Pledge-and-Review Bargaining

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Abstract

This paper analyzes a bargaining game that is new to the literature, but that is inspired by real-world international negotiations. With so-called pledge-and-review bargaining, as stipulated in the Paris climate agreement, each party submits an intended nationally determined contribution (INDC), quantifying a cut in its own emission level. Thereafter, the set of pledges must be unanimously ratified. The procedure is repeated periodically, as newly developed technology makes earlier pledges obsolete. I first develop a dynamic model of the pledge-and-review game before deriving four main results: (1) If there is some uncertainty on the set of pledges that is acceptable (for example because of unknown discount factors), each equilibrium pledge is approximated by the *asymmetric Nash Bargaining Solution*. The weights placed on the others' payoffs reflect the underlying uncertainty. The weights vary from pledge to pledge, and the set of equilibrium pledges is inefficient. (2) When this result is embedded in a simple dynamic climate change model, I show that each party contributes too little to the public good, the incentive to develop new technology is weak, and the optimal commitment period length is (therefore) long. (3) This result is overturned when each party can decide whether or not to participate: The undemanding pledge-and-review process motivates a larger number of parties to participate. This increases aggregate contributions and technology investments, and the optimal commitment period length is (therefore) short. (4) When the parties can choose between alternative bargaining games in advance, the (broad but shallow) pledge-and-review game is preferred when there is a larger number of potential participants. However, pledge-and-review is preferred too often (seldom) by technology leaders (laggards), relative to what is socially efficient. The results are consistent with several crucial differences between the climate agreements signed in Kyoto (1997) and Paris (2015) and they rationalize the development from the former to the latter.

Key words: Dynamic games, bargaining games, the Nash Program, climate change, the Paris Agreement, the Kyoto Protocol

1 Introduction

-The pledge-and-review strategy is completely inadequate.

Christian Gollier and Jean Tirole
The Economist (guest blog)
June 1st, 2015

Pledge-and-review bargaining refers to the structure of the negotiation process adopted in Paris, December, 2015. Each party in the negotiation process was first asked to submit an "intended nationally determined contribution" (INDC). After the INDC's had been announced by all parties, the parties were expected to ratify the treaty. The INDC's should specify cuts in the emissions of greenhouse gases being effective from 2020 to 2025 (or to 2030), and every five years the parties shall review and make new pledges for another five-year period.

This negotiation structure is remarkably different from the one used under the Kyoto Protocol of 1997. There, a "top-down" approach was used to ensure the parties made legally binding commitments to cut emissions by (on average) five percent compared to the 1990-levels. By comparison, pledge-and-review has been referred to as a "bottom-up" approach since countries will themselves determine how much to cut nationally, without making these cuts conditional on other countries' emission cuts. No wonder, then, that economic theorists question the effectiveness of the pledge-and-review bargaining game.

Interestingly, the Paris agreement differs from the Kyoto Protocol in several other ways, as well. First, while the second commitment period under the Kyoto Protocol was eight years (2012-2020), each commitment period under Paris will be only five years. Second, while only 35 countries faced binding emission cuts under Kyoto, the Paris agreement has been signed by nearly every country in the world. At the same time, both types of agreements share the emphasis on emission cuts rather than specifying national investments in environmentally friendly technology, for example, although the importance of developing such new technology has been emphasized in every recent treaty text.

The purpose of this paper is to propose a framework for studying pledge-and-review bargaining and to use it to shed light on the development from the Kyoto-style agreement to Paris.

The next section describes a bargaining game that is new to the theoretical literature, although it is based on actual (Paris-style) negotiations. With perfect information, the unique and trivial equilibrium of the described bargaining game will coincide with the non-cooperative (or "business-as-usual") outcome, where every party simply contributes so as to maximize its own utility. With sufficiently important shocks on the other parties' willingness to decline and delay ratification, I show that each party's equilibrium contribution level coincides with the quantity that maximizes an asymmetric Nash product, where the weights on other parties' payoffs reflect the extent of uncertainty and how shocks are correlated. (The weights also reflect differences in expected discount rates in an intuitive way.) The weights on other parties' payoffs are less than $1/2$ for single-peaked and symmetric shock distributions, and they are (close to) zero when the variance of the shocks is small. These small weights on others' payoffs makes pledge and review quite different from the (symmetric) Nash Bargaining Solution often used to describe Kyoto-style negotiations.

The subsequent sections investigate the effects of the small weights associated with pledge and review. Naturally, emission cuts are smaller and (thus) investments in new technology less when the weights are small, for any fixed number of parties. Since technology then develops slowly, it is not important to revise the cuts very frequently, and the optimal commitment period is longer.

All these conclusions are reversed, however, when the decision to participate in the agreement is endogenized. Since not much is expected from the participating countries (when the weights on others' payoffs are small), it is not that costly to participate and the equilibrium coalition size is larger. The larger number of parties makes the aggregate cuts more ambitious, the aggregate investments larger, and the optimal commitment period shorter, Section 4 shows.

The outcome under pledge-and-review is clearly quite different than the outcome under the (symmetric) Nash Bargaining Solution, often used to describe the outcome of the Kyoto Protocol. Section 5 compares the outcomes under the two alternative bargaining games when the participation decision is taken into account. According to the results described so far, the "shallow but broad" pledge-and-review game is superior because a larger number of potential parties volunteer to participate. In reality, there is a limited number ($\overline{n}$) of potential members and if this upper boundary is binding, then the comparison is less clear. Further, when potential parties are heterogeneous, in that a number ($\underline{n}$) of them will participate regardless of the game, then the "deep and narrow" coalition under the Nash Bargaining Solution can be quite attractive. My final result states that pledge-and-review is preferred if and only if $\overline{n}$ is large while $\underline{n}$ is small.

This result is in line with the development from Kyoto to Paris: In the 1990s, there were a large number of developing countries that could not be expected to contribute much to a global climate policy. Over the last twenty years, some of these have become emerging economies that potentially has an important role to play. This implies that the number of relevant potential parties, $\overline{n}$, has increased. During the same period, eight of the countries that initially signed the Kyoto Protocol have declared that they do not intend to make commitments in the second commitment period of Kyoto: Belarus, Kazakhstan, Ukraine, Japan, New Zealand, Russia, Canada, and USA. This can be interpreted as a smaller $\underline{n}$. Both these developments make pledge-and-review relatively better, according to my theory. Section 5 discusses this in detail, and explain how different groups of countries may have different opinions on exactly when to switch to pledge-and-review.

The pledge-and-review bargaining game has not been analyzed in the theoretical literature, as far as I know. By showing that this bargaining game implements an asymmetric (or "generalized") Nash Bargaining Solution (NBS) for each party's contribution, I contribute to the 'Nash Program', aimed at finding noncooperative games implementing cooperative solution concepts. The Nash demand game, first described by Nash (1953), intended to implement the Nash Bargaining Solution, axiomatized by Nash (1950). There is a large subsequent literature investigating the extent to which the Nash demand game implements the Nash Bargaining Solution (Binmore, 1992; Abreu and Gul, 2000; Kambe, 2000), and some contributions also allow for uncertainty, as I do here (Binmore, 1987; Carlsson, 1991; Andersson et al., 2017).[1]

The alternating offer bargaining game by Rubinstein (1982) also implements the Nash Bargaining Solution, as shown by Binmore et al. (1986), and asymmetric discount rates rationalizes the *asymmetric* Nash Bargaining Solution (axiomatized by Harsanyi and Selten, 1972; Kalai, 1977; Roth, 1979). Although there can be multiple equilibria with more than two players (Sutton, 1986; Osborne and Rubinstein, 1990), the weights in the asymmetric NBS may then also depend on recognition probabilities (Miyakawa, 2008; Britz et al. 2010; Laruelle and Valenciano, 2008), in addition to the discount rates (Kawamori, 2014).[2]

The next section contributes to this literature by showing that also 'pledge-and-review' bargaining implements the asymmetric NBS for each party's contribution. However, the weights are shown to vary from one party's contribution level to another's, so the set of contributions is not Pareto optimal. The weights will reflect differences in the discount rates (as in some of the papers already mentioned), but also the extent of uncertainty in shocks and the correlation in shocks across the parties.

[The relevant literature in environmental economics will be discussed in the next version of this draft.]

2 A Model of Pledge-and-Review Bargaining

This section describes a novel bargaining game, not yet analyzed in the literature, and characterizes its outcome. The section may be read independently from the other sections, as the model here may have several other applications than the climate negotiations motivating the subsequent sections.[3] The main new feature of the game is that each party proposes only one single aspect of the agreement, although payoffs depend on the entire vector of aspects.

There are n parties, each endowed with a payoff function $U_i : \mathbb{R}^n \to \mathbb{R}$, $i \in \{1, ...n\}$. The bargaining game starts when each party i simultaneously proposes its own contribution $x_i \in \mathbb{R}$, before observing the

[1] Also the Nash bargaining solution with endogenous threats has been given noncooperative foundations in dynamic games (Abreu and Pearce, 2007 and 2015).

[2] There are also papers showing how the NBS is implemented exactly in other ways, either by a specific game (Howard, 1992) or in a matching context (Cho and Matsui, 2013).

[3] For example, the bargaining game could be appropriate when a number of business partners are negotiating a package deal, and each partner has expertise on and is making the proposal on one single aspect of the package (such as quality, price, delivery time, etc).

vector of proposed contributions, $\mathbf{x} = (x_1, ..., x_n)$. Thereafter, each party must decide whether to accept (or ratify) the proposed agreement, $\mathbf{x}$. If one or more party declines $\mathbf{x}$, the game restarts in the next period, i.e., after some delay, $\Delta > 0$. If everyone accepts, every party i receives the payoff $U_i(\mathbf{x})$ and the game ends.

I assume U_i to be concave and continuously differentiable, and both U_i and x_i are measured relative to the default outcome (which is therefore normalized to zero). In addition, I will start out by assuming $\partial U_i(\cdot)/\partial x_i < 0$ for $x_i > 0$, and $\partial U_j(\cdot)/\partial x_i > 0$, $j \neq i$, so that the x_i's can be interpreted as contributions to a public good. However, the Appendix proves Theorem 2 and a version of Theorem 1 without these additional assumptions: see the below 'Remark on generality'.

Party i's discount factor between time t and $t + \Delta$ is $\delta_{i,t}^{\Delta}$, but it will be more convenient to refer to the "discount rate" $\rho_{i,t} \equiv \left(1 - \delta_{i,t}^{\Delta}\right)/\Delta$.[4] Thus, i receives $\left(1 - \rho_{i,t}\Delta\right) U_i(\mathbf{x}^*)$ by declining an offer if $\mathbf{x}^*$ can be expected next period. Given $\mathbf{x}^*$, i prefers to accept $\mathbf{x}$ now if:

$$U_i(\mathbf{x}) \geq \left(1 - \rho_{i,t}\Delta\right) U_i(\mathbf{x}^*). \tag{1}$$

I will restrict attention to stationary subgame-perfect equilibria (SPEs). If information were perfect, it is easy to see that $\mathbf{x}^*$ could be a stationary SPE only if x_i^* were equal to the noncooperative level $x_i^* = \arg\max_{x_i} U_i\left(x_i, \mathbf{x}_{-i}^*\right)$, if $U_j(\mathbf{x}^*) > 0 \forall j$. For any other equilibrium candidate, i could always suggest an x_i slightly different from x_i^* without violating (1) if just $\rho_{j,t} > 0 \forall j$. Therefore, with the assumptions added above, the "trivial equilibrium" $\mathbf{x}^* = \mathbf{0}$ is unique. This observation confirms the pessimism associated with pledge and review, as described in the Introduction.

In reality, party i is unlikely to know precisely the condition under which an offer will be accepted. One way of modelling this uncertainty is to assume that the discount rates for the next period are not known (to anyone) at the time at which the offers are made. After all, a country's impatience when it comes to ratifying a treaty may depend on a range of temporary domestic policy or economy issues. To capture this, write $\rho_{i,t} = \theta_{i,t}\rho_i$, where ρ_i is i's expected discount rate while $\theta_{i,t}$ is a shock with mean 1. The shocks are jointly distributed with pdf $f(\theta_{1,t}, ..., \theta_{n,t})$ on support $\prod_i \left[0, \overline{\theta}_i\right]$, i.i.d. at each time t, and the marginal distribution of $\theta_{i,t}$ is $f_i(\theta_{i,t}) = \int_{\Theta_{-i}} f(\theta_{1,t}, ..., \theta_{n,t})$, where $\Theta_{-i} \equiv \prod_{j \neq i} \left[0, \overline{\theta}_j\right]$. The $\theta_{i,t}$'s are realized and observed by everyone after the offers but before acceptance decisions are made.[5]

After learning $\theta_{i,t}$, i accepts $\mathbf{x}$ if and only if:

$$U_i(\mathbf{x}) \geq \left(1 - \theta_{i,t}\rho_i\Delta\right) U_i(\mathbf{x}^*) \Rightarrow \theta_{i,t} \geq \frac{U_i(\mathbf{x}^*) - U_i(\mathbf{x})}{\rho_i \Delta U_i(\mathbf{x}^*)}. \tag{2}$$

When $\theta_{i,t}$ is drawn from a continuous distribution, the probability that i accepts will be continuous in x_i. As the following result will show, this continuity can motivate larger contributions: $\mathbf{x}^*$ can be sustained as a "nontrivial" stationary SPE if the marginal benefit for i by reducing x_i slightly is outweighed by the risk that at least one party might be sufficiently patient to decline the offer and wait for $\mathbf{x}^*$.

Theorem 1. *If $\mathbf{x}^*$ is a nontrivial stationary SPE in which $U_i(\mathbf{x}^*) > 0 \forall i$, then, for every $i \in N$:*

$$x_i^* \leq \arg\max_{x_i} \prod_{j \in N} \left(U_j\left(x_i, \mathbf{x}_{-i}^*\right)\right)^{w_j^i}, \text{ where } \frac{w_j^i}{w_i^i} = \frac{\rho_i}{\rho_j} f_j(0) \, \mathrm{E}\left(\theta_{i,t} \mid \theta_{j,t} = 0\right), \forall j \neq i. \tag{3}$$

The colored area in Figure 1 illustrates the set of equilibria when $n = 2$ and $u_i = x_j - cx_i^2/2$. There are multiple equilibria, since the inequality in (3) can be strong: The reasoning above does not limit how

[4] If the real discount rate is $\widetilde{\rho}_{i,t}$, the discount factor is $e^{-\widetilde{\rho}_{i,t}\Delta} = \delta_{i,t}^{\Delta}$, so $\rho_{i,t} = \left(1 - e^{-\widetilde{\rho}_{i,t}\Delta}\right)/\Delta$, which approaches $\widetilde{\rho}_{i,t}$ when $\Delta \to 0$. I thus refer to $\rho_{i,t}$ as the discount rate even though the identity holds only in the limit.

[5] This is not unreasonable: (i) Technically, instead of letting $\Delta > 0$ be the delay between rejections and new offers, Δ can be the delay between offers and acceptance decisions, if we assume that new offers can be made as soon as earlier offers are rejected. (ii) Since there is (then) a lag between offers and acceptance decisions, it is natural that policy makers in the meanwhile learn about how urgent it is for them to conclude the negotiations, or about the attention they instead have to give to other policy and economic issues.

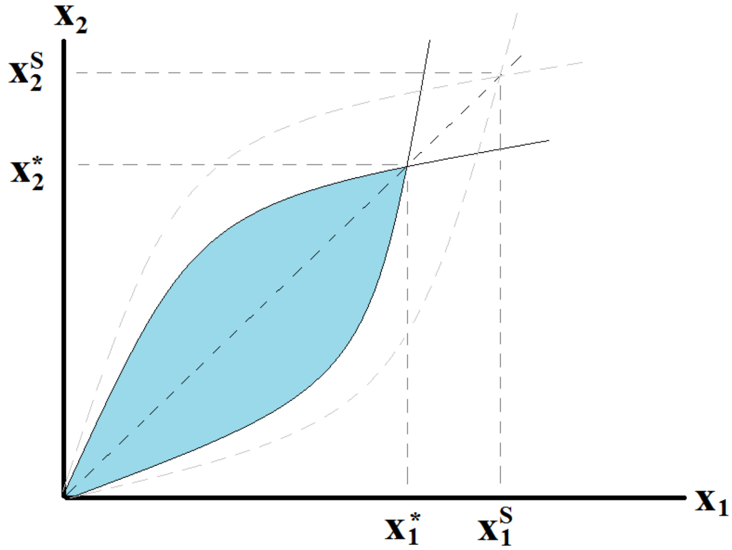

Figure 1: *There are multiple equilibrium contribution levels (unless there are trembles), but they are all smaller than the efficient levels ($\mathbf{x}^S$).*

small the equilibrium x_i^*'s can be, as there is no point for i to contribute more than x_i^*, whatever the equilibrium $\mathbf{x}^*$ is. (The stationary equilibrium $\mathbf{x}^*$ will always be accepted when parties are impatient).

The inequality in (3) must hold with equality if we introduce some small chance that even $\mathbf{x}^*$ will be declined: this will be the consequence of introducing small trembles either in the parties' actions—*or* in the support for the $\theta_{i,t}$'s:

(A1) Assume that when the intended offers are given by $\mathbf{x}$, $\mathbf{x} + \boldsymbol{\epsilon}_t^k$ is realized, where $\boldsymbol{\epsilon}_t^k$ is a vector of n shocks, each i.i.d. over time with mean zero and $\mathrm{E}\left(\boldsymbol{\epsilon}_t^k\right)^2 \to 0$ as $k \to 0$.

Or:

(A2) Assume that the support of $\theta_{j,t}$ is $\left[\underline{\theta}_j^k, \overline{\theta}_j\right]$ (instead of $\left[0, \overline{\theta}_j\right]$) where $\underline{\theta}_j^k < 0$ and $\underline{\theta}_j^k \uparrow 0 \forall j$ as $k \to 0$.[6]

Theorem 2. *If, under either (A1) or (A2), $\mathbf{x}^*(k)$ is a nontrivial stationary SPE, then (3) holds with equality for $x_i^* = \lim_{k\to 0} x_i^*(k)$, for every $i \in N$.*

As a comparison, note that if $\mathbf{x}$ were given by an Asymmetric Nash Bargaining Solution, then $\mathbf{x}$ could be described as:

$$x_i^A = \arg\max_{x_i} \prod_{j\in N} \left(U_j\left(x_i, \mathbf{x}_{-i}^A\right)\right)^{w_j} = \arg\max_{x_i} \sum_j w_j \frac{U_j\left(x_i, \mathbf{x}_{-i}^A\right)}{U_j\left(\mathbf{x}^A\right)}.$$

Here, each x_i^A maximizes the Asymmetric Nash product, and thus a weighted sum of utilities, where the weights w_j's are exogenously given. In this case, the set of x_j^A's will be Pareto optimal.

Also when (3) binds in the pledge-and-review bargaining game, the outcome for x_i^* maximizes an asymmetric Nash product, but the weights vary with i and thus the set of x_i^*'s is not Pareto optimal. In particular, if every $w_j^i/w_i^i < 1$, it is possible to make every party better off by increasing all the contributions relative to $\mathbf{x}^*$.

[6] The interpretation of a negative discount rate may be that, in some circumstances, a party prefers to delay signing agreements due to other urgent economic/policy issues that requires the decision makers' attention. It is required that the lower boundaries, the $\underline{\theta}_j^k$'s, approach zero in the limit (as $k \to 0$), since otherwise there will be delay on the equilibrium path.

The theorems also endogenize the weights and show how they depend on three thing. First, the weights on j's utility is larger if j is expected to be patient relative to i. This is natural (and in line with other bargaining papers, as discussed in the Introduction): When j is patient, j is more tempted to reject an offer that is worse than what one can expect in the next period, and thus i finds it too risky to reduce x_i, especially when i is quite impatient and dislikes delay.

Second, the weight on j's payoff is larger when there is a lot of uncertainty regarding j's shock. Of importance is especially the (marginal) likelihood that j's discount rate is close to 0, so that even a small reduction from x_i^* involves some risk that j will decline.

Third, if the shocks are correlated, then the weight on j's payoff is less for a small $\mathrm{E}(\theta_{i,t} \mid \theta_{j,t}=0)$, which measures i's expected shock (on the discount rate) given that j's shock is small. Intuitively, if i can be expected to have a small discount rate exactly when j has, then it matters less that j declines an offer in this circumstance. When the delay matters less, i does not find it necessary to offer a lot. A party i will therefore pay more attention to the payoffs of those other parties who face shocks that are less correlated with i's shock. In sum, each party pledges to contribute an amount that puts some weight on the utility of other parties, but only to the extent that one is uncertain about the other's willingness to accept.

The theorems provide several important corollaries:

(1) If $f_j(\cdot)$ is single-peaked and symmetric, then clearly $f_j(0) \leq 1/2$.[7] In this situation, the weights on other parties is less than 1/2 of the weight on i when x_i is proposed, if discount rates are equal and shocks not negatively correlated.

(2) If uncertainty vanishes, such that the pdf $f_j(.)$ concentrates around its mean, $f_j(0) \to 0$ and x_i^* must approach the level in the trivial equilibrium as when there is no bargaining.

(3) In the completely symmetric case (with symmetric discount rates and utility functions), Theorem 2 predicts that the nontrivial x_i^*'s are simply given by:

$$x_i^* = \arg\max_{x_i} U_i\left(x_i, \mathbf{x}_{-i}^*\right) + w \sum_{j \neq i} U_j\left(x_i, \mathbf{x}_{-i}^*\right), \tag{4}$$

where $w = f_j(0)\mathrm{E}(\theta_{i,t} \mid \theta_{j,t}=0)\, \forall i, j$.

These observations are in stark contrast to the symmetric Nash Bargaining Solution, predicting that the x_i's would follow from (4) with $w = 1$, as illustrated by x_1^S and x_2^S in Figure 1 for the example in which $n = 2$ and $u_i = x_j - cx_i^2/2$.

For this example, it is also easy to check that all equilibria satisfying (3) with strict inequalities are Pareto dominated by the equilibrium where the inequalities bind if just $w < \sqrt{3} - 1 \approx 0,73$. Thus, focusing on equilibria that are not Pareto dominated can in some cases replace assumptions (A1) and (A2).[8]

Remark on generality. Above, it was assumed that $\partial U_i(\cdot)/\partial x_i < 0$ and $\partial U_j(\cdot)/\partial x_i > 0$, $j \neq i$. Although these assumptions simplify the expression of Theorem 1, a more general version of Theorem 1 is stated and proven in the Appendix and the additional assumptions are not needed for Theorem 2. Further, if the trivial equilibrium, $\mathbf{x}^b$, characterized by $x_i^b = \arg\max_{x_i} U_i\left(x_i; \mathbf{x}_{-i}^b\right) \forall i$, is such that $U_i\left(\mathbf{x}^b\right) > 0$ for some i, then it ceases to exist with assumptions (A1) or (A2) and, therefore, the word "nontrivial" in Theorem 2 is, in this case, redundant.

Remark on sufficiency. Condition (3) is necessary for $\mathbf{x}^*$ to be an equilibrium, but it may not be sufficient. Whether the second-order condition for an optimal deviation x_i^i holds globally depends on the pdf f. If $n = 2$, a sufficient condition for the second-order condition to hold is that f_j is weakly increasing, as when $\theta_{j,t}$ is uniformly distributed, for example.

3 Application to International Treaties

To better understand the implications of pledge-and-review, this section embeds the above bargaining solution into a simple but reasonable climate change model. For a start, the set of participants is taken as

[7] To see this, note that if $f_j(0) > 1/2$, then, when $f_j(\cdot)$ is single-peaked and symmetric around the mean of one, $\int_0^2 f_j(\theta_j)\, d\theta_j > 1$, which is impossible for a pdf $f_j(\cdot)$.

[8] I thank Asher Wolinsky for making this observation.

exogenous, but this set is endogenized in Section 4. Section 5 compares the outcome of pledge-and-review to the outcome under a more traditional (conditional offer) bargaining game, applicable to the Kyoto agreement, in order to determine when pledge-and-review is preferred.

3.1 A Climate Change Model

In a reasonable climate change model, the parties (i.e., countries) will over time emit as well as invest in new technology. Chapter 16 of the Stern Review (2007) pointed out that new technology would be crucial to mitigate climate change. At the same time, §114 of the 2010 Cancun Agreement states that "technology needs must be nationally determined, based on national circumstance and priorities." The 2015 Paris Agreement follows this tradition of letting countries decide on technology themselves. Thus, to be consistent with past and present climate change negotiations, levels of emissions, or emission cuts, are here assumed to be negotiable (and contractible), while technology investments are not.

Equilibrium investments will depend on the negotiated levels of abatement, since the technology level is assumed to be a strategic substitute to emitting. Thus, the technology can be interpreted as environmentally friendly or "green" technology, such as the stock of renewable energy sources (windmills, solar technology, etc.).[9]

The per-period utility is assumed to be concave in a party's level of energy consumption. The energy consumption consists of energy from fossil fuels, $g_{i,t}$, plus the energy from the stock of renewable energy sources, $Y_{i,t}$. The bliss level of energy consumption is given by some constant, $\overline{Y}_i$. Both the constant $\overline{Y}_i$ and the initial technology level, $Y_{i,0}$, may differ between the parties. Fossil fuel leads to greenhouse gas emission, whose marginal cost is given by parameter c, and k measures the marginal cost of investing in renewable energy sources:

$$\widetilde{u}_{i,t} = -\frac{b}{2}\left[\overline{Y}_i - (g_{i,t} + Y_{i,t})\right]^2 - c\sum_{j\in N} g_{j,t} - ky_{i,t}. \tag{5}$$

The technology stock $Y_{i,t}$ depends on the stock at time $t-1$, plus investments made at time $t-\Delta_y$, meaning that the time it takes to develop new technology is $\Delta_y \geq 1$:

$$Y_{i,t} = Y_{i,t-1} + y_{i,t-\Delta_y}.$$

It is easy to see that the noncooperative MPE, without bargaining, referred to as the "business as usual" equilibrium, implies that the emission level is $g_i^{bau} = \overline{Y}_i - Y_{i,t} - c/b$ and $y_{i,t} = 0$ if $k > c\delta^{\Delta_y}/(1-\delta)$, which is henceforth assumed.[10] Thus, by measuring emission cuts relative to the business as usual level, $x_i \equiv g_i^{bau} - g_i$, we can write:

$$u_{i,t} \equiv \widetilde{u}_{i,t} - \widetilde{u}_{i,t}^{bau} = c\sum_j x_j - \frac{b}{2}\left(x_i + \frac{c}{b} - \sum_{\tau=0}^{t-\Delta_y} y_{i,\tau}\right)^2 + \frac{c^2}{2b} - ky_{i,t}. \tag{6}$$

The timing is thus the following. First, the n parties negotiate cuts in the emission levels, as described in the above pledge-and-review game. Thereafter, in every period each party i decides on its investment level, $y_{i,t}$, and emission levels subject to the negotiated cap. The length of the commitment period is referred to as T, and the commitment x_i puts a constant cap on g until T. After T, the parties bargain according to pledge-and-review, once again. The optimal T is described below.

9 Party i's per-period utility is measured by $\widetilde{u}_{i,t}$ and i seeks to maximize $\sum_{t=0}^{\infty} \delta^t \widetilde{u}_{i,t}$ at time 0. Thus, even if a party's impatience was allowed to be stochastic and uncertain during the bargaining process (this was assumed to make the parties' acceptance decisions uncertain), I henceforth assume the parties apply the same constant and deterministic (expected) discount factor when they decide on the long-lasting investment levels. This is natural, since the uncertainty in the willingness to accept bargaining offers could be related to policy makers' need to give attention to other urgent policy or economic issues. These shocks cannot be predicted in advance and for long-term technology investments, one will apply the expected discount factors (since the utility is linear in the discount factor, and there is no risk aversion w.r.t. them).

10 If instead $k < c\delta^{\Delta_y}/(1-\delta)$, there would be no limit to how much each party would like to invest since both the investment cost and the benefit of abatements are linear functions. These linearities do not create any problem in the present analysis, however, as long as $k > c\delta^{\Delta_y}/(1-\delta)$.

Remark on pollution stock: This model can easily allow for environmental harm caused by a stock of greenhouse gases instead of by per-period emissions. Suppose the pollution stock is $G_t = q_G G_{t-1} + \sum_j g_{j,t}$, depreciating at rate $1 - q_G \in [0, 1]$. If parameter $C > 0$ measures the per-period marginal environmental harm from the stock G_t, then the per-period utility can be written exactly as in (5) when $c \equiv C/(1 - q_G \delta)$ represents the present-discounted cost of emitting one more unit into the atmosphere.

Other extensions: Section 5 discusses how the model easily can be extended to situations in which investment costs are nonlinear, the technology is "brown" (complementary to emissions rather that being a substitute), technology depreciates over time, investments are made by firms rather than governments, and if the negotiated contribution levels are allowed to be functions of time.

3.2 Equilibrium investments

Before describing the equilibrium x_i's, it is useful to derive the equilibrium investment levels as a function of these commitments. Clearly, negotiating an emission cut $x_i \equiv \overline{Y}_i - Y_{i,0} - c/b - g_i$ is equivalent to negotiating an emission cap g_i, once $Y_{i,0}$ is sunk. Since every party has the same utility function (6) as a function of x_i and of the $y_{i,t}$'s, the parties have symmetric continuation value functions, $U_i(x_i, \mathbf{x}_{-i}) = \sum_{t=0}^{\infty} \delta^t u_i$, and we can therefore apply (4), which followed as a corollary from Theorem 2. Consequently, the equilibrium x_i's will be the same for every i and it will also be independent of $Y_{i,0}$. A larger $Y_{i,0}$ is simply having the effect that i's emission level will be lower and the marginal present discounted value of this is $c/(1-\delta)$ to each party. Analogously, the terminal value at time T of any investment $y_{i,t}$, $t \in (0, T - \Delta_y)$, will be $cy_{i,t}/(1-\delta) < k/\delta^{\Delta_y}$. After the contribution level x_i has been negotiated, party i thus invests to maximize the present-discounted value:

$$\max_{\{y_{i,t} \geq 0\}_t} \sum_{t=0}^{T-1} \delta^t \left[c \sum_j x_j - \frac{b}{2} \left(x_i + \frac{c}{b} - \sum_{\tau=0}^{t-\Delta_y} y_{i,\tau} \right)^2 + \frac{c^2}{2b} - k y_{i,t} \right] + \delta^T \frac{c y_{i,t}}{1-\delta}.$$

This optimal control theory problem has the simple solution that party i invests only in the very beginning of the commitment period:

$$y_{i,0} = \max \left\{ 0, x_i - \frac{(1-\delta) k - \delta^{\Delta_y} c}{b \left(\delta^{\Delta_y} - \delta^T \right)} \right\}. \tag{7}$$

Investments are thus larger if c is large and k is small and, importantly, when the pledge x_i is large. Thus, larger negotiated contributions lead to larger investments and this has an effect that outlives the commitment period, which itself ends at time T. Of course, these equilibrium investment levels are taken into account when the x_i's are negotiated.

3.3 Equilibrium contributions

The positive externality of one party's contribution level on another is simply $c/(1-\delta)$. The length of the commitment period is unimportant for this externality because, as implied by (7), a larger contribution level has a long-lasting effect through the investment level in technology. A party's cost of pledging contributions are convex in $x_{i,t}$, however, and it equals $bx_i + c$ until time Δ_y, when the new technology has been developed. Therefore:

$$\begin{aligned} \frac{\partial U_{j,t}}{\partial x_i} &= \frac{c}{1-\delta} \text{ and} \\ \frac{\partial U_{i,t}}{\partial x_i} &= \frac{c}{1-\delta} - \frac{1-\delta^{\Delta_y}}{1-\delta} (bx_i + c) - k. \end{aligned}$$

According to the Theorem above, pledge-and-review bargaining implies that i proposes x_i while placing weight w on every other party's utility. This implies that:

$$\begin{aligned}
&\frac{\partial U_{i,t}}{\partial x_i}+w\sum_{j\setminus i}\frac{\partial U_{j,t}}{\partial x_i}=0\Rightarrow \\
&\frac{c}{1-\delta}-\frac{1-\delta^{\Delta_y}}{1-\delta}\left(bx_i+c\right)-k+w\left(n-1\right)\frac{c}{1-\delta}=0\Rightarrow \\
&x_i=\frac{\left(n-1\right)w+\delta^{\Delta_y}}{1-\delta^{\Delta_y}}\frac{c}{b}-\frac{1-\delta}{1-\delta^{\Delta_y}}\frac{k}{b}. \qquad (8)
\end{aligned}$$

Consequently, a smaller w implies smaller contributions and this, in turn, reduces investments in green technology.

3.4 Commitment period

The larger is the length of the commitment period, T, the larger are the equilibrium investments, according to (7). The intuition for this result is the standard hold-up problem: If the next bargaining stage (at T) is close, then each party invests less because the other parties will expect larger contributions from a party when this is inexpensive. The hold-up problem is thus an argument in favor of a larger T (as in Harstad, 2016).

After the investments are sunk, however, it would be better for all the parties to start the pledge-and-review bargaining game again, to take advantage of the newly developed technology. When T is committed to in advance, the optimal T trades off the positive effect on investments and the benefit that newly developed technology can strengthen the commitments sooner when T is small. The benefit of reducing T is therefore smaller when technological investments are unimportant: the optimal T is thus larger when w is small, it can be shown [proof to be added].

Suppose the parties determine T before making the pledges. Since the parties are symmetric when it comes to the $x_{i,t}$'s and the investment costs, they all agree on the optimal T, and the negotiation game when it comes to T is straightforward.

Proposition 1. *(Consequences of pledge-and-review bargaining) A smaller w reduces contributions, investments, and welfare, but increases the length of the commitment period.*

4 Endogenous Participation

While the number of participants has so far been taken as exogenous, this section endogenizes the coalition size. The standard way of endogenizing the coalition size is to follow the approach developed by Carlo Carraro, Scott Barrett, etc. [Literature review to be added.] In this literature, the game begins with a participation stage at which every potential party decides whether or not to participate in the coalition. These decisions are made simultaneously and everyone expects that the coalition that forms will play the game as described above.

After the coalition has been formed, the free-riders will simply follow their business-as-usual strategy, while the coalition members will contribute according to (8). Since the coalition members will contribute more than the level that would maximize their own utility, there is a cost of participating in the coalition, and this cost must be smaller than the benefit of participating for a member to be willing to participate. The benefit of participating is that the other participants will take into account (some of) the utility of a larger number of coalition members. Thus, i finds it attractive to participate if and only if:[11]

$$\frac{b}{2}\frac{1-\delta^{\Delta_y}}{1-\delta}\left[\left(x_i+\frac{c}{b}-y_{i,0}\right)^2-\frac{c^2}{b^2}\right]+ky_{i,0}-\frac{c}{1-\delta}x_i\leq\frac{c}{1-\delta}\frac{\left(n-1\right)w}{1-\delta^{\Delta_y}}\frac{c}{b}.$$

[11] Here it is assumed that a party takes T as given when deciding on whether to participate. This holds if n and T are required to be best responses to each other, i.e., they are decided on at the same time, before the pledges are negotiated. In Battaglini and Harstad (2016), however, the parties commit to participate or free-ride before T is decided on.

So, this inequality must hold for the equilibrium n, referred to as $n(w)$, but the inequality must fail for $n(w)+1$. Since the equilibrium x_i is linear in n, and since i faces a convex cost of contributing, it is less attractive to participate when n is large. There is therefore a unique $n(w)$ such that i prefers to participate when $n < n(w)$ but not when $n > n(w)$.

If w is smaller, x_i is smaller, and it is less costly to participate in the coalition. The equilibrium $n(w)$ is thus a decreasing function of w, if there are no constraints on n. In this case, the product $(n(w)-1)w$ must stay constant if w varies. To see this, note that at the equilibrium coalition size, a party is indifferent between whether to participate or free-ride. For a constant $(n-1)w$, x_i is constant and thus the cost of participating remains unchanged. The benefit of participating is that the $n-1$ other parties will increase their x_j's by an amount that is proportional to w if i participates. Thus, for a constant $(n-1)w$, the benefit as well a the cost stays constant, and thus a party remains indifferent between whether or not to participate.

Since $(n-1)w$ stays constant as w is reduced, x_i is also constant, and so is $y_{i,t}$ (conditional on T). Since the individual contributions are invariant in w while the coalition size is larger, overall welfare will be larger when w is small. For these reasons, Proposition 1 is overturned.

Proposition 2. *The equilibrium coalition size $n(w)$ decreases in w and $(n(w)-1)w$ is invariant in w. Thus, when n is endogenous, Proposition 1 is reversed: A smaller w increases aggregate contributions, investments, and welfare, but reduces the length of the commitment period.*

[Proof to be added.]

5 Choice of Bargaining Game: From Kyoto to Paris

The previous section showed that pledge-and-review, characterized by a small w, can be better than traditional conditional-offer bargaining because the coalition size is larger when w is smaller. As argued above, conditional-offer bargaining, as permitted under the Kyoto Protocol, is likely to implement the (symmetric) Nash Bargaining Solution, i.e., the situation in which $w = 1$. This is better than pledge-and-review when n is taken as exogenously given, but not when n is endogenous, according to the results above. In the present model, a smaller w is always better since the larger n more than compensates for the smaller weight each party puts on another party's payoff.

In reality, there are several reasons for why n does not increase so fast, when w declines, that $(n-1)w$ remains unchanged. First, the world consists of a finite number of countries, $\overline{n}$. When w is so small that everyone participates, $\overline{n} \leq n(w)$, then a further reduction in w is harmful for everyone, just as described by Proposition 1. According to this argument, overall welfare (and aggregate contributions and investments) is single-peaked in w and maximized when everyone is just willing to participate: $\overline{n} = n(w)$.

Second, countries are more heterogeneous in reality than the model above has permitted. If the willingness to participate had varied across countries, the countries that would benefit most from participating would be the first ones to participate if w is reduced. Countries that benefit less from participating would not be indifferent and a significant reduction in w would therefore be necessary for them to be willing to participate. Thus, $(n-1)w$ will increase in w when countries are heterogeneous. Also in this case, there will be a real trade-off when choosing between a deep-and-narrow agreement (where w is large and n is small) or a shallow-but-broad agreement (where w is small but n is large).

There are several ways of letting the parties be heterogeneous so that the trade-off above can be captured in the model. The most straightforward way is to simply let a set of parties, $\underline{N}$, with number $\underline{n} = |\underline{N}| \leq \overline{n}$, be committed to participate regardless of w. The reason for why these countries are committed can be outside of the model, but one may think off existing international treaties on non-climate issues such as international trade or regulatory politics. In particular, the countries in the European Union have proven to be committed to climate negotiations regardless of the treaty details. Clearly, when exactly $\underline{n}$ participates, because $n(w) \leq \underline{n}$, everyone's welfare is higher if w is larger.

It holds also more generally that pledge-and-review, associated with a smaller w, is preferred if and only if $\underline{n}$ is small while $\overline{n}$ is large.

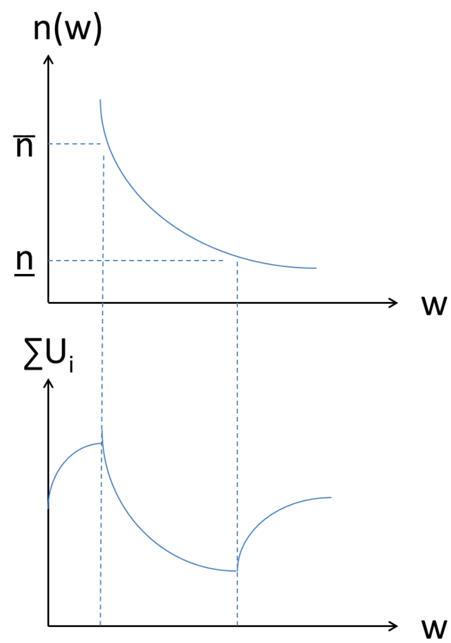

Figure 2: *n and $\sum U_i$ are strictly decreasing in w only when $n(w) \in (\underline{n}, \overline{n})$.*

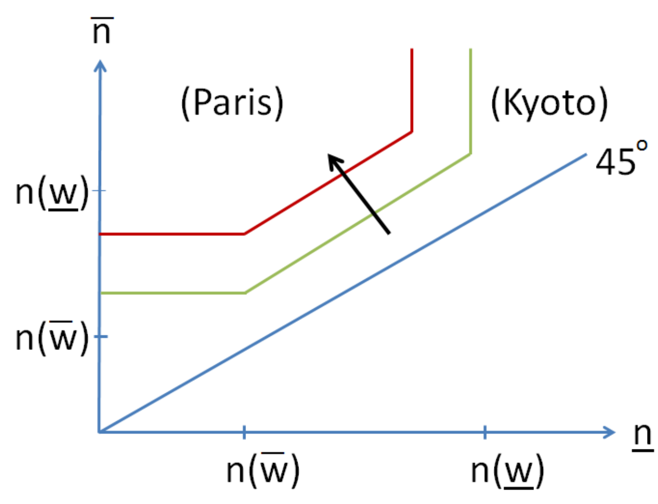

Figure 3: *Committed parties prefer pledge-and-review above the green line; uncommitted countries prefer pledge-and-review only above the red line.*

Proposition 3. *Consider a "shallow" agreement characterized by $\underline{w}$, a "deep" agreement characterized by $\overline{w} > \underline{w}$, and an arbitrary subgroup $S \subset N$.*

(1) S prefers the shallow to the deep agreement if $\overline{n}$ is large while $\underline{n}$ is small:

(i) If $\underline{n} < n(\overline{w}) < n(\underline{w}) \leq \overline{n}$, then the shallow agreement is always preferred by every group $S \subset N$.

(ii) If $\overline{n} < n(\overline{w})$ or $n(\underline{w}) < \underline{n}$, then the deep agreement is always preferred by every group $S \subset N$.

(iii) If $n(\overline{w}) < \underline{n} < n(\underline{w}) \leq \overline{n}$, then there exists a constant $\underline{n}_S$ such that the shallow agreement is preferred by group $S \subset N$ if and only if $\underline{n} \leq \underline{n}_S$.

(iv) If $\underline{n} < n(\overline{w}) \leq \overline{n} < n(\underline{w})$, then there exists a constant $\overline{n}_S$ such that the shallow agreement is preferred by group $S \subset N$ if and only if $\overline{n} \geq \overline{n}_S$.

(v) If $n(\overline{w}) < \underline{n} \leq \overline{n} < n(\underline{w})$, then there exists a linearly increasing function f_S such that the shallow agreement is preferred by group $S \subset N$ if and only if $\overline{n} \geq f_S(\underline{n})$.

(2) Furthermore, $\underline{N}$ is more likely to prefer the shallow agreement than is $\overline{N} \equiv N \backslash \underline{N}$: $\overline{n}_{\underline{N}} < \overline{n}_{\overline{N}}$, $f_{\underline{N}}(\cdot) < f_{\overline{N}}(\cdot)$, and $\underline{n}_{\overline{N}} > \underline{n}_{\underline{N}}$.

[Explicit equations for the thresholds, and proofs, can and will be added in the next draft.]

A figure can illustrate part (1), (i)-(v), for an arbitrary group $S \subset N$. If there is a larger number of potential parties in the world, or if fewer countries are committed to participate even when w is large, then we may move in the direction of the arrow in the figure, so that the "shallow" agreement becomes preferred even though the "deep" agreement was preferred for a smaller number of potential parties, or for a larger number of committed parties.

One may argue that both these developments are in line with changes in world politics the last few decades. Today we have a larger number of countries that are emerging economies, although they up to recently were developing countries that could not be expected to contribute (much) to an international climate change treaty. During the same period, eight of the original Annex I countries, who initially signed the Kyoto Protocol, announced that they would not make commitments under the Kyoto Protocol's second commitment periods: Belarus, Kazakhstan, Ukraine, Japan, New Zealand, Russia, Canada, and USA. The switch from a deep conditional-offer type of treaty to pledge-and-review is thus consistent with the theory of this paper.

Part (2) of Proposition 3 states that the committed countries prefer to switch to a shallow agreement for a larger set of parameters than do the uncommitted countries. In fact, even if the switch to pledge-and-review reduces aggregate contributions, so that the new members are clearly worse off, the original set of members might prefer to switch to pledge-and-review because the cost of contributing is spread across a larger set of parties. This result implies that if the original set of committed parties decide on the (next) type of treaty, they may switch to a shallow agreement even if a deep agreement would have been better if the uncommitted parties' payoffs were taken into account. Similarly, if it is the new potential members who are pivotal in the decision on treaty design, they will accept pledge-and-review too late, or too seldom, relative to the decision that would be optimal if the committed members' payoffs had been taken into account.

6 Extensions

The model above is simple and can [and will] be extended in a number of directions. With the exception of the results describing how T is affected by w, all the above results continue to hold under the following extensions.

Gradual investments with nonlinear costs. When the investment cost is linear in the invested level, as assumed above, then a party will invest immediately and once and for all, as soon as a contribution pledge is made. If the cost of investment were convex in the level of investment, however, then a party would prefer to smooth out investments over time. If the investment cost function is quadratic, then it is easy to show that investments (at every point in time) will be linear in the pledged contribution, and the equilibrium contribution will be linear in w, just as above. Also in this model, $(n-1)w$ stays unchanged as w changes when participation is endogenous. All the above results continue to hold, with the exception that I have been unable to determine how the optimal length of the commitment period varies with w.

Brown technology. The technology level above was referred to as "green," since more investments reduced the cost of abatements. It is straightforward to instead include a "brown" type of technology, that is complementary to emissions of greenhouse gases. Such a technology would be a strategic substitute to contributions to abatements, and one could assume that a party could downsize the stock of brown technology by $y_{i,t}$ at some cost. With this, the analysis above will essentially be unchanged if $Y_{i,t}$ is interpreted as cumulated reductions in the technology stock.

Committing to a path of contributions. Above, it has been assumed that the parties pledge contributions that stay constant until time T, when new pledges are negotiated. If, instead, parties negotiated a path of contribution levels, then there would be no reason to reduce T and the optimal T would be infinite.[12] Of course, the optimal T would stay finite if one introduced, for example, uncertainties regarding the future optimal abatement level (as in Harstad, 2016).

Firms investing. If firms invest in technology rather than countries, and if countries are unable to regulate the firms' investments, then all results continue to hold except that the optimal commitment period length is reduced to one. The reason for this is that firms would not invest less when the next bargaining game is close in time, since it would be countries and not firms that would be subject to the hold-up problem described above. However, if each government can subsidize/tax the firms' investments, then the government can implement its preferred choice of investment, as described in Section 3, and all the results above stay unchanged.

7 Concluding Remarks

This paper provides an analysis of a pledge-and-review bargaining game. The novelty of this bargaining game is that each party proposes how much to contribute to the agreement, unconditional on what other parties pledge, before the vector of contributions must be agreed to by all the parties. If there is some uncertainty regarding what other parties are willing to accept, for example due to shocks in the short-term discount rate, then contributions will be larger if there is a substantial variance in these shocks. It has been shown that each party's contribution level is as described by an asymmetric Nash Bargaining Solution. Since the weights vary from pledge to pledge, the collection of pledges is not Pareto optimal.

This pledge-and-review game has been associated with the 2015 Paris Agreement on climate change. Not surprisingly, this system has been criticized because it neither requires nor motivates parties to contribute significantly to abatements. As a consequence, abatement levels will be small and so will investments in new and "green" technology, and thus the optimal commitment period is long since it is not important to update the agreement soon when the level of technological progress is slow. These conclusions are reversed when the coalition size is endogenous: then, the shallowness of pledge-and-review makes it affordable for a larger number of countries to participate, and this effect dominates the fact that each party puts a smaller weight on the utility of another party. These results fit well with the Paris agreement: Compared to the Kyoto Protocol, the number of signatories is much larger, and the length of the commitment period (5 years) is smaller than the last commitment period of the Kyoto Protocol (2012-2020).

By comparison, pledge-and-review is preferred in this model if the number of potential parties is large while relatively few parties are committed to participate regardless of the treaty details. Compared to the 1990s, there is today a larger number of economies that can be serious contributors to a climate agreement, at the same time as there are fewer countries that commit to participate regardless of the treaty design. The developments since the 1990s can thus rationalize the fact that a conditional-offer type of agreement was signed in 1997 (i.e., the Kyoto Protocol) while in 2015 the world community preferred the more shallow pledge-and-review system.[13]

[12] Such a model would require the investment cost function to be convex. If the investment cost function were linear, and contributions could be functions of time, one would get a bang-bang result in that it would be optimal to commit to infinitely large contribution levels.

[13] While the theory above can rationalize the development from Kyoto to Paris, the results also indicates that the switch from a narrow-and-broad to a shallow-and-broad system takes place too often or too early, if the switch is decided on by the original participants, while it takes place too seldom or too late if the new members are pivotal when deciding on the system.

8 Appendix

Proof of Theorem 1.

As advertised in Section 2, the following version of Theorem 1 is here proven without the additional assumptions $\partial U_i(\cdot)/\partial x_i < 0$ for $x_i > 0$, and $\partial U_j(\cdot)/\partial x_i > 0$, $j \neq i$.

Theorem 1^G. *If x^* is a nontrivial stationary SPE in which $U_i(\mathbf{x}^*) > 0 \forall i$, then, for every $i \in N$, we have:*

(i) if $\frac{\partial U_i(\mathbf{x}^*)}{\partial x_i} \leq 0$,

$$-\frac{\partial U_i(\mathbf{x}^*)}{\partial x_i} \leq \sum_{j \setminus i} \max\left\{0, \frac{\partial U_j(\mathbf{x}^*)/\partial x_i}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\} f_j(0)\,\mathrm{E}(\theta_{i,t} \mid \theta_{j,t} = 0)\,\rho_i \Delta U_i(\mathbf{x}^*); \tag{9}$$

(ii) if $\frac{\partial U_i(\mathbf{x}^*)}{\partial x_i} > 0$,

$$\frac{\partial U_i(\mathbf{x}^*)}{\partial x_i} \leq \sum_{j \setminus i} \max\left\{0, -\frac{\partial U_j(\mathbf{x}^*)/\partial x_i}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\} f_j(0)\,\mathrm{E}(\theta_{i,t} \mid \theta_{j,t} = 0)\,\rho_i \Delta U_i(\mathbf{x}^*).$$

Note that with the additional assumptions $\partial U_i(\cdot)/\partial x_i < 0$ for $x_i > 0$, and $\partial U_j(\cdot)/\partial x_i > 0$, $j \neq i$, the first-order condition of (3) is equivalent to (9).

(i) First, note that in any stationary SPE we must have $U_i(\mathbf{x}^*) \geq 0 \forall i$, since otherwise a party with $U_i(\mathbf{x}^*) < 0$ would reject $\mathbf{x}^*$ in order to obtain the default payoff, normalized to zero. We will search for nontrivial equilibria in which $U_i(\mathbf{x}^*) > 0 \forall i$.

A stationary equilibrium $\mathbf{x}^*$, such that $U_j(\mathbf{x}^*) > 0 \forall j$, is accepted with probability 1 when $\rho_{j,t} \geq 0$. Therefore, i will never offer $x_i > x_i^*$ when $\frac{\partial U_i(\mathbf{x}^*)}{\partial x_i} \leq 0$, so to check when $\mathbf{x}^*$ is an equilibrium, it is sufficient to consider a deviation by i, $\mathbf{x}^i$, such that $x_i^i < x_i^*$ while $x_j^i = x_j^*$, $j \neq i$.

Acceptable offers. Let $p(\mathbf{x}^i; \mathbf{x}^*)$ be the probability that at least one $j \neq i$ rejects $\mathbf{x}^i$, and $p_{-j}(\mathbf{x}^i; \mathbf{x}^*)$ the probability that at least one party other than j and i rejects $\mathbf{x}^i$.

Since party j's discount factor is $\delta_{j,t}^{\Delta} \equiv 1 - \rho_{j,t}\Delta = 1 - \theta_{j,t}\rho_j\Delta$, $j \neq i$ rejects $\mathbf{x}^i$ iff:

$$\begin{aligned}
\left(1 - p_{-j}(\mathbf{x}^i)\right) U_j(\mathbf{x}^i) + p_{-j}(\mathbf{x}^i)\left(1 - \rho_{j,t}\Delta\right) U_j(\mathbf{x}^*) \quad &< \quad \left(1 - \rho_{j,t}\Delta\right) U_j(\mathbf{x}^*) \Rightarrow \\
\theta_{j,t} < \widetilde{\theta}_j(\mathbf{x}^i) \equiv \max\left\{0, \frac{U_j(\mathbf{x}^*) - U_j(\mathbf{x}^i)}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\}&.
\end{aligned}$$

When the joint pdf of shocks $\boldsymbol{\theta}_t = (\theta_{1,t}, ..., \theta_{n,t})$ is represented by $f(\boldsymbol{\theta}_t)$, the probability that every $j \neq i$ accepts $\mathbf{x}^i$ can be written as:

$$\begin{aligned}
1 - p(\mathbf{x}^i) &= G\left(\widetilde{\theta}_1(\mathbf{x}^i), ..., \widetilde{\theta}_{i-1}(\mathbf{x}^i), \widetilde{\theta}_{i+1}(\mathbf{x}^i), ...\widetilde{\theta}_n(\mathbf{x}^i)\right) \\
&\equiv \int_0^{\overline{\theta}_i} \left[\int_{\widetilde{\theta}_1(\mathbf{x}^i)}^{\overline{\theta}_1} ... \int_{\widetilde{\theta}_{i-1}(\mathbf{x}^i)}^{\overline{\theta}_{i-1}} \int_{\widetilde{\theta}_{i+1}(\mathbf{x}^i)}^{\overline{\theta}_{i+1}} ... \int_{\widetilde{\theta}_n(\mathbf{x}^i)}^{\overline{\theta}_n} f(\boldsymbol{\theta}_t)\, d\boldsymbol{\theta}_{-i,t}\right] d\theta_i,
\end{aligned}$$

which is a function of $n-1$ thresholds. By taking the derivative w.r.t. x_i^i and using the chain rule,

$$-\frac{\partial p(\mathbf{x}^i)}{\partial x_i} = \sum_{j \setminus i} -\max\left\{0, \frac{\partial U_j(\mathbf{x}^i)/\partial x_i}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\} G_j'\left(\widetilde{\theta}_1(\mathbf{x}^i), ..., \widetilde{\theta}_{i-1}(\mathbf{x}^i), \widetilde{\theta}_{i+1}(\mathbf{x}^i), ...\widetilde{\theta}_n(\mathbf{x}^i)\right), \tag{10}$$

and, at the equilibrium, $\mathbf{x}^i = \mathbf{x}^*$,

$$\frac{\partial p(\mathbf{x}^*)}{\partial x_i} = \sum_{j \setminus i} \max\left\{0, \frac{\partial U_j(\mathbf{x}^*)/\partial x_i}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\} G_j'(\mathbf{0}) = -\sum_{j \setminus i} \max\left\{0, \frac{\partial U_j(\mathbf{x}^*)/\partial x_i}{\rho_j \Delta U_j(\mathbf{x}^*)}\right\} f_j(0), \tag{11}$$

where, as written in the text already, $f_j(0)$ is the marginal distribution of $\theta_{j,t}$ at $\theta_{j,t}=0$.

Equilibrium offers. When proposing x_i, party i's problem is to choose $x_i \leq x_i^*$ so as to maximize

$$\left(1-p\left(\mathbf{x}^i\right)\right)U_i\left(\mathbf{x}^i\right)+p\left(\mathbf{x}^i\right)\left(1-\mathrm{E}\theta_{i,t}^R\rho_i\Delta\right)U_i\left(\mathbf{x}^*\right), \tag{12}$$

where $\mathrm{E}\theta_{i,t}^R$ is the expected $\theta_{i,t}$ conditional on being rejected (this will be more precise below).

To derive the first-order condition w.r.t. x_i^i, suppose i considers a small (marginal) reduction in x_i relative to x_i^*, given by $dx_i = x_i^i - x_i^* < 0$. If accepted, this gives i utility $U_i\left(\mathbf{x}^i\right) \approx U_i\left(\mathbf{x}^*\right)+dx_i\partial U_i\left(\mathbf{x}^*\right)/\partial x_i > U_i\left(\mathbf{x}^*\right)$, but it is rejected with probability

$$\frac{\partial p\left(\mathbf{x}^*\right)}{\partial x_i}dx_i = -\sum_{j\setminus i}\max\left\{0,\frac{\partial U_j\left(\mathbf{x}^*\right)/\partial x_i}{\rho_j\Delta U_j\left(\mathbf{x}^*\right)}\right\}dx_i f_j(0),$$

where each of the $n-1$ terms represents the probability that $\theta_{j,t}$ is so small that j rejects if x_i is modified by dx_i, i.e., $\Pr\left(\theta_{j,t}\leq\widehat{\theta}_j\right)$ for $\widehat{\theta}_j \equiv \frac{\partial U_j\left(\mathbf{x}^i\right)/\partial x_i}{\rho_j\Delta U_j\left(\mathbf{x}^*\right)}\left|dx_i\right|$. Naturally, the probability that more than one party has such a small shock vanishes when $\left|dx_i\right|\to 0$ since f is assumed to have no mass point.

In combination, the reduction in x_i is not beneficial to i iff:

$$\begin{aligned}
&\left(1-\frac{\partial p\left(x^*\right)}{\partial x_i}dx_i\right)\left(U_i\left(\mathbf{x}^*\right)+dx_i\frac{\partial U_i\left(\mathbf{x}^*\right)}{\partial x_i}\right) \\
-\sum_{j\setminus i}\max\left\{0,\frac{\partial U_j\left(\mathbf{x}^*\right)/\partial x_i}{\rho_j\Delta U_j\left(\mathbf{x}^*\right)}\right\}&dx_i f_j(0)\,U_i\left(\mathbf{x}^*\right)\left(1-E\left(\theta_{it}\mid\theta_{jt}\leq\widehat{\theta}_{j,t}\right)\rho_i\Delta\right)\leq \quad U_i\left(\mathbf{x}^*\right),
\end{aligned} \tag{13}$$

where (11) has been substituted into the second line of (13), and where $\mathrm{E}\left(\theta_{i,t}\mid\theta_{j,t}\leq\widehat{\theta}_j\right)$ follows from Bayes' rule:

$$\mathrm{E}\left(\theta_{i,t}\mid\theta_{j,t}\leq\widehat{\theta}_j\right)\equiv\frac{\int_0^{\widehat{\theta}_j}\int_{\Theta_{-j}}\theta_{i,t}f\left(\boldsymbol{\theta}_t\right)d\theta_j}{\int_0^{\widehat{\theta}_j}\int_{\Theta_{-j}}f\left(\boldsymbol{\theta}_t\right)d\theta_j},\text{ and }\mathrm{E}\left(\theta_{i,t}\mid\theta_{j,t}=0\right)\equiv\lim_{dx_i\uparrow 0}\frac{\int_0^{\widehat{\theta}_j}\int_{\Theta_{-j}}\theta_{i,t}f\left(\boldsymbol{\theta}_t\right)d\theta_j}{\int_0^{\widehat{\theta}_j}\int_{\Theta_{-j}}f\left(\boldsymbol{\theta}_t\right)d\theta_j},$$

and, as already defined, $\Theta_{-j}\equiv\prod_{k\neq j}\left[0,\overline{\theta}_k\right]$ and $\widehat{\theta}_j\equiv\frac{\partial U_j\left(\mathbf{x}^*\right)/\partial x_i}{\rho_j\Delta U_j\left(\mathbf{x}^*\right)}\left|dx_i\right|$.

When both sides of (13) are divided by $\left|dx_i\right|$ and $dx_i\uparrow 0$, (13) can be rewritten as the first-order condition (9).

The proof of part (ii) is analogous and thus omitted. *QED*

Proof of Theorem 2.

(ii) A continuum of $\mathbf{x}^*$'s can satisfy the equilibrium condition in Theorem 1, since it is not necessary for i to improve an offer relative to $\mathbf{x}^*$ when $p\left(\mathbf{x}^*\right)=0$. The idea of assumption (A1) or (A2) is to introduce trembles such that $p\left(\mathbf{x}^*\right)>0$ and thus we must check that i cannot benefit from marginally increasing *or* decreasing x_i^i from x_i^* to reduce $p\left(\mathbf{x}^i\right)$. With (A1) or (A2), i will strictly benefit from $dx_i>0$ when (3) is strict, and thus it must hold with equality at $\mathbf{x}^*$.

(A1) The vector $\boldsymbol{\epsilon}_t$ is i.i.d. over time according to some cdf, $H(\cdot)$. (For simplicity, I omit the superscript k.) When j considers whether to accept $U_j\left(\mathbf{x}^i+\boldsymbol{\epsilon}_t\right)$, j faces the continuation value $V_j\left(\mathbf{x}^*\right)$ by rejecting, where $V_j\left(\mathbf{x}^*\right)$ takes into account that $\mathbf{x}^*$ may be rejected in the future (if the future $\epsilon_{i,t'}$'s are sufficiently small).[14] The shocks, combined with the possibility to reject, imply that $V_j\left(\mathbf{x}^*\right)>0$ even if $U_j\left(\mathbf{x}^*\right)=0$, so there is no need to assume $U_j\left(\mathbf{x}^*\right)>0\forall j$.

[14] It will be the *combination* of the $\epsilon_{i,t}$'s and the $\theta_{j,t}$'s that determines whether j rejects x^*: let $\Phi_A\left(\mathbf{x}^*\right)$ be the set of $\boldsymbol{\epsilon}_{i,t}$'s and $\theta_{j,t}$'s such that every j accepts $\mathbf{x}^*$, while $\Phi_R\left(\mathbf{x}^*\right)$ is the complementary set. We then have $p\left(\mathbf{x}^*\right)=\Pr\left\{(\boldsymbol{\epsilon},\theta)\in\Phi_R\left(\mathbf{x}^*\right)\right\}$ and:

$$V_j\left(\mathbf{x}^*\right)=\mathrm{E}_{\boldsymbol{\epsilon}_{j,t}:(\boldsymbol{\epsilon},\theta)\in\Phi_A\left(\mathbf{x}^*\right)}\left(1-p\left(\mathbf{x}^*\right)\right)U_j\left(\mathbf{x}^*+\boldsymbol{\epsilon}_t\right)+p\left(\mathbf{x}^*\right)V_j\left(\mathbf{x}^*\right)\mathrm{E}_{\theta_{j,t}:(\boldsymbol{\epsilon},\theta)\in\Phi_R\left(\mathbf{x}^*\right)}\left(1-\theta_{j,t}\rho_j\Delta\right),$$

where the two expectations are taken over the set of parameters leading to acceptance vs. rejections, respectively.

With this, party $j \neq i$ rejects $\mathbf{x}^i$ if and only if:

$$\left(1-p_{-j}\left(\mathbf{x}^i\right)\right) U_j\left(\mathbf{x}^i+\boldsymbol{\epsilon}_t\right)+p_{-j}\left(\mathbf{x}^i\right)\left(1-\rho_{j,t}\Delta\right) V_j\left(\mathbf{x}^*\right)<\left(1-\rho_{j,t}\Delta\right) V_j\left(\mathbf{x}^*\right) \Rightarrow$$

$$1-\theta_{j,t}\rho_j\Delta>\frac{U_j\left(\mathbf{x}^i+\boldsymbol{\epsilon}_t\right)}{V_j\left(\mathbf{x}^*\right)} \Rightarrow \theta_{j,t}<\widetilde{\theta}_j\left(\mathbf{x}^i\right) \equiv \frac{V_j\left(\mathbf{x}^*\right)-U_j\left(\mathbf{x}^i+\boldsymbol{\epsilon}_t\right)}{\rho_j\Delta V_j\left(\mathbf{x}^*\right)}.$$

So, the probability that every $j \neq i$ accepts is:

$$\begin{aligned} 1-p\left(\mathbf{x}^i\right) &= \int_{\boldsymbol{\epsilon}} G\left(\widetilde{\theta}_1\left(\mathbf{x}^i\right),...,\widetilde{\theta}_{i-1}\left(\mathbf{x}^i\right),\widetilde{\theta}_{i+1}\left(\mathbf{x}^i\right),...\widetilde{\theta}_n\left(\mathbf{x}^i\right)\right) dH(\boldsymbol{\epsilon}) \\ &\equiv \int_{\boldsymbol{\epsilon}}\int_0^{\overline{\theta}_i}\left[\int_{\widetilde{\theta}_1(\mathbf{x}^i)}^{\overline{\theta}_1}...\int_{\widetilde{\theta}_{i-1}(\mathbf{x}^i)}^{\overline{\theta}_{i-1}}\int_{\widetilde{\theta}_{i+1}(\mathbf{x}^i)}^{\overline{\theta}_{i+1}}...\int_{\widetilde{\theta}_n(\mathbf{x}^i)}^{\overline{\theta}_n} f\left(\boldsymbol{\theta}_t\right) d\boldsymbol{\theta}_{-i,t}\right] d\theta_i dH(\boldsymbol{\epsilon}) \Rightarrow \end{aligned}$$

$$-\frac{\partial p\left(\mathbf{x}^i\right)}{\partial x_i}=\int_{\boldsymbol{\epsilon}}\sum_{j\backslash i}-\frac{\partial U_j\left(\mathbf{x}^i+\boldsymbol{\epsilon}\right)/\partial x_i}{\rho_j\Delta V_j\left(\mathbf{x}^*\right)} G_j'\left(\widetilde{\theta}_1\left(\mathbf{x}^i\right),...,\widetilde{\theta}_{i-1}\left(\mathbf{x}^i\right),\widetilde{\theta}_{i+1}\left(\mathbf{x}^i\right),...\widetilde{\theta}_n\left(\mathbf{x}^i\right)\right) dH(\boldsymbol{\epsilon}). \quad (14)$$

The condition under which i does not benefit from a marginal change dx_i is given by the analogously modified (13),[15] but now, since this inequality is continuous at $dx_i = 0$, it must hold whether dx_i is positive or negative; it thus has to hold with equality; and it thus holds with equality regardless of whether i could benefit from $dx_i > 0$ or $dx_i < 0$ (so, we do not need the assumptions $\partial U_i(\cdot)/\partial x_j > 0$ for $j \neq i$ and < 0 for $j = i$).

When we let $\boldsymbol{\epsilon}$ vanish ($\mathrm{E}\left(\boldsymbol{\epsilon}_t^k\right)^2 \to \mathbf{0}$ when $k \to 0$), we get $p(\mathbf{x}^*) \to 0$ and $V_j(\mathbf{x}^*) \to U_j(\mathbf{x}^*)$, and then the condition simplifies to

$$-\frac{\partial U_i\left(\mathbf{x}^*\right)}{\partial x_i}=\sum_{j\backslash i}\frac{\partial U_j\left(\mathbf{x}^*\right)/\partial x_i}{\rho_j\Delta U_j\left(\mathbf{x}^*\right)} f_j(0)\,\mathrm{E}\left(\theta_{i,t}\mid\theta_{j,t}=0\right)\rho_i\Delta U_i\left(\mathbf{x}^*\right),$$

which is the first-order condition of

$$\arg\max_{x_i}\prod_{j\in N}\left(U_j\left(x_i,\mathbf{x}_{-i}^*\right)\right)^{w_j^i},$$

when $\frac{w_j^i}{w_i^i}=\frac{\rho_i}{\rho_j}f_j(0)\mathrm{E}(\theta_{i,t}\mid\theta_{j,t}=0)$, $\forall j\neq i$.

(A2) The proof when $\underline{\theta}_j < 0$ is analogous: Now, $p(\mathbf{x}^*) = 1 - G(\mathbf{0}) > 0$ and $p(\mathbf{x}^*)$ is continuous at $x_i^i = x_i^*$. Thus, the (modified) version of (13) must hold with equality, and it boils down to (3) holding with equality since, when $\underline{\theta}_j \uparrow 0 \forall j$, then $p(\mathbf{x}^*) \to 0$. *QED*

The other proofs are rather standard and will be added to the paper soon.

[15] The modified version of (13) can be written as:

$$\begin{aligned} &\left(1-p\left(\mathbf{x}^*\right)-\frac{\partial p\left(\mathbf{x}^*\right)}{\partial x_i}dx_i\right)\mathrm{E}_{\boldsymbol{\epsilon}_{i,t}:(\boldsymbol{\epsilon},\theta)\in\Phi_A(\mathbf{x}^*)}\left(U_i\left(\mathbf{x}^*+\boldsymbol{\epsilon}\right)+\frac{\partial U_i\left(\mathbf{x}^*+\boldsymbol{\epsilon}\right)}{\partial x_i}dx_i\right) \\ &+\int_\epsilon\sum_{j\backslash i}\left[\frac{\partial U_j\left(\mathbf{x}^*+\boldsymbol{\epsilon}\right)/\partial x_i}{\rho_j\Delta V_j\left(\mathbf{x}^*\right)}dx_i\int_0^{\overline{\theta}_i}G_j'\left(\frac{V_1\left(\mathbf{x}^*\right)-U_1\left(\mathbf{x}^*+\boldsymbol{\epsilon}\right)}{\rho_1\Delta V_1\left(\mathbf{x}^*\right)},\frac{V_2\left(\mathbf{x}^*\right)-U_2\left(\mathbf{x}^*+\boldsymbol{\epsilon}\right)}{\rho_2\Delta V_2\left(\mathbf{x}^*\right)},...;\theta_i\right)d\theta_i\right]. \\ &U_i\left(\mathbf{x}^*\right)\mathrm{E}_{\theta_{i,t}:(\boldsymbol{\epsilon},\theta)\in\Phi_R(\mathbf{x}^*)}\left(1-\theta_{i,t}\rho_i\Delta\right)\leq U_i\left(\mathbf{x}^*\right). \end{aligned}$$

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