# Shareholder Heterogeneity, Asymmetric Information, and the Equilibrium Manager

M. Bianchi*, R.-A. Dana** & E. Jouini**

MAD/Stat. - Toulouse School of Economics
*Toulouse School of Economics
** Université Paris-Dauphine, PSL University

# Introduction

- Since Berle and Means (32) and Alchian (68), conflicts of interest between shareholders and managers are usually analyzed with

# Introduction



- Since Berle and Means (32) and Alchian (68), conflicts of interest between shareholders and managers are usually analyzed with
  - a representative shareholder

# Introduction



- Since Berle and Means (32) and Alchian (68), conflicts of interest between shareholders and managers are usually analyzed with
  - a representative shareholder
  - partial equilibrium and principal/agent approach (Jensen and Meckling, 1976)

# Introduction



- Since Berle and Means (32) and Alchian (68), conflicts of interest between shareholders and managers are usually analyzed with
  - a representative shareholder
  - partial equilibrium and principal/agent approach (Jensen and Meckling, 1976)
- Model of heterogeneous shareholders : weigh differently scenarios and rank differently projects and alternatives

# Introduction



- Since Berle and Means (32) and Alchian (68), conflicts of interest between shareholders and managers are usually analyzed with
  - a representative shareholder
  - partial equilibrium and principal/agent approach (Jensen and Meckling, 1976)
- Model of heterogeneous shareholders : weigh differently scenarios and rank differently projects and alternatives
- J&M76 : "The 'behavior' of the firm is like the behavior of a market, that is, the outcome of a complex equilibrium process (...) in which the conflicting objectives of individuals are brought into equilibrium"

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?
    - Which compensation schemes lead to such an alignment?

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?
    - Which compensation schemes lead to such an alignment?
- Manager's objectives are in line with those of the shareholders iff

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?
    - Which compensation schemes lead to such an alignment?
- Manager's objectives are in line with those of the shareholders iff
  - she does not trade,

# Equilibrium Approach (BDJ, 2019)



- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?
    - Which compensation schemes lead to such an alignment?
- Manager's objectives are in line with those of the shareholders iff
  - she does not trade,
  - compensation is inear,

# Equilibrium Approach (BDJ, 2019)

- A **general equilibrium** model :
  - Shareholders trade taking firm's decisions into account
  - Firm's decisions taken by a utility maximizing manager
  - Main questions :
    - How to ensure that manager's and shareholders' objectives are in line?
    - How do their beliefs are related?
    - Which compensation schemes lead to such an alignment?
- Manager's objectives are in line with those of the shareholders iff
  - she does not trade,
  - compensation is inear,
  - she has the same belefs as the *representative* shareholder.

# Extending BDJ 2019



- Manager's contract : compensation schem AND space of allowed transactions,

# Extending BDJ 2019



- Manager's contract : compensation schem AND space of allowed transactions,
- Constraints on insider trading are endogeneous and part of the equilibrium description,

# Extending BDJ 2019



- Manager's contract : compensation schem AND space of allowed transactions,
- Constraints on insider trading are endogeneous and part of the equilibrium description,
- Main results

# Extending BDJ 2019



- Manager's contract : compensation schem AND space of allowed transactions,
- Constraints on insider trading are endogeneous and part of the equilibrium description,
- Main results
  - Trading constraints restore equilibrium regarless of manager's beliefs

# Extending BDJ 2019



- Manager's contract : compensation schem AND space of allowed transactions,
- Constraints on insider trading are endogeneous and part of the equilibrium description,
- Main results
    - Trading constraints restore equilibrium regarless of manager's beliefs
    - We characterize the associated compensation scheme,

# Extending BDJ 2019

- Manager's contract : compensation schem AND space of allowed transactions,
- Constraints on insider trading are endogeneous and part of the equilibrium description,
- Main results
  - Trading constraints restore equilibrium regarless of manager's beliefs
  - We characterize the associated compensation scheme,
  - Qualitative properties valid irrespective of the exact form of beliefs heterogeneity.

# Consumption/Production Space



- One consumption good. One trading and consumption date $T$.

# Consumption/Production Space

- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$

# Consumption/Production Space



- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,

# Consumption/Production Space

- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$

# Consumption/Production Space

- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$
- $y(\omega)$ / $x(\omega)$ prod. / consumption in state $\omega$

# Consumption/Production Space

- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X^{'} = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$
- $y(\omega)$ / $x(\omega)$ prod. / consumption in state $\omega$
- For $Y$ convex,

# Consumption/Production Space



- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$
- $y(\omega)$ / $x(\omega)$ prod. / consumption in state $\omega$
- For $Y$ convex,
  - $N_Y(y) = \{p \in X' : p \cdot (y' - y) \le 0, \forall\, y' \in Y\}$

# Consumption/Production Space



- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$
- $y(\omega)$ / $x(\omega)$ prod. / consumption in state $\omega$
- For $Y$ convex,
  - $N_Y(y) = \{p \in X' : p \cdot (y' - y) \leq 0, \forall\, y' \in Y\}$
  - $\mathrm{Eff}^+(Y) = \{y \in Y : \exists p \in N_Y(y), p >> 0\}$

# Consumption/Production Space



- One consumption good. One trading and consumption date $T$.
- Probability space $(\Omega, \mathcal{F}, P)$
- $X = L^r(\Omega, \mathcal{F}, P)$, $X' = L^{r'}(\Omega, \mathcal{F}, P)$, $\frac{1}{r} + \frac{1}{r'} = 1$,
- Contingent plans $y$ and $x$
- $y(\omega)$ / $x(\omega)$ prod. / consumption in state $\omega$
- For $Y$ convex,
  - $N_Y(y) = \{p \in X' : p \cdot (y' - y) \leq 0, \forall\, y' \in Y\}$
  - $\mathrm{Eff}^+(Y) = \{y \in Y : \exists p \in N_Y(y), p >> 0\}$
  - $Y$ is smooth if $\forall y \in \mathrm{Eff}^+(Y)$, $N_Y(y)$ is a half-line

# The Firm



**Assumption (P)**

1. $Y = K - X_+$ *where* $K \subset X_+$,

# The Firm

**Assumption (P)**

1. *$Y = K - X_+$ where $K \subset X_+$,*
2. *$Y$ is closed and smooth,*

# The Firm



**Assumption (P)**

1. $Y = K - X_+$ *where* $K \subset X_+$,
2. $Y$ *is closed and smooth,*
3. *If* $y \in \mathrm{Eff}^+(Y)$, *the random variable* $y$ *has a density* $h_y$ *with* $h_y > 0$, $\mu-$*a.e. on* $(0, \infty)$ *(full support and no atoms).*

# The Firm

**Assumption (P)**

1. $Y = K - X_+$ *where* $K \subset X_+$,
2. $Y$ *is closed and smooth,*
3. *If* $y \in \mathrm{Eff}^+(Y)$, *the random variable* $y$ *has a density* $h_y$ *with* $h_y > 0$, $\mu-$*a.e. on* $(0, \infty)$ *(full support and no atoms).*

**Example (typical)**

Terminal values of a production process governed by a Brownian motion $W$ and a control $\theta$ that determines drift and volatility

$$K = \left\{ y_T^\theta : d\ln y_t^\theta = \mu\left(\theta_t\right) dt + \theta_t dW_t \right\},$$

where $\mu$ is real valued.

# Information Asymmetry



- Manager chooses $y$ and observes realizations $y(\omega)$

# Information Asymmetry

- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$

# Information Asymmetry



- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$
- If $y \in \mathrm{Eff}^+(Y)$, $y(\omega)$ does not permit to identify $y$ nor to exclude some $y$s (same support assumption)

# Information Asymmetry

- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$
- If $y \in \mathrm{Eff}^+(Y)$, $y(\omega)$ does not permit to identify $y$ nor to exclude some $y$s (same support assumption)
- Plans in $\mathrm{Eff}^+(Y)$ only differ by the probabilities of outcomes (Magill-Quinzii, 2009)

# Information Asymmetry



- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$
- If $y \in \mathrm{Eff}^+(Y)$, $y(\omega)$ does not permit to identify $y$ nor to exclude some $y$s (same support assumption)
- Plans in $\mathrm{Eff}^+(Y)$ only differ by the probabilities of outcomes (Magill-Quinzii, 2009)
- $M$ can manipulate shareholders' information by announcing $y$ and implementing $y'$

# Information Asymmetry



- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$
- If $y \in \mathrm{Eff}^{+}(Y)$, $y(\omega)$ does not permit to identify $y$ nor to exclude some $y$s (same support assumption)
- Plans in $\mathrm{Eff}^{+}(Y)$ only differ by the probabilities of outcomes (Magill-Quinzii, 2009)
- $M$ can manipulate shareholders' information by announcing $y$ and implementing $y'$
- Contracts necessarily contingent to the realizations $\Longrightarrow$ possible manipulation of prices/allocations

# Information Asymmetry



- Manager chooses $y$ and observes realizations $y(\omega)$
- Shareholders only observe realizations $y(\omega)$
- If $y \in \mathrm{Eff}^+(Y)$, $y(\omega)$ does not permit to identify $y$ nor to exclude some $y$s (same support assumption)
- Plans in $\mathrm{Eff}^+(Y)$ only differ by the probabilities of outcomes (Magill-Quinzii, 2009)
- $M$ can manipulate shareholders' information by announcing $y$ and implementing $y'$
- Contracts necessarily contingent to the realizations $\Longrightarrow$ possible manipulation of prices/allocations

**Problem**

**How to lead the manager to truthfully implement the announced plan?**

# Market Timing



- Shareholders hire a manager and propose a contract $(\Phi, W)$

# Market Timing



- Shareholders hire a manager and propose a contract $(\Phi, W)$
- Manager announces a production plan $y$

# Market Timing



- Shareholders hire a manager and propose a contract $(\Phi, W)$
- Manager announces a production plan $y$
- $S$ and $M$ announce contingent demand functions

# Market Timing



- Shareholders hire a manager and propose a contract $(\Phi, W)$
- Manager announces a production plan $y$
- $S$ and $M$ announce contingent demand functions
- Exchange equilibrium prices emerge; contingent contracts are established

# Market Timing

- Shareholders hire a manager and propose a contract $(\Phi, W)$
- Manager announces a production plan $y$
- $S$ and $M$ announce contingent demand functions
- Exchange equilibrium prices emerge; contingent contracts are established
- $M$ implements a plan; contracts are settled (physical trades occur)

# Market Timing



- Shareholders hire a manager and propose a contract $(\Phi, W)$
- Manager announces a production plan $y$
- $S$ and $M$ announce contingent demand functions
- Exchange equilibrium prices emerge; contingent contracts are established
- $M$ implements a plan; contracts are settled (physical trades occur)

**Problem**

**How to lead the manager to propose shareholders' preferred plan?**

# The Manager



- Belief $Q^m$ with $M^m \stackrel{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma}x^\gamma$, $\gamma < 1$,

# The Manager



- Belief $Q^m$ with $M^m \stackrel{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma}x^\gamma$, $\gamma < 1$,
- Utility function $U_m(x) = E\left[M^m u(x)\right]$,

# The Manager

- Belief $Q^m$ with $M^m \stackrel{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma}x^\gamma$, $\gamma < 1$,
- Utility function $U_m(x) = E\left[M^m u(x)\right]$,
- Compensation scheme $\Phi : X_+ \to X_+$, $\Phi(y)(\omega) = \phi(y(\omega))$,

# The Manager



- Belief $Q^m$ with $M^m \overset{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma}x^\gamma$, $\gamma < 1$,
- Utility function $U_m(x) = E\left[M^m u(x)\right]$,
- Compensation scheme $\Phi : X_+ \to X_+$, $\Phi(y)(\omega) = \phi(y(\omega))$,
- For $y \in Y$, $\mathcal{C}(y) = \{c : c(\omega) = C(y(\omega)\}$,

# The Manager



- Belief $Q^m$ with $M^m \overset{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma}x^\gamma$, $\gamma < 1$,
- Utility function $U_m(x) = E\left[M^m u(x)\right]$,
- Compensation scheme $\Phi : X_+ \to X_+$, $\Phi(y)(\omega) = \phi(y(\omega))$,
- For $y \in Y$, $\mathcal{C}(y) = \{c : c(\omega) = C(y(\omega)\}$,
- Space of allowed transactions $W$, $\mathcal{C}^m(y) = (\phi(y) + W) \cap \mathcal{C}(y)$.

# The Manager

- Belief $Q^m$ with $M^m \stackrel{def}{=} \frac{dQ^m}{dP}$, CRRA function, $u(x) = \frac{1}{\gamma} x^\gamma$, $\gamma < 1$,
- Utility function $U_m(x) = E\left[M^m u(x)\right]$,
- Compensation scheme $\Phi : X_+ \to X_+$, $\Phi(y)(\omega) = \phi(y(\omega))$,
- For $y \in Y$, $\mathcal{C}(y) = \{c : c(\omega) = C(y(\omega)\}$,
- Space of allowed transactions $W$, $\mathcal{C}^m(y) = (\phi(y) + W) \cap \mathcal{C}(y)$.

**Assumption (F) :** $\forall y \in K$, $\forall w \in W$, $M_m \phi(y)^{\gamma-1} w \in L^1$ and $M_m \phi(y)^\gamma \in L^1$.

# The Shareholders



- $N$ shareholders, same $u$ as the manager,

# The Shareholders

- $N$ shareholders, same $u$ as the manager,
- Beliefs $Q^i$, density $\frac{dQ^i}{dP} = M^i$

# The Shareholders



- $N$ shareholders, same $u$ as the manager,
- Beliefs $Q^i$, density $\frac{dQ^i}{dP} = M^i$
- Utility functions, $U_i(x) = E\left[M^i u(x)\right]$ ,

# The Shareholders

- $N$ shareholders, same $u$ as the manager,
- Beliefs $Q^i$, density $\frac{dQ^i}{dP} = M^i$
- Utility functions, $U_i(x) = E\left[M^i u(x)\right]$,
- Shares $\nu_i > 0$ of the firm,

# The Shareholders

- $N$ shareholders, same $u$ as the manager,
- Beliefs $Q^i$, density $\frac{dQ^i}{dP} = M^i$
- Utility functions, $U_i(x) = E\left[M^i u(x)\right]$,
- Shares $\nu_i > 0$ of the firm,
- Indirect utilities

$$
\begin{aligned}
V^i(y,q) &= \max\left\{ U^i(c) : \begin{array}{c} q\cdot c \le \nu^i\, q\cdot(y-\Phi(y)) \\ \text{and } c\in\mathcal{C}(y)) \end{array} \right\} \\
V_m(y,q) &= \max\left\{ U_m(c) : \begin{array}{c} q\cdot c \le q\cdot\Phi(y) \\ \text{and } c\in\mathcal{C}^m(y)) \end{array} \right\}
\end{aligned}
$$

# The M-S Equilibrium Concept



M. Bianchi*, R.-A. Dana** & E. Jouini**



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq v^i \hat{q}\cdot(\hat{y} - \Phi(\hat{y})) \text{ and } c \in \mathcal{C}(y)} U^i(c)$

# The M-S Equilibrium Concept



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \nu^i \hat{q}\cdot(\hat{y}-\Phi(\hat{y})) \text{ and } c \in \mathcal{C}(y)} U^i(c)$
2. $\hat{c}_m = \widehat{C}_m(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \hat{q}\cdot\Phi(\hat{y}) \text{ and } c \in \mathcal{C}^m(y)} U_m(c)$

# The M-S Equilibrium Concept



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \nu^i \hat{q}\cdot(\hat{y}-\Phi(\hat{y})) \text{ and } c \in \mathcal{C}(y)} U^i(c)$
2. $\hat{c}_m = \widehat{C}_m(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \hat{q}\cdot\Phi(\hat{y}) \text{ and } c \in \mathcal{C}^m(y)} U_m(c)$
3. $\sum_i \hat{c}^i + \hat{c}_m = \hat{y}$ (market clearing)

# The M-S Equilibrium Concept



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \nu^i \hat{q}\cdot(\hat{y}-\Phi(\hat{y})) \text{ and } c\in\mathcal{C}(y)} U^i(c)$
2. $\hat{c}_m = \widehat{C}_m(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \hat{q}\cdot\Phi(\hat{y}) \text{ and } c\in\mathcal{C}^m(y)} U_m(c)$
3. $\sum_i \hat{c}^i + \hat{c}_m = \hat{y}$ (market clearing)
4. $V_m(\hat{y}, \hat{q}) = \max_{y\in Y} V_m(y, \hat{q})$ (best production plan)

# The M-S Equilibrium Concept



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \nu^i \hat{q}\cdot(\hat{y}-\Phi(\hat{y})) \text{ and } c\in\mathcal{C}(y)} U^i(c)$
2. $\hat{c}_m = \widehat{C}_m(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \hat{q}\cdot\Phi(\hat{y}) \text{ and } c\in\mathcal{C}^m(y)} U_m(c)$
3. $\sum_i \hat{c}^i + \hat{c}_m = \hat{y}$ (market clearing)
4. $V_m(\hat{y}, \hat{q}) = \max_{y\in Y} V_m(y, \hat{q})$ (best production plan)
5. $V^i(\hat{y}, \hat{q}) = \max_{y\in Y} V^i(y, \hat{q})$ (best production plan)

# The M-S Equilibrium Concept



## Definition

Compensation scheme $\Phi$, subspace $W$, production process $\hat{y}$, contingent contracts $\left((\widehat{C}^i)_i, \widehat{C}_m\right)$, and price process $\hat{q}$ s.t.

1. $\hat{c}^i = \widehat{C}^i(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \nu^i \hat{q}\cdot(\hat{y}-\Phi(\hat{y})) \text{ and } c\in\mathcal{C}(y)} U^i(c)$
2. $\hat{c}_m = \widehat{C}_m(\hat{y}) = \arg\max_{\hat{q}\cdot c \leq \hat{q}\cdot\Phi(\hat{y}) \text{ and } c\in\mathcal{C}^m(y)} U_m(c)$
3. $\sum_i \hat{c}^i + \hat{c}_m = \hat{y}$ (market clearing)
4. $V_m(\hat{y}, \hat{q}) = \max_{y\in Y} V_m(y, \hat{q})$ (best production plan)
5. $V^i(\hat{y}, \hat{q}) = \max_{y\in Y} V^i(y, \hat{q})$ (best production plan)
6. $U_m(\widehat{C}_m(\hat{y})) = \max_{y\in Y} U_m(\widehat{C}_m(y))$ (truthful implementation)

# Link with Production Equilibrium

## Theorem

*Let* $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *be a m-s equilibrium then*

1. *The manager does not trade, i.e.* $\hat{c}_m = \phi(\hat{y})$*,*

# Link with Production Equilibrium



## Theorem

*Let* $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *be a m-s equilibrium then*

1. *The manager does not trade, i.e.* $\hat{c}_m = \phi(\hat{y})$,
2. $U_m(\phi(\hat{y})) > U_m(\phi(y))$ *for all* $y \in Y$,

# Link with Production Equilibrium



M. Bianchi*, R.-A. Dana** & E. Jouini**



## Theorem

*Let $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ be a m-s equilibrium then*

1. *The manager does not trade, i.e. $\hat{c}_m = \phi(\hat{y})$,*
2. *$U_m(\phi(\hat{y})) > U_m(\phi(y))$ for all $y \in Y$,*
3. *$((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ is a production equilibrium associated to $Y^\phi = \{y - \phi(y) : y \in Y\}$ that fulfills $\hat{q} \in \mathcal{C}(\hat{y})$, we denote by $\tilde{N}$ the belief of the associated rep. agent,*

# Link with Production Equilibrium



**Theorem**

*Let* $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *be a m-s equilibrium then*

1. *The manager does not trade, i.e.* $\hat{c}_m = \phi(\hat{y})$,
2. $U_m(\phi(\hat{y})) > U_m(\phi(y))$ *for all* $y \in Y$,
3. $((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ *is a production equilibrium associated to* $Y^\phi = \{y - \phi(y) : y \in Y\}$ *that fulfills* $\hat{q} \in \mathcal{C}(\hat{y})$, *we denote by* $\tilde{N}$ *the belief of the associated rep. agent,*
4. $\phi'(\hat{y}) M^m u'(\phi(\hat{y})) \| (1 - \phi'(\hat{y})) \tilde{N} u'(\hat{y} - \phi(\hat{y}))$,

## Link with Production Equilibrium



### Theorem

*Let* $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *be a m-s equilibrium then*

1. *The manager does not trade, i.e.* $\hat{c}_m = \phi(\hat{y})$,
2. $U_m(\phi(\hat{y})) > U_m(\phi(y))$ *for all* $y \in Y$,
3. $((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ *is a production equilibrium associated to* $Y^\phi = \{y - \phi(y) : y \in Y\}$ *that fulfills* $\hat{q} \in \mathcal{C}(\hat{y})$, *we denote by* $\tilde{N}$ *the belief of the associated rep. agent,*
4. $\phi'(\hat{y}) M^m u'(\phi(\hat{y})) \| (1 - \phi'(\hat{y}))\tilde{N} u'(\hat{y} - \phi(\hat{y}))$,
5. $((\phi, W'), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *is a m-s equilibrium for any* $W' \subseteq W$ *(in particular,* $(\phi, \{0\})$ *m-s equilibrium).*

# Link with Production Equilibrium



## Theorem

*Let* $((\phi, W), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *be a m-s equilibrium then*

1. *The manager does not trade, i.e.* $\hat{c}_m = \phi(\hat{y})$,
2. $U_m(\phi(\hat{y})) > U_m(\phi(y))$ *for all* $y \in Y$,
3. $((\hat{c}^i)_i, \hat{q}, \hat{y} - \phi(\hat{y}))$ *is a production equilibrium associated to* $Y^\phi = \{y - \phi(y) : y \in Y\}$ *that fulfills* $\hat{q} \in \mathcal{C}(\hat{y})$, *we denote by* $\tilde{N}$ *the belief of the associated rep. agent,*
4. $\phi'(\hat{y}) M^m u'(\phi(\hat{y})) \| (1 - \phi'(\hat{y}))\tilde{N} u'(\hat{y} - \phi(\hat{y}))$,
5. $((\phi, W'), (\hat{c}^i)_i, (\hat{c}_m), \hat{q}, \hat{y})$ *is a m-s equilibrium for any* $W' \subseteq W$ *(in particular,* $(\phi, \{0\})$ *m-s equilibrium).*

## Theorem

*1. 2. and 3. characterize m-s equilibria.*

# Equilibrium Contract



## Corollary

*1. If $W = X$, $\phi$ is linear. 2. If $\phi$ is linear, $M^m = \tilde{N}$. 3. If $M^m \neq \tilde{N}$, $W \subsetneq X$.*

# Equilibrium Contract



**Corollary**

*1. If $W = X$, $\phi$ is linear. 2. If $\phi$ is linear, $M^m = \tilde{N}$. 3. If $M^m \neq \tilde{N}$, $W \subsetneq X$.*

Let $\tilde{y} = \hat{y} - \phi(\hat{y})$ (net production) and $h : \mathbb{R}_+ \to \mathbb{R}_+$ s.t.

$$h(\tilde{y}) = \frac{E\left[\tilde{N} \middle| \tilde{y}\right]}{E\left[M^m \middle| \tilde{y}\right]}.$$

4. becomes

$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}h(z-\phi(z)), \text{ for all } z.$$

# Equilibrium Contract



## Corollary

*1. If $W = X$, $\phi$ is linear. 2. If $\phi$ is linear, $M^m = \tilde{N}$. 3. If $M^m \neq \tilde{N}$, $W \subsetneq X$.*

Let $\tilde{y} = \hat{y} - \phi(\hat{y})$ (net production) and $h : \mathbb{R}_+ \to \mathbb{R}_+$ s.t.

$$h(\tilde{y}) = \frac{E\left[\tilde{N} \middle| \tilde{y}\right]}{E\left[M^m \middle| \tilde{y}\right]}.$$

4. becomes

$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}h(z-\phi(z)), \text{ for all } z.$$

## Problem

*Seems standard differential equation, but fixed point problem : $\phi$ depends on $h$, $h$ on $\tilde{y}$ and $\tilde{y}$ on $\phi$.*

# Common Beliefs



$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}, \text{ for all } z.$$

# Common Beliefs



$$\phi'(z)\phi(z)^{\gamma-1} = \mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}, \text{ for all } z.$$

## Corollary

*If $M^i = M^m$ for all $i$, at a m-s equilibrium*

1. *if $\gamma > 0$, $\phi$ is linear,*
2. *if $\gamma < 0$, $\phi(z)^\gamma = C + \mu\,(z - \phi(z))^\gamma$ for $C$ given and $\mu > 0$ (when $C = 0$, $\phi$ is linear).*

# Common Beliefs cont'd



$$\phi(z)^{\gamma} = C + \mu\left(z - \phi(z)\right)^{\gamma} \text{ for } C \text{ given and } \mu > 0$$

1. The manager has a reservation utility level $U^*$

# Common Beliefs cont'd



$$\phi(z)^\gamma = C + \mu\,(z - \phi(z))^\gamma \text{ for } C \text{ given and } \mu > 0$$

1. The manager has a reservation utility level $U^*$
2. The shareholders minimize $\hat{q} \cdot \phi(\hat{y})$

# Common Beliefs cont'd



$$\phi(z)^\gamma = C + \mu\,(z - \phi(z))^\gamma \text{ for } C \text{ given and } \mu > 0$$

1. The manager has a reservation utility level $U^*$
2. The shareholders minimize $\hat{q} \cdot \phi(\hat{y})$

**Corollary**

*Let $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ production equilibrium associated to $Y$. If $U(\tilde{y}) > U^*$, $\phi$ is linear ($C = 0$ and $\phi(z) = \frac{\mu^{1/\gamma}}{1+\mu^{1/\gamma}} z$) and $\mu$ given by*

$$\frac{1}{\gamma}\left(\frac{\mu^{1/\gamma}}{1+\mu^{1/\gamma}}\right)^\gamma E\left[\tilde{y}^\gamma\right] = U^* \text{ or } \mu = \left(\frac{\left(\frac{\gamma U^*}{E[\tilde{y}^\gamma]}\right)^{1/\gamma}}{1 - \left(\frac{\gamma U^*}{E[\tilde{y}^\gamma]}\right)^{1/\gamma}}\right)^\gamma$$

# Unique shareholder or identical shareholders



- The shareholder(s) believe(s) $\ln \tilde{y} \sim \mathcal{N}(a, \sigma^2)$

# Unique shareholder or identical shareholders



- The shareholder(s) believe(s) $\ln \tilde{y} \sim \mathcal{N}(a, \sigma^2)$
- The manager believes $\ln \tilde{y} \sim \mathcal{N}(b, \sigma^2)$

$$h(z - \phi(z)) = \frac{\exp - \frac{(\ln z - b)^2}{\sigma^2}}{\exp - \frac{(\ln z - a)^2}{\sigma^2}} = \beta z^{\alpha}$$

# Unique shareholder or identical shareholders

- The shareholder(s) believe(s) $\ln \tilde{y} \sim \mathcal{N}(a, \sigma^2)$
- The manager believes $\ln \tilde{y} \sim \mathcal{N}(b, \sigma^2)$

$$h(z-\phi(z)) = \frac{\exp -\frac{(\ln z - b)^2}{\sigma^2}}{\exp -\frac{(\ln z - a)^2}{\sigma^2}} = \beta z^{\alpha}$$

- $\beta = \exp(\frac{a^2-b^2}{\sigma^2})$ and $\alpha = 2\frac{b-a}{\sigma^2}$ is positive if themanager is more pessimistic than the shareholder(s)

$$\begin{aligned}
\phi'(z)\phi(z)^{\gamma-1} &= \beta\mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}z^{\alpha} \\
\phi'(z) &= \frac{\beta\mu(z-\phi(z))^{\gamma-1}z^{\alpha}}{\phi(z)^{\gamma-1}+\beta\mu(z-\phi(z))^{\gamma-1}z^{\alpha}}
\end{aligned}$$

# Unique shareholder or identical shareholders

- The shareholder(s) believe(s) $\ln \tilde{y} \sim \mathcal{N}(a, \sigma^2)$
- The manager believes $\ln \tilde{y} \sim \mathcal{N}(b, \sigma^2)$

$$h(z-\phi(z)) = \frac{\exp -\frac{(\ln z - b)^2}{\sigma^2}}{\exp -\frac{(\ln z - a)^2}{\sigma^2}} = \beta z^{\alpha}$$

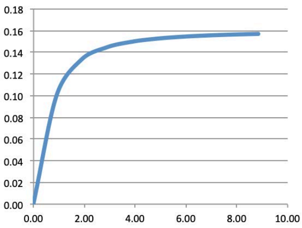
- $\beta = \exp(\frac{a^2-b^2}{\sigma^2})$ and $\alpha = 2\frac{b-a}{\sigma^2}$ is positive if themanager is more pessimistic than the shareholder(s)

$$\begin{aligned} \phi'(z)\phi(z)^{\gamma-1} &= \beta\mu(1-\phi'(z))(z-\phi(z))^{\gamma-1}z^{\alpha} \\ \phi'(z) &= \frac{\beta\mu(z-\phi(z))^{\gamma-1}z^{\alpha}}{\phi(z)^{\gamma-1}+\beta\mu(z-\phi(z))^{\gamma-1}z^{\alpha}} \end{aligned}$$

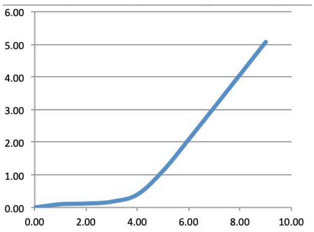
- We take $\gamma = -1$, $\phi' = \frac{\nu\phi^2 z^{\alpha}}{(z-\phi)^2+\nu\phi^2 z^{\alpha}}$

# Unique shareholder or identical shareholders cont'd

Not so pessimistic manager




Very pessimistic manager



# From Gross to Net Production

- Let $z_G$ (resp. $z_N$) be the gross (resp. net) production

$$z_N = z_G - \phi(z_G) \text{ and } z_G = (\mathrm{Id} - \phi)^{-1}(z_N)$$

# From Gross to Net Production



- Let $z_G$ (resp. $z_N$) be the gross (resp. net) production

$$z_N = z_G - \phi(z_G) \text{ and } z_G = (\mathrm{Id} - \phi)^{-1}(z_N)$$

- Compensation as a function of the net production is given by

$$\psi = \phi \circ (\mathrm{Id} - \phi)^{-1} \text{ and } \phi = \psi \circ (\mathrm{Id} + \psi)^{-1}.$$

# From Gross to Net Production

- Let $z_G$ (resp. $z_N$) be the gross (resp. net) production
$$z_N = z_G - \phi(z_G) \text{ and } z_G = (\mathrm{Id} - \phi)^{-1}(z_N)$$
- Compensation as a function of the net production is given by
$$\psi = \phi \circ (\mathrm{Id} - \phi)^{-1} \text{ and } \phi = \psi \circ (\mathrm{Id} + \psi)^{-1}.$$
- If $\mathcal{Y}$ net production set and $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ production equilibrium, the differential equation above becomes
$$\psi'(z)\psi(z)^{\gamma-1} = \mu z^{\gamma-1} h(z), \text{ for all } z.$$

# From Gross to Net Production



- Let $z_G$ (resp. $z_N$) be the gross (resp. net) production

$$z_N = z_G - \phi(z_G) \text{ and } z_G = (\mathrm{Id} - \phi)^{-1}(z_N)$$

- Compensation as a function of the net production is given by

$$\psi = \phi \circ (\mathrm{Id} - \phi)^{-1} \text{ and } \phi = \psi \circ (\mathrm{Id} + \psi)^{-1}.$$

- If $\mathcal{Y}$ net production set and $((\hat{c}^i)_i, \hat{q}, \tilde{y})$ production equilibrium, the differential equation above becomes

$$\psi'(z)\psi(z)^{\gamma-1} = \mu z^{\gamma-1} h(z), \text{ for all } z.$$

**Theorem**

*If $\hat{q} \in \mathcal{C}(\tilde{y})$, $M^m \in \mathcal{C}(\tilde{y})$ and $u \circ \psi$ strictly concave, then $((\phi, \{0\}), (\hat{c}^i)_i, \phi(\hat{y}), \hat{q}, \hat{y})$ with $\phi = \psi \circ (\mathrm{Id} + \psi)^{-1}$ and $\hat{y} = \tilde{y} + \psi_\mu(\tilde{y})$ is a m-s equilibrium for $Y = \mathcal{Y} + \psi(\mathcal{Y})$.*

# Heterogeneous Beliefs

## Definition

Agent $i$ is more optimistic than $j$ (with respect to $y$) if

$$\lim_{y \to \infty} \frac{M^i}{M^j} = \infty \text{ and } \lim_{y \to 0} \frac{M^i}{M^j} = 0.$$

# Heterogeneous Beliefs

## Definition

Agent $i$ is more optimistic than $j$ (with respect to $y$) if

$$\lim_{y \to \infty} \frac{M^i}{M^j} = \infty \text{ and } \lim_{y \to 0} \frac{M^i}{M^j} = 0.$$

**Assumption** There exists $i$ more optimistic than the manager and $j$ more pessimistic than the manager.

# Heterogeneous Beliefs

## Definition

Agent $i$ is more optimistic than $j$ (with respect to $y$) if

$$\lim_{y\to\infty}\frac{M^i}{M^j}=\infty \text{ and } \lim_{y\to 0}\frac{M^i}{M^j}=0.$$

**Assumption** There exists $i$ more optimistic than the manager and $j$ more pessimistic than the manager.

## Corollary

*The compensation should verify*

$$\frac{\phi(z)}{z}\longrightarrow_{0,\infty} 0 \textit{ and } \frac{\psi(z)}{z}\longrightarrow_{0,\infty} 0 \textit{ for } \gamma<0,$$
$$\frac{\phi(z)}{z}\longrightarrow_{0,\infty} 1 \textit{ and } \frac{\psi(z)}{z}\longrightarrow_{0,\infty} \infty \textit{ for } \gamma>0.$$

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\},$

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\},$
- $\widetilde{x} \sim \mathcal{N}(0,1),\ m(\theta) = a - b\left(\theta - \theta_0\right)^2,\ a,\ b,\ \theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.

# Example



- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)

# Example



- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\},$
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
  - The production equilibrium is fully calculable

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
  - The production equilibrium is fully calculable
  - If $W = X$, the m-s equilibrium exists only if the manager overestimate the risk with respect to the shareholders.

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{\exp\left(m(\theta)t + \theta\sqrt{T}\widetilde{x}\right) : \theta \geq 0\right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
  - The production equilibrium is fully calculable
  - If $W = X$, the m-s equilibrium exists only if the manager overestimate the risk with respect to the shareholders.
  - If $W = \{0\}$

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
  - The production equilibrium is fully calculable
  - If $W = X$, the m-s equilibrium exists only if the manager overestimate the risk with respect to the shareholders.
  - If $W = \{0\}$
    - 2-parameters family of solutions,

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\},$
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
    - The production equilibrium is fully calculable
    - If $W = X$, the m-s equilibrium exists only if the manager overestimate the risk with respect to the shareholders.
    - If $W = \{0\}$
        - 2-parameters family of solutions,
        - $u \circ \psi_{(\mu,C)}$ is concave,

# Example

- $\mathcal{Y}_{a,b,\theta_0} = \left\{ \exp\left( m(\theta)t + \theta\sqrt{T}\widetilde{x} \right) : \theta \geq 0 \right\}$,
- $\widetilde{x} \sim \mathcal{N}(0,1)$, $m(\theta) = a - b\left(\theta - \theta_0\right)^2$, $a$, $b$, $\theta_0$ given
- We take $\gamma = -1$ and we have 2 agents.
- Agent 1 (resp 2) believes $\widetilde{x} \sim \mathcal{N}(\delta, 1)$ (resp. $\widetilde{x} \sim \mathcal{N}(\delta, 1)$)
  - The production equilibrium is fully calculable
  - If $W = X$, the m-s equilibrium exists only if the manager overestimate the risk with respect to the shareholders.
  - If $W = \{0\}$
    - 2-parameters family of solutions,
    - $u \circ \psi_{(\mu,C)}$ is concave,
    - $\phi_{(\mu,C)} = \psi_{(\mu,C)} \circ \left(\mathrm{Id} + \psi_{(\mu,C)}\right)^{-1}$ is the equ. compensation for $Y = \mathcal{Y}_{a,b,\theta_0} + \psi_{(\mu,C)}(\mathcal{Y}_{a,b,\theta_0})$.

# Illustration

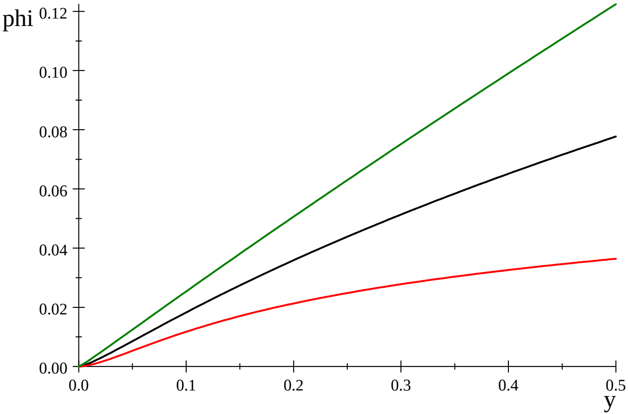



Figure: $\delta = 1/4$ (green), $\delta = 1/2$ (black) and $\delta = 3/4$ (red)

# Restricted Stock Compensation and Distortion from Linearity



- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m\left[\widetilde{x}\right] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders

# Restricted Stock Compensation and Distortion from Linearity



- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m[\widetilde{x}] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders
- An RSC manager should overestimate the variance of $\widetilde{x}$ in order to take the right decision

# Restricted Stock Compensation and Distortion from Linearity



- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m[\widetilde{x}] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders
- An RSC manager should overestimate the variance of $\widetilde{x}$ in order to take the right decision
- Optimal compensation is concave for large $y$

# Restricted Stock Compensation and Distortion from Linearity



- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m[\widetilde{x}] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders
- An RSC manager should overestimate the variance of $\widetilde{x}$ in order to take the right decision
- Optimal compensation is concave for large $y$
- Puzzling since it is often advocated that convexity is needed : stock options

# Restricted Stock Compensation and Distortion from Linearity

- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m\left[\widetilde{x}\right] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders
- An RSC manager should overestimate the variance of $\widetilde{x}$ in order to take the right decision
- Optimal compensation is concave for large $y$
- Puzzling since it is often advocated that convexity is needed : stock options
- Main argument: 1. firm is risk neutral 2. $m$ is risk averse 3. $m$ should be compensated for taking risk

# Restricted Stock Compensation and Distortion from Linearity



- An RSC manager that **implements** $y^*$ would perceive $\mathrm{VAR}^m[\widetilde{x}] > 1$ while $\mathrm{VAR}[\widetilde{x}] = 1$ for both shareholders
- An RSC manager should overestimate the variance of $\widetilde{x}$ in order to take the right decision
- Optimal compensation is concave for large $y$
- Puzzling since it is often advocated that convexity is needed : stock options
- Main argument: 1. firm is risk neutral 2. $m$ is risk averse 3. $m$ should be compensated for taking risk
- But shareholders are risk averse and risk aversion is reflected by equilibrium prices

# Conclusion





- Novel and different insights on the shape of the optimal contract for the manager

# Conclusion

- Novel and different insights on the shape of the optimal contract for the manager
- Restrictions should be imposed to the manager : a rationale for the prohibition of insider trading

# Conclusion



- Novel and different insights on the shape of the optimal contract for the manager
- Restrictions should be imposed to the manager : a rationale for the prohibition of insider trading
- We qualify the view that agency conflicts are minimized when the manager owns a substantial part of the firm's shares, which has motivated the rise in stock compensation : should vary with the level of production,

# Conclusion

- Novel and different insights on the shape of the optimal contract for the manager
- Restrictions should be imposed to the manager : a rationale for the prohibition of insider trading
- We qualify the view that agency conflicts are minimized when the manager owns a substantial part of the firm's shares, which has motivated the rise in stock compensation : should vary with the level of production,
- The compensation rate should induce the manager to overweight the occurrence of extreme realizations in contrast to the argument in favor of compensations -such as call options- which encourage risk taking