The martingale optimal transport problem

A martingale transport between μ_1 and μ_2 is the law of a martingale $(X_i)_{i=1,2}$ with $X_i \sim \mu_i$, for $i = 1, 2$. The designation 'transport' for a joint law originates from the optimal transport problem, introduced by Gaspard Monge in 1781, where a heap of soil distributed as μ_1 (the 'déblais') has to be transported according to a distribution μ_2 (the 'remblais') in an optimal way. The word 'optimal' refers to the fact that we look for minimizing $\mathbb{E}(f(X,Y))$ where f is a real-valued cost function, typically the distance $|Y - X|$. The martingale version was introduced in 2011 in two parallel papers by Beiglböck, Henry-Labordère and Penkner (*Model-independent bounds for option prices—a mass transport approach*) and Galichon, Henry-Labordère and Touzi (*A stochastic control approach to no-arbitrage bounds given marginals, with an application to lookback options*) in relation with the robust model-independant pricing of options. In the martingale version of the transport problem the additional martingale constraint $\mathbb{E}(X_2|X_1) = X_1$ is mandatory and f is rather seen as a reward function to maximize than a cost to minimize.

During the talk, we will review some examples, techniques and features of (optimal) martingale transport. Some of the results to be presented were obtained in collaboration with Mathias Beiglböck (Vienna).