"The large space of information structures",
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Abstract : We revisit the question of modeling incomplete information among 2 players, with the idea that a piece of information is only useful if it is valuable, i.e. if it helps to have better payoffs in games. How large is the set of possible situations ?

K being the finite set of possible states of nature (or unknown parameters), an information structure is defined as a probability distribution u with finite support over $K \times \mathbb{N} \times \mathbb{N}$ with the interpretation that : u is publicly known by the players, (k, c, d) is selected according to u, then c (resp. d) is announced to Player 1 (resp. Player 2). Given a payoff structure g, composed of matrix games indexed by the state, the value of the incomplete information game defined by u and g is denoted $\mathrm{val}(u, g)$. We evaluate the distance $d(u, v)$ between 2 equivalent information structures u and v by the supremum of $|\mathrm{val}(u, g) - \mathrm{val}(v, g)|$ for all g with payoffs in $[-1, 1]$, and study the associated metric space.

We first provide a tractable characterization of $d(u, v)$, as the minimal distance between 2 convex sets in a finite-dimensional space, and recover the characterization of Peski (2008) for $u \succeq v$, generalizing to 2 players Blackwell's comparison of experiments via garblings. We then present the links with the universal belief space, and finally show the existence of a sequence of information structures, where players acquire more and more information, and of $\varepsilon > 0$ such that any two elements of the sequence have distance at least ε : having more and more information may lead nowhere. As a consequence, the completion of our metric space is not compact, hence not homeomorphic to the set of consistent probabilities over the states of the world *à la* Mertens and Zamir. This example answers by the negative the second (and last unsolved) problem posed by J.F. Mertens in "Repeated Games", ICM 1986.