# A Theory of Falling Growth and Rising Rents

Philippe Aghion,   Antonin Bergeaud

Timo Boppart,   Peter J. Klenow,   Huiyu Li*

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### Abstract

Growth has fallen in the U.S., while firm concentration and profits have risen. Meanwhile, labor's share of national income is down, mostly due to the rising market share of low labor share firms. We propose a theory for these trends in which the driving force is falling firm-level costs of spanning multiple markets, perhaps due to accelerating IT advances. In response, the most efficient firms spread into new markets, thereby generating a temporary burst of growth. Because their efficiency is difficult to imitate, less efficient firms find their markets more difficult to enter profitably and innovate less. Even the most efficient firms do less innovation eventually because they are more likely to compete with each other if they try to expand further.

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# 1.  Introduction

Recent studies have documented the following patterns in the U.S. economy over the past several decades:[1]

1.  Falling "long run" growth (interrupted by a temporary burst of growth)

2.  Falling labor share due to rising revenue shares of low labor share firms

3.  Rising firm concentration within industries at the national level

In this paper we construct a theory of endogenous growth with heterogeneous firms which speaks to these facts. There are two main sources of heterogeneity in our model.  First, *product quality* which improves endogenously on each product line through innovation and creative destruction. Second, *process efficiency*, which is time-invariant and is unevenly distributed across firms in the economy.  High process efficiency firms command a higher markup than low productivity firms, conditional on having the same product quality advantage over their competitors.

A possible source of persistent differences in process efficiency across firms is their organizational capital.  Firms such as Walmart and Amazon have established successful business models and logistics that are evidently hard to copy. Both firms experienced considerable expansion into new geographic and product markets over the past two decades.  Similarly, Amazon and Microsoft have acquired dominant positions in cloud storage and computing due to their logistical advantage over potential competitors.  Such firms have achieved a level of process efficiency which is arguably harder to reverse engineer and build upon than quality, which is more observable.

Our story is that the IT (Information Technology) wave in the 1990s has allowed high productivity firms to extend their boundaries — to expand over a wider set of product lines.  We model the IT wave as a downward shift in the

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[1]We discuss papers presenting evidence on these patterns in the next section.

overhead cost $c(n)$ of running $n$ product lines. This cost is assumed to be convex in $n$, which puts a brake on the quality innovation (creative destruction) efforts of high process efficiency firms. Since high productivity firms enjoy higher profits per product line, while sharing the same overhead cost function and cost of R&D inputs as low productivity firms, the downward shift in the overhead cost schedule will allow high productivity firms to expand to a higher fraction of lines. The expansion of high productivity firms fuels a temporary surge in aggregate productivity growth — both because they innovate to take over more markets (bringing quality improvements) and because they apply their superior process efficiency to those additional markets. However, in the long run the fall in overhead cost may lead to a productivity slowdown at the same time as it increases aggregate markups and reduces the aggregate labor share.

Since high productivity firms have higher markups and lower labor shares on average across their product lines, their expansion into more lines will indeed result in an increase in the aggregate markup and a reduction of the aggregate labor share. This is entirely driven by firm composition rather than within-firm changes. Within-firm average markups can actually fall, as the quality leader on a product line is more likely to face a high process efficiency incumbent, limiting their markups whether they are a high or low process efficiency firm.

And the expansion of high productivity firms into more lines eventually deters innovation. This is because innovating on a line where the incumbent firm has high productivity yields lower profits than when the incumbent firm is a low productivity firm. This results in lower within-firm markups. Both high and low productivity firms eventually curtail their efforts at creative destruction, knowing they will face stiffer competition. This can outweigh the positive direct effect of a downward shift in the overhead cost on R&D incentives, such that long run innovation and productivity growth may fall. A drop in long run growth leads to a lower pace of job reallocation, which is tied

to creative destruction.

We calibrate our model to gauge the strength of the model's mechanism. We choose parameter values to fit the pre-IT revolution period (1949–1995) on the level of concentration, productivity growth, aggregate markup, the real interest rate, the intangible investment rate and the correlation across firms between their labor share and their sales share. We then scale down the overhead cost to match the between component of the decline in labor share. With such decline in overhead costs, the model can generate the full extent of the slowdown growth seen in recent years and two-thirds of the decline in aggregate labor share.

Most closely related to our paper are Akcigit and Ates (2019) and Liu, Mian and Sufi (2019), who both study declining growth and rising concentration. We differ from these papers by emphasizing the IT wave and its effect on high productivity versus low productivity firms as the driving force. These papers emphasize declining imitation rates or declining real interest rates as driving forces behind the productivity slowdown. Our contribution is to develop a growth model with persistent firm heterogeneity which generate opposite trends for labor share and markups within versus across firms.

Our paper also relates to Chatterjee and Eyigungor (2017) and Hopenhayn et al. (2018), which study the rise in concentration; to recent papers on declining labor share, in particular Karabarbounis and Neiman (2013, 2018), Barkai (2016), Koh, Santaeulalia-Llopis and Zheng (2016), Eggertsson, Mehrotra, Singh and Summers (2016), Kehrig and Vincent (2017), Autor, Dorn, Katz, Patterson and Van Reenen (2017), Martinez (2017), and Farhi and Gourio (2018). Whereas Autor et al. (2017) looked at labor share in U.S. Census data, Baqaee and Farhi (2017) and De Loecker and Eeckhout (2017) estimate markups in Compustat firms. These latter papers decompose the recent evolution of of the aggregate markup into within-firm and between-firm components. They find the dominant contributor to be the rising market share of high markup firms. We contribute to this literature by providing a

theoretical framework that links the rise in concentration and the rise in average markups (similarly, decline in aggregate labor share) to the slowdown in U.S. growth in recent decades.

The rest of the paper is organized as follows. Section 2. describes the empirical patterns documented by other studies that motivate our modeling effort. Section 3. lays out our model. We first solve analytically for the steady state and perform some comparative statics. We then solve numerically for the transitional dynamics showing that a reduction in overhead cost can lead to a short run boost in productivity growth followed by a slowdown. In section 4. we calibrate the model to see whether it can generate a realistic decline in long run growth. Section 5. concludes.

## 2.  Stylized facts

**Fact 1: Falling "long run" growth (after a burst of growth)**    Figure 1 presents U.S. annual TFP growth over subperiods from the U.S. Bureau of Labor Statistics (BLS). Note that the BLS attempts to net out the contribution of both physical and human capital growth to output growth. The BLS sometimes subtracts contributions from R&D and other intellectually property investments; we consistently included this portion in residual TFP growth as part of what we are trying to explain.

The Figure shows growth accelerating from its 1949–1995 average of 1.8% per year to 2.8% per year from 1996–2005, before falling to just 1.1% per year from 2006–2017. Fernald, Hall, Stock and Watson (2017) argue that the recent slowdown is statistically significant and predates the Great Recession. Syverson (2017) and Aghion, Bergeaud, Boppart, Klenow and Li (2019) contend that the slowdown is real and unlikely to be fully attributable to growing measurement errors.

Figure 1: U.S. productivity growth rate
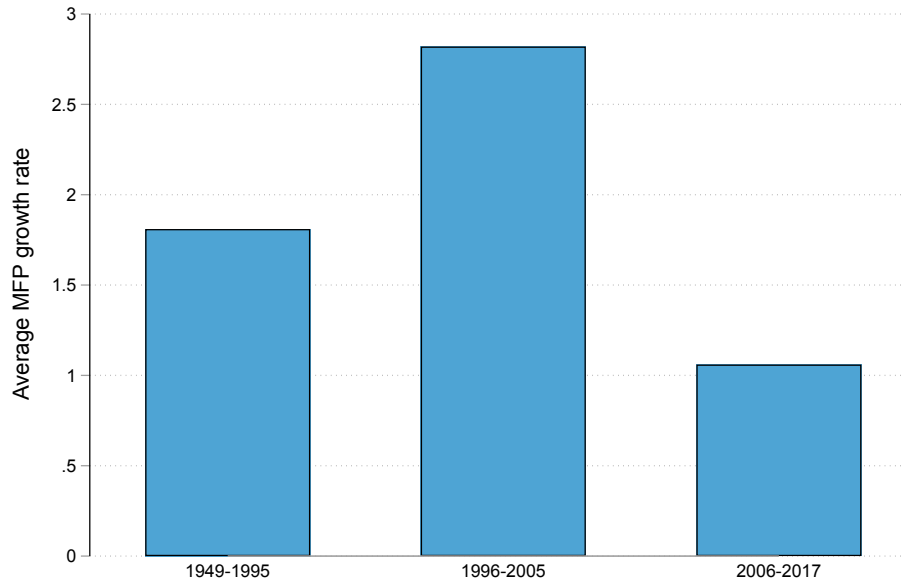
**Fact 2: Falling labor share (mostly due to composition)**    Figure 2 shows that, according to the BLS, the aggregate U.S. labor share of output in the nonfarm business sector fell about six % points in the last two decades.[2] Autor, Dorn, Katz, Patterson and Van Reenen (2017) find declining labor share in a number of Census sectors, but most sharply in manufacturing. Table 1 reproduces their statistics on the cumulative change in labor share for six Census sectors in recent decades. Finance is the contrarian, with rising labor share. In five of the six sectors the sales shares shifted to low labor share firms, so that the "between" component pushed labor share downward notably.    And within-firm labor shares actually rose in all sectors but manufacturing.

In the business cycle literature, labor share is often used as an inverse

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[2]Since this is the business sector, it is not affected by the Rognlie (2016) critique that the rise of housing is exaggerating the decline in labor share.

measure of price-cost markups. See Karabarbounis (2014) and Bils, Klenow and Malin (2018). Thus one interpretation of falling labor share due to composition effects is that markups are rising due to composition effects. De Loecker and Eeckhout (2017) and Baqaee and Farhi (2017) document precisely that, though with a broader measure of variable inputs that adds intermediates to labor costs. A competing interpretation is that the elasticity of output with respect to capital has risen. Barkai (2016), Gutiérrez and Philippon (2016, 2017), and Farhi and Gourio (2018) argue against this interpretation and in favor of rising markups on the grounds that the investment rate and capital-output ratio have not risen.[3]

### Figure 2: U.S. labor share
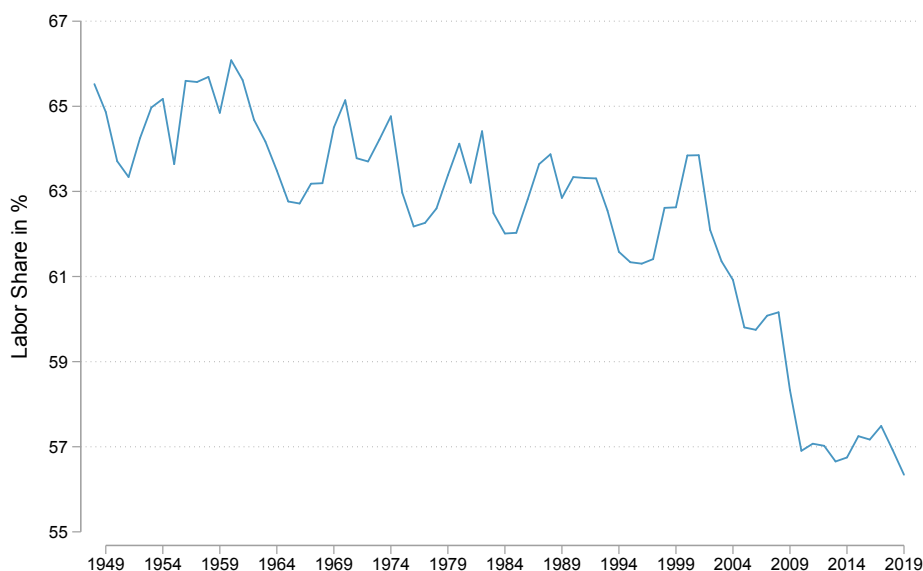


**Source**: BLS. The figure plots the aggregate labor compensation of all employed persons as a share of aggregate output for the nonfarm business sector. The unit is percentage points.

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[3]Koh, Santaeulalia-Llopis and Zheng (2016) and Traina (2018) argue that labor share has not fallen and markups have not increased if one adds intangibles investments such as R&D and marketing. These expenditures are arguably not part of variable costs, in which case their rise may be compatible with rising markups. Moreover, Autor et al. (2017) document falling payroll relative to sales; sales, unlike value added, should not be affected by whether intangibles are expensed or treated as part of value added.

Table 1: Cumulative change in labor share over given period (ppt)

|  | 1982–2012 | | | | 92–12 | 92–07 |
|---|---|---|---|---|---|---|
|  | MFG | RET | WHO | SRV | FIN | UTL |
| $\triangle \frac{\text{Payroll}}{\text{Sales}}$ | -7.01 | -0.79 | 0.19 | -0.19 | 3.25 | -1.89 |
| Within | -1.19 | 3.74 | 4.01 | 2.43 | 6.29 | 0.58 |
| Between | -4.97 | -4.03 | -4.38 | -0.44 | -3.62 | -2.39 |

**Source**: Table 5 in Autor et al. (2017). This is a Melitz-Polanec decomposition of the change in the labor share. The unit is percentage points.

**Fact 3: Rising concentration** Table 2, which is also based on Autor, Dorn, Katz, Patterson and Van Reenen (2017), presents the average 5-year change in top 4 or top 20 firm concentration measures in 4-digit NAICS. These results are, again, from firm-level data in U.S. Census years. Across the six sectors, the top 4 firm shares increase from 0.4 to 2.5 percentage points per five-year period, while the top 20 firm shares increase between 0.8 and 3.6 percentage points per year. Concentration increased the most rapidly in retail and finance, and least rapidly in manufacturing and wholesale.

The rise in concentration in Table 2 is at the national level. In contrast, Rossi-Hansberg, Sarte and Trachter (2018) and Rinz (2018) find that local concentration declined. One explanation for the diverging trends is that the largest firms grew by adding establishments in new locations. Figure 3 shows cumulative growth of the number of establishments per firm, by firm size bins, in the Business Dynamic Statistics from the Census Bureau. The red line is the growth of establishments for the largest firms. It shows that, between 1990 and 2014, the largest firms expanded by adding establishments. The average number of establishments rose for smaller firms too but not as quickly as for the largest firms. In a parallel study, Cao, Hyatt, Mukoyama and Sager (2019) document a similar pattern in the Quarterly Census of Employment and Wages data and Rinz (2018) documents increasing number of markets with at least

one establishment belonging to a top 5 firm. To the extent that growth in the number of establishments is connected to growth in the number of products or markets, this evidence suggests that the rise in national concentration may not reflect an increase in market power of the largest firms.

Table 2: Average 5-year change in national concentration (ppt)

|  | 1982–2012 | | | | 92–12 | 92–07 |
|---|---|---|---|---|---|---|
|  | MFG | RET | WHO | SRV | FIN | UTL |
| Δ Top 4 firms sales share | 0.7 | 2.5 | 0.4 | 0.7 | 2.1 | 1.9 |
| Δ Top 20 firms sales share | 0.8 | 2.7 | 1.0 | 1.0 | 3.6 | 1.2 |

**Source**: Table 1 of Autor et al. (2017). Averages across 4-digit industries, with the industries weighted by industry sales shares.

Figure 3: Establishments per firm by firm size
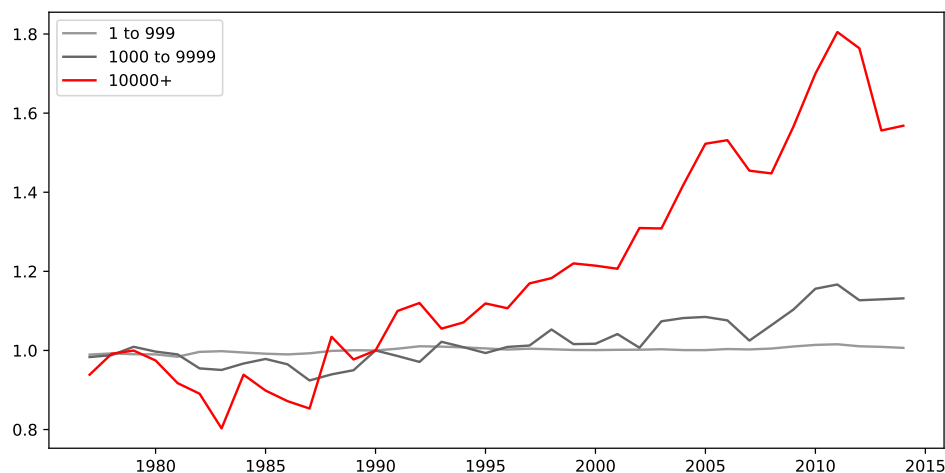


**Source**: U.S. Census Bureau Business Dynamic Statistics. The graph plots the number of establishments per firm within employment bins relative to 1990.

**IT as a driving force** We focus on changes in IT as a possible driver of the patterns described above for three reasons. First, Figure 4 displays growth rate of multi-factor productivity for IT-producing, IT-intensive and non-IT-intensive industries, classified based on Fernald (2015).[4] The figure shows a burst of growth for the IT-intensive sectors in early 2000s after a burst of growth for IT-producing sectors in the second half of 1990. In contrast, the non-IT-intensive sectors did not experience a burst of growth. When comparing the beginning and the end the series the overall productivity slowdown is also more pronounced for the IT-intensive sectors.

Second, using the same allocation of sectors across three groups, we plot the average labor share respectively for IT producing, IT intensive and non IT intensive sectors in Figure 5. Except for a short spike in the early 2000s, we see that the labor share is declining in all case, but the magnitude of this decline is particularly large for IT producing and IT intensive sectors.

Third, price declines for IT goods accelerated sharply for a decade from the mid-1990s to the mid-2000s. See Figure 6. This is in the middle of the period of rising concentration.

Fourth, Crouzet and Eberly (2018) and Bauer, Boussard and Laskhari (2018) document that bigger firms invest a higher share of their sales in intangibles and IT, respectively. Lower costs of IT seems to benefit larger firms more. The former evidence is for U.S. firms and the latter for French firms.

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[4]Industries called "IT producing" are computer and electronic, computer system design and publishing industries. The other not IT producing industries are ranked based on the average value of their IT capital relative to value added over the years 1987 to 1999 and then split into two categories: IT intensive and non IT intensive such that the share of total value added in the two groups is roughly the same. We consider 3 digit sectors spanning the entire business sectors excluding finance and aggregate the MFP growth rates using value added weights.

## Figure 4: Productivity growth by IT intensity
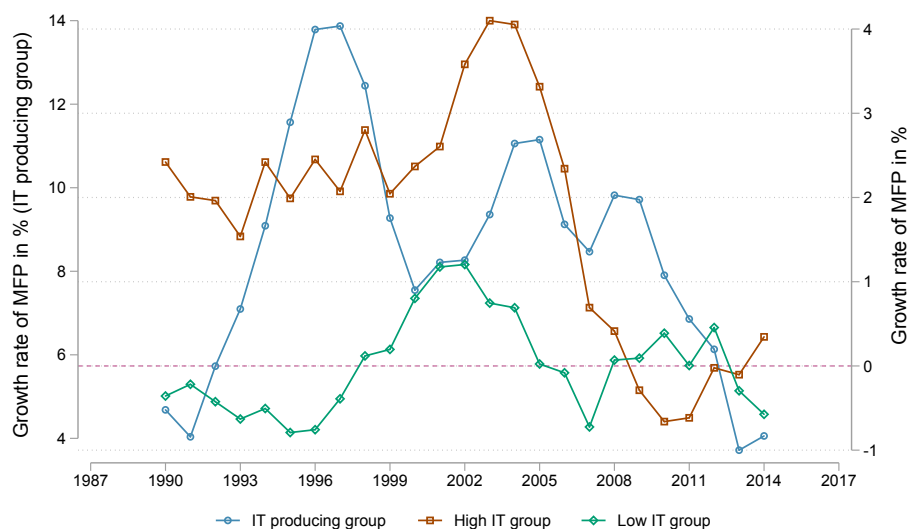


**Source**: Update of Fernald (2015) Fig 6A. % per year, 5-year moving average. MFP data come from the BLS multifactor productivity series. We calculate yearly productivity growth rate by adding R&D and IP contribution to BLS MFP and then converting the sum to labor augmenting form. Growth rates are aggregated using value added weights constructed from the same source.

## Figure 5: Labor share by IT intensity
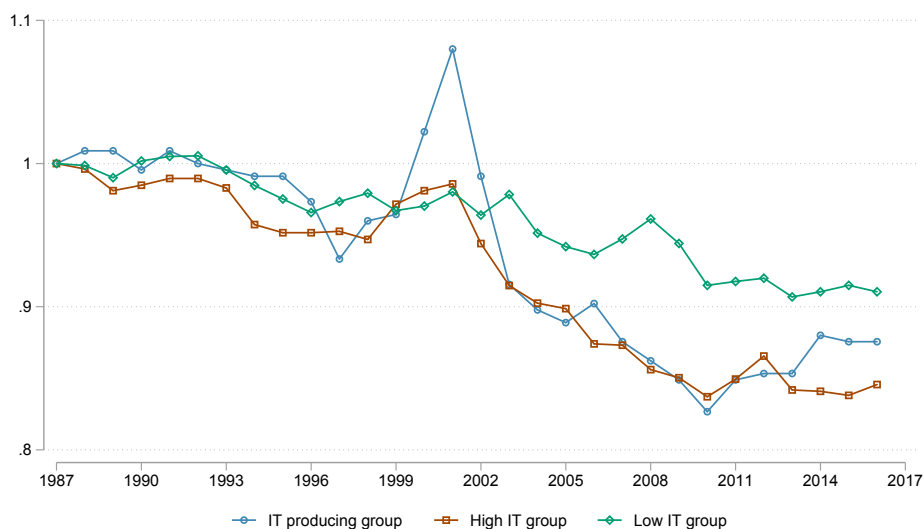


**Source**: Labor share is taken from the BLS production tables. IT groups are the same as in Figure 4. Labor share is standardized to 1 for each group in 1987.

## Figure 6: Relative price of IT
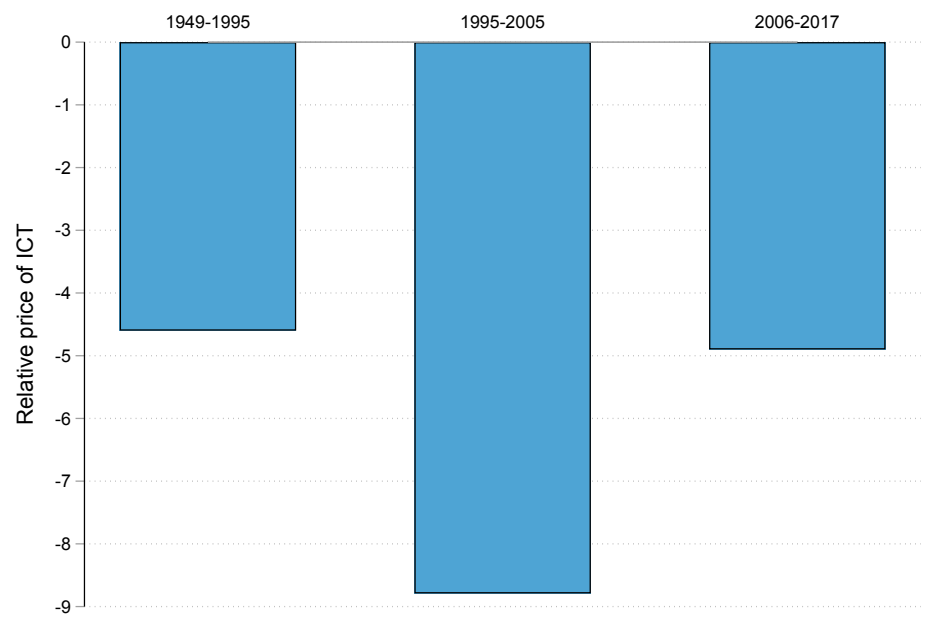


**Source**: BEA. % Change per year in the price of IT relative to the GDP deflator.

# 3. Model

The above evidence on the opposite trends in between vs. within firm labor income shares, asks for a model with persistent firm heterogeneity. Moreover, in order to account for the observed burst and then slowdown in productivity growth an model of endogenous growth is needed. In this section we lay out a model which combines these two elements to speak to the facts in Section 2.. The goal is to build a parsimonious theoretical building block which has quantitative bite, can be generalized in various ways and which ultimately can be used for policy analysis.

## 3.1. Preferences

The household side is relatively standard. Time is discrete and the economy is populated by a representative household who chooses consumption $C$ to maximizes the preferences

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log\left(C_t\right),$$

subject to

$$a_{t+1} = (1 + r_t)a_t + w_t L - C_t, \tag{1}$$

a standard no-Ponzi game condition and a given initial wealth level $a_0 > 0$. Here $a$ denotes wealth, $r$ the interest rate, $w$ the wage rate and $L$ is the labor endowment that is inelastically supplied to the labor market.

The Euler equation resulting from household's optimization is given by

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_{t+1}). \tag{2}$$

## 3.2.  Production of final output

A final output good is produced competitively out of a unit interval of intermediate inputs according to the following Cobb-Douglas technology

$$Y = \exp\left(\int_0^1 \log\left[q(i)y(i)\right]di\right).\tag{3}$$

Here $y(i)$ denotes the quantity and $q(i)$ the quality of product $i$. This structure yields demand for each product $i$ as

$$y(i) = \frac{YP}{p(i)},\tag{4}$$

where we defined the aggregate price index

$$P \equiv \exp\left(\int_0^1 \log\left[p(i)/q(i)\right]di\right),\tag{5}$$

which we will in the following normalize to one in each period. As we will see below, each line $i$ will be operated by one firm $j$ at a point in time. In the following we will denote the quality on line $i$ in the period when firm $j$ operates that line as $q(i, j)$.

## 3.3.  Production and market structure for intermediate inputs

There are $J$ firms indexed by $j$. In the following we assume that $J$ is "large" such that firms take the aggregate price index in the economy as given. Each firm $j$ has the knowledge to produce quality $q(i, j) \geq 0$ in a specific market $i \in [0, 1]$. There are two sources of heterogeneity across firms: (i) heterogeneity in the firm-market specific quality $q(i, j)$ which evolves endogenously as a result of innovation in the quality dimension; (ii) permanent heterogeneity in firm-specific process efficiency.

We first describe the heterogeneity in the process efficiency. There is a firm-specific level of process efficiency denoted by $\varphi(j)$. A firm with process

efficiency $\varphi(j)$ can produce in any line $i$ with the following linear technology

$$y(i,j) = \varphi(j) \cdot l(i,j), \tag{6}$$

where $l(i,j)$ denotes labor used by firm $j$ to produce in line $i$ and $y(i,j)$ denotes the output of this firm in this line. We assume that the heterogeneity in process efficiency is persistent over time reflecting, e.g., differences in organizational capital that is hard to copy. This heterogeneity in process efficiency will lead in our model to persistent differences in revenue TFP, labor income shares, and markups across firms.

Note that the linear technology in (6) applies irrespective of the specific quality $q(i,j)$ firm $j$ produces in line $i$. In addition to the heterogeneity in process efficiency, firms differ in their product quality. We will explain below how the distribution of quality of the different lines across firms changes endogenously due to innovations. But for the static firm problem here we just assume that in a period $t$ each firm $j$ can produce at some line-specific quality $q(i,j)$. Labor is fully mobile such that the wage rate equalizes across firms. Hence, the marginal cost of firm $j$ per output unit of line $i$ is given by $w/\varphi(j)$, or the marginal cost per quality-adjusted unit of line $i$, $q(i,j)y(i,j)$, is equal to $\frac{w}{q(i,j)\varphi(j)}$.

## 3.4. Pricing

In each market all the firms engage in Bertrand competition. This means that only the firm with the highest quality-adjusted productivity $q(i,j) \cdot \varphi(j)$ is active in a given market $i$ and sets the price such that the firm with the second highest quality-adjusted productivity finds it not profitable to be active. In the following we denote the index of the leading firm in line $i$ by $j(i)$ and the second-best firm by $j'(i)$. Hence, the quality-adjusted productivity of the "leader" is given by $q(i,j(i)) \cdot \varphi(j(i))$ and the same quality-adjusted productivity of the second-best firm in line $i$ is given by $q(i,j'(i)) \cdot \varphi(j'(i))$. The price setting behavior of this

leader is constrained by the second-best firm and the leader will set its quality-adjusted price equal to the quality-adjusted marginal cost of the second-best firm. Formally, we then have

$$\frac{p(i, j(i), j'(i))}{q(i, j(i))} = \frac{w}{q(i, j'(i)) \cdot \varphi(j'(i))}. \tag{7}$$

Note that the equilibrium price in line $i$ depends on the process efficiency of both the leader and the follower as well as the quality difference between the two. We define the markup in line $i$, $\mu(i)$, as the ratio between the price of a unit divided by the marginal cost of the producer. The markup is then given by

$$\mu(i, j(i), j'(i)) \equiv \frac{p(i, j(i), j'(i))}{w/\varphi(j(i))} = \frac{q(i, j(i)) \cdot \varphi(j(i))}{q(i, j'(i)) \cdot \varphi(j'(i))}. \tag{8}$$

The markup of a product increases in the quality gap $\frac{q(i,j(i))}{q(i,j'(i))}$ as well as in the process efficiency gap $\frac{\varphi(j(i))}{\varphi(j'(i))}$ between the leader and the second-best firm. All else equal, the product level markup is increasing with the process efficiency of the leading firm $\varphi(j(i))$ and decreasing in the process efficiency of the second-best firm $\varphi(j'(i))$. Within a firm the markup differs across product lines but a firm with a higher process efficiency will charge on average a higher markup.

Given the markup $\mu(i)$ the operating profits in a period of the leader in line $i$ follows directly. Combining the pricing with the demand (48), using the definition of the numéraire gives for the operating profits (before overhead) $Y\left(1 - \frac{1}{\mu(i)}\right)$.

## 3.5. Innovation and productivity growth

The quality distribution evolves endogenously over time as a result of innovation. Any firm $j$ can engage in R&D activity to acquire a patent to produce in a new randomly drawn line at higher quality. More specifically, by investing $x_t(j)\psi_c Y_t$ units of final output in R&D in period $t$, $x_t(j)$ product lines are randomly drawn among the lines in which firm $j$ is currently not actively

producing. In such a randomly drawn line $i$ the highest existing quality $q(i)$ is multiplied by a factor $\gamma > 1$ and the innovating firm $j$ obtains a perpetual patent to produce at this higher quality level from the next period $t + 1$ onward.

The initial distribution of quality levels across lines and firms is exogenously given. In each line the firm with the highest quality will face competition by a firm with lower quality by a factor $\gamma$.

We assume that a period is short enough such that no two innovations arrive in the same line in a given period. If we denote the innovation rate of firm $j$ in period $t$ by $x_t(j)$ the aggregate rate of creative destruction is given by

$$z_{t+1} = \sum_{j=1}^{J} x_t(j),\tag{9}$$

i.e., for any given line an innovation arrives in period $t + 1$ with probability $z_{t+1}$. This endogenous quality improvement due to creative destruction is the source of long-run growth in this model.

## 3.6.  Markups with binary process efficiency levels

For simplicity we assume in the following two types of firms. A fraction $\phi$ of all firms are of high process efficiency type $\varphi_H$ whereas the remaining fraction $1 - \phi$ of all firms is of low process efficiency type $\varphi_L$. We denote the productivity differential by $\Delta \equiv \varphi_H/\varphi_L > 1$. So in total there are $\phi J$ high productivity producers and $(1 - \phi)J$ low productivity producers. In this baseline model we further assume $\gamma > \Delta$ so that the firm with the highest quality is always the active leader irrespective of whether she is of high or low process efficiency.

Then, given the two process efficiency levels (high and low) there are four potential cases of markups and operating profits in a given line:

1.  A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a high productivity second-

best firm $\varphi(j'(i)) = \varphi_H$ in line $i$. In this case we have

$$\mu(i) = \gamma, \tag{10}$$

and the profits are $Y\left(1 - \frac{1}{\gamma}\right)$.

2. A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a low productivity second-best firm $\varphi(j'(i)) = \varphi_L$ in line $i$. In this case we have

$$\mu(i) = \Delta\gamma, \tag{11}$$

and profits of $Y\left(1 - \frac{1}{\Delta\gamma}\right)$.

3. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a high productivity second-best firm $\varphi(j'(i)) = \varphi_H$ in line $i$. In this case we have

$$\mu(i) = \frac{\gamma}{\Delta}, \tag{12}$$

and operating profits in this line $i$ are $Y\left(1 - \frac{\Delta}{\gamma}\right)$.

4. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a low productivity second-best firm $\varphi(j'(i)) = \varphi_L$ in line $i$. In this case we have

$$\mu(i) = \gamma, \tag{13}$$

and profits are $Y\left(1 - \frac{1}{\gamma}\right)$.

## 3.7.  Boundary of the firm

Given the constant cost of acquiring a line through innovation and the fact that firms with a high process efficiency make higher expected operating profits in an additional line, high productivity firms have a higher incentive to undertake R&D activity. To prevent the high productivity firms from taking

over all lines and to have instead a well defined boundary of the firm, we assume that firms have to pay an additional per-period overhead cost which is a convex function of the number of lines in which they own the highest quality patent. More specifically, suppose the number of lines firm $j$ owns the highest quality patent is denoted by $n(j)$. We assume a quadratic per-period overhead cost in terms of final output in this number of lines $n(j)$, namely

$$\frac{1}{2}\psi_o n(j)^2 Y, \tag{14}$$

with $\psi_o > 0$. The convexity of the overhead cost in the number of product lines $n(j)$, gives rise to a natural boundary of the firm. High productivity firms will typically operate more lines than low productivity firms, but no firm (type) will operate all lines.

It may be worthwhile to briefly compare our model to Klette and Kortum (2004) which serves as a benchmark in this literature. Here we assume a linear cost of innovating on a new line and convex overhead cost. Consequently, the (expected) marginal value of an additional line in a firm is decreasing in the number of lines, $n(j)$, and this diminishing marginal value defines a natural boundary of the firm. By contrast, Klette and Kortum (2004) assumes a convex cost of acquiring extra product lines through creative destruction, and a non-diminishing value of additional lines.

Our model allows us to do comparative statics with respect to the scalar $\psi_o$, which affects the boundary of firms without altering the technology for undertaking innovations. With IT improvements in mind, we lower $\psi_o$ permanently (for all firms) and study its effect on concentration, labor share and growth during the transition as well as in the new steady state. Another difference with Klette and Kortum (2004), is that we assume that firms operate on a continuum of lines, so that the law of large numbers applies. One consequence is that there is no firm exit in our baseline model.[5]

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[5]In a model extension we consider (gross) entry and exit too.

## 3.8. Labor income shares

This simple version of the model abstracts from physical capital and therefore labor is the only factor in variable production. Furthermore we assumed both R&D expenditure and overhead costs are denominated in final output and are treated as investment as opposed to intermediate inputs. These last two assumptions are made to avoid a mechanical effect of the firm size distribution (and overhead cost) and the overall level of R&D activity on the labor income share. Hence in this framework the aggregate labor income share is simply determined by the distribution of markups across lines.

Because of the Cobb-Douglas technology in final output production, the revenue from each product is equal to $Y$. Then the total variable cost in a line $i$ is equal to

$$wl(i) = \frac{Y}{\mu(i)}.$$

Integrating both sides of the above equation over all $i$ yields

$$wL = Y \int_0^1 \frac{1}{\mu(i)} di.$$

Dividing the above two equations by each other we then get the cost (or employment) share of product line $i$ as

$$\frac{l(i)}{L} = \frac{1}{\mu(i)} \frac{1}{\int_0^1 \frac{1}{\mu(\iota)} d\iota}. \tag{15}$$

The relative cost per line, $l(i)/L$, is inversely proportional to the markup factor per line. This comes from revenue being equalized across lines due to the Cobb-Douglas technology in final production.

Finally, the aggregate labor income share is given by

$$1 - \alpha \equiv \frac{wL}{Y} = \frac{1}{\int_0^1 \mu(i)l(i)/L \, di} = \int_0^1 \mu(i)^{-1} \, di. \tag{16}$$

This is identical to the inverse of the average cost-weighted markup factor. There is no physical capital in this model and the profit share and the labor income share add up to one. However, the aggregate labor share depends non-trivially upon the full distribution of markups across lines. This distribution is determined by the types of the leader and second-best firm across lines.

What about the labor income share at the firm level? Consider firm $j$ with $n(j)$ lines that faces in a fraction $h(j)$ of these lines a high type second-best firm and in the remaining fraction $1 - h(j)$ a low productivity second-best firm. Note that the firm's labor income share in a line $i$ is simply given by $\frac{1}{\mu(i)}$. If firm $j$ is itself of high type, its overall labor income share is given by

$$1 - \alpha(j) = h(j)\frac{1}{\gamma} + (1 - h(j))\frac{1}{\gamma\Delta}. \tag{17}$$

In contrast, if firm $j$ is low type its overall labor income share is given by

$$1 - \alpha(j) = h(j)\frac{\Delta}{\gamma} + (1 - h(j))\frac{1}{\gamma}. \tag{18}$$

Faced by the same share of high type competitors $h(j)$, firms with a higher process efficiency have a lower labor income share (as they can on average charge a higher markup). Hence the model will generate persistent differences in the labor income share across firms.[6] However, since the composition of competitors $h(j)$ is endogenous the model is flexible enough to also generate changes over time in the labor income share *within* firms.

## 3.9. Dynamic firm problem

There are two individual state variables in the firm problem: the number of lines a firm $j$ operates, $n(j)$, and the fraction of high productivity second-best firms firm $j$ faces in these lines $h(j)$. Each firm then chooses in how many new

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[6]See Hsieh and Klenow (2009) and David and Venkateswaran (forthcoming) for evidence on persistent difference in revenue per worker.

lines to innovate upon, $x_t$, to maximize the net present value of future profits. Let us denote by $\pi_H$ and $\pi_L$ the per-period profits of a firm after overhead relative to total output, $Y$, of a high and low type firm operating $n(j)$ and facing competition of high second-best firms in a fraction of $h(j)$ of them. Formally, we have

$$\pi_H(n(j), h(j)) = n(j) - \frac{n(j)h(j)}{\gamma} - \frac{n(j)\left(1 - h(j)\right)}{\gamma\Delta} - \frac{1}{2}\psi_o n(j)^2,$$

and

$$\pi_L(n(j), h(j)) = n(j) - \frac{n(j)h(j)\Delta}{\gamma} - \frac{n(j)\left(1 - h(j)\right)}{\gamma} - \frac{1}{2}\psi_o n(j)^2.$$

Note these are profits divided by output $Y_t$; they only depend on the individual states $n(j)$ and $h(j)$ and are otherwise time invariant. Letting $S_t$ denote the aggregate fraction of lines operated by high productivity firms, the problem of a firm of type $j = H, L$ can be written as

$$V_0 = \max_{\{x_t, n_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Y_t \left[\pi_j(n_t, h_t) - x_t \psi_c\right] \prod_{s=0}^{t} \left(\frac{1}{1 + r_s}\right) \tag{19}$$

subject to

$$n_{t+1} = n_t(1 - z_{t+1}) + x_t, \tag{20}$$

$$h_{t+1}n_{t+1} = h_t n_t(1 - z_{t+1}) + S_t x_t, \tag{21}$$

and a given initial $n_0$ and $h_0$. For completeness there are also a non-negativity constraints $x_t \geq 0$. The constraint (20) states that the number of product lines of a firm tomorrow is equal to the newly added lines $x$ plus the number of lines today times one minus the rate of creative destruction in the economy, $z$. The second constraint (21) states that the number of lines in which the firm faces a high type second-best firm is equal to the number of such lines today times $1 - z$ plus the number of newly added lines times the average fraction of lines operated by high productivity type firms today $S_t$. When optimizing the firm takes the path of output $Y_t$, the interest rate $r_t$, the rate of creative destruction

$z_{t+1}$, and the aggregate fraction of lines operated by high productivity firms $S_t$ as given.

## 3.10.  Market clearing and resource constraints

We close the model with the following market clearing conditions that hold each period. First, final output will be used for consumption, $C$, total overhead cost, $O$, and total R&D expenditure, $Z$, or formally

$$Y = C + O + Z, \tag{22}$$

where

$$O = \sum_{j=1}^{J} \frac{1}{2} \psi_o n(j)^2 Y, \tag{23}$$

and

$$Z = \sum_{j=1}^{J} x(j) \psi_c Y. \tag{24}$$

Labor is only used as a variable input by the producer of different intermediate inputs. Labor market clearing implies

$$L = \sum_{j=1}^{J} \int_0^1 l(j,i)\,di,$$

where $l(j,i)$ denotes labor used by firm $j$ that operates $i$.  Furthermore, asset market clearing requires

$$\sum_{j=1}^{J} V_t(j) = a_t. \tag{25}$$

In addition, we have the equations defining the aggregate share of lines operated by high types as[7]

$$S_t = \sum_{j=1}^{\phi J} n_t(j), \tag{26}$$

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[7]Here we assume that the high productivity type firms are indexed by $j = 1, 2, \ldots, \phi J$.

with an accounting equation that states that all lines are operated by some firm, namely,

$$1 = \sum_{j=1}^{J} n_t(j), \tag{27}$$

and finally an equation that relates output to the distribution of process efficiency, quality levels and markups

$$Y_t = Q_t \frac{\varphi_L \Delta^{S_t} \exp\left[-\int_0^1 \log\left(\mu_t(i)\right) di\right]}{\int_0^1 \left(\mu_t(i)\right)^{-1} di} L, \tag{28}$$

where $Q_t = \exp\left[\int_0^1 \log\left(q_t(i,j)\right) di\right]$ denotes the "average" quality level.

An equilibrium in this economy is then a path of allocations and prices that jointly solve the household's problem, the firms' problems, and is consistent with the market clearing and accounting equations stated above.

There is no free entry in the model and the number of firms is fixed, so firms' profits from selling at a markup over marginal cost will exceed the total investments in R&D and overhead. We call profits after R&D investment and overhead cost "rents".

Since output is a function of the full distribution of markups across product lines the equilibrium path is in general a function of the entire initial distribution of product lines $n(j)$ and the level of competition $h(j)$ across all firms. We can assume that all firms of the same type start out with the same level of $n_0(j)$ and $h_0(j)$.[8] Since the law of large number applies firms of the same type will then be identical along the entire equilibrium path and therefore only two firm problems—one for a high type and one for a low type—need to be solved. The aggregate state vector can then be summarized by $S$, and the shares $h_L$ and $h_H$ of high second-best firms in lines operated respectively by low and high productivity firms.

With the two "representative" types of firms output can be expressed in

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[8]This assumption will be fulfilled if the economy starts from an initial steady state.

terms of the aggregate state variables $(S, h_L, h_H)$. We have $\exp\left[-\int_0^1 \log\left(\mu_t(i)\right) di\right] = \Delta^{(1-S_t)h_{Lt}-S_t(1-h_{Ht})}/\gamma$ and we get for the aggregate labor share

$$\int_0^1 (\mu_t(i))^{-1} di = \frac{1}{\gamma}\left[S_t h_{Ht} + (1-S_t)(1-h_{Lt}) + S_t(1-h_H)\frac{1}{\Delta} + (1-S_t)h_{Lt}\Delta\right].$$

As a consequence, aggregate output can be expressed as

$$
\begin{aligned}
Y_t &= Q_t \cdot L \cdot \varphi_L \Delta^{S_t} \cdot \frac{\Delta^{(1-S_t)h_{Lt}-S_t(1-h_{Ht})}}{S_t h_{Ht} + (1-S_t)(1-h_{Lt}) + S_t(1-h_{Ht})\frac{1}{\Delta} + (1-S_t)h_{Lt}\Delta} \\
&= Y(S_t, h_{Ht}, h_{Lt}). \qquad\qquad (29)
\end{aligned}
$$

Output is therefore the product of four terms. The first term $Q_t$ captures the effect of "average quality". Output is of course also linear in the aggregate labor force $L$. The third term captures the aggregate level of process efficiency (if $S_t = 0$ the productivity is all determined by the low type $\varphi_L$, in contrast if $S_t = 1$ the productivity is all determined by $\varphi_H = \varphi_L\Delta$). Finally, the fourth term on the right-hand side captures the average distortion due to markup dispersion. If $S_t = 1 = h_{Ht}$ or $S_t = 0 = h_{Lt}$ this distortion term is equal to 1 (no dispersion of markups since all markups are equal to $\gamma$ across all lines). In all other cases this third term is smaller than one. We call this third term allocative efficiency.

In Section 3.14. we characterize and numerically solve for the transitional path of the economy. However, before we do so we focus on the steady state this economy converges to as time goes to infinity. We will show below that this steady state takes a very tractable functional form and can be analytically solved for. We then discuss how a permanent drop in $\psi_o$ (triggered by improvements in IT) affects market concentration, labor income shares (within firms as well as on the aggregate), and productivity growth in the long run.

## 3.11. Steady state definition

We define this steady state equilibrium in the following way:

**Definition 1** *A steady state is an equilibrium path along which the interest rate and the gross growth rate of output remain constant, equal to $r^\star$ and $g^\star$, in which a constant fraction of the lines, $S^\star$, is provided by high productivity producers.*

In a steady state the number of products must be equalized across all firms of the same type and must remain constant over time. The number of products however will differ between firms of different types. So all high productivity type firms have $n(j)^\star = n_H^\star$ and all low type firms have $n(j)^\star = n_L^\star$. For the number of lines within firm to be constant, the R&D activity of each firm must be proportional to its number of products, i.e., $x(j)^\star = n(j)^\star z^\star$, where $z^\star$ is the aggregate rate of creative destruction in steady state. Since all firms draw new lines from a stationary distribution, the fraction of high productivity type second-best firms faced is equalized across firms and we have

$$h(j)^\star = S^\star, \ \forall j. \tag{30}$$

Since the markup distribution is stationary in steady state output $Y_t$ is proportional to the average quality $Q_t$ (see equation (28)). Consequently we have

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t} = \gamma^{z^\star} \equiv g^\star. \tag{31}$$

Finally, since total overhead, $O$, total R&D expenditure, $Z$, all grow at the same gross rate $g^\star$ also consumption has to grow at this rate $g^\star$ (see (22)). Then, the Euler equation determines the steady state interest rate as

$$r^\star = \frac{g^\star}{\beta} - 1. \tag{32}$$

Next, we show that solving for the steady state boils down to solving for the quadruple $S^\star, n_L^\star, n_H^\star$, and $z^\star$.

## 3.12.   Steady state characterization

Let us denote by $v$ the value of a firm relative to total output, i.e., $v \equiv V/Y$. In steady state (with $h(j)^\star = S^\star$) the number of products per firm $n$ becomes the only state variable so that we can write $v = v(n)$. All high productivity firms then solve the Bellman equation

$$v_H(n) = \max_{n' \geq 0} \{\pi_H(n, S^\star) - (n' - n(1 - z^\star))\psi_c + \beta v_H(n')\},$$

where we denote its solution as $n' = f_h(n)$.

Similarly, all low productivity firms solve

$$v_L(n) = \max_{n' \geq 0} \{\pi_L(n, S^\star) - (n' - n(1 - z^\star))\psi_c + \beta v_L(n')\},$$

and we denote the solution as $n' = f_L(n)$.

The two accounting equations (26) and (27) become in steady state

$$S^\star = n_H^\star \phi J, \tag{33}$$

and

$$n_H^\star \phi J + n_L^\star (1 - \phi) J = 1. \tag{34}$$

Finally, in steady state we must have

$$n_H^\star = f_H(n_H^\star) \tag{35}$$

and

$$n_L^\star = f_L(n_L^\star). \tag{36}$$

These equations fully characterize the steady state.

We formally characterize the steady state solution in the next proposition.

**Proposition 1** *The steady state is a quadruple* $(n_H^\star, n_L^\star, S^\star, z^\star)$ *that fulfills*

$$\phi J n_H^\star = S^\star \ and \ (1 - \phi) J n_L^\star + \phi J n_H^\star = 1, \tag{37}$$

*as well as the following research arbitrage equations for high and low productivity firms respectively:*

$$\psi_c = \frac{1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta) - \psi_o n_H^\star}{1/\beta - 1 + z^\star}; \tag{38}$$

$$\psi_c = \frac{1 - S^\star\Delta/\gamma - (1 - S^\star)/\gamma - \psi_o n_L^\star}{1/\beta - 1 + z^\star}. \tag{39}$$

**Proof.** The first-order condition for the high type Bellman equation is simply

$$\psi_c = \beta \frac{\partial v_H(n')}{\partial n'}. \tag{40}$$

Using the envelope theorem we get

$$\frac{\partial v_H(n)}{\partial n} = 1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta) - \psi_o n + (1 - z^\star)\psi_c. \tag{41}$$

Using the fact that $n' = n_H^\star$ in steady state equilibrium yields the research arbitrage equation of the high type firm. The research arbitrage equation of the low type firm is derived in an analog way. ■

The intuition for the two research arbitrage equations is straightforward: The optimality condition states that the marginal cost of innovating in a line, $\psi_c$ is in equilibrium equal to the marginal (expected) value of having an additional line. This marginal value consists in the case of the high type firm of the marginal profit $1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta)$ minus the marginal overhead cost $\psi_o n$ divided by the denominator $1/\beta - 1 + z^\star$ because there is time discounting and because there is a probability $z^\star$ of loosing the additional line again in each future period.

Equations (37)–(39) are four equations in the four unknowns $(n_H^\star, n_L^\star, S^\star, z^\star)$ and can be solved explicitly. All the other endogenous variables then follow immediately. We have the following steady state results:

**Proposition 2** *In steady state (i) the share of lines operated by high productivity firms is equal to*

$$S^\star = \phi + \frac{(\Delta - 1)\phi + 1}{(\psi_0 \Xi - 1)(\Delta - 1)}, \tag{42}$$

*where* $\Xi \equiv \frac{\Delta\gamma}{J(\Delta-1)^2\phi(1-\phi)}$.

*(ii) High productivity firms operate more lines than low productivity types, i.e.,*

$$n_H^\star > n_L^\star. \tag{43}$$

*(iii) The labor income share of a high type firm is given by*

$$1 - \alpha_H^\star = S^\star \frac{1}{\gamma} + (1 - S^\star)\frac{1}{\gamma\Delta} \tag{44}$$

*which is strictly smaller than the labor income share of a low type firm*

$$1 - \alpha_L^\star = S^\star \frac{\Delta}{\gamma} + (1 - S^\star)\frac{1}{\gamma}. \tag{45}$$

**Proof.** The solution for $S^\star$ follows immediately from the system (37)–(39). For the difference in the number of products we get

$$n_H^\star - n_L^\star = \frac{(S^\star - \phi)}{J\phi(1-\phi)} > 0.$$

The labor income shares follow from (17), (18) and (30). ∎

In this tractable model one can explicitly solve for $S^\star$. In steady state $S^\star$ can be viewed as a summary statistic of market concentration. It is worthwhile to note is that all the endogenous steady state variables only depend on the ratio $\frac{\psi_o}{J}$ and not on the individual level of $\psi_o$ or $J$.

High process efficiency firms can (on average) charge higher markups. Consequently their incentive to undertake R&D is higher and they run into a steeper area of the convex overhead cost, i.e., operate more line than low process efficiency firms. A corollary of this is that $S^\star > \phi$ since high productivity firms have larger sales than low productivity firms. High productivity firm will also differ in employment but the employment difference is smaller than the sales difference because high productivity firms charge higher markups and hence have lower labor share.

## 3.13. Steady state effects as $\psi_o$ decreases

In this section we consider the effects of a permanent reduction in the overhead cost $\psi_o$. How does the new steady state compare to the old one? We are particularly interested in the changes in the following endogenous variables: (i) market concentration, $S^\star$, (ii) the labor income share at the aggregate level as well as within firms, (iii) and the long-run growth rate.

The next proposition states the comparative static effect with respect to concentration.

**Proposition 3** *Concentration $S^\star$ increases monotonically as $\psi_o$ decreases.*

**Proof.** The comparative static effect follows directly from the expression (42). ∎

The intuition that a fall in $\psi_o$ increases $S^\star$ is the following: With a lower $\psi_o$ a larger size gap $n_H^\star - n_L^\star$ is needed to yield the same difference in the marginal overhead cost between high and low productivity firms. Consequently, high process efficiency firms will operate more lines whereas low productivity firms shrink in size as $\psi_o$ decreases; therefore market concentration goes up.

In the next proposition we turn to the labor income share.

**Proposition 4** *As $\psi_o$ decreases (i) the labor income share within firms increases, (ii) the reallocation of market shares goes in the opposite direction, (iii) as a consequence the aggregate labor income share may increase or decrease.*

**Proof.**    For the within part note that both (44) and (45) are monotonically increasing in $S^\star$ (and $S^\star$ increases as $\psi_o$ falls as demonstrated in Proposition 3). The reallocation effect is simply that, as $S^\star$ increases, sales share of high productivity firms goes up and these firms charge on average higher markups and have a lower labor income share than low productivity firms (comparing (44) with (45)). ∎

The model thus makes very sharp predictions about the labor income shares at the aggregate vs. micro levels. As $S^\star$ increases due to the drop in $\psi_o$, all firms are more likely to face a high productivity firm as second-best competitor on any given line. As a consequence *within* firm the labor income share *increases* (see (44) and (45)) as within firms the markup charged decreases. However, there is sales reallocation across firms that goes the opposite direction. As $S^\star$ increases the high productive firms with a lower labor income share expand and the low productivity firms contract. This between firm effect pushes the aggregate labor income share downwards. As emphasized in Section 2. these within and between firm effects that go in opposite directions are a very salient feature of the U.S. micro data.

In this simple model it is easy to show that the between firm effect dominates so that the aggregate labor income share falls as $\psi_o$ decreases if and only if $S^\star > 1/2$.

Finally, let us analyze the comparative static effects of a reduction in $\psi_o$ on the long-run growth rate in the next proposition.

**Proposition 5** *There are two counteracting effects of a reduction in $\psi_o$ on the long-run growth rate $g^\star$.*

A decrease in $\psi_o$ has a direct positive effect on the incentive to innovate: as the overhead cost decreases the marginal value of operating an additional line increases. However, there is a general equilibrium effect that goes in the opposite direction. Namely by increasing $S^\star$ the reduction in $\psi_o$ reduces the expected markup a firm gets on an additional line (as the risk of facing a high

process efficiency type second best firm went up). This general equilibrium effect decreases the incentive to undertake R&D. Which of the two effects dominates depends on the precise parameters. In Section 4. below we show that the model indeed predicts a productivity slowdown for realistic parameter values. Our theory then predicts that this productivity slowdown should be accompanied by a decreasing rate of creative destruction and consequently by less churning in the labor market as well as a fall in the interest rate.

Overall, qualitatively our theory can generate a productivity slowdown, rising concentration, and opposite changes in the labor income shares within firms and between firms as the outcome of a drop in $\psi_o$ presumably triggered by IT improvements. The next step is to gauge the quantitative size of these effects in a simple calibration. This we will undertake in Section 4..

However, before turning to the calibration we now briefly look at the transition dynamics (showing that our model can generate a burst in productivity growth followed by a long-run slowdown) and we discuss various potential generalizations and extensions of our theoretical framework.

## 3.14. Transition dynamics

### 3.14.1. Burst in growth during transition

So far we have compared between steady states. In the data we saw that the productivity slowdown after the mid-2000s was preceded by a ten-year burst in productivity growth. It is easy to show that, as $\psi_o$ falls, our theory will also generate a burst in productivity growth along the transition. The reason for the burst in growth along the transition is twofold: (i) The general equilibrium force that decreases the incentive to innovate — stiffer competition as $S_t$ increases — is only realized over time. Hence on impact, as $\psi_o$ decreases, the incentive to do R&D increases unambiguously for all firms and therefore quality growth will increase initially; and (ii) the new steady state with a higher $S^\star$ exhibits higher average process efficiency because the efficient firms

operate a larger fraction of the product lines. This static efficiency gain must be realized along the transition, leading yet again to high growth along the transition.

### 3.14.2. Numerical illustration

To illustrate, the possibility of a productivity burst followed by a slowdown, we compute the transition dynamics for a simple parametrization.[9] We set the model parameters $\phi = 0.008$, $\gamma = 1.277$, $\psi_c = 1.014$, $\beta = 0.957$, $\Delta = 1.215$ and the initial $\psi_o^0 = 0.002$, in order to match moments we will discuss in Table 3 below. We choose the change in the overhead cost $\psi_o$ to exactly match the decline in the observed growth rate.

Figure 7 displays the share of lines operated by the high type firms ($S_t$) and the rate of creative destruction ($z_{t+1}$) after the overhead cost parameter $\psi_o$ declines in year 0. $S_t$ rises sharply and converges to the new steady state after around 10 years. On impact, there is a sharp increase in the rate of creative destruction, as all firms (but especially the high process efficiency ones) invest more in R&D. But the rise in creative destruction is short-lived: the rate of creative destruction converges to its new, lower steady state level after around 10 years.

Compared to $S_t$ and $z_{t+1}$, the transition of the shares of *second-best* producers $h_H$ and $h_L$ is much slower (see Figure 8). It takes over 100 years (!) for this distribution to converge to the new steady state. The identity of second-best producers affects the distribution of markups. Hence the slow transition of labor shares (Figure 9), which are inversely related to markups in our model.

Figure 10 plots allocative efficiency and process efficiency at the aggregate level, as defined in (29), along the transition. Allocative efficiency rises slightly (about a quarter of a percent) because markup dispersion falls as the most productive firms grab a dominant share of products. Process efficiency rises by

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[9]See Appendix B for a description of the computational method we employ.

Figure 7: Transition dynamics of $S_t$ and $z_{t+1}$
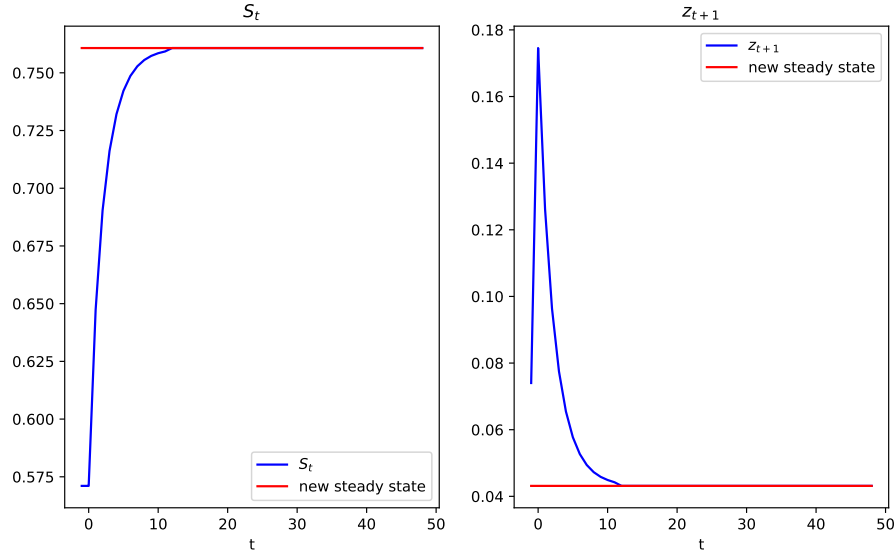


Figure 8: Transition dynamics of $h_t$
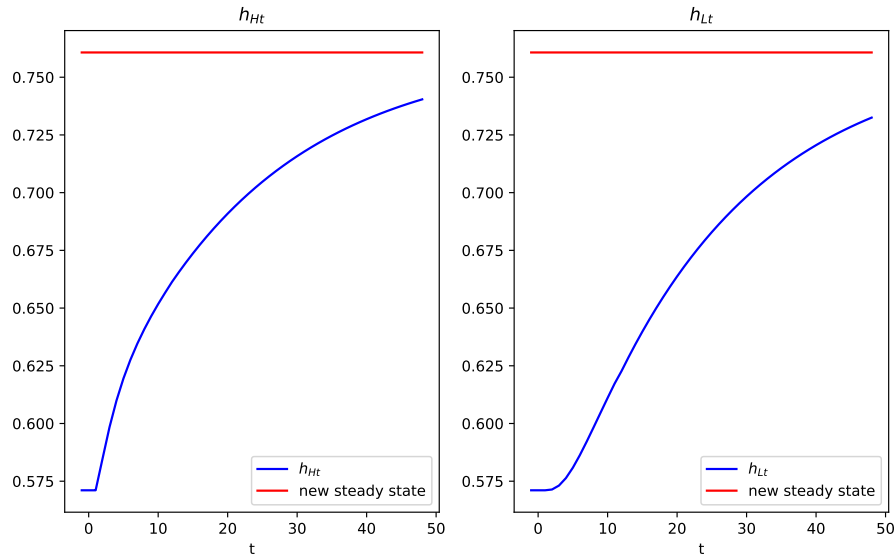


4% for the same reason. Allocative efficiency depends on the distribution of the *second-best* producers and hence converges to the new steady state slowly. In contrast, process efficiency depends only on the share of lines operated by the high productivity firms ($S_t$) and converges quickly to the new steady state.

Figure 9: Labor share along the transition



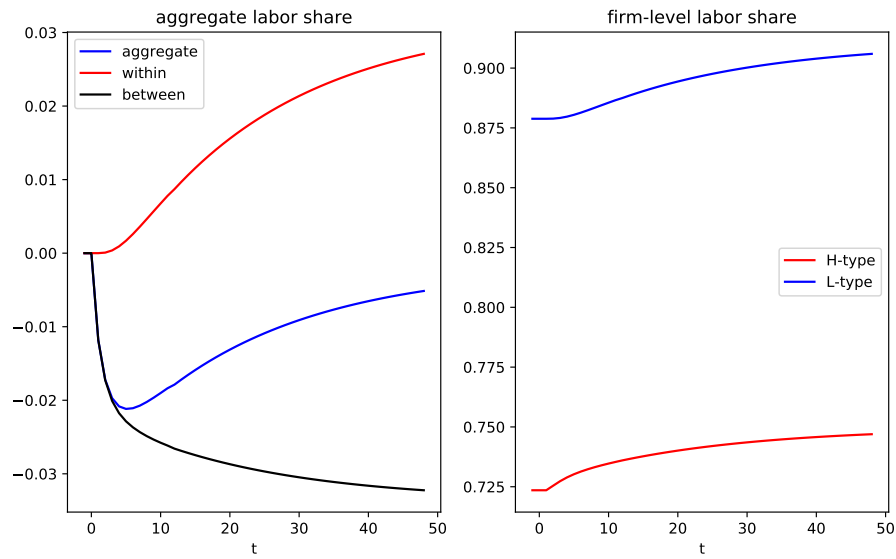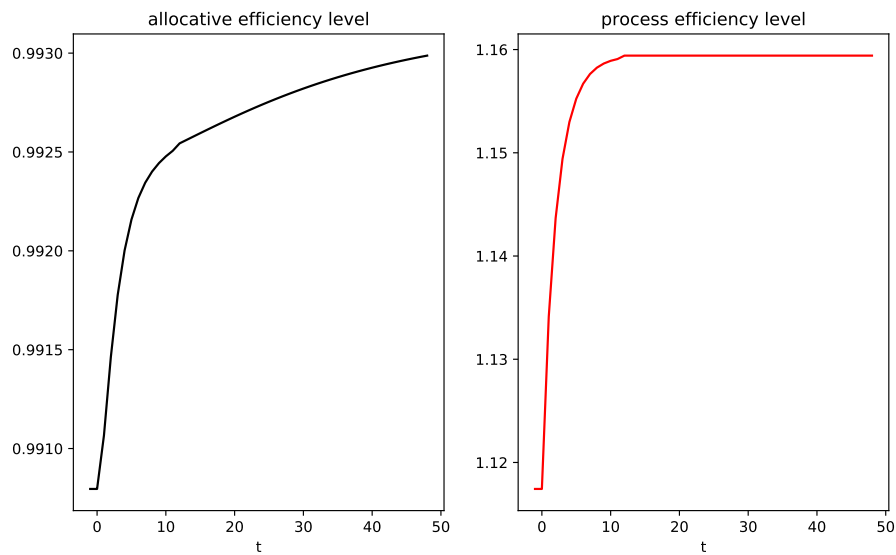Figure 10: Process and allocative efficiency along the transition



Finally, Figure 11 compares the path of output and consumption following the reduction in $\psi_o$ with their initial steady state path (with an unchanged $\psi_o$). Following the drop in $\psi_o$ output grows faster for the first five years but grows more slowly thereafter. Consumption drops sharply in the first period as firms

increase their R&D and overhead investments. Consumption then recovers and is above the initial steady state path for more than a decade, due to the temporary burst in growth. Eventually the slowdown in innovation and growth takes its toll and consumption falls below its old steady state trajectory.

Figure 11: Output and consumption along the transition
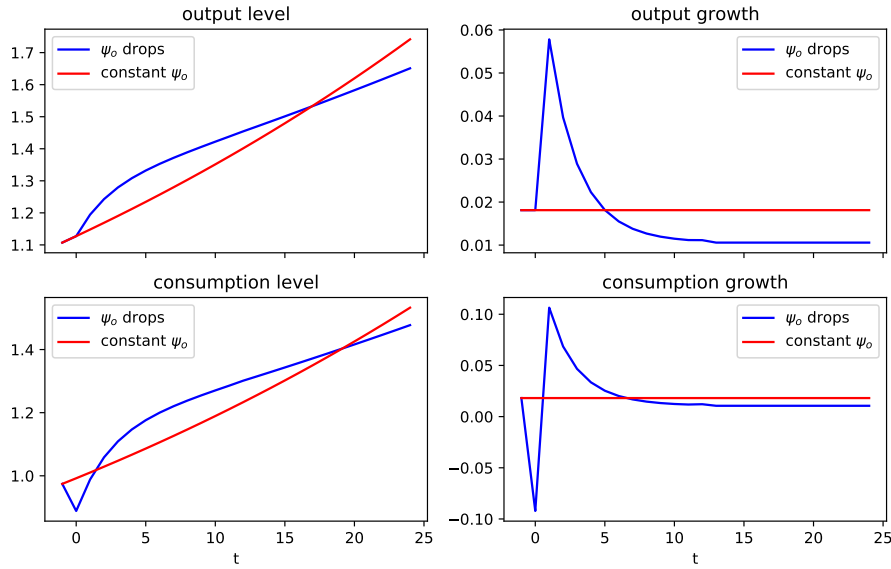


The drop in $\psi_o$ raises consumption growth in the short run but reduces consumption growth in the long run. Hence it is natural to ask whether present discounted welfare is higher or lower because of the drop in $\psi_o$. Recall that utility from a consumption path is given by

$$U(\{C_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \ln C_t.$$

The change in welfare can therefore be evaluated in (permanent) consumption-equivalent terms, $\lambda$, using

$$U(\{(1+\lambda)C_t^{old}\}_t) = \frac{\ln(1+\lambda)}{1-\beta} + U(\{C_t^{old}\}_t) = U(\{C_t^{new}\}_t).$$

where $\{C_t^{new}\}_t$ and $\{C_t^{old}\}_t$ are paths of consumption with and without a drop

in $\psi_o$. We obtain $\lambda = -5.1\%$, so that the decline in $\psi_o$ *lowers* welfare by about 5%. This numerical example illustrates that, despite the permanent boost in allocative and process efficiency and the temporary boost in innovation, the drop in long-run innovation dominates so that overall welfare is reduced.

## 3.15. Theoretical extensions

The baseline model we laid out here is kept very parsimonious to show the minimum ingredients need to speak to the empirical facts in Section 2. However, this tractable model can be augmented and generalized in various ways in order to make the theory more quantitative and without changing the key mechanism at work. Here we elaborate on some potential extensions.

The binary process efficiency is imposed to keep the structure as simple as possible. It is straightforward to generalize it for instance to a continuous distribution with and upper and lower bound. Then in general the whole (stationary) distribution will matter for the steady state and a simple sufficient statistic like $S^\star$ will not exist anymore.

One could also allow for some transition matrix between the process efficiency levels. It is not important that this variable is here assumed to be permanent. What is however important is that there is some persistence in the $\varphi(j)$ differences.

Another generalization we have analyzed is to relax the assumption $\gamma > \Delta$. With $\gamma < \Delta$ high productivity firms are less likely to be replaced by creative destruction since they remain the leader even if a low productivity type innovated upon them in the quality space. This then leads to a more dispersed markup distribution even with just two type of process efficiency. For instance with $\gamma^2 > \Delta$, high productivity firms can have a markup factor in a given line of either $\gamma$, $\Delta\gamma$, or $\Delta/\gamma$ whereas the low productivity type firms can have a markup of $\gamma$ or $\gamma^2/\Delta$.

The quadratic functional form of the overhead function gives rise to this

simple linear-quadratic dynamic programming problem with a closed form solution. This property is maintained by adding an additional linear effect of $n(j)$ on overhead cost. The overhead function could be generalized to any convex function.

Other generalizations are likewise straightforward, such as CES aggregation instead of Cobb-Douglas aggregation, or having CRRA preferences rather than log utility. Appendix A incorporates these more general cases. With a CES elasticity of substitution $\sigma > 1$ the high productivity firm facing a low productivity second-best firm may no longer be constrained and instead simply charge the monopoly markup $\frac{\sigma}{\sigma-1}$. In this event, the labor share within high productivity firms will decrease less as $\psi_o$ decreases.

Since all firms operate an interval of lines of measure $n(j)$ firms will not lose all lines at once and consequently there is no firm exit in equilibrium. However we also analyzed a variant of the model where there are additional "small" firms that operate only one line. These firms exit when creative destruction occurs in the one line they operate. Then, as the rate of creative destruction decreases with the productivity slowdown so will gross firm exit and entry.

We also considered a version of our model where firms can create new varieties. Then, as the span of control increases, more varieties are created. As a result R&D expenditures per variety fall, reinforcing the productivity slowdown of our baseline theory.

The baseline model here abstracts from physical capital. It is however straightforward to include physical capital by assuming a Cobb-Douglas production function for the variable output. The model would then predict that the physical capital share declines together with the labor income share (and the profit share goes up).

Finally, we analyzed a version of our model in which we allow for mergers and acquisitions. This extended model predicts increased M&A activity during the transition. Moreover, allowing for M&A magnifies the long-term productivity slowdown as a reduction $\psi_o$ results in even larger increase in $S^\star$.

# 4. Calibration

While the Cobb-Douglas model is useful for illustration, it features limit pricing in all product lines and ties markup to the inverse of labor share. This implies implausibly large markup. Hence we calibrate a more general model with CES production and CRRA utility function. We lay out the general model in Appendix A.

We assess the quantitative importance of the overhead cost mechanism by comparing steady states where the only difference is the overhead cost parameter $\psi_o$. We define the initial steady state period as 1949–1995 and the new steady state period as 2006–2017. We calibrate six parameter values in the model to match six moments in the initial period (or the subset with available data). We then vary $\psi_o$ to match the change in the between component of labor share between the initial and new steady state periods. We evaluate the fit of the model by comparing the model with the data on changes in concentration, productivity growth, aggregate labor share, and the intangible investment share.[10]

The six moments from the initial period we use to calibrate are: 1) top 10% concentration (share of sales going to the largest 10% of firms) within industries over 1987–1992 from Autor et al. (2017); 2) the average annual rate of productivity growth over 1949–1995 from the BLS MFP dataset; 3) the aggregate markup from Hall (2018); 4) the real interest rate from Farhi and Gourio (2018); 5) the intangible investment share of output from Corrado et al. (2012); and 6) the correlation across firms between their labor share and their sales share from Autor et al. (2019).

We match the initial steady state growth rate and the level of the aggregate markup exactly, but give equal weights to all other moments since we do not fit them perfectly. The calibrated parameters are: 1) the initial overhead parameter $\psi_o^0$; 2) the share of high productivity firms $\phi$; 3) the quality stepsize $\gamma$; 4) the R&D

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[10]In the model intangible investments is assumed to be the sum of R&D investments and overhead cost.

cost parameter $\psi_c$; 5) the discount factor $\beta$; and 6) the process efficiency gap $\Delta$ between high-type and low-type firms. We set the elasticity of substitution $\sigma$ to 4 (Redding and Weinstein, 2019) and the CRRA parameter $\theta$ to 2 (Hall, 2009).

Table 3 displays the calibrated parameter values. First, the concentration level is sensitive to the share of high productivity firms $\phi$. If $\phi$ is close to 1, the top 10% share is close to 10%. Lower $\phi$, combined with a sufficiently high $\Delta$, help to match the top 10% concentration in the data. We obtain $\phi = 3.2\%$. These high efficiency firms enjoy an oversized market share because they have about 34% higher process efficiency ($\Delta = 1.34$). Next, the quality step $\gamma$ is sensitive to the growth target, with a higher growth target leading to a higher $\gamma$ estimate. We calibrate it to 1.468. For a given growth rate of the economy, the real interest rate decreases with the discount factor $\beta$. We calibrate $\beta$ to 0.978. $\psi_o$ affects the aggregate markup through the share of products produced by high productivity firms. We calibrate it to 0.020. Finally, the intangible share, which includes R&D investment, helps to pin down $\psi_c$, which scales the cost of R&D.

Table 3: Baseline Parameter Values

| Calibrated | | |
|---|---|---|
| Definition | Parameter | Value |
| 1. share of H-type firms | $\phi$ | 0.032 |
| 2. quality step | $\gamma$ | 1.468 |
| 3. discount factor | $\beta$ | 0.978 |
| 4. initial overhead cost | $\psi_o^0$ | 0.020 |
| 5. R&D costs | $\psi_c$ | 1.665 |
| 6. productivity gap | $\Delta$ | 1.341 |

| Assigned | | |
|---|---|---|
| Definition | Parameter | Value |
| 7. CES | $\sigma$ | 4 |
| 8. CRRA | $\theta$ | 2 |

Table 4 displays the targets and model fit under the calibrated parameters in Table 3. By construction, we fit the productivity growth rate and markup exactly. We fit the real interest rate, intangible share and labor share/sales correlation very closely. We undershoot the concentration by about 10 percentage points. Nonetheless the model generates high level of concentration with over half of sales accruing to the top 10% of firms. Despite its simplicity, the model is able to mimic the features of the data very well.

Table 4: Baseline calibration

| Targeted | Target | Model |
|---|---|---|
| 1. top 10% concentration 1987–1992 | 67.5 | 57.2 |
| 2. productivity growth 1949–1995 | 1.81 | 1.81 |
| 3. aggregate markup | 1.27 | 1.27 |
| 4. real interest rate | 6.1 | 5.9 |
| 5. intangible share | 10.4 | 9.3 |
| 6. labor share and size relation | -1.10 | -1.09 |

**Source:** 1 and 6: Autor et al. (2019). 2: BLS MFP series. 3: Hall (2018). 4: Farhi and Gourio (2018). 5: Corrado et al. (2012).

Table 5 displays the moments in the new steady state when $\psi_o$ is lowered to match the between component of the change in labor share. The between component declined by about 11% relative to the initial level of labor share. The model matches this decline when $\psi_o$ declines by 65%. With this change in $\psi_o$, the model explains more than the full extent of the fall in productivity growth and about two-thirds of the decline in aggregate labor share. The model does not explain the full extent of the fall in labor share because the within component of labor share rose more than in the data (8 vs. 6 ppt). As in the data, the output share of intangibles rises in the model, though slightly less than the data (1.1 vs. 1.5 ppt). One dimension the model is significantly different from the data is the rise in concentration. Concentration rose by about 5 ppt in the data while it rose by 35 ppt in the model. Figure 12 displays

the change in the key moments as $\psi_o$ decreases.

Table 5: Effect of changing $\psi_o$

| Targeted | Date | Model |
|---|---|---|
| Between change in labor share (%) | -11.6 | -11.6 |
| % change in $\psi_o$ | 65.0% | |
| | | |
| **Untargeted** | **Data** | **Model** |
| 1. 2006–17 productivity growth rate (ppt) | 1.06 | 0.86 |
| 2. change in aggregate labor share (%) | -5.7 | -3.6 |
| 3. within change in labor share (%) | 5.9 | 8.0 |
| 4. change in intangible share (ppt) | 1.5 | 1.1 |
| 5. change in concentration (ppt) | 5.3 | 35.1 |

**Source:** 1: BLS MFP series. 2, 3 and 5: Autor et al. (2019), BLS KLEMS. 4: Corrado et al. (2012).

To clarify the mechanism in our model, Table 6 displays values of selected endogenous variables in the initial and new steady state. The decline in overhead costs encourages productive firms to expand, increasing the share of products and sales of the high efficiency firms (higher $S^\star$ and $\tilde{S}^\star$). This reaction leads to a rise in overhead costs as a share of output despite the downward shift in the overhead cost curve (due to the decrease in $\psi_o$). Hence, rising overhead spending are behind the rising intangible share (as seen in Table 5) whereas R&D spending falls. The rising overall intangible share implies that the drop in the aggregate labor income share would be smaller if overhead and R&D expenses were treated as intermediate inputs as opposed to investments. This is in line with the finding in Koh et al. (2016).

With the rise in $S^\star$, within firm markup *declines* for the low productivity firms because these firms are more likely to produce a product where the next best producer is a high productivity producer. Within firm markup stays constant for the high productivity firms because they are not subject to limit
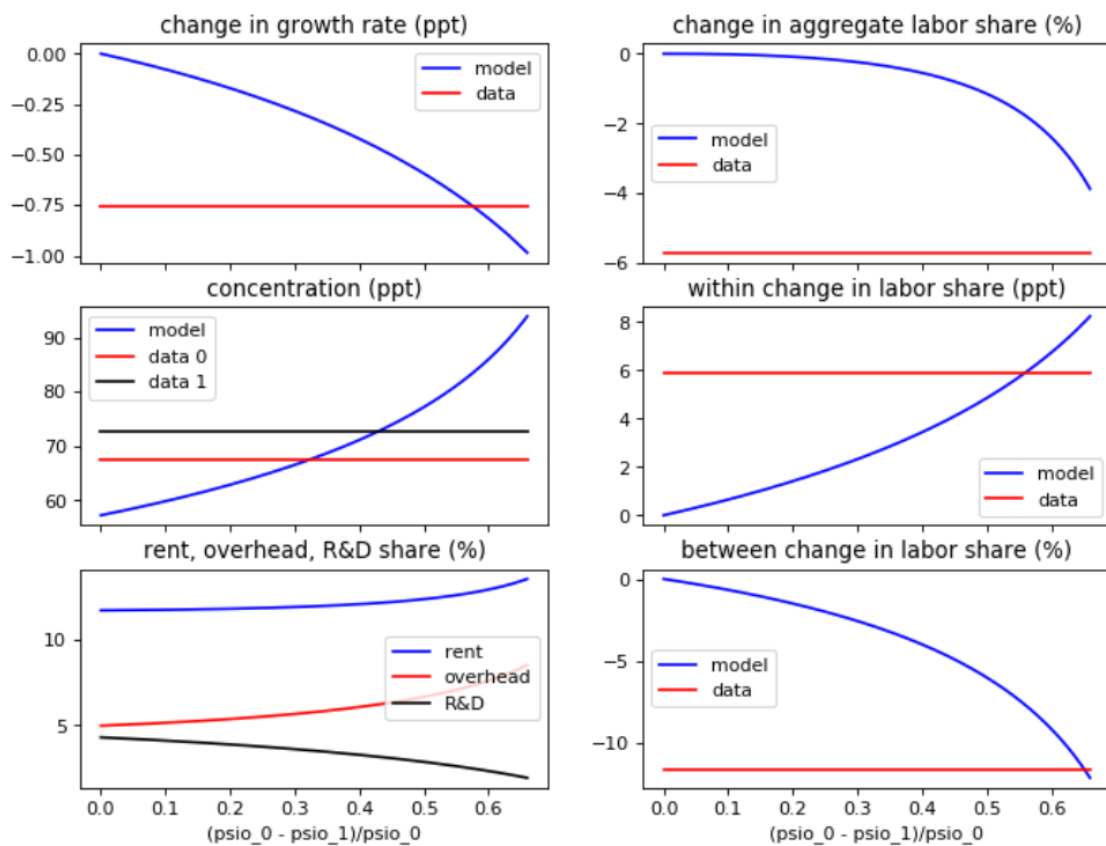
Figure 12: Change in key moments as $\psi_o$ declines

pricing under the calibrated parameters.[11]

Figure 13 display the shift in the markup distribution. It shows that production reallocates to the high productivity firms who have higher markups. This reallocation generates a rise in the aggregate markup and rent amidst falling within firm markup.

On the margin, the expected markup from innovating also declines as firms are more likely to innovate on a product produced by a high productivity producer. This decline discourages firms from innovating. The firms reduce their R&D expenditures, leading to a lower rate of creative destruction in the equilibrium (lower $z^\star$) and hence lower growth. This lower growth in turn translates into lower interest rate in the new steady state. Thus, growth rate is lower in the new steady state even though rent and aggregate markup is higher.

Table 6: Initial vs. new steady state

|  | Initial | New |
|---|---|---|
| 1. creative destruction rate ($z^\star$) | 2.58 | 1.20 |
| 2. % of H-type products ($S^\star$) | 39.0 | 88.8 |
| 3. % of H-type sales ($\tilde{S}^\star$) | 54.0 | 91.8 |
| 4. markup of H-type firms | 1.33 | 1.33 |
| 5. markup of L-type firms | 1.19 | 1.11 |
| 6. aggregate markup | 1.27 | 1.31 |
| 7. R&D/PY | 4.3 | 2.0 |
| 8. overhead/PY | 5.0 | 8.3 |
| 9. rent/PY | 11.7 | 13.4 |
| 10. real interest rate | 5.9 | 3.9 |

Finally, recall that job reallocation across firms and establishments as well as entry and exit rates are trending down in the data, as shown in Figure 14 and 15. How might our model speak to this? Job reallocation across firms occurs

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[11]The fact that the decrease in the within labor income share is particularly pronounced for firms with a falling sales share is in line with the shift share result in Kehrig and Vincent (2017).
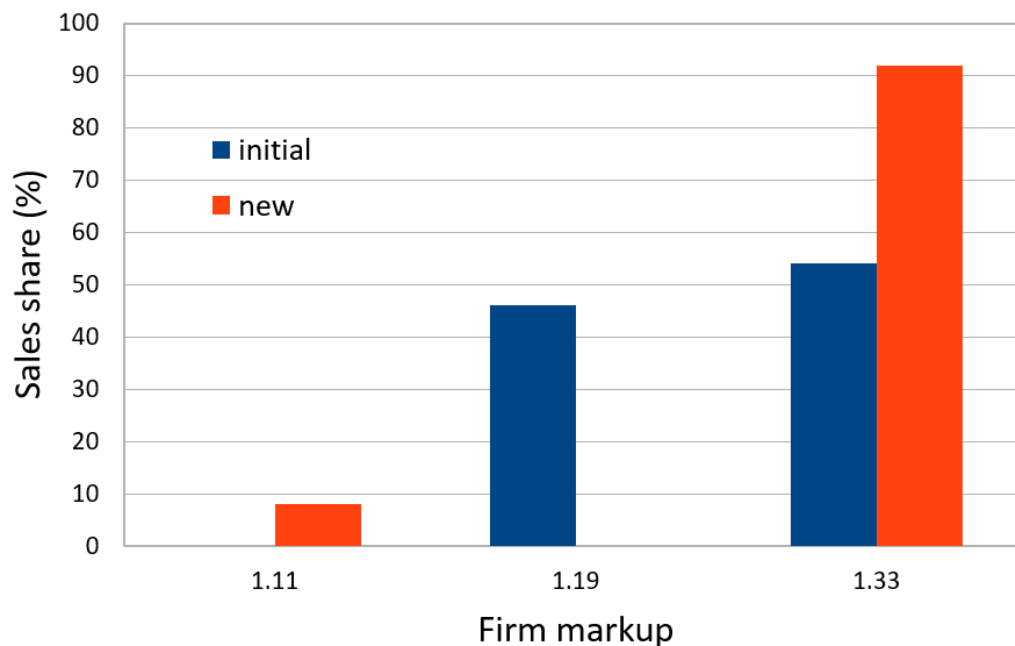
Figure 13: Change in the distribution of markups

when a firm's employment level rises (gross job creation) or falls (gross job destruction). In the data, this reallocation is partially due to firm entry and exit, which our baseline model does not have. But a significant component of job reallocation in the data is across surviving firms. In our model, firms add and subtract products from their portfolio due to creative destruction by themselves and their competitors. For simplicity our firms have a continuum of products, so this should ebb and flow nets out in steady state. But it is a short leap to a model in which firms have a finite number of products so that their employment levels rise and fall. See Garcia-Macia et al. (2018) for just such an analysis. Our model may speak more directly to job reallocation across establishments, if one makes the strong assumption that each plant is associated with given product line produced by the firm. Then plant entry and exit in the data can be compared to the rate of creative destruction in our model. As our model features falling long run growth, it implies falling long run job reallocation associated with product turnover.

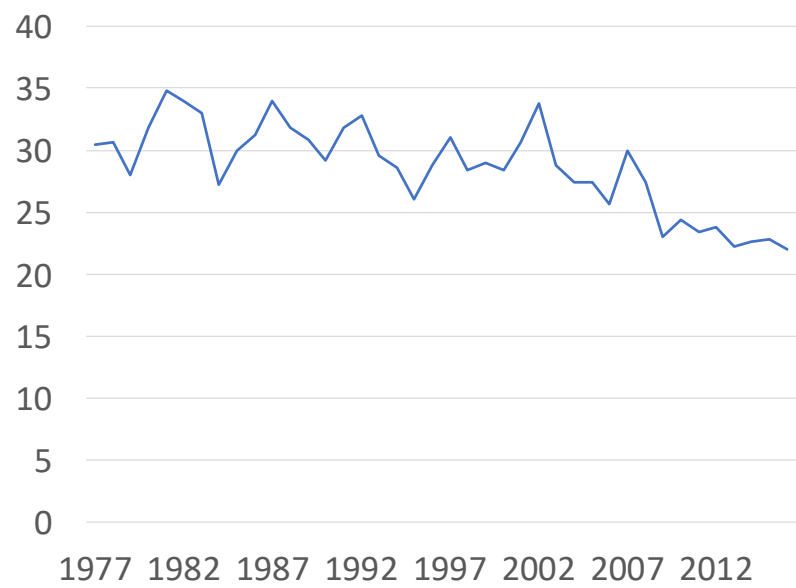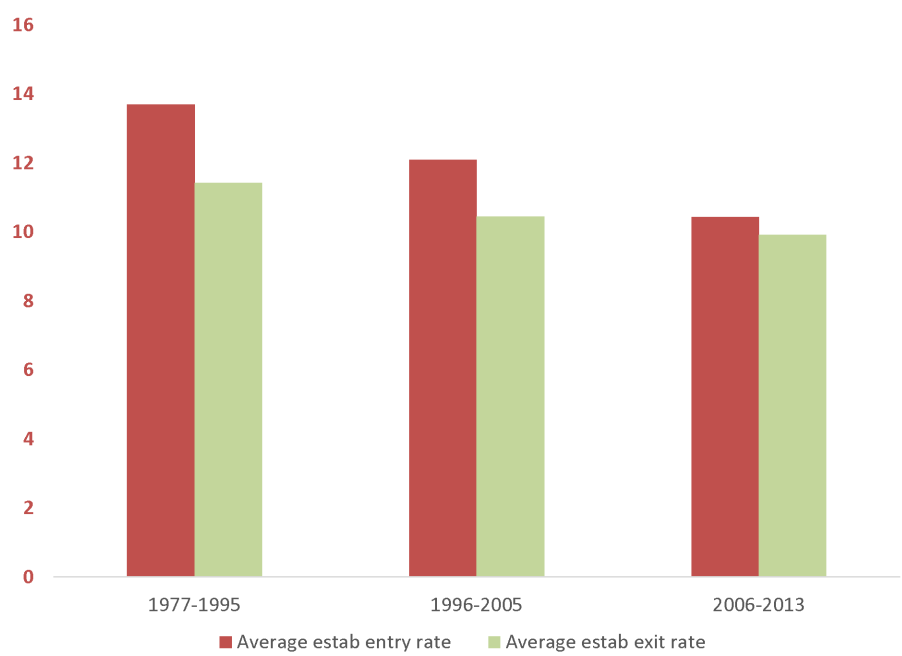Figure 14: Falling job reallocation rate 1977–2016



Figure 15: Entry and exit rates of establishments

# 5. Conclusion

We provide a new theoretical framework that can potentially account for a significant portion of the U.S. growth experience over the past 30 years: (i) a decline in the labor income share (driven by resource reallocation across firms as opposed to a declining labor income share within firms), (ii) a productivity slowdown (after a burst in productivity growth), (iii) rising concentration at the national level; and (iv) falling job reallocation rates. We argue that a significant part of these phenomena can be explained by IT improvements in the mid-1990s to mid-2000s which increased the optimal boundary of (especially) the most efficient firms. In our theory, these firms enjoy higher markups; when they expand their reach into more markets, they raise average markups and lower the aggregate labor share. They expand by innovating on more product lines, bringing a temporary surge of growth. Within-firm markups eventually fall for both high and low productivity firms, as they are more likely to face high productivity competitors. This force ultimately drags down innovation, growth, and job reallocation.

In this paper we focused our analysis on the overhead cost parameter $\psi_o$. However, the model lends itself to richer comparative static and transitional analyses. In particular it is straightforward to explore the steady state effects of changes in the efficiency gap $\Delta$, the innovation size $\gamma$, or the innovation cost $\psi_c$. We see it as a virtue of our model that the within vs. between firm effects of such changes can be easily analyzed.

Another next step is to explore the cross-industry predictions of our theory and see if they hold up in the data. In particular, we might look at whether more intensively IT-using industries experienced bigger increases in concentration (paired with declining labor share, and a more pronounced boom-bust cycle of productivity growth. IT can be used for more than just overhead, so we will try to gauge the overhead component of IT by the difference in IT investment rates of large versus small firms within industries.

Also, we are keen on exploring tax and subsidy policies in our quantitative framework. The decentralized equilibrium is suboptimal in due to markup dispersion across products as well as knowledge spillovers across firms (quality innovations build on previous innovations by other firms). It is possible that falling overhead costs reduce welfare, and that a welfare-improving policy response might be to (counterintuitively!) constrain the expansion of the most efficient firms. Conversely, we may find that the temporary surge in productivity more than justifies the lower long run growth prospects.

Finally, our framework is well suited discussing competition policy and its relation with the productivity slowdown. Above we already discussed the implications of allowing for M&As but other dimensions of competition policy as data access or firm breakup can be naturally considered through the lenses of our model. These and other extensions of the analysis in this paper are left for future research.

# A    Appendix: CRRA-CES generalization

In this appendix we derive the key steady state equations for the slightly generalized model used in the calibration in Section 4. with a CES production structure and CRRA preferences.

## CRRA preferences

Instead of log preferences we generalize the utility function to be of the CRRA class

$$U_0 = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta} - 1}{1 - \theta}.$$

Then the resulting Euler equation from household's optimization is given by

$$\frac{C_{t+1}}{C_t} = [\beta(1 + r_{t+1})]^{\frac{1}{\theta}}. \tag{46}$$

Hence, the steady state relationship between gross growth rate $g^\star$ and the interest rate is given by

$$g^\star = [\beta(1 + r^\star)]^{\frac{1}{\theta}}.$$

## CES production

Instead of the Cobb-Douglas technology we assume a more general CES technology in final production

$$Y = \left( \int_0^1 [q(i)y(i)]^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}. \tag{47}$$

Here $y(i)$ denotes the quantity and $q(i)$ the quality of product $i$. This new structure yields as demand for product $i$

$$y(i) = q(i)^{\sigma-1} \left( \frac{P}{p(i)} \right)^{\sigma} Y, \tag{48}$$

where we have the new aggregate price index given by

$$P = \left( \int_0^1 [p(i)/q(i)]^{1-\sigma} \, di \right)^{\frac{1}{1-\sigma}}, \tag{49}$$

which we can again normalize to one in each period.

## Solving for the steady state in this more general model

The rest of the model is unchanged. In particular we still have two process efficiency types and the productivity differential is captured by $\Delta$. We now solve for the steady state in this model.

Together with the definition of the numéraire the demand (48) gives for the operating profits in a period (before overhead cost)

$$Y P \left( \frac{P}{p(i)/q(i)} \right)^{\sigma - 1} \left( 1 - \frac{1}{\mu(i)} \right). \tag{50}$$

With $\sigma > 1$ (which is the empirically relevant case we will focus on in the following) there is an optimal markup factor of $\frac{\sigma}{\sigma-1}$. So depending whether the fringe of the second-best firm is binding or not we have the following three cases of markups in a line $i$:

1. In the case of a high type (H) facing a L second-best firm

$$\mu_{HL} = \min \left\{ \gamma \Delta, \frac{\sigma}{\sigma - 1} \right\}, \tag{51}$$

2. In the case that both leader and second-best firm are of the same type

$$\mu_{HH} = \mu_{LL} = \min \left\{ \gamma, \frac{\sigma}{\sigma - 1} \right\}, \tag{52}$$

3. In the case that a low type (L) facing a high type (H) second-best firm

$$\mu_{LH} = \min\left\{\frac{\gamma}{\Delta}, \frac{\sigma}{\sigma - 1}\right\}. \tag{53}$$

With the CES structure the demand for a product line $i$ depends also on the particular quality of this line (relative to the other lines). But because there is no possibility to target the innovation activity to particular lines and all firms draw in steady state repetitively from the same distribution, the steady state quality level in line $i$ is uncorrelated with the identity of the leading or second-best firm (and therefore uncorrelated with the markup). Since the law of large number applies each firm has in steady state in a given period $t$ the same distribution of quality levels across the different lines.

In the following let us summarized the "average quality" by

$$Q_t = \left(\int_0^1 [q_t(i)]^{\sigma - 1}\, di\right)^{\frac{1}{\sigma - 1}}. \tag{54}$$

Since the quality of a line is independent of the markup in a line we can write the aggregate price index, (49), as

$$P = 1 = \frac{\tilde{P}_t}{Q_t} \tag{55}$$

where

$$\tilde{P}_t = w_t \left(\int_0^1 [\mu(i)/\varphi(j(i))]^{1-\sigma}\, di\right)^{\frac{1}{1-\sigma}}.$$

In steady state we have

$$\tilde{P}_t = \frac{w_t}{\varphi_L}\left[(S^\star)^2 \left(\frac{\mu_{HH}}{\Delta}\right)^{1-\sigma} + S^\star(1 - S^\star)\left(\frac{\mu_{HL}}{\Delta}\right)^{1-\sigma} + S^\star(1 - S^\star)\mu_{LH}^{1-\sigma} + (1 - S^\star)^2 \mu_{LL}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

The profit in a given line is given by (50). The sum of operating profits (before overhead cost) of a high type firm that is active in $n(j)$ lines and is facing in a

52

fraction $S^\star$ of them a high second-best firm is given by[12]

$$n(j)Y\left[S^\star\frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HH}w_t}{\varphi_L\Delta}\right)^{\sigma-1}}\left(1-\frac{1}{\mu_{HH}}\right)+(1-S^\star)\frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HL}w_t}{\varphi_L\Delta}\right)^{\sigma-1}}\left(1-\frac{1}{\mu_{HL}}\right)\right]=n(j)Y\cdot\tilde{\pi}_H.$$

Where we define $\tilde{\pi}_H$ accordingly (to be equal to the term squared brackets). Similarly, the sum of operating profits before overhead of a L-type firm having $n(j)$ lines and facing in a fraction $S^\star$ of them a high second-best firm is

$$n(j)Y\left[S^\star\frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LH}w_t}{\varphi_L}\right)^{\sigma-1}}\left(1-\frac{1}{\mu_{LH}}\right)+(1-S^\star)\frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LL}w_t}{\varphi_L}\right)^{\sigma-1}}\left(1-\frac{1}{\mu_{LL}}\right)\right]=n(j)Y\cdot\tilde{\pi}_L.$$

Where we also define $\tilde{\pi}_L$ accordingly. Hence, the new profit functions of H and L types relative to GDP, $Y$, become in steady state:

$$\pi_H(n)=n\tilde{\pi}_H-\frac{1}{2}\psi_o n^2,$$

and

$$\pi_L(n)=n\tilde{\pi}_L-\frac{1}{2}\psi_o n^2.$$

The steady state is then characterized as before just with these new profit functions and some different relationship of $\beta$, $r^\star$ and $g^\star=\frac{Q_t}{Q_{t-1}}$.

## Steady state characterization

Let us denote the value of a firm $V$ relative to total output $Y$ by $v$. In steady state (with $h(j)^\star=S^\star$) the number of products per firm becomes the only individual state variable and we can write $v(n)$. All high productivity then solve

$$v_H(n)=\max_{n'\geq0}\{\pi_H(n,S^\star)-(n'-n(1-z^\star))\psi_c+\frac{g^\star}{1+r^\star}v_H(n')\}.$$

---

[12]Note that the $Q$ terms cancels out since the quality distribution in each H/L combination is identical to the aggregate $Q$.

Similarly, all low productivity firms solve

$$v_L(n) = \max_{n' \geq 0} \{\pi_L(n, S^\star) - (n' - n(1 - z^\star))\psi_c + \frac{g^\star}{1 + r^\star}v_L(n')\}.$$

The household's Euler equation yields $\frac{g^\star}{1+r^\star} = \beta(g^\star)^{1-\theta}$ and we have

$$g^\star = \frac{Y_t}{Y_{t-1}} = \frac{Q_t}{Q_{t-1}} = \left[1 + z^\star(\gamma^{\sigma-1} - 1)\right]^{\frac{1}{\sigma-1}}.$$

The two accounting equations are again

$$S^\star = n_H^\star \phi J, \tag{56}$$

and

$$n_H^\star \phi J + n_L^\star (1 - \phi)J = 1. \tag{57}$$

Finally, in steady state we must have

$$n_H^\star = f_H(n_H^\star), \tag{58}$$

and

$$n_L^\star = f_L(n_L^\star), \tag{59}$$

where $f_H(\cdot)$ and $f_L(\cdot)$ are the policy functions of the high and low types. These equations fully characterize the steady state values of $S^\star$, $z^\star$, $n_H^\star$ and $n_L^\star$.

The solution is given in the following proposition.

**Proposition 6** *The steady state is a $(n_H^\star, n_L^\star, S^\star, z^\star)$ combination that fulfills*

$$\phi J n_H^\star = S^\star \textbf{ and } (1 - \phi)J n_L^\star + \phi J n_H^\star = 1, \tag{60}$$

*as well as the following research arbitrage equations for high and low*

*productivity firms respectively:*

$$\psi_c = \frac{\tilde{\pi}_H - \psi_o n_H^\star}{\frac{1}{\beta(1-z^\star+z^\star\gamma^{\sigma-1})^{\frac{1-\theta}{\sigma-1}}} - 1 + z^\star}, \tag{61}$$

$$\psi_c = \frac{\tilde{\pi}_L - \psi_o n_L^\star}{\frac{1}{\beta(1-z^\star+z^\star\gamma^{\sigma-1})^{\frac{1-\theta}{\sigma-1}}} - 1 + z^\star}. \tag{62}$$

This is again a system of four equations in four unknowns which can be solved.

## Derivation of expression for concentration and labor share

In this more general model with a CES production function there is now a difference between the fraction of lines provided by high productivity firm, $S^\star$, and the sales weight of high productivity firm in the aggregate economy which we denote by $\tilde{S}^\star$. Total sales of a firm of high type is in steady state

$$\int_0^{n_H^\star} p(i)y(i)di = n_H^\star Y \left[ S^\star \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HH}w_t}{\varphi_L\Delta}\right)^{\sigma-1}} + (1 - S^\star) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{HL}w_t}{\varphi_L\Delta}\right)^{\sigma-1}} \right].$$

Sales of a firm of low type is given by

$$\int_0^{n_L^\star} p(i)y(i)di = n_L^\star Y \left[ S^\star \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LH}w_t}{\varphi_L}\right)^{\sigma-1}} + (1 - S^\star) \frac{\tilde{P}_t^{\sigma-1}}{\left(\frac{\mu_{LL}w_t}{\varphi_L}\right)^{\sigma-1}} \right].$$

As a consequence the sales share of high types in the total economy can be written as

$$\tilde{S}^\star = \frac{S^\star \left[ S^\star \left(\frac{\mu_{HH}}{\Delta}\right)^{1-\sigma} + (1 - S^\star)\left(\frac{\mu_{HL}}{\Delta}\right)^{1-\sigma} \right]}{S^\star \left[ S^\star \left(\frac{\mu_{HH}}{\Delta}\right)^{1-\sigma} + (1 - S^\star)\left(\frac{\mu_{HL}}{\Delta}\right)^{1-\sigma} \right] + (1 - S^\star)\left[ S^\star \mu_{LH}^{1-\sigma} + (1 - S^\star)\mu_{LL}^{1-\sigma} \right]}.$$

Finally, let us derive the expressions for the labor income shares with a CES

production function. The firm level labor share for high type is

$$1 - \alpha_H = \frac{\int_0^{n_H^\star} wl(i)di}{\int_0^{n_H^\star} p(i)y(i)di} = \frac{S^\star \mu_{HH}^{-\sigma} + (1 - S^\star)\mu_{HL}^{-\sigma}}{S^\star \mu_{HH}^{1-\sigma} + (1 - S^\star)\mu_{HL}^{1-\sigma}}. \tag{63}$$

The firm level labor share for low-type is given by

$$1 - \alpha_L = \frac{\int_0^{n_L^\star} wl(i)di}{\int_0^{n_L^\star} p(i)y(i)di} = \frac{S^\star \mu_{LH}^{-\sigma} + (1 - S^\star)\mu_{LL}^{-\sigma}}{S^\star \mu_{LH}^{1-\sigma} + (1 - S^\star)\mu_{LL}^{1-\sigma}}. \tag{64}$$

The aggregate labor share is the sales weighted average of the firm labor shares

$$1 - \alpha = \tilde{S}^\star [1 - \alpha_H] + (1 - \tilde{S}^\star)[1 - \alpha_L]. \tag{65}$$

The within change in labor share is the unweighted average of the change in within firm labor share. We target the within change as a fraction of the initial labor share

$$\frac{\phi(1 - \alpha_{H,1} - (1 - \alpha_{H,0})) + (1 - \phi)(1 - \alpha_{L,1} - (1 - \alpha_{L,0}))}{\tilde{S}_0^\star(1 - \alpha_{H,0}) + (1 - \tilde{S}_0^\star)(1 - \alpha_{L,0})}, \tag{66}$$

where $0$ denotes the initial and $1$ the new steady state.

# B   Appendix: solving transition dynamics

This section lays out the numerical method used to compute the transition dynamics in section 3.14.2. Let $n_t$ be the number of product a firm holds, $h_t$ be the share of those products with a high productivity second-best producer and define $m_t := n_t h_t$. Given initial $n_0, m_0$ and path of prices and aggregate variables, each firm of type $j = H, L$ solves

$$
\begin{aligned}
V(n_0, m_0) &= \max_{\{n_s, m_s\}_{s=1}^{\infty}} \{\pi_j(n_0, m_0) - (n_1 - n_0(1 - z_1))\psi_c\} \frac{Y_0}{Q_0} \qquad (67) \\
&+ \frac{\gamma^{z_1}}{1 + r_1} \{\pi_j(n_1, m_1) - (n_2 - n_1(1 - z_2))\psi_c\} \frac{Y_1}{Q_1} \\
&+ \frac{\gamma^{z_1}}{1 + r_1} \frac{\gamma^{z_2}}{1 + r_2} \{\pi_j(n_2, m_2) - (n_3 - n_2(1 - z_3))\psi_c\} \frac{Y_2}{Q_2} \\
&\;\;\vdots \\
&+ \prod_{\tau=1}^{t} \frac{\gamma^{z_\tau}}{1 + r_\tau} \{\pi_j(n_t, m_t) - (n_{t+1} - n_t(1 - z_{t+1}))\psi_c\} \frac{Y_t}{Q_t} \\
&\;\;\vdots
\end{aligned}
$$

subject to

$$
m_0 \equiv n_0 h_0 = n_0 S_0 \qquad (68)
$$

$$
m_t = m_{t-1}(1 - z_t) + S_{t-1}(n_t - (1 - z_t)n_{t-1}), \quad t = 1, 2, \cdots \qquad (69)
$$

$$
n_t \geq n_{t-1}(1 - z_t), \quad t = 1, 2, \cdots \qquad (70)
$$

where

$$
\pi_H(n_t, m_t) = m_t \left(1 - \frac{1}{\gamma}\right) + (n_t - m_t)\left(1 - \frac{1}{\Delta\gamma}\right) - \psi_o \frac{1}{2} n_t^2. \qquad (71)
$$

$$
\pi_L(n_t, m_t) = m_t \left(1 - \frac{\Delta}{\gamma}\right) + (n_t - m_t)\left(1 - \frac{1}{\gamma}\right) - \psi_o \frac{1}{2} n_t^2. \qquad (72)
$$

This is the only place where the $j$ index shows up (since the profit functions are different between the high and low type firms).

We can iterate backwards to express (69) as a function of all past $n$ choices

$$
\begin{aligned}
m_t &= (m_0 - S_0 n_0) \prod_{b=1}^{t}(1 - z_b) + S_{t-1} n_t + \sum_{a=1}^{t-1}(S_{a-1} - S_a) n_a \prod_{b=1}^{t-a}(1 - z_{a+b}) \quad (73) \\
&= S_{t-1} n_t + \sum_{a=1}^{t-1}(S_{a-1} - S_a) n_a \prod_{b=1}^{t-a}(1 - z_{a+b}) \quad \forall t = 1, 2, \cdots \quad (74)
\end{aligned}
$$

We denote this function $m_l(\{n_s\}_{s=1}^{l})$. The second equality follows from $h_{0j} = S_0$.

From this we derive the derivative of $m_{t+k}$ with respect to $n_t$ (we abbreviate $j$ subscript since the expression is the same).

$$
\frac{\partial m_{t+k}(\{n_s\}_{s=1}^{t+k})}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1} & \text{if } k = 0 \\ (S_{t-1} - S_t) \prod_{b=1}^{k}(1 - z_{t+b}) & \text{if } k > 0 \end{cases} \quad (75)
$$

This is the effect of increasing the number of products in period $t$ by one unit on the number of products facing high type second-best firm in period $t + k$ while holding the number of product in all periods other than $t$ constant. Adding a product in $t$ adds $(1 - z_{t+1})$ products in $t + 1$. $x_{t+1}$ therefore needs to drop by $(1 - z_{t+1})$ to keep $n_{t+1}$ constant. This drop in addition to holding $x_\tau, \tau > t + 2$ constant ensures the number of products in periods $\tau > t$ constant.

What is the effect on $m_{t+k}$? Adding a product in $t$ adds $S_{t-1}(1 - z_{t+1})$ products with a high-type follower in $t+1$ while lowering $x_{t+1}$ by $(1 - z_{t+1})$ in $t+1$ reduces high type follower by $S_t(1 - z_{t+1})$. The net effect on $m_{t+1}$ is $(S_{t-1} - S_t)(1 - z_{t+1})$. This change decays at rate of survival from creative destruction $\prod_{b=1}^{k}(1 - z_{t+b})$. Hence what matters for $m_{t+k}$, $k > 1$ is the difference in the composition of the pool these products are drawn from, i.e., the difference between $S_t$ and $S_{t-1}$. If $S_t = S_{t-1}$ an increase in $n_t$ has no effect on $m_{t+k}$, $k > 0$. If $S_t > S_{t-1}$, the change shrinks the number of products with high-type followers. Vice versa for $S_t < S_{t-1}$.

Substituting (75) into (71) and (72) and taking derivatives with respect to $n$

yields

$$\frac{\partial \pi_{t+k,H}(n_{t+k}, m_{t+k}(\{n_s\}_{s=1}^{t+k}))}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1}\frac{1-\Delta}{\Delta\gamma} + 1 - \frac{1}{\Delta\gamma} - \psi_o n_t & \text{if } k = 0 \\ \frac{1-\Delta}{\Delta\gamma}(S_{t-1} - S_t)\prod_{b=1}^{k}(1 - z_{t+b}) & \text{if } k > 0 \end{cases} \tag{76}$$

$$\frac{\partial \pi_{t+k,L}(n_{t+k}, m_{t+k}(\{n_s\}_{s=1}^{t+k}))}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1}\frac{1-\Delta}{\gamma} + 1 - \frac{1}{\gamma} - \psi_o n_t & \text{if } k = 0 \\ \frac{1-\Delta}{\gamma}(S_{t-1} - S_t)\prod_{b=1}^{k}(1 - z_{t+b}) & \text{if } k > 0 \end{cases} \tag{77}$$

It is useful to rewrite the value function (67) before taking first-order conditions. First, we substitute in the household's Euler equation to express the discount factors as

$$\prod_{t=a}^{b} \frac{\gamma^{z_t}}{1 + r_t} = \beta^{b-a+1} \frac{y_{a-1} c_{a-1}}{y_b c_b} \tag{78}$$

where $y_t \equiv Y_t/Q_t$ and $c_t$ denotes consumption share of output $C_t/Y_t$:

$$\begin{aligned} c_t \equiv \frac{C_t}{Y_t} &= 1 - \frac{O_t}{Y_t} - \frac{Z_t}{Y_t} \\ &= 1 - \left(\phi n_{tH}^2 + (1 - \phi)n_{tL}^2\right)\frac{\psi_o J}{2} - \psi_c z_{t+1} \\ &= 1 - \left(\frac{S_t^2}{\phi} + \frac{(1 - S_t)^2}{1 - \phi}\right)\frac{\psi_o}{2J} - \psi_c z_{t+1} = c(S_t, z_{t+1}) \end{aligned} \tag{79}$$

Substituting this expression into the objective function (67), dividing by $y_0$ and rearranging reformulates the problem as

$$\max_{\{n_t\}_{t=1}^{\infty}} \quad \pi_j(n_0, m_0) + n_0(1 - z_1)\psi_c \tag{80}$$

$$+ \sum_{t=1}^{\infty} \beta^t \frac{c_0}{c_t}\left\{\pi_j(n_t, m_t(\{n_s\}_{s=1}^{t})) + \psi_c n_t\left[(1 - z_{t+1}) - \frac{c_t}{c_{t-1}\beta}\right]\right\}$$

The first-order conditions of the above objective function with respect to

$n_t, t = 1, 2, ...$ are

$$\frac{\partial \pi_j(n_t, m_t(n_t))}{\partial n_t} = \psi_c \left[ \frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1}) \right] + f_j \frac{1 - \Delta}{\Delta \gamma} (S_t - S_{t-1}) \sum_{a=t+1}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1 - z_{t+b}) \tag{81}$$

where $f_j = \Delta$ if $j = L$ and $f_j = 1$ otherwise.

Define

$$d_t := \sum_{a=t+1}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1 - z_{t+b}) \tag{82}$$

We can show

$$d_{t-1} = \sum_{a=t}^{\infty} \beta^{a-t+1} \frac{c_{t-1}}{c_a} \prod_{b=1}^{a-t+1} (1 - z_{t-1+b}) \tag{83}$$

$$= \beta(1 - z_t) \frac{c_{t-1}}{c_t} \sum_{a=t}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1 - z_{t+b}) \tag{84}$$

$$= \beta(1 - z_t) \frac{c_{t-1}}{c_t} (1 + d_t) \tag{85}$$

Substituting (82) and (85) into (81) yields the following set of equations for each period $t = 1, 2, \cdots$

$$\frac{\partial \pi_H(n_{tH}, m_{tH}(n_{tH}))}{\partial n_{tH}} = \psi_c \left[ \frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1}) \right] + \frac{1 - \Delta}{\Delta \gamma} (S_t - S_{t-1}) d_t \tag{86}$$

$$\frac{\partial \pi_L(n_{tL}, m_{tL}(n_{tL}))}{\partial n_{tL}} = \psi_c \left[ \frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1}) \right] + \frac{1 - \Delta}{\gamma} (S_t - S_{t-1}) d_t \tag{87}$$

$$d_t = d_{t-1} \frac{1}{\beta(1 - z_t)} \frac{c_t}{c_{t-1}} - 1 \tag{88}$$

$$h_{tH} = (h_{t-1,H} - S_{t-1}) \frac{S_{t-1}}{S_t} (1 - z_t) + S_{t-1} \tag{89}$$

$$h_{tL} = (h_{t-1,L} - S_{t-1}) \frac{1 - S_{t-1}}{1 - S_t} (1 - z_t) + S_{t-1} \tag{90}$$

Substituting in equilibrium condition for $n_t$, we have additionally

$$\frac{\partial \pi_H(n_{tH}, m_{tH}(n_{tH}))}{\partial n_{tH}} = S_{t-1} \frac{1 - \Delta}{\gamma \Delta} + 1 - \frac{1}{\Delta \gamma} - \psi_o \frac{S_t}{\phi J} \tag{91}$$

and

$$\frac{\partial \pi_L(n_{tL}, m_{tL}(n_{tL}))}{\partial n_{tL}} = S_{t-1}\frac{1-\Delta}{\gamma} + 1 - \frac{1}{\gamma} - \psi_o\frac{(1-S_t)}{(1-\phi)J} \tag{92}$$

## B1. Forward iteration algorithm

Given $(d_{t-1}, z_t, S_{t-1}, h_{t-1,H}, h_{t-1,L})$, equations (86) to (90) solves for $(d_t, z_{t+1}, S_t, h_{tH}, h_{tL})$. We can multiply (86) by $\Delta$ and subtract (87) to eliminate the $d_t$ term on the RHS. This yields

$$\Delta - 1 - \Delta\psi_o\frac{S_t}{\phi J} + \psi_o\frac{1-S_t}{(1-\phi)J} = (\Delta-1)\psi_c\left[\frac{c_t}{c_{t-1}\beta} - (1-z_{t+1})\right] \tag{93}$$

Substituting in $c(S_t, z_{t+1})$ from (79) yields

$$\frac{c_{t-1}\beta}{(\Delta-1)\psi_c}\left[\Delta - 1 - \Delta\psi_o\frac{S_t}{\phi J} + \psi_o\frac{1-S_t}{(1-\phi)J}\right]$$

$$= \left[1 - \left(\frac{S_t^2}{\phi} + \frac{(1-S_t)^2}{1-\phi}\right)\frac{\psi_o}{2J} - c_{t-1}\beta + (c_{t-1}\beta - \psi_c)z_{t+1}\right]$$

$$z_{t+1} = \frac{\frac{c_{t-1}\beta}{(\Delta-1)\psi_c}\left[\Delta - 1 - \Delta\psi_o\frac{S_t}{\phi J} + \psi_o\frac{1-S_t}{(1-\phi)J}\right] - \left[1 - \left(\frac{S_t^2}{\phi} + \frac{(1-S_t)^2}{1-\phi}\right)\frac{\psi_o}{2J} - c_{t-1}\beta\right]}{c_{t-1}\beta - \psi_c}$$

$$\equiv z(S_t, S_{t-1}, z_t)$$

We can substitute this into (86) to get an equation with $S_t$ the only unknown

$$\frac{\partial \pi_H(n_{tH}, m_{tH}(n_{tH}))}{\partial n_{tH}} = \frac{\Delta\frac{\partial \pi_H(n_{tH}, m_{tH}(n_{tH}))}{\partial n_{tH}} - \frac{\partial \pi_L(n_{tL}, m_{tL}(n_{tL}))}{\partial n_{tL}}}{\Delta - 1}$$

$$+ \frac{1-\Delta}{\Delta\gamma}(S_t - S_{t-1})\left(\frac{d_{t-1}}{\beta(1-z_t)}\frac{c(S_t, z(S_t, S_{t-1}, z_t))}{c_{t-1}} - 1\right)$$

This is a quadratic function in $S_t$ which built-in solvers easily minimize. We choose the solution that is between the old and new steady state $S$.

We initiate the algorithm with a guess for $(z_1, d_0)$ and set $S_0 = h_{0H} = h_{0L} = S_{old}^\star$. The algorithm has an outer loop and an inner loop. The inner loop holds $d_0$ fixed and iterates on $z_1$. It iterates on (86) to (90) until $S_t$ is close the new steady state $S_{new}^\star$. Then it uses bisection to update the guess of $z_1$. Namely, it increases

$z_1$ if the last value of $z$ is lower than the new steady state $z^\star_{new}$ and reduce $z_1$ otherwise.

The inner loop yields a path of $(z_{t+1}, S_t)$ that converges to $(z^\star_{new}, S^\star_{new})$ holding fixed $d_0$. The path implies a path for $d_t$ that may not converge to the steady state value of $d^\star_{new} := \frac{\beta(1-z^\star_{new})}{1-\beta(1-z^\star_{new})}$. The outer loop uses bisection to update $d_0$ until $d_t$ also converges. Namely, it reduces $d_0$ if the inner loop overshoots and increases $d_0$ otherwise.

We stop the algorithm when $(d_t, z_{t+1}, S_t)$ all approximately converge to the new steady state. Suppose this happens after $T$ periods. Then we set $(d_t, z_{t+1}, S_t)$ for $t > T$ to their new steady state values and iterate forward until $(h_{tH}, h_{tL})$ converges to the new steady state. We do not keep on iterating on $(d_t, z_{t+1}, S_t)$ until $(h_{tH}, h_{tL})$ converges because (88) is not stable outside of its fixed point. As machine precision does not allow the algorithm to reach the exact fixed point, $d_t$ eventually explodes as we iterate forward.

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