

Trading Complex Risks

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Abstract

This paper studies how complexity impacts markets' ability to aggregate information and distribute risks. I amend fundamental asset pricing theory to reflect agents' imperfect knowledge about complex dividend distributions and test its clear-cut predictions in the laboratory. Market equilibria corroborate complexity-averse trading behavior. Despite being overpriced, markets efficiently share complex risks between buyers and sellers. While complexity induces noise in individual trading decisions, market outcomes remain theory-consistent. This striking feature reconciles with a random choice model, where bounds on rationality are reinforced by complexity. By adjusting for estimation biases, traders reduce the variation in market-clearing prices of complex risks.

Keywords: Complexity, risk sharing, information aggregation, bounded rationality

JEL Classification: G12, G14, G41

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That economic decisions are made without certain knowledge of the consequences is pretty self-evident.

Kenneth J. Arrow

The essence of the situation [the problems of life] is action according to opinion, of greater or less foundation and value, neither entire ignorance nor complete and perfect information, but partial knowledge.

Frank H. Knight

1. Introduction

Risk is ubiquitous to decision-making in financial markets, where investors' trading decisions directly affect their financial well-being. In their seminal work, [Debreu \(1959\)](#) and [Arrow \(1964\)](#) provide an elegant theory of value and choice under risk in the context of perfect and complete markets. In sharp contrast to this theoretical benchmark, the inherent complexity of real-world markets only allows for an imperfect measurement of financial risks ([Knight, 1921](#)) with varying levels of confidence ([Keynes, 1921](#)). Thus, it may come as little surprise that the predictions by Debreu and Arrow's theory are generally rejected in the field. Acknowledging this particular discrepancy between theory and reality, I ask the following three questions: (i) How powerful is the neoclassical theory in describing market outcomes, i.e., prices *and* allocations, if one accounts for the complex risk structure of financial assets? (ii) Can such an amended theory improve our understanding of the highly non-trivial process that transforms individual trading decisions into collective market outcomes? (iii) How does complexity impact financial markets' ability to aggregate information and distribute risk? I try to answer these questions in two steps.

I begin with the theory. For the most simple two-state setting, I extend traditional consumption-based asset pricing theory to reflect agents' *partial* knowledge about the distribution of future dividends ([Knight, 1921](#)). The source of complexity that impairs information quality is exogenous to the model. To increase generality, I incorporate both kinked as well as smooth complexity preferences by applying two canonical decision theories under ambiguity ([Ghirardato, Maccheroni, and Marinacci, 2004](#); [Klibanoff, Marinacci, and Mukerji, 2005](#)). In a Walrasian market, both preference classes provide similar qualitative implications regarding the trading of complex risks: In the absence of aggregate uncertainty, competitive market prices are sensitive to mispricing, whereas risk allocations are relatively more robust to incorrect beliefs. Intuitively, the latter is the result of increased risk sharing incentives in the face of imperfect information. Contrary to the

above decision theories, subjective expected utility (Savage, 1954), i.e., *the* theoretical foundation of choice under uncertainty, implies relatively *less* efficient risk sharing under partial knowledge.

Clearly, whether preferences known to explain behaviour under (pure) ambiguity preserve any explanatory power (over subjective expected utility) for *partially* measurable risks remains an empirical question. In a second step, I therefore test the theory's clear-cut predictions in the laboratory, constructing a one-to-one replication of the underlying model economy. The lab is the sole environment that enables me to simultaneously control for both individual beliefs and strategic uncertainty, a virtual impossibility in the context of field data. In my main treatment, I introduce complexity in traded risks by relying on the seminal description of financial risks by Bachelier (1900) and Black and Scholes (1973).

My findings shed light on the above questions. First, in the presence of complex risks, asset market equilibria corroborate complexity-averse trading behavior. In line with markets' awareness of traders' imperfect information, complexity reduces the price elasticity of risk-minimizing supply and demand schedules. Complex risks are generally overpriced, suggesting that individuals overestimate the drift relative to the volatility component of financial risks. However, the reduction in price sensitivity overcomes the sizeable variation in subjective beliefs, allowing complex risks to be shared almost perfectly between buyers and sellers. This striking feature of market equilibrium under partial knowledge demonstrates the explanatory power of a conditional rational choice paradigm that accounts for informational imperfections.

Second, at the individual level, complexity causes more mistakes in trading decisions, where mistakes are defined as adopting strategies that are strictly dominated in terms of their risk-return profile.¹ Both frequencies and distributions of dominated actions confirm that individual trading strategies become increasingly noisy under more complex risks. Crucially, as the number of participants becomes larger, this noise cancels out in equilibrium and theory-consistent prices and optimal risk allocations prevail. This aggregation result can be explained by a random choice model in which the relative likelihood of a given action is increasing in its anticipated utility.²

Third, combining complexity aversion with random choice provides an effective setting to investigate how complex risks generically impact market outcomes. While complexity

¹ Conditions for strict dominance are derived under perfect and imperfect information about dividends. In the latter case, the conditions hold for both kinked (Ghirardato, Maccheroni, and Marinacci, 2004) and smooth (Klibanoff, Marinacci, and Mukerji, 2005) preferences.

² See Section 3 for a formal definition of anticipated utility under imperfect information.

aversion facilitates robust risk sharing, its implication on the informativeness of prices is per se ambiguous. However, I find evidence that individual trading behavior exhibits self-awareness of prevailing estimation biases. More specifically, the bigger the distance between individual point estimates and the true dividend probability, the less price sensitive the submitted supply and demand curves. Eventually, self-awareness translates into collective awareness which decreases the variation in market-clearing prices relative to the variation in subjective beliefs. This showcases the stabilizing effect of discontinuous trading periods during times of high price uncertainty. In general, markets' ability to aggregate subjective beliefs into prices is therefore determined by the trade-off between reduced price sensitivity and amplified bounded rationality.

Finally, my empirical findings have several implications for the experimental and theoretical asset pricing literature: (i) Extending the finding in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), I show that, in absence of complex risks, second-order stochastic dominance is sufficient for generating competitive market outcomes in line with rational choice.³ (ii) In the presence of complex risks, individual behavior is highly heterogeneous, i.e., trading strategies implied by subjective expected utility as well as kinked and smooth ambiguity preferences are observed. (iii) Notwithstanding individual heterogeneity and complexity-induced bounded rationality, incorporating partial knowledge into a neoclassical asset pricing model convincingly explains market equilibria under imperfect information. Hence, subjective expected utility à la [Savage \(1954\)](#) is insufficient to understand market behavior if traders have access to *partial* knowledge only. Moreover, my bottom-up approach demonstrates how aggregate stability results from individual heterogeneity. This stands in contrast to the rational of representative agent models motivated by specific singular behavior.

The merits of taking the study of how (complex) individual trading translates into to market outcomes to the lab are manifold. By design, the lab allows for the construction of complete markets and—by comparing market-clearing prices to random price draws—provides a direct test of their competitiveness. Also, the experimenter can exercise full control over each market participants' information set and how their individual decisions interact towards equilibrium. Crucial for any study of complex information, the laboratory environment offers the unique virtue of measuring subjective beliefs (expectations). This generally constitutes an impracticality when confronted with real-world data. Most importantly, the treatment effect under investigation can be analyzed in isolation, while

³ In the setting studied by [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), rational choice is implied by first order stochastic dominance.

controlling for any kind of endogeneity concerns.

The obvious benchmark for this study is the case in which traders face ‘simple’ risks and hence everybody has perfect information about the objective dividend distribution. In my experiment, complexity is introduced by linking risky dividends directly to the payoff of a digital option, i.e., making dividends depending on the realizations of a particular ‘reference path’. In this case, the reference path follows a geometric Brownian motion. More specifically, participants are provided with the parameters of the reference path in addition to a dynamic visualization of its past trajectory. Thus, the presence of complex risks requires them to deductively determine dividend distributions by solving a stochastic differential equation. Although solvable by hand for certain parameters, deriving the problem’s closed-form solution proves infeasible for most participants.⁴ The advantage of this implementation is the simple structure of the complicated but yet well-defined task at heart. The problem’s comprehensible form together with its visualization allows participants to appraise—with more or less certainty—the *apparently* objective dividend risk.⁵ Note, the objectivity of the complex dividend distribution is a necessary condition for contrasting the empirical data to any kind of theoretical benchmark.

In the absence of perfect information, participants acquire a more or less precise estimate of the relevant dividend distribution, i.e., are faced with a smaller or wider set of possible priors. Being aware of the incompleteness of their knowledge, “it would be irrational for an individual who has poor information about her environment to ignore this fact and behave as though she were much better informed” (Epstein and Schneider, 2010, p. 5). Thus, I theoretically study trading decisions under complex risks as a departure from subjective expected utility by applying two seminal ambiguity models: a generalization of the multiple-priors model by Gilboa and Schmeidler (1989), and the smooth ambiguity model by Klibanoff, Marinacci, and Mukerji (2005). While the former implies kinked ambiguity preferences, the later allows for smooth ambiguity effects.

In my setting, the main implication of both models is intuitive. If agents are averse

⁴ This is not surprising given the means at hand and the limited time available during the experiment. Presenting participants with an obviously solvable but complicated problem represents the design’s integral treatment.

⁵ Although this notion of complexity is arguably specific, it naturally extends to real-world financial markets’ perceived risk structure. There is a vast scientific literature on various notions of complexity. In computer science and machine learning one distinguishes, e.g., between computational complexity (required resources), sample complexity (minimum number of draws), and Kolmogorov complexity (minimum descriptive length) of problem solving. Interestingly enough, recent contributions in decision science provide evidence for commonalities between the human brain and computer algorithms solving and reacting to problems with varying levels of complexity (see, e.g., Bossaerts and Murawski (2016)).

to perceived ambiguity, they, *ceteris paribus*, prefer to avoid being exposed to imperfectly understood risks. When starting from a zero ambiguity exposure, this leads to a no-trade interval.⁶ For nonzero endowments in the risky asset, as pointed out by [Dow and da Costa Werlang \(1992\)](#), engaging in trade is generally optimal. In my model economy, incentives to trade stem from nontradable but *hedgeable* consumption risk. In short, under both models, agents' price sensitivity of their perfect hedging strategy decreases in the presence of complex risks. Intuitively, being completely hedged insures not only against risk but also against potential complexity-induced ambiguity. The main difference between the two models lies in their implied conditions for mispricing. Within the smooth ambiguity model, incorrect beliefs immediately impact equilibrium prices, whereas this does not unconditionally hold for the multiple-priors model. As noted above, my experiment finds evidence for both preference classes.

To compare the above documented complexity effects to those induced by *the* canonical ambiguity instrument in experimental economics, I additionally study both individual behavior and market allocations in an [Ellsberg \(1961\)](#) environment. My results indicate that complex risks have similar but more pronounced implications on individual trading and market outcomes than ambiguity induced by conventional Ellsberg urns.

The remainder of the paper is organized as follows. [Section 2](#) reviews the literature. [Section 3](#) introduces the model economy and develops the necessary theory for generating predictions about trading both simple and complex risks. [Section 4](#) describes the experimental design. [Section 5](#) confronts the theoretical predictions with the data. [Section 6](#) concludes.

2. Literature

This paper relates to four distinctive strands of the literature. First, my design directly builds on the experimental setup proposed by [Biais, Mariotti, Moinas, and Pouget \(2017\)](#). Relying on a two-state world with two nonredundant assets (a risk-free bond and a risky stock), it offers the simplest possible setting to test the rational paradigm of general equilibrium asset pricing theory. Controlling for participants competitive behavior, they find market outcomes to be consistent with the theory of complete and perfect markets: On average, (simple) risk is perfectly shared and only aggregate risk is priced. Therefore, [Biais, Mariotti, Moinas, and Pouget's \(2017\)](#) parameter-free test of the most fundamental

⁶ For example, this phenomenon serves [Dimmock, Kouwenberg, Mitchell, and Peijnenburg \(2016\)](#) in explaining known household portfolio puzzles, e.g., the equity home bias.

theory constitutes the ideal benchmark upon which the trading of simple and complex risks can be compared.⁷ Moreover, its simple market-clearing pricing scheme based on individual supply and demand functions can be controlled for any kind of strategic uncertainty. This constitutes an impracticality in the context of the continuous double auction that is most commonly used in experimental asset market studies.

Second, the herein presented analysis naturally relates to experimental studies on trading ambiguous or complex assets. Implementing a continuous double auction of state-contingent claims based on an Ellsberg urn, [Bossaerts, Ghirardato, Guarnaschelli, and Zame \(2010\)](#) analyze how participants' ambiguity aversion affects asset prices and final portfolio holdings. Similar to the no-trade result, they find that, for certain subsets of prices, ambiguity-averse agents prefer to hold nonambiguous portfolios. Furthermore, [Bossaerts, Ghirardato, Guarnaschelli, and Zame \(2010\)](#) show how, in the presence of aggregate risk, sufficiently ambiguity-averse investors indirectly impact asset prices by altering the per capita risk to be shared among marginal investors.

[Asparouhova, Bossaerts, Eguia, and Zame \(2015\)](#) show how ambiguity preferences can explain asset prices under asymmetric reasoning. They consider a continuous double auction of arrow securities, where, midway through the auction, agents are confronted with an involved updating problem regarding the relative likelihood of the underlying states.⁸ In line with [Fox and Tversky's \(1995\)](#) comparative ignorance proposition, [Asparouhova, Bossaerts, Eguia, and Zame \(2015\)](#) argue that agents perceive irreconcilable post-updating market prices as ambiguous. Hence, if ambiguity-averse, agents who apply incorrect reasoning become price-insensitive. Consistent with ambiguity aversion, the more price-sensitive agents there exist, the less severe is the experimentally documented mispricing.

[Carlin, Kogan, and Lowery \(2013\)](#) study how computational complexity alters bidding behavior in a *deterministic* environment.⁹ They find higher complexity to increase volatility, lower liquidity, and decrease trade efficiency, i.e., to reduce gains from trade. Moreover, [Carlin, Kogan, and Lowery \(2013\)](#) provide evidence that, additionally to any noise arising from estimation errors, traders' bidding strategies are influenced by a complexity-induced adverse selection problem. Intuitively, given traders' private values of the tradable asset

⁷ For their most general predictions, [Biais, Mariotti, Moinas, and Pouget \(2017\)](#) only rely on first order stochastic dominance. When allowing for deviations from their symmetric payoff distribution, my analysis assumes expected utility maximization instead.

⁸ The updating task in [Asparouhova, Bossaerts, Eguia, and Zame's \(2015\)](#) experimental design is an adaptation of the famous 'Monty Hall problem'.

⁹ In the experimental design by [Carlin, Kogan, and Lowery \(2013\)](#) participants trade different assets whose values have to be determined deductively by solving systems of linear equations, where the authors differentiate between simple and complex computational problems.

are affiliated, the fear of winner’s curse, i.e., to systematically lose by trading against a better informed counterparty, leads traders to submit more conservative price quotes.

Third, a growing literature investigates the drivers and implications of financial complexity both from a theoretical as well as an empirical perspective. [Ellison \(2005\)](#) and [Gabaix and Laibson \(2006\)](#) demonstrate theoretically that inefficient levels of financial complexity can prevail in a competitive equilibrium. [Carlin \(2009\)](#) finds that financial complexity increases with the degree of competition among financial institutions. [Carlin and Manso \(2011\)](#) show how educational initiatives aiming to foster financial literacy may eventually cause welfare diminishing obfuscation, i.e., the strategic acceleration of complexity by financial service providers to preserve industry rents (see [Ellison and Ellison \(2009\)](#)). From an investor’s view, [Brunnermeier and Oehmke \(2009\)](#) discuss three different ways to deal with complexity: (i) applying separation results, (ii) relying on models, or (iii) via standardization. [Arora, Barak, Brunnermeier, and Ge \(2011\)](#) illustrate how computationally complex derivatives may worsen asymmetric information costs.

[C el erier and Vall e \(2017\)](#) empirically test the implications of the [Carlin \(2009\)](#) model and indeed find complexity to be increasing in issuer competition. Furthermore, several studies analyze the steadily growing market for complex securities, in particular their pricing, historical performance, as well as the characteristics of the involved issuers and investors ([Henderson and Pearson \(2011\)](#), [Ghent, Torous, and Valkanov \(2017\)](#), [Griffin, Lowery, and Saretto \(2014\)](#), [Sato \(2014\)](#), and [Amromin, Huang, Sialm, and Zhong \(2011\)](#)). Relying on expected utility theory, [Hens and Rieger \(2008\)](#) moreover reject the often-claimed market completing effect of structured products. Hence, there exists both theoretical and accumulating empirical evidence that financial institutions rely on a continuing increase in complexity to shield industry rents from competitors and learning by investors rather than to create higher quality products. My paper complements this literature by investigating the effects of rising complexity on agents’ trading behavior, deliberately abstracting from financial innovation’s potential market completion role and the strategic use of complexity to mitigate competition.¹⁰

¹⁰ In reality, most markets, including those for financial assets, can hardly be characterized as being complete in a static sense, i.e., in the absence of retrading opportunities. Hence, the financial innovation industry’s touted services towards market completion have to be evaluated against dynamic completeness as developed in [Kreps \(1982\)](#) and [Duffie and Huang \(1985\)](#). Assuming dynamic completeness, the existence of a Radner equilibrium ([Radner, 1972](#)) crucially depends on agents’ ability of *perfect foresight*, i.e., to perfectly forecast today all future prices depending on information revealed tomorrow. [Asparouhova, Bossaerts, Nilanjan, and Zame \(2016\)](#) experimentally show how the inability of perfect foresight can cause considerable deviations from equilibrium prices. Thus, one reasonable concern implied by the increasing complexity of traded risks is that agents lacking the required resources to fully understand their complicated nature may fail to correctly forecast future price movements.

Finally, this paper also relates to an emerging literature comparing individuals' preferences towards pure Ellsberg-like ambiguity and complex risk(s), where, as in this paper, the latter is uniquely defined by an objective probabilistic structure. The findings in [Halevy \(2007\)](#) give support to a close relation between individuals' ability to correctly reduce compound lotteries and their attitudes to pure ambiguity. The vast majority of participants (95%) who failed to disentangle compound objective lotteries, displayed nonambiguity-neutral behavior.

In their recent paper, [Armantier and Treich \(2016\)](#) provide strong empirical evidence for “a tight link between attitudes toward ambiguity and attitudes toward complex risk” ([Armantier and Treich, 2016](#), p. 5). In their ambiguity treatment, participants are confronted with lotteries whose outcomes depend on draws from an opaque Ellsberg urn, while complex risks are represented by lotteries that get settled by simultaneous draws from multiple transparent urns. Based on estimated certainty equivalents for both lottery types, [Armantier and Treich \(2016\)](#) elicit ambiguity as well as complex risk premiums. They find a strong positive correlation between the two premiums across participants.

3. Theory

This section introduces the simple model economy for which I study individual trading behavior conditional on agents' information quality. If risks are simple, implications of varying risk-preferences are analyzed within the classical framework of expected utility. In contrast, if risks are complex, individual preferences are adjusted to account for agents' imperfect information. The former case provides a clear-cut benchmark for investigating the latter. At the end of [Section 3.3](#), I provide a summary of the theory's general predictions in contrast to subjective expected utility.

3.1. Model

I start from the simple setting of [Biais, Mariotti, Moinas, and Pouget \(2017\)](#). In the two-period interpretation of this trading economy, $t \in \{1, 2\}$, uncertainty gets resolved in $t = 2$, where there exist two possible states of the world, $\Omega = \{u, d\}$. The probability of reaching state u is denoted by π , i.e. $\mathbb{P}(\omega=u) = \pi$ and $\mathbb{P}(\omega=d) = 1 - \pi$, respectively. Contrary to [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), I allow for any nontrivial binary

payoff distribution $\pi \in (0, 1)$.¹¹ This generalization is crucial, given agents' subjective beliefs in the context of complex risks.

The economy offers access to a complete asset market, where shares of a risky asset (stock) can be traded in exchange for units in the risk-free asset (numéraire). The stock pays a state-dependent dividend X per share in $t = 2$, but nothing beforehand. The dividend fully transfers the stock's final value to shareholders, i.e., after dividends have been paid, all shares expire worthless (no continuation value). Without loss of generality, I assume that $X(u) > X(d)$. The time difference between $t = 1$ and $t = 2$ is considered to be very short, allowing to abstract from any time discounting. Therefore, in-between periods, the risk-free asset simply serves as pure storage device (cash) that does not pay any interests.

There is an infinite number of agents populating the economy. I denote the unbounded set of agents by \mathcal{I} . Agent $i \in \mathcal{I}$ is endowed with nonnegative holdings in the risk-free asset B_i , S_i shares of the risky stock, and some state-contingent nontradable income $I_i(\omega)$ in $t = 2$. Moreover, every agent belongs to one of two types, i.e., either she is allowed to buy (potential buyer) or to sell (potential seller) shares. There are as many buyers as sellers and their respective endowments are identical within each type. Every agent only cares about her utility of consumption $C_i(\omega)$ in $t = 2$, where consumption equals the sum of final holdings in the risk-free asset, dividend payments, and nontradable income. In the first period, potential buyers and sellers are able to trade shares via a *call*-mechanism to maximize their increasing utility from consumption $U_i(C_i)$ in the second period.¹²

Importantly, agents' nontradable income is set to exactly *offset* the aggregate consumption risk generated by the stock's dividend payments. For constant aggregate wealth (across states), I show in the following that, under risk aversion, the unique rational expectation equilibrium is independent from heterogeneous attitudes towards simple consumption risks. In particular, under simple risks, the stock market-clearing price and quantity remain unaffected by the shape of agents' utility functions $(U_i)_{i \in \mathcal{I}}$. If, despite the income $I_i(\omega)$, aggregate risk prevailed, market equilibrium would necessarily reflect

¹¹ Biais, Mariotti, Moinas, and Pouget (2017) only consider the symmetrical case, i.e., $\pi = 1/2$. Imposing symmetry has the advantage of delivering robust predictions even under the inapplicability of expected utility theory, i.e., by only assuming the absence of first order stochastically dominated actions.

¹² To trade, agents submit either demand or supply schedules for a closed discrete set of prices. In the spirit of a Walrasian clearinghouse (Friedman (2018)), the call-mechanism then maximizes trade by minimizing the gap between demand and supply.

agents' (average) risk preferences.¹³

3.2. Trading Simple Risks: The Expected Utility Benchmark

In the presence of simple risks, the probability π is common knowledge, and thus, agents possess perfect information about the stock's payoff distribution. According to classical consumption-based asset pricing theory, the stochastic discount factor then corresponds to the representative agent's marginal rate of intertemporal substitution. In [Biais, Mariotti, Moinas, and Pouget's \(2017\)](#) simple economy, with consumption restricted to $t = 2$, expected utility theory implies an equilibrium stock price P equal to the stock's normalized expected payoff weighted by her marginal utilities across states.¹⁴ If $(I_i(\omega))_{i \in \mathcal{I}}$ eliminates aggregate consumption risk, the interconnectedness between P and marginal utilities disappears, allowing for predictions robust to any set of increasing utility functions $(U_i)_{i \in \mathcal{I}}$. Next, I generalize the analysis in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), who focus on $\pi \neq 1/2$ only, to any possible dividend distribution.

Recalling the two-state nature of the economy in $t = 2$, one can write agent i 's expected utility from consumption as

$$\begin{aligned} E[U_i(C_i(\omega))] &= \pi U_i(C_i(u)) + (1 - \pi) U_i(C_i(d)) \\ &= \pi U_i\left(\mu_i + \sqrt{\frac{1 - \pi}{\pi}} \sigma_i\right) + (1 - \pi) U_i\left(\mu_i - \sqrt{\frac{\pi}{1 - \pi}} \sigma_i\right), \end{aligned} \quad (1)$$

where $\mu_i \equiv \pi C_i(u) + (1 - \pi) C_i(d)$ and $\sigma_i^2 \equiv \pi(1 - \pi) (C_i(u) - C_i(d))^2$. Thus, any agent's expected utility can be rewritten as a function of the probability π , her expected consumption, and the standard deviation of consumption across states.

¹³ [Constantinides \(1982\)](#) shows that if agents with different risk attitudes all maximize expected utility subject to a common prior, equilibrium prices can always be rationalized in a representative agent framework. Hence, in the absence of complex risks, market equilibrium can be explained by the risk preferences of this representative agent.

¹⁴ When deciding on her optimal trading strategy Q in $t = 1$, the representative agent solves the following problem (where $Q > 0$ implies buying)

$$\max_Q E[U_i(C_i(\omega))] \quad \text{s.t.} \quad C_i(\omega) = (S_i + Q)X(\omega) + (B_i - QP) + I_i(\omega),$$

maximizing her expected utility from consumption in $t = 2$ subject to her budget constraint (neglecting any borrowing constraints). Hence, the first order condition yields

$$P = E\left[\frac{U'_i(C_i(\omega))}{E[U'_i(C_i(\omega))]} X(\omega)\right].$$

In the absence of aggregate risk, i.e., if aggregate wealth $\Sigma_i (S_i X(\omega) + I_i(\omega))$ is constant across states, there must exist a tradable quantity \widehat{Q} at which every seller and buyer is perfectly hedged against any consumption risk in $t = 2$.¹⁵ If agents are risk-averse, i.e., whenever U_i is strictly concave for every agent i , there exists a unique equilibrium for the call-mechanism in $t = 1$.

Proposition 1. *If U_i is differentiable and strictly concave $\forall i \in \mathcal{I}$, and there exists a tradable quantity \widehat{Q} such that every seller and buyer is perfectly hedged, i.e., $\sigma_i = 0 \forall i \in \mathcal{I}$, then seller i 's supply and buyer j 's demand curve for the risky asset have the unique intersection point $(E[X], \widehat{Q}) \forall \{i, j\} \in \mathcal{I} \times \mathcal{I}$.*

Proof. For proof see Appendix A. □

The driving force behind Proposition 1 is the strict concavity of the utility functions, i.e., agents' aversion to consumption risk. To see this, it is helpful to separately consider the shape of both seller i 's supply and buyer j 's demand curve for the risky stock.

First, note that for a price equal to the stock's expected dividend, seller i 's expected consumption in Eq. (1) is independent of the number of shares sold. Since seller i is risk-averse, for $P = E[X]$, she will therefore always decide to sell exactly \widehat{Q} shares and thereby be perfectly hedged against future fluctuations in consumption. However, for $P < E[X]$ ($P > E[X]$), her expected consumption *only* increases, if she sells less (more) than \widehat{Q} shares. Because she is only willing to bear risk, i.e., deviate from selling \widehat{Q} shares, if appropriately compensated in return, her supply curve *must* lie somewhere in the lower left and upper right quadrant of the price-quantity space shown in Subfigure (a) of Figure 1.

Second, note that for $P = E[X]$, similarly buyer j 's expected consumption in Eq. (1) is independent of the number of shares bought. Given her risk-aversion, she chooses to buy exactly \widehat{Q} shares for $P = E[X]$, and more (less) than \widehat{Q} shares if $P < E[X]$ ($P > E[X]$), as illustrated in Subfigure (b) of Figure 1. Thus, when there is no aggregate risk, seller i 's supply and buyer j 's demand curve exhibit the unique intersection point $(E[X], \widehat{Q})$ as depicted in Subfigure (c).

Interestingly enough, depending on the shape of U_i , a large opposite income effect can dominate the corresponding substitution effect of a given price change. Hence, seller i 's supply or buyer j 's demand curve can effectively be nonmonotonic within the respective dominating quadrants of the PQ -plane. The following remark provides an example of a nonmonotonic supply curve.

¹⁵ Recall that endowments only differ between types.

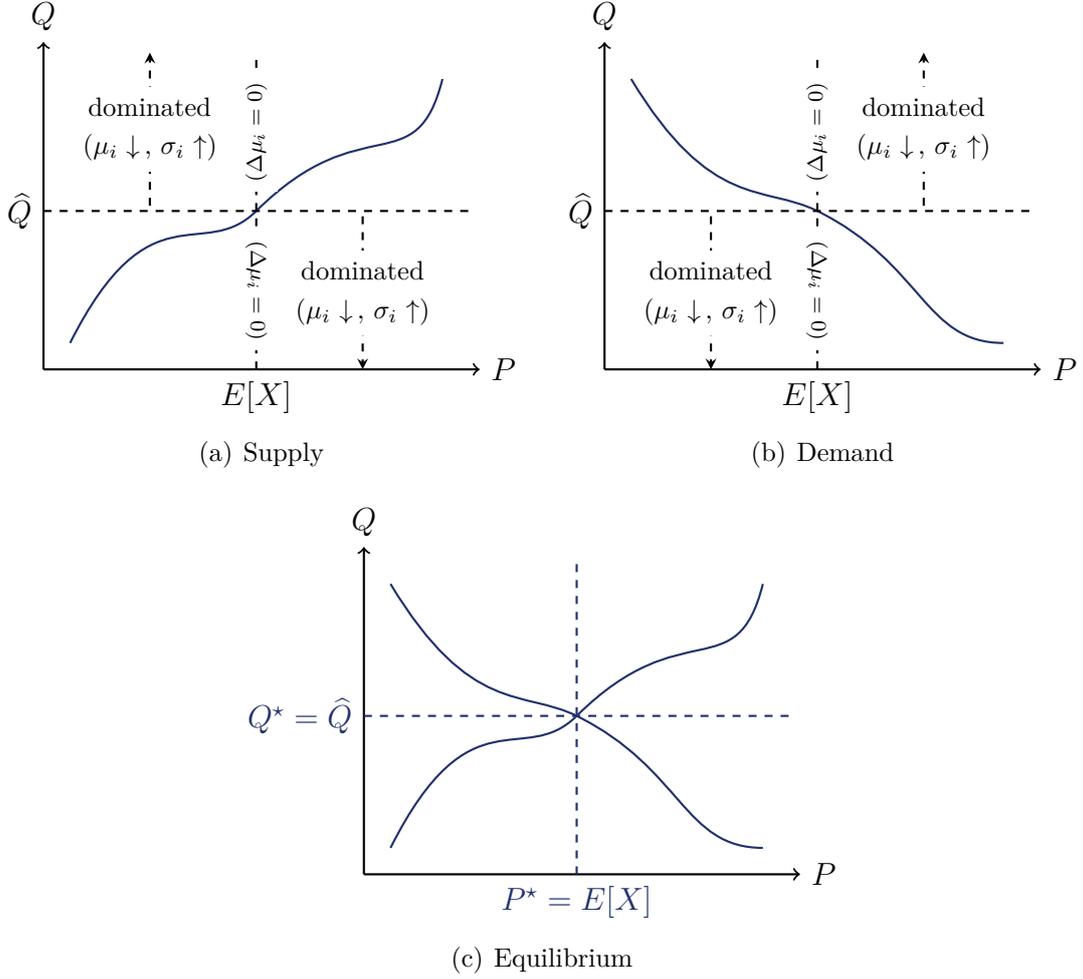


FIGURE 1. TRADING EQUILIBRIUM FOR SIMPLE RISKS

Notes: This figure shows the unique equilibrium for risk-averse agents in the absence of aggregate consumption risk.

Remark 1. Suppose, seller i 's utility function is defined piecewise as follows

$$U_i(C) = \begin{cases} c_1 \frac{C^{1-\epsilon}}{1-\epsilon}, & \text{for } 0 \leq C < \bar{C}, \\ c_2 - e^{-\alpha C}, & \text{for } \bar{C} \leq C, \end{cases}$$

where $\alpha > \epsilon > 0$ and ϵ small, and c_1 and c_2 are positive constants such that U_i is differentiable $\forall C \geq 0$. For certain parameter pairs (α, π) , seller i 's supply curve can be nonmonotonic over a nonempty subset of P .

Proof. For proof see Appendix A. □

Figure D.1 in the Appendix D shows an example of a nonmonotonic supply curve for similar parameter values as in the actual experiment.¹⁶

Absence of Risk Aversion

In case agents are *not* averse to consumption risk, for all $P \neq E[X]$, an even stricter separation between dominating and dominated strategies than shown in Figure 1 applies. From the proof of Proposition 1 it directly follows that whenever U_i is either linear or convex, seller i always strictly prefers to sell zero shares for $P < E[X]$. In contrast, for $P > E[X]$, her expected utility is maximized if and only if she sells her full initial endowment in shares. The symmetric behavior applies to risk-neutral and risk-loving buyers, respectively.

For $P = E[X]$, risk-neutral agents are indifferent between trading \hat{Q} shares or any other quantity, whereas risk-loving agents are indifferent between trading zero shares or the maximum number possible. In summary, as long as they do not consistently choose among their set of indifferent strategies in an *asymmetric* manner, the equilibrium in Figure 1 remains unaffected by a nonzero mass of nonrisk-averse agents.

3.3. Trading Complex Risks: Heterogeneous Complexity Preferences

When agents' information about the distribution of $X(\omega)$ is imperfect, I consider the associated consumption risk to be (more) complex. In the presence of such complex risks, 'rationality' in decision making requires some form of acknowledgment of the information's inherent degree of (im)precision. The literature provides a vast number of models intended to account for individuals' degree of confidence in their relative likelihood estimates. In the following, I analyze individual trading of complex risks within two classes of seminal ambiguity models: multiple-priors utility and the 'smooth ambiguity' model proposed by Klibanoff, Marinacci, and Mukerji (2005). In the former, agents' information quality has a first-order effect on their trading decision (change in mean), whereas for the latter, lower information precision increases the *total* amount of perceived 'risk' (see Epstein and Schneider (2010)). For multiple-priors utility, there exists a direct mapping to rank-dependent expected utility, which I briefly discuss.

¹⁶ The intuition behind this exemplary nonmonotonicity effect is simple. For every seller and any given Q , both $C(d)$ and $C(u)$ are strictly increasing in $P > 0$. If prices are high enough, seller i 's higher CARA coefficient α , relevant for $C(\omega) > \bar{C}$, can dominate her lower CRRA coefficient ϵ . Thus, for even higher prices, she is willing to bear less and less risk, causing her supply curve to decrease until it eventually reaches \hat{Q} , thereby completely eliminating her consumption risk.

Multiple-priors Utility

Agents facing complex risks are unable to determine π with certainty, but rather have to consider several payoff distributions possible. Hence, intuitively, when making their trading decisions, they are guided by a set of potential probability laws. I denote agent i 's subjective set of possible priors on the state space Ω by \mathcal{C}_i .

Based on this idea of multiple priors, [Gilboa and Schmeidler \(1989\)](#) axiomatize a multiple-priors maxmin decision rule that assumes infinite ambiguity-aversion. In order to allow for a full spectrum of ambiguity preferences, I employ the generalization proposed by [Ghirardato, Maccheroni, and Marinacci \(2004\)](#), the so-called α -maxmin model, instead. Assuming the set \mathcal{C}_i of subjective priors to be convex, agent i 's utility from consumption in $t = 2$ is then given by

$$U_i(C_i(\omega)) = \alpha_i \min_{\pi \in \mathcal{C}_i} \left(E[U_i(\pi)] \right) + (1 - \alpha_i) \max_{\pi \in \mathcal{C}_i} \left(E[U_i(\pi)] \right), \quad (2)$$

where $E[U_i(\pi)] \equiv \pi U_i(C_i(u)) + (1 - \pi) U_i(C_i(d))$, U_i , as before, is a differentiable and strictly concave utility function, and $\alpha_i \in [0, 1]$. First, note the straightforward interpretation of [Eq. \(2\)](#). On the one hand, the cardinality or wideness of \mathcal{C}_i measures agent i 's ambiguity perception: The bigger her set of subjective priors, the more ambiguity she perceives. On the other hand, her preferences towards ambiguity are expressed by α_i : If $\alpha_i > 1/2$, she puts more weight on the minimal expected utility, implying ambiguity-aversion. In contrast, if $\alpha_i < 1/2$ ($\alpha_i = 1/2$), then she is ambiguity-loving (ambiguity-neutral). For their axiomatization, [Gilboa and Schmeidler \(1989\)](#) assume maximal ambiguity-aversion, i.e., $\alpha_i = 1$. Second, whenever \mathcal{C}_i is a singleton, [Eq. \(2\)](#) reduces to [Eq. \(1\)](#) with subjective probability π_i , i.e., [Eq. \(2\)](#) converges to subjective expected utility as $\mathcal{C}_i \rightarrow \pi_i$. For ease of notation, I furthermore rely on the following definition:

$$E_i[X] := \alpha_i E_i[\underline{X}] + (1 - \alpha_i) E_i[\overline{X}], \quad (3)$$

where $E_i[\underline{X}] \equiv E^{\underline{\pi}_i}[X]$ with $\underline{\pi}_i := \arg \min_{\pi \in \mathcal{C}_i} \mu_i(\pi)$, and $E_i[\overline{X}] \equiv E^{\overline{\pi}_i}[X]$ with $\overline{\pi}_i := \arg \max_{\pi \in \mathcal{C}_i} \mu_i(\pi)$.

When risks are complex, agents perceive ambiguity regarding the probability π . In order to analyze individual trading behavior within the α -maxmin model, a case-by-case analysis is required, whereby agent i can behave differently from agent j in two dimensions: First, agent i is either averse to ($\alpha_i > 1/2$) or even favors ($\alpha_i \leq 1/2$) perceived ambiguity. Second, she can either have correct or incorrect beliefs about the true payoff probability

TABLE I. AGENT TYPES WITH MULTIPLE-PRIORS UTILITY

		<i>Beliefs about π</i>	
		Correct ($\pi \in \mathcal{B}_i$)	Incorrect ($\pi \notin \mathcal{B}_i$)
<i>Ambiguity-averse</i>	Yes ($\alpha_i > 1/2$)	Type AC	Type AI
	No ($\alpha_i \leq 1/2$)	Type NC	Type NI

Notes: In the presence of complexity-induced ambiguity, I distinguish between four different types of agents with multiple-priors utility. Agent i can either be ambiguity-averse or does not dislike ambiguity. Additionally, she can either apply correct or incorrect reasoning when processing her imperfect information about π .

π . More precisely, I classify agent i as having incorrect beliefs, if π is not sufficiently close to the midpoint of her set of priors, i.e., if $\pi \notin \mathcal{B}_i \subset \mathcal{C}_i$, where \mathcal{B}_i itself depends on her ambiguity-aversion:

$$\mathcal{B}_i = \begin{cases} [\pi_M - \Delta(2\alpha_i - 1), \pi_M + \Delta(2\alpha_i - 1)], & \text{for } \alpha_i > \frac{1}{2}, \\ \pi_M & \text{for } \alpha_i \leq \frac{1}{2}, \end{cases} \quad (4)$$

where π_M denotes the midpoint of \mathcal{C}_i with length (or maximum difference) 2Δ . We note that $\mathcal{B}_i \rightarrow \mathcal{C}_i$ as $\alpha_i \rightarrow 1$ and $\mathcal{B}_i \rightarrow \pi_M$ as $\alpha_i \rightarrow 1/2$. Table I summarizes the four possible combinations of different types.

Price Sensitivity

To deduce the effect(s) of complexity-driven ambiguity on agents' trading behavior, the different types presented in Table I have to be considered separately. I start with the first row of Table I. If aggregate endowments are constant, any risk-averse agent, as shown above, prefers to trade exactly \hat{Q} shares for $P = E[X]$. Now, given their distaste for the perceived ambiguity regarding π , agents of type AC and AI eventually both prefer to trade \hat{Q} for prices significantly different from $E[X]$. More precisely, for any given degree of risk-aversion, the subset of prices for which they wish to be perfectly hedged against consumption risk is increasing in both their ambiguity aversion and ambiguity perception.

Proposition 2. *In the presence of perceived ambiguity and if there exists a tradable quantity \hat{Q} such that $\sigma_i = 0 \forall i \in \mathcal{I}$, then agents of types AC and AI exhibit constant supply or demand curves over closed subsets of P . Their absolute price elasticity is a decreasing function in both α_i and the cardinality/length of \mathcal{C}_i .*

Proof. For proof see Appendix A. □

In case of no aggregate risk, it holds for any *seller* i that $\underline{\pi}_i < \bar{\pi}_i$ for $Q < \hat{Q}$ and $\underline{\pi}_i > \bar{\pi}_i$ for $Q > \hat{Q}$, respectively. Intuitively, if seller i is hedged against varying consumption by selling exactly \hat{Q} shares, her expected consumption μ_i decreases in $1 - \pi_i$ (π_i) whenever she sells less (more) than \hat{Q} shares. Analogously, for any *buyer* j it holds that $\underline{\pi}_j > \bar{\pi}_j$ for $Q < \hat{Q}$ and $\underline{\pi}_j < \bar{\pi}_j$ for $Q > \hat{Q}$, respectively. These shifts in relative size of $\underline{\pi}_i$ and $\bar{\pi}_i$ around \hat{Q} in combination with ambiguity-aversion are the driving force behind Proposition 2.

To foster the reader's intuition, the result in Proposition 2 is illustrated in Figure 2 from the perspective of an ambiguity-averse seller—the analogous reasoning also applies to any ambiguity-averse buyer. First, due to seller i 's risk-aversion, it can be shown that for $P = E_i[X]$, selling exactly \hat{Q} shares strictly dominates trading any other quantity of the risky asset. Moreover, given Eq. (2), she is only willing to sell less than \hat{Q} shares for prices strictly below $E_i[X]$ (see proof of Proposition 2). This is illustrated in Subfigure (a) of Figure 2. Analogously, seller i only agrees to sell more than \hat{Q} shares in return for $P > E_i[X]$ (see Subfigure (b)). Second, due to the above discussed order effect of $\underline{\pi}_i$ and $\bar{\pi}_i$, it follows that the lower price bound L in Subfigure (a) and the upper price bound U in Subfigure (b) do *not* coincide. Therefore, putting everything together, the piecewise constant supply curve depicted in Subfigure (c) prevails, where seller i 's supply of the risky asset is constant over the closed subset $[L, U]$.

In comparison to the analysis under simple risks in Section 3.2, a nice and intuitive interpretation of Proposition 2 emerges. Since agents of types AC and AI are averse to ambiguity, selling or buying \hat{Q} shares becomes even more attractive compared to situations with objective payoff distributions. By trading exactly \hat{Q} units of the risky asset, agents not only are able to avoid risk, but additionally to dispose any exposure to perceived ambiguity. Trading \hat{Q} shares hence simultaneously corresponds to the perfect hedging strategy against both risk and ambiguity. In return for this dual insurance, agents are willing to forego potential gains from trade.

I now turn to the second row in Table I. For nonambiguity-averse agents, there are two cases to be distinguished between. First, if α_i equals $1/2$, agent i is ambiguity-neutral. For a seller with $\alpha_i = 1/2$, L and U in Figure 2 coincide, i.e., under complex risks, she behaves as a *subjective* expected utility-maximizer. The analogous argument applies for an ambiguity-neutral buyer. Second, if $\alpha_i < 1/2$, agent i is ambiguity-loving. The same reasoning as in the proof of Proposition 2 implies that for an ambiguity-loving seller, it holds that $L > U$. Hence, when risks are complex, there exists a certain price between U and L for which she is indifferent between gaining exposure to ambiguity from selling less

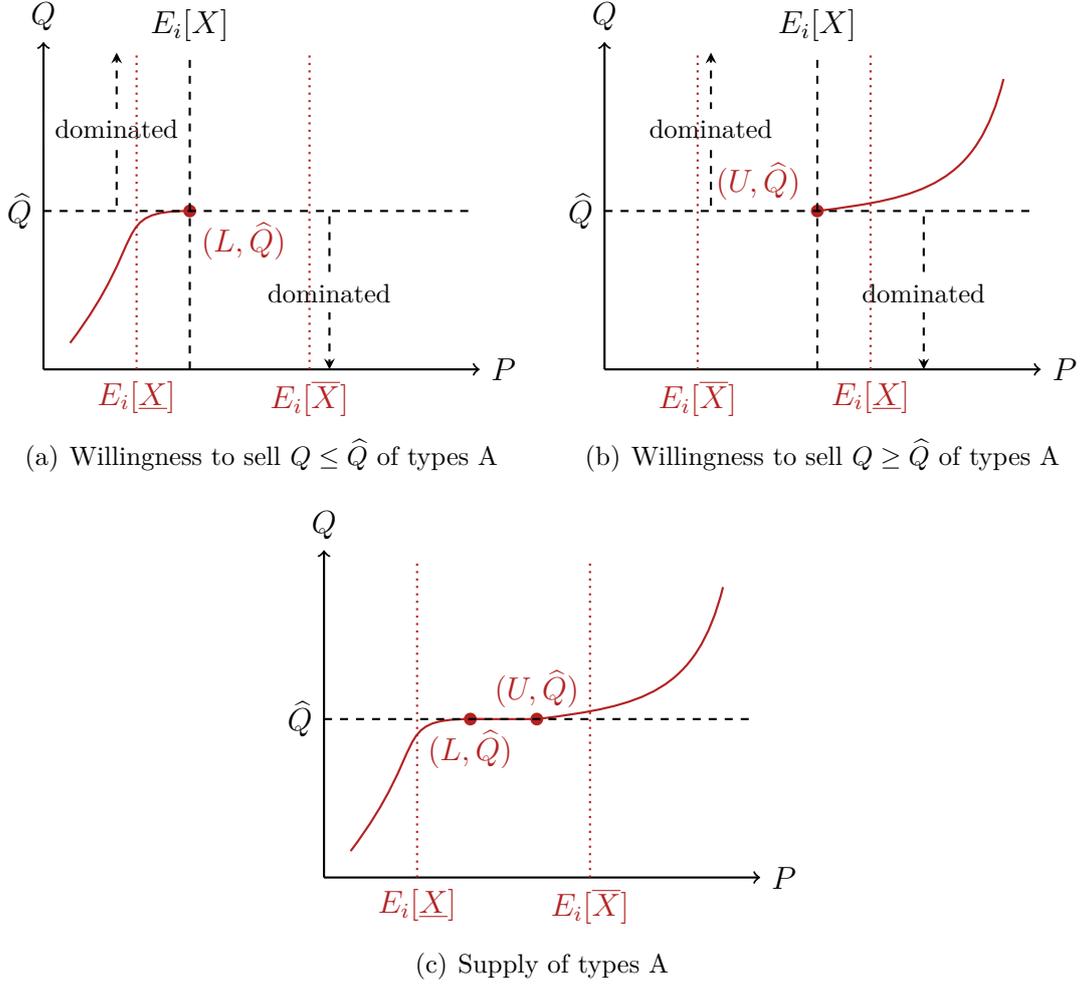


FIGURE 2. SUPPLY CURVE OF AMBIGUITY-AVERSE SELLER WITH MULTIPLE-PRIORS

Notes: This figure shows the piecewise flat supply curve for complex risks implied by the α -maxmin model (Eq. (2)) for a risk-averse and ambiguity-disliking seller i .

or more than \hat{Q} shares. At or precisely beyond this threshold, her supply curve therefore exhibits a discontinuity, i.e., jumping from strictly below to strictly above \hat{Q} .¹⁷ For prices below and above the threshold, her supply curve's price elasticity increases in comparison to simple risks. Again, the analogous argument can be made for an ambiguity-loving buyer.

¹⁷ This can be interpreted as the natural counterpart of ambiguity-averse sellers' piecewise flat supply curves.

Mispricing and Suboptimal Risk Sharing

How complex risks are priced and shared in equilibrium, crucially depends on agents' beliefs regarding π . If aggregate wealth is constant, the risky asset is mispriced whenever the market-clearing price deviates from its expected dividend.¹⁸ Hence, whenever the market-clearing quantity (per capita) is different from \widehat{Q} , consumption risk is only suboptimally shared between risk-averse buyers and sellers. I therefore subsequently refer to the market-clearing price and quantity for simple risks, i.e., $(E[X], \widehat{Q})$, as *benchmark equilibrium*.

Nonambiguity-loving agents ($\alpha_i \geq 1/2$) with correct beliefs ($\pi \in \mathcal{B}_i$) never cause any mispricing or incomplete risk sharing, simply because their supply or demand curves always contain the benchmark equilibrium (see above). Due to the jump of their supply (demand) curve between U and L , an ambiguity-loving seller (buyer) almost surely never chooses to sell (buy) \widehat{Q} shares at $P = E[X]$, independently of her beliefs regarding π . While it is clear why ambiguity-neutral agents with incorrect beliefs provoke mispricing and suboptimal risk sharing (due to their piecewise constant supply and demand curves) this is, however, less clear for ambiguity-averse agents whose subsets of beliefs \mathcal{B}_i does not contain π .

Proposition 3. *In the presence of perceived ambiguity and if there exists a tradable quantity \widehat{Q} such that $\sigma_i = 0 \ \forall i \in \mathcal{I}$, then any nonzero mass of type AI sellers (buyers) moves aggregate supply (demand) away from the benchmark equilibrium under simple risks.*

Proof. For proof see Appendix A. □

Figure 3 illustrates the mechanics behind Proposition 3 for the simplified case of only three sellers and buyers, respectively. Subfigure (a) depicts the exemplary supply curves (for a given discrete price grid) for three different types of sellers. Assuming type NC to be ambiguity-neutral, she rationally chooses—in line with her correct beliefs—to sell \widehat{Q} shares for $P = E[X]$. Because of AC's pronounced ambiguity-aversion, her supply curve is constant over a considerable subset of prices (delimited by circles in Subfigure (a)). Importantly, since $\pi \in \mathcal{B}_{AC}$, the constant part still contains the benchmark equilibrium. In contrast, the constant piece of type AI's supply curve (delimited by squares) does *not* include the point $(E[X], \widehat{Q})$. Hence, neither the length of \mathcal{C}_{AI} nor the degree of her ambiguity-aversion $\alpha_{AI} > 1/2$ are sufficiently large to prevent that $\pi \notin \mathcal{B}_{AI}$ (see Eq. (4)).

¹⁸ The absence of aggregate risk in combination with a complete market allows for perfect risk sharing.

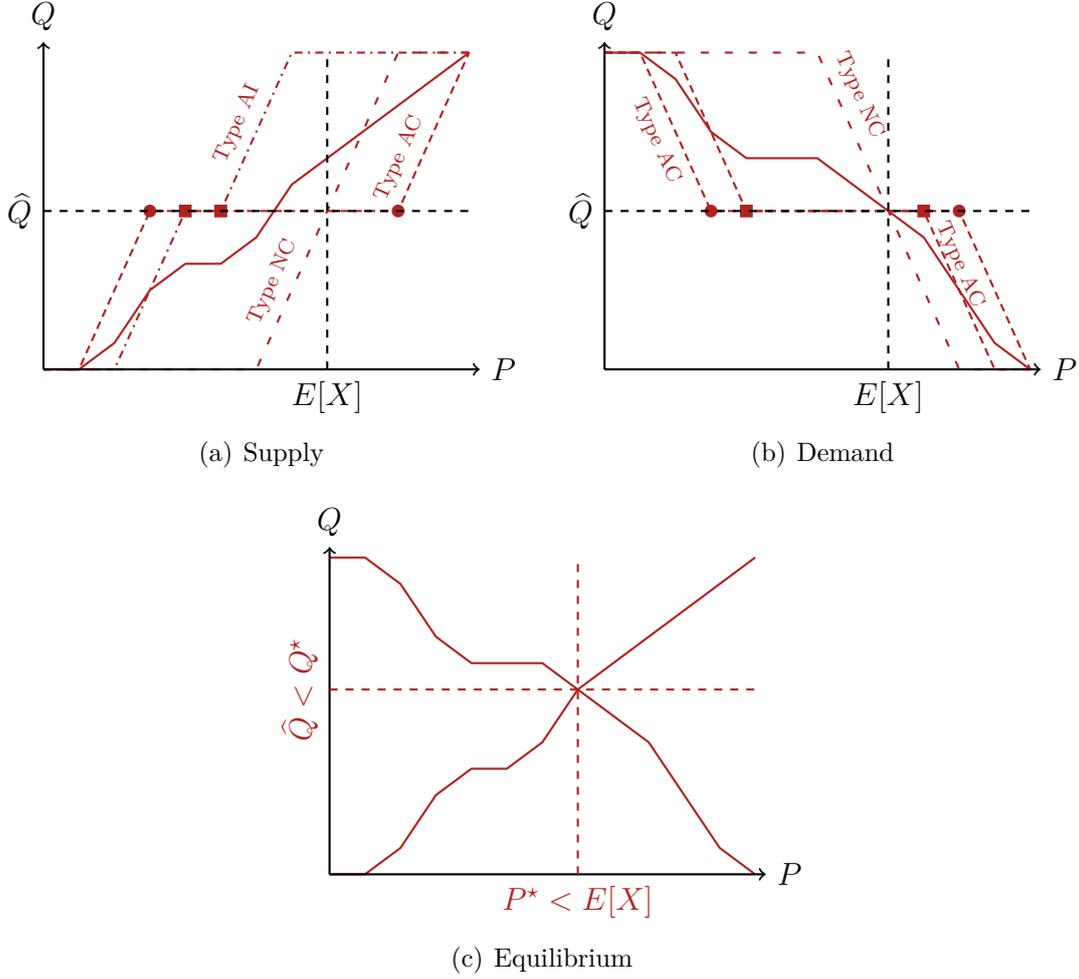


FIGURE 3. EQUILIBRIUM ANALYSIS FOR COMPLEX RISKS UNDER MULTIPLE-PRIORS UTILITY

Notes: For the α -maxmin model (Eq. (2)), this figure illustrates how ambiguity-averse agents with incorrect beliefs can cause mispricing and suboptimal risk sharing of complex risks in equilibrium (Proposition 3). Subfigure (a) shows three exemplary supply curves of one ambiguity-neutral (type NC) and two ambiguity-averse (type AC and AI) sellers. All exemplary buyers in Subfigure (b) are assumed to be nonambiguity-loving and to have correct beliefs. Subfigure (c) finally shows, how the incorrect beliefs of seller AI cause mispricing and incomplete risk sharing of complex risks in equilibrium. Due to the absence of aggregate consumption risk, both distortions are unambiguously defined and measurable.

Therefore, due to her incorrect beliefs, she pulls the *average* supply curve (solid line) away from the benchmark equilibrium.

For simplicity, all three buyers in Subfigure (b) are assumed to hold correct beliefs such that their demand curves all contain the benchmark equilibrium. This ensures that

any mispricing and incomplete risk sharing in equilibrium is solely driven by the AI-type seller’s supply curve in Subfigure (a). The solid line constitutes the resulting *average* demand curve. Finally, Subfigure (c) depicts the market-clearing price P^* and quantity Q^* (per capita) that corresponds to the intersection of the average supply and demand curves. Due to seller AI’s underestimation of π , the market-clearing price is smaller than the stock’s expected dividend, implying mispricing equal to $|P^* - E[X]|$. Furthermore, the average market-clearing quantity of shares is greater than \widehat{Q} , i.e., in equilibrium, agents do not share complex risks perfectly.

Intuitively, Proposition 3 establishes a condition under which ambiguity-induced price insensitivity is sufficiently large to offset any equilibrium effects of incorrect beliefs about complex risks. Given the midpoint of agent i ’s set of priors \mathcal{C}_i , the more ambiguity-averse she is, i.e., the larger her α_i , the wider becomes the subset of payoff distributions \mathcal{B}_i for which incorrect beliefs do not cause any deviations from the benchmark equilibrium. Note that for any $\alpha_i < 1$, the subset \mathcal{B}_i in Eq. (4) is strictly smaller than \mathcal{C}_i . Thus, as long as agent i is not maximally ambiguity-averse, requiring the true payoff distribution π to be contained in \mathcal{C}_i is not sufficient for precluding differences between simple and complex equilibria.

Another implication of the multiple-priors model’s constant supply (demand) curve is the arising possibility of multiple equilibria. In an economy with heterogeneous agents (with respect to their beliefs as well as their preferences towards risk and ambiguity), multiple equilibria are nevertheless unlikely to prevail. For instance, if the supply curve of a given mass of sellers equals \widehat{Q} for a nonsingleton subset of prices, a nonzero mass of sellers whose supply is not constant over the same subset is sufficient for the average supply curve to be nonconstant.

From Multiple-priors to Rank-dependent Expected Utility

Since the seminal work by [Tversky and Kahneman \(1992\)](#), cumulative prospect theory has become the most prominent alternative to expected utility for modeling decision making under uncertainty. Therefore, a reasonable question to ask is how trading decisions under complex risks of agents with rank-dependent utility differ from the above analysis? For binary acts, e.g., the herein considered risky asset, [Chateauneuf, Eichberger, and Grant \(2007\)](#) show that ‘neo-additive’ decision weights allow for a one-to-one correspondence from α_i and \mathcal{C}_i in Eq. (2) to (i) a likelihood sensitivity index and (ii) a pessimism

(optimism) index as generally used in rank-dependent expected utility models.¹⁹

Smooth Ambiguity Preferences

Proposition 2's somehow extreme result of (local) perfect price inelasticity is clearly linked to the kinked preferences induced by the maxmin property of Eq. (2). To emphasize the generalizability of its main implications, I now analyze individual trading behavior under the 'smooth ambiguity' model by [Klibanoff, Marinacci, and Mukerji \(2005\)](#). Adopting the above notation, agent i 's utility from consumption in $t = 2$ can then be written as

$$\mathcal{U}_i(C_i(\omega)) = \int_{\Delta(\Omega)} \phi_i \left(E[U_i(\tilde{\pi})] \right) d\mu_i(\tilde{\pi}), \quad (5)$$

where $\Delta(\Omega)$ is the simplex of all possible payoff distributions on Ω , μ_i is agent i 's subjective probability measure on $\Delta(\Omega)$, and ϕ_i is a continuous, strictly increasing, real-valued function.

Eq. (5) has an intuitive interpretation: On the one hand, the more payoff distributions exhibit a nonzero probability mass under μ_i , the bigger agent i 's set of possible priors. On the other hand, the curvature of $\phi_i(\cdot)$ expresses her ambiguity preferences: As for utility functions for simple risks, concavity of $\phi_i(\cdot)$ implies ambiguity-averse, linearity ambiguity-neutral, and convexity ambiguity-loving preferences. Hence, similar to the α -maxmin model in Eq. (2), the smooth ambiguity model allows for a separation between the level of perceived ambiguity as well as agent i 's general preferences towards it. For ease of notation and analog to Eq. (3), I rely on the following definition:

$$E_i[X] := \int_{\Delta(\Omega)} E_{\tilde{\pi}}[X] d\mu_i(\tilde{\pi}), \quad (6)$$

where $E_{\tilde{\pi}}[X]$ denotes the expected payoff of the risky asset based on $\mathbb{P}(\omega=u) = \tilde{\pi}$ and $\mathbb{P}(\omega=d) = 1 - \tilde{\pi}$, respectively.

Proposition 4. *Let $\mu_i(\tilde{\pi})$ be the normalized Lebesgue measure on agent i 's set of possible priors $[\underline{\pi}_i, \bar{\pi}_i] \subset [0, 1]$, i.e., $\mu_i(\tilde{\pi}) := 1/(\bar{\pi}_i - \underline{\pi}_i) d\tilde{\pi} \quad \forall \tilde{\pi} \in [\underline{\pi}_i, \bar{\pi}_i]$. In the presence of perceived ambiguity, if there exists a tradable quantity \hat{Q} such that $\sigma_i = 0 \quad \forall i \in \mathcal{I}$, then*

- (i) *agent i 's price elasticity is an increasing function in the second order derivative of $\phi_i(\cdot)$.*

¹⁹ In rank-dependent expected utility models, the likelihood sensitivity index measures the steepness of the probability weighting function and the optimism (pessimism) index its intersection point with the 45-degree line.

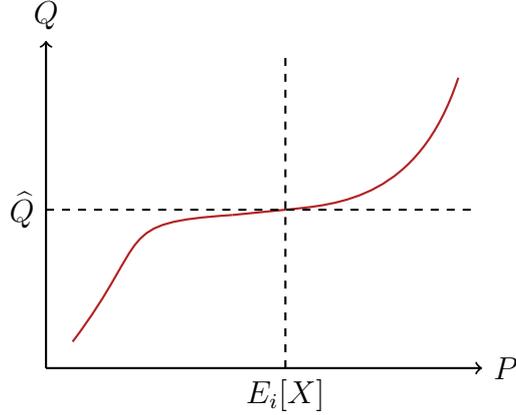


FIGURE 4. SUPPLY CURVE OF AMBIGUITY-AVERSE SELLER WITH SMOOTH PREFERENCES

Notes: This figure shows the decreased price elasticity of the supply curve for complex risks implied by the smooth ambiguity model (Eq. (5)) for a risk-averse and ambiguity-disliking seller i .

- (ii) any nonzero mass of sellers (buyers) for whom $\frac{\pi_i + \bar{\pi}_i}{2} \neq \pi$ moves aggregate supply (demand) away from the benchmark equilibrium under simple risks.

Proof. For proof see Appendix A. □

As implied by the proof of Proposition 4, with utility as in Eq. (5), any agent's supply (demand) curve goes through $(\hat{Q}, E_i[X])$. Thus, independently of her ambiguity preferences, she always finds it optimal to sell (buy) \hat{Q} shares for a price P equal to her subjective expected payoff per share given her subset of priors.

For prices below and above $E_i[X]$, Figure 4 exemplarily illustrates how imperfect information about π affects an ambiguity-averse seller's supply curve. The demand curve for any ambiguity-averse buyer behaves analogously. If, under complex risks, seller i dislikes any perceived ambiguity regarding π , selling \hat{Q} shares generally becomes more attractive than under simple risks. Due to her smooth distaste for ambiguity, i.e., the concavity of $\phi_i(\cdot)$, she smoothly decreases her supply's price elasticity for prices different from $E_i[X]$, as displayed in Figure 4. However, in contrast to Figure 2, her supply curve never becomes perfectly inelastic for any interior nonempty subset of prices.

In case seller i is ambiguity-loving, i.e., $\phi_i(\cdot)$ is convex, the slope of her supply curve amplifies when moving from simple to complex risks. Comparing Figure 4 to Subfigure (a) in Figure 1 moreover shows how increasing complexity under Eq. (5) manifests itself similarly as a shift in sellers' risk aversion under Eq. (1): If seller i is ambiguity-averse,

she is always willing to accept a lower μ_i in return for a *gradual* reduction in σ_i .

For equally probable priors, the second part of Proposition 4 states that whenever there is a critical mass of agents for whom π is different from their respective midpoint of priors, they shift aggregate supply (demand) away from the benchmark equilibrium. Under the smooth ambiguity model, the pricing and allocation of complex risks is therefore more sensitive to agents' ex-ante beliefs than under kinked ambiguity-preferences. For smooth preferences, ambiguity-induced price insensitivity can never offset a critical mass' distorting equilibrium effects of incorrect beliefs, no matter how small the respective deviations from π .

In contrast to the multiple-priors model, the pricing of complex risks by ambiguity-averse agents with smooth preferences is more sensitive to incorrect beliefs. Under the multiple-priors model, the necessary mispricing condition requires the exclusion of the true probability π from a *set* of priors, i.e., $\pi \notin \mathcal{B}_i$, instead of 'only' a pointwise deviation.

Summary

In general, for both kinked and smooth ambiguity preferences, complexity has (qualitatively) similar implications for individual trading behavior and aggregate market outcomes. This is illustrated in Figure 5. If averse to complexity-induced ambiguity, the price sensitivity of agents with nonsingleton priors *decreases* under complex risks. In the presence of incorrect beliefs, these agents can cause *mispricing* and potentially trade towards *suboptimal* risk allocations. However, their reduced price sensitivity is likely to mitigate averse effects on risk sharing. This is intuitive, since, under complexity, ambiguity-averse agents always prefer to trade towards lower consumption risk for a wider range of prices. Importantly, the latter is not true under subjective expected utility (Savage, 1954).²⁰

To summarize, decision theory under ambiguity implies the following two predictions regarding the trading of complex risks:

P1: Mispricing–Equilibrium prices are a function of subjective beliefs.

P2: Robust risk sharing–Equilibrium allocations are *less* sensitive to incorrect beliefs relative to subjective expected utility.

²⁰ Under subjective expected utility, the trading of complex risks can be modeled according to Proposition 1, while simply accounting for subjective beliefs π_i . Hence, there does not exist any a priori mechanism that decreases the sensitivity of either equilibrium prices nor risk allocation with respect to subjective beliefs.

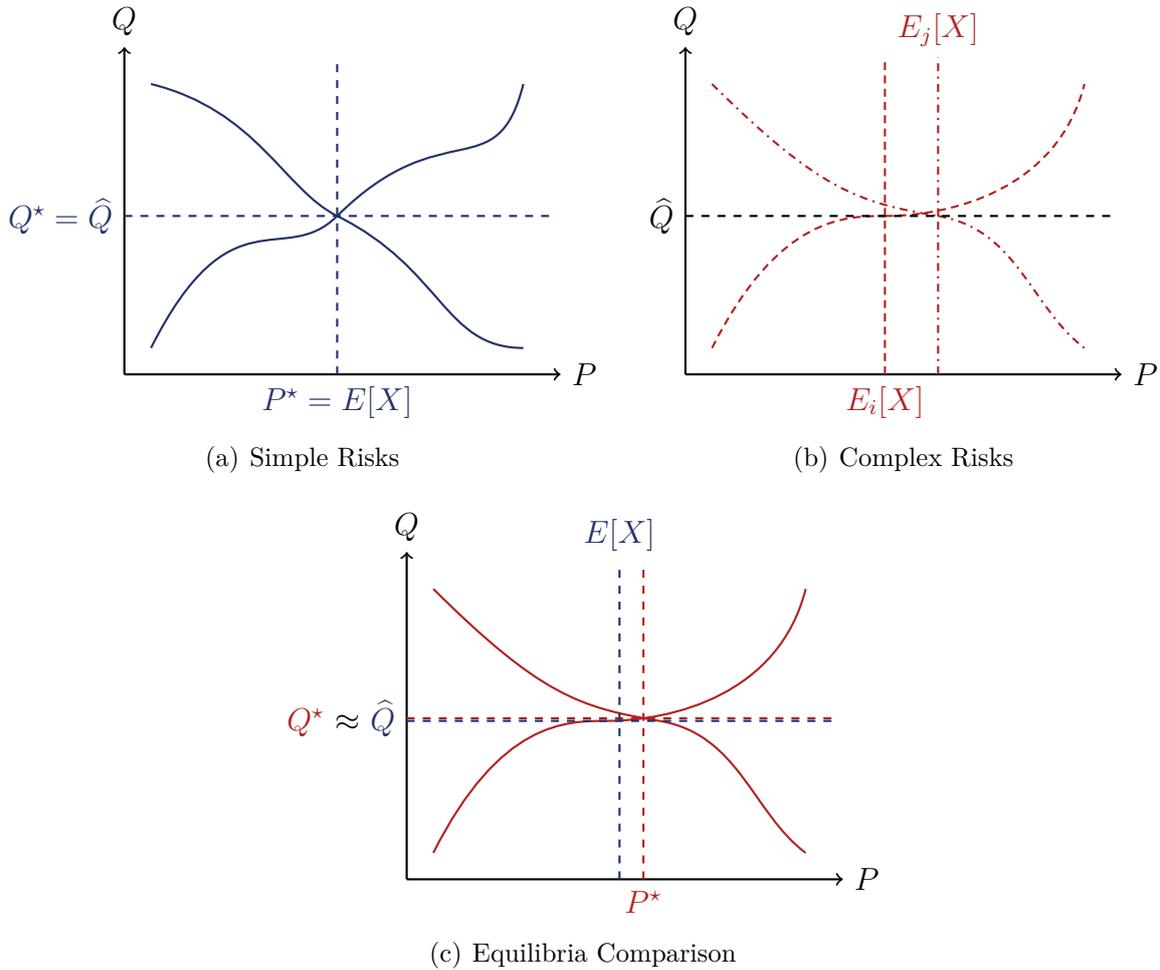
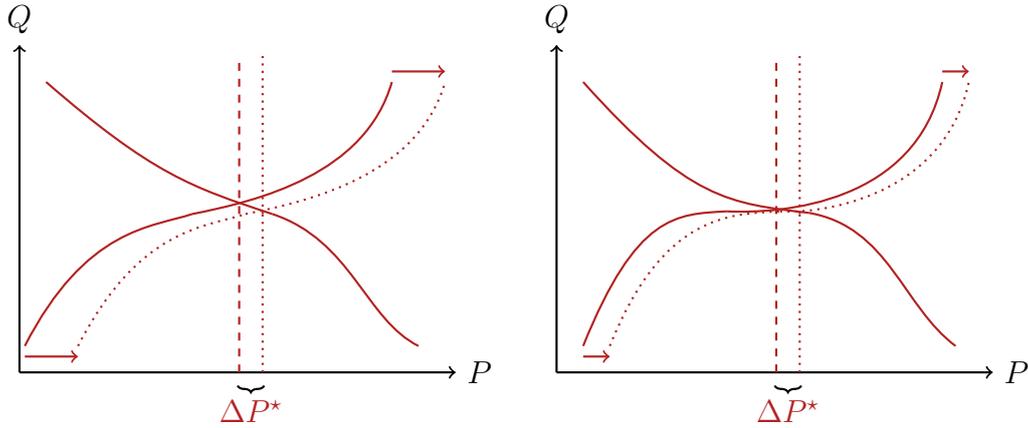


FIGURE 5. GENERAL COMPARISON BETWEEN EQUILIBRIA FOR SIMPLE VERSUS COMPLEX RISKS

Notes: This figure summarizes the main implications of trading simple versus complex risks. Subfigure (a) shows the uniquely defined equilibrium for simple risks. Subfigure (b) shows exemplary supply and demand curves for complex risks of two traders with different subjective beliefs. Subfigure (c) illustrates (i) that, in the presence of complexity, incorrect beliefs can cause mispricing, whereas (ii) the local reduction in price sensitivity mitigates their adverse effect on risk sharing.

Finding conclusive evidence in favour or against the above predictions remains an empirical matter. Crucially, doing so requires compliance with the underlying model assumptions.

Finally, note that, in contrast to risk allocations, the impact of incorrect subjective beliefs on equilibrium prices can be *reinforced* by a complexity-induced decrease in price sensitivity. Figure 6 illustrates how a (relatively) large and small supply shift can lead to



(a) Price impact under low price sensitivity (b) Price impact under very low price sensitivity

FIGURE 6. PRICE IMPACT OF INCORRECT BELIEFS ABOUT COMPLEX RISKS

Notes: This figure illustrates the ambiguous price impact of incorrect subjective beliefs under varying levels of price (in)sensitivity. Subfigure (a) shows the price impact from a large shift of a relatively more price-sensitive supply curve. Subfigure (b) shows the price impact from a small shift of a less price-sensitive supply curve. Due to the lower price sensitivity of the demand curve in Subfigure (b), the two price impacts exactly coincide.

identical price impacts in case of an offsetting difference in demand price (in)sensitivities. However, a negative²¹ (positive) correlation between the magnitude of estimation errors and individual price sensitivities clearly decreases (increases), *ceteris paribus*, the mispricing effect of the former. An empirical correlation analysis should therefore provide valuable insights into the price stability of complex risks in equilibrium.

3.4. Price-taking Behavior, Asymmetric Information, and Strategic Uncertainty

Before turning to the experimental test of the above theory, three potentially interfering effects need to be addressed more carefully. First, my model economy assumes infinitely many agents. When implementing it in the laboratory, complying with this particular assumption constitutes an apparent impossibility. I meet this practical constraint by running all sessions with a relatively high number of at least 16 participants.²² Moreover, I

²¹ As opposed to the supply shifts in Figure 6.

²² This minimum number is in line with the average number of 17.6 participants per session in Biais, Mariotti, Moinas, and Pouget (2017).

alternate between two different pricing schemes: market-clearing—as persistently assumed above—and random price draws (see below). Comparing participants’ supply and demand functions between these two pricing schemes allows me to control for their price-taking behavior.

Second, and more importantly, depending on how agents self-assess their information processing capabilities relative to others, they might perceive considerable information asymmetries in the presence of complex risks. In a [Grossman and Stiglitz \(1980\)](#) rational-expectation equilibrium, market-clearing prices imperfectly reflect informed traders’ costly information about the risky stock’s expected payoff. Applied to my setting, there exists a dominant strategy for (completely) uninformed agents whose implications are in line with the ambiguity preference-based theory above: Agents who perceive themselves as uninformed (i.e., face too high information processing costs) and *simultaneously* believe markets to generate, at least partially, informative prices should always submit perfectly inelastic supply (demand) functions, i.e., $Q_i(P) = \hat{Q} \forall P$.

Somewhat similar to [Grossman and Stiglitz \(1980\)](#), I require some unobservable heterogeneity in agents’ information processing abilities (costs) to prevent market-clearing prices to be fully informative.²³ Otherwise, given the implied *conditionality* of agents’ supply (demand) functions on market-clearing prices, no one would have an incentive to engage in processing complex information in the first place. Thus, [Grossman and Stiglitz’s \(1980\)](#) informational efficiency paradox would prevail.

Third, any further potential implications caused by strategic uncertainty must be accounted for. In a trading game such as the one considered herein, agent i generally faces strategic uncertainty about the behavior of the remaining $-i$ traders. Whenever agent i forms subjective beliefs about her opponents’ actions, these beliefs—whether rationalizable or not—may affect her trading decisions ex-ante.

Alternating between market-clearing and random price draws not only allows for testing the price-taking hypothesis, but additionally enables me to control for *any* potential effects from either perceived asymmetric information or strategic uncertainty.

²³ Whenever agent i believes that there is a nonzero mass of agents submitting supply (demand) functions based on relatively less informative beliefs, she finds herself better off trading according to her own more informative beliefs.

4. Experiment

In this section, I first present the parameterization of the model economy that balances tradable and nontradable income such to eliminate aggregate risk. This is followed by a motivation of the main design feature of my experiment, i.e., the creation of simple and complex risks in the laboratory. Second, I provide a detailed overview of the conducted sessions, including summary statistics and randomizations checks.

4.1. Design and Parameterization

The selection process of the model parameters is twofold. On the one hand, the distribution of the stock's binary dividend needs to be fixed. In order to control for a natural focal point effect, I alternate between two values of π , i.e., $\pi \in \{1/3, 1/2\}$. Furthermore, to simplify calculations of expected payoffs, I set the stock's dividend $X(\omega)$ equal to ECU 150 (experimental currency units) in state u and ECU 0 in state d , respectively.

On the other hand, agents' endowments need to be as such that aggregate consumption is constant across states. Table II presents the endowments for both sellers and buyers that independently apply at the beginning of every trading round. Note, in the presence of equally many sellers and buyers, consumption risk is zero on the aggregate level. In particular, if any seller i and any buyer j agree to trade $\hat{Q} = 2$ shares at a price per share of P , both are perfectly hedged with constant consumption equal to ECU $300 + 2P$ and ECU $600 - 2P$, respectively. The symmetry between sellers' and buyers' potential overall consumption is intentional. When comparing local sensitivities between their supply and demand, symmetry arguments allow me to isolate and solely analyze preference-driven differences.²⁴

Complex versus Simple Risks in the Laboratory

When implemented in the laboratory, complex risks need to satisfy two necessary conditions to generate data that can be analyzed in the light of the above theory:

- (i) complex risks have to follow an *objective* underlying probability distribution, and
- (ii) participants have to be aware of the problem's well-defined nature and the existence of its *unique* solution.

²⁴ Despite the symmetry in total consumption, endowment effects and reference-dependent preferences (see, e.g., [Kahneman, Knetsch, and Thaler \(1991\)](#)) could still be at play. However, I find no evidence of this in my experimental data.

TABLE II. ENDOWMENTS FOR SELLERS AND BUYERS

	Seller	Buyer
Stock	4	0
Bond	0	300
Cont. income $I(\omega)$		
State u : $I(u)$	0	0
State d : $I(d)$	300	300
Agg. wealth	constant	

Notes: This table shows the respective endowments for sellers and buyers that apply at the beginning of every independent trading round. All figures except the number of shares are in experimental currency units (ECU). The state-contingent nontradable income $I(\omega)$ exactly offsets the aggregate risk from stock endowments.

Moreover, when aiming for informative empirical data, the (imperfect) information about complex risks should:

- (iii) not be too complex, i.e., imposing nontrivial restrictions on participants' sets of priors, but still be complex enough such that subjective priors neither are singletons.

I argue that the following implementation satisfies (i), (ii), and (iii). Consider the geometric Brownian motion shown in Subfigure (a) of Figure 7. In the 'complex risk treatment', participants were provided with both the *dynamic visualization* of a reference path between $t = 0$ and $t = 1$, as well as the *formal specification* of the stochastic differential equation governing its evolution. To map this continuous process S_t into the required binary payoff distribution,²⁵ a simple threshold approach was applied. More specifically, whenever the reference path in $t = 2$ was greater or equal than a predefined threshold L , i.e., if $S_2 \geq L$, the risky stock paid a dividend $X(u)$ equal to 150 and zero otherwise. As demonstrated in Appendix B, the problem of determining $\mathbb{P}(S_2 \geq L)$ in Figure 7 can be solved with a back-of-the-envelope calculation applying Itô calculus. Essentially, this implementation of complex risks links the stock's dividend to the payoff of a digital option in a Black and Scholes (1973) model.

Before submitting their respective supply (demand) functions during the first stage of trading complex risks, participants were presented the type of information displayed in Subfigure (a) of Figure 7. Simultaneously, they were given the possibility to repeatedly observe the reference path's dynamic evolution between $t = 0$ and $t = 1$. Across complex

²⁵ Not to be confused with seller i 's share endowment S_i in Section 3.

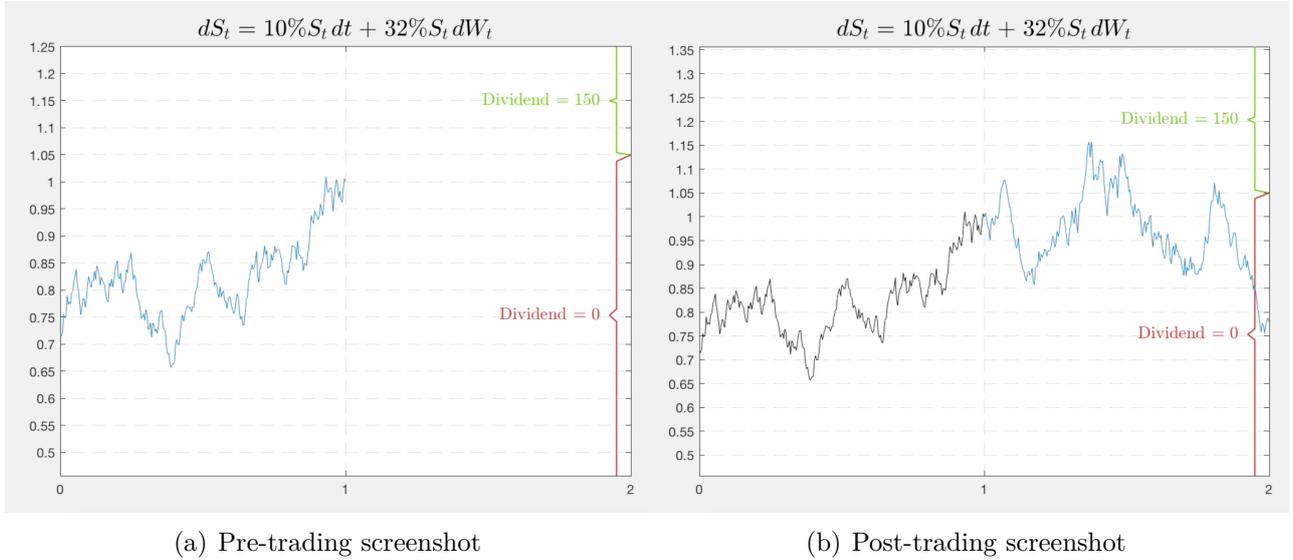


FIGURE 7. COMPLEX RISKS IN THE LABORATORY

Notes: This figure shows the information about complex risks participants were provided with during the experiment. For the first stage, Subfigure (a) presents an example of the information displayed on participants’ screens when asked to enter their supply (demand) schedules. Whenever the blue reference path ends up in the green region, the stock pays a dividend per share equal to ECU 150 (experimental currency units) and zero otherwise. Given the here considered parameterization, Appendix B shows that the former probability equals $1/2$. For the second stage, Subfigure (b) presents a possible realization of the process and the stock’s corresponding payoff per share.

trading rounds, two different parameterizations of S_t were used—one for $\pi = 1/3$ and one for $\pi = 1/2$, respectively—whereas the realized path was unique to every round. At the second stage, participants were informed about their number of shares sold (bought) and were presented with the realization of S_2 as shown in Subfigure (b).

For submitting their supply (demand) schedules, participants faced—similar as in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#)—a predefined price vector. The increment of the uniformly spaced price vector was set to five ECU, i.e., for every $P \in \{0, 5, 10, \dots, 145, 150\}$, participants were asked to choose the preferred number of shares to be sold (bought).²⁶ These quantities were reported with a precision of two decimal places.

To test the above theoretical predictions more precisely, it is helpful to control for participants’ subjective beliefs about complex risks. This is achieved in the following way. During the first stage of complex trading rounds, participants were additionally

²⁶ In order to minimize the number of necessary keyboard entries, the decision process was divided into two substages (see the experimental instructions in Appendix E for details).

asked to provide their point estimate regarding the stock’s expected payoff per share.²⁷ Independently of subjective preferences, the so elicited point estimates allow to anchor participants’ individual sets of priors.

In contrast, during the first stage of the ‘simple risks treatment’, participants *knew* the exact probability of the stock paying a dividend equal to 150. For the case where $\pi = 1/2$, participants were confronted with an urn containing 15 green and 15 red balls, as depicted in Subfigure (a) of Figure 8. At the second stage, the color of one randomly drawn ball was revealed. Whenever this ball happened to be green, the stock paid a dividend per share equal to ECU 150 and zero otherwise.²⁸ Finally, as a control treatment, the tradable risks of the last trading round were purely ambiguous. Instead of a ‘transparent urn’, participants were confronted with the [Ellsberg \(1961\)](#)-like urn shown in Subfigure (b), whose composition of green and red balls was unknown.

For both treatments, two different pricing schemes were applied: market clearing versus random price draws. Whereas the former maximizes trade by minimizing the difference between supply and demand,²⁹ the latter randomly picks one price from the given price vector, each with equal probability.³⁰

4.2. Sessions Structure, Incentivization, and Participant Summary Statistics

Table III provides an overview of the six sessions conducted in the ‘Laboratory for Experimental and Behavioral Economics’ at the University of Zurich during fall 2016.³¹ The number in parenthesis indicates the number of participants in a given session.³² Each session consisted of ten independent trading rounds. All participants only participated in one session. For every single trading round, Table III lists the actual payoff distribution, the nature of the underlying consumption risk, simple (S) versus complex (C), and the applied pricing scheme, market clearing (MC) versus random price draw (random). A

²⁷ Depending on participants’ respective preferences, the risky asset’s expected payoff under complex risks is either defined by the mean of Eq. (3) for trading more or less than \hat{Q} shares, or by Eq. (6), respectively.

²⁸ Similarly, for $\pi = 1/3$, the presented urn contained ten green and 20 red balls.

²⁹ To prevent any effects due to anticipated rationing, following [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), all orders at the market-clearing price were fully executed.

³⁰ The inherent logic of the random price draw is equivalent to the standard mechanism proposed by [Becker, DeGroot, and Marschak \(1964\)](#).

³¹ The experiment was fully computerized using z-Tree ([Fischbacher, 2007](#)).

³² To be eligible, participants were required to have some basic finance knowledge (i.e., major or minor in finance, internship or other work experience in the field of finance, or free-time trading experience).

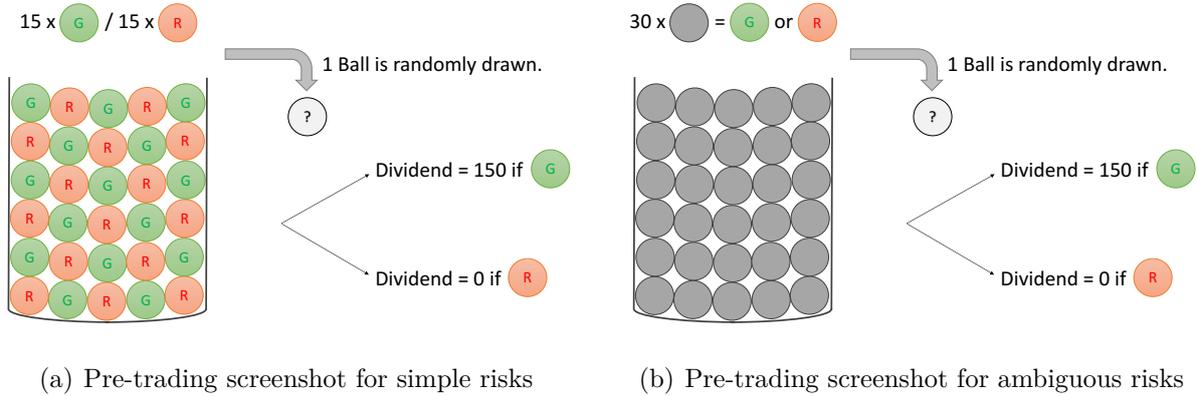


FIGURE 8. SIMPLE AND AMBIGUOUS RISKS IN THE LABORATORY

Notes: This figure shows the information about simple and ambiguous risks participants were provided with at the first stage during the respective trading rounds of the experiment. Whenever the randomly drawn ball is green, the stock pays a dividend per share equal to ECU 150 (experimental currency units) and zero otherwise. In contrast to simple risks in Subfigure (a), the distribution of green and red balls in Subfigure (b) is arbitrary.

‘high’ (‘low’) π refers to an integer parameterization of the stochastic reference path S_t that results in a probability $\mathbb{P}(S_2 \geq L)$ of 84.21% (15.89%). ‘P’ denotes a practice round. To control for potential ‘comparative ignorance effects’ (see [Fox and Tversky \(1995\)](#)), the sequential ordering of simple and complex risks was reversed between the first three and the last three sessions.

In each session, after the ten trading rounds shown in [Table III](#), participants were additionally presented with two lotteries, each based on one of the two urns in [Figure 8](#). For both lotteries, their certainty equivalents were elicited via [Abdellaoui, Baillon, Placido, and Wakker’s \(2011\)](#) computerized iterative choice list method.³³ Importantly, the lotteries’ payoffs were chosen such that they exactly matched the range of possible consumption levels in each of the previous trading rounds (see [Figure D.2](#) in [Appendix D](#)). Overall, one session lasted approximately 90 minutes.

At the end of every session, one out of the seven nonpractice trading rounds *or* one of the two lottery outcomes was randomly chosen, each with equal probability. Participants then were paid either their final wealth of the selected trading round or the outcome of the selected lottery (divided by twelve in either case). Additionally, if their point estimate regarding π was correct (within $\pm 3\%$), they earned an extra three Swiss francs, whenever

³³ No risk or ambiguity aversion is assigned to participants with multiple switching points for either the simple (one) or the ambiguous lottery (two).

TABLE III. SESSIONS OVERVIEW

Round	Session 1 (#16)			Session 2 (#18)			Session 3 (#16)		
	π	Type	Pricing	π	Type	Pricing	π	Type	Pricing
1	1	C (P)	MC	1	C (P)	MC	1	C (P)	MC
2	high	C (P)	random	high	C (P)	random	high	C (P)	random
3	low	C (P)	MC	low	C (P)	MC	low	C (P)	MC
4	1/2	C	MC	1/3	C	random	1/3	C	MC
5	1/3	C	MC	1/2	C	random	1/3	C	random
6	1/2	C	random	1/3	C	MC	1/2	C	MC
7	1/3	C	random	1/2	C	MC	1/2	C	random
8	1/2	S	MC	1/2	S	random	1/2	S	MC
9	1/3	S	random	1/3	S	MC	1/3	S	random
10	ambig	A	MC	ambig	A	random	ambig	A	MC

Round	Session 4 (#16)			Session 5 (#16)			Session 6 (#16)		
	π	Type	Pricing	π	Type	Pricing	π	Type	Pricing
1	1/2	S (P)	MC	1/2	S (P)	MC	1/2	S (P)	MC
2	9/10	S (P)	random	9/10	S (P)	random	9/10	S (P)	random
3	1/2	S	MC	1/2	S	random	1/2	S	MC
4	1/3	S	random	1/3	S	MC	1/3	S	random
5	high	C (P)	MC	high	C (P)	MC	high	C (P)	MC
6	1/2	C	MC	1/3	C	random	1/3	C	MC
7	1/3	C	MC	1/2	C	random	1/3	C	random
8	1/2	C	random	1/3	C	MC	1/2	C	MC
9	1/3	C	random	1/2	C	MC	1/2	C	random
10	ambig	A	MC	ambig	A	random	ambig	A	MC

Notes: This table provides an overview of the six conducted sessions. Each session consisted of ten independent trading rounds. The number in parenthesis indicates the number of participants in a given session. For every session, the first column lists the actual payoff distribution, the second column the nature of the underlying consumption risk (simple (S) versus complex (C)), and the third column the applied pricing scheme (market clearing (MC) versus random price draw (random)). The ‘high’ (‘low’) π refers to an integer parameterization of the geometric Brownian motion that implies a 84.21% (15.89%) probability of a dividend per share equal to 150. Trading rounds with a ‘P’ in parenthesis are practice rounds.

the corresponding trading round was selected for payment. On average, participants received 38.40 Swiss francs, with a maximum of CHF 50 and a minimum of CHF 25.

Recruited participants were students from either the University of Zurich or ETH Zurich, majoring in economics, business, mathematics, physics, engineering, or computer science, respectively. Their respective role of either a buyer or a seller was randomly assigned at the beginning of the experiment and thereafter retained throughout all trading rounds. The instructions provided to participants acting as sellers are provided in

TABLE IV. SUMMARY STATISTICS AND RANDOMIZATION CHECK

Variable	Total sample ($N = 98$)	Sellers ($N = 49$)	Buyers ($N = 49$)	p -value
Age	23.674 (3.008)	23.837 (3.287)	23.510 (2.724)	0.689
Gender	0.337 (0.475)	0.286 (0.456)	0.388 (0.492)	0.393
UZH (ETH)	0.582 (0.496)	0.653 (0.481)	0.510 (0.505)	0.219
# semesters	3.806 (2.827)	3.633 (2.928)	3.980 (2.742)	0.365
Knowledge BM	0.459 (0.501)	0.367 (0.487)	0.551 (0.503)	0.105
Risk aversion	0.060 (0.265)	0.087 (0.294)	0.035 (0.232)	0.328
CRRA- γ	0.684 (3.358)	1.045 (4.415)	0.323 (1.740)	0.335
Ambiguity aversion	0.101 (0.245)	0.067 (0.225)	0.133 (0.262)	0.133

Notes: This table reports means and standard deviations (in parenthesis) in the total sample and across sellers and buyers, respectively. p -values for the null hypothesis of perfect randomization are listed in the last column (Wilcoxon signed rank tests for interval variables and [Yates \(1934\)](#)’ corrected χ^2 tests for binary variables). ‘Age’ is reported in years. ‘Gender’ and ‘UZH’ are dummy variables indicating female participants and students from the University of Zurich (versus ETH). ‘# semesters’ denotes the number of completed semesters. ‘Knowledge BM’ is a dummy variable equal to one for participants who have heard about the mathematical object ‘Brownian motion’ before. Risk aversion is measured as the normalized difference ($\in [-1, 1]$) between the simple lottery’s expected payoff and participants’ respective certainty equivalents. CRRA- γ denotes the corresponding constant relative risk aversion coefficient. Ambiguity aversion is measured as the individual differences in certainty equivalents between the simple and ambiguous lottery.

Appendix E.³⁴

Table IV presents the average values (proportions) of certain socioeconomic variables collected via a short questionnaire following the main experiment. Risk aversion is measured as the normalized difference between the simple lottery’s expected payoff and participants’ respective certainty equivalents. A value of one (minus one) denotes maximum

³⁴ Analogous instructions were provided to participants acting as buyers and are available upon request.

(minimum) risk aversion,³⁵ a value of zero implies risk-neutrality. The total sample’s average risk aversion of 0.060 corresponds to a constant relative risk aversion (CRRA) coefficient of 0.684.³⁶ Ambiguity aversion is defined as the individual differences in certainty equivalents between the simple and ambiguous lottery. Hence, a positive value indicates ambiguity aversion. A standard randomization check reveals no significant indications of an unbalanced sample.³⁷

5. Results

This section presents the analysis of the experimental data. I investigate trading decisions and outcomes both at the aggregate and the individual level. For the latter, I construct two different measures of price sensitivity based on the theory in Section 3. The increasing discrepancy between individual behavior and aggregate outcomes for simple relative to complex risks is linked to varying bounds on quasi-rational choice. Finally, I investigate markets’ effectiveness in aggregating traders’ imperfect information about complex risks.

5.1. Aggregate Market Outcomes

For both dividend distributions and for both types of risks, Figure 9 plots all individual and the corresponding average supply and demand curves. This first glance of the empirical results showcases three important findings: First, despite extensive heterogeneity in individual behavior, average supply and demand curves are smooth and well-behaved. Second, average market-clearing prices are close to expected payoffs for simple risks and almost identical to average estimations thereof for complex risks. Third, notwithstanding the vast variation in subjective beliefs (see below), average market-clearing allocations of complex risks are generally closer to the perfect hedging quantity than under simple risks.

Figure 10 shows average supply and demand curves across sessions with identical ordering of simple versus complex risks. Independently of whether participants first trade simple or complex risks, the price sensitivity of average supplies and demands locally decreases around (average) expected payoffs under complexity.

³⁵ According to Abdellaoui, Baillon, Placido, and Wakker’s (2011) iterative choice list method.

³⁶ My estimate of average relative risk aversion is in line with the experimental literature: see Holt and Laury (2002) for binary lotteries, Goeree, Holt, and Palfrey (2002) for private value auctions, Goeree, Holt, and Palfrey (2003) for generalized matching pennies games, and Goeree and Holt (2004) for one-shot matrix games. Similarly, Biais, Mariotti, Moinas, and Pouget (2017) find the representative investor’s CRRA coefficient to approximately equal 0.5.

³⁷ Throughout the entire paper, I report two-sided p -values.

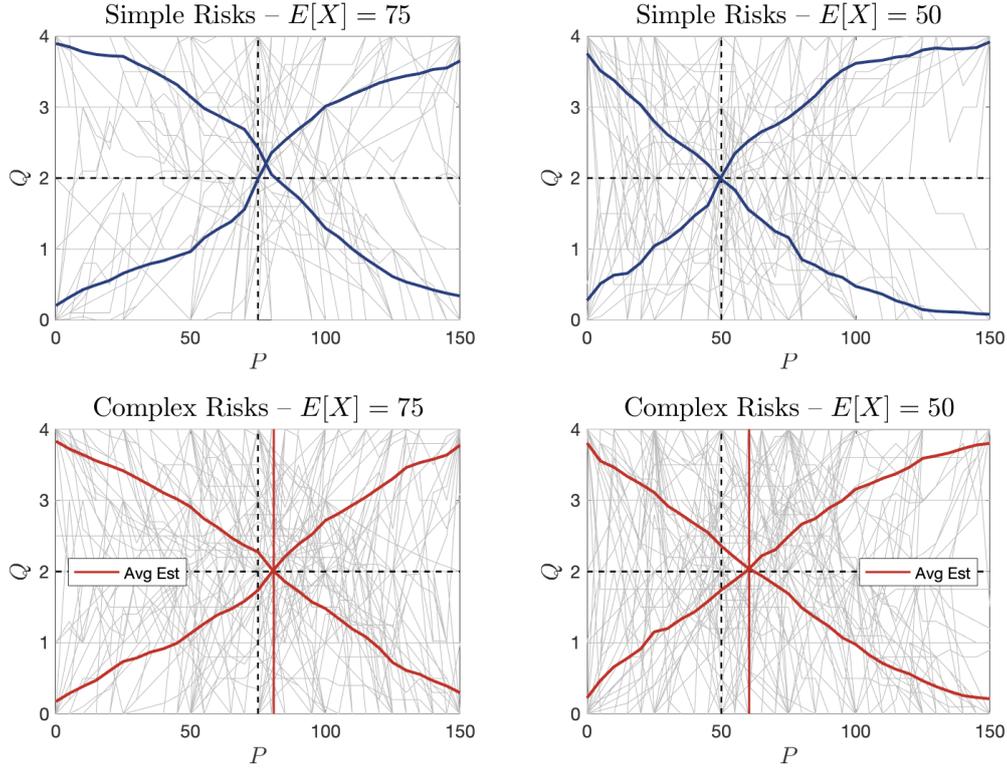


FIGURE 9. FROM INDIVIDUAL TO AVERAGE SUPPLY AND DEMAND

Notes: This figure shows individual and average (across participants and sessions) supply and demand curves for trading rounds with simple and complex risks. In the top (bottom) row, averages are computed for simple (complex) risks. In the left (right) column, averages are computed across trading rounds where π equals $1/2$ ($1/3$). The dotted horizontal line indicates the perfect hedging quantity. The dotted vertical line indicates the risky asset's expected payoff. The solid vertical line indicates participants' average point estimate of the risky asset's expected payoff under complex risks.

In accordance with Figure 10, Table V reports average market-clearing prices and quantities as well as average point estimates of expected payoffs for complex risks. Moreover, columns four and five of Table V list the degree of mispricing and suboptimal risk sharing as defined in Section 3. Unsurprisingly, mispricing is clearly less pronounced under simple than under complex risks. Complex risks are overpriced by approximately 8% (20%) for $\pi = 1/2$ ($\pi = 1/3$). This indicates that participants generally overestimate the impact of the reference path's positive drift relative to its volatility.³⁸ On average, both simple and complex risks are well shared. Strikingly, in three out of four cases, the degree

³⁸ Such misestimation is in line with participants failing to adjust for the second order effect from the Brownian motion's nonzero quadratic variation.

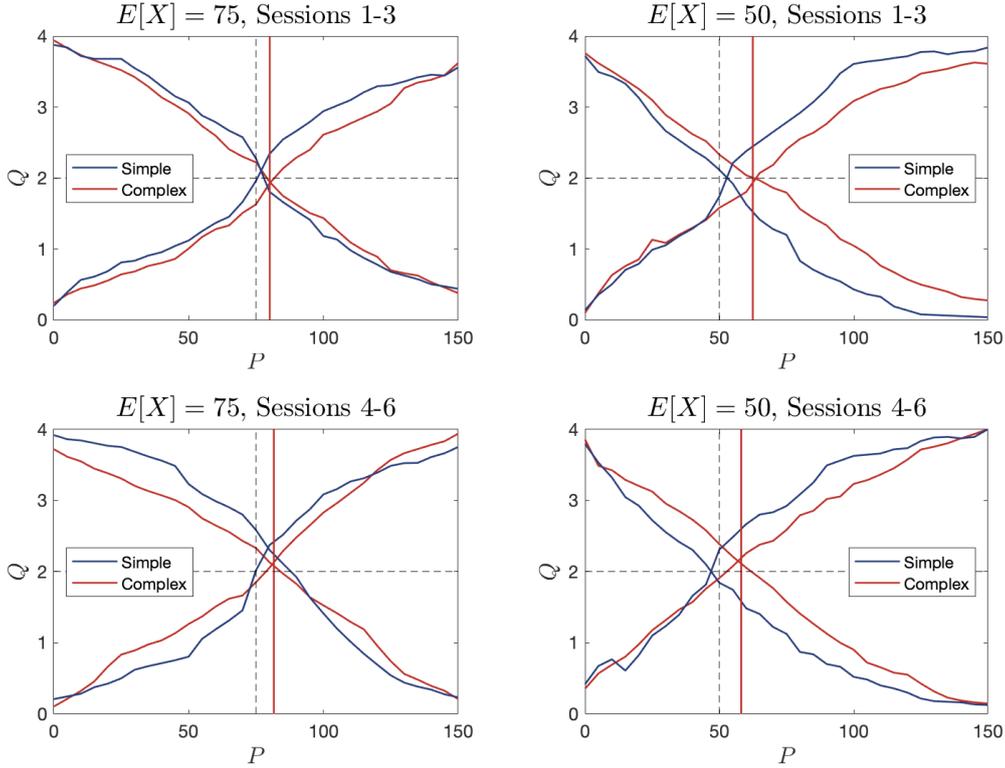


FIGURE 10. AVERAGE SUPPLY AND DEMAND

Notes: This figure shows the average supply and demand curves across participants and trading rounds. In the top (bottom) row, averages are computed across sessions where complex (simple) trading rounds are followed by simple (complex) trading rounds. In the left (right) column, averages are computed across trading rounds where π equals $1/2$ ($1/3$). The dotted horizontal line indicates the perfect hedging quantity. The dotted vertical line indicates the risky asset's expected payoff. The solid vertical line indicates participants' average point estimate of the risky asset's expected payoff under complex risks.

of risk sharing between buyers and sellers is higher or equal under the latter. This is remarkable given the extensive variation in subjective beliefs about complex risks (see Figure D.3 in Appendix D).

To better visualize local differences in price sensitivity, I adjust the average supply and demand curves under complex risks to control for subjective beliefs. Essentially, the price grid in Figure 10, against which individual curves are plotted, is adapted such that average payoff estimates coincide with true expectations (see Appendix C for details). Figure D.4 in Appendix D presents the adjusted supply and demand curves under complex risks. In contrast to Figure 10, it allows for a direct comparison of price sensitivities. For prices

TABLE V. AVERAGE MARKET-CLEARING PRICES AND QUANTITIES

	Market clearing		Avg Point Estimate	Mispricing	Suboptimal risk sharing
	P^*	Q^*	$\text{Avg}(E_i[X])$	$ P^* - E[X] $	$ Q^* - \hat{Q} $
<i>Simple risks</i>					
$\pi = 1/2$					
Sessions 1-3	76.87	2.10	–	1.87	0.10
Sessions 4-6	79.47	2.33	–	4.47	0.33
$\pi = 1/3$					
Sessions 1-3	52.82	2.01	–	2.82	0.01
Sessions 4-6	46.89	2.00	–	3.11	0.00
<i>Complex risks</i>					
$\pi = 1/2$					
Sessions 1-3	80.30	1.94	80.10	5.30	0.06
Sessions 4-6	80.98	2.10	81.62	5.98	0.10
$\pi = 1/3$					
Sessions 1-3	63.47	1.99	62.46	13.47	0.01
Sessions 4-6	56.98	2.15	58.13	6.98	0.15

Notes: This table reports average market-clearing prices and quantities across sessions with identical ordering of trading rounds involving simple and complex risks, respectively. Moreover, average point estimates of expected payoffs as well as the measures of mispricing and suboptimal risk sharing as defined in Section 3 are listed in columns three to five.

close to but below $E[X]$, all four supply curves for simple risks lie below the respective supply curves for complex risks, only to cross the latter for prices close to but (generally) higher than $E[X]$ (dotted vertical lines in Figure D.4). The opposite holds true for the two demand curves where π equals $1/2$ (left column of Figure D.4). For π equal to $1/3$, demand curves coincide for very low prices, but are higher in the case of complex risks for prices around and above $E[X]$.

For a more systematic investigation of the supply and demand functions depicted in Figure D.4, I plot the respective averages across all sessions (to reduce noise) together with their corresponding error bounds, indicating standard errors of the mean. The resulting supply and demand curves are shown in Figure 11. The above described pattern now manifests itself more clearly. For π equal to $1/2$ (left column of Figure 11), the average supply (demand) for simple risks crosses the respective supply (demand) for complex

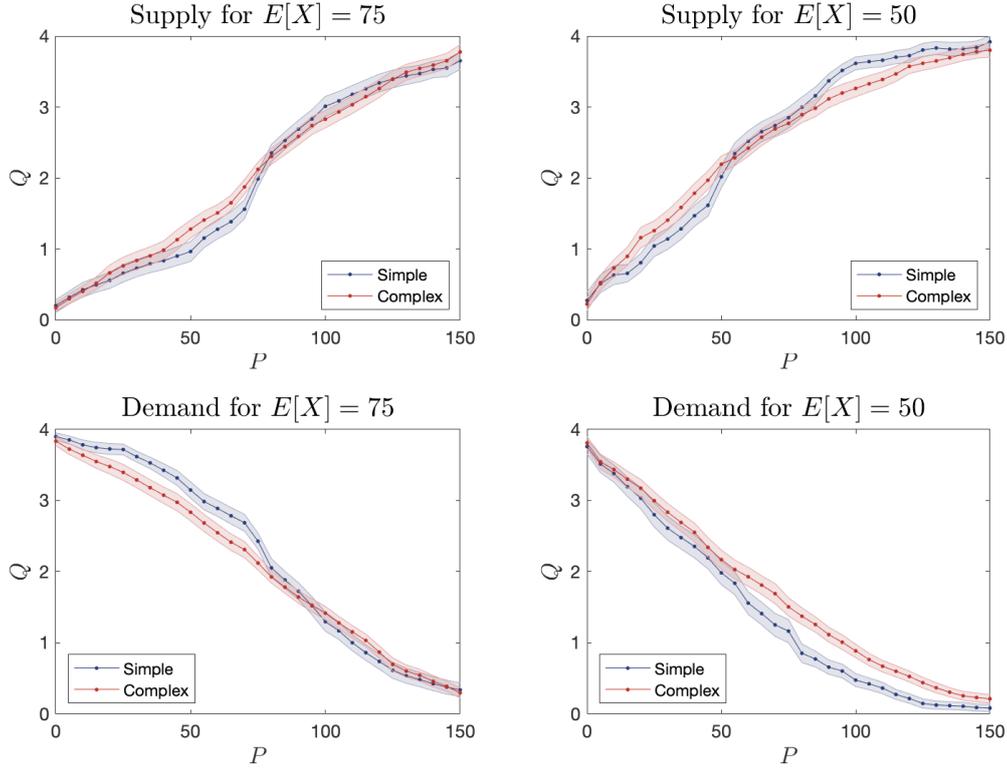


FIGURE 11. DIFFERENCES IN AVERAGE SUPPLY AND DEMAND FOR SIMPLE AND COMPLEX RISKS

Notes: This figure shows the average adjusted supply and demand curves across participants and trading rounds. Average curves for complex risks are adjusted as described in Appendix C to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across trading rounds where π equals $1/2$ ($1/3$).

risks from below (above). For π equal to $1/3$, the same is true for sellers, whereas for buyers, average demands converge at a price close to the risky stock's expected payoff. Furthermore, in all four cases, there is a clear difference in sensitivity for prices close to expected payoffs.

I test the statistical significance of the differences in Figure 11 by conducting a Wilcoxon signed-rank test, where, in the case of complex risks, I use interpolated quantities. The results are plotted in Figure D.5 in Appendix D. As conjectured, the average supply curves are statistically different for prices below and above expected payoffs. In case of π equal to $1/2$, the same statistically significant hump-shaped pattern around $E[X]$ is observed for average demand curves. For π equal to $1/3$, respective p -values are close to

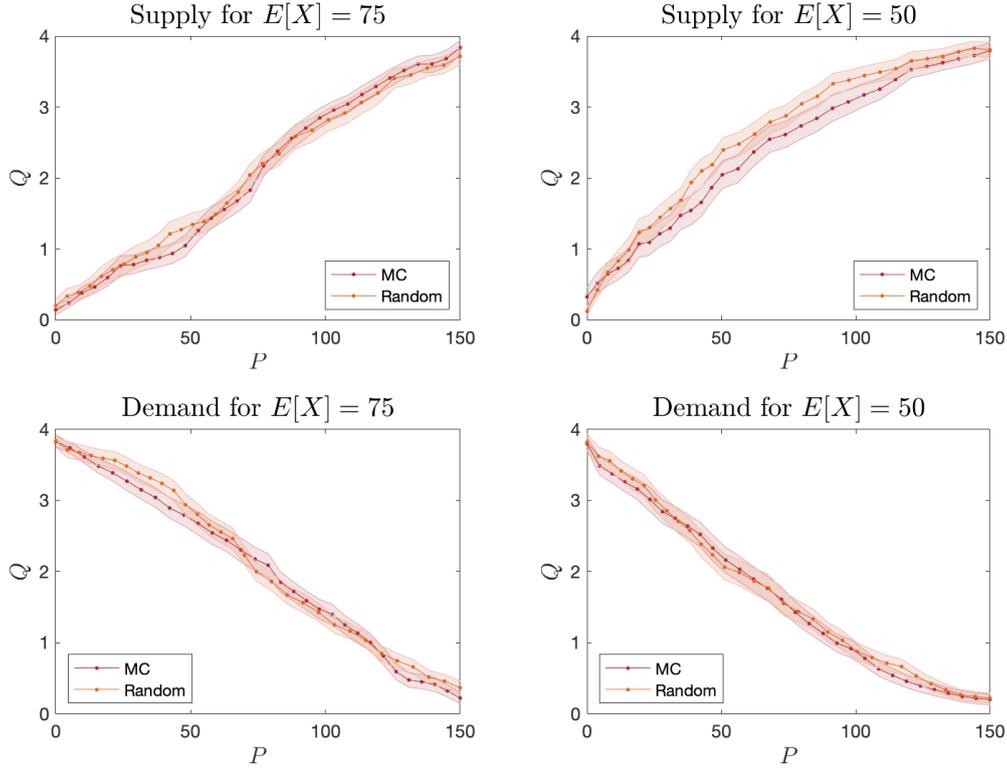


FIGURE 12. AVERAGE SUPPLY AND DEMAND ACROSS PRICING SCHEMES

Notes: This figure shows the average adjusted supply and demand curves for complex risks across participants and the two different pricing schemes: market clearing (MC) and random price draws (random). Average curves are adjusted as described in Appendix C to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across complex trading rounds where π equals $1/2$ ($1/3$).

0.1 below $E[X]$, temporarily increase around $E[X]$, and decrease again sharply to values close to zero thereafter.

Naturally, an analogous analysis lends itself to contrast participants' behavior between the two applied pricing mechanisms: market clearing and random price draws. Figure 12 presents the respective supply and demand curves averaged across complex trading rounds with equal pricing schemes. Overall, average supplies and demands for complex risks look very similar between the two pricing mechanisms. The p -values of the corresponding Wilcoxon signed-rank test are plotted in Figure D.6 in Appendix D. The patterns in Figure D.6 indicate that there exists no statistical evidence against the null hypothesis of a globally (across pricing schemes) adopted price-taking behavior. Hence, markets behave

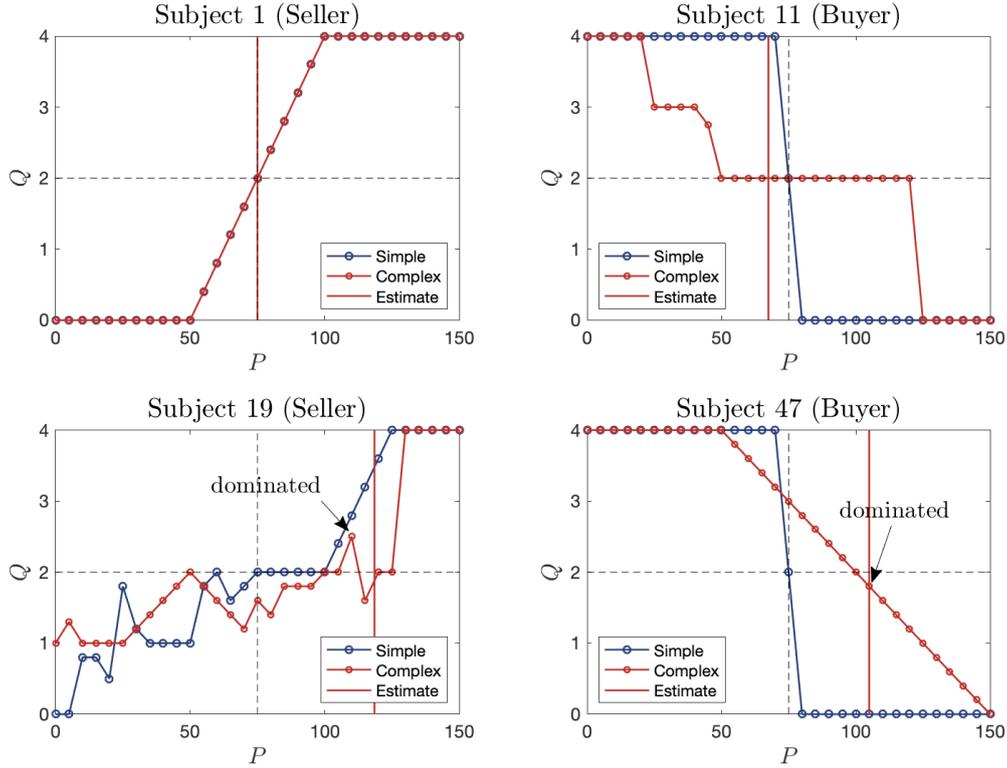


FIGURE 13. SELECTED INDIVIDUAL SUPPLY AND DEMAND CURVES

Notes: This figure shows selected individual supply and demand curves for four different participants. Only trading rounds with π equal to $1/2$ are shown. The solid vertical line indicates participants' point estimate of the risky asset's expected payoff under complex risks.

competitively and neither asymmetric information nor strategic uncertainty affects local price sensitivity under complex risks.

5.2. Individual Behavior

Aggregate market outcomes appear to corroborate the predictions from theory: Equilibrium quantities are less price-sensitive under complex than simple risks, thereby mitigating the suboptimality in the allocation of the former. However, as Figure 13 illustrates by using data from four selected participants, individual behavior is very heterogeneous. For instance, subject 1's trading decisions are unaffected by complexity, whereas the behavior under complex risks displayed by participants 11 and 19 indicates kinked and smooth preferences, respectively. Additionally, participant 19's supply and participant 47's demand curves foreshadow an increased error-proneness under complex risks. Therefore,

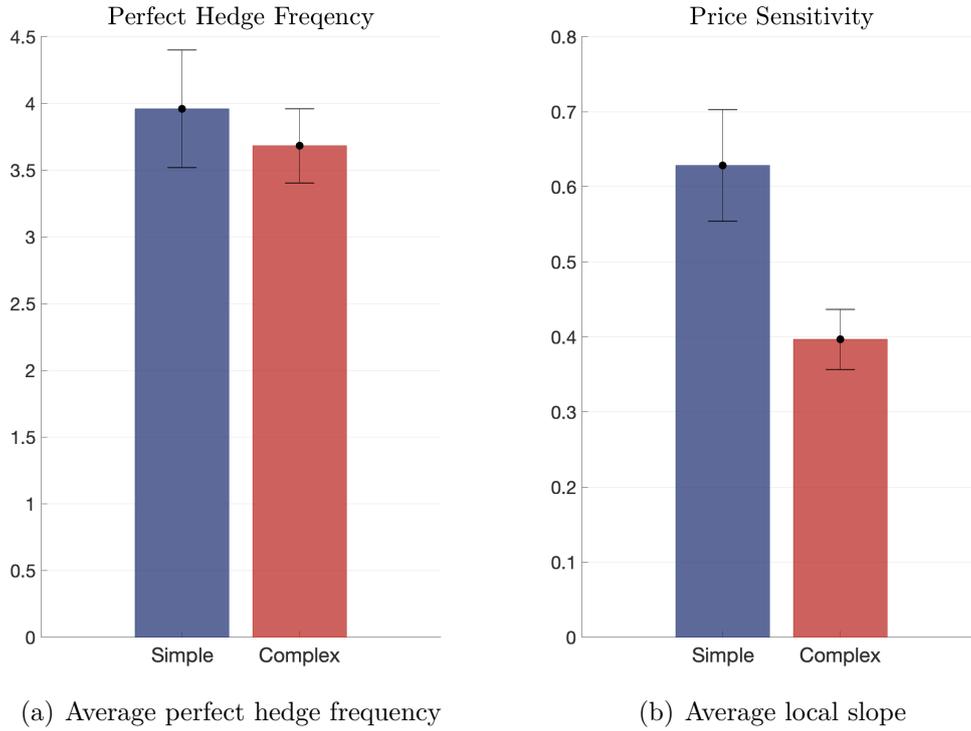


FIGURE 14. INDIVIDUAL MEASURES OF PRICE SENSITIVITY

Notes: This figure shows average individual trading behavior under simple and complex risks across all participants. Subfigure (a) plots the average number of prices for which participants adopt the perfect hedging strategy, i.e., choosing to trade \hat{Q} shares (see Eq. (7)). Subfigure (b) plots the average slope of participants' supply and demand curves at their individual point estimates of the risky asset's expected payoff (see Eq. (8)). Error bars indicate standard errors of the mean.

and to ensure that the previous results are not simply due to averaging, I now turn to the testing of the theory's general predictions at the individual level. To do so, I propose two different measures for the local price sensitivity of individual supply and demand curves.

First, starting from a quantity perspective, I count for each participant i the number of prices for which her submitted supply (demand) schedule equals the perfect hedging quantity \hat{Q} shares, i.e.,

$$HF_i := |\{Q = \hat{Q}\}_i|, \quad (7)$$

where bars denote the cardinality of the considered set.

Second, starting from a pricing perspective, I compute the slope of each participant

i 's supply (demand) function at her individual point estimate $E_i[X]$, i.e.,

$$Slope_i := \Delta Q_i(E_i[X]), \quad (8)$$

where for sellers

$$\Delta Q_i(E_i[X]) \hat{=} Q_i(P_{l+2}) - Q_i(P_l),$$

and for buyers

$$\Delta Q_i(E_i[X]) \hat{=} Q_i(P_{f-2}) - Q_i(P_f),$$

respectively, with P_l (P_f) denoting the last (first) price strictly below (above) seller (buyer) i 's point estimate $E_i[X]$. The ' ± 2 ' in the index ensures that $P_l < E_i[X] < P_{l+2}$ for sellers and $P_{f-2} < E_i[X] < P_f$ for buyers, respectively. Note, if sellers' (buyers') supply (demand) curves are upward (downward) sloping, $Slope_i$ is expected to be positive. By construction, it can only be computed if P_{l+2} (P_f) is smaller or equal to the maximum price of 150. Under simple risks it holds of course that $E_i[X] = E[X] \forall i \in \mathcal{I}$. Moreover, $Slope$ can be interpreted as a less extreme measure of price sensitivity than HF , where the latter only accounts for perfect price inelasticity, and thus requires kinked preferences.

Figure 14 displays the between-treatment average values of HF and $Slope$ across all participants. Subfigure (a) plots the average frequency with which the perfect hedging strategy is adopted (relative to the total cardinality of the price grid). The average number of prices for which participants choose to trade exactly \hat{Q} shares decrease by 0.278 under complex relative to simple risks (p -value = 0.580, t -test). Average slopes of pooled supply and demand curves are plotted in Subfigure (b). Price sensitivity locally decreases by 0.232 when moving from complex to simple risks (p -value = 0.003, t -test).

The results presented in Figure 14 are somewhat inconclusive. While, from a 'slope perspective', the empirical evidence is in line with the general theoretical predictions, from a 'pure quantity perspective', no significant increase in the average frequency of the perfect hedging strategy is observed. One can think of two possible reasons: (i) participants exhibit smooth ambiguity preferences instead of multiple-priors utility, or (ii) participants more frequently fail to trade in their best interest when risks are complex.

The second argument requires some more elaboration. A priori the theory does not provide any reason, why agents failing to trade \hat{Q} shares at a price equal to $E_i[X]$ should adopt the perfect hedging strategy more frequently under complex relative to simple risks. Put differently, increasing complexity of traded risks may tighten the bounds on traders' rationality, thereby abating the explanatory power of the theory based on kinked preferences.

Complexity Bounds on Rationality

In order to control for varying bounds on rationality, I follow [Biais, Mariotti, Moinas, and Pouget \(2017\)](#) by contrasting individual trading data to a setting of bounded rationality in the spirit of a [McKelvey and Palfrey \(1995, 1998\)](#) quantal response model.³⁹ As proposed by [Luce \(1959\)](#), I hereafter assume that agent i 's trading decisions follow a random choice model. Specifically, for a given price P and under slight misuse of notation, her probability density of trading Q_j shares under *simple* risks is given by

$$f_i(Q_j|P) = \frac{\psi_i E[U_i(Q_j|P)]}{\int \psi_i E[U_i(Q|P)] dQ}, \quad (9)$$

where $\psi_i(\cdot)$ denotes an increasing differentiable function and Q runs from zero to the maximum number of tradable shares.

Since $\psi_i(\cdot)$ is increasing in $E[U_i(Q_j|P)]$, [Eq. \(9\)](#) implies that the likelihood with which agent i decides to sell (buy) Q_j shares is also increasing in $E[U_i(Q_j|P)]$. In other words, the higher the expected utility from trading Q_j shares for a price P , the greater the probability that agent i actually ends up doing so. Hence, the lower the slope of $\psi_i(\cdot)$, the more frequently she deviates from her optimal strategy, i.e., the more severe the bounds on her rationality.

As in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), applying bounded rational behavior as formalized in [Eq. \(9\)](#) to the above theory of trading simple risks imposes the following three implications:⁴⁰

- S1 For $P = E[X]$, the distribution of supplied and demanded shares has a unique mode at \hat{Q} .
- S2 For $P < E[X]$, the distribution of supplied and demanded shares is asymmetric around \hat{Q} and decreasing above (below) \hat{Q} for sellers (buyers).
- S3 For $P > E[X]$, the distribution of supplied and demanded shares is asymmetric around \hat{Q} and decreasing below (above) \hat{Q} for sellers (buyers).

³⁹ A somewhat stricter caveat as in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#) applies: In my competitive setting with sufficiently imperfect price information (see [Section 3.4](#)), agents solely trade according to their own beliefs. By differentiating between market-clearing and random pricing, I control for any deviations from such individual behavior.

⁴⁰ In contrast to [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), risk-aversion, i.e., the accordance of agents' expected utilities with second order stochastic dominance, is a necessary condition for the decreasing quantity distributions for prices different than $E_i[X]$. If $\pi = 1/2$, which always holds in [Biais, Mariotti, Moinas, and Pouget \(2017\)](#), first order stochastic dominance is sufficient.

According to Proposition 1, every risk-averse agent whose rationality is bounded as in Eq. (9) most likely chooses to trade \hat{Q} shares for $P = E[X]$. Similarly, such agents more often adopt nondominated instead of dominated strategies. Moreover, for a given price, the bigger the distance between a dominated quantity and the corresponding set of dominating trades, the less frequently should they opt for the former. Because of the randomness implied by Eq. (9), all three implications are convergence results. Hence, if actual behavior is indeed governed by Eq. (9), whether the limited number of participants in my sample suffices to yield according results remains again an empirical question.

Accordingly, under *complex* risks, a boundedly rational agent i decides to trade Q_j shares for a price P with probability density

$$\underline{f}_i(Q_j|P) = \frac{\psi_i \mathcal{U}_i(Q_j|P)}{\int \psi_i \mathcal{U}_i(Q|P) dQ}, \quad (10)$$

where $\mathcal{U}_i(\cdot)$ denotes agent i 's utility from consumption under her subjective beliefs, assuming either multiple-priors utility (Eq. (2)) or smooth ambiguity preferences (Eq. (5)).

Proposition 5. *The supply and demand curves of infinitely many, ambiguity-averse, and boundedly rational agents who trade simple risks according to Eq. (9) and complex risks according to Eq. (10), respectively, generate the following three distributional properties:*

- C1 For $P = E_i[X]$, the distributions of supplied and demanded shares exhibit a unique mode at \hat{Q} under both $\{f_i\}_{i \in \mathcal{I}}$ and $\{\underline{f}_i\}_{i \in \mathcal{I}}$, where the former is less dispersed than the latter.*
- C2 For $P < E_i[X]$, the distribution of supplied and demanded shares is less asymmetric around \hat{Q} and decreases (increases) less rapidly above (below) \hat{Q} for sellers (buyers) under $\{f_i\}_{i \in \mathcal{I}}$ than under $\{\underline{f}_i\}_{i \in \mathcal{I}}$.*
- C3 For $P > E_i[X]$, the distribution of supplied and demanded shares is less asymmetric around \hat{Q} and increases (decreases) less rapidly below (above) \hat{Q} for sellers (buyers) under $\{f_i\}_{i \in \mathcal{I}}$ than under $\{\underline{f}_i\}_{i \in \mathcal{I}}$.*

Proof. For proof see Appendix A. □

Intuitively, the lower price sensitivity under complex risks implies more frequent deviations from the optimal trading strategy relative to simple risks. Analogously to S1–S3, the three implications C1–C3 require sufficient convergence in distribution, which is only guaranteed as the number of observations approaches infinity.

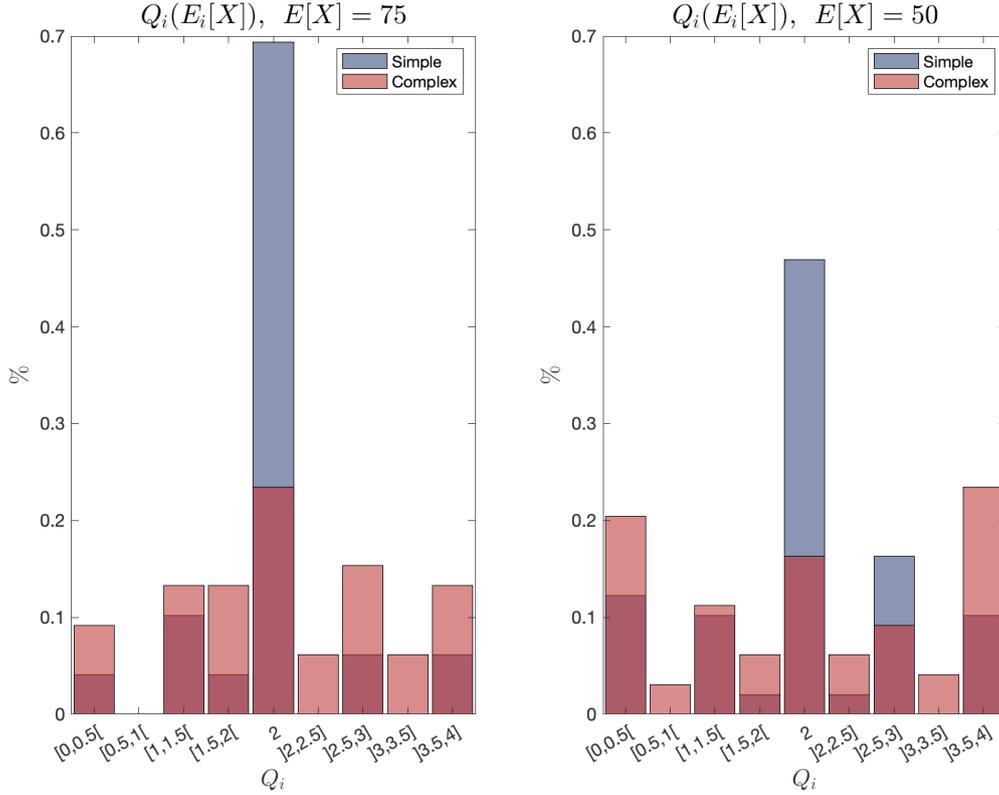


FIGURE 15. SUPPLY DISTRIBUTION FOR PRICES EQUAL TO EXPECTED PAYOFFS

Notes: This figure shows the number of shares supplied by sellers for prices equal to (estimated) expected payoffs. The empirical distributions are computed across participants and sessions. The left (right) plot contrasts average distributions between simple and complex trading rounds with π equal to $1/2$ ($1/3$). If, under complex risks, sellers' point estimate $E_i[X]$ lies between two elements of the predefined price vector, linearly interpolated quantities are reported.

Figure 15 presents the supply distribution for $P = E_i[X]$. While integer numbers are more frequently supplied than fractions of shares, all distributions are roughly symmetric around $\hat{Q} = 2$, constituting the clear mode under simple risks and complex risks with $\pi = 1/2$ (left plot in Figure 15). When moving from simple to complex risks, the frequency of the perfect hedging strategy decreases sharply, i.e., from 0.694 to 0.235 for $\pi = 1/2$ (p -value = 0.000, t -test) and from 0.469 to 0.163 for $\pi = 1/3$ (p -value = 0.000, t -test). In the case of $\pi = 1/3$, the frequencies of the most extreme deviations from \hat{Q} increase considerably under complex risks. These results are in line with both implications S1 and C1.

Figure 16 shows the distribution of shares supplied for $P < E_i[X]$ and $P > E_i[X]$, respectively. Under both simple and complex risks, supplies of less (more) than \hat{Q} shares

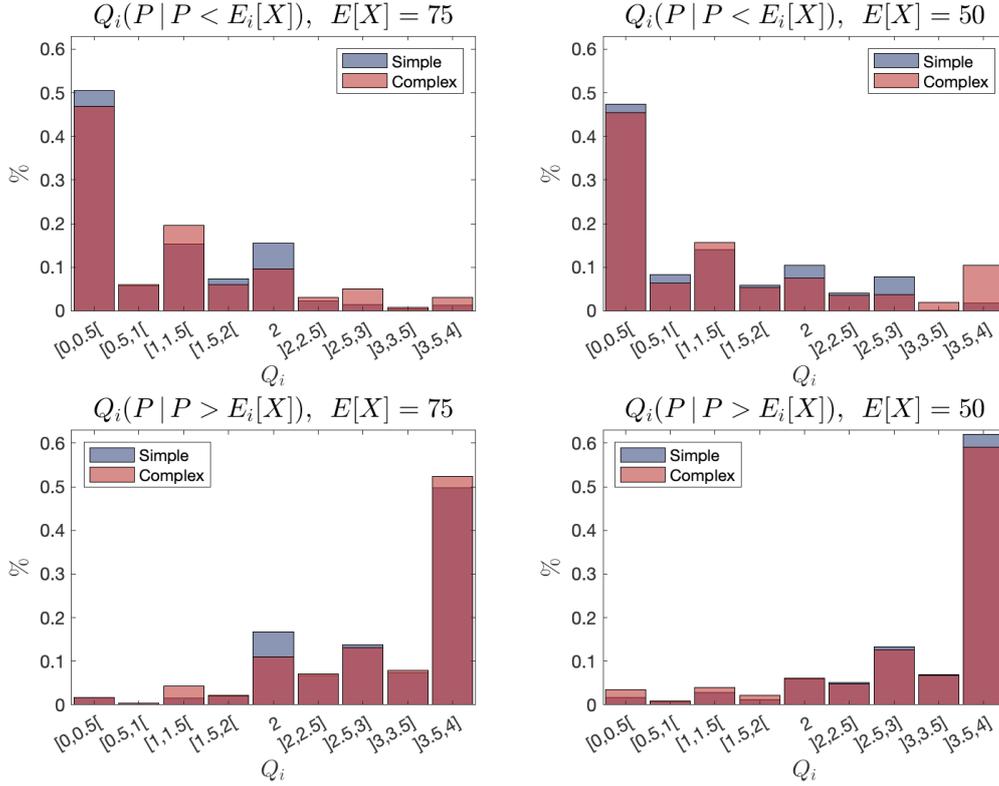


FIGURE 16. SUPPLY DISTRIBUTION FOR PRICES DIFFERENT FROM EXPECTED PAYOFFS

Notes: This figure shows the number of shares supplied by sellers for prices different from expected payoffs. The empirical distributions between simple and complex risks are computed across participants and sessions. In the top (bottom) row, total supplies for prices below (above) $E_i[X]$ are reported. The left (right) column shows average supply distributions across trading rounds with π equal to $1/2$ ($1/3$).

clearly occur most often for low (high) prices. Additionally, except for complex risks with $\pi = 1/3$ (upper right plot in Figure 16), the frequency of supplying more (less) than \hat{Q} shares is decreasing (increasing) in Q_i for $P < E_i[X]$ ($P > E_i[X]$), with generally lower frequency levels under simple risks. The supply distributions presented in Figure 16 reconcile well with the above proposed implications. First, participants more often choose nondominated instead of dominated actions, where the occurrence of the latter is decreasing in their inferiority (see implications S2–S3). Second, under complex risks, participants deviate more frequently from utility-maximizing actions than under simple risks (C2–C3). The analogous analysis of the corresponding demand distributions (see Figure D.7 and Figure D.8 in Appendix D) reveals similar evidence in support of S1–S3 and C1–C3 for buyers.

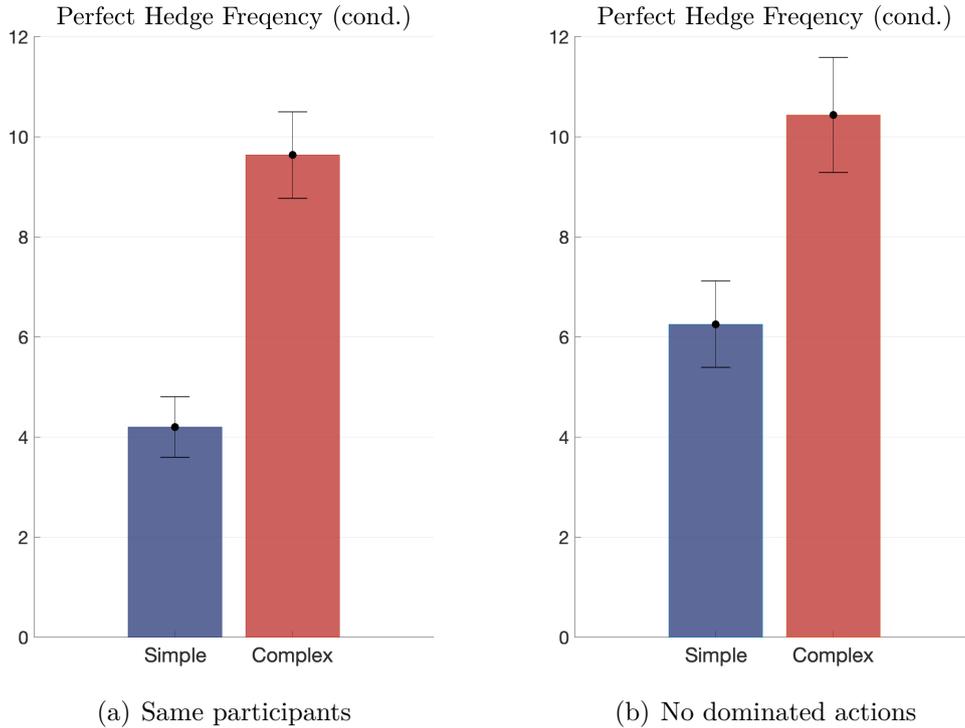


FIGURE 17. CONDITIONAL FREQUENCY OF PERFECT HEDGING STRATEGY

Notes: This figure shows the average frequency of the perfect hedging strategy under simple and complex risks, conditional on rational trading behavior. Both subfigures plot conditional average cardinalities of the subsets of prices for which participants supply (demand) \hat{Q} shares (see Eq. (7)). In Subfigure (a), averages are only based on participants who, under complex risks, supply (demand) \hat{Q} shares at $P = E_i[X]$. In Subfigure (b), the average value for simple risks is computed across all nondominated (see Figure D.9) supply (demand) curves. The corresponding average for complex risks is determined across all nondominated supply (demand) curves with \hat{Q} at $P = E_i[X]$. Error bars indicate standard errors of the mean.

In summary, my empirical findings reconcile well with the random choice models postulated in Eq. (9) and Eq. (10): Complexity tightens the bounds on risk-averse agents' rational behavior, where, rationality under complex risks is defined in line with decision theory under ambiguity. A simple counting exercise further underpins this hypothesis. Figure D.9 in Appendix D shows the distributions of dominated action frequencies between risk types. As expected, participants more frequently fall for dominated trading strategies if risks are complex. Although, as can be deduced from Figure D.10 in Appendix D, some limited learning takes place over the course of trading complex risks.

Once varying levels of rationality are controlled for, the inconclusiveness regarding the above two price sensitivity measures disappears. Figure 17 shows the between-treatment

average values of HF , where two different rationality conditions are applied. In Subfigure (a), averages are only computed for participants who prefer to be perfectly hedged for $P = E_i[X]$ under complex risks. For these participants, the average number of prices for which they supply (demand) \hat{Q} shares increases by 5.436 under complex relative to simple risks (p -value = 0.000, t -test).

In contrast, Subfigure (b) plots averages computed across potentially different participants: Under simple risks, only nondominated supply and demand curves (as presented in Figure D.9) are considered. Accordingly, the corresponding average for complex risks is solely based on nondominated supply and demand curves equal to \hat{Q} at $P = E_i[X]$. Relying on these conditions, the average cardinality of the set of prices for which the perfect hedging strategy is adopted increases by 4.180 under complex risks (p -value = 0.004, t -test). Hence, for both cases in Figure 17, the conditional HF measure relates strikingly well with the theoretical predictions implied by kinked ambiguity preferences.

Regression Analysis

For a controlled regression analysis, I additionally include the remaining data from each session's last trading round (see Table III), where tradable risks are based on the draw from the nontransparent Ellsberg urn depicted in Figure 7. Since participants' beliefs in these rounds are ambiguous, classifying individual trades into dominated and nondominated actions is no longer possible.

Columns I and II in Table VI reports GLS coefficient estimates of the following pooled regression model:

$$\begin{aligned} Slope_{ir} = & \beta_0 + \beta_1 Complexity_r + \beta_2 Ambiguity_r + \beta_3 RA_i \\ & + \beta_4 (AA_i \times Complexity_r) + \beta_5 (AA_i \times Ambiguity_r) + \mathbf{bX}_{ir} + u_i + \epsilon_{ir}, \end{aligned} \quad (11)$$

where the dependent variable is the slope of participant i 's supply (demand) curve (see Eq. (8)) in trading round r . $Complexity_r$ and $Ambiguity_r$ are dummy variables indicating trading rounds with complex and ambiguous risks, respectively. RA_i and AA_i measure individual risk and ambiguity aversion (see Table III). Finally, \mathbf{X}_{ir} contains trading round-specific, socio-economic, as well as further individual control variables, u_i is a participant random effect, and ϵ_{ir} denotes the idiosyncratic error term. Robust standard errors clustered at the participant level are reported in parenthesis.

The reported coefficients in columns I and II in Table VI affirm the findings reported in Subfigure (b) of Figure 14. Complex risks significantly decrease the slope of local supply

TABLE VI. REGRESSION ANALYSIS

	I	II	III	IV	V	VI
Dependent variable	Slope	Slope	HF	HF	Slope \times Complexity	Slope \times Complexity
Complexity	-0.236 ^a (0.062)	-0.315 ^a (0.086)				
Complexity cond.			0.857 ^a (0.125)	1.219 ^a (0.156)		
Complexity \times EstError					-0.730 ^b (0.288)	-0.723 ^b (0.304)
Ambiguity		0.001 (0.120)		0.571 ^a (0.155)		
RA		-0.254 (0.230)		0.813 ^a (0.259)		-0.087 (0.221)
AA \times Complexity		-0.053 (0.127)		0.150 (0.323)		-0.054 (0.163)
AA \times Ambiguity		-0.136 (0.267)		0.237 (0.469)		
Order \times Complexity		0.121 (0.093)		-0.450 ^b (0.195)		-0.026 (0.089)
Gender		-0.283 ^a (0.098)		0.388 ^a (0.147)		-0.214 ^a (0.082)
Constant	0.627 ^a (0.084)	-0.333 (0.671)	1.210 ^a (0.092)	1.840 ^b (0.788)	0.513 ^a (0.093)	-0.716 (0.682)
Controls	No	Yes	No	Yes	No	Yes
<i>N</i>	686	665	686	665	392	380

Notes: This table reports coefficient estimates from linear GLS (columns I, II, IV, and V) and Poisson (columns III and IV) regressions with random effects at the participant level. The dependent variables ‘Slope’ and ‘HF’ measure the slope of individual supply (demand) curves and the perfect hedge frequency as defined in Eq. (8) and Eq. (7), respectively. The dependent variable in the last two columns corresponds to the interaction between the former and the dummy variable ‘Complexity’. ‘Complexity’ indicates trading rounds with complex risks. ‘Complexity cond.’ is a dummy variable indicating trading rounds with complex risks for which participants are perfectly hedged at prices equal to expected payoffs. ‘Complexity \times EstError’ is the interaction between ‘Complexity’ and participants estimation error, i.e., the absolute distance between individual point estimates and true probabilities. ‘Ambiguity’ is a dummy variable indicating trading rounds with ambiguous risks. ‘RA’ measures participants’ risk aversion as the normalized difference between the expected payoff of the simple lottery and their respective certainty equivalents. ‘AA \times Complexity’ (‘AA \times Ambiguity’) controls for the effect of ambiguity aversion (‘AA’) in trading rounds with complex (ambiguous) risks, where ambiguity aversion is measured as the difference between participants’ certainty equivalents for the simple and the ambiguous lottery. ‘Order \times Complexity’ is the interaction between the dummy variable ‘Order’, indicating sessions where complex risks were preceded by simple risks, and ‘Complexity’. ‘Gender’ is a dummy variable indicating female participants. ‘Controls’ comprise participants’ age, university affiliation, and number of completed semesters. Furthermore, ‘Controls’ contain participants’ self-evaluated understanding and difficulty level of the task (measured by integers from one to five) and two additional dummy variables controlling for their familiarity and knowledge regarding the Brownian motion. Numbers in parenthesis denote robust standard errors. Superscripts ^a and ^b indicate statistical significance at the 1% and 5%-level, respectively.

(demand) by approximately 0.315 on average (p -value = 0.000, t -test). While the estimate of β_2 is essentially zero, the estimates of β_3 , β_4 , and β_5 all exhibit the expected (negative) sign, but lack statistical significance. Moving from the univariate model in column I to the full model in column II, the number of observations slightly decreases. This is due to the removal of one participant (two participants) with multiple switching points for the multiple price lists used to elicit RA_i (AA_i).⁴¹

Given that HF (see Eq. (7)) is a count variable, columns III and IV report the corresponding coefficient estimates from a Poisson regression with participants' perfect hedge frequency as dependent variable. In contrast to Eq. (11), the main independent variable 'Complexity cond.' is a conditional indicator of complex trading rounds for which participants choose to be perfectly hedged at prices equal to expected payoffs. In line with Figure 17, 'rational' participants on average increase their perfect hedge frequency by a ratio of more than three ($e^{1.219}$, p -value = 0.000, t -test) under complex risks. For more risk-averse participants, the number of prices for which they choose to adopt the perfect hedging strategy also increases significantly. Furthermore, the average perfect hedge frequency is significantly higher during trading rounds with ambiguous risks (Ellsberg urn).

Columns V and VI finally only focus on trading rounds with complex risks. Implementing an otherwise identical regression model as described in Eq. (11), the slope of participants' supply (demand) curve is regressed on their individual estimation errors under complex risks. The latter is defined as the absolute distance between participants' point estimate and the true probability (both expressed in decimal numbers) of a high dividend payment. I find evidence that participants' price sensitivity is negatively correlated with their estimation error (-0.723, p -value = 0.017, t -test). Put differently, the bigger participants' mistake in estimating complex risks, the less price sensitive their submitted supply (demand) curve. The impact of this negative correlation on price stability is analyzed in more detail below.

Overall, female participants' price sensitivity reacts stronger to complex risks under which they more often follow the perfect hedging strategy. This somewhat contrasts the findings in Borghans, Heckman, Golsteyn, and Meijers (2009) that men require higher compensation for the introduction of ambiguity than do women. Moreover, introducing ambiguity via a standard Ellsberg urn significantly increases the perfect hedge frequency. This reassures my design's effectiveness in translating ambiguity preferences into measurable theory-consistent trading outcomes. More interestingly, however, complex risks lead

⁴¹ The same holds true for all fully controlled models in Table VI (even columns).

to a more pronounced decrease in price sensitivity than pure ambiguity. Hence, while ambiguity preference-based theories appear to explain individual behavior under complex risks reasonably well, relying on pure ambiguity only would underestimate the latter’s impact on market outcomes.

Given the fundamental difference regarding the existence of a uniquely defined risk structure, it is not surprising that the magnitudes of participants’ reactions to complex and ambiguous risks are different. Even though their beliefs under pure ambiguity are unknown, a similar analysis as presented in Figure 15 lends itself as a *simplified* comparison of relative bounded ‘rationality’. For both simple and ambiguous risks, Figure D.11 in Appendix D presents the joint distributions of supplied and demanded shares at a price of ECU 75. Assuming participants adopt the natural reference point (fifty-fifty) under pure ambiguity, there is no evidence that pure ambiguity affects participants’ ‘rationality’.

5.3. Market’s Effectiveness in Aggregating Complex Information

In light of the attained insights regarding individual trading behavior, I can now return to an equilibrium perspective asking a final overarching question: How well are financial markets suited to cope with complexity? In particular, are they capable of efficiently allocating complex risks at informative prices in a reliable way? I investigate this question by dissecting both the equilibration process of the above asset markets and their outcomes.

Figure 18 displays bootstrapped distributions of aggregate market outcomes. All densities are based on ten thousand resamples of 49 individual supply and demand functions.⁴² For any given resample, average supply and demand are crossed and the linearly interpolated market-clearing price P^* and average quantity Q^* deduced.

Comparing estimated densities between simple and complex risks unveils three striking characteristics of the market equilibrium. First, and not surprisingly, both distributions of P^* under simple risks are closer to and more centered around the risky asset’s true expected payoff than those under complex risks. Second, and contrary to market-clearing prices, the centers of both Q^* distributions under complex risks are remarkably close to the perfect hedging quantity ($\hat{Q} = 2$). In case of $\pi = 1/2$ (lower left plot in Figure 18), complex risks are even more efficiently shared between buyers and sellers than simple risks. Both observations are in line with actual market outcomes reported in Table V.

Third and foremost, the increase in variation for simple relative to complex risks is much larger for market-clearing prices than for average market-clearing quantities. Given

⁴² This ensures an identical resampling size under simple and complex risks.

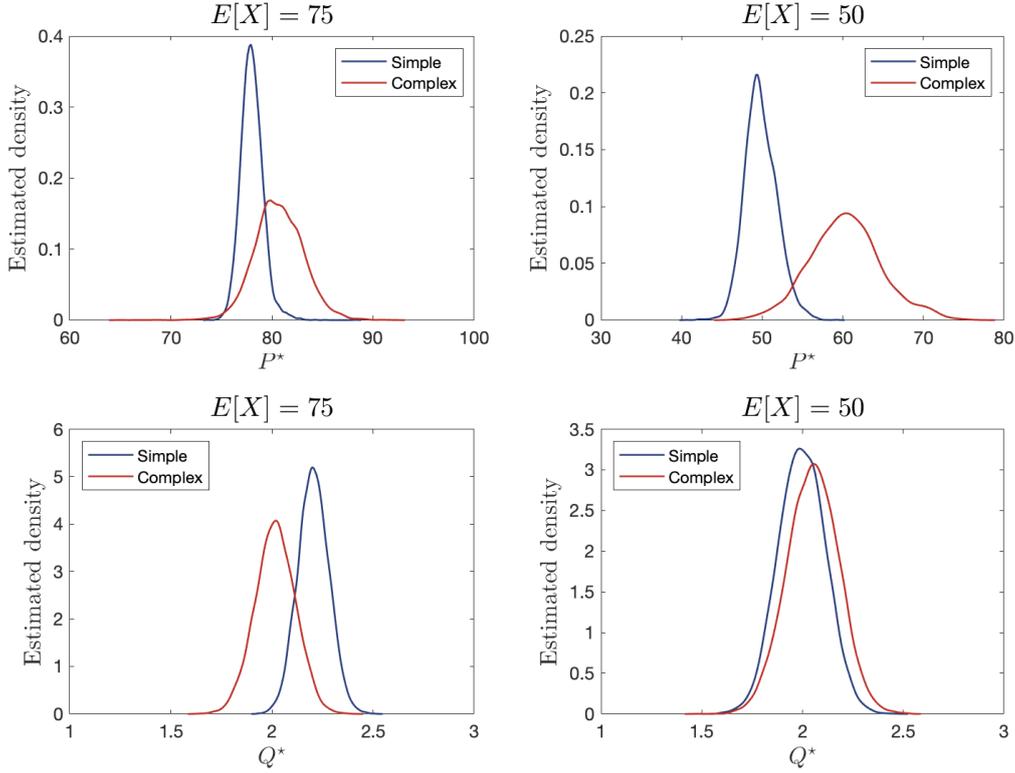


FIGURE 18. BOOTSTRAPPED EQUILIBRIUM DISTRIBUTIONS

Notes: This figure shows bootstrapped densities of market-clearing prices and quantities for simple and complex risks. Every average supply and demand curve is based on resampling 49 individual supply and demand schedules (same resampling size under simple and complex risks). For each pair of averaged supply and demand, the linearly interpolated market-clearing price and average quantity are computed. Repeating this procedure ten thousand times yields the depicted estimated densities of equilibrium prices (top row) and average quantities (bottom row). The left (right) column shows bootstrapped densities for trading rounds with π equal to $1/2$ ($1/3$).

their predicted decrease in supply and demand sensitivity, this observation aligns well with the above ambiguity preference-based theories. Figure D.12 in Appendix D furthermore illustrates how these relative variations in P^* and Q^* depend on the underlying resampling size. All variability ratios are considerably stable in the number of traders. At the maximum resampling size, both standard deviations of P^* under complex risks are still more than twice as high as under simple risks. In contrast, standard deviations of average market-clearing quantities are consistently much closer for simple and complex risks. In the limit, the variation in Q^* under complex risks only exceeds the one under simple risks by approximately 30% for $\pi = 1/2$ and less than 10% for $\pi = 1/3$. Hence, throughout this

‘equilibration path’, the variation in markets’ overall risk sharing ability are remarkably similar for simple and complex risks.

To directly evaluate markets’ ability to reliably aggregate agents’ subjective information about complex risks into equilibrium prices, the actual variation in subjective beliefs has to be accounted for. The above distributions of P^* under complex risks do not yet control for the dispersion of participants’ heterogeneous beliefs regarding the risky asset’s true expected payoff. Therefore, I propose the following ratio as a measure of relative price stability:

$$P^*\text{-Stability} = \sqrt{\frac{\text{Var}(P_c^*)}{\text{Var}(P_s^*) + \text{Var}(E_c^*[X])}}, \quad (12)$$

where P_s^* (P_c^*) denotes the bootstrapped market-clearing price for simple (complex) risks, and $E_c^*[X]$ indicates resampled participants’ average estimate of $E[X]$ under complex risks. When comparing variations in P_c^* to those in P_s^* , Eq. (12) thus controls for the fluctuations of participants’ point estimates by accounting for the variations in $E_c^*[X]$.⁴³

The question whether the ratio in Eq. (12) is eventually greater or smaller than unity, i.e., whether markets reliably aggregate complex information or not, can only be answered empirically. From a theoretical perspective, however, the answer is: *it depends*. The decisive factor is whichever of the following trade-off effects dominates: increased severity of bounded rationality versus reduced price sensitivity. In the absence of both effects, the ratio in Eq. (12) should equal one. Whenever risk-averse agents’ behave fully rationally, P_s^* coincides with $E[X]$ and is thus deterministic. Moreover, if agents are neutral to complexity-induced ambiguity, $\text{Var}(P_c^*)$ exactly corresponds to $\text{Var}(E_c^*[X])$, since the market-clearing price equals the average of agents’ point estimates.

In the presence of ambiguity aversion and a thereby implied decrease in local price sensitivity under complex risks, $\text{Var}(P_c^*)$ may fall below $\text{Var}(E_c^*[X])$, thereby pushing Eq. (12) downwards.⁴⁴ However, as shown in Figure 6, the effect of an *unconditional* decrease in price sensitivity on pricing stability may be ambiguous. This is because a given shift in supply (demand) has a *bigger* impact on the equilibrium *price*, if demand

⁴³ Note that due to the balanced resampling size (see above), the variance of $E_c^*[X]$ is nonzero (twice as many trading rounds with complex than simple risks).

⁴⁴ For a simple example, consider two different agents with multiple-priors utility (see Table I): an ambiguity-neutral seller and an infinitely ambiguity-averse buyer. Facing complex risks, the seller shall believe that $E[X] \in [a, b]$ with uniform probability, while the buyer believes that $E[X]$ is uniformly distributed over $[\frac{a+b}{2}, b]$, where $a < b$. Hence, the seller’s supply curve goes through $(\frac{a+b}{2}, \hat{Q})$, whereas the buyer’s demand curve is completely flat at \hat{Q} over the nonempty subset of prices $[\frac{a+b}{2}, b]$. Therefore, the unique trading equilibrium equals $(\frac{a+b}{2}, \hat{Q})$ with $P_c^* = \frac{a+b}{2}$ and $E_c^* = \frac{3a+5b}{8}$. However, if instead the buyer believes that $E[X] \in [a, \frac{a+b}{2}]$, P_c^* would still equal $\frac{a+b}{2}$, but E_c^* would jump to $\frac{5a+3b}{8}$.

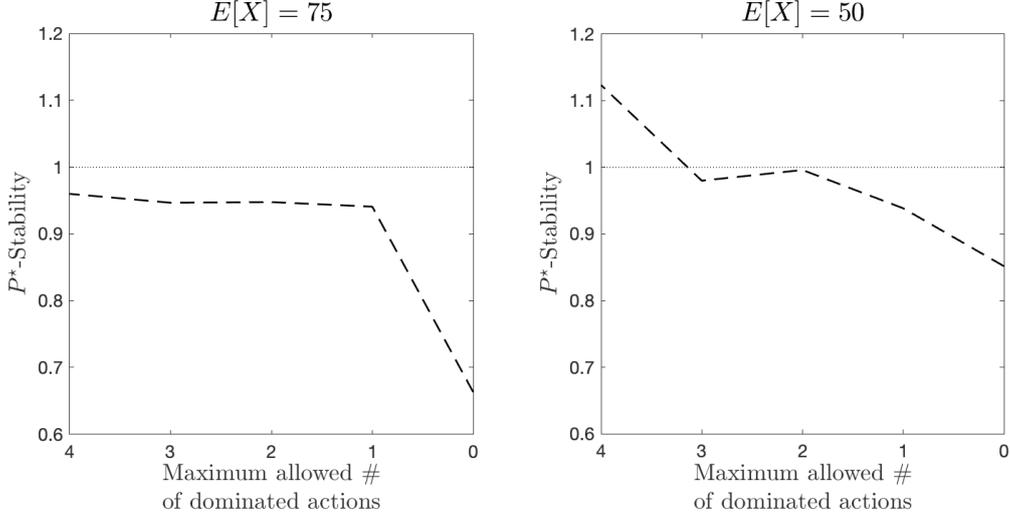


FIGURE 19. RELATIVE VARIABILITY OF MARKET-CLEARING PRICES

Notes: Conditioning on participants' maximum number of dominated actions (see Figure D.9), this figure shows the ratio

$$P^*\text{-Stability} = \sqrt{\frac{\text{Var}(P_c^*)}{\text{Var}(P_s^*) + \text{Var}(E_c^*[X])}},$$

where P_s^* (P_c^*) denotes the market-clearing price for simple (complex) risks, and $E_c^*[X]$ indicates participants' average estimate of $E[X]$ under complex risks. Both estimates P_s^* and P_c^* are bootstrapped based on resampling and averaging individual supply and demand schedules. For each pair of averaged supply and demand, the linearly interpolated market-clearing price is computed. The respective resample size is set to the minimum number of sellers or buyers who satisfy the given rationality condition (maximum allowed number of dominated actions). This procedure is repeated ten thousand times. The left (right) plot shows standard deviation ratios for trading rounds with π equal to $1/2$ ($1/3$).

(supply) is *less* price sensitive. Hence, to reliably generate lower pricing variability, a *negative* correlation between price sensitivity and extreme subjective beliefs is needed. In contrast, more severe bounds on rationality under complex risks generally increase the noise in P_c^* relative to P_s^* , which ultimately pushes Eq. (12) upwards.

Both trade-off effects are present in the data. Figure 19 shows the respective values of Eq. (12), conditional on participants' maximum number of dominated actions (see Figure D.9). Unconditionally, the price stability ratio for $\pi = 1/2$ lies below one (0.813), whereas for $\pi = 1/3$ it exceeds one (1.170). This is in line with the observations from Figure 15 and Figure 16, as well as Table VI: Relative to π equal to one half, the number

of strongly dominated actions is substantially higher for π equal to one third, implying more severe bounds on participants' rationality in the latter case. As shown in the right plot of Figure 19, the ratio for $\pi = 1/3$ is decreasing in the strictness of the applied rationality constraint. In other words, the negative correlation between participants' price sensitivity and their individual estimation errors begins to take over, driving down the variability of equilibrium prices. Focusing solely on participants who mostly abstain from dominated actions, the price stability ratio eventually also falls below unity.

To sum up, markets prove to be notably efficient in pricing and sharing complex risks, despite increased noise levels in individual trading behavior. Reliable pricing is achieved because participants whose beliefs are further away from the truth become less price-sensitive. Eventually, beyond binding limits to bounded rationality, information aggregation is impaired, while efficient risk sharing nevertheless prevails.

6. Concluding Remarks

This paper studies the trading of complex but purely objective risks in a competitive asset market. Relying on decision theory under ambiguity, I provide a novel perspective on agents' trading behavior in the presence of imperfectly understood uncertainty. In his seminal work, [Ellsberg \(1961\)](#) characterizes ambiguity as “a quality depending on the amount, type, and ‘unanimity’ of information, and giving rise to one’s degree of ‘confidence’ in an estimate of relative likelihoods” ([Ellsberg, 1961](#), p. 657). Based on Ellsberg’s original interpretation, I advocate for a bridging of decision theories under ambiguity with models of financial markets. The former can viably assist the latter in explaining the transformation of increasingly complex information into prices.

In the absence of aggregate risk, the controlled setting of [Biais, Mariotti, Moinas, and Pouget \(2017\)](#) offers an ideal starting point to distinctively test for complexity’s impact on individual trading decisions and aggregate market outcomes. Starting from the implications of [Debreu \(1959\)](#) and [Arrow’s \(1964\)](#) general equilibrium theory under simple risks, ambiguity models provide significant insights regarding both adopted trading strategies and achieved equilibrium allocations under complex risks.

I find asset markets to prove remarkably effective in pricing complex risks and even more robust in sharing them optimally across risk-averse investors. However, the former quality crucially depends on the severity by which complexity curtails agents' rationality under the perceived ambiguity of complex risks. Strikingly, individual trading behavior appears to exhibit self-awareness of prevailing estimation biases, which helps to reduce

the variation in market-clearing prices.

Moreover, my findings demonstrate that aggregate stability results from individual heterogeneity in preferences. However, despite being relatively common at the individual level, kinked ambiguity preferences can not be inferred from any of the aggregate market outcomes. This raises questions about the generalizability of the many representative agent models that are omnipresent in modern asset pricing theory.

Finally, I argue that my findings shed new light on the question why modern exchanges rely on *discontinuous* trading periods in times of high valuation uncertainty, e.g., during preopening hours. Importantly, my argument is complementary to the endogenous concentration of liquidity and informed traders in the presence of ex-ante information asymmetries (Admati and Pfleiderer, 1988) and the facilitation of coordination through binding orders (Biais, Bisière, and Pouget, 2013).

The closest paper that looks at asymmetric reasoning in a *continuous* open book system is Asparouhova, Bossaerts, Eguia, and Zame (2015). In their setting, after observing dissenting opening prices, the trading decisions of the vast majority of participants—more than 90% on average—become independent of past prices. Intuitively, the more traders discard their own information, the higher the documented degree of mispricing. Absent of the authority of public prices, I find that equilibrium prices reliably reflect the average beliefs of *all* participants. Thus, discontinuous trading over periodically updated price thresholds, as, e.g., in the Nasdaq opening cross, could facilitate robust price discovery.

References

- Abdellaoui, M., A. Baillon, L. Placido, and P. Wakker, 2011, The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation, *American Economic Review* 101, 695–723.
- Admati, Anat R, and Paul Pfleiderer, 1988, A Theory of Intraday Patterns: Volume and Price Variability, *The Review of Financial Studies* 1, 3–40.
- Amromin, G., J. Huang, C Sialm, and E. Zhong, 2011, Complex Mortgages, *NBER Working Paper (17315)*.
- Armantier, O., and N. Treich, 2016, The Rich Domain of Risk, *Management Science* 62, 1954–1969.
- Arora, Sanjeev, Boaz Barak, Markus Brunnermeier, and Rong Ge, 2011, Computational Complexity and Information Asymmetry in Financial Products, *Communications of the ACM* 54, 101–107.

- Arrow, Kenneth J, 1964, The Role of Securities in the Optimal Allocation of Risk-bearing, *Review of Economic Studies* 31, 91–96.
- Asparouhova, E., P. Bossaerts, J. Eguia, and W. R. Zame, 2015, Asset Pricing and Asymmetric Reasoning, *Journal of Political Economy* 123, 66–122.
- Asparouhova, E., P. Bossaerts, R. Nilanjan, and W. R. Zame, 2016, “Lucas” in the Laboratory, *Journal of Finance* 71, 2727–2780.
- Bachelier, Louis, 1900, Théorie de la spéculation, in *Annales scientifiques de l’École normale supérieure* , vol. 17 pp. 21–86.
- Becker, Gordon M, Morris H DeGroot, and Jacob Marschak, 1964, Measuring Utility by a Single-response Sequential Method, *Behavioral science* 9, 226–232.
- Biais, Bruno, Christophe Bisière, and Sebastien Pouget, 2013, Equilibrium Discovery and Preopening Mechanisms in an Experimental Market, *Management Science* 60, 753–769.
- Biais, B., T. Mariotti, S. Moinas, and S. Pouget, 2017, Asset Pricing and Risk Sharing in a Complete Market: An experimental Investigation, *Working Paper*.
- Black, Fischer, and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637–654.
- Borghans, Lex, James J Heckman, Bart HH Golsteyn, and Huub Meijers, 2009, Gender Differences in Risk Aversion and Ambiguity Aversion, *Journal of the European Economic Association* 7, 649–658.
- Bossaerts, P., P. Ghirardato, S. Guarnaschelli, and W. R. Zame, 2010, Ambiguity in Asset Markets: Theory and Experiment, *Review of Financial Studies* 23, 1325–59.
- Bossaerts, P., and C. Murawski, 2016, How Humans Solve Complex Problems: The Case of the Knapsack Problem, *Scientific Reports* 6, 34851.
- Brunnermeier, K., and M. Oehmke, 2009, Complexity in Financial Markets, *Working paper*.
- Carlin, B., 2009, Strategic Price Complexity in Retail Financial Markets, *Journal of Financial Economics* 91, 278–87.
- , S. Kogan, and R. Lowery, 2013, Trading Complex Assets, *Journal of Finance* 68, 1937–60.
- Carlin, B., and G. Manso, 2011, Obfuscation, Learning, and the Evolution of Investor Sophistication, *Review of Financial Studies* 24, 755–85.
- Célérier, Claire, and Boris Vallée, 2017, Catering to Investors Through Security Design: Headline Rate and Complexity, *The Quarterly Journal of Economics* 132, 1469–1508.

- Chateauneuf, Alain, Jürgen Eichberger, and Simon Grant, 2007, Choice under Uncertainty with the Best and Worst in Mind: Neo-additive Capacities, *Journal of Economic Theory* 137, 538–567.
- Constantinides, G., 1982, Intertemporal Asset Pricing with Heterogeneous Consumers, and Without Demand Aggregation, *Journal of Business* 55, 253–67.
- Debreu, Gerard, 1959, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium* . No. 17 (Yale University Press).
- Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg, 2016, Ambiguity Aversion and Household Portfolio Choice Puzzles: Empirical Evidence, *Journal of Financial Economics* 119, 559 – 577.
- Dow, James, and Sergio Ribeiro da Costa Werlang, 1992, Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio, *Econometrica* pp. 197–204.
- Duffie, Darrell, and Chi-Fu Huang, 1985, Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-lived Securities, *Econometrica* pp. 1337–1356.
- Ellison, G., 2005, A Model of Add-On Pricing, *Quarterly Journal of Economics* 120, 585–637.
- , and S. Ellison, 2009, Search, Obfuscation, and Price Elasticities on the Internet, *Econometrica* 77, 427–52.
- Ellsberg, D., 1961, Risk, Ambiguity, and the Savage Axioms, *Quarterly Journal of Economics* 75, 643–69.
- Epstein, Larry G., and Martin Schneider, 2010, Ambiguity and Asset Markets, *Annual Review of Financial Economics* 2, 315–346.
- Fischbacher, U., 2007, z-Tree: Zurich toolbox for ready-made economic experiments, *Experimental Economics* 10, 171–78.
- Fox, Craig R, and Amos Tversky, 1995, Ambiguity Aversion and Comparative Ignorance, *Quarterly Journal of Economics* 110, 585–603.
- Friedman, Daniel, 2018, The Double Auction Market Institution: A Survey, in *The Double Auction Market* . pp. 3–26 (Routledge).
- Gabaix, X., and D. Laibson, 2006, Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets, *Quarterly Journal of Economics* 121, 505–40.
- Ghent, Andra C, Walter N Torous, and Rossen I Valkanov, 2017, Complexity in Structured Finance, *The Review of Economic Studies* 86, 694–722.
- Ghirardato, P., F. Maccheroni, and M. Marinacci, 2004, Differentiating Ambiguity and Attitude, *Journal of Economic Theory* 118, 133–73.

- Gilboa, I., and D. Schmeidler, 1989, Maxmin Expected Utility with a Non-unique Prior, *Journal of Mathematical Economics* 18, 141–53.
- Goeree, Jacob K, and Charles A Holt, 2004, A Model of Noisy Introspection, *Games and Economic Behavior* 46, 365–382.
- , and Thomas R Palfrey, 2002, Quantal Response Equilibrium and Overbidding in Private-Value Auctions, *Journal of Economic Theory* 104, 247–272.
- , 2003, Risk Averse Behavior in Generalized Matching Pennies Games, *Games and Economic Behavior* 45, 97–113.
- Griffin, John, Richard Lowery, and Alessio Saretto, 2014, Complex Securities and Underwriter Reputation: Do Reputable Underwriters Produce Better Securities?, *The Review of Financial Studies* 27, 2872–2925.
- Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, *American Economic Review* 70, 393–408.
- Halevy, Yoram, 2007, Ellsberg Revisited: An Experimental Study, *Econometrica* 75, 503–536.
- Henderson, B., and N. Pearson, 2011, The Dark Side of Financial Innovation: A Case Study of the Pricing of a Retail Financial Product, *Journal of Financial Economics* 100, 227–47.
- Hens, T., and M. Rieger, 2008, The Dark Side of the Moon: Structured Products from the Customer’s Perspective, *Working Paper*.
- Holt, Charles A, and Susan K Laury, 2002, Risk Aversion and Incentive Effects, *American Economic Review* 92, 1644–1655.
- Kahneman, Daniel, Jack L Knetsch, and Richard H Thaler, 1991, Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias, *Journal of Economic Perspectives* 5, 193–206.
- Keynes, J. M., 1921, *A Treatise on Probability* (London: Macmillan).
- Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji, 2005, A Smooth Model of Decision Making under Ambiguity, *Econometrica* 73, 1849–1892.
- Knight, F. H., 1921, *Risk, Uncertainty and Profit* (Boston, New York, Houghton Mifflin Company).
- Kreps, David M, 1982, Multiperiod Securities and the Efficient Allocation of Risk: A Comment on the Black-Scholes Option Pricing Model, in *The Economics of Information and Uncertainty* . pp. 203–232 (University of Chicago Press).

- Luce, R Duncan, 1959, *Individual Choice Behavior: A Theoretical Analysis* (John Wiley and sons).
- McKelvey, Richard D, and Thomas R Palfrey, 1995, Quantal Response Equilibria for Normal Form Games, *Games and Economic Behavior* 10, 6–38.
- , 1998, Quantal Response Equilibria for Extensive Form Games, *Experimental Economics* 1, 9–41.
- Radner, Roy, 1972, Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets, *Econometrica* 40, 289–303.
- Sato, Y, 2014, Opacity in Financial Markets, *Review of Financial Studies* 27, 3502–46.
- Savage, L. J., 1954, *Foundations of Statistics*. (Wiley).
- Tversky, Amos, and Daniel Kahneman, 1992, Advances in Prospect Theory: Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty* 5, 297–323.
- Yates, F., 1934, Contingency Tables Involving Small Numbers and the χ^2 Test, *Supplement to the Journal of the Royal Statistical Society* 1, 217–235.

INTERNET APPENDIX

Appendix A: Proofs

Proof of Proposition 1. Here I prove the case if agent i is a seller. In the case of a buyer, the analogous reasoning applies. Relying on the identities in Eq. (1), any agent i 's expected utility from consumption can be rewritten as (neglecting the subscript i)

$$\begin{aligned} E[U(C(\omega))] &= \pi U\left(\mu + \sqrt{\frac{1-\pi}{\pi}}\sigma\right) + (1-\pi)U\left(\mu - \sqrt{\frac{\pi}{1-\pi}}\sigma\right) \\ &\triangleq f(\mu, \sigma, \pi), \end{aligned}$$

and since U is increasing it follows that

$$\begin{aligned} \frac{\partial f}{\partial \mu} &= \pi U'\left(\mu + \sqrt{\frac{1-\pi}{\pi}}\sigma\right) + (1-\pi)U'\left(\mu - \sqrt{\frac{\pi}{1-\pi}}\sigma\right) \\ &> 0, \end{aligned} \tag{A.1}$$

and from decreasing marginal utility from consumption that

$$\begin{aligned} \frac{\partial f}{\partial \sigma} &= \pi U'\left(\mu + \sqrt{\frac{1-\pi}{\pi}}\sigma\right) \sqrt{\frac{1-\pi}{\pi}} \\ &\quad + (1-\pi)U'\left(\mu - \sqrt{\frac{\pi}{1-\pi}}\sigma\right) \left(-\sqrt{\frac{\pi}{1-\pi}}\right) \\ &= \sqrt{\pi(1-\pi)}U'\left(\mu + \sqrt{\frac{1-\pi}{\pi}}\sigma\right) - \sqrt{\pi(1-\pi)}U'\left(\mu - \sqrt{\frac{\pi}{1-\pi}}\sigma\right) \\ &< 0. \end{aligned} \tag{A.2}$$

When selling Q shares for a price equal to P , the seller's consumption in $t = 2$ equals

$$C(u) = (S - Q)X(u) + (B + QP) + I(u) \tag{A.3}$$

in state u , and

$$C(d) = (S - Q)X(d) + (B + QP) + I(d) \tag{A.4}$$

in state d .

Let us now denote the expected asset payoff $E[X]$ by P^* , i.e.,

$$P^* := \pi X(u) + (1 - \pi)X(d).$$

Furthermore, I define \widehat{Q} as the quantity for which $\sigma^2 = 0$, i.e.,

$$\begin{aligned} \sigma^2 = 0 &\Leftrightarrow C(u) = C(d) \\ &\Leftrightarrow (S - Q)X(u) + I(u) = (S - Q)X(d) + I(d) \\ &\Leftrightarrow \widehat{Q} = S + \frac{I(u) - I(d)}{X(u) - X(d)}, \end{aligned} \tag{A.5}$$

where I assume that $\widehat{Q} > 0$. From the definition of μ together with (Eq. (A.3)) and (Eq. (A.4)) it follows that

$$\frac{\partial \mu}{\partial Q} = \pi(P - X(u)) + (1 - \pi)(P - X(d)),$$

and thus

$$\frac{\partial \mu}{\partial Q} \begin{cases} < 0 & \text{if } P < P^*, \\ = 0 & \text{if } P = P^*, \\ > 0 & \text{if } P > P^*. \end{cases} \tag{A.6}$$

First, strict concavity now implies

$$\begin{aligned} E[U(C(\omega))] &< U\left(\pi\mu + \sqrt{\pi(1-\pi)}\sigma + (1-\pi)\mu - \sqrt{(1-\pi)\pi}\sigma\right) \\ &= U(\mu), \end{aligned}$$

hence, from (Eq. (A.5)) and (Eq. (A.6)) it follows that, $\forall \pi \in (0, 1)$, (P^*, \widehat{Q}) *strictly dominates* all other points on the line (P^*, Q) .

Second, (Eq. (A.1)) & (Eq. (A.6)) together with (Eq. (A.2)) & (Eq. (A.5)) imply that

- (i) for any given price $P < P^*$, any point in the upper left quadrant of Subfigure (a) of Figure 1 is *strictly dominated* by (P, \widehat{Q}) ;
- (ii) for any given price $P > P^*$, any point in the lower right quadrant of of Subfigure (a) of Figure 1 is *strictly dominated* by (P, \widehat{Q}) .

Hence, $\forall \pi \in (0, 1)$, the seller's supply curve has to lie somewhere in the lower left and upper right quadrant and has to go through the point (P^*, \widehat{Q}) . This completes the proof.

□

Proof of Remark 1. Since ϵ can be arbitrarily small, I directly consider the limit $\epsilon \rightarrow 0$, i.e., $\lim_{\epsilon \rightarrow 0} U_i(C) = c_1 C$, for $0 \leq C \leq \bar{C}$. The corresponding first and second derivatives of $U_i(C)$ are

$$\lim_{\epsilon \rightarrow 0} U_i'(C) = c_1, \quad \text{and} \quad \lim_{\epsilon \rightarrow 0} U_i''(C) = 0.$$

For $C \geq \bar{C}$, the respective derivatives are

$$U_i'(C) = \alpha e^{-\alpha C}, \quad \text{and} \quad U_i''(C) = -\alpha^2 e^{-\alpha C}.$$

The following conditions ensure the differentiability of $U_i(C)$ at \bar{C} :

$$c_1 \bar{C} = c_2 - e^{-\alpha \bar{C}} \quad \Leftrightarrow \quad c_2 = c_1 \bar{C} + e^{-\alpha \bar{C}}, \quad (\text{A.7})$$

$$c_1 = \alpha e^{-\alpha \bar{C}}. \quad (\text{A.8})$$

Given Eq. (A.3) and Eq. (A.4), the FOC for $E[U(C(\omega))]$ with respect to Q implies

$$\pi U'(C(u))(P - X(u)) + (1 - \pi)U'(C(d))(P - X(d)) = 0. \quad (\text{A.9})$$

Taking the first derivative of the LHS of Eq. (A.9) with respect to P yields

$$\pi U'(C(u)) + (1 - \pi)U'(C(d)) + \pi U''(C(u))Q(P - X(u)) + (1 - \pi)U''(C(d))Q(P - X(d)).$$

Since $\frac{\partial}{\partial Q}(\text{LHS of Eq. (A.9)}) < 0 \quad \forall (P, Q) \in \mathbb{R}_{\geq 0}^2$, agent i 's supply curve is decreasing in P if $\frac{\partial}{\partial P}(\text{LHS of Eq. (A.9)}) < 0$. For the here considered utility function, this is the case whenever

$$c_1 < \frac{1 - \pi}{\pi} \alpha e^{\alpha C(d)} (\alpha Q(P - X(d)) - 1).$$

Together with Eq. (A.8), this implies that for high enough prices, i.e., if

$$P > X(d) + \frac{1 + \frac{\pi}{1-\pi} \alpha e^{-\alpha(\bar{C}-C(d))}}{\alpha Q},$$

seller i 's supply curve can be *locally* decreasing in P . This completes the proof. □

Proof of Proposition 2. Here I prove the case if the ambiguity-averse agent i , i.e., $\alpha_i > 1/2$, is a seller. In the case of a buyer, the analogous reasoning applies. Relying on the identities in Eq. (1), any agent i 's utility from consumption according to the α -maxmin in Eq. (2)

can be rewritten as (neglecting the subscript i)

$$\begin{aligned}
\mathcal{U}(C(\omega)) &= \alpha \min_{\pi \in \mathcal{C}} (E[U(\pi)]) + (1 - \alpha) \max_{\pi \in \mathcal{C}} (E[U(\pi)]) \\
&= \alpha \left(\underline{\pi} U \left(\underline{\mu} + \sqrt{\frac{1 - \underline{\pi}}{\underline{\pi}}} \underline{\sigma} \right) + (1 - \underline{\pi}) U \left(\underline{\mu} - \sqrt{\frac{\underline{\pi}}{1 - \underline{\pi}}} \underline{\sigma} \right) \right) \\
&\quad + (1 - \alpha) \left(\bar{\pi} U \left(\bar{\mu} + \sqrt{\frac{1 - \bar{\pi}}{\bar{\pi}}} \bar{\sigma} \right) + (1 - \bar{\pi}) U \left(\bar{\mu} - \sqrt{\frac{\bar{\pi}}{1 - \bar{\pi}}} \bar{\sigma} \right) \right),
\end{aligned}$$

where $\underline{\pi} = \arg \min_{\pi \in \mathcal{C}} E[U(\pi)]$ ($\bar{\pi} = \arg \max_{\pi \in \mathcal{C}} E[U(\pi)]$) and $\underline{\mu}$ ($\bar{\mu}$) and $\underline{\sigma}$ ($\bar{\sigma}$) denote expected consumption and standard deviation of consumption according to $\underline{\pi}$ ($\bar{\pi}$). For $Q \neq \hat{Q}$, i.e., for strictly positive $\underline{\sigma}$ and $\bar{\sigma}$, it directly follows from U 's strict concavity that

$$\mathcal{U}(C(\omega)) < \alpha U(\underline{\mu}) + (1 - \alpha) U(\bar{\mu}) < U(\alpha \underline{\mu} + (1 - \alpha) \bar{\mu}).$$

Eq. (A.3) and Eq. (A.4) imply

$$\begin{aligned}
\alpha \underline{\mu} + (1 - \alpha) \bar{\mu} &= \alpha \left(\underline{\pi} ((S - Q)X(u) + (B + QP) + I(u)) + \right. \\
&\quad \left. (1 - \underline{\pi}) ((S - Q)X(d) + (B + QP) + I(d)) \right) \\
&\quad + (1 - \alpha) \left(\bar{\pi} ((S - Q)X(u) + (B + QP) + I(u)) + \right. \\
&\quad \left. (1 - \bar{\pi}) ((S - Q)X(d) + (B + QP) + I(d)) \right) \\
&= \dots \text{ terms indep. from } Q \dots + Q \left(P - \left(\alpha E^{\underline{\pi}}[X] + (1 - \alpha) E^{\bar{\pi}}[X] \right) \right).
\end{aligned}$$

Hence, if $P = \alpha E^{\underline{\pi}}[X] + (1 - \alpha) E^{\bar{\pi}}[X]$, denoted by \tilde{P} hereafter, the linear combination of expected consumption (for constant $\underline{\pi}$ and $\bar{\pi}$) does not change for different quantities of shares sold. Therefore, for \tilde{P} , it is optimal for the seller to exactly sell \hat{Q} share and get the constant utility $U(\alpha \underline{\mu} + (1 - \alpha) \bar{\mu}) = U(\alpha \underline{\mu}) = U((1 - \alpha) \bar{\mu})$.

In general, it holds that

$$\begin{aligned}
\mathcal{U}(C(\omega)) &= \alpha \left(\underline{\pi} U(C(u)) \right) + (1 - \underline{\pi}) U(C(d)) \\
&\quad + (1 - \alpha) \left(\bar{\pi} U(C(u)) + (1 - \bar{\pi}) U(C(d)) \right),
\end{aligned}$$

and, for any given price, the corresponding FOC reads

$$\begin{aligned}\frac{\partial \mathcal{U}}{\partial Q} &= \alpha \left(\underline{\pi} U'(C(u))(P - X(u)) + (1 - \underline{\pi}) U'(C(d))(P - X(d)) \right) \\ &\quad + (1 - \alpha) \left(\bar{\pi} U'(C(u))(P - X(u)) + (1 - \bar{\pi}) U'(C(d))(P - X(d)) \right) \\ &= 0.\end{aligned}\tag{A.10}$$

As shown, for \tilde{P} , it is optimal to sell \hat{Q} shares. Hence, the question now is, for what prices it is optimal to sell less (more) than \hat{Q} ? Or, put differently, starting from \tilde{P} per share, below (above) which price does it become beneficial to sell less (more) than \hat{Q} shares?

Since when selling \hat{Q} shares $C(u) = C(d)$, Eq. (A.10) yields

$$\left. \frac{\partial \mathcal{U}}{\partial Q} \right|_{Q=\hat{Q}} = 0 \quad \Leftrightarrow \quad P = \tilde{P}.$$

I denote by $\tilde{P}(\hat{Q} \downarrow) = L$ ($\tilde{P}(\hat{Q} \uparrow) = U$) the lowest (highest) price for which the seller prefers to sell \hat{Q} shares. Because $\underline{\pi} < \bar{\pi}$ whenever the seller considers to sell less than \hat{Q} , and $\underline{\pi} > \bar{\pi}$ whenever she thinks about selling more than \hat{Q} , it follows that $L < U$.

Therefore, in summary, seller i 's supply curve is constant over the closed subset $[L, U] \subset P$ and the difference $U - L$ becomes larger as her \mathcal{C} becomes wider and/or as $\alpha \rightarrow 1$. This completes the proof. \square

Proof of Proposition 3. Whenever there is a nonzero mass of ambiguity-averse agents whose supply (demand) curves do not go through the benchmark equilibrium $(E[X], \hat{Q})$, they draw average supply (demand) away from the latter. Given the result in Proposition 2, this clearly occurs if either

$$L > E[X] \quad \text{or} \quad U < E[X].\tag{A.11}$$

For ambiguity-averse agents, L is always strictly smaller than U , hence, the two cases in Eq. (A.11) are mutually exclusive.

I begin with the first inequality in Eq. (A.11). For any ambiguity-averse seller i it holds that (neglecting the subscript i)

$$L = \alpha E^{\underline{\pi}}[X] + (1 - \alpha) E^{\bar{\pi}}[X],$$

where $\alpha > 1/2$ and $\underline{\pi} < \bar{\pi}$ since L denotes the lower price limit below which she prefers to

sell *less* than \widehat{Q} shares. Thus, denoting by π_M the midpoint and by 2Δ the length of the seller's set of priors,⁴⁵ the inequality $L > E[X]$ can be written as

$$\begin{aligned} E[X] &< L \\ E[X] &< \alpha((\pi_M - \Delta)X(u) + (1 - (\pi_M - \Delta))X(d)) \\ &\quad + (1 - \alpha)((\pi_M + \Delta)X(u) + (1 - (\pi_M + \Delta))X(d)) \\ \pi X(u) + (1 - \pi)X(d) &< \pi'X(u) + (1 - \pi')X(d), \end{aligned} \quad (\text{A.12})$$

where $\pi' := \pi_M - \Delta(2\alpha - 1)$. By the analogous argument and relying on the same notation, it follows that the second inequality in Eq. (A.11) is equivalent to

$$\begin{aligned} E[X] &> U \\ \pi X(u) + (1 - \pi)X(d) &> \pi''X(u) + (1 - \pi'')X(d), \end{aligned} \quad (\text{A.13})$$

whereas now $\pi'' := \pi_M + \Delta(2\alpha - 1)$.

Together, Eq. (A.12) and Eq. (A.13) imply that

$$L < E[X] < U \quad \Leftrightarrow \quad \pi' < \pi < \pi'', \quad (\text{A.14})$$

where $\pi' < \pi''$ because $\alpha > 1/2$. Hence, whenever $\pi \notin \mathcal{B}$ as defined in Eq. (4), the seller's supply curve draws average supply away from the benchmark equilibrium. Because of perfect symmetry, the same condition simultaneously holds for any ambiguity-averse buyer. This completes the proof. \square

Proof of Proposition 4. I first prove (ii). Eq. (5) can be written as (neglecting the subscript i)

$$\mathcal{U}(C(\omega)) = \frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} \phi(E[U(\tilde{\pi})]) d\tilde{\pi}.$$

For any given price, the FOC with respect to Q reads

$$\frac{\partial \mathcal{U}}{\partial Q} = \frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} \phi'(E[U(\tilde{\pi})]) \left(\tilde{\pi} \frac{\partial}{\partial Q} U(C(u)) + (1 - \tilde{\pi}) \frac{\partial}{\partial Q} U(C(d)) \right) d\tilde{\pi} = 0. \quad (\text{A.15})$$

Eq. (A.3) and Eq. (A.4) imply

$$\frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} \phi'(E[U(\tilde{\pi})]) \left(\tilde{\pi} U'(C(u))(P - X(u)) + (1 - \tilde{\pi}) U'(C(d))(P - X(d)) \right) d\tilde{\pi} = 0.$$

⁴⁵ Alternatively, for a discrete set of priors, 2Δ refers to the difference $\max(\mathcal{C}) - \min(\mathcal{C})$.

For $Q = \widehat{Q}$ the agent bears no consumption risk, i.e., $C(u) = C(d) \forall \tilde{\pi}$ and $E[U] \perp \tilde{\pi}$. At $Q = \widehat{Q}$, Eq. (A.15) therefore becomes

$$\begin{aligned}
\frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} \tilde{\pi}(P - X(u)) + (1 - \tilde{\pi})(P - X(d)) d\tilde{\pi} &= 0 \quad \Leftrightarrow \\
\frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} P d\tilde{\pi} &= \frac{1}{\bar{\pi} - \underline{\pi}} \int_{\underline{\pi}}^{\bar{\pi}} \tilde{\pi}X(u) + (1 - \tilde{\pi})X(d) d\tilde{\pi} \quad \Leftrightarrow \\
P &= \frac{1}{2} \frac{\bar{\pi}^2 - \underline{\pi}^2}{\bar{\pi} - \underline{\pi}} X(u) + \left(1 - \frac{1}{2} \frac{\bar{\pi}^2 - \underline{\pi}^2}{\bar{\pi} - \underline{\pi}}\right) X(d) \quad \Leftrightarrow \\
P &= \frac{\bar{\pi} - \underline{\pi}}{2} X(u) + \left(1 - \frac{\bar{\pi} - \underline{\pi}}{2}\right) X(d). \tag{A.16}
\end{aligned}$$

Hence, any seller's (buyer's) supply (demand) curve only goes through the benchmark equilibrium $(\widehat{Q}, E[X])$, if the RHS of Eq. (A.16) equals the stock's expected dividend, i.e.,

$$\frac{\bar{\pi} - \underline{\pi}}{2} X(u) + \left(1 - \frac{\bar{\pi} - \underline{\pi}}{2}\right) X(d) = E[X] \quad \Leftrightarrow \quad \pi = \frac{\underline{\pi} + \bar{\pi}}{2}.$$

Thus, whenever π does not correspond to the midpoint of her set of priors $[\underline{\pi}, \bar{\pi}]$, she induces mispricing and suboptimal risk sharing of complex risks.

I hereafter prove (i) for the case where the considered nonzero mass of agents are sellers. In the case of buyers, the analogous reasoning applies. For a given seller i and price P per share, let $Q_i^*(P)$ denote the number of shares satisfying Eq. (A.15). Taking the first order derivative of the second integrand in Eq. (A.15) with respect to $\tilde{\pi}$ yields (neglecting again the subscript i)

$$\begin{aligned}
\frac{\partial}{\partial \tilde{\pi}} \left(\tilde{\pi} U'(C(u))(P - X(u)) + (1 - \tilde{\pi}) U'(C(d))(P - X(d)) \right) &= \\
\underbrace{U'(C(u))(P - X(u))}_{>0} - \underbrace{U'(C(d))(P - X(d))}_{>0} &< 0, \tag{A.17}
\end{aligned}$$

i.e., is always strictly negative for $X(d) \leq P \leq X(u)$.

Regarding the first integrand in Eq. (A.15), there are three different cases. First, if seller i is ambiguity-neutral, i.e., if $\phi'(\cdot)$ is a positive constant, only the second integrand is relevant for determining the optimal number of shares to be sold at P , hereafter denoted by $Q_N^*(P)$. Second, if seller i is ambiguity-averse, i.e., if $\phi'(\cdot)$ is a decreasing function, then the first integrand becomes relevant for determining $Q_A^*(P)$. Third, if she is ambiguity-loving, her increasing function $\phi'(\cdot)$ conversely affects $Q_L^*(P)$.

Because Eq. (A.17) strictly decreases at a constant rate over $[\underline{\pi}, \bar{\pi}]$, Eq. (A.15) can only

hold for $Q_N^*(P)$, if the second integrand changes its sign between $\underline{\pi}$ and $\bar{\pi}$. For $Q < \hat{Q}$, it holds that

$$\frac{\partial}{\partial \tilde{\pi}} E[U(\tilde{\pi})] = U(C(u)) - U(C(d)) > 0 \quad \forall Q < \hat{Q},$$

i.e., whenever seller i is ambiguity-averse, the first integrand in Eq. (A.15) is a strictly decreasing function over $[\underline{\pi}, \bar{\pi}]$. Hence, for $Q_A^*(P)$ the second integrand in Eq. (A.15) needs to switch its sign for a smaller $\tilde{\pi} \in [\underline{\pi}, \bar{\pi}]$, relative to $Q_N^*(P)$, in order to satisfy the first order condition.

Taking the first order derivative of the second integrand in Eq. (A.15) with respect to Q yields

$$\begin{aligned} \frac{\partial}{\partial Q} \left(\tilde{\pi} U'(C(u))(P - X(u)) + (1 - \tilde{\pi}) U'(C(d))(P - X(d)) \right) = \\ \tilde{\pi} \underbrace{U''(C(u))(P - X(u))^2}_{<0} + (1 - \tilde{\pi}) \underbrace{U''(C(d))(P - X(d))^2}_{<0} < 0, \end{aligned}$$

i.e., is always strictly negative for any risk-averse seller. It therefore follows that $Q_A^*(P) > Q_N^*(P)$, i.e., that $Q_A^*(P)$ is closer to \hat{Q} than $Q_N^*(P)$. Since, for any ambiguity-loving seller, the first integrand in Eq. (A.15) then is a strictly increasing function over $[\underline{\pi}, \bar{\pi}]$, the analogous reasoning implies $Q_L^*(P) < Q_N^*(P)$. Thus, the distance between \hat{Q} and $Q_L^*(P)$ is larger than between $Q_N^*(P)$ and \hat{Q} . Finally, the symmetric argument for $Q^*(P) > \hat{Q}$ yields $Q_A^*(P) < Q_N^*(P) < Q_L^*(P)$. This completes the proof. \square

Proof of Proposition 5. Here I prove the distributional results for sellers (supply side). In the case of buyers, the analogous reasoning applies. From the proof of Proposition 1 it follows that for every risk-averse seller i

$$\arg \max_Q f_i(Q|P=E[X]) = \hat{Q}.$$

From the proofs of Proposition 2 and Proposition 4 it follows that for every ambiguity-averse seller i

$$\arg \max_Q \underline{f}_i(Q|P=E_i[X]) = \hat{Q}.$$

Hence, for $P = E_i[X]$, the law of large numbers implies a unique mode at $Q = \hat{Q}$ under both simple and complex risks.

Since $E_i[X]$ is subjective under complex risks, I assume $\pi = \frac{\underline{\pi} + \bar{\pi}}{2}$ in order to compare the *relative* sensitivity of $\frac{\partial E[U_i]}{\partial Q}$ and $\frac{\partial \mathcal{U}_i}{\partial Q}$ for given prices. For multiple-priors utility with

$\alpha_i > 1/2$, it follows from Eq. (A.1) and Eq. (A.10) that for $P = E_i[X]$

$$\left| \frac{\partial E[U_i]}{\partial Q} \right| > \left| \frac{\partial \mathcal{U}_i}{\partial Q} \right| \quad \forall Q \neq \widehat{Q} \quad (\text{A.18})$$

and for $P < E_i[X]$ ($P > E_i[X]$)

$$\left| \frac{\partial E[U_i]}{\partial Q} \right| > \left| \frac{\partial \mathcal{U}_i}{\partial Q} \right| \quad \forall Q > \widehat{Q} \quad (\forall Q < \widehat{Q}). \quad (\text{A.19})$$

To see this, consider the following case. If at a fixed price $P < E_i[X] \leq X(u)$ and a fixed quantity $Q_0 > \widehat{Q}$ it holds that $\frac{\partial E[U_i]}{\partial Q} > 0$, then $Q_0 < Q^*$ and

$$\left| \underbrace{\pi U'(C(u))(P - X(u))}_{\text{part 1}} \right| < \left| \underbrace{(1 - \pi) U'(C(d))(P - X(d))}_{\text{part 2}} \right|. \quad (\text{A.20})$$

Recall that, in general, $\underline{\pi} < \bar{\pi}$ if $C(u) > C(d)$ and $\underline{\pi} > \bar{\pi}$ if $C(u) < C(d)$. Since, for seller i , $Q_0 > \widehat{Q}$ implies $C(u) < C(d)$, it follows in this case that $\underline{\pi} > \bar{\pi}$. Hence, $\alpha_i > 1/2$ in Eq. (A.10) puts a relatively higher weight on part 1 in Eq. (A.20) than on part 2, which in turn implies the corresponding inequality in Eq. (A.19). Analogous reasoning applies to the other cases.

Similarly, for smooth ambiguity preferences, strict concavity of $\phi_i(\cdot)$ in Eq. (5) implies a lower sensitivity with respect to Q of Eq. (10) relative to Eq. (9).

Since, for every seller i with fixed $\psi_i(\cdot)$, the area under both pdfs f_i and \underline{f}_i has to equal unity, it needs to hold that

$$\begin{aligned} f_i(\widehat{Q}|P=E[X]) &> \underline{f}_i(\widehat{Q}|P=E[X]) \\ f_i(\overline{Q}|P=E[X]) &< \underline{f}_i(\overline{Q}|P=E[X]) \quad \text{and} \quad f_i(\underline{Q}|P=E[X]) < \underline{f}_i(\underline{Q}|P=E[X]) \\ f_i(\overline{Q}|P<E[X]) &< \underline{f}_i(\overline{Q}|P<E[X]) \quad \text{and} \quad f_i(\underline{Q}|P>E[X]) < \underline{f}_i(\underline{Q}|P>E[X]) \end{aligned}$$

for $\overline{Q} \gg \widehat{Q}$ and $\underline{Q} \ll \widehat{Q}$ distant enough from \widehat{Q} . C1–C3 then follow from the law of large numbers. This completes the proof. □

Appendix B: Determining π in the Presence of Complex Risks

Starting point is the SDE of the geometric Brownian motion in Figure 7, i.e.,

$$dS_t = 10\%S_t dt + 32\%S_t dW_t,$$

where W_t is a standard Brownian motion. Applying Itô to $f := \ln(S_t)$ yields

$$S_2 = \exp \left\{ \left(10\% - \frac{32\%^2}{2} \right) + 32\%(W_2 - W_1) \right\}.$$

Hence,

$$\mathbb{P}(S_2 \geq 1.05) = \mathbb{P} \left(W_2 - W_1 \leq \underbrace{\left(\ln(1.05) - 10\% + \frac{32\%^2}{2} \right)}_{\approx 0} \frac{1}{32\%} \right).$$

Recalling that the increment $W_2 - W_1$ has a standard normal distribution,⁴⁶ it follows that $\mathbb{P}(S_2 \geq 1.05)$ corresponds to $1/2$.⁴⁷

Appendix C: Adjustment of average supply and demand curves according to subjective beliefs

For a given case, I denote by $\bar{E}_S[X]$ sellers' average point estimate of the risky asset's expected payoff under complex risks. In order to account for deviations of $\bar{E}_S[X]$ from $E[X]$, the following linear transformation is applied to the predefined price vector used to elicit sellers' supply functions:

$$adj(P) = \begin{cases} P - (\bar{E}_S[X] - E[X]) \frac{P - X(d)}{E_S[X] - X(d)}, & \text{for } X(d) \leq P < \bar{E}_S[X] \\ P - (\bar{E}_S[X] - E[X]) \frac{X(u) - P}{X(u) - \bar{E}_S[X]}, & \text{for } \bar{E}_S[X] \leq P \leq X(u). \end{cases}$$

Furthermore, let \bar{Q}_S denote the linearly interpolated average supply curve and $\bar{Q}_{S,adj}$ the corresponding curve plotted against $adj(P)$ instead of P . It then still holds that $\bar{Q}_{S,adj}$ spans from $X(d)$ to $X(u)$, but simultaneously that $\bar{Q}_{S,adj}(E[X]) = \bar{Q}_S(\bar{E}_S[X])$. The exact

⁴⁶ This information was provided as part of the instructions.

⁴⁷ Strictly speaking, it holds that $\mathbb{P}(S_2 \geq 1.05) = 0.49999$. Linearly approximating $\ln(1.05)$ by 0.05 implies $\mathbb{P}(S_2 \geq 1.05) = 0.50150$.

same linear transformation with $\bar{E}_B[X]$ instead of $\bar{E}_S[X]$, with B for buyers, is also used to adjust average demand curves under complex risks.

Appendix D: Additional Figures

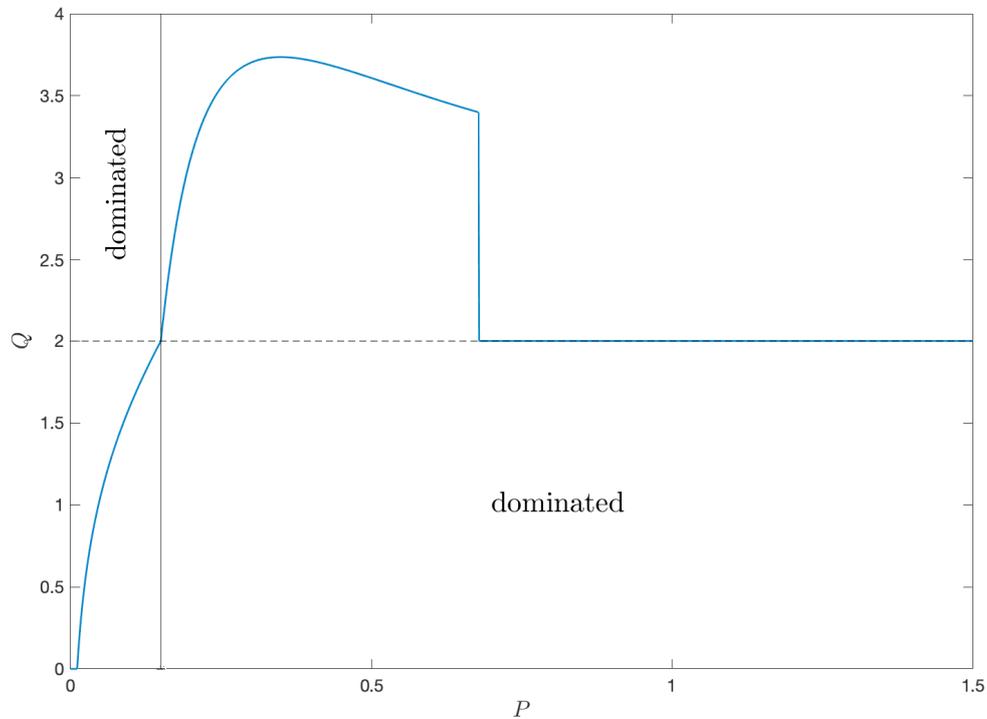


FIGURE D.1. EXAMPLE OF NONMONOTONIC SUPPLY CURVE

Notes: Supply curve for seller i with utility function as defined in Remark 1. Parameters: $X(u) = 1.5$, $X(d) = 0$, $\pi = 1/10$, $\epsilon = 0$, $\alpha = 1$, $\bar{C} = 3 + 2\pi X(u)$, $E_i = 4$, $I_i(u) = 0$, $I_i(d) = 3$.

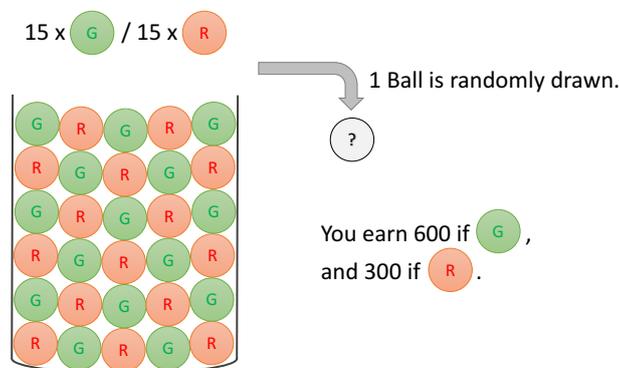


FIGURE D.2. LOTTERY BASED ON URN WITH SIMPLE RISKS

Notes: This figure shows the lottery based on the urn with simple risks. Whenever the randomly drawn ball is green, the lottery pays ECU 600 (experimental currency units) and ECU 300 if it is red. participants' respective certainty equivalents were elicited via [Abdellaoui, Baillon, Placido, and Wakker's \(2011\)](#) iterative choice list method.

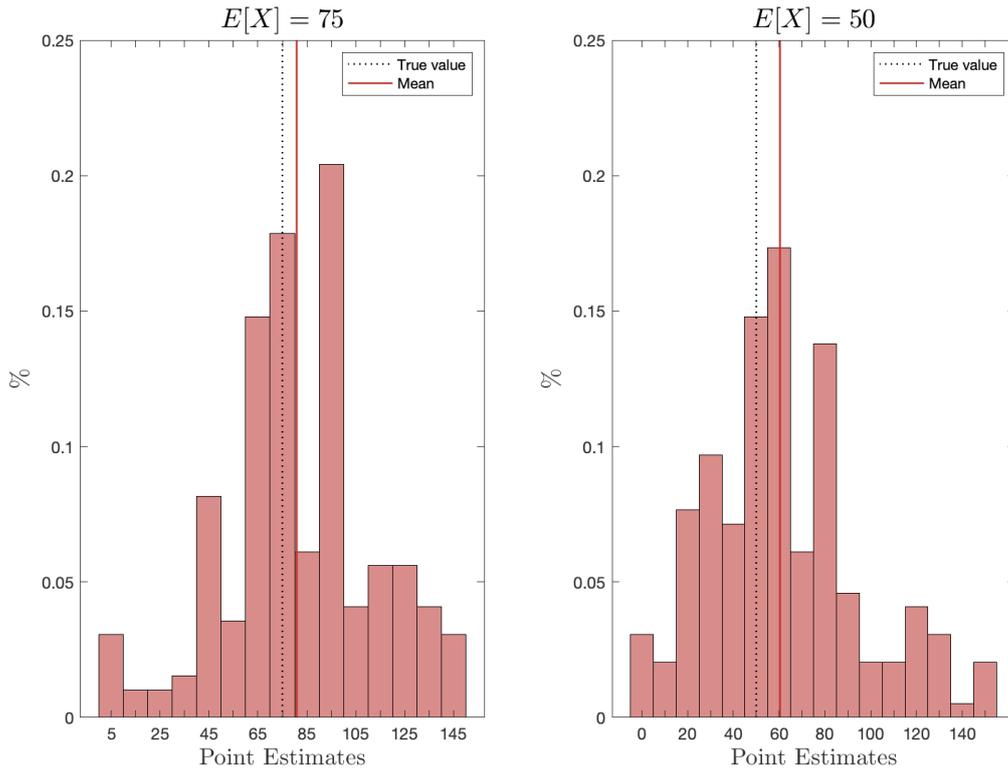


FIGURE D.3. DISTRIBUTION OF EXPECTED PAYOFFS UNDER COMPLEX RISKS

Notes: This figure plots the distribution of participants' point estimates of the risky asset's expected payoff under complex risks. In the left (right) column, the distribution is based on trading rounds where π equals $1/2$ ($1/3$). The dotted horizontal line indicates the true expected payoff. The dotted vertical line indicates the mean point estimate across participants.

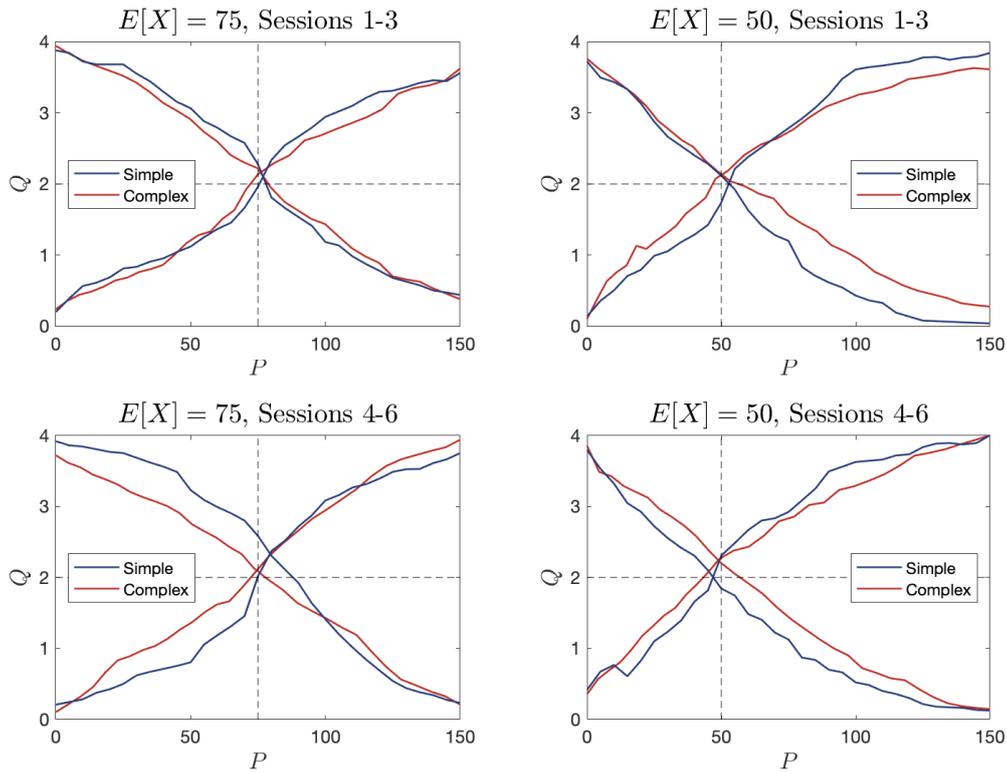


FIGURE D.4. AVERAGE SUPPLY AND DEMAND ADJUSTED FOR SUBJECTIVE BELIEFS

Notes: This figure shows the average adjusted supply and demand curves across participants and trading rounds. Average curves for complex risks are adjusted as described in Appendix C to account for deviations of average beliefs from the true underlying payoff distribution. In the top (bottom) row, averages are computed across sessions where complex (simple) trading rounds are followed by simple (complex) trading rounds. In the left (right) column, averages are computed across trading rounds where π equals $1/2$ ($1/3$). The dotted horizontal line indicates the perfect hedging quantity. The dotted vertical line indicates the risky asset's expected payoff.

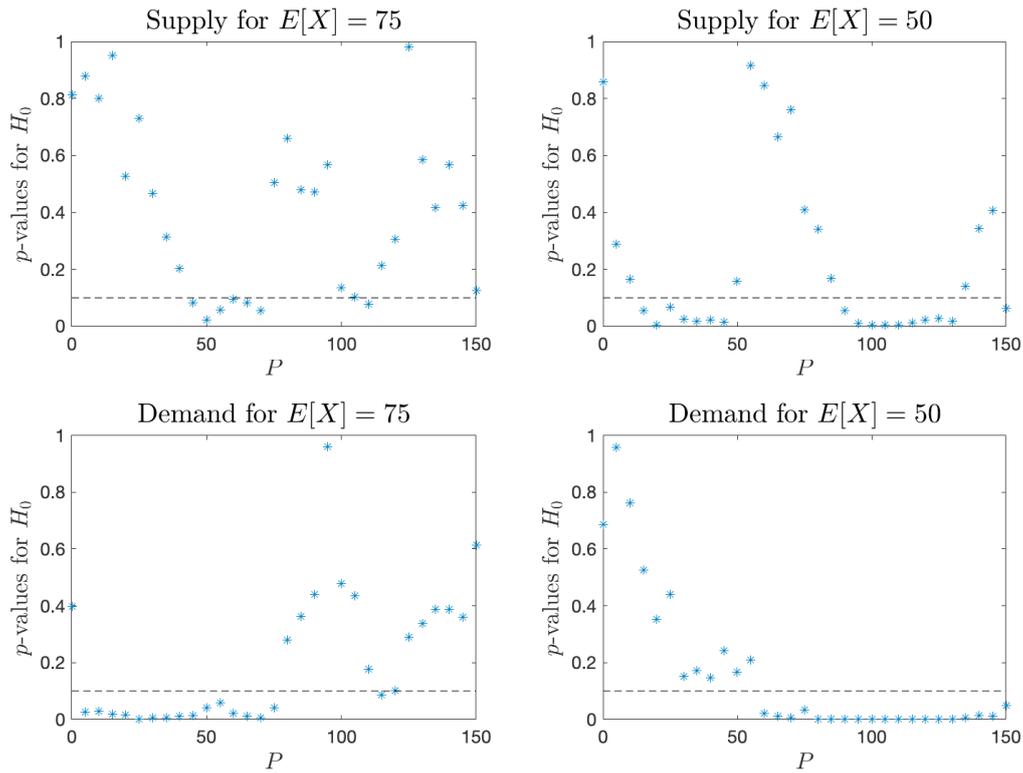


FIGURE D.5. TESTING FOR DIFFERENCES IN PRICE SENSITIVITY

Notes: This figure reports the p -values of a Wilcoxon signed-rank test of the differences between average supply (demand) curves for simple and complex risks. Averages are computed across participants and trading rounds. Average curves for complex risks are adjusted as described in Appendix C and linearly interpolated to allow for a direct comparison with simple risks. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across trading rounds where π equals $1/2$ ($1/3$). The dotted line indicates a p -value equal to 10%.

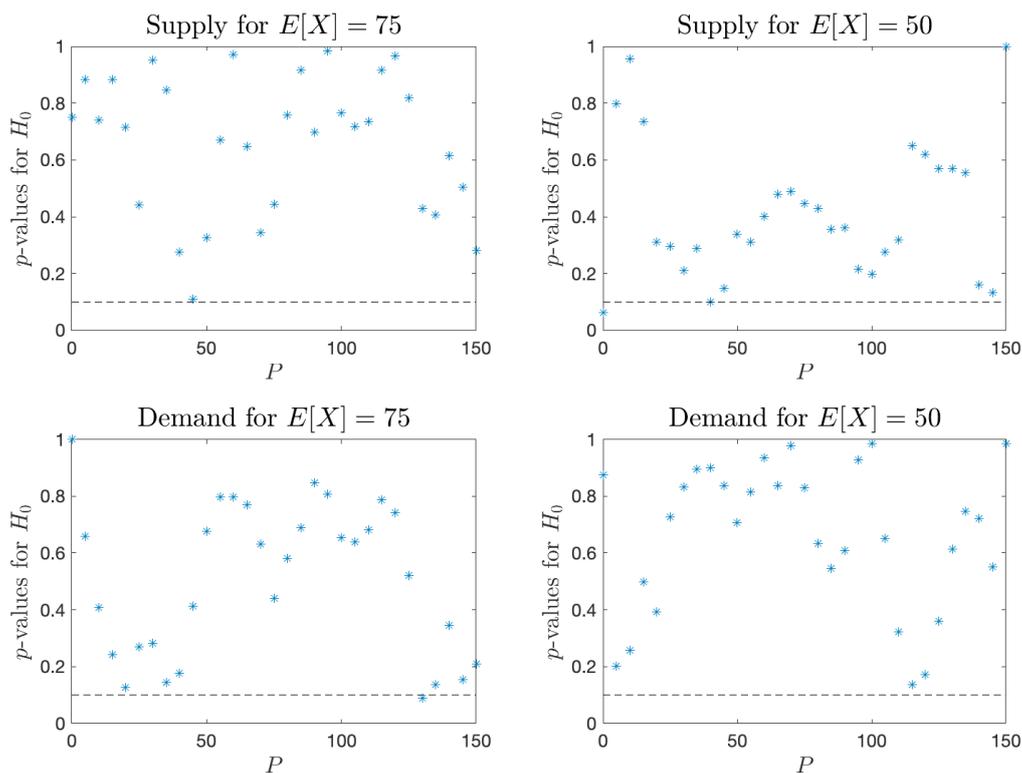


FIGURE D.6. TESTING FOR PRICE-TAKING BEHAVIOR UNDER COMPLEX RISKS

Notes: This figure reports the p -values of a Wilcoxon signed-rank test of the differences between average supply (demand) curves for complex risks under market clearing and random price draws. Averages are computed across participants and complex trading rounds. Average curves are adjusted as described in Appendix C and linearly interpolated to allow for a direct comparison with simple risks. In the top (bottom) row, average supply (demand) curves are computed across all sessions. In the left (right) column, averages are computed across complex trading rounds where π equals $1/2$ ($1/3$). The dotted line indicates a p -value equal to 10%.

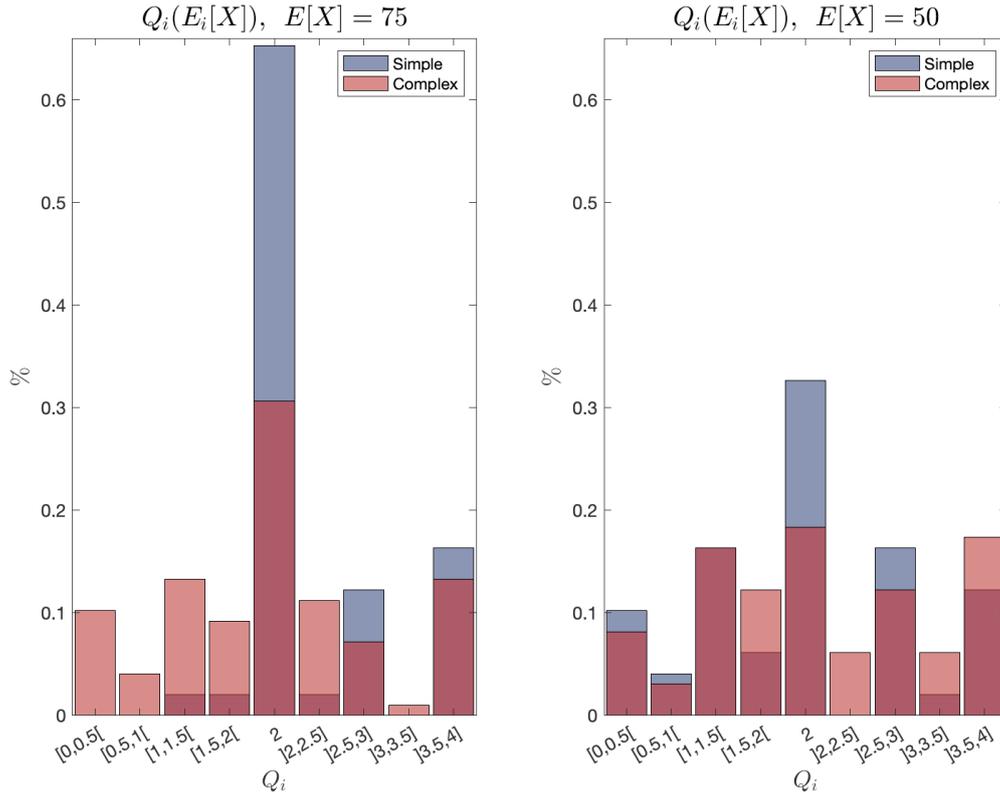


FIGURE D.7. DEMAND DISTRIBUTION FOR PRICES EQUAL TO EXPECTED PAYOFFS

Notes: This figure shows the number of shares demanded by buyers for prices equal to (estimated) expected payoffs. The empirical distributions are computed across participants and sessions. The left (right) plot contrasts average distributions between simple and complex trading rounds with π equal to $1/2$ ($1/3$). If, under complex risks, buyers' point estimate $E_i[X]$ lies between two elements of the predefined price vector, linearly interpolated quantities are reported.

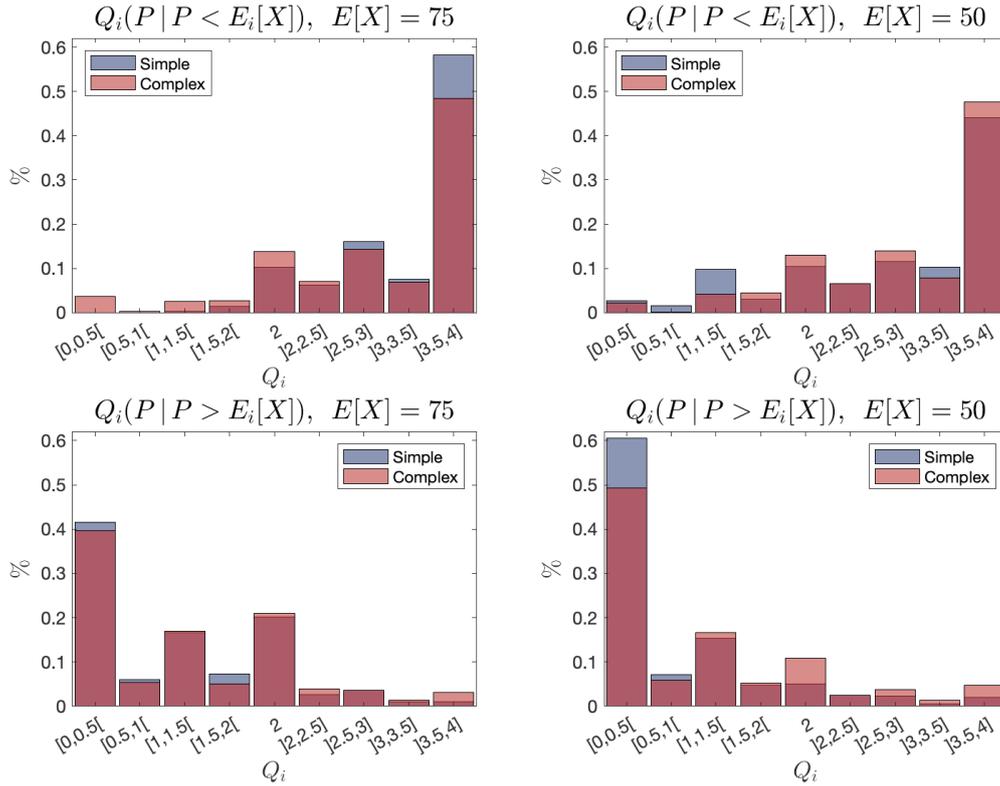


FIGURE D.8. DEMAND DISTRIBUTION FOR PRICES DIFFERENT FROM EXPECTED PAY-OFFS

Notes: This figure shows the number of shares demanded by buyers for prices different from expected payoffs. The empirical distributions between simple and complex risks are computed across participants and sessions. In the top (bottom) row, total demands for prices below (above) $E_i[X]$ are reported. The left (right) column shows average demand distributions across trading rounds with π equal to $1/2$ ($1/3$).

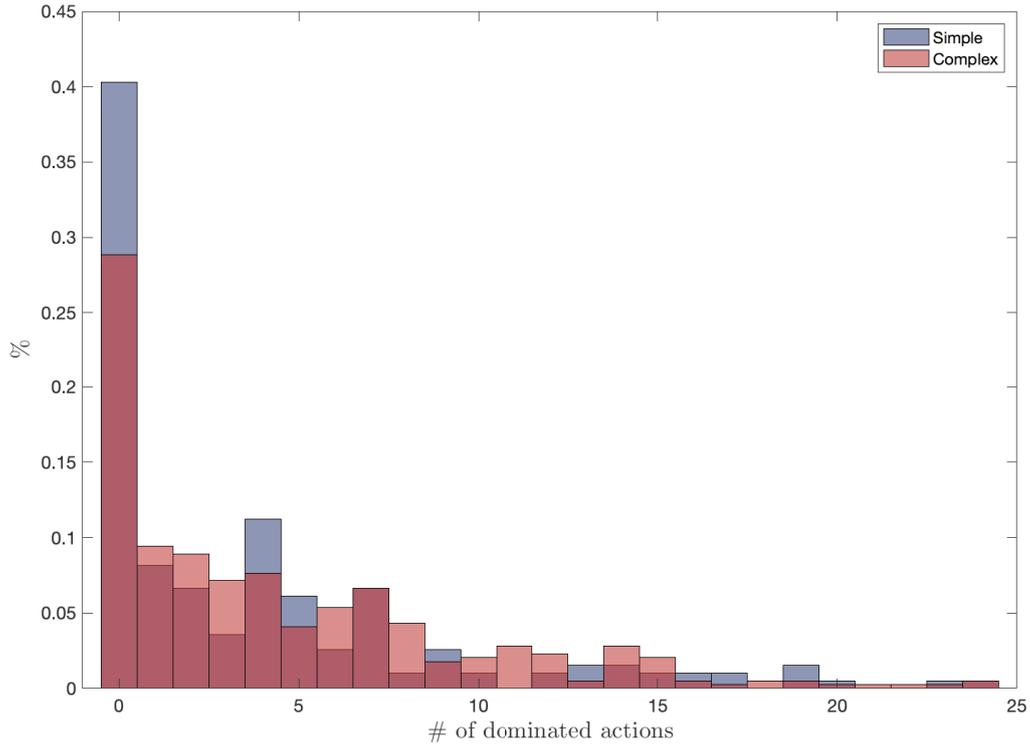


FIGURE D.9. FREQUENCY OF DOMINATED TRADING STRATEGIES

Notes: This figure shows the distributions of dominated action frequencies across all participants. Under simple risks, dominated actions correspond to offered (demanded) quantities above (below) \hat{Q} shares for $P < E[X]$ and vice versa for $P > E[X]$. Under complex risks, dominated actions correspond to offered (demanded) quantities above (below) \hat{Q} shares for $P < E_i[X]$ and vice versa for $P > E_i[X]$. Note that in the presence of complex risks, the price thresholds depend on participants' individual point estimates.

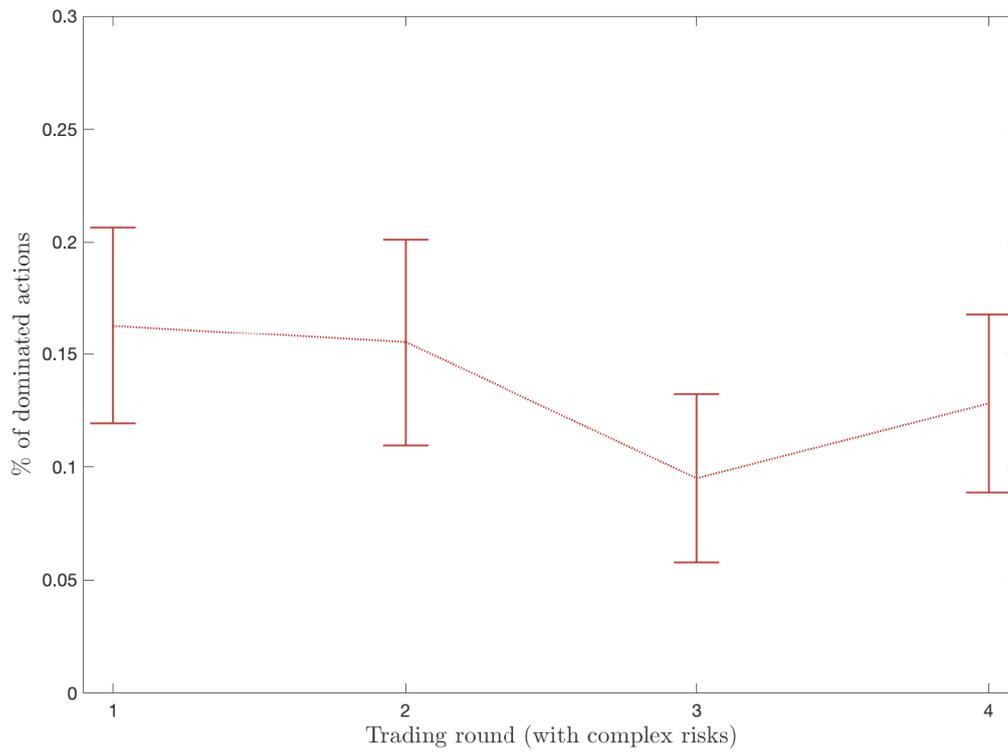


FIGURE D.10. LEARNING UNDER COMPLEX RISKS

Notes: This figure shows the evolution of the average percentage of dominated trading strategies (see Figure D.9) over the four trading rounds with complex risks (see Table III). Error bars indicate standard errors of the mean.

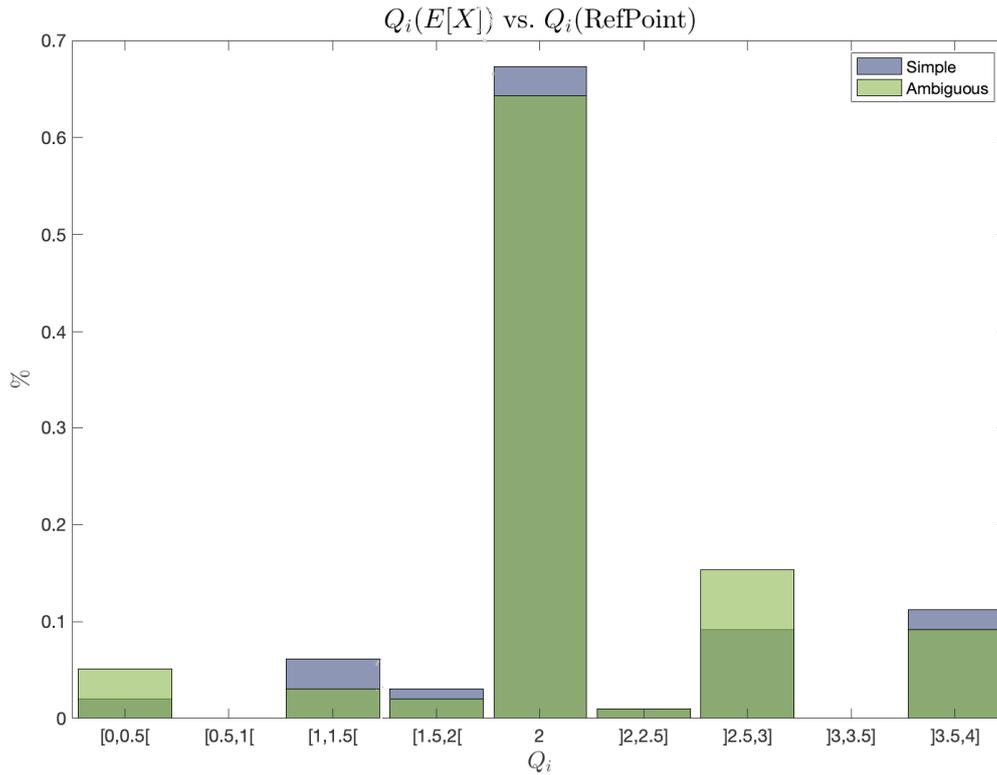


FIGURE D.11. DISTRIBUTION FOR PRICES EQUAL TO EXPECTED PAYOFF (REFERENCE POINT)

Notes: This figure shows empirical distributions of supplied and demanded shares at a fixed price of ECU 75. Percentages are computed across participants and sessions. For simple risks, only the trading round with π equal to $1/2$ is considered. For ambiguous risks, a price of ECU 75 corresponds to the natural reference point, assuming that participants believe in a fifty-fifty likelihood under pure ambiguity.

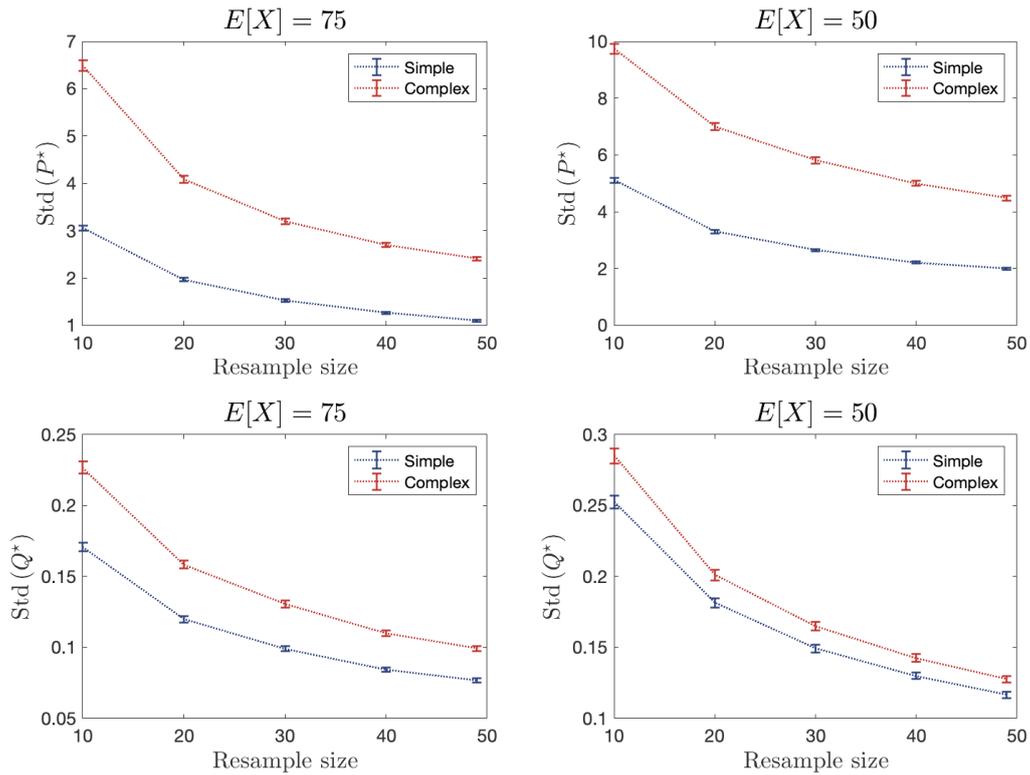


FIGURE D.12. EQUILIBRATION VARIABILITY

Notes: This figure shows bootstrapped standard deviation estimates of market-clearing prices and quantities for simple and complex risks. Average supply and demand curves are determined for different resample sizes. For each pair of averaged supply and demand, linearly interpolated market-clearing prices and quantities are computed. Repeating this procedure ten thousand times yields the depicted standard deviation estimates of equilibrium prices (top row) and quantities (bottom row). The left (right) column shows bootstrapped moment estimates for trading rounds with π equal to $1/2$ ($1/3$). Error bars indicate 99%-confidence intervals.

Appendix E: Experimental Instructions

The instructions for participants acting as sellers are provided on the following pages. Analogous instructions were distributed to participants acting as buyers and are available upon request.

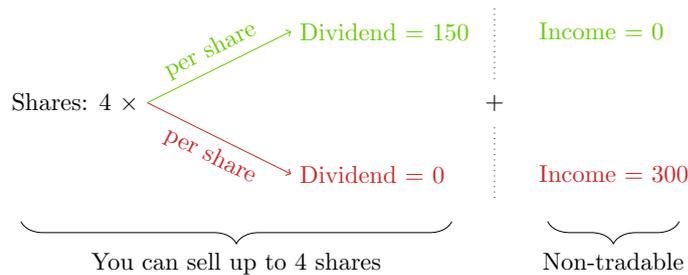
Instructions I/II

Welcome to this experiment at the Department of Banking and Finance, University of Zurich. This is the first out of 2 instruction sheets. Please read each sheet very carefully. Fully understanding the instructions will allow you to perform better on the task, thereby earning more money. Raise your hand if you have any questions or as soon as you have read everything and are ready to continue.

1 Situation

The experiment consists of a sequence of 7 trading rounds. In each trading round the same number of buyers and sellers are present. You are a **seller**. Your role will not change throughout the experiment.

At the beginning of every round, you will receive a fresh supply of 4 shares of a given security. During each round you can sell between 0 and 4 of these shares. The security either pays a dividend per share equal to 150 or 0. Besides this dividend per share, the security does not pay anything else (no capital gains). Additionally, you are provided with some non-tradable income: whenever the security happens to pay a dividend of 150 per share, you receive 0, and if it does not pay anything (dividend of 0), you receive 300. This additional income **does not depend** on how many shares you are selling. The following graph summarizes your holdings at the beginning of every round:



Your wealth at the end of each round is the sum of received proceeds from trading, collected dividends, and additional income. It is not carried over to the subsequent round, this means you always start out with 4 shares. At the end of every round, the trading outcome, realized dividends, and your respective wealth are displayed.

2 Trading

Trading happens in 2 phases. First, you have to select how many shares you want to sell in case the price equals 0, 25, 50, 75, 100, 125, or 150. The computer then linearly fills up your selling quantities for the remaining 5-unit steps between 0 and 150 (5, 10, 15, ...). Second, you are asked to make **further adjustments** until you end up with the **exact** quantities you want to sell for any given price. Note, quantities can be entered with up to 2 decimal places of precision.

The price determination method of the current round is always displayed in the upper right corner of your screen. There are two ways how prices are determined. If there is **market clearing**, the computer sets the price such that the number of traded shares is maximized. Alternatively, the computer will choose the price randomly (**random price**) with equal probabilities across the full list of given prices.

You will now go through a first practice round. This practice round will not impact your payment.

Instructions II/II

Please read this sheet very carefully. Raise your hand if you have any questions or as soon as you have read everything and answered the comprehension questions at the end.

3 How Dividends Are Determined

The computer randomly determines whether the security is going to pay a dividend or not. However, the information about the structure that governs the computer's random choices varies between trading rounds. There are 2 different cases:

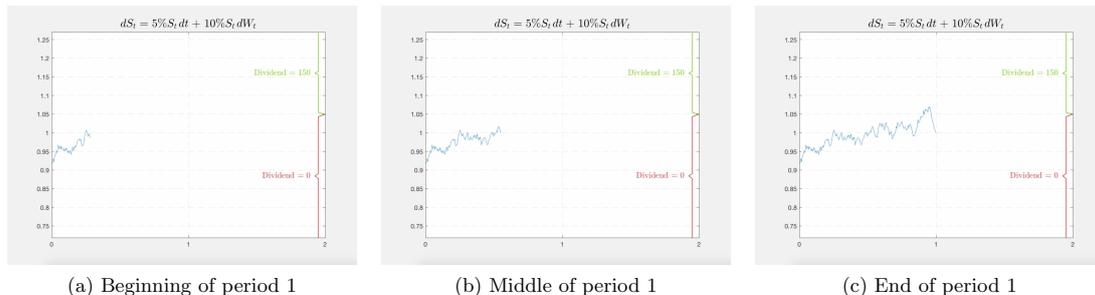
1. *Urn.*—The computer draws 1 ball out of an urn with 30 balls. The balls are either green or red, the respective composition is revealed at the beginning of the trading round. Whenever the color of the drawn ball is green, the security pays a dividend equal to 150 per share (and 0 if red).

2. *Simulated reference path.*—The computer simulates the evolution of a reference path over 2 time periods, but only the first period will be displayed. Whenever the path ends up above a certain limit, the security pays a dividend equal to 150 per share (and 0 if the path ends up below this limit). The only purpose of this path is to determine whether the security pays a dividend or not.

What you will see.—You are provided with a formal description of the reference path S_t , where the random component is denoted by W_t . W_t follows a normal distribution with mean equal to 0 and variance equal to the corresponding change in time. For example, the full description of the path S_t could look like this

$$dS_t = 5\%S_t dt + 10\%S_t dW_t,$$

where dS_t denotes the change of S_t over the very small (infinitesimal) time change of length dt . Additionally, you will see a video of the path S_t between time 0 and the end of period 1:



The difference of the random component W_t between the ends of period 1 and 2, $W_2 - W_1$, follows a normal distribution with mean 0 and variance 1. For simplicity, every path is scaled such that $S_1 = 1$.

Based on this information you can assess the probability of the dividend being equal to 150 (\rightarrow path ends up in the green region at time 2).

4 List of Lotteries, Questionnaire, and Payment

After the 7 trading rounds, you have to repeatedly choose 1 out of 2 options for 2 lists of lotteries. For both lists, the computer randomly selects and plays 1 of your chosen options. Finally, you will be asked

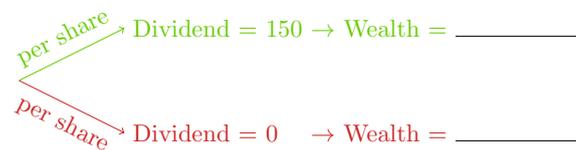
to fill-in a short questionnaire.

Your final payment will be determined as follows:

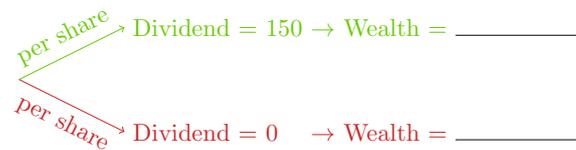
1. The computer randomly picks 1 out of the 7 trading rounds or 1 of the 2 lottery outcomes with equal probability ($\frac{1}{9}$ for each). It is therefore critical that you concentrate on **every round**. You will be paid either your wealth at the end of the selected trading round or the outcome of the selected lottery, both in CHF divided by 12.
2. In all rounds with simulated reference paths, you are asked to submit your best guess regarding the probability of the dividend being equal to 150. If your guess is correct (within +/- 3%), you earn an additional 3 CHF whenever this round is selected for payment.

5 Comprehension Questions

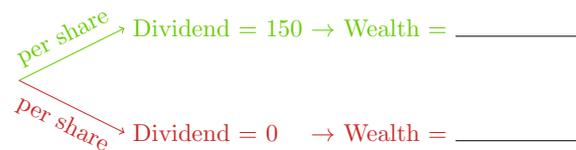
- (1) Assume you have sold 4 shares at a price of 50 per share, what is your wealth in the 2 scenarios?



- (2) Assume you have sold 4 shares at a price of 150 per share, what is your wealth in the 2 scenarios?



- (3) Assume you have sold 2 shares at a price of 50 per share, what is your wealth in the 2 scenarios?



- (4) Does the **difference** between your wealth in the green and the red scenario depend on...?

The paid price The number of sold shares

- (5) What is the difference between your wealth in the 2 scenarios, if you exactly sell 2 shares?

Difference = _____

Raise your hand after you have answered the comprehension questions. After double-checking, you will go through 2 last practice rounds. These practice rounds will not impact your payment.