Structural Stochastic Volatility*

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Abstract

We use a local (in time) expansion of the characteristic function of the equity process in continuous time to derive short-maturity option prices. The prices, along with data on short-maturity options, are employed to jointly identify equity characteristics (spot volatility, spot leverage and spot volatility of volatility) which have been the focus of separate strands of the literature. We show that the proposed identification method yields measurements which are statistically accurate and economically revealing. Interpreting equity as a call option on asset values, all equity characteristics should depend on fundamental state variables, such as the variance of the firm's assets and the extent of the firm's financial leverage. Among other findings, consistent with economic logic, we document a strong link between spot leverage (the generally-negative correlation between equity returns and spot volatility) and financial leverage (the firm's debt-to-equity ratio), a relation invariably found to be elusive in the data. We conclude that the economic content of option-implied measurements can be put to work to study the structural drivers of equity (and debt) return dynamics from a novel vantage point.

Keywords: short-maturity options, spot volatility, spot volatility of volatility, spot leverage.

JEL classification: C51, C52, G12.

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1 Introduction

Reduced-form stochastic volatility models are ubiquitous in the vast asset pricing literature (see, e.g., Andersen and Benzoni, 2014, for a review). Such models (for instance, the "affine" specification of Duffie, Pan, and Singleton, 2000) generally rely on parametric specifications of the equity return dynamics.[1] The adopted parametrizations are intended to make the model easy to specify, estimate, and deploy in the pricing of structured products. In spite of the usefulness of reduced-form models, however, the economic interpretation of the processes governing equity return dynamics continues to be elusive. The economics of stochastic volatility and, by extension, of the equity dynamics is the subject of this paper.

We make four contributions. First, we employ a *local* expansion of the characteristic function of a general Brownian semi-martingale recently suggested by Bandi and Renò (2019) to derive a novel closed-form pricing formula for *short-maturity options*. As documented by Andersen, Fusari, and Todorov (2017), short-maturity options have seen an ever increasing interest from investors since the introduction by the Chicago Board Options Exchange (CBOE) of the so-called "weekly options" in 2005. We exploit the increased liquidity of this new market for the effective identification of equity characteristics. Specifically, the proposed pricing formula, coupled with the estimation technique recently developed by Andersen, Fusari, and Todorov (2015a) and Andersen, Fusari, Todorov, and Varneskov (2019), allows us to *jointly* recover the option-implied dynamics of the equity *spot* volatility, the *spot* volatility of volatility, and *spot* leverage. Identification relies on the above quantities not varying under measure change (from statistical to risk-adjusted). In essence, not only do option prices contain information about the prices of risk, they are also informative about the risks themselves, as emphasized by Andersen, Fusari, and Todorov (2015b) in other contexts.

It is well-known that identifying time-varying *spot* (or local, in time) quantities is a nuanced econometric problem with a well-defined bias/variance trade-off. Localization in time is critical to avoid biases and capture genuine time variation. Excessive localization will, however, lead to noisy estimates due to a necessarily reduced sample size. Yet, the measurement of spot quantities is helpful for risk assessments and proper risk management in that it amounts to the identification of the time series of potentially-priced quantities of risk. Not surprisingly, separate interest in spot volatility, spot volatility of volatility and spot leverage has lead to three separate strands of the literature (which we review in Section 2). In our first contribution, we address the three strands of

[1] Nonparametric exceptions are contained in the work of Kanaya and Kristensen (2016) and Bandi and Renò (2018).

the literature in a cohesive methodological framework. Specifically, exploiting the available cross section of short-maturity options at every point in time and the newly-proposed short-maturity option prices, we *jointly* identify the three quantities for all time periods of interest. Joint option-based identification will be shown to be particularly beneficial for spot volatility of volatility and spot leverage in that it does not rely on the first-stage estimation of spot volatility, a necessary input in the implementation of high-frequency sample counterparts. As a consequence, we show that the informational content of short-term options, when suitably extracted, translates into estimates with favorable statistical properties as compared to sample analogues constructed using high-frequency equity prices only. It will also be shown to yield economically revealing measurements. The economic drivers of reduced-form equity characteristics represent the substantive core of the remainder of the paper, to which we now turn.

In our second contribution, consistent with the original intuition in Merton (1974), we view equity as a call option written on the firm's assets with strike price given by the firm's debt. Within a generalized Merton model which (differently from the original specification in Merton, 1974) allows for time-varying asset volatility (correlated with asset returns) and jumps in asset values, we derive the implied dynamics of the processes driving both the firms' equity returns and their volatility. Specifically, we make explicit the mapping between the reduced-form characteristic of the equity process (i.e., spot volatility, spot volatility of volatility and spot leverage) and the structural state variables of the model (e.g., asset value and volatility of the asset values). In particular, because the "moneyness" of equity, viewed as a call option on the assets, is one-to-one with the debt-to-equity ratio (i.e., financial leverage), the model provides clear implications for the relation between reduced-form risk quantities and structural sources of risk, as captured by the firm's relative (to assets) debt.

Third, we study the empirical validity of the model's predictions using the reduced-form (option-implied) estimates of the equity characteristics along with accounting data on corporate debt for a rich cross section of companies representing an array of industries (from financials, to energy, to health care and so on). Unconditionally (i.e., across firms), we find a strong relation between financial leverage and option-implied estimates with signs which are consistent with the model's predictions (positive for spot volatility and spot volatility of volatility and negative for spot leverage). Time-series findings (averaged across firms) confirm and reinforce these predictions. So does a more detailed sectorial analysis. Because the model predicts that, in low financial leverage firms, the dependence between characteristics of the equity process and financial leverage should be milder, we also investigate companies in the bottom 25th percentile of the cross-sectional distribution of financial leverage. The data, along with the option-implied estimates, support this model implication as well. Importantly, we show that the use of spot estimates obtained from high-frequency nonparametric sample analogues would be - particularly in the case of volatility of volatility and spot leverage - excessively noisy and, therefore, less revealing about their structural drivers. This observation explains the elusive nature of, e.g., the relation between spot leverage and financial leverage in the existing literature (see, e.g., Figlewski and Wang, 2000, Hens and Steude, 2009, Hasanhodzic and Lo, 2019, and the references therein) and justifies our emphasis on the discipline imposed by semi-parametric option-implied identification.

Our fourth contribution is an alternative take on the study of the integration between equity and bond markets and their relative pricing. Just like equity may be viewed as a call option on the firm's assets, the value of debt is naturally interpreted as that of a zero-coupon bond net of of the value of the corresponding put option on asset values. Thus, credit spreads should be a function of the same state variables which drive reduced-form equity characteristics like spot volatility, spot volatility of volatility and spot leverage. In particular, even though the equity characteristics depend on the firm's structural riskiness (as represented by its financial leverage), they should contain information about the model's state variables that goes beyond that in financial leverage. In this sense, they should also play a role in explaining corporate bond pricing beyond financial leverage. This is, again, what the data suggests. Both cross-sectionally and in the time series, we find that credit spreads are *jointly* explained by financial leverage and equity characteristics. As found previously, the use of nonparametric high-frequency measurements in place of the proposed option-implied estimates would obfuscate considerably the reported findings.

Fig. 1 offers a visualization of our approach. In a nutshell, we exploit a new market of liquid exchange-traded derivatives (short-term options) to estimate equity characteristics (Sections 3, 4 and 5). Because equity can be viewed as a claim on the firm's assets, the equity characteristics can be mapped into the fundamentals of the firm (Sections 6 and 8). This map justifies further validating the structural drivers of the equity characteristics by examining the integration between liquid, exchange-traded, claims (equity) and illiquid, over-the-counter, claims (corporate debt) on the same assets (Section 9).

The paper proceeds as follows. Section 2 discusses the existing literature and clarifies the paper's positioning. In Section 3 we introduce a new pricing formula for short-maturity options. Section 4 is about identification of the equity characteristics by means of short-maturity option data. For each day in the sample, the equity characteristics are inferred by matching the empirical Black-Scholes implied volatilities for a cross-section of options with limited time-to-maturity to the

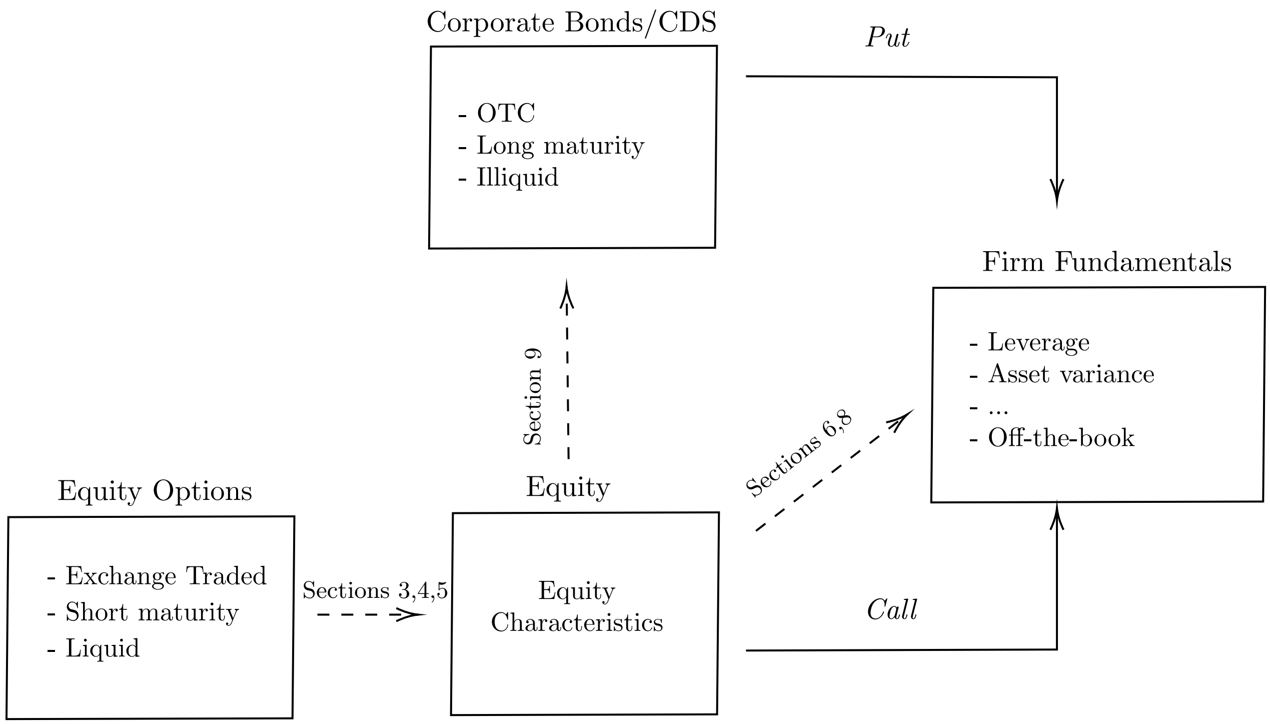


Figure 1: **A diagram of the paper's structure.**

theoretical implied volatilities from the model proposed in Section 3. Section 5 studies the accuracy of the procedure and compares it to that of methods of inference based on sample counterparts constructed using high-frequency price data. In Section 6 we derive theoretical implications for equity dynamics from a Merton's style structural model with stochastic volatility in the unobservable asset process and jumps in asset values. The model leads to explicit functional forms mapping the equity characteristics into structural state variables. These relations are at the core of our empirical work. Section 7 presents the data and provides details about the merging of short-maturity option price information and necessary (for the purposes of our analysis) accounting information. Empirical findings supporting structural interpretations of the equity characteristics are reported in Sections 8 and 9. Section 10 concludes by providing directions for future work. The Appendix contains technical details (including additional simulations and supporting empirical analysis regarding specific corporate sectors).

2 Positioning in the existing literature

A burgeoning literature has focused on the identification of a variety of measures of "variation" using high-frequency price data (a rich discussion is provided by Aït-Sahalia and Jacod, 2014).

The literature has studied the identification of volatility measures (like spot volatility)[2] as well as the identification of quantities which depend on (innovations to) volatility (like spot leverage[3] and spot volatility of volatility[4]).

While spot volatility can be rather successfully identified using nonparametric high-frequency sample analogues, quantities which depend on *estimates* of spot volatility (like estimates of spot leverage and estimates of spot volatility of volatility) are more delicate. Aït-Sahalia, Fan, and Li (2013) are emphatic about this observation in the case of spot leverage. They write: " ... even in idealized situations, the bias (in leverage estimation) is large, and attempts to correct for the latency of the volatility, or for the presence of market microstructure noise, do not improve matters."

A natural solution, one which we adopt in this paper, is to use methods of identification which do not require preliminary estimates of spot volatility and instead hinge on the joint estimation of all quantities of interest. To this extent, we follow the logic in Andersen, Fusari, and Todorov (2015a) and Andersen, Fusari, and Todorov (2015b), whose focus is on spot volatility estimation,[5] and use panels of option over each day to jointly identify the relevant state variables. In agreement with our desire for robustness in the estimation of quantities which depend on spot volatility estimates, we show that the use of option information is particularly suitable for the purpose of identifying spot leverage and spot volatility of volatility. Their option-based estimation, in fact, dispenses with the first-stage evaluation of spot volatility as a necessary input to define sample analogues.

While the computation of sample counterparts based on high-frequency asset prices amounts to a fully nonparametric procedure, the method we suggest is semi-parametric in nature because of its reliance on parametric assumptions on the jump sizes. The usefulness of parametric restrictions on the jump sizes for the identification of otherwise unrestricted (i.e., nonparametric) stochastic volatility models for equity returns has been highlighted in other contexts (Bandi and Renò, 2016, and Bandi and Renò, 2018)

The literature on option pricing and related inferential issues is broad and well-established (see, e.g., Garcia, Ghysels, and Renault, 2010). Interesting recent work has exploited expansions of the implied volatility surface either in the maturity dimension (Medvedev and Scaillet, 2007) or in the maturity and long-moneyness dimension (Aït-Sahalia, Li, and Li, 2019) in order to calibrate

[2]See, e.g., Fan and Wang, 2008, Mykland and Zhang, 2008, Kristensen, 2010, Zu and Boswijk, 2014, Mancini, Mattiussi, and Renò, 2015, Bandi and Renò, 2018, Bibinger, Hautsch, Malec, and Reiss, 2019, and the references therein.

[3]See, e.g., Bandi and Renò, 2012, Aït-Sahalia, Fan, and Li, 2013, Wang and Mykland, 2014, Wang, Mykland, and Zhang, 2017, Kalnina and Xiu, 2017, Aït-Sahalia, Fan, Laeven, Wang, and Yang, 2017, and the references therein

[4]See, e.g., Vetter et al., 2015, Sanfelici, Curato, and Mancino, 2015, Barndorff-Nielsen and Veraart, 2012, and the references therein.

[5]See, also, the work of Todorov (2019) on spot volatility estimation from option data.

or estimate, respectively, stochastic volatility models. We do not expand the implied volatility surface. The procedure identifies the state variables by matching (implied volatilities from) traded prices to (implied volatilities from) new theoretical prices which we derive a from a local (in maturity) expansion of the price process' characteristic function (c.f. Bandi and Renò, 2019 for the characteristic function's expansion). While of independent interest, the local nature of the characteristic function's expansion makes it particularly suitable for the valuation of short-maturity derivatives. In this paper, we employ it in order to price short-maturity options and exploit the recent surge in their trading, as documented by Andersen, Fusari, and Todorov (2017), for the effective identification of the equity characteristics and their dynamics.

The mapping between reduced-form estimates of the equity characteristics and their structural drivers is central to our work. Efforts to provide *structural* interpretations of *reduced-form* models for equity and bond returns begin with Merton (1974) and hinge on the idea that the value of equity is that of a call option written on the firm's assets with strike price given by the firm's debt. Similarly, the value of debt is that of a zero-coupon bond net of the value of the corresponding put option on asset values. To the best of our knowledge, the existing finance literature has almost exclusively focused on the latter implication. Because the value of corporate debt net of the value of a zero-coupon bond can be expressed as an implied put written on asset values, (by simply taking logs) credit spreads can be linked to asset dynamics through the implied (put) option value. This observation has spurred a considerable amount of work on the pricing of risk in the corporate bond market.[6]

The first implication (i.e., equity is a call on the assets) is less explored. The work of Choi and Richardson (2016), and Engle and Siriwardane (2017) are relevant exceptions related to the present paper. Both Choi and Richardson (2016) and Engle and Siriwardane (2017) focus on the link between volatility and financial leverage using discrete GARCH-type volatility models. Differently from their work, we operate in the context of a flexible continuous-time model. By a simple application of Itô's Lemma, the dynamics of all equity processes (and, in particular, of spot volatility, spot volatility of volatility and spot leverage, i.e., our objects of interest) are shown to be functions of the unknown state variables (e.g. volatility of the assets and financial leverage). This mapping gives us a theoretical framework to evaluate structural relations, like the dependence between spot leverage and financial leverage (Black, 1976), which have been elusive in the literature (see, e.g., Figlewski and Wang, 2000, Hens and Steude, 2009, Hasanhodzic and Lo, 2019, and the

[6]See, e.g., Eom, Helwege, and Huang (2004), Chen, Collin-Dufresne, and Goldstein (2008), Schaefer and Strebulaev (2008), Huang and Huang (2012), Du, Elkamhi, and Ericsson (2018), Culp, Nozawa, and Veronesi (2018), Huang, Shi, and Zhou (2019), and the references therein.

references therein). Empirically, using the proposed option-implied estimates, we suggest that this dependence is stronger than previously reported.

Finally, regarding the study of the structural determinants of corporate bond returns, our interest in this paper is *not* in spanning the space of possible drivers, something which has been done successfully in existing work, cited above. Our interest is, instead, in using corporate bond returns has a lens to interpret the economic significance of the option-implied equity characteristics. Because these characteristics should be a function (like bond returns) of the asset dynamics, our documented relation between bond returns and equity characteristics speaks to the existence of common structural drivers. These common drivers are further evidence for hard-to-detect structural interpretations of equity. Importantly, they are also evidence for a different take on the integration between the corporate bond market and the more liquid equity market.

3 Pricing short-maturity options

We write the equity value as E_t. We begin by expressing the dynamics of the equity log-return process ($d\log E_t = de_t$) under the statistical measure ($\mathbb{P}$) as

$$de_t = de_t^d + \underbrace{c_e dN_t^e}_{dJ_t^e}, \tag{1}$$

where e^d is a diffusive component and J^e is a discontinuous finite-variation component. The diffusive dynamics are given by

$$de_t^d = \left(\mu_t^e - \frac{\sigma_t^2}{2}\right) dt + \sigma_t dW_t^e \tag{2}$$

$$d\sigma_t = \alpha_t dt + \beta_t dW_t^\sigma, \tag{3}$$

where W^e and W^σ are correlated Brownian motions with $corr(dW_t^e, dW_t^\sigma) = \rho_t dt$. The spot volatility ($\sigma_t$), the spot volatility of volatility (β_t) and spot leverage (ρ_t) are the equity characteristics which represent the objects of our interest.

The discontinuous dynamics are modeled by way of an independent (of the Brownian motions) compound Poisson process, J^e, with Gaussian jump sizes, c_e.[7] The conditional mean and the standard deviation of the jump sizes are $\mu_{j,t}$ and $\sigma_{j,t}$, respectively. The infinitesimal intensity of the jumps is $\lambda_t dt$.

[7]Gaussianity is a classical assumption on the distribution of the jump sizes in log returns, one which can be very easily relaxed. Any parametric assumption on the density of the jump sizes is, in fact, allowed as long as the corresponding characteristic function is known.

Define $z = iu\sigma_t\sqrt{\tau}$, for notational simplicity. Following Bandi and Renò (2019), the characteristic function (under the risk-adjusted measure ($\mathbb{Q}$)) of the log-return diffusive component, i.e., $\mathbb{C}^d(u,\tau) = \mathbb{E}_t^{\mathbb{Q}}[e^{iu(\log E^d_{t+\tau} - \log E^d_t)}]$ with $u \in \mathbb{C}$ and $\tau = T - t$, can be expressed as a (small τ) expansion given by

$$\mathbb{C}^d(u,\tau) = e^{iu(r_t - \delta_t - \frac{\sigma_t^2}{2})\tau} e^{\frac{z^2}{2}} \left(1 + z^3 \frac{\beta_t \rho_t}{2\sigma_t}\sqrt{\tau} + \frac{1}{2} z^2 \frac{\alpha_t}{\sigma_t}\tau + \frac{1}{8}\frac{\beta_t^2}{\sigma_t^2} z^2 (2 - \rho_t^2 z^2(-z^2 - 4))\tau\right), \tag{4}$$

c.f. Appendix B, where r_t is the risk-free rate and δ_t is the dividend yield.

We note that the characteristic function's expansion of the *standardized* diffusive price process under the risk-adjusted measure (i.e., $\frac{(\log E^d_{t+\tau} - \log E^d_t) - (r_t - \delta_t - \frac{\sigma_t^2}{2})\tau}{\sigma_t\sqrt{\tau}}$) is second order in $\sqrt{\tau}$. A first-order expansion would allow us to identify σ_t and $\beta_t\rho_t$, but not β_t and ρ_t separately. The reported second-order expansion permits identification of all equity characteristics.

In light of the Gaussian assumption on the jump sizes, the characteristic function of the jump component in $\log E_{t+\tau} - \log E_t$ (with risk-adjusted parameters) is given by:

$$\mathbb{C}^j(u,\tau) = e^{\tau\lambda_t(e^{iu\mu_{j,t} - \frac{1}{2}u^2\sigma_{j,t}^2} - 1 - u\bar{\mu}_{j,t})}, \tag{5}$$

where $\bar{\mu}_{j,t}$ is a jump compensator expressed as $e^{\mu_{j,t} + \frac{1}{2}\sigma_{j,t}^2} - 1$.

Following Dubinsky, Johannes, Kaeck, and Seeger (2018), if an earning announcement is scheduled before the expiration of the option, we can augment the equity return process with a jump component associated with the announcement. Scheduled earning announcements are modeled as deterministic in their arrivals and stochastic in their sizes. Assuming that each earning announcement generates a Gaussian jump with zero mean and standard deviation σ_{ea}, the associated characteristic function over $\tau = T - t$, inclusive of a convexity adjustment, can be expressed as:

$$\mathbb{C}^{ea}(u,\tau) = e^{\sum_{i=N_t^d+1}^{N_T^d} - \frac{iu\sigma_{i,ea}^2}{2} - \frac{u^2\sigma_{i,ea}^2}{2}}, \tag{6}$$

where N_t^d counts the earning announcements before time t. Due to our use of short-term instruments, the number of earning announcements over τ, per option, is at most one in our sample. As a consequence, we only estimate one size variance, i.e., $\sigma_{i,ea}^2 = \sigma_{ea}^2$.

Because of the independence between e^d and J^e (and the sizes of potential earning announcements), an assumption which is standard in the literature, we may write the complete log-return characteristic function as

$$\mathbb{E}_t^{\mathbb{Q}}[e^{iu(\log E_{t+\tau} - \log E_t)}] = \mathbb{C}^d(u,\tau) \times \mathbb{C}^j(u,\tau) \times \mathbb{C}^{ea}(u,\tau), \tag{7}$$

which is a sort of Lévy-Khintchine representation.

Short-maturity option prices are now obtained by Fourier inversion of the characteristic function as in the work of Heston (1993), Bakshi and Madan (2000) and many others.[8]

4 Identifying time-varying equity characteristics

The diffusive component of the data generating process in Eqs. (2) and (3) is nonparametric in the sense that no parametric structure is imposed on the equity characteristics of interest, i.e., σ_t, β_t, and ρ_t. Because of the imposition of a parametric assumption on the distribution of the jump sizes, we will define the full model in Eq. (1) as being "semiparametric" in nature. The assumed semiparametric model subsumes most of the specifications commonly used in continuous-time finance. For example, spot leverage, ρ_t, is allowed to be time-varying (c.f., Bandi and Renò, 2012).

Estimation of the model is based on a panel of *short-maturity options* written on equity. High-frequency data on equity prices may - however - be used to regularize the criterion, as we discuss is Section 8.

We denote the time-t prices of European OTM options on equity by $O_{t,k,\tau}$. Option prices are expected discounted payoffs under the risk-adjusted measure:

$$O_{t,k,\tau} = \begin{cases} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} r_s\,ds}\,(E_{t+\tau}-K)^+\right] & \text{if } K > F_{t,t+\tau} \\ \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} r_s\,ds}\,(K-E_{t+\tau})^+\right] & \text{if } K \leq F_{t,t+\tau}\ , \end{cases} \tag{8}$$

where τ is the *tenor* or time-to-maturity, K is the strike price, $F_{t,t+\tau}$ is the futures price of the underlying asset at time t for date $t+\tau$ and $k = \log(K/F_{t,t+\tau})$. As is common, we quote option prices in terms of their Black-Scholes implied volatility (BSIV). The option BSIV is denoted by $\kappa_{t,k,\tau}$.

Given r_t and δ_t, the theoretical time-t price of an option with tenor τ and log-moneyness k depends on the value of the state vector $\mathbf{S}_t = (\sigma_t, \rho_t, \beta_t, \alpha_t, \lambda_t, \mu_{j,t}, \sigma_{j,t}, \sigma_{ea})$: we denote the implied BSIV by $\widetilde{\kappa}_{t,k,\tau}(\mathbf{S}_t)$. Theoretical prices are computed as in Section 3. We estimate the model using the M_t options with tenor shorter than one month which are available at the end of the trading day. We employ the following criterion:

$$\{\widehat{\mathbf{S}}_t\}_{t=1}^T = \underset{\{\mathbf{S}_t\}_{t=1}^T}{\operatorname{argmin}} \left\{ \sum_{j:\tau_j \leq 31} \frac{\left(\kappa_{t,k_j,\tau_j} - \widetilde{\kappa}_{k_j,\tau_j}(\mathbf{S}_t)\right)^2}{M_t} \right\}. \tag{9}$$

As in Andersen, Fusari, and Todorov (2015a), a critical aspect of the objective function in Eq. (9) is the mean squared distance between theoretical and empirical BSIVs. Minimizing the mean

[8] We use the Fourier-cosine method of Fang and Oosterlee (2008).

squared error leads to the joint estimation of the state vector $\mathbf{S}_t$ and, in particular, of the equity characteristics of interest.

Turning to identification, Fig. 2 shows that options with different levels of moneyness load in distinct ways on different equity characteristics. Specifically, the left panel documents that at-the-money options are particularly revealing about the level of spot volatility (i.e., $\frac{\partial \kappa}{\partial \sigma_t} > 0$, for $K = e_t$). The effect of σ_t on the implied volatility surface quickly dissipates for strikes progressively out-of-the-money. From the middle panel, we observe that options that are exactly at-the-money do not depend on ρ_t (i.e., $\frac{\partial \kappa}{\partial \rho_t} = 0$, for $K = e_t$). However, mildly out-of-the-money puts and calls load on ρ_t with opposite signs: it is the slope of the implied volatility (around) at-the-money which provides identification for spot leverage. Finally, the right panel shows that the identification of spot volatility of volatility derives from both at-the-money and out-of-the-money options, with the loading being a function of spot leverage and its sign. In essence, Fig. 2 illustrates that a cross section of options can jointly identify all equity characteristics.

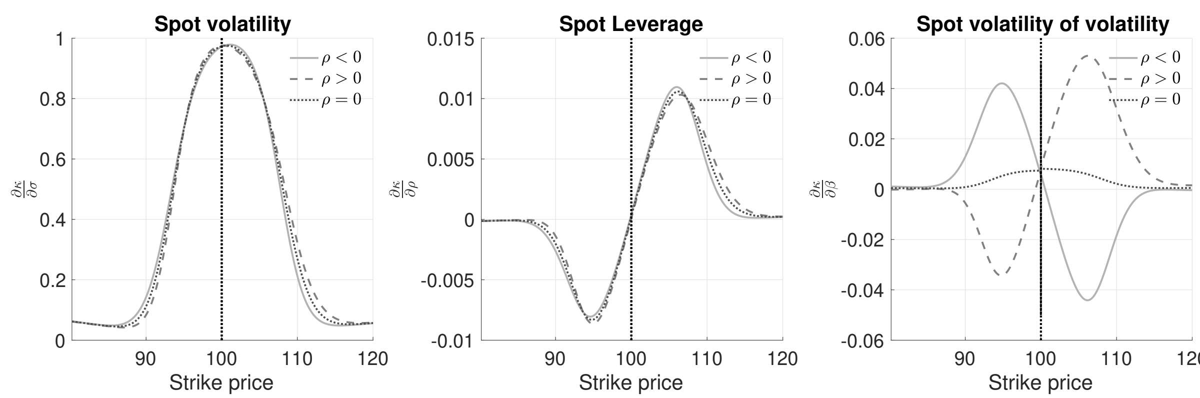


Figure 2: **Identification of the equity characteristics from options**. *From the left to the right panel, we plot the derivative of the implied volatility with respect to spot volatility, spot leverage, and spot volatility of volatility. We compute option prices with parameters as in Subsection 5.1. We report results for* $\rho_t = -0.46$*,* $\rho_t = 0.46$*, and* $\rho_t = 0$*. The option maturity is set equal to 12 calendar days, in line with the average maturity used in the empirical analysis (see Table 3). The vertical line in all three panels corresponds to the spot value of equity, i.e.* e_t*.*

5 Option-implied identification: performance

Comparing option-implied estimates to estimates based on high-frequency price data can hardly be done using asymptotic arguments. While the former rely on cross-sectional information across levels of option moneyness, the latter hinge on time-series information over high-frequency prices on the options' underlying. The result is different asymptotic conceptual frameworks.

Yet, it is important to evaluate the identification potential that the recent surge in the liquidity of short-maturity options has brought about. In the same vein, it is important to assess the ability of the proposed (local) pricing expansion to exploit such potential. To this extent, we focus on *finite sample* performance in realistic scenarios allowing for frictions in option prices.

Subsection 5.1 focuses on the accuracy of the estimates of the equity characteristics across different (short) times to maturity. Subsection 5.2 compares the estimated dynamics of the option-implied equity characteristic to the dynamics implied from nonparametric high-frequency sample analogues.

5.1 Accuracy of short-tenor identification

We work with the double-jump model popularized by Duffie, Pan, and Singleton (2000) and adopted by Broadie, Chernov, and Johannes (2007), among many others. The model is able to reproduce different degrees of skewness and excess kurtosis via stochastic volatility and return/volatility co-jumps. The risk-adjusted dynamics are given by

$$\begin{aligned} de_t &= \left(r_t - \delta_t - \frac{\sigma_t^2}{2}\right) dt + \sigma_t dW_t^e + dJ_t^e \\ d\sigma_t^2 &= \kappa_d \left(\overline{v} - \sigma_t^2\right) dt + \sigma_d \sqrt{\sigma_t^2} dW_t^\sigma + dJ_t^\sigma, \end{aligned} \tag{10}$$

where (W_t^e, W_t^σ) is a two-dimensional Brownian motion with (constant) instantaneous correlation ρ and (J_t^e, J_t^σ) is an independent (of the Brownian motion) bivariate compound Poisson process with intensity λ. The marginal distribution of the volatility jump sizes, c_σ, is exponential with mean μ_σ, while the distribution of the jump sizes in the logarithmic prices, c_e, is Gaussian with mean $\mu_j + \rho_j c_\sigma$ and standard deviation σ_j, conditional on c_σ. The addition of volatility jumps will be shown not to be critical for our purposes. We return to this observation below.

We use the parameter estimates from Broadie, Chernov, and Johannes (2007). However, differently from Broadie, Chernov, and Johannes (2007), we allow for correlated jumps in volatility and returns (i.e., we set $\rho_j = -0.5$).[9] Finally, for simplicity, we also set r_t and δ_t equal to zero. Table 1 reports the values of all other parameters.

[9] Bandi and Renò (2018) estimate a strongly negative jump-induced leverage effect.

Parameter	Value	Parameter	Value
ρ	-0.4600	λ	1.0080
$\overline{v}$	0.0144	μ_j	-0.0501
κ_d	4.0320	σ_j	0.0751
σ_d	0.2000	μ_σ	0.0930
		ρ_j	-0.5000

Table 1: *Parameter values. With the exception of ρ_j, the numbers are from Broadie, Chernov, and Johannes (2007).*

In order to design simulations that are as close as possible to our data (c.f. Table 3), we consider $N = 11$ options on an equi-spaced grid within the log-moneyness range $[-4, 2] \cdot \sigma^{\mathrm{ATM}}\sqrt{\tau}$, where σ^{ATM} is the at-the-money Black-Scholes implied volatility (ATM-BSIV). Because we are not interested in dynamics in this subsection, but solely in the accuracy of the estimates as a function of tenor, we price a single cross section of options over a single day. We choose the spot volatility as being equal to the long-run mean of the volatility process (i.e., $\sigma_t^2 = 0.0144 = \overline{v}$). The model-generated option prices are then transformed into BSIVs ($\kappa_{k,\tau}$).

We allow for possible observation error in $\kappa_{k,\tau}$. Specifically, we assume that the observed BSIVs $\widehat{\kappa}_{k,\tau}$ are given by:

$$\widehat{\kappa}_{k,\tau} \;=\; \kappa_{k,\tau} + \epsilon_{k,\tau}, \qquad \epsilon_{k,\tau} \;=\; \sigma_{k,\tau}\,\zeta_k, \quad k = 1, ..., N. \tag{11}$$

In order to select the volatility of the error term $\epsilon_{k,\tau}$, we follow Andersen, Fusari, Todorov, and Varneskov (2020) and set $\sigma_{k,\tau} = 0.25\,\psi_{k,\tau}\,\kappa_{k,\tau}$, with $\psi_{k,\tau}$ denoting an estimate from a kernel regression of the relative bid-ask spreads of the SPX options on their volatility-adjusted log-strikes. Finally, given the evidence of mild positive error dependence in Andersen, Fusari, Todorov, and Varneskov (2020), we specify the mean-zero errors $\{\zeta_k\}_{k=1,\ldots,N}$ as being AR(1) processes with autoregressive parameter 0.5.

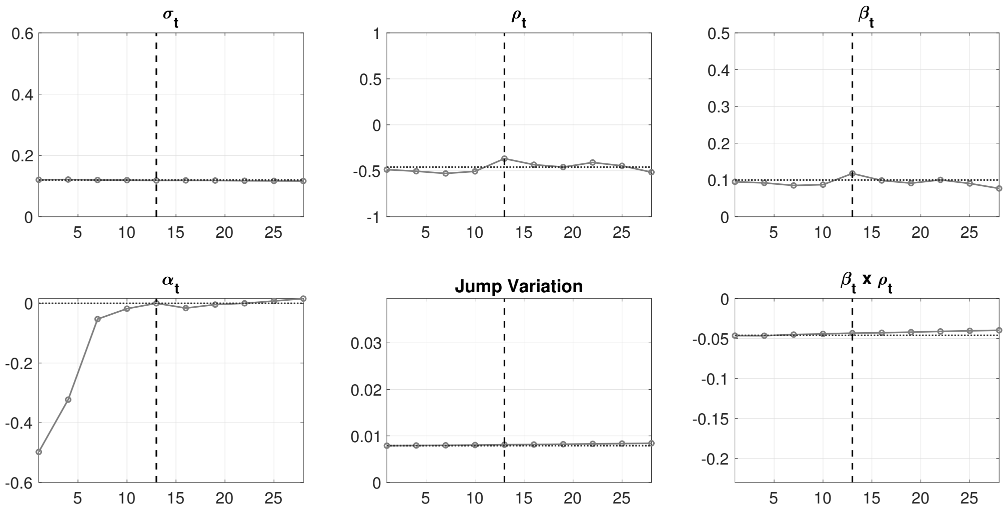
We estimate the model using the objective function in Eq. (9).[10] The exercise is conducted by varying the tenor of the full cross section of options from one day to thirty days.

Fig. 3 provides a visual representation without measurement error in option prices. In the absence of volatility jumps (i.e., setting $\mu_\sigma = 0$), spot volatility and jump variation are almost perfectly recovered, regardless of the option's tenor (Panel (a)). The quantities β_t and ρ_t are estimated rather precisely with only a small positive bias. Adding jumps in volatility (as in Panel (b)) mainly affects the identification of the jump variation, which is now estimated with a sizable

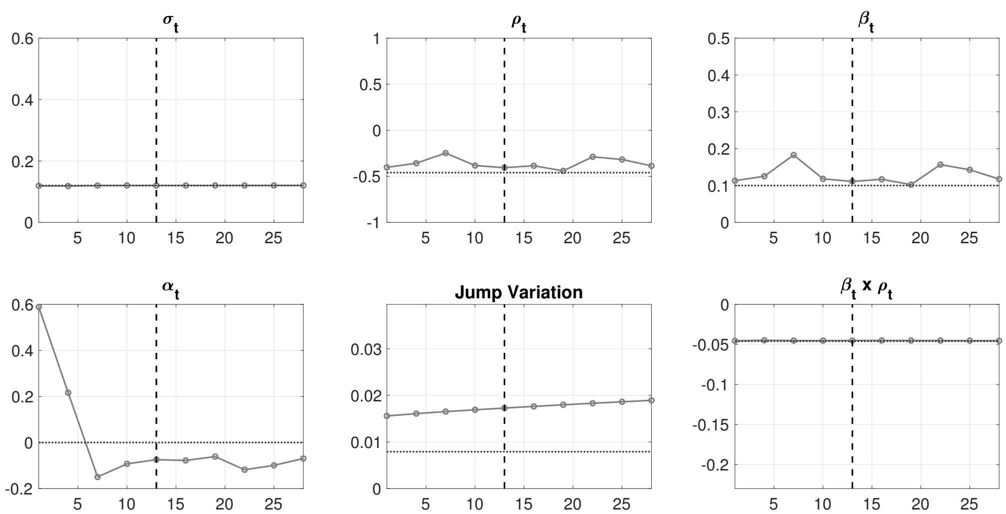
[10]The minimization is carried out using the Nelder-Mead simplex algorithm in the NLopt library (`https://nlopt.readthedocs.io`).

upward bias.

We perform the same analysis but add observation error to the options BSIVs, as described in Eq. (11) (c.f. Fig. 4). In this case, we report the median of the estimates across 1000 Monte Carlo draws. Spot volatility is estimated almost exactly. Once more, β_t and ρ_t are also estimated rather precisely, with only a slight positive bias. Similarly to the case without observation error, the introduction of volatility jumps (in Panel (b)) significantly biases the estimation of the jump variation.

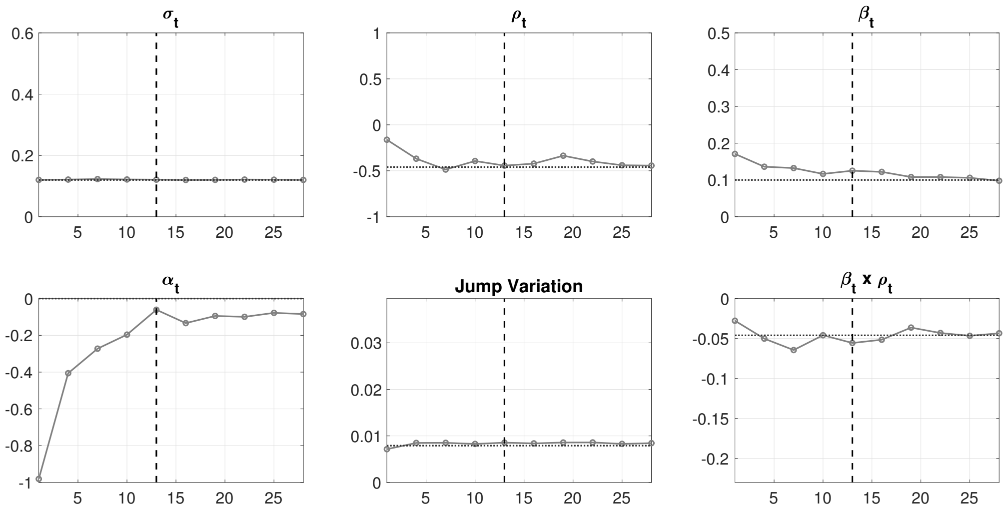


(a) **No noise, no volatility jumps**

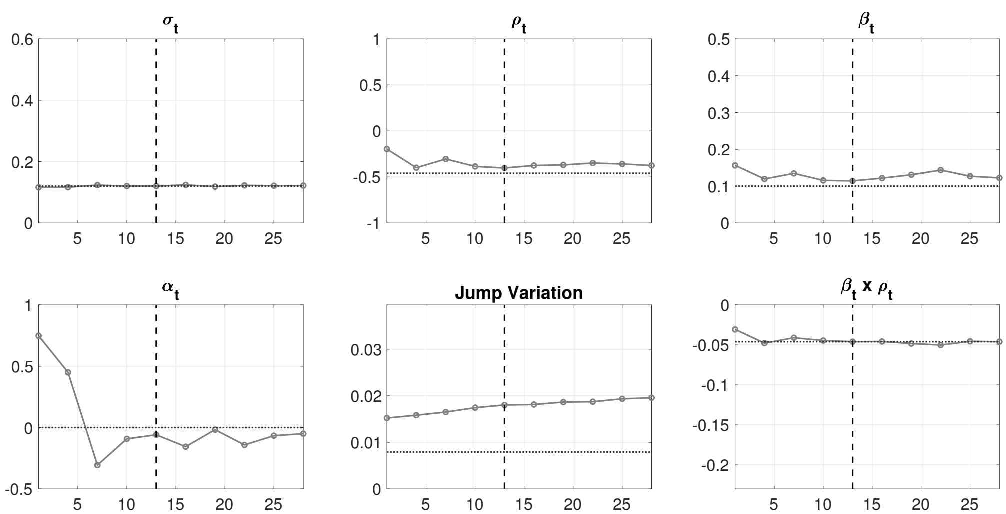


(b) **No noise, volatility jumps**

Figure 3: **Short-tenor identification** *The left, middle, and right panels show the estimated coefficients as a function of the time-to-maturity of the cross section of options used in the estimation. The instantaneous jump variation is computed as* $2\lambda(e^{\mu_j+0.5\sigma_j^2}-1-\mu_j)$*. Option prices are generated according to the model in Eq.* (10).



(a) **Noise, no volatility jumps**



(b) **Noise, volatility jumps**

Figure 4: **Short-tenor identification.** *The left, middle, and right panels show the estimated coefficients as a function of the time-to-maturity of the cross section of options used in the estimation. The instantaneous jump variation is computed as* $2\lambda(e^{\mu_j+0.5\sigma_j^2}-1-\mu_j)$. *Option prices are generated according to the model in Eq.* (10). *Observation error in option prices is added as in Eq.* (11).

5.2 Option-implied estimates versus high-frequency estimates

Because the equity characteristics do not vary under change of measure, they can be estimated either by using time-series observations on the underlying asset or by using a panel of options. In this subsection, we compare the estimation accuracy of the aforementioned strategies over time.

We work, again, with the double-jump stochastic volatility model in Eq. (10). Because we are interested in dynamics, we let both σ_d (the parameter controlling the independent variation of the spot volatility of volatility with respect to spot volatility) and ρ be time-varying. We write $\rho_{d,t}$ and $\sigma_{d,t}$ and assume that the evolution of both $\rho_{d,t}$ and $\sigma_{d,t}$ is that of an AR(1) process with a half-life of one year.

For simplicity in comparing alternative inferential methods, we assume that e_t belongs to the same parametric model class under the statistical ($\mathbb{P}$) and the risk-adjusted ($\mathbb{Q}$) measures. Table 2 reports the full set of parameter values.

Under $\mathbb{P}$				Under $\mathbb{Q}$			
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$\rho_{d,t}$	-	λ	1.0080	$\rho_{d,t}$	-	λ	1.0080
$\overline{v}$	0.0144	μ_j	-0.0284	$\overline{v}$	0.0144	μ_j	-0.0501
κ_d	4.0320	σ_j	0.0490	κ_d	4.0320	σ_j	0.0751
$\sigma_{d,t}$	-	μ_σ	0.0315	$\sigma_{d,t}$	-	μ_σ	0.0930
ρ_d: AR(1) with half-life 1 year				σ_d: AR(1) with half-life 1 year			
$\rho_{d,0}$: -0.46				$\sigma_{d,0}$: 0.2			

Table 2: *Parameter values (from Broadie, Chernov, and Johannes, 2007).*

Given the parameters in Table 2, we simulate a price trajectory that spans an horizon of ten years. We generate prices at 5-minute intervals (i.e., $M = 85$ observations over a 7-hour trading day). A 5-minute frequency is believed to translate into sufficient reduction in market microstructure noise for stocks similar to the ones we use in our empirical investigation.

At the end of each day, we price, again, 11 options on an equi-spaced log-moneyness grid covering the range $[-4, 2] \cdot \sigma^{\text{ATM}}\sqrt{\tau}$. We set the maturity of the options to be equal to 12 calendar days. This results in a time-varying range of moneyness, depending on the level of volatility, which mimics the features of the option data we use (c.f. Table 3). As in Subsection 5.1, we transform the model-generated option prices into BSIVs ($\kappa_{t,k,\tau}$) and allow for observation error.

Finally, we estimate σ_t, $\rho_{d,t}$ and β_t using two sets of data: short-maturity option data *only* and high-frequency price data *only*.

Regarding the first strategy, for which we denote the estimates with a "hat", we implement the procedure in Section 4 by estimating the equity characteristics at the end of each trading day. We then take averages of these (annualized) estimates within a month.

Consider the second strategy, for which we denote the estimates with a "tilde". Using intra-day prices, we estimate - over each day - the diffusive variance, i.e., σ_t^2, by employing the threshold bipower variation introduced by Corsi, Pirino, and Renò (2010):

$$\widetilde{\sigma}_t^2 = \sum_{j=1}^{M} |\Delta_{t,j} e|^2 I_{\{|\Delta_{t,j} e|^2 \leq \vartheta_j\}}, \tag{12}$$

where $|\Delta_{t,j} e|$ is the absolute value of a log-price increment on day t over the j-th 5-minute interval (with $j = 1, ..., M$), I_A is the indicator function over the set A, and ϑ_j is a threshold which depends on the value of volatility immediately preceding the j-th time interval (we refer to Corsi, Pirino, and Renò, 2010, for details on the threshold choice).

Given $\widetilde{\sigma}_t^2$, we compute monthly averages of (annualized) high-frequency estimates of the equity characteristics:

$$\widetilde{\sigma}_{m,t}^2 = \frac{252}{21} \sum_{t=1}^{t+21} \widetilde{\sigma}_t^2, \tag{13}$$

$$\widetilde{\beta}_{m,t}^2 = \frac{252}{21} \sum_{t=1}^{t+21} \left(\Delta_t \widetilde{\sigma}_t^2\right)^2, \tag{14}$$

$$\widetilde{\rho}_{m,t} = \frac{\frac{252}{21} \sum_{t=1}^{t+21} (\Delta_t \widetilde{\sigma}_t^2 - \overline{\Delta_t \widetilde{\sigma}_t^2})(\Delta_t e_t - \overline{\Delta_t e_t})}{\sqrt{\frac{252}{21} \sum_{t=1}^{t+21} (\Delta_t \widetilde{\sigma}_t^2 - \overline{\Delta_t \widetilde{\sigma}_t^2})^2} \sqrt{\frac{252}{21} \sum_{t=1}^{t+21} (\Delta_t e_t - \overline{\Delta_t e_t})^2}}. \tag{15}$$

Monthly averages will also be used in the empirical work in Section 8. Their calculation is standard in the asset pricing literature and allows us to better align equity information to accounting information (which, as discussed more thoroughly in Section 7, is rendered monthly by linear interpolation of quarterly observations).

We emphasize that, while the option-implied estimates are genuinely "spot", the high-frequency estimates cannot be. The choice of a sampling horizon, say ϕ, however small, such that high-frequency price observations are collected over $(t - \phi, t)$, is an inevitable aspect of the construction of high-frequency price-based *spot* estimators. In this paper, we choose a day ($\phi = \Delta_t$) in the construction of the main input, i.e., spot variance (c.f. Eq. (12)). We will see that this choice results in average high-frequency estimates $\widetilde{\sigma}_t^2$ close to the true value.

Fig. 5 visualizes our findings. Both options and high-frequency data are able to recover the trajectory of spot volatility with high precision. However, when focusing on the estimation of spot leverage and spot volatility of volatility, the high frequency-based estimates show a sizable positive bias while the option-based estimates display a remarkable level of accuracy.[11]

Two observations are in order. First, as mentioned in Section 2, the literature has emphasized the empirical challenges related to leverage identification using price data (see, e.g., Aït-Sahalia, Fan, and Li, 2013). The same challenges plague the nonparametric estimation of volatility of volatility. In both case, the first-stage estimation of spot volatility, and its associated estimation error, leads to biased and/or excessively noisy second-stage estimates. By not relying on the first-stage estimation of spot volatility, option-based methods yield a cleaner signal, one which we exploit below to study the structural drivers of equity. Second, microstructure noise is an additional (to the first-stage estimation of spot volatility) source of bias in the estimation of spot leverage and spot volatility of volatility. While the use of 5-minute data would reduce microstructure noise considerably in the data, it would not eliminate it completely (and would lead to enhanced estimation error). From a bias standpoint, the reported results (which account explicitly for measurement error in option prices but do not account for microstructure noise in the high-frequency prices of the underlying) can, therefore, be viewed as being conservative.

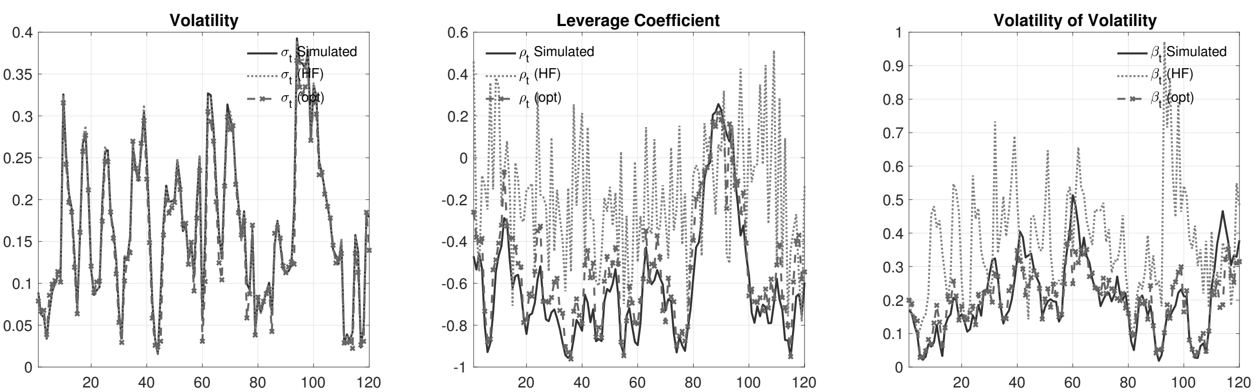


Figure 5: **Option-implied vs. high-frequency estimates**. *We plot mean values of equity characteristic estimates based on simulated option and price data.*

In Fig. 6, we report *infeasible* high-frequency versions of the estimators in Eqs. (13)-(15) constructed by replacing $\widetilde{\sigma}_t^2$ with the unobservable true spot variance σ_t^2 sampled both daily and every 5-minutes. Using daily spot variance observations produces estimates of the leverage coefficient (middle panel) which are significantly biased while the estimates of the volatility of volatility (right panel) are fairly precise. When using 5-minute spot variance observations, the estimated trajec-

[11] Rather than $\widetilde{\beta}_{m,t}^2$, in the third panel we plot estimates of the component of the volatility of variance independent of the spot variance itself, i.e., $\sigma_{d,t}^2$. Each estimated (monthly) value is defined as $\widetilde{\sigma}_{d,t}^2 = \frac{252}{21} \sum_{t=1}^{t+21} \Delta_t^2 \widetilde{\sigma}_t^2$.

tories of both leverage and volatility of volatility are close to the true ones. However, they are affected by a few occasional spikes produced by the jumps in volatility.

As shown in Subsection 5.1, the volatility jumps hardly affect the option-based estimates. In essence, while the price jump dynamics and the diffusive dynamics have an important impact on option prices (and option-based identification) for out-of-the-money and at-the-money options respectively, the impact of volatility jumps is extremely limited for our purposes. For parsimony, consistent with the specification in Eq. (1), we dispense with them in what follows.

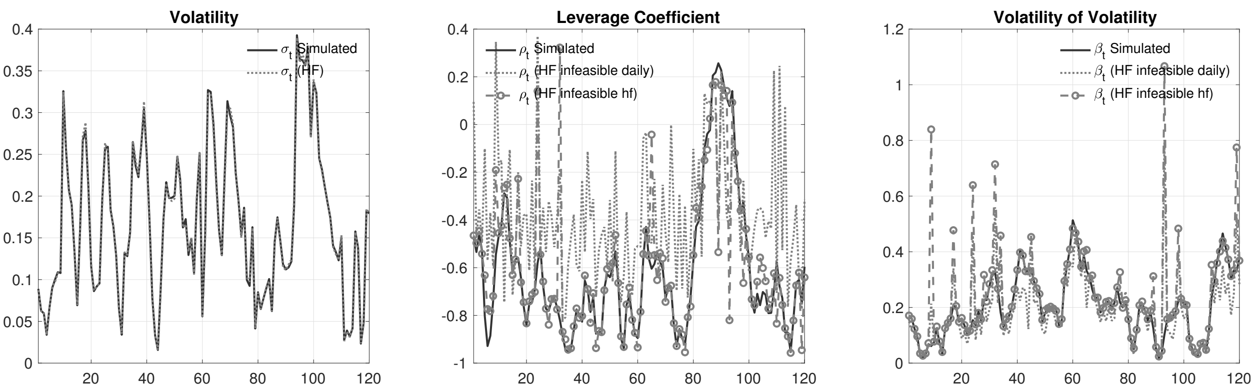


Figure 6: **Infeasible high-frequency estimates**. *We plot mean values of equity characteristic estimates based on simulated price data.*

6 A structural model of equity

The emphasis of the stochastic volatility literature has been on the estimation of reduced-form models of equity. Our aim is to use the estimated reduced-form equity characteristics to study the structural determinants of the equity dynamics. While this objective necessarily requires modeling the asset dynamics, any empirically-reasonable model specification could be employed to derive testable theoretical links between equity and its structural drivers (as in Proposition 1).

In what follows, we use a specification which has proved successful in recent work focused on the term structure of credit spreads (c.f. Du, Elkamhi, and Ericsson (2018)). Specifically, we assume the firm's asset dynamics are given by the following affine stochastic volatility model with jumps:

$$dA_t = \mu A_t dt + A_t \sqrt{V_t^A} dW_t^A + J_t^A A_t dN_t \tag{16}$$

$$dV_t^A = \kappa(\theta - V_t^A)dt + \eta\sqrt{V_t^A} dW_t^V, \tag{17}$$

where W_t^A and W_t^V are correlated Brownian motions with

$$corr(dW_t^A, dW_t^V) = \rho_A dt,$$

N_t is a Poisson counting process with intensity λ_A and J_t^A is a jump size in asset values. The assumed specification adds stochastic volatility and jumps to the Merton (1974) model. Both features are warranted. In the absence of stochastic volatility, equity's spot leverage is constant and equal to 1,[12] which is at odds with its time-variation and negativity in equity data.[13] Without jumps in assets, equity would not have jumps in returns, which is - again - contrary to empirical evidence. It is, for instance, contrary to the importance of return jumps in the valuation of out-of-the-money options.

Equity is a residual interest in the (asset) value of the firm. Following Merton (1974), the equity value is, therefore, modeled as the value of a call option on the firm's asset value with strike price given by the face value of debt, i.e., B_t: $E_t := E(A_t, V_t^A)$. We may write

$$E_t := E(A_t, V_t^A) = A_t F_1 - B_t F_2, \tag{18}$$

where F_1 and F_2 are Heston-style (Heston, 1993) probabilities for call pricing and B_t is the face value of debt.

We now use the "physicist" notation and write, e.g., E_A to represent the derivative of E_t with respect to A_t. In other words, we remove the subscript t from all sensitivities.

Proposition 1. The model in Eq. (16) and Eq. (17) implies that:

1. Equity *spot volatility* can be expressed as:

$$\sigma_t = \sqrt{\frac{V_t^A}{E_t^2}\left(A_t^2 E_A^2 + \eta^2 E_V^2 + 2\rho_A \eta E_A E_V A_t\right)}. \tag{19}$$

2. Equity *spot volatility of volatility* can be expressed as:

$$\beta_t = \sqrt{\frac{V_t^A}{4\sigma_t^2}\left(A_t^2 \Lambda_A^2 + \eta^2 \Lambda_V^2 + 2A_t \eta \rho_A \Lambda_A \Lambda_V\right)}, \tag{20}$$

   where Λ_A and Λ_V are defined in Appendix C as functions of E_A, E_V, E_{AA}, E_{VA}, E_{VV} and the model parameters.

[12] This is easily seen by setting η equal to zero in Eq. (21) of Proposition 1. More generally, all implied values of the equity characteristics in Merton (1974) model can be found by setting η equal to zero in Proposition 1.

[13] The importance of stochastic volatility in asset values for capturing credit risk has been emphasized by, e.g., Huang, Shi, and Zhou (2019).

3. *Spot leverage* can be expressed as:

$$\rho_t = \frac{V_t^A}{2E_t\sigma_t^2\beta_t}\left(\Lambda_A E_A A_t^2 + \Lambda_V E_V \eta^2 + \rho_A \eta A_t \left(\Lambda_A E_V + \Lambda_V E_A\right)\right). \tag{21}$$

Proof. See Appendix C.

Given Eq. (19), Eq. (20) and Eq. (21), it is apparent that the equity characteristics depend on the Greeks (delta, E_A, vega, E_V, gamma, E_{AA}, vanna, E_{VA}, and volga, E_{VV}) of the implied call option on asset values.

In order to visualize the relation between the equity characteristics and the state variables of the structural model (namely, asset value and asset volatility), we implement Eqs. (19), (20), and (21) with parameter values taken from Du, Elkamhi, and Ericsson (2018).[14] It is clear from Eq. (18) that the equity "moneyness" is a one-to-one function of financial leverage. Because we are interested in the link between equity characteristics and financial leverage, we do not report graphs with respect to A_t but, rather, with respect to $L_t = B_t/A_t$, where B_t denotes, as earlier, the face value of the firm's debt (c.f., Fig. 7)

The top left panel displays a non-linear, positive relation between the firm's financial leverage and spot volatility: as financial leverage increases, the riskiness of equity - as represented by spot volatility - increases too. We recall that, while the positive link between spot volatility and financial leverage is a textbook implication of a Modigliani and Miller economy, its empirical validation (on which we focus in Section 8) has resulted in mixed findings.[15] The second and the third top panels document two findings which appear to be less understood: both spot leverage and spot volatility of volatility increase, in absolute value, when financial leverage increases. The relation between spot leverage (resp. spot volatility of volatility) and financial leverage is negative (resp. positive). The bottom panels show the maps between σ_t, ρ_t, β_t and asset volatility.

In essence, Fig. 7 illustrates that financial leverage and asset volatility (and their variation over time) should affect the dynamics of the processes driving stochastic volatility models of equity. In turn, the dependence on underlying state variables is expected to generate strong correlations between the equity characteristics, something which is coherent with our empirical findings, to which we now turn.

[14] In their specification, the jump sizes, J_t^A, are Gaussian. We use the parameter values corresponding to the 50th percentile in their Table VII.

[15] Choi and Richardson (2016) have recently been successful at linking financial leverage to volatility. Chun, Kim, Morck, and Yeung (2008) and Brandt, Brav, Graham, and Kumar (2009), *inter alia*, provide negative results.

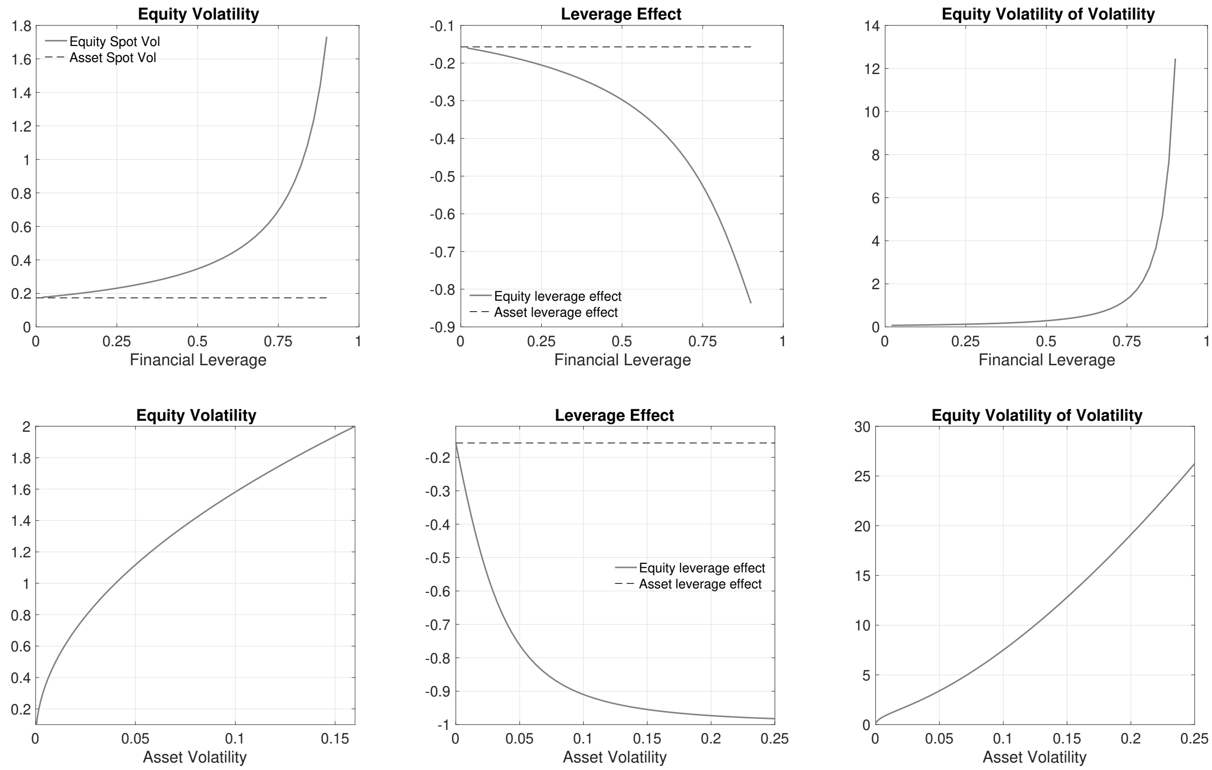


Figure 7: **The map between structural and reduced-form quantities.** *The figure shows the relation between financial leverage (L_t) and asset volatility (V_t^A) and the reduced-form characteristics of the stochastic volatility model in Eq. (1). The asset volatility and the correlation between asset returns and asset volatility are set equal to 0.2 and -0.5, respectively.*

7 The data

We use data from four sources. Daily option data come from OptionMetrics' IvyDB. Intra-day price observations are from TickData. We use Compustat to gather firms' accounting information. Finally, we collect CDS data from Markit from January 2006 to December 2014.[16]

For each ticker of the S&P500 constituents between 2006 and 2015, we obtained all of the available options in the OptionMetrics database. For each option contract, OptionMetrics provides the strike price, the time-to-maturity, the best ask and the best bid price at settlement, among other contract characteristics. The BSIV is computed by OptionMetrics using the binomial model.[17]

[16]As in Kelly, Manzo, and Palhares, 2019, we retain the contracts with the most common default definition at each point in time: the MR clause from January 1, 2002 to March 15, 2009, the XR clause from March 15, 2009 to September 22, 2014, and the XR14 clause from September 23, 2014 onward. For each firm, we create monthly observations by averaging the available quotes over each month.

[17]Even if options on single names are American options, given that our analysis relies on short-maturity contracts,

We apply the following filters. First, we only retain options with time-to-maturity less than or equal to 31 calendar days. Second, we discard options without positive open interest and positive volume. Also, we discard all the options for which the ratio of the ask price over the bid price exceeds ten. Third, we remove observations with missing values for implied volatility. Fourth, for each day and for each option's time-to-maturity, we infer the underlying forward price using put-call parity. Fifth, for each available strike, we keep only either the corresponding put or the corresponding call, with a preference for out-of-the-money contracts, if available. We only retain a given cross section if it has at least five different strike prices. Finally, for each contract, we re-compute the BSIV.

Because the BSIV of equity options is greatly affected by the timing of the earning announcements (as shown by Dubinsky, Johannes, Kaeck, and Seeger, 2018), we collect the scheduled announcement dates from Compustat (variable name: `rdq`) and price the announcements by virtue of $\mathbb{C}^{ea}(u,\tau)$ (which is defined in Section 3).

	Q05	Q25	Q50	Q75	Q95
Panel A:	Average implied volatility				
2006-2015	0.23	0.29	0.34	0.43	0.54
2006-2010	0.32	0.39	0.46	0.53	0.75
2011-2015	0.21	0.26	0.30	0.36	0.43
Panel B:	Average maturity (in days)				
2006-2015	10.11	11.13	11.95	12.76	15.21
2006-2010	16.52	17.05	17.44	17.76	18.38
2011-2015	9.04	11.33	12.34	13.94	15.83
Panel C:	Average number of options				
2006-2015	8.25	9.80	10.91	11.56	12.33
2006-2010	7.95	8.53	9.32	10.03	11.14
2011-2015	8.31	9.35	10.07	10.94	13.12

Table 3: **Summary statistics for the options data**. *The table reports the average implied volatility, the average maturity, and the average number of options over the sub-samples 2006-2010 and 2011-2015. Q05, Q25, Q50, Q75, Q95 are the 5th, 25th, 50th, 75th, and 95th percentiles, respectively. We compute the percentiles for each underlying over the specific sub-sample and we average them.*

Table 3 reports summary statistics for the available option data. From Panel A, we gauge that the implied volatility of equity options is usually around 30% but varies significantly across different firms and different time periods. This can be inferred by looking at two sub-samples. The

we treat them as European. The early exercise premium in known to be negligible over short horizons.

first one ranges from 2006 to 2010. This time period includes the 2008/2009 financial crisis and is associated with average implied volatilities as high as 75%. The second time period, spanning the years 2011-2015, displays more moderate implied volatility values. Regarding the available strike prices, we have an average of 11 contracts for each cross section (c.f. Panel C). This figure does not change significantly across the two sub-samples.

In terms of the options time-to-maturity, the average tenor is around 12 calendar days (c.f. Panel B). The second sub-sample has generally shorter maturity options thanks to the introduction of weekly options (in 2005) on some of the firms considered in our study. Fig. 8 is explicit in representing the drastic increase in the liquidity of short-term options since 2005. Such an increase is a key source of identification for our purposes. Section 5 showed that the short-tenor expansion in Section 3 is effective in leading to accurate prices over the average time to maturity in our data (i.e., 12 days) when using a number of options similar to the average figure in the data (i.e., 11).

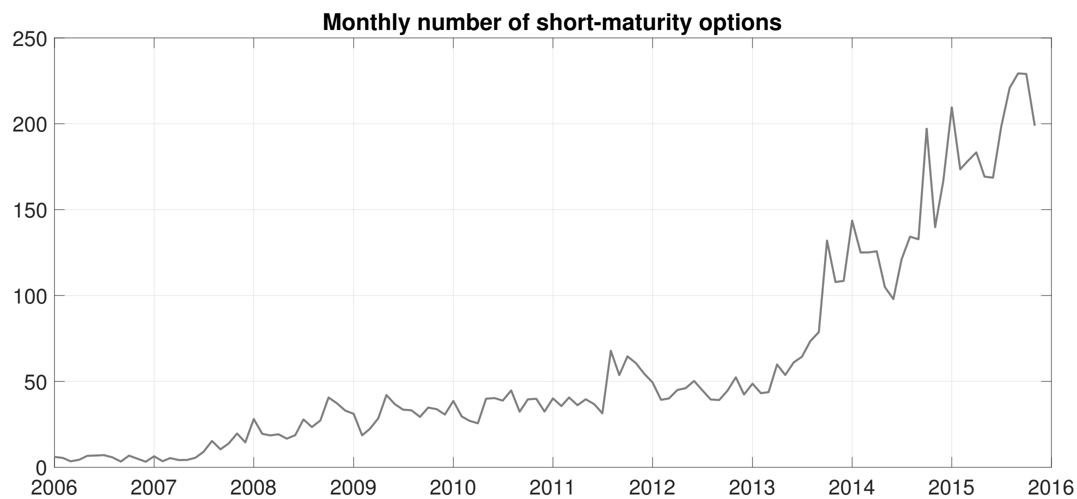


Figure 8: **Short-maturity options.** *The figure reports the average monthly number of options with less than 12 days to maturity across the 130 firms used in our empirical work.*

For each ticker, we also collect information about the capital structure of the corresponding firm from Compustat. Specifically, we construct a quarterly measure of financial leverage as the ratio between total debt and total assets. Total debt is computed as the sum of current liabilities (`dlcq`) and long-term debt (`dlttq`) while total asses are computed as the sum of total debt and the value of equity (given by the product of the number of common shares outstanding (`cshoq`) and the price per share (`prccq`)):

$$L_t = \frac{\text{dlcq} + \text{dlttq}}{(\text{dlcq} + \text{dlttq}) + (\text{prccq} \times \text{cshoq})}. \tag{22}$$

This definition of financial leverage is standard (see, e.g., Kelly, Manzo, and Palhares, 2019). Fig. 9 reports the cross-sectional empirical distribution of (average) financial leverage for our firms.

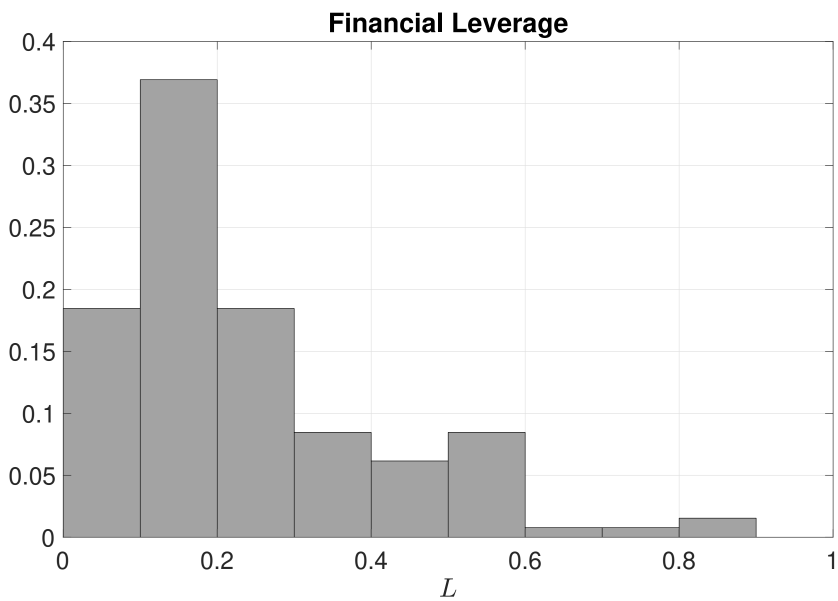


Figure 9: **Unconditional distribution of financial leverage**. *Leverage is defined in Eq.* (22). *The figure shows the distribution of financial leverage across firms. For each firm, we compute the average of L_t over the same horizon for which we have option data.*

.

Importantly, we linearly interpolate the quarterly data from Compustat in order to obtain *monthly* financial leverage series. The monthly financial leverage series are, for each firm, associated with *monthly* averages of option-implied and high-frequency estimates of the equity characteristics, whose construction is detailed in Subsection 5.2.

After matching the four data sources, we obtain a final sample which comprises 130 companies over the 2006-2015 period. Table 14 in the Appendix reports the full list of firms included in the final sample.

8 The map between equity characteristics and financial leverage

For each firm in our sample, we estimate the equity characteristics following a slight modification of the procedure described in Section 4. Specifically, we minimize the same criterion as in Section

4 but add to it a term penalizing deviations between the option-implied spot volatility estimates and their nonparametric high-frequency counterparts. The criterion is:

$$\{\widehat{\mathbf{S}}_t\}_{t=1}^T \;=\; \underset{\{\mathbf{S}_t\}_{t=1}^T}{\text{argmin}} \left\{ \sum_{j:\tau_j \le 31} \frac{\left(\kappa_{t,k_j,\tau_j} \;-\; \widetilde{\kappa}_{k_j,\tau_j}(\mathbf{S}_t)\right)^2}{M_t} \;+\; \gamma\left(\sqrt{\widetilde{\sigma}_t^2} - \sigma_t\right)^2 \right\}, \tag{23}$$

where $\widetilde{\sigma}_t^2$ is defined in Eq. (12). We set $\gamma = 0.05$.

We stressed previously that, differently from (spot leverage and spot volatility of volatility) estimates based on nonparametric sample analogues, the option-based criterion does not require preliminary first-stage estimates of (innovations in) spot volatility as a *necessary* input. The presence of a regularization term (constructed using high-frequency estimates of spot volatility) does not invalidate this argument. As in Section 4, the term is - in fact - not needed. In order to provide a clean comparison between nonparametric estimates obtained from high-frequency price data and option-implied estimates, it was not used in the simulations reported in Section 5. The term is solely added here for increased robustness in empirical work. We discuss two instances in which its presence may be beneficial. First, around scheduled earning announcements, the at-the-money implied volatility is known to diverge (c.f., Dubinsky, Johannes, Kaeck, and Seeger, 2018). While the penalization term would regularize our estimates in this case, the explicit presence of a term capturing scheduled announcements in the proposed pricing formula (c.f., $\mathbb{C}^{ea}(u,\tau)$ in Section 3), reduces the need for regularization. The second example of a situation in which regularization may help is more relevant for the current paper. There are two parameters which control the implied volatility term structure, namely spot volatility (σ_t^2) and the volatility mean reversion (α_t). The latter can be well identified by using multiple cross sections. Using only one cross section per day, as in this paper, may require additional information to identify α_t, which we provide through the penalty.[18]

In Fig. 10, we report the monthly time series of estimated equity characteristics for our selected 130 S&P500 firms. Fig. 10 plots the median values of the three estimated processes, along with the Q3-Q1 interquartile range.[19]

In agreement with the implications of the structural model, spot volatility, spot leverage, and spot volatility show persistent time variation and are highly correlated. While spot volatility and spot volatility of volatility increase during the financial crisis, the correlation between diffusive

[18] The simulations in Section 4, which did not contain a penalty, resulted in inaccurate α_t estimates (c.f., Fig. 3 and 4). While α_t is *not* our focus, we are opting here for superior robustness in the empirical work.

[19] The monthly option characteristics are obtained as in Subsection 5.2. We retain only firms-month observations if there are at least 5 days of estimation over the month (i.e., if there are at least 5 days that passed the data filtering procedure, thereby resulting in a sufficient number of options to perform estimation).

shocks to equity and shocks to volatility (i.e., spot leverage) becomes more negative. The negative correlation between spot volatility and spot leverage supports the market-based evidence in Bandi and Renò (2012) and extends it to individual stocks.

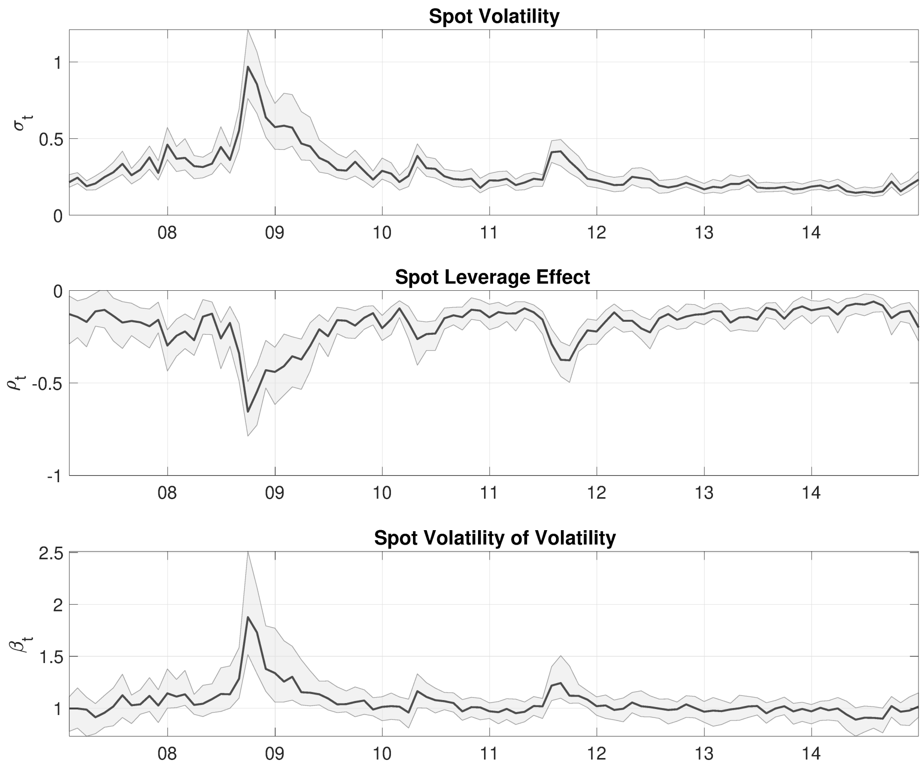


Figure 10: **Estimated equity characteristics.** *The top, middle, and bottom panels show the monthly time series of the estimated option-implied spot volatility, spot leverage, and spot volatility of volatility. The solid lines represent median values across the selected 130 firms. The three shaded areas are Q3-Q1 interquartile ranges.*

In Fig. 11, we display scatter plots of the median values of L_t with respect to the corresponding median values of the three equity characteristics. The dependencies we document are consistent with the theoretical implications of the model in Section 3: both spot volatility and spot volatility of volatility correlate positively with financial leverage, the spot leverage process being, instead, inversely related to financial leverage. We note that, *even* in the absence of measurement error in financial leverage and in the estimates of the equity characteristics, the structural model in Eq. (16) and Eq. (17) implies that the relation between the equity characteristics and financial leverage should not be exact. Because of the presence of an additional (empirically-warranted) state variable, i.e., the variance of the assets, we should expect to report dispersion of the data around solid lines like in the four panels of Fig. 11 (c.f. Appendix D).

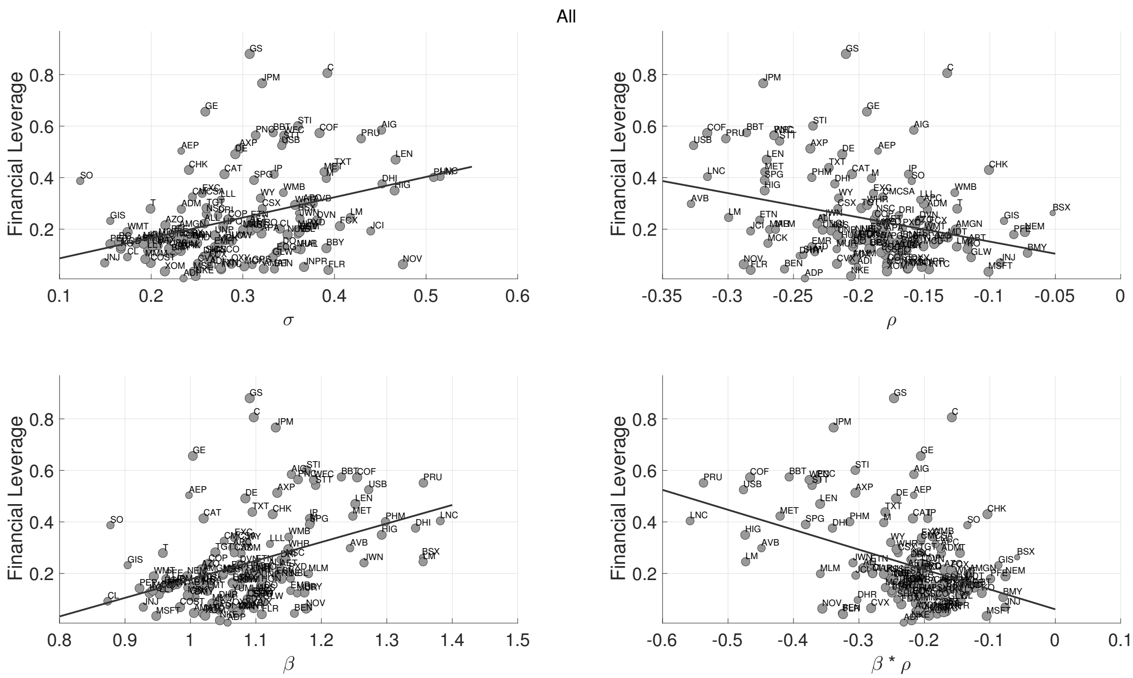


Figure 11: **Scatter plots of financial leverage vs. the option-implied measures.** *Each dot in the panels corresponds to one firm. For each firm we consider the median value of* L_t*,* σ_t*,* ρ_t*, and* β_t *over the 2007-2015 sample period.*

We expand on these results by regressing financial leverage on the equity characteristics.[20] The findings are in Table 4. Once more, all equity characteristics relate to financial leverage in ways that are consistent with a structural justification of equity as a call option on an unobservable asset process with stochastic volatility. The cross-sectional t-statistics from regressions of financial leverage on spot volatility, spot volatility of volatility and spot leverage are 4.45, 5.22 and -3.58, respectively, with R^2 values of 0.13, 0.18 and 0.09.

The association between spot leverage and financial leverage, in particular, is strongly negative. This association has long been hypothesized (hence, the terminology "leverage effect" to define ρ_t) but has turned out to be empirically rather elusive (see, e.g., the negative views in Figlewski and Wang, 2000, Hens and Steude, 2009, and Hasanhodzic and Lo, 2019). We are able to uncover it because of the precision with which spot leverage is identified from short-maturity option prices.

[20]In the regressions we only consider firms for which we have at least 60 monthly observations (i.e., 5 years - not necessary consecutive, as we do not do any prediction, but just contemporaneous). Changing (i.e., increasing or decreasing) this threshold, however, does not modify the reported results.

This is in contrast with the information that one would derive from the noisier (and generally more biased) estimates that one would obtain from using high frequency asset prices. We will return to this observation below.

	Financial Leverage								
σ	4.45	-	-	3.05	0.63	-	0.55	-	1.63
ρ	-	-3.58	-	-1.69	-	-0.96	-0.90	-	-
β	-	-	5.22	-	2.62	3.75	2.17	-	-
β x ρ	-	-	-	-	-	-	-	-5.17	-2.96
R^2	0.13	0.09	0.18	0.15	0.18	0.18	0.18	0.17	0.19

Table 4: **Cross-sectional regressions.** *Financial leverage is regressed on option-implied measures of spot volatility, spot leverage and spot volatility of volatility. For each firm, we use the median value of each quantity over the available sample. The table reports regressions t-statistics and R^2s.*

	Financial Leverage								
σ	3.47	-	-	2.67	2.71	-	2.20	-	2.42
ρ	-	-1.76	-	-0.36	-	-1.30	-0.36	-	-
β	-	-	1.71	-	0.13	1.30	0.18	-	-
β x ρ	-	-	-	-	-	-	-	-2.04	-0.07
R^2	0.25	0.08	0.08	0.28	0.26	0.13	0.29	0.10	0.29

Table 5: **Time-series regressions.** *For each firm in the sample, financial leverage is regressed on contemporaneous option-implied measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are averages across the available firms.*

When regressing cross-sectionally financial leverage on all equity characteristics jointly, the signs do not change but spot volatility and spot leverage lose some of their statistical power. This is not surprising in light of the strong dependence between alternative equity characteristics.

We now turn to the time-series relation between financial leverage and the option-implied measures. Table 5 reports the average results of 130 individual monthly time-series regressions in which L_t is regressed on contemporaneous values of the equity characteristics. The time-series findings continue to confirm the implications of the structural model, with spot volatility and spot volatility of volatility positively correlated with financial leverage and spot leverage negatively correlated with financial leverage. The significance of spot leverage and spot volatility of volatility (and, to some extent, of spot volatility as well) is, in these regressions, slightly reduced. Because of time-series correlation, it is now spot volatility which dominates in a specification in which financial leverage is regressed on all equity characteristics jointly.

In Appendix D we simulate from the structural model in Eq. (16) and Eq. (17) in order to reproduce empirically-realistic heterogeneity in the level of financial leverage across 130 firms. Appendix D further supports our findings regarding the cross-sectional and time-series dependence between equity characteristics and financial leverage. Among other issues, it confirms the lower signal in time-series regressions

Next, we divide the companies by sector. Detailed results are provided in Appendix E. Consumer Discretionary (20 companies) and Health Care (18 companies) companies broadly confirm our findings both cross-sectionally and in the time series. For Financials (19 companies) and Industrials (7 companies), the reported results are weaker cross-sectionally but are strong in the time series. Information Technology (11 companies), Consumer Staple (9 companies), Energy (21 companies), and Materials (18 companies), taken as separate groups, behave in ways which are, along some dimension, less consistent with the model. The small sample size of single sectors, however, suggest caution.[21]

Finally, adopting the logic in Hasanhodzic and Lo (2019), we consider companies with either no financial leverage or very limited levels of it. We choose the bottom 25th percentile of the financial leverage distribution, as reported in Fig. 9. We note that the model specification in Section 6 implies a rather flat relation between equity characteristics and financial leverage for low values of financial leverage (Fig. 7). The data are consistent with this implication. Table 6 and Table 7 show that, both cross-sectionally and in the time series, the dependence between equity characteristics and financial leverage is considerably muted and, often, of the wrong sign.

	Financial Leverage								
σ	-0.53	-	-	-0.53	-0.54	-	-0.58	-	-0.53
ρ	-	1.07	-	0.91	-	1.11	0.99	-	-
β	-	-	0.49	-	1.46	1.77	2.38	-	-
β x ρ	-	-	-	-	-	-	-	1.13	0.97
R^2	0.01	0.04	0.01	0.04	0.09	0.15	0.23	0.05	0.05

Table 6: **Cross-sectional regressions (low leverage firms).** *For firms in the bottom 25th percentile of the financial leverage distribution, financial leverage is regressed on contemporaneous option-implied measures measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s.*

[21] We do not report sector specific results for the Communication Services (1 company), Utilities (3 companies), and Real Estate (3 companies) sectors because of the extremely limited sample.

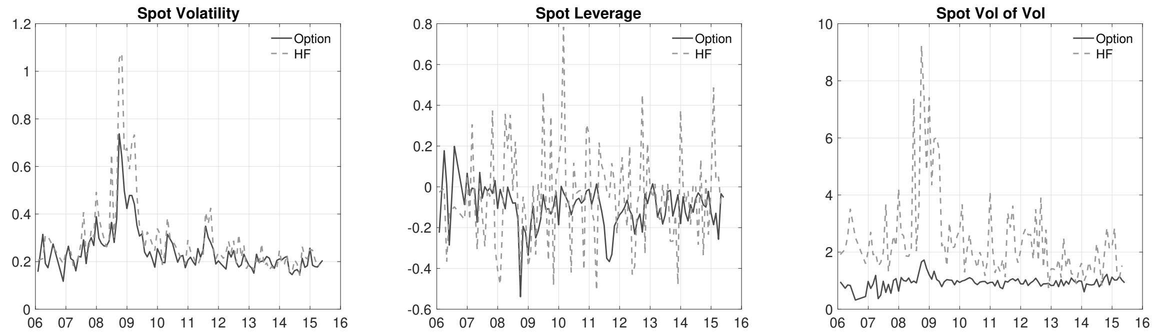
	Financial Leverage								
σ	-0.88	-	-	-0.89	-0.89	-	-0.89	-	-0.59
ρ	-	-0.14	-	-0.73	-	-0.16	-0.74	-	-
β	-	-	-0.07	-	0.47	0.14	0.47	-	-
β x ρ	-	-	-	-	-	-	-	0.04	-0.85
R^2	0.16	0.04	0.04	0.21	0.18	0.08	0.23	0.04	0.21

Table 7: **Time-series regressions (low leverage firms).** *For firms in the bottom 25th percentile of the financial leverage distribution, financial leverage is regressed on contemporaneous option-implied measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are the averages across firms.*

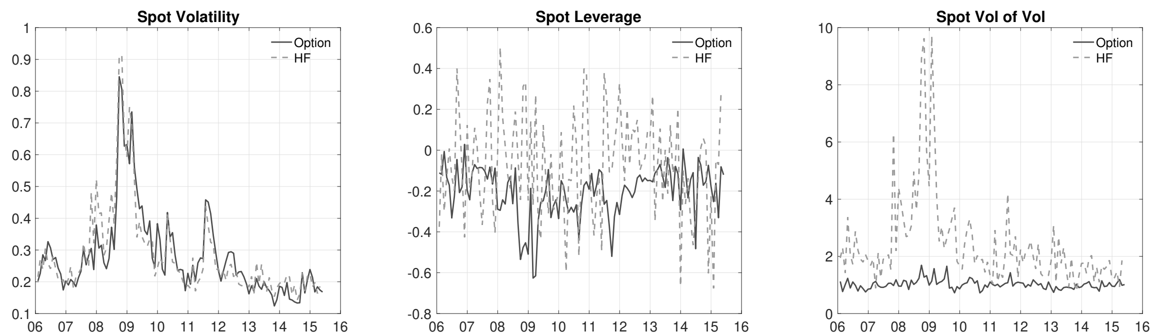
8.1 Using high-frequency price data: are the structural implications supported?

In light of the evidence reported in Section 5 regarding bias and noise in high-frequency estimates of spot volatility of volatility and spot leverage, it is now informative to evaluate the relation between financial leverage and equity characteristics using high-frequency estimates.

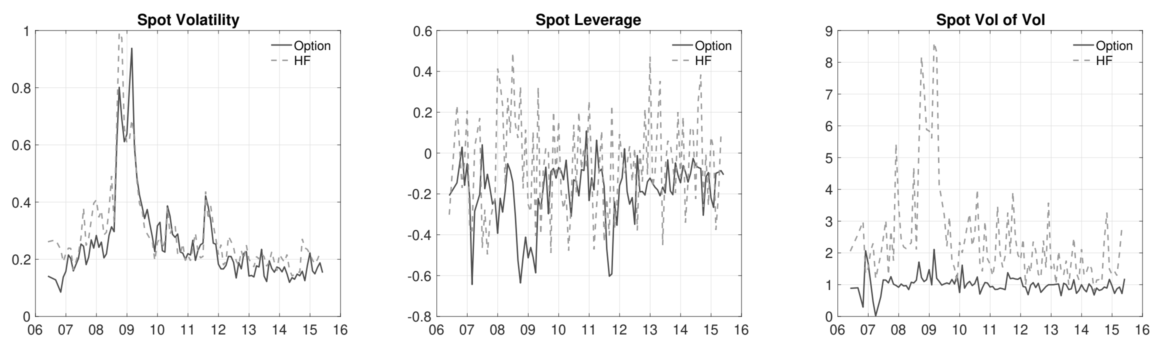
We begin with a visual representation of the equity characteristics estimates for individual stocks. In Fig. 12, Panels (a)-(c), we report results for Microsoft (MSFT), Caterpillar (CAT), and General Electric (GE). Consistent with our discussion in Section 5, while the spot volatility estimates are rather similar (in spite of being less noisy using options), the spot leverage estimates and the spot volatility of volatility estimates are considerably different across identification methods. In particular, as documented previously using simulations (see Fig. 5), the spot volatility of volatility estimates obtained from high-frequency prices tend to closely mirror the corresponding spot volatility estimates (as evidenced, e.g., by the run up around the financial crisis). Given the simulations in Section 5, it is hard to view this phenomenon - which is bound to blur the independent (from spot volatility) information in spot volatility of volatility regarding dependence on structural drivers - as being genuine. The spot volatility of volatility estimates implied from option prices are not affected by this issue.



(a) **Microsoft**



(b) **Caterpillar**



(c) **General Electric**

Figure 12: **Option-implied vs. high-frequency estimates**. *We plot mean values of equity characteristic estimates based on option data and high-frequency price data.*

We conclude by reporting the equivalent of Tables 4 and 5 using high-frequency estimates in the regressions in place of estimates extracted from option prices.

	Financial Leverage								
σ	0.64	-	-	0.58	-0.19	-	-0.19	-	0.53
ρ	-	0.48	-	0.39	-	0.26	0.27	-	-
β	-	-	1.19	-	1.02	1.12	0.97	-	-
β x ρ	-	-	-	-	-	-	-	1.19	1.13
R^2	0.00	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01

Table 8: **Cross-sectional regressions (high-frequency data).** *Financial leverage is regressed on option-implied measures of spot volatility, spot leverage and spot volatility of volatility. For each firm we use the median value of each quantity over the available sample. The table reports regressions t-statistics and R^2s.*

	Financial Leverage								
σ	2.89	-	-	2.88	1.63	-	1.62	-	2.85
ρ	-	0.07	-	-0.00	-	-0.03	-0.01	-	-
β	-	-	2.12	-	0.25	2.11	0.25	-	-
β x ρ	-	-	-	-	-	-	-	-0.05	-0.03
R^2	0.22	0.01	0.15	0.22	0.23	0.16	0.24	0.02	0.22

Table 9: **Time-series regressions (high-frequency data).** *For each firm in the sample, financial leverage is regressed on contemporaneous option-implied measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are averages across the available firms.*

The cross-sectional relation between the quantities of interest is now obfuscated by the presence of estimation noise. None of the equity characteristics is statistically significant and the sign on spot leverage is inconsistent with theory. In the time series, we still find a positive and significant association between financial leverage and spot volatility and between financial leverage and spot volatility of volatility. The latter result, however, is an immediate by-product of the fact that the spot volatility of volatility is spuriously estimated to be very similar to spot volatility. Once we control for spot volatility, the significance of spot volatility of volatility decreases considerably. Consistent with the cross-sectional analysis, the leverage coefficient is also found to be insignificant in the time series. As earlier, its sign is incompatible with structural justifications.

9 Evidence from credit spreads

A mature literature has successfully used structural modeling to justify the use of balance sheet information in the evaluation of corporate spreads (c.f., Eom, Helwege, and Huang (2004), Chen, Collin-Dufresne, and Goldstein (2008), Schaefer and Strebulaev (2008), Huang and Huang (2012), Du, Elkamhi, and Ericsson (2018), Culp, Nozawa, and Veronesi (2018), Huang, Shi, and Zhou (2019), and the references therein.)

Our interest in this section is *not* in improving on the modeling of credit spreads, something which would require a richer treatment better left for future work. Rather, it is to use credit spreads as an economic lens to further evaluate the informational content of the model-implied equity characteristics. Our logic is simple. Because of the structural model in Section 6, the equity characteristics should be a function of all state variables, not only of equity "moneyness" or financial leverage, something which we explored for a rich cross section of assets in the previous section. Given this premise, we should expect the equity characteristics to have *residual* (with respect to financial leverage) explanatory power for credit spreads (c.f., Fig. 1).

We test this hypothesis (and, as an implication, the validity of structural justifications of equity) by running regressions of 5-year credit spreads on financial leverage and the option-implied estimates. Again, we do so both cross-sectionally (Table 10) and in the time series (Table 11).[22]

Importantly, in bivariate regressions of credit spreads on financial leverage and equity characteristics (considered one-by-one), we find that both financial leverage and the corresponding characteristic are highly statistically significant. The signs on financial leverage, spot volatility and spot volatility of volatility are, as expected, positive. The sign on spot leverage is, instead, negative. While financial leverage is a key driver, the equity characteristics improve model specification, particularly in the time-series regressions, thereby providing incremental information about the unobserved state variables. Not surprisingly, adding equity characteristics to each specification generally leads to the replication of (structural) information and, therefore, to some characteristics being driven out. Spot volatility, in particular, appears to be robust to the inclusion of additional characteristics.

As in the previous section, these findings change drastically when using equity characteristics estimated using high-frequency sample analogues (c.f. Table 12 and Table 13). In the cross section, none of the equity characteristics contributes to the explanatory power of financial leverage. In the time series, high-frequency spot volatility is accurate enough as to provide dynamic signal,

[22]Because the Markit data is through December 2014, these regressions cover a slightly shorter horizon (2006-2014) than the horizon covered by the regressions in Section 8 (2006-2015).

thereby resulting in statistically-significant (positive) partial effects. This is in line with Zhang, Zhou, and Zhu (2009). The same applies to spot volatility of volatility but, as pointed out earlier, the dynamics of high-frequency estimates of spot volatility of volatility tend to spuriously mimick the volatility dynamics. Unsurprisingly, in fact, when controlling for spot volatility, spot volatility of volatility loses its significance.

If bond and equity are viewed as claims on the assets, we should expect (forms of) integration in their pricing. The analysis in this section supports this logic by showing how accurately-estimated features of the equity process can be used as mediating variables (proxying for unobserved state variables) in the pricing of non-equity claims on the same assets.

	5-year credit spreads									
L	10.79	9.49	9.69	8.57	9.35	8.95	8.51	8.93	8.62	8.96
σ	-	8.89	-	-	7.97	4.60	-	4.61	-	6.78
ρ	-	-	-3.17	-	-0.01	-	0.09	0.47	-	-
β	-	-	-	7.25	-	1.50	6.26	1.57	-	-
β x ρ	-	-	-	-	-	-	-	-	-4.99	-0.83
R^2	0.48	0.68	0.51	0.63	0.68	0.68	0.63	0.68	0.56	0.68

Table 10: **Cross-sectional regressions.** *The 5-year credit spread is regressed on financial leverage and option-implied measures of spot volatility, spot leverage and spot volatility of volatility. For each firm, we use the median value of each quantity over the available sample. The table reports regressions t-statistics and R^2s.*

	5-year credit spreads									
L	7.22	5.95	6.78	6.81	5.79	5.86	6.54	5.71	6.72	5.77
σ	-	5.85	-	-	4.68	4.82	-	3.99	-	3.84
ρ	-	-	-2.87	-	-0.55	-	-2.35	-0.56	-	-
β	-	-	-	2.59	-	0.31	2.05	0.33	-	-
β x ρ	-	-	-	-	-	-	-	-	-3.85	-0.63
R^2	0.37	0.56	0.45	0.43	0.58	0.57	0.49	0.59	0.48	0.58

Table 11: **Time-series regressions.** *For each firm in the sample, the 5-year credit spread is regressed on contemporaneous financial leverage and option-implied measures measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are averages across the available firms.*

	5-year credit spreads									
L	10.79	10.72	10.75	10.64	10.68	10.59	10.61	10.56	10.64	10.58
σ	-	0.23	-	-	0.09	-0.34	-	-0.36	-	0.17
ρ	-	-	1.08	-	1.06	-	0.96	0.96	-	-
β	-	-	-	0.74	-	0.78	0.55	0.65	-	-
β x ρ	-	-	-	-	-	-	-	-	0.61	0.59
R^2	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48

Table 12: **Cross-sectional regressions (high-frequency data).** *The 5-year credit spread is regressed on financial leverage and high-frequency measures of spot volatility, spot leverage and spot volatility of volatility. For each firm, we use the median value of each quantity over the available sample. The table reports regressions t-statistics and R^2s.*

	5-year credit spreads									
L	7.22	6.33	7.19	6.56	6.32	6.29	6.53	6.28	7.19	6.33
σ	-	6.14	-	-	6.11	3.74	-	3.73	-	6.06
ρ	-	-	0.15	-	0.25	-	0.13	0.24	-	-
β	-	-	-	4.30	-	0.41	4.28	0.40	-	-
β x ρ	-	-	-	-	-	-	-	-	0.15	0.36
R^2	0.37	0.59	0.38	0.51	0.59	0.60	0.52	0.60	0.38	0.60

Table 13: **Time-series regressions (high-frequency data).** *For each firm in the sample, the 5-year credit spread is regressed on contemporaneous financial leverage and high-frequency measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are the averages across the available firms.*

10 Conclusions

A rich and insightful body of work has focused on the estimation of continuous-time stochastic volatility models for equity with an emphasis on specific quantities of risk, such as spot volatility, spot volatility of volatility and spot leverage.

To the best of our knowledge, this is the first paper that performs joint identification of the fundamental characteristics of the equity process and maps them to structural determinants of equity, like the firm's debt-to-assets ratio.

A key to our method is the recent surge in the liquidity of traded options with a short tenor. We have shown that the increased availability of daily cross sections of short-maturity options spanning a spectrum of moneyness levels provides new opportunities for *joint* identification. We have also shown that the use of a novel pricing formula for short-maturity contracts is instrumental in extracting information from recorded (short-maturity) option prices while avoiding the two-stage estimation that one would typically implement in the case of spot volatility of volatility and spot leverage estimation. The proposed formula is explicit in mapping equity characteristics into features of the implied volatility surface, such as level, slope and convexity, thereby delivering identification for all three characteristics of interest jointly.

Much remains to be done. Two examples of promising directions are the following. First, the recent availability of high-frequency option data may permit implementation of the proposed methods at several instants during the day. Suitable local averages of the resulting estimates would likely result in even more accurate measurements of the equity characteristics, at every point in time and over a day. Second, and more interestingly in our view, because the reduced-form equity dynamics can be readily expressed as functions of the dynamics of the asset values, accurate identification of the equity characteristics gives us a way to filter out (*i*) the parameters and (*ii*) the states of the underlying asset process. This procedure is economically revealing for two reasons. First, accounting information provides the *book* value of assets. While book values are routinely used as proxies of unobservable market values - a logic which was exploited in this paper as well - filtering would deliver *market* values. Second, it would deliver market values at frequencies which align with those of the estimated equity characteristics. As such, the resulting frequencies would be considerably higher than those imposed by the accounting standards in the reporting of book-value financial statements. The end result would be the ability to evaluate quantities which depend on the availability of market values of assets over short time horizons: from (close to) real-time probabilities of default to the marking-to-market of securities (among which equity and corporate bonds) representing claims on the same assets. We leave this line of inquiry for future investigations.

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A Companies

No.	Ticker	Starting date	Ending date	N	Leverage	Name
1	ABT	31-Mar-2006	30-Jun-2015	107	0.14	ABBOTT LABORATORIES
2	ADI	31-Oct-2007	30-Jun-2015	58	0.05	ANALOG DEVICES
3	ADM	30-Apr-2006	30-Jun-2015	92	0.28	ARCHER-DANIELS-MIDLAND CO
4	ADP	31-Mar-2007	30-Jun-2015	70	0.01	AUTOMATIC DATA PROCESSING
5	AEP	31-Mar-2007	28-Feb-2014	60	0.50	AMERICAN ELECTRIC POWER CO
6	AET	31-Jan-2006	30-Jun-2015	98	0.22	AETNA INC
7	AIG	31-Jan-2006	30-Jun-2015	95	0.58	AMERICAN INTERNATIONAL GROUP
8	ALL	31-Jan-2006	30-Jun-2015	103	0.22	ALLSTATE CORP
9	AMAT	31-Jan-2006	30-Jun-2015	110	0.05	APPLIED MATERIALS INC
10	AMGN	31-Jan-2006	30-Jun-2015	113	0.20	AMGEN INC
11	APA	31-Jan-2006	30-Jun-2015	109	0.18	APACHE CORP
12	APC	31-Jan-2006	30-Jun-2015	107	0.30	ANADARKO PETROLEUM CORP
13	AVB	31-Oct-2006	30-Jun-2015	77	0.30	AVALONBAY COMMUNITIES INC
14	AXP	28-Feb-2006	30-Jun-2015	111	0.51	AMERICAN EXPRESS CO
15	AZO	31-Jan-2006	30-Jun-2015	113	0.22	AUTOZONE INC
16	BA	31-Jan-2006	30-Jun-2015	113	0.15	BOEING CO
17	BAX	31-Mar-2006	30-Jun-2015	97	0.14	BAXTER INTERNATIONAL INC
18	BBT	31-Jul-2007	30-Jun-2015	92	0.57	BB&T CORP
19	BBY	31-Jan-2006	30-Jun-2015	113	0.13	BEST BUY CO INC
20	BEN	31-Jan-2006	31-Jan-2014	86	0.05	FRANKLIN RESOURCES INC
21	BMY	31-Jul-2006	30-Jun-2015	101	0.11	BRISTOL-MYERS SQUIBB CO
22	BSX	31-Jan-2006	31-May-2015	39	0.26	BOSTON SCIENTIFIC CORP
23	C	31-Jan-2006	30-Jun-2015	108	0.81	CITIGROUP INC
24	CAH	31-Mar-2006	30-Jun-2015	53	0.16	CARDINAL HEALTH INC
25	CAT	31-Jan-2006	30-Jun-2015	113	0.41	CATERPILLAR INC
26	CHK	31-Jan-2006	30-Jun-2015	110	0.43	CHESAPEAKE ENERGY CORP
27	CI	31-Jan-2006	30-Jun-2015	104	0.20	CIGNA CORP
28	CL	30-Jun-2006	30-Jun-2015	80	0.09	COLGATE-PALMOLIVE CO
29	CMCSA	30-Apr-2006	30-Jun-2015	85	0.32	COMCAST CORP
30	COF	31-Jan-2006	30-Jun-2015	113	0.57	CAPITAL ONE FINANCIAL CORP
31	COP	31-Jan-2006	30-Jun-2015	112	0.24	CONOCOPHILLIPS
32	COST	31-Jan-2006	30-Jun-2015	113	0.07	COSTCO WHOLESALE CORP
33	CSCO	28-Feb-2006	30-Jun-2015	108	0.10	CISCO SYSTEMS INC
34	CSX	30-Apr-2006	30-Jun-2015	87	0.28	CSX CORP
35	CVX	31-Jan-2006	30-Jun-2015	110	0.07	CHEVRON CORP
36	DE	31-Jan-2006	30-Jun-2015	112	0.49	DEERE & CO
37	DHI	31-May-2006	30-Jun-2015	92	0.37	D R HORTON INC
38	DHR	28-Feb-2007	30-Jun-2015	64	0.10	DANAHER CORP
39	DIS	30-Apr-2006	30-Jun-2015	95	0.16	DISNEY (WALT) CO
40	DO	31-Jan-2006	30-Jun-2015	113	0.14	DIAMOND OFFSHRE DRILLING INC
41	DRI	31-May-2008	30-Jun-2015	79	0.25	DARDEN RESTAURANTS INC
42	DVN	31-Jan-2006	30-Jun-2015	113	0.23	DEVON ENERGY CORP
43	EMR	31-Jan-2006	30-Jun-2015	82	0.13	EMERSON ELECTRIC CO
44	EOG	31-Jan-2006	30-Jun-2015	112	0.11	EOG RESOURCES INC
45	ESRX	31-Jan-2006	30-Jun-2015	112	0.16	EXPRESS SCRIPTS HOLDING CO
46	ESV	31-Jan-2006	30-Jun-2015	101	0.19	ENSCO PLC
47	ETN	30-Apr-2006	30-Jun-2015	87	0.24	EATON CORP PLC
48	EXC	31-Mar-2007	30-Jun-2015	90	0.34	EXELON CORP
49	FCX	31-Jan-2006	30-Jun-2015	113	0.21	FREEPORT-MCMORAN INC
50	FDX	31-Jan-2006	30-Jun-2015	113	0.08	FEDEX CORP
51	FLR	28-Feb-2006	30-Jun-2015	106	0.04	FLUOR CORP
52	GD	31-Jan-2006	30-Jun-2015	80	0.11	GENERAL DYNAMICS CORP

53	GE	31-May-2006	30-Jun-2015	105	0.66	GENERAL ELECTRIC CO
54	GIS	31-Aug-2008	30-Jun-2015	71	0.23	GENERAL MILLS INC
55	GLW	31-Jan-2006	30-Jun-2015	99	0.09	CORNING INC
56	GPS	31-Dec-2006	30-Jun-2015	77	0.06	GAP INC
57	GS	31-Jan-2006	30-Jun-2015	113	0.88	GOLDMAN SACHS GROUP INC
58	HAL	31-Jan-2006	30-Jun-2015	113	0.12	HALLIBURTON CO
59	HD	31-Jan-2006	30-Jun-2015	112	0.15	HOME DEPOT INC
60	HIG	31-Jan-2006	30-Jun-2015	106	0.35	HARTFORD FINANCIAL SERVICES
61	HON	31-Jan-2006	30-Jun-2015	108	0.16	HONEYWELL INTERNATIONAL INC
62	HPQ	31-Jan-2006	30-Jun-2015	113	0.21	HP INC
63	HUM	31-Mar-2006	30-Jun-2015	104	0.16	HUMANA INC
64	IBM	31-Jan-2006	30-Jun-2015	113	0.16	INTL BUSINESS MACHINES CORP
65	INTC	31-Jan-2006	30-Jun-2015	110	0.04	INTEL CORP
66	IP	31-Mar-2006	30-Jun-2015	92	0.41	INTL PAPER CO
67	JCI	28-Feb-2007	30-Jun-2015	84	0.19	JOHNSON CONTROLS INTL PLC
68	JNJ	31-Jan-2006	30-Jun-2015	92	0.07	JOHNSON & JOHNSON
69	JNPR	31-Jan-2006	30-Jun-2015	111	0.05	JUNIPER NETWORKS INC
70	JPM	31-Jan-2006	30-Jun-2015	110	0.77	JPMORGAN CHASE & CO
71	JWN	28-Feb-2007	30-Jun-2015	91	0.24	NORDSTROM INC
72	KO	31-Jan-2006	30-Jun-2015	106	0.12	COCA-COLA CO
73	KSS	28-Feb-2006	30-Jun-2015	104	0.20	KOHL'S CORP
74	LEN	31-Jan-2006	30-Jun-2015	109	0.47	LENNAR CORP
75	LLL	31-Mar-2006	30-Nov-2014	66	0.31	L3 TECHNOLOGIES INC
76	LLY	30-Nov-2006	30-Jun-2015	84	0.12	LILLY (ELI) & CO
77	LM	31-Jan-2006	31-Jul-2014	91	0.25	LEGG MASON INC
78	LMT	31-Mar-2006	30-Jun-2015	103	0.13	LOCKHEED MARTIN CORP
79	LNC	31-Aug-2007	30-Jun-2015	84	0.40	LINCOLN NATIONAL CORP
80	LOW	31-Jan-2006	30-Jun-2015	107	0.15	LOWE'S COMPANIES INC
81	M	31-Jan-2006	30-Jun-2015	105	0.38	MACY'S INC
82	MAR	31-Aug-2007	30-Jun-2015	88	0.20	MARRIOTT INTL INC
83	MCD	31-Jan-2006	30-Jun-2015	110	0.13	MCDONALD'S CORP
84	MCK	30-Apr-2006	30-Jun-2015	91	0.15	MCKESSON CORP
85	MDT	31-Jan-2006	30-Jun-2015	108	0.17	MEDTRONIC PLC
86	MET	28-Feb-2007	30-Jun-2015	94	0.42	METLIFE INC
87	MLM	31-Jan-2006	30-Jun-2015	83	0.20	MARTIN MARIETTA MATERIALS
88	MMM	31-Jan-2006	30-Jun-2015	105	0.08	3M CO
89	MO	31-Jan-2006	30-Jun-2015	113	0.15	ALTRIA GROUP INC
90	MON	31-Jan-2006	30-Jun-2015	109	0.06	MONSANTO CO
91	MRK	31-Jan-2006	30-Jun-2015	113	0.12	MERCK & CO
92	MRO	31-Jan-2006	30-Jun-2015	112	0.21	MARATHON OIL CORP
93	MSFT	31-Jan-2006	30-Jun-2015	108	0.04	MICROSOFT CORP
94	MUR	30-Jun-2006	28-Feb-2015	74	0.12	MURPHY OIL CORP
95	NBL	31-Jan-2006	31-May-2015	79	0.18	NOBLE ENERGY INC
96	NEM	31-Jan-2006	30-Jun-2015	113	0.19	NEWMONT MINING CORP
97	NKE	31-Jan-2006	30-Jun-2015	108	0.02	NIKE INC
98	NOV	31-Jan-2006	30-Jun-2015	113	0.06	NATIONAL OILWELL VARCO INC
99	NSC	31-Mar-2006	30-Jun-2015	89	0.26	NORFOLK SOUTHERN CORP
100	NUE	31-Jan-2006	30-Jun-2015	113	0.18	NUCOR CORP
101	OXY	31-Jan-2006	30-Jun-2015	112	0.07	OCCIDENTAL PETROLEUM CORP
102	PEP	30-Apr-2006	30-Jun-2015	105	0.14	PEPSICO INC
103	PFE	31-Jan-2006	30-Jun-2015	94	0.18	PFIZER INC
104	PG	30-Jun-2006	30-Jun-2015	99	0.15	PROCTER & GAMBLE CO
105	PHM	31-Jan-2006	30-Jun-2015	93	0.40	PULTEGROUP INC
106	PNC	30-Apr-2006	30-Jun-2015	102	0.56	PNC FINANCIAL SVCS GROUP INC
107	PRU	30-Sep-2006	30-Jun-2015	99	0.55	PRUDENTIAL FINANCIAL INC
108	PXD	31-Jan-2006	30-Jun-2015	82	0.20	PIONEER NATURAL RESOURCES CO

109	RTN	31-Mar-2006	30-Jun-2015	99	0.14	RAYTHEON CO
110	SHW	31-Jan-2006	30-Jun-2015	71	0.10	SHERWIN-WILLIAMS CO
111	SO	31-Mar-2009	30-Jun-2015	73	0.39	SOUTHERN CO
112	SPG	31-Dec-2006	30-Jun-2015	101	0.39	SIMON PROPERTY GROUP INC
113	STI	31-Mar-2006	30-Jun-2015	101	0.60	SUNTRUST BANKS INC
114	STT	31-Jul-2007	30-Jun-2015	95	0.54	STATE STREET CORP
115	T	31-Aug-2006	30-Jun-2015	105	0.28	AT&T INC
116	TGT	31-Jan-2006	30-Jun-2015	113	0.28	TARGET CORP
117	TXN	31-Jan-2006	30-Jun-2015	112	0.05	TEXAS INSTRUMENTS INC
118	TXT	30-Apr-2006	30-Jun-2015	101	0.44	TEXTRON INC
119	UNH	31-Jan-2006	30-Jun-2015	104	0.20	UNITEDHEALTH GROUP INC
120	UNP	31-Mar-2006	30-Jun-2015	111	0.18	UNION PACIFIC CORP
121	UPS	31-Jan-2006	30-Jun-2015	111	0.12	UNITED PARCEL SERVICE INC
122	USB	30-Nov-2006	30-Jun-2015	93	0.52	U S BANCORP
123	UTX	30-Apr-2006	30-Jun-2015	98	0.16	UNITED TECHNOLOGIES CORP
124	WFC	30-Apr-2006	30-Jun-2015	98	0.56	WELLS FARGO & CO
125	WHR	31-Jan-2006	30-Jun-2015	113	0.29	WHIRLPOOL CORP
126	WMB	31-Jan-2006	30-Jun-2015	82	0.34	WILLIAMS COS INC
127	WMT	31-Jan-2006	30-Jun-2015	113	0.19	WALMART INC
128	WY	30-Nov-2006	30-Jun-2015	94	0.32	WEYERHAEUSER CO
129	XOM	31-Jan-2006	30-Jun-2015	113	0.04	EXXON MOBIL CORP
130	YUM	31-May-2007	30-Jun-2015	87	0.12	YUM BRANDS INC

Table 14: **List of firms in the sample.** *Each row shows the ticker, the start and the ending date of our sample, the number of monthly observations, the average value of financial leverage, and the full name of each company. Financial leverage is computed from Compustat as the ratio between the total debt (given by the sum of debt in current liabilities (`dlcq`) and long-term debt (`dlttq`)) over the sum of total debt and value of the equity (given by the product of the number of common shares outstanding (`cshoq`) and the price per share (`prccq`)).*

B The characteristic function of the diffusive process

Define $\log E^d_{t+\tau} - \log E^d_t = X_t$ and $\mu_t = r_t - \delta_t - \frac{1}{2}\sigma_t^2$, to simplify the notation. Write

$$\begin{aligned}
\mathbb{C}^d(u,\tau) &= \mathbb{E}_t[e^{iuX_t}] \\
&= \frac{1}{\sqrt{2\pi\sigma_t^2\tau}} \int e^{iuX_t} e^{-\frac{1}{2}\left(\frac{X_t-\mu_t\tau}{\sigma_t\sqrt{\tau}}\right)^2} dX_t \\
&= \frac{1}{\sqrt{2\pi}} \int e^{iu(\mu_t\tau+\sigma_t\sqrt{\tau}Z_t)} e^{-\frac{1}{2}Z_t^2} dZ_t \\
&= e^{iu\mu_t\tau} \underbrace{\frac{1}{\sqrt{2\pi}} \int e^{iu\sigma_t\sqrt{\tau}Z_t} e^{-\frac{1}{2}Z_t^2} dZ_t}_{\mathbb{E}_t[e^{ivZ_t}]} .
\end{aligned}$$

Now, defining $v = u\sigma_t\sqrt{\tau}$, the term with an under-brace becomes the characteristic function of the *standardized* process (Z_t), for which we use the (local, in τ) expansion in Theorem 1 of Bandi and Renò (2019). In the main text, we use the notation $z = iv$.

C Proof of Proposition 1

We recall that equity values depend on two state variables, the value of the assets and their variance, i.e., $E_t := E(A_t, V_t^A)$. Using Itô's Lemma here, and repeatedly below, we have

$$dE_t = \text{drift}_E + E_A A_t \sqrt{V_t^A} dW_t^A + E_V \eta \sqrt{V_t^A} dW_t^V + \underbrace{\left(E(A_t(1+J_t^A), V_t^A) - E_t\right)}_{J_t^E} dN_t.$$

In terms of logarithmic equity values (or continuously-compounded returns), after defining $e_t = \log E_t$, we obtain

$$de_t = \text{drift}_e + E_A \frac{1}{E_t} A_t \sqrt{V_t^A} dW_t^A + E_V \frac{1}{E_t} \eta \sqrt{V_t^A} dW_t^V + \left(\log(E_t + J_t^E) - \log(E_t)\right) dN_t,$$

where $\text{drift}_e = \text{drift}_E - \frac{1}{2} V_t^E dt$, with V_t^E defined next. The continuous equity spot variance $V_t^E := V^E(A_t, V_t^A)$ is, now,

$$V_t^E := V^E(A_t, V_t^A) = \frac{V_t^A}{E_t^2} \left(A_t^2 E_A^2 + \eta^2 E_V^2 + 2\rho_A \eta E_A E_V A_t\right).$$

The term *spot volatility* naturally refers to the square root of V_t^E, i.e., $\sigma_t := \sigma(A_t, V_t^A) = \sqrt{V_t^E}$. We can now write

$$dV_t^E = \text{drift}_V + \Lambda_A A_t \sqrt{V_t^A} dW_t^A + \Lambda_V \eta \sqrt{V_t^A} dW_t^V + \underbrace{\left(V(A_t(1+J_t^A), V_t^A) - V_t\right)}_{J_t^V} dN_t,$$

with

$$\Lambda_A = 2\frac{V_t^A}{E_t^2} \left(A_t E_A^2 + A_t^2 E_A E_{AA} + \eta^2 E_V E_{VA} + \eta\rho_A \left(E_A E_V + A_t E_V E_{AA} + A_t E_A E_{VA}\right)\right) - 2\frac{V_t^E}{E_t} E_A,$$

and

$$\Lambda_V = 2\frac{V_t^A}{E_t^2} \left(A_t^2 E_A E_{VA} + \eta^2 E_V E_{VV} + \eta\rho_A A_t \left(E_V E_{VA} + E_A E_{VV}\right)\right) - 2\frac{V_t^E}{E_t} E_V + \frac{V_t^E}{V_t^A}.$$

In terms of volatility, we have

$$d\sigma_t = \text{drift}_\sigma + \frac{1}{2\sqrt{V_t^E}} \Lambda_A A_t \sqrt{V_t^A} dW_t^A + \frac{1}{2\sqrt{V_t^E}} \Lambda_V \eta \sqrt{V_t^A} dW_t^V + \left(\sqrt{V_t^E + J_t^V} - \sqrt{V_t^E}\right) dN_t,$$

where $\text{drift}_\sigma = \text{drift}_V - \frac{1}{2\sqrt{V_t^E}} \beta_t^2$, with β_t^2 defined next. The variance of volatility can be expressed as

$$\beta_t^2 := \beta^2(A_t, V_t^A) = \frac{V_t^A}{4V_t^E} \left(A_t^2 \Lambda_A^2 + \eta^2 \Lambda_V^2 + 2A_t \eta \rho_A \Lambda_A \Lambda_V\right).$$

Hence, the term *spot volatility of volatility* refers to the square root of β_t^2, i.e., $\beta_t := \beta(A_t, V_t^A) = \sqrt{\beta_t^2}$. Finally, the *leverage effect* is defined as the instantaneous correlation between spot volatility and log returns:

$$\rho_t = \rho(A_t, V_t^A) = \frac{V_t^A}{2E_t\sqrt{V_t^E}} \frac{\Lambda_A E_A A_t^2 + \Lambda_V E_V \eta^2 + \rho_A \eta A_t \left(\Lambda_A E_V + \Lambda_V E_A\right)}{\sqrt{V_t^E}\beta_t}.$$

Regarding jumps, we have

$$\lambda_E = \lambda_\sigma = \lambda_A.$$

Thus, the return and variance jumps are joint. For small jumps, using a Taylor expansion, we have

$$J_E = E(A_t(1 + J_t^A), V_t^A) - E_t = E_A A_t J_t^A.$$

Hence,

$$J_e = \log(E_t + J_E) - \log(E_t) \approx \frac{1}{E_t} A_t E_A J_t^A.$$

Similarly,

$$J_V = V(A_t(1 + J_t^A), V_t^A) - V_t = V_A A_t J_t^A.$$

Thus,

$$J_\sigma = \sqrt{V_t^E + J_t^V} - \sqrt{V_t^E} \approx \frac{1}{2\sigma_t} A_t V_A J_t^A.$$

D Structural Model: Monte Carlo Evidence

In order to shed more light on our findings, in this Appendix we replicate our empirical analysis using simulated data from the model in Eq. (16) and Eq. (17).

We set up the Monte Carlo experiment as follow. First, we consider the same number of firms as in Section 8: 130. Second, we fix the face value of the debt of each firm at \$50 and endow each firm with an initial asset value which is equi-spaced between \$100 and \$600. As we will show below, the assumed heterogeneity in initial asset values will lead to a realistic cross-sectional distribution of financial leverage. Third, we simulate 10 years of daily observations using parameter values taken from Du, Elkamhi, and Ericsson (2018).[23] Fourth, we winsorize each trajectory at \$51 in order to prevent the firm's assets to go below the face value of debt. Finally, we sample the simulated data at the monthly frequency. Given simulated trajectories from the structural model, the equity characteristics are computed as in Proposition 1 in the main text.

Fig. D.1 compares the cross-sectional distribution of (average) financial leverage in the data (left panel) with the one generated by Monte Carlo simulation (right panel). Along this dimension,

[23]We use the estimated parameters corresponding to the 50th quantile of the firms' distribution in their Table VII.

the simulated data replicates the empirical counterpart rather well. Most of the firms have financial leverage between zero and 30% and only a few exhibit financial leverage levels in excess of 60%.

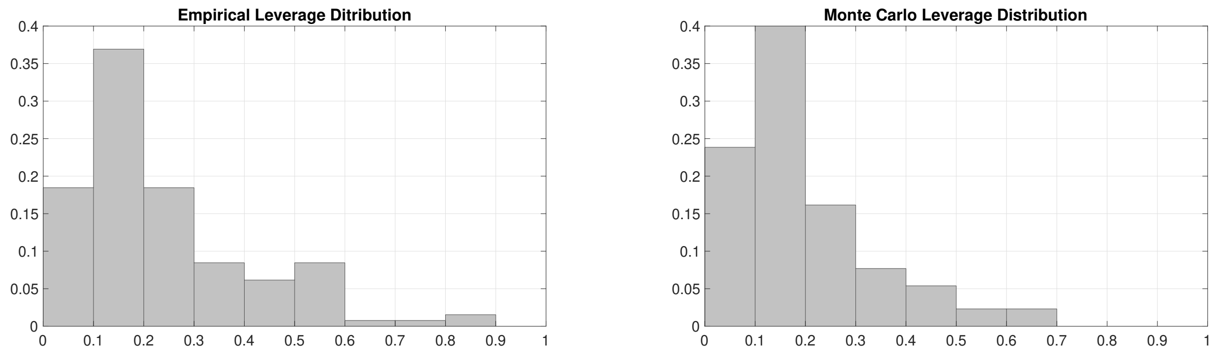

Figure D.1: *Comparison between the cross-sectional distribution of (average) financial leverage in the data (left panel) and in Monte Carlo simulations (right panel).*

Fig. D displays scatter plots of the equity characteristics with respect to financial leverage. Because the simulated cross-sectional distribution of financial leverage is realistic, in Fig. D we focus on empirically-meaningful levels of firm-level riskiness. Importantly, the errors around non-linear least-squares (solid) lines are simply due to the presence of an additional state variable in the determination of the equity characteristics, i.e., the volatility of the assets. In essence, the specification in Eq. (16) and Eq. (17) implies that this second state variable should be expected to blur the relation between equity characteristics and financial leverage both in the simulations and in the data. In this sense, Fig. D is directly comparable to, and nicely justifies, Fig. 11 in the main text. The relations in Fig. 11 should, however, be further contaminated by additional measurement errors discussed below.

Turning now to regressions, a few observations can be drawn from Tables 15 and 16. First, consistent with data, all equity characteristics uniquely contribute to explain level and time variation of financial leverage (with signs that are consistent with data). Second, spot leverage slightly dominates spot volatility and spot volatility of volatility both in terms of R^2 and in terms of t-statistics.

When comparing these simulation results to the empirical results reported in the main text, one should notice that, in the data, (1) financial leverage is measured with an error[24] and (2) both spot leverage and spot volatility of volatility are estimated with less precision than spot volatility

[24] There are two sources of measurement error. Compustat provides book values, rather than market values, and the monthly measures are obtained by linearly interpolating quarterly measures.

(c.f., Subsection 5.2).

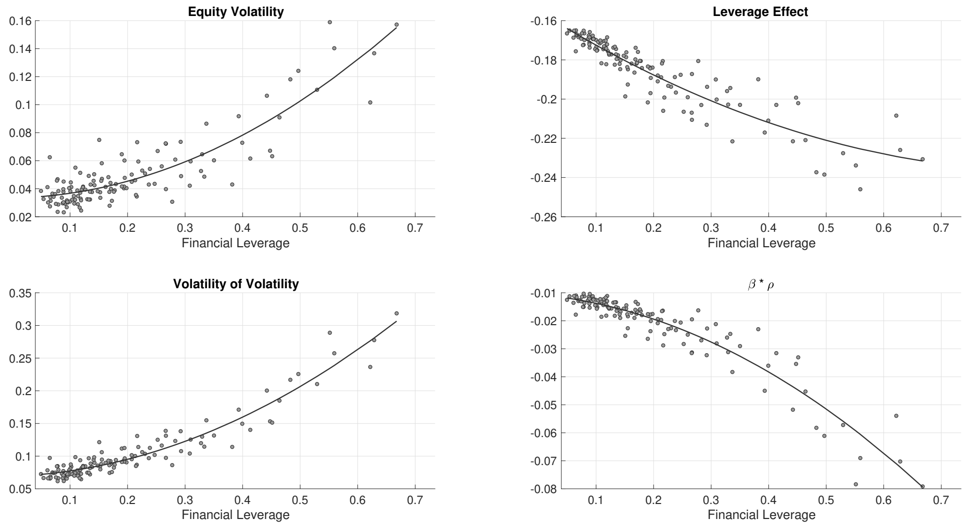

Figure D.2: *The simulated relation between equity characteristics and financial leverage.*

	Financial Leverage								
σ	18.87	-	-	23.31	37.60	-	80.05	-	38.28
ρ	-	-24.62	-	-8.26	-	-29.82	-28.38	-	-
β	-	-	28.33	-	19.52	7.80	37.05	-	-
β x ρ	-	-	-	-	-	-	-	-27.46	-19.99
R^2	0.74	0.83	0.86	0.83	0.93	0.88	0.99	0.85	0.94

Table 15: **Cross-sectional regressions (Monte Carlo).** *We regress financial leverage on option-implied measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and* R^2*s.*

	Financial Leverage								
σ	1.83	-	-	2.72	2.20	-	3.26	-	2.48
ρ	-	-3.07	-	-10.63	-	-3.50	-10.96	-	-
β	-	-	2.81	-	4.03	-0.62	5.87	-	-
β x ρ	-	-	-	-	-	-	-	-2.78	-8.01
R^2	0.08	0.11	0.11	0.61	0.21	0.21	0.66	0.11	0.38

Table 16: **Time-series regressions (Monte Carlo).** *We regress financial leverage on option-implied measures of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and* R^2*s. Reported numbers are the averages across 130 firms.*

E Analysis by Sector

	Financial Leverage								
Panel A: Materials									
σ	0.97	-	-	0.73	1.17	-	0.25	-	0.84
ρ	-	0.51	-	0.14	-	4.00	2.90	-	-
β	-	-	1.20	-	1.35	4.52	3.68	-	-
β x ρ	-	-	-	-	-	-	-	0.23	0.04
R^2	0.16	0.05	0.22	0.16	0.42	0.84	0.85	0.01	0.16
Panel B: Industrial									
σ	1.01	-	-	1.91	1.21	-	2.00	-	2.03
ρ	-	0.40	-	1.65	-	0.29	1.59	-	-
β	-	-	-0.46	-	-0.82	-0.36	-0.76	-	-
β x ρ	-	-	-	-	-	-	-	0.18	1.73
R^2	0.05	0.01	0.01	0.18	0.08	0.02	0.20	0.00	0.19
Panel C: Energy									
σ	-0.21	-	-	1.97	-0.62	-	0.98	-	1.97
ρ	-	4.16	-	4.91	-	5.04	5.03	-	-
β	-	-	0.49	-	0.76	2.18	1.28	-	-
β x ρ	-	-	-	-	-	-	-	3.29	4.07
R^2	0.00	0.52	0.01	0.62	0.04	0.63	0.66	0.40	0.53
Panel D: Consumer Discretionary									
σ	3.80	-	-	2.78	0.65	-	0.41	-	0.95
ρ	-	-2.09	-	-0.19	-	-0.62	-0.36	-	-
β	-	-	3.94	-	0.99	3.01	1.01	-	-
β x ρ	-	-	-	-	-	-	-	-3.83	-1.01
R^2	0.45	0.19	0.46	0.45	0.48	0.47	0.48	0.45	0.48
Panel E: Consumer Staple									
σ	0.82	-	-	1.64	-0.63	-	-0.11	-	1.47
ρ	-	1.50	-	2.12	-	2.27	1.94	-	-
β	-	-	1.42	-	1.23	2.21	1.12	-	-
β x ρ	-	-	-	-	-	-	-	0.47	1.30
R^2	0.09	0.24	0.22	0.48	0.27	0.58	0.58	0.03	0.29
Panel F: Heath Care									
σ	4.52	-	-	5.59	1.23	-	1.91	-	5.63
ρ	-	0.36	-	2.34	-	-0.01	1.42	-	-
β	-	-	4.86	-	1.72	4.67	0.22	-	-
β x ρ	-	-	-	-	-	-	-	-0.09	2.35
R^2	0.56	0.01	0.60	0.68	0.63	0.60	0.68	0.00	0.68
Panel G: Financials									
σ	-0.44	-	-	-0.25	0.98	-	0.97	-	0.30
ρ	-	1.53	-	1.44	-	0.46	-0.49	-	-
β	-	-	-1.67	-	-1.87	-0.76	-1.20	-	-
β x ρ	-	-	-	-	-	-	-	1.61	1.52
R^2	0.01	0.12	0.14	0.12	0.19	0.15	0.20	0.13	0.14
Panel H: Information Technology									
σ	-0.51	-	-	-0.52	-0.93	-	-1.20	-	-0.47
ρ	-	0.00	-	0.18	-	0.15	0.79	-	-
β	-	-	0.60	-	0.98	0.58	1.23	-	-
β x ρ	-	-	-	-	-	-	-	-0.26	-0.21
R^2	0.03	0.00	0.04	0.03	0.13	0.04	0.20	0.01	0.03

Table 17: **Cross-sectional regressions.** *For each sector we regress financial leverage on option-implied estimates of spot volatility, spot leverage and spot volatility of volatility. Each panel reports t-statistics and* R^2*s.*

	Financial Leverage								
Panel A: Materials									
σ	2.33	-	-	2.05	1.95	-	1.67	-	1.31
ρ	-	-0.30	-	-0.89	-	-0.31	-0.92	-	-
β	-	-	0.88	-	0.66	0.96	0.69	-	-
β x ρ	-	-	-	-	-	-	-	-0.96	-1.10
R^2	0.22	0.05	0.05	0.25	0.24	0.08	0.27	0.07	0.27
Panel B: Industrial									
σ	4.41	-	-	3.63	3.88	-	3.21	-	3.31
ρ	-	-2.30	-	-0.44	-	-2.06	-0.47	-	-
β	-	-	1.65	-	0.02	1.38	0.10	-	-
β x ρ	-	-	-	-	-	-	-	-2.68	-0.34
R^2	0.23	0.07	0.05	0.25	0.25	0.11	0.27	0.09	0.26
Panel C: Energy									
σ	-0.13	-	-	-0.94	-0.26	-	-1.10	-	-1.29
ρ	-	-0.96	-	-1.71	-	-0.99	-1.71	-	-
β	-	-	0.12	-	0.24	0.06	0.53	-	-
β x ρ	-	-	-	-	-	-	-	-0.79	-1.82
R^2	0.11	0.06	0.03	0.16	0.12	0.08	0.17	0.05	0.16
Panel D: Consumer Discretionary									
σ	6.89	-	-	5.87	5.93	-	5.05	-	5.41
ρ	-	-2.83	-	0.16	-	-2.42	0.12	-	-
β	-	-	2.63	-	0.20	2.40	0.27	-	-
β x ρ	-	-	-	-	-	-	-	-3.45	0.54
R^2	0.36	0.14	0.09	0.39	0.37	0.20	0.41	0.16	0.39
Panel E: Consumer Staple									
σ	0.73	-	-	0.57	0.46	-	0.40	-	0.34
ρ	-	-0.55	-	-0.50	-	-0.49	-0.56	-	-
β	-	-	0.71	-	0.45	0.61	0.36	-	-
β x ρ	-	-	-	-	-	-	-	-0.53	-0.60
R^2	0.13	0.02	0.05	0.14	0.15	0.07	0.16	0.03	0.15
Panel F: Heath Care									
σ	0.93	-	-	0.94	0.66	-	0.70	-	0.88
ρ	-	-0.20	-	0.17	-	-0.08	0.20	-	-
β	-	-	0.69	-	0.39	0.65	0.41	-	-
β x ρ	-	-	-	-	-	-	-	-0.38	0.05
R^2	0.14	0.04	0.03	0.17	0.16	0.06	0.18	0.04	0.18
Panel G: Financials									
σ	9.18	-	-	7.06	6.77	-	5.98	-	7.18
ρ	-	-4.20	-	0.59	-	-2.18	0.62	-	-
β	-	-	4.96	-	-0.65	3.17	-0.58	-	-
β x ρ	-	-	-	-	-	-	-	-4.75	2.15
R^2	0.46	0.19	0.22	0.47	0.47	0.28	0.49	0.22	0.50
Panel H: Information Technology									
σ	-2.69	-	-	-2.91	-2.90	-	-2.98	-	-3.00
ρ	-	0.15	-	-0.89	-	0.17	-0.83	-	-
β	-	-	-0.21	-	0.58	-0.34	0.36	-	-
β x ρ	-	-	-	-	-	-	-	0.43	-0.86
R^2	0.16	0.03	0.04	0.22	0.19	0.06	0.24	0.03	0.24

Table 18: **Time-series regressions.** *For each sector we regress financial leverage on option-implied estimates of spot volatility, spot leverage and spot volatility of volatility. The table reports t-statistics and R^2s. Reported numbers are the averages across firms within each sector.*