ADAPTIVENESS OF THE EMPIRICAL DISTRIBUTION OF RESIDUALS IN SEMI-PARAMETRIC CONDITIONAL LOCATION SCALE MODELS

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Abstract. This paper addresses the problem of deriving the asymptotic distribution of the empirical distribution function $\widehat{F}_n$ of the residuals in a general class of time series models, including conditional mean and conditional heteroscedaticity, whose independent and identically distributed errors have unknown distribution F. We show that, for a large class of time series models (including the standard ARMA-GARCH), the asymptotic distribution of $\sqrt{n}\{\widehat{F}_n(\cdot) - F(\cdot)\}$ is impacted by the estimation but does not depend on the model parameters. It is thus neither asymptotically estimation free, as is the case for purely linear models, nor asymptotically model dependent, as is the case for some nonlinear models. The asymptotic stochastic equicontinuity is also established. We develop consequences of these results for the estimation and tests of the innovations quantiles, which have useful applications in econometrics and finance. The paper also contains a numerical study based on simulated and real data.