Off-Balance Sheet Funding, Voluntary Support and Investment Efficiency*

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Abstract

Financing an investment off-balance sheet gives a bank the option, but not the obligation, to voluntarily support debt repayments using its on-balance sheet funds when the investment fails. Such flexibility, which is absent with on-balance sheet funding, allows the bank to signal information about the quality of its future projects, improving investment efficiency. Yet, off-balance sheet funding reduces the bank's skin-in-the-game and effort incentives. Off-balance sheet funding with voluntary support is optimal for activities that are rapidly growing or negatively correlated with existing assets. The model yields testable predictions on the relationship between off-balance sheet debt spreads and sponsors' characteristics.

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Keywords: off-balance sheet funding, voluntary support, signaling, optimal funding mode

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1 Introduction

An important fraction of banks' assets and activities is funded through sponsored off-balance sheet entities such as securitization vehicles and money market funds (MMFs). While theories of corporate structure have emphasized the role of limited liability at the subsidiary level (e.g. Flannery, Houston, and Venkataraman, 1993; Chemmanur and John, 1996; Kahn and Winton, 2004; and Dell'Ariccia and Marquez, 2010), banks frequently provide *voluntary support* to their sponsored entities that goes beyond their contractual obligations. The voluntary provision of recourse to the originator's balance sheet has been documented to be a widespread feature of securitization (e.g. Higgins and Mason, 2004, Gorton and Souleles, 2007, Vermilyea et al., 2008). Instances of voluntary support have heightened during the 2007-09 financial crisis amid surging default risks. Examples include Bear Stearns' rescue of two of its hedge funds in July 2007, sponsor banks' rescue of their SIVs in December 2007 (e.g. Acharya, Schnabl, and Suarez, 2013) as well as numerous instances of support in the money market industry (e.g. Brady, Anadu, and Cooper, 2012; Kacperczyk and Schnabl, 2012).

Regulators' concern about the potential costs for banks of providing support has motivated the introduction in the aftermath of the crisis of prohibitions or limitations to those voluntary transactions in many jurisdictions (e.g. as part of the Dodd-Frank in the US, and of the Financial Services (Banking Reform) Act 2013 in the UK). Yet, a better understanding of the rationale for off-balance sheet funding with voluntary support is necessary in order to evaluate the impact of regulatory interventions. In this paper we address the following questions. First, what determines banks' incentives to voluntarily provide support ex post, and their decisions to raise off-balance sheet funding ex ante? Second, what are the consequences of policy interventions that limit the provision of voluntary support?

This paper builds a model of the funding choice of a firm, which we refer to as a bank, with repeated investment opportunities. The bank can finance its investments with either *on-balance sheet* or *off-balance sheet* short-term debt. The latter funding mode consists of the creation of a new off-balance sheet legal entity, a vehicle, which holds the right to the project pay-offs and issues debt against them. Crucially, compared to on-balance sheet funding, off-balance sheet funding gives the bank the option, but not the obligation, to use pay-offs from its other on-balance sheet assets to repay the debt when the project fails. Such flexibility, which is absent with on-balance sheet funding, is valuable as it allows the bank to signal to investors its private information about the quality of its future investment opportunities, improving investment efficiency. However, the possibility of not repaying the off-balance sheet debt reduces the bank's skin-in-the-game and its incentives to exert effort. Off-balance sheet funding emerges as the optimal funding mode if the signaling value of voluntary support outweighs the losses stemming from the reduction on effort it induces. Our model delivers a rich set of novel empirical predictions regarding the use of off-balance sheet funding with voluntary support and the relationship between off-balance sheet debt spreads and sponsor banks' characteristics. The paper also yields policy implications on the regulation of voluntary support.

Formally, we develop a model of a bank with some asset-in-place and access to two consecutive investment opportunities. At the initial date, the bank can invest in a project with positive net present value (NPV), and can exert costly unobservable effort to further improve the probability that the project succeeds. At the time the second project is undertaken, however, there could be asymmetric information about its NPV. More precisely, we assume that upon the realization of a systematic shock, the bank's first project fails and the second project may become bad and have negative NPV. The bank privately learns whether its second project is good or bad. The modelling assumptions capture the association between negative economic shocks and uncertainty regarding future investment opportunities.[1]

In order to finance the investments, the bank can obtain funds at each of the investment dates from competitive investors. We assume that external funds can be obtained with either on-balance sheet or off-balance sheet debt. In either case, we assume that the debt issued is short-term and matures when the project pay-off is realized.

We first show that, if the first project was financed with off-balance sheet debt and it fails due to a systematic shock, the bank with a good second investment opportunity has incentives to use the pay-off of its asset-in-place to voluntarily provide support to the off-balance sheet debtholders. The reason is that the provision of voluntary support is interpreted as a signal of the quality of the bank's future investment opportunity and helps

[1] A large literature in macroeconomics presents robust evidence that uncertainty rises sharply in recessions, e.g. Jaroda, Ludvigson and Ng (2015).

to reduce the cost of external funds.[2] The result is consistent with the interpretation of commentators and regulators that voluntary support is driven by the banks' reputational concerns.[3] Specifically, the signaling properties of voluntary support in our model result from the higher sensitivity of the good bank's profits to changes in the cost of debt funding for the second project, as this bank type repays debt with a higher probability.

We then focus on how the bank's financing mode affects its effort choice for the first project. As in Holmstrom and Tirole (1997), the non-observability of the bank's effort leads to a moral hazard problem that inefficiently reduces effort when external funding is required. We show that the effort level achieved under off-balance sheet funding is lower than that under on-balance sheet funding. The reason is that the later funding mode obliges the bank to use the pay-off from its asset-in-place for debt repayments, while the former does not, so that the bank has more skin-in-the game when it relies on on-balance sheet funding. Interestingly, the provision of voluntary support increases the bank's skin-in-the-game when it relies on off-balance sheet funding, which increases effort to a level that is closer (but still lower) than that induced by on-balance sheet funding. Voluntary support thus partially mitigates the moral hazard problem associated with off-balance sheet funding.

The bank's optimal financing mode for the first project takes the two considerations above into account. On the one hand, the bank anticipates that off-balance sheet funding gives rise to signaling through voluntary support, which improves the efficiency of investment in the second project. On-balance sheet funding precludes signaling possibilities, as it leaves the bank no discretion but to use its funds to repay debtholders. On the other hand, off-balance sheet debt reduces the bank's skin-in-the-game and leads to lower and more inefficient levels of effort. Since investors are competitive, the gains and losses associated with the two

[2] Voluntary support amounts to a transfer to outstanding debtholders in order to affect future investors' perception on the bank's quality. In this respect, voluntary support constitutes a "money burning" signal. For examples of money burning signals, see, e.g., Milgrom and Roberts (1986), Daniel and Titman (1995), and Bagwell and Bernheim (1996). Note that an original feature of voluntary support as a money burning signal in the context of our model is that since investors anticipate the bank's support decisions, any money "burnt" to provide support ex post is priced in when the off-balance sheet debt is issued at the initial date.

[3] An example of the rating agency view is: "In effect, there is moral recourse since the failure to support the securitization may impair future access to the capital market." FitchIBCA (1999). The following quote on HSBC's rescue of its two Structured Investments Vehicles (SIVs) during the past financial crisis provides another illustration: "A huge SIV failure, especially if it triggered losses for the holders of its commercial paper, would be a reputational black eye."Financial Times, November 28, 2007.

investment inefficiencies just described translate into gains and losses for the bank owners from an ex ante perspective, so that the bank's optimal funding mode results from the trade-off between the two.

Our model yields novel empirical predictions. A first set of predictions relate to the determinants of the emergence of off-balance sheet funding. We find that banks with high growth activities are more likely to finance them off-balance sheet. In the context of the model, when the scale of the second investment opportunity is larger, the efficiency gain from being able to signal the quality of that investment through voluntary support is higher, making off-balance sheet funding more likely to emerge. Consistent with our prediction, Almazan, Martín-Oliver, and Saurina (2015) present evidence of more intense use of securitization by banks with stronger growth opportunities.[4] We also find that off-balance sheet funding is more likely to emerge for activities that are negatively correlated with the bank's core activities. The reason is that the bank's ability to signal strength through support when its project fails depends on its own funds under that contingency, which are larger when the bank's core activities are negatively correlated with the new investment.

The paper also delivers novel predictions that can be tested with market data. First, the provision of voluntary support conveys positive information on the bank. Consistent with this prediction, Higgins and Mason (2004) show that such events are associated with improved short-term stock price performance and long-term financial performance. Second, the spread of off-balance sheet debt decreases with the financial strength of the sponsor. In the model, an increase in the expected pay-off of the asset-in-place leads a bank that relies on off-balance sheet debt to provide more support in expected terms, reducing the spread on its off-balance sheet debt. This prediction is consistent with rating agencies' view (Moody's Investor Service, 2006), and has also been documented in Gorton and Souleles (2006) for the credit card backed securities market. Third, the spread of off-balance sheet debt decreases with the expected growth of off-balance sheet activities. This is because banks with large future investment opportunities are expected to provide larger support to their

[4]In a different but related context, Cerutti, Dell'Ariccia and Martinez-Peria (2007) find that banks, when expanding to a foreign country, are more likely to establish branches when the operations are small, and more likely to set up a subsidiary when seeking to penetrate the foreign market. Notice that the limited liability protecion of banks in regards of their subsidiaries render this form of expansion similar to off-balance sheet funding. That is not the case when expansion is undertaken through branches.

outstanding off-balance sheet debt investors. Finally, the spread of off-balance sheet debt depends negatively on the correlation between the banks' on-balance sheet and off-balance sheet assets. The reason is that when there is positive correlation between the two assets the bank has low capability to provide support to off-balance sheet debtholders when support is needed, that is, when the assets baking the claims perform badly.

We next use the model to analyze the effects of the regulatory interventions in many jurisdictions that limit the provision of voluntary support. In a first exercise, we consider the ex ante implications for the bank's profits (which coincide with aggregate surplus) of the introduction at the initial date of a ban on the provision of voluntary support. This prohibition renders off-balance sheet funding sub-optimal, as a bank that relies on off-balance sheet funding can no longer signal information to investors, but nevertheless suffers from more severe moral hazard problems. If the ban on voluntary support induces the bank to switch from off- to on-balance sheet funding, it reduces the bank's profits and surplus in the economy.

Even if allowing voluntary support increases the bank's profits from an ex ante perspective, a bank regulator might be concerned that the ex post provision of support reduces bank capitalization. In a second exercise, we analyze the effect on the bank's profits of the unexpected ex post introduction of a ban on voluntary support when the first project fails. Notice that from an ex ante perspective voluntary support does not directly affect the bank's expected surplus since these voluntary transfers are priced by the investors in off-balance sheet debt, but from an ex post perspective support amounts to money burning and directly leads to a reduction on the bank's net worth. We find that such ex post negative effect is more than overcome by the investment efficiency gains induced by the provision of voluntary support, so that the ex post introduction of a ban on support would also be detrimental for banks.[5] The results point to the absence of a time consistency problem in permitting banks to provide voluntary support to their off-balance sheet funding structures.

Finally, we demonstrate that the use of voluntary support to signal quality is robust to the presence of other signaling devices. In an extension of the baseline model, we allow the

[5]This is in contrast to other signaling games, in which some equilibria are Pareto dominated by the pooling outcome that arises when agents are not allowed to send signals. A classical example of this is a version of Spence (1973) in which education does not increase productivity.

bank access to a storage technology. This implies that, upon first project failure, the bank can store the pay-off of its asset-in-place and use it to (partially) self-finance the second investment. This gives rise to an alternative means of signaling strength via self-financing, similarly as in Leland and Pyle (1977). The extension shows that voluntary support remains a valuable signaling device that is used in equilibrium, although sometimes in conjunction with self-financing.

Related literature The main intuition of this paper is related to Boot, Greenbaum, and Thakor (1993), who show that unenforceable financial contracts grant the issuer discretion whether or not to satisfy them, which fosters reputation building. The authors then show that this may lead in equilibrium to the emergence of unenforceable contracts in a context in which enforceable contracts are feasible. Similarly, our paper shows that funding an investment off-balance sheet gives the bank discretion over whether or not to voluntarily provide support. This in turn allows the bank to signal information to outside investors and is valuable for the bank from an ex ante perspective.

This paper belongs to the theoretical literature that analyzes the emergence of a shadow banking system that relies on voluntary support from sponsoring institutions.[6] Ordoñez (2018) develops a model of off-balance sheet funding in which providing voluntary support has reputation implications, focusing on its negative consequence in terms of fragility. By contrast, we emphasize the positive consequence of such reputation implications in terms of improved investment efficiency.[7] There are other contributions in which voluntary support is not driven by reputational considerations. Gorton and Souleles (2007) and Kuncl (2015) show that voluntary support arises as a form of collusion between the originator banks and investors in off-balance sheet vehicles in a repeated interaction context. Kobayashi and Osano (2012) build a model in which banks voluntarily satisfy implicit guarantees on short-term funded vehicles to avoid the costly liquidation of their long-term assets in some states of the world. From an ex ante perspective implicit guarantees dominate explicit ones because they

[6] Other theories of shadow banking in which voluntary support considerations are absent include Parlour and Plantin (2008), Dang, Gorton and Holmström (2012) and Gennaioli, Shleifer and Vishny (2013).

[7] Segura (2017) develops a signaling model of voluntary support in which a sponsor bank rescues its vehicle in distress to avoid that investors run on the bank's short-term liabilities. Yet, the paper does not analyze the bank's ex ante decision to create a vehicle.

lead the bank to liquidate the vehicle assets when it is efficient to do so. Parlatore (2016) builds a model of delegated portfolio management in which the sponsor obtains fees that are proportional to the market price of assets under management and the incentives to provide support depend on these fees.

A number of theoretical papers analyze the moral hazard problem associated with securitization, including Gorton and Pennacchi (1995), Parlour and Plantin (2008), Chemla and Henessy (2014), Rajan, Seru, and Vig (2010) and Vanasco (2017).[8] These papers typically focus on the trade-off between incentive costs associated with off-balance sheet activities and some exogenous gains from off-balance sheet funding. Our main contribution to this literature is highlighting the endogenous gains from off-balance sheet funding afforded by the possibility to signal quality through voluntary support. A second contribution is showing that voluntary support provision partially restores the bank's skin-in-the-game and thus alleviates the moral hazard problem associated with off-balance sheet funding.

Our paper relates more broadly to the literature on optimal corporate organizational structure that explores intragroup support. Friedman, Johnson and Mitton (2003) argue that controlling shareholders have incentives to use their private funds to temporarily "prop up" troubled group affiliates in order to preserve the option to expropriate their future profits. Riyanto and Toolsema (2008) and Luciano and Nicodano (2014) argue that the parent company may provide voluntary support to solvent subsidiary debtholders with insufficient cash flows in order to avoid costly bankruptcy. By contrast, voluntary support in our model is driven by reputation concerns of the parent firm.

Our paper is also related to the literature on the emergence of securitized instruments with explicit guarantees. Benveniste and Berger (1987) argue that securitization with explicit recourse improves risk sharing among investors with heterogeneous risk aversion. Greenbaum and Thakor (1987) explore the trade off between securitization with explicit credit enhancement, in which the flexibility in the ex ante choice of the enhancement level allows the bank to alleviate information asymmetry, and deposit funding that allows more efficient risk

[8]There is also a large empirical literature on the importance of moral hazard problems associated with the originate-to-distribute intermediation model (e.g. Berndt and Gupta (2009) for the syndicated loan market, and Dell'Ariccia, Igan, and Laeven (2012), Elul (2009), Jaffee et al. (2009), Keys et al. (2010), and Mian and Sufi (2009) for the mortgage market)

sharing.

The rest of the paper is organized as follows. Section 2 presents the model set-up. Section 3 proceeds by backward induction to determine the bank's optimal funding structure at the initial date. Section 4 discusses the empirical predictions (Section 4.1), the policy implications (Section 4.2) and provides an extension of the baseline model to allow for storage possibilities (Section 4.3). Section 5 presents the conclusions of our work. The proofs of the formal results of the paper can be found in the Appendix.

2 The model

There are four dates $t \in \{0, 1, 2, 3\}$ and two classes of risk-neutral agents: a bank and investors. There is no time discounting, and in the baseline model there is no access to a storage technology.[9] The bank has an asset-in-place which pays off at $t = 1$ but has no funds at $t = 0$. The bank has investment opportunities at $t = 0$ and at $t = 2$ whose pay-offs realize at $t + 1$. The investors are deep-pocketed and competitive.

Investment opportunities The first investment opportunity arises at $t = 0$ and requires one unit of funds. Its pay-off at $t = 1$ is either $R_1 = R$ in case of success, or $R_1 = 0$ in case of failure. We assume that the first investment is good (g), and succeeds with probability p_g such that:

Assumption 1 $p_g R > 1$.

The assumption states that investment in the first project has positive NPV, so that the bank always undertakes it.

We assume that the project failure can be due to either systematic reasons, with probability $q < 1 - p_g$, or idiosyncratic reasons, with probability $1 - p_g - q$. The bank can exert unobservable effort to increase the probability of success and reduce that of idiosyncratic failure. Intuitively, the systematic shock captures macroeconomic factors that affect the investment return beyond the bank's control.

[9] We extend the model to allow for a storage technology in Section 4.3 and show that our main results continue to hold.

Specifically, by exerting effort $e \in [0,1]$ at private cost $c(e)$, the bank increases the first project's probability of success to $p_g + me$ and reduces that of idiosyncratic failure to $1 - p_g - q - me$, where m is a strictly positive constant. One interpretation of the bank's effort might be the intensity of its monitoring or screening of borrowers and the parameter m can be interpreted as capturing the marginal value of effort. We restrict m to satisfy $m \leq 1 - p_g - q$, which ensures that idiosyncratic failure probability is non-negative. We assume that the effort cost function is increasing and convex in the effort level, which ensures a unique interior optimal effort choice by the bank. Formally:

Assumption 2 *(i)* $c(0) = 0$, *(ii)* $c'(0) = 0$ *and* $c'(1) > mR$, *and (iii)* $c''(e) > 0$.

We denote by $\sigma = S$ the state of the economy at $t = 1$ in which the first project fails due to systematic reasons and refer to it as the systematic state, and with $\sigma = \overline{S}$ its complementary state, which includes the project's success and its idiosyncratic failure, and refer to it as the non-systematic state. We assume that at $t = 1$ both the project return R_1 and whether or not it has systematically failed σ are public information.

The second investment opportunity arises at $t = 2$ and is of scale $I > 0$. That is, it requires $I > 0$ units of investment, and pays off at $t = 3$ either $R_3 = RI$ in case of success, or $R_3 = 0$ in case of failure. Its success probability depends on the bank type $j \in \{g, b\}$ at $t = 1$, which is privately observed by the bank at that date. A good (g) bank's investment succeeds with probability p_g, whereas a bad (b) bank's investment succeeds with probability $p_b < p_g$. For simplicity, we assume that the second bank's investment does not require effort.[10] Moreover, we assume that a b bank's investment has negative NPV:

Assumption 3 $1 > p_b R$.

The bank type depends on whether or not there is systematic failure of the first project at $t = 1$. If there is no systematic failure ($\sigma = \overline{S}$) the bank type is g with certainty (and a fortiori there is no asymmetric information on the bank type). However, if the first project fails due to systematic reasons ($\sigma = S$), the bank type is g with probability $\alpha \in (0, 1)$ and b otherwise. We assume that:

[10]Introducing an effort decision for a good second investment similar to that of the first project does not affect our results.

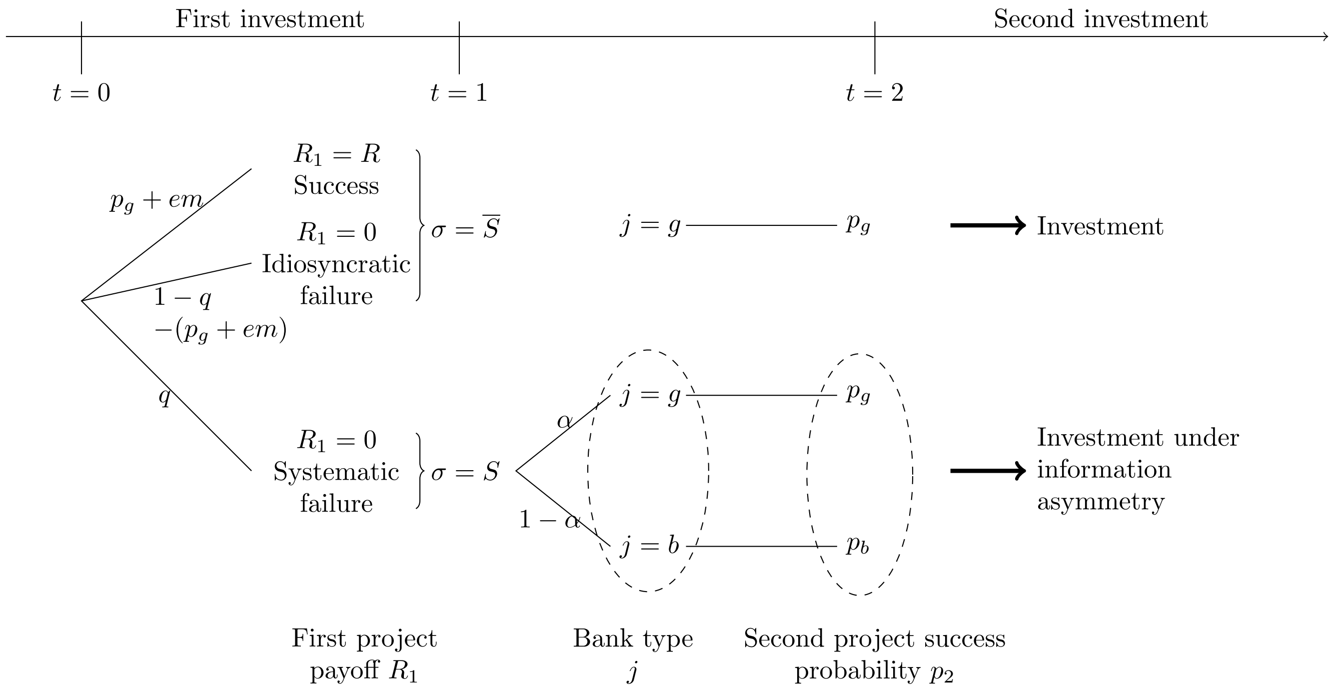

Figure 1: Distribution of the second project's success probability p_2. Dashed circles represent the information set of uninformed investors.

Assumption 4 $\alpha < \bar{\alpha} \equiv \frac{1-p_bR}{p_gR-p_bR}$.

Note that $\bar{\alpha}$ is defined to satisfy the equality $[\bar{\alpha}p_g+(1-\bar{\alpha})p_b]R = 1$ and thus this assumption states that an imperfectly informed investor who believes a bank is of type g with probability α is not willing to finance the bank's second project.[11] Figure 1 describes the distribution of bank types and the investors' information sets at $t = 1$ for all possible contingencies.

Let us comment on two features generated by our assumptions. First, asymmetric information regarding the quality of the second investment opportunities may arise at $t = 1$. This assumption is key to give the bank incentives at $t = 0$ to set-up financing structures that afford for signaling opportunities at $t = 1$, which constitutes the focus of this paper. Second, the effort choice for the first project at $t = 0$ does not affect the probability of systematic failure and thus neither the quality distribution of the second project. This assumption, which may be justified by the evidence that following negative macroeconomic shocks uncertainty rises sharply (e.g. Jurado, Ludvigson and Ng, 2015), is mainly adopted

[11]All the results and intuitions in the paper are valid in the case $\alpha \geq \bar{\alpha}$ but some of the analytical expressions we derive might change. For the sake of simplicity we thus focus only on the $\alpha < \bar{\alpha}$ case.

for analytical tractability. Our results would still hold in a context in which there were no systematic failure of the first project ($q = 0$) and, instead, asymmetric information were to arise following idiosyncratic failure of that project.[12]

Asset-in-place The pay-off at $t = 1$ of the bank's asset-in-place is $Y_\sigma < 1$, where $\sigma \in \{S, \overline{S}\}$ denotes the first project's systematic and non-systematic states. The dependence of the asset-in-place pay-off on the state σ allows to model different correlation patterns between the bank's asset-in-place and the first project. In fact, R_1 and Y_σ are positively correlated if $Y_S < Y_{\overline{S}}$, uncorrelated if $Y_S = Y_{\overline{S}}$, and negatively correlated if $Y_S > Y_{\overline{S}}$.

Financing choices In order to invest in the projects, the bank must raise external funds from investors. We assume external funding can be raised with either *on-balance sheet* or *off-balance sheet* one-period debt. Under on-balance sheet funding, the bank issues one-period debt backed by the return of the project *and* any other funds in the bank at the time the project return is realized.

Under off-balance sheet funding, the project is financed with one-period debt that is backed *only* by the project return. This is achieved by the following off-balance sheet funding structure. First, the bank incurs the unit cost of investing in the project and sets up a one-period-lived separate legal entity, which we refer to as off-balance sheet vehicle. Second, the bank sells the project to the vehicle at a price of one unit of funds that the bank uses to pay the investment cost. Third, the vehicle finances the project purchase by issuing one-period debt backed by its only asset, that is, by the project, and the bank keeps the residual claim of the vehicle. Crucially, since the vehicle is a legal entity separated from the bank whose only asset is the project, the bank is not contractually obliged to use the return of its asset-in-place to repay the vehicle debt when the project pay-off is zero. Rather, the bank has discretion whether or not to use its own funds to make such repayment. That is, the bank may choose to *voluntarily provide support*.[13] We show in Section 3.1 that in presence of asymmetric information the provision of voluntary support to off-balance sheet debt at

[12]Under such assumptions some of the analytical derivations we conduct in Section 3 would be more complicated because of the effect of effort choice at $t = 0$ on the quality of second investment opportunities.

[13]Our modelling of the securitization process captures its key features in practice. For more institutional details, see Gorton and Souleles (2007) and Gorton and Metrick (2013).

$t = 1$ constitutes a signal of quality that reduces funding costs for the second project.

More precisely, the sequence of decisions is as follows. At $t = 0$, the bank decides whether to finance its project on-balance sheet or off-balance sheet. The competitive investors set the required promise repayment D_1 to provide the unit of funds demanded by the bank or its vehicle and the bank incurs the investment cost. After that the bank exerts effort e.

At $t = 1$, the pay-off R_1 of the first investment and the pay-off Y_σ of the asset-in-place are realized. If $R_1 = R$, investors are repaid D_1 regardless of the financing mode. Under off-balance sheet funding, the residual pay-off $R - D_1$ of the vehicle is paid to the bank.

Instead, if $R_1 = 0$, the debt repayment depends on the nature $\sigma \in \left\{S, \overline{S}\right\}$ of the failure and the financing mode. Under on-balance sheet funding, the bank is obliged to use the asset-in-place return to repay D_1. If $Y_\sigma < D_1$ the bank is not able to repay the debt in full. For simplicity, we assume that existing debtholders allow the bank to write down the $D_1 - Y_\sigma$ units of unrepaid debt promise, so that the bank does not default and can undertake the second investment opportunity.[14] By contrast, under off-balance sheet funding, the bank is not obliged to use its asset-in-place to repay the off-balance vehicle debtholders. Depending on the nature σ of the failure (systematic or not) and its type, the bank decides the amount $s \in [0, \min\{Y_\sigma, D_1\}]$ of its own funds to use to repay (part of) the D_1 promise to the off-balance sheet debt investors. We refer to s as the voluntary *support* decision of the bank and assume that it is observable by all investors.

Since there is no access to a storage technology, at the end of $t = 1$ the bank pays out any remaining funds to its owners as profits.[15]

At $t = 2$, each type of bank decides whether to invest in its second project. Since the bank has no asset-in-place paying-off at $t = 3$ the financing mode choice is irrelevant at this date. If the bank decides to invest, competitive investors (who have all the public information

[14]The assumption removes the possibility that bank default prevents the bank from investing in its second project, which could constitute an additional cost associated with on-balance sheet funding. Allowing for potentially costly bank default would only provide another reason for the bank to rely on off-balance sheet funding in addition to that associated with the possibility to provide voluntary support, which is the focus of this paper.

[15]Notice that when asymmetric information arises at $t = 1$, the assumption that there is no access to a storage technology rules out the possibility for the bank to send signals at $t = 2$. This restrictive assumption allows us to highlight in the baseline model the signaling role of voluntary support. In Section 4.3 we introduce a zero net return storage technology and show that all our main results remain valid.

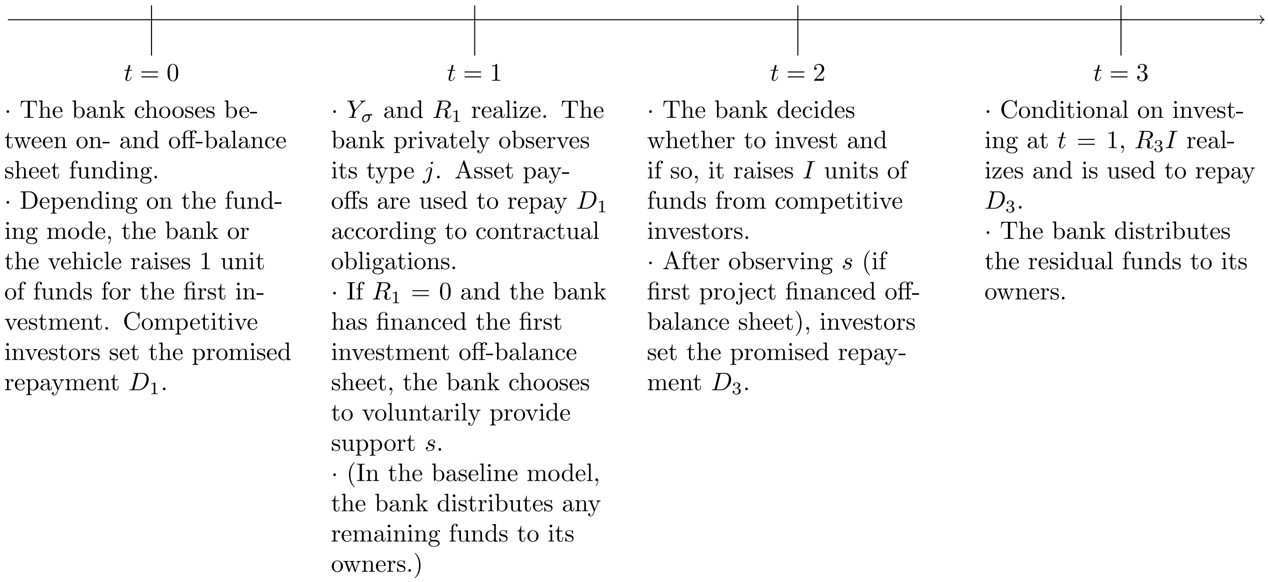

Figure 2: Sequence of decisions and events

on the history of the economy) set their required promised debt repayment $D_3 \in [0, RI]$ in exchange for the I units of funds demanded by the bank, or refuse to provide funding if they expect not to be able to break-even.

Conditional on investment at $t = 2$, the pay-off R_3 of the project is realized at $t = 3$ and used to repay D_3. Any remaining funds in the bank at $t = 3$ are distributed to its owners as profits.

The sequence of decisions and events is summarized in Figure 2. When asymmetric information arises at $t = 1$ we use the concept of perfect Bayesian equilibrium to determine the subsequent decisions of each bank type and investors and use the D1 refinement of Cho and Kreps (1987) to refine investors' off-equilibrium beliefs.[16]

First-best benchmark Before concluding this section, we compute the expected net present value from the bank investments and asset-in-place in a first-best benchmark. Recall that Assumptions 1 and 3 imply that it is efficient to undertake the first project and the second project ony if the bank is of type g. Besides, taking into account that effort increases the

[16]This is a commonly used refinement. In the context of financing decision under asymmetric information, see, for example, Nachman and Noe (1994) and DeMarzo and Duffie (1999). We summarize how to apply this refinement at the beginning of the Appendix.

success probability of the first project at a constant rate m and has no effect on the quality of the second investment, Assumption 2 implies that the first-best effort choice, which we denote by e^{FB}, is uniquely determined by:

$$c'(e^{FB}) = mR. \tag{1}$$

The expression equalizes the marginal cost and benefit of effort.

The expected net present value from the bank's asset-in-place and its investments under efficient decisions, which we denote by V^{FB}, is thus immediately given by:

$$\begin{aligned} V^{FB} = \underbrace{E[Y_\sigma]}_{\text{asset-in-place exp. pay-off}} &+ \underbrace{\left[(p_g R - 1) + \int_0^{e^{FB}} (mR - c'(e))\, de\right]}_{\text{first project NPV}} \\ &+ \underbrace{\left[\underbrace{(1-q)\,(p_g R - 1)\, I}_{\text{after success or idios. default } (\sigma=\overline{S})} + \underbrace{q\alpha(p_g R - 1) I}_{\text{after systematic default } (\sigma=S)}\right]}_{\text{second project expected NPV}}. \end{aligned} \tag{2}$$

3 Equilibrium analysis

We solve the model by backward induction. We first consider the bank's investment in the second project following the systematic failure of the first project, and highlight the signaling possibilities associated with the provision of voluntary support in case the first project were funded off-balance sheet. We then analyze how the bank's funding choice for the first project affects the effort decision in that project. We finally use these results to determine the optimal funding choice at $t = 0$.

3.1 Second investment: The signaling value of voluntary support

In this section, we analyze how following the systematic failure of the first project at $t = 1$ the possibility to voluntarily provide support affects the bank's investment in the second project. Suppose that the first project fails for systematic reasons at $t = 1$. The bank privately learns its type $j \in \{g, b\}$ while investors believe that the bank is of type g with probability α. At $t = 2$ the second investment is undertaken under asymmetric information. We describe next how the initial funding mode choice affects investment under such contingency.

No voluntary support possibility Suppose that the bank finances the first project on-balance sheet and the project fails due to systematic reasons at $t = 1$. The bank is contractually obliged to use the return Y_S of its asset-in-place to repay the debt promise D_1.[17] Since repayment is compulsory, it does not reveal information about the bank type and investors' belief about the bank quality is α at $t = 2$. The investment of each bank type is characterized as follows.

Lemma 1 *Suppose the bank has financed the first project off-balance sheet and the project fails due to systematic reasons at $t = 1$. There exists a unique equilibrium of the second investment game at $t = 2$ with the property that the good bank invests with probability 1. In that equilibrium, the bad bank invests with probability $\pi^{On} < 1$, given by*

$$\pi^{On} = \frac{\alpha}{1-\alpha}\frac{p_g R - 1}{1 - p_b R}.$$

All other equilibria are pay-off equivalent, in which each bank type obtains zero expected profit.

The lemma focuses on the equilibrium of the investment under asymmetric information game in which the g bank invests with probability one. The equilibrium is characterized by the probability π^{On} that the b bank also invests. Notice that π^{On} is such that the updated belief on the quality of a bank that invests leads investors to ask a promised repayment in exchange of their funds amounting to $D_3 = RI$. As a result, the b bank is indifferent between investing or not, which sustains the equilibrium. For reasons that will become clear below, we focus without loss of generality in the rest of the paper on this equilibrium of the second investment game under asymmetric information when on-balance sheet funding is used.

Voluntary support possibility Suppose that the bank has financed the first project off-balance sheet and at $t = 1$ the project fails due to systematic reasons ($\sigma = S$). The return of the bank's asset-in-place is Y_S and the bank is not contractually obliged to use it to repay the off-balance sheet debt promise D_1. Yet, after privately observing its type, the bank can voluntarily provide support, which consists of a payment $s \leq Y_S$ the bank makes to the

[17] Since it is necessarily the case that the debt promise repayment D_1 satisfies $D_1 \geq 1$ and by assumption $Y\sigma < 1$, the bank exhausts the return from its asset-in-place to partially repay the debt promise. Moreover, recall that in Section 2 we have assumed that any unpaid on-balance sheet debt at $t = 1$ is written down and the bank can continue to $t = 2$.

off-balance sheet debt investors.[18] At $t = 2$, the bank chooses whether or not to invest. If it invests, investors update their belief $\hat{\alpha}$ on the bank type based on its voluntary support decision s at $t = 1$, and set a competitive promised repayment $D_3(\hat{\alpha})$ to provide the I units of funds necessary for investment, which satisfies

$$D_3(\hat{\alpha}) = \frac{I}{\hat{\alpha} p_g + (1 - \hat{\alpha}) p_b}. \tag{3}$$

Notice that $D_3(\hat{\alpha})$ is decreasing in $\hat{\alpha}$, reflecting that investors provide better funding terms to a bank whose perceived quality is better. Whenever $D_3(\hat{\alpha}) > R$, we interpret that the investors refuse to provide funding and investment does not take place.

Conditional on investing in the second project when asymmetric information persists, the expected profits as of $t = 1$ of a bank of type j for given $(s, \hat{\alpha})$ are

$$\Pi_{j,1}(s, \hat{\alpha}) = (Y_S - s) + p_j[RI - D_3(\hat{\alpha})]. \tag{4}$$

The first term in this expression accounts for the funds the bank pays out to its owners at the end of $t = 1$, which are reduced due to the provision of voluntary support. The second term captures that the bank obtains a profit at $t = 3$ equal to the expected value of the residual claim on the second project, $p_j[RI - D_3(\hat{\alpha})]$.

The presence of asymmetric information gives the bank incentives to incur the cost of providing voluntary support to the outstanding off-balance sheet investors, if by doing so it is able to improve its perceived quality and thus the funding terms for its second investment. Indeed, voluntary support is a credible signal of information in our model. To see this, notice that, for a bank of type $j \in \{g, b\}$ the ratio of the marginal cost of providing support to its marginal benefit satisfies

$$-\frac{\partial \Pi_{g,1}(s, \hat{\alpha})/\partial s}{\partial \Pi_{g,1}(s, \hat{\alpha})/\partial \hat{\alpha}} = \frac{1}{-p_g \frac{\partial D_3(\hat{\alpha})}{\partial \hat{\alpha}}} < \frac{1}{-p_b \frac{\partial D_3(\hat{\alpha})}{\partial \hat{\alpha}}} = -\frac{\partial \Pi_{b,1}(s, \hat{\alpha})/\partial s}{\partial \Pi_{b,1}(s, \hat{\alpha})/\partial \hat{\alpha}}. \tag{5}$$

The ratio of marginal cost to benefit from providing support is lower for a g bank, which can be interpreted as the "single-crossing" condition conferring voluntary support the properties of a signal of good quality. The reason is that the expected profit of a g bank is more sensitive

[18] Notice that the amount of support provision is also constrained to be less than the promised repayment of the off-balance sheet debt, i.e. $s \leq D_1$. However, since $Y_S < 1 \leq D_1$, the constraint $s \leq D_1$ is implied by the constraint $s \leq Y_S$.

to the reduction in the promised debt repayment D_3 induced by an improvement in $\hat{\alpha}$, as the g bank satisfies the promise with a higher probability. Notice that voluntary support is a *money burning* signal sent at $t = 1$, as it amounts to a transfer from equityholders to existing debtholders.

The following proposition describes the equilibrium of the signaling game.

Proposition 1 *Suppose the bank has financed the first project off-balance sheet and the project fails due to systematic reasons at $t = 1$. There exists a unique equilibrium of the subsequent support and investment game, characterized as follows:*

- *The good bank voluntarily provides support $s^{Off}(Y_S, I) = \min\{Y_S, \overline{Y}_S(I)\}$ at $t = 1$ and invests in the second project, where $\overline{Y}_S(I) > 0$. Moreover, $\overline{Y}_S(I)$ and thus $s^{Off}(Y_S, I)$ are increasing in I.*

- *The bad bank mimics the good bank in the support decision at $t = 1$ with probability $\pi^{Off}(Y_S, I)$. The bad bank invests in the second project if and only if it mimics the good bank at $t = 1$. Moreover, $\pi^{Off}(Y_S, I) = 0$ if and only if $Y_S \geq \overline{Y}_S(I)$, and $\pi^{Off}(Y_S, I)$ is decreasing in Y_S and increasing in I.*

The proposition describes how the g bank uses the return of its asset-in-place at $t = 1$ following systematic failure of the firs project to provide voluntary support to off-balance sheet debtholders in order to separate from the b bank and in this way improve its perceived quality and the external funding terms at $t = 2$. When the bank's own funds are low ($Y_S < \overline{Y}_S$), the g bank exhausts them to provide support ($s^{Off} = Y_S$) but full separation is nevertheless not feasible. A higher pay-off from the asset-in-place gives the g bank more resources to provide voluntary support, and separation across the bank types increases (π^{Off} decreases). When the bank's own funds are sufficiently large ($Y_S \geq \overline{Y}_S$), mimicking becomes too costly for the b bank and the g bank is able to achieve full separation through voluntary support. Since providing support is a money burning signal that is also costly for the g bank, this bank chooses the minimum amount of support necessary to achieve full separation ($s^{Off} = \overline{Y}_S$) and the remaining funds are distributed to its owners.

The scale of the second investment I also shapes the equilibrium voluntary support and degree of separation. Suppose that the second project scale is low relative to the pay-off from the asset-in-place conditional on systematic failure of the project ($Y_S > \overline{Y}_S(I)$), so that the g bank is able to achieve full separation through voluntary support without exhausting its own funds ($\pi^{Off} = 0$ and $s^{Off} < Y_S$). As second project scale increases, the gains for the b bank from mimicking the g bank and providing support at $t = 1$ increase, so that the g bank has to increase its provision of voluntary support s^{Off} to achieve full separation. For a sufficiently large scale (I such that $Y_S < \overline{Y}_S(I)$), the mimicking benefits for the b bank are so important that full separation is not anymore feasible despite the use of the entire return of the asset-in-place to provide support ($\pi^{Off} > 0$ and $s^{Off} = Y_S$). Further increases reduce the degree of separation between the bank types and also induce the exhaustion of own funds by banks that provide voluntary support (π^{Off} increases and $s^{Off} = Y_S$).

Finally, a comparison between Lemma 1 and Proposition 1 allows us to assess the effect of the possibility to provide voluntary support on second investment efficiency. Focusing as we did in Lemma 1 on the equilibrium of the second investment game under asymmetric information in which the g bank invests with probability one, the probability that the b bank undertakes the second project is lower under off-balance sheet funding than under on-balance sheet funding. This shows that the signaling possibilities under off-balance sheet funding through voluntary support mitigate information asymmetry and improve the efficiency of the second investment. The next corollary formally states this result, which will play a key role in the analysis of the optimal funding mode at $t = 0$ in Section 3.3.

Corollary 1 *The probability that the bad bank undertakes its project at* $t = 2$ *is lower under off-balance sheet funding than under on-balance sheet funding. That is,* $\pi^{Off}(Y_S, I) < \pi^{On}$.

3.2 First investment effort

We now move to the first investment and analyze how the bank's effort decision depends on the funding mode.

On-balance sheet funding Suppose the first project is funded on-balance sheet. Let $D_1 \in [1, R]$ be the competitive promised repayment at $t = 1$ required by the investors to provide the unit of funds and e the bank's effort choice in the first investment.

Taking into account that the bank's asset-in-place pays off Y_σ at $t = 1$ and that the bank is contractually obliged to use it if necessary to satisfy debt repayments, the promise D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + qY_S + (1 - q - p_g - me)Y_{\overline{S}} = 1. \tag{6}$$

The expression captures that if the project succeeds the promise D_1 is repaid in full, while if the project fails the bank repays $Y_S < 1 \leq D_1$ if failure is due to systematic reasons (with probability q) and $Y_{\overline{S}} < 1 \leq D_1$ if it is due to non-systematic reasons (with probability $1-q-p_g-me$). Notice that the break-even condition determining D_1 depends on the bank's effort e. Since this variable is not observable, investors have expectations on the effort e that will be chosen by the bank that in equilibrium have to be satisfied. We next move to the analysis of that choice.

Recall that the bank's effort choice only affects the probabilities that the first project either succeeds or fails due to idiosyncratic reasons, which implies that effort does not affect the bank's profits from its second investment project. As a result, for a given D_1 the bank chooses effort e to maximize its expected profits from the *first investment* net of effort costs, which are given by

$$(p_g + me)\left(R + Y_{\overline{S}} - D_1\right) - c(e). \tag{7}$$

Notice that the expression takes into account that if $R_1 = R$ then the asset in place returns $Y_{\overline{S}}$, and if $R_1 = 0$ then the asset-in-place return Y_σ is used entirely to (partially) repay D_1. The bank's optimal effort choice thus satisfies the following first order condition that equalizes the marginal cost and the marginal benefit of effort:

$$c'(e) = m\left(R + Y_{\overline{S}} - D_1\right). \tag{8}$$

Comparing with the condition for the first-best effort level in (1) and using that $D_1 \geq 1 > Y_{\overline{S}}$, we have that the bank's effort choice is below its first-best level.

Equation (6) and condition (8) jointly determine D_1 and e. Assumption 2 ensures that a solution to that system of equations always exists. In some cases though, there could be multiple solutions due to the strategic complementarities between the D_1 and e choices. Specifically, self-fulfilling bad equilibria with a low level of effort can arise: if investors expect the bank to exert low effort then they ask a high D_1 and (8) then implies that the bank finds it optimal to choose a low e. We follow Stiglitz and Weiss (1981) who propose a refinement to eliminate this type of equilibrium multiplicity by exploiting Bertrand competition among investors, and select the solution to (8) with the lowest required repayment D_1 and thus highest effort e. This is also the most efficient equilibrium.

The following lemma formalizes our discussion above and characterizes the bank's optimal effort choice with on-balance sheet funding.

Lemma 2 *Suppose the bank invests in the first project on-balance sheet. The bank's optimal effort choice is strictly below the first-best level and is given by the largest solution to the equation*

$$c'(e) = m\left(R - \frac{1 - E[Y_\sigma]}{p_g + me}\right).$$

Moreover, the optimal effort, which we denote by $e^{On}(E[Y_\sigma])$*, is increasing in* $E[Y_\sigma]$ *and* $\lim_{E[Y_\sigma]\to 1} e^{On}(E[Y_\sigma]) = e^{FB}$.

The lemma states that the bank's optimal effort choice is below the first-best level. Inefficiently low effort arises because the debt contract cannot be made contingent on the (unobservable) effort choice, so that part of the value from effort is appropriated after the debt has been issued by debtholders. Besides, the bank's optimal effort depends on the expected pay-off of the asset-in-place but not on its distribution. This is because an increase in $Y_{\overline{S}}$ and decrease in Y_S so that $E[Y_\sigma]$ remains constant has two opposing effects that offset each other. First, an increase in $Y_{\overline{S}}$ has a direct positive effect on effort because it increases the residual claim of the bank under the project success. Second, such a mean-preserving change has an indirect negative effect on effort because it reduces the asset-in-place expected pay-off conditional on the project failure, which increases the promised return D_1 required by investors and thus reduces the residual claim of the bank under the under the project success. We prove that the net effect on the difference $Y_{\overline{S}} - D_1$, which determines the optimal effort level by (8), depends only on $E[Y_\sigma]$. Notice in particular that the bank's effort under on-balance sheet funding does not depend, for given asset-in-place expected pay-off, on whether the asset-in place and the project pay-offs are positively or negatively correlated ($Y_{\overline{S}} > Y_S$ and $Y_{\overline{S}} < Y_S$, respectively).

Moreover, Lemma 2 states that the bank's optimal effort e^{On} is increasing in $E[Y_\sigma]$, since the bank' contractual obligation to use its asset-in-place pay-off to repay the on-balance sheet debt when the project fails increases the ex-post value from effort for the bank. In fact, as $Y_{\overline{S}}, Y_S \to 1$ (so that $E[Y_\sigma] \to 1$), the on-balance sheet debt becomes riskless and we have that $e^{On} \to e^{FB}$. The results show that under on-balance sheet funding the bank's asset-in-place serves as skin-in-the-game that improves the bank's incentives to exert effort.

Off-balance sheet funding Suppose instead that the bank finances the first project off-balance sheet. Let $D_1 \in [1, R]$ be the competitive promised repayment at $t = 1$ required by the investors to provide the unit of funds and e the bank's effort choice in the first investment.

Taking into account that the bank is not contractually obliged to use the asset-in-place pay-off to repay debt but may voluntarily do so following the systematic failure of the project, the promise D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + q\left[\alpha + (1-\alpha)\pi^{\mathit{Off}}\right] s^{\mathit{Off}} = 1, \tag{9}$$

where we have dropped the dependence of $\pi^{\mathit{Off}}, s^{\mathit{Off}}$ on (Y_S, I) for brevity. Let us highlight the reason for the differences between the second term in the LHS of the condition above and its counterpart for on-balance sheet debt in (6). If $R_1 = 0$, there is some repayment to the debtholders only if the project failure is due to systematic reasons ($\sigma = S$, with probability q), which gives rise to information asymmetry and signaling needs through voluntary support. In this case, each of the bank types provides voluntary support as described in Proposition 1. Furthermore, we note that (9) implies that any transfer from equityholders to debtholders at $t = 1$ in the form of voluntary support is priced in by off-balance sheet debt investors at $t = 0$, so that voluntary support is a money burning signal ex post, but it is not dissipative from an ex ante perspective.

As in the on-balance sheet funding case, the bank's effort choice only affects its net profits from the first project following either success or idiosyncratic failure. Therefore for a given D_1, the bank chooses effort e to maximize its expected profits from the first investment in the contingencies in which the project either succeeds or fails due to idiosyncratic reasons ($\sigma = \overline{S}$), net of effort costs, which are given by

$$(p_g + me)\left(R + Y_{\overline{S}} - D_1\right) + (1 - q - p_g - me)Y_{\overline{S}} - c(e). \tag{10}$$

The key difference between the above expression and its counterpart under on-balance sheet funding in (7) is that, following the idiosyncratic failure of the first project (with probability $1-q-p_g-me$), the bank does not repay the off-balance sheet debt and pays out the return $Y_{\overline{S}}$ of the asset-in-place to its shareholders.[19] The bank's optimal effort choice thus satisfies the following first order condition:

$$c'(e) = m\left(R - D_1\right). \tag{11}$$

Comparing the optimal effort condition above to that for on-balance sheet funding in (8), we can see that the bank's lack of obligation to repay the off-balance sheet debt using the retunr of its asset-in-place makes the bank less liable in case the project fails. This leads to lower incentives to exert effort under off-balance sheet funding.

As in the case under on-balance sheet funding, D_1 and e are jointly determined by (9) and (11). We maintain the assumption that, in case of multiplicity of solutions, the one with the lowest required repayment D_1 is selected. The following lemma characterizes bank's optimal effort choice with off-balance sheet funding.

Lemma 3 *Suppose the bank invests in the first project off-balance sheet. The bank's optimal effort choice is strictly below that under on-balance sheet funding and is given by the largest solution to the equation*

$$c'(e) = m\left(R - \frac{1-q\left[\alpha+(1-\alpha)\pi^{Off}(Y_S,I)\right]s^{Off}(Y_S,I)}{p_g+me}\right).$$

Moreover, the optimal effort, which we denote by $e^{Off}(Y_S,I)$, does not depend on $Y_{\overline{S}}$ and is increasing in Y_S and I.

The lemma states that the bank's optimal effort under off-balance sheet funding is lower than under on-balance sheet funding. This highlights that the lack of obligation to use the

[19] For completeness, the bank's expected profit from the first investment when the project fails due to systematic reasons, a contingency whose probability does not depend on the bank's effort choice and thus does not enter (10), is given by

$$q\left(\left[\alpha+(1-\alpha)\pi^{Off}\right]\left(Y_S - s^{Off}\right) + (1-\alpha)\left(1-\pi^{Off}\right)Y_S\right).$$

asset-in-place pay-off for off-balance sheet debt repayments reduces the bank's skin-in-the-game incentives to exert effort when this funding form is used.

Interestingly, voluntary support provision by the bank to its off-balance sheet debtholders leads to *endogenous* skin-in-the-game, and through this cannel affects the effort level. Lemma 3 describes how the endogenous skin-in-the-game and effort are affected by some exogenous variables. First, if the pay-off Y_S of the asset-in-place conditional on systematic failure of the project increases, a g bank at $t = 1$ is able to provide more voluntary support (s^{Off} increases, from Proposition 1), and the ex ante value of the implicit guarantees on off-balance sheet debt increases.[20] The bank has thus more skin-in-the-game and its effort e^{Off} at $t = 0$ increases. Second, an increase in the scale I of the second investment opportunity increases the benefits for the bank from an improvement in its perceived quality by investors. As a result, mimicking incentives of a b bank at $t = 1$ become larger, leading to lower separation despite the higher support provided by a g bank (π^{Off} and s^{Off} increase, from Proposition 1). The associated increase in the value of implicit guarantees makes the bank more liable in case of project failure and increases the effort level e^{Off}. Finally, the lemma also states that the bank's effort does not depend on the return $Y_{\overline{S}}$ of the asset-in-place conditional on the non-systematic state of the project. The reason is that such return is never used to repay off-balance sheet debtholders as it is realized when either the project succeeds or it fails due to idiosyncratic reasons and there are no signaling motives to to provide support.

Let us highlight a final implication of Lemma 3: an increase in Y_S and decrease in $Y_{\overline{S}}$ that preserves $E[Y_\sigma]$ leads to an increase in the bank's optimal effort. In particular, for a given asset-in-place expected pay-off, the bank's effort is higher if the the asset-in place and the project pay-offs are negatively correlated ($Y_{\overline{S}} < Y_S$) than if they are positively correlated ($Y_{\overline{S}} > Y_S$).

3.3 First investment's optimal funding mode

The results in Sections 3.1 and 3.2 imply that, compared to on-balance sheet funding, off-balance sheet funding improves the second investment efficiency due to the signaling role of

[20]The increase in Y_S also leads to a reduction in the mimicking probability π^{Off} of the b bank, which partially mitigates but does not offset the increase in the value of the implicit guarantees implied by the increase in s^{Off}.

voluntary support, but leads to lower effort incentives for the first investment. This gives rise to a trade-off in the bank's optimal funding mode for the first investment, which we analyze in this section.

We start by computing the overall expected bank profits from the two investment under each funding mode. Let $k \in \{On, Off\}$ denote one of the funding modes, and (e^k, π^k) the equilibrium effort level and the subsequent probability that the b bank invests in the second investment following the systematic failure of the first investment and in case information asymmetry persists at $t = 2$, for the funding mode k. The bank's expected profits for the funding mode k are thus given by

$$\Pi_0(e^k, \pi^k) \equiv V^{FB} - \underbrace{\int_{e^k}^{e^{FB}} (mR - c'(e))\, de}_{\text{first project inefficiency}} - \underbrace{q(1-\alpha)\pi^k(1-p_b R)I}_{\text{second project inefficiency}}, \tag{12}$$

where we have used the fact that investors (in both projects) are competitive and thus the banks' profits are equal to the total NPV of the two projects. The interpretation of the profit decomposition is as follows. The first term, whose expression is in (2), captures the expected net value created by the bank in the first-best. The next two terms account for how the inefficiencies associated with the moral hazard problem in effort choice in the first project and the asymmetric information frictions in the second investment reduce the bank's expected profits relative to those in the first-best. Specifically, the second term includes the losses due to the inefficiently low level of effort in the first project ($e^k < e^{FB}$). The third term includes the expected losses from a $t = 0$ perspective implied by investment in negative NPV b projects at $t = 2$. In particular, q accounts for the probability that investment at $t = 2$ is undertaken under asymmetric information, $1-\alpha$ corresponds to the probability that in such contingency the bank is of type b, π^k is the probability that the b bank invests and $(1 - p_b R)I$ are the expected losses generated by such investment.

The bank's choice of optimal funding mode is determined by the difference in the bank's expected profits between off- and on-balance sheet funding. Using (12), this difference is given by

$$\begin{aligned}\Delta\Pi_0 &\equiv \Pi_0(e^{\mathit{Off}}, \pi^{\mathit{Off}}) - \Pi_0(e^{On}, \pi^{On}) \\ &= \underbrace{q(1-\alpha)\left(\pi^{On} - \pi^{\mathit{Off}}(Y_S, I)\right)(1-p_b R)I}_{\text{second project efficiency gains (>0)}} - \underbrace{\int_{e^{\mathit{Off}}(Y_S,I)}^{e^{On}(E[Y_\sigma])} (mR - c'(e))\, de}_{\text{first project inefficiency losses (>0)}}, \end{aligned} \tag{13}$$

where in the last expression we have made explicit the dependence of the equilibrium variables on $Y_S, E[Y_\sigma]$, and I. This expression decomposes the expected profit difference as the gains and losses associated with the use of off-balance sheet funding and its impact in the efficiency of the two investments. The first term captures the gains from off-balance sheet funding which amount to the possibility to provide voluntary support following the systematic failure of the first project and in this way reduce investment in bad projects at $t = 2$ (recall that $\pi^{On} > \pi^{\mathit{Off}}$, from Corollary 1). The second term instead includes the losses created by off-balance sheet funding due to the lower effort it leads to (recall that $e^{\mathit{Off}} < e^{On}$, from Lemma 3).

We derive from the properties of the profit difference function $\Delta\Pi_0$ in (13) with respect to $m, Y_S, E[Y_\sigma]$, and I the main formal result of the paper, which characterizes how the optimal funding mode depends on the most relevant parameters in the model.

Proposition 2 *There exists a threshold $\overline{I} \in \mathbf{R}^+ \cup \{\infty\}$, such that off-balance sheet funding is strictly optimal if and only if $I > \overline{I}$. Moreover, there exists $\overline{m} > 0$ such $\overline{I} \neq \infty$ for all $m < \overline{m}$. Finally, $\overline{I}$ is decreasing in Y_S holding $E[Y_\sigma]$ fixed, and increasing in $E[Y_\sigma]$ holding Y_S fixed.*

This proposition characterizes the optimal funding mode for the first project. Figure 3 illustrates the results in the bi-dimensional space (m, I) that captures the value of effort in the first project (m) and the scale of the second project (I). As second project scale increases, the efficiency gains in the second project investment induced by the voluntary support option embedded in off-balance sheet funding are larger ($\Delta\Pi_0$ is increasing in I despite $\pi^{\mathit{Off}}(Y_S, I)$ being increasing in I). As a result, when the second project scale is above a threshold $\overline{I}$ (which could be infinity) then off-balance sheet funding dominates, while if it is below then on-balance sheet funding dominates. The Proposition also states that when the effort value

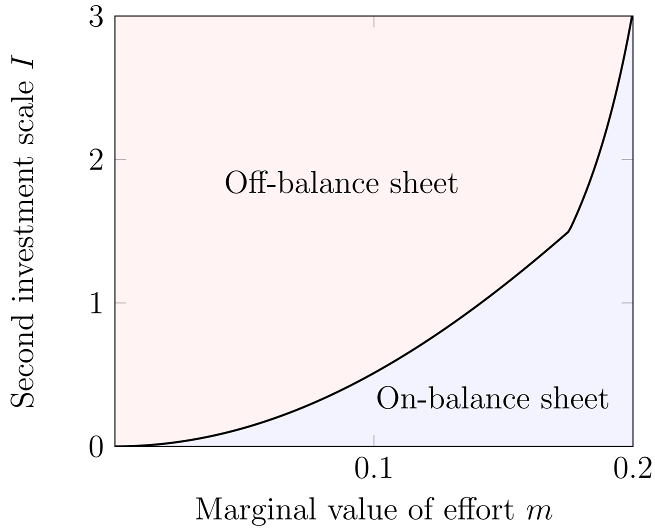

Figure 3: The bank's optimal funding mode for the first investment in the (m, I) plane. The parameters used in this plot are $R = 2$, $p_g = 0.6$, $p_b = 0.4$, $\alpha = 0.4$, $q = 0.2$, $Y_S = Y_{\overline{S}} = 0.2$ and $c(e) = 0.3e^2$.

in the first project m is sufficiently low then the second project scale threshold $\overline{I}$ is finite, so that the region in the (m, I) plane in which off-balance sheet funding is optimal has positive measure. The reason is that when the value of effort is small, then the costs from the reduced effort induced by off-balance sheet funding due to the lower skin-in-the-game implied by this funding form (second term of $\Delta\Pi_0$ in (13)), are dominated by the second project efficiency gains allowed by the possibility to signal quality through voluntary support associated with off-balance sheet funding (first term of $\Delta\Pi_0$ in (13)).

Proposition 2 also states that, for a given expected return of the asset-in-place, the second project scale threshold $\overline{I}$ that defines the optimality frontier between the two funding modes is decreasing in the return Y_S of the asset-in-place conditional on systematic failure of the first project. This means that the region in Figure 3 in which off-balance sheet funding is optimal expands as Y_S increases holding $E[Y_\sigma]$ fixed. The reason is twofold. First, the higher return Y_S allows a bank that relies on off-balance sheet funding to achieve larger separation through voluntary support, so that the efficiency gains in the investment in that project afforded by this funding mode increase (first term of $\Delta\Pi_0$ in (13) increases in Y_S). Second, the increase in Y_S also augments the expected voluntary support the bank grants if it uses off-balance sheet funding and this endogenous increase in skin-in-the-game reduces the costs from off-balance sheet funding stemming from the moral hazard in effort (second term of $\Delta\Pi_0$ in (13) decreases in absolute value in Y_S holding $E[Y_\sigma]$ fixed).

The final statement in Proposition 2 is that, for a given return of the asset-in-place conditional on systematic failure of the first project, the second project scale threshold $\overline{I}$ that defines the optimality frontier between the two funding modes is increasing in the expected pay-off of the asset-in-place. In other words, the region in Figure 3 in which off-balance sheet funding is optimal shrinks as the return $Y_{\overline{S}}$ of the asset-in-place conditional on the non-systematic state of the project increases. The reason is that the increase in $Y_{\overline{S}}$ improves effort under on-balance sheet funding while it has no effect whatsoever if off-balance sheet funding is used.

4 Discussion and extensions

In this section we explore the empirical predictions of the model, discuss some policy implications of the paper, and analyze the robustness of our results so some extensions of the baseline model.

4.1 Empirical predictions

In this section, we discuss the applications of our model, collect its main predictions and, where possible, provide supporting evidence.

Our model directly applies to banks' funding through sponsored off-balance sheet entities such as securitization vehicles and money market funds. We have discussed evidence for banks' provision of voluntary support to these off-balance sheet entities during the financial crisis and the concerns these actions raised on regulatory authorities in the Introduction. Our model also applies to a banks' decision to expand in a foreign country through a branch, which would correspond to on-balance sheet funding in our model as a branch's assets and liabilities are integrated in the parent bank's balance sheet, or a subsidiary, which would correspond to off-balance sheet funding in our model as the subsidiary is an independent legal entity and the parent bank is protected by limited liability should the subsidiary's be unable to satisfy its debt liabilities. The reputational risk associated with letting a subsidiary fail has long been acknowledged by policy makers and during the financial crisis there were several examples of voluntary rescue of bank subsidiaries by their parent banks motivated by reputational concerns.[21]

More broadly, our model also applies to nonfinancial firms' decision to finance investments through a subsidiary versus directly within the parent company.[22] Major rating agencies in fact recognize the potential voluntary support provision by the parent company to its subsidiaries for nonfinancial firms. For example, Morningstar (2016) states that,

> "even in the absence of cross defaults or guarantees, the parent typically has powerful incentives for supporting its subsidiaries: commercial reputation. [...] Allowing the default of a material subsidiary risk calling into question the parent's willingness as well as its capacity to adhere to its contractual obligations."

The existence of such implicit guarantee in groups is also made explicit through "comfort letters" written by the parent to the subsidiary's lenders, assuring lenders that they would receive assistance in distress. However, these letters are legally unenforceable (Boot, Greenbaum and Thakor, 1993).

The model assumes that investors rationally anticipate the banks' voluntary support decisions, so that they price them. This is confirmed by Moody's (1997) statement:

> "Part of the reason for the favorable pricing of the [SPVs'] securities is the perception on the part of many investors that originators (i.e., the 'sponsors' of the securitizations) will voluntarily support—beyond that for which they are contractually obligated—transactions in which asset performance deteriorates significantly in the future. Many originators have, in fact, taken such actions in the past' (p. 40)."

We next present our model's novel empirical predictions relating to both financial and nonfinancial firms.

[21]Swedish banks provided voluntary support to their Baltic subsidiaries and Austrian and Italian banks did so for their subsidiaries in central and eastern European countries (Fiechter et al., 2011). An earlier example is the recapitalization by by Portugal's Banco Espiritu Santo of its Brazilian subsidiary Banco Boavista Interatlantico following the devaluation of the Brazilian real in 1999.

[22]Kolasinsky (2009) finds that 13% of all U.S. nonfinancial corporate public debt was issued by subsidiaries.

Implication 1: High growth activities are more likely to be financed off-balance sheet. In our model, off-balance sheet funding affords the bank means to signal positive information about its future investment opportunities. This implies that banks with higher growth activities (higher I) are more likely to use an off-balance sheet structure to finance them (Proposition 2). This prediction is consistent with Almazan, Martín-Oliver, and Saurina (2015), who find that banks with stronger growth opportunities make more intense use of securitization. In addition, Cerutti, Dell'Ariccia and Martinez-Peria (2007) show that banks, when expanding to a foreign country, are more likely to establish branches when the foreign operation is small, and more likely to set up a subsidiary when seeking to penetrate the foreign market and establish large retail operations.

Implication 2: Investments with negative correlation with the bank's core activities are more likely to be financed off-balance sheet. In the model, the bank's ability to signal its quality through voluntary support when its project fails and asymmetric information becomes a concern, depends on its own funds under that contingency. As a result, investment opportunities that are negatively correlated with the bank's existing assets (high Y_S, for given $E[Y_\sigma]$) are more likely to be financed off-balance sheet (Proposition 2). In the context of the bank's organizational structure choice when entering a foreign market, Dell'Ariccia and Marquez (2010) similarly predict that negative correlation between the home and host economies is more likely to lead the bank to expand through subsidiaries. In contrast to our model in which all the liabilities issued by the bank are fairly priced, the mechanism in that paper stems from the bank's incentives to maximize the value of subsidies associated with deposit insurance.

Implication 3: Voluntary support events convey positive information about banks' future investment opportunities. Our first formal result shows that voluntary support emerges endogenously as banks with good future investment opportunities wish to signal strength (Proposition 1). Consistent with this prediction, Higgins and Mason (2004) show that such events are associated with improved short-term stock price performance and long-term financial performance.

Implication 4: Positive shocks to the banks' on-balance sheet assets should reduce the spread of their outstanding off-balance sheet debt. Consider a proportional increase or decrease in the return Y_σ of the asset-in-place in the two $t = 1$ states $\sigma = S, \overline{S}$. Notice that this is a shock to the bank's asset-in-place without affecting the fundamental asset held by the off-balance sheet vehicle. The model predicts that such an increase would raise the expected value of the voluntary support given by the bank to its off-balance sheet debtholders, leading to a reduction in the off-balance sheet debt spread (Proposition 1). This is consistent with rating agencies' views and has been supported by empirical evidence. Moody's Investor Service (2006) concludes that lower-rated sponsors are associated with higher ABS spreads and weaker credit performance. Gorton and Souleles (2007) find that credit card backed securities sponsored by riskier sponsors command higher yields, suggesting that the market recognizes sponsor risk as determinants of security risk.[23]

Implication 5: The off-balance sheet debt spread should decrease with the expected growth of off-balance sheet activities. The larger the size of the future investment opportunities (higher I) the larger the expected value of the voluntary support provided by the bank to its off-balance sheet debtholders in the model (Proposition 1). The reason is that the bank has more incentives to signal strength through support to obtain cheap external funding in the future when financing needs are expected to be larger. The model thus predicts that the spread of newly issued or outstanding off-balance sheet debt should be lower for banks whose off-balance sheet activities are expected to increase more. This implication can potentially be tested by exploiting the unanticipated freeze of private mortgage securitization in 2007 (see, Kruger (2018)), which can be interpreted as an exogenous shock to future off-balance sheet activities.[24] Our model predicts that the spread of outstanding off-balance sheet debt of exposed banks should increase following the shock, and the more so for banks whose securitization activities were growing faster.

Implication 6: The off-balance sheet debt spread should increase with the correlation between on-balance sheet and off-balance sheet activities. A reduction on the asset-in-place

[23]It is also possible to extend the intuition of our model to multiple vehicles, and predict a propagation effect *between* different vehicles. Specifically, an increase in the repayment of the project of one vehicle reduces the need for voluntary support to that vehicle and thus increases the amount of funds available to provide voluntary support to other vehicles. Using data from Korean chaebol groups, Bae, Cheon and Kang (2008) show that the announcement of increased earnings by one affiliate has a positive effect on the value of other affiliates.

[24]Kruger (2018) exploits this unanticipated freeze of private mortgage securitization to provide evidence on the effect of securitization on foreclosure and modification.

return under the systematic failure of the project, Y_S, accompanied by an increase in the asset-in-place return when there is no systematic failure, $Y_{\overline{S}}$, such that expected return $E[Y_\sigma]$ remains fixed, corresponds to an increase in the correlation between the two assets in the model. Our results predict that this change should lead the spread of off-balance sheet debt to decrease because the expected value of voluntary support increases (Proposition 1).

4.2 Ban on voluntary support

Concerns about the potential negative impact of voluntary support on bank capitalization and its eventual cost for the taxpayer have motivated regulatory changes that limit or prohibit such transactions between depository institutions and their sponsored off-balance sheet entities in the shadow banking system. Examples include the Volcker Rule (Dodd-Frank Act 2010) in the US,[25] the proposals of the Vickers Commission enacted by the Financial Services (Banking Reform) Act 2013 in the UK,[26] and, more recently, the guidelines issued by the Basel Committee on the regulation of step-in risk in securitization (BCBS, 2017).[27]

In this section we investigate the effect in the context of our model of the introduction of a ban on the voluntary provision of support on aggregate surplus and bank profits. We conduct our analysis from two perspectives depending on *when* the ban is introduced: either ex ante *at* $t = 0$, or ex post and unexpectedly *at* $t = 1$. Our results show that limiting voluntary support has a detrimental effect on aggregate surplus and bank profits from either perspective.

Ex ante ban on voluntary support Consider the introduction at $t = 0$ of a ban on voluntary support provision at $t = 1$. The ban on voluntary support eliminates the second

[25]The Volcker Rule, which went into effect on April 1 2014, prohibits banking entities from engaging in proprietary trading and from acquiring ownership interests in funds, as well as from entering into transaction with funds for which they serve as investment advisers and in particular to rescue them.

[26]The Financial Services (Banking Reform) Act 2013 limits the exposure of depository institutions to other financial entities within the same bank holding company (BHC). In particular, transactions between a regulated commercial bank and entities within the BHC will have to be conducted in market terms, which rules out voluntary support to these entities when they suffer financial distress.

[27]The Basel Committee defines step-in risk as the risk that a bank provides financial support to an unconsolidated entity that is facing distress, over and above any contractual obligation to provide such support. Entities where step-in risk is identified face costly compliance consequences, such as consolidation, liquidity ratio adjustments, and Pillar 2 capital add-on.

project efficiency gains associated with off-balance sheet funding in the no ban economy. As a result, on-balance sheet funding is always strictly optimal because it leads to better effort incentives for the first project.[28] Besides, aggregate surplus from $t = 0$ perspective, which coincides with the bank's expected profits, is strictly reduced relative to that in the no ban (baseline) economy analyzed in Section 3 whenever off-balance sheet funding is optimal in that economy.

Ex post ban on voluntary support We have just highlighted that an ex-ante ban on voluntary support reduces bank's value because voluntary support allows the bank to improve second investment efficiency. Yet, from an ex post perspective voluntary support has a money burning feature that could be detrimental for the banks' profits despite being good for investment efficiency. In fact, it is well known that the equilibrium of some signaling games is Pareto dominated by the outcome when agents are not allowed to send signals.[29] From a policy perspective, the question of the effect as of $t = 1$ of the introduction of a ban is of the maximum importance as it addresses a potential time consistency problem faced by a bank regulator whose objective is to maximize the bank's profits. Since at $t = 0$ the regulator finds it optimal to allow support, should an unexpected ban on support at $t = 1$ increase the bank's expected profits from that date onwards, the regulator would have incentives at $t = 1$ to prohibit those actions, and a time consistency problem would arise. We analyze next whether that potential problem arises.

Suppose that the optimal financing mode at $t = 0$ in the (no ban) baseline economy is off-balance sheet funding. Following the systematic failure of the first project at $t = 1$, the g bank provides voluntary support amounting to $s^{Off} > 0$, whereas the b bank mimics with probability $\pi^{Off} \in [0, 1)$, where s^{Off} and π^{Off} are defined in Proposition 1.

If the regulator were to ban at $t = 1$ the provision of support, the transfer of s^{Off} units of funds from the bank to the off-balance sheet debt investors would not be realized, which

[28]Notice that the ban on support reduces even further the bank's skin-in-the-game when it relies on off-balance sheet funding and increases the first project efficiency costs implied by this funding mode.

[29]A classical example of this arises in a simple version of the signaling model in Spence (1973) in which education does not increase productivity. When the probability that the worker has high productivity is sufficiently high, the separating equilibrium in which that worker type gets education is dominated by the pooling outcome that would result from a prohibition on investing in education.

would increase the bank's profits at $t = 1$. However, limiting the use of voluntary support would reduce investment efficiency due to information asymmetry and the expected profits at $t = 3$ of an average bank. The following proposition shows that the investment efficiency effect dominates:

Proposition 3 *If off-balance sheet funding is optimal in the (no ban) baseline economy, then the ex post unexpected introduction of a ban on voluntary support at the time of a systematic failure of the first project strictly reduces the expected overall profits from $t = 1$ of a good bank, has no effect on those of a bad bank and strictly reduces aggregate surplus.*

We discuss next the intuition for this result. Suppose off-balance sheet funding is optimal in the baseline economy and let s^{Off} be the voluntary support provided by a g bank. Since support is from a $t = 1$ perspective a costly money burning signal, in the equilibrium of the game following the systematic failure of the first project the b bank is indifferent between providing support or not (see Proposition 1). The expected profits for this bank type from $t = 1$ onwards are thus equal to the asset-in-place pay-off, Y_S. If an ex post ban on support were introduced, the bank would distribute Y_S as profits at $t = 1$ and a b bank would not obtain subsequently any other profits.The ex post ban has thus no effect on the expected overall profits from $t = 1$ of a b bank. The single-crossing condition derived in (5) then implies that the expected overall profits from $t = 1$ of a g bank are strictly reduced when an ex post ban is introduced. We conclude that the potential time consistency problem in the authorization of support provision faced by a bank regulator does in fact not emerge.

4.3 Access to storage technology

In the baseline model, we have assumed that the bank has no access to a storage technology. In this section, we relax this restriction, and show that our main results on the signaling value of voluntary support and the optimal funding form continue to hold.

Allowing the bank access to a storage technology enriches the mechanisms in the baseline model along two dimensions. First, the bank may store any remaining funds at $t = 1$ after obligatory or voluntary repayments to debtholders, and invest some of them into the second project at $t = 2$. In presence of asymmetric information, such (partial) self-financing constitutes an alternative means to signal strength for the bank, potentially reducing the reliance on voluntary support. Second, the bank may want to raise more funds at $t = 0$ than what is necessary for investment, as the additional funds can be stored until $t = 1$ and enhance its capability to signal strength when the second investment is undertaken under asymmetric information.[30]

For technical reasons we introduce an additional ingredient in this extension. Following the systematic failure of the first project at $t = 1$, which creates information asymmetry at that date, there is a small probability $\tau > 0$ that a transparency shock that exogenously reveals the bank type $j \in \{g, b\}$ is realized at $t = 2$ just before investment takes place. This small but positive probability that information is exogenously revealed allows us to ensure uniqueness of equilibrium in presence of the alternative signaling device considered in this extension. For the rest of this section, we consider the limit case $\tau \to 0$ so that asymmetric information persists with probability tending to one following the systematic shock (as in the baseline model) and the parameter τ will play no further role.[31]

In the rest of the section we show how the main formal results in the equilibrium analysis of the baseline model extend to this set-up.

4.3.1 Second investment and voluntary support

We first consider the bank's investment in the second project following the systematic failure of the first project. If the first project was financed on-balance sheet, the access to a storage technology does not affect the investment efficiency in the second project because the bank is contractually obliged to exhaust all its funds at $t = 1$ to repay the debt.

Suppose that the bank has financed the first project off-balance sheet raising $d_0 \geq 1$ units

[30] In practice, this is achieved by setting the price at which the off-balance sheet vehicle purchases the first project from the bank at $d_0 \geq 1$. The off-balance sheet vehicle then raises d_0 by issuing debt backed by the project. Therefore at $t = 0$, the vehicle receives zero net cash flow, while the bank receives a net cash flow equal to $d_0 - 1$, where 1 is the initial investment in the first project and d_0 is the proceeds from selling the project to the off-balance sheet vehicle. The net cash flow $d_0 - 1$ can be interpreted as a set-up fee charged by the bank to the vehicle.

[31] More generally, if the probability $\tau \in (0, 1)$ of the transparency shock does not tend to zero, this parameter could be economically interpreted as a measure of transparency as it captures how easy it is for investors to obtain information about the quality of the new investment opportunities of the bank once uncertainty about them arises. All the formal results in this section, which are presented for the case $\tau \to 0$, can be extended to the case of a fixed $\tau > 0$ as is shown in the proofsof these results in the Appendix.

of funds from competitive investors. Following the systematic failure of the first project, even though the bank is not contractually obliged to use its own funds $d_0 - 1 + Y_S$ at $t = 1$ to repay the off-balance sheet debt holders, it can voluntarily provide support $s \leq d_0 - 1 + Y_S$ to the off-balance sheet debtholders. The remaining funds $d_0 - 1 + Y_S - s$ are stored until $t = 2$ and at that date the bank decides whether or not to invest and, if so, how much of its own funds $i \leq \min\{I, d_0 - 1 + Y_S - s\}$ to use for investment. After observing the history of the bank's decisions (s and i) and updating their belief $\widehat{\alpha}$ on the bank's type, competitive investors set a promised repayment $D_3(i, \hat{\alpha})$ to provide the $I - i$ units of funds necessary for investment.

It can be shown that the use of self-financing i satisfies an analogous single-crossing condition to that for voluntary support in (5) so that this action also constitutes a signal of quality. The bank thus faces a bi-dimensional signaling problem: Each of the bank types decides how to use its limited internal funds to signal quality with a combination of voluntary support (at $t = 1$) and self-financing (at $t = 2$). The following characterizes the equilibrium of this bi-dimensional signaling game.

Proposition 4 *Suppose the bank has financed the first project off-balance sheet with an initial funding amount d_0 and the project fails due to systematic reasons at $t = 1$. There exists a unique equilibrium of the subsequent support and investment game, characterized as follows:*

- *The good bank voluntarily provides support $\widetilde{s}^{Off}$ at $t = 1$ and invests with as much remaining own funds as possible at $t = 2$. Moreover, $\widetilde{s}^{Off} = d_0 - 1 + Y_S$ if and only if $d_0 - 1 + Y_S \leq \overline{Y}_S(I)$, where $\overline{Y}_S(I)$ is given by Proposition 1.*

- *The bad bank mimics the good bank in the support decision at $t = 1$ with probability $\widetilde{\pi}^{Off}$. The bad bank invests in the second project if and only if it mimics the good bank at $t = 1$. Moreover, $\widetilde{\pi}^{Off} = \pi^{Off}(d_0 - 1 + Y_S, I)$, where $\pi^{Off}(\cdot)$, is given by Proposition 1. In particular, $\widetilde{\pi}^{Off}$ is decreasing in d_0.*

The proposition shows that the main results on the signaling value of voluntary support in Proposition 1 remain valid. The g bank provides voluntary support in equilibrium to signal strength even in the presence of self-financing as an alternative signaling opportunity and the b bank mimics with a probability that coincides with that in the baseline model for the same amount of available funds at $t = 1$.[32] Moreover, the g bank exhausts all its available funds to provide support ($\widetilde{s}^{Off} = d_0 - 1 + Y_S$) when its available funds at $t = 1$ are not sufficient to achieve full separation (recall from Proposition 1 that $\widetilde{\pi}^{Off} > 0$ iff $d_0 - 1 + Y \leq \bar{Y}_S(I)$).

A final important result in Proposition 4 is that separation across bank types increases in the initial funding amount d_0, as the access to storage allows the bank to carry those funds until $t = 1$ and use them for signaling purposes at that date. This suggests that if the bank uses off-balance sheet funding it might decide to set $d_0 > 1$.

4.3.2 First investment and optimal funding mode

The bank's optimal choice between on- and off-balance sheet funding then results from a similar trade-off between the efficiency gains efficiency gains and costs in the two investments as in the baseline model. The characterization of the optimal funding mode is given by the following proposition.

Proposition 5 *There exists a threshold $\widetilde{I} \in \mathbf{R}^+ \cup \{\infty\}$, such that off-balance sheet funding is strictly optimal if and only if $I > \widetilde{I}$. Moreover, there exists $\widetilde{m} > 0$ such that $\widetilde{I} \neq \infty$ for all $m < \widetilde{m}$. Finally, the threshold $\widetilde{I}$ satisfies $\widetilde{I} < \overline{I}$ if and only if $Y_S < \overline{Y}_S(I)$, where $\overline{I}$ is given by Proposition 2 and $\overline{Y}_S(\cdot)$ is given by Proposition 1.*

The first part of the proposition shows that the main results of the baseline model in Proposition 2 hold in this extension: Off-balance sheet funding is optimal when the scale of the second project I is sufficiently large and/or the marginal value of effort m is low.

Proposition 5 also shows that access to a storage technology may increase or decrease the threshold scale of the second project $\overline{I}$ below which off-balance sheet funding is optimal, depending on the pay-off of the asset-in-place Y_S and the scale of the second investment. When Y_S is low such that $Y_S < \overline{Y}_S(I)$, in the baseline model the bank is unable to achieve full separation even when it exhausts all pay-offs from the asset-in-place to provide voluntary support. Access to a storage technology enables the bank to raise additional funds $d_0 > 1$

[32]Notice that following the first project failure, in the baseline model the bank funds at $t = 1$ amount to Y_S while in this extension they are increased to $d_0 - 1 + Y_S$.

at $t = 0$ and carry $d_0 - 1$ until $t = 1$ for signaling purposes. This increases the degree of separation of bank types and improves the bank's second investment efficiency. The increased amount of voluntary support also constitutes additional skin-in-the-game and enhances the bank's effort incentives for the first project. Overall, access to a storage technology in this case increases the bank's investment efficiency in both projects and thus expected profits under off-balance sheet funding. When Y_S is high such that $Y_S > \overline{Y}_S(I)$, however, the bank is able to achieve full separation with pay-offs from the asset-in-place through voluntary support. Access to a storage technology enables the bank to reduce (ex post) costly voluntary support, while still achieving full separation across bank types by signaling with a combination of voluntary support and self-financing. While this does not affect the investment efficiency of the second project, reduced voluntary support decreases the bank's skin-in-the-game and thus effort in the first project. Therefore in this case, access to a storage technology decreases the bank's investment efficiency and thus expected profits under off-balance sheet funding.

5 Conclusion

In this paper we develop a model of a bank's choice between on- and off-balance sheet debt financing that emphasizes the value of the flexibility granted by the latter to use own funds to voluntarily support debt repayments. When a project failure leads to asymmetric information about future investment quality, the bank voluntarily uses its own funds to support repayments to existing off-balance sheet debtholders to signal quality. We show that such flexibility improves future investment efficiency. By contrast, the contractual obligation to use own funds to repay debt when the funding is on-balance sheet does not allow for such signaling possibilities but increases skin-in-the-game and improves incentives to exert effort. The bank funding mode thus trades-off the future investment efficiency benefits implied by off-balance sheet funding and the costs from lower incentives to exert effort on current investment under this funding form.

The paper adds to the corporate structure literature that has usually emphasized the role of limited liability at the subsidiary level. Our model brings a novel perspective to this issue in which the flexibility associated with the *option* not to repay off-balance sheet promises *but* the possibility of doing so is the source of value for the firm and the determinant of the choice of off-balance sheet funding. Our theory is consistent with the numerous instances of support beyond contractual obligations by banks to their sponsored shadow banking entities observed during the crisis. Besides, the paper contributes to the literature on incentive problems associated with off-balance sheet funding by showing that endogenous voluntary support provision partially restores the bank's skin-in-the-game and thus alleviates the moral hazard problem associated with this funding form.

The paper yields some novel empirical predictions. On the one hand, off-balance sheet funding is more likely to emerge for high growth activities and for investments that are negatively correlated with existing on-balance sheet assets. In both cases the value of the signaling opportunities associated with voluntary support is higher. On the other hand, the model predicts a positive reaction of the bank's stock price to voluntary support, and the off-balance sheet debt spread to respond negatively to positive shocks to its sponsor bank's on-balance sheet assets and to depend positively on the correlation between the sponsor's assets and those backing the off-balance sheet debt.

The paper also analyzes the impact of the introduction in many jurisdictions in the aftermath of the crisis of regulatory restrictions on the provision of voluntary support with the aim of avoiding the costs for the banks and, eventually the taxpayer, of those actions. In the context of our model, the prohibition of voluntary support to off-balance sheet entities reduces the bank's capability to signal quality and worsens investment efficiency. If the prohibition is introduced ex-ante, this leads to a reduction on the bank's profits, which coincide with aggregate surplus. Even more, if the prohibition is unexpectedly introduced ex post it still leads to a reduction on the bank's profits (and expected surplus) even though from an ex post perspective voluntary support constitutes a money burning signal. Although there could be other reasons justifying the introduction of prohibitions on voluntary support, most notably the attempt to avoid that depository institutions extend their access to the safety net to their sponsored shadow entities, our results highlight that these restrictions might have the unintended consequence of exacerbating information asymmetry problems and lowering investment efficiency. Extending our framework in order to include these considerations is a possible avenue for future research.

Appendix

The D1 refinement of Cho and Kreps (1987) We use the D1 refinement of Cho and Kreps (1987) to refine the set of Perfect Bayesian Equilibria. A brief summary of how to apply this concept in our model in which there are only two types $j \in \{g, b\}$ is as follows. Given an equilibrium, for any off-equilibrium action a taken by a bank of type j, let $\Lambda_j(a)$ denote the set of beliefs held by investors, such that their best response leads to a pay-off for the bank strictly higher than the equilibrium pay-off. If there exists a type $j' \in \{g, b\}$ and $j' \neq j$ such that $\Lambda_j(a) \subsetneq \Lambda_{j'}(a)$, then the off-equilibrium belief associated with the action a must assign probability 1 to type j'.

Proof of Lemma 1 Suppose the bank has financed the first project on-balance sheet and the project fails due to systematic reasons at $t = 1$. Since there is no storage technology, the bank arrives at $t = 2$ with no funds. The investors' belief about the bank's quality is α. Each bank type chooses among the set of actions $i \in A_2 \equiv \{i_1, i_0\}$, where $i = i_1$ means the bank invests and $i = i_0$ means the bank does not.

An equilibrium (possibly in mixed strategies) consists of the probability of investment π_j for each bank type j and investors' belief $\hat{\alpha}$ about the probability that the bank is of type g conditional on investment, such that the banks' actions are optimal given investors' belief and investors' belief is given by Bayes rule if $\pi_g + \pi_b > 0$ and is robust to the D1 refinement otherwise.

The expected profits as of $t = 2$ of a bank of type j that invests, given the investors' belief $\hat{\alpha}$, are given by

$$\Pi_{j,2}(i, \hat{\alpha}) = \begin{cases} p_j \left[RI - D_3(\hat{\alpha}) \right], & \text{if } i = i_1 \text{ and } D_3(\hat{\alpha}) \leq RI, \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

where $D_3(\hat{\alpha})$ is given by (3). The second case in (14) represents the profits of the bank when it does not invest in the second project ($i = i_0$) or when the bank chooses to invest but investors refuse to provide financing ($i = i_1$ and $D_3(\hat{\alpha}) > RI$).

To prove this lemma, we first note that $\pi_g = 1$ and $\pi_b = \pi^{On}$, where π^{On} is given in Lemma 1, is indeed an equilibrium. This equilibrium satisfies $\Pi_{g,2} = \Pi_{b,2} = 0$.

Next, we show that any other equilibria are pay-off equivalent. To see this, we show that $D_3(\hat{\alpha}) \geq R$ in any equilibrium. Conjecture by way of contradiction an equilibrium in which $D_3(\hat{\alpha}) < R$. This implies $\pi_j = 1$ as $\Pi_{j,2}(\hat{\alpha}) > 0$. This then implies that $\hat{\alpha} = \alpha < \bar{\alpha}$, contradicting the supposition that $D_3(\hat{\alpha}) < R$ by Assumption 4. $D_3(\hat{\alpha}) \geq R$ then implies that $\Pi_{g,2} = \Pi_{b,2} = 0$, which is pay-off equivalent to the equilibrium with $\pi_g = 1$.□

Proof of Proposition 1 Suppose the bank has financed the first project on-balance sheet and the project fails due to systematic reasons at $t = 1$. Each bank chooses among the set of actions $A_1 \equiv \{(s, i) \in [0, Y_S] \times A_2\}$, where i denotes whether the bank invests at $t = 2$.

An equilibrium (possible in mixed strategies) consists of a set $(\pi_j(a))_{a \in A_1}$ for each bank type j describing the probability $\pi_j(a)$ with which type j plays action a and subject to $\int_{A_1} \pi_j(a) da = 1$, a set of investors' beliefs $(\alpha(a))_{a \in A_1}$ about the probability that the bank is of type g conditional on action a, such that the banks' actions are optimal given investors' beliefs and investors' beliefs are given by Bayes rule if $\pi_g(a) + \pi_b(a) > 0$ and are robust to the D1 refinement otherwise.

The expected profits as of $t = 1$ for a bank of type j that provides support s at $t = 1$ and chooses i at $t = 2$, given the investors' belief $\hat{\alpha}$, are given by

$$\Pi_{j,1}(s, i, \hat{\alpha}) = \begin{cases} (Y_S - s) + p_j \left[RI - D_3(\hat{\alpha}) \right], & \text{if } i = i_1 \text{ and } D_3(\hat{\alpha}) \leq RI, \\ (Y_S - s), & \text{otherwise}, \end{cases} \tag{15}$$

where $D_3(\hat{\alpha})$ is given by (3). Notice that the difference between this expression and the expression given by (4) is that (4) represents the bank's $t = 1$ expected profits conditional on investment at $t = 2$.

We now prove this proposition through the series of claims below.

Claim 1 *In equilibrium, if a bank plays action (s, i), where $s > 0$, with positive probability, then $i = i_1$.*

Proof. We prove this claim by contradiction. Suppose a bank plays action (s, i), where $s > 0$, with positive probability and the claim is not true. This implies that the bank plays action (s, i_0) with positive probability and the bank's equilibrium pay-off is equal to $\Pi_{j,1}(s, i_0) = Y_S - s < \Pi_{j,1}(0, i_0) = Y_S$, a contradiction. ∎

Claim 2 *In equilibrium, the b bank plays with positive probability only actions that are either played with positive probability by the g bank, or $(s, i) = (0, i)$ and receives $\Pi_{b,1}(0, i, \alpha(0, i)) = Y_S$.*

Proof. Suppose the b bank chooses with positive probability some action (s, i) such that $\pi_g(s, i) = 0$. Then by Bayes rule, $\alpha(s, i) = 0$. From Assumption 3, (3) and (15) we have $\Pi_{b,1}(s, i, 0) = Y_S - s < \Pi_{b,1}(0, i_0, \alpha(0, i_0)) = Y_S$ for all $s > 0$. This implies that, if the b bank plays with positive probability an action (s, i) that is not played by the g bank, then the $(s, i) = (0, i)$ and the b bank receives an equilibrium pay-off of $\Pi_{b,1}(s, i, \alpha(s, i)) = Y_S$. ∎

Claim 3 *If in equilibrium* $\int_{A_2} \pi_b(0,i)di < 1$ *then the* g *bank plays with probability one the action* $(s,i) = (Y_S, i_1)$.

Proof. We prove this claim by contradiction. If the claim is not true, then Claims 1 and 2 imply that there exists (s, i_1) with $s < Y_S$ such that $\pi_b(s,i_1) > 0$ and $\pi_g(s,i_1) > 0$. This implies that $\alpha(s,i_1) \in (0,1)$, and the equilibrium profits $\pi^*_{j,1}$ must satisfy

$$\Pi^*_{g,1} = \Pi_{g,1}(s,i_1,\alpha(s,i_1)) \text{ and } \Pi^*_{b,1} \geq \max\left\{\Pi_{b,2}(s,i_1,\alpha(s,i_1)), \Pi_{b,2}(0,i_0,\alpha(0,i_0))\right\}, \quad (16)$$

where the inequality follows because in an equilibrium, the b bank plays with positive probability only actions that provide it with higher profits, among (s,i_1), $(0,i_0)$, or another action played by the g bank with positive probability.

We now prune the supposed equilibrium by constructing a profitable deviation for the g bank under the D1 refinement. Consider an off-equilibrium $s' = s + \epsilon$ for some small $\epsilon > 0$ such that $s' \leq Y_S$. The set of beliefs that make deviation strictly optimal for the j bank is defined as

$$\Lambda_j(s',i_1) = \left\{\hat{\alpha} \in [0,1] : \Pi_{j,1}(s',i_1,\hat{\alpha}) > \Pi^*_{j,1}\right\}. \quad (17)$$

Using (16) and the fact that $\frac{\partial \Pi_{j,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$, the deviation set $\Lambda_b(s',i_1)$ satisfies

$$\Lambda_b(s',i_1) \subseteq \hat{\Lambda}_b(s',i_1) \equiv \{\hat{\alpha} \in [0,1] : \Pi_{b,1}(s',i_1,\hat{\alpha}) > \max\{\Pi_{b,1}(s,i_1,\alpha(s,i_1)), Y\}\}. \quad (18)$$

We now show that $\hat{\Lambda}_b(s',i_1) \subsetneq \Lambda_g(s',i_1)$, which then implies that $\Lambda_b(s',i_1) \subsetneq \Lambda_g(s',i_1)$. First, if $\hat{\Lambda}_b(s',i_1) \neq \emptyset$, then $\Pi^*_{b,1} \geq Y$ implies that $D_3(\hat{\alpha}) < R$ for all $\hat{\alpha} \in \hat{\Lambda}_b(s',i_1)$, so that we have the following property for all $\hat{\alpha} \in \hat{\Lambda}_b(s',i_1)$:

$$\Pi_{b,1}(s',i_1,\hat{\alpha}') \geq \Pi_{b,1}(s,i_1,\hat{\alpha}) \Rightarrow \Pi_{g,1}(s',i_1,\hat{\alpha}') \geq \Pi_{g,1}(s,i_1,\hat{\alpha}). \quad (19)$$

This implies that $\hat{\Lambda}_b(s',i_1) \subseteq \Lambda_g(s',i_1)$. Further, $p_b R < 1$ implies that $\hat{\Lambda}_b(s',i_1) \subseteq (0,1]$. This and the fact that $\frac{\partial \Pi_{j,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$ then imply that $\hat{\Lambda}_b(s',i_1) \subsetneq \Lambda_g(s',i_1)$. Second, if $\hat{\Lambda}_b(s',i_1) = \emptyset$, then we also have $\hat{\Lambda}_b(s',i_1) \subsetneq \Lambda_g(s',i_1)$ because $\Lambda_g(s',i_1) \neq \emptyset$. To see this, notice that we have $\frac{\partial \Pi_{j,1}(s,i,\hat{\alpha})}{\partial s} < 0$. Yet, since $\alpha(s,i_1) < 1$, for ϵ sufficiently small, under the deviation (s',i_1) we have $\Lambda_g(s',i_1) \neq \emptyset$.

The D1 refinement then implies that $\alpha(s',i_1) = 1$ and thus $\Pi_{g,1}(s',i_1,1) > \Pi_{g,1}(s,i_1,\alpha(s,i_1))$ for ϵ sufficiently small as argued above. The g bank would find it optimal to deviate, a contradiction. ∎

Claim 4 *If in equilibrium* $\int_{A_2} \pi_b(0,i)di = 1$ *then the* g *bank plays with probability one the action* (s,i_1), *where* s *is the minimum value such that* $\Pi_{b,1}(s,i_1,1) \leq Y_S$.

Proof. we prove this claim by contradiction. Suppose that $\int_{A_2} \pi_b(0,i)di = 1$ and the claim is not true. That is, there exists $s' < s$ such that $\Pi_{b,1}(s', i_1, 1) \leq Y_S$. There are two possibilities. If (s', i_1) is played with positive probability, then $\alpha(s', i_1) = 1$. But since $\frac{\partial \Pi_{j,1}(s,i,\hat{\alpha})}{\partial s} < 0$, $s' < s$ implies that $\Pi_{g,1}(s', i_1, 1) > \Pi_{g,1}(s, i_1, 1)$, a contradiction. If (s', i_1) is not played with positive probability, then we have $\Lambda_b(s', i_1) = \emptyset$, since the b bank's equilibrium pay-off is equal to Y_S. Since $\frac{\partial \Pi_{j,1}(s,i,\hat{\alpha})}{\partial s} < 0$ implies that $\Lambda_g(s', i_1) \neq \emptyset$, the D1 refinement then implies that $\alpha(s', i_1) = 1$ and the g bank would find it optimal to deviate, a contradiction. ∎

Summing, up, Claims 1–4 imply that equilibria are characterized by the probability that the b bank mimics the pure strategy action played by the g bank. For simplicity, let us denote by s^{Off} the support given by the g bank in equilibrium and define $\pi^{\mathit{Off}} \equiv \pi_b(s^{\mathit{Off}}, i_1)$. We use the results in the previous claims without explicit reference in the rest of the proof.

Let us first characterize the existence of a separating equilibrium, i.e. an equilibrium with $\pi^{\mathit{Off}} = 0$ (or equivalently $\int_{A_2} \pi_b(0,i)di = 1$). In a separating equilibrium, we have $\alpha(s^{\mathit{Off}}, i_1) = 1$. Separation is sustained in equilibrium if and only if the incentive compatibility constraint for the b bank not to mimic at $t = 1$ is satisfied, i.e. $\Pi_{b,1}(s^{\mathit{Off}}, i_1, 1) \leq Y_S$, which is the case if and only if $s^{\mathit{Off}} \geq \overline{Y}_S(I)$, where $\overline{Y}_S(I)$ is defined by

$$\Pi_{b,1}(\overline{Y}_S(I), i_1, 1) = \overline{Y}_S(I) \qquad \Leftrightarrow \qquad \overline{Y}_S(I) = \frac{p_b}{p_g}(p_g R - 1) I. \tag{20}$$

Since $s \in [0, Y_S]$, a separating equilibrium exists if and only if $Y_S \geq \overline{Y}_S(I)$. Notice that $\overline{Y}_S(I)$ is increasing in I.

By construction, for $Y_S \geq \overline{Y}_S(I)$, there is a unique separating equilibrium such that $s^{\mathit{Off}} = \overline{Y}_S(I)$. Conversely, if a separating equilibrium exists, then $Y_S \geq \overline{Y}_S(I)$.

Next, we characterize the semi-pooling equilibria, i.e. equilibria with $\pi^{\mathit{Off}} \in (0,1)$. In such an equilibrium, we have $s^{\mathit{Off}} = Y_S$ and $\alpha(s^{\mathit{Off}}, i_1) = \frac{\alpha}{\alpha + (1-\alpha)\pi^{\mathit{Off}}(Y_S, I)}$. This is an equilibrium if and only if the b bank is indifferent between (s^{Off}, i_1) and $(0, i)$, i.e. $\Pi_{b,1}(s^{\mathit{Off}}, i_1, \alpha(s^{\mathit{Off}}, i_1)) = Y_S$, which can be written as

$$p_b \left[R - \frac{1}{\alpha^{\mathit{Off}} p_g + (1 - \alpha^{\mathit{Off}}) p_b} \right] I = Y_S, \quad \text{where } \alpha^{\mathit{Off}} = \frac{\alpha}{\alpha + (1-\alpha)\pi^{\mathit{Off}}}. \tag{21}$$

Notice that if a solution $\pi^{\mathit{Off}} \in (0,1)$ to the equality above exists then it is unique and $Y_S < \overline{Y}_S(I)$. Conversely, for $Y_S < \overline{Y}_S(I)$, Assumption 4 implies that there exists a solution $\pi^{\mathit{Off}} \in (0,1)$ to the equality above. In particular, this means that there cannot exist a pooling equilibrium.

To summarize, there exists a unique equilibrium, in which the g bank plays (s^{Off}, i_1), where $s^{\mathit{Off}}(Y_S, I) = \min\{Y_S, \overline{Y}_S(I)\}$ is increasing in Y_S and increasing in I. The b bank mimics with probability $\pi^{Off}(Y_S, I)$, where $\pi^{Off}(Y_S, I) > 0$ if and only if $Y_S < \overline{Y}_S(I)$. Finally, π^{Off} is decreasing in Y_S and increasing in I.$\square$

Proof of Corollary 1 A comparison between π^{On} given by Lemma 1 and $\pi^{Off}(Y_S, I)$ given by (21) implies that $\pi^{Off}(Y_S, I) < \pi^{On}$.$\square$

Proof of Lemma 2 This lemma follows from its preceding discussion. $\square$

Proof of Lemma 3 The characterization of $e^{Off}(Y_S, I)$ follows from its preceding discussion. To derive the properties of $e^{Off}(Y_S, I)$, notice that $e^{Off}(Y_S, I)$ is increasing in each of its arguments if and only if $T(Y_S, I)$ is increasing in the same argument, where $T(Y_S, I)$ is the expected value of voluntary repayment conditional on the systematic state ($\sigma = S$, which occurs with probability q) and is given by

$$T(Y_S, I) \equiv \left[\alpha + (1-\alpha)\pi^{Off}(Y_S, I)\right] s^{Off}(Y_S, I). \tag{22}$$

Using Proposition 1, consider the following two cases. If $Y_S \geq \overline{Y}_S(I)$, then

$$T(Y_S, I) = \alpha \overline{Y}_S(I). \tag{23}$$

The properties of $\overline{Y}_S(I)$ stated in Proposition 1 then imply that $T(Y_S, I)$ is independent of Y_S and increasing in I. If $Y_S < \overline{Y}_S(I)$, then

$$T(Y_S, I) = \left[\alpha + (1-\alpha)\pi^{Off}(Y_S, I)\right] Y_S, \tag{24}$$

where $\pi^{Off}(Y_S, I)$ is defined by (21). We thus have

$$\begin{aligned} \frac{\partial T(Y_S, I)}{\partial Y_S} &= \alpha + (1-\alpha)\pi^{Off}(Y_S, I) + (1-\alpha)\frac{\partial \pi^{Off}(Y_S, I)}{\partial Y_S} Y_S \\ &= \left[\alpha + (1-\alpha)\pi^{Off}(Y_S, I)\right] \\ &\quad - \left[\alpha \frac{\frac{p_g}{p_b} Y_S}{(1-p_b R)I + Y_S} + (1-\alpha)\pi^{Off}(Y_S, I)\frac{Y_S}{(1-p_b R)I + Y_S}\right] > 0, \end{aligned} \tag{25}$$

where the inequality follows because $Y_S < \overline{Y}_S(Y_S, I)$ implies that $\frac{\frac{p_g}{p_b} Y_S}{(1-p_b R)I + Y_S} < 1$. Moreover, the properties of $\pi^{Off}(Y_S, I)$ stated in Proposition 1 imply that $T(Y_S, I)$ increasing in I.

To summarize, $e^{Off}(Y_S, I)$ is increasing in Y_S and I.$\square$

Proof of Proposition 2 We first show that $\Delta\Pi_0(E[Y_\sigma], Y_S, I)$ given by (13) is increasing in I. Consider again two cases. If $Y_S \geq \overline{Y}_S(I)$,

$$\frac{\partial \Delta\Pi_0(E[Y_\sigma], Y_S, I)}{\partial I} = q(1-\alpha)\pi^{On}(1-p_bR) + \frac{\partial e^{Off}(Y_S, I)}{\partial I}\left[mR - c'^{Off}(Y_S, I))\right] > 0. \quad (26)$$

because $e^{Off}(Y_S, I)$ is increasing in I as stated in Proposition 1. For $Y_S < \overline{Y}_S(I)$,

$$\begin{aligned}\frac{\partial \Delta\Pi_0(E[Y_\sigma], Y_S, I)}{\partial I} = q(1-\alpha)(1-p_bR)\Bigg(&\left[\pi^{On} - \pi^{Off}(Y_S, I)\right]\frac{Y_S}{(1-p_bR)I + Y_S} \\ &+\pi^{On}\frac{1}{1-\alpha}\frac{1-\left[\alpha p_g + (1-\alpha)p_b\right]R}{(1-p_bR)I + Y_S}I\Bigg) \\ &+\frac{\partial e^{Off}(Y_S, I)}{\partial I}\left[mR - c'^{Off}(Y_S, I))\right] > 0. \end{aligned} \quad (27)$$

It then follows that there exists a threshold $\overline{I} \in \mathbf{R}^+ \cup \{\infty\}$, such that $\Delta\Pi_0(E[Y_\sigma], Y_S, I) > 0$ if and only if $I > \overline{I}$. Notice that $\overline{I} > 0$ because $\Delta\Pi_0(E[Y_\sigma], Y_S, 0) < 0$.

Next, we consider how $\overline{I}$ depends on m. As $m \to 0$, $\Delta\Pi_0(\Delta\Pi_0(E[Y_\sigma], Y_S, I) > 0$. This implies that there exists $\overline{m} > 0$, such that $\overline{I} \neq \infty$ for all $m < \overline{m}$.

Finally, we show that $\overline{I}$ is decreasing in Y_S and increasing in $E[Y_\sigma]$. This follows because

$$\frac{\partial \Delta\Pi_0(E[Y_\sigma], Y_S, I)}{\partial Y_S} = -q(1-\alpha)\frac{\partial \pi^{Off}(Y_S, I)}{\partial Y_S}(1-p_bR)I + \left[mR - c'(e^{Off}(Y_S, I))\right]\frac{\partial e^{Off}(Y_S, I)}{\partial Y_S} \geq 0, \quad (28)$$

$$\frac{\partial \Delta\Pi_0(E[Y_\sigma], Y_S, I)}{\partial E[Y_\sigma]} = -\left[mR - c'(e^{On}(E[Y_\sigma]))\right]\frac{\partial e^{On}(E[Y_\sigma]}{\partial E[Y_\sigma]} \leq 0, \quad (29)$$

where we have used the facts that $\frac{\partial \pi^{Off}(Y_S,I)}{\partial Y_S} \leq 0$ by Proposition 1, $\frac{\partial e^{Off}(Y_S,I)}{\partial Y_S} \geq 0$ by Lemma 3 and $\frac{\partial e^{On}(E[Y_\sigma]}{\partial E[Y_\sigma]} \geq 0$ by Lemma 2.$\square$

Proof of Proposition 3 Suppose the bank has financed the first investment off-balance sheet and the first project fails due to systematic reasons. If voluntary support is allowed ex post, the g bank provides voluntary support $s^{Off}(Y_S, I)$ defined in Proposition 1.

In equilibrium, the b bank mimics with probability $\pi^{Off}(Y_S, I) \in [0, 1)$ and therefore its expected profit as of $t = 1$ is equal to Y_S, that is,

$$\Pi_{b,1}(s^{Off}, i_1, \alpha^{Off}) = (Y_S - s^{Off}) + p_b\left[RI - D_3(\alpha^{Off})\right] = Y_S, \quad (30)$$

where $\alpha^{Off} = \frac{\alpha}{\alpha + (1-\alpha)\pi^{Off}}$. Recall that this is because of the indifference condition for the b bank to mimic if $Y_S \leq \overline{Y}_S(I)$, and is because of the optimal choice of $s^{Off} = \overline{Y}_S(I)$ if

$Y_S > \overline{Y}_S(I)$. Using the equality above, the g bank's equilibrium expected profits as of $t = 1$ satisfy

$$\begin{aligned}\Pi_{g,1}(s^{Off}, i_1, \alpha^{Off}) &= (Y_S - s^{Off}) + p_g \left[RI - D_3(\alpha^{Off})\right] \\ &= Y_S + \frac{p_g - p_b}{p_b} s^{Off}. \end{aligned} \tag{31}$$

Suppose an ex post ban on voluntary support is introduced, given the optimal ex-ante funding choice described above. The subsequent investment game at $t = 2$ should information asymmetry persist is as described in Lemma 1. In particular, the b bank invests with probability $\pi^{On} \in (0, 1)$. This implies that the b bank is indifferent between investing in this case or not, resulting in expected profits as of $t = 1$ equal to

$$\Pi_{b,1}^{Ban} = Y_S + p_b \left[RI - D_3(\alpha)\right] = Y_S. \tag{32}$$

Using the equality above, we have that the expected profits for the g bank satisfy

$$\Pi_{g,1}^{Ban} = Y_S < \Pi_{g,1}(s^{Off}, i_1, \alpha^{Off}). \tag{33}$$

We thus conclude that an ex post unexpected ban on voluntary support keeps the expected profits of the b bank constant while strictly reduces those for the g bank.□

Proof of Proposition 4 While the Section 4.3 focuses on the limit as $\tau \to 0$, in this proof we derive the results for any given $\tau \in (0, 1)$. The results given by the proposition are then obtained by taking the limit as $\tau \to 0$.

Suppose the bank has financed the first project off-balance sheet with an initial funding amount d_0 and the project fails due to systematic reasons at $t = 1$. For the ease of notation, let $w_1 \equiv d_0 - 1 + Y_S$ denote the amount of the bank's own funds at $t = 1$. Each bank chooses among the set of actions $\widetilde{A}_1 \equiv \{(s, i) \in [0, w_1] \times \widetilde{A}_2 : s + i \leq w_1\}$, where $i \in \widetilde{A}_2 \equiv [0, w_1] \cup \{i_0\}$ denotes the bank's action at $t = 2$ conditional on information asymmetry persists (with probability $1 - \tau$). In particular, $i \in [0, w_1]$ means that the bank invests in this contingency with i amount of self-financing, raising the remaining $I - i$ from outside investors, while i_0 means that the bank does not invest in this contingency. We extend the natural order in the interval $[0, w_1]$ to the set $\widetilde{A}_2$ by assuming that for any $i \in [0, w_1]$ we have $i > i_0$. Notice that the bank's set of actions $\widetilde{A}_1$ does not include the bank's investment decision conditional on the transparency shock realizing at $t = 2$ (with probability τ). This is because, in this contingency, the bank invests if and only if it is of type g, irrespective of how much self-financing it contributes, since externally financing is always fairly priced. The formal equilibrium definition is analogous to that in the proof of Proposition 1.

The expected profits as of $t = 1$ for a bank of type j that provides support s at $t = 1$ and chooses i when information asymmetry persists at $t = 2$, given the investors' belief $\hat{\alpha}$, are given by

$$\widetilde{\Pi}_{j,1}(s, i, \hat{\alpha}) = \begin{cases} \tau\left[w_1 - s + \max\{(p_j R - 1)I, 0\}\right] & \\ \quad (1-\tau)\left[w_1 - s - i + p_j(RI - \widetilde{D}_3(i, \hat{\alpha}))\right], & \text{if } i \in [0, w_1] \text{ and } \widetilde{D}_3(i, \hat{\alpha})) \leq RI, \\ w_1 - s + \tau \max\{(p_j R - 1)I, 0\}, & \text{otherwise,} \end{cases} \tag{34}$$

where $\widetilde{D}_3(i, \hat{\alpha}))$ for $i \in [0, w_1]$ is given by

$$\widetilde{D}_3(i, \hat{\alpha})) = \frac{I - i}{\hat{\alpha} p_g + (1 - \hat{\alpha}) p_b}. \tag{35}$$

We first establish the following claims.

Claim 5 *In equilibrium, if a g bank plays action (s, i) with positive probability, then $i = \min\{w_1 - s, I\}$.*

Proof. We prove this claim by contradiction.Suppose there exists $i \in [0, \min\{w_1 - s, I\}) \cup \{i_0\}$ such that $\pi_g(s, i) > 0$. We thus have that the equilibrium profits $\widetilde{\Pi}^*_{j,1}$ must satisfy

$$\widetilde{\Pi}^*_{g,1} = \widetilde{\Pi}_{g,1}(s, i, \alpha(s, i)) \text{ and } \widetilde{\Pi}^*_{b,1} \geq \max\{\widetilde{\Pi}_{b,1}(s, i, \alpha(s, i)), \widetilde{\Pi}_{j,1}(s, i_0, \alpha(s, i_0))\}, \tag{36}$$

where the inequality follows because the b bank may play another action in equilibrium which gives it a strictly higher pay-off.

We now prune the supposed equilibrium by constructing a profitable deviation for the g bank under the D1 refinement. Consider an off-equilibrium deviation by the bank to contribute some $i' \in (i, \min\{w_1 - s, I\}]$. The set of beliefs that make deviation strictly optimal for the j bank is defined as:

$$\widetilde{\Lambda}_j(s, i') = \left\{\hat{\alpha} \in [0, 1] : \widetilde{\Pi}_{j,1}(s, i', \hat{\alpha}) > \widetilde{\Pi}^*_{j,1}\right\}. \tag{37}$$

Using (37) and the fact that $\frac{\partial \widetilde{\Pi}_{j,1}(s,i,\hat{\alpha})}{\partial \hat{\alpha}} > 0$, the deviation set $\widetilde{\Lambda}_b(s, i')$ satisfies

$$\widetilde{\Lambda}_b(s, i') \subset \hat{\widetilde{\Lambda}}_b(s, i') \equiv \left\{\hat{\alpha} \in [0, 1] : \widetilde{\Pi}_{b,1}(s, i', \hat{\alpha}) > \max\{\widetilde{\Pi}_{b,1}(s, i, \alpha(s, i)), w_1 - s\}\right\}. \tag{38}$$

We now show that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$, which then implies that $\widetilde{\Lambda}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$. First, if $\hat{\widetilde{\Lambda}}_b(s,i') \neq \emptyset$, then $\widetilde{\Pi}^*_{b,2} \geq w_1 - s$ implies that $\widetilde{D}_3(i',\hat{\alpha}) \leq R$ for all $\hat{\alpha} \in \hat{\widetilde{\Lambda}}_b(s,i')$, so that we have the following property for all $\hat{\alpha}$ in $\hat{\widetilde{\Lambda}}_b(s,i')$:

$$\widetilde{\Pi}_{b,1}(s,i',\hat{\alpha}') \geq \widetilde{\Pi}_{b,1}(s,i,\hat{\alpha}) \Rightarrow \widetilde{\Pi}_{g,1}(s,i',\hat{\alpha}') \geq \widetilde{\Pi}_{g,1}(s,i,\hat{\alpha}). \tag{39}$$

This implies that $\hat{\widetilde{\Lambda}}_b(s,i') \subseteq \widetilde{\Lambda}_g(s,i')$. Further, $p_b R < 1$ implies that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq (0,1]$. This and the fact that $\frac{\widetilde{\Pi}_{j,1}(s,i,\hat{\alpha})}{\partial\hat{\alpha}} > 0$ then imply that $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$. Second, if $\hat{\widetilde{\Lambda}}_b(s,i') = \emptyset$, then we also have $\hat{\widetilde{\Lambda}}_b(s,i') \subsetneq \widetilde{\Lambda}_g(s,i')$, because $p_g R > 1$ and $1 \in \widetilde{\Lambda}_g(s,i') \neq \emptyset$.

The D1 refinement then implies that $\alpha(s,i') = 1$, but then using that $\alpha(s,i') \geq \alpha(s,i)$, $\frac{\partial\widetilde{\Pi}_{b,1}(s,i,\hat{\alpha})}{\partial\hat{\alpha}} > 0$ and $\frac{\partial\widetilde{\Pi}_{b,1}(s,i,\hat{\alpha})}{\partial i} > 0$ we have that

$$\widetilde{\Pi}_{g,1}(s,i',\alpha(s,i')) \geq \widetilde{\Pi}_{g,1}(s,i',\alpha(s,i)) > \widetilde{\Pi}_{g,1}(s,i,\alpha(s,i)) = \widetilde{\Pi}^*_{g,1}, \tag{40}$$

a contradiction. ∎

Claim 6 *In equilibrium, the b bank plays with positive probability only actions that are either played with positive probability by the g bank, or $(s,i) = (0,i)$ and receives $\widetilde{\Pi}_{b,1}(0,i,\alpha(0,i)) = w_1$.*

Proof. Proof analogous to that of Claim 2. ∎

Claim 7 *If in equilibrium $\int_{\widetilde{A}_2} \pi_b(0,i)di < 1$ then the g bank plays with probability one the action $(s,i) = (w_1,0)$.*

Proof. The proof of this claim is analogous to that of Claim 3. The logic is that, if this claim is not true, then we can construct a profitable deviation for the g bank (s',i') where $s'^* + \epsilon$ and $i' = i - \epsilon \geq 0$ for some sufficiently small ϵ. ∎

Claim 8 *If in equilibrium $\int_{\widetilde{A}_2} \pi_b(0,i)di = 1$ then the g bank plays with probability one the action (s,i), where $i = \min\{w_1 - s, I\}$ and s is the minimum value such that $\widetilde{\Pi}_{b,1}(s,i,1) \leq w_1$.*

Proof. Proof analogous to that of Claim 4. ∎

Analogous to the baseline model, Claims 5–8 imply that equilibria are characterized by the probability that the b bank mimics the pure strategy action played by the g bank. Let us denote by $\widetilde{s}^{Off}$ the support given by the g bank in equilibrium and define $\widetilde{i}^{Off} = w_1 - \widetilde{s}^{Off}$ and $\widetilde{\pi}^{Off} = \pi_b(\widetilde{s}^{Off}, \widetilde{i}^{Off})$. We use the results in the previous claims without explicit reference in the rest of the proof.

Let us first characterize the existence of a separating equilibrium, i.e. an equilibrium with $\widetilde{\pi}^{Off} = 0$ (or equivalent, $\int_{\widetilde{A}_2} \pi_b(0,i)di = 1$). In a separating equilibrium, we have $\alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off}) = 1$. Separation is sustained in equilibrium if and only if the incentive compatibility constraint for the b bank not to mimic at $t = 1$ is satisfied, i.e. $\widetilde{\Pi}_{b,1}(\widetilde{s}^{Off}, \widetilde{i}^{Off}, 1) \leq w_1$, which is the case if and only if $\widetilde{s}^{Off} \geq \overline{s}(w_1, \tau, I)$, where $\overline{s}(w_1, \tau, I)$ is defined by

$$\widetilde{\Pi}_{b,1}(\overline{s}(w_1,\tau,I), \min\{w_1 - \overline{s}(w_1,\tau,I), I\}, 1) = w_1 \quad \Leftrightarrow$$
$$\tau\left[w_1 - \overline{s}(w_1,\tau,I)\right] + (1-\tau)p_b\left[RI - \frac{I - w_1 + \overline{s}(w_1,\tau,I)}{p_g}\right] = w_1. \tag{41}$$

Notice that $\overline{s}(w_1, \tau, I)$ is continuous and decreasing in w_1. $\widetilde{s}^{Off} < w_1$ thus implies that a separating equilibrium exists if and only if $w_1 \geq \widetilde{Y}_S(\tau, I)$, where $\widetilde{Y}_S(\tau, I)$ is defined by

$$\widetilde{\Pi}_{b,1}(\widetilde{Y}_S(\tau,I), 0, 1) = \widetilde{Y}_S(\tau,I) \qquad \Leftrightarrow \qquad \widetilde{Y}_S(\tau,I) = (1-\tau)\frac{p_b}{p_g}(p_g R - 1)I. \tag{42}$$

Notice that there exists $\overline{w}_1(\tau, I) > \widetilde{Y}_S(\tau, I)$, such that $\overline{s}(w_1, \tau, I) > 0$ if and only if $w_1 < \overline{w}_1(\tau, I)$, where $\overline{w}_1(\tau, I)$ is given by $\overline{s}(\overline{w}_1(\tau, I), \tau, I) > 0$, or

$$p_b\left[RI - \frac{I - \overline{w}_1(\tau,I)}{p_g}\right] = \overline{w}_1(\tau,I). \tag{43}$$

By construction, for $w_1 \geq \widetilde{Y}_S(\tau, I)$, there is a unique separating equilibrium such that $\widetilde{s}^{Off} = \max\{\overline{s}(w_1, \tau, I), 0\}$. Conversely, if a separating equilibrium exists then $w_1 \geq \widetilde{Y}_S(\tau, I)$.

Next, we characterize the semi-pooling equilibrium, i.e., equilibria with $\widetilde{\pi}^{Off} \in (0, 1)$. In such an equilibrium, we have $\widetilde{s}^{Off} = w_1$, $\widetilde{i}^{Off} = 0$ and $\alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off}) = \frac{\alpha}{\alpha + (1-\alpha)\widetilde{\pi}^{Off}}$. This is an equilibrium if and only if the b bank is indifferent between $(\widetilde{s}^{Off}, \widetilde{i}^{Off})$ and $(0, i)$, i.e. $\widetilde{\Pi}_{b,1}(\widetilde{s}^{Off}, \widetilde{i}^{Off}, \alpha(\widetilde{s}^{Off}, \widetilde{i}^{Off})) = w_1$, which implies that $\widetilde{\pi}^{Off}(w_1, \tau, I)$ is defined by

$$(1-\tau)p_b\left[R - \frac{1}{\alpha^{Off} p_g + (1-\alpha^{Off})p_b}\right]I = Y_S, \quad \text{where } \alpha^{Off} = \frac{\alpha}{\alpha + (1-\alpha)\pi^{Off}}. \tag{44}$$

To summarize, there exists a unique equilibrium, in which the g bank plays $(\widetilde{s}^{Off}, \widetilde{i}^{Off})$, where

$$\widetilde{s}^{Off}(w_1,\tau,I) = \begin{cases} w_1, & \text{if } w_1 \leq \widetilde{Y}_S(\tau,I), \\ \overline{s}(w_1,\tau,I), & \text{if } w_1 \in [\widetilde{Y}_S(\tau,I), \overline{w}_1(\tau,I)], \\ 0, & \text{if } w_1 \geq \overline{w}_1(\tau,I), \end{cases} \quad \text{and} \quad \widetilde{i}^{Off} = \min\{w_1 - \widetilde{s}^{Off}, I\}. \tag{45}$$

The b bank mimics with probability $\widetilde{\pi}^{Off}(w_1, \tau, I)$, where $\widetilde{\pi}^{Off} > 0$ if and only if $w_1 < \widetilde{Y}_S(\tau, I)$.

Focusing on the case of $\tau \to 0$, we have the result stated in this proposition. First, notice that $\lim_{\tau \to 0} \widetilde{Y}(\tau, I) = \overline{Y}(I)$, where $\overline{Y}(\tau, I)$ is defined by Proposition 1. We thus have that, for $\tau \to 0$, $\widetilde{s}^{Off} = w_1 = d_0 - 1 + Y_S$ if and only if $w_1 = d_0 - 1 + Y_S \leq \overline{Y}_S(I)$. Second, for $\tau \to 0$, $\widetilde{\pi}^{Off} = \pi^{Off}(d_0 - 1 + Y_S, I)$, where $\pi^{Off}(\cdot)$ is defined by Proposition 1. This implies that $\widetilde{\pi}^{Off}$ is decreasing in d_0. $\square$

Proof of Proposition 5 As in the proof of Proposition 4, we derive the results for any given $\tau \in (0, 1)$ in this proof. The results given by this proposition are then obtained by taking the limit as $\tau \to 0$.

We first characterize the bank's effort choice for the first investment e and investment decision at $t = 2$ under on-balance sheet funding in the following claim.

Claim 9 *Suppose the bank invests in the first project on-balance sheet and chooses the amount of funds $d_0 \geq 1$ to raise from investors at $t = 0$. The bank's optimal effort choice is given by $e^{On}(E[Y_\sigma])$, where $e^{On}(E[Y_\sigma])$ is defined in Lemma 2. If the project fails due to systematic reasons at $t = 1$ and information asymmetry persists at $t = 2$, the bank's investment decision is as described in Lemma 1.*

Proof. Suppose the first project is funded on-balance sheet and the bank raises $d_0 \geq 1$ units of funds from investors at $t = 0$. Let $D_1 \leq R$ denote the competitive promised repayment required by the investors. Taking into account that the bank's asset-in-place pays off Y_σ at $t = 1$ and that the bank is contractually obliged to use it if necessary to satisfy debt repayments, the promise D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + q \min\{d_0 - 1 + Y_S, D_1\} + (1 - q - p_g - me) \min\{d_0 - 1 + Y_{\overline{S}}, D_1\} = d_0. \tag{46}$$

Notice that (46) and $Y_\sigma < 1$ imply $D_1 > d_0 - 1 + Y_\sigma$. This implies that if $R_1 = 0$, the bank exhausts its own funds to repay the on-balance sheet debt holders. As a result, for a given D_1, the bank chooses effort e to maximize

$$(p_g + me)(d_0 - 1 + Y_{\overline{S}} + R - D_1) - c(e). \tag{47}$$

Notice that this expression is analogous to (7) in the baseline model. The bank's optimal effort choice thus satisfies the following first order condition

$$c'(e) = m(d_0 - 1 + Y_{\overline{S}} + R - D_1). \tag{48}$$

Equation (46) and (48) jointly determine D_1 and e. After some algebraic manipulation, the solution is identical to that described in Lemma 2 and does not depend on d_0.

Finally, since the equilibrium repayment D_1 satisfies $D_1 > d_0 - 1 + Y_\sigma$, the bank has no funds after repaying the debt holders if the first project fails at $t = 1$. If the failure is due to systematic reasons and information asymmetry persists at $t = 2$, it follows that the bank's investment decision is as described in Lemma 1. ∎

Next, we consider off-balance sheet funding. In the following claims, we characterize first the bank's effort choice for the first investment e for a given initial funding amount d_0, then the bank's optimal choice of initial funding amount.

Claim 10 *Suppose the bank invests in the first project off-balance sheet. For a given initial funding amount $d_0 \geq 1$, the bank's optimal effort choice is given by the largest solution to the equation*

$$c'(e) = m\left(R - \frac{1 - q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}\right]\widetilde{s}^{Off}}{p_g + me}\right),$$

where $w_1 = d_0 - 1 + Y_S$ is the bank's own funds at $t = 1$ following the systematic failure of the first project. Moreover, the optimal effort, which we denote by $\widetilde{e}^{Off}(w_1, \tau, I)$, is increasing in I and strictly so if and only if $w_1 < \widetilde{Y}_S(\tau, I)$. Finally, $\widetilde{e}^{Off}(w_1, \tau, I)$ is strictly increasing in w_1 if and only if $w_1 < \widetilde{Y}_S(\tau, I)$, where $\widetilde{Y}_S(\tau, I)$ is defined in the proof of Proposition 4.

Proof. We first derive the bank's optimal effort choice. Taking into account that the bank provides voluntary support following the systematic failure of the project as described in Proposition 4, the promised repayment D_1 satisfies the following break-even condition:

$$(p_g + me)D_1 + q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}\right]\widetilde{s}^{Off} = d_0, \tag{49}$$

where $\widetilde{\pi}^{Off}$ and $\widetilde{s}^{Off}$ are defined in the proof of Proposition 4. Moreover, for a given D_1, the bank chooses effort e to maximize

$$(p_g + me)(d_0 - 1 + Y_{\overline{S}} + R - D_1) + (1 - q - p_g - me)(d_0 - 1 + Y_{\overline{S}}) - c(e). \tag{50}$$

This expression is analogous to (10) in the baseline model. The bank's optimal effort choice thus satisfies the same first order condition as in the baseline model, given by (11). Equation (49) and (11) jointly determine D_1 and e. After some algebraic manipulation, the solution is as described in this claim.

We now consider the properties of $\widetilde{e}^{Off}(w_1, \tau, I)$, using the properties of $\widetilde{s}^{Off}(w_1, \tau, I)$ and $\widetilde{\pi}^{Off}(w_1, \tau, I)$ given in the proof of Proposition 4. Consider the following two cases. If $w_1 < \widetilde{Y}_S(\tau, I)$, $\widetilde{s}^{Off}(w_1, \tau, I) = w_1$ and $\widetilde{\pi}^{Off}(w_1, \tau, I)$ is strictly decreasing in w_1 and increasing in I. Therefore $\widetilde{e}^{Off}(w_1,\tau,I)$ is strictly increasing in w_1 in I. If $w_1 \geq \widetilde{Y}_S(\tau,I)$, $\widetilde{s}^{Off}(w_1,\tau,I) = \max\{\overline{s}(w_1,\tau,I),0\}$ and $\widetilde{\pi}^{Off}(w_1,\tau,I) = 0$. Notice that $\overline{s}(w_1,\tau,I)$ given by (41) is decreasing in w_1. Therefore $\widetilde{e}^{Off}(w_1,\tau,I)$ is decreasing in w_1. To summarize, $\widetilde{e}^{Off}(w_1,\tau,I)$ is increasing in I and strictly so if and only if $w_1 < \widetilde{Y}_S(\tau,I)$; $\widetilde{e}^{Off}(w_1,\tau,I)$ is also strictly increasing in w_1 if and only if $w_1 < \widetilde{Y}_S(\tau,I)$. ∎

Claim 11 *Suppose the bank invests in the first project off-balance sheet. The bank optimally chooses an initial funding amount* $d_0^{Off}(Y_S,\tau,I) = \widetilde{w}_1(Y_S,\tau,I)+1-Y_S$*, where* $\widetilde{w}_1(Y_S,\tau,I) \geq Y_S$*, with equality if and only if* $Y_S \geq \widetilde{Y}_S(\tau,I)$*.*

Proof. The bank chooses the initial funding amount d_0 to maximize the expected profits as of $t=0$, given by

$$\widetilde{\Pi}_0(\widetilde{e}^{Off},\widetilde{\pi}^{Off}) \equiv V^{FB} - \int_{\widetilde{e}^{Off}}^{e^{FB}} (mR - c'(e))de - (1-\tau)q(1-\alpha)\widetilde{\pi}^{Off}(1-p_bR)I. \tag{51}$$

This expression is analogous to (12) in the baseline model, with the only difference being that second project inefficiency occurs only when the information asymmetry persists until $t=2$ (with probability $1-\tau$).

Since $\widetilde{e}^{Off}(w_1,\tau,I)$ and $\widetilde{\pi}^{Off}(w_1,\tau.I)$ only indirectly depend on d_0 through w_1, choosing the optimal initial funding amount d_0^{Off} is equivalent to choosing the optimal $\widetilde{w}_1^{Off} \geq Y_S$. The optimal funding can then be residually derived as $d_0^{Off} = \widetilde{w}_1^{Off} + 1 - Y_S$.

Using the properties of $\widetilde{e}^{Off}(w_1,\tau,I)$ derived in the proof of Proposition 4 and the properties of $\widetilde{e}^{Off}(w_1,\tau,I)$ given by Claim 10, we have that $\Pi_0(\widetilde{e}^{Off},\widetilde{\pi}^{Off})$ is increasing in w_1 if and only if $w_1 \leq \widetilde{Y}_S(\tau,I)$. This implies that $\widetilde{w}_1^{Off} = Y_S$ for all $Y_S \geq \widetilde{Y}_S(\tau,I)$.

If $Y_S < \widetilde{Y}_S(\tau,I)$, then $w_1 = \widetilde{Y}_S(\tau,I)$ is feasible if and only if there exists $D_1 < R$ that satisfies (49), or equivalently,

$$\left[p_g + me^{Off}(\widetilde{Y}_S(\tau,I),\tau,I)\right]R + q\alpha\widetilde{Y}_S(\tau,I) \geq \widetilde{Y}_S(\tau,I) + 1 - Y_S. \tag{52}$$

Notice that the right hand side of the above expression is decreasing in Y_S. Further, (52) is satisfied with strict inequality for $Y_S = \widetilde{Y}_S(\tau,I)$. Therefore there exists $\underline{Y}_S(\tau,I) < \widetilde{Y}_S(\tau,I)$, such that $w_1 = \widetilde{Y}_S(\tau,I)$ is feasible if and only if $Y_S \in [\underline{Y}_S(\tau,I),\widetilde{Y}_S(\tau,I)]$. In this case, the bank optimally chooses $\widetilde{w}_1^{Off} = \widetilde{Y}_S(\tau,I)$.

If $Y_S < \underline{Y}_S(\tau,I)$, $\Pi_0(\widetilde{e}^{Off},\widetilde{\pi}^{Off})$ is increasing in w_1 for all feasible w_1, while all feasible w_1 satisfy $w_1 < \widetilde{Y}_S(\tau,I)$ as argued above. Therefore the bank chooses the maximum feasible w_1, denoted by $\overline{w}_1^{Off}(Y_S,\tau,I)$, which is the largest solution to

$$\left[p_g + m\widetilde{e}^{Off}(w_1,\tau,I)\right]R + q\left[\alpha + (1-\alpha)\widetilde{\pi}^{Off}(w_1,\tau,I)\right]w_1 = w_1 + 1 - Y_S. \tag{53}$$

To summarize, the bank's optimal initial funding amount is given by $d_0^{\mathit{Off}}(Y_S,\tau,I) = \widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I) + 1 - Y_S$, where

$$\widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I) = \begin{cases} \overline{w}_1^{\mathit{Off}}(Y_S,\tau,I), & \text{if } Y_S \le \underline{Y}_S(\tau,I), \\ \widetilde{Y}_S(\tau,I), & \text{if } Y_S \in [\underline{Y}_S(\tau,I), \widetilde{Y}_S(\tau,I)], \\ Y_S, & \text{if } Y_S \ge \widetilde{Y}_S(\tau,I). \end{cases} \tag{54}$$

∎

We can now characterize the bank's choice of optimal funding mode, which is determined by the difference in the bank's expected profits between off- and on-balance sheet funding. Using the results of Claims 9–11, this is given by

$$\begin{aligned}\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\tau,I) &\equiv \widetilde{\Pi}_0(\widetilde{e}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I),\widetilde{\pi}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I)) - \Pi_0(e^{On}(E[Y_\sigma]),\pi^{On}) \\ &= q(1-\tau)(1-\alpha)\left(\pi^{On} - \widetilde{\pi}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I)\right)(1-p_bR)I \\ &\qquad - \int_{\widetilde{e}^{\mathit{Off}}(\widetilde{w}_1^{\mathit{Off}}(Y,\tau,I),\tau,I)}^{e^{On}(Y)} (mR - c'(e))de.\end{aligned} \tag{55}$$

Notice that $\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\tau,I)$ is strictly increasing in I. To see this, consider the following three cases. If $Y_S \le \underline{Y}_S(\tau,I)$, then (53) implies that $\widetilde{w}_1^{\mathit{Off}}(Y_S,\tau,I) = \overline{w}_1^{\mathit{Off}}(Y_S,\tau,I)$ is increasing in I. Using the properties of $\widetilde{e}^{\mathit{Off}}(w_1,\tau,I)$ stated in Claim 10, we have that

$$\begin{aligned}\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma],Y_S,\tau,I)}{\partial I} &= q\alpha\frac{(1-\tau)\frac{p_g-p_b}{p_b}\left[\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I)\right]^2}{\left[(1-\tau)(1-p_bR)I + \overline{w}_1^{\mathit{Off}}(Y_S,\tau,I)\right]^2} \\ &\quad - q(1-\tau)(1-\alpha)\frac{\partial\widetilde{\pi}^{\mathit{Off}}(\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I)}{\partial w_1}\frac{\partial\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I)}{\partial I}(1-p_bR)I \\ &\quad + \left[\frac{\widetilde{e}^{\mathit{Off}}(\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I)}{\partial I} + \frac{\widetilde{e}^{\mathit{Off}}(\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I),\tau,I)}{\partial w_1}\frac{\partial\overline{w}_1^{\mathit{Off}}(Y_S,\tau,I)}{\partial I}\right] \\ &\qquad \times (mR - c'(\widetilde{e}^{\mathit{Off}})) > 0,\end{aligned} \tag{56}$$

If $Y_S \in [\underline{Y}_S(\tau, I), \widetilde{Y}_S(\tau, I)]$, we have

$$\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I) = q(1-\tau)(1-\alpha)\pi^{On}(1-p_bR)I - \int_{\widetilde{e}^{Off}(\overline{Y}(\tau,I),\tau,I)}^{e^{On}(Y)} (mR - c'(e))de, \tag{57}$$

$$\begin{aligned}\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I)}{\partial I} &= q(1-\tau)(1-\alpha)\pi^{On}(1-p_bR) \\ &+ \left[\frac{\widetilde{e}^{Off}(\widetilde{Y}_S(\tau, I), \tau, I)}{\partial I} + \frac{\widetilde{e}^{Off}(\widetilde{Y}_S(\tau, I), \tau, I)}{\partial w_1}\frac{\partial \widetilde{Y}_S(\tau, I)}{\partial I}\right](mR - c'(\widetilde{e}^{Off})) > 0\end{aligned} \tag{58}$$

If $Y_S \geq \widetilde{Y}_S(\tau, I)$, we have

$$\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I) = q(1-\tau)(1-\alpha)\pi^{On}(1-p_bR)I - \int_{\widetilde{e}^{Off}(Y_S,\tau,I)}^{e^{On}(Y)} (mR - c'(e))de, \tag{59}$$

$$\frac{\partial\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I)}{\partial I} = q(1-\tau)(1-\alpha)\pi^{On}(1-p_bR) + \frac{\widetilde{e}^{Off}(Y_S, \tau, I)}{\partial I}(mR - c'(\widetilde{e}^{Off})) > 0. \tag{60}$$

It then follows that there exists a threshold $\widetilde{I} \in \mathbf{R}^+ \cup \{\infty\}$, such that $\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I) > 0$ if and only if $I > \widetilde{I}$. Notice that $\widetilde{I} > 0$ because $\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, 0) < 0$.

Next, we consider how $\widetilde{I}$ depends on m. As $m \to 0$, $\Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I) > 0$ for all I. This implies that there exists $\widetilde{m} > 0$, such that $\widetilde{I} \neq \infty$ for all $m < \widetilde{m}$.

Finally, we consider, as $\tau \to 0$, how $\widetilde{I}$ compares to $\overline{I}$ given by Proposition 1. Comparing the results Claims 9–11 to those of Proposition 1 and Lemma 3, we have that

$$\lim_{\tau\to 0} \widetilde{\pi}^{Off}(\widetilde{w}_1^{Off}(Y_S, \tau, I), \tau, I) \begin{cases} < \pi^{Off}(Y_S, I), & \text{if } Y_S < \overline{Y}_S(I), \\ = \pi^{Off}(Y_S, I), & \text{if } Y_S \geq \overline{Y}_S(I), \end{cases} \tag{61}$$

and that

$$\lim_{\tau\to 0} \widetilde{e}^{Off}(\widetilde{w}_1^{Off}(Y_S, \tau, I), \tau, I) \begin{cases} > e^{Off}(Y_S, I), & \text{if } Y_S < \overline{Y}_S(I), \\ = e^{Off}(Y_S, I), & \text{if } Y_S = \overline{Y}_S(I), \\ < e^{Off}(Y_S, I), & \text{if } Y_S > \overline{Y}_S(I). \end{cases} \tag{62}$$

This implies that $\lim_{\tau\to 0} \Delta\widetilde{\Pi}_0(E[Y_\sigma], Y_S, \tau, I) > \Delta\Pi_0(E[Y_\sigma], Y_S, I)$ if and only if $Y_S < \overline{Y}_S(I)$. Equivalently, $\widetilde{I} < \overline{I}$ if and only if $Y_S < \overline{Y}_S(I)$. $\square$

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