Investment Dynamics and Cyclical Redistribution*

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Demand for durable goods and residential investment is strongly pro-cyclical. Workers employed in durable industries are imperfectly insured against these fluctuations, leading to distributional consequences during booms and busts. This paper studies the interaction between cyclical durable demand and redistribution of labor income. I explore this feedback loop within a heterogeneous agent New Keynesian (HANK) model with multiple sectors and lumpy durable adjustment. Crucially, lumpy adjustment at the micro level generates non-linearities at the macro level: the average marginal propensity to spend on durable goods varies with the size of income shocks. As a result, sectoral redistribution of labor incomes has aggregate effects. I find that the interaction between cyclical investment and redistribution amplifies the aggregate response of durable spending during booms and dampens it during recessions. The lumpy nature of durable adjustment entirely accounts for this non-linear effect.

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1 Introduction

Purchases of durable goods and residential investment are strongly pro-cyclical. Workers employed in the industries producing these goods are imperfectly insured against the fluctuations in durable expenditure, leading to distributional consequences during booms and busts (Guvenen et al. (2017)). Microeconomic evidence suggests that these income changes have a high pass-through to durable spending (Dynarski and Gruber (1997); Browning and Crossley (2009)). These two facts suggest the presence of a feedback loop that has not been explored in the literature: the cyclicality of durable expenditure induces a redistribution of labor income between sectors; in turn, this redistribution feeds back into the composition of spending and durable expenditure.

In this paper, I study this feedback loop and its role in the amplification of aggregate shocks. My starting point is a general equilibrium model of lumpy durable demand with incomplete markets (Berger and Vavra (2015)) and sticky prices. The main novelty lies in the presence of distributional effects. I recognize that consumption and investment goods are produced in different sectors. Redistribution stems from two features. First, durable spending is more elastic to aggregate shocks. This translates into a more cyclical demand for labor demand in durable industries. Second, labor mobility is restricted between sectors. Households employed in the durable sector fail to relocate and are disproportionately affected by these cyclical fluctuations.

Cyclical income inequality has aggregate effects in my model. Investment is infrequent and discontinuous at the microeconomic level.[1] This form of discontinuities can produce non-linearities at the macroeconomic level.[2] I show that the response of aggregate durable spending to income shocks is non-linear in their size. In other words, the average marginal propensity to spend (MPC) on durable goods depends on the magnitude of these shocks. During expansions, households employed in the durable sector experience a disproportionate increase in income, and their MPCs on durables rise endogenously. As a result, cyclical income inequality amplifies the increase in durable spending during expansions. On the contrary, it dampens its decline during recessions.

My first contribution is to clarify the sources of the aggregate non-linearities in durables demand, and quantify their importance even without redistribution. I proceed in two steps. In the first step, I explore the theoretical determinants of this non-linearity using my

[1] See Bertola and Caballero (1990) for a review on investment subject to lumpy adjustment. Fixed adjustment costs are essential to explain the micro behavior of durable investment (Eberly (1994); Attanasio (2000)), and its aggregate time series properties (Caballero (1990, 1993, 1994)).

[2] See Caballero et al. (1997) and Caballero and Engel (1999) in the context of capital investment, and Alvarez et al. (2017b) in the context of price setting.

model of lumpy adjustment. Non-convex adjustment costs lead to a standard inaction region for durable adjustment. Durable adjustment occurs along two margins: an extensive margin, which controls the propensity of households to exit their inaction region and pay the adjustment cost; and an intensive margin, which determines their MPC conditional on adjustment. The contribution of each of these margins to aggregate non-linearities depends on two objects: the shape of the distribution of idiosyncratic characteristics at the edge of the inaction region; and the monotonicity and concavity of durable spending conditional on adjustment. Imposing restrictions that resemble those obtained in numerical simulations, I show that adjustment along the extensive margin *amplifies* expansionary income shocks as their magnitude increases: more and more households adjust their stock of durables. On the contrary, it *dampens* contractionary shocks. Lumpy adjustment plays a central role in this non-linear effect. Models of smooth or time-dependent adjustment, where only the intensive margin operates, would actually predict the *opposite* effect.[3]

In the second step, I calibrate my model and use it to quantify the importance of these aggregate non-linearities. I simulate the response of durable investment to income shocks, varying their sign and magnitude. I find that the average MPC on durable goods increases with income changes. The degree of non-linearity is economically significant. For instance, for negative income shocks of the magnitude experienced by durable workers during the Great Recession, the response of durable investment is roughly 20% lower than would be predicted without non-linearities. The opposite is true for positive shocks of the same magnitude. Decomposing the responses into extensive and intensive margins, I find that the former dominates quantitatively. In other words, lumpy adjustment entirely accounts for these aggregate non-linearities.

My theoretical findings are in line with the existing evidence on the consumption response to transfers. Johnson et al. (2006) estimate negligible spending multipliers on durable goods in response to the the 2001 stimulus payment, while Souleles (1999) and Parker et al. (2013) estimate strong responses of durable investment following springtime tax refunds and the 2008 stimulus payment, respectively. Parker et al. (2013) attribute this difference to the size of the transfers. My model replicates this finding qualitatively and produces an average MPC that increases with the size of the tax rebate.

My second contribution is to explore the interaction between cyclical income inequality and lumpy durable investment. Durable spending is strongly pro-cyclical, which redistributes labor income between sectors in general equilibrium. To assess the role of redistribution, I compare the aggregate response of durable investment in my model to

[3] In this case, MPCs are declining in wealth when markets are incomplete: expansionary shocks are dampened as their size increases, and contractionary shocks are amplified.

that obtained in a counterfactual economy where fiscal policy undoes this redistribution and provides cross-sectoral insurance. I show that a cyclical redistribution of income amplifies the effect of expansionary shocks and dampens contractionary shocks when the average MPC on durable investment is increasing with income changes.

An "earnings heterogeneity channel" (Auclert (2019); Werning (2015); Patterson (2019)) emerges endogenously in my setting. However, my model is set up so that there is no *ex ante* heterogeneity in the average MPC on durables across sectors. Redistribution of income between sectors is neutral for first order deviations from the steady state. Instead, I focus on non-linear amplification effects away from the stationary equilibrium. Redistribution drives a wedge across sectors in the households' propensity to adjust their stock of durables. In other words, MPCs on durable goods are heterogeneous *ex post* across sectors. This heterogeneity ensures that cyclical income inequality is non-neutral, and amplifies the response of durable spending during expansions and attenuates it during recessions. I show that lumpy investment plays a crucial role in this form of non-linear amplification.

Finally, I quantify the role of redistribution in general equilibrium. I embed my model of lumpy durable investment in a multi-sector heterogeneous agent New Keynesian framework (HANK). There are two sectors, which respectively produce a durable investment good, and a non-durable consumption good. There is no labor mobility between sectors, so that households employed in the durable sector are more exposed to aggregate shocks. As a consequence, labor income is endogenously redistributed across sectors during booms and busts. Financial markets are incomplete, which limits risk sharing across households employed in different sectors.[4] I calibrate my model by targeting a set of micro and macro moments. Following Berger and Vavra (2015), I focus on productivity shocks. I find that redistribution affects both the magnitude, and the timing of the aggregate response of durable investment to aggregate shocks.

Related literature. My paper lies at the intersection of several strands of the literature on durable investment, heterogeneous agents, and redistribution.

First, my paper contributes to an extensive literature on lumpy durable investment.[5] Durable adjustment is infrequent and discontinuous at the microeconomic level. In the closely related context of capital adjustment and price setting, these discontinuities tend to produce non-linearities at the macroeconomic level. In particular, Caballero et al. (1997)

[4] Participation in financial markets is limited (Mankiw and Zeldes (1991); Heaton and Lucas (2000)), and households fail to hedge against (sector-specific) aggregate risk (Massa and Simonov (2006)).

[5] Seminal contributions on the subject include Arrow (1968), Nickell (1974), Pindyck (1988), Bar-Ilan and Blinder (1987) and Grossman and Laroque (1990). Again, see Bertola and Caballero (1990) for a review.

and Caballero and Engel (1999) find that firms' capital investment responds proportionately more to large shocks than smaller ones when adjustment is lumpy.[6] Building on this insight, I explore the aggregate non-linearities produced by a canonical model of lumpy durable investment with uninsured idiosyncratic risk and borrowing constraints. I find that expansionary income shocks are amplified as their size increases, while contractionary shocks are dampened. I trace this asymmetry back to the monotonicity of durable investment conditional on adjustment, and the shape of the distribution of liquid assets around the durable adjustment thresholds. I then explore the implications of this non-linearity in the presence of redistribution of labor income.

My quantitative model of durable demand builds directly on the one developed by Berger and Vavra (2015). My focus is complementary to theirs. They investigate the role of lumpy adjustment for the cyclical response to aggregate shocks. I extend their setting by adding redistributional effects. I recognize that durable investment goods and non-durable consumption goods are produced in different sectors. Households employed in the durable sector are more exposed to aggregate shocks, thus they claim a higher share of labor income during booms and a lower share during busts. In turn, this redistribution of labor income interacts with lumpy adjustment and the aggregate non-linearities it generates.

Models of lumpy adjustment typically produce state-contingent responses to aggregate shocks. In particular, Berger and Vavra (2015) find that the response of durable investment to aggregate shocks is pro-cyclical.[7] This pro-cyclicality is consistent with the non-linearity that I document: an incremental income shock following a period of expansion is akin to a discrete aggregate shock in my model. I argue that the amplification is controlled in both cases by the same structural objects.

Second, my paper fits into a growing literature on heterogeneous agent New Keynesian (HANK) models. My approach reconciles two separate strands of this literature: one interested in the role of durable spending and one focused on redistribution of labor income. Falling into the first category, Guerrieri and Lorenzoni (2017) explore the effect of deleveraging shocks in the presence of durable investment with convex adjustment costs. In a recent paper, McKay and Wieland (2019) quantify the effect of monetary policy and forward guidance in a model of lumpy durable investment. I introduce a role for cyclical income redistribution by assuming that labor markets are sector-specific. Households are

[6] On price setting, Alvarez et al. (2017b) show that the degree of monetary non-neutrality in menu costs models is non-linear in the size of monetary shocks, and Alvarez et al. (2017a) find empirical support for this property.

[7] See Bachmann et al. (2013) and Winberry (2019) in the context of capital investment, and Dotsey et al. (1999) and Burstein (2006) in the context of price setting.

either employed in the durable or non-durable sector and cannot relocate between them. Durable spending is strongly pro-cyclical, which induces a redistribution of labor income across sectors during booms and busts.

By introducing distributional concerns into a model of durable investment, I draw a connection to the existing literature on redistribution in heterogeneous agent models. Auclert (2019), Werning (2015) and Patterson (2019) explore the role of earnings heterogeneity for the marginal response to aggregate shocks in general equilibrium. My paper complements their work by exploring the non-linear effects of redistribution. In my model, redistribution is neutral for local deviations from the stationary equilibrium. This allows me to focus on higher-order effects. I find that redistribution amplifies expansionary shocks and dampens contractionary shocks. I show that lumpy investment is central to this non-linear amplification.

More broadly, my paper speaks to a literature on business cycle fluctuations in multi-sector economies. A first branch of this literature studies cyclical changes in the composition of spending and their implications for aggregate labor demand. In particular, Bils et al. (2013) and Jaimovich et al. (2019) investigate the interaction between cyclical spending on durable and luxury goods and the relatively low capital intensity in these sectors.[8] A second strand of this literature focuses on real rigidities and amplfication stemming from heterogeneous frequencies of price adjustment across sectors.[9] I set up my model to abstract from these considerations, and isolate the role of redistribution and the non-linearities it produces.

Layout. I start by introducing a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy investment in Section 2. I discuss the sources of non-linearity inherent to this model in Section 3, and explore the implications of income redistribution in this setting. Section 4 describes the calibration of the model and the empirical targets. In Section 5, I quantify the degree of non-linearity in my calibrated model. I explore the general equilibrium interaction between income redistribution and aggregate non-linearities in Section 6. Section 7 concludes. The appendix contains the proofs, and complementary quantitative and empirical results.

[8] Alonso (2016) quantifies this effect in a heterogeneous agent model.

[9] References include Carvalho (2006), Nakamura and Steinsson (2010) and Pastén et al. (2018).

2 A Multi-Sector Model with Lumpy Investment

I introduce a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy durable investment. There are two sectors producing a consumption good and a durable investment good, respectively. Each household is employed in a given sector and is unable to relocate betweem them.[10] The demand side of the economy builds on the canonical model of durable demand with incomplete markets from Berger and Vavra (2015). The supply side is standard. Prices are sticky à la Calvo (1983), which leads to sector-specific Phillips curves. I describe the environment below. For concision, I only include here the main expressions. Appendix A provides a full description of the full model.

2.1 Environment

Time is discrete, and there is no aggregate uncertainty.[11] Periods are indexed by $t \in \{0, 1, \ldots\}$. The two goods are indexed by $h \in \mathcal{H} \equiv \{c, d\}$. Sector $h = c$ produces the non-durable consumption good, and sector $h = d$ produces the durable investment good.

Households. The economy is inhabited by a continuum of mass 1 of households. Each household is assigned permanently to a given sector h. Households are characterized by four idiosyncratic states: their financial asset holdings (a), their holdings of durable goods (d), their idiosyncratic labor supply shock (ζ), and the sector they are employed in (h). The mass of households in each sector is denoted by $\boldsymbol{\mu} \equiv \{\mu^h\}_h$.

Households consume durable and non-durable goods. Preferences are represented by

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t \frac{u\left(c_t, d_t\right)^{1-\sigma}}{1-\sigma}\right]$$

with discount factor $\beta \in (0, 1)$, and inverse elasticity of substitution $\sigma > 0$. Intratemporal preferences exhibit constant elasticity of substitutiton:

$$u\left(c, d\right) = \left[\vartheta^{\frac{1}{\nu}} c^{\frac{\nu-1}{\nu}} + (1-\vartheta)^{\frac{1}{\nu}} d^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

with share parameter $\vartheta \in (0, 1)$ and elasticity of substitution $\nu > 0$.

[10] There is evidence of some cyclical reallocation between durable and non-durable sectors over the cycle (Loungani and Rogerson (1989)). The magnitudes are relatively small, however, compared to the relative income changes between sectors. Durable employment is unevenly spread across the U.S., and geographic mobility is limited at the business cycle frequencies (Yagan (2014); Kaplan and Schulhofer-Wohl (2017)).

[11] I focus on the effect of one-time, unanticipated but persistent shocks.

After observing their idiosyncratic labor income, households decide whether to adjust their stock of durable goods. Adjustment entails a non-convex cost Γ_t. Following Berger and Vavra (2015) and Kaplan et al. (2017), I assume that durable adjustment costs are proportional to the nominal value of the undepreciated stock of durable goods. If households don't adjust, they pay a maintenance cost, i.e. an investment required to repair or operate the existing stock of durables.[12] This maintenance corresponds to a share $\iota \in [0,1]$ of the current depreciation of their stock of capital. Summing up, the non-convex adjustment costs are

$$\Gamma_t\left(d', d\right) = \begin{cases} (1-\delta)\, P_t^d \gamma d & \text{if } d' \neq (1-(1-\iota)\,\delta)\, d \\ 0 & \text{otherwise} \end{cases}$$

for some adjustment cost $\gamma \geq 0$, where d' denotes the new stock of durables and $\delta \in (0,1)$ denotes the depreciation rate. Here, $\mathbf{P}_t \equiv \left\{P_t^h\right\}_h$ denotes goods prices. I suppose that these adjustment costs take the form of services (real estate, moving, etc.) provided by the non-durable sector, while maintenance takes the form of investment goods (new windows, tires, etc.) purchased from the durable sector.

Firms. Firms have access to technologies with decreasing returns:

$$y = F_t^h\left(l\right) \tag{2.1}$$

for some concave $F_t^h : [0,1] \to \mathbb{R}_+$ that allows for time-varying producivity.[13] Here, l denotes firms' individual labor demand.

Nominal rigidities. Prices are flexible at the stationary equilibrium. Firms set prices to maximize profits, subject to technology and given wages. On the contrary, prices are sticky à la Calvo (1983) along the transition path. The idiosyncratic reset probability is denoted by $1-\lambda^h$ in each sector, with $\lambda^h \in [0,1]$. The (implicit) elasticity of substitution across varieties, within each sector, is denoted by $\varepsilon > 1$.

Households supply labor inelastically in their industry of employment. They are compensated in proportion to their idiosyncratic labor supply. Wages are flexible at the stationary equilibrium, but rigid along the transition path: $W_t^h = W^h$. Labor is demand-determined in this case. As a result, earnings in the durable sector are more elastic to

[12] Maintenance is a standard feature of lumpy adjustment models (Bachmann et al. (2013); Berger and Vavra (2015)). It decreases the effective depreciation rate in the case of no adjustment. Fixing the depreciation rate, adjustment costs and idiosyncratic risk, a higher maintenance decreases the average frequency of adjustment.

[13] Implicitly, I suppose that capital is firm-specific and fixed in the short-run.

aggregate shocks in general equilibrium. The fiscal authority can potentially use lump sum taxes to provide cross-sectoral insurance against these aggregate shocks. Profits are redistributed symmetrically across households. Summing up, total individual (gross) incomes are

$$e_t^h(\zeta) = \frac{1}{\mu^h}\zeta\left(\mathcal{Y}_t^h - T_t^h\right) + \pi_t, \tag{2.2}$$

where $\boldsymbol{\mathcal{Y}}_t \equiv \left\{\mathcal{Y}_t^h\right\}_h$ denotes the sector-specific wage bills, $\mathbf{T}_t \equiv \left\{T_t^h\right\}_h$ denotes lump sum taxes and π_t denotes aggregate profits. The idiosyncratic process for labor supply follows a Markov chain with transition kernel Σ on some set $S \subset \mathbb{R}_{++}$, with $\mathbb{E}[\zeta] = 1$ and independence across households.

Policy. The government can potentially use lump sum taxes $\mathbf{T}_t$ to provide cross-sectoral insurance. It maintains an exogeneous and constant debt level $B > 0$ and sets linear taxes on income $\boldsymbol{\tau}_t$ to balance its flow budget. Monetary policy implements a standard Taylor rule:

$$i_t = \max\left\{r + \varphi^\pi\left(\Pi_t - 1\right), 0\right\}, \tag{2.3}$$

where Π_t denotes gross inflation[14] and r denotes the nominal interest rate at the stationary equilibrium.

2.2 Households' Optimization

The households' problem can be formulated recursively. The durable adjustment choice solves:

$$\mathcal{V}_t^h(a, d, \zeta) = \max_{\mathcal{A}\in\{0,1\}}\left\{V_t^h(a, d, \zeta; \mathcal{A})\right\}, \tag{2.4}$$

where $\mathcal{A} \in \{0, 1\}$ denotes the adjustment decision and $V_t^h(\cdot; \mathcal{A})$ denotes the continuation values associated to each adjustment option.

The value associated with no adjustment is

$$\begin{aligned} V_t^h(a, d, \zeta; 0) = \max_{\{c,a'\}} & \frac{u(c, d^\star)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a', d^\star, \zeta')\middle|\zeta\right] \\ \text{s.t. } & P_t^c c + P_t^d \iota\frac{\delta}{1-(1-\iota)\delta}d^\star + a' \leq (1-\tau_t)e_t^h(\zeta) + (1+r_{t-1})a \\ & a' \geq 0, \end{aligned} \tag{2.5}$$

[14] See Appendix A.1 for the definition of the price index.

with $d^\star \equiv (1-(1-\iota)\,\delta)\,d$. Here, $e_t^h(\zeta)$ denotes labor income, and r_{t-1} denotes the nominal interest rate. Earnings are indexed by the idiosyncratic labor supply and the industry of employment.

Similarly, the value associated with adjustment is

$$
\begin{aligned}
V_t^h(a,d,\zeta;1) = \max_{\{c,a',d'\}} \; & \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d',\zeta') \middle| \zeta\right] \\
\text{s.t. } & P_t^c c + P_t^d\left(d' - (1-\delta)\,d\right) + \Gamma_t(d',d) + a' \\
& \leq \left(1-\tau_t^h\right) e_t^h(\zeta) + (1+r_{t-1})\,a \\
& a' \geq 0,
\end{aligned}
\tag{2.6}
$$

2.3 Earnings and Insurance

Households' gross earnings (2.2) consist of labor income, aggregate profits, and transfers from the government. Wage bills are given by

$$\mathcal{Y}_t^h \equiv W^h \hat{\mu}_t^h, \tag{2.7}$$

where demands for labor $\hat{\boldsymbol{\mu}}_t \equiv \left\{\hat{\mu}_t^h\right\}_h$ satisfy

$$F^h\left(\hat{\mu}_t^h\right) \equiv \Omega_t^h \left(\frac{1}{P_t^h}\right)^{-\varepsilon} Y_t^h \tag{2.8}$$

Here $\mathbf{Y}_t$ denotes aggregate demand for each good, and Ω_t^h denotes the productivity distorsion associated with price dispersion.[15] Aggregate profits are

$$\pi_t \equiv \sum_h \left(P_t^h Y_t^h - \mathcal{Y}_t^h\right) \tag{2.9}$$

In the absence of insurance from fiscal policy, the strong pro-cyclical in the demand for durable investment Y_t^d translates into a high cyclicality of labor incomes $e_t^h(\zeta)$ in that sector, by (2.2) and (2.7)–(2.8). This leads to a redistribution of income between sectors.

I contrast two regimes to assess the role of redistribution. The first regime is one where fiscal policy is passive and labor income is endogenously redistributed between sectors. That is,

$$\mathbf{T}_t = 0 \tag{2.10}$$

[15] See Appendix A.1 for the definition of this productivity distorsion.

for each period t.

The second regime (denoted by $\star$) corresponds to a counterfactual economy where fiscal policy undoes this redistribution of labor income using lump sum taxes. Specifically,

$$\frac{\mathcal{Y}_t^d - \mu^d T_t^{d,\star}}{\mathcal{Y}_t^c - \mu^c T_t^{c,\star}} = \frac{\mathcal{Y}^d}{\mathcal{Y}^c}, \tag{2.11}$$

with budget balance $\sum_h \mu^h T_t^{h,\star} \equiv 0$, where $\boldsymbol{\mathcal{Y}} \equiv \{\mathcal{Y}^h\}_h$ denotes wage bills at the stationary equilibrium. This second regime rules out distributional effects and effectively corresponds to the case considered in the literature on lumpy durable investment.

2.4 Market Clearing

Markets for goods clear:

$$Y_t^c = \sum_h \mu^h \int \left[c_t^h(a,d,\zeta) + \Gamma_t\left(d_t^{h,\prime}(a,d,\zeta), d\right)\right] d\Lambda_t^h \tag{2.12}$$

$$Y_t^d = \sum_h \mu^h \int \left[d_t^{h,\prime}(a,d,\zeta) - (1-\delta)\, d\right] d\Lambda_t^h, \tag{2.13}$$

where c_t^h and $d_t^{h,\prime}$ denote the solution to (2.5)–(2.6), with $d_t^{h,\prime} \equiv d^\star$ when there is no adjustment. Here, $\boldsymbol{\Lambda}_t \equiv \{\Lambda_t^h\}_h$ denotes the conditional distributions of idiosyncratic states within each sector.

At the stationary equilibrium, wages are flexible and the labor markets clear in the two sectors: $\boldsymbol{\mu} = \hat{\boldsymbol{\mu}}$. Then,[16]

$$Y_t^h = F^h\left(\mu^h\right), \tag{2.14}$$

where μ^h denotes the exogeneous mass of households located in sector h. However, the market clearing condition (2.14) typically does not hold along the transition path since wages are fixed. Labor demands are demand-determined:

$$\boldsymbol{\mu} = \mathbf{Z}_t \circ \hat{\boldsymbol{\mu}}_t \tag{2.15}$$

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \leq Z_t^h < +\infty$ along the transition path, for each sector h and period t. That is, there is rationing of labor incomes in response to

[16] There is no price dispersion within each sector at the stationary equilibrium, so that $\hat{\boldsymbol{\mu}}$ denotes both aggregate and individual labor demands at the steady state.

aggregate shocks.

Equilibrium. An equilibrium in my economy consists of sequences of policy functions for consumption and assets, distributions of idiosyncratic states, and prices, incomes and taxes such that households and firms optimize, the government balances its budget (and potentially provides insurance) and monetary policy implements a Taylor rule. I provide a formal definition of an equilibrium in Appendix A.2. The economy is initially at its non-inflationary stationary equilibrium.

The comparative statics of interest is a one-time, unanticipated, and persistent aggregate productivity shock. I assume that this productivity shock is symmetric between the durable and non-durable sectors, so there is no redistribution of labor incomes in partial equilibrium. Instead, redistribution is *induced* in general equilibrium by the pro-cyclicality of durable investment. Financial markets are effectively incomplete with respect to this aggregate productivity shock.

3 Non-Linearity and Redistribution

I introduced a multi-sector model with lumpy investment in the previous section. Durable investment plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between sectors. Second, its lumpiness implies that the average MPC on durable goods depends on the magnitude of income shocks. In this section, I discuss each of these roles, and I show that their interaction acts as a non-linear propagation channel. In Section 3.1, I explore how lumpy adjustment at the micro level shapes non-linearities at the macro level. In Section 3.2, I clarify the role of durable investment as a source of labor income redistribution. I then explore the interaction between these two roles and its aggregate implications in Section 3.3. I draw a connection to the existing literatures on redistribution with heterogeneous agents in Section 3.4. Finally, I briefly review supporting evidence in Section 3.5.

3.1 Lumpy Investment and Aggregate Non-Linearity

Durable adjustment is infrequent and discontinuous in my model. I am interested in the aggregate implications of these microeconomic discontinuities. For now, I abstract from redistribution, so I drop the sector index h.[17]

[17] For notational convenience, I abstract from the masses $\boldsymbol{\mu}$ in (2.2).

I assume that the economy is initially at its stationary equilibrium.[18] I consider an exogeneous one-time, unanticipated change in aggregate income in period $t = 0$:

$$\mathcal{Y}_0 = (1 + \Delta)\,\mathcal{Y}$$

for some $\Delta \in \mathbb{R}$, where $\mathcal{Y}$ denotes the aggregate wage bill at the stationary equilibrium. My focus is on the non-linearity of aggregate investment with respect to the size and sign of this income shock.

Aggregate durable investment in the first period as a function of the aggregate income shock is[19]

$$\begin{aligned} I(\Delta) \equiv & \int \underbrace{\mathcal{A}(a,d,\zeta)}_{\text{Hazard}} \cdot \underbrace{(d^\star(a,d,\zeta) - (1-\delta)\,d)}_{\text{Gap to target}} \underbrace{d\Lambda(a - \zeta\Delta\mathcal{Y}, d, \zeta)}_{\text{Density}} \\ & + \int [1 - \mathcal{A}(a,d,\zeta)] \underbrace{\iota\delta d}_{\text{Maintenance}} \; d\Lambda(a - \zeta\Delta\mathcal{Y}, d, \zeta), \end{aligned} \tag{3.1}$$

where $\mathcal{A}(\cdot)$ denotes the durable adjustment hazard and $d^\star(\cdot)$ denotes the adjustment target that solves (2.6). The first integral in (3.1) captures investment by households who pay the fixed adjustment cost, while the second integral captures maintenance by those who do not.

The model described in Section 2 generates a standard inaction region for durable adjustment. That is, the adjustment hazard takes the form of a step function. In my calibrated model, upward adjustment is the most relevant margin at the stationary distribution.[20] I focus on this case for illustration. The adjustment hazard satisfies

$$\mathcal{A}(a,d,\zeta) = \begin{cases} 1 & \text{if } a \geq \bar{a}(d,\zeta) \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

for some threshold $\bar{a}(\cdot)$ that depends on durable holdings and idiosyncratic labor supply.

I am interested in the non-linear properties of the impulse response of aggregate durable

[18] State-contingency is an intrinsic property of models of lumpy investment (Bachmann et al. (2013); Berger and Vavra (2015); Winberry (2019)). Responses to aggregate shocks are typically larger following an expansion, compared to a contraction. See Section 5.3 for a discussion of the connection between state-contingency and non-linearity, and the interaction between redistribution and state-contingency.

[19] The integral is over the idiosyncratic states (a, d, ζ).

[20] Downward changes account for roughly 1% of adjustments at the stationary equilibrium. They could potentially play a more important role in the presence of deleveraging shocks (Guerrieri and Lorenzoni (2017)).

investment to an income shock:[21]

$$\hat{I}(\Delta) \equiv \frac{I(\Delta)}{\mathcal{I}} - 1, \tag{3.3}$$

where $\mathcal{I}$ denotes aggregate investment at the stationary equilibrium.

Fixing the adjustment hazard, the impulse response given by (3.1) and (3.3) is determined by adjustment along two margins: the *extensive* margin, i.e. the households' marginal propensity to adjust; and the *intensive* margin, i.e. their marginal propensity to invest conditional on adjustment. The following result decomposes the response of durable investment into these two margins. For the sake of exposition, I assume that the adjustment target $d^{\star}(\cdot, d, \zeta)$ is smooth and increasing, and that the distribution of liquid assets conditional on durable holdings and labor supply admits a smooth density $d\Lambda(a|d,\zeta)$.[22] I define $\Lambda^{\star} \equiv \text{marg}_{d,\zeta}\Lambda$. Finally, I consider discrete but plausibly small shocks.[23]

Proposition 1. *The response of durable investment to a positive income shock $\Delta > 0$ can be decomposed as follows:*

$$\hat{I}(\Delta) \equiv \underbrace{\Sigma_1(\Delta)}_{\textit{Extensive margin}} + \underbrace{\Sigma_2(\Delta)}_{\textit{Intensive margin}} + \underbrace{\varsigma(\Delta)}_{\textit{Residual}} \tag{3.4}$$

with

$$\begin{aligned}
\Sigma_1(\Delta) \equiv & \frac{1}{\mathcal{I}} \int \left\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) \right. \\
& \left. + \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} \left(a + \zeta\Delta\mathcal{Y} - \bar{a}(d,\zeta)\right) d\Lambda(a|d,\zeta) \right\} d\Lambda^{\star} \\
\Sigma_2(\Delta) \equiv & \frac{1}{\mathcal{I}} \int \mathcal{A}(a,d,\zeta) \left[d^{\star}(a+\zeta\Delta\mathcal{Y}, d, \zeta) - d^{\star}(a,d,\zeta)\right] d\Lambda(a,d,\zeta)
\end{aligned}$$

for some $\kappa(d,\zeta) > 0$ and some residual $\varsigma(\Delta)$ that satisfies $\lim_{\Delta\to 0} \frac{\varsigma(\Delta)}{\Delta} = 0$. The analogous decomposition for a negative income shock $\Delta < 0$ is provided in Appendix B.

[21] In the context of price setting, Caballero and Engel (2007) show that the aggregate response in the Caplin and Spulber (1987) model coincides with the average response across time at the firm-level, when the economy is stationary. In other words, my exercise captures the "average" degree of non-linearity over time in the household-level response of durable investment.

[22] For expositional purposes, I further assume that the conditional distribution $d\Lambda(\cdot|d,\zeta)$ has full support on $[0, a^{\star}]$ with $a^{\star} > \bar{A} \equiv \sup_{(d,\zeta)} \bar{a}(d,\zeta)$, so that some households adjust for each (d,ζ).

[23] Specifically, I assume that $\Delta \in \left[-\sup_{(d,\zeta)} \frac{1}{\zeta}\frac{a^{\star}-\bar{a}(d,\zeta)}{\mathcal{Y}}, \inf_{(d,\zeta)} \frac{1}{\zeta}\frac{\bar{a}(d,\zeta)}{\mathcal{Y}}\right]$ to avoid cases where either no household, or all households adjust. In the following examples, I assume some monotonicity properties around the durable adjustment threshold. These properties might not hold globally. I assume that Δ is sufficient small so that they apply.

Proof. See Appendix B. $\square$

The extensive margin consists of two terms. The first term captures *discontinuities* at the microeconomic level. Households who pay the fixed cost adjust their stock by a discrete amount.[24] In turn, the mass of households who adjust depends on the shape of the distribution of liquid assets. The second term reflects heterogeneity among households who pay the fixed cost after the income shock: those that were initially further away from their adjustment threshold invest relatively less when adjusting their stock of durables. The intensive margin captures the decreasing propensity to invest conditional on adjustment, due to precautionary savings (Carroll and Kimball (1996); Bertola et al. (2005)). The residual plays a negligible role in my numerical simulations.

The following three examples clarify the contribution of each term in (3.4) to the impulse response of aggregate durable investment, and its non-linearity. For illustration, I focus on positive income shocks. The opposite case is symmetric. I collect all derivations in Appendix B.3.

Example 1 (Extensive Margin I)**.** I first focus on adjustment at the extensive margin. Specifically, I illustrate the role of the shape of the distribution of liquid assets, i.e. the first term in $\Sigma_1(\Delta)$. I suppose that the adjustment target $d^\star(\cdot)$ is constant at some level $\bar{d} > 0$.[25] In this case, there is no adjustment at the intensive margin, households adjust to a common level, and the residual is zero:

$$\Sigma_2(\Delta) = 0 \quad , \quad \kappa(d,\zeta) = 0 \quad \text{and} \quad \varsigma(\Delta) = 0$$

The impulse response (3.4) satisfies

$$\hat{I}(\Delta) = \frac{1}{\mathcal{I}} \int \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta)\, d\Lambda^\star \tag{3.5}$$

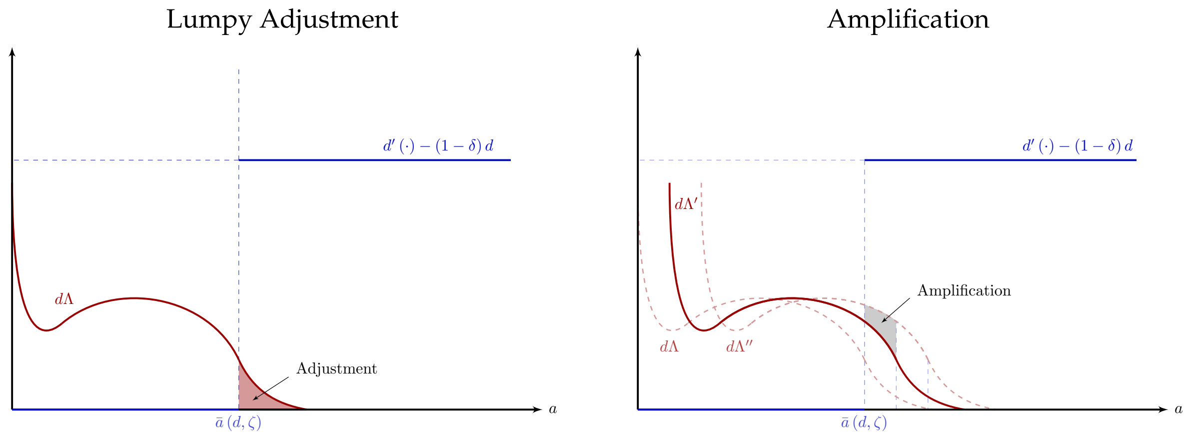
The response of durable investment depends on the slope of the density of liquid assets around the thresholds. If $d\Lambda(\cdot|d,\zeta)$ is uniform, then $\hat{I}(\cdot)$ is linear in the size of the shock: there is neither amplification, nor dampening. Quantitatively, the relevant case is one where this density is *decreasing* around the thresholds (Section 5.2). Then, the impulse response of aggregate durable investment $\hat{I}(\cdot)$ is convex: more and more households exit their inaction region for expansionary shocks; less and less do so for contractionary shocks. In other words, positive income shocks are *amplified* as their size increases, and

[24] See Baumol (1952), Tobin (1956) and Scarf (1960) for seminal contributions the subject.
[25] That is, the durable adjustment cost satisfies $\Gamma_t(d',d) = \mathbf{1}_{\{d'=\bar{d}\}}\gamma + (1-\mathbf{1}_{\{d'=\bar{d}\}})M$ with $M \to +\infty$.

negative shocks are *dampened*. That is, the impulse response of aggregate durable investment $\hat{I}(\Delta)$ is *convex* in the income shock.

Figure 3.1 illustrates this non-linear amplification. The left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red),[26,27] together with the durable adjustment threshold $\bar{a}(d,\zeta)$. I implicitly fix some level of durable holdings and labor supply. A positive income shock $\Delta > 0$ shifts the density of cash-on-hand to the right, as shown in the right panel. When the density of liquid assets is decreasing at the adjustment threshold, positive income shocks are amplified: more and more households adjust at the extensive margin. The opposite happens for negative shocks.

Figure 3.1: Extensive Margin I



Example 2 (Extensive Margin II)**.** I focus again on adjustment at the extensive margin, but I now highlight the role of the adjustment target. For illustration, I assume that the adjustment target $d^{\star}(\cdot)$ is linear, with $\kappa(d,\zeta)$ denoting the corresponding slope. To abstract from the effect illustrated in the previous example, I suppose that the density $d\Lambda(a|d,\zeta)$ is uniform over $[0,a^{\star}]$ with $\bar{a}(\cdot) < a^{\star} < +\infty$. In this case, adjustment at the intensive margin is linear, and the residual is zero: $\varsigma(\Delta) = 0$. The impulse response (3.4) satisfies

$$\hat{I}(\Delta) = \theta\Delta + \left[\frac{1}{2}\frac{1}{\mathcal{I}}\frac{1}{a^{\star}}\int \kappa(d,\zeta)(\zeta\mathcal{Y})^2\, d\Lambda^{\star}\right]\Delta^2 \tag{3.6}$$

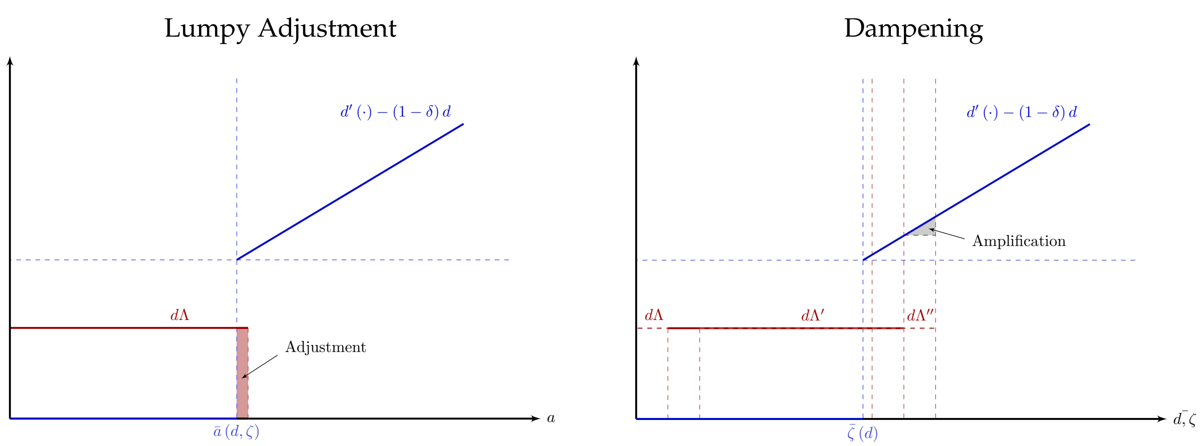
for some $\theta > 0$. Again, this form of adjustment at the extensive margin contributes to a *convex* impulse response of aggregate durable investment $\hat{I}(\cdot)$.

[26] The borrowing constraint $a \geq 0$ acts as a reflection barrier. Hence the mass at $a = 0$.

[27] The model is set in discrete time. Therefore, there is a positive mass of households at the stationary equilibrium outside of the inaction region. These households adjust in the current period, which depletes their stock of liquid assets.

Figure 3.2 illustrates this case. As in the previous example, the left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red), and the durable adjustment threshold. A positive income shock $\Delta > 0$ shifts the density of cash-on-hand to the right, as shown in the right panel. Positive income shocks are *amplified* as their size increases: households who decide to adjust do so increasing amounts. On the contrary, the effect of negative shocks is *dampened* as their magnitude increases.

Figure 3.2: Extensive Margin II



Example 3 (Intensive Margin)**.** Finally, I focus on the intensive margin, i.e. the one that would operate in a frictionless model. I assume that the adjustment target $d^\star(\cdot)$ is concave due to precautionary savings and binding borrowing constraints. By definition, the adjustment hazard and the thresholds satisfy $\mathcal{A}(a,d,\zeta) \equiv 1$ and $\bar{a}(d,\zeta) \equiv 0$ in this case. The residual is zero: $\varsigma(\Delta) = 0$. Then, the impulse response writes:

$$\hat{I}(\Delta) = \frac{1}{\mathcal{I}} \int \left[d^\star(a + \zeta\Delta\mathcal{Y}, d, \zeta) - d^\star(a,d,\zeta)\right] d\Lambda(a,d,\zeta)$$

The response of durable investment $\hat{I}(\Delta)$ is *concave*, i.e. it inherits the shape of the durable adjustment target. The effect of positive income shocks is *dampened* as their size increases, while the effect of negative shocks is *amplified*.

Marginal propensities to spend. In Examples 1–2, durable investment reacts proportionetaly more (less) to large positive (negative) shocks. In other words, the average MPC increases with income changes. Lumpy adjustment, i.e. state-dependent adjustment, plays a central role in this non-linear effect. In models with frictionless adjustment, convex adjustment costs, or time-dependent adjustment, or even purely non-durable consumption, only the intensive margin operates. In this case, the average MPC decreases with the size of the income shock due to precautionary savings, as illustrated in Example 3. The relative importance of these two effects, and the effective degree of non-linearity are quantitative questions. In Section 5, I use my calibrated model to implement the decomposition from Proposition 1. I find that adjustment at the extensive margin dominates. That is, the average MPC increases with income changes.

3.2 Redistribution in General Equilibrium

I now explore the role of durable investment for the redistribution of labor income in general equilibrium. Keeping with the rest of the paper, the comparative statics is a one-time, unexpected, and persistent aggregate productivity shock. I assume that this productivity shock is not targeted toward any particular sector, so that redistribution of labor income is entirely induced by endogeneous changes in households' spending. That is, $A_t^h / A^h = \hat{A}_t$ for each sector h, with

$$\log\left(\hat{A}_t\right) = \rho^A \log\left(\hat{A}_{t-1}\right) + \psi_t \tag{3.7}$$

for some persistence $\rho^A \in (0,1)$ and some innovation $\psi_0 \in \mathbb{R}$ in the first period, with $\psi_t \equiv 0$ for each consecutive period $t \geq 1$.

Following a productivity shock, the pro-cyclicality of durable spending induces a redistribution of labor income. I impose some restrictions to avoid introducing additional sources of redistribution. In particular, I assume that technologies and price stickiness satisfy a certain degree of symmetry across sectors.[28]

Assumption 1 (Technologies)**.** *Technologies are isoelastic and the elasticity is symmetric across sectors: $F_t^h(l) = A_t^h l^\alpha$, for some $\alpha \in (0,1)$ and some vector of productivities $\mathbf{A}_t \equiv \left\{A_t^h\right\}_h$. Productivities at the stationary equilibrium $\mathbf{A}$ are such that $W^c = W^d$, i.e. wages are symmetric across sectors.*

Assumption 2 (Price setting)**.** *Price stickiness is symmetric across sectors: $\lambda^h = \lambda \in [0,1)$.*

To highlight the role of durable investment for the redistribution of labor income, I first establish a benchmark where the two goods are non-durable. That is, there is full depreciation: $\delta = 1$. The relative demand for the two goods is acyclical in this case, by homotheticity of the intratemporal preferences. Under Assumptions 1 and 2, there

[28] This symmetry is somewhat restrictive empirically. In particular, the labor share is higher in the durable good sector, prices of durable goods tend to be more flexible (Bils and Klenow (2004)). These sources of heterogeneity and their role over the business cycle have been studied separately (Jaimovich et al. (2019); Pastén et al. (2018)). I choose to abstract from the sources of heterogeneity, to isolate the role of redistribution induced by cyclical durable investment.

is no redistribution of labor income and thus no role for insurance from fiscal policy. I formalize this point in Proposition 2 (Appendix B.2). This benchmark case clarifies that, in my setting, durability is *necessary* for income redistribution to take place.[29] Furthermore, it confirms that Assumptions 1 and 2 are as neutral as possible: the supply side does not induce any redistribution *per se*. I maintain these two assumptions throughout the paper.

Having established this benchmark, I now assume partial depreciation: $\delta \in (0,1)$. Durable spending is more cyclical than non-durable spending, which induces a redistribution of labor income across sectors. This redistribution depends on the elasticity of durable investment with respect to income and price changes. In particular, these elasticities are governed by: the elasticity of substitution between durables and non-durables ν; the adjustment cost γ; and the maintenance parameter ι.[30] In virtually every realistic calibration, durable investment is more cyclical than non-durable consumption. I assume that this is the case.

Assumption 3 (Monotonicity)**.** *The parameters characterizing the income fluctuations problem (2.4)–(2.6) are such that relative demand for durable invesment* $\left(d_t^h(\cdot) - (1-\delta)\, d\right) / c^h(\cdot)$ *and non-durable consumption* $c^h(\cdot)$ *increase with liquid assets and decrease with the nominal interest rate. Furthermore, the price elasticity of* $d^h(\cdot)$ *and* $c^h(\cdot)$ *is larger than* 1.

To understand the pro-cyclicality of relative spending on durable goods, consider a representative-agent version of the model presented in Section 2.[31] In this case,

$$\frac{d_t - (1-\delta)\, d_{t-1}}{c_t} = \frac{1-\vartheta}{\vartheta} \underbrace{\left(\frac{P_t^c}{P_t^d - \frac{(1-\delta) P_{t+1}^d}{1+r_t}}\right)^{\nu}}_{\text{User cost}} - \underbrace{(1-\delta)\, \frac{d_{t-1}}{c_t}}_{\text{Stock / flow}}$$

at optimum. A productivity shock of the form (3.7) affects the relative demand for the durable investment good through two channels. First, it raises incomes in general equilibrium. Non-durable consumption and the *stock* of durable consumption increase proportionately. Mechanically, the *flow* of durable investment is much more volatile. In the first place, this increase in incomes is induced by a fall in the real interest rate, when monetary policy is sufficiently responsive ($\varphi^{\pi} > 1$). This decrease in the households' *user cost* of

[29] This result is not specific to productivity shocks: the same obtains for monetary policy shocks, deleveraging shocks or symmetric tax rebates across sectors.

[30] For instance, durable demand is inelastic to income changes as $\gamma \to +\infty$. Similarly, the interest rate elasticity of durable demand is zero as $\nu \to 0$.

[31] Specifically, there is no idiosyncratic risk ($\Sigma \to_d 0$) and durable adjustment is frictionless ($\gamma = 0$).

durables tilts spending in favor of durable investment[32]. This second effect is modulated by the elasticity of substitution between durables and non-durables (ν).

3.3 Non-Neutrality and Amplification

In Sections 3.1–3.2, I clarified the dual role of lumpy durable investment in my model: its pro-cyclicality induces income redistribution; and micro discontinuities imply macro non-linearities. I now explore the interaction between these two properties, and the non-linear amplification it generates.

To assess the role of redistribution, I contrast the aggregate response of my economy under the two regimes (2.10)–(2.11) for fiscal policy. In the first regime, income redistribution takes place between sectors and fiscal policy provides no cross-sectoral insurance. In the second regime, fiscal policy undoes this redistribution and provides full (aggregate) insurance. For tractability, I focus on the general equilibrium response in period $t = 0$, keeping the sequence of labor incomes, aggregate profits, taxes, interest rates and sector-specific inflation $X_t \equiv (\boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \mathbf{T}_t, r_t, \boldsymbol{\Pi}_t)$ fixed for each period $t > 1$. I also abstract from changes in the relative price of durable goods[33] by assuming that technologies are linear: $\alpha \equiv 0$. For illustration, I build on Example 2, where only the extensive margin operates, i.e. I implicitly assume that the role of precautionary savings is negligible. Again, I include the derivations in Appendix B.3.

Example 4 (Non-neutrality)**.** Consider an expansionary productivity shock: $\psi_0 > 0$. From the firms' price setting problem, current prices and inflation decrease: $P_0^h / P^h = 1/\psi_0$.[34] Monetary policy is accomodative and reduces the interest rate $r_0 < r$. Spending on both goods increases in partial and general equilibrium, by Assumption 3. However, $\mathcal{Y}_0^d / \mathcal{Y}^d > \mathcal{Y}_0^c / \mathcal{Y}^c$, i.e. labor income is redistributed in favor of the durable sector.

Now, suppose that fiscal policy undoes this redistribution by providing aggregate insurance, using lump sum taxes (2.11),

$$T_0^d = -\frac{\mu^c}{\mu^d} T_0^c > 0$$

These transfers reduce the dispersion in the distribution of labor incomes. Under the

[32] This second channel is particularly powerful in typical models of investment. Existing estimate suggest that this effect is overstated (Hall (1977); Shapiro (1986); McKay and Wieland (2019)).

[33] The relative price of durable goods is mostly acyclical in the data (Pistaferri (2016); Cantelmo and Melina (2018); McKay and Wieland (2019)).

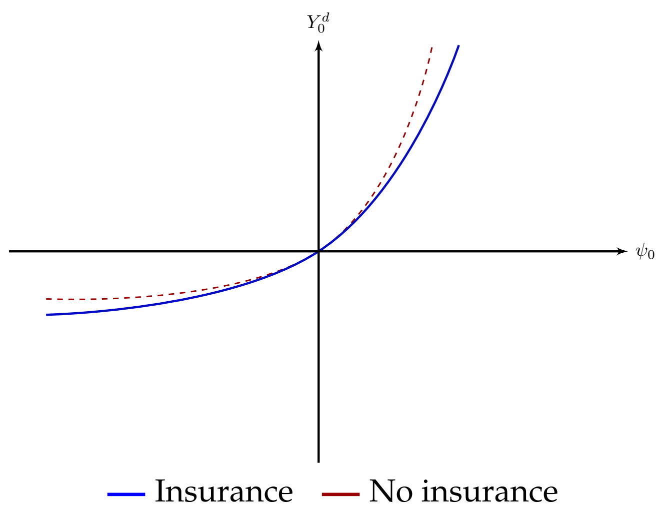
[34] Variables at the stationary equilibrium are not indexed by time.

assumptions of Example 2, aggregate (sectoral) durable investment

$$Y_0^{d,h} \equiv \int \left[d_0^{h,\prime} (a, d, \zeta) - (1 - \delta) d \right] d\Lambda_0^h$$

is *convex* in (sectoral) incomes,[35] for each sector h. By Jensen's inequality, a lower dispersion depresses aggregate durable investment $Y_0^d \equiv \sum_h \mu^h Y_0^{d,h}$. Put it differently, redistribution *amplifies* the effect of an expansionary productivity shocks, compared to a benchmark with full aggregate insurance. On the contrary, redistribution *dampens* the effect of contractionary shocks. Figure 3.3 illustrates this result. The case without insurance is depicted in red (dashed). The case with full aggregate insurance is plotted in blue (solid).

Figure 3.3: Impulse Response of Aggregate Durable Investment



3.4 Sufficient Statistics

The effect of income redistribution in my model can be understood through a sufficient statistics approach. Focusing on the first period $t = 0$, let

$$Y_0^{d,h} \left(\mathcal{Y}_0^h - \mu^h T_0^h \right) \equiv \int \left[d_0^{h,\prime} (a, d, \zeta) - (1 - \delta) d \right] d\Lambda_0^h$$

denote aggregate durable investment in each sector as a function of the aggregate wage bills $\boldsymbol{\mathcal{Y}}_0$ and transfers $\mathbf{T}_0$.[36] Assuming differentiability, define

$$\mathrm{MPC}_0^{d,h} \left(\mathcal{Y}_0^h - \mu^h T_0^h \right) \equiv \frac{d}{d\mathcal{Y}} Y_0^{d,h} \left(\mathcal{Y}_0^h - \mu^h T_0^h \right) \tag{3.8}$$

[35] Incomes are implicit to the time index.
[36] Again, the dependance on incomes on the right-hand-side is implicit to the time index.

i.e. the average marginal propensity to spend (MPC) on durables in each sector.[37] My focus is on non-linear effects, i.e. *endogeneous* changes in MPC.

Let lump sum taxes $\mathbf{T}$ satisfy:

$$\frac{\mathcal{Y}_0^d - \mu^d T_0^d}{\mathcal{Y}_0^c - \mu^c T_0^c} = \frac{\mathcal{Y}_0^d}{\mathcal{Y}_0^c} + \omega \left(\frac{\boldsymbol{\mathcal{Y}}^d}{\boldsymbol{\mathcal{Y}}^c} - \frac{\mathcal{Y}_0^d}{\mathcal{Y}_0^c} \right) \tag{3.9}$$

where $\boldsymbol{\mathcal{Y}}$ denotes the wage bills at the stationary equilibrium. This formulation nests the two cases of interest: no insurance ($\omega = 0$); and full aggregate insurance ($\omega = 1$). Finally, fix some productivity shock ψ_0, and let $\mathcal{Y}_0^h(\psi_0)$ denote the corresponding equilibrium wage bills under the regime with no insurance.

The shock ψ_0 induces a redistribution of labor income. I am interested in the effect of insurance from fiscal policy that undoes this redistribution. Let

$$Y_0^d(\omega;\psi_0) \equiv \sum_h \mu^h Y_0^{d,h}\left(\mathcal{Y}_0^h(\psi_0) - \mu^h T_0^h(\omega;\psi_0)\right) \tag{3.10}$$

denote aggregate demand for durable investment, where taxes $\mathbf{T}_0(\cdot)$ solve (3.9). For the sake of exposition, I focus on the effect of insurance in partial equilibrium, starting from the no insurance benchmark ($\omega = 0$).[38]

My model is set up so that

$$\left.\frac{d}{d\psi_0} Y_0^d(1;\psi_0)\right|_{\psi_0=0} = \left.\frac{d}{d\psi_0} Y_0^d(0;\psi_0)\right|_{\psi_0=0}$$

i.e. redistribution of labor income is neutral for first order deviations from the stationary equilibrium. This explains the tangency point at the origin in Figure 3.3. By Assumption 1, the distribution of liquid assets is symmetric across sectors at the stationary equilibrium.[39] Therefore, $\text{MPC}_0^{c,h}$=$\text{MPC}_0^{d,h}$ at the steady state. In other words, the "earnings heterogeneity channel" studied by Auclert (2019) and Patterson (2019) does not operate in my model for first order deviations from the steady state. This property allows me to isolate the non-linear effects of income redistribution.

[37] Note that (3.8) corresponds to the *average* MPC on durables in each sector, which differs from individual MPCs. Individual MPC are not well-defined at the adjustment threshold (3.2).

[38] This explains why wage bills are not indexed by the degree of insurance ω in (3.10).

[39] I focus on one-time, unanticipated shocks (Guerrieri and Lorenzoni (2017); Kaplan et al. (2018)). If households employed in the durable sector anticipate aggregate risk, they could in theory accumulate higher precautionary savings. However, there is only limited empirical support for this prediction (Skinner (1988)), which can partly be attributed to heterogeneity in risk-aversion across sectors (Schulhofer-Wohl (2011)). Patterson (2019) finds that heterogeneity in MPCs across sectors plays a relatively minor role in the general equilibrium amplification of aggregate shocks.

Consider now a discrete productivity shock $\psi_0 \in \mathbb{R}$. Starting from the no insurance case $(\omega = 0)$, suppose that fiscal policy increases the degree of cross-sectoral insurance. From (3.8) and (3.10),

$$\frac{d}{d\omega} Y_0^d(\omega;\psi_0)\bigg|_{\omega=0} = \sum_h \mu^h \frac{d}{d\omega} T_0^h(\omega;\psi_0)\bigg|_{\omega=0} \times \mathrm{MPC}_0^{d,h}\left(\mathcal{Y}_0^h(\cdot) - \mu^h T_0^h(\cdot)\right)\bigg|_{\omega=0}$$

For instance, consider an expansionary shock $(\psi_0 > 0)$. Incomes expand proportionately more in the durable sector. Fiscal policy partly offsets this redistribution by taxing the households employed in the durable sector: $T_0^d(\cdot) = -\mu^c/\mu^d T_0^c(\cdot) > 0$, from (3.9). These transfers are non-neutral when $\mathrm{MPC}_0^{d,h}$ is heterogeneous across sectors *away* from the stationary equilibrium.

In the context of Example 1 or 2, aggregate durable investment is a strictly *convex* function of (sectoral) incomes. That is, $\mathrm{MPC}_0^{d,h}(\cdot)$ is strictly *increasing* in cash-on-hand, so that

$$\mathrm{MPC}_0^{d,d}(\cdot)\Big|_{\omega=0} > \mathrm{MPC}_0^{d,c}(\cdot)\Big|_{\omega=0} \Rightarrow \frac{d}{d\omega} Y_0^d(\omega;\psi_0)\bigg|_{\omega=0} < 0$$

That is, insurance mitigates the expansion. In other words, redistribution *amplifies* the response of aggregate durable investment to an expansionary shocks, compared to a full aggregate insurance benchmark. Symmetrically, redistribution *dampens* this response following a contractionary shock. Taken together, these observations explain why the response without insurance defines the *upper envelope* of the response with insurance in Figure 3.3.

3.5 Supporting Evidence

I conclude this section by reviewing supporting evidence on the interaction between the cyclicality of durable investment and redistribution.

Redistribution. Purchases of durable goods and residential investment are strongly procyclical in the data (Kydland and Prescott (1982); Baxter (1996)). Hall (2005) and Bils et al. (2013) document a substantial pass-through to durable employment during booms and busts. Using administrative U.S. data, Guvenen et al. (2017) confirm that expansions and recessions have distributional consequences between durable and non-durable sectors.[40]

I illustrate the importance of this sectoral income redistribution in two contexts: the Great Recession; and in response to well-identified, exogeneous shocks. To account for

[40] They estimate a contemporaneous elasticity of individual income with respect to GDP of roughly 2 for durable industries.

cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).[41] The PSID is a representative panel survey of U.S. households conducted annually during 1968-1997, and bi-annually since then. The sample consists of male household's heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use (real) gross labor incomes as an empirical counterpart to labor earnings in my model.[42] I describe the data, the sample selection, and the specifications in Appendices C.2–C.3.

The left panel of Figure 3.4 plots the time series of real spending during the Great Recession for the two categories of interest: durables and residential investment; and non-durables and services. Durable spending fell by roughly 25% over this period, compared to 5% for non-durable spending. The right panel plots the time series of mean labor income for households employed in the corresponding sectors in 2008.[43] The recession led to a substantial redistribution of labor income between sectors. The pass-through was incomplete, however. Labor incomes in the durable sector decreased by 13% over two years, compared to a 4% decline in the non-durable sector.

For robustness, I estimate the sector-specific response of labor incomes to well identified, exogeneous shocks. The leading example in my paper is an unanticipated productivity shocks. Fluctuations in measured productivity are potentially endogeneous, however.[44] To address this concern, I focus instead on narratively-identified, exogeneous policy shocks. I use the series of exogeneous tax changes constructed by Romer and Romer (2010),[45] which are sufficiently persistent to be aggregated at the low-frequency of the PSID data.

The left panel of Figure 3.5 plots the response of spending on durables and non-durables following a one-standard deviation contractionary Romer and Romer (2010) tax shock. The right panel reports the response of labor incomes in the corresponding industries. The pattern is similar as in the Great Recession: durable investment is strongly

[41] Other datasets commonly used for the study of income dynamics – such as the Current Population Survey (CPS), the Survey of Income and Program Participation (SIPP), or the National Longitudinal Survey of Youth (NLSY) – either have a shorter panel dimension, or their sample is not as representative of the U.S. population as the PSID's.

[42] I also report the responses for family income in Appendix C.4 to account for unemployment insurance and intra-household risk-sharing. The conclusions are very similar in this case.

[43] Specifically, I allocate households to either the durable or non-durable sectors based on their industry of employment in 2008 (i.e. 2009 PSID wave). Fixing this cross-section, I compute mean labor income for each year.

[44] See Chari et al. (2007) and Buera and Moll (2015), among others. Productivity shocks identified via Structural Vector Auto-Regression (SVAR) might not be exogeneous either (Ramey (2011)).

[45] I prefer tax changes to government spending shocks (Ramey (2011)). Those are typically targeted toward a particular sector, which mechanically induces redistribution.

pro-cyclical, which leads to a substantial redistribution of labor incomes.

Non-linearity. In my model, the average MPC out of an income shock depends on the magnitude of this shock (Section 3.1). This property ensures that cyclical income inequality has aggregate effects. The monotonicity in the average MPC depends on the relative strength of the margins identified in Section 3.1. The empirical evidence on the consumption response to tax shocks provides some guidance.

In the context of the 2001 stimulus payment, Johnson et al. (2006) estimate negligible spending multipliers on durable goods. On the contrary, Souleles (1999) finds that the response of spending to springtime tax refunds is almost entirely driven by durable investment (and purchases of vehicles in particular), while Parker et al. (2013) document sizeable multipliers following the 2008 stimulus payment.[46] Parker et al. (2013) attribute this difference to the size of the transfers.[47] Table 3.1 lists the average transfer size and the average MPC on durable goods for these episodes. The average MPC is insignificant for a $500 average transfer, but accounts for roughly half of the spending response for the range $\$1,000$ to $\$2,500$.

Building on the discussion of Section 3.1, these estimates suggest that adjustment at the extensive margin dominates. In Section 5.1, I show that my calibrated model produces an increasing average MPC to tax rebates, consistently with this empirical evidence.

Table 3.1: Marginal Propensities to Spend on Durables (1 quarter)

	Johnson et al. (2006)	Parker et al. (2013)	Souleles (1999)
Type of transfer	Stimulus ('01)	Stimulus ('08)	Rebates (spring)
Average amount	\$480	$\$1,000$	$\$2,500$
Average MPC (D)	$\sim 0\%$	$\sim 50\%$	$\sim 55\%$

4 Calibration

The remainder of the paper quantifies the mechanisms documented in Section 3. I first parametrize the model using a mix of external and internal calibration. Following Berger

[46] Parker et al. (2013) find that their estimates of MPCs on durable goods are less precise than for non-durable goods. The corresponding figure in Table 3.1 corresponds to the average between the lower and upper bounds that they obtained for durables (p. 2531).

[47] In particular (p. 2532): "For instance, some prior research finds that larger payments can skew the composition of spending towards durables, which is consistent with our findings given that the 2008 stimulus payments were on average about twice the size of the 2001 rebates."

Figure 3.4: Great Recession

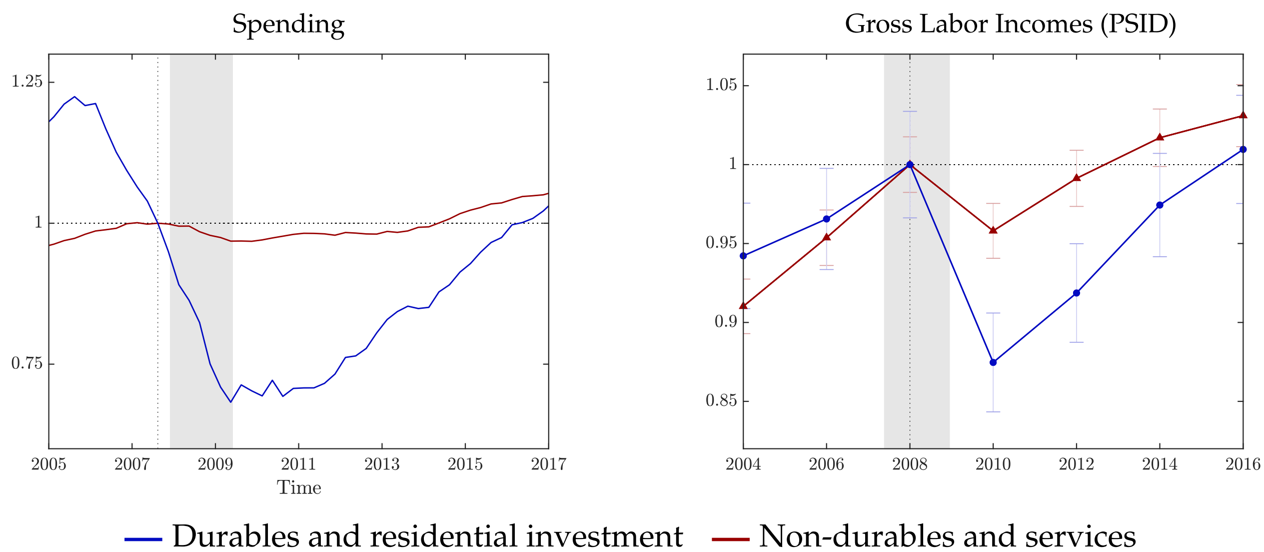
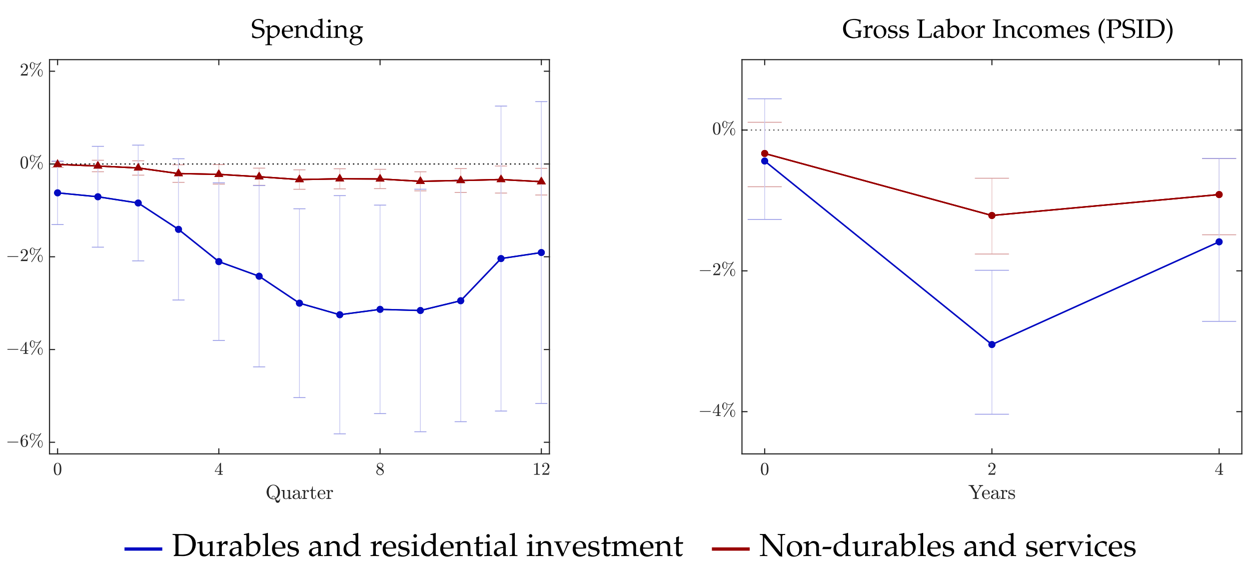


Figure 3.5: Response to Exogeneous Tax Increase (Romer and Romer (2010))



and Vavra (2015), I adopt a broad definition of durable goods that includes residential investment and consumer durables.

I calibrate five parameters internally: the discount factor (β), the preference parameter for non-durable goods (ϑ), the durable adjustment costs (γ), the exogeneous supply of liquidity (B), and the relative productivity in the durable sector $\left(A^d\right)$. I calibrate the remaining parameters externally, using standard values in the literature. I first review the external calibration, before discussing the targeted moments and the fitted parameters. Table 4.2 describes the parametrization. Data sources are listed in Appendix C.1.

4.1 External Calibration

I set the inverse elasticity of intertemporal substitution to $\sigma = 4$. This value, while large compared to typical calibrations, is commonly used in models with durable goods (Guerrieri and Lorenzoni (2017); McKay and Wieland (2019)), which predict a high elasticity of durable investment to interest rate changes. I choose an elasticity of substitution between durables and non-durables of $\nu = 0.75$, as in Barsky et al. (2016).[48] I choose a maintenance parameter of $\iota = 0.5$, which lies between the estimates of Berger and Vavra (2015) $(\iota = 0.8)$ and McKay and Wieland (2019) $(\iota = 0.35)$.

The stock of durable goods depreciates at roughly 2% $(\delta = 0.018)$. The income process (in log) follows an AR(1) process with persistence $\hat{\rho} = 0.975$ and standard deviation of innovations $\hat{\sigma} = 0.1$ to match the evidence of Floden and Lindé (2001). I set the mass of households in the durable sector to $\mu^d = 0.23$, based on data from the Current Employment Statistics. Based on Assumptions 1 and 2, I assume a certain degree of symmetry between sectors. Technologies are isoelastic and the elasticity is symmetric across sectors. I normalize productivity to 1 in the non-durable sector. Labor receives roughly 2/3 of revenues at the stationary equilibrium $(\alpha = 0.3)$. Similarly, price stickiness is symmetric across sectors. The average duration between price resets is 4 quarters $(\lambda = 0.75)$. I set the elasticity of substitution across varieties to $\varepsilon = 10$, which is standard in the literature. Finally, I assume that monetary policy is responsive to inflation $(\varphi = 1.25)$.

4.2 Internal Calibration

The remaining parameters $\left(\beta, \vartheta, \gamma, B, A^d\right)$, together with the interest rate (r) and the price of the durable good $\left(P^d\right)$ at the stationary equilibrium, are the implicit solution to seven

[48] The literature has historically used a unitary elasticity, based on the estimates of Ogaki and Reinhart (1998) and Piazzesi and Schneider (2007). However, values below unity are often used to dampen the interest rate elasticity of durable demand (Barsky et al. (2016); McKay and Wieland (2019)).

restrictions: two equilibrium conditions, and five empirical moments. The moments I target, their values and the sources I use are listed in Table 4.1[49]. Appendix A.4 describes the calibration strategy in more details.

I choose the discount factor β to target a real interest rate at the stationary equilibrium of 2.5%, which corresponds to the average value since 1970. I set the preference parameter for non-durable goods ϑ to obtain a ratio of investment to consumption of roughly 18%. I adjust the durable adjustment parameter γ to target an annual frequency of adjustment of 12%, as in Berger and Vavra (2015). I obtain $\gamma = 0.032$, which corresponds to the standard value of 5% when expressed in terms of the *numéraire* (Díaz and Luengo-Prado (2010); Berger and Vavra (2015)). The exogeneous supply of liquidity (B) targets a ratio of liquidity to annual GDP of 1.4, following McKay et al. (2016)[50]. Finally, I choose the relative productivity in the durable goods sector A^d to target symmetric wages across sectors at the stationary equilibrium.

Table 4.1: Targeted Moments

Target	Value	Source
Real interest rate (A)	0.025	FRED and NIPA
Ratio of investment to consumption	0.18	NIPA
Frequency of durable adjustment (A)	0.10	Berger and Vavra (2015)
Liquidity supply to GDP (A)	1.4	McKay et al. (2016)
Relative wages	1	Normalization

5 Lumpy Investment and Non-Linearity

In this section, I assess the degree of non-linearity produced by the demand block in my model, i.e. the households' income fluctuations problem (2.4)–(2.6). As discussed in Section 3.2, this non-linearity shapes the non-neutrality of income redistribution. In Section 5.1, I quantify this non-linearity by simulating the partial equilibrium response of durable spending to income shocks. In Section 5.2, I implement the decomposition from Proposition 1 in my calibrated model. Finally, I relate my findings to the literature on state-contingent responses with lumpy adjustment in Section 5.3.

[49] In Table 4.1, "relative" refers to the ratio of the value for durable to that for non-durable.

[50] Liquidity includes: deposits, government-issued securities, corporate bonds and equitities and mutual fund shares.

Table 4.2: Calibration

Parameter	Description	Calibration	Source / Target
Preferences			
β	Discount factor	0.985	Internal calibration
ν	Elasticity of substitution	0.75	See text
ϑ	Non-durable parameter	0.540	Internal calibration
σ	EIS (inverse)	4	See text
ε	Elasticity of substitution (inverse)	10	Kaplan et al. (2018)
Durable goods			
δ	Depreciation rate	0.018	Berger and Vavra (2015)
γ	Adjustment cost	0.032	Internal calibration
Liquidity			
B	Ratio of bond supply to GDP	1.49	Internal calibration
Income process			
$\hat{\rho}$	Persistence	0.967	Floden and Lindé (2001)
$\hat{\sigma}^2$	Standard deviation	0.13	Floden and Lindé (2001)
Production			
A^d	Relative productivity (durable)	0.490	Internal calibration
α	Decreasing returns	0.3	Berger and Vavra (2015)
Prices and policy			
λ	Calvo parameter	0.75	Schorfheide (2008)
φ	Taylor rule coefficient (inflation)	1.25	Kaplan et al. (2018)

5.1 Non-Linearity

I perform two comparative statics exercises to quantify the degree of non-linearity of aggregate durable spending. First, I simulate the effect of transitory tax rebates. Then, I consider a persistent aggregate income shock, and I vary the sign and magnitude of this shock.

Tax rebates. The existing evidence suggests that the average MPC is increasing in the size of the income shock (Section 3.5). My model replicates this empirical finding. Specifically, I compute the response of durable spending to a one-time, unexpected fiscal transfer. The income shock takes place in the first quarter, and is fully transitory. I repeat this exercise for each of the amounts listed in Table 3.1.[51] The response of durable investment is persistent in my model, so that I compute propensities to spend over 4 quarters[52]. Table 5.1 reports the average MPC on durable investment for each of these episodes.

Table 5.1: Marginal Propensities to Spend on Durables (4 quarters)

	$500	$1,000	$2,500
MPC	0.13	0.17	0.26

Two observations are in order. First, the *level* of MPC on durable investment is substantially smaller in my model (Table 5.1), than in the data (Table 3.1). This property can be explained by how I parametrize the model. I choose a relatively low elasticity of substitution between durables and non-durables to dampen the excess interest rate elasticity of durable investment. The households' stock of durables increases on impact and depletes slowly over time. To avoid large departures between durable and non-durable consumption, households save a large share of the transfer to finance non-durable consumption over the medium term.

More importantly, the overall pattern is consistent with the data: the average MPC is increasing with the size of the transfer. The average MPC out of a $2,500 transfer is twice as large as that out of a $500 transfer.

Persistent income shocks. I now consider the effect of an aggregate, persistent income shock. I abstract from redistribution for now. The wage bills satisfy: $\mathcal{Y}_t^h/\mathcal{Y}^h = \hat{\mathcal{Y}}_t$, with $\hat{\mathcal{Y}}_t = \psi_0\rho^t$ for some persistence $\rho \in (0,1)$. I am interested in the non-linear properties of the

[51] A transfer of $500 amounts to roughly 3% of quarterly incomes ($16,500 in Kaplan et al. (2018)).

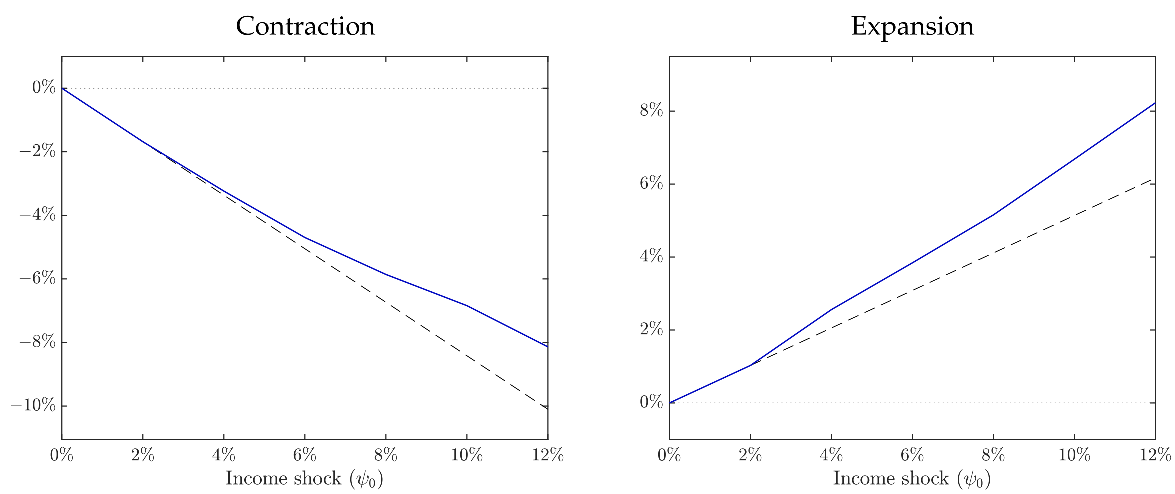
[52] Specifically, I cumulate the individual responses over 4 quarters. See Appendix A.5 for details. The average MPC on impact features the same non-linearity.

aggregate response of durable investment as I vary ψ_0. I set ρ to obtain a half-life of 6 quarters and replicate the behavior of real filtered GDP since 1960.

Figure 5.1 plots the cumulative response of aggregate durable investment over 6 quarters[53] in terms of $|\psi_0|$.[54] The left panel corresponds to contractionary shocks, and the right panel to expansionary shocks. The dashed lines extrapolate the response associated to $\psi_0 = \pm 0.02$.

I find that expansionary shocks are amplified, while contractionary shocks are dampened. This effect is economically significant. For instance, shocks of the magnitude experienced by durable workers during the Great Recession produce a response of durable spending that is roughly 25% lower than would be predicted without non-linearities. Building on the analyis in Section 3.1, these findings suggests that the lumpy investment (extensive margin) dominates precautionary savings (intensive margin) as a source of non-linearity.

Figure 5.1: Impulse Response of Durable Investment (6 quarters)



For reference, I report the same responses for non-durable consumption in Figure A.1 (Appendix A.6). The difference is striking: there is no significant amplification or dampening, and the responses are very symmetric with respect to positive and negative shocks. This is another indication that lumpy adjustment is a key determinant of the non-linearity in durable spending. I confirm this conclusion in the next section by implementing the decomposition from Proposition 1.

[53] The response of durable investment is hump-shaped. Choosing a horizon of 6 quarters ensures that the window includes this hump for each value of ψ_0 that I consider. The cumulative responses over one and two years are very similar. After two years, the responses are negligible.

[54] I annualize these cumulative responses by diving them by the number of quarters over which I cumulate.

5.2 Decomposition

To explain these findings, I examine the two sources of non-linearity identified in Section 3.1: the extensive and intensive margins of durable adjustment.

The extensive margin of adjustment is controlled by two objects: the slope of the durable adjustment target; and the slope of the density of the distribution of liquid assets around the adjustment thresholds. In turn, the intensive margin is shaped by the concavity of the durable investment target. Figure A.2 (Appendix A.6) plots these objects for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). A casual inspection suggests that the extensive and intensive margins should operate as in Examples 1-3. In particular, the durable adjustment target is *increasing* but *concave*. The density of liquid assets is typically *decreasing* around the adjustment threshold.[55] That is, the extensive margin of adjustment predicts an amplification of expansionary shocks, and a dampening of contractionary shocks. The intensive margin predicts the opposite.

I implement the decomposition from Proposition 1 to quantify the contribution of each margin. Specifically, I compute the three terms in (3.4), i.e. the extensive and intensive margins and the residual, for a one-time, transitory shock Δ.[56] Figure 5.2 plots each of these objects. Adjustment at the extensive margin dominates quantitatively and entirely accounts for the non-linear response of durable investment in my model. In other words, lumpy adjustment is responsible for the aggregate non-linearity documented in Section 5.1, while precautionary savings plays a minor role.

5.3 State-Contingency

I conclude this section by drawing a connection between the non-linearity that I document and the state-contingency of impulse responses in presence of lumpy adjustment (Bachmann et al. (2013); Berger and Vavra (2015); Winberry (2019)). I clarify the link between these two properties, and I explore the implications of cyclical income redistribution for the degree of state-contingency of impulse responses.

State-contingency. For expositional purposes, I suppose there is a single source of aggregate disturbance $\{\xi_t\}_t$, which follows a Markov process of order 1. I am interested in the

[55] This pattern is not uniform, however. In particular, the threshold is close tot he mode of the distribution of liquid assets at the bottom of the wealth distribution.

[56] This transitory shock is annualized with the same persistence as in Section 5.1. On the contrary, the shocks associated to Figure 5.1 are persistent. This explains the discrepancies with Figure 5.2 in terms of magnitudes and degree of non-linearity.

impulse response of aggregate durable investment, conditional on the state of the economy. Specifically, this response is parametrized by two aggregate states: the exogeneous disturbance (ξ_t) and the distribution of idiosyncratic states ($\boldsymbol{\Lambda}_t$). The impulse response to an innovation z in ξ_t in period t is

$$\mathcal{R}_t\left(z\right) = \mathcal{I}\left(\xi_t + z, \boldsymbol{\Lambda}_t\right) - \mathcal{I}\left(\xi_t, \boldsymbol{\Lambda}_t\right),$$

where $\mathcal{I}\left(\cdot\right)$ denotes aggregate durable investment. In the context of Section 5.1, $\{\xi_t\}_t$ corresponds to a persistent sequence of transfers. In this case, the impulse response $\mathcal{R}_t\left(\cdot\right)$ reflects the average MPC on durables following a transitory income shock.

Using a version of the income fluctuations problem (2.4)–(2.6) with aggregate risk, Berger and Vavra (2015) find that the response of durable investment to (income) changes is *pro-cylical*, i.e. the effect of shocks is amplified during expansions compared to contractions. There are three possible causes for this state-dependency: the aggregate disturbance ξ_t itself; the (marginal) distribution of liquid assets $\text{marg}_a \boldsymbol{\Lambda}_t$; or the (marginal) distribution of durable holdings $\text{marg}_d \boldsymbol{\Lambda}_t$. The distribution of durable holdings is unlikely to be responsible for the pro-cyclicality of the response of durable investment: durable holdings are already high at the peak of a boom, which should actually mitigate the response of investment.

The effect of the two other states can be understood as a manifestation of the nonlinearity that I focus on. Dependence on the aggregate disturbance ξ_t is a form of nonlinearity, by definition. Similarly, shifts in the (marginal) distribution of liquid assets reflect successive income changes in the preceeding periods. In both cases, the nature of the state-contingency is controlled by the margins of adjustment identified in Section 3.1. A pro-cyclical response suggests that the extensive margin dominates, which is consistent with my findings.

For illustration, I compute the response of aggregate investment to a one-time, transitory income shock following a period of expansion or contraction. The initial boom or bust results from a persistent, unanticipated income change. Specifically, aggregate incomes satisfy $\mathcal{Y}_t/\mathcal{Y} = \hat{\mathcal{Y}}_t$, with $\hat{\mathcal{Y}}_t = \psi_0 \rho^t$ for some persistence $\rho \in (0,1)$. I choose ρ so that the half-life of the shock is 6 quarters, and ψ_0 to obtain an annualized response of aggregate income of 3% to mimick a typical boom or bust. A transitory, unanticipated income shock takes place after 8 quarters. It takes the form of an exogeneous \$500 transfer, as in Kaplan et al. (2018). Figure 5.3 plots the state-contingent response to this transitory income shock (solid curves). The response on impact is *larger* during a boom than a bust.[57]

[57] The difference between the impact responses is relatively small in magnitude, reflecting the size of the income shock (\$500).

Figure 5.2: Decomposition from Proposition 1

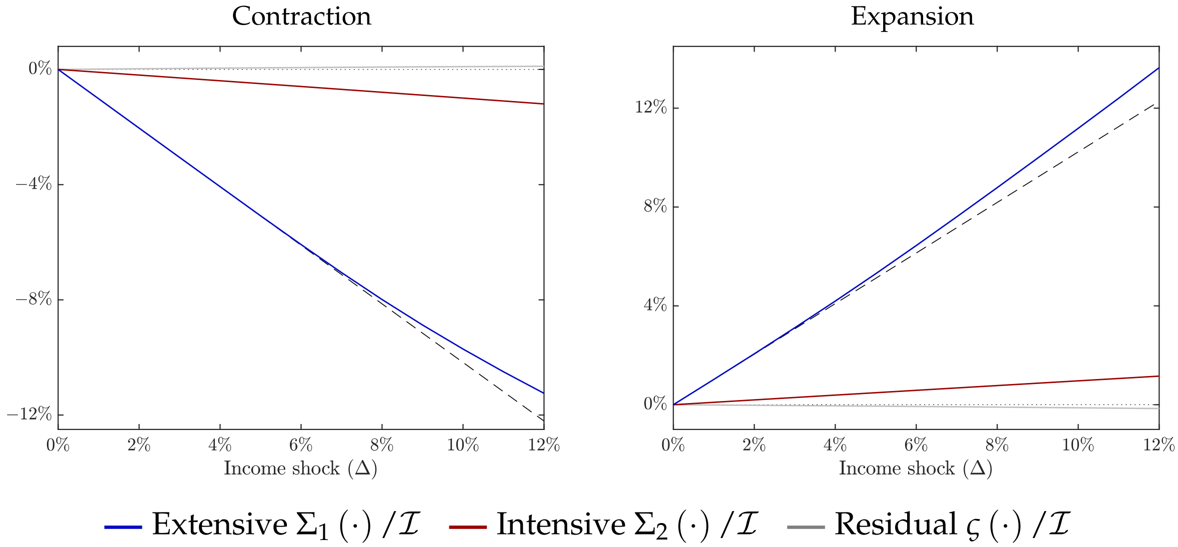
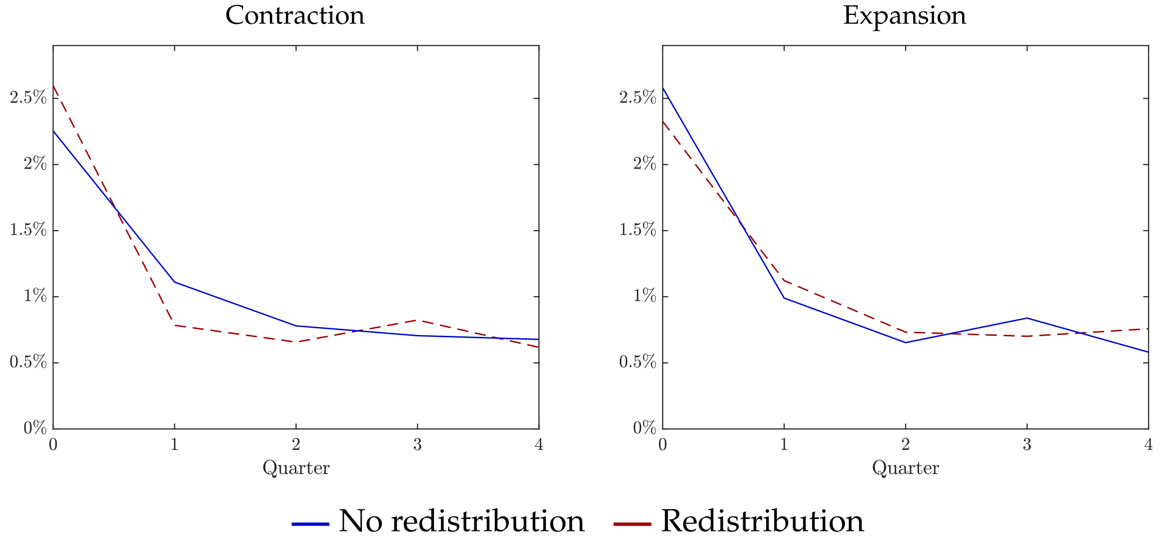


Figure 5.3: State-Contingency



Redistribution. I suppose now that the initial expansion or recession has an asymmetric incidence across sectors. Specifically, sector-specific wage bills are given by:

$$\frac{\mathcal{Y}_t^d}{\mathcal{Y}^d} = \left(\frac{\mathcal{Y}_t}{\mathcal{Y}}\right)^{\theta} \quad \text{and} \quad \sum_h \mu^h \mathcal{Y}_t^h = \mathcal{Y}_t \tag{5.1}$$

for some elasticity $\theta \geq 1$. I calibrate this elasticity using the estimates of Guvenen et al. (2017) based on U.S. administrative data. I set $\theta = 2$, which corresponds to the average income elasticity in construction and durable manufacturing, weighted by relative employment in these industries.

Figure 5.3 plots the corresponding state-dependent responses in red (dashed curves). In this example, I find that the initial redistribution reduces the degree of state-contingency. State-contingency itself reflects the monotonicity of the *slope* of the impulse of durable investment to income shocks (or MPC). In turn, whether redistribution affects the degree of state-contingency should depend on the convexity of this slope. Table 5.1 suggest that the average MPC on durable goods is increasing in the size of the income shock, but concave.[58] This is consistent with a dampening of the degree of state-contingency. In the particular instance of Figure 5.3, the response on impact during an expansion can actually be lower than during a contraction. This reversal does not seem to be a general property, however, and depends on the exact magnitudes of the transitory shock and the initial expansion or contraction.[59]

6 General Equilibrium

In the previous section, I showed that lumpy investment at the microeconomic level produces non-linearities at the aggregate level. I now explore the implications of these non-linearities in presence of cyclical income inequality.

I simulate the response of my general equilibrium model (Section 2) to aggregate disturbances. Following Berger and Vavra (2015), I focus on productivity shocks of the form (3.7). An increase in productivity reduces current inflation. Monetary policy is responsive

[58] Adjustment at the extensive margin reflects two effects (Proposition 1). In Example 1, the first effect contributes to a concave average MPC if the density of the distribution of liquid assets around the adjustment threshold is itself concave. A casual inspection of this density (Figure A.2 in Appendix A.6) suggests that this is the case. In Example 2, the second effect predicts a linear average MPC.

[59] In this particular instance, this reversal can be explained by two features of my model. First, the non-durable sector $h = c$ employs roughly 80% of households. These households have a low income elasticity, from (5.1). Second, impulse responses are mildly asymmetric between expansionary and contractionary shocks for small deviations from the stationary equilibrium. This asymmetry is reflected in the difference in the slopes of the responses in Figure 5.1.

$(\varphi^{\pi} > 1)$, which leads to a decrease in the real interest rate. Durable investment responds more strongly than non-durable consumption. As a result, labor income is redistributed in favor of the households employed in the durable sector.

I find that cyclical income inequality affects both the magnitude, and the timing of the aggregate response of durable investment to productivity shocks. I will add the corresponding plots to my draft soon.

7 Conclusion

In this paper, I study the implications of cyclical income inequality for the dynamics of aggregate durable investment. I explore this question using a multi-sector heterogeneous agent (HANK) model with lumpy durable investment. Durable demand plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between durable and non-durable sectors. Second, lumpy durable adjustment at the micro level produces non-linearities at the macro level. As a result, income redistribution has aggregate effects. I show that cyclical income inequality contributes to an amplification of the response of durable investment during expansions, and a dampening during recessions. In numerical exercises, I confirm that the response of durable investment is non-linear in income changes. I find that the magnitudes are economically significant. Finally, I simulate the response of durable investment to productivity shocks in general equilibrium. I find that cyclical income inequality affects both the magnitude, and the timing of the response of aggregate durable spending.

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A Quantitative Appendix

In this Appendix, I present the full model and describe the approach used to simulate and calibrate the model. Section A.1 describes the full model. Section A.2 defines an equilibrium. Section A.3 provides the algorithm used to solve for the stationary equilibrium and the transition dynamics. I discuss the calibration strategy in Section A.4. Finally, Section A.5 provides details about the numerical implementation.

A.1 Environment

For concision, Section 2 provided a partial description of the environment. For reference, I now present the full model.

Timing. Periods are indexed by $t \in \{0, 1, \ldots\}$. Each period effectively consits of two sub-periods, indexed by $t.0$ $(-)$ and $t.1$ $(+)$. Households make their adjustment decisions at $t.0$, and their consumption, saving and investment decision at $t.1$. Households are indexed by financial asset holdings (a), their holdings of durable goods (d), their idiosyncratic labor supply shock (ζ) and the sector they are employed in (h) in period $t.0$. In addition, they are indexed by their adjustment choice $(\mathcal{A})$ in period $t.1$[60]. The conditional distributions of idiosyncratic states within each sector, at the begining of each subperiod, are denoted by $\mathbf{\Lambda}^{-}_{t-1}$ and $\mathbf{\Lambda}^{+}_{t}$. All agents have perfect foresight, and all (aggregate) information is revealed at the begining of the first sub-period.

Households. The households' value function in period $t.0$ satisfies:

$$\mathcal{V}^h_t\left(a, d, \zeta\right) = \max_{\mathcal{A} \in \{0,1\}} \left\{ V^h_t\left(a, d, \zeta; \mathcal{A}\right) \right\} \tag{A.1}$$

The adjustment choice satisfies:

$$\mathcal{A}^h_t\left(a, d, \zeta\right) \equiv \begin{cases} 1 & \text{if } V^h_t\left(a, d, \zeta; 1\right) > V^h_t\left(a, d, \zeta; 0\right) \\ 0 & \text{otherwise} \end{cases} \tag{A.2}$$

The continuation value functions in period $t.1$ associated to adjustment and no adjustment write:

[60] This additional state variable is for notational convenience. Financial assets are a sufficient statistics for this adjustment.

$$V_t^h(a,d,\zeta;0) = \max_{\{c,a'\}} \frac{u(c,d^\star)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d^\star,\zeta')\middle|\zeta\right] \tag{A.3}$$

$$\text{s.t. } P_t^c c + P_t^d \iota \frac{\delta}{1-(1-\iota)\delta} d^\star + a' \leq (1-\tau_t)\, e_t^h(\zeta) + (1+r_{t-1})\, a$$

$$a' \geq 0$$

$$V_t^h(a,d,\zeta;1) = \max_{\{c,a',d'\}} \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d',\zeta')\middle|\zeta\right] \tag{A.4}$$

$$\text{s.t. } P_t^c c + P_t^d\left(d' - (1-\delta)\, d\right) + \gamma P_t^d (1-\delta)\, d + a'$$

$$\leq (1-\tau_t)\, e_t^h(\zeta) + (1+r_{t-1})\, a$$

$$a' \geq 0$$

with $d^\star \equiv [1-(1-\iota)\,\delta]\, d$, where $e_t^h(\zeta) \equiv \frac{1}{\mu_t^h}\zeta\left(\mathcal{Y}_t^h - T_t^h\right) + \pi_t$ denotes incomes and $\boldsymbol{\mu}$ denotes the mass of households in each sector. The distributions in period $t.0$ and $t.1$ evolve as follows:

$$\Lambda_t^{h,+}(a,d,\zeta,\mathcal{A}) = \Lambda_{t-1}^{h,-}(a,d,\zeta) \times \begin{cases} \mathcal{A}_t^h(a,d,\zeta) & \text{if } \mathcal{A}=1 \\ 1-\mathcal{A}_t^h(a,d,\zeta) & \text{otherwise} \end{cases} \tag{A.5}$$

and

$$\Lambda_t^{h,-}(a',d',\zeta') = \sum_{\mathcal{A}} \sum_{\Omega_t^h(a',d';\mathcal{A})} \Lambda_t^{h,+}(a,d,\zeta,\mathcal{A})\, \Sigma\left(\log(\zeta')\,|\zeta\right) \tag{A.6}$$

with $\Omega_t^h(a^\star,d^\star;\mathcal{A}) \equiv \left\{(a,d,\zeta)\,|a_t^{h,\prime}(\cdot;\mathcal{A}) = a^\star, d_t^{h,\prime}(\cdot;\mathcal{A}) = d^\star\right\}$, where $a_t^{h,\prime}$ and $d_t^{h,\prime}$ denote the solution to (A.1)–(A.4), with $d_t^{h,\prime} \equiv d^\star$ when no adjustment. I define c_t^h similarly.

Firms. Prices are sticky along the transition path:

$$\left(P_t^h\right)^{1-\varepsilon} = \lambda^h\left(P_{t-1}^h\right)^{1-\varepsilon} + \left(1-\lambda^h\right)\left(P_t^{\star,h}\right)^{1-\varepsilon} \tag{A.7}$$

with initial condition $P_{-1}^h \equiv P^h$, i.e. prices at the stationary equilibrium. Reset prices $\mathbf{P}_t^\star$ satisfy:

$$P_t^{\star,h} = \left[\frac{1}{1-\alpha}\frac{\varepsilon}{\varepsilon-1}\frac{G_t^h}{H_t^h}\right]^{\frac{1-\alpha}{(1-\alpha)(1-\varepsilon)+\varepsilon}} \tag{A.8}$$

in each sector h, with

$$G_t^h = \bar{W}^h \left[\left(\frac{1}{P_t^h} \right)^{-\varepsilon} \frac{Y_t^h}{A^h} \right]^{\frac{1}{1-\alpha}} + \lambda^h \frac{1}{1+r_t} G_{t+1}^h \tag{A.9}$$

$$H_t^h = \left(\frac{1}{P_t^h} \right)^{-\varepsilon} Y_t^h + \lambda^h \frac{1}{1+r_t} H_{t+1}^h \tag{A.10}$$

for each $t \in \{0, 1, \ldots, T-1\}$, and

$$G_T^h = \frac{1+r}{1+r-\lambda^h} \bar{W}^h \left[\left(\frac{1}{P^h} \right)^{-\varepsilon} \frac{Y^h}{A^h} \right]^{\frac{1}{1-\alpha}}$$
$$H_T^h = \frac{1+r}{1+r-\lambda^h} \left(\frac{1}{P_h} \right)^{-\varepsilon} Y^h$$

for each $h \in \{c, d\}$, where W_t^h denotes nominal nominal wages, and $(\mathbf{Y}_t, \mathbf{P}, r)$ denotes aggregate demands for each good, prices and the nominal interest rate at the steady state.

Policy. Depending on the regime of interest (insurance, or not), lump sum taxes satisfy:

$$\mathbf{T}_t = 0 \quad \text{or} \quad \frac{\mathcal{Y}_t^d - \mu^d T_t^d}{\mathcal{Y}_t^c - \mu^c T_t^c} = \frac{\mathcal{Y}^d}{\mathcal{Y}^c} \tag{A.11}$$

with $\sum_h \mu^h T_t^h \equiv 0$. The government's flow budget constraint is

$$\tau_t^h \sum_h \mu^h \int e_t^h(\zeta)\, d\Lambda_t^{h,+} = -r_{t-1} B \tag{A.12}$$

Monetary policy implements a Taylor rule:

$$i_t = \max\{r + \varphi^\pi (\Pi_t - 1), 0\} \tag{A.13}$$

Here, $\Pi_t - 1 \equiv \Delta \log(\hat{P}_t)$ denotes the inflation rate, where

$$\hat{P}_t \equiv \left[\vartheta (P_t^c)^{1-\nu} + (1-\vartheta) \left(P_t^d\right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{A.14}$$

denotes the CES ideal price index.

Market clearing. Markets for goods clear:

$$Y_t^c = \sum_h \mu^h \int \left[c_t^h\left(a,d,\zeta\right) + \Gamma_t\left(d_t^{h,\prime}\left(a,d,\zeta\right),d\right)\right] d\Lambda_t^{h,+} \tag{A.15}$$

$$Y_t^d = \sum_h \mu^h \int \left[d_t^{h,\prime}\left(a,d,\zeta\right) - \left(1-\delta\right)d\right] d\Lambda_t^{h,+} \tag{A.16}$$

The TFP component associated to price dispersion satisfies:

$$\left(\Omega_t^h\right)^{\frac{1}{1-\alpha^h}} \equiv \lambda^h\left(\Omega_{t-1}^h\right)^{\frac{1}{1-\alpha^h}} + \left(1-\lambda^h\right)\left(P_t^{h,\star}\right)^{-\frac{\varepsilon}{1-\alpha^h}} \tag{A.17}$$

with $\Omega_{-1}^h \equiv \left(P^h\right)^{-\varepsilon}$. Demands for labor satisfy

$$\hat{\mu}_t^h \equiv \hat{\Omega}_t^h\left[\frac{1}{A^h}\left(\frac{1}{P_t^h}\right)^{-\varepsilon} Y_t^h\right]^{\frac{1}{1-\alpha}} \tag{A.18}$$

with $\hat{\Omega}_t^h \equiv \left(\Omega_t^h\right)^{\frac{1}{1-\alpha^h}}$. Wage bills and aggregate profits write

$$\mathcal{Y}_t^h = \bar{W}^h\hat{\mu}_t^h \tag{A.19}$$

$$\pi_t = \sum_h\left(P_t^hY_t^h - \mathcal{Y}_t^h\right), \tag{A.20}$$

where $\bar{\mathbf{W}}$ denotes rigid wages. In turn,

$$\boldsymbol{\mu} = \mathbf{Z}_t \circ \hat{\boldsymbol{\mu}}_t \tag{A.21}$$

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \leq Z_t^h < +\infty$ along the transition path, for each sector h and period t.

A.2 Definition of Equilibrium

Definition 1. An *equilibrium* in my economy consists of a sequence policy functions for consumption and assets $\left\{c_t\left(\cdot\right), a_t'\left(\cdot\right), d_t'\left(\cdot\right)\right\}_{t\geq 0}$, a sequence of distributions of idiosyncratic states $\left\{\boldsymbol{\Lambda}_{t-1}^-, \boldsymbol{\Lambda}_t^+\right\}_t$, sequences for the nominal interest rates $\left\{r_{t-1}\right\}_t$, sectoral price indices $\left\{\mathbf{P}_{t-1}\right\}_t$, reset prices $\left\{\mathbf{P}_t^\star\right\}_t$, wages, wage bills and profits $\left\{\mathbf{W}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t\right\}_t$, the TFP component associated to price dispersion $\left\{\boldsymbol{\Omega}_{t-1}\right\}_t$ and linear taxes $\left\{\tau_t\right\}_t$ such that: (a) the policy functions solve (A.1)–(A.4), given interest rates, prices and wage bills; (b) reset prices satisfy (A.8)–(A.10), given outputs and wages; (c) the distributions of idiosyncratic states evolve according to (A.5)–(A.6); (d) sectoral prices and the TFP component associated to price dispersion satisfy (A.7) and (A.17); (e) monetary policy implements the Taylor rule (A.13)–(A.14), given prices; (f) fiscal policy sets linear taxes to satisfy its flow budget constraint (A.12); and (g) the market clearing conditions for good (A.15)–(A.16) and labor (A.21) hold.

A.3 Numerical Solution

A.3.1 Stationary Equilibrium

I solve numerically for the stationary equilibrium by iterating on the nominal interest rate $\bar{r}$, the (relative) price of the durable good $\bar{P}^d$, the vector of wage bills, profits[61] and linear taxes $\bar{\mathbf{X}} \equiv \left(\bar{\boldsymbol{\mathcal{Y}}}, \bar{\pi}, \bar{\tau}\right)$[62]. The price of the non-durable good is used as a *numéraire* $\bar{P}^c \equiv 1$ at the stationary equilibrium, and the TFP component associated to price dispersion satisfies $\bar{\Omega}^h \equiv \left(\bar{P}^h\right)^{-\varepsilon}$. I proceed in three steps.

Step 0 (Initial conditions)**.** I choose a set of initial conditions for $\left(\bar{r}, \bar{P}^d\right)$ and $\bar{\mathbf{X}}$. The procedure described in Section A.4 provides a good guess.

Step 1 (Households)**.** Let

$$\bar{\mathcal{V}}^h\left(a, d, \zeta\right) \equiv \mathbb{E}\left[\mathcal{V}^h\left(a, d, \zeta'\right)\middle|\,\zeta\right] \tag{A.22}$$

for each sector h. Starting from a guess $\bar{\mathcal{V}}_{-1}$, I first solve the problems (A.3)–(A.4) to obtain policy functions (c_0, a_0', d_0')[63], and the value functions associated to each adjustment choice. I obtain the adjustment flows $\left(\mathcal{A}_0\right)$ from (A.3), and the expected value functions $\bar{\mathcal{V}}_0^h$ from (A.2) and (A.22). If

$$\left\|\bar{\mathcal{V}}_0 - \bar{\mathcal{V}}_{-1}\right\|_{+\infty} < \epsilon^V$$

for some tolerance $\epsilon^V > 0$, I set $\hat{V}^\star \equiv \hat{V}_0$ and $x^\star \equiv x_0$, for each policy function $x \in \{c, a', d', \mathcal{M}\}$. Otherwise, I repeat this procedure after updating $\bar{\mathcal{V}}_{-1}$ with $\bar{\mathcal{V}}_0$.

Starting with a guess for the initial distribution $\boldsymbol{\Lambda}_{-1}^-$, I iterate on (A.5)–(A.6) using the

[61] Profits are non-zero at the stationary equilibrium, due to decreasing returns.
[62]
[63] See Section A.5 for details.

policy functions obtained previously to obtain distributions Λ_0^- and Λ_0^+. If

$$\left\|\mathbf{\Lambda}_0^- - \mathbf{\Lambda}_{-1}^-\right\|_{+\infty} < \epsilon^\Lambda$$

for some tolerance $\epsilon^\Lambda > 0$, I set $\mathbf{\Lambda}^{-,\star} \equiv \mathbf{\Lambda}_0^-$ and $\mathbf{\Lambda}^{+,\star} \equiv \mathbf{\Lambda}_0^+$. Otherwise, I repeat this procedure after updating $\mathbf{\Lambda}_{-1}^-$ with $\mathbf{\Lambda}_0^-$.

Finally, aggregate demands $\mathbf{Y}^\star$ for each good are obtained from (A.15)–(A.16) using the policy functions $(c^\star, d'^{,\star})$ and the distribution $\hat{\Lambda}^{+,\star}$. Similarly, aggregate savings writes:

$$S^\star \equiv \sum_h \mu^h \int a'^{,h,\star}(a,d,\zeta)\, d\Lambda^{h,+,\star} \tag{A.23}$$

Step 2 (Firms and market clearing)**.** From (A.18) and (A.7)–(A.8), I obtain the wages that insure labor market clearing in each sector:

$$\frac{W^{h,\star}}{\bar{P}^h} = A^h \left(1-\alpha^h\right) \frac{\varepsilon - 1}{\varepsilon} \left(\mu^h\right)^{-\alpha}$$

Similarly, demand for labor in each sector satisfies

$$\hat{Y}^{h,\star} = A^h \left(\hat{\mu}^{h,\star}\right)^{1-\alpha} \tag{A.24}$$

since $\bar{\Omega}^h \equiv \left(\bar{P}^h\right)^{-\varepsilon}$ at the stationary equilibrium.

I then compute the associated wage bills and aggregate profits:

$$\begin{aligned} \mathcal{Y}^{h,\star} &= W^{h,\star}\mu^{h,+,\star} \\ \pi^\star &= \sum_h \left(\bar{P}^h \hat{Y}^{h,\star} - \mathcal{Y}^{h,\star}\right) \end{aligned}$$

Finally, I obtain linear taxes from (A.12).

Step 3 (Convergence)**.** Finally, I check whether the vector of prices $\left(\bar{r}, \bar{P}^d\right)$ insures market clearing. If,

$$|S^\star| < \epsilon^S \quad \text{and} \quad \left|\mu^d - \hat{\mu}^d\right| < \epsilon^d$$

for some tolerances $\epsilon^S, \epsilon^d > 0$, with aggregate savings given by (A.23) and labor demand given by (A.24), then the policy functions $(c^\star, a'^{,\star}, d'^{,\star}, \mathcal{A}^\star)$, the distributions $\left(\mathbf{\Lambda}^{-,\star}, \mathbf{\Lambda}^{+,\star}\right)$, the vector of prices $\left(\bar{r}, \bar{P}^d\right)$, and wage bills, profits and linear taxes $\bar{\mathbf{X}}$ form a (stationary) equilibrium.[64] Let $\boldsymbol{\mathcal{V}}^\star$ denote the value functions at the stationary equilibrium in this case. Otherwise, I update the vector of prices $\left(\bar{r}, \bar{P}^d\right)$ to reduce excess demand and I repeat Step 1 onward. In this case, I also update the vector of wage bills, profits and linear taxes $\bar{\mathbf{X}}$ using a weighted average of the initial guess $\bar{\mathbf{X}}$, and the values computed above $\mathbf{X}^\star$.

A.3.2 Transition Dynamics

The comparative statics of interest is a one-time, unanticipated innovation in aggregate productivity. The shock is symmetric across sectors. Specifically,

$$\log\left(A_t^h\right) = \rho^A \log\left(A_t^h\right) + \psi_t$$

for each sector h, with $\psi_0 \in \mathbb{R}$ and $\psi_t \equiv 0$ for each period $t \geq 1$. I solve numerically for the transition dynamics by iterating on the sequence for $\mathbf{X}_t \equiv (r_t, \mathbf{P}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \boldsymbol{T}_t)$, where $\boldsymbol{T}_t$ corresponds to the lump sum taxes that satisfy (A.11). I proceed in four steps.

Step 0 (Initial and terminal conditions)**.** I set $\mathbf{X}_t = \mathbf{X}^\star$, for each period t, as an initial guess, where $\mathbf{X}^\star$ denotes the vector at the stationary equilibrium. Fix $\boldsymbol{\Lambda}_{-1}^- \equiv \boldsymbol{\Lambda}^{-,\star}$ and $\mathcal{V}_{T+1} \equiv \mathcal{V}^\star$ as initial and terminal conditions for the distribution of idiosyncratic states and the value function.

Step 1 (Households)**.** First, I iterate backward on the functional equation (A.1)–(A.4), for $t \in \{0, \ldots, T\}$, to obtain a sequence of policy functions $\{c_t, a_t', d_t', \mathcal{A}_t\}_t$. Then, I iterate forward on the transition kernel (A.5)–(A.6) using these policy functions, for $t \in \{0, \ldots, T\}$, to obtain a sequence of distributions $\left\{\boldsymbol{\Lambda}^-, \boldsymbol{\Lambda}_t^+\right\}_t$. Finally, I compute the sequence of aggregate demands for each goods $\left\{Y_t^c, Y_t^d\right\}_t$ using (A.15)–(A.16).

Step 2 (Firms and market clearing)**.** I iterate backward on (A.9)–(A.10), for $t \in \{0, \ldots, T\}$, to obtain the sequence of reset prices $\{\mathbf{P}_t^\star\}_t$. Then, I iterate forward on (A.7) and (A.17) to obtain a new sequence of sectoral price indices $\{\mathbf{P}_t^{\text{new}}\}_t$ and a sequence of TFP distorsions $\{\boldsymbol{\Omega}_t\}_t$. Labor demands are computed from (A.18), using the sequence $\left\{Y_t^c, Y_t^d\right\}_t$ from Step 1 and the initial sequence of prices $\{\mathbf{P}_t\}_t$. Finally, I compute new sequences for the wage bills $\{\boldsymbol{\mathcal{Y}}_t^{\text{new}}\}_t$ and aggreagte profits $\{\boldsymbol{\pi}_t^{\text{new}}\}_t$ from (A.19)–(A.20).

Step 3 (Policy)**.** I obtain new sequences for lump sum taxes $\{\boldsymbol{T}_t^{\text{new}}\}_t$ and linear taxes

[64] Note that the market clearing conditions for labor in the non-durable sector is implied by (A.21) and (A.23) when $S^\star = 0$.

$\{\tau_t^{\text{new}}\}_t$ from (A.11)–(A.12). Similarly, I compute the new sequence of nominal interest rate $\{i_t^{\text{new}}\}_t$ set by the monetary policy rule (A.13)–(A.14).

Step 4 (Convergence)**.** Finally, I check whether the sequence of endogeneous variables $\mathbf{X}_t \equiv (r_t, \mathbf{P}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \boldsymbol{T}_t)$ insures market clearing. If,

$$\|x^{\text{new}} - x\|_\infty < \epsilon^x$$

for some tolerance $\epsilon^x > 0$, for each $x \in \{r, \mathbf{P}, \boldsymbol{\mathcal{Y}}, \pi, \tau, \mathbf{T}\}$, then the policy functions $\{c_t, a_t', d_t', \mathcal{A}_t\}_t$, the distributions $\{\boldsymbol{\Lambda}^-, \boldsymbol{\Lambda}_t^+\}_t$, and the prices and incomes $\{\mathbf{X}_t\}_t$ form an equilibrium. Otherwise, I update the sequence $\{\mathbf{X}_t\}_t$ using a weighted average of $\{\mathbf{X}_t\}_t$ and $\{\mathbf{X}_t^{\text{new}}\}_t$[65] and I repeat Step 1 onward.

A.4 Calibration Strategy

The internal calibration consists of solving for the vectors of parameters $\left(\beta, \vartheta, \gamma, B, A^d\right)$ — i.e. the discount factor, the preference parameter on non-durables, the (non-convex) durable adjustment cost, the supply of bonds, and the relative productivity in the durable sector — and prices $\left(\bar{r}, \bar{P}^d\right)$ — i.e. the real interest rate and the relative price of the durable good at the stationary equilibrium — that solve the following restrictions: the two market clearing conditions for labor (A.21); and the five empirical moments listed in Table 4.1. I proceed in two steps. First, I use the empirical targets and the restrictions in the model to pin down the parameters $\left(A^d, B\right)$ and the prices$\left(\bar{r}, \bar{P}^d\right)$. Then, I iterate over the remaining parameters to insure market clearing, and match the rest of the targets.

Step 1. The first empirical moment in Table 4.1 directly pins down $\bar{r}$. From (A.18) and (A.21),

$$\mu_t^h \equiv \left(\frac{1}{A^h} Y_t^h\right)^{\frac{1}{1-\alpha}} \tag{A.25}$$

for each sector h, at the stationary equilibrium. The second moment in Table 4.1 and (A.25) thus pin down A^d. Similarly, from (A.8),

$$\frac{\bar{W}^h}{\bar{P}^h} = A^h \left(1 - \alpha^h\right) \frac{\varepsilon - 1}{\varepsilon} \left(\mu^h\right)^{-\alpha} \tag{A.26}$$

for each sector h. The last moment in Table 4.1 and (A.26) pin down $\bar{P}^d$, given the *numéraire*

[65] To insure convergence of the algorithm, the corresponding weights a decreasing exponentially in the period t.

$\bar{P}^c \equiv 1$ and the vector of productivities $\mathbf{A}$ previously solved for. Aggregate output writes:

$$\bar{Y} \equiv \sum_h \bar{P}^h \bar{Y}^h \quad \text{with} \quad \bar{Y}^h = A^h \left(\mu^h\right)^{1-\alpha} \tag{A.27}$$

for each sector h, by definition of the production technologies. The fourth moment in Table 4.1 and (A.27) pin down B, given the vectors of prices $\bar{\mathbf{P}}$ and productivities $\mathbf{A}$ obtained above.

Step 2. Finally, I iterate on the remaining parameters $(\beta, \vartheta, \gamma)$ to satisfy the rest of the restrictions. Specifically, I adjust β until aggregate savings (A.23) are zero. Similarly, I use ϑ to clear the market for durable goods (A.21), with labor demand given by (A.16) and (A.18). Finally, I vary γ until the aggregate frequency of adjustment

$$\mathcal{A}^T \equiv \sum_h \mu^h \int \mathcal{A}^h (a, d, \zeta) \, d\bar{\Lambda}^h$$

matches the third moment in Table 4.1.

A.5 Numerical Implementation

I describe below the computation of the impulse responses in Section 5, and the numerical implementation of the algorithms described in Appendices A.3.1 and A.3.2.

Grids and transition. The value functions are approximated on discrete grids for financial assets and durable goods, consisting of 100 points each. The distribution of financial assets and durable goods is discretized on grids consisting of 300 points. I interpolate policy functions linearly between grid points, and use a generalization of Young (2010)'s non-stochastic stimuation method with multiple assets when using the policy functions to iterate on the distribution of assets. For accuracy, the grid for financial assets is more dense in the neighborhood of the borrowing constraint. Finally, I discretize the income process on a 7-point grid using the method of Rouwenhorst (1995). I set $T = 40$ (quarters) when solving for the transition dynamics in partial equilibrium (Section 5), and $T = 225$ in general equilibrium (Section 6).

Numerical solution. The households' problems (A.3)–(A.4) are non-convex due to fixed adjustment costs. To solve for the optimal policy, I first perform a grid search on a fine grid to locate a candidate for the global maximum. I then use the gradient-free, simplex algorithm (Nelder-Meade) implemented in Matlab's fminsearch to solve for a local optimum, using this candidate as an initial condition. I use a tolerance of 10^{-5} to solve for the value function at the stationary equilibrium, and one of 10^{-17} for the stationary distribution (Step 1 in Appendix A.3.1).

Impulse responses. The computation of the responses to income shocks in Sections 5.1 and 5.3 follows Step 1 in Appendix A.3.2. In particular, let $\mathcal{I}_t\left(\{\mathcal{Y}_t\}_t\right)$ denote aggregate investment (A.16) in terms of the sequence of wage bills. The other parameters entering the households' income fluctuations problem and the distribution of idiosyncratic states are implicitly fixed at their stationary equilibrium level. The impulse responses associated to two sequences $\{\mathcal{Y}_t, \mathcal{Y}'_t\}_t$ defined as follows:

$$\mathcal{R}_t\left(\cdot\right) \equiv \mathcal{I}_t\left(\{\mathcal{Y}'_s\}_s\right) - \mathcal{I}_t\left(\{\mathcal{Y}_s\}_s\right)$$

In Section 5.1, I am interested in on the non-linear response to income shocks. This corresponds to $\mathcal{Y}_s = \bar{\mathcal{Y}}$ for each period s, and some mean-reverting sequence $\{\mathcal{Y}'_s\}_s$. In Section 5.3, I focus instead on the state-contingency. In this case, I fix some sequence $\{\mathcal{Y}_s\}_s$ and consider a perturbation of the form $\mathcal{Y}'_s \equiv \mathcal{Y}_s + \boldsymbol{\eta}_s$, for some mean-reverting sequence $\{\boldsymbol{\eta}_s\}_s$.

Marginal propensities to spend. My model is calibrated at the quarterly frequency. In Section 5.1, I compute annual marginal propensities to spend. To do so, I randomly sample 500,000 households from the stationary distribution. I then perform Monte Carlo simulations over the corresponding periods, with and without transitory income shocks. The individual propensities to spend are defined as the difference between the cumulative spending in the two scenarios, normalized by the size of the income shock. I then average these spending multipliers over households.

Distance to threshold. In Section 5.2, I report the distribution of distances to the adjustment threshold $a - \bar{a}\left(\cdot\right)$ at the stationary equilibrium. As mentioned above, the grids I use for the financial assets is more dense in the neighborhood of the borrowing constraint. Using this grid to compute the distribution of distances to threshold would mechanically over-weight households at the bottom of the wealth distribution. To address this issue, I re-interpolate the policy functions and the distribution of idiosyncratic states on a linearly-spaced grids consisting of 500 points.

Decomposition. I implement the decomposition of Proposition 1 in Section 5.2. I compute the extensive margin $\Sigma_1(\Delta)$ in three steps. First, I approximate:

$$\hat{d}(a,d,\zeta) \equiv \bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d + \kappa(d,\zeta)(a - \bar{a}(\cdot))$$

Specifically, I filter the adjustment target with using a rolling average, interpolate the resulting vector on a fine grid in the neighborhood of the adjustment threshold, estimate the slope by ordinary least squares and extrapolate on the grid for financial assets. Second, I recover the density $d\Lambda(a,d,\zeta)$ using the (discretized) distribution of idiosyncratic states and the non-regular grid for financial assets. Finally, I integrate numerically over the range $[\bar{a}(\cdot) - \zeta\Delta\mathcal{Y}, \bar{a}(\cdot)]$, using the density $d\Lambda(a,d,\zeta)$, and sum up across durable holdings and idiosyncratic labor supply (d,ζ). I repeat this computation using actual investment as $\hat{d}(a,d,\zeta)$, i.e. without linearizing around the adjustment threshold, and define the residual $\varsigma(\Delta)$ as the difference between the resulting response and $\Sigma_1(\Delta)$. The computation of the intensive margin $\Sigma_2(\Delta)$ is straighforward.

A.6 Complementary Numerical Results

Non-durable consumption. Figure A.1 plots the response of non-durable consumption to aggregate income shocks. This response is almost perfectly linear, which suggests that precautionary savings plays a minor role in shaping the non-linear response of durable investment.

Sources of non-linearity. Figure A.2 plots the density of the distribution of liquid assets (normalized by quarterly GDP) and the adjustment threshold, and the durable investment target for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). The dashed lines denote the durable adjustment thresholds. The density of liquid assets is typically decreasing around the adjustment threshold. As expected, the durable adjustment target is increasing, and concave due to precautionary savings. For reference, Figure A.3 plots the distribution of distances to the adjustment threshold $a - \bar{a}(\cdot)$ (again, normalized by quarterly GDP) at the stationary equilibrium. A large fraction of households lies relatively closely to their adjustment threshold.

Figure A.1: Impulse Response of Non-Durable Consumption (6 quarters)

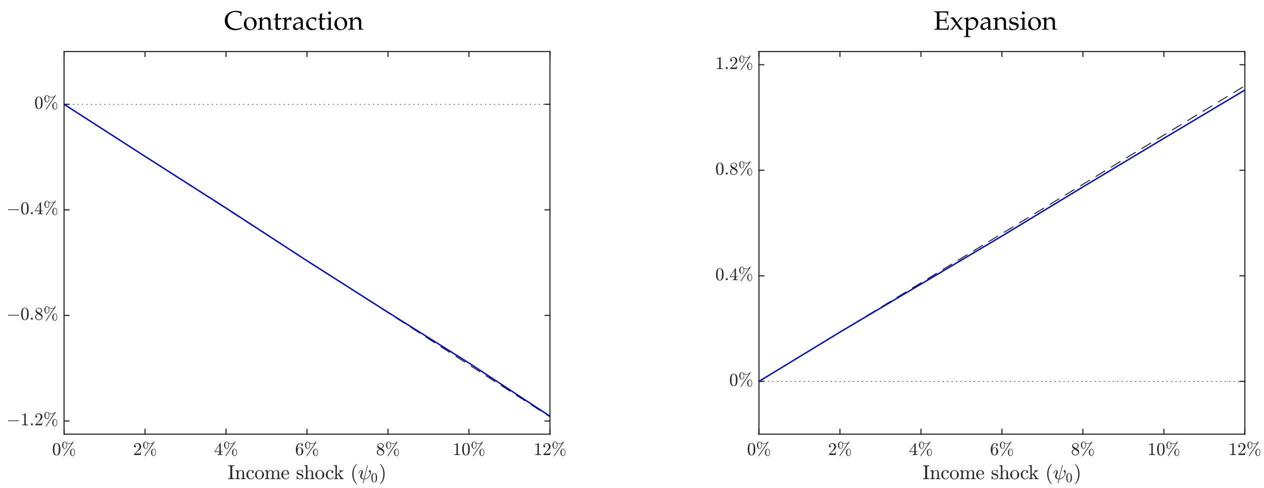


Figure A.2: Sources of Non-Linearity (median labor supply)

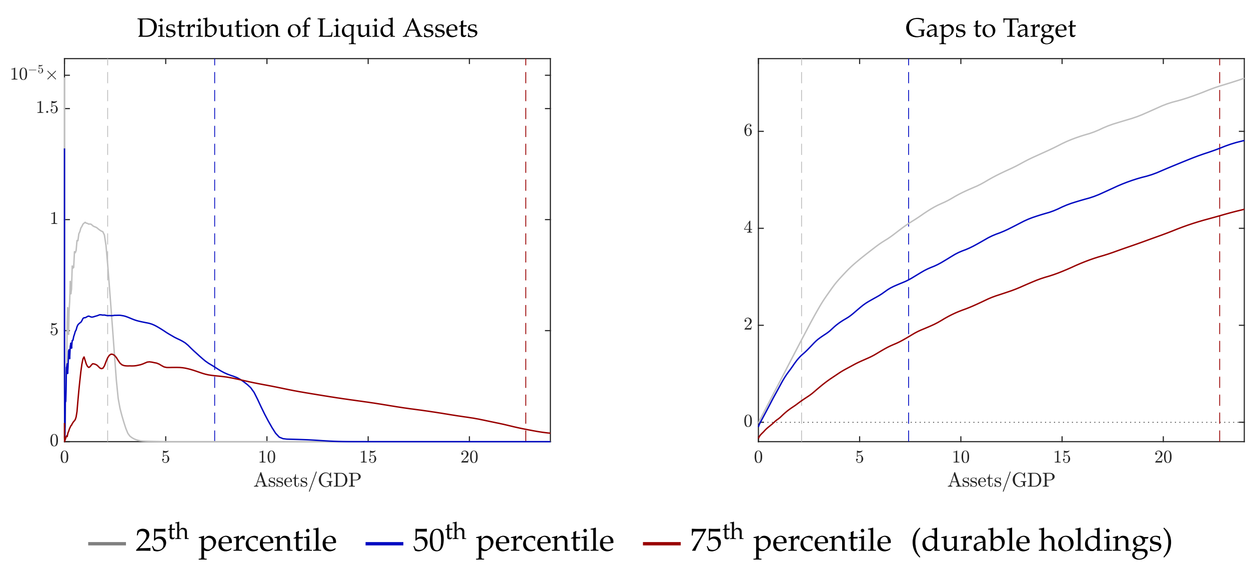
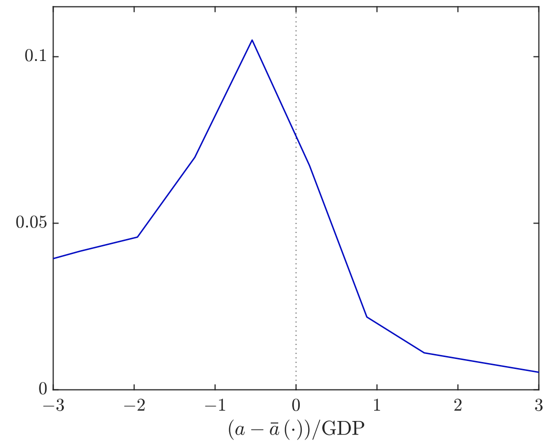


Figure A.3: Distribution of Distance to Adjustment threshold



B Omitted Proofs, Results and Derivations

B.1 Decomposition

I provide a decomposition of the response of durable investment in income shocks in Section 3.1. Expression (3.4) in Proposition 1 applies to positive income shocks. I first provide the analogous decomposition for negative income shocks, before proving these results.

Proposition 1 (cont'd). The response of durable investment to a negative income shock $\Delta < 0$ can be decomposed as follows:

$$\hat{I}(\Delta) \equiv \underbrace{\Sigma_1'(\Delta)}_{\text{Extensive margin}} + \underbrace{\Sigma_2'(\Delta)}_{\text{Intensive margin}} + \underbrace{\varsigma'(\Delta)}_{\text{Residual}} \tag{B.1}$$

with

$$\begin{aligned}\Sigma_1'(\Delta) \equiv & \frac{1}{\mathcal{I}} \int \Bigg\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) \\ & + \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} (a - \bar{a}(d,\zeta))\, d\Lambda(a|d,\zeta) \Bigg\}\, d\Lambda^{\star} \\ \Sigma_2'(\Delta) \equiv & \frac{1}{\mathcal{I}} \int \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^{\star}(a+\zeta\Delta\mathcal{Y}, d, \zeta) - d^{\star}(a,d,\zeta)\right] d\Lambda(a,d,\zeta)\, d\Lambda^{\star}\end{aligned}$$

for some $\kappa(d,\zeta) > 0$ and some residual $\varsigma'(\Delta)$ that satisfies $\lim_{\Delta\to 0} \frac{\varsigma'(\Delta)}{\Delta} = 0$.

Proof of Proposition 1. From (3.1)–(3.3),

$$\hat{I}(\Delta) \equiv \frac{1}{\mathcal{I}}\left[\int\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\Lambda\left(a-\zeta\Delta\mathcal{Y}\middle|\,d,\zeta\right)d\Lambda^{\star}\right.$$
$$\left.+\int\int_{0}^{\bar{a}(d,\zeta)}\delta d\imath d\Lambda\left(a-\zeta\Delta\mathcal{Y},d,\zeta\right)d\Lambda^{\star}-\mathcal{I}\right]$$

where $\Lambda^{\star}\equiv \mathrm{marg}_{d,\zeta}\Lambda$ denotes the marginal distribution of durable holdings and producitivty. By definition, aggregate durable investment at the stationary equilibrium satisfies: $\mathcal{I}\equiv I(0)$. Thus,

$$\hat{I}(\Delta) \equiv \frac{1}{\mathcal{I}}\left[\int\underbrace{\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\hat{\Lambda}\left(a-\zeta\Delta\mathcal{Y}\middle|\,d,\zeta\right)}_{\equiv\Omega_1}d\Lambda^{\star}\right.$$
$$\left.+\int\underbrace{\int_{0}^{\bar{a}(d,\zeta)}\delta d\imath d\hat{\Lambda}\left(a-\zeta\Delta\mathcal{Y}\middle|\,d,\zeta\right)}_{\equiv\Omega_2}d\Lambda^{\star}\right] \tag{B.2}$$

where $\hat{\Lambda}\left(a-\zeta\Delta\mathcal{Y}\middle|\,d,\zeta\right)\equiv\Lambda\left(a-\zeta\Delta\mathcal{Y}\middle|\,d,\zeta\right)-\Lambda\left(a\middle|\,d,\zeta\right)$, with an abuse of notation.

Using the change of variable $a'\equiv a-\zeta\Delta\mathcal{Y}$, the first term in (B.2) writes:

$$\Omega_1=\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{+\infty}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-(1-\delta)\,d\right)d\Lambda\left(a\middle|\,d,\zeta\right)$$
$$-\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\Lambda\left(a\middle|\,d,\zeta\right)$$

Then,

$$\Omega_1=\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-d^{\star}(a,d,\zeta)\right)d\Lambda\left(a\middle|\,d,\zeta\right)d\Lambda^{\star}$$
$$+\underbrace{\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-(1-\delta)\,d\right)d\Lambda\left(a\middle|\,d,\zeta\right)}_{\equiv\hat{\Omega}_1} \tag{B.3}$$

The first integral in (B.3) corresponds to the *intensive* margin of adjustment in Proposition 1. The second integral contributes to the *extensive* margin, together with the term Ω_2 in (B.2).

This second integral can be decomposed as follows:

$$\begin{aligned}\hat{\Omega}_1 = & \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-\bar{d}\left(d,\zeta\right)\right) d\Lambda\left(a|\, d,\zeta\right) \\ & + \left(\bar{d}\left(d,\zeta\right)-\left(1-\delta\right)d\right) \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\, d,\zeta\right)\end{aligned}$$

where $\bar{d}\left(d,\zeta\right) \equiv d^\star\left(\bar{a}\left(d,\zeta\right),d,\zeta\right)$ denotes investment at the threshold. By assumption, $d^\star\left(\cdot\right)$ is smooth. Define $\kappa\left(d,\zeta\right) \equiv \frac{\partial}{\partial a} d^\star\left(a,d,\zeta\right)\Big|_{a=\bar{a}(d,\zeta)} > 0$. Then,

$$\begin{aligned}\hat{\Omega}_1 \equiv & \kappa\left(d,\zeta\right) \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(a+\zeta\Delta\mathcal{Y}-\bar{a}\left(d,\zeta\right)\right) d\Lambda\left(a|\, d,\zeta\right)+\varsigma\left(\Delta,d,\zeta\right) \\ & + \left(\bar{d}\left(d,\zeta\right)-\left(1-\delta\right)d\right) \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\, d,\zeta\right)\end{aligned} \tag{B.4}$$

where $\varsigma\left(\Delta,d,\zeta\right)$ is defined residually.

Again, using a change of variable, the second term in (B.2) writes:

$$\begin{aligned}\Omega_2 &= \delta d\iota \left[\int_0^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|\, d,\zeta\right) - \int_0^{\bar{a}(d,\zeta)} d\Lambda\left(a|\, d,\zeta\right)\right] \\ &= -\delta d\iota \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\, d,\zeta\right)\end{aligned} \tag{B.5}$$

since $\Lambda\left(\cdot|\, d,\zeta\right)$ has positive support, from the household's income fluctuations problem (2.5)–(2.6).

Note that the previous expressions apply to both positive income shock $\Delta > 0$ and negative income shocks $\Delta < 0$. However, the attribution of each term to the extensive and intensive margins differs between these two cases. I thus treat them separately.

Case 1: $\Delta < 0$. The expression (3.4) in the text follows by collecting the terms from (B.2)–(B.4) and (B.5), and by definition of the hazard (3.2) and the marginal distribution $\Lambda^\star$. Finally, $\varsigma\left(\Delta\right)$ is a second-order term, by Taylor's Theorem.

Case 2: $\Delta < 0$. Note that

$$\begin{aligned}\Sigma_2\left(\Delta\right) = & \frac{1}{\mathcal{I}} \int \Bigg\{ \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} \left[d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-d^\star\left(a,d,\zeta\right)\right] d\Lambda\left(a|\, d,\zeta\right) \\ & + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-d^\star\left(a,d,\zeta\right)\right] d\Lambda\left(a|\, d,\zeta\right) \Bigg\} d\Lambda^\star\end{aligned}$$

Equivalently,

$$\Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} \left[d^\star(a+\zeta\Delta\mathcal{Y}, d, \zeta) - \bar{d}(d,\zeta) \right] d\Lambda(a|\, d, \zeta) + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[\bar{d}(d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y}, d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star \tag{B.6}$$

Then,

$$\Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \kappa(d,\zeta) \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} \zeta\Delta\mathcal{Y} d\Lambda(a|\, d,\zeta) + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y}, d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star + \hat{\varsigma}(\Delta) \tag{B.7}$$

for some second-order term $\hat{\varsigma}(\Delta)$, by definition of $\kappa(d,\zeta)$ and $\bar{d}(d,\zeta)$. Therefore, summing up $\Sigma_1(\cdot)$ and $\Sigma_2(\cdot)$ in the expression (3.4) in the text:

$$\Sigma_1(\Delta) + \Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d \right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) + \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} (a - \bar{a}(d,\zeta))\, d\Lambda(a|d,\zeta) + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y}, d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star + \hat{\varsigma}(\Delta) \tag{B.8}$$

Finally, the expression (B.1) is obtained from (B.8), collecting the terms appropriately. □

B.2 Benchmark Case

In Section 3.2, I discuss a benchmark where the two goods are non-durable: $\delta = 1$, i.e. full depreciation. In this case, no redistribution of labor income takes place across sectors and there is no role for insurance from fiscal policy. For concision, no formal statement is provided in the text. I do so in this section instead.

Proposition 2. *Let Assumptions 1-2 hold. Consider a persistent, unanticipated productivity shock of the form (3.7), for any $\psi_0 \in \mathbb{R}$. Then, taxes under the insurance regime (2.11) satisfy:*

$$\mathbf{T}^\star = 0$$

Consequently, the response of durable investment satisfies:

$$\frac{Y_0^d}{Y^d} = \frac{Y_0^{d,\star}}{Y^{d,\star}}$$

for each period $t \geq 0$, where $Y_0^{d,(\star)}$ denotes aggregate durable investment (2.13) under the regimes (2.10) and (2.11).

Proof of Proposition 2. First, I guess and verify that outputs in the economy without insurance ($\mathbf{T} \equiv 0$) satisfy:

$$\frac{Y_t^c}{Y^c} = \frac{Y_t^d}{Y^d} \tag{B.9}$$

By homotheticity,

$$c_t^h(a, d, \zeta) = c_t^\star e_t^h(a, d, \zeta) \quad \text{and} \quad d_t^{h,\prime}(a, d, \zeta) = d_t^\star e_t^h(a, d, \zeta) \tag{B.10}$$

with full depreciation. Here, $(c_t^\star, d_t^\star)$ denotes the (dual) cost-minimizing bundle that achieves $u(c, d') \geq 1$, and $e_t^h(a, d, \zeta)$ solves the following income fluctuations problem[66]:

$$\begin{aligned} V_t^h(a, \zeta) = \max_{\{e, a'\}} & \frac{e^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^h(a', \zeta') \middle| \zeta \right] \\ \text{s.t. } & \hat{P}_t e + a' \leq (1 - \tau_t) e_t^h(\zeta) + (1 + r_{t-1}) a \\ & a' \geq 0 \end{aligned}$$

where $\hat{P}_t$ denotes the CES ideal price index (A.14). In particular, $(c_t^\star, d_t^\star)$ is homogeneous of degree 0 in prices $\mathbf{P}_t$.

From the firm's price setting (A.8)–(A.10), the law of motion of price indices (A.7), the guess (B.9), and by Assumptions 1-2,

$$\frac{P_t^c}{P^c} = \frac{P_t^d}{P^d} \tag{B.11}$$

so that $(c_t^\star, d_t^\star) = (c^\star, d^\star)$, i.e. the relative demand is unchanged compared to the stationary equilibrium. Thus, the guess (B.9) is verified, from the market clearing conditions (A.15)–(A.16). The remaining equilibrium restrictions are satisfied.

Furthermore, note that:

$$\frac{\mathcal{Y}_t^c}{\bar{Y}^c} = \frac{\mathcal{Y}_t^d}{\bar{Y}^d} \tag{B.12}$$

[66] Note that the state variable d is redundant when $\delta = 1$, and households do not incur a fixed costs when adjusting their consumption of the good $h = d$.

since wages are rigid, using the definitions of labor demands (A.17)–(A.18), and the wage bills (A.19). Then,

$$T_t^{\star} = 0 \tag{B.13}$$

under the regime with insurance, for each period $t \geq 0$, by definition of taxes (A.11) and given the wage bills (B.12). □

B.3 Omitted Derivations

Derivations for Example 1. The impulse response satisfies:

$$\hat{I}\left(\Delta\right) = \frac{1}{\mathcal{I}} \int \left[\bar{d}\left(d,\zeta\right) - \left(1 - \delta + \iota\delta\right) d\right] \Psi\left(\Delta\right) d\Lambda^{\star} \tag{B.14}$$

with

$$\Psi\left(\Delta\right) \equiv \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda\left(a|d,\zeta\right) \tag{B.15}$$

Let

$$\eta\left(\Delta,\Delta'\right) \equiv \Psi\left(\Delta'\right) + \Psi\left(\Delta\right) - 2\Psi\left(\frac{\Delta+\Delta'}{2}\right) \tag{B.16}$$

with $\Delta' > \Delta$ without loss of generality.

From (B.15),

$$\begin{aligned}
\eta\left(\Delta,\Delta'\right) &= 2\int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda\left(a|d,\zeta\right) + \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - 2\int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|d,\zeta\right) \\
&= \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - 2\int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) \\
&= \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - \int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right)
\end{aligned}$$

Using the change of variable $a' \equiv a + \zeta\frac{\Delta'-\Delta}{2}\mathcal{Y}$ for the first integral,

$$\eta\left(\Delta,\Delta'\right) = \int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\hat{\Lambda}\left(a|\,d,\zeta\right) \tag{B.17}$$

where $\hat{\Lambda}\left(a|\,d,\zeta\right) \equiv \Lambda\left(a - \zeta\frac{\Delta'-\Delta}{2}\mathcal{Y}\middle|\,d,\zeta\right) - \Lambda\left(a|\,d,\zeta\right)$, with an abuse of notation. Then, $\eta\left(\Delta,\Delta'\right) > 0$ since the density $d\Lambda\left(a|d,\zeta\right)$ is decreasing at the adjustment threshold $\bar{a}\left(\cdot\right)$, by assumption. The convexity of the impulse response follows from (B.14)–(B.17).

Derivations for Example 2. From (3.4) and by definition of the adjustment threshold (3.2), the extensive margin satisfies:

$$\begin{aligned}\Sigma_1(\Delta) =& \frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \Big\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right]\zeta\Delta\mathcal{Y} \\ & + \kappa(d,\zeta)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(a+\zeta\Delta\mathcal{Y}-\bar{a}(d,\zeta)\right) da \\ & + \int_{\bar{a}(d,\zeta)}^{a^\star} \left(d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - d^\star(a,d,\zeta)\right) da \Big\}\, d\Lambda^\star \end{aligned} \tag{B.18}$$

since the (conditional) distribution of liquid assets is uniform on $[0, a^\star]$ and shocks are sufficiently small, by assumption. By linearity of the adjustment target,

$$\Sigma_1(\Delta) = \theta\Delta + \frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \kappa(d,\zeta)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(a+\zeta\Delta\mathcal{Y}-\bar{a}(d,\zeta)\right) da d\Lambda^\star \tag{B.19}$$

where $\kappa(d,\zeta)$ denotes the slope of the adjustment target $d^\star(\cdot)$, with

$$\theta \equiv \frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d + \kappa(d,\zeta)\left(a^\star - \bar{a}(d,\zeta)\right)\right]\zeta\mathcal{Y} d\Lambda^\star$$

Integrating the second term in (B.19),

$$\Sigma_1(\Delta) = \theta\Delta + \left[\frac{1}{2}\frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \kappa(d,\zeta)\left(\zeta\mathcal{Y}\right)^2 d\Lambda^\star\right]\Delta^2 \tag{B.20}$$

The convexity of the impulse response immediately follows from (B.20), since $\kappa(d,\zeta) > 0$ by assumption.

Derivations for Example 4. Consider an expansionary productivity shock $\psi_0 > 0$. From (3.7), and the firms' price setting problem (A.8)–(A.10),

$$\frac{P_0^h}{P^h} = \frac{1}{\psi_0} \tag{B.21}$$

since technologies are linear ($\alpha \equiv 1$) by assumption, wages are rigid along the transition path, and the sequence $\{X_t\}_{t>1}$ is held constant. From the definition of price indices (A.7) and (A.14), inflation in period $t = 0$ is negative:

$$\Pi_0' < \Pi_0$$

From the monetary policy rule (A.13),

$$r_0 < r \tag{B.22}$$

since monetary policy is responsive, by assumption.

Note that the increase in productivity lowers the demand for labor in partial equilibrium, from (A.18). However, profits adjust so that labor incomes are unchanged on impact, from (A.19)–(A.20). By Assumption 3, the decrease in prices (B.21) and in the nominal interest rate (B.22) raises revenues (and thus income) in each sector, but proportionately more in the durable sector. From (A.12), distortionary taxes decrease $\tau_0 < \tau$ which further contributes to an increase in liquid assets for households. Iterating on the market clearing conditions (2.12)–(2.13) and using Assumption 3,

$$\mathcal{Y}_0^d / \mathcal{Y}^d > \mathcal{Y}_0^c / \mathcal{Y}^c > 1 \tag{B.23}$$

in general equilibrium, since prices are inelastic to output ($\alpha \equiv 0$).

Starting from this equilibrium, suppose now that fiscal policy provides full aggregate insurance. From (A.11) and (B.23), taxes satisfy:

$$T_0^d = -\frac{\mu^c}{\mu^d} T_0^c > 0 \tag{B.24}$$

Under the assumptions of Example 2, aggregate (sectoral) durable investment

$$Y_0^{d,h} \equiv \int \left[d_0^{h,\prime}\left(a, d, \zeta\right) - \left(1 - \delta\right) d \right] d\Lambda_0^h$$

is *convex* in (sectoral) incomes[67], for each sector h. Then, the redistribution from fiscal policy (B.24) reduces aggregate durable investment in partial equilibrium, using (B.23). Aggregate non-durable consumption is unchanged, since the role of precautionary savings is negligible, by assumption. Therefore, incomes in durable sector sector while those in the non-durable sector are unchanged in partial equilibrium, from the market clearing conditions (2.12)–(2.13) and the definition of incomes (A.18)–(A.20). Again, iterating on the market clearing conditions (2.12)–(2.13) and using Assumption 3, aggregate durable investment is lower than in the economy without insurance ($\mathbf{T} \equiv 0$).

[67] Incomes are implicit to the time index.

C Empirical Appendix

C.1 Aggregate Series

The series for the nominal interest rate, household consumption and investment expenditures, and employment are listed in Table C.1. I define durable spending as the sum of household's expenditures on durable goods and residential investment. Non-durable spending consists of the sum of households' expenditures on non-durable goods and services, minus housing and financial / insurance services. I define durable-employment as the sum of employment in: construction, durable manufacturing, wholesale trade and durable retail for durables, and repair and maintenance. Non-durable employment consits of the sum of employment in: non-durable manufacturing, wholesale trade and durable retail for durables, information, professional and business services, leisure and hospitality, and other services (except repair and maintenance). I exclude wholesale trade, retail trade (for data availability reasons) from either employment category. I exclude financial services and public administration.

Table C.1: Data Sources

Series	Seasonal Adjst	Source
Nominal Interest Rate		Fed Board H.15
Effective Fed funds rate	No	(line 1)
Deflator		NIPA 1.1.4.
PCE	Yes	(line 2)
Personal Consumption Expenditures		NIPA 1.1.3
Real	Yes	(lines 4-6)
Residential Investment		NIPA 1.1.3
Real	Yes	(lines 14)
Employment		CES (National)
All employees	Yes	(see text)
Population		NIPA 2.1
Total	No	(line 40)

C.2 PSID Data

In Section 3.5, I provide supporting evidence on the interaction between cyclical durable investment and redistribution. In particular, I confirm that labor income decreases proportionally more in durable sectors, compared to non-durable sectors, following a contractionary tax shock. To account for cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).

Industry classification. Following Berger and Vavra (2015), I adopt a broad definition of durable goods when calibrating the quantitative model in Section 4. This definition includes both consumer durables, and residential investment. Consistently, I classify industries as either durable, or non-durable when using the PSID. Durable industries consists of construction, and durable manufacturing. Non-durable industries include non-durable manufacturing, and all services except public administration and the military, and finance[68].

Income data. My preferred measure of incomes corresponds to (pre-tax) labor income, deflated using a price index for total consumption expenditure. I also use family money income to account for unemployment insurance and intra-household risk sharing.

Sample selection. I use the bi-annual PSID waves from 1968 to 2015[69]. The sample consists of male household's heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use PSID longitudinal weights.

C.3 Response to Fiscal Shock

Figure 3.5 in the text plots the sector-specific impulse responses of individual income to a Romer and Romer (2010) tax shock, using PSID data. I specify the following moment condition:

$$X^{h,j}_{t+s} - X^{h,j}_{t-1} = \alpha^{h,j}_s + \psi^h_s s^\star_t + \mathbf{Z}^h_{t-1}\theta^h_s + \eta^{h,j}_{t,s} \quad \text{for each} \quad s \in \{0, 1, \ldots, S\} \tag{C.1}$$

together with the standard orthogonality condition. Here, $h \in \{c, d\}$ denotes the sector of employment in the previous period $t-1$, j indexed individuals, and s indexes the horizon of the impulse response. The variable X_t denotes labor income (in log), $s^\star_t$ corresponds to the external instrument of Romer and Romer (2010), and $\mathbf{Z}^h_t$ denotes the set of control

[68] Financial activities and real estate are inter-related, making their classification ambiguous. Public administration and the military spending / employment have no immediate counterpart in my model. Excluding these two industries is conservative with respect to the mechanism I am interested in since spending and employment are less cyclical for these two industries than for private industries.

[69] The industry classification in the PSID changed in 2017. I do not exploit this latest wage to avoid measurement issues.

variables. The coefficients of interest are $\{\hat{\psi}_s^h\}_{h,s}$, i.e. the cumulative impulse responses for each sector at various horizons. I specify (C.1) at the bi-annual frequency of the PSID. The set of control variables includes a sector-specific cubic time trends and 12 lags of the fiscal policy shock. Confidence intervals (90%) are bootstrapped (200 replications) to account for heteroskedasticity and serial correlation.

C.4 Complementary Empirical Evidence

The measure of income that I use in the text is (real) labor income from the PSID. For robustness, I also use family income in to account for unemployment insurance and intra-household risk sharing. Figure C.1 plots the corresponding series. The pattern is very similar: aggregate shocks lead to a redistribution of income between durable and non-durable sectors.

Figure C.1: Family Income (PSID)