On the efficient growth rate of carbon price under a carbon budget*

Christian Gollier
Toulouse School of Economics, University of Toulouse-Capitole

September 8, 2018

Abstract

When an intertemporal carbon budget is imposed to fight climate change, abating emissions earlier has a social rate of return that is equal to the growth rate of the marginal abatement cost, i.e., of the carbon price. I use a normative version of asset pricing theory to determine the efficient level of the growth rate of expected carbon price in this Hotelling's framework under uncertainty. When future marginal abatement costs are negatively correlated with aggregate consumption, an immediate vigorous reduction in emissions provides a hedge against the macroeconomic risk borne by the representative agent. The growth rate of expected carbon price should therefore be smaller than the interest rate in that case, and the initial carbon price should be large. The opposite is true when this correlation is positive, and the Hotelling's rule applies as a limit case with independence. We calibrate a simple two-period version of the model by introducing infrequent macroeconomic catastrophes à la Barro in order to fit the model to observed assets pricing in the economy. From this numerical exercise, we recommend a growth rate of expected carbon price around 3.5% per year (plus inflation), which is much larger than the 1% equilibrium interest rate in our economy.

Keywords: Uncertain mitigation cost, marginal abatement cost, Hotelling's rule, consumption-based CAPM.

JEL codes: Q54, D81, G12.

*I want to thank Ottmar Edenhofer, Simon Quemin, Alain Quinet, Kai Lessmann, Chuck Mason and Cees Withagen for particularly helpful comments, together with seminar participants at PIK (Potsdam) and at the 5th annual Conference of FAERE. The research leading to this paper has received indirect funding from Engie, EDF, Total, SCOR through TSE research contracts, and from the Chair "Sustainable Finance and Responsible Investments" at TSE.

1 Introduction

Global warming is a global externality that should be managed by imposing a uniform carbon price equaling the discounted marginal climate damage of carbon dioxide.[1] Because ecological and economic systems are expected to be more stressed in the future, the estimate of this social cost of carbon is expected to grow over time.[2] For example, using a 3% discount rate, the U.S. administration published a scientific report (Interagency Working Group on the Social Cost of Greenhouse Gases (2013, Revised August 2016)) based on this cost-benefit approach that recommends a price of 42 dollars per ton of CO_2 in 2020, growing to 69 dollars in 2050. This yields a growth rate of 0.7% per year.

Given the overwhelming difficulty to estimate the flow of marginal damages generated by greenhouse gases, this cost-benefit approach to carbon pricing tends to be replaced by a cost-efficiency approach in which a concentration target is exogenously determined.[3] Given the current state of the atmosphere, this concentration target can be translated into an exogenous intertemporal carbon budget. Determining the optimal timing to consume this carbon budget is a problem equivalent to the Hotelling's problem of extracting a non-renewable resource (Hotelling (1931)). This implies that, under certainty, the price of carbon should grow at the interest rate. The intuition of this result is simple: If future marginal abatement costs are perfectly known today, reallocating the carbon budget intertemporally is a safe. In particular, a marginal increase in the abatement effort today that is compensated by an equivalent reduction of the abatement in the future[4] is a safe investment project that we hereafter call "abatement frontloading". It costs the marginal abatement cost today and that generates a future benefit equaling the future marginal abatement cost. Thus, it yield a return that is equal to the growth rate of the marginal abatement cost, i.e., of the carbon price. The evaluation and implementation decision of this safe investment project should be treated as for any other safe projects in the economy. In particular, its equilibrium return, which is the growth rate of carbon price, should be equal to the interest rate. And the discount rate that should be used to evaluate the social benefit of the intertemporal reallocation of the carbon budget should also be the interest rate. These are the two faces of the same coin, since entrepreneurs and investors active on the market for emission permits will use the interest rate as the discount rate to evaluate the abatement frontloadingstrategy if they are confronted to a cost of capital equaling the interest rate. Private and public interests will be aligned.[5]

But in the long term, the evolution of abatement costs is fundamentally uncertain, together with the evolution of the carbon budget and with the emissions in the business-as-usual

[1] See for example Gollier and Tirole (2015) for a recent policy paper on this issue.

[2] Van der Ploeg and Withagen (2014) provide an analysis of the determinants of the growth rate of the social cost of carbon in a Ramsey growth model with an exhaustible fossil resource, an infinitely elastic supply of renewables and a convex climate damage function under certainty.

[3] For a discussion about the uncertainty surrounding the climate damage function, see for example Pindyck (2013). The use of a carbon budget has also been promoted by incentive schemes based on emissions targets, such as the Kyoto Protocole and the various markets for carbon permits that exist now around the world.

[4] In this introduction, we assume for simplicity that there is no natural decay of carbon dioxide in the atmosphere.

[5] Chakravorty et al. (2006) and Chakravorty et al. (2008) reexamined the Hotelling's problem of optimal extraction of nonrenewable resources with a carbon budget under certainty when different resources differ in their extraction costs and pollution intensities.

scenario. Nobody really knows today what will be the cost of abatement associated to wind or solar energy in 30 years. And deep uncertainties surround future electricity storage technologies and nuclear fusion for example. The extraordinary large uncertainty surrounding the emergence of economically viable renewable systems of energy is an inherent dimension of the energy transition. One should also recognize that scientific discoveries in the future could induce us to revise the carbon budget downwards or upwards. This implies that modelers around the world face enormous challenges to implement this cost-efficiency approach to carbon pricing. In this paper, we focus the analysis on how uncertainty affect the efficient rate of growth of the carbon price. Proponents of the precautionary principle claim that this uncertainty should induce us to implement strong immediate actions to reduce emissions. This suggests that uncertainty should push carbon prices up in the short run, thereby allowing for a reduction of the growth rate of carbon price to satisfy the carbon budget.[6]

Because abatement frontloading is risky when the evolution of abatement cost is uncertain, asset pricing theory tells us that its expected return should not necessarily be equal to the interest rate at equilibrium. A positive or negative risk premium should be included to the growth rate of carbon price in order to take account of the impact of the climate policy on the macroeconomic risk. Suppose for example that marginal abatement costs are negatively correlated with aggregate consumption. In that context, the future benefit of abating early is also negatively linked to economic growth, and performing this investment reduces the macroeconomic risk. Fighting climate change early has the extra benefit to hedge the macro risk in that case. This climate policy should be promoted by imposing a large initial carbon price, and a low growth rate of this expected price. The main message of this paper is that the growth rate of expected carbon price should be smaller (larger) than the interest rate whenever the marginal abatement cost is negatively (positively) correlated to aggregate consumption. This policy provides the right price signal for private investors in renewables technologies to take account of the impact of their decisions on social welfare, as is the case on efficient financial markets for other investment projects.

Our approach to this problem is based on the Consumption-based Capital Asset Pricing Model (CCAPM) of Breeden (1979), Lucas (1978) and Rubinstein (1976) to assure the coherence between carbon pricing and the pricing of other assets in the economy. We solve the classical asset pricing puzzles (Mehra and Prescott (1985), Weil (1989) and Kocherlakota (1996)) by introducing catastrophes in the growth process, as suggested by Barro (2006). In this framework, we show that the beta of abatement frontloading is the income-elasticity of marginal abatement costs. Multiplying this beta by the equilibrium aggregate risk premium tells us by how much the growth rate of expected carbon price should differ from the equilibrium interest rate.

It remains thus to characterize the determinants of this "carbon beta".[7] We show in the second part of this paper that the sign of this carbon beta is generally ambiguous. But three elements matter to determine this sign. Suppose first that the evolution of green technologies is the main source of long-term uncertainty in the economy. In that case, assuming a positive

[6] Another reason for doing more efforts earlier is that governments face a credibility issue about their long-term commitments. Laffont and Tirole (1996) take this question seriously by proposing a commitment device based on forward financial contracts.

[7] Dietz et al. (2018) estimated another concept of a climate beta that based on a cost-benefit analysis as in the DICE model. The approach used in this paper is alternatively based on a cost-efficiency analysis in which the carbon budget is exogenously fixed.

link between total and marginal abatement costs, the occurrence of an optimistic scenario with strong green innovations will imply at the same time a low marginal abatement cost and a large aggregate consumption (because of the low cost of fighting climate change). Thus, when the main source of uncertainty is technological, a negative carbon beta prevails, yielding an efficient growth rate of expected carbon price below the interest rate.

Consider a second context in which the main source of uncertainty is about economic growth. Assuming a convex abatement function, the occurrence of a large growth scenario implies more emissions under business-as-usual. This implies more abatement efforts and thus a larger marginal abatement cost. In this second context, one should expect a positive correlation between marginal abatement cost and aggregate consumption. This positive correlation is in line for example with the crash of the price of emission permits observed on the EU-ETS market during the subprime and eurozone crises during the last decade. The traditional explanation is based on the glut of carbon credits due to the low emission intensity during these crises.[8] Under such a positive correlation, one should impose a growth rate of expected carbon price larger than the interest rate (and a low initial carbon price). In the third context, suppose that the carbon budget is the main source of uncertainty. If it happens that the carbon budget is larger than expected, the marginal abatement cost will be low (because of the convexity of the abatement function) and consumption will be large (because of the low total abatement cost). This negative carbon beta implies that one should impose a growth rate of expected carbon price smaller than the interest rate in that case.

2 The efficient growth rate of expected carbon price

In this section, we examine a simple dynamic model of exogenous growth and technological uncertainty. Consider an economy with a fixed carbon budget. Suppose that this carbon budget has been allocated intertemporally in an optimal way. We determine the properties of the schedule of carbon prices that supports this optimum. In the spirit of the CCAPM, suppose that the economy has a representative agent whose rate of pure preference for the present is ρ. The von Neumann-Morgenstern utility function u of the representative agent is increasing and concave. Along the optimal path, the consumption per capita $C_t|_{t\geq 0}$ evolves in a stochastic way.

In the constellation of investment opportunities existing in the economy, consider a marginal project that yields a cost I_0 today and generates a single benefit B_t at date t, where B_t is potentially uncertain and statistically related to stochastic process governing aggregate consumption. At the margin, investing in this project raises the discounted expected utility of the representative agent by

$$\Delta V = -I_0 u'(C_0) + e^{-\rho t} E[B_t u'(C_t)] = u'(C_0) \times NPV, \tag{1}$$

with

$$NPV = -I_0 + e^{-r_t t} E[B_t] \tag{2}$$

and

$$e^{-r_t t} = e^{-\rho t} \frac{E[B_t u'(C_t)]}{u'(C_0) E[B_t]}. \tag{3}$$

[8] Although the estimated correlation between carbon prices on the EU-ETS market and quartely growth rate of GDP in the EU28 is positive, it is not statistically significant.

This means that the increase in the representative agent's intertemporal welfare generated by the project is proportional to its Net Present Value (NPV) when using the appropriate risk-adjusted discount rate r_t to discount the project's expected future benefit. This supports the use of the NPV rule to evaluate the investment project. The efficient discount rate defined by equation (3) depends upon the risk profile of the project and its maturity. Notice also that along the optimal path, all socially desirable investments must have been implemented so that $NPV = 0$ is an equilibrium condition, yielding the property that $\exp(r_t t)$ be equal to EB_t/I_0, which is the expected gross rate of return of the project. In other words, the socially desirable discount rate of an asset must also be its expected rate of return at equilibrium. Because entrepreneurs implementing the project must compensate stakeholders by offering this return in expectation, this induces them to invest in it only if its expected return is larger than r_t. This is equivalent to using r_t as the discount rate to evaluate the project. This is an illustration of the first theorem of welfare economics.

We apply this classical justification of the efficient risk-adjusted discount rates and of equilibrium expected returns to three specific assets. Consider first the case in which the future benefit is certain, or more generally when it is independent of C_t. This yields the risk-free discount rate r_{ft}, i.e., the interest rate, which is defined as follows:

$$\exp(-r_{ft}t) = \exp(-\rho t)\frac{E[u'(C_t)]}{u'(C_0)}. \tag{4}$$

Similarly, the risk-adjusted discount rate r_{ct} to discount a claim on aggregate consumption must satisfy the following efficiency condition:

$$\exp(-r_{ct}t) = \exp(-\rho t)\frac{E[C_t u'(C_t)]}{u'(C_0)E[C_t]}. \tag{5}$$

The systematic risk premium π_t is the extra expected rate of return of a claim on aggregate consumption over the interest rate that must compensate agents who accept to bear the macroeconomic risk:

$$\pi_t = r_{ct} - r_{ft}. \tag{6}$$

It is easy to check that π_t is positive at equilibrium under risk aversion.

Finally, consider an investment consisting in abatement frontloading: One increases the abatement effort today by one ton of carbon dioxide. This allows for abating $\exp(-\delta t)$ less tons of carbon dioxide in t years, where δ is the rate of decay of carbon dioxide. This implies that the concentration of CO_2 is unaffected by this intertemporal reallocation at any time after date t. Because the initially optimal allocation satisfies the carbon budget constraint, this new allocation does also satisfy this constraint. Let $A'_t|_{t\geq 0}$ denote the dynamics of marginal abatement costs along the optimal allocation of climate efforts. This investment yields an initial cost $I_0 = A'_0$ and generates a future benefit $B_t = \exp(-\delta t)A'_t$. Therefore, it is socially desirable that this benefit be discounted at rate g satisfying the following condition:

$$\exp(-g_t t) = \exp(-(\delta + \rho)t)\frac{E[A'_t u'(C_t)]}{u'(C_0)E[A'_t]}. \tag{7}$$

If the climate policy is decentralized through a market for emission permits, marginal abatement costs will be equalized across firms and individuals, and will be equal to the equilibrium carbon price p_t. Remember now that g_t can also be interpreted as the equilibrium expected return: $\exp(g_t t)$ must be equal to EA'_t/A'_0, i.e., to Ep_t/p_0. This means that g_t is the efficient growth rate of expected carbon price.

Combining equations (4), (7) and $\exp(g_t t) = Ep_t/p_0$ yields

$$\frac{Ep_t}{p_0} = \exp((r_{ft}+\delta)t)\frac{E[A'_t]E[u'(C_t)]}{E[A'_t u'(C_t)]},$$

or, equivalently,

$$\frac{1}{t}\log\left(\frac{Ep_t}{p_0}\right) = r_{ft}+\delta+\frac{1}{t}\log\left(\frac{E[A'_t]E[u'(C_t)]}{E[A'_t u'(C_t)]}\right). \tag{8}$$

The left-hand side of this equality is the annualized growth rate of expected carbon price between dates 0 and t. Suppose first that A'_t is constant, or more generally, statistically independent of C_t. In that case, the last term in the right side of this equality vanishes. This implies that the efficient growth rate of (expected) carbon price must be equal to the sum of the interest rate and the rate of natural decay of carbon dioxide in the atmosphere. This is the well-known Hotelling's rule adapted to carbon pricing under a fixed intertemporal carbon budget (Schubert (2008)).

More generally, equation (8) tells us that g_t is larger or smaller than $r_{ft}+\delta$ depending upon whether the last term in the right-hand side of this equality is positive or negative. At this stage, let us characterize the uncertainty by a set of possible states of nature and an associated probability distribution on these states. A random variable is a function of the state of nature. Two random variables (X, Y) are said to be comonotone iff for any pair (s, s') of states of nature, $(X(s)-X(s'))(Y(s)-Y(s'))$ is non-negative. Comonotonicity is a strong form of positive correlation. Anti-comonotonicity is defined symmetrically. Suppose now that A'_t and C_t are comonotone. Because u' is decreasing, this implies that A'_t and $u'(C_t)$ are anti-comonotone. By the covariance rule (Gollier (2001), Proposition 15), this implies that $E[A_t u'(C_t)]$ is smaller than $E[A'_t]E[u'(C_t)]$. This implies that the last term of the right-hand side of the above equation is positive. This means in turn that the growth rate of expected carbon price is larger than $r_{ft}+\delta$. This demonstrates the following result.

Proposition 1. *The growth rate of the expected carbon price that supports the optimal temporal allocation of abatement efforts is larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the marginal abatement cost and aggregate consumption are (anti-)comonotone.*

From the social point of view, facing a positive correlation between marginal abatement costs and aggregate consumption is good news. It means that the worst-case scenarios in terms of abatement costs arise when aggregate consumption is large, i.e., when the marginal abatement effort has a smaller utility impact. This means abating more in the future reduces the macroeconomic risk. It raises the collective willingness to postpone abatement efforts. In a decentralized economy, this is translated into increasing the incentive to do so by selecting a growth rate of expected carbon price larger than the sum of the interest rate and the rate of natural decay. From the individual point of view, as long as some abatement efforts are implemented today, investors who implement the abatement frontloading must be compensated for the fact that the benefit of doing so (compared to late renewable adopters) has a positive beta, in the sense that the return of this investment is smaller when other assets also perform poorly in the economy. Because the return of abatement frontloading is the growth rate of carbon price, this compensation takes the form of a growth rate of expected carbon price larger than the sum of the interest rate and the rate of natural decay.

Let us now consider the following special case. Suppose that relative risk aversion is a constant γ. Suppose also that aggregate consumption and marginal abatement costs evolve according to the following stochastic process:

$$\begin{aligned} dc_t &= \mu_c dt + \sigma_c dz_t && (9) \\ da'_t &= \mu_p dt + \phi\sigma_c dz_t + \sigma_w dw_t, && (10) \end{aligned}$$

with $c_t = \log C_t$ and $a'_t = \log A'_t$, and where z_t and w_t are two independent standard Wiener processes.[9] This means that the logarithm of aggregate consumption and marginal costs are jointly normally distributed. Parameters μ_c and σ_c are respectively the trend and the volatility of consumption growth. The trend of growth of the marginal abatement cost, and thus of the carbon price, is given by parameter μ_p. The volatility of the marginal abatement cost has an independent component σ_w and a component coming from its correlation with economic growth. Notice that ϕ can be interpreted as the elasticity of marginal abatement costs to unanticipated changes in aggregate consumption.

The following proposition can be interpreted as an application of the Consumption-based CAPM. We provide a formal proof of this proposition in the Appendix.

Proposition 2. *Suppose that relative risk aversion is constant and that the logarithms of aggregate consumption and marginal abatement costs follow a bivariate Brownian process. Then, the growth rate of the expected carbon price that supports the optimal temporal allocation of abatement efforts must be equal to the sum of three terms:*

- δ*: the rate of natural decay of greenhouse gas in the atmosphere;*
- r_f*: the interest rate in the economy;*
- $\phi\pi$*: the abatement risk premium, which is the product of the income-elasticity of marginal abatement cost by the aggregate risk premium in the economy.*

In short, we have that

$$g = \delta + r_f + \phi\pi, \tag{11}$$

where the interest rate r_f and the aggregate risk premium π are characterized in the Appendix. This result tells us that the CCAPM risk premium for carbon permits holds with a CCAPM "carbon beta" being equal to the income-elasticity ϕ of the marginal abatement cost. This is related to Proposition 1 in which the statistical relationship between marginal abatement costs and aggregate consumption is summarized by ϕ. An immediate consequence of Proposition 2 is described in the following corollary.

Corollary 1. *Under the assumptions of Proposition 2, the growth rate of expected carbon price should be larger (smaller) than the sum of the interest rate and the rate of decay of carbon dioxide if the income-elasticity of marginal abatement costs is positive (negative).*

[9] Without loss of generality, I normalize C_0 and A'_0 to unity.

Under the stochastic process (9)-(10), the estimation of the key parameter ϕ is rather simple. Indeed, this system implies that

$$\Delta \log(A'_t) = a + \phi \Delta \log(C_t) + \varepsilon_t, \tag{12}$$

where $\Delta \log(A'_t)$ and $\Delta \log(C_t)$ are respectively changes in log marginal cost and in log consumption, and ε_t is an independent noise that is normally distributed. This means that the OLS estimator of the slope of this linear equation is an unbiaised estimator of the income-elasticity of the marginal abatement cost that must be used to determine the efficient growth rate of expected carbon price. The beauty of Proposition 2 compared to Proposition 1 is to provide a quantification of the effect of uncertainty on the efficient growth rate of the expected carbon price. When the assumptions of Proposition 2 are not satisfied, equation (11) can be obtained as an approximation where ϕ is the local estimation of the income-elasticity of the marginal abatement cost along the optimal path.

3 The determinants of the income-elasticity of marginal abatement costs in a simple two-period model

Proposition 2 provides a simple characterization of the efficient growth rate of expected carbon price that relies on the income-elasticity of the marginal abatement cost. In this section, we explore the determinants of this income-elasticity. Because the current and future marginal abatement costs depend upon which intertemporal abatement strategy is used, this characterization requires solving the intertemporal carbon allocation problem. This cannot be easily done in a continuous-time framework. In this section, we solve this problem in a simple two-period framework. Suppose that the carbon budget constraint covers only two periods, $t = 0$ and 1. The production of the consumption good is denoted Y_0 and Y_1 for periods 1 and 2 respectively, where Y_1 is uncertain in period 0. The carbon intensity of the economy in the business-as-usual scenario in period t is denoted $Q_t \geq 0$, so that $Q_t Y_t$ tons of carbon dioxide are emitted in period t under this scenario. The country is committed not to exceed a total emission target T for the two periods. As stated for example in the Paris Agreement, the long-term carbon budget allocated to the countries could be modified depending upon new scientific information about the intensity of the climate change problem for example. In our model, this means that, in period 0, there may be some uncertainty about what the intertemporal carbon budget T will be in the future.

Compared to the business-as-usual scenario, the country must choose how much to abate in each period. Let K_t denote the number of tons of carbon dioxide abated in period t, so that one can write the carbon budget constraint as follows:

$$e^{-\delta} (Q_0 Y_0 - K_0) + Q_1 Y_1 - K_1 \leq T, \tag{13}$$

where δ is the rate of natural decay of carbon dioxide in the atmosphere. We hereafter assume that this carbon budget constraint is always binding, so that we can rewrite the abatement in period 1 as a function of the other variables:

$$K_1 = K_1(K_0, Y_1, T) = e^{-\delta} (Q_0 Y_0 - K_0) + Q_1 Y_1 - T. \tag{14}$$

Because Y_1 and T are potentially uncertain, so is the abatement effort K_1 in period 1 that will be necessary to satisfy the intertemporal carbon budget constraint.

Abating is costly. Let $A_0(K_0)$ and $A_1(K_1, \theta)$ denote the abatement cost function in periods 0 and 1 respectively. We assume that A_t is an increasing and convex function of K_t. In order to allow for technological uncertainty, A_1 is a function of parameter θ, which is unknown in period 0. Consumption in period t is $C_t = Y_t - A_t$.

The problem of the social planner is thus to select the abatement strategy (K_0, K_1) to maximize the intertemporal welfare function subject to the carbon budget constraint:

$$\max_{K_0,K_1} \quad H(K_0, K_1) = u(Y_0 - A_0) + e^{-\rho} E[u(Y_1 - A_1)] \quad s.t. \quad (14). \tag{15}$$

The first-order condition of this problem is written as follows:

$$A_0' u'(Y_0 - A_0) = e^{-\rho-\delta} E\left[A_1' u'(Y_1 - A_1)\right], \tag{16}$$

where A_t' denote the partial derivative of the total abatement cost function with respect to abatement K_t.

We know from Proposition 1 that the growth rate of the expected carbon price is larger (smaller) than the interest rate when the marginal abatement cost and aggregate consumption are (anti-)comonotone. In the remainder of this section, we examine various special cases that highlight some of the factors that determine whether the growth rate of expected carbon price should be larger or smaller than the interest rate plus the rate of natural decay of carbon dixiode. Suppose first that the only source of uncertainty in the economy is related to the exogenous growth of production Y_1. In particular, this means that θ is certain, i.e., there is no uncertainty about the green technological progress. It also means that there is no uncertainty about the intertemporal carbon budget allocated to the country. The only source of correlation between A_1' and C_1 comes from the fact that both random variables covary with Y_1. In that case, we have that

$$\frac{\partial A_1'}{\partial Y_1} = Q_1 A_1''(K_1, \theta),$$

which is positive. We also have that

$$\frac{\partial C_1}{\partial Y_1} = 1 - Q_1 A_1'(K_1, \theta).$$

We hereafter assume that $Q_1 A_1'$ is smaller than unity. Although it is restrictive, this condition is intuitive, since it means that more production growth cannot be a bad news, in spite of the increased abatement effort necessary to compensate the extra emission generated by this production. $Q_1 A_1'$ is the increased abatement cost necessary to compensate for the increased production growth in the business-as-usual scenario. This condition states that production growth always increases consumption, even after taking account of the increased abatement effort to compensate for it under the intertemporal carbon constraint. Thus, under this condition, the marginal abatement cost and aggregate consumption are comonotone. Using Proposition 1, this demonstrates the following proposition.

Proposition 3. *Suppose that the growth of aggregate production Y_1 is the only source of uncertainty in the economy, and that $Q_1 A_1'$ is smaller than unity. Then, it is socially desirable that the growth rate of expected carbon price be larger than the sum of the interest rate and the rate of decay of CO_2.*

A similar exercise can be done in a context where the only source of uncertainty is related to the intertemporal budget constraint T. In that case, a larger budget T implies a smaller abatement effort, and thus a larger share of production available for consumption rather than for abatement efforts. At the same time, because of the convexity of the cost function, the marginal abatement cost is smaller. Thus, aggregate consumption and marginal abatement cost are anti-comonone. This yields the following result.

Proposition 4. *Suppose that the intertemporal carbon budget T is the only source of uncertainty in the economy. Then, it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of CO_2.*

Suppose finally that the only source of uncertainty is about θ, which is related to the speed of green technological progress. Suppose that an increase in θ implies a reduction in both the total and the marginal abatement costs, i.e., that for all (K_1, θ),

$$\frac{\partial A_1(K_1,\theta)}{\partial \theta} \leq 0 \quad and \quad \frac{\partial^2 A_1(K_1,\theta)}{\partial K_1 \partial \theta} \leq 0. \tag{17}$$

A possible illustration is when marginal abatement cost is an uncertain constant, i.e., when $A_1(K_1,\theta)$ is equal to $\alpha + g(\theta)K_1$ with $g' \leq 0$, a case examined by Baumstark and Gollier (2010). In that case, a small θ means at the same time a large marginal abatement cost and a large total abatement cost, and thus a low aggregate consumption. Thus, A_1' and C_1 are anti-comonotone, thereby demonstrating the following proposition.

Proposition 5. *Suppose that the speed of green technological progress θ is uncertain. If total and marginal abatement costs are comonotone (condition (17)), it is socially desirable that the growth rate of expected carbon price be smaller than the sum of the interest rate and the rate of decay of CO_2.*

Up to this point, we only characterized the impact of uncertainty on the optimal growth rate of the carbon price. A more complete analysis would be to characterize its effect on the optimal abatement effort in the first period. This is a more difficult question. In order to address it, let us simplify the problem by assuming that the marginal abatement cost in period 1 is constant but potentially uncertain: $A_1(K_1,\theta) = \theta K_1$. In that case, aggregate consumption in period 1 equals

$$C_1 = Y_1 - \theta\left(e^{-\delta}\left(Q_0Y_0 - K_0\right) + Q_1Y_1 - T\right).$$

Observe that in that case, the first period abatement K_0 has a role similar to saving in the standard consumption-saving problem. Each ton of CO_2 "saved" in the first period generates an increase in consumption by $R = \exp(-\delta)\theta$ in the second period, where R can be interpreted as the rate of return of savings. Suppose first that θ is certain. It is well-known in that case that the uncertainty affecting future incomes raises optimal (precautionary) saving if and only if the individual is prudent (Drèze and Modigliani (1972), Leland (1968), Kimball (1990)).[10] Applying this result to our context directly yields the following proposition. Notice that because the marginal abatement cost is certain, it must grow at the interest rate in this case.

[10] An individual is prudent if and only if the third derivative of u is positive.

Proposition 6. *Suppose that $A_1(K_1, \theta) = \theta K_1$ and that the marginal abatement cost θ is a known constant. Increasing risk on future production Y_1 or on the intertemporal carbon budget T increases the initial abatement effort K_0 if and only if the representative agent is prudent.*

When the marginal abatement cost is uncertain, the future return of abating more today becomes uncertain in that case. By risk aversion, this reinforces the willingness to abate in the first period because it also reduces the risk borne in the second period. because of this second effect, prudence is sufficient but not necessary in this case.

Proposition 7. *Suppose that $A_1(K_1, \theta) = \theta K_1$ and that the marginal abatement cost θ is the only source of uncertainty. Increasing the risk affecting θ raises the initial abatement effort K_0 if the representative agent is prudent.*

Proof: Consider two random variables, θ_1 and θ_2, where θ_2 is riskier than θ_1 in the sense of Rothschild and Stiglitz (1970). Let $G_i(K_0) = H_i(K_0, K_1(K_0, Y_1, T))$ denote the corresponding objective function, as described by (15). Let K_{0i} denote the optimal initial abatement under distribution θ_i of the marginal abatement cost. The optimal abatement effort K_{01} under the initial uncertainty θ_1 satisfies the first-order condition

$$A_0'(K_{01})u'\left(Y_0 - A_0(K_{01})\right) = \beta E\left[\theta_1 u'\left(Y_1 - \theta_1 K_{11}\right)\right], \tag{18}$$

where K_{11} is the optimal abatement effort in period 1 under the initial risk θ_1, i.e., $K_{11} = K_1(K_{01}, Y_1, T)$. Because G_2 is concave in K_0, we obtain that K_{02} is larger than K_{01} if and only if $G_2'(K_{01})$ is positive. Using condition (18), this condition can be written as follows:

$$E\left[\theta_2 u'\left(Y_1 - \theta_2 K_{11}\right)\right] \geq E\left[\theta_1 u'\left(Y_1 - \theta_1 K_{11}\right)\right]. \tag{19}$$

This is true for any Rothschild-Stiglitz risk increase if and only if function v is convex, where $v(\theta)$ equals $\theta u'(Y_1 - \theta K_{11})$ for all θ in the joint support of θ_1 and θ_2. It is easy to check that

$$v''(\theta) = -2K_{11}u''(Y_1 - \theta K_{11}) + \theta K_{11}^2 u'''(Y_1 - \theta K_{11}). \tag{20}$$

Because K_{11} is positive and u'' is negative, we see that v is convex when u''' is positive. ■

4 Calibration

In this section, we calibrate the two-period model described in the previous section. A standard approach to climate policy in the western world is based on the hypothesis that the energy transition should be performed within the next 3 decades in order to remain below the 2°C objective. We follow for example Metcalf (2018) to decompose the next 3 decades into two periods of 15 years, 2021-2035 and 2036-2050. We examine the case of the European Union (EU-28). We hereafter describe the calibration of this model. We assume a rate of pure preference for the present equaling $\rho = 0.5\%$ per year, and a constant relative risk aversion of $\gamma = 3$.

4.1 Economic growth

The current annual GDP of EU-28 is around 19,000 billions USD (GUS\$). Assuming an annual growth rate of 1.4% per year over the period 2021-2035 yields a total production for this first period estimated at Y_0=315,000 billions USD. The production Y_1 of the second period is uncertain. A key element of this paper is that the recommended returns of green investments are compatible with the equilibrium returns of other assets in the economy, and with intertemporal social welfare. However, as is well-known, the CCAPM model that we use in this paper has been unable to predict observed asset prices when beliefs are normally distributed as assumed in Section 2. This model yields an interest rate that is too large and an aggregate risk premium that is too low.[11] In this paper, we use the resolution of these asset pricing puzzles that has been proposed by Barro (2006), who recognized the possibility of infrequent large recessions that are not well represented in U.S. growth data. We follow the calibration proposed by Martin (2013). The production in the first period of 15 years is normalized to unity. The change in log production during the second subperiod is equal to the sum of 15 independent draws of an annual growth rate x_i whose distribution compounds two normally distributed random variables:

$$\begin{aligned}
\log\left(\frac{Y_1}{Y_0}\right) &= \sum_{i=1}^{15} x_i && (21)\\
x_i &\sim (h_{bau}, 1-p; h_{cat}, p) && (22)\\
h_{bau} &\sim N(\mu_{bau}, \sigma^2_{bau}) && (23)\\
h_{cat} &\sim N(\mu_{cat}, \sigma^2_{cat}). && (24)
\end{aligned}$$

With probability $1-p$, the annual growth rate is drawn from a "business-as-usual" normal distribution with mean μ_{bau} and volatility σ^2_{bau}. But with a small probability p, the annual growth rate is drawn from a "catastrophic" normal distribution with a large negative μ_{cat} and a large volatility σ^2_{cat}. In Table 1, we describe the value of the parameters of the model that are used as a benchmark. The order of magnitude of the parameters of the production growth process is in the range of what has been considered by Barro (2006) and Martin (2013). The annual probability of a macroeconomic catastrophe is fixed at 1.7%, and the expected annual drop in production is assumed to be 35% in that case. The calibration of these parameters is detailed in Table 1. It yields an annual trend of growth of 1.37% and an expected production of $Y_1 = 387,000$ billions USD (GUS\$)in the second period.

4.2 Emissions and decay

The EU-28 currently emits 4.4 $GtCO_2e$ per year. Under the Business-As-Usual (BAU), we assume that this flow is maintained over each of the 15 years of the first period, implying 66 $GtCO_2e$ emitted in this scenario. When compared to the production Y_0 estimated above, this yields a carbon intensity of $Q_0 = 2.10 \times 10^{-4}$ $GtCO_2e$/GUS\$. Even without any mitigation policy, the world economy have benefitted from a natural reduction of the energy intensity of its global production over the recent decades. According to Clarke et al. (2014), the average of decline of the energy intensity has been approximately 0.8% per year. This is why we assume in this calibration exercise that the carbon intensity in the second period goes down

[11]See for example Kocherlakota (1996) and Cochrane (2017).

to $Q_1 = 1.85 \times 10^{-4}$ GtCO$_2$e/GUS\$ in the BAU. This implies an expected total emission of around 72 GtCO$_2$e in the second period in the BAU.

There exists an intense debate about the half-life of carbon dioxide in the atmosphere, and thus on its rate of natural decay. It appears that the carbon cycle is highly no linear, and involves complex interactions between the atmosphere and different layers of the oceans. The existing literature on the half-life of carbon dioxide offers a wide range of estimates, from a few years to several centuries.[12] We conservatively assume a rate of natural decay of CO_2 in the atmosphere of 0.5% per year. This implies a total expected emission net of the natural decay for the European Union over the period 2021-2050 in the BAU around 133 GtCO$_2$e.

4.3 Carbon budget

In the fifth report of the IPCC (Clarke et al. (2014)), it is estimated that a 450ppm concentration of greenhouse gases should not be exceeded in order to achieve the goal of not exceeding a 2°C increase in temperature compared to the pre-industrial age. That implied a remaining carbon budget around 950 GtCO$_2$e. Given that we have emitted around 40 Gt of greenhouse gases per year since then, we assume that this global carbon budget has now been reduced to 750 Gt. There is a debate about how to share this total carbon budget among the different countries. Let us take the conservative (and ethically sounded) approach of sharing the budget on a per capita basis. Because the European Union is home for roughly 7% of the world population, we assume that EU-28 should be allocated a carbon budget of approximately 50 GtCO$_2$e. Let us further assume that four-fifth of this budget could be consumed between 2021 and 2050. This gives an expected carbon budget for EU-28 for that period equalling μ_T=40 GtCO$_2$e. Compared to the global emission of 133 GtCO$_2$e, this represents a global abatement effort of 93 GtCO$_2$e, or a reduction of more than 70% of the global BAU emissions in the EU-28 during the next 3 decades.

There is of course much uncertainty about what will be the actual carbon budget that will emerge from the international negotiations in the next 3 decades. We model this uncertainty by assuming that T is normally distributed with mean μ_T and standard deviation $\sigma_T = 10$ GtCO$_2$e.

4.4 Abatement costs

We assume that the abatement cost function is quadratic:

$$A_t(K_t) = a_t K_t + \frac{1}{2} b K_t^2. \tag{25}$$

An important element of our model is related to how the marginal abatement cost (MAC) changes with the ambition of the mitigation policy. The answer to this question is given by the MAC slope coefficient b, which tells us by how much the marginal abatement cost increases when the abatement effort increases by 1 Gt of CO$_2$e. The researchers behind the MIT Emissions Prediction and Policy Analysis (EPPA, Morris et al. (2012)) have developed computable general equilibrium models with a very detailed energy sector. They have estimated the shadow price of carbon associated to various carbon budgets for different regions of the world, thereby generating regions-specific MAC curves. We used their analysis of the

[12]For a survey on this matter, see Archer et al. (2009).

parameter	value	description
ρ	0.5%	annual rate of pure preference for the present
γ	3	relative risk aversion
Y_0	315,000	production in the first period (in GUS$)
p	1.7%	annual probability of a macroeconomic catastrophe
μ_{bau}	2%	mean growth rate of production in a business-as-usual year
σ_{bau}	2%	volatility of the growth rate of production in a business-as-usual year
μ_{cat}	-35%	mean growth rate of production in a catastrophic year
σ_{cat}	25%	volatility of the growth rate of production in a catastrophic year
δ	0.5%	annual rate of natural decay of CO_2 in the atmosphere
Q_0	2.10×10^{-4}	carbon intensity of production in period 0 (in $GtCO_2e$/GUS$)
Q_1	1.85×10^{-4}	carbon intensity of production in period 0 (in $GtCO_2e$/GUS$)
μ_T	40	expected carbon budget (in $GtCO_2e$)
σ_T	10	standard deviation of the carbon budget (in $GtCO_2e$)
b	1.67	slope of the marginal abatement cost functions (in GUS$/$GtCO_2e^2$)
a_0	23	marginal cost of abatement in the BAU, first period (in GUS$/$GtCO_2e$)
μ_θ	2.30	expected future log marginal abatement cost in BAU
σ_θ	1.21	standard deviation of future log marginal abatement cost in BAU

Table 1: Benchmark calibration of the two-period model.

MAC curve for the European Union in 2020 to estimate that the MAC increases by 25 USD whenever the annual abatement effort is increased by 1 $GtCO_2e$. Expressed for a period of 15 years, this suggests $b = 1.67$ GUS$/$GtCO_2e^2$. We assume that b is certain and constant over time.

Parameter a_t measures the MAC along the BAU scenario. For the first period, we estimate it by price of carbon permit in the summer of 2018 on the EU-ETS market, around 23 GUS$/$GtCO_2e$. The full full elimination of the 66 $GtCO_2e$ emitted in the first period would cost around 5,000 GUS$, or 1.6% of GDP in the first period.

The MAC in the BAU in the second period is uncertain. Anticipating green innovations would suggest using a_1 smaller than a_0, at least in expectation. By how much smaller remains an open question. In order to estimate the degree of uncertainty that surrounds abatement costs in the second period of our analysis, we have used a set of AIM models scrutinized by the Working Group III for the Fifth Report of the IPCC (Clarke et al. (2014)). In the associated database, we have collected 374 estimations of the carbon price estimates for 2030 that are in line with the objective of not exceeding 450ppm over the century. These estimates differ by the IAM model used for the estimation, and by the assumed technological progresses available at that time horizon. We depict the histogram of these MAC estimates for 2030 in Figure 1. The distribution of these estimates is heavily skewed to the right, which suggests using a lognormal distribution for $a_1 = \theta$. The standard deviation of the log MAC in this sample is equal to $\sigma_\theta = 1.21$.[13] In this benchmark calibration, we assume that $\log(\theta)$ is lognormally distributed with mean $\mu_\theta = 2.30$ and $\sigma_\theta = 1.21$. This yields an expected MAC

[13]Because these estimates are based on an ambitious abatement target, the mean value of the carbon price in this sample is not useful for the estimation of the expected MAC in the BAU.

variable	value	description
K_0	31	optimal abatement in the first period (in $GtCO_2e$)
$E[K_1]$	66	optimal expected abatement in the second period (in $GtCO_2e$)
p_0	75	optimal carbon price in the first period (in US$/$tCO_2e$)
g	3.47%	annualized growth rate of expected carbon price
r_f	1.14%	annualized interest rate
π	2.42%	annualized systematic risk premium
ϕ	1.04	OLS estimation of the income-elasticity of the marginal abatement cost

Table 2: Description of the optimal solution in the benchmark case.

in the BAU around 18 GUS$/$GtCO_2e$. The 20% reduction in the expected MAC under the BAU between the two periods measures the green innovations that are expected to emerge in the next 15 years. The standard deviation of the future MAC at the BAU is equal to 38 US$/$tCO_2e$, which is in the range of the MAC uncertainty measured by Kuik et al. (2009) for a time horizon of 15 years.[14]

4.5 Results

We solved the first-order condition (16) numerically by using the Monte-Carlo method. We draw 100.000 random triplets (Y_1, θ, T) that we approximated the expectation of the right-hand side of this equality by an equally weighted sum of this random sample. In Table 2, we describe the optimal solution of this problem under the calibration of the parameters described in Table 1. We obtain equilibrium asset prices that are in line with the observed real interest rate of 1% and systematic risk premium of 2% that have been observed in the United States during the last century (Kocherlakota (1996)). The optimal abatement is much larger in the second period than in the first one. This is partly due to the anticipation of a larger price of carbon in the second period. In expectation, the annualized growth rate of the carbon price equals 3.47%. This is much larger than the sum of the natural rate of decay of CO_2 and the interest rate, which is equal to 1.64%. This is due to the fact that at the optimum, the marginal abatement cost is positively correlated with aggregate consumption, as shown in Figure 2. In fact, the OLS estimation of the income-elasticity of the marginal abatement cost is $\phi \simeq 1.04$.[15]

As observed by Metcalf (2018), Aldy (2017) and M. et al. (2017), carbon price predictability is the most important feature of a climate policy for the business community as it plans long-term investments in line with the energy transition. For example, Metcalf (2018) proposes to fix the annual growth rate of carbon price at 4% (plus inflation) as long as the path of emissions is in line with the objective. However, under uncertainty, the efficient growth rate of carbon price must be uncertain in this model because the resolution of the uncertainty affecting economic growth, green innovations and the carbon budget needs to be translated

[14] These authors performed a meta-analysis of MAC estimates in the literature, and observed a standard deviation of MAC of 27.9 and 52.9 euors per tCO_2e respectively for 2025 and 2050.

[15] Because consumption and marginal abatement costs are not log normal, equation (29) cannot be used to estimate the optimal growth rate of expected carbon price. If we use it as an approximation with $\delta = 0.5\%$, $r_f = 1.14$, $\pi = 2.42\%$ and $\phi = 1.04$, we would obtain $g \simeq 4.16\%$.

variable	benchmark	no catastrophe	no macro risk	no tech risk	no budget risk
K_0	31	26	26	28	31
$E[K_1]$	66	69	69	69	67
p_0	75	67	66	70	74
g	3.47%	4.61%	4.77%	3.77%	3.60%
r_f	1.14%	4.31%	4.49%	1.04%	1.12%
π	2.42%	0.13%	0.00%	2.51%	2.42%
ϕ	1.04	0.66	-25	1.04	0.96

Table 3: Sensitivity analysis. The "no catastrophe" context is obtained by shifting the probability of catastrophe p to zero, and by reducing the trend of growth to μ_{bau} to 1.37% to preserves the expected growth rate of production as in the benchmark. The "no macro risk" context combines these changes with the shift of the volatility σ_{bau} to zero. In the "no tech risk" context, we switched σ_θ to zero compared to the benchmark. In the "no budget risk" case, we reduced σ_T to zero compared to the benchmark.

into a variable carbon price in the second period. We represented the distribution of the annualized growth rate of carbon price in Figure 3. Its standard deviation is equal to 2.4% per annum. Contrary to the above-mentioned view, it is desirable that this risk be borne by the business community. It reflects the uncertainties of the social benefits of the climate policy. Investment decisions in energy transition should take account of these uncertainties. The attractiveness of green investments should be based on their expected return rather than its reduced risk, something that cannot be guaranteed under a rigid carbon budget.

4.6 Sensitivity analysis

Table 3 provides some information about the sensitivity of our results to the intensity of the exogenous risk of the model. In the third column entitled "no catastrophe", we have solved the model by using the benchmark calibration except for the probability of catastrophe p to has been switched to zero. Because that change has the undesirable consequence to increase the expected growth of production, we have reduced the trend of growth μ_{bau} to 1.37% in order to leave $E[Y_1]$ unchanged. This has the effect to raise the interest rate and to reduce the systematic risk premium to unrealistic levels. This observation justifies our choice of introducing macroeconomic catastrophes à la Barro in our calibration. The consequence of reducing the macro risk is to make the growth rate of expected carbon price smaller than the sum of the interest rate and the rate of natural decay, as suggested by our theoretical results. However, because the systematic risk premium is marginal in the absence of catastrophe, the difference between the two is small, as suggested by equation (29), which is an approximation in this non-gaussian calibration.

In the last two columns of Table 3, we document the results of simulations in which risks on technological progress θ and on the carbon budget T are respectively switched off. Because these effects are marginal, these results suggest that the main argument for a departure of the Hotelling's rule $g = \delta + r_f$ comes from the macroeconomic uncertainty, not from technological risks or from carbon budget risks.

5 Conclusion

In a decentralized economy, fighting climate change should be implemented by providing the right price signals to consumers and investors. The level of the carbon price determines the intensity of the effort to reduce emissions, whereas the growth rate of this price determines the allocation of this effort over time. Under an intertemporal carbon budget constraint, assuming no decay of carbon dioxide in the atmosphere, the growth rate of the carbon price is also the return of the action to reduce emissions earlier. In the benchmark case in which marginal abatement costs are certain, such frontloading of climate effort is a safe investment strategy that should be remunerated at the interest rate. This observation supports the celebrated Hotelling's result that carbon price should grow at the interest rate, under certainty. This implies that green projects will be evaluated by the private sector by using the interest rate as the rate at which future benefits of these projects should be discounted.

In this paper, we have examined the impact of the deep uncertainties surrounding the energy transition on this Hotelling's result. If marginal abatement costs are negatively correlated with aggregate consumption, frontloading climate efforts provides a hedge against the macroeconomic risk, because the saved future mitigation costs are larger when future aggregate consumption is smaller. Mitigation frontloading has a negative CCAPM beta in that case. Because of their insurance benefits, investors will be too eager to invest in these technologies if the growth rate of expected carbon price is equal to the interest rate. This cannot be an equilibrium. At equilibrium, the growth rate of expected carbon price must be made smaller than the interest rate. This risk-adjustment is socially desirable. It induces investors to use a smaller discount rate to evaluate mitigation investments, which reflects the collective insurance benefit of frontloading. On the contrary, when marginal abatement costs are positively correlated with aggregate consumption, this growth rate must be made larger than the interest rate. It is socially desirable that investors use such a discount rate larger than the interest rate because mitigation frontloading raises the macroeconomic risk in that case, which yields a positive CCAPM beta of mitigation frontloading.

The CCAPM beta of mitigation frontloading is equal to the income-elasticity of marginal abatement costs. We identified different channels that influence this beta. The negative beta channel is active when the main source of the uncertainty comes from innovations in green technologies. In hat case, a strong wave of innovations will drive marginal abatement cost downwards and it will at the same time increases consumption net of the mitigation effort. A positive beta channel is active when the main source of uncertainty comes from economic growth. When growth is larger than expected, aggregate consumption will be larger and marginal abatement cost will also be larger, because the mitigation effort must be increased to compensate for the larger emissions generated by the expanded production of goods and services. When these two sources of uncertainty are combined, the sign of the CCAPM beta of mitigation frontloading is thus generically ambiguous.

References

Aldy, J. (2017): "Designing and updating a U.S. carbon tax in an uncertain world," *Harvard Environmental Law Review*, 41, 28–40.

Archer, D., M. Eby, V. Brovkin, A. Ridgwell, L. Cao, U. Mikolajewicz, K. Caldeira, K. Matsumoto, G. Munhoven, A. Montenegro, and K. Tokos (2009): "Atmospheric lifetime of fossil fuel carbon dioxide," *Annual Review of Earth and Planetary Sciences*, 37, 117–134.

Barro, R. (2006): "Rare Disasters and Asset Markets in the Twentieth Century," *Journal of Finance*, 121, 823–866.

Baumstark, L. and C. Gollier (2010): "La dynamique de la valeur carbone dans un cadre incertain," TSE Working paper.

Breeden, D. (1979): "An intertemporal asset pricing model with stochastic consumption and investment opportunities,," *Journal of Financial Economics*, 7, 265–296.

Chakravorty, U., B. Magné, and M. Moreaux (2006): "A Hotelling model with a ceiling on the stock of pollution," *Journal of Economic Dynamics and Control*, 30, 2875–2904.

Chakravorty, U., M. Moreaux, and M. Tidball (2008): "Ordering the extraction of polluting non renewable resources," *American Economic Review*, 98, 1128–1144.

Clarke, L., K. Jiang, K. Akimoto, B. M., G. Blanford, K. Fisher-Vanden, J.-C. Hourcade, V. Krey, E. Kriegler, A. Loschel, D. McCollum, S. Paltsev, S. Rose, P. R. Shukla, M. Tavoni, B. C. C. van der Zwaan, and D. van Vuuren (2014): "Assessing transformation pathways," in *Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, ed. by O. Edenhofer, R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, B. Kriemann, J. Savolainen, S. Schlomer, C. von Stechow, T. Zwickel, and J. Minx, Cambridge, UK and New York, NY, USA: Cambridge University Press.

Cochrane, J. (2017): "Macro-finance," *Review of Finance*, 945–985.

Dietz, S., C. Gollier, and L. Kessler (2018): "The climate beta," *Journal of Environmental Economics and Management*, 87, 258–274.

Drèze, J. and F. Modigliani (1972): "Consumption decisions under uncertainty," *Journal of Economic Theory*, 5, 308–335.

Gollier, C. (2001): *The economics of risk and time*, MIT Press.

Gollier, C. and J. Tirole (2015): "Negotiating effective institutions against climate change," *Economics of Energy and Environmental Policy*, 4, 5–27.

Hotelling, H. (1931): "The economics of exhaustible resources," *Journal of Political Economy*, 39, 137–175.

Interagency Working Group on the Social Cost of Greenhouse Gases (2013, Revised August 2016): “Technical update of the social cost of carbon for regulatory impact analysis under executive order 12866,” Tech. rep., Washington: Executive Office of the President, Available: https://obamawhitehouse.archives.gov/sites/default/files/omb/assets/inforeg/technical-update-social-cost-of-carbon- for-regulator-impact-analysis.pdf.

Kimball, M. (1990): “Precautionary saving in the small and in the large,” *Econometrica*, 58, 53–73.

Kocherlakota, N. (1996): “The equity premium: It’s still a puzzle,” *Journal of Economic Literature*, 34, 42–71.

Kuik, O., L. Brander, and S. Tol (2009): “Marginal abatement costs of greenhouse gas emissions: A meta-analysis,” *Energy Policy*, 37, 1395–1403.

Laffont, J. and J. Tirole (1996): “Pollution Permits and Compliance Strategies,” *Journal of Public Economics*, 62, 85–125.

Leland, H. (1968): “Savings and uncertainty: The precautionary demand for savings,” *Quarterly Journal of Economics*, 45, 621–636.

Lucas, R. (1978): “Asset prices in an exchange economy,” *Econometrica*, 46, 1429–46.

M., H., G. Metcalf, and C. Roberton (2017): “Adding Quantity Certainty to a Carbon Tax: The Role of a Tax Adjustment Mechanism for Policy Pre-Commitment,” *Harvard Environmental Law Review*, 41, 41–57.

Martin, I. (2013): “Consumption-based asset pricing with higher cumulants,” *Review of Economic Studies*, 80, 745–773.

Mehra, R. and E. C. Prescott (1985): “The Equity Premium: A Puzzle,” *Journal of Monetary Economics*, 15, 145–161.

Metcalf, G. (2018): “An emissions assurance mechanism: Adding environmental certainty to a carbon tax,” Resources for the future Report.

Morris, J., S. Paltsev, and J. Reilly (2012): “Marginal abatement costs and marginal welfare costs for greenhouse gas emissions reductions: Results from the EPPA model,” *Environmental Modeling and Assessment*, 17, 325–336.

Pindyck, R. (2013): “Climate change policy: What do the models tell us?” *Journal of Economic Literature*, 51, 860–872.

Rothschild, M. and J. Stiglitz (1970): “Increasing Risk I: A definition,” *Journal of Economic Theory*, 2, 225–243.

Rubinstein, M. (1976): “The valuation of uncertain income streams and the pricing of options,” *The Bell Journal of Economics*, 7, 407–425.

Schubert, K. (2008): "La valeur carbone: Niveau initial et profil temporel optimaux," Paris School of Economics.

Van der Ploeg, F. and C. Withagen (2014): "Growth, renewables, and the optimal carbon tax," *International Economic Review*, 55, 283–311.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," *Journal of Monetary Economics*, 24, 401–421.

Appendix: Proof of Proposition 2

Under these two assumptions, combining equation (7) with the property that at equilibrium $\exp(g_t t)$ equals EA'_t/A'_0 implies the following equation:

$$1 = e^{-(\rho+\delta)t} E\left[\frac{A'_t u'(C_t)}{A'_0 u'(C_0)}\right] = e^{-(\rho+\delta)t} E\left[\exp(a'_t - \gamma c_t)\right].$$

Notice that our assumptions implies that $a'_t - \gamma c_t$ is normally distributed with mean $\mu_x - \gamma\mu_c$ and variance $(1-\gamma\phi)^2\sigma_c^2 + \sigma_w^2$. By Stein's Lemma, the above condition can then be rewritten as follows:

$$1 = \exp\left(\left(-\rho - \delta + \mu_p - \gamma\mu_c + 0.5(\phi-\gamma)^2\sigma_c^2 + 0.5\sigma_w^2\right)t\right),$$

or, equivalently,

$$\mu_p + 0.5\phi^2\sigma_c^2 + 0.5\sigma_w^2 = \delta + \rho + \gamma\mu_c - 0.5\gamma^2\sigma_c^2 + \phi\gamma\sigma_c^2. \tag{26}$$

In this economy, the following standard CCAPM formula for the risk-free interest rate can be derived from equation (4):

$$r_{ft} = r_f = \gamma + \gamma\mu_c - 0.5\gamma^2\sigma_c^2. \tag{27}$$

The systematic risk premium π_t is given by equation (6). Using Stein's Lemma twice to estimate r_{ct} given by equation (5) yields the following result:

$$\pi_t = \pi = \gamma\sigma_c^2. \tag{28}$$

Notice also that, using Stein's Lemma again, we have that the expected marginal abatement cost satisfies the following condition:

$$E\frac{A'_t}{A'_0} = E\exp\left(a'_t\right) = \exp\left(\left(\mu_p + 0.5\phi^2\sigma_c^2 + 0.5\sigma_w^2\right)t\right).$$

This implies that the growth rate g of expected marginal abatement cost is a constant given by

$$g = \frac{dEA'_t/dt}{EA'_t} = \mu_p + 0.5\phi^2\sigma_c^2 + 0.5\sigma_w^2.$$

Because in a decentralized economy, the marginal abatement cost is equal to the price of carbon in all states of nature and at all dates, g can also be interpreted as the growth rate of expected carbon price. Combining these properties implies that one can rewrite condition (26) as follows:

$$g = \delta + r_f + \phi\pi. \tag{29}$$

This concludes the proof of Proposition 2. ■

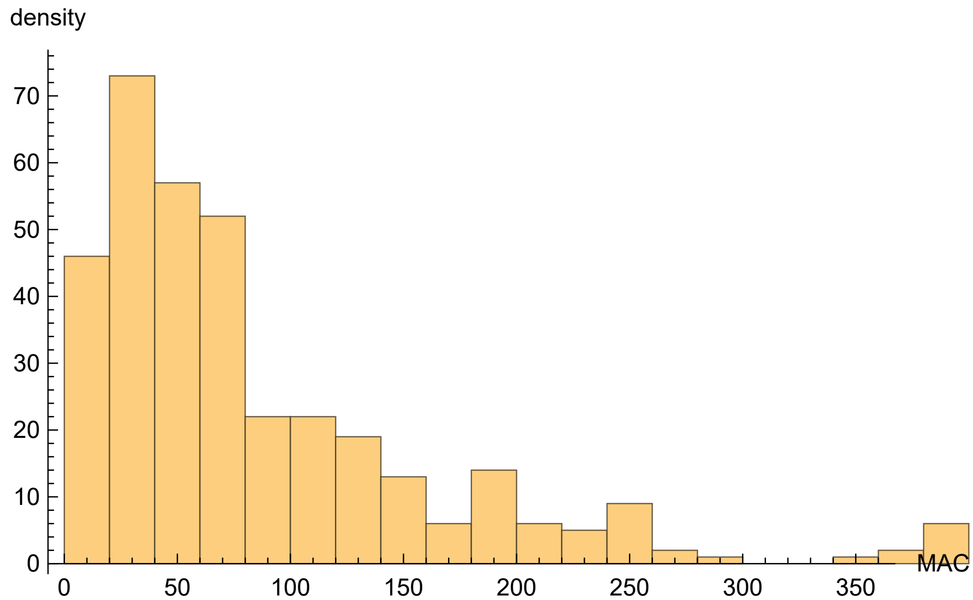

Figure 1: Histogram of the world marginal abatement costs for 2030 extracted from the IPCC database (https://tntcat.iiasa.ac.at/AR5DB). We have selected the 374 estimates of carbon prices (in US\$2005/tCO_2) in 2030 from the IAM models of the database compatible with a target concentration of 450ppm.

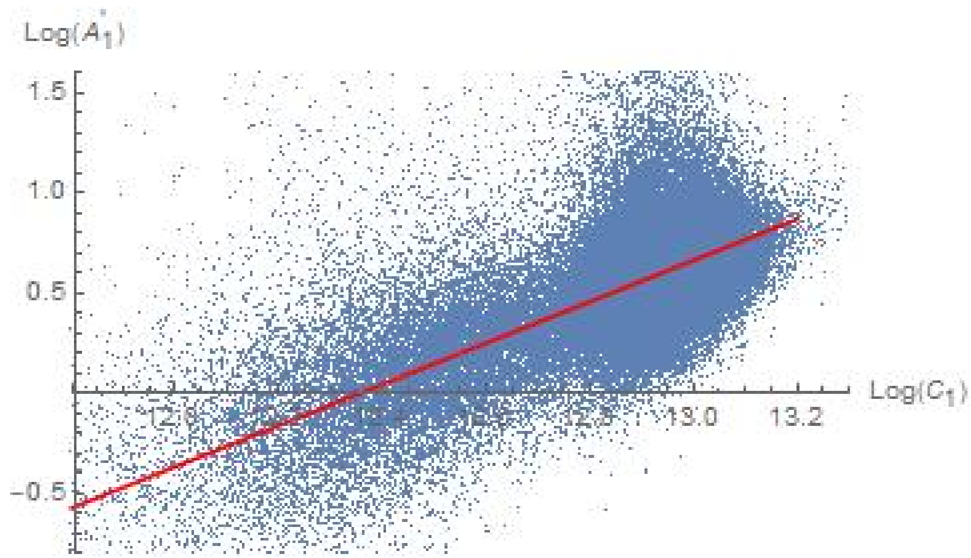

Figure 2: Monte-Carlo simulation under the benchmark case. We used 100.000 draws of the triplets (Y_1, θ, T) to estimate the optimal abatement strategy. The figure illustrates the positive statistical relation between log consumption growth and the log marginal abatement costs (and thus log carbon price) in the second period. The red curve depicts the OLS estimation in log-log, yielding $\log(A_1') = -12.8 + 1.04 \log(C_1) + \varepsilon$.

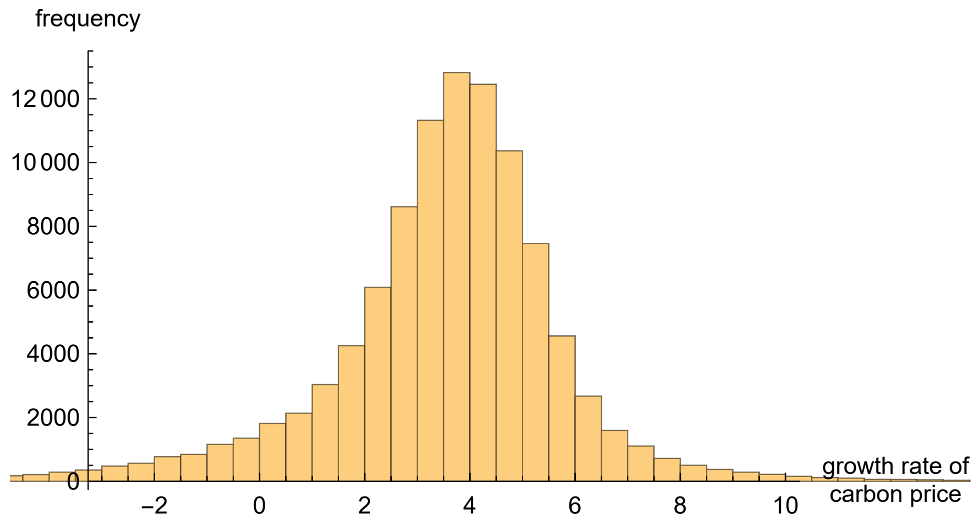

Figure 3: Empirical probability distribution of the annualized growth rate of carbon price under the optimal abatement strategy in the benchmark calibration of the two-period model. The Monte-Carlo simulation uses a sample of 100.000 draws of the triplet (Y_1, θ, T). The growth rate is in percent per year. The mean growth rate is 3.47% and the standard deviation is equal to 2.4%.