Competition among Renewables*

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Abstract

We analyze an auction model in which firms' production capacities are private information. The results shed light on the nature of the strategic interaction between renewable generators, whose available capacities are subject to random shocks. In equilibrium, firms bid above marginal costs, with markups decreasing in their realized capacities. Capacity withholding is not optimal, unless a single firm has excess capacity to cover total demand. Hence, supply functions shift outwards and downwards at times when there is more renewable energy, with market prices smoothly converging towards marginal costs. We also analyze the effects of switching from a uniform to a discriminatory auction format, and the effects of fragmenting the market structure.

Keywords: electricity, renewables, competition, auctions.

1 Introduction

Ambitious environmental targets, together with decreasing investment costs, have fostered the rapid deployment of renewable energy around the world. Installed renewable capacity has more than doubled over the last ten years, and it is expected to further increase during the coming decade.[1] Indeed, Europe expects that over two thirds of its electricity generation will come from renewable resources by 2030, with the goal of achieving a carbon-free power sector by 2050 (European Commission (2012)). Likewise, California has recently mandated that 100% of its electricity will come from clean energy

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[1]The International Renewable Energy Agency estimates that compliance with the 2017 Paris Climate Agreement will require overall investments in renewables to increase by 76% in 2030, relative to 2014 levels.

sources by 2045. This global trend begs the question: how will electricity markets perform in the future once renewables become the major energy source?

Whereas competition among conventional fossil-fuel generators is by now well understood (e.g. Borenstein (2002); von der Fehr and Harbord (1993) Green and Newbery (1992), among others), much less is known about competition among wind and solar producers (which we broadly refer to as *renewables*).[2] Competition-wise, there are two key differences between these two technologies. First, the marginal costs of conventional plants depend on their efficiency rate as well as on the price at which they buy the fossil fuel. In contrast, the marginal costs of renewable generation equal zero as they produce electricity out of freely available natural resources (e.g. wind or sun). Second, the available capacity of conventional plants is well known as they tend to be available at all times (absent rare outages). In contrast, the availability of renewable plants is uncertain as it depends on weather conditions that are difficult to forecast. Hence, the move from fossil fuel generation towards renewables will imply a change in paradigm. Whereas the previous literature has analyzed environments where marginal costs are private information but production capacities are publicly known, the relevant setting will soon be one in which marginal costs are known (and zero) but firms' capacities become private information. By changing the type of private information held by firms, renewables fundamentally change the nature of strategic interaction among electricity producers.

In this paper we build a theoretical model of competition among renewable producers that captures this new paradigm in electricity markets: firms' marginal costs are known but their production capacities are private information.[3] Each firm competes by submitting a price-quantity pair (i.e., an inverted-L supply function), indicating the maximum quantity that it is willing to produce and the minimum price at which it is willing to do so. The auctioneer calls firms to produce in increasing price order until total demand is

[2]Strictly speaking, there are renewable energies other than wind and solar. However, these two are the dominant ones. Not all renewable technologies share the same properties as wind and solar. For instance, hydro electricity is storable (in contrast to wind and solar which are flow technologies); the available capacity of biomass is known very much like a thermal plant, etc.

[3]A firm might also find it difficult to perfectly forecast its own availability. However, it knows exactly how much output it has offered to the market, which is what matters to determine dispatched output and prices.

satisfied, and pays all accepted output at the highest accepted price offer.[4] If the output offered by the renewable producers is not enough, the auctioneer calls conventional producers to cover the residual demand. Thus, the marginal cost of the conventional producers (which for simplicity are assumed to behave competitively) serves as an implicit price cap to the offers made by the renewable producers. Importantly, firms can exercise market power by either bidding above marginal costs and/or withholding output, i.e., offer to produce below their available capacity.

Equilibrium outcomes differ as compared to when (i) capacities and costs are known (as in von der Fehr and Harbord (1993), Fabra et al. (2006), de Frutos and Fabra (2012)) or when (ii) capacities are known but costs are private information (as in Holmberg and Wolak (2018)). When production capacities are known, uniform-price auctions can give rise to very high market prices. In equilibrium, one firm bids high while the others bid low enough so as to make undercutting by the high bidder unprofitable. In this paper we show that this logic extends to settings in which available capacities are random but become common knowledge before firms submit their firms (i.e., in the absence of private information). In this case, firms can use their capacity realizations to symmetrically correlate their high and low bids, thus allowing them to evenly share profits.

These seemingly-collusive equilibria are ruled out when capacities are private information. Simply put, firms cannot condition their strategies on each-others' capacities. Instead, as we show in this paper, bidding functions are decreasing in realized capacities (i.e. the higher a firm's realized capacity, the lower the price at which it is willing to supply it). In a symmetric equilibrium, the price offers of the renewable producers range from the marginal costs of the conventional technology (when the realized capacity is at the lower bound) to their own marginal costs (when the realized capacity reaches the upper bound). These equilibrium bid functions reflect the standard trade-off faced by competing firms when choosing prices: reducing the price would allow the firm to sell more (quantity effect), but it would also reduce the market price if all rivals bid below (price effect). The fact that larger firms benefit more from the quantity effect, explains

[4]This mimics the actual uniform-price auction format used in most electricity markets in practice.

why the equilibrium bid functions are decreasing in the firm's realized capacity. Interestingly, we show that firms never find it optimal to exercise market power by withholding output, unless their available capacity is enough to cover total demand. In the latter case, there exists an equilibrium in which large firms withhold output so as to produce slightly below total demand. This allows them to bid low so as to sell almost total demand, while still receiving the higher price set by the rival.

In sum, expected market prices are strictly below the marginal costs of the conventional producers but strictly above the marginal costs of renewables. This suggests that renewable energy mitigates market power, even if it does not fully eliminate it. This result is reminiscent to the literature on Treasury auctions (LiCalzi and Pavan, 2005) which shows that introducing noise in the demand function rules out the seemingly collusive equilibria that arise otherwise (Back and Zender, 2001).

We perform a comparative statics exercise with respect to increases in the installed renewable capacity, changes in demand, and improvements in the precision of the capacity forecast. In particular, we show that the price depressing effect of renewables is non-linear in the amount of installed renewable capacity. At the initial stages of the deployment process, an additional unit of renewable capacity leaves the price almost always unchanged at the marginal cost of the conventional technology; hence, the price impact of renewables is expected to be small. However, in more advanced stages, an additional unit of renewable capacity might displace the conventional technologies more often, thus leading to sharp reductions in the market price. As the investment in renewables increases, the probability that the realized capacity of a single firm exceeds demand is larger, and hence the mass that is placed at marginal costs goes up. Eventually, when the capacity of both firms always exceeds demand, Bertrand competition drives prices down to marginal cost. Hence, the price impact of renewables becomes small again.

We extend our baseline model in several directions. First, we study the effects of firms' entry and changes in market structure. We show that an increase in the number of (symmetric) firms brings about a standard pro-competitive effect: the more firms there are, the bigger is the market share that a firm gains by slightly undercutting the rivals (quantity effect) and the smaller is the residual demand that a firm serves if it is outbid (price effect). Both effects lead to more competitive outcomes. If we leave total capacity fixed and consider splitting it among a larger number of firms, an information-related anti-competitive effect arises as smaller firms face more uncertainty regarding the rivals' capacities. Although this mitigates the former pro-competitive effect, the price-depressing effect of fragmenting the market structure dominates.

We also characterize equilibrium bidding in discriminatory auctions (that pay all winning firms their price offers). Similarly to Holmberg and Wolak (2018), we show that in this format firms submit higher bids than in uniform price auctions: firms are simply discouraged from bidding low prices as they are paid as they bid. Thus, the choice of the auction format introduces a trade-off, as uniform price auctions induce more competitive bidding but pay winning bidders the highest accepted bid. In contrast with Holmberg and Wolak (2018), we show that there is no revenue equivalence across auction formats.[5] The main force driving our different predictions is that equilibrium quantities are affected by private information on capacities, but not by private information on costs. Numerical solutions indicate that the discriminatory auction typically leads to higher prices, for example, when capacity availability arises from a Beta distribution.

Related Literature

Holmberg and Wolak (2018) analyze competition in electricity markets in a model in which firms are privately informed about their marginal costs, rather than about their capacities. In their model, private information moves supply functions up or down (when realized costs are either high or low) while firms' quantity offers remain unchanged (given that capacities are assumed to be known and fixed).[6] This is unlike our model in which private information on capacities moves the supply functions up and to the left (when

[5] They find revenue equivalence across auction formats for the case of independent costs. When marginal costs are positively affiliated, they also find, similarly to us, that the discriminatory auction pays firms more.

[6] More precisely, they assume that each firm might be unaware about its own capacity when it submits its bids. However, expected capacities are uncorrelated to the cost realization. As a result, the equilibrium outcomes are the same as if capacities were fixed and known at the time when firms submit their bids.

realized capacities are low) or down and to the right (when realized capacities are high). In our model, because we allow for capacity withhoding, changes in private information also leave supply functions unchanged when firms find it optimal to behave as if their realized capacity was slightly below total demand.

Vives (2011) studies a model of competition where firms offer linear supply functions under imperfect information about marginal costs. Since firms' marginal costs are correlated, the market price aggregates firms' private information. Firms' willingness to learn about their own costs from the market price induces them to submit steeper supply functions, yielding less competitive outcomes. This effect is not present in our model since we are dealing with a private value setup. In this respect, our model is less general than Vives (2011)'s but it allows us to consider constant marginal costs up to capacity, which is suitable for the case of renewable energy. This cost function contains kinks that would make the linear supply-function approach intractable.

Other recent papers have also analyzed competition among renewables (Acemoglu et al. (2017) and Kakhbod et al. (2018)). While they share with us the fact that capacities are random, they differ from our approach in that they assume Cournot competition, thus restricting firms to exerting market power only through capacity withholding. In this paper we show that if firms are allowed to choose both prices and quantities, they exercise market power by raising prices above marginal costs when they are capacity constrained to serve total demand, and only resort to capacity withholding otherwise.

Acemoglu et al. (2017)'s analysis applies to markets at an earlier stage of renewables deployment, with firms owning a portfolio of conventional and renewable plants. They show that the common ownership of these two technologies mitigates the price depressing effect of renewables as firms withhold more output from their conventional plants when there is more renewable generation. This effect is not present in our model since renewable capacity is often enough to cover total demand. Furthermore, in order to focus on the strategic interaction among renewables, we assume that conventional power plants are owned by independent producers.[7]

Kakhbod et al. (2018)'s focus is on the impact on market outcomes of the heterogeneity in stochastic renewable availability across locations. They show that firms withhold more output when they are closely located (i.e., when their output is more positively correlated). This implies that the market fails to induce investments at the optimal locations.

Even though we have motivated our model in the context of electricity markets, it is applicable to other relevant contexts. Notably, in the context of treasury auctions it is typically assumed that bidders are privately informed about their valuation up to a fixed number of units (Hortasu and McAdams, 2010). Our model would allow us to extend those analyses to situations in which this limit is private information (e.g. bidders might have different hedging needs that are unknown to their rivals) as long as it is positively related to the bidders' valuations.

The reminder of the paper is structured as follows. In Section 2 we describe the basic model. In Section 3 we characterize the bidding equilibria when firms' capacities are private information and compare equilibrium outcomes with the ones that emerge when capacities are publicly known. In Section 4 we interpret the model in the context of electricity markets. Section 5 discusses extensions, including the discriminatory auction and the effect of changing the number of firms. Section 6 concludes.

2 The Model

Two firms $i = 1, 2$ compete in a market to supply a perfectly price-inelastic demand, denoted as $\theta > 0$. Firms are capacity constrained. The capacity of firm i, denoted $k_i > 0$, is assumed to be random. In particular, $k_i \in \left[\underline{k}, \bar{k}\right]$ for $i = 1, 2$ is distributed according to the function $G(k_i)$ with density $g(k_i) > 0$ in the whole interval. The capacity of both firms is assumed to be independently distributed. For simplicity, we assume that firm i can observe its own capacity but not that of its rival, i.e., available capacities are private

[7] If we allowed the independent conventional producers to exercise market power, an increase in renewables would reduce the residual demand faced by the conventional producers, thus reducing their incentives to increase prices.

information. Each firm can produce at a constant marginal cost $c \geq 0$ up to its capacity.

Firms compete on the basis of the bids submitted to an auctioneer. Each firm simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply the whole of its realized capacity, $b_i \in [0, P]$, $i = 1, 2$, where P denotes the "market reserve price". This price can be interpreted as the marginal cost of the next available technology, as long as it is offered competitively, or as a regulated price cap.

The auctioneer ranks firms according to their price offers, and calls them to produce in increasing rank order. In particular, if firms submit different bids, the low-bidding firm is ranked first while the high-bidding firm is ranked second. If firms submit equal bids, firm i is ranked first with probability $\alpha(k_i, k_j)$ and it is ranked second with probability $1 - \alpha(k_i, k_j)$. We assume a symmetric function $\alpha(k_i, k_j) = \alpha(k_j, k_i) \in (0, 1)$ so that when their capacities are equal, $\alpha(k, k) = 1/2$.[8] If firm i is ranked first it produces $q_i = \min\{\theta, k_i\}$, while if it is ranked second it produces $q_i = \max\{0, \min\{\theta - k_j, k_i\}\}$.

Firms receive a uniform price per unit of output, p, which is set equal to the highest accepted price offer.[9] If both firms jointly have enough capacity to cover total demand, $p = b_j$ if $b_i \leq b_j$ and $k_i < \theta$; $p = b_i$ otherwise. If both firms' aggregate capacity is below demand, $k_i + k_j < \theta$, the market price is set at P.

The profits made by each firm are computed as the product of their per unit profit margin $(p - c)$ and their output q_i. As explained before, both p and q_i are a function of demand θ, the bids submitted by both firms (b_i, b_j), and their realized capacities (k_i, k_j). Firms, which are assumed to be risk neutral, bid so as to maximize their individual expected profits, given their realized capacities.

Throughout the paper we solve for the symmetric Bayesian Nash Equilibria (BNE) in pure-strategies, and whenever these do not exist, we allow for mixed strategies.

[8] We do not need to specify $\alpha(k_i, k_j)$ outside of the diagonal as it is inconsequential for equilibrium bidding.

[9] In section 5.1 we characterize the equilibrium under a discriminatory auction, in which firms are paid according to their bid.

3 Equilibrium Characterization

In this section we characterize the symmetric Bayesian Nash Equilibria (BNE) of the game in which capacities are private information. As a benchmark, we start by discussing the case in which both capacity realizations are known to both firms.

3.1 Known capacities

Suppose that firms observe realized capacities prior to submitting their bids. Accordingly, firms' bids can be conditioned on realized capacities, i.e., $b_i(k_i, k_j)$ for $i = 1, 2$, and $j \neq i$. We focus on the characterization of the symmetric equilibria of the game, i.e., $b_i(k', k'') = b_j(k', k'')$ for any pair (k', k'').

Building on Fabra et al. (2006), the game with known capacities allows for potentially many symmetric pure-strategy equilibria, all of which are ex-ante outcome equivalent, i.e., the equilibrium market price is the same and firms make identical expected profits. The next proposition characterizes such equilibrium outcome and provides equilibrium bidding profiles that give rise to it. The results are illustrated in Figure 1.

Proposition 1. *For given realized capacities (k_i, k_j), define $k^+ \equiv \max(k_i, k_j)$ and $k^- \equiv \min(k_i, k_j)$. Accordingly, let $b_i(k^+, k^-) = b^+$ and $b_i(k^-, k^+) = b^-$.*

(i) If $k^- \geq \theta$, in any pure-strategy equilibrium, $p^* = c$. This outcome is sustained by both firms bidding at marginal cost.

(ii) If $k^- < \theta < k^- + k^+$ and $k^- < k^+$, in any pure-strategy equilibrium, $p^* = P$. This outcome is sustained by pure-strategy equilibria with $b^+ = P$ and $b^- \leq \underline{b} = c + (P - c)\frac{\theta - k^-}{k^+}$.

(iii) If $k^- < \theta < k^- + k^+$ and $k^- = k^+$, there does not exist a pure-strategy equilibrium. At the unique symmetric mixed-strategy equilibrium, firms choose prices in $[\underline{b}, P]$, resulting in $E[p^*] < P$.

(iv) If $\theta \geq k^- + k^+$, in any pure-strategy equilibrium, $p^* = P$. Any bid profile allows both firms to sell all their capacity at P.

To shed light on Proposition 1, it is useful to define the notion of *pivotality*: a firm is pivotal if it faces a positive residual demand regardless of the bid of its rival, i.e., firm i is pivotal if $\theta - k_j > 0$. Indeed, the equilibrium characterization critically depends on whether firms are pivotal or not. When none of the firms is pivotal (as in part (i) of the Proposition), total demand is served by the firm with the lowest bid. Standard Bertrand arguments imply that the equilibrium market price equals marginal costs, c. In contrast, when at least one of the two firms is pivotal, the equilibrium market price is (almost always) equal to P.

When at least one firm is pivotal but there is excess capacity overall (parts (ii) and (iii)), there exist multiple pure-strategy equilibria in all of which the market price is set at P by a pivotal firm.[10] The rival firm bids low enough (at or below some threshold $\underline{b}$) so as to make it unprofitable for the high bidder to undercut it. In equilibrium, the low bidder makes higher profits than the high bidder, as the former sells at capacity while the latter sells the residual demand. Hence, whenever the two firms are pivotal, they face a coordination problem as they would both prefer to act as the low bidder. If their realized capacities are asymmetric, firms can use them as a correlation device. For instance, there exists a symmetric equilibrium in which the small (large) firm plays the role of the low (high) bidder.[11] Even if this equilibrium involves asymmetric bidding, it is ex-ante symmetric as both firms are equally likely to be either the small or the large firm.

However, if firms' realized capacities are symmetric (as in part (iii) of the Proposition), firms can no longer use their capacities to designate who bids low and who bids high. The alternative would have both firms submitting equal bids, but this cannot be sustained in equilibrium as firms would have incentives to undercut each other. Hence, the unique symmetric equilibrium involves mixed-strategy pricing, with firms randomizing bids in the support $[\underline{b}, P]$.[12] Even though the expected market price falls below P, the probability

[10] In this region, there also exists a symmetric mixed strategy equilibrium. This equilibrium is Pareto dominated by the pure-strategy equilibria.

[11] There is a continuum of such equilibria, but they are all payoff equivalent.

[12] Trivially, if firms could resort to an external correlation device, they could always correlate their roles of high and low bidder with no need to play mixed strategies.

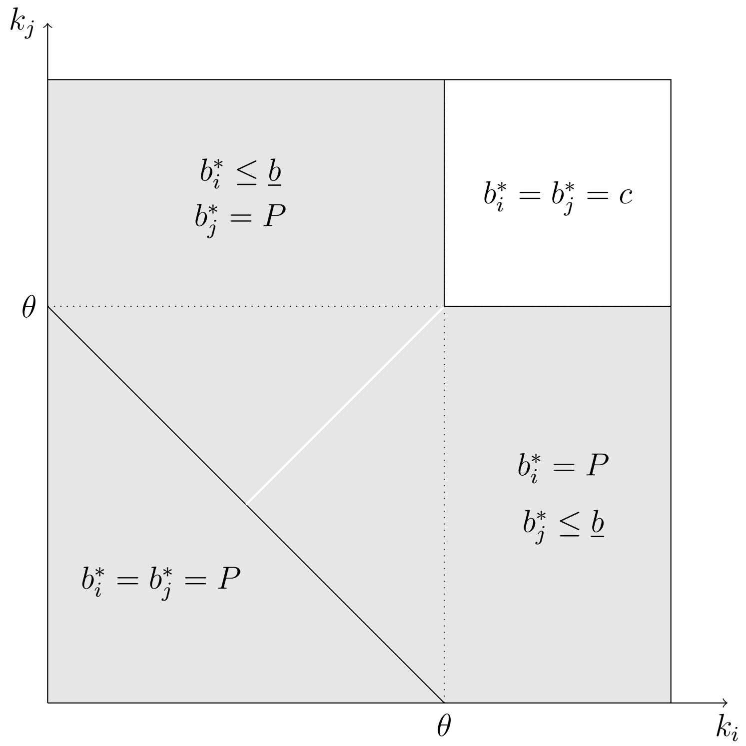

Figure 1: Equilibrium bids when capacities are known. The shaded region corresponds to an equilibrium price $p^* = P$.

that this situation occurs is zero (as it requires both capacity realizations to be equal).

Finally, if aggregate capacity is no greater than total demand (part (*iv*) of the Proposition), the result is trivial since there is no competition: both firms sell total capacity at P regardless of their bids.

3.2 Capacities are private information

In general, when firms' capacities are private information, the equilibrium characterized above no longer exists. Simply put, a firm's bidding strategy cannot be conditioned on the rival's capacity. There are only two trivial cases in which the equilibrium with known and unknown capacities coincide. First, when $\underline{k} > \theta$, firms know that none of them is ever pivotal, regardless of their realized capacities. Hence, as in part (*i*) of Proposition 1, equilibrium prices are driven down to marginal costs. Second, when $\bar{k} < \theta/2$, firms know that aggregate capacity is never enough to cover total demand. Again, as in part (*iv*) of Proposition 1, both firms sell all their capacity at P with no need to compete.

In this section, we characterize the symmetric Bayesian Nash Equilibria of the game in the remaining non-trivial cases. We first focus on the cases in which (i) firms always have enough renewable capacity to cover total demand, $\underline{k} > \theta/2$, and (ii) they are always pivotal with certainty, $\bar{k} < \theta$. We refer to this as the baseline model. We next relax these assumptions to allow for cases in which (i) renewable capacity is not always enough, $\underline{k} \leq \theta/2$; and (ii) firms' pivotality status is uncertain, $\bar{k} \geq \theta$.

3.2.1 Baseline model

We start by identifying two key properties that any equilibrium must satisfy.

Lemma 1. *Assume $\theta/2 < \underline{k} < \bar{k} < \theta$. In this case,*

(i) All Nash Equilibria must be in pure strategies.

(ii) The optimal bid of firm i, $b_i(k_i)$, must be non-increasing in k_i.

The first part of the lemma rules out non-degenerate mixed strategy equilibria. The underlying reason is simple: a firm's profits at a mixed-strategy equilibrium depend on its realized capacity, which is non-observable by the rival. If the competitor randomizes its bids in a way that makes the firm indifferent between two different bids for a given capacity realization, the same randomization cannot make the firm indifferent for other capacity realizations as well. It follows that the equilibria must involve pure-strategies.

The second part of the above lemma rules out bids that are increasing in the firm's capacity. When a firm considers whether to reduce its bid marginally, two effects are at play for a given bid of the rival: a profit gain due to the output increase (*quantity effect*), and a profit loss due to the reduction in the market price (*price effect*). On the one hand, the *quantity effect* is increasing in the firm's capacity, as if its bid falls below the rival's, it would sell at capacity rather than just the residual demand. On the other hand, the *price effect* is independent of the firm's capacity as, contingent on bidding higher than the rival, the firm always sells the residual demand. Combining these two effects, the incentives to bid low are (weakly) increasing in the firm's capacity, giving rise to optimal bids that are non-increasing in k_i.[13]

[13]The incentives to bid low are strictly increasing in the firm's capacity if marginally reducing the bid implies a strictly positive probability of increasing the firm's output, i.e., a strictly positive *quantity effect*. This need not be the case if the equilibrium is asymmetric.

Building on this result, we now turn to characterizing the symmetric Bayesian Nash equilibria of the game. The first thing to note is that, at a symmetric equilibrium, the optimal bid must be *strictly* decreasing in k_i. The logic is simple: standard Bertrand competition arguments imply that equilibrium bids never contain flat regions, as firms would have incentives to slightly undercut such bids. This allows us to invert $b_j(k_j)$ to write the expected profits of firm i when firm j bids according to a candidate symmetric equilibrium, as follows

$$\pi_i(b_i; k_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)(\theta - k_j)g(k_j)dk_j.$$

When $k_j < b_j^{-1}(b_i)$, firm i has the low bid and hence sells up to capacity at the price set by firm j. Otherwise, firm i serves the residual demand and sets the market price at b_i.

Maximizing profits with respect to b_i and applying symmetry we can characterize the optimal bid at a symmetric equilibrium.

Proposition 2. *Assume $\theta/2 < \underline{k} < \bar{k} < \theta$. There exists a unique symmetric Bayesian Nash equilibrium in pure-strategies: each firm $i = 1, 2$ chooses a bid that is strictly decreasing in k_i according to*[14]

$$b^*(k_i; \underline{k}, \bar{k}) = c + (P - c) \exp\left(-\omega(k_i)\right), \tag{1}$$

where

$$\omega(k_i) = \int_{\underline{k}}^{k_i} \frac{(2k - \theta)g(k)}{\int_k^{\bar{k}} (\theta - k_j)g(k_j)dk_j} dk,$$

with $b^*(\underline{k}; \underline{k}, \bar{k}) = P$ and $b^*(\bar{k}; \underline{k}, \bar{k}) = c$.

The optimal bid adds a markup above marginal costs that is strictly decreasing in k_i, as illustrated in Figure 2. In order to provide some intuition, it is useful to implicitly re-write the optimal bid as follows

$$\omega'(k_i) = \frac{(2k_i - \theta)g(k_i)}{\int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j}$$

[14] Since this bid is optimal for all k_i in $\underline{k}, \bar{k}$, we make this interval explicit as an argument of the optimal bid. This notation will be useful to simplify the rest of the analysis.

On the left-hand side, $\omega'(k_i)$ captures the incentives to marginally reduce the bid, which in turn reflect the trade-off between the *quantity effect* and the *price effect*, as described before. These effects are represented by the ratio on the right-hand side of the above equation.

The price effect (on the denominator), or price loss from marginally reducing the bid, is relevant only when the firm is setting the market price, i.e., when the rival firm's capacity is above k_i. In this case, reducing the bid implies that the firm keeps on selling the expected residual demand, $\int_{k_i}^{\bar{k}}(\theta - k_j)g(k_j)dk_j$, but at a lower market price. The quantity effect (on the numerator), or output gain from marginally reducing the firm's bid, is relevant only when the two firms tie in prices, i.e., when both firms have the same capacity k_i, an event that occurs with probability $g(k_i)$. In this case, reducing the bid implies that the firm sells all its capacity rather than just the residual demand, i.e. its output jumps up by $k_i - (\theta - k_i) = 2k_i - \theta$.

The quantity effect is weighted by two forces. On the one hand, the quantity effect is more relevant when the rival's bid function is flatter, since a given reduction in the firm's bid makes it more likely that the firm will sell at capacity. On the other hand, the quantity effect is less relevant when the mark-up on the increased sales is smaller. Indeed, the ratio of the quantity and price effects can also be written as

$$-\frac{b'^*(k_i)}{b^*(k_i) - c} = \frac{(2k_i - \theta)g(k_i)}{\int_{k_i}^{\bar{k}}(\theta - k_j)g(k_j)dk_j}$$

The optimal bid starts at P for the lowest possible capacity realization and ends at c for the largest one. When $k_i = \underline{k}$, firm j is bigger by construction, so firm i sells the residual demand and sets the market price with probability one. A bid below P could never be part of an equilibrium as firm i could sell the same output at a higher price by bidding at P. When $k_i = \bar{k}$, firm j is smaller by construction, so firm i never sets the market price. Hence, the firm's bid has no impact on the price and only the quantity effect matters. Therefore, a bid above c could never be part of an equilibrium as firm i could expect to sell more output at the same price by bidding at c.

Note that firms make higher profits when their realized capacities are larger. The

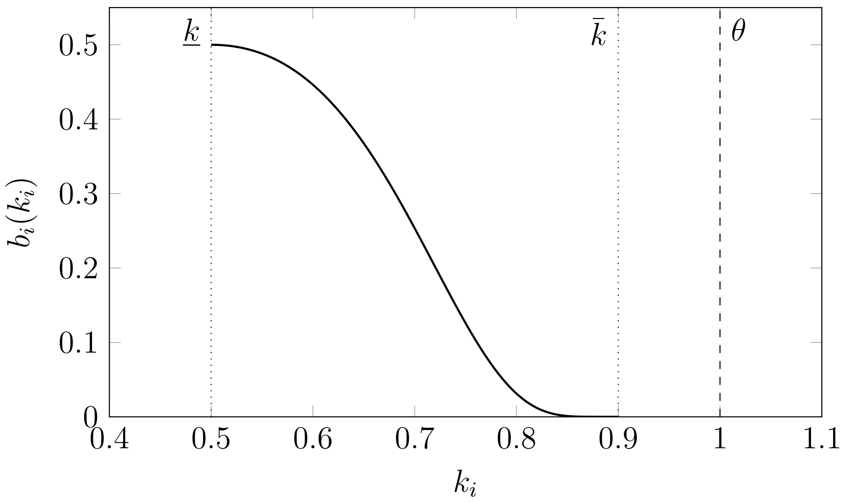

Figure 2: Equilibrium bids when $k_i \sim U[0.5, 0.9]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

reason is simple. If a firm with capacity $k' > k$ bids at $b(k)$ it makes more profits than a firm with capacity k: if it has the low bid, it sells $k' > k$ at the rival's price, whereas if it has the high bid it serves the residual demand at $b^*(k)$ in either case. By revealed preference, equilibrium profits at k' must exceed those at k.[15]

It is useful to compare this equilibrium with the one obtained in the game with known capacities (Proposition 1). As in that case, for given realized capacities, the equilibrium involves asymmetric bidding while it still preserves ex-ante symmetry (as firms' capacities are symmetrically distributed). However, unlike that case, firms cannot be certain as to whether the rival firm's capacity is larger or smaller than its own, and hence do not know whether the rival is bidding below or above. In other words, uncertainty over capacities induces uncertainty over the prices charged by the rival, just *as if* the rival were playing a mixed strategy. Thus, the standard price *versus* quantity trade-off emerges, ruling out an equilibrium market price equal to P.

As a result, while the expected equilibrium price equals P with known capacities, it is strictly below P when capacities are private information. Therefore, the price comparison across the two games is unambiguous.

Proposition 3. *Assume $\theta/2 < \underline{k} < \bar{k} < \theta$. Expected equilibrium market prices are lower*

[15]Furthermore, if we allowed firms to choose both prices as well as quantities, this shows that firms never want to mimic smaller types, i.e., they never want to withhold capacity. It follows that the same equilibrium would prevail as the one presented here.

in the game in which capacities are private information as compared to the game in which capacity realizations are known.

3.2.2 When renewables are not always enough

Allowing for $\underline{k} < \theta/2$ implies that the capacity of both firms might not always be enough to cover total demand. If that is the case, both firms sell at capacity regardless of their bids. In this section we show that the equilibrium characterized above naturally extends to encompass this case.

In particular, as shown in the next Proposition, the optimal bid is flat at P up to $k_i = \theta/2$ and it subsequently takes the same shape as in Proposition 2.

Proposition 4. *Assume $\underline{k} < \theta/2 < \bar{k} < \theta$. There exists a unique symmetric Bayesian Nash equilibrium in pure-strategies. Each firm $i = 1, 2$ chooses a bid that is non-increasing in k_i. If $k_i < \theta/2$, the optimal bid is P. Otherwise, the optimal bid is $b^*(k_i; \theta/2, \bar{k})$ as defined in* (1)*, where $b^*(\theta/2; \theta/2, \bar{k}) = P$ and $b^*(\bar{k}; \theta/2, \bar{k}) = c$.*

The equilibrium strategy calls firms to bid at P whenever their realized capacities are at or below $\theta/2$. To understand this result, assume that firm i's realized capacity is exactly equal to $\theta/2$. If the rival's capacity was at or below $\theta/2$, reducing the price below P would not allow firm i to increase its profits as both firms would sell all capacity at P regardless of their bids. In contrast, if the rival's capacity was above $\theta/2$, firm i would serve the residual demand at its own bid. Hence, it would be optimal for firm i to raise the market price up to P, as stated in the Proposition. A similar reasoning implies that bidding at P is optimal when the firm's capacity is below $\theta/2$.

Figure 3 depicts the equilibrium bid in this case. As compared to Figure 2, the optimal bid simply adds a flat region at P for all $k_i \leq \theta/2$.

3.2.3 When firms are not certain to be pivotal

Allowing for $\bar{k} \geq \theta$ implies that, for certain realizations, the capacity of a single firm is enough to cover total demand. In other words, firms are no longer certain to be pivotal. This has a dramatic impact on the bidding incentives. First, conditionally on having the

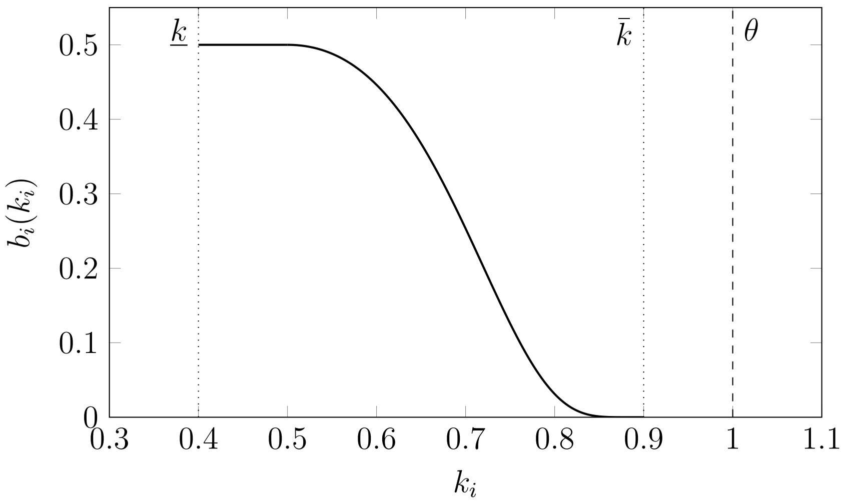

Figure 3: Equilibrium bids when $k_i \sim U[0.4, 0.9]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

high bid, a firm produces nothing if its rival's capacity is at or above θ. Second, the bid of a firm whose capacity exceeds θ is payoff relevant even if it has the low bid, as in this case the firm serves total demand at its own bid. The former effect intensifies competition, whereas the second induces firms to charge higher prices.

To describe more clearly how the equilibrium bids change when $\bar{k} \geq \theta$, we split the equilibrium characterization in two cases, depending on whether the firm's realized capacity is above or below θ.

Lemma 2. *Assume $\bar{k} \geq \theta$. There does not exist a Bayesian Nash Equilibrium in pure strategies. Furthermore, in any equilibrium, for $k_i \geq \theta$, firm i randomizes its bid in a support $\left[\underline{b}, \bar{b}\right]$ independently of k_i, where $\underline{b} > c$ and $\bar{b} < P$.*

If $k_i \geq \theta$, firm i is never capacity constrained. Since its expected profits do not depend on its realized capacity, its optimal bid at a candidate pure-strategy equilibrium is the same for all capacity realizations above θ. However, this would give rise to ties with positive probability and it is thus ruled out by standard Bertrand-Edgeworth arguments. More specifically, ties cannot be part of an equilibrium as firms would be better off by slightly undercutting any price above marginal costs in order to sell more output with only (if any) a slight reduction in the price. Furthermore, tying at marginal cost is ruled out as firms could make positive profits by selling the expected residual demand at P. Thus, at a symmetric equilibrium, firms must randomize their bids for all capacity realizations above θ.

The previous argument implies, of course, that the symmetric Bayesian Nash Equilibrium of the game must be in mixed strategies, as at least for $k_i \geq \theta$ firms randomize their bids. One distinctive feature of this equilibrium is that the upper bound of the price support does not go all the way up to P. The reason is that when firms have a capacity $k_i \geq \theta$, they face a downward sloping residual demand, induced by the downward sloping bid function of the rival when its capacity realization is below θ. We now turn to characterizing the equilibrium in this case. The next proposition describes the optimal bid, and Figure 4 illustrates it.

Proposition 5. *Assume* $\bar{k} \geq \theta$. *In the unique symmetric Bayesian Nash Equilibrium, if* $k_i < \theta$, *the optimal bid for firm* i *is*

(i) $b^*(k_i; \underline{k}, \theta)$ *for* $k_i \in \left[\underline{k}, \hat{k}\right]$ *and* $k_i \in \left[\tilde{k}, \theta\right)$, *as defined in* (1).

(ii) $\hat{b}(k_i; \underline{k}, \theta)$ *for* $k_i \in \left(\hat{k}, \tilde{k}\right)$, *strictly decreasing in* k_i *and strictly lower than* $b^*(k_i; \underline{k}, \theta)$.

(iii) $b_i \sim F(b_i)$ *with density* $f(b_i)$ *in a support* $[\underline{b}, \bar{b}]$.

The thresholds $\hat{k}$ *and* $\tilde{k}$ *are implicitly defined as* $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ *and* $b^*(\tilde{k}; \underline{k}, \theta) = \underline{b}$, *where* $\underline{b}$ *and* $\bar{b}$ *are defined in Lemma 2.*

The optimal bid when k_i belongs to either $\left[\underline{k}, \hat{k}\right]$ or $\left[\tilde{k}, \theta\right)$ is similar to the one in the baseline model. The sole difference is that, from firm i's point of view, firm j's relevant capacities now range from $\underline{k}$ to θ given that firm i's profits are constant when $k_j \geq \theta$. In particular, for such capacities, firm j randomizes its bid in the support $(\underline{b}, \bar{b})$ and, thus, prices are limited above by $b^*(\hat{k}; \underline{k}, \theta) = \bar{b}$ and below by $b^*(\tilde{k}, \underline{k}, \theta) = \underline{b}$. Hence, if $k_j \geq \theta$, firm i does not produce anything if k_i belongs to $\left[\underline{k}, \hat{k}\right]$, while firm i sells at capacity at the price set by firm j if k_i belongs to $\left[\tilde{k}, \theta\right)$. It follows that firm i's marginal profits are zero whenever $k_j \geq \theta$, and hence its bidding incentives are equal to those in the base line model with $\bar{k}$ arbitrarily close to θ.

This result is in contrast to the case where $k_i \in \left[\hat{k}, \tilde{k}\right)$. For these realizations, firm i might have the low or the high bid depending on the bid chosen by firm j when playing its

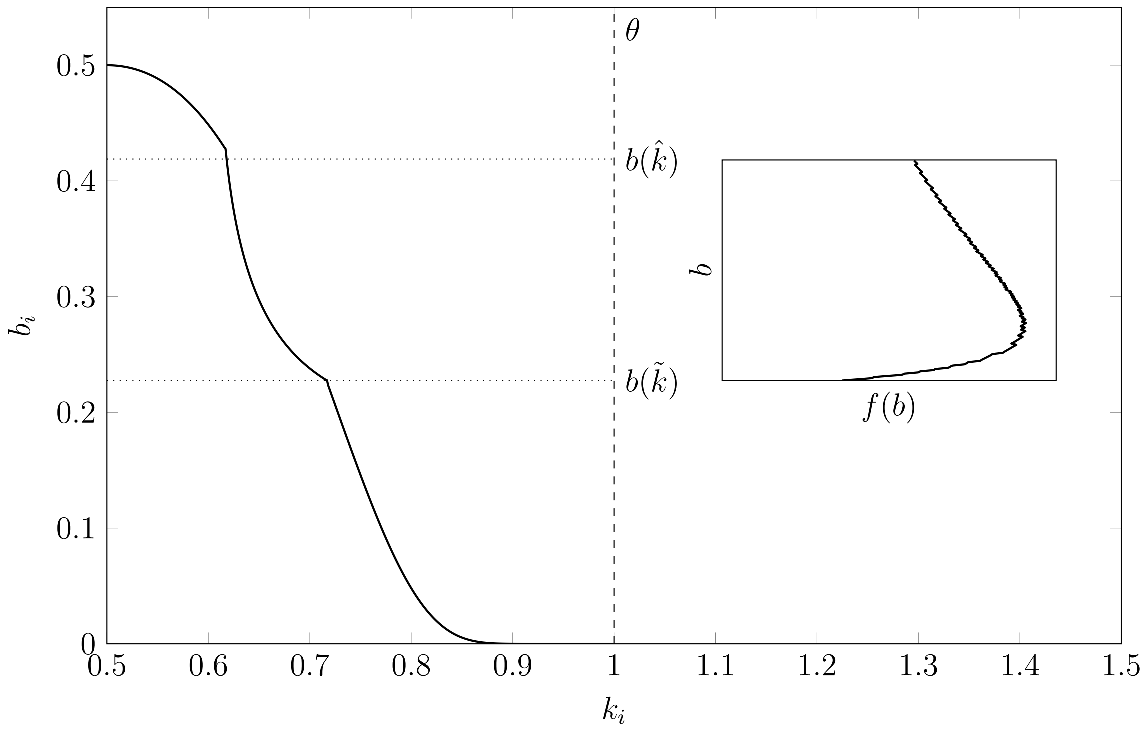

Figure 4: Equilibrium bids and probability density when $k_i \sim U[0.5, 1.1]$, with $\theta = 1$, $c = 0$, and $P = 0.5$.

mixed strategy. In particular, firm i's incentives to bid low are now stronger as compared to the baseline model, given that by reducing the b it can outbid the rival for a larger range of capacity realizations, including $k_j \geq \theta$.

The previous equilibrium bidding function is not monotonic in k_i, particularly around θ. The optimal bid converges to c to the left of θ as the firm is certain to be selling at capacity at the price set by the rival. In contrast, the bid jumps above c when $k_i > \theta$, as the firm is aware that its bid is always payoff relevant.[16]

Allowing $\bar{k}$ to increase above θ shows how the equilibrium bid schedules approach the competitive outcome. Suppose that capacities are uniformly distributed in $\left[\underline{k}, \bar{k}\right]$, and consider moving the whole capacity support to the right. For capacity realizations above θ, the equilibrium mixed strategy puts increasingly more weight on the lower bound of the price support, which converges towards c. In turn, the range $\left(\hat{k}, \tilde{k}\right)$ widens up. This

[16]We do not allow firms to withhold capacity, which might become a concern, particularly, when $k_i > \theta$. It is easy to see that firms are indifferent between offering any capacity above or at θ. With capacity withholding over this range, firms would have a distribution with a mass point at θ but it would not affect the results in any way. Matters might be different if firms have incentives to withhold capacity and offer a capacity slightly below θ at marginal cost. In this case, the firm would sell more (it would produce almost total demand with probability one) but possibly at a lower price.

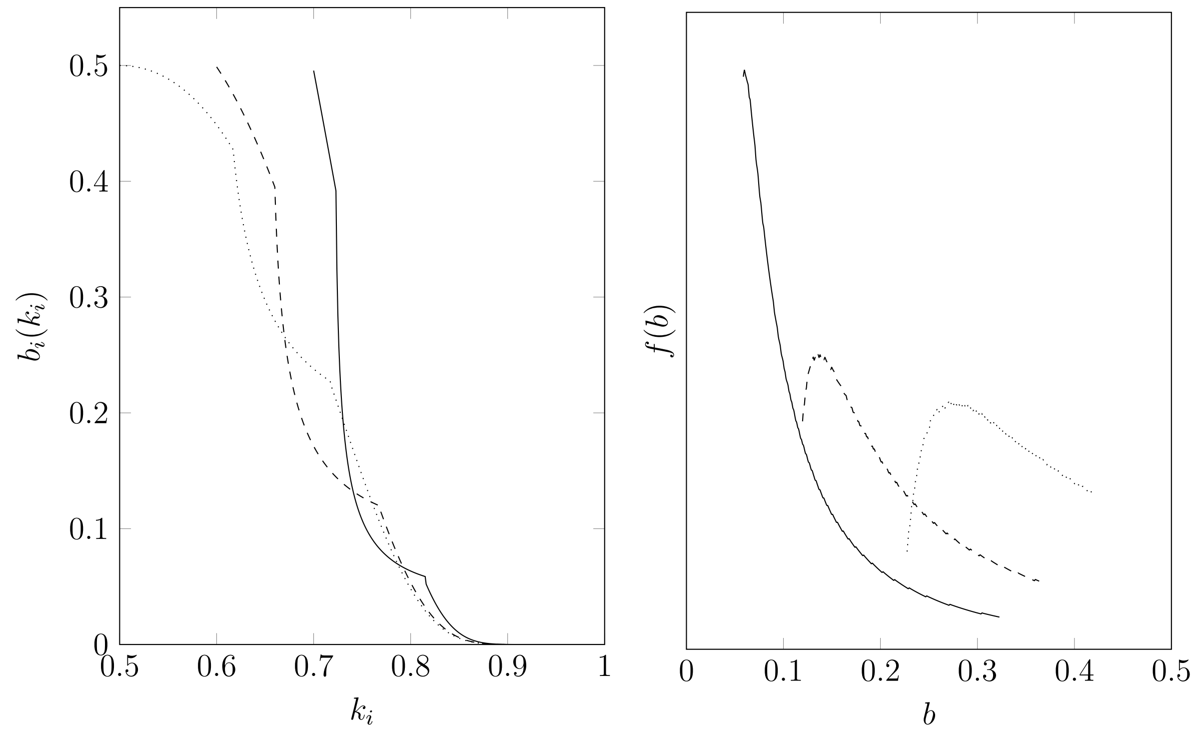

Figure 5: Equilibrium bids and probability density when $k_i \sim U[0.5, 1.1]$ (dotted), $k_i \sim U[0.6, 1.2]$ (dashed) and $k_i \sim U[0.7, 1.3]$ (solid), with $\theta = 1$, $c = 0$, and $P = 0.5$.

process continues until $\underline{k}$ reaches θ, in which case the equilibrium bid functions become flat at marginal costs. Figure 5 depicts this process of convergence towards the competitive outcome.

4 Interpreting the model

In the description of the model, the private information regarding k_i, for $i = 1, 2$, stems from a capacity realization that is known to firm i but not to firm $j \neq i$. That is, the distribution $G(k_i)$ reflects the private information of firm i. Although we have cast the model as appropriate to describe the electricity market, the same setup can be applied to other markets. In this section we discuss some other interpretations of the model. We also show how the setup can accommodate some important institutional features that are specific the electricity market.

Our setup directly applies to other markets where agents bid for multiple units of a good in an auction or in a marketplace. In our private-values setting, however, the uncertainty does not reflect private information regarding the valuation (or cost) of those agents as in most papers of the auction literature, but rather, the quantity that they are willing to buy or sell. This applies to a variety of examples: how many financial derivatives a bank is willing to buy depends on its hedging needs, which in turn depends on how many loans and deposits it has; how much oil an oil producer is willing to sell depends on how much oil there remains in its well; how many available Uber cars there are depends on how many drivers are on service, net of those who are already occupied; how many rooms a hotel is willing to offer online depends on how many rooms have already been booked through other channels; how much cloud computing space a firm is willing to offer depends on how much excess capacity it has above its own data needs; or how much olive oil a firm is willing to sell depends on whether its harvest was good or bad.

Regarding the application of our model to the electricity market some important comments are in order. First, our model implicitly assumes that firms own a unique plant. Obviously, the results would be unchanged if k_i were interpreted as the total production and $G(k_i)$ as the distribution of this production, accounting for the potential correlation between the capacity of each plant. Second, our model setup assumes that firms know exactly their available capacity before bids are placed. This assumption can be understood as a simplification made to capture the asymmetry in the information available to the competitors. In practice, however, firms bid in the day-ahead market, offering at a given hour of the following day a fixed production k_i at a price b_i. In this market, the equilibrium price is computed ahead of production using all firms' committed quantities. This is what the model aims to explain.

Since the realized capacity is unknown to firms one day in advance, this quantity will typically differ from the one that a firm has offered. Other markets close to the time of production allow firms to buy or sell energy in order to adjust their total quantity to the committed production. In general, participation in these markets leads to less favorable prices for the firms, which means that in the day-ahead market they typically have incentives to bid a production that corresponds to their expected available capacity. This quantity is estimated using the firm's own weather forecast model, which is unavailable to the competitor. In this context, the distribution $G(k_i)$ can be interpreted as the uncertainty that the competitor has on the forecast of the firm, which is based on the local information it has gathered.

Finally, system operators might provide some information on the expected aggregate production. This source of information can be accommodated in the model by assuming that the expected available capacity of firm i, k_i, has a common component θ and an specific one $\alpha_i + \varepsilon_i$. The parameter α_i is a known component specific of plant i. The system operator publishes a forecast of θ, $E(\theta)$, and firms use their weather forecast to obtain their estimate of ε_i. This means that the expected available capacity of firm i can be written as

$$k_i = \alpha_i + E[\theta] + \varepsilon_i.$$

Firm j, however, only observes the public information and its best estimate corresponds to

$$E[k_i | \Omega_j] = \alpha_i + E[\theta] + E[\varepsilon_i] = \alpha_i + E[\theta],$$

where Ω_j indicates the information available to j. Hence, the function G can be interpret as the distribution of ε_i shifted by $\alpha_i + E[\theta]$ and $k_i \in [\underline{k}, \bar{k}]$ is equivalent to $k_i \in [\alpha_i + E[\theta] + \underline{\varepsilon}, \alpha_i + E[\theta] + \bar{\varepsilon}]$.

5 Extensions

In this section we consider three extensions to the baseline model. We allow for affiliated capacities, we characterize and compare the equilibria with the one that would arise if firms were paid according to their own bids (discriminatory auction), and we extend the equilibrium to allow for an arbitrary number of firms.

5.1 Discriminatory Auctions

In this section we characterize equilibrium bidding under the discriminatory auction in which each firm is paid according to its own bid.

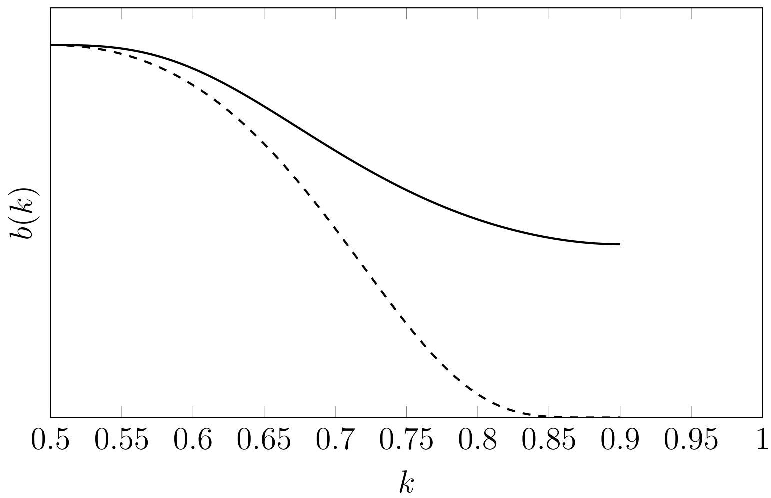

Figure 6: Comparison between the optimal bid function under the uniform auction (dashed line) and the discriminatory auction (solid line). Parameter values: $k_i \sim U[0.5, 0.9]$, $c = 0$, $P = 0.5$ and $\theta = 1$.

Proposition 6. *Assume $\theta/2 < \underline{k} < \bar{k} < \theta$. In the discriminatory auction, there exists a unique symmetric Pure Strategy Equilibrium: each firm $i = 1, 2$ chooses a bid that is strictly decreasing in k_i according to the function*

$$b_d^*(k_i; \underline{k}, \bar{k}) = c + (P - c)\exp\left(-\omega_d\left(k_i\right)\right),$$

where

$$\omega_d\left(k_i\right) = \int_{\underline{k}}^{k_i} \frac{(2k-\theta)g(k)}{kG(k) + \int_k^{\bar{k}}(\theta - k_j)g(k_j)dk_j}dk,$$

with $b_d^*(\underline{k}) = P$.

Firms now face stronger incentives to increase their bids. Unlike the uniform auction, a higher bid under the discriminatory auction always translates into a higher price, even when the firm bids below the rival. In other words, the price effect is always stronger. In particular, in this case for $k = \overline{k}$ the optimal bid is strictly above marginal cost, unlike in the uniform auction.

Since firms submit higher bids under the discriminatory auction, it follows that the highest accepted bid is higher than under the uniform auction. However, this does not necessarily imply that firms make higher profits under the discriminatory auction. The reason is that they receive their own bid rather than the highest.

When costs rather than capacities are private information Holmberg and Wolak (2018) provide a *revenue equivalence* result between the two auction formats. We now show that this is not the case when there is private information regarding the capacity.

Lemma 3. *The uniform and the discriminatory auction are not revenue equivalent.*

Suppose that the private information is on cost, as in Holmberg and Wolak (2018). In the symmetric equilibrium, small changes in costs affect prices, but due to the Envelope Theorem, this effect on profits is zero. Furthermore, contingent on having either the low or the high cost, the quantity is independent of the private information. Thus, to the extent that the probability that two firms have the same cost is zero, the cost ranking and quantities are not affected. Hence, revenue stays unchanged. The only effect of private information on profits is through changes in the costs of production, but this effect is the same across auction formats.

Matters are different when private information is on capacities, as changes in k affect firms' profits through their profit margin. In turn, since this profit margin is not the same across the two auctions, the impact of private information on revenues differs depending on the auction format in place (Lemma 3). Beyond this, it is difficult to arrive at general conclusions regarding the revenue ranking across auctions. However, numerical results suggest that the less competitive bidding under the discriminatory auction dominates over the fact that the uniform auction pays the highest bid to all units. Thus, at leasts when capacities are distributed according to a Beta distribution (Figure 6), the discriminatoy auction results in higher payments to firms.

5.2 N Firms

In this section we extend our equilibrium analysis to allow for an arbitrary number of firms, $N \geq 2$. This increase in the number of firms allows us to carry out two kinds of exercises: the effects of entry and the effects of changes in the market structure. Regarding the first, an increase in the number of firms brings in new capacity into the market. In the second case, a fixed distribution of capacity is allocated among a different

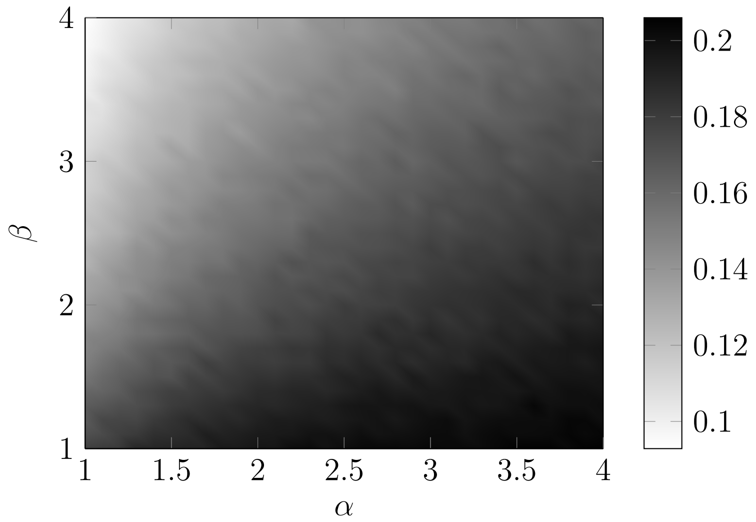

Figure 7: Difference in equilibrium prices between the discriminatory and the uniform auction when $k \sim Beta(\alpha, \beta)$ in the support $[0.5, 0.7]$.

number of firms.

For this purpose, we need some extra notation. Let k_j be the minimum capacity among those of firm i's rivals, i.e., $k_j = \min\{..., k_{i-1}, k_{i+1}\}$. Its cumulative distribution function and density are

$$\begin{aligned} \Phi(k_j) &= 1 - (1 - G(k_j))^{N-1} \\ \varphi(k_j) &= (N-1)\, g(k_j)\,(1 - G(k_j))^{N-2} \end{aligned}$$

We are going to focus on the case in which firms are pivotal. As a result, all firms but the one with the highest bid (and the smallest capacity) will sell at capacity. This means that from the point of view of firm i the N-firm problem can be reinterpreted as if it was only facing the smallest competitor.

The following result extends Proposition 2 to N firms.

Proposition 7. *Assume $\frac{\theta}{N} < \underline{k} < \bar{k} < \frac{\theta}{N-1}$. There exists a unique symmetric Pure Strategy Equilibrium: each firm $i = 1, ..., N$ chooses a bid that is strictly decreasing in k_i according to the function*

$$b^*(k_i) = c + (P - c)\exp\left(-\omega\left(k_i; \underline{k}, \bar{k}, N\right)\right),$$

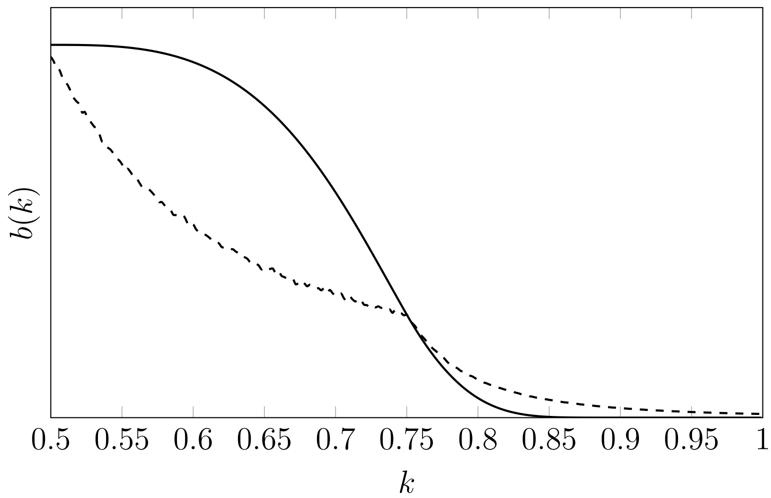

Figure 8: Comparison between $N = 2$ and $N = 4$. The dashed line corresponds to the maximum of the bids of two firms when $N = 4$ and their capacities are uniformly distributed, $k_i \sim U[0.25, 0.5]$. The solid line is the equilibrium bid of a firm when $N = 2$ and each firm has two plants that have the previous capacity distribution. The rest of parameters are $c = 0$, $P = 0.5$ and $\theta = 1$.

where

$$\omega\left(k_i; \underline{k}, \bar{k}, N\right) = \int_{\underline{k}}^{k_i} \frac{\left(2k + \int_k^{\bar{k}} (N-2)\, kg(k)dk - \theta\right) \varphi(k)}{\int_k^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right) \varphi(k_j)dk_j} dk,$$

with $b^*(\underline{k}) = P$ *and* $b^*(\bar{k}) = c$.

As compared to the solution in the duopoly case, N enhances the quantity effect because the loss in production from marginally increasing the bid is higher the more competitors there are in the market. At the same time the price effect is reduced because the firm only benefits from increasing the bid through the residual demand, which is now smaller. Both effects imply that the optimal bid goes down with N and so does the equilibrium price.

The model also allows us to understand what is the effect of changing market structure for a given total capacity distribution. Suppose that N is the number of plants and consider a situation in which we move from single-plant firms to a fewer number of firms owning multiple plants. In this case there are two different effects. As the number of firms decreases and each becomes bigger, they tend to behave less competitively and increase their bids. However, a second effect arises, as a decrease in the number of firms changes the distribution of the rivals' capacity. The capacity of a firm owning multiple plants tends to take an intermediate value since smaller realizations of one plant are compensated with larger realizations of another. Instead, in the single plant case what matters is the realization of the smallest capacity which is smaller than the average capacity. Firms face multiple plant competitors need to behave more aggressively as they expect them to submit lower bids.

Figure 8 provides an example of how the previous two forces shape the equilibrium with $N = 2$ as compared to the case with $N = 4$. The dashed line corresponds to the case with $N = 4$ and capacities are uniformly distributed between $U[0.25, 0.5]$, as described at the beginning of this section. The dashed line displays, $\hat{b}(k)$, the average highest bid of the two of these firms given a total capacity k defined as

$$\hat{b}(k) = \int_{2\underline{k}}^{2\overline{k}} \max[b(x), b(k-x)]g(x)g(k-x)dx.$$

The solid line is the equilibrium bid of a firm that owns two of the plants and competes with another firm that owns the other two. Hence, the difference between both cases is only due the ownership of the plants and not the distribution of the capacity available. This example illustrates how an increase in the number of firms translates in lower bids only when capacity is low, whereas the reverse is true when firms have a large aggregate capacity. The equilibrium price is lower when N increases.

6 Concluding Remarks

In this paper we have analyzed equilibrium bidding in multi-unit auctions when bidders' production capacities are private information. Furthermore, we have allowed changes in capacity to shape the bidding functions, both through changes in the prices and the quantities offered by firms. This is unlike other papers in the literature which typically assume that the private information is on costs (or bidders' valuations) and which, with few exceptions, do not allow bidders to act on both the price and quantity dimensions. We have shown that the nature of private information and the strategies available to firms have a key impact on equilibrium behaviour.

We have motivated the model in the context of electricity markets, in which renewable technologies have constant and zero marginal cost but their capacity is subject to fluctuations that are difficult to forecast by competitors. However, its approach and insights are also applicable to other multi-unit auction settings in which the quantity dimension (e.g. bidders' maximum willigness to buy or sell) need not be common knowledge, as typically assumed. Treasury auctions and the auctions for emissions permits are two examples, among others.

Understanding competition among renewable producers is key to predict the performance of future electricity markets as conventional technologies are being substituted by renewables. The profitably of current investments critically depends on the future prices at which firms expect to sell their renewable generation. Whereas in the past firms were often paid according to fixed prices (i.e., the so-called feed-in tariffs), the European Commission (in line with other jurisdictions) is now advocating to expose renewables to the time-varying prices set in wholesale electricity markets. Thus, to assess whether market revenues will be enough to induce the desired investments, regulators need to understand firms' price setting incentives if such investments indeed take place. Concerns over the profitability of renewable investments, have led regulators in several countries to pay them some sort of market premia that are set through auctions. Likewise, firms need to understand the future market performance in order to determine how to bid in the auctions for the new investments.

The standard approach to assess these issues has been to assume that renewable producers offer their output in the wholesale market at marginal costs. However, this approach tends to overestimate the price-depressing effect of renewables, therefore underestimating firms' investment incentives. As we have shown in this paper, since strategic producers add a markup to their marginal costs, the price impact of renewables is smoothed out, thus making investments more profitable than would otherwise be assumed under perfect competition. Likewise, the assumption of either competitive or strategic bidding also has implications for the expected price volatility, ultimately affecting investment incentives of risk averse firms. While in this paper we do not model investment decisions, analyzing equilibrium market outcomes for given capacities is a necessary first step to characterize equilibrium investment in the future.

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A Proofs

Proof of Proposition 1: See Fabra et al. (2006). Unlike their paper, the fact that capacities are random and observable allows to symmetrize the equilibrium through perfect correlation between the two asymmetric pure strategy equilibria. □

Proof of Lemma 1: For part (i) of the lemma, suppose that firm j chooses a bid according to a distribution $F_j(b, k_j)$ in the interval c to P. Denote this mixed strategy as $\sigma_j(k_j)$. The probability that the bid of firm i, b_i is lower than the bid of j, $h(b_i, \sigma_j(k_j))$ is non-increasing in b_i.

Profits for firm i can be written as

$$\pi_i(b_i, \sigma_j(k_j), k_i) = \int_{k_j} \left[(\sigma_j(k_j) - c)k_i h(b_i, \sigma_j(k_j)) + (b_i - c)(\theta - k_j)(1 - h(b_i, \sigma_j(k_j)))\right] g(k_j) dk_j.$$

Towards a contradiction, consider two bids b_i and $b_i' > b_i$ for which firm i randomizes. Then, it must be that firm i is indifferent and, thus,

$$\pi_i(b_i', \sigma_j(k_j), k_i) - \pi_i(b_i, \sigma_j(k_j), k_i) = \int_{k_j} \{k_i \left[b_j(k_j) - c\right] \left[h(b_i', \sigma_j(k_j)) - h(b_i, \sigma_j(k_j))\right] \quad (2)$$
$$+(\theta - k_j)\left[(b_i' - c)(1 - h(b_i', \sigma_j(k_j))) - (b_i - c)(1 - h(b_i, \sigma_j(k_j)))\right]\} g(k_j) dk_j = 0.$$

Since j cannot condition its strategy on k_i, then $\sigma_j(k_j)$ must be such that the previous expression holds for all k_i. Hence, either $b_j(k_j) = c$ for all k_j, in which case $\sigma_j(k_j)$ would be a degenerate mixed strategy, or

$$\int_{k_j} \left[h(b_i', \sigma_j(k_j)) - h(b_i, \sigma_j(k_j))\right] g(k_j) dk_j = 0.$$

Because the function h is non-increasing in the bid of firm i, this expression can only hold if $h(b_i, \sigma_j(k_j)) = h(b_i', \sigma_j(k_j))$ for all k_j. But in this case, from equation (2), $\pi_i(b_i', \sigma_j(k_j), k_i) > \pi_i(b_i, \sigma_j(k_j), k_i)$ leading to a contradiction.

Regarding part (ii) of the lemma, using the previous result we can focus on firm j choosing a pure strategy. As a result, it is enough to show that the function $\pi_i(b_i, b_j(k_j), k_i)$ has non-increasing differences in b_i and k_j. Using the previous expression and taking the derivative with respect to k_i we have

$$\frac{\partial\left[\pi_i(b_i', b_j(k_j), k_i) - \pi_i(b_i, b_j(k_j), k_i)\right]}{\partial k_i} = \int_{k_j} \left[b_j(k_j) - c\right] \left[h(b_i', b_j(k_j)) - h(b_i, b_j(k_j))\right] g(k_j) dk_j \leq 0.$$

In words, larger firms gain (weakly) less from increasing their bids. Hence, the optimal bid function is non-increasing in k_i. $\square$

Proof of Proposition 2: First, we show that in a symmetric equilibrium the bid of firm i has to be strictly decreasing in k_i. Towards a contradiction, suppose that this is not the case. From Lemma 1, this implies that there is a region $[k_a, k_b]$ such that both firms choose the same bid, $b_i = b_j$. At least one of the firms would not sell all its capacity for some capacity realizations. The standard Bertrand argument implies that this cannot be part of an equilibrium, as the firm that sells below capacity could increase its profits by slightly undercutting the competitor in order to sell at capacity.

Suppose, therefore, that b_j is strictly decreasing in k_j. As a result, the optimal bid for firm i can be characterized as $b_i \in \arg\max_{b_i} \pi_i(k_i; b_i, b_j(k_j))$ where

$$\pi_i(k_i; b_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)(\theta - k_j)g(k_j)dk_j. \quad (3)$$

Firm i has the low bid when $k_j < b_j^{-1}(b_i)$ and hence sells up to capacity at the price set by firm j. Otherwise, it sets the market price since it has the high bid.

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{\partial \pi_i}{\partial b_i} = b_j^{-1\prime}(b_i)g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j)g(k_j)dk_j = 0. \quad (4)$$

Furthermore, around the candidate equilibrium, the profit function is strictly concave.[17]

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$\frac{1}{b_i'(k_i)} g(k_i)(b_i(k_i) - c)(2k_i - \theta) + \int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j = 0. \quad (5)$$

The first term is negative and the second term is positive, so an interior solution exists. Indeed, the first order condition takes the form

$$b_i'(k_i) + a(k_i)b_i(k_i) = ca(k_i)$$

[17] The proof is available from the authors upon request.

where

$$a(k) = \frac{(2k-\theta)g(k)}{\int_k^{\bar{k}}(\theta-k_j)g(k_j)dk_j}. \tag{6}$$

If we multiply both sides by $e^{\int_{\underline{k}}^{k} a(s)ds}$ and integrate from $\underline{k}$ to k_i we obtain

$$\int_{\underline{k}}^{k_i}\left(e^{\int_{\underline{k}}^{k} a(s)ds} b_i'(k) + a(k)e^{\int_{\underline{k}}^{k} a(s)ds} b_i(k)\right) dk_i = c\int_{\underline{k}}^{k_i} a(k_i)e^{\int_{\underline{k}}^{k} a(s)ds} dk_i.$$

We can now evaluate the integral as

$$e^{\int_{\underline{k}}^{k} a(k)dk} b_i(k)\Big]_{\underline{k}}^{k_i} = ce^{\int_{\underline{k}}^{k} a(s)ds}\Big]_{\underline{k}}^{k_i}.$$

This results in

$$e^{\int_{\underline{k}}^{k_i} a(k)dk} b_i(k_i) - b_i(\underline{k}) = ce^{\int_{\underline{k}}^{k_i} a(k)dk} - c.$$

Solving for $b_i(k_i)$ we obtain

$$b_i(k_i) = c + Ae^{-\int_{\underline{k}}^{k_i} a(k)dk} = c + Ae^{-\omega(k_i)},$$

where $A \equiv b_i(\underline{k}) - c$ and $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk$,

A necessary condition for this to constitute an equilibrium is that equilibrium profits are at or above the minmax, which the firm can obtain by bidding at P. Hence, a necessary and sufficient condition for equilibrium existence is that

$$\pi_i(k_i) \geq (P-c)(\theta - E[k]) \tag{7}$$

for all k_i. Since equilibrium profits are increasing in k_i, whereas the minmax is independent of k_i, it follows that this condition is binding for $k_i = \underline{k}$. Note that the profits of $k_i = \underline{k}$ can be written as

$$\pi_i(\underline{k}) = (b_i(\underline{k}) - c)(\theta - E[k])$$

Hence, to satisfy condition (7), we must have $b_i(\underline{k}) = P$, implying $A = P - c$. Since any lower A would violate condition (7), the equilibrium is unique:

$$b_i^*(k_i) = c + (P-c)e^{-\omega(k_i)}.$$

Finally, we need to verify that the candidate equilibrium, indeed, maximizes profits for each of the firms. From the first order condition in (4) we can compute the second derivative of the profit function of firm i, when firm j uses a bidding function $b_j(k_j)$ as

$$\frac{g(b_j^{-1}(b_i))}{b_j'(k_j)}\left(-\frac{b_j''(k_j)}{\left(b_j'(k_j)\right)^2}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'(k_j)}\frac{g'(b_j^{-1}(b_i))}{g(b_j^{-1}(b_i))}(b_i-c)(k_i+b_j^{-1}(b_i)-\theta)\right.$$
$$\left.+(k_i+b_j^{-1}(b_i)-\theta)+\frac{1}{b_j'(k_j)}(b_i-c)-(\theta-b_j^{-1}(b_i))\right).$$

Once we substitute the candidate equilibrium $b_i(k)=b_j(k)$ the previous expression becomes

$$\frac{\partial^2\pi_i}{\partial^2 b_i(k_i)}=\frac{g(k_i)}{b^{*\prime}(k_i)}\frac{1}{a(k_i)}<0.$$

While this rules out local deviations, we still need to verify that firms do not have incentives to bid at c. □

Proof of Proposition 3: The proof follows the same logic as in Fabra et al. (2006). The difference between the two games is that in Fabra et al. (2006) capacities are known but fixed, so that firms cannot use their capacities as a correlation device. □

Proof of Proposition 4: We first show that the profit function is submodular. We need to distinguish three cases. First, if $k_i<\theta-\bar{k}$, profits are always $(P-c)\,k_i$ since there is not enough aggregate capacity. Since profits do not depend on b_i, the cross derivative of profits with respect to b_i and k_i is zero.

Second, if $\theta-\bar{k}<k_i<\theta-\underline{k}$, profits are

$$\pi_i(b_i,b_j,k_i)=\int_{\underline{k}}^{\theta-k_i}(P-c)\,k_i g(k_j)\,dk_j$$
$$+\int_{\theta-k_i}^{\bar{k}}((b_j-c)\,k_i h(b_i,b_j(k_j))+(b_i-c)(\theta-k_j)(1-h(b_i,b_j(k_j))))\,g(k_j)\,dk_j$$

Taking derivatives,

$$\frac{\partial\pi_i(b_i,b_j,k_i)}{\partial b_i}=\int_{\theta-k_i}^{\bar{k}}((b_j-c)\,k_i-(b_i-c)(\theta-k_j))\,h'(b_i,b_j(k_j))$$
$$+(\theta-k_j)(1-h(b_i,b_j(k_j))))\,g(k_j)\,dk_j$$

$$\frac{\partial\pi_i(b_i,b_j,k_i)}{\partial b_i\partial k_i}=\int_{\theta-k_i}^{\bar{k}}(b_j-c)\,h'(b_i,b_j(k_j))\,g(k_j)\,dk_j$$
$$+(b_j(\theta-k_i)-b_i)\,k_i h'(b_i,b_j(k_j))+k_i(1-h_i(b_i,\theta-k_i))\leq 0.$$

Note that the second and third terms are evaluated at $k_j = \theta - k_i$. The second term is non-positive given that for these values $\theta > k_i + k_j$ so that $b_j(\theta - k_i) = P \geq b_i$ and $h(b_i, b_j(k_j))$ is non-increasing. The last term is zero since, again $\theta > k_i + k_j$, so that $h_i(b_i, \theta - k_i) = 1$. Hence, the profit function is submodular.

Last, if $k_i > \theta - \underline{k}$, the proof is as in the proof of Lemma 1.

Now, we characterize the symmetric equilibrium. Consider equilibria with the following shape:

$$\tilde{b}_j(k_j) = \begin{cases} P & \text{if} \quad k_j < \tilde{k} \\ b_j(k_j) & \text{if} \quad k_j \geq \tilde{k} \end{cases}$$

where $b_j(k_j)$ is strictly decreasing k_j (recall that Bertrand arguments rule out flat regions).

We first show that $\tilde{k}$ cannot be lower than $\theta/2$. Argue by contradiction and suppose $\tilde{k} < \theta/2$, or rearranging, $\tilde{k} < \theta - \tilde{k}$. This cannot part of a symmetric equilibrium since for $k_i \in \left(\tilde{k}, \theta - \tilde{k}\right)$ aggregate capacity is not enough to cover total demand, implying that the best response of firm i includes P, a contradiction.

We now show that $\tilde{k}$ cannot be greater than $\theta/2$. Argue by contradiction and suppose $\tilde{k} > \theta/2$. This cannot be part of a symmetric equilibrium since for $k_i \in \left(\theta/2, \tilde{k}\right)$ firm i would be better off undercutting P. If the rival firm's capacity falls in the interval $k_j \in \left(\theta - k_i, \tilde{k}\right)$, the two firms would tie at P and each would sell below capacity. By slightly undercutting P, expected profits would increase by $(P - c)\left(G\left(\tilde{k}\right) - G(\theta - k_i)\right)\left(k_i - \frac{\theta}{2}\right)$. It follows that we must have $\tilde{k} = \theta/2$.

In turn, note that this implies that we only have ties at P when both firms have capacity below $\theta/2$ so that aggregate capacity is not enough to cover total demand. Otherwise, when at least one firm is selling below capacity, we never have ties at P.

Expected profits are

$$\pi_i(b_i, b_j, k_i) = (P - c)\, k_i G(\theta - k_i) + \int_{\max(\theta - k_i, \underline{k})}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)(\theta - k_j) g(k_j) dk_j$$

As compared to the profit expression in (3), there are only two differences: the first term, which reflects profits when aggregate capacity is not enough to cover total demand; and the lower limit of the integral in the second term, which now becomes $\max(\theta - k_i, \underline{k})$

instead of $\underline{k}$. None of these two differences affect marginal profits, which remain as in (5). If $k_i \leq \theta/2$, the first term in the first order condition (5) is always positive, hence it is optimal to bid at P. If $k_i > \theta/2$, the first term of the first order condition (5) is negative and the second term is positive, so an interior solution exists.

The proof follows the same steps as in the proof of Proposition 2, simply replacing $\underline{k}$ with $\max(\underline{k}, \theta/2)$ in all expressions. In this case, minmax profits depend on k_i. In particular, they are given by

$$\pi_i(b_i, b_j, k_i) = (P-c)\, k_i G(\theta - k_i) + \int_{\max(\theta - k_i, \underline{k})}^{\bar{k}} (P-c)(\theta - k_j) g(k_j) dk_j.$$

Hence, to rule out deviations to P, we now need to prove that minmax profits increase less in k_i as compared to equilibrium profits. First, suppose $k_i \leq \theta/2$. Equilibrium profits are exactly equal to the minmax as in equilibrium the firm bids at P. Second, suppose $\theta/2 < k_i < \theta - \underline{k}$. The derivative of the minmax with respect to k_i is

$$(P-c) G(\theta - k_i).$$

The derivative of profits is

$$(P-c) G(\theta - k_i) + \int_{\theta - k_i}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j$$

The other terms of the derivative cancel out because of continuity and because of the envelope theorem. Clearly, this derivative is greater than that of the minmax.

Last, suppose $k_i > \theta - \underline{k}$. The derivative of the minmax is

$$(P-c)\left(G(\theta - k_i) - g(\theta - k_i) k_i\right).$$

The derivative of profits is

$$(P-c)\left(G(\theta - k_i) - g(\theta - k_i) k_i\right) + \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c) k_i g(k_j) dk_j.$$

Again, this derivative is greater than that of the minmax.

It follows that deviations to P are not profitable since equilibrium profits are always strictly greater than the minmax, except for $k_i \leq \theta/2$ when equilibrium profits are exactly equal to the minmax. □

Proof of Lemma 2: We first prove the non-existence of a pure-strategy equilibrium when $k_i \geq \theta$. By way of contradiction, assume that there exists one. Following the same steps as in the proof of Lemma 1, it is easy to show that it must be non-increasing in k_i. Suppose, therefore, that b_j is non-increasing in k_j. As a result, the optimal bid for firm i can be characterized as $b_i \in \arg\max_{b_i} \pi_i(k_i; b_i, b_j(k_j))$.

If $b_i > b_j(\theta)$, expected profits are

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j\right).$$

Instead, if $b_i \leq b_j(\theta)$,

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c)\int_{\underline{k}}^{b_j^{-1}(b_i)} \theta g(k_j)dk_j.$$

In both cases, profit functions do not depend on k_i. Therefore, the optimal bid is the same for all $k_i \geq \theta$. Thus, at the candidate pure strategy equilibrium, $b^*(k_i) = b^*(\theta)$ for all $k_i \geq \theta$.

However, this is ruled out by standard Bertrand-Edgeworth arguments. First, if $b^*(\theta) > c$, firm i would have incentives to slightly undercut $b^*(\theta)$. If $k_j \geq \theta$, this would allow firm i to serve total demand, rather than a share of it, at only a slightly lower price, with almost no effect on firm i's profits if $k_j < \theta$. Second, if $b^*(\theta) = c$, the market price would always be c. Hence, firm i would make zero profits regardless of k_j and would rather deviate to P in order to make positive profits over the expected residual demand. It follows that the equilibrium must involve mixed strategies. Standard arguments imply that firms choose prices in a compact support $\left[\underline{b}, \bar{b}\right]$. □

Proof of Proposition 5: A symmetric Bayesian Nash Equilibrium must have the following properties. First, using the same arguments in Proposition 2, the optimal bid must be strictly decreasing in k_i for $k_i < \theta$. Second, from Lemma 2 it must involve a mixed strategy when $k_i \geq \theta$.

To make things simpler, we first assume $\bar{b} \leq b(\underline{k})$ and $\underline{b} \geq b(\theta)$. At the end of the proof we will show that this assumption must hold in equilibrium. We define $\tilde{k} = b_j^{-1}(\bar{b})$ and $\hat{k} = b_j^{-1}(\underline{b})$. Since $b_j(k_j)$ is decreasing, it follows that $\left[\tilde{k}, \hat{k}\right] \subseteq [\underline{k}, \theta]$. We consider four capacity regions:

Region I. If $k_i \in \left[\underline{k}, \tilde{k}\right]$, expected profits are

$$\pi_i(b_i; k_i, b_j(k_j)) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j$$

Firm i has the low bid when $k_j < b_j^{-1}(b_i)$ and, hence sells up to capacity at the price set by firm j. Otherwise, either it sells the residual demand and sets the price or, if $k_j > theta$ the rival will serve all the market.

Taking derivatives, we obtain a similar First Order Condition as in equation (5), with the only difference that $\bar{k}$ is replaced by θ. Hence, the solution is the same as in Proposition 2, with the only difference that $\bar{k}$ is replaced by θ in equation (6). Hence, the optimal bid in this region is

$$b^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}, \tag{8}$$

where $\omega(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk$, and

$$a(k) = \frac{(2k - \theta)g(k)}{\int_{\underline{k}}^{\theta}(\theta - k_j)g(k_j)dk_j}. \tag{9}$$

Using the optimal bid in (8), for given $\bar{b}$, $\tilde{k}$ is implicitly defined by

$$b^*\left(\tilde{k}\right) = \bar{b}.$$

Region II. If $k_i \in \left[\tilde{k}, \hat{k}\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i; k_i, b_j(k_j)) \quad &= \quad \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\theta} (b_i - c)(\theta - k_j)g(k_j)dk_j \\ &\quad + (1 - G(\theta)) \int_{b_i}^{\bar{b}} (b_j - c)k_i f_j(b_j)\, db_j.\end{aligned}$$

The profit expression now adds a third term as the firm will serve all its capacity at the price set by the rival whenever $k_j \geq \theta$ and $b_i < b_j$.

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\frac{1}{b_j'(k_j)} g(b_j^{-1}(b_i))(b_i - c)(k_i + b_j^{-1}(b_i) - \theta) + \int_{b_j^{-1}(b_i)}^{\theta} (\theta - k_j)g(k_j)dk_j - (1 - G(\theta))(b_i - c)k_i f_j(b_i) = 0.$$

This expression is similar to equation (5), where $\bar{k}$ replaces θ, plus an additional third term, which is negative. If follows that the optimal bid that solves the above equation is lower than the optimal bid in the baseline case.

Using symmetry, the optimal bid is the solution to

$$\left(1-\frac{(1-G(\theta))(b(k)-c)kf(b(k))}{(2k-\theta)g(k)}a(k)\right)b'(k)+a(k)b(k)=ca(k),$$

where $a(k)$ is defined as in equation (9). Note that if $G(\theta)=1$ we would obtain the same solution as in the baseline model. Since we now have $G(\theta)<1$, the solution is lower.

Region III. If $k_i \in \left[\hat{k},\theta\right]$, expected profits are

$$\begin{aligned}\pi_i(b_i;k_i,b_j(k_j)) &= \int_{\underline{k}}^{b_j^{-1}(b_i)}(b_j(k_j)-c)k_i g(k_j)dk_j+\int_{b_j^{-1}(b_i)}^{\theta}(b_i-c)(\theta-k_j)g(k_j)dk_j \\ &\quad +(1-G(\theta))\int_{\underline{b}}^{\bar{b}}(b_j-c)k_i f_j(b_j)\,db_j.\end{aligned}$$

The first-order condition that characterizes the optimal bid of firm i is the same as in Region I as the last term does not depend on b_i. Hence, the solution is also given by expressions (8) and (9). Hence, (8), for given $\underline{b}$, $\hat{k}$ is implicitly defined by $b^*(\hat{k},\underline{k},\theta)=\underline{b}$.

Region IV. Last, consider $k_i \in \left[\theta,\bar{k}\right]$. Expected profits are given by,

$$\pi_i(b_i;k_i,b_j(k_j))=(b_i-c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)}\theta g(k_j)dk_j+\int_{b_j^{-1}(b_i)}^{\theta}(\theta-k_j)g(k_j)dk_j+(1-F_j(b_i))(1-G(\theta))\theta\right) \tag{10}$$

As argued above, this profit function does not depend on k_i, so the optimal bid must be constant in k_i.

At the upper bound of the support, $F_j\left(\bar{b}\right)=1$. Hence, $\bar{b}$ maximizes

$$\begin{aligned}\pi_i(\bar{b},k_i) &= (\bar{b}-c)\left(\int_{\underline{k}}^{b_j^{-1}(\bar{b})}\theta g(k_j)dk_j+\int_{b_j^{-1}(\bar{b})}^{\theta}(\theta-k_j)g(k_j)dk_j\right) \\ &= \left(b^*\left(\tilde{k}\right)-c\right)\left(\theta G(\theta)-\int_{\tilde{k}}^{\theta}k_j g(k_j)dk_j\right)\end{aligned}$$

Taking derivatives with respect to $\bar{b}$,

$$\theta G(\theta)-\int_{b_j^{-1}(\bar{b})}^{\theta}k_j g(k_j)dk_j+(\bar{b}-c)\frac{1}{b_j'(k_j)}g(b_j^{-1}(\bar{b}))b_j^{-1}(\bar{b})=0$$

Using the definition of $\tilde{k}$ above, it can be re-written as

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j + (\bar{b} - c) \frac{1}{b_j^{*\prime}(\tilde{k})} g(\tilde{k})\tilde{k} = 0.$$

From the analysis of the case with certain pivotality we know that

$$b_i'(k_i) + a(k_i) b_i(k_i) = c a(k_i)$$

so that

$$b_j'(k_j) = -\left(b_i(k_i) - c\right) a(k_i)$$

Hence,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j - (\bar{b} - c) \frac{1}{\left(b^*(\tilde{k}) - c\right) a(\tilde{k})} g(\tilde{k})\tilde{k}$$

Since $b^*(\tilde{k}) = \bar{b}$,

$$\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j - \frac{g(\tilde{k})\tilde{k}}{a(\tilde{k})} = 0$$

Using the expression for $a(k)$ in equation (9),

$$\theta G\left(\tilde{k}\right) - \frac{\theta - \tilde{k}}{2\tilde{k} - \theta} \int_{\tilde{k}}^{\theta} \left(\theta - k_j\right) g(k_j) dk_j = 0,$$

which defines $\tilde{k}$. Note that we must have an interior solution, $\tilde{k} \in (\underline{k}, \theta)$. For $\tilde{k} = \underline{k}$, the first term is zero so the left hand side would be negative; whereas for $\tilde{k} = \theta$, the second term is zero so the left hand side would be positive.

At the lower bound of the support, $F_j\left(\underline{b}\right) = 1$. Expected profits are

$$\begin{aligned} \pi_i(\underline{b}, k_i) &= \left(\underline{b} - c\right) \left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) \\ &= \left(b\left(\hat{k}\right) - c\right) \left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) \end{aligned}$$

Since the firm must be indifferent between all the prices in the support, profits at the lower and upper bounds must be equal,

$$\left(\bar{b} - c\right) \left(\theta G(\theta) - \int_{b_j^{-1}(\bar{b})}^{\theta} k_j g(k_j) dk_j\right) = \left(\underline{b} - c\right) \left(\theta - \int_{b_j^{-1}(\underline{b})}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

Using the definitions for $\bar{b}$ and $\underline{b}$,

$$\left(b^*\left(\tilde{k}\right) - c\right) \left(\theta G(\theta) - \int_{\tilde{k}}^{\theta} k_j g(k_j) dk_j\right) = \left(b^*\left(\hat{k}\right) - c\right) \left(\theta - \int_{\hat{k}}^{\theta} k_j g(k_j) dk_j\right) = \pi^*$$

which defines $\hat{k}$. Hence, equilibrium profits are well defined $\underline{\pi}$ and we can treat them like a constant.

By the above equality, when $\bar{k}$ is just above θ, $\tilde{k}$ is arbitrarily close to $\hat{k}$. Instead, when $\underline{k}$ is so large that $G(\theta) = 0$, then $b\left(\hat{k}\right) = c$.

Now, we can use the above expression for equilibrium profits to solve for $F(b)$ in equation (10),

$$F(b) = \frac{1}{(1 - G(\theta))\,\theta}\left(\theta - \int_{b^{*-1}(b)}^{\theta} kg(k)dk - \frac{\pi^*}{(b-c)}\right)$$

where $b^*(k)$ is defined above by expressions (8) and (9).

Computing the density,

$$f(b) = \frac{1}{(1 - G(\theta))\,\theta}\left(\frac{\pi^*}{(b-c)^2} + \frac{k}{b^{*\prime}(k)}g(k)\right).$$

□

Proof of Proposition 6: Expected profits under the discriminatory auction are given by:

$$\pi_i(k_i; b_i, b_j(k_j)) = (b_i - c)\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j)g(k_j)dk_j\right). \quad (11)$$

Maximization with respect to b_i implies,

$$\left(\int_{\underline{k}}^{b_j^{-1}(b_i)} k_i g(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (\theta - k_j)g(k_j)dk_j\right) + (b_i - c)b_j^{-1\prime}(b_i)\left(g(b_j^{-1}(b_i))(k_i + b_j^{-1}(b_i) - \theta)\right) = 0.$$

Under symmetry, $b_j(k) = b_i(k)$. Accordingly, we can rewrite the expression as

$$k_i G(k_i) + \int_{k_i}^{\bar{k}} (\theta - k_j)g(k_j)dk_j + (b_i - c)\frac{1}{b_i'(k_i)}g(k_i)(2k_i - \theta) = 0$$

This expression is the similar as equation (5) for the uniform auction, but is has an additional term, $k_i G(k_i)$, reflecting the fact that the firm is always paid according to its bid, also when it is the large firm and hence has the low bid. The rest of the proof follows the same steps as the proof of Proposition 2. □

Proof of Lemma 3: Suppose that profits under the uniform and the discriminatory auction, determined by equations (3) and (11), are equal for all k_i. Since the bid of the discriminatory auction is higher than the uniform one, $b_u^*(k_i) < b_d^*(k_i)$, where we denote the latter as b_u^*, it then follows that the first term of the two profit equations must be such that

$$\int_{\underline{k}}^{k_i} (b^u(k_j) - c)k_i g(k_j)dk_j > \int_{\underline{k}}^{k_i} (b^d(k_i) - c)k_i g(k_j)dk_j$$

which implies

$$\int_{\underline{k}}^{k_i} \left(b^u(k_j) - b^d(k_i)\right) g(k_j)dk_j > 0.$$

It can be verified that the previous expression is equal to $\frac{\partial \pi_u(k)}{\partial k} - \frac{\partial \pi_d(k)}{\partial k}$. Using the Fundamental Theorem of Calculus and the fact that $\pi_u(\underline{k}) = \pi_d(\underline{k})$ it follows that $\pi_u(k) > \pi_d(k)$ which contradicts that profits are equal and, hence, there is no revenue equivalence.

$\square$

Proof of Proposition 7: Profits for firm i are:

$$\pi_i(k_i) = \int_{\underline{k}}^{b_j^{-1}(b_i)} (b_j(k_j) - c)k_i\varphi(k_j)dk_j + \int_{b_j^{-1}(b_i)}^{\bar{k}} (b_i - c)\left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j$$

The first-order condition that characterizes the optimal bid of firm i can be written as

$$\begin{aligned}\frac{\partial \pi_i}{\partial b_i(k_i)} &= b_j^{-1\prime}(b_i)\varphi(b_j^{-1}(b_i))(b_i - c)\left(k_j + \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk + b_j^{-1}(b_i) - \theta\right) \\ &+ \int_{b_j^{-1}(b_i)}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j = 0.\end{aligned}$$

Under symmetry, $b_j(k) = b_i(k)$, we can rewrite the expression as

$$\begin{aligned}\frac{\partial \pi_i}{\partial b_i(k_i)} &= \frac{1}{b_i'(k_i)}\varphi(k_i)(b_i(k_i) - c)\left(2k_i + \int_{k_i}^{\bar{k}} (N-2)\, kg(k)dk - \theta\right) \\ &+ \int_{k_i}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j = 0\end{aligned}$$

Reorganizing it,

$$b_i'(k_i) + b_i(k_i)a(k_i) = ca(k_i)$$

where $a(k_i)$ does not depend on b_i,

$$a(k_i) = \frac{\left(2k_i + \int_{k_i}^{\bar{k}} (N-2)\, kg(k)dk - \theta\right)\varphi(k_i)}{\int_{k_i}^{\bar{k}} \left(\theta - k_j - \int_{k_j}^{\bar{k}} (N-2)\, kg(k)dk\right)\varphi(k_j)dk_j}.$$

Hence, the solution is the same as above:

$$b_i^*(k_i) = c + (P - c)\, e^{-\omega(k_i)}$$

where $\omega\,(k_i) \equiv \int_{\underline{k}}^{k_i} a(k)dk.$ $\square$