weak in recessions, arguing that monetary policy is "pushing on a string" (Tenreyro and Thwaites, 2016, Alpanda et al., 2019, Jordà et al., 2019). The mechanism that generates this effect is as follows: in expansions, the level of productivity is high, which incentivizes firms to connect to more suppliers and thus creates stronger downstream propagation of price rigidity due to amplified complementarities in price setting. On the other hand, in recessions firms decide to disconnect from suppliers, effectively making the economy more money neutral, as complementarities in price setting weaken.

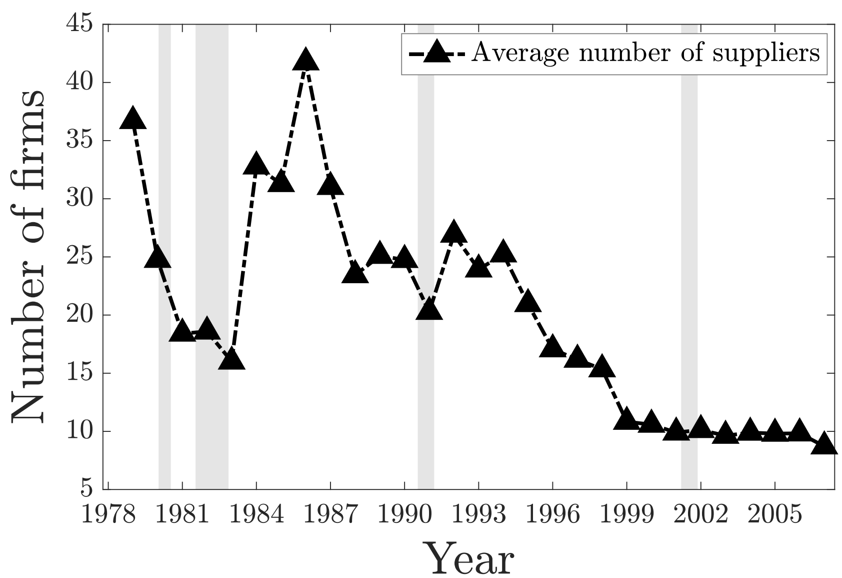
Second, the strength of monetary transmission is *path dependent* in my model, as any monetary interventions have a stronger effect on GDP in the aftermath of previous loose monetary policy. This is consistent with the findings of Jordà et al. (2019) regarding stronger response of GDP to monetary policy shocks in states of the world where inflation is high. Under nominal rigidities, prolonged episodes of loose monetary policy erode real prices charged by supplier firms, which incentivizes firms to connect to more suppliers and hence makes the network denser and complementarities in price setting stronger. As a result, either contractionary or expansionary monetary shocks would

Figure 1: **The importance of intermediate inputs over time**

(a) Expenditure on intermediate inputs as share of total output (US)



(b) Average number of supplier firms (US public companies)



Notes*: panel (a) documents the ratio of total expenditure on intermediate inputs and total output in the United States (1963-2013), based on data published by the US Bureau of Economic Analysis (BEA); panel (b) uses Compustat-based data on firm-to-firm input-output linkages (1979-2007) constructed by Atalay et al. (2013) in order to estimate the average number of suppliers of a publicly listed company in a given year by assuming that the number of suppliers follows a Pareto distribution, whose tail parameter is then estimated following Gabaix and Ibragimov (2011).*

have a stronger effect of GDP in the aftermath of previous episodes of loose policy. Of course, the converse is also true: any shocks would have a weaker effect on real GDP following prolonged episodes of tight monetary policy.

Third, large shocks exhibit different transmission compared to small shocks, and such *size dependence* occurs even if the probability of price adjustment is fully time-dependent. The latter is consistent with econometric findings in Alvarez et al. (2017) and Ascari and Haber (2019), who find that large nominal shocks have a disproportionally smaller effect on real variables, compared to small nominal shocks. In my model, large monetary expansions make the network denser, and hence have a disproportionally *larger* positive effect on real GDP compared to small monetary expansions. As a result, achieving a given degree of real GDP expansion require a (weakly) smaller monetary intervention. On the other hand, large monetary contractions break the network, and hence have a disproportionally *smaller* negative effect on real GDP compared to small monetary contractions. A corollary of the above is that achieving a given degree of real GDP contraction requires a (weakly) larger monetary intervention.

Fourth, I provide novel model-free econometric evidence on network cyclicality, conditional on identified technology and monetary shocks, which supports my key new theoretical channel. As a first exercise, I use sectoral data from US Bureau of Economic Analysis (BEA) to construct annual time series of intermediates intensities for 65 sectors of the US economy. Consistently with my model, I find that intermediates intensity rises following a positive technology shock, and following a monetary easing. Moreover, I find evidence of cascade effects: large shocks, both technology and monetary, lead to disproportionally larger magnitudes of network responses, once again consistently with my model. One disadvantage of using sectoral data is that it does not allow to disentangle intensive and extensive margins of network adjustment, whereas it is the latter that is specific to my model. I therefore provide additional evidence using data firm-level linkages in Compustat, as constructed by Atalay et al. (2011). My findings using firm-level data fully confirm the findings using sectoral data.

Contribution to the literature. This paper makes a contribution to at least two strands of the literature. First, I contribute to the relatively recent literature on *endogenous* production networks in macroeconomics, by developing the first model featuring both *nominal rigidities* and endogenous network formation. Previous macroeconomic studies have focused either on environments with flexible prices (Carvalho and Voigtländer, 2015, Oberfield, 2018 Acemoglu and Azar, 2020) or on finding the social planner's solution to the network formation problem (Taschereau-Dumouchel, 2019). On other other hand, the literature on production networks and nominal rigidities has only considered environments with sticky prices and *exogenously* given production networks (Nakamura and Steinsson, 2010, Carvalho et al., 2018, Pasten et al., 2019, Smets et al., 2019).

Second, I also contribute to the literature on non-linearities in monetary transmission, which has so far been predominantly empirical. In particular, Tenreyro and Thwaites (2016), Jordà et al. (2019) and Alpanda et al. (2019) estimate that monetary policy is much less effective in recessions compared to expansions. As for path dependent effects, those are found empirically in Berger et al.

(2018) and Jordà et al. (2019). With regard small and large nominal shocks, Alvarez et al. (2017) and Ascari and Haber (2019), who find that large nominal shocks have a disproportionally smaller effect on real variables, compared to small nominal shocks. Recent theoretical contributions include Santoro et al. (2014), Eichenbaum et al. (2018) and McKay and Wieland (2019).

The rest of the paper is structured as follows. Section 2 sets out the general theoretical framework. Section 3 considers an analytically tractable version of the model, and presents the key theoretical results. Section 4 presents econometric evidence that supports the key mechanisms of the model. Section 5 concludes.

2 A general theory formulation

This section sets out a general formulation of my theoretical framework, which combines two environments previously considered in the literature. On the one hand, the model features a multi-sector roundabout production structure, with firms' pricing subject to sector-specific nominal rigidities, in spirit of Carvalho et al. (2018) and Pasten et al. (2019). The key additional feature is the fact that input-output linkages are formed *endogenously* by firms' desire to optimize their production costs, in a manner introduced by Acemoglu and Azar (2020) in a flexible-price environment. As a result, the equilibrium production network responds to shocks, both real and monetary, allowing for the degree of complementarity in price setting, and hence the degree of monetary non-neutrality, to be state-dependent in my model.

2.1 Model primitives

The model is set in discrete time, with outcomes in $t = 0$ being exogenously given, and outcomes in $t \geq 1$ being determined endogenously by agents' decisions. There are three types of agents in my model. First, there is a continuum of infinitely lived households. Second, there is a continuum of monopolistically competitive firms, owned by the households, and each firms belongs to one, and only one, of the K sectors; I denote the set of all firms in sector k by $\Phi_k, \forall k = 1, 2, ..., K$. Third, there is a government, comprising of a central bank which conducts monetary policy, and a fiscal authority which collects taxes from firms and rebates them to households.

A crucial feature of my economy is presence of a roundabout production structure across sectors, where the input-output linkages are formed endogenously through each firm's choice of set of suppliers, denoted by $S_k \subseteq \{1, 2, ..., K\}, \forall k, \forall j \in \Phi_k$. For every choice of supplier sectors, there is a given level of productivity, as pinned down by a predetermined sector-specific mapping $\mathcal{A}_{k,0} : S_k \rightarrow \mathcal{R}^+, \forall k$, and $\mathcal{A}_0 \equiv [\mathcal{A}_{1,0}(.), \mathcal{A}_{2,0}(.), ..., \mathcal{A}_{K,0}(.)]'$. Importantly, $\mathcal{A}_0$ is ex ante known by the agents in my economy, and they expect it to remain unchanged forever. For any two mappings $\mathcal{A}_0$, $\mathcal{A}'_0$, the convention is that $\mathcal{A}'_0 \geq \mathcal{A}_0$ if and only if $\mathcal{A}'_{k,0}(S_k) \geq \mathcal{A}_{k,0}(S_k), \forall S_k, \forall k$.

As for the nominal side of the economy, the agents are aware of the initial level of money supply $\mathcal{M}_0$, and ex ante anticipate it to stay at that level forever. At the beginning of the first period, they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$, and face no uncertainty beyond that point. The central bank credibly commits to a set of policies that maintain equilibrium quantity of money at the exogenously given level $\mathcal{M}_t$ in every period $t \geq 1$.

2.2 Firms: production and *endogenous* choice of suppliers

On the production side, there are K sectors, indexed by $k = 1, 2, ..., K$ with a measure one of firms in each sector; let Φ_k denote the set of all firms in sector k. The production function of firm $j \in \Phi_k$ is given by:

$$Y_{kt}(j) = e(\Omega, S_{kt})\mathcal{A}_{k,0}(S_{kt})N_{kt}(j)^{1-\sum_{r\in S_{kt}}\omega_{kr}} \prod_{r\in S_{kt}} Z_{krt}^{\omega_{kr}}(j) \tag{1}$$

where $\omega_{kr} = [\Omega]_{kr}$ are cost shares, which could be interpreted as entries of the input-output matrix, $e(\Omega, S_k) \equiv (1 - \sum_{r\in S_k}\omega_{kr})^{-(1-\sum_{r\in S_k}\omega_{kr})} \prod_{r\in S_k}\omega_{kr}^{-\omega_{kr}}$ is a normalization term, $S_k \subseteq \{1, 2, ..., K\}$ is the set of sectors, whose firms supply inputs to firms in sector k, $\mathcal{A}_{k,0}(.)$ is a mapping from the chosen level of suppliers to the associated level of productivity; $N_{kt}(j)$ is the labor input of firm j in sector k, whereas $Z_{krt}(j)$ are purchases of intermediate inputs from sector r, which in turn is an aggregator of purchases from all firms in sector r: $Z_{krt}(j) \equiv \left(\int_{\Phi_r} Z_{krt}(j,j')^{\frac{\theta-1}{\theta}} dj'\right)^{\frac{\theta}{\theta-1}}$, $\theta > 1$.

Given the above production structure, the marginal cost of all firms in sector k, conditional on a particular set of suppliers, is given by:

$$MC_{kt}(j) = MC_{kt} = \frac{1}{\mathcal{A}_{k,0}(S_{kt})} W_t^{1-\sum_{r\in S_{kt}}\omega_{kr}} \prod_{r\in S_{kt}} P_{rt}^{\omega_{kr}}, \quad \forall k \tag{2}$$

where W_t is the wage paid to labor, and P_{rt} is the sector price index of sector r.

The choice of suppliers is chosen optimally to minimize the marginal cost in every period:

$$S_{kt}^{*} \in \arg\min_{S_{kt}} MC_{kt}(S_t, P_t), \quad \forall k \tag{3}$$

where $S_t \equiv [S_{1t}, ..., S_{Kt}]'$ and $P_t \equiv [P_{1t}, ..., P_{Kt}]'$. The above minimization problem highlights the trade-off faced by firms when choosing the optimal set of suppliers: on the one hand, firms want to buy inputs from sectors whose combination delivers a high level of productivity, while at the same time avoiding those that charge high prices for their output.

2.3 Firms: pricing under nominal rigidities

Price stickiness in my economy is modeled as a modified version of Calvo (1983), where in periods $1 \leq t \leq (T-1)$ firms face a constant and sector-specific probability of price non-adjustment α_k, whereas in periods $t \geq T$ firms face no nominal rigidities. The cut-off period $T > 1$ is known by all agents in the economy; naturally, as $T \to \infty$ the price setting problem collapses back to the standard Calvo (1983) pricing.

More precisely, in any period $1 \leq t \leq (T-1)$ a firm in sector k has probability $(1-\alpha_k)$ of setting its price equal to its optimal value. The optimal price at time $1 \leq t \leq (T-1)$ is chosen to maximize expected future discounted nominal profits:

$$\max_{P_{kt}(j)} \sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} \left[P_{kt}(j) Y_{k,t+s}(j) - (1+\tau_k) MC_{kt} Y_{k,t+s}(j) \right], \tag{4}$$

subject to the production function in (1); $F_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$ and is defined in the next subsection; τ_k is a tax imposed by the government, revenue from which is rebated to households. The first order condition for the optimal price for any firm in sector k, $\tilde{P}_{kt}$ is given by:

$$\tilde{P}_{kt}(j) = \tilde{P}_{kt} = (1+\mu_k) \frac{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s} MC_{k,t+s}}{\sum_{s=0}^{T-t-1} \alpha_k^s F_{t,t+s} P_{k,t+s}^{\theta} Y_{k,t+s}}, \quad 1 \leq t \leq T-1, \quad \forall k \tag{5}$$

where $(1+\mu_k) \equiv (1+\tau_k)\frac{\theta}{\theta-1}$ is the steady-state markup. On the other hand, in any period $t \geq T$ there are no nominal rigidities, firms' maximize contemporaneous profits, and optimally set $P_{kt} = (1+\mu_k) MC_{kt}, t \geq T$. Given that the optimal price is identical for all firms within a sector, sectoral price index can be obtained by aggregation using the ideal sectoral price index $P_{kt} \equiv \left(\int_{\Phi_k} P_{kt}(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \forall k$:

$$P_{kt} = \begin{cases} \left[\alpha_k P_{k,t-1}^{1-\theta} + (1-\alpha_k)(\tilde{P}_{kt})^{1-\theta} \right]^{\frac{1}{1-\theta}}, & 1 \leq t \leq (T-1); \\ \\ (1+\mu_k) MC_{kt}, & t \geq T. \end{cases} \tag{6}$$

2.4 Households

A continuum of infinitely lived households populates our economy and owns all the firms. Markets are assumed to be complete, so a full set of Arrow-Debreu securities is available. The representative household makes choices to maximize the lifetime utility:

$$\max_{\{C_{t+s}, N_{t+s}, B_{t+s+1}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - \gamma N_{t+s} \right] \tag{7}$$

subject to

$$P_t^c C_t + [F_{t,t+1} B_{t+1}] \leq B_t + W_t N_t + \sum_{k=1}^{K} \Pi_{kt} + T_t, \quad \forall t \tag{8}$$

where C_t and P_t^c are the composite consumption good and the consumption price index (defined below) respectively, N_t is labor supply, B_{t+1} is the payoff of securities purchased at time t, $F_{t,t+1}$ is the price of those securities at time t, Π_{kt} denotes aggregate nominal profits of firms in sector k, β is the discount factor for future utility and T_t are lump-sum transfers from the government, γ parameterizes disutility of labor supply.[1]

The composite consumption index C_t is an aggregator for the final consumption of goods produced in the different sectors of our economy, so that $C_t \equiv \prod_{k=1}^{K} \omega_{ck}^{-\omega_{ck}} C_{kt}^{\omega_{ck}}$, where ω_{ck} is the relative weight consumers put on goods produced in sector k, $\sum_{k=1}^{K} \omega_{ck} = 1$; C_{kt} is in turn an aggregator for the final consumption of goods produced by firms that belong to sector k: $C_{kt} \equiv \left(\int_{\Phi_r} C_{kt}(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$.

The households' maximization problem delivers standard first-order conditions, namely an equation for the stochastic discount factor $F_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-1} \frac{P_t^c}{P_{t+s}^c}$, and the equation for consumption-labor supply choice $C_t^{-1} = \gamma \frac{P_t^c}{W_t}, \quad \forall k$, where the consumption price index is given by $P_t^c = \prod_{k=1}^{K} P_{kt}^{\omega_{ck}}$.

2.5 Monetary policy

Purchases of final goods are subject to a cash-in-advance constraint, so that $P_t^c C_t = \mathcal{M}_t$. Agents are aware of the initial level of money supply $\mathcal{M}_0$, and ex ante anticipate it to stay at that level forever. In period $t = 1$ they discover the future path of money supply $\{\mathcal{M}_t\}_{t=1}^{\infty}$ and therefore any $\mathcal{M}_t \neq \mathcal{M}_0$ constitutes a monetary shock at time t to the agents. I assume that the central bank makes a credible commitment at $t = 1$ to make sure the equilibrium quantity of money follows the exogenous path $\{\mathcal{M}_t\}_{t=1}^{\infty}$; although I focus on monetary policy implemented by making sure money supply follows a given path in equilibrium, it could also be implemented by appropriately varying the nominal interest rate in my economy considered in the cashless limit.

2.6 Market clearing and equilibrium

In addition to the optimality conditions, budget constraints and the policy rule above, equilibrium in my economy is also characterized by market-clearing conditions in the asset market: $B_t = 0$; the the labor market: $N_t = \sum_{k=1}^{K} \int_{\Phi_k} N_{kt}(j) dj$; and the goods markets: $Y_{kt}(j) = C_{kt}(j) + \sum_{r=1}^{K} \int_{\Phi_r} Z_{rkt}(j', j) dj', \quad \forall k, \forall j \in \Phi_k$. All in all, equilibrium in my economy can be summarized as follows:

[1]The assumption of log utility of consumption and linear disutility of labor is made for analytical tractability. Moreover, such preference formulation is the baseline choice in other workhorse models of monetary policy and production networks, such as Nakamura and Steinsson (2010).

Definition 1 (Equilibrium). *Equilibrium is a collection of prices, allocations and networks, which given the exogenous series of money supply* $\{\mathcal{M}_t\}_{t=1}^{\infty}$*, the initial state vector* $(\mathcal{A}_0, \mathcal{M}_0)$ *and the vector of initial prices* $[P_{1,0}, ..., P_{K,0}]'$*, satisfy agent optimization and market clearing in every time period.*

3 An analytically tractable version

In this section I consider an analytically tractable version of the model obtained under $T = 2$, so that nominal rigidities are only present in the first period. Such simplification allows to formally characterize propagation of monetary shocks to real variables, under different initial states of productivity and initial levels of money supply. Formal propositions establish that small monetary shocks, which do not affect the shape of the network, have an impact on real final consumption that is (weakly) procyclical and (weakly) larger if the initial money supply is high. Further, large monetary expansions have more than proportional positive effect on aggregate consumption than small monetary expansions; on the other hand, large monetary contractions have less than proportional negative effect on aggregate consumption than small monetary contractions.

3.1 Equilibrium in the simplified version

In this section I focus on the version of my model where $T = 2$, so that nominal rigidities are only present at $t = 1$, and show that such simplification allows to analytically characterize transmission of monetary shocks to real variables, conditional on a particular initial state of the world $(\mathcal{A}_0, \mathcal{M}_0)$. The assumption is formally documented below:

Assumption 1. *In the firms' pricing problem* $T = 2$*, so that nominal rigidities are only present at* $t = 1$*, and prices are fully flexible for all* $t \geq 2$*.*

One implication of the above assumption is that the optimal reset price at $t = 1$ is given by $\tilde{P}_{k1} = (1 + \mu_k) MC_{k1}, \forall k$, delivering a very simple and tractable expression for the sectoral price index in the first period, namely $P_{k1} = \left[\alpha_k P_{k0}^{1-\theta} + (1 - \alpha_k)\{(1 + \mu_k) MC_{k1}\}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \forall k$. Moreover, Assumption 1 implies that money is neutral for all $t \geq 2$; given that my interest is in the transmission of monetary shocks to real variables, in this section I am only going to focus on outcomes at $t = 1$. For notational simplicity, I am therefore dropping time subscripts for variables at $t = 1$ for the remainder of this section.

As a result of money neutrality for all $t \geq 2$, the only change in money supply relevant for real variables is that between money supply at $t = 1 (\mathcal{M})$, and its baseline level $(\mathcal{M}_0)$. For the ease of interpretation of my results, I am introducing the following notion of a *monetary shock*:

Definition 2 (Monetary shock). *Let $\varepsilon^m \equiv \ln(\mathcal{M}/\mathcal{M}_0)$, where $\mathcal{M}$ is money supply at $t = 1$ and $\mathcal{M}_0$ is the baseline level of money supply, be the monetary shock in my economy.*

3.2 Baseline ($\varepsilon^m = 0$)

The baseline equilibrium in current simplified setting is given by the equilibrium evaluated under $\varepsilon^m = 0$. Below I formally analyze how different initial conditions $(\mathcal{A}_0, \mathcal{M}_0)$ affect the baseline allocation, and, even more importantly, the baseline equilibrium production networks. Before providing formal analytcial results, I build intuition using a simple intuition example with only two sectors.

Example 1. *Consider two sectors, where for simplicity firms are not allowed to trade with other firms in the same sector. For simplicity, let $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \varepsilon^a, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \varepsilon^a$, the sectoral shares be given by $\omega_{12} = \omega_{c1} = 0.5, \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, so that all pricing distortions are removed, and $\theta \to 1^+$, which allows us to obtain closed-form expressions for equilibrium sectoral prices and quantities. Under the above parameters, we can summarize the real marginal costs associated with different supplier choices as follows:*

	$S_2 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1, -1)$	$(-1, -\varepsilon^a - \frac{1}{2})$
$S_1 = \{2\}$	$(-\varepsilon^a - \frac{1}{4}m_0 - \frac{1}{4}, -1)$	$\left(\frac{2}{7}\{-5\varepsilon^a - m_0\}, \frac{2}{7}\{-6\varepsilon^a - 0.5m_0\}\right)$

As can be seen in Figure 2, conditional on $m_0 = 0$, the optimal network in a recessionary episode caused by low productivity is empty. As we increase productivity to 0.65 the sticky price Sector 2 now optimally decides to purchase from Sector 1, but not the other way round. Finally, as we further increase productivity to 0.8, the optimal network is the full network. Overall, one can see that initial states with higher productivity deliver (weakly) denser baseline networks. In Figure 3 we instead fix the productivity level at $\varepsilon^a = 0$, and consider variations in the initial level of money supply. As before, under a "tight" initial level $m_0 = 0$, the optimal network is empty. As we *increase the initial level of money to $m_0 = 4$ the flexible sector 1 would like to buy from the sticky sector 2, but not the other way round. Finally, in the "loose" money state, where $m_0 = 8$, the optimal network is full. Overall, initial states with higher money supply deliver (weakly) denser baseline networks.*

Having established the intuition in a simple two-sector example, one can now formalize the above results regarding densities of baseline networks in a general setting. First, we introduce the following general notion of a positive technology shock:

Definition 3 (Positive technology shock; Acemoglu and Azar, 2020)**.** *A positive technology shock is a change from $\mathcal{A}_0$ to $\mathcal{A}_0'$, such that $\mathcal{A}_0' \geq \mathcal{A}_0$ and $MC_k(S_{k,0}, \mathcal{A}_{k,0}(S_{k,0}), P)$ is quasi-submodular in $(S_{k,0}, \mathcal{A}_{k,0}(S_{k,0})), \forall k, \forall P$.*

Given the formal definition of a positive technology shock above, we can now formally establish the comparative statics for expansionary and recessionary baseline networks:

Lemma 1 (Expansionary vs Recessionary Baseline)**.** *For any two initial technology mappings such that $\overline{\mathcal{A}_0}$ is a positive technology shock over $\underline{\mathcal{A}_0}$ it holds that:*

$$S_k^*(\overline{\mathcal{A}_0}, \mathcal{M}_0) \supseteq S_k^*(\underline{\mathcal{A}_0}, \mathcal{M}_0) \qquad C_k^*(\overline{\mathcal{A}_0}, \mathcal{M}_0) \geq C_k^*(\underline{\mathcal{A}_0}, \mathcal{M}_0), \ \forall k$$

so that initial states with higher productivity, ceteris paribus, deliver (weakly) denser baseline networks and higher final consumption.

The intuition behind the above result is very simple. Higher baseline productivity directly (weakly) decreases the sectoral marginal costs, which in turn lowers sectoral prices and makes it (weakly) more attractive for sectors to adopt more suppliers, as it would further lower their marginal costs. Lower eventual marginal costs imply lower equilibrium prices, which through the cash-in-advance constraint imply higher final consumption.

As for different baselines featuring high and low initial money supply, a similar formal result can be established:

Lemma 2 (Loose vs Tight Money Baseline)**.** *For any two initial levels money supply such that $\overline{\mathcal{M}_0} > \underline{\mathcal{M}_0}$:*

$$S_k^*(\mathcal{A}_0, \overline{\mathcal{M}_0}) \supseteq S_k^*(\mathcal{A}_0, \underline{\mathcal{M}_0}) \qquad C_k^*(\mathcal{A}_0, \overline{\mathcal{M}_0}) \geq C_k^*(\mathcal{A}_0, \underline{\mathcal{M}_0}), \ \forall k$$

so that initial states with higher money supply, ceteris paribus, deliver (weakly) denser baseline networks and higher final consumption.

As for higher initial level of money supply, under nominal rigidities it lowers the real sectoral prices, as they do not adjust one-for-one with money supply, which in turn makes it more attractive to adopt more suppliers to minimize the marginal cost. Given that the eventual price change is less than one-for-one relative to the change in the money supply, the cash-in-advance constraint delivers higher baseline final consumption.

3.3 Propagation of a monetary shock

Having established properties of the baseline, we now consider deviations from the baseline, driven by a monetary shock ε^m. The mechanics of a monetary shock in terms of its effect on the equilibrium network and allocations are summarized below:

Lemma 3 (Comparative statics after a monetary shock)**.** *A positive monetary shock $\varepsilon^m > 0$, such that $\mathcal{M} > \mathcal{M}_0$, is (weakly) expansionary and makes the equilibrium network (weakly) denser:*

$$S_k^*(\mathcal{A}_0, \mathcal{M}) \supseteq S_k^*(\mathcal{A}_0, \mathcal{M}_0) \qquad C_k^*(\mathcal{A}_0, \mathcal{M}) \geq C_k^*(\mathcal{A}_0, \mathcal{M}_0), \ \forall k$$

The opposite holds for a negative monetary shock $\varepsilon^m < 0$, such that $\mathcal{M} < \mathcal{M}_0$.

The intuition behind this result, and the mechanics of a monetary shock's propagation, is very similar to that of different baseline levels of money supply. Namely, conditional on a given initial state, an expansionary monetary shock lowers real sectoral prices, as they do not adjust one-for-one with money supply, which (weakly) encourages adoption of more suppliers for cost-minimization purposes.

The above lemma establishes that a positive monetary shock weakly expands the equilibrium network, and vice versa. We can therefore distinguish between shocks that do not change the shape of the network, and the ones that do:

Definition 4 (Small monetary shock)**.** *Define a monetary shock ε^m to be* ***small*** *with respect to the initial state $(\mathcal{A}_0, \mathcal{M}_0)$ if and only if it leaves the equilibrium network unchanged:*

$$S_k^*(\mathcal{A}_0, \mathcal{M}) = S_k^*(\mathcal{A}_0, \mathcal{M}_0), \ \forall k$$

Otherwise, define the monetary shock to be ***large*** *with respect to the initial state $(\mathcal{A}_0, \mathcal{M}_0)$.*

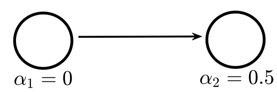
Figure 2: **Baseline networks across recessions and expansions (two sectors)**

Recession: $\varepsilon^a = 0$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1.00, -1.00)$	$(-1.00, -0.50)$
$S_2 = \{2\}$	$(-0.25, -1.00)$	$(0.00, 0.00)$

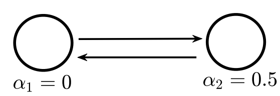
Normal: $\varepsilon^a = 0.65$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1.00, -1.00)$	$(-1.00, -1.15)$
$S_2 = \{2\}$	$(-0.90, -1.00)$	$(-0.92, -1.11)$



Expansion: $\varepsilon^a = 0.80$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1, -1)$	$(-1.00, -1.20)$
$S_2 = \{2\}$	$(-1.05, -1.00)$	$(-1.14, -1.37)$



Notes: *the figure uses the analytically tractable version of our model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \varepsilon^a, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \varepsilon^a$, the sectoral shares be given by $\omega_{12} = \omega_{c1} = 0.5, \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$. Throughout all examples, I set $m_0 = 0$.*

In the next two subsections we study properties of impulse response functions following small monetary shocks in initial states with different levels of productivity and money supply. The section after compares properties of large and small monetary shocks' propagation to real variables.

3.4 Expansions vs. recessions

Panel (a) of Figure 4 considers three initial states associated with different levels of productivity in the context of our earlier two-sector examples. We subject each of the initial states with a small monetary expansion, which, by definition, does not change the initial network. As one can see, in the recessionary state with the empty initial network, only the sticky sector 2 responds to the shock; in the normal state the situation is unchanged, as sector 1 does not buy anything from sector 2, and hence inherits no stickiness. However, in expansion, where both sectors buy from each other, both sectors see their final consumption rise. We can see that the magnitude of both sectoral and aggregate final consumption's response to a monetary shock is weakly larger in more expansionary states.

The following proposition formalizes this result regarding monetary *cycle dependence*:

Proposition 1 (IRF in Expansion and Recession)**.** *Consider a monetary shock ε^m that is* ***small*** *with respect to both $(\underline{\mathcal{A}_0}, \mathcal{M}_0)$ and $(\overline{\mathcal{A}_0}, \mathcal{M}_0)$, where $\overline{\mathcal{A}_0} \geq \underline{\mathcal{A}_0}$; further let $\hat{\mathbb{C}}_k(\mathcal{A}_0, \mathcal{M}_0; \varepsilon^m)$ be a first order approximation of $\log C_k^*(\mathcal{A}_0, \mathcal{M})$ around $\log C_k^*(\mathcal{A}_0, \mathcal{M}_0)$, $\mathcal{A}_0 \in \{\underline{\mathcal{A}_0}, \overline{\mathcal{A}_0}\}$, at $P_{k,0} = (1+\mu_k) MC_k^*(\mathcal{A}_0, \mathcal{M}_0), \forall k$. It can be shown that:*

$$|\hat{\mathbb{C}}(\overline{\mathcal{A}_0}, \mathcal{M}_0; \varepsilon^m) - \hat{\mathbb{C}}(\underline{\mathcal{A}_0}, \mathcal{M}_0; \varepsilon^m)| = \left\{\mathcal{L}(S^*(\overline{\mathcal{A}_0}, \mathcal{M}_0)) - \mathcal{L}(S^*(\underline{\mathcal{A}_0}, \mathcal{M}_0))\right\} A|\varepsilon^{\mathfrak{m}}| \geq 0$$

where $\mathcal{L}(S)$ is the Leontief Inverse associated with network S:

$$\mathcal{L}(S) \equiv [I - (1-A)\Omega(S)]^{-1}$$

and $\hat{\mathbb{C}} \equiv [\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, ..., \hat{\mathbb{C}}_K]', A = diag[\alpha_1, \alpha_2, ..., \alpha_K]', [\Omega(S)]_{kr} = \mathbf{1}_{r \in S_k} \omega_{kr}, |\varepsilon^{\mathfrak{m}}| \equiv [|\varepsilon^m|, ..., |\varepsilon^m|]'$. Hence, the magnitude of impulse response of final consumption to a small monetary shock is (weakly) ***procyclical****.*

Intuitively, states with higher level of productivity deliver (weakly) denser equilibrium networks, which strengthens complementarities in price setting, and hence the degree of monetary

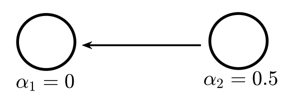
Figure 3: **Baseline networks across initial states with tight and loose money (two sectors)**

Tight money: $m_0 = 0$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1.00, -1.00)$	$(-1.00, -0.50)$
$S_2 = \{2\}$	$(-0.25, -1.00)$	$(0.00, 0.00)$

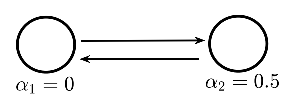
Normal money: $m_0 = 4$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1.00, -1.00)$	$(-1.00, -0.50)$
$S_2 = \{2\}$	$(-1.25, -1.00)$	$(-1.14, -0.57)$



Loose money: $m_0 = 8$

	$S_1 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1.00, -1.00)$	$(-1.00, -0.50)$
$S_2 = \{2\}$	$(-2.25, -1.00)$	$(-2.28, -1.14)$



Notes*: the figure uses the analytically tractable version of our model under $T = 2$, calibrated for $K = 2, \omega_{kk} = 0, \forall k$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ and $m_0 = \log \mathcal{M}_0$, the production technology be given by $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \varepsilon^a, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \varepsilon^a$, the sectoral shares be given by $\omega_{12} = \omega_{c1} = 0.5, \omega_{21} = \omega_{c1} = 0.5$, and Calvo parameters by $\alpha_1 = 0, \alpha_2 = 0.5$. Finally, assume that $\tau_k = -1/\theta$, and $\theta \to 1^+$. Throughout all examples, I set $\varepsilon_0^a = 0$.*

non-neutrality as sectors inherit stickiness from more suppliers. Hence, the magnitude of impulse response of final consumption to a monetary shock is (weakly) larger in states with higher productivity.

3.5 Loose vs. tight initial money supply

Panel (b) of Figure 6 considers three initial states with different levels of money supply, in the context of our earlier two-sector example, each perturbed by a small monetary expansion. One can see that in the tight money state only the sticky sector 2; in the normal money state sector 1 buys from sector 2 and inherits stickiness, hence responding to the shock. Finally, in the loose money state both sectors buy from each other and respond by even more than in the normal money state. Overall, one can see that both sectoral and aggregate consumption respond weakly stronger in states with looser money.

The proposition bellow formalizes this results regarding monetary *path dependence*:

Proposition 2 (IRF under Loose and Tight money)**.** *Consider a monetary shock ε^m that is* ***small*** *with respect to both $(\mathcal{A}_0, \underline{\mathcal{M}_0})$ and $(\mathcal{A}_0, \overline{\mathcal{M}_0})$, where $\overline{\mathcal{M}_0} \geq \underline{\mathcal{M}_0}$; further let $\hat{\mathbb{C}}_k(\mathcal{A}_0, \mathcal{M}_0; \varepsilon^m)$ be a first order approximation of $\log C^*_k(\mathcal{A}_0, \mathcal{M})$ around $\log C^*_k(\mathcal{A}_0, \mathcal{M}_0)$, $\mathcal{M}_0 \in \{\underline{\mathcal{M}_0}, \overline{\mathcal{M}_0}\}$. It can be shown that:*

$$|\hat{\mathbb{C}}(\mathcal{A}_0, \overline{\mathcal{M}_0}; \varepsilon^m) - \hat{\mathbb{C}}(\mathcal{A}_0, \underline{\mathcal{M}_0}; \varepsilon^m)| = \left\{\tilde{\mathcal{L}}(S^*(\mathcal{A}_0, \overline{\mathcal{M}_0})) - \tilde{\mathcal{L}}(S^*(\mathcal{A}_0, \underline{\mathcal{M}_0}))\right\} A|\varepsilon^{\mathbb{m}}| \geq 0$$

where $\tilde{\mathcal{L}}(S)$ is the adjusted Leontief Inverse associated with network S:

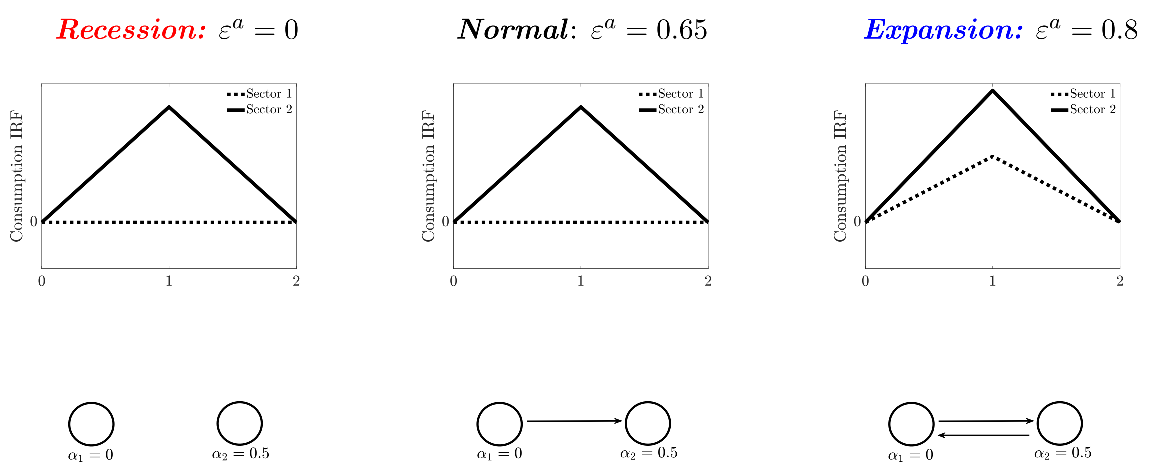
$$\tilde{\mathcal{L}}(S) \equiv [I - (1-A)\Gamma\Omega(S)]^{-1}[I - (I-A)\Gamma]$$

and $\hat{\mathbb{C}} \equiv [\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, ..., \hat{\mathbb{C}}_K]', A = diag[\alpha_1, \alpha_2, ..., \alpha_K]', [\Omega(S)]_{kr} = \mathbf{1}_{r \in S_k}\omega_{kr}, |\varepsilon^{\mathbb{m}}| \equiv [|\varepsilon^m|, ..., |\varepsilon^m|]', \Gamma = diag[\gamma_1, \gamma_2, ..., \gamma_K]', \gamma_k \equiv \frac{((1+\mu_k)MC_k)^{1-\theta}}{P_{k,0}^{1-\theta} + ((1+\mu_k)MC_k)^{1-\theta}}, \forall k$. Hence, the magnitude of impulse response of final consumption to a small monetary shock is (weakly) ***higher under loose money****.*

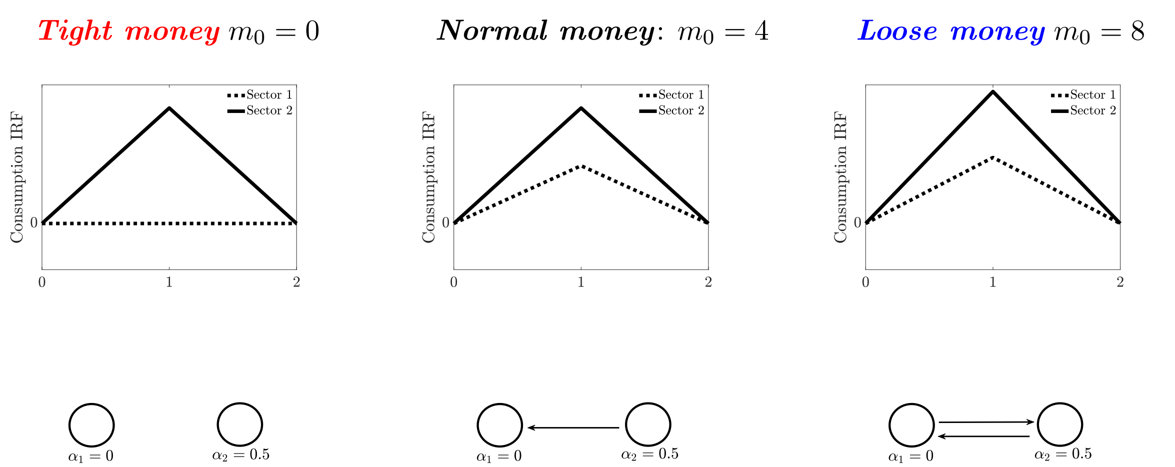
Intuitively, states with higher initial level of money deliver (weakly) denser equilibrium networks, which strengthens complementarities in price setting, and hence the degree of monetary non-neutrality as sectors inherit stickiness from more suppliers. Hence, the magnitude of impulse response of final consumption to a monetary shock is (weakly) larger in states with higher initial level of money supply.

Figure 4: **Final consumption following a small monetary expansion (two sectors)**

(a) Responses of final consumption to a small monetary expansion under recessions and expansions



(b) Responses of final consumption to a small monetary expansion under tight and loose initial money



Notes: *the figure uses the analytically tractable version of our model under* $T = 2$, *calibrated for* $K = 2, \omega_{kk} = 0, \forall k$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ *and* $m_0 = \log \mathcal{M}_0$, *the production technology be given by* $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \varepsilon^a, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \varepsilon^a$, *the sectoral shares be given by* $\omega_{12} = \omega_{c1} = 0.5, \omega_{21} = \omega_{c1} = 0.5$, *and Calvo parameters by* $\alpha_1 = 0, \alpha_2 = 0.5$. *Finally, assume that* $\tau_k = -1/\theta$, *and* $\theta \to 1^+$. *Throughout exercises in Panel (a), I set* $m_0 = 0$ *throughout exercises in Panel (b), I set* $\varepsilon^a_0 = 0$.

3.6 Large monetary shocks

This subsection describes the propagation of large monetary shock, which have the power to change the shape of the network. We then compare their transmission to that of small monetary shocks, considered in the previous two subsections.

Panel (a) of Figure 6 considers large monetary shocks in the context of our earlier two-sector example. In particular, start from the initial state $(a_0, m_0) = (0, 0)$, we subject the two-sector economy to a sequence of monetary shocks $\varepsilon^m \in (0, 8)$. One can see that shocks smaller than 3 do keep the initially empty network unchanged, and the changes in the aggregate consumption impulse response are exactly proportional to the size of the shock. However, as we consider larger expansions, they turn out to be large enough to expand the network buy encouraging the flexible sector 1 to buy from the sticky sector 2, also increasing the slope of the relationship between the monetary shock size and consumption response.

As can ben seen from the above example, large monetary expansions cause more than proportional increase of aggregate consumption (GDP) than small expansions. The proposition below formalizes this result:

Proposition 3 (Large monetary expansion)**.** *Let $E_+^m > 0$ be a large expansionary monetary shock, and $\varepsilon_+^m > 0$ be a small expansionary monetary shock, both with respect to $(\mathcal{A}_0, \mathcal{M}_0)$; further, denote $S_{E_+} \equiv S^*\left(\mathcal{A}_0, \mathcal{M}_0 \exp(E_+^M)\right)$ and $S_0 \equiv S^*\left(\mathcal{A}_0, \mathcal{M}_0 \exp(\varepsilon_+^m)\right) = S^*(\mathcal{A}_0, \mathcal{M}_0)$. It can be shown that:*

$$\mathcal{L}(S_0)A(\mathbb{E}_+^m - \varepsilon_+^m) + h.o.t. \;\leq\; \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; E_+^m) - \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; \varepsilon_+^m) \;\leq\; \mathcal{L}(S_{E_+})A(\mathbb{E}_+^m - \varepsilon_+^m) + h.o.t.$$

Hence, large monetary expansions have a ***more than proportional effect on GDP*** *than small monetary expansions.*

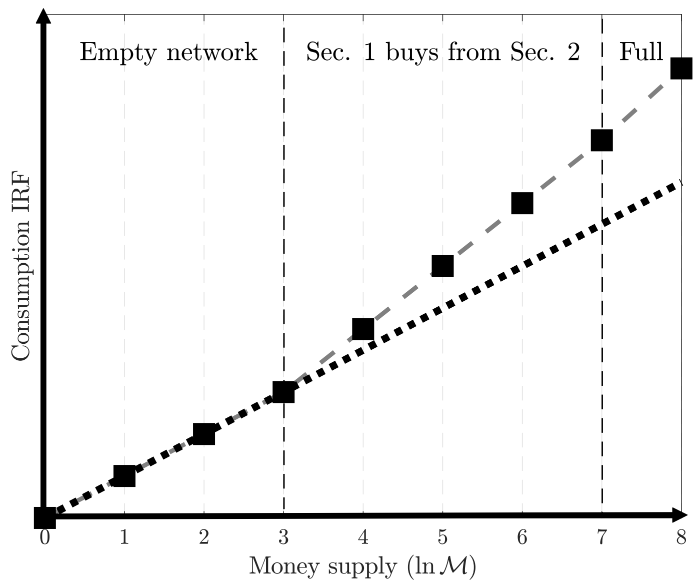
A direct corollary of the above proposition is that achieving a given consumption expansion under endogenous networks requires a weakly smaller expansion in money supply than under fixed networks.

In Panel (b) of Figure 6 considers large monetary contractions in the context of the same two-sector example. Starting from the initial state where $(a_0, m_0) = (0, 8)$ initial reductions in money supply leave the initially full network unchanged and the magnitude of aggregate consumption contraction is exactly proportional to the size of the monetary shock. However, larger contractions break the network, in this case by discouraging sector 2 from buying from sector 1, and lower the slope of the relationship between the monetary shock size and consumption response.

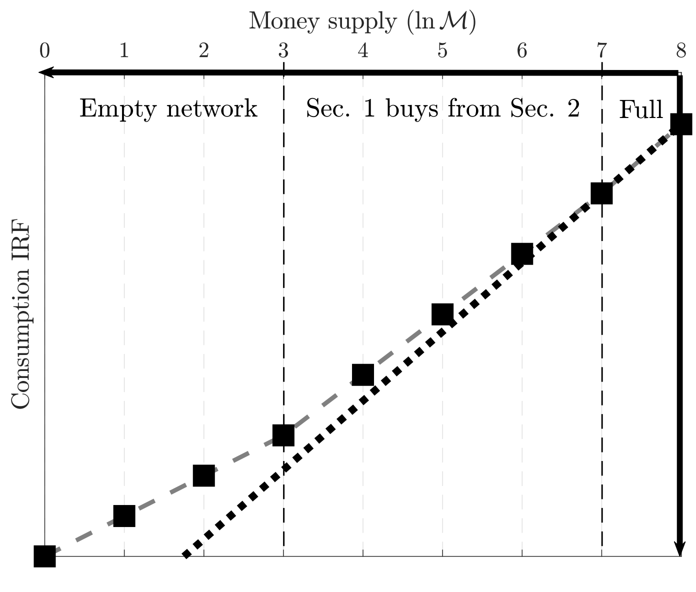
As can ben seen from the above example, large monetary contractions cause less than proportional drop of aggregate consumption (GDP) than small contractions. The proposition below formalizes this result:

Figure 5: **Aggregate consumption response to large monetary shocks (two sectors)**

(a) Large monetary expansions



(b) Large monetary contractions



Notes: *the figure uses the analytically tractable version of our model under* $T = 2$, *calibrated for* $K = 2, \omega_{kk} = 0, \forall k$, $a_{k,0} \equiv \log \mathcal{A}_{k,0}, \forall k$ *and* $m_0 = \log \mathcal{M}_0$, *the production technology be given by* $a_{1,0}(\varnothing) = 0, \quad a_{1,0}(\{2\}) = \varepsilon^a, a_{2,0}(\varnothing) = 0, \quad a_{2,0}(\{1\}) = \varepsilon^a$, *the sectoral shares be given by* $\omega_{12} = \omega_{c1} = 0.5, \omega_{21} = \omega_{c1} = 0.5$, *and Calvo parameters by* $\alpha_1 = 0, \alpha_2 = 0.5$. *Finally, assume that* $\tau_k = -1/\theta$, *and* $\theta \to 1^+$. *Throughout exercises I set* $\varepsilon^a_0 = 0, m_0 = 0$.

Proposition 4 (Large monetary contraction)**.** *Let $E_{-}^{m} < 0$ be a large contractionary monetary shock, and $\varepsilon_{-}^{m} < 0$ be a small contractionary monetary shock, both with respect to $(\mathcal{A}_0, \mathcal{M}_0)$; further, denote $S_{E_-} \equiv S^*\left(\mathcal{A}_0, \mathcal{M}_0 \exp(E_{-}^{M})\right)$ and $S_0 \equiv S^*\left(\mathcal{A}_0, \mathcal{M}_0 \exp(\varepsilon_{-}^{m})\right) = S^*(\mathcal{A}_0, \mathcal{M}_0)$. It can be shown that:*

$$\mathcal{L}(S_{E_-})A(\varepsilon_{-}^{m} - \mathbb{E}_{-}^{m}) + h.o.t. \;\leq\; \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; E_{+}^{m}) - \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; \varepsilon_{+}^{m}) \;\leq\; \mathcal{L}(S_0)A(\varepsilon_{-}^{m} - \mathbb{E}_{-}^{m}) + h.o.t.$$

Hence, large monetary contractions have a ***less than proportional effect on GDP*** *than small monetary contractions.*

A direct corollary of the above proposition is that achieving a given consumption contraction under endogenous networks requires a weakly larger contraction in money supply than under fixed networks.

4 Empirical evidence

Having established the theoretical results, I now turn to empirical assessment of the key mechanisms. In this section I use both sectoral and firm-level data on input-output linkages to estimate network cyclicality both unconditionally and conditional on identified technology and monetary shocks. I show that both sectoral and firm-level data strongly supports key theoretical predictions.

4.1 Evidence using sector-level data (BEA KLEMS)

In this subsection I use sector-level data on intensity of intermediates intensity in order to perform model-free evaluation of the theoretical model regarding business cycle fluctuations in the shape of the production network. In particular, I use US data on sector-level employee compensation and expenditure on intermediate inputs, published by the US Bureau of Economics Analysis (BEA), to construct the share of total costs going to intermediates for 65 three-digit sectors of the US economy at annual frequency between 1987-2017. Such measure maps directly into the model-based measure of sector-specific intermediates intensity in a particular time period, given by $\delta_{k,t} \equiv \sum_{r \in S_{k,t}} \omega_{kr}, \forall k$.

Once those measures have been constructed, I use the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of sectoral intermediates use intensity to productivity and monetary shocks:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H shock_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \qquad H = 0, 1, ..., \overline{H} \tag{9}$$

where $\delta_{k,t}$ is intermediate inputs intensity of sector k, $shock_t$ is identified exogenous productivity or monetary shock, $x_{k,t-1}$ is a vector of control variables and $\alpha_{k,H}$ represents horizon-specific sectoral fixed effects. The estimated value of β_H gives the horizon-specific response of intermediate inputs intensity to an identified exogenous shock; according to our theory, intermediate inputs intensity should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*, and the specification above allows to test such predictions in a transparent model-free using horizon-by-horizon fixed-effects regressions.

Another feature of theory is that the response of intermediates intensities to business cycle shocks can be non-linear, both along the dimension of sign and size of the shock. In order to test such predictions, we extend our specification above to include quadratic and cubic shocks:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta^l_H shock_t + \beta^q_H shock^2_t + \beta^c_H shock^3_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \qquad H = 0, 1, ..., \overline{H} \quad (10)$$

where it follows that $\beta^q_H > 0$ indicates that positive shocks make the impulse response more positive, whereas β^c_H indicates that larger shocks make the response more positive.

In the baseline results presented in this subsection, we use identified annual productivity shocks from Fernald (2014) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

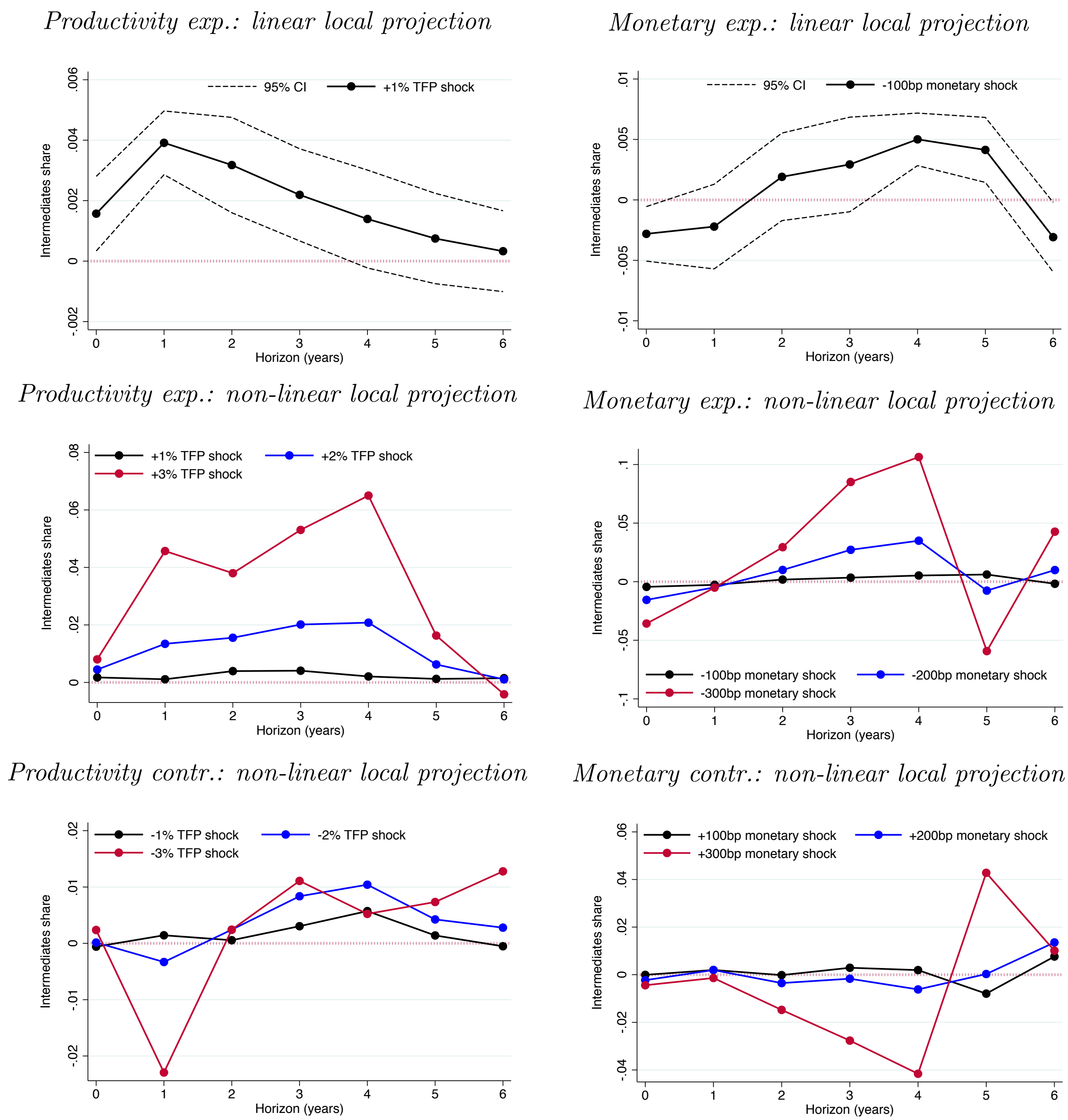
The top panel of Figure 7 shows estimation results for the linear local projection. As one can see, following a positive 1 per cent shock to aggregate productivity, the intermediates intensity responds positively on impact, reaching peak increase of 0.004 after one year, and then gradually declines back to zero; such positive response is consistent with our theory. As for a monetary expansion, a surprise one-time 100 bp easing leads to a slight decline in the intermediates intensity, after which in gradually increases, reaching a peak of 0.005 increase after four years; as we can see, the response is consistent with our theory at longer horizons.

The bottom two panels of Figure 6 show results from non-linear local projection estimation, which suggest substantial non-linearities along both the sign and size dimension. In particular, while a 2 per cent increase in aggregate productivity leads to a peak 0.02 increase in intermediates intensity, a 3 per cent expansion leads to a more than 0.06 increase in intermediates intensity. Moreover, one can also see that productivity contractions are substantially less powerful at moving intermediates intensity compared to expansions. Similar patterns hold for monetary shocks: while a -200 bp easing leads to a peak effect of around 0.03 on intermediates intensity, a 300 bp easing leads to a much larger 0.10 increase; monetary contractions seem to also be less powerful than monetary expansions: a 300 bp tightening leads to only -0.04 fall in intermediates intensity.

4.2 Evidence using firm-level data (Compustat)

In this subsection I use firm-level data on the number of suppliers of US publicly listed firms in order to perform model-free evaluation of the theoretical model regarding business cycle fluctuations in

Figure 6: **IRFs of intermediate inputs share to productivity and monetary shocks**



***Notes**: the figure shows estimated impulse response functions of intermediates intensity to productivity and monetary shocks using specifications in (9) and (10). Productivity shocks come from Fernald (2014), monetary shocks come from Romer and Romer (2004).*

the number of suppliers. In particular, I use the dataset constructed by Atalay et al. (2011) using US Compustat data, which contains the *indegree* (number of supplier firms) for a large number of US publicly listed firms.

Using the indegree measure, I once again follow the local projection approach of Jordà (2005) in order to estimate horizon-specific impulse responses of sectoral intermediates use intensity to productivity and monetary shocks:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta_H shock_t + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \qquad H = 0, 1, ..., \overline{H} \tag{11}$$

where $indeg_{k,t}$ is indegree (number of suppliers) of firm j in year t, $shock_t$ is identified exogenous productivity or monetary shock, $x_{j,t-1}$ is a vector of control variables and $\alpha_{j,H}$ represents horizon-specific firm fixed effect. The estimated value of β_H gives the horizon-specific response of the number of suppliers to an identified exogenous shock; according to our theory, the number of suppliers should, *ceteris paribus*, rise following expansionary productivity and monetary shocks, and *vice versa*, and the specification above allows to test such predictions in a transparent model-free using horizon-by-horizon fixed-effects regressions.

Another feature of theory is that the response of the number of suppliers to business cycle shocks can be non-linear, both along the dimension of sign and size of the shock. In order to test such predictions, we extend our specification above to include quadratic and cubic shocks:

$$indeg_{j,t+H} = \alpha_{j,H} + \beta^l_H shock_t + \beta^q_H shock^2_t + \beta^c_H shock^3_t + \gamma_H x_{j,t-1} + \varepsilon_{j,t+H}, \qquad H = 0, 1, ..., \overline{H} \tag{12}$$

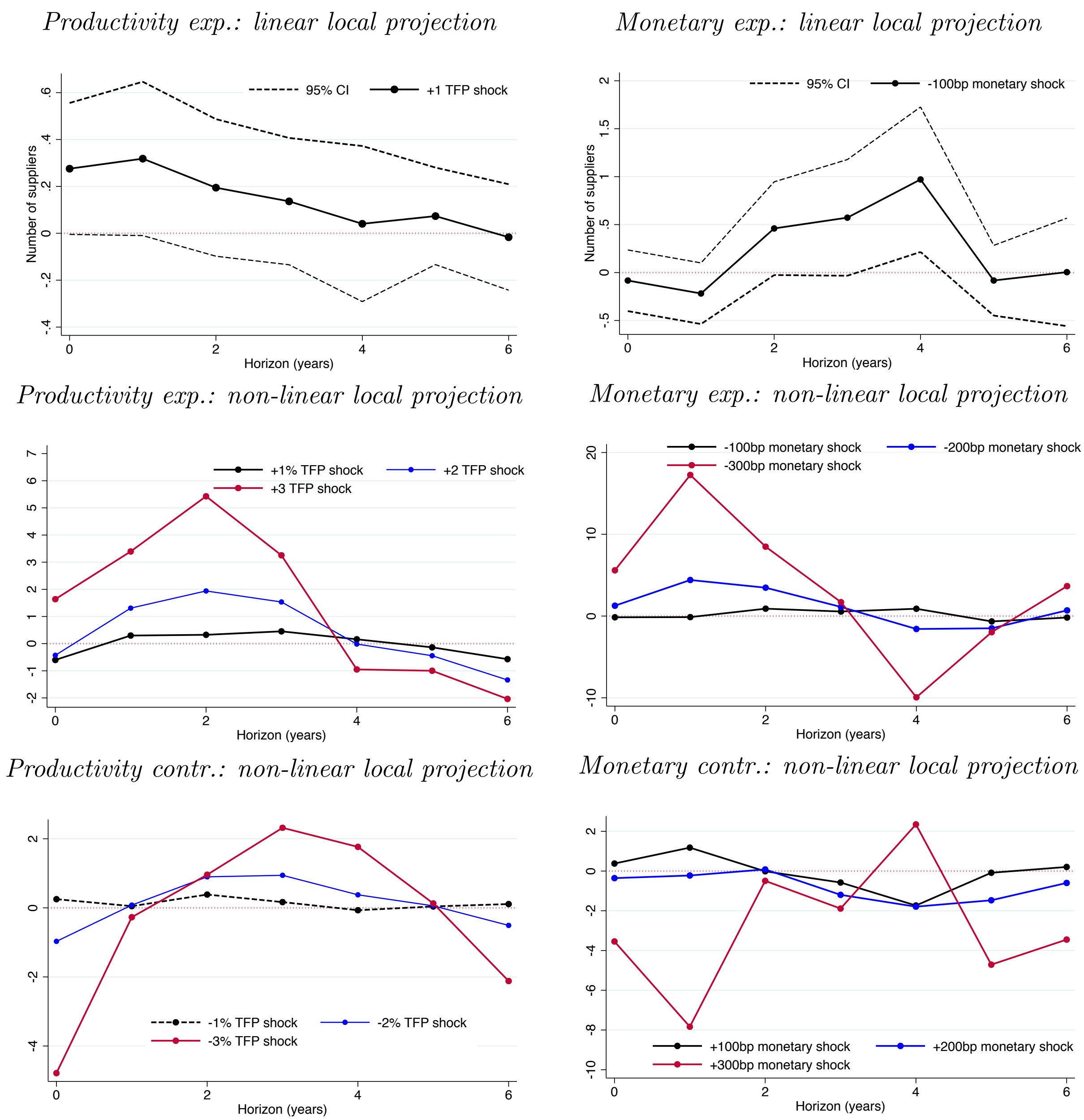
where it follows that $\beta^q_H > 0$ indicates that positive shocks make the impulse response more positive, whereas β^c_H indicates that larger shocks make the response more positive.

In the baseline results presented in this subsection, I once again use identified annual productivity shocks from Fernald (2014) and identified annual monetary shocks from Romer and Romer (2004) that have been extended by Wieland and Yang (2016).

The top panel of Figure 7 shows estimation results for the linear local projection. As one can see, following a positive 1 per cent shock to aggregate productivity, the number of suppliers responds positively on impact, reaching peak increase of 0.35 after one year, and then gradually declines back to zero; such positive response is consistent with our theory. As for a monetary expansion, a surprise one-time 100 bp easing leads to a slight decline in the intermediates intensity, after which in gradually increases, reaching a peak of 1.00 increase after four years; as we can see, the response is consistent with our theory at longer horizons.

The bottom two panels of Figure 7 show results from non-linear local projection estimation, which suggest substantial non-linearities along both the sign and size dimension. In particular, while a 2 per cent increase in aggregate productivity leads to a peak 2 suppliers' increase, a 3 per cent expansion leads to a more than 5 supplier increase. Moreover, one can also see that produc-

Figure 7: **IRFs of number of suppliers to productivity and monetary shocks**



Notes: *the figure shows estimated impulse response functions of the number of suppliers to productivity and monetary shocks using specifications in (11) and (12). Productivity shocks come from Fernald (2014), monetary shocks come from Romer and Romer (2004).*

tivity contractions are substantially less powerful at moving the number of suppliers compared to expansions. Similar patterns hold for monetary shocks: while a -200 bp easing leads to a peak effect of around 5 extra suppliers, a 300 bp easing leads to a much larger increase of almost 17 extra suppliers; monetary contractions seem to also be less powerful than monetary expansions: a 300 bp tightening leads to only a fall of 8 fewer suppliers.

5 Conclusion

This paper develops a novel sticky-price New Keynesian model, where input-output linkages across sectors are determined endogenously through firms' optimizing decisions. The model generates comparative statics of the shape of the production network that are consistent with empirical evidence: in periods of low productivity and following strong monetary tightenings the network gets sparse and vice versa.

Crucially, I show that my novel sticky-price model with endogenous production networks can jointly rationalize empirically observed non-linearities associated with monetary transmission. First, the strength of monetary transmission to real GDP weakens in recessionary episodes associated with low productivity, as the network is sparse and downstream propagation of price rigidity is weak. In this sense, the effect of monetary policy on real variables is procyclical. Second, monetary policy transmission get stronger following periods of previous loose monetary policy, as the latter erodes real prices charged by supplier firms, and makes the network denser and strategic complementarities in price setting stronger. Finally, large monetary expansions make the network denser and hence have a more than proportional positive effect on GDP relative to small expansions; on the other hand, large contractions make the network sparser and hence have a less than proportional negative effect on real GDP, relative to small contractions.

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