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Ramsey Taxation in the Global Economy*

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ABSTRACT

How should fiscal and trade policy be set cooperatively when government expenditures must be financed with distorting taxes? We study this question in canonical dynamic general equilibrium models of international trade using both the Ramsey and Mirrlees approaches. We show that free trade and unrestricted capital mobility are optimal. One way to implement efficient outcomes is to tax final consumption goods and labor income. We study alternative tax systems and show that taxing returns on assets held by households at a uniform rate and not taxing corporate income yields efficient outcomes. We argue that border adjustments that exempt exports from and includes imports in the tax base are desirable. Destination and residence based tax systems are desirable compared to origin and source based systems.
Keywords: Capital income tax; free trade; value-added taxes; border adjustment; origin- and destination-based taxation; production efficiency

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1. Introduction

The design of the international tax system, in the face of increased globalization, is a pressing concern for policy makers. The European Union, for example, has conducted extensive discussions on the feasibility and desirability of tax harmonization and has also discussed the possibility of forming a fiscal union. Awareness of the ways in which fiscal policy can be used to mimic tariffs, and in this sense can interfere with the free flow of goods and services, is increasing. Recent discussion on the tax reform package in the United States, for example, focused on the extent to which border adjustments were being proposed to mimic tariffs.[1] These concerns of policy makers imply that free trade agreements may well need to be supplemented with agreements on fiscal policy.

In this paper we ask how fiscal and trade policy should be set when countries can choose these policies cooperatively. Standard dynamic general equilibrium models of international trade with well functioning markets imply that free trade in goods and services and unrestricted capital mobility lead to efficient outcomes. With lump-sum taxes available to finance government expenditures, free trade is desirable. We analyze a model in which government expenditures must necessarily be financed with distorting taxes and ask how fiscal and trade policies should be jointly set.

We study this question as well as related ones in the workhorse model of dynamic international trade with multiple countries due to Backus, Kehoe and Kydland (1994), though, as we argue, the results hold in a variety of other environments. We take the Ramsey approach to optimal taxation, in that the tax system is exogenously given, and study cooperative Ramsey equilibria. We consider taxes widely used in practice in developed economies. We begin with a benchmark system that taxes consumption, labor income and international trade.

We show that, if governments can make lump sum transfers to each other, every point on the Pareto frontier has production efficiency. In this sense, we prove an analog of the second welfare theorem for multi-country environments with distorting taxes. We show that any point on the Pareto frontier can be implemented by setting all trade taxes to zero, choosing the optimal time path of consumption and labor income taxes and setting

[1] See Auerbach, Devereux, Keen, and Vella (2017) for a policy evaluation of the recent destination-based cash flow tax proposal.

government to government transfers appropriately. In this sense, our theorem implies that free trade is optimal.

We go on to prove the analog of the first welfare theorem. We show that there is a point on that Pareto frontier in which government to government transfers are zero. This results implies that if countries have chosen an allocation associated with this point, then even if they are prevented from making transfers to each other, no Pareto improvement is possible.

We show that adding other widely used taxes, such as taxes on corporate income and returns to household assets, as well as value-added taxes does not change the Pareto frontier. For a variety of reasons, including minimizing administration and compliance costs, countries may prefer tax systems other than our benchmark system. Motivated by these considerations, we begin by considering a system that only taxes labor income, corporate income and household asset income. We show that any point in the Pareto frontier can be implemented by setting the tax on corporate income to zero and choosing the other two taxes appropriately. In this setting, free trade continues to be optimal in the sense that if we allow for trade taxes, they would be optimally set to zero. We show that it is optimal to tax all types of household asset income at the same rate. We go on to show that tax systems which allow only labor and corporate income to be taxed cannot, in general, implement outcomes on the Pareto frontier.

These results are quite different from those in a closed economy. There, a system with only labor and corporate income taxes can always implement outcomes on the Pareto frontier. Since a uniform tax on household asset income is a residence based tax system while a tax on corporate income is source based, our analysis implies that residence based tax systems have advantages over source based systems.

Many countries use value-added taxes. The tax base of a value-added tax with border adjustment excludes revenues from exports and includes expenditures on imports. We show that such a tax, referred to as VAT with BA, is equivalent to a consumption tax. Thus, a system that has this tax together with a tax on labor income can implement any point on the Pareto frontier and trade taxes are not needed. Next, we consider a system in which the value-added tax has no border adjustment (VAT without BA). Here the tax base includes revenues from exports and excludes expenditures on imports. We show that a system that has only a VAT without BA and a labor income tax cannot achieve the Pareto frontier if, in the benchmark system, optimal consumption tax rates vary over time. Since, in general consumption taxes vary over time, a system with a VAT without BA and a labor income tax cannot implement outcomes on the Pareto frontier. In this sense, systems that allow for border adjustments are desirable.

These results shed light on apparent differences between the literature in public finance (see Auerbach, Devereux, Keen, and Vella (2017)) and that in international trade (see Grossman (1980), Feldstein and Krugman (1990), Costinot and Werning (2018)) on the desirability of border adjustments. The public finance literature has argued that border adjustments are desirable while the international trade literature has argued that they are irrelevant. The international trade literature effectively considers uniform tax systems in the sense that the VAT tax rate is the same for all goods. We can think of our dynamic economy as a static economy with an infinite number of goods. If, in the benchmark system, optimal consumption tax rates are constant over time, then the associated VAT tax rate is the same for all goods so that, regardless of border adjustments, systems with VAT and labor income taxes can implement outcomes on the Pareto frontier. If, in the benchmark system, optimal consumption taxes vary over time, then the associated VAT rate is different for different goods an the international trade results no longer apply. Our results help reconcile these differences and suggest that, in general, border adjustments are desirable. Barbiero, Farhi, Gopinath, and Itskhoki (2017) show that permanent changes in border adjustments are irrelevant if they are unanticipated, while they are not if anticipated. The difference between the two exercises is that the first change is uniform while the second is not.

The analysis of border adjustments helps compare destination-based with origin-based taxes on goods and services. A tax system is destination-based if tax rates at the destination of use are independent of the origin of production (a tax system is origin-based if the tax rates are independent of destination of use). VAT with BA are destination-based and VAT without BA are origin-based. Thus our results suggest that destination based systems have advantages over origin-based systems.

Finally, we argue that our results generalize to other international trade models, such as Obstfeld and Rogoff (1995), Stockman and Tesar (1995), Eaton and Kortum (2002), and to models of optimal non-linear taxation which build on Mirrlees (1971), Atkinson and Stiglitz (1976), among others.

For ease of exposition, we study a deterministic model. It is straightforward to extend the analysis to stochastic models in which productivity, government consumption and other shocks generate fluctuations in the aggregates. All our results continue to hold in the stochastic model. In such models, optimal consumption tax rates will typically vary with the underlying state, even in the stochastic steady state. These fluctuations may be large if the underlying shocks are large. This observation strengthens the case for the desirability of household asset taxes over taxation of corporate income and for the desirability of border adjustments in VAT systems.

A related paper to ours is Keen and Wildasin (2004). They argue that if governments cannot make transfers to each other,[2] then Pareto efficient allocations will typically not have production efficiency.[3] In our analogue of the first welfare theorem, we do not allow for transfers either. In this environment we show that there is a Ramsey equilibrium which is Pareto efficient and has production efficiency. Figure 1 illustrates the difference in the results. This figure shows the utility possibilities frontiers with two countries for environments with and without transfers. Note that the utility possibility frontier without transfers lies strictly below the one with transfers, except for point A. This observation is Keen and Wildasin result. Our analog of the first welfare theorem refers to point A on the figure.

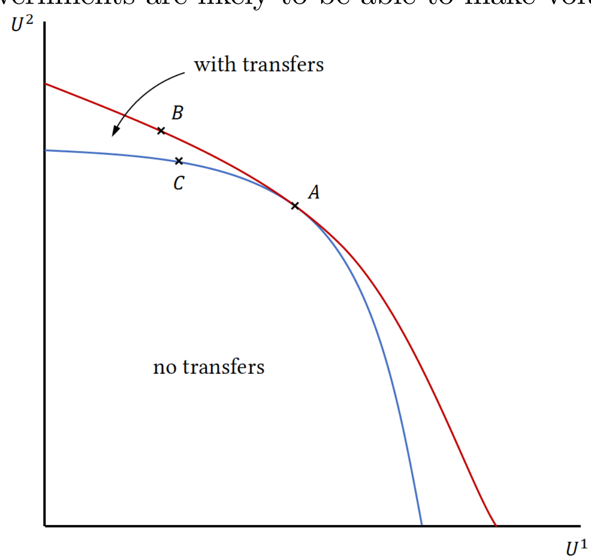
Note that Keen and Wildasin's result applies even to pure exchange economies without government expenditures or international trade. In such economies, the utility possibility frontier without transfers lies strictly below that with transfers, except for one point. In such economies, the standard first welfare theorem asserts that a planner cannot use transfers or price distortions to Pareto improve any competitive equilibrium. The logic behind Keen and Wildasin result is the same as the one behind the familiar result that if a country has

[2] They show that transfers can be replicated with an export subsidy and a tariff in a model with more goods than countries, which is generally the case in dynamic economies.

[3] According to Keen and Wildasin (2004) the three tenets of optimal international taxation do not apply, namely the optimality of free trade, the superiority of destination-based taxation of goods, and of residence-based taxation of capital income.

monopoly power and cannot extract transfers from the other country, it should levy a tariff. To see the similarity, consider a planner whose welfare function consists of maximizing only the welfare of, say, country 1 and cannot make transfers. This planner clearly solves exactly the same problem as the government in the optimal tariff environment.

In any event, we argue that it is reasonable to suppose that governments can make transfers. To understand our argument, consider point B on this figure which illustrates an outcome that is consistent with Keen and Wildasin's result. This outcome has production inefficiency. Starting at point B, the two countries have strong incentives to move to a Pareto dominating point like point C in the figure. Moving to point C clearly requires the countries to give up the taxes and subsidies that distort production, and in return the governments will make transfers to each other. We can think of compelling reasons for cooperating countries to prefer direct transfers to inefficiencies in production induced by taxes and subsidies. We think of the restrictions on tax policies in Ramsey problems as arising from private information on the characteristics of individuals. These considerations limit the kinds of tax instruments that governments can use and often rule out lump sum taxes. When it comes to governments, private information considerations are, in our judgement, likely to be much less important. Thus, governments are likely to be able to make voluntary transfers to one another.



The paper is organized as follows. In Section 2, we present the two-country economy model with consumption, labor income and trade taxes. We compute optimal Ramsey allocations and show that trade and capital mobility should not be restricted. In Section 3, we consider alternative tax systems that implement the same Ramsey optimal allocation. We first consider a common tax on all household asset income, together with a corporate income tax (Section 3.1). We also discuss alternative ways of taxing consumption through value-added taxes with and without border adjustment (Sections 3.2 and 3.3). In Section 4 we argue that the results extend to other models of international trade and non-linear taxation. Section 4 concludes.

2. A two-country economy

The model is that in Backus, Kehoe and Kydland (1994) with distorting taxes. There are two countries indexed by $i = 1, 2$. The preferences of a representative household in each country are over consumption c_{it}, labor n_{it}, and government consumption, g_{it},

$$(1) \quad U^i = \sum_{t=0}^{\infty} \beta^t \left[u^i \left(c_{it}, n_{it} \right) + h^i \left(g_{it} \right) \right].$$

We assume that u^i satisfies the usual properties and h^i is an increasing, concave, differentiable function. We assume that the total endowment of time is normalized to be one. Allowing for government consumption to be chosen endogenously ensures that an equilibrium always exists. For much of what follows we assume that government consumption, g_{it}, is exogenously given.

Each country, $i = 1, 2$, produces a country specific intermediate good, y_{it}, according to a production technology given by

$$(2) \quad y_{i1t} + y_{i2t} = y_{it} = F^i \left(k_{it}, n_{it} \right),$$

where y_{ijt} denotes the quantity of intermediate goods produced in country i and used in country $j = 1, 2$, k_{it} is the capital stock, n_{it} is labor input and F^i is constant returns to scale. Note that the first subscript denotes the location of production and the second subscript denotes the location of use. The intermediate goods produced by each country are used to produce a country specific final good that can be used for private consumption, c_{it}, public consumption, g_{it}, and investment, x_{it}, according to

(3) $\quad c_{it} + g_{it} + x_{it} \leq G^i\left(y_{1it}, y_{2it}\right),$

where G^i is constant returns to scale. Capital accumulates according to the law of motion

(4) $\quad x_{it} = k_{it+1} - (1-\delta)\, k_{it}.$

Note that in this economy only intermediate goods are traded across countries and final goods are not.

If lump sum taxes and transfers across countries are available, the allocations on the Pareto frontier satisfy the following efficiency: conditions,

(5) $$-\frac{u^i_{ct}}{u^i_{nt}} = \frac{1}{G^i_{i,t} F^i_{nt}},$$

(6) $$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1} F^i_{kt+1},$$

(7) $$\frac{G^1_{1,t}}{G^1_{2,t}} = \frac{G^2_{1,t}}{G^2_{2,t}},$$

(8) $$\frac{G^1_{j,t}}{G^1_{j,t+1}}\left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{j,t}}{G^2_{j,t+1}}\left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right],\ j = 1 \text{ or } 2$$

which, together with the resource constraints, characterize the Pareto frontier. Note that these conditions imply the intertemporal consumption efficiency condition

(9) $$\frac{u^1_{c,t}}{\beta u^1_{c,t+1}} = \frac{u^2_{c,t}}{\beta u^2_{c,t+1}} \frac{G^2_{j,t}/G^1_{j,t}}{G^2_{j,t+1}/G^1_{j,t+1}},\ j = 1, 2.$$

The conditions above mean that there are no intratemporal wedges (conditions (5)), no intertemporal wedges ((conditions (8)), and no production distortions (conditions (6) and (7)). We say that an allocation is *statically production efficient* if it satisfies (7), *dynamically production efficient* if it satisfies (8) and simply *production efficient* if it satisfies both. Static production efficiency requires that the marginal rates of technical substitution for the two intermediate goods are equated across countries. Dynamic production efficiency requires that capital (technically investment) be allocated so as to equate the effective rate of return on capital across the two countries.

We can use the intratemporal and intertemporal conditions, (5) and (6), to write the intertemporal condition for labor,

$$(10)\quad \frac{u^i_{nt}}{\beta u^i_{n,t+1}} = \frac{G^i_{i,t}F^i_{nt}}{G^i_{i,t+1}F^i_{n,t+1}}\left[1-\delta+G^i_{i,t+1}F^i_{kt+1}\right],\ i=1,2.$$

We explicitly characterize this intertemporal labor margin because we are interested in understanding when it is optimal not to distort this margin.

A. Equilibria with consumption, labor income and trade taxes

Consider now the economy with distorting taxes. Each government finances public consumption and initial debt with proportional taxes on consumption and labor income, τ^c_{it} and τ^n_{it}, trade taxes and a tax on initial wealth, l_{i0}. The trade taxes consist of an export tax, τ^e_{ijt}, levied on exports shipped from country i to country j, and a tariff, τ^y_{ijt}, levied on imports shipped from country i to country j.

Each country has two representative firms. The *intermediate good firm* in each country uses the technology in (2) to produce the intermediate good using capital and labor, purchases investment goods, and accumulates capital according to (4). Let V_{i0} be the value of the firm in period zero after the dividend paid in that period, d_{i0}. The intermediate good firms maximize the value of dividends

$$(11)\quad V_{i0}+d_{i0}=\sum_{t=0}^{\infty}Q_t\left[p_{iit}y_{iit}+\left(1-\tau^e_{ijt}\right)p_{ijt}y_{ijt}-w_{it}n_{it}-q_{it}x_{it}\right],\ i\neq j,$$

subject to (2) and (4). Here p_{ijt} is the price of the intermediate good produced in country i and sold in country j at t, w_{it} is the wage rate, and q_{it} is the price of the final good, all in units of a common world numeraire. The intertemporal price Q_t is the price of the numeraire at time t in units of the numeraire at zero ($Q_0=1$). Note that we assume that the intertemporal prices Q_t are the same in both countries. This assumption captures the idea that world capital markets are fully integrated.

If we define r^f_{t+1} to be the return on one period bonds in units of the numeraire between period t and $t+1$, then it must be the case that

$$\text{(12a)} \quad \frac{Q_t}{Q_{t+1}} = 1 + r^f_{t+1}, \text{ for } t \geq 0.$$

The assumption that Q_t is the same in both countries is equivalent to the assumption that interest rate parity holds.

The *final goods firm* of country i chooses the quantities of intermediate goods to maximize the value of dividends

$$\sum_{t=0}^{\infty} Q_t \left[q_{it} G^i \left(y_{iit}, y_{jit} \right) - p_{iit} y_{iit} - \left(1 + \tau^y_{jit} \right) p_{jit} y_{jit} \right], i \neq j.$$

Household The household's problem is to maximize utility (1) subject to the budget constraint

$$\text{(13)} \quad \sum_{t=0}^{\infty} Q_t \left[q_{it} \left(1 + \tau^c_{it} \right) c_{it} - \left(1 - \tau^n_{it} \right) w_{it} n_{it} \right] \leq \left(1 - l_{i0} \right) a_{i0},$$

with

$$a_{i0} = V_{i0} + d_{i0} + Q_{-1} b_{i0} + \left(1 + r^f_0 \right) f_{i0},$$

where a_{i0} denotes net holdings of assets by the household of country i, $Q_{-1} b_{i0}$ denotes holdings of domestic public debt in units of the numeraire, inclusive of interest, and $\left(1 + r^f_0 \right) f_{i0}$ denotes holdings of claims on households in the other country, in units of the numeraire, also inclusive of interest. Without loss of generality, households within a country hold claims to the firms in that country as well as the public debt of the government of that country. Note again that the assumption that Q_t is the same in both countries captures the idea that consumers can freely trade in international asset markets. We explore restrictions on such trade below.

Government We allow for initial lump sum transfers across governments denoted by T_{i0}. The budget constraint of the government of country i is given by

$$
\begin{aligned}
(14) \qquad & \sum_{t=0}^{\infty} Q_t \left[\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{1t} + \tau_{jit}^y p_{jit} y_{jit} + \tau_{ijt}^e p_{ijt} y_{ijt} - q_{it} g_{it}\right] \\
= \; & Q_{-1} b_{i0} - T_{i0} - l_{i0} a_{i0}, \;\; i \neq j.
\end{aligned}
$$

Since transfers made by one government are received by the other, we have

$$
(15) \quad T_{10} + T_{20} = 0.
$$

Combining the budget constraints of the government and the household (with equality) in each country, we obtain the following balance of payments condition

$$
(16) \quad \sum_{t=0}^{\infty} Q_t \left[p_{ijt} y_{ijt} - p_{jit} y_{jit}\right] = -\left(1 + r_0^f\right) f_{i,0} - T_{i0},
$$

for $i, j = 1, 2$ and $i \neq j$, and with $\left(1 + r_0^f\right) f_{1,0} + \left(1 + r_0^f\right) f_{2,0} = 0$.

A *competitive equilibrium* consists of an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$, prices and initial dividends, $\{q_{it}, p_{ijt}, w_{it}, Q_t, V_{i0}, d_{i0}\}$, and policies $\left\{\tau_{it}^c, \tau_{it}^n, , \tau_{ijt}^y, \tau_{ijt}^e, l_{i0}, T_{i0}, g_{it}\right\}$, given k_{i0}, $Q_{-1} b_{i0}$, $\left(1 + r_0^f\right) f_{i0}$ such that households maximize utility subject to their budget constraints, firms maximize value, the balance of payments conditions (16) hold, and markets clear in that (2), (3), and (4) hold, and (15) is satisfied.

Note that we have not explicitly specified the governments' budget constraints because they are implied by the other constraints.

We say that an allocation $\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ is *implementable* if it is part of a competitive equilibrium.

Next, we characterize the competitive equilibrium. To do so, note that the first-order conditions of the household's problem include

$$
(17) \quad -\frac{u_{c,t}^i}{u_{n,t}^i} = \frac{(1 + \tau_{it}^c)\, q_{it}}{(1 - \tau_{it}^n)\, w_{it}},
$$

(18) $u^i_{c,t} = \dfrac{Q_t q_{it}\left(1+\tau^c_{it}\right)}{Q_{t+1} q_{it+1}\left(1+\tau^c_{it+1}\right)} \beta u^i_{c,t+1},$

for all $t \geq 0$, where $u^i_{c,t}$ and $u^i_{n,t}$ denote the marginal utilities of consumption and labor in period t. Note that (18) can be used to recover the familiar interest rate parity condition

$$\frac{u^1_{c,t}\left(1+\tau^c_{1t+1}\right)}{\beta u^1_{c,t+1}\left(1+\tau^c_{1t}\right)} = \frac{u^2_{c,t}\left(1+\tau^c_{2t+1}\right)}{\beta u^2_{c,t+1}\left(1+\tau^c_{2t}\right)} \frac{e_{t+1}}{e_t},$$

where e_t denotes the price of the final goods in country 2 in units of goods in country 1, namely the real exchange rate.

The first-order conditions of the firms' problems are, for all $t \geq 0$, $p_{iit} F^i_{n,t} = w_{it}$,

(19) $\dfrac{Q_t}{Q_{t+1}} = \dfrac{p_{iit+1}}{q_{it}} F^i_{k,t+1} + \dfrac{q_{it+1}}{q_{it}}\left(1-\delta\right),$

where $F^i_{n,t}$ and $F^i_{k,t}$ denote the marginal products of capital and labor in period t,

(20) $p_{iit} = \left(1-\tau^e_{ijt}\right) p_{ijt}$, $i, j = 1, 2$, and $i \neq j$,

(21) $q_{it} G^i_{i,t} = p_{iit}$, $i = 1, 2$,

(22) $q_{it} G^i_{j,t} = \left(1+\tau^y_{jit}\right) p_{jit}$, $i, j = 1, 2$, and $i \neq j$.

Combining the household's and firm's equilibrium conditions, it can be shown that the value of the firm in (11) is

(23) $V_{i0} + d_{i0} = q_{i0}\left[1-\delta+G^i_{i,0} F^i_{k,0}\right] k_{i0}.$

We can obtain the familiar condition that the returns on capital adjusted for the real exchange rates are equated across countries. To obtain this condition, note that (19) and (21) can be combined to obtain

$$G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta = \frac{e_{t+1}}{e_t}\left(G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right).$$

The first-order conditions can be rearranged as

$$(24)\quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{(1+\tau^c_{it})}{(1-\tau^n_{it})\,G^i_{i,t}F^i_{n,t}}$$

$$(25)\quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta\right],$$

$$(26)\quad \frac{G^1_{2,t}}{G^1_{1t}} = \frac{(1+\tau^y_{21t})\,(1+\tau^y_{12t})}{(1-\tau^e_{21t})\,(1-\tau^e_{12t})}\frac{G^2_{2t}}{G^2_{1,t}}$$

$$(27)\quad \frac{(1+\tau^y_{12t})\,/\,(1-\tau^e_{12t})}{\left(1+\tau^y_{12t+1}\right)/\left(1-\tau^e_{12t+1}\right)}\frac{G^1_{1t}}{G^1_{1t+1}}\left[G^1_{1,t+1}F^1_{k,t+1}+1-\delta\right] = \frac{G^2_{1,t}}{G^2_{1,t+1}}\left[G^2_{2,t+1}F^2_{k,t+1}+1-\delta\right]$$

Comparing these conditions with the ones for the Pareto frontier with lump sum taxation, (5), (6), (8), and (7), we have that the consumption and labor taxes create an intratemporal wedge in (24), and that time varying consumption taxes create intertemporal wedges in (25). The consumption and labor income taxes do not affect the production efficiency conditions (26) and (27). Trade taxes distort static production efficiency, and, if they vary over time, distort dynamic production efficiency.

Using conditions (24) and (25), we can write

$$(28)\quad \frac{u^i_{n,t}}{\beta u^i_{n,t+1}} = \frac{(1-\tau^n_{it})}{\left(1-\tau^n_{it+1}\right)}\frac{G^i_{i,t}F^i_{n,t}}{G^i_{i,t+1}F^i_{n,t+1}}\left[G^i_{i,t+1}F^i_{k,t+1}+1-\delta)\right],$$

which makes clear how the taxes affect the labor intertemporal margin. We can also use (25) and (27) to write the intertemporal consumption condition

$$(29)\quad \frac{(1+\tau^y_{12t})\,/\,(1-\tau^e_{12t})}{\left(1+\tau^y_{12t+1}\right)/\left(1-\tau^e_{12t+1}\right)}\frac{\left(1+\tau^c_{1t+1}\right)/\left(1+\tau^c_{2t+1}\right)}{(1+\tau^c_{1t})\,/\,(1+\tau^c_{2t})}\frac{u^1_{c,t}}{\beta u^1_{c,t+1}} = \frac{u^2_{c,t}}{\beta u^2_{c,t+1}}\frac{G^2_{1,t}/G^1_{1t}}{G^2_{1,t+1}/G^1_{1t+1}}$$

that makes clear how time-varying ratios of consumption and trade taxes distort this intertemporal margin.

Implementability

In order to characterize the Ramsey equilibrium, we begin by characterizing the set of implementable allocations for a given path of government consumption, $\{g_{it}\}$. An allocation

$\{c_{it}, n_{it}, y_{ijt}, k_{it+1}, x_{it}\}$ and period zero policies and prices, $\{l_{i0}, \tau^c_{i0}, T_{i0}, q_{i0}\}$, given $\{k_{i0}, b_{i0}, f_{i0}\}$ is implementable as a competitive equilibrium if and only if they satisfy the resource constraints (2), (3), (4), and the implementability conditions

$$(30) \quad \sum_{t=0}^{\infty} \left[\beta^t u^i_{c,t} c_{it} + \beta^t u^i_{n,t} n_{it}\right] = \mathcal{W}_{i0},$$

where

$$(31) \quad \mathcal{W}_{i0} = (1 - l_{i0}) \frac{u^i_{c,0}}{(1+\tau^c_{i0})} \left[\left(1 - \delta + G^i_{i,0} F^i_{k,0}\right) k_{i0} + Q_{-1} \frac{b_{i0}}{q_{i,0}} + \left(1 + r^f_0\right) \frac{f_{i,0}}{q_{i,0}}\right].$$

The proof of the following proposition is standard and it is omitted.

Proposition 1 (Characterization of the implementable allocations): Any implementable allocation and period zero policies and prices satisfy the implementability constraints (30), and the resource constraints (2), (3), (4). Furthermore, if an allocation satisfies these conditions for some period zero policies and prices, then it is implementable by a tax system with consumption and labor income taxes.

B. Cooperative Ramsey equilibria

Here we ask how fiscal policy and trade policy should be conducted when governments can cooperate in setting these policies. We assume that governments can make lump sum transfers to each other but that taxes on households and firms must be linear. We show that free trade and unrestricted capital mobility are optimal even when governments must raise revenues using distorting taxes. Specifically, we show that any point on the Pareto frontier has free trade and unrestricted capital mobility, as long as governments can choose transfers appropriately. We go on to show that even if governments are prevented from making transfers a cooperative outcome with free trade and unrestricted capital mobility cannot be Pareto improved.

We assume that households in each country must be allowed to keep an exogenous value of initial wealth $\bar{\mathcal{W}}_i$, measured in units of utility. Specifically, we impose the following restriction on the policies:

$$(32)\quad \mathcal{W}_{i0} = \bar{\mathcal{W}}_i,$$

which we refer to as the *wealth restriction in utility terms*. With this restriction, policies, including initial policies, can be chosen arbitrarily, but the household must receive a value of initial wealth in utility terms of $\bar{\mathcal{W}}_i$. Chari et al. (2018) offer a rationalization and a defense of restrictions of this kind in a closed economy (see also Armenter (2008) for an analysis in a closed economy with such a restriction).[4]

Formally, a (*cooperative) Ramsey equilibrium* is a competitive equilibrium that is not Pareto dominated by any other competitive equilibrium. The *Ramsey allocation* is the associated implementable allocation.

Ramsey problem

Since a competitive equilibrium is summarized by the implementability constraint and the resource constraints, it immediately follows that the cooperative Ramsey equilibrium must solve the following programming problem. This problem is to choose allocations and period zero policies to maximize a weighted sum of utilities of the households of the two countries,

$$(33)\quad \omega^1 U^1 + \omega^2 U^2$$

with weights $\omega^i \in [0, 1]$, subject to implementability, (30) , the resource constraints,

$$(34)\quad c_{it} + g_{it} + k_{it+1} - (1 - \delta)\, k_{it} \leq G^i \left(y_{1it}, y_{2it}\right),$$

and (2), and the wealth restriction in utility terms, (32.

Note that the wealth restriction constraint can be dropped since initial taxes can be chosen to satisfy it. We assume throughout that for any given welfare weights the solution

[4] Clearly if the wealth restriction does not have to be satisfied, it immediately follows that it is possible to implement the lump-sum tax allocation as the Ramsey equilibrium.

of the Ramsey problem is unique.

Next we state and prove the analog of the second welfare theorem for our economy. To do so it is convenient to define

$$v^i\left(c_{it}, n_{it}; \varphi^i\right) = \omega^i u^i\left(c_{it}, n_{it}\right) + \varphi^i\left[u^i_{c,t} c_{it} + u^i_{n,t} n_{it}\right],$$

where φ^i is the multiplier of the implementability condition (30). The Ramsey problem, then, reduces to

$$\text{Max} \sum_{i=1}^{2} \omega^i \left[\sum_{t=0}^{\infty} \beta^t \left[v^i\left(c_{it}, n_{it}; \varphi^i\right) + h^i\left(g_{it}\right)\right] - \varphi^i \bar{\mathcal{W}}_i\right]$$

subject to the resource constraints (2), (34).

Proposition 2 (Optimality of free trade): For any welfare weights, there exist transfers across governments such that production efficiency is optimal. .

Proof: The solution of the Ramsey problem satisfies

(35) $$-\frac{v^i_{c,t}}{v^i_{n,t}} = \frac{1}{G^i_{i,t} F^i_{n,t}},\ i = 1, 2$$

(36) $$\frac{v^i_{c,t}}{\beta v^i_{c,t+1}} = 1 - \delta + G^i_{i,t+1} F^i_{kt+1},\ i = 1, 2$$

(37) $$\frac{G^1_{j,t}}{G^1_{j,t+1}}\left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{j,t}}{G^2_{j,t+1}}\left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right],\ j = 1 \text{ or } 2$$

(38) $$\frac{G^1_{1,t}}{G^2_{1,t}} = \frac{G^1_{2,t}}{G^2_{2,t}}.$$

These equations imply an analog of the intertemporal consumption efficiency condition

(39) $$\frac{v^1_{c,t}}{\beta v^1_{c,t+1}} = \frac{v^2_{c,t}}{\beta v^2_{c,t+1}} \frac{G^2_{j,t}/G^1_{j,t}}{G^2_{j,t+1}/G^1_{j,t+1}},\ j = 1, 2.$$

In the appendix, we report the first-order conditions for the optimal levels of government consumption.

Conditions (37) and (38) are the production efficiency conditions (8) and (7). It follows that the solution of the Ramsey problem must have production efficiency Q.E.D.

One way of implementing the efficient allocation is to set all tariffs and export taxes to zero, and use only consumption and labor income taxes. In this sense Proposition 2 implies that free trade is optimal in a cooperative Ramsey equilibrium.

This observation implies the corollary that even if trade taxes are unavailable and only consumption and labor income taxes are available the Ramsey outcome is unaffected.

Corollary (irrelevance of trade taxes): The Ramsey outcome in an economy in which consumption, labor income and trade taxes are available coincides with that in which only consumption and labor income taxes are available.

The logic behind Proposition 2 can be extended to show that restrictions on capital mobility are not efficient. To see this result, consider, for example, constraints on the amount of foreign assets that residents of country i can hold. These constraints can be represented as adding constraints on the household problem of the form

$$f_{it} \leq \bar{f}_{it}.$$

Using the evolution of the household wealth, they can alternatively be represented as

$$(40) \qquad \sum_{s=0}^{\infty} Q_{t+s} \left[q_{it+s} \left(1 + \tau_{it+s}^{c} \right) c_{it+s} - \left(1 - \tau_{it+s}^{n} \right) w_{it+s} n_{it+s} \right] \leq V_{it} + d_{it} + Q_{t-1} b_{it} + \left(1 + r_0^f \right) \bar{f}_{it}, \ t \geq 1$$

These are additional constraints to the Ramsey problem that can be written in terms of the allocations. Thus, in the solution to the cooperative Ramsey problem it is optimal to ensure that these additional constraints are never binding. The same logic applies to any other restrictions on capital mobility, including taxes on capital flows. In this sense, the logic behind Proposition 2 implies that unrestricted capital mobility is optimal in a cooperative Ramsey equilibrium.

In order to further characterize the optimal wedges, it is useful to write

$$v_{c,t}^{i} = u_{c,t}^{i} \left[\omega^{i} + \varphi^{i} \left[1 - \sigma_t^{i} - \sigma_t^{cni} \right] \right]$$

$$v_{n,t}^{i}=u_{n,t}^{i}\left[\omega^{i}+\varphi^{i}\left[1+\sigma_{t}^{ni}-\sigma_{t}^{nci}\right]\right],$$

where

$$\sigma_{t}^{i}=-\frac{u_{cc,t}^{i}c_{it}}{u_{c,t}^{i}},\ \sigma_{t}^{ni}=\frac{u_{nn,t}^{i}n_{it}}{u_{n,t}^{i}},\ \sigma_{t}^{nci}=-\frac{u_{nc,t}^{i}c_{it}}{u_{n,t}^{i}},\ \sigma_{t}^{cni}=-\frac{u_{cn,t}^{i}n_{it}}{u_{c,t}^{i}}$$

are own and cross elasticities that are only functions of consumption and labor at time t.

Note also that if consumption and labor are constant over time, then the relevant elasticities are also constant, so $v_{c,t}^{i}$ and $v_{n,t}^{i}$ are proportional to $u_{c,t}^{i}$ and $u_{n,t}^{i}$, respectively. We say that a competitive equilibrium has *no intertemporal distortions* from period s onwards if the allocations satisfy (6) and (10) for all $t\geq s$. Since the relevant elasticities are constant in the steady state we have the following proposition.

Proposition 3 (No intertemporal distortions in the steady state): If the Ramsey equilibrium converges to a steady state, it is optimal to have no intertemporal distortions asymptotically.

For standard macro preferences,

$$(41)\quad U^{i}=\sum_{t=0}^{\infty}\beta^{t}\left[\frac{c_{t}^{1-\sigma^{i}}-1}{1-\sigma^{i}}-\eta_{i}\frac{n_{t}^{1+\sigma^{ni}}}{1+\sigma^{ni}}\right],$$

following the logic in Chari et. al. (2018), the Ramsey solution has no intertemporal distortions for all $t\geq 0$ and can be implemented with consumption or labor taxes that are constant over time, but possibly different across countries.

We turn next to proving the analog of the first welfare theorem. In order to do so, we assume that the initial debts of the governments are zero, $b_{i0}=0$ and initial net foreign asset positions are also zero, $f_{i0}=0$. We will show that there is a pair of welfare weights, ω^{1} and ω^{2}, such that the government-to-government transfers are zero. Without loss of generality, let $\omega^{1}=\omega\in[0,1]$, and $\omega^{2}=1-\omega$. Let $T^{i}(\omega)$.denote the transfers to country i under the Ramsey allocation associated with welfare weight ω.

Proposition 4 (Optimality of production efficiency with zero transfers): Assume $b_{i0}=f_{i0}=0$, for $i=1,2$. There exists a weight $\omega\in[0,1]$ such that transfers are zero, $T^{i}(\omega)=0$, $i=1,2$.

Proof: Since, by assumption, the solution to the Ramsey problem is unique, it follows that $T^i(\omega)$, $i = 1, 2$ are continuous functions. In the appendix, we show that $T^1(0) \leq 0$ and $T^1(1) \geq 0$. The result follows from the intermediate value theorem Q.E.D.

Remark: The same theorem holds with more than two countries. In this case we can apply the argument in Negishi (1972) to prove the result.

This proposition implies that a cooperative Ramsey allocation in an environment where governments cannot make transfers to each other cannot be Pareto improved.

C. Allowing for distributional considerations

In the model above we abstract from the distributional effects of policies within each country. In this section we briefly address those considerations, allowing for the possibility that different agents may be affected differently by trade policies.[5] For simplicity, we consider only two worker types with equal mass. The production function in country i is described by

$$y_{i1t} + y_{i2t} = y_{it} = F^i\left(k_{it}, n^a_{it}, n^b_{it}\right),$$

where n^a_{it} and n^b_{it} are the labor hours of agents a and b in country i. Notice that with this production function the relative wages of the two agents are endogenous and are a function of trade policies. A special case in which the relative wage is exogenous is when the two agent types only differ in their efficiency units but are perfect substitutes in production, as in $F^i\left(k_{it}, n^a_{it} + \eta_i n^b_{it}\right)$. The preferences of type a agents are

$$U^{ia} = \sum_{t=0}^{\infty} \beta^t \left[u^{ia}\left(c^a_{it}, n^a_{it}\right) + h^i\left(g_{it}\right)\right],$$

and similarly for type b agents

An allocation in this economy consists of consumption and labor allocations for each household a and b,$\{c^a_{it}, n^a_{it}\}$ and $\left\{c^b, n^b_{it}\right\}$ and aggregate allocations for each country $\{y_{ijt}, k_{it+1}, x_{it}\}$. The market clearing condition for the final good is

$$c^a_{it} + c^b_{it} + g_{it} + x_{it} \leq G^i\left(y_{1it}, y_{2it}\right),$$

[5] Details are available upon request.

and the capital accumulation equation is (4).

We start by allowing for consumption and labor income taxes that are agent specific. The intermediate good firm now must pay wages to a and b types according to

$$p_{iit}F^{i}_{n^a,t} = w^{a}_{it} \text{ and } p_{iit}F^{i}_{n^b,t} = w^{b}_{it}.$$

The other conditions of the competitive equilibrium are straightforward generalizations of the homogeneous agent case.

In this case it is straightforward to show that the implementability conditions are the analogous ones to those in the representative agent model (see Chari et al. (2016)). Here, whether or not we impose the wealth constraint, the Ramsey allocation need not coincide with the lump-sum allocation. The reason is that a planner who wishes to redistribute to one of the two types may choose to use tax distortions to accomplish such redistribution (see Werning (2007)). Nevertheless, the results in Proposition 2 still hold, so that free trade and unrestricted capital mobility are optimal.

If, instead, the tax rates on the two agents are restricted to be the same, then there are additional implementability conditions. In particular the following conditions have to be imposed:

$$-\frac{u^{i}_{c^a,t}F^{i}_{n^a,t}}{u^{i}_{n^a,t}} = -\frac{u^{i}_{c^b,t}F^{i}_{n^b,t}}{u^{i}_{n^b,t}}$$

and

$$\frac{u^{i}_{c^a,t}}{\beta u^{i}_{c^a,t+1}} = \frac{u^{i}_{cb,t}}{\beta u^{i}_{c^b,t+1}}.$$

With these extra restrictions it is no longer the case that the result in Proposition 2 is generally true. If the preferences of the two agent types are the same and their labor inputs are perfect substitutes in the sense that the production function is given by $F^{i}\left(k_{it}, n^{a}_{it} + \eta_i n^{b}_{it}\right)$, then the results in Proposition 2 go through, even if the tax rates are required to be the same.

3. Alternative implementations

Thus far we have considered tax systems which include taxes on consumption, labor income, and trade. Here, we discuss a variety of other tax systems, including taxes on the income from different assets and value-added taxes. Our analysis is motivated by the observation that these alternative tax systems are widely used in practice. We show that no tax system can yield higher welfare than the tax system with only consumption and labor income taxes. We show that a variety of tax systems can implement the Ramsey allocation associated with those taxes. Furthermore, some tax systems do yield lower welfare.

A. Taxes on corporate income and asset returns

Here, we consider a tax system which consists of taxes on labor income, corporate income, and on the returns of households holdings of assets. We assume a residence-based system in which the tax rates on returns from different assets are the same. We show that the Ramsey outcome can be implemented with zero taxation of corporate income and suitably chosen taxes on household asset income.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium.

Firms The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$, where Q_t is the pre-tax discount factor, given by $(12a)$. Dividends, d_{it}, in units of the numeraire, are given by

$$(42)\quad d_{it} = p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^{k}\left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right],$$

where τ_{it}^{k} is the tax rate on corporate income net of depreciation.

The first-order conditions of the firm's problem are now $p_{iit}F_{n,t}^{i} = w_{it}$ together with

$$(43)\quad \frac{Q_t q_{it}}{Q_{t+1}q_{it+1}} = 1 + \left(1 - \tau_{it+1}^{k}\right)\left(\frac{p_{it+1}}{q_{it+1}}F_{k,t+1}^{i} - \delta\right).$$

Substituting for d_{it} from (42) and, using the firm's first-order conditions, it is easy to show that the present value of the dividends at time zero in units of the numeraire is given by

$$V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it} = \left[1 + \left(1 - \tau_{i0}^k\right)\left(\frac{p_{i0}}{q_{i0}} F_{ik,0} - \delta\right)\right] p_{i0} k_{i0}. \tag{44}$$

The problem of the final good firm is as before.

Households Here we explicitly allow for sequential trading households. In each period, households choose consumption, labor supply, and holdings of domestic and foreign bonds and equity in domestic firms. For simplicity, we assume that households cannot hold foreign equity. It is possible to show that the Ramsey allocations are the same if households can hold foreign equity.

The tax base for each unit of bond income, expressed in real terms, is given by $\left(r_t^f - (q_{it} - q_{it-1})/q_{it-1}\right)$. This way of defining the base ensures that taxes are levied only on real income. The tax base for equity income taxation, is given by $(d_{it} + V_{it} - V_{it-1} - (q_{it} - q_{it-1}) V_{it-1}/q_{it-1})$ per share. Note that this tax base includes dividends received in the current period and accrued capital gains generated by changes in the price of equity as well as an adjustment to ensure that the base is expressed in real terms.

The flow of funds constraint in period $t \geq 1$, for the household in country i in units of the numeraire is then given by

$$\begin{aligned} & q_{it} c_{it} + b_{it+1} + f_{it+1} + V_{it} s_{it+1} \\ = \; & \left(1 - \tau_{it}^n\right) w_{it} n_{it} + \left[1 + r_t^f - \tau_{it}\left(r_t^f - \frac{q_{it} - q_{it-1}}{q_{it-1}}\right)\right](f_{it} + b_{it}) \\ & + (V_{it} + d_{it}) s_{it} - \tau_{it}\left(d_{it} + V_{it} - V_{it-1} - \frac{(q_{it} - q_{it-1}) V_{it-1}}{q_{it-1}}\right) s_{it}. \end{aligned} \tag{45a}$$

The period zero constraint needs to be adjusted by the wealth tax and is given in the appendix. Note that world capital markets are integrated in the sense that pre tax returns on bonds are the same in the two countries.

The household's problem is to maximize utility (1), subject to $(45a)$, the relevant budget constraint at period zero and no-Ponzi-scheme conditions, $\lim_{T\to\infty} Q_{iT+1} b_{iT+1} \geq 0$, and $\lim_{T\to\infty} Q_{iT+1} f_{iT+1} \geq 0$, where Q_{it}/Q_{it+1} is the return on bonds net of taxes,

$$\text{(46a)}\quad \frac{Q_{it}}{Q_{it+1}} = (1-\tau_{it+1})\left(1+r^f_{t+1}\right) + \tau_{it+1}\frac{q_{it+1}}{q_{it}} \text{ with } Q_{i0} = 1.$$

The first-order conditions of the household's problem in each country are, for $t \geq 0$, (17) with $\tau^c_{it} = 0$, and

$$\text{(47)}\quad u^i_{c,t} = \frac{Q_{it} q_{it}}{Q_{it+1} q_{it+1}} \beta u^i_{c,t+1},$$

together with the arbitrage conditions that the after tax return on bonds and equity must be equated

$$\text{(48a)}\quad \frac{Q_{it}}{Q_{it+1}} = \frac{(V_{it+1} + d_{it+1}) - \tau_{it+1}\left(V_{it+1} - V_{it} + d_{it+1} - \frac{q_{it+1}-q_{it}}{q_{it}} V_{it}\right)}{V_{it}}.$$

Using the no-Ponzi- scheme condition, the budget constraints of the household, (45a) and the period zero budget constraint, can be consolidated into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it}\left[q_{it}c_{it} - (1-\tau^n_{it})\, w_{it} n_{it}\right] = (1-l_{i0})\, a_{i0},$$

where, using (44) as well as $s_0 = 1$, the initial asset holdings can be written as

$$\begin{aligned} a_{i0} \;&=\; Q_{i-1} b_{i0} + (1-\tau_{i0})\, q_{i0}\left[k_0 + \left(1-\tau^k_{i0}\right)\left(G^i_{i,0} F_{ik,0} - \delta\right) k_{i0}\right] + \tau_{i0}\frac{q_{i0} V_{i-1}}{q_{i-1}} \\ &\quad + \left[1 + r^f_0 - \tau_{i0}\left(1 + r^f_0 - \frac{q_{i0}}{q_{i-1}}\right)\right] f_{i0} \end{aligned}$$

It is straightforward to show that the consolidated budget constraint reduces to the same implementability constraint, (30) with

$$\begin{aligned} \mathcal{W}_{i0} \;&=\; (1-l_{i0})\, u^i_{c,0}\,(1-\tau_{i0})\left[1 + \left(1-\tau^k_{i0}\right)\left(G^i_{i,0} F_{ik,0} - \delta\right)\right] k_0 + \\ &\quad (1-l_{i0})\, u^i_{c,0}\left[\frac{\tau_{i0} V_{i-1}}{q_{i-1}} + Q_{-1}\frac{b_{i0}}{q_{i,0}} + \left[1 + r^f_0 - \tau_{i0}\left(1 + r^f_0 - \frac{q_{i0}}{q_{i-1}}\right)\right]\frac{f_{i0}}{q_{i,0}}\right]. \end{aligned}$$

The Ramsey problem can then be thought of as maximizing (33) subject to the implementability constraint (30) with the wealth restriction (32) and the resource constraints. In the Appendix, we prove that policies can be suitably chosen so that the solution of this problem can be implemented as a competitive equilibrium. These policies typically require asset taxes. These asset taxes stand in for time-varying consumption taxes.

Using Corollary 1, it then follows that the Ramsey allocation in the economy with consumption and labor income taxes coincides with the one in an economy with the taxes considered here.

Next we show that it is optimal to set corporate income taxes to zero and that, in general, asset taxes are needed to implement the Ramsey outcome. Using the first-order conditions of the firms, it is straightforward to show that any competitive equilibrium satisfies static production efficiency. Next we turn to conditions under which dynamic production efficiency holds. Using (43) for both countries, as well the final good firms' conditions we obtain a version of the interest rate parity condition,

$$\begin{aligned} (49) \qquad & \frac{G^1_{j,t}}{G^1_{j,t+1}}\left[1+\left(1-\tau^k_{1t+1}\right)\left(G^1_{1,t+1}F^1_{k,t+1}-\delta\right)\right] \\ = \; & \frac{G^2_{j,t}}{G^2_{j,t+1}}\left[1+\left(1-\tau^k_{2t+1}\right)\left(G^2_{2,t+1}F^2_{k,t+1}-\delta\right)\right], \text{ for } j=1,2. \end{aligned}$$

Clearly, setting both corporate income taxes to zero ensures dynamic production efficiency.

In order to show that asset taxes are needed, we use the households and firms first-order conditions to obtain

$$(50) \quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{\left(1-\tau^n_{it}\right)G^i_{i,t}F^i_{n,t}}.$$

and

$$(51) \quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1+\left(1-\tau_{it+1}\right)\left(1-\tau^k_{it+1}\right)\left(G^i_{i,t+1}F^i_{k,t+1}-\delta\right).$$

In general, the solution to the Ramsey problem requires time varying intertemporal distortions. Thus, implementing the Ramsey outcome with the system considered here, requires asset taxes given that the corporate income tax is set to zero. If we set the asset taxes to zero, it is in general not possible to choose the corporate income tax rates in both countries to satisfy both (49) and (51) at the Ramsey allocation.

We summarize these results in the following proposition:

Proposition 5 (Common tax on domestic equity and foreign returns) : The Ramsey outcome can be implemented with labor income taxes and asset taxes only and setting the corporate income taxes to zero. In general, the Ramsey outcome cannot be implemented with labor income taxes and corporate income taxes only.

These results are quite different from those in a closed economy. In a closed economy, household asset taxes and corporate income taxes distort capital accumulation in the same way. Thus it is possible to support the Ramsey allocations with labor income taxes and corporate income taxes, or, equivalently, with labor income and household asset taxes. In the open economy, a system with corporate income taxes distorts dynamic production efficiency, by distorting the allocation of capital across countries in addition to distorting capital accumulation. In this sense, a system with corporate income taxes is dominated by a system with household asset taxes.

Note that we have assumed that the tax rates on domestic and foreign asset income are the same. If these tax rates are allowed to be different, then it is straightforward to prove that in the Ramsey equilibrium it is optimal to set these tax rates to be the same.

Thus far, we have considered a decentralization in which investment decisions are made by firms. Much of the macroeconomics literature considers decentralizations in which investment decisions are made by households and firms simply rent capital and labor from households. It is possible to show that with this decentralization the same Ramsey outcomes can be supported by a tax system under which households assets are taxed at a rate that may vary across countries but is uniform across assets types.

Our analysis allows for a comparison of residence-based and source-based tax systems. In our model, a residence-based system is one in which all household asset income is taxed at a rate that is independent of where the income is generated, but can depend on where the household resides. A source-based system is one in which income is taxed where it is generated, namely at a point of production. A corporate income tax is an example of a source-based system. Since we have argued that household asset taxes have advantages over corporate income taxes, we have shown that residence-based tax systems have advantages over source-based systems.

B. Border-adjusted value-added taxes and labor income taxes

Consider next an economy in which consumption taxes are replaced by value-added taxes levied on firms with border adjustment. Border adjustment means that firms in a country do not pay value-added taxes on exports and cannot deduct imports. Taxes on assets are set to zero, but labor income taxes are not. The value-added taxes are denoted by τ^v_{it}. We refer to the system with value-added taxes with border adjustment as a *VA with BA system.*

The intermediate good firm now maximizes

$$(52)\quad \sum_{t=0}^{\infty} Q_t \left[(p_{i1t}y_{i1t} + p_{i2t}y_{i2t}) - w_{it}n_{it} - q_{it}x_{it}\right] - \sum_{t=0}^{\infty} Q_t \tau^v_{it} \left[p_{iit}y_{iit} - q_{it}x_{it}\right]$$

subject to (2) and (4), where p_{ijt} is the price of the intermediate good produced in country i and sold in country j. Note that the final good firm pays taxes on the good sold domestically, but not when it is exported. In this sense, there is a border adjustment.

The final goods firm now maximizes

$$(53)\quad \sum_{t=0}^{\infty} Q_t \left[q_{it}G^i\left(y_{1it}, y_{2it}\right) - p_{1it}y_{1it} - p_{2it}y_{2it}\right] - \sum_{t=0}^{\infty} Q_t \tau^v_{it} \left[q_{it}G^i\left(y_{1it}, y_{2it}\right) - p_{iit}y_{iit}\right].$$

This firm is able to deduct the input produced domestically, but not the one imported. Also in this sense, there is a border adjustment. The household problem is the same as above, except that the consumption taxes are set to zero.

The first-order conditions of the household's problem now include

$$(54)\quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{q_{it}}{\left(1-\tau^n_{it}\right) w_{it}},\ t \geq 0$$

and

$$(55)\quad u^i_{c,t} = \frac{Q_t q_{it}}{Q_{t+1} q_{it+1}} \beta u^i_{c,t+1},\ t \geq 0.$$

The first-order conditions of the firms' problems for an interior solution are

(56) $p_{iit}\left(1-\tau_{it}^{v}\right)F_{n,t}^{i}=w_{it}$

(57) $Q_{t}q_{it}\left(1-\tau_{it}^{v}\right)=Q_{t+1}p_{iit+1}\left(1-\tau_{it+1}^{v}\right)F_{k,t+1}^{i}+Q_{t+1}q_{it+1}\left(1-\tau_{it+1}^{v}\right)(1-\delta))$

(58) $p_{iit}\left(1-\tau_{it}^{v}\right)=p_{ijt}$, for $j\neq i$

(59) $q_{it}G_{i,t}^{i}=p_{iit}$

(60) $q_{it}\left(1-\tau_{it}^{v}\right)G_{j,t}^{i}=p_{jit}$, for $j\neq i$.

We can manipulate the households and firms conditions to obtain

(61) $-\dfrac{u_{c,t}^{i}}{u_{n,t}^{i}}=\dfrac{1}{\left(1-\tau_{it}^{n}\right)\left(1-\tau_{it}^{v}\right)G_{i,t}^{i}F_{n,t}^{i}},\ t\geq 0,$

(62) $u_{c,t}^{i}\left(1-\tau_{it}^{v}\right)=\left(1-\tau_{it+1}^{v}\right)\beta u_{c,t+1}^{i}\left[G_{i,t+1}^{i}F_{k,t+1}^{i}+1-\delta\right)\right],\ t\geq 0$

together with (7) and (8). Comparing these four equilibrium conditions with the corresponding ones in the economy with consumption, labor income and trade taxes, (24) - (27), we see that the equilibrium conditions in this economy with VAT with BA coincide with the conditions in that economy if trade taxes are set to zero and

(63) $1+\tau_{it}^{c}=\dfrac{1}{1-\tau_{it}^{v}}.$

We have proved the following proposition:

Proposition 6 (Value-added taxes with border adjustment): A value-added tax system with border adjustment is equivalent to a system that taxes consumption and labor and has no tariffs.

Since the Ramsey allocation can be implemented by a system that taxes only consumption and labor, this proposition implies that the Ramsey allocations can be implemented by a value-added tax system with border adjustments. In this sense, a value-added tax system with border adjustments has desirable features.

C. Value-added taxes without border adjustment: The role of tariffs

Consider next an economy just like the one in the previous section, except that value-added taxes are levied on firms without border adjustment. This means that the taxation of intermediate goods will be source-based. We will show that this system without tariffs cannot in general implement the Ramsey allocation. If we add suitably chosen import and export tariffs then the Ramsey allocation can indeed be implemented. We refer to the system with value-added taxes without border adjustment and with tariffs as a *VA without BA system.*

The intermediate goods firm in country 1 now maximizes

$$\sum_{t=0}^{\infty} Q_t \left[(1-\tau_{1t}^{v}) \left(p_{11t} y_{11t} + (1-\tau_{12t}^{e}) p_{12t} y_{12t} - q_{1t} x_{1t} \right) - w_{1t} n_{1t} \right]$$

subject to (2) and (4).

The final goods firm in country 1 now maximizes

$$\sum_{t=0}^{\infty} Q_t (1-\tau_{1t}^{v}) \left[q_{1t} G^1 (y_{11t}, y_{21t}) - p_{11t} y_{11t} - (1+\tau_{21t}^{y}) p_{21t} y_{21t} \right].$$

Firms in country 2 solve similar problems.

The first-order conditions of the firms' problems for an interior solution are $p_{iit}(1-\tau_{it}^{v}) F_{n,t}^{i} = w_{it}$, (20) - (22) and

$$Q_t q_{it} (1-\tau_{it}^{v}) = Q_{t+1} p_{iit+1} \left(1-\tau_{it+1}^{v}\right) F_{k,t+1}^{i} + Q_{t+1} q_{it+1} \left(1-\tau_{it+1}^{v}\right) (1-\delta))$$

Using these conditions, the equilibrium conditions become (61), (62) and

$$(64) \quad \frac{(1+\tau_{21t}^{y})}{(1-\tau_{21t}^{e})} \frac{G_{2,t}^{2}}{G_{2,t}^{1}} = \frac{(1-\tau_{12t}^{e})}{(1+\tau_{12t}^{y})} \frac{G_{1,t}^{2}}{G_{1,t}^{1}},$$

$$(65) \quad \frac{(1+\tau_{12t}^{y})}{(1-\tau_{12t}^{e})} \frac{\left(1-\tau_{12t+1}^{e}\right)}{\left(1+\tau_{12t+1}^{y}\right)} \frac{\left(1-\tau_{1t+1}^{v}\right)}{(1-\tau_{1t}^{v})} \frac{G_{1,t}^{1}}{G_{1,t+1}^{1}} \left[G_{1,t+1}^{1} F_{k,t+1}^{1} + 1 - \delta\right]$$
$$= \frac{\left(1-\tau_{2t+1}^{v}\right)}{(1-\tau_{2t}^{v})} \frac{G_{1,t}^{2}}{G_{1,t+1}^{2}} \left[G_{2,t+1}^{2} F_{k,t+1}^{2} + 1 - \delta\right].$$

Using (62) and (65), we obtain the analog to (29),

$$(66)\quad \frac{\left(1+\tau_{12t}^{y}\right)/\left(1-\tau_{12t}^{e}\right)}{\left(1+\tau_{12t+1}^{y}\right)/\left(1-\tau_{12t+1}^{e}\right)}\frac{u_{c,t}^{1}}{\beta u_{c,t+1}^{1}}=\frac{u_{c,t}^{2}}{\beta u_{c,t+1}^{2}}\frac{G_{1,t}^{2}/G_{1t}^{1}}{G_{1,t+1}^{2}/G_{1t+1}^{1}}.$$

Condition (65) implies that if trade taxes are constrained to be zero in both countries it is not possible to implement the Ramsey outcome for general preferences. The reason is that, in general, the Ramsey outcome implies that relative VATs in the two countries vary over time in the sense that $\left(1-\tau_{1t+1}^{v}\right)/\left(1-\tau_{2t+1}^{v}\right)\neq\left(1-\tau_{1t}^{v}\right)/\left(1-\tau_{2t}^{v}\right)$. Without trade taxes (65) is inconsistent with the dynamic production efficiency condition (8).

Once we allow for tariffs, then it is possible to implement the Ramsey outcome. To ensure static production efficiency, tariffs have to compensate each other so that (64) coincides with (7). To ensure dynamic production efficiency the tariffs have to suitability vary over time so as to undo the distortions arising from time varying VATs.

The VAT without BA, if $\left(1-\tau_{1t}^{v}\right)/\left(1-\tau_{2t}^{v}\right)$ is time-varying, distorts the allocation of capital across countries and does not distort the intertemporal allocation of consumption across agents in (66). Instead, the VAT with BA does not distort the allocation of capital and distorts the intertemporal allocation of consumption in (29). The Ramsey allocation has no distortions in the allocation of capital across countries and has distortions in the allocation of consumption across countries. Trade taxes can correct for these inefficiencies.

One implementation of the Ramsey outcome has

$$(67)\quad \frac{1-\tau_{1t}^{v}}{1-\tau_{2t}^{v}}=\frac{1-\tau_{21t}^{e}}{1+\tau_{21t}^{y}}=\frac{1+\tau_{12t}^{y}}{1-\tau_{12t}^{e}}.$$

It is straightforward to verify that with these policies it is possible to implement the Ramsey allocation. This implementation has an effective export subsidy on good 2 and an effective import tax on good 1, of the same magnitude of the ratio of the two VATs $\left(1-\tau_{1t}^{v}\right)/\left(1-\tau_{2t}^{v}\right)$. Trade taxes chosen in this fashion do not distort static production efficiency, and they correct for the dynamic production inefficiencies induced by time-varying VATs.

We state these results in the following proposition

Proposition 7 (Value-added taxes without border adjustment): Suppose trade taxes are constrained to be zero in both countries. Then, for general preferences, the Ramsey allocation cannot be implemented with a tax system with labor income taxes and value-added taxes without border adjustment. If trade taxes are unconstrained, then the Ramsey allocation can be implemented with consumption taxes replaced by value-added taxes and tariffs.

For standard macro preferences, VATs are constant over time in each country and therefore there is no need for tariffs. Border tax adjustments in this case are irrelevant.

Proposition 7 is connected to results in international trade. Some of the literature in international economics (Grossman (1980), Feldstein and Krugman (1990), Barbiero et. al. (2018), Costinot and Werning (2018) has argued that VAT systems with border adjustment are equivalent to VAT systems without such adjustment, holding trade taxes constant. The version of the result applicable to our analysis (see Grossman (1980)) is that a uniform value-added tax with border adjustment is equivalent to a uniform value-added tax without border adjustments, in the sense that, taking international prices as given, an individual country can achieve the same allocations with either system. (The theorem requires qualifications regarding the availability of initial wealth taxes to ensure that the government's budget is balanced and international lump sum transfers to ensure that the balance of payments condition is satisfied.)

The key requirement in Grossman's version of the theorem is that value-added taxes are the same across all goods. If value-added taxes differ across goods, then the two systems are not in general equivalent. We can think of our dynamic economy as a static economy with an infinite number of goods. Suppose that the dynamic economy has constant value-added taxes over time. Then, in the reinterpreted static economy, value-added taxes are the same across all goods. Inspecting the marginal conditions with BA, namely (7) and (8), and those without BA, namely (64) and (65), we see that the same allocations can be supported by a VAT with BA and a VAT without BA with no tariffs in either case. Suppose next that in the dynamic economy value-added taxes vary over time, so that in the reinterpreted static economy value-added taxes are different across goods. Then, inspecting the same conditions, we see that the two systems are not equivalent in the absence of tariffs.

Our results can also be used to compare destination- versus source-based systems. To see this comparison, note that a destination-based system is one where tax rates do not depend on origin, and an origin-based system is one where tax rates do not depend on destination. In the case of value-added taxes with border adjustment, the goods leave the country untaxed and are taxed in the destination country at the single value-added tax rate in the destination country. In this sense, the VAT system with border adjustment is a destination-based system. With value-added taxes without border adjustments, goods are taxed at the single rate of the origin country, so that a VAT system without border adjustment is an origin-based system. Our results imply that if countries are restricted not to impose trade taxes, then a destination-based system dominates an origin-based system.

D. Lerner symmetry

The arguments in the previous section make clear that any competitive equilibrium allocation in a VAT system without BA and no trade taxes can be implemented in a VAT system with BA with the same VAT rates and trade taxes chosen according to (67). The trade taxes are an effective import tariff and an export subsidy of the same magnitude. The results regarding the conditions under which VAT with and without border adjustments are equivalent are related to the Lerner symmetry. This symmetry asserts that for an individual country taking international prices as given, import taxes are equivalent to export taxes. An alternative way to state this is that import tariffs together with export subsidies of equivalent magnitude are neutral. We state a Lerner symmetry theorem formally in Lemma 1, below. In proving these results we use only the properties that any competitive equilibrium must satisfy and do not use any properties of the Ramsey allocation.

Lemma 1 (Lerner symmetry) The competitive equilibrium allocations of an economy with trade taxes given by τ^e_{12t} and τ^y_{21t} coincide with the competitive equilibrium allocations with trade taxes $\hat{\tau}^e_{12t}$ and $\hat{\tau}^y_{21t}$ satisfying $(1 - \hat{\tau}^e_{12t}) = \kappa\,(1 - \tau^e_{12t})$ and $(1 + \hat{\tau}^y_{21t}) = \kappa\,(1 + \tau^y_{21t})$, for $\kappa > 0$, provided initial wealth taxes or international transfers are chosen appropriately.

The proof of the lemma is in the appendix. This change in trade taxes raises all domestic prices in the world numeraire proportionately, so that $\hat{p}_{11t} = \kappa p_{11t}$, $\hat{q}_{1t} = \kappa q_{1t}$,

$\hat{w}_{1t} = \kappa w_{1t}$.

The key idea is that a proportional change in import tariffs and export subsidies leaves domestic relative prices unaffected, although the change requires a change in the level of prices denominated in terms of the world numeraire. For this reason, the real value of outstanding assets may change, requiring changes in wealth taxes or international transfers.

No change in the level of domestic prices may be needed if those prices are denoted in terms of a domestic numeraire, rather than a world numeraire. The proportional change in trade taxes would require a change in the exchange rate between the world numeraire and the domestic numeraire. We turn now to a formal proof of this suggestion.

Let tildes denote prices in terms of a domestic numeraire. Let E_t denote the exchange rate between the domestic and the world numeraire measured as units of domestic numeraire per world numeraire. Then , for example $\tilde{p}_{11t} = E_t p_{11t}$. In the Appendix we prove the following lemma.

Lemma 2 (Exchange rate adjustment). Consider a competitive equilibrium of an arbitrary economy. Consider now an alternative economy with the same international prices in world numeraire, Q_t, p_{12t}, p_{21t}, same domestic prices in domestic currency $\tilde{q}_{1t}, \tilde{p}_{11t}, \tilde{w}_{1t}$, in which allocations, domestic policies and the exchange rate are denoted with carets. Suppose now policies in the alternative economy satisfy $1 - \hat{\tau}^e_{12t} = \kappa\left(1 - \tau^e_{12t}\right)$ and $1 + \hat{\tau}^y_{21t} = \kappa\left(1 + \tau^y_{21t}\right)$. There is an equilibrium in the alternative economy, with the same allocations and domestic policies, and with exchange rates given by $\hat{E}_t = E_t/\kappa$, provided initial wealth taxes or international transfers are chosen appropriately.

Lemma 2 states that uniform changes in import tariffs and export subsidies have no effects on allocations or domestic prices. The required changes on prices in the world numeraire are accomplished entirely by changes in the nominal exchange rate.

In Appendix B we show that if foreign assets are denominated in the world numeraire, only the initial wealth tax may have to be adjusted. There is no need to adjust international transfers to satisfy the balance of payments condition for country $i = 1$ in (16). If, instead, domestic and foreign assets of country 1, b_{i0} and f_{10}, are denominated in the domestic numeraire, there is no need to adjust the initial wealth tax, but international transfers may need to be adjusted.

Thus far, in this section, we have considered uniform changes in trade taxes. Next, consider an alternative economy in which trade taxes in period t are given by $1 - \hat{\tau}^{e}_{12t} = \kappa_t \left(1 - \tau^{e}_{12t}\right)$ and $1 + \hat{\tau}^{y}_{21t} = \kappa_t \left(1 + \tau^{y}_{21t}\right)$. In the Appendix, we show that κ_t is not neutral if $\kappa_t \neq \kappa_s$ for some t and s.

The results in this section shed some light on results in the literature. For example, Barbiero et. al. (2018) show that, in an economy with sticky prices and no capital, permanent changes in tax systems similar to the ones studied here have no effects on allocations. This result is similar to our result that uniform changes in trade taxes have no real effects. They also show that anticipated changes in tax systems have real effects. This result is similar to our result that nonuniform changes in trade taxes may lead to changes in allocations. For another example, Costinot and Werning (2018) show that uniform changes in trade taxes have no effect on allocations.

4. Remarks on the generality of the results

In this section we argue that our results generalize to other models of international trade and models of non-linear taxation.

Other models of international trade Thus far, for concreteness, we have focused attention on one widely used model of international trade, namely that in Backus, Kydland and Kehoe (1994). This focus allowed us to derive explicit expressions for the optimal wedges and allowed for a detailed analysis of alternative tax systems. Here we show that Proposition 2 continues to hold in other widely used models of international trade, including Obstfeld and Rogoff (1995), Stockman and Tesar (1995), Eaton and Kortum (2002). We conjecture that the analogue of the other propositions would also hold in these other models, but a detailed analysis in the plethora of these models is beyond the scope of this paper. Proposition 2 is of particular interest because it addresses the central question at the intersection of public finance and international trade: Is it desirable to interfere with international trade when governments must finance government consumption with distorting taxes. The other propositions are also of interest because they address a variety of issues at the forefront of policy discussions.

In many models of international trade, a vector of final consumption goods is produced using labor and possibly initial capital. The models differ in terms of details regarding the technology including the use of intermediate goods. The technology in these models can be written as

$$Y = \{(C_{it})_{t=0}^{\infty}, (N_{it})_{t=0}^{\infty}, k_{i0}\}$$

where C_{it} and N_{it} denote vectors of different types of consumption goods and different types of labor in country i in period t, and k_{i0} denotes the initial capital stock in country i. The technology set Y has constant returns to scale. For convenience we will say that an allocation is resource feasible if it is in the technology set. Preferences of households in each country are given by

$$U^i = \sum_{t=0}^{\infty} \beta^t \left[u^i (C_{it}, N_{it})\right].$$

Suppose now that the government in each country can levy taxes on each type of consumption good and on the income of each type of labor input and that these tax rates can be different across consumption goods and labor types. Suppose also that the government in each country is allowed to levy taxes on initial wealth and is required to respect an initial wealth constraint as in the benchmark model above, and that governments can make lump sum transfers to each other. Consider the competitive equilibria of this model. It is possible to show that any such equilibrium allocation must satisfy an implementability condition of the form (30). Since any allocation must also be resource feasible, it follows that no Ramsey outcome can yield higher welfare than the allocation that maximizes a weighted average of country welfare subject to implementability and the resources constraints. Clearly, the solution to this problem is at the boundary of the production set and is therefore production efficient. It is possible to support this outcome as a competitive equilibrium by (possibly different) taxes on each consumption good and each type of labor. Thus, this outcome is a Ramsey equilibrium.

This result shows that proposition 2 generalizes to environments with trade in final goods as in Obstfeld and Rogoff (1995) and to environments with traded and non-traded final goods as in Stockman and Tesar (1995) and environments with a detailed structure of intermediate and final goods as in Eaton and Kortum (2002). Proposition 2 does not generalize to trade models in which firms have monopoly power as in Helpman and Krugman (1985) and Melitz (2003) or in which there are externalities as in Alvarez, Buera and Lucas (2013). These environments require corrective tax and subsidy instruments even absent the need to finance government expenditures with distorting taxes.

Non-linear taxation Here we briefly show how Proposition 2 generalizes to environments with non-linear taxation. We consider a Mirrlees-like environment in which the government can choose a non-linear tax function to finance government expenditures and to redistribute across households. Consider a version of our benchmark model with a continuum of households in each country in the unit interval. Household k in country i is indexed by a parameter θ_i^k. This parameter is constant over time and determines the effective units of labor supplied by household k in country i. Specifically, a household of type θ_i^k that supplies n_t units of labor, supplies $l_t = \theta_i^k n_t$ units of effective labor. The distribution of household types is given by $H_i\left(\theta_i^k\right)$.

The cooperative planner observes consumption and effective labor by each household but not the household type. An allocation in this economy consists of allocations for each household $\left\{c_t\left(\theta_i^k\right), l_t\left(\theta_i^k\right)\right\}$ and aggregate allocations for each country $\{y_{ijt}, k_{it+1}, x_{it}\}$. The resource constraints are the analogs of (2) and (3),

$$(68)\quad y_{i1t} + y_{i2t} = y_{it} = F^i\left(k_{it}, \int l_t\left(\theta_i^k\right) dH_i\left(\theta_i^k\right)\right),$$

$$(69)\quad \int c_t\left(\theta_i^k\right) dH_i\left(\theta_i^k\right) + g_{it} + x_{it} \leq G^i\left(y_{1it}, y_{2it}\right),$$

and (4). The utility of household of type θ_i^k is given by

$$(70)\quad U^i\left(\theta_i^k\right) = \sum_{t=0}^{\infty} \beta^t \left[u^i\left(c_t\left(\theta_i^k\right), \frac{l_t\left(\theta_i^k\right)}{\theta_i^k}\right) + h^i\left(g_{it}\right)\right].$$

An allocation is *incentive compatible*, if

$$(71)\quad \sum_{t=0}^{\infty} \beta^t u^i \left(c_t \left(\theta_i^k\right), l_t \left(\theta_i^k\right)/\theta_i^k\right) \geq \sum_{t=0}^{\infty} \beta^t u^i \left(c_t \left(\hat{\theta}_i^k\right), l_t \left(\hat{\theta}_i^k\right)/\theta_i^k\right)$$

for all θ_i^k, $\hat{\theta}_i^k$. An allocation is *incentive feasible* if it is incentive compatible and resource feasible in that it satisfies the resource constraints.

An allocation is a cooperative Mirrlees outcome if it maximizes

$$\omega^1 \int U^1 \left(\theta_1^k\right) dJ_1 \left(\theta_1^k\right) + \omega^1 \int U^2 \left(\theta_2^k\right) dJ_2 \left(\theta_2^k\right)$$

over the set of incentive feasible allocations where $J_i \left(\theta_i^k\right)$ is a distribution that represents a combination of the underlying distribution H and Pareto weights over households of different types.

Suppose now that the preferences of households are a variant of the standard macro form, in that

$$(72)\quad u^i \left(c_t \left(\theta_i^k\right), l_t \left(\theta_i^k\right)/\theta_i^k\right) = u^i \left(c_t \left(\theta_i^k\right)\right) - \eta_i \frac{\left(l_t \left(\hat{\theta}_i^k\right)/\theta_i^k\right)^{1+\sigma^{ni}}}{1+\sigma^{ni}}.$$

It is straightforward to show that the Mirrleesian allocation can be supported as a competitive equilibria with nonlinear taxes. Using the same logic as in Atkinson and Stiglitz (1976), Golosov, Kocherlakota, Tsyvinski (2003) and Werning (2007), we have the following proposition.

Proposition 8 (Production efficiency and no intertemporal distortions): The Mirrleesian outcomes satisfy production efficiency so that free trade and unrestricted capital mobility are optimal. Furthermore, if household preferences satisfy (72), then it is optimal to have no intertemporal distortions.

In this formulation, workers differ from each other on a single dimension, namely the parameter θ_i^k, that determines the effective units of labor supplied by a worker. If they differ along multiple dimensions, say because they differ in their comparative advantage in working in the various sectors, then, the planning problem becomes a multidimensional screeing problem and the analysis becomes more complicated. See Hosseini and Shourideh (2018) and Costinot and Werning (2018) for analyses of optimal trade taxation with restricted systems.

5. Concluding remarks

We characterize cooperative Ramsey allocations in the global economy. We show that free trade and unrestricted capital mobility are optimal. In the benchmark model, Ramsey allocations can be supported by time-varying taxes on consumption and labor income. We study alternative implementations of the Ramsey allocation including taxation of equity returns, foreign asset returns as well as corporate income. We show that it is optimal to tax all types of household assets at the same country-specific rate and not to tax corporate income. We show that border adjustments are desirable if in the benchmark model it is optimal to have time-varying consumption taxes. We clarify apparently conflicting views in the public finance and trade literatures regarding the desirability of border adjustments. We show that our results hold in a variety of trade models and we extend our results to non-linear tax systems.

References

[1] Alvarez, Fernando E., Francisco J. Buera, Robert E. Lucas, Jr., 2013, "Idea Flows, Economic Growth and Trade," NBER Working Paper 19667.

[2] Armenter, Roc, 2008, "A Note on Incomplete Factor Taxation," Journal of Public Economics 92 (10-11), 2275–2281.

[3] Atkeson, Andrew, V. V. Chari, and Patrick J. Kehoe, 1999, "Taxing Capital Income: A Bad Idea," Federal Reserve Bank of Minneapolis Quarterly Review 23 (3), 3–18.

[4] Atkinson, Anthony B. and Joseph E. Stiglitz, 1972, "The Structure of Indirect Taxation and Economic Efficiency," Journal of Public Economics 1 (1), 97–119.

[5] Atkinson, Anthony B. and Joseph E. Stiglitz, 1976, "The Design of Tax Structure: Direct versus Indirect Taxation." Journal of Public Economics 6 (1-2), 55-75.

[6] Auerbach, Alan, Michael P. Devereux, Michael Keen, and John Vella, 2017, "Destination-Based Cash Flow Taxation," Oxford University Center for Business Taxation WP 17/01.

[7] Backus, David K., Patrick J. Kehoe and Finn E. Kydland, 1994, "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?," American Economic Review 84 (1), 84-103.

[8] Barbiero, Omar, Emmanuel Farhi, Gita Gopinath, and Oleg Itskhoki, 2017, "The Economics of Border Adjustment Tax," mimeo, Harvard University.

[9] Bassetto, Marco, and Jess Benhabib, 2006, "Redistribution, Taxes, and the Median Voter," Review of Economic Dynamics 9(2), 211-223.

[10] Chamley, Christophe, 1986, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," Econometrica 54 (3), 607–622.

[11] Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe, 1994, "Optimal Fiscal Policy in a Business Cycle Model," Journal of Political Economy 102 (4), 617–52.

[12] Chari, V. V., Juan Pablo Nicolini, and Pedro Teles, 2016, "Optimal Capital Taxation Revisited," mimeo, Federal Reserve Bank of Minneapolis.

[13] Costinot, Arnaud and Iván Werning, "The Lerner Symmetry Theorem: Generalizations, Qualifications, and Implications for Border Tax Adjustment", 2017, mimeo, MIT.

[14] Costinot, Arnaud and Iván Werning, 2018, Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation, mimeo, MIT.

[15] Coleman, Wilbur John II, 2000, "Welfare and Optimum Dynamic Taxation of Consumption and Income," Journal of Public Economics 76 (1), 1–39.

[16] Diamond, Peter A and James A. Mirrlees, 1971, "Optimal Taxation and Public Production I: Production Efficiency," American Economic Review 61 (1), 8–27.

[17] Dixit, Avinash, 1985, "Tax Policy in Open Economies," in Handbook of Public Economics, Vol. 1, ed. Alan Auerbach and Martin Feldstein. Amsterdam: North-Holland.

[18] Eaton, Jonathan and Samuel Kortum, 2002, "Technology, Geography and Trade," Econometrica 70 (5), 1741–1779

[19] Feldstein, Martin S., and Paul R. Krugman, 1990, "International Trade Effects of Value-Added Taxation," in Taxation in the Global Economy, ed. Assaf Razin and Joel Slemrod, University of Chicago Press, 263–282.

[20] Golosov, Mikhail, Narayana Kocherlakota and Aleh Tsyvinski, 2003, "Optimal Indirect and Capital Taxation," Review of Economic Studies 70 (3), 569-587.

[21] Grossman, Gene M., 1980, "Border Tax Adjustments: Do They Distort Trade?," Journal of International Economics, 10(1), 117–128.

[22] Helpman, and Paul Krugman, 1985, "Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy." The MIT Press, Cambridge, MA.

[23] Hosseini, Roozbeh and Ali Shourideh, 2018, Inequality, Redistribution and Optimal Trade Policy: A Public Finance Approach, mimeo, Carnegie Mellon University.

[24] Judd, Kenneth L., 1985, "Redistributive taxation in a simple perfect foresight model," Journal of Public Economics 28 (1), 59–83.

[25] Judd, Kenneth L., 1999, "Optimal Taxation and Spending in General Competitive Growth Models," Journal of Public Economics 71 (1), 1–26.

[26] Judd, Kenneth L., 2002, "Capital-Income Taxation with Imperfect Competition," American Economic Review, Papers and Proceedings 92 (2), 417–421.

[27] Lucas, Robert E., Jr. and Nancy L. Stokey, 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital," Journal of Monetary Economics 12 (1), 55–93.

[28] Obstfeld, Maurice and Kenneth Rogoff, 1995, "Exchange Rate Dynamics Redux," Journal of Political Economy 103 (3) , 624-660

[29] Melitz, Marc J., 2003, "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," Econometrica 71 (6), 1695–1725.

[30] Negishi Takashi, 1972, "General equilibrium theory and international trade." North-Holland Publishing Company, Amsterdam.

[31] Stockman, Alan C. and Linda L. Tesar, 1995, "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining,"The American Economic Review 85 (1), 168-185.

[32] Straub, Ludwig and Iván Werning, 2015, Positive Long Run Capital Taxation: Chamley-Judd Revisited, mimeo, MIT.

[33] Werning, Iván, 2007, "Optimal Fiscal Policy with Redistribution," Quarterly Journal of Economics 122 (3), 925-967.

A. Appendix A: Optimality of production efficiency with zero transfers

In this appendix, we show that there is a cooperative Ramsey solution implemented with zero transfers across countries. We use consumption and labor income taxes, set trade taxes to zero, and solve for the optimal level of government consumption. Note that (14) can be written as

$$\left[\sum_{t=0}^{\infty} Q_t q_{it} g_{it} + Q_{-1} b_{i0}\right] - \left[\sum_{t=0}^{\infty} Q_t \left(\tau_{it}^c q_{it} c_{it} + \tau_{it}^n w_{it} n_{it}\right) + l_{i0} a_{i0}\right] = T_{i0}.$$

The Ramsey problem is to maximize

$$\omega^1 U^1 + \omega^2 U^2$$

subject to the conditions

$$\sum_{t=0}^{\infty} \left[\beta^t u_{c,t}^i c_{it} + \beta^t u_{n,t}^i n_{it}\right] \geq \mathcal{W}_{i0}$$

$$c_{it} + g_{it} + k_{it+1} - (1-\delta) k_{it} \leq G^i \left(y_{1it}, y_{2it}\right)$$

$$y_{i1t} + y_{i2t} \leq F^i \left(k_{it}, n_{it}\right)$$

Let λ^i, ε_{it}, and δ_{it} be the multipliers on these three conditions.

Proposition: Let $\mathcal{W}_{10} = 0$. Then there exits a weight ω^1 small enough such that $T_{10} < 0$.

Pf: The first-order conditions of the Ramsey problem include

$$\omega^1 h'(g_{1t}) = \varepsilon_{1t}$$

Thus, as $\omega^1 \to 0$, $g_{1t} \to 0$ for all t.

Preliminary result 1.

The first-order conditions for an interior solution are

$$\omega^1 \beta^t u^1_{ct} + \lambda^1 \beta^t u^1_{ct} + \lambda^1 \beta^t \left[u^1_{cct} c_{1t} + u^1_{cnt} n_{1t} \right] = \varepsilon_{1t}$$

$$\omega^1 \beta^t u^1_{nt} + \lambda^1 \beta^t u^1_{nt} + \lambda^1 \beta^t \left[u^1_{nct} c_{1t} + u^1_{nnt} n_{1t} \right] = -\delta_{1t} F^1_{nt}$$

$$\varepsilon_{1t} G^1_{1t} = \delta_{1t}$$

$$\varepsilon_{1t} G^1_{2t} = \delta_{2t}$$

$$\varepsilon_{2t} G^2_{1t} = \delta_{1t}$$

$$\varepsilon_{2t} G^2_{2t} = \delta_{2t}$$

$$\varepsilon_{1t} = \varepsilon_{1t+1} (1 - \delta) + \delta_{1t+1} F^1_{kt}$$

Now, replace δ_{1t} and multiply the FOC by quantities

$$\omega^1 \beta^t u^1_{ct} c_{1t} + \lambda^1 \beta^t u^1_{ct} c_{1t} + \lambda^1 \beta^t \left[u^1_{cct} c^2_{1t} + u^1_{cnt} n_{1t} c_{1t} \right] = \varepsilon_{1t} c_{1t}$$

$$\omega^1 \beta^t u^1_{nt} n_{1t} + \lambda^1 \beta^t u^1_{nt} n_{1t} + \lambda^1 \beta^t \left[u^1_{nct} c_{1t} n_{1t} + u^1_{nnt} n^2_{1t} \right] = -\varepsilon_{1t} G^1_{1t} F^1_{nt} n_{1t}$$

add them up

$$\beta^t \left[u^1_{ct}c_{1t} + u^1_{nt}n_{1t}\right]\left[\omega^1 + \lambda^1\right] + \lambda^1\beta^t\left[u^1_{cct}c^2_{1t} + 2u^1_{cnt}n_{1t}c_{1t} + u^1_{nnt}n^2_{1t}\right] = \varepsilon_{1t}\left[c_{1t} - G^1_{1t}F^1_{nt}n_{1t}\right]$$

and add over time

$$\left[\omega^1 + \lambda^1\right]\sum_{t=0}^{\infty}\beta^t\left[u^1_{ct}c_{1t} + u^1_{nt}n_{1t}\right] + \lambda^1\sum_{t=0}^{\infty}\beta^t\left[u^1_{cct}c^2_{1t} + 2u^1_{cnt}n_{1t}c_{1t} + u^1_{nnt}n^2_{1t}\right] = \sum_{t=0}^{\infty}\varepsilon_{1t}\left[c_{1t} - G^1_{1t}F^1_{nt}n_{1t}\right]$$

Note that, since the multiplier λ^1 is non-negative and the function u is concave, the term

$$\lambda^1\sum_{t=0}^{\infty}\beta^t\left[u^1_{cct}c^2_{1t} + 2u^1_{cnt}n_{1t}c_{1t} + u^1_{nnt}n^2_{1t}\right]$$

is negative.[6] It follows that

$$(73)\quad \left[\omega^1 + \lambda^1\right]\mathcal{W}_{i0} > \sum_{t=0}^{\infty}\varepsilon_{1t}\left[c_{1t} - G^1_{1t}F^1_{nt}n_{1t}\right]$$

Preliminary result 2.

We relate the term in the right hand side,

$$\sum_{t=0}^{\infty}\varepsilon_{1t}\left[c_{1t} - G^1_{1t}F^1_{nt}n_{1t}\right]$$

to a term involving the present value of trade balances.

Due to constant returns to scale, Euler theorem implies

$$(74)\quad c_{1t} + g_{1t} + k_{1t+1} - (1-\delta)k_{1t} = G^1(y_{11t}, y_{21t}) = G^1_{1t}y_{11t} + G^1_{2t}y_{21t}$$

$$(75)\quad y_{11t} + y_{12t} = F^1(k_{1t}, n_{1t}) = F^1_{kt}k_{1t} + F^1_{nt}n_{1t}$$

[6] The non-neagativity of the multiplier is directly implied by the Khun-Tucker conditions once we allow each government to make non-negative lump sum transfers to the private agents. We omitted those transfers from the problem for simplicity.

The trade balance (in units of the intermediate good produced in country 1) satisfies

$$y_{21t}q_{2t} = y_{12t}q_{1t} - TB_{1t}q_{1t}$$

or, dividing by q_{1t},

$$y_{21t}\frac{q_{2t}}{q_{1t}} = y_{12t} - TB_{1t}$$

But in a Ramsey allocation $\frac{q_{2t}}{q_{1t}} = \frac{G_2^1}{G_1^1}$ so

$$y_{21t}\frac{G_{2t}^1}{G_{1t}^1} = y_{12t} - TB_{1t}$$

Replacing in (74) above,

$$\begin{aligned} c_{1t} + g_{1t} + k_{1t+1} - (1-\delta)\, k_{1t} &= G_{1t}^1 y_{11t} + G_{1t}^1 y_{12t} - G_{1t}^1 TB_{1t} \\ &= G_{1t}^1 (y_{11t} + y_{12t}) - G_{1t}^1 TB_{1t} \end{aligned}$$

and using (75)

$$c_{1t} + g_{1t} + k_{1t+1} - (1-\delta)\, k_{1t} = G_{1t}^1 F_{kt}^1 k_{1t} + G_{1t}^1 F_{nt}^1 n_{1t} - G_{1t}^1 TB_{1t}$$

so

$$c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t} = G_{1t}^1 F_{kt}^1 k_{1t} - G_{1t}^1 TB_{1t} - g_{1t} - [k_{1t+1} - (1-\delta)\, k_{1t}]\,.$$

Multiplying each term by ε_{1t} and adding up for all t,

$$\sum_{t=0}^{\infty} \varepsilon_{1t} \left[c_{1t} - G_{1t}^1 F_{nt}^1 n_{1t}\right] = \sum_{t=0}^{\infty} \varepsilon_{1t} \left[G_{1t}^1 F_{kt}^1 k_{1t} - G_{1t}^1 TB_{1t} - g_{1t} - [k_{1t+1} - (1-\delta)\, k_{1t}]\right]$$

Recall that the first-order condition with respect to k_{1t+1} implies

$$-\varepsilon_{1t} + \left[G_{1t+1}^1 F_{kt+1}^1 + (1-\delta)\right] \varepsilon_{1t+1} = 0,$$

So we obtain the preliminary result 2.

$$(76)\quad \sum_{t=0}^{\infty}\varepsilon_{1t}\left[c_{1t}-G_{1t}^{1}F_{nt}^{1}n_{1t}\right]=-\sum_{t=0}^{\infty}\varepsilon_{1t}\left[G_{1t}^{1}TB_{1t}+g_{1t}\right]+\left[G_{10}^{1}F_{k0}^{1}+(1-\delta)\right]\varepsilon_{10}k_{10}$$

Proof: Using (76) into (73), and noting that when $\omega^1 \to 0$, $g_{1t} \to 0$ for all t, we obtain

$$\left[\omega^{1}+\lambda^{1}\right]\mathcal{W}_{10}>-\sum_{t=0}^{\infty}\varepsilon_{1t}G_{1t}^{1}TB_{1t}-\left[G_{10}^{1}F_{k0}^{1}+(1-\delta)\right]\varepsilon_{10}k_{10}$$

or

$$(77)\quad \sum_{t=0}^{\infty}\varepsilon_{1t}G_{1t}^{1}TB_{1t}=\sum_{t=0}^{\infty}\delta_{1t}TB_{1t}>-\left[\omega^{1}+\lambda^{1}\right]\mathcal{W}_{10}+\left[G_{10}^{1}F_{k0}^{1}+(1-\delta)\right]\varepsilon_{10}k_{10}$$

As we assumed that $\mathcal{W}_{10}=0$, it follows that

$$\sum_{t=0}^{\infty}\delta_{1t}TB_{1t}>\left[G_{10}^{1}F_{k0}^{1}+(1-\delta)\right]\varepsilon_{10}k_{10}$$

As the right hand side is positive, this equation implies that

$$\sum_{t=0}^{\infty}\delta_{1t}TB_{1t}>0,$$

The δ_{1t} are the multipliers of constraints

$$(\delta_{1t})y_{11t}+y_{12t}\leq F^{1}\left(k_{1t},n_{1t}\right)$$

which is the value for the planner of the intermediate goods. Because of production efficiency, the private and social values of the intermediate goods are the same, so the present value of the trade balance is positive which means that the transfer is negative.

Remark: Equation (76) makes clear that, given that $\omega^1 \to 0$, a weaker sufficient condition is

$$-\left[\omega^{1}+\lambda^{1}\right]\mathcal{W}_{10}+\left[G_{10}^{1}F_{k0}^{1}+(1-\delta)\right]\varepsilon_{10}k_{10}\geq 0$$

or

$$\lambda^1 \mathcal{W}_{10} \leq \left[G^1_{10} F^1_{k0} + (1-\delta)\right] \varepsilon_{10} k_{10}.$$

which is weaker than the one assumed in the proposition. This condition, however, involves multipliers, which are endogenous.

To understand the role of restricting the value for $\mathcal{W}_{10}$, imagine that it takes a value that is higher than the present value of current plus all future national incomes in country 1, when all taxes are set to zero and all government expenditures are set to zero. Any feasible allocation therefore requires transfers of resources from country 2 to country 1, independently of the values of the weights ω^i. This logic also makes clear that there are high enough values for $\mathcal{W}_{10}$ and $\mathcal{W}_{20}$, such that the set of implementable allocations is empty.

This far we have focused on interior allocations. It is possible to extend the proof to situations in which the solution is at corner, details are available upon request.

B. Appendix B: Border adjustments and Lerner symmetry

Lemma 1 We start by proving Lemma 1. Consider that country 1 introduces an import tariff and an export tax on all goods. The conditions for the household and firms in country 1 are

$$(78)\quad -\frac{u^1_{c,t}}{u^1_{n,t}} = \frac{(1+\tau^c_{1t})\, q_{1t}}{(1-\tau^n_{1t})\, w_{1t}},$$

$$(79)\quad \frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{Q_t q_{1t}}{Q_{t+1} q_{1t+1}} \frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$(80)\quad F^1_{n,t} = \frac{w_{1t}}{p_{11t}}$$

$$(81)\quad \frac{Q_t}{Q_{t+1}} = \frac{p_{11t+1}}{q_{1t}} F^1_{k,t+1} + \frac{q_{1t+1}}{q_{1t}}(1-\delta)$$

$$(82)\quad G^1_{1,t} = \frac{p_{11t}}{q_{1t}}$$

$$(83)\quad p_{11t} = (1-\tau^e_{12t})\, p_{12t}$$

$$(84)\quad q_{1t} G^1_{2,t} = (1+\tau^y_{21t})\, p_{21t}$$

The proof of Lemma 1 follows by inspecting the first-order conditions above, (78) through (84), as well as the household budget constraints written as (30) and (31) satisfied with an appropriate choice of $\hat{l}_{10}$,

$$\mathcal{W}_{10} = \left(1 - \hat{l}_{10}\right) \frac{u^1_{c,0}}{(1+\tau^c_{10})} \left[\left(1 - \delta + G^1_{1,0} F^1_{k,0}\right) k_{10} + Q_{-1} \frac{b_{10}}{\hat{q}_{10}} + \left(1 + r^f_0\right) \frac{f_{1,0}}{\hat{q}_{10}}\right].$$

The higher price of the final good in country 1 (and the price of the imported good after the tariff together with the price of the exported good after the subsidy) reduces the value of domestic and foreign assets, so that the government must compensate that with a lower tax on initial wealth $\hat{l}_{10}$.There is no need to adjust transfers to satisfy the Balance of Payments condition for country $i = 1$ in (16).

Lemma 2 Let tildes denote prices in terms of domestic currency. Let E_t denote domestic currency per numeraire. Then, for example $\tilde{p}_{11t} = E_t p_{11t}$. Now when we multiply all the trade policy terms by κ, it is equivalent to letting $\hat{E}_t = \frac{E_t}{\kappa}$, (if $\kappa > 1$, the domestic currency appreciates) and leaving all domestic prices denoted in domestic currency unaffected.

Then, conditions (78) through (84) can be written as

$$-\frac{u^1_{c,t}}{u^1_{n,t}} = \frac{(1+\tau^c_{1t})\,\tilde{q}_{1t}}{(1-\tau^n_{1t})\,\tilde{w}_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{Q_t}{Q_{t+1}} \frac{\tilde{q}_{1t}}{\tilde{q}_{1t+1}} \frac{e_{t+1}}{e_t} \frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{\tilde{w}_{1t}}{\tilde{p}_{11t}}$$

$$\frac{Q_t}{Q_{t+1}} = \frac{\tilde{p}_{11t+1}}{\tilde{q}_{1t}} \frac{e_t}{e_{t+1}} F^1_{k,t+1} + \frac{\tilde{q}_{1t+1}}{\tilde{q}_{1t}} \frac{e_t}{e_{t+1}} (1-\delta)$$

$$G^1_{1,t} = \frac{\tilde{p}_{11t}}{\tilde{q}_{1t}}$$

$$\tilde{p}_{11t} = E_t\,(1-\tau^e_{12t})\,p_{12t}$$

$$\tilde{q}_{1t} G^1_{2,t} = E_t\,(1+\tau^y_{21t})\,p_{21t}$$

The proof of Lemma 2 follows by inspecting the first-order conditions above, as well as the household budget constraints written as (30) and (31) satisfied with an appropriate choice of $\hat{l}_{10}$, as long as foreign assets are denominated in the world numeraire, so as to satisfy

$$\mathcal{W}_{10} = \left(1 - \hat{l}_{10}\right) \frac{u^1_{c,0}}{(1+\tau^c_{10})} \left[\left(1 - \delta + G^1_{1,0} F^1_{k,0}\right) k_{10} + Q_{-1} \frac{b_{10}}{\tilde{q}_{10}} + \left(1 + r^f_0\right) \frac{f_{1,0}}{\tilde{q}_{10}} \frac{e_0}{\kappa}\right].$$

There is no need to adjust transfers to satisfy the Balance of Payments condition for country $i = 1$ in (16).

Suppose now net foreign assets were denominated in the domestic numeraire. The value of initial wealth is given by

$$\mathcal{W}_{10} = (1 - l_{10}) \frac{u^1_{c,0}}{(1+\tau^c_{10})} \left[\left(1 - \delta + G^1_{1,0} F^1_{k,0}\right) k_{10} + Q_{-1} \frac{b_{10}}{\tilde{q}_{1,0}} + \left(1 + r^f_0\right) \frac{f_{1,0}}{\tilde{q}_{1,0}}\right].$$

Note that in this case there is no change in the real value of domestic public debt and foreign assets, so that here is no need to change l_{10}. On the other hand, there is a need to change the level of international transfers, since the balance of payments condition is now

$$\sum_{t=0}^{\infty} Q_t \left[p_{12t} y_{12t} - p_{21t} y_{21t}\right] = -\left(1 + r^f_0\right) \frac{f_{1,0}\kappa}{E_0} - \hat{T}_{10}.$$

Since the foreign assets are denominated in domestic currency, they are now worth more in units of foreign currency, and country 1 would have to receive lower transfers.

Non-uniform changes in trade taxes We start by taking international prices p_{21t}, p_{12t}, and Q_t and allocations as given. We multiply the trade taxes in country 1, $(1+\tau^y_{21t})$ and $(1-\tau^e_{12t})$, by $\kappa_t > 0$. The equilibrium conditions become

$$-\frac{u^1_{c,t}\left(1-\tau^n_{1t}\right)}{u^1_{n,t}\left(1+\tau^c_{1t}\right)} = \frac{q_{1t}}{w_{1t}},$$

$$\frac{u^1_{c,t}}{(1+\tau^c_{1t})} = \frac{q_{1t} Q_t}{q_{1t+1} Q_{t+1}} \frac{\beta u^1_{c,t+1}}{\left(1+\tau^c_{1t+1}\right)},$$

$$F^1_{n,t} = \frac{w_{1t}}{p_{11t}}$$

$$\frac{Q_t}{Q_{t+1}} = \frac{p_{11t+1}}{q_{1t}} F^1_{k,t+1} + \frac{q_{1t+1}}{q_{1t}} (1-\delta)$$

$$\frac{1}{(1-\tau^e_{12t})\, p_{12t}} = \frac{\kappa_t}{p_{11t}}$$

$$G^1_{1,t} = \frac{p_{11t}}{q_{1t}}$$

$$\frac{G^1_{2,t}}{p_{21t}\,(1+\tau^y_{21t})} = \frac{\kappa_t}{q_{1t}}$$

In order for κ_t to be neutral, it must be that $\frac{\kappa_t}{q_{1t}}$, $\frac{\kappa_t}{p_{11t}}$, $\frac{q_{1t}}{w_{1t}}$ and $\frac{q_{1t}}{q_{1t+1}}$.are kept constant This can only happen if $\kappa_t = \kappa$.

Changes in trade taxes may also be neutral, if both countries change them in particular ways. To see this, let both countries multiply $\left(1+\tau^y_{jit}\right)$ and $\left(1-\tau^e_{ijt}\right)$ by κ_{it}, for $i = 1, 2$ and $j \neq i$. The equilibrium conditions can be written as

$$-\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{(1+\tau^c_{it})}{(1-\tau^n_{it})\, G^i_{i,t} F^i_{n,t}}$$

$$\frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = \frac{(1+\tau^c_{it})}{\left(1+\tau^c_{it+1}\right)} \left[G^i_{i,t+1} F^i_{k,t+1} + 1 - \delta\right],$$

$$\frac{G^1_{2,t}}{G^1_{1t}} = \frac{\kappa_{1t}\,(1+\tau^y_{21t})}{\kappa_{1t}\,(1-\tau^e_{12t})} \frac{\kappa_{2t}\,(1+\tau^y_{12t})}{\kappa_{2t}\,(1-\tau^e_{21t})} \frac{G^2_{2t}}{G^2_{1,t}}$$

$$\frac{\kappa_{2t}\,(1+\tau^y_{12t})}{\kappa_{2t+1}\left(1+\tau^y_{12t+1}\right)} \frac{\kappa_{1t+1}\left(1-\tau^e_{12t+1}\right)}{\kappa_{1t}\left(1-\tau^e_{12t+1}\right)} \frac{G^1_{1t}}{G^1_{1t+1}} \left[G^1_{1,t+1} F^1_{k,t+1} + 1 - \delta\right] = \frac{G^2_{1,t}}{G^2_{1,t+1}} \left[G^2_{2,t+1} F^2_{k,t+1} + 1 - \delta\right]$$

If the adjustments are such that $\frac{\kappa_{1t+1}}{\kappa_{1t}} = \frac{\kappa_{2t+1}}{\kappa_{2t}}$, the policy is neutral.. The nominal intertemporal price, $\frac{Q_t}{Q_{t+1}}$,is adjusting by the same amount $\frac{\kappa_{1t+1}}{\kappa_{1t}}$.

C. Appendix C: Taxes on assets

In this appendix we show that it is possible to implement the solution of the Ramsey problem in Section 3.1 as a competitive equilibrium.

We consider a system with income taxation of labor and assets including a corporate income tax. We consider a common tax on the household's returns from foreign assets and on equity returns including capital gains.

We now describe the problems of the firms and the household in each country and define a competitive equilibrium. We maintain the assumption that ownership of firms is domestic, but we will see that this is without loss of generality.

Firm The representative intermediate good firm in each country produces and invests in order to maximize the present value of dividends, $V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it}$. Dividends, in units of the numeraire, d_{it}, are given by

$$(85) \quad d_{it} = p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - \tau_{it}^{k}\left[p_{it}F(k_{it}, n_{it}) - w_{it}n_{it} - q_{it}\delta k_{it}\right] - q_{it}\left[k_{it+1} - (1-\delta)k_{it}\right],$$

where τ_{it}^{k} is the tax rate on capital income net of depreciation.

The first-order conditions of the firm's problem are now $p_{iit}F_{n,t}^{i} = w_{it}$ together with

$$(86) \quad \frac{Q_t q_{it}}{Q_{t+1}q_{it+1}} = 1 + \left(1 - \tau_{it+1}^{k}\right)\left(\frac{p_{it+1}}{q_{it+1}}F_{k,t+1}^{i} - \delta\right).$$

Substituting for d_{it} from (85) and, using the firm's first-order conditions, it is easy to show that the present value of the dividends at time zero in units of the numeraire is given by

$$(87) \quad V_{i0} + d_{i0} = \sum_{t=0}^{\infty} Q_t d_{it} = \left[1 + \left(1 - \tau_{i0}^{k}\right)\left(\frac{p_{i0}}{q_{i0}}F_{ik,0} - \delta\right)\right] p_{i0}k_{i0}.$$

The problem of the final good firm is as before.

Households The flow of funds constraint in period t for the household in country i in units of the numeraire is given by

$$\begin{aligned}(88a) \quad & q_{it}c_{it} + b_{it+1} + f_{it+1} + V_{it}s_{it+1} \\ = \; & \left(1 - \tau_{it}^{n}\right) w_{it}n_{it} + \left[1 + r_t^{f} - \tau_{it}\left(r_t^{f} - \frac{q_{it} - q_{it-1}}{q_{it-1}}\right)\right](b_{it} + f_{it}) \\ & + \left(V_{it} + d_{it}\right)s_{it} - \tau_{it}\left(d_{it} + V_{it} - V_{it-1} - \frac{\left(q_{it} - q_{it-1}\right)V_{it-1}}{q_{it-1}}\right)s_{it}.\end{aligned}$$

In period 0, the constraint is

$$(89)\quad \begin{aligned} & q_{i0}c_{i0}+b_{i1}+f_{i1}+V_{i0}s_{i1} \\ = \; & \left(1-\tau_{i0}^{n}\right)w_{i0}n_{i0}+\left(1-l_{i0}\right)\left[1+r_{0}^{f}-\tau_{i0}\left(r_{0}^{f}-\frac{q_{i0}-q_{i-1}}{q_{i-1}}\right)\right]\left(b_{i0}+f_{i0}\right) \\ & \left(1-l_{i0}\right)\left[\left(V_{i0}+d_{i0}\right)s_{i0}-\tau_{i0}\left(d_{i0}+V_{i0}-V_{i-1}-\frac{\left(q_{i0}-q_{i-1}\right)V_{i-1}}{q_{i-1}}\right)s_{i0}\right]. \end{aligned}$$

Dividends and capital gains are taxed at rate τ_{it} with an allowance for numeraire inflation. Returns on domestic and foreign bonds are also taxed at the same rate, τ_{it}, also with an allowance for numeraire inflation.

The household's problem is to maximize utility (1), subject to $(88a)$, (89), and no-Ponzi-scheme conditions, $\lim_{T\to\infty} Q_{iT+1}b_{iT+1}\geq 0$, and $\lim_{T\to\infty} Q_{iT+1}f_{iT+1}\geq 0$ with

$$(90a)\quad \frac{Q_{it}}{Q_{it+1}}=\left(1-\tau_{it+1}\right)\left(1+r_{t+1}^{f}\right)+\tau_{it+1}\frac{q_{it+1}}{q_{it}} \text{ with } Q_{i0}=1$$

The first-order conditions of the household's problem in each country are, for $t\geq 0$, the analog of (17),

$$(91)\quad u_{c,t}^{i}=\frac{Q_{it}q_{it}}{Q_{it+1}q_{it+1}}\beta u_{c,t+1}^{i},$$

and

$$(92a)\quad \frac{Q_{it}}{Q_{it+1}}=\frac{\left(V_{it+1}+d_{it+1}\right)-\tau_{it+1}\left(V_{it+1}-V_{it}+d_{it+1}-\frac{q_{it+1}-q_{it}}{q_{it}}V_{it}\right)}{V_{it}},$$

Condition $(92a)$ implies that

$$1+r_{t+1}^{f}=\frac{V_{it+1}+d_{it+1}}{V_{it}}.$$

This condition on the two returns can be written, using $1+r_{t+1}^{f}=\frac{Q_t}{Q_{t+1}}$, as

$$Q_tV_{it}=Q_{t+1}V_{it+1}+Q_{t+1}d_{it+1}.$$

Imposing that $\lim_{T\to\infty} Q_{T+1}V_{iT+1} = 0$, then

$$V_{it} = \sum_{s=0}^{\infty} \frac{Q_{t+1+s}}{Q_t} d_{it+1+s}.$$

The present value of dividends for the households of country i is a different expression from the expression above because they pay taxes on the asset income. Using (92a), we have that

$$V_{i0} = \sum_{t=0}^{\infty} \left(1 - \hat{\tau}^a_{it+1}\right) Q_{it+1} d_{it+1},$$

where $1-\hat{\tau}^a_{it+1} = \Pi^t_{s=0}\left(1-\hat{\tau}_{is+1}\right)$, and $1-\hat{\tau}_{it+1} = \frac{(1-\tau_{it+1})}{\left(1-\tau_{it+1}\frac{q_{it+1}Q_{it+1}}{q_{it}Q_{it}}\right)}$. The values are the same since $\left(1-\hat{\tau}^a_{it+1}\right) Q_{it+1} = Q_{t+1}$. This condition is obtained from (90a).

The value of the firm for the households in country i including the dividends in period 0 is

$$\begin{aligned} (96) \qquad & V_{i0} + d_{i0} - \tau_{i0}\left(V_{i0} + d_{i0} - \frac{q_{i0}V_{i-1}}{q_{i-1}}\right) \\ = \quad & \left(1-\tau_{i0}\right)\left(V_{i0}+d_{i0}\right) + \tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}. \end{aligned}$$

Notice that the market price of the firm before dividends, $V_{i0} + d_{i0}$, is a linear function of the value for the firm for the households of each country, so that the solution of the maximization problem of the firm also maximizes shareholder value. That would also be the case if the stocks were held by the households of the foreign country. This means that the restriction that firms are owned by the domestic households is without loss of generality.

Using the no-Ponzi-games condition, the budget constraints of the household, (88a) and (89), can be consolidated into the single budget constraint,

$$\sum_{t=0}^{\infty} Q_{it}\left[q_{it}c_{it} - \left(1-\tau^n_{it}\right) w_{it}n_{it}\right] = \left(1 - l_{i0}\right) a_{i0},$$

where

$$(97)\quad a_{i0}=(1-\tau_{i0})(V_{i0}+d_{i0})+\tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}}+\left[1+r_0^f-\tau_{i0}\left(1+r_0^f-\frac{q_{i0}}{q_{i-1}}\right)\right](b_{i0}+f_{i0}).$$

Using (87) as well as $s_0=1$, the initial asset holdings in (97) can be written as

$$\begin{aligned} a_{i0} &= (1-\tau_{i0})\,q_{i0}\left[1+\left(1-\tau_{i0}^k\right)\left(G_{i,0}^i F_{ik,0}-\delta\right)\right]k_{i0}+\tau_{i0}\frac{q_{i0}V_{i-1}}{q_{i-1}} \\ &\quad +\left[1+r_0^f-\tau_{i0}\left(1+r_0^f-\frac{q_{i0}}{q_{i-1}}\right)\right](b_{i0}+f_{i0}) \end{aligned}$$

The interest rate parity condition is obtained from

$$\frac{Q_t}{Q_{t+1}}=\frac{q_{it+1}}{q_{it}}\left[1+\left(1-\tau_{it+1}^k\right)\left(\frac{p_{it+1}}{q_{it+1}}F_{k,t+1}^i-\delta\right)\right]$$

for $i=1,2$, or

$$\frac{q_{1t+1}}{q_{1t}}\left[1+\left(1-\tau_{1t+1}^k\right)\left(\frac{p_{1t+1}}{q_{1t+1}}F_{k,t+1}^1-\delta\right)\right]=\frac{q_{2t+1}}{q_{2t}}\left[1+\left(1-\tau_{2t+1}^k\right)\left(\frac{p_{2t+1}}{q_{2t+1}}F_{k,t+1}^2-\delta\right)\right].$$

Using the first order conditions of the firms to replace the relative prices of the intermediate and final goods, it follows that

$$\begin{aligned} (98)\qquad & \frac{G_{j,t}^1}{G_{j,t+1}^1}\left[1+\left(1-\tau_{1t+1}^k\right)\left(G_{1,t+1}^1F_{k,t+1}^1-\delta\right)\right] \\ = \; & \frac{G_{j,t}^2}{G_{j,t+1}^2}\left[1+\left(1-\tau_{2t+1}^k\right)\left(G_{2,t+1}^2F_{k,t+1}^2-\delta\right)\right],\ \text{for } j=1,2. \end{aligned}$$

To get production efficiency, that is, to satisfy (8), we need either to set the two tax rates to zero or to pick τ_{1t+1}^k and τ_{2t+1}^k according to

$$\begin{aligned} & \tau_{1t+1}^k\left(G_{1,t+1}^1F_{k,t+1}^1-\delta\right) \\ = \; & \tau_{2t+1}^k\left(G_{1,t+1}^1F_{k,t+1}^1-\delta-\left(\frac{G_{j,t+1}^1/G_{j,t+1}^2}{G_{j,t}^1/G_{j,t}^2}-1\right)\right),\ \text{for } j=1,2. \end{aligned}$$

Using the intertemporal condition of the household (91), and

$$\frac{Q_{it}}{Q_{it+1}} = (1 - \tau_{it+1}) \frac{Q_t}{Q_{t+1}} + \tau_{it+1} \frac{q_{it+1}}{q_{it}}$$

obtained from (90a), together with $\frac{Q_t}{Q_{t+1}} = 1 + r^f_{t+1}$, and combining it with the firm's condition (86), together with the first order conditions of firms production decisions, we obtain

$$\text{(100)} \quad \frac{u^i_{c,t}}{\beta u^i_{c,t+1}} = 1 + (1 - \tau_{it+1}) \left(1 - \tau^k_{it+1}\right) \left(G^i_{i,t+1} F^i_{k,t+1} - \delta\right).$$

The marginal conditions in this economy can be summarized by

$$\text{(101)} \quad -\frac{u^i_{c,t}}{u^i_{n,t}} = \frac{1}{(1 - \tau^n_{it}) G^i_{i,t} F^i_{n,t}},$$

the intertemporal condition (100), the interest rate parity condition (98), and condition (7), for all $t \geq 0$.

The Ramsey allocation can be implemented with a (possibly time-varying) common tax on home and foreign assets. Corporate income taxes in both countries either must be set to zero or must be set according to the difference in real returns in the goods of the two countries to ensure production efficiency. For standard macro preferences, all the taxes on assets are set to zero and the labor income tax is constant over time. In this economy with a common tax on domestic equity and foreign returns, firms use a common price to value dividends. If relaxed, the restriction that firms are owned by the domestic residents would not change the implementable allocations.