

# Learning while Trading: Experimentation and Coasean Dynamics\*

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## Abstract

I study a dynamic bilateral bargaining problem with incomplete information where better outside opportunities may arrive during negotiations. Gains from trade are uncertain. In a good-match market environment, outside opportunities are not available. In a bad-match market environment, superior outside opportunities stochastically arrive for either or both parties. The two parties begin their negotiations with the same belief on the type of the market environment. As arrivals are public information, learning about the market environment is common. One party, the seller, makes price offers at every instant to the other party, the buyer. The seller has no commitment power and the buyer is privately informed about his own valuation. This gives rise to rich bargaining dynamics. In equilibrium, there is either an initial period with no trade or trade starts with a burst. Afterwards, the seller screens out buyers one by one as uncertainty about the market environment unravels. Delay is always present, but it is inefficient only if valuations are interdependent. Whether prices increase or decrease over time depends on which party has a higher option value of learning. The seller exercises market power. In particular, when the seller can clear the market in finite time at a positive price, prices are higher than the competitive price. However, market power need not be at odds with efficiency. Applications include durable-good monopoly without commitment, wage bargaining in markets for skilled workers, and takeover negotiations.

**Keywords:** Bargaining; Coase Conjecture; Learning; Market Experimentation; Delay; Independent vs Interdependent Values; Dynamic Games in Continuous Time.

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# 1 Introduction

Bargaining is ubiquitous. Many economic interactions involve negotiations on a variety of issues. For example, prices of commodities are often the outcome of negotiations between the concerned parties, wages are set as an arrangement between firms and workers, and takeovers require an agreement over the price of the transaction. As such, bargaining relationships are the cornerstone of many theory of markets, from industrial organization to labor economics. Bargaining games with incomplete information, though, are often treated as bilateral monopolies. Yet, in many bargaining situations, better opportunities may become available to either or both parties – parties routinely investigate what their best outside options are, as a large literature on search documents. The goal of this paper is to characterize bargaining dynamics in such an environment in which parties may want to wait in order to learn about their best opportunities during negotiations.

I study a one-sided incomplete information bargaining game between a seller and a privately informed buyer. I show that the option value of learning about the existence of better opportunities is of first-order importance in shaping the bargaining relationship. It affects the timing of sales, the dynamics of prices, surplus division, and the seller's ability to exercise market power. Trade begins with a burst or after a silent period with no agreement. Afterwards, the seller screens out buyers one-by-one as uncertainty unravels. Delay is always present, but not necessarily inefficient. Inefficiently timed sales only occur if valuations are interdependent. Price dynamics depend on which party has a higher option value of learning. When the seller can clear the market in finite time at a positive price, prices are higher than the competitive price. Market power, however, need not be at odds with efficiency.

For concreteness, consider a capacity-constrained supplier of a leading technology making price offers to a downstream buyer who is privately informed about its own valuation for the technology. If the two firms were to negotiate in isolation, they could do no better than reaching an immediate agreement, at least in terms of surplus to share. In innovative industries, however, the supplier's competitors may develop a new disruptive technology in the future. The downstream buyer has then the option to wait for a new technology to arrive and so may only be willing to accept favorable trading conditions. Uncertainty about the outside option may also be present for the supplier. At some point, a new buyer with a higher valuation for the technology or without the time to engage in lengthy negotiations may approach the supplier. In this case, the supplier has the chance to conclude a favorable deal, whereas the downstream buyer loses the opportunity to trade. Parties are unlikely to know for sure whether such opportunities will arrive. The two parties, however, may learn by waiting. For instance, as time elapses with no breakthrough from the supplier's competitor, the original parties revise downwards their beliefs about the chances of such an R&D success. They reach a corresponding conclusion about the existence of buyers interested in the technology if no new buyer shows up. This snapshot of economic activity raises a number of questions.

How do parties choose their bargaining posture in the face of market uncertainty? How do strategies depend on which party hopes for better outside opportunities to arrive? Does the option to wait for the uncertainty to unravel lead to inefficiently late agreements? Or, rather, does the threat to leave the negotiation empty-handed leads the supplier (resp., buyer) to propose (resp., accept) a particularly favorable (resp., unfavorable) deal to (resp., from) the counterparty, to the effect that negotiations conclude inefficiently early?<sup>1</sup> Will the supplier exercise market power as uncertainty unravels in its favor? How will the supplier price his technology over time? How do learning about market opportunities and learning about the downstream buyer's private valuation interact in equilibrium?

In this paper, I develop a framework to answer all these questions. Formally, I study a dynamic bargaining game between two risk-neutral players: a long-lived seller (he) and a long-lived buyer (she). Time is continuous and the time horizon is infinite. The seller has an indivisible durable good (or asset) to sell. His valuation for the good is normalized to zero. The buyer has a privately known type which represents her (positive) valuation for the good. Gains from trade are ex ante uncertain. Negotiations take place in a market that can be of two types. In a market of type 0, no outside opportunity is available. In a market of type 1, superior opportunities arise stochastically on either or both sides according to a Poisson process with commonly known intensity. Bargaining begins with a common prior about the type of the market. The arrival of a superior outside opportunity is public information and concludes the game. Thus, learning about the market type is common. As time elapses with no event occurring, the two parties' belief that the market is of type 1 drifts downwards and an agreement with the current trading partner becomes more attractive. The seller has no commitment power and makes price offers to the buyer at every instant. The buyer accepts or rejects.

Without arrivals (i.e., when it is common knowledge that the market is of type 0) the model reduces to the standard bargaining game with one-sided incomplete information. By the classic Coase Conjecture argument (see [Coase \(1972\)](#); [Stokey \(1981\)](#), [Bulow \(1982\)](#), [Fudenberg, Levine and Tirole \(1985\)](#), [Gul, Sonnenschein and Wilson \(1986\)](#), and [Ausubel and Deneckere \(1989\)](#) formalize the insight), the seller prices at the lowest buyer valuation as soon as negotiations begin; thus, trade occurs “in the twinkling of an eye”, with the seller being unable to extract any rent from the transaction, and the market outcome is efficient. The intuition for the result is simple. For any given price, high valuation buyers are more likely to purchase than low valuation buyers, leading to negative selection in the demand pool. Accordingly, the seller cuts its price over time. Forward-looking consumers expect prices to fall, so they are unwilling to pay a high price in the first place. The seller's inability to commit thus leads its later selves to exert a negative externality on its former selves, reducing its overall profit to the lowest buyer valuation.

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<sup>1</sup>When the option value of learning is high enough, the efficient benchmark is not immediate agreement but rather calls for an optimal degree of delay.

Equilibrium bargaining dynamics drastically change when learning about the availability of superior outside opportunities is taken into account. Such a natural extension of the baseline model allows me to gain new insights on the original problem and to establish a set of novel results. To begin, I show that trade occurs over time in equilibrium. The seller serves different (groups of) buyer types at different points in time and charges them different prices. In particular, trade starts with a burst or following a silent period with no agreement. Afterwards, the seller slowly screens out buyer types one by one while uncertainty about outside opportunities unravels. Additionally, learning endogenously gives rise to either negative or positive selection in the demand pool in equilibrium. The seller begins screening the buyer types either from the top or from the bottom of their distribution, depending on which types have a lower option value of waiting to learn. Absent outside opportunities or learning about their existence, the two parties would either trade immediately, as the market opens, or never reach an agreement.

When parties have the option to learn whether better opportunities are available, immediate agreement is in general not efficient. Under efficiency, trade occurs when the joint benefit of market experimentation, as measured by the sum of the two players' option value from waiting, equals its joint cost, as measured by the foregone gains from trade in terms of discounting. The optimal delay is different for different buyer types. Thus, periods with no trade, as well as bursts of trade, followed by periods where different types trade one by one, are possible as efficient outcomes.

Whether players' incentives point towards inefficient hurry or inefficient delay is unclear. I show that delay is always present in equilibrium. Learning alone, however, only accounts for delay, but not for inefficiently timed agreements. In particular, inefficiently early transactions never occur.<sup>2</sup> Inefficiently late agreements, instead, only arise when the seller's payoff from the outside opportunity is correlated with the buyer type. This dependence endogenously creates a bargaining environment with interdependent values, as in [Evans \(1898\)](#), [Vincent \(1989\)](#), and [Deneckere and Liang \(2006\)](#). The main economic intuition behind the delay is similar to that in those papers.

These results yield three main takeaways. First, they provide a novel and particularly natural rationale for both equilibrium delay and non-trivial trading dynamics in one-side incomplete information bargaining. As striking as it is, the Coase Conjecture is at odds with how negotiations often occur in practice.<sup>3</sup> Second, as inefficient delay only arises when additional frictions are present in the trading environment (namely, interdependent valuations), the Coasean force towards efficiency remains overwhelming when parties are learning about the bargaining environment. Third, the result questions the view that long disputes result in inefficient outcomes: in markets with search and learning, examples of which are countless,

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<sup>2</sup>Except in an extension of the main model, which I discuss at the end of the paper.

<sup>3</sup>For instance, [Ausubel, Cramton and Deneckere \(2002\)](#) argue that the Coase Conjecture "has the unfortunate implication that real bargaining delays can only be explained by either exogenous limitations on the frequency with which bargaining partners can make offers, or by significant differences in the relative degree of impatience between the bargaining parties."

this need not be true.

My model predicts that there is price discrimination in equilibrium. Prices gently decrease or increase over time depending on which party has the higher option value of learning. In particular, if better opportunities may only arrive for the buyer, the price schedule is increasing. If, instead, better opportunities may only arrive for the seller, the price schedule decreases over time. If outside opportunities may arrive to both players, price dynamics may become non-monotone.

I show that the seller exercises market power if he has the option to clear the market in finite time at positive prices. Notably, prices are higher than the competitive price and the seller's payoff is higher than what he would get if he were: *(i)* awaiting for the possible arrival of a superior opportunity; *(ii)* unable to screen using prices; *(iii)* selling to a market in which all buyers had the lowest valuation. Substantial market power is also present when the seller has the option to wait and learn whether superior outside opportunities are available to him. In this case, when valuations are interdependent, prices are higher than the seller's expected present payoff from waiting even without the option to clear the market in finite time at positive prices.

My framework has a number of applications. A prominent one sees the bargaining game as the problem of a monopolist who is selling a perfectly divisible and infinitely durable good to a demand curve of atomless buyers and is unable to commit to future prices. The connection obtains because to every actual buyer type in the durable goods model, there corresponds an equivalent potential buyer type in the bargaining model. With this interpretation, my insights shed new light on the determinants of market power in monopolistic industries. Other applications include takeover negotiations, wage bargaining in markets for skilled workers, and negotiations in the housing market.

My contribution lies also on the methodological side. Posing the model in continuous time allows for additional economic insights. In particular, continuous time captures the idea that there are no institutional frictions in the bargaining protocol (in addition to incomplete information). Thus, my analysis clearly distinguishes the effect of learning about the bargaining environment on equilibrium outcomes from that of other frictions in the protocol. Additionally, optimality conditions, equilibrium strategies, and equilibrium outcomes have a relatively simple characterization in continuous time. These conditions are described by means of Hamilton-Jacobi-Bellman equations and (solutions to) partial and ordinary differential equations with a clear economic intuition. Closed-form solutions and relatively simple expressions for all the relevant outcomes of the game open the doors to comparative statics as well as to empirical studies and more applied research.<sup>4</sup>

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<sup>4</sup>Fuchs and Skrzypacz (2010, 2013a), Ortner (2017), and Daley and Green (2017) achieve a similar simplification. I refer to Section 1.1 for further discussion.

## 1.1 Related Literature

This paper joins a recent literature that explicitly models stochastic features of the environment where the bargaining game with incomplete information takes place. [Fuchs and Skrzypacz \(2010\)](#), [Huang and Li \(2013\)](#), [Daley and Green \(2017\)](#), [Ortner \(2017\)](#), and [Ishii, Öry and Vigier \(2017\)](#) are the contributions closest to mine. [Huang and Li \(2013\)](#) analyze a discrete-time bargaining game where a seller makes all price offers to a privately informed buyer. A new buyer with a higher valuation for the seller’s good may arrive in the future. As time elapses with no arrival, the two players revise downwards their common belief about the existence of such a buyer. Their main result shows that prices fluctuate in equilibrium. This is so because the seller posts a price at the very beginning of each time period and, in discrete-time, he has to commit to that price for the whole period. I can specialize my model to capture their setup by assuming independent private values, that there is a gap between the seller and the lowest buyer’s valuation, and that outside opportunities, if existing, are only available to the seller. In this case, I show that price fluctuations disappear in continuous time. The price, instead, gently declines over time as the seller becomes more pessimistic about his outside opportunities. An interpretation of the difference in our findings is that price fluctuations are not driven exclusively by the option value of learning; rather, they result from the combined effect of learning with that of other frictions in the protocol (e.g., some form of commitment power).

[Fuchs and Skrzypacz \(2010\)](#) study a one-sided incomplete information bargaining game where a new trader arrives according to a Poisson process. There are two main differences between their setup and mine. First, in my setting there is uncertainty about the market type; in contrast, in their setup outside options arrive stochastically, but are known to exist (i.e., it is common knowledge that the market is of type 1). Thus, learning about the bargaining environment and learning about the buyer’s private information do not interact in their model. Second, in their setup arrivals do not correspond to superior, but rather to alternative, trading opportunities. Therefore, the efficient benchmark calls for immediate agreement, and not for optimal market experimentation.<sup>5</sup> These contrasts lead to distinct insights and equilibrium dynamics. [Fuchs and Skrzypacz \(2010\)](#) show that trade occurs over time only if valuations are interdependent. With independent private values, instead, trade occurs either immediately or never (see also [Inderst \(2008\)](#)). In contrast, when learning about the market environment interacts with learning about parties’ private information, I show that the seller serves different buyer types at different points in time and charges them different prices even when private valuations are independent. A common insight of our models, however, is that interdependent valuations are necessary for inefficiently timed transactions. [Fuchs and Skrzypacz \(2010\)](#) also propose the following generalization of the Coase Conjecture: although there is inefficient

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<sup>5</sup>In addition, they write the model in discrete time and then study the atomless continuous-time limit of stationary perfect Bayesian equilibria. In contrast, I pose the game directly in continuous time and develop a suitable framework for equilibrium analysis.

delay and the price does not drop immediately to zero, the Coasean dynamics force down the seller's profit to his outside option. In my setting, this insight only holds if the seller cannot clear the market in finite time at a positive price. In contrast, when the seller has the option to do so, I show that he prices above his marginal cost, may exercise substantial market power, and his payoff is larger from that he would obtain by simply awaiting for the possible arrival of an outside opportunity. The result holds independently of whether private valuations are interdependent or not.<sup>6</sup>

[Daley and Green \(2017\)](#) propose a one-sided incomplete information bargaining model with news. Their setup differs from mine in two relevant aspects. First, in their setting news are about the informed party's private information, and not about the bargaining environment. Second, the social value of waiting for news is nil, and so efficiency calls for immediate agreement.<sup>7</sup> They show that the uninformed party's ability to leverage public information to extract more surplus from the transaction is remarkably limited. They suggest a novel interpretation of the Coase Conjecture: because of his perfect lack of commitment, the uninformed party derives no benefit from the ability to screen using prices. The equilibrium may involve delay of trade and positive profits, depending on the environment, but the uninformed party's payoffs must be exactly what it would receive if it were unable to make offers at all. In contrast, I show the the seller's payoff exceeds what he would get if he were unable to screen using prices when he has the option to clear the market in finite time at a positive price.

[Ortner \(2017\)](#) studies the problem of a durable-good monopolist who lacks commitment power and whose marginal cost of production varies stochastically over time. He suggests a generalization of the Coase Conjecture according to which the monopolist seller earns the same profit as he would earn if he were selling to a market in which all consumers had the lowest valuation. I show that the seller's profit is larger than this lower bound when he has the option to clear the market at a positive price in finite time. [Ortner \(2017\)](#) also shows that the seller exercises market power if the distribution of buyers valuations is discrete but is unable to do so when there is a continuum of types. In contrasts, my findings on market power do not rely on the distribution of buyers valuations being discrete.

[Ishii et al. \(2017\)](#) study wage bargaining between a worker and two firms, with public learning about worker-specific productivity. Firms make take-it-or-leave-it offers over time, and hiring is irreversible. Search frictions delay the arrival of one firm, the entrant, while informational frictions prevent the incumbent from observing the entrant's arrival. They show that the combined effect of search and informational frictions induces unraveling in all equilibria: parties reach inefficiently early agreements and the average talent of hired workers is lower than socially optimal. They also show that without market frictions, or when the search friction is present whereas the informational friction is not, there is no unraveling

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<sup>6</sup>My paper also adds to the literature studying the role of outside options or the arrival of new traders in bargaining games with asymmetric information. Notable contributions are [Fudenberg, Levine and Tirole \(1987\)](#), [Samuelson \(1992\)](#), [Chang \(2015\)](#), and [Hwang \(2016\)](#)

<sup>7</sup>In addition, in [Daley and Green \(2017\)](#) learning about private information occurs via the observation of a public Brownian news process, and not in a Poisson bandit framework.

in equilibrium. In my model, one can interpret the arrival of the outside opportunity as a breakthrough in some underlying (on-the-market) search activity that parties are engaged in in parallel to their negotiations. With this interpretation, my results say that search and learning do not give rise to inefficient bargaining outcomes, unless they are paired with an additional friction in the environment, which in my model takes the form of interdependent valuation. At a high level, this insight is evocative of the one of [Ishii et al. \(2017\)](#), who show that learning and the search friction do not give rise to inefficiencies, unless they are paired with the informational friction.

My work relates to the recent contribution by [Nava and Schiraldi \(2016\)](#), who analyze the problem of a durable-good monopolist who sells multiple varieties of a good. They show that, in such settings, the seller regains the ability to command strictly positive profits. They propose a novel interpretation of the Coase Conjecture by arguing that the force driving any Coasean equilibrium is market-clearing, and not competition or efficiency. This is so because any market-clearing price (that is, any price at which all consumers are willing to buy) provides a credible commitment to the monopolist (as it is no longer compelled to lower prices). Although our settings are very distinct and they do not model any dynamic feature of the bargaining environment, their result is reminiscent of my finding that a monopolist seller exercise market power any time he has the option to clear the market at a positive price in finite time. Similarly to me, they find that profit-maximizing market-clearing prices are neither competitive nor efficient.

From a methodological viewpoint, I add to the work that deals with the technical complexities that arise when modeling bargaining games in continuous time. In particular, I build on [Daley and Green \(2017\)](#) and [Ortner \(2017\)](#) to develop an ad hoc equilibrium notion for the game I study by introducing strategy restrictions directly into the equilibrium definition. Although not being fully Nash, the equilibrium concept captures the key features of any Perfect Bayes (stationary) analysis of the discrete-time analog of the model.<sup>8</sup>

More broadly, my work relates to the literature on equilibrium delay in bargaining (beyond interdependent values). For example, delay occurs in a model with two-sided private information about fundamentals and overlap in values (e.g., [Cramton \(1984\)](#), [Chatterjee and Samuelson \(1987\)](#), and [Cho \(1990\)](#)), with reputational concerns (e.g., [Abreu and Gul \(2000\)](#), [Compte and Jehiel \(2002\)](#), and [Atakan and Ekmekci \(2014\)](#)), with higher order uncertainty ([Feinberg and Skrzypacz \(2005\)](#)), with disagreement about continuation play ([Yildiz \(2004\)](#)), with externalities ([Jehiel and Moldovanu \(1995\)](#)), with the possibility that players can commit to not responding to offers ([Admati and Perry \(1987\)](#) and [Freshtman and Seidmann \(1993\)](#)), or when outside options are history-dependent ([Compte and Jehiel \(2004\)](#)). Efficient delay may emerge in bargaining games with complete information where the size of the cake and

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<sup>8</sup>Other notable contributions on bargaining games in continuous time, although less immediately connected to my work, are [Perry and Reny \(1993\)](#), [Sákovics \(1993\)](#), [Ambrus and Lu \(2015\)](#), and [Ortner \(2016\)](#). More broadly, my paper relates to literature that uses continuous-time techniques to study strategic interactions.

the identity of the proposer evolves stochastically over time (Merlo and Wilson (1995) and Merlo and Wilson (1998)). (Inefficient) delay when players may receive new information while bargaining also arises in the complete information setting of (Avery and Zemsky (1994)). In my environment, there is incomplete information and the identity of the proposer is fixed; in a similar spirit, however, parties do not trade as long as the option value of waiting for news is, in expectation, sufficiently larger than the surplus from the transaction.

My paper also relates to the literature that checks the robustness of the Coase's insight or identifies different ways in which a dynamic monopolist can exercise market power. For instance, a monopolist could relax its commitment problem and increase its profit by renting the good rather than selling it (Bulow (1982)), by introducing best-price provisions (Butz (1990)), or by introducing new updated versions of the durable good over time (Levinthal and Purohit (1989), Waldman (1993, 1999), Choi (1994), Fudenberg and Tirole (1998), and Lee and Lee (1998)). Instead, the Coase Conjecture would fail altogether when the monopolist faces capacity constraints (Kahn (1986) and McAfee and Wiseman (2008)), with entry of new buyers (Sobel (1991)), when buyers' valuations are subject to idiosyncratic stochastic shocks (Biehl (2001), Deb (2014), and Garrett (2016)), when buyers can exercise an outside option (Board and Pycia (2014)), when goods depreciate over time (Bond and Samuelson (1987)), and when demand is discrete (Bagnoli, Salant and Swierzbinski (1989), von der Fehr and Kuhn (1995), and Montez (2013)). I identify a novel and particularly natural setting where a monopolist seller can exercise market power.

Finally, I model learning and market experimentation building on the exponential bandit framework pioneered by Keller, Rady and Cripps (2005).

## 1.2 Road Map

Section 2 describes the general model and formalizes the equilibrium notion. Section 3 presents two benchmark cases: efficient trade and the bargaining game without arrivals. Sections 4-6 contain the main results. To gain insight, I develop the analysis in three steps. Section 4 characterizes equilibrium bargaining dynamics when superior opportunities, if existing, are only available to the buyer. Section 5 analyzes bargaining dynamics when superior opportunities, if existing, are only available to the seller. Section 6 builds on the insights developed in the two previous sections and extends the analysis to the general bargaining game where superior opportunities, if existing, stochastically arise on both sides of the transaction. Section 7 contains extensions and robustness checks. It also presents the other relevant benchmark case: the bargaining game with arrivals but no learning about the market type. Section 8 concludes. The main text contains a detailed account of the equilibrium characterization and develops the relevant intuition. The more technical proofs, as well as additional details of the formal analysis, are in Appendices A-D.

## 2 The Model

This section is divided into two parts. In the first part, I present the general bargaining game and its benchmark specifications. I also discuss the main assumptions of the model in detail. In the second part, I formalize players' strategies and the equilibrium notion.

### 2.1 The Bargaining Game

There are two players, a seller and a buyer. The seller has an indivisible durable good (or asset) to sell. His valuation for the good is normalized to zero. The buyer has a privately known type  $v \in [\underline{v}, \bar{v}]$  that represents his valuation for the good. I assume  $\bar{v} > \underline{v} \geq 0$ . Type  $v$  is distributed according to a c.d.f.  $F$ , which is an atomless distribution with full support and density  $f$ . Following the standard terminology in the literature, if  $\underline{v} > 0$  (resp.,  $\underline{v} = 0$ ), I refer to the model as the “gap” case (resp., “no gap” case) bargaining game. Hereafter, I use the words type and valuation interchangeably.

Time, denoted by  $t$ , is continuous. The game starts at time zero and has a potentially infinite horizon:  $t \in \mathbb{R}_+ \cup \{+\infty\}$ . Players are long-lived, risk-neutral expected utility maximizers with common discount rate  $r > 0$ . The seller has no commitment power and makes a price offer  $p_t$  to the buyer at every instant  $t$ . If the buyer accepts the price offer  $p_t$  at time  $t$ , trade is executed and the game ends. If so, the seller's payoff is  $e^{-rt} p_t$  and the buyer's payoff is  $e^{-rt}(v - p_t)$ .

Negotiations take place in a market of type  $m \in \{0, 1\}$ . If  $m = 1$ , an event stochastically occurs according to a Poisson process with intensity  $\lambda > 0$ . If  $m = 0$ , such event never occurs. The type of the market is unknown to both players, who share a common prior  $\mu^0 \in (0, 1)$  on  $m = 1$  at the beginning of the game. The arrival of the event at time  $t$  is public and concludes the game with (possibly) type-dependent payoffs  $e^{-rt} O^S(v)$  for the seller and  $e^{-rt} O^B(v)$  for the buyer. For now, think of the event as a reduced-form of some continuation play, and of  $O^S(v)$  and  $O^B(v)$  as the reduced-form payoffs associated to it. More structure will be imposed momentarily. The joint surplus conditional on the arrival of the event is  $O(v) := O^S(v) + O^B(v)$ .

The model has *independent private values* if the function  $O^S(v)$  is constant. When  $O^S(v)$  is not constant in the buyer type, the correlation of the seller's outside option with the buyer valuation endogenously creates a bargaining environment with *interdependent values*.

For future reference, define

$$\underline{O}^S(k) := \int_{\underline{v}}^k O^S(v) [f(v)/F(v)] dv = \mathbb{E}[O^S(v) \mid v \leq k]$$

as the seller's expected payoff conditional on the arrival of the event and the buyer type being

distributed according to the right-truncation of  $F$  over  $v \in [\underline{v}, k]$ ; moreover, let

$$\bar{O}^S(k) := \int_k^{\bar{v}} O^S(v) [f(v)/(1 - F(k))] dv = \mathbb{E}[O^S(v) \mid v \geq k]$$

be the seller's expected payoff conditional on the arrival of the event and the buyer type being distributed according to the left-truncation of  $F$  over  $v \in [k, \bar{v}]$ .

Either a transaction or the occurrence of an event conclude the game. Thus, whenever in the paper I refer to time  $t$ , I do so with the understanding that the game is still in place by then.

Since the arrival of the event is public, the seller and the buyer always share the same belief about the market type. To derive the law of motion of the common belief, suppose the two players start with the belief  $\mu_t$  at time  $t$  and no event occurs in the interval  $[t, t + dt)$ . By Bayes' rule, the updated belief at the end of the time interval is

$$\mu_t + d\mu_t = \frac{\mu_t(1 - \lambda dt)}{1 - \mu_t + \mu_t(1 - \lambda dt)}.$$

Simplifying, we obtain that, as long as no event occurs, the common belief changes by  $d\mu_t = -\lambda\mu_t(1 - \mu_t)dt$ ; its law of motion is described by the ordinary differential equation (henceforth, ODE)

$$\dot{\mu}_t = -\lambda\mu_t(1 - \mu_t), \tag{1}$$

with solution

$$\mu_t = \frac{\mu^0 e^{-\lambda t}}{\mu^0 e^{-\lambda t} + (1 - \mu^0)}. \tag{2}$$

Thus, if the event does not occur, the common belief on  $m = 1$  drifts downwards over time. Once the event occurs, instead, the belief jumps to 1 and the game ends.

For future reference, let  $(\Omega_A, \mathcal{A}, \mathbb{P}_A)$  be the (sufficiently rich) probability space where the Poisson process describing the arrival of the event when  $m = 1$  is defined. Moreover, let  $\mathbb{A} := (\mathcal{A}_t)_{t \geq 0}$  be the natural filtration of  $(\Omega_A, \mathcal{A}, \mathbb{P}_A)$  associated to the process. The filtration  $\mathbb{A}$  is complete and right-continuous. Moreover, denote with  $t_\mu$  the time at which the common belief  $\mu_t$  equals  $\mu \in [0, \mu^0]$ , with the convention that  $t_0 = +\infty$ . Since  $\mu_t$  strictly decreases over time,  $t_\mu$  is well-defined.

The heuristic timeline within a single ‘‘period’’  $[t, t + dt)$  is the following:

- (i) The period begins with a common belief  $\mu_t$  on  $m = 1$ .
- (ii) If the market is of type  $m = 1$ , the event occurs with instantaneous probability  $\lambda dt$ , terminating the game, and players collect payoffs; with complementary probability, no event occurs. If the market is of type  $m = 0$ , no event occurs. From the agents' viewpoint, the event occurs with subjective probability  $\mu_t \lambda dt$ .

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<sup>9</sup>Here,  $e^{-\lambda t}$  is the probability that the event has not occurred by time  $t$  if the market is of type  $m = 1$ .

- (iii) If no event occurs, the seller makes a price offer  $p_t$ , which the buyer accepts or rejects:
  - (a) If the buyer accepts, the game ends and players collect payoffs.
  - (b) If the buyer rejects, players update their belief about the bargaining environment to  $\mu_t + d\mu_t$ , and the game moves to the next period.<sup>10</sup>

The arrival of the event represents a *superior* opportunity arising for either or both players. In a real-world application, the event may correspond to one of the two parties finding an alternative trading partner or a more satisfactory use of his resources, thus disappearing from the original negotiation. It may also capture the arrival of a new agent (privately) offering better terms of trade to one of the two players. The event may represent a breakthrough in some underlying (on-the-market) search activity that parties are engaged in in parallel to their negotiations. Alternatively, it may correspond to a major technological step forward rendering obsolete the object that is originally for sale. Many natural interpretations of the event (and the associated learning process) are possible. The focus of this paper, however, is not to model explicitly what the event stands for or the strategic interaction it gives rise to. Rather, it is to investigate how search for superior deals and learning about their existence during negotiations affect bargaining dynamics. In this spirit, I replace the event with the payoffs  $O^S(v)$  and  $O^B(v)$  that would arise upon its arrival. Assumption 1 below introduces the relevant restrictions on the payoffs  $O^S(v)$  and  $O^B(v)$ . These restrictions are minimal and only reflect the motivation of the paper. Therefore, the framework is flexible enough to capture a variety of applications.<sup>11</sup>

**Assumption 1.** *Throughout the paper, I assume the following.*

A1  $O^S(v)$  and  $O^B(v)$  are non-negative differentiable functions.

A2  $O(v) > v$  for all  $v \in [\underline{v}, \bar{v}]$ .

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<sup>10</sup>The results of the paper hold unchanged under the following alternative specification of the timing within a single “period”  $[t, t + dt)$ :

- (i) The period begins with a common belief  $\mu_t$  on  $m = 1$ .
- (ii) The seller makes a price offer  $p_t$ , which the buyer accepts or rejects.
- (iii) If the buyer accepts, the game ends and players collect payoffs.
- (iv) If the buyer rejects:
  - (a) If the market is of type  $m = 1$ , the event occurs with probability  $\lambda dt$ , terminating the game, and players collect payoffs; with complementary probability, no event occurs.
  - (b) If the market is of type  $m = 0$ , no event occurs.

From the agents’ viewpoint, the event occurs with subjective probability  $\mu_t \lambda dt$ . If no event occurs, players update their belief about the bargaining environment to  $\mu_t + d\mu_t$  and the game moves to the next period.

Under the timing convention I adopt, however, the notation and the analysis are cleaner.

<sup>11</sup>The same argument justifies the assumption that the arrival of the event is public and concludes the game. In particular, this paper neither studies the role of transparency of outside options on bargaining dynamics (see, e.g., [Hwang and Li \(2017\)](#)), nor how negotiations evolve in the shadow of existing outside options that players may decide to exercise (see, e.g., [Lee and Liu \(2013\)](#) and [Board and Pycia \(2014\)](#)).

A3  $O^B(v)/v$  is either strictly monotone on  $(\underline{v}, \bar{v}]$ , or identically equal to zero;  $O^S(v)/v$  is either strictly monotone on  $(\underline{v}, \bar{v}]$ , or identically equal to zero.

A4  $v/O(v)$  is strictly monotone on  $[\underline{v}, \bar{v}]$ .

A5 Learning is non-trivial. That is, if  $v/O(v)$  is strictly increasing (resp., decreasing), there exists  $v^* \in (\underline{v}, \bar{v}]$  (resp.,  $v^* \in [\underline{v}, \bar{v})$ ) such that

$$\frac{\mu^0 \lambda}{\mu^0 \lambda + r} O(v) > v \quad \text{for all } v < v^* \quad (\text{resp., for all } v > v^*).$$

In part A1, differentiability is a technical requirement, but neither carries relevant economic meaning nor imposes restrictions affecting the insights of the model.<sup>12</sup>

Part A2 says that the joint surplus associated to the arrival of the event is larger than the joint surplus from the transaction. That is, in a market of type  $m = 1$  there are superior opportunities available to the two players, at least in terms of total surplus to share. The insights of the paper remain unchanged if  $O(v) > v$  only holds for buyer types in a positive-measure subset of  $[\underline{v}, \bar{v}]$ . Assuming  $O(v) > v$  for all  $v$  only simplifies the exposition.

To understand A3, observe that the ratio  $O^B(v)/v$  captures how attractive to the buyer of type  $v$  the outside alternative is, if available, compared to the seller's good. Therefore,  $O^B(v)/v$  is a measure of how eager to trade the buyer of type  $v$  is. When  $O^B(v)/v$  is strictly decreasing (resp., increasing), lower (resp., higher) types are more eager to trade immediately. In particular, the strict monotonicity of  $O^B(v)/v$  guarantees that some version of the *skimming property* holds (see below).

Similarly, the ratio  $O^S(v)/v$  describes how attractive to the seller the outside alternative is compared to an agreement with the buyer of type  $v$ . So,  $O^S(v)/v$  is a measure of how eager to trade with the buyer of type  $v$  the seller is. The strict monotonicity of  $O^S(v)/v$  is necessary for the existence of regions where the seller smoothly screens out buyer types in equilibrium. I discuss in Section 7.3 bargaining dynamics when  $O^S(v)/v$  is monotone, but not strictly so.

Part A4 simplifies the exposition, but does not affect the main results. In particular, A4 only fails in “pathological” cases when A3 holds.

Part A5 is central to the paper. It says that at time zero the social value of waiting for new developments is larger than the value of trading, at least for a positive measure of buyer types. Thus, gains from trade are ex ante uncertain. The uncertainty, however, unravels over time if players postpone reaching an agreement and engage in market experimentation. Therefore, some delay in the transaction may be efficient. Importantly, players may individually learn over time that there are gains from trade, but this fact does not necessarily become common

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<sup>12</sup>Relaxing the assumption that  $O^B(v)$  is non-negative may generate inefficiently early transactions in equilibrium. Although this is an interesting extension of the model, it is omitted from the current version of the paper.

knowledge. Whether it does in equilibrium depends on the specific assumptions that are imposed on the model, and affects trading dynamics and other equilibrium outcomes. I will discuss this point extensively in the next sections.

Assumption A5 is silent with respect to the two parties' individual incentives to actually postpone or advance the transaction in time. By A4,  $v/O(v)$  is strictly monotone on  $[\underline{v}, \bar{v}]$ . In particular, when  $v/O(v)$  is increasing (resp., decreasing), the social value of waiting for new developments is greater the lower (resp., higher) the buyer's valuation is. If  $v/O(v)$  is increasing (resp., decreasing) and A5 holds with  $v^* = \bar{v}$  (resp.,  $v^* = \underline{v}$ ), immediate trade is never (i.e., for no buyer type) efficient.

Finally, I make the standard assumption that the game is common knowledge among the players.

The analysis will make clear the precise role of the restrictions in Assumption 1. Meanwhile, note that they are natural in the settings this paper aims to model.

In the next sections I characterize bargaining dynamics in three benchmark specifications of the general model. Each of them corresponds to a different market configuration.

**Buyers' Market.** By setting  $O^S(v) := 0$  for all  $v \in [\underline{v}, \bar{v}]$ , the seller does not reap any benefit from the potential arrival of the event. This is an extreme form of a buyers' market, where better opportunities, if existing, only benefit the buyer.

**Sellers' Market.** By setting  $O^B(v) := 0$  for all  $v \in [\underline{v}, \bar{v}]$ , the buyer does not reap any benefit from the potential arrival of the event. This is an extreme form of a sellers' market, where better opportunities, if existing, only benefit the seller.

**General Market.** In a general market, post-arrival payoffs are non-trivial for both parties. One way to think of a general market is to assume that the arrival of the event alters both parties payoffs at the same time. Alternatively, one may assume that, upon arrival, the event is favorable to the seller with probability  $\alpha$  and to the buyer with probability  $1 - \alpha$ . Since players are risk neutral, this is a parsimonious way to model the possibility that either side of the transaction may benefit from the existence of better deals independently of the other side.

### 2.1.1 The Benefits of Continuous Time

I formulate the model directly in continuous time for two reasons. First, continuous time captures the idea that there are no institutional frictions in the bargaining protocol, in addition to private information, and that the seller loses all his commitment power.<sup>13</sup> As a consequence, my analysis clearly disentangles the role of learning on bargaining dynamics from that of other frictions in the trading environment. Second, equilibrium strategies in discrete-time bargaining games are in general analytically intractable. In contrast, they are

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<sup>13</sup>The two interpretations are mathematically equivalent

easier to characterize in continuous time. Moreover, continuous-time methods are particularly suitable to perform the option value calculations that arise when studying learning problems of this kind. As a result, I will be able to describe optimality conditions, as well as equilibrium strategies and outcomes, by means of Hamilton-Jacobi-Bellman equations and (solutions to) partial and ordinary differential equations which carry a clear economic intuition. Closed-form solutions and relatively simple expressions for equilibrium outcomes open the doors to comparative statics. In addition, when the model is specialized to particular applications, they yield sharp predictions for empirical studies and more applied research.

## 2.2 Strategies and Equilibrium Notion

### Preliminaries

There are well-known technical difficulties that arise when modeling games in continuous time (see, in particular, [Simon and Stinchcombe \(1989\)](#) and [Bergin and MacLeod \(1993\)](#)). To address these issues, I introduce an ad hoc equilibrium concept for the bargaining game I study. My equilibrium notion, which builds on [Daley and Green \(2017\)](#) and [Ortner \(2017\)](#), captures a set of basic properties that would hold in any perfect Bayesian equilibrium of a discrete-time counterpart of the model. I collect these features in the following remarks.

**Remark 1.** In equilibrium, the buyer solves an optimal stopping problem. Given his type, the evolution of the common belief on the market type, the seller’s pricing rule, and conditional on the event not having occurred, the buyer decides when to accept the offer and conclude the bargaining process.

**Remark 2.** In equilibrium, the buyer types remaining at the end of each time “period” are a truncated sample of the original distribution. This is the so called *skimming property*. The ratio  $O^B(v)/v$  is a measure of how eager to trade the buyer of type  $v$  is and, by Assumption 1-A3, it is strictly monotone. When  $O^B(v)/v$  is strictly decreasing, it is more costly for the high types to delay trade than it is for the low types. Thus, at the end of each time “period”, the pool of remaining buyer types is a right-truncation of the original type distribution, implying that *negative selection* (in  $v$ ) occurs in equilibrium. In contrast, when  $O^B(v)/v$  is strictly increasing, it is more costly for the low types to delay trade than it is for the high types. Thus, at the end of each time “period”, the pool of remaining buyer types is a left-truncation of the original type distribution, implying that *positive selection* arises in equilibrium.<sup>14</sup>

**Remark 3.** The current truncation of the original type distribution describes the seller’s current belief on the buyer type. Therefore, given (ii), the type defining the current truncation (hereafter, the *cutoff type*), together with the current belief on the market type, describe the payoff-relevant state of the game. These are natural state variables on which the seller can

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<sup>14</sup>Under Assumption 1-A3, the skimming property follows by standard arguments, which I thus omit. See, for instance, [Fudenberg et al. \(1985\)](#).

condition his strategy. In particular, this is so when focusing on stationary strategies (as I will), where the seller conditions his price offer at each point in time only on the current cutoff type and the current belief.

**Remark 4.** To any given equilibrium price history, there corresponds a history of realized cutoff types. Thus, along the equilibrium path the seller can be thought of as choosing his own future beliefs on buyer types (as described by the future path of cutoff types) as a function of his current belief (as described by the current cutoff type), instead of choosing a price schedule.

**Remark 5.** At each point in time, the willingness to pay of the buyer of type  $v$  is the difference between his valuation and the current present discounted value of waiting for the outside alternative. At  $t = t_\mu$  this difference is

$$v - \frac{\mu\lambda O^B(v)}{\lambda + r}. \quad (3)$$

In any discrete-time analog of the model, it is straightforward to show that if the seller proposes at time  $t = t_\mu$  a price that is smaller than (3) for *all* buyer types that have not traded by time  $t_\mu$ , then all remaining types accept the price offer and the game concludes.<sup>15</sup>

Such a time and price combination need not exist in my model. Suppose  $\underline{v} = 0$  and that  $O^B(v)/v$  is strictly decreasing, so that negative selection arises in equilibrium. In this case, at each time  $t \in \mathbb{R}_+$ , (3) is strictly negative for a positive-measure subset of buyer types. Therefore, there is no finite time at which the seller can make a non-negative price offer that all remaining buyer types accept.<sup>16</sup>

In the two next subsections I build on the previous insights to introduce the equilibrium notion. Formalizing players' strategies and equilibrium conditions consistently, however, will require the introduction of some technical concepts and notation.

## Equilibrium Conditions

To begin, I lay out the components of and requirements for equilibrium.

### Consistency – Part 1

The first consistency condition is simple: The two players share a common belief about the market type; if neither the event nor trade has occurred by time  $t$ , the common belief is derived by Bayes rule given the information available at the time. Formally, we have the following.

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<sup>15</sup>I refer to [Fudenberg and Tirole \(1991\)](#) for an argument along these lines. A similar reasoning applies here, once appropriately extended to the dynamic setting I consider.

<sup>16</sup>A negative price offer can never be part of an equilibrium.

**Equilibrium Condition 1** (Consistency – Part 1). For all  $t \in \mathbb{R}_+$ , the two players share a common belief  $\mu_t$  on  $m = 1$ . The process  $(\mu_t)_{t \geq 0}$  is a time-homogeneous  $\mathcal{A}_t$ -Markov process adapted to  $\mathbb{A}$ . It is càdlàg and piecewise differentiable, with a random jump time  $T$  defined by the arrival of the event, and satisfies the ODE in (1) for all  $t < T$ , with initial condition  $\mu_0 = \mu^0$ ; the common belief  $\mu_t$  on  $m = 1$  is given by (2) for all  $t < T$ , with  $\mu_t = 1$  for  $t \geq T$ .<sup>17</sup>

From the viewpoint of the two players, the arrival of the event follows a Poisson process with subjective intensity function  $t \mapsto \lambda_t := \mu_t \lambda$ , where  $\mu_t$  is the solution to the ODE in (1). I denote with  $T_\mu$  the first jump time associated to this Poisson process with initial condition  $\mu_0 = \mu$ . The distribution of  $T_\mu$  is uniquely determined by the law of the process  $(\mu_t)_{t \geq 0}$  and  $T_\mu$  is  $\mathcal{A}_t$ -measurable for all  $t \geq 0$ .

## Stationarity

To define continuous-time strategies consistently, I introduce the auxiliary filtered probability space  $(\Omega_F, \mathcal{F}, \mathbb{F}, \mathbb{P}_F)$  where: (i) the underlying probability space  $(\Omega_F, \mathcal{F}, \mathbb{P}_F)$  is complete; (ii) the filtration  $\mathbb{F} := (\mathcal{F}_t)_{t \geq 0}$  of the probability space  $(\Omega_F, \mathcal{F}, \mathbb{P}_F)$  is both complete and right-continuous. I assume that the filtration  $\mathbb{F}$  is independent of all other stochastic elements in the model (namely, the buyer type and the arrival of the event).<sup>18</sup> Moreover, let  $\{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$  be the class of monotone càdlàg stochastic processes adapted to the filtration  $\mathbb{F}$  describing the possible paths of the cutoff type, one path for each initial cutoff type  $k \in [\underline{v}, \bar{v}]$ .

Hereafter, whenever  $(K_t, \mu_t) = (k, \mu) \in [\underline{v}, \bar{v}] \times [0, \mu^0]$ , I say that the game is in *state*  $(k, \mu)$  at time  $t$ . Since the arrival of the event concludes the game, any state  $(k, \mu)$  with  $\mu = 1$  does not play any role in the analysis.

In keeping with the literature, I focus on behavior that is stationary, using the current cutoff type and the current belief about the type of the market as state variables. Stationarity requires that, as long as the game is still in place, both the current price offer and the evolution of the cutoff type depend only on the current state of the game. Formally, we have the following.

**Equilibrium Condition 2** (Stationarity). The seller's price offer in state  $(k, \mu)$  is given by  $P(k, \mu)$ , where  $P: [\underline{v}, \bar{v}] \times [0, \mu^0] \rightarrow \mathbb{R}$  is a Borel-measurable function, and  $\{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$  is a class of time-homogeneous  $\mathcal{F}_t$ -Markov process.

Define the filtered probability space  $(\Omega, \mathcal{G}, \mathbb{G}, \mathbb{P}) := (\Omega_F \times \Omega_A, \mathcal{F} \otimes \mathcal{A}, (\mathcal{F}_t \otimes \mathcal{A}_t)_{t \geq 0} \mathbb{P}_F \times \mathbb{P}_A)$ , and let  $\mathcal{G}_t := \mathcal{F}_t \otimes \mathcal{A}_t$  for all  $t \geq 0$ . By the previous discussion,  $\{(K_t, \mu_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$  is

<sup>17</sup>However, as the arrival of the event concludes the game, any time  $t > T$  does not play any role in the analysis.

<sup>18</sup>The filtration  $\mathbb{F}$  acts as a public correlation device. Although the scope for randomization is limited in my model, the filtration allows me to define strategies and the equilibrium notion without incurring in the usual non-existence problems that arise when modeling games in continuous time.

a class of processes defined over the probability space  $(\Omega, \mathcal{G}, \mathbb{P})$  and adapted to the filtration of  $\mathbb{G}$ .

By consistency – part 1 and stationarity,  $\{(K_t, \mu_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$  is a class of time-homogeneous  $\mathcal{G}_t$ -Markov processes.

### The Buyer's Problem

The buyer takes the price offer function  $P$  as given and his belief about the market type follow the process  $(\mu_t)_{t \geq 0}$ . A pure strategy for the buyer of type  $v$  is then a  $\mathcal{G}_t$ -adapted stopping time  $\tau^v: \Omega \rightarrow \mathbb{R}_+ \cup \{+\infty\}$ . Since in equilibrium a skimming property always holds, it is without loss of generality to restrict attention to pure strategies. Note that  $\tau^v$  does not specify how to handle off-path price offers. This will be addressed by Equilibrium Condition 6.

Let  $\mathcal{T}$  be the set of all  $\mathbb{G}$ -adapted stopping times. Given any price offer function  $P$  and process  $((K_t, \mu_t))_{t \geq 0}$ , the buyer of type  $v$  faces the following optimal stopping problem (at the beginning of the game):

$$\sup_{\tau \in \mathcal{T}} \mathbb{E}^v \left[ \mathbb{1}_{\{\tau < T_{\mu_0}\}} e^{-r\tau} (v - P(K_\tau, \mu_\tau)) + \mathbb{1}_{\{\tau \geq T_{\mu_0}\}} e^{-rT_{\mu_0}} O^B(v) \right], \quad (\text{BP}^v)$$

where  $\mathbb{E}^v$  is the expectation with respect to the law of the process  $((K_t, \mu_t))_{t \geq 0}$  induced by  $\{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$ , conditional on  $(K_{0-}, \mu_{0-}) = (\bar{v}, \mu^0)$  (resp.,  $(K_{0-}, \mu_{0-}) = (\underline{v}, \mu^0)$ ) in the case of negative (resp., positive) selection and the buyer type being  $v$ . Here, the first term in the expectation reflects the surplus from trading before the arrival of the event, and the second part stands for the possibility that the event occurs before time  $\tau$ .

**Equilibrium Condition 3** (Buyer's Optimality). Let  $\tau^v$  the the stopping time chosen by the buyer of type  $v$ . Given the price offer function  $P$  and the law of the process  $((K_t, \mu_t))_{t \geq 0}$ ,  $\tau^v$  solves the optimal stopping problem (BP<sup>v</sup>).

Given stationarity, the buyer's value function depends only on the current state. In particular, the expected payoff of the buyer of type  $v$  when the game is in state  $(k, \mu)$ , denoted by  $B^v(k, \mu)$ , is

$$B^v(k, \mu) = \mathbb{E}_{(k, \mu)}^v \left[ \mathbb{1}_{\{\tau^v < T_\mu\}} e^{-r\tau^v} (v - P(K_{\tau^v}, \mu_{\tau^v})) + \mathbb{1}_{\{\tau^v \geq T_\mu\}} e^{-rT_\mu} O^B(v) \right],$$

where  $\mathbb{E}_{(k, \mu)}^v$  is the expectation with respect to the law of the process  $((K_t, \mu_t))_{t \geq 0}$  induced by  $\{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$ , conditional on  $(K_{0-}, \mu_{0-}) = (k, \mu)$  and the buyer type being  $v$ . Hereafter, when the random jump time  $T_\mu$  appears as the argument of the expectation  $\mathbb{E}_{(k, \mu)}^v$ , it has to be interpreted as the first jump time of a Poisson process with intensity function  $t \mapsto \lambda_t := \mu_t \lambda$ , where  $\mu_t$  is given by (2).

## Consistency – Part 2

If neither the event nor trade has occurred by time  $t$ , the seller's belief about the buyer type is conditioned on the fact that the buyer has rejected all past offers. This belief is summarized by the current cutoff type, where “cutoff type at time  $t$ ” should be interpreted to mean before observing the buyer's decision at time  $t$ . The second consistency condition simply requires that the cutoff type is derived from the buyer's optimal strategy.

**Equilibrium Condition 4** (Consistency – Part 2). For all  $t < T$ :

(a) If  $O^B(v)/v$  is strictly decreasing (negative selection),

$$K_t = \underline{v} + \int_{\underline{v}}^{\bar{v}} \mathbb{1}_{\{\tau^v \geq t\}} dv. \quad (4)$$

(b) If  $O^B(v)/v$  is strictly increasing (positive selection),

$$K_t = \bar{v} - \int_{\underline{v}}^{\bar{v}} \mathbb{1}_{\{\tau^v \geq t\}} dv. \quad (5)$$

## Option for Immediate Trade

The next condition says that if the seller offers a price that is smaller than the willingness to pay of *all* buyer types that have not yet traded, then all remaining types accept the price offer and the game concludes.

**Equilibrium Condition 5** (Option for Immediate Trade). Let  $(K_t, \mu_t) = (k, \mu)$ . Under negative (resp, positive) selection, if

$$P(k, \mu) \leq v - \frac{\mu \lambda O^B(v)}{\lambda + r},$$

for all  $v \in [\underline{v}, k]$  (resp.,  $v \in [k, \bar{v}]$ ), then  $\tau^v = t$  for all  $v \in [\underline{v}, k]$  (resp.,  $v \in [k, \bar{v}]$ ).

## The Seller's Problem

Given stationarity, the seller's value function depends only on the current state. In particular, the seller's expected payoff when the game is in state  $(k, \mu)$ , denoted by  $S(k, \mu)$ , is

$$S(k, \mu) = \mathbb{E}_{(k, \mu)} \left[ \int_0^{T_\mu} e^{-rt} P(K_t, \mu_t) dF(K_t) + e^{-rT_\mu} \underline{Q}^S(K_T) \right], \quad (6)$$

where  $\mathbb{E}_{(k, \mu)}$  is the expectation with respect to the law of the process  $((K_t, \mu_t))_{t \geq 0}$ , induced by  $\{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$ , conditional on  $(K_{0-}, \mu_{0-}) = (k, \mu)$ .

Instead of writing the seller's problem in terms of price offers, I will write it as an impulse control problem over the seller's beliefs about the buyer type.<sup>19</sup> In this case, the seller's problem is to choose a stopping time, denoted by  $T^S$ , at which he exercises the option for immediate trade, and a process  $(Q_t)_{t \geq 0}$  for the intensity of trade at time  $t \leq T^S$ . One way to interpret this exercises is to think of the seller as setting quantities instead of prices. The intensity of trade at time  $t$ ,  $dQ_t$ , determines the belief about the buyer type conditional on rejection according to condition (4) or (5), depending on whether negative or positive selection arises in equilibrium. Therefore, the price at time  $t$  must be the payoff of the cutoff type at time  $t$  conditional on accepting the offer; that is,

$$P(k, \mu) = k - B^{v=k}(k, \mu).^{20} \quad (7)$$

Implicitly, (7) assumes that the seller can resolve buyers' indifference in his favor. In other words,  $P(k, \mu)$  is not only the price offer in state  $(K_t, \mu_t) = (k, \mu)$ , but also the reservation price strategy of the buyer with type  $k$ .

I refer to the pair  $(T^S, (Q_t)_{t \geq 0})$  as a policy. A policy is *feasible* if  $T^S$  is a  $\mathcal{G}_t$ -measurable stopping rule and:

- (a) If  $O^B(v)/v$  is strictly decreasing (negative selection),  $(Q_t)_{t \geq 0}$  is a non-negative, non-increasing, and  $\mathcal{G}_t$ -measurable process.
- (b) If  $O^B(v)/v$  is strictly increasing (positive selection),  $(Q_t)_{t \geq 0}$  is a non-negative, non-decreasing, and  $\mathcal{G}_t$ -measurable process.

Let  $\Gamma$  denote the set of feasible policies. Moreover, define  $T_\mu^{\min} := \min \{T_\mu, T^S\}$ ,  $O_A^S(Q_{T_\mu^{\min}}) := \underline{O}^S(Q_{T_\mu^{\min}})$  if negative selection arises in equilibrium, and  $O_A^S(Q_{T_\mu^{\min}}) := \overline{O}^S(Q_{T_\mu^{\min}})$  if positive selection arises in equilibrium.

**Equilibrium Condition 6** (Seller's Optimality). For any  $(k, \mu) \in [\underline{v}, \bar{v}] \times [0, \mu^0]$ ,  $S(k, \mu)$ , as defined by (6), satisfies

$$S(k, \mu) = \sup_{(T^S, (Q_t)_{t \geq 0}) \in \Gamma} \mathbb{E}_{(k, \mu)}^Q \left[ \int_0^{T_\mu^{\min}} e^{-rt} P(Q_t, \mu_t) dF(Q_t) + e^{-rT_\mu^{\min}} O_A^S(Q_{T_\mu^{\min}}) \right], \quad (8)$$

where  $\mathbb{E}_{(k, \mu)}^Q$  is the expectation with respect to the law of the process  $((Q_t, \mu_t))_{t \geq 0}$  induced by  $\{(Q_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}$ , conditional on  $(Q_{0-}, \mu_{0-}) = (k, \mu)$ .

Modeling the game in continuous time and writing the seller's problem as an impulse control problem greatly simplifies the analysis. The impulse-control formulation allows me to

<sup>19</sup>For a standard reference on impulse control problems, see [Harrison \(2013\)](#).

<sup>20</sup>Formally dealing with continuation play following deviations from  $P$  poses well-known existence problems in a continuous-time setting (see again [Simon and Stinchcombe \(1989\)](#) and [Bergin and MacLeod \(1993\)](#)). Thus, it would require a substantially more complicated set of available strategies for the seller.

identify three qualitatively different dynamics: instants when the probability mass of trade is infinitesimal, so that the seller screens buyer types one by one; bursts of trade, so that the seller screens through a positive mass of buyer types in an instant; and periods of silent trade, where no buyer type trades.

## Regular Stationary Equilibria (RSE)

**Definition 1** (Stationary Equilibrium). *A stationary equilibrium is a quadruple*

$$\left( (\mu_t)_{t \geq 0}, \{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}, (\tau^v)_{v \in [\underline{v}, \bar{v}]}, P \right)$$

that satisfies Equilibrium Conditions 1-6.

I focus on a subset of stationary equilibria with the property that the seller alternates between periods of sufficiently gradual trade and a few instants with bursts of trade.

**Definition 2** (Regular Stationary Equilibrium). *A stationary equilibrium*

$$\left( (\mu_t)_{t \geq 0}, \{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}, (\tau^v)_{v \in [\underline{v}, \bar{v}]}, P \right)$$

is regular if

$$K_t = K_t^{abs} + K_t^{jump},$$

where  $K_t^{abs}$  is absolutely continuous in  $t$  and  $K_t^{jump}$  is a step function with finitely many jumps. The acronym RSE denotes a regular stationary equilibrium.

Hereafter, I use the term equilibrium to mean regular stationary equilibrium, thus omitting the “regular stationary” qualifier.

Before discussing the equilibrium restriction, it is convenient to introduce the following terminology.

**Definition 3.** *Let  $s, \underline{s}, \bar{s} \in \mathbb{R}_+$ , with  $\underline{t} < \bar{t}$ . In the RSE*

$$\left( (\mu_t)_{t \geq 0}, \{(K_t)_{t \geq 0}, k \in [\underline{v}, \bar{v}]\}, (\tau^v)_{v \in [\underline{v}, \bar{v}]}, P \right)$$

- (i) *Trade is smooth over the time interval  $[\underline{s}, \bar{s}]$  if  $K_t$  is absolutely continuous over  $[\underline{s}, \bar{s}]$ ; trade is silent over the time interval  $[\underline{s}, \bar{s}]$  if, in addition,  $K_{\bar{s}} - K_{\underline{s}} = 0$ ;*
- (ii) *There is a burst of trade at time  $s$  if  $K_u^{jump} \neq K_s^{jump}$  for all  $u \in \mathbb{R}_+$  with  $u < s$ .*

Note that, from the viewpoint of formalities, I consider silent trade as a special case of smooth trade. If trade is smooth over the time interval  $(\underline{s}, \bar{s})$  and  $s \in (\underline{s}, \bar{s})$ , I refer to  $\dot{K}_s \in (-\infty, +\infty)$  as the *speed* of trade at time  $s$ . If there is a burst of trade at time  $t$ , I write  $\dot{K}_t \in \{-\infty, +\infty\}$ .

One may ask how restrictive it is to confine attention to stationary equilibria that are regular. By the skimming property, the function  $K_t$  is monotone. Thus, it has a Lebesgue decomposition of the form  $K_t = K_t^{abs} + K_t^{Jump} + K_t^{sing}$ , where  $K_t^{abs}$  is an absolutely continuous function (in  $t$ ),  $K_t^{Jump}$  is a piecewise constant jump function, and  $K_t^{sing}$  is a singular continuous function (i.e., a non-constant continuous function with first derivative equal to zero almost everywhere). Imposing regularity on  $K_t$  introduces two additional restrictions. First, it says that there are only finitely many jumps, implying that there are only finitely many bursts of trade in equilibrium. Second, it says that the continuous part of  $K_t$  is sufficiently smooth, implying that over a time interval of smooth trade the buyer sees the price changing gradually over time, rather than  $K_t$  only moving in twitches. The restriction is weaker than taking the continuous-time *atomless* limit of a selection of stationary equilibria of the corresponding discrete-time model. Such an exercise, in fact, would exclude bursts of trade (see, e.g., [Fuchs and Skrzypacz \(2010\)](#)). I will discuss in due time when (if so) the restriction to regular stationary equilibria may rule out interesting dynamics.

### 3 Benchmarks

In this section, I first characterize the efficient trade dynamics, where the transaction occurs so as to maximize the total surplus. Then, I describe the equilibria of the bargaining game where the event never occurs (i.e., the case where it is common knowledge that the market is of type  $m = 0$ ). The two cases serve as natural benchmarks for the analysis to come. The other relevant benchmark is the bargaining game with stochastic arrival of the event but no learning (i.e., the case where it is common knowledge that the market is of type  $m = 1$ ). This case is analyzed in [Section 7](#).

#### 3.1 Efficient Trade Dynamics

To characterize the efficient trade dynamics consider a social planner who wishes to maximize the sum of the two players' payoffs from the transaction. Suppose that the planner knows the buyer's valuation  $v$  for the object. The planner has to choose the surplus-maximizing time at which parties stop searching for better opportunities and the seller serves the buyer. The planner's optimal stopping problem can be written as a dynamic programming problem where the planner's current belief on  $m = 1$  serves as state variables. Formally, we have the following.

Fix a buyer type  $v \in [\underline{v}, \bar{v}]$  and let  $\mu$  be the planner's current belief on  $m = 1$ . The expected total surplus from delaying trade so as to learn about the market environment, which I denote by  $L^v(\mu)$ , is expressed using the continuous-time recursion

$$L^v(\mu) = \mu \lambda O(v) dt + e^{-r dt} \mathbb{E}[L^v(\mu + d\mu) | \mu]. \quad (9)$$

Here, the first term on the right-hand side is the expected instantaneous surplus from waiting for news, where the event with associated joint payoff  $O(v)$  occurs with subjective instantaneous probability  $\mu\lambda dt$ ; the second term is the discounted expected continuation surplus from waiting for news.<sup>21</sup> As to the latter, with subjective probability  $\mu\lambda dt$  the event occurs and the game ends, so that the expected surplus from waiting jumps to  $L(v, 1) = 0$ ; with subjective probability  $\mu(1 - \lambda dt) + (1 - \mu) = 1 - \mu\lambda dt$ , no event occurs and, assuming that the function  $L^v(\mu)$  is differentiable with respect to its second argument, the expected surplus from waiting changes to  $L^v(\mu) + L_2^v(\mu)d\mu = L^v(\mu) - \lambda\mu(1 - \mu)L_2^v(\mu)dt$ . Here,  $L_2^v(\mu)$  denotes the partial derivative of  $L^v(\mu)$  with respect to its second argument. Using these expectations, together with  $1 - rdt$  as an approximation to  $e^{-rdt}$  as  $dt \rightarrow 0$ , I replace the second term in equation (9), simplify, and rearrange, to obtain that  $L^v(\mu)$  satisfies the first-order ODE

$$(\mu\lambda + r)L^v(\mu) + \lambda\mu(1 - \mu)L_2^v(\mu) = \mu\lambda O(v).$$

This has the solution

$$L^v(\mu) = \frac{\mu\lambda O(v)}{\lambda + r} + C(1 - \mu)\left(\frac{1 - \mu}{\mu}\right)^{r/\lambda}, \quad (10)$$

where  $C$  is the constant of integration.<sup>22</sup> The closed-form solution to the ODE shows that  $L^v(\mu)$  is differentiable with respect to its second argument, and so it was legitimate to assume differentiability.

At any belief  $\mu$  on  $m = 1$ , the total surplus from the transaction with the buyer is  $v$ . Thus, by imposing the optimality conditions  $L^v(\mu) = v$  (value matching) and  $L_2^v(\mu) = 0$  (smooth pasting) we obtain

$$\mu\lambda(O(v) - v) = rv \quad (11)$$

or, equivalently,

$$\mu = \frac{rv}{\lambda(O(v) - v)}. \quad (12)$$

Denote the right-hand side of (12) by  $\hat{\mu}(v)$ , i.e.,  $\hat{\mu}(v) := rv/\lambda(O(v) - v)$ . If  $\hat{\mu}(v) < \mu^0$ , then  $\hat{\mu}(v)$  is the belief at which the planner stops waiting for the arrival of the event and the transaction takes place. If instead  $\hat{\mu}(v) \geq \mu^0$ , it is never efficient to engage in market experimentation, and the transaction takes place at belief  $\mu^0$ . That is, if we denote with  $\mu(v)$  the efficient trading belief with the buyer, who has type  $v$ , we have

$$\mu(v) = \min\{\mu^0, \hat{\mu}(v)\}.$$

Optimality of the planner's strategy follows by standard verification arguments. To justify

<sup>21</sup>Henceforth, I refer to subjective instantaneous probabilities as subjective probabilities, omitting the instantaneous qualifier.

<sup>22</sup>I refer to [Polyanin and Zaitsev \(2003\)](#) for the closed-form solutions to the ODEs that appear in the paper.

smooth pasting, it is enough to determine the constant of integration  $C$  from value matching and to check that  $\mu(v)$  maximizes  $L^v(\mu)$  with respect to  $\mu$ .

The left-hand side of (11) is the expected value of a jump in the joint surplus from  $v$  to  $O(v)$  should the event occur. This is the flow benefit of market experimentation at belief  $\mu$ . The right-hand side is the flow cost, due to discounting, of postponing a transaction whose joint value is  $v$ . This is the cost of market experimentation. Condition (11), which describes the efficient benchmark, is thus intuitive: as long as there is sufficient optimism on the market being of type  $m = 1$ , the parties engage in costly experimentation. As soon as the benefit of waiting for the event equates the cost of delaying trade, however, the transaction occurs.

The previous characterization can be used to determine the buyer types that trade at belief  $\mu$  (equivalently, at time  $t_\mu$ ). These are all types  $v \in [\underline{v}, \bar{v}]$  such that  $\mu\lambda(O(v) - v) = rv$  or, equivalently,

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{v}{O(v)}. \quad (13)$$

By Assumption 1-A4, the ratio  $v/O(v)$  is strictly monotone, implying that at most one  $v \in [\underline{v}, \bar{v}]$  solves (13) for  $t_\mu > 0$ . The next proposition summarizes.

**Proposition 1** (Efficient Trade Dynamics). *Suppose trade is efficient. Then:*

- (i) *At time  $t = 0$ , trade occurs with all buyer types  $v \in [\underline{v}, \bar{v}]$  for which  $\mu^0\lambda O(v) \leq (\mu^0\lambda + r)v$ . At time  $t_\mu > 0$  trade occurs with the buyer type  $v \in [\underline{v}, \bar{v}]$  satisfying*

$$\mu\lambda(O(v) - v) = rv \quad (14)$$

*or, equivalently,*

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{v}{O(v)}. \quad (15)$$

- (ii) *The efficient trading time with the buyer of type  $v \in [\underline{v}, \bar{v}]$ , denoted by  $t(v)$ , is*

$$t(v) = \begin{cases} 0 & \text{if } v \geq \frac{\mu^0\lambda O(v)}{\mu^0\lambda + r} \\ T_v & \text{if } v < \frac{\mu^0\lambda O(v)}{\mu^0\lambda + r} \end{cases}, \quad (16)$$

*where  $T_v$  solves  $\mu_t\lambda O(v) = (\mu_t\lambda + r)v$  for  $t$ .*

The next observations, which follow immediately from Proposition 1, describe the main properties of the efficient trade dynamics.

- (i) The left-hand side of (15) strictly decreases over time. Thus, when  $v/O(v)$  is strictly increasing, learning endogenously gives rise to negative selection in the buyer type distribution. If, instead,  $v/O(v)$  is strictly decreasing, learning endogenously gives rise to positive selection in the buyer type distribution.

- (ii) Trade may begin with a burst or after a silent period. There is a burst of trade at  $t = 0$  if  $v \geq \mu^0 \lambda O(v) / (\mu^0 \lambda + r)$  holds for buyers types in a positive-measure subset of  $[\underline{v}, \bar{v}]$ . Trade begins silently if, instead,  $v < \mu^0 \lambda O(v) / (\mu^0 \lambda + r)$  for all buyer types. After trade begins, it proceeds smoothly until the end, as  $v/O(v)$  is strictly monotone. Finally, trade may also begin smoothly (and proceed so afterwards), This happens when  $\bar{v} = \mu^0 \lambda O(\bar{v}) / (\mu^0 \lambda + r)$  (resp.,  $\underline{v} = \mu^0 \lambda O(\underline{v}) / (\mu^0 \lambda + r)$ ) and negative (resp., positive) selection occurs under efficiency- The latter case is non-generic in the space of parameters.
- (iii) The market may or may not clear in finite time. In particular, under positive selection all trade occurs in finite time, the last instant of trade being at time  $T_{\bar{v}}^e$  at which

$$\frac{\mu_{T_{\bar{v}}^e} \lambda}{\mu_{T_{\bar{v}}^e} \lambda + r} = \frac{\bar{v}}{O(\bar{v})}.$$

Under negative selection, whether the market clears in finite time depends on whether the lowest buyers valuation is strictly positive (“gap” case) or not (“no gap” case). If  $\underline{v} > 0$ , the last instant of trade is at the finite time  $T_{\underline{v}}^e$  at which  $\mu_{T_{\underline{v}}^e} / (\mu_{T_{\underline{v}}^e} \lambda + r) = \underline{v} / O(\underline{v})$ ; if, instead  $\underline{v} = 0$ , the market does not clear in finite time, as the right-hand side of (15) is equal to zero when evaluated at  $\underline{v}$ , whereas its left-hand side is strictly positive for any finite time.

The rich trading dynamics that emerge under efficiency are entirely driven by on-the-market search and learning. In fact, without arrivals (i.e.,  $\mu^0 = 0$ ) or without learning (i.e.,  $\mu^0 = 1$ ) the efficient benchmark is trivial and interesting dynamics are absent. In particular, if there are no arrivals, under efficiency all buyer types trade at time  $t = 0$ . If, instead, there is no learning, efficiency prescribes that trade occurs at time  $t = 0$  with all buyer types  $v$  such that  $v \geq \lambda O(v) / (\lambda + r)$ , and never occurs with all types  $v$  such that  $v < \lambda O(v) / (\lambda + r)$  (when  $v < \lambda O(v) / (\lambda + r)$ , it is efficient to wait for the total surplus to grow).

### 3.2 The Bargaining Game without Arrivals

Assume that  $\mu^0 = 0$  (or, equivalently,  $\lambda = 0$ ), but that the bargaining game remains otherwise unchanged. In this case, the event never arrives and learning plays no role. Suppose that time is continuous but that the seller makes price offers at times  $t = 0, \Delta, 2\Delta, \dots$ <sup>23</sup> In this case, the model reduces to the canonical bargaining game with one-sided incomplete information studied in the seminal contributions of [Stokey \(1981\)](#), [Bulow \(1982\)](#), [Fudenberg et al. \(1985\)](#), [Gul et al. \(1986\)](#), and [Ausubel and Deneckere \(1989\)](#). They show that in any stationary

<sup>23</sup>The assumption of discrete time is made for a direct comparison with the existing literature, which considers the discrete-time game and takes the limit of the period length to zero. The same observation applies to the arguments in Sections 4 and 5, where I describe bargaining dynamics with stochastic arrival of the event but no learning.

perfect Bayesian equilibrium of the game, as  $\Delta \rightarrow 0$  (i.e., in the frictionless, continuous-time limit of arbitrarily frequent offers, where the seller loses all his commitment power):

- (i) The initial price offer converges to  $\underline{v}$ , the lowest buyer type;
- (ii) The expected time to trade converges to zero, and so there is no delay in equilibrium;
- (iii) The market outcomes is efficient;
- (iv) All buyer types are served at the same time and at the same price;
- (v) The seller's profit converges to  $\underline{v}$  and so the seller is unable to extract rents from the buyer with higher valuations.

These are the classic Coase Conjecture dynamics, named so after [Coase \(1972\)](#).<sup>24</sup> The result holds because a monopolist seller lacking the ability to commit to future prices faces the competition of its own future selves thereby dissipating all of its own monopoly power. After selling the initial quantity, the monopolist would necessarily benefit by lowering prices so as to sell to consumers with lower valuations who did not yet purchase. Thus, prices would decline after each sale. Forward-looking consumers expecting prices to fall, would then be unwilling to pay the initial high price. Consequently, if the time between offers were to vanish, the opening price would converge to the lowest buyer type and the competitive quantity would be sold at the opening of the market.

In the next sections I show that in a market with search and learning trade dynamics fundamentally differ from the Coase Conjecture benchmark. (Some) Coasean forces, however, will still be present; I explain how they generalize to or need to be reinterpreted in the current environment.

## 4 Bargaining in a Buyers' Market

In this section, I characterize bargaining dynamics in a buyers' market. In this market configuration  $O^S(v) := 0$  holds for all  $v \in [\underline{v}, \bar{v}]$ ; that is, better opportunities, if existing, are only available to the buyer.

To characterize a RSE in a buyers' market I proceed by construction. I begin by assuming that time intervals with smooth trade arise in equilibrium and identify bargaining dynamics in such intervals. By the definition of RSE, one such interval occurs unless all types trade in a single instant in equilibrium. Then, I characterize the unique candidate RSE. I show that

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<sup>24</sup>In the “gap” case (i.e.,  $\underline{v} > 0$ ), for any  $\Delta > 0$ , there exists a unique perfect Bayesian equilibrium (generically), and this equilibrium is stationary. In the “no gap” case (i.e.,  $\underline{v} = 0$ ), a stationary equilibrium exists, but there may be perfect Bayesian equilibria which are not stationary. The Coase Conjecture fails when consumers use non-stationary strategies (see [Ausubel and Deneckere \(1989\)](#)). As I focus on the stationary equilibria of my model, I compare my results to those that arise in the stationary equilibria of prior models.

bargaining dynamics in the candidate equilibrium are mostly determined by smooth trade, and establish when trade is smooth and silent and when, instead, it occurs in a burst. To identify the unique candidate equilibrium I only rely on necessary conditions for optimality. A verification argument shows that the candidate is indeed an equilibrium. I follow the same steps to characterize a RSE in Sections 5 and 6. Thus, I detail the procedure in Section 4 but omit from the main text the parts of the analysis that repeat in the next sections.

I present the formalities under the assumption that  $O^B(v)/v$  is strictly decreasing, so that negative selection occurs in equilibrium. I also discuss the case of positive selection, but I leave the formal analysis for the appendix, as it is similar to that for the case of negative selection. The same observation applies to the exposition in Section 6.<sup>25</sup>

## 4.1 Equilibrium Characterization in a Buyers' Market

### The Sellers' Problem

Suppose that  $O^B(v)/v$  is strictly decreasing, and that  $(K_t, \mu_t) = (k, \mu)$  for some  $t$  in the interior of a smooth-trade time interval. In this case, the seller's expected payoff satisfies the Hamilton-Jacobi-Bellman (hereafter, HJB) equation

$$rS(k, \mu) = \sup_{\dot{K} \in (-\infty, 0]} \left\{ -\mu\lambda S(k, \mu) + [P(k, \mu) - S(k, \mu)] \frac{f(k)}{F(k)} (-\dot{K}) + S_1(k, \mu)\dot{K} + S_2(k, \mu)\dot{\mu} \right\}, \quad (17)$$

where  $S_1(k, \mu)$  (resp.,  $S_2(k, \mu)$ ) denotes the partial derivative of  $S(k, \mu)$  with respect to its first (resp., second) argument.<sup>26</sup> Condition (17) has a direct interpretation. The left-hand side is the seller's expected equilibrium payoff expressed in flow terms. The right-hand side represents the possible sources of the flow: upon arrival of the event, which happens with a subjective probability flow  $\mu\lambda$ , the game ends with the seller forgoing  $S(k, \mu)$ . With a flow probability  $[f(k)/F(k)](-\dot{K})$  the buyer accepts the current offer,  $P(k, \mu)$ , which also ends the game. Finally, if the game does not end immediately, the continuation payoff changes by  $S_1(k, \mu)\dot{K} + S_2(k, \mu)\dot{\mu}$ .

The right-hand side of (17) is linear in  $\dot{K}$ . This linearity is the source of Coasean dynamics when outside alternatives or learning about their existence are absent. In that case, for any non-decreasing price offer function, the seller wants to run down the demand function as fast as possible. In a buyers' market, if anything, this incentive is even stronger. This is so because the arrival of the event prevents the seller from concluding any transaction. As the

<sup>25</sup>In Section 5,  $O^B(v) := 0$  holds for all  $v \in [\underline{v}, \bar{v}]$ , so that only negative selection arises in equilibrium.

<sup>26</sup>I provide below a closed-form expression for  $S(k, \mu)$  which is differentiable. Thus, it is legitimate to assume differentiability. Hereafter, I omit to state this argument when taking partial derivatives of a payoff function whose closed-form solution can be derived from the analysis.

equilibrium analysis will show, what prevents the seller from running down the demand curve instantaneously is the buyers' option value of waiting for new developments, which excludes instantaneous agreement at time zero with all types at any non-negative price.

The linearity in  $\dot{K}$  of the right-hand side of (17) implies that the sum of the coefficients on  $\dot{K}$  must be non-negative on the interior of a smooth-trade time interval. In fact, if the coefficients on  $\dot{K}$  in (17) added up to something negative, the seller would maximize his payoff by trading as fast as possible, that is by setting  $\dot{K} = -\infty$ , which is incompatible with smooth trade. Thus, the coefficients on  $\dot{K}$  either add up to zero or to something strictly positive. Now, note that the seller finds it optimal to set  $\dot{K} = 0$  (i.e., not to trade at all) if the coefficients on  $\dot{K}$  add up to something strictly positive. Lemma 8 in Appendix A.1 uses the necessary conditions for the buyers' problem (see below) to show that  $\dot{K} = 0$  cannot occur after parties begin to trade. Thus, for now suppose that  $\dot{K} < 0$ ; in this case, the coefficients on  $\dot{K}$  must add up to zero, which means that the seller must be indifferent between speeds of trade.

Setting the coefficients on  $\dot{K}$  to zero in (17) yields the partial differential equations (hereafter, PDEs)

$$S_1(k, \mu) = \left[ P(k, \mu) - S(k, \mu) \right] \frac{f(k)}{F(k)}, \quad (18)$$

$$S_2(k, \mu)\dot{\mu} = (\mu\lambda + r)S(k, \mu), \quad (19)$$

which describe the seller's best response problem in the interior of a smooth-trade time interval of any candidate equilibrium when  $\dot{K} < 0$ . The PDE in (19) has general solution

$$S(k, \mu) = \mathcal{K}_S \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (20)$$

where  $\mathcal{K}_S$  is the constant of integration. By (20),

$$S_1(k, \mu) = 0. \quad (21)$$

Now, as  $f(k)/F(k) > 0$  by assumption, (18) and (21) yield

$$P(k, \mu) = S(k, \mu).$$

The next lemma, which characterizes smooth-trade prices and their properties in a buyers' market when  $\dot{K} < 0$ , follows.

**Lemma 1** (Smooth Trade Prices in a Buyers' Market – Negative Selection). *Suppose  $O^B(v)/v$  is strictly decreasing. In a buyers' market, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} < 0$ :*

- (i) *Prices are determined by the seller's indifference between speeds of trade.*

(ii) The price offer function satisfies the PDE

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu), \quad (22)$$

where  $P_2(k, \mu)$  denotes the partial derivative of  $P(k, \mu)$  with respect to its second argument, and

$$P(k, \mu) = S(k, \mu). \quad (23)$$

(iii) Moreover,

$$P(k, \mu) = S(k, \mu) = \mathcal{K} \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (24)$$

where  $\mathcal{K}$  is the constant of integration. Thus, the price offer function is independent of the distribution of buyer types and of the buyer type that trades at any given instant.

### The Buyer's Problem

The speed of smooth trade is determined by the buyer's indifference between accepting and rejecting the smooth-trade price. Let  $k$  be the buyer type that trades at time  $t_\mu$ . His expected payoff when the game is in state  $(k, \mu)$  satisfies the HJB equation

$$rB^k(k, \mu) = \mu\lambda \left( O^B(k) - B^k(k, \mu) \right) - \left[ B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} \right] \quad (25)$$

where  $B_1^k(k, \mu)$  (resp.,  $B_2^k(k, \mu)$ ) is the partial derivative of  $B^k(k, \mu)$  with respect to its first (resp., second) argument. Again, condition (25) has a direct interpretation. The left-hand side is the buyer's expected equilibrium payoff expressed in flow terms. The right-hand side represents the possible sources of the flow: upon arrival of the event, which happens with a subjective probability flow  $\mu\lambda$ , the game ends with the buyer earning  $O^B(k)$  and forgoing  $B^k(k, \mu)$ . If the game does not end immediately, the continuation payoff changes by  $-\left[ B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} \right]$ . In the HJB equation in (25) there is no term corresponding to the flow payoff from accepting the offer,  $k - P(k, \mu)$ , and foregoing  $B^k(k, \mu)$ . This is so because, by Equilibrium Condition 6, the equilibrium price at any time is equal to the payoff of the cutoff type at that time conditional on accepting the price offer:  $P(k, \mu) = k - B^k(k, \mu)$ . That  $k - P(k, \mu) - B^k(k, \mu) = 0$  follows.

Taking the total derivative of the equilibrium condition

$$B^k(k, \mu) = k - P(k, \mu) \quad (26)$$

with respect to time yields

$$B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} = P_1(k, \mu)\dot{K} + P_2(k, \mu)\dot{\mu} = P_2(k, \mu)\dot{\mu}, \quad (27)$$

where the second equality follows from Lemma 1, which shows that smooth trade prices do

not depend on the buyer type that trades at any given instant. Replacing (26) and (27) into the HJB equation in (25) we obtain

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - (\mu\lambda + r)k + \mu\lambda O^B(k). \quad (28)$$

Together, the equilibrium necessary conditions (22) and (28) yield

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^B(k)},$$

which is the efficiency condition.

By Lemma 8 in Appendix A.1, trade proceeds smoothly after it begins. So we are left with only two candidate equilibria. One in which all types trade at the same instant, and one in which trade is efficient. Lemma 9 in Appendix A.1 shows that the seller has always a profitable deviation from any price schedule that sustains instantaneous trade with all buyer types. Thus, this case can never be part of an equilibrium. The next result follows.

**Lemma 2** (Efficient Trade in a Buyers' Market – Negative Selection). *Suppose  $O^B(v)/v$  is strictly decreasing. In a buyers' market, there exists a unique candidate RSE. In this RSE, trade is efficient. At time  $t = 0$ , trade occurs with all buyer types  $k \in [\underline{v}, \bar{v}]$  for which  $\mu^0\lambda O^B(k) \leq (\mu^0\lambda + r)k$ . At time  $t_\mu > 0$  trade occurs with the buyer type  $k \in [\underline{v}, \bar{v}]$  satisfying*

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^B(k)}.$$

*Trade begins with a burst or after a silent period. After trade begins, it proceeds smoothly until the end.*

As a last step, it remains to determine the exact price schedule in the unique candidate RSE. For future reference, I define the following objects. The relevant objects are defined for both positive and negative selection. Hereafter, I keep focusing on the case of negative selection, but I state the general result in Section 4.2 for both cases. I refer to Appendix A.2 for the formal analysis of the case of positive selection.

**Definition 4.** *Let  $T_{\underline{v}}^{bm}$  (resp.,  $T_{\bar{v}}^{bm}$ ) be time at which trade stops in equilibrium under negative (resp., positive) selection in a buyers' market. That is,  $T_{\underline{v}}^{bm}$  and  $T_{\bar{v}}^{bm}$  satisfy*

$$\frac{\mu_{T_{\underline{v}}^{bm}}\lambda}{\mu_{T_{\underline{v}}^{bm}}\lambda + r} = \frac{\underline{v}}{O^B(\underline{v})} \quad \text{and} \quad \frac{\mu_{T_{\bar{v}}^{bm}}\lambda}{\mu_{T_{\bar{v}}^{bm}}\lambda + r} = \frac{\bar{v}}{O^B(\bar{v})}.$$

*Moreover, define pointwise the two functions  $\underline{\Omega}^{bm}(\mu)$  and  $\bar{\Omega}^{bm}(\mu)$  as*

$$\underline{\Omega}^{bm}(\mu) := \left( \frac{\mu_{T_{\underline{v}}^{bm}}}{\mu} \right)^{r/\lambda} \left( \frac{1 - \mu}{1 - \mu_{T_{\underline{v}}^{bm}}} \right)^{(\lambda+r)/\lambda} \quad (29)$$

and

$$\underline{\Omega}^{bm}(\mu) := \left( \frac{\mu T_{\underline{v}}^{bm}}{\mu} \right)^{r/\lambda} \left( \frac{1 - \mu}{1 - \mu T_{\underline{v}}^{bm}} \right)^{(\lambda+r)/\lambda}. \quad (30)$$

Note that  $T_{\underline{v}}^{bm} < +\infty$  if and only if  $\underline{v} > 0$ . That is, the market clears in finite time if and only if  $\underline{v} > 0$ . When  $\underline{v} > 0$ , along the equilibrium path the seller knows that only the buyer of type  $\underline{v}$  has still to trade at time  $T_{\underline{v}}^{bm}$ . So, the asymmetric information disappears and the seller can charge the buyer of type  $\underline{v}$  his willingness to pay. Therefore, he exercises the option of immediate trade at time  $T^S = T_{\underline{v}}^{bm}$  (Equilibrium Condition 5) by setting

$$P(\underline{v}, \mu T_{\underline{v}}^{bm}) = \underline{v} - \frac{\lambda \mu T_{\underline{v}}^{bm} O(\underline{v})}{\lambda + r} > 0. \quad (31)$$

Now, (31) can be used as the terminal condition to provide an expression for the equilibrium price schedule. From (24), (29), and (31) we get

$$P(k, \mu) = S(k, \mu) = \underline{\Omega}^{bm}(\mu) \left( \underline{v} - \frac{\lambda \mu T_{\underline{v}}^{bm} O(\underline{v})}{\lambda + r} \right).$$

If, instead,  $\underline{v} = 0$ , the seller cannot exercise the option for immediate trade at a finite time by charging a non-negative price. Thus, he cannot do better than setting  $P(k, \mu) = 0$  in equilibrium. The analysis of positive selection is similar to the one for the case of negative selection with  $\underline{v} > 0$ . I refer to Appendix A.2 for the formalities. The next result follows.

**Lemma 3** (Price Dynamics in a Buyers' Market – Negative Selection). *Suppose  $O^B(v)/v$  is strictly decreasing. In a buyers' market, in the unique candidate RSE, the price offer function satisfies:*

(a) If  $\underline{v} = 0$ ,  $P(k, \mu) = S(k, \mu) = 0$ .

(b) If  $\underline{v} > 0$ ,

$$P(k, \mu) = S(k, \mu) = \underline{\Omega}^{bm}(\mu) \left( \underline{v} - \frac{\lambda \mu T_{\underline{v}}^{bm} O(\underline{v})}{\lambda + r} \right).$$

A standard verification argument show that the unique candidate RSE is indeed an equilibrium.

## 4.2 Bargaining Dynamics in a Buyers' Market

Before discussing bargaining dynamics in a buyers' market, I summarize the equilibrium characterization in the next theorem.

**Theorem 1** (Bargaining Dynamics in a Buyers' Market). *There exists a unique RSE of the bargaining game in a buyers' market. In this RSE:*

(i) Trade is efficient.

(ii) Trade begins with a burst or after a silent period. Next, it proceeds smoothly until the end.

(iii) Under negative selection:

(a) If  $\underline{v} = 0$ , the market does not clear in finite time and  $P(k, \mu) = S(k, \mu) = 0$ .

(b) If  $\underline{v} > 0$ , the market clears at the finite time  $t = T_{\underline{v}}^{bm}$  and

$$P(k, \mu) = S(k, \mu) = \underline{\Omega}^{bm}(\mu) \left( \underline{v} - \frac{\lambda \mu T_{\underline{v}}^{bm} O(\underline{v})}{\lambda + r} \right).$$

Thus, the price offered strictly increases over time.

(iv) Under positive selection, the market clears at the finite time  $t = T_{\bar{v}}^{bm}$  and

$$P(k, \mu) = S(k, \mu) = \bar{\Omega}^{bm}(\mu) \left( \bar{v} - \frac{\lambda \mu T_{\bar{v}}^{bm} O(\bar{v})}{\lambda + r} \right).$$

Thus, the price offered strictly increases over time.

(v) The market clears in finite time if and only if market clearing prices are strictly positive.

In this case, prices are higher than the competitive price.

The analysis of the bargaining game in a buyers' market delivers several insights. Trade is not immediate, but rather occurs over time, with the seller serving different (groups of) buyer types at different times. Additionally, the market may or may not clear in finite time.

Whether different types trade at different prices depends on how the two parties' learning about the market environment interacts with the seller's learning about the buyer's private information. The seller may or may not exercise market power. The price schedule coincides with the competitive one (which is identically equal to zero) when the seller cannot credibly commit to clear the market in finite time at a positive price. In this case, all buyer types trade at the same price. When, instead, the seller can exercise the option to clear the market in finite time at a positive price, prices are higher than under competition. The terms of trade remain favorable to the privately informed party, but the seller extracts part of the surplus from the transaction by slowly increasing the price offer over time in order to exploit the buyer's cost of waiting for superior outside opportunities and learn about their existence. What provides the seller with a credible commitment not to lower further the price offer is the fact that, when negative selection occurs and  $\underline{v} > 0$ , or when positive selection occurs, at some finite future date it becomes common knowledge that there exist gains from trade and the information asymmetry vanishes. Interestingly, the seller is able to screen using prices by

conditioning his price offer only on the common belief about the market type, which evolves exogenously over time, and not directly on the current cutoff type.

Market power, however, does not prevent the bargaining outcome from being efficient. In fact, although trade occurs over time and the seller gains from the ability to screen using prices, the Coasian force towards efficient agreements remains overwhelming.

## 5 Bargaining in a Sellers' Market

In this section, I characterize bargaining dynamics in a sellers' market. In this market configuration  $O^B(v) := 0$  holds for all  $v \in [\underline{v}, \bar{v}]$ ; that is, better opportunities, if existing, are only available to the seller. To ease the exposition, I split the analysis into two parts. First, I consider the case of independent private values (IPV), where  $O^S(v) := O^S$  for all  $v \in [\underline{v}, \bar{v}]$  and some real constant  $O^S$  satisfying Assumption 1. Then, I study bargaining dynamics when valuations are interdependent (IV), where the seller's payoff conditional on the arrival of the event depends on the buyers' type.

### 5.1 Equilibrium Characterization in a Sellers' Market – Independent Private Values (IPV)

#### The Sellers' Problem

Suppose that  $(K_t, \mu_t) = (k, \mu)$  for some  $t$  in the interior of a smooth-trade time interval. The seller's expected payoff satisfies the HJB equation

$$rS(k, \mu) = \sup_{\dot{K} \in (-\infty, 0]} \left\{ \mu \lambda [O^S - S(k, \mu)] + [P(k, \mu) - S(k, \mu)] \frac{f(k)}{F(k)} (-\dot{K}) + S_1(k, \mu) \dot{K} + S_2(k, \mu) \dot{\mu} \right\}. \quad (32)$$

The interpretation of (32) is analogous to that of equation (17). The only difference is in the first term on the right-hand side of the HJB equation. Now, upon arrival of the event, the seller still forgoes  $S(k, \mu)$ , but also earns  $O^S$ . Intuitively, the availability of superior opportunities (if  $m = 1$ ) provides a counterbalance for the seller's temptation to run down the demand curve, leading to a strictly downward-sloping price offer function  $P(k, \mu)$ . The analysis will show that this intuition is correct.

With independent private valuation the seller's payoff upon the arrival of the event does not depend on the buyer type. As I will show in the next section, this affects the speed at which the seller screens out buyer types in equilibrium. It also changes in part the analysis, and so it is convenient to study the two cases separately.

Again, as the right-hand side of (32) is linear in  $\dot{K}$ , the sum of the coefficients on  $\dot{K}$  must

be non-negative on the interior of a smooth-trade time interval. The seller finds it optimal to set  $\dot{K} = 0$  (i.e., not to trade at all) if the coefficients on  $\dot{K}$  add up to something strictly positive. Lemma 15 in Appendix B.2 uses the necessary conditions for the buyers' problem (see below) to show that  $\dot{K} = 0$  cannot occur after parties begin to trade. Thus, for now suppose that  $\dot{K} < 0$ ; in this case, the coefficients on  $\dot{K}$  must add up to zero, which means that the seller must be indifferent between speeds of trade.

Setting the coefficients on  $\dot{K}$  to zero in (32) yields the PDEs

$$S_1(k, \mu) = \left[ P(k, \mu) - S(k, \mu) \right] \frac{f(k)}{F(k)}, \quad (33)$$

$$S_2(k, \mu)\dot{\mu} = (\mu\lambda + r)S(k, \mu) - \mu\lambda O^S, \quad (34)$$

which describe the seller's best response problem in the interior of a smooth-trade time interval of any candidate equilibrium when  $\dot{K} < 0$ . The PDE in (34) has general solution

$$S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \mathcal{K}_S \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (35)$$

where  $\mathcal{K}_S$  is the constant of integration. By (35),

$$S_1(k, \mu) = 0. \quad (36)$$

Now, as  $f(k)/F(k) > 0$  by assumption, (33) and (36) yield

$$P(k, \mu) = S(k, \mu).$$

The next lemma, which characterizes smooth-trade prices and their properties in a buyers' market when  $\dot{K} < 0$ , follows.

**Lemma 4** (Smooth Trade Prices in a Sellers' Market – IPV). *Suppose private values are independent. In a sellers' market, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} < 0$ :*

(i) *Prices are determined by the seller's indifference between speeds of trade.*

(ii) *The price offer function satisfies the PDE*

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - \mu\lambda O^S. \quad (37)$$

and

$$P(k, \mu) = S(k, \mu). \quad (38)$$

(iii) *Moreover,*

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \mathcal{K} \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (39)$$

where  $\mathcal{K}$  is the constant of integration. Thus, the price offer function is independent of the distribution of buyer types and of the buyer type that trades at any given instant.

### The Buyer's Problem

Again, the speed of smooth trade is determined by the buyer's indifference between accepting and rejecting the smooth-trade price. Let  $k$  be the buyer type that trades at time  $t_\mu$ . His expected payoff when the game is in state  $(k, \mu)$  satisfies the HJB equation

$$rB^k(k, \mu) = -\mu\lambda B^k(k, \mu) - \left[ B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} \right] \quad (40)$$

The interpretation of (40) is analogous to that of equation (25). The only difference is in the first term of the right-hand side of the HJB equation. Now, upon arrival of the event, the buyer still forgoes  $B^k(k, \mu)$ , but does earn anything in expectation, as the event is only favorable to the seller.

Taking the total derivative of the equilibrium condition

$$B^k(k, \mu) = k - P(k, \mu) \quad (41)$$

with respect to time yields

$$B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} = P_1(k, \mu)\dot{K} + P_2(k, \mu)\dot{\mu} = P_2(k, \mu)\dot{\mu}, \quad (42)$$

where the second equality follows from Lemma 4, which shows that smooth trade prices do not depend on the buyer type that trades at any given instant. Replacing (41) and (42) into the HJB equation in (40) yields

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - (\mu\lambda + r)k. \quad (43)$$

Together, the equilibrium necessary conditions (37) and (43) yield

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^S},$$

which is the efficiency condition.

By Lemma 15 in Appendix B.2, trade proceeds smoothly after it begins. So we are left with only two candidate RSEs. One in which all types trade at the same instant, and one in which trade is efficient. Lemma 17 in Appendix B.2 shows that the seller has always a profitable deviation from any price schedule that sustains instantaneous trade with all buyer types. Thus, this case can never be part of an equilibrium. The next result follows.

**Lemma 5** (Efficient Trade in a Sellers' Market – IPV). *Suppose private values are independent. In a sellers' market, there exists a unique candidate RSE. In this RSE, trade is efficient. At time  $t = 0$ , trade occurs with all buyer types  $k \in [\underline{v}, \bar{v}]$  for which  $\mu^0 \lambda O^S \leq (\mu^0 \lambda + r)k$ . At time  $t_\mu > 0$  trade occurs with the buyer type  $k \in [\underline{v}, \bar{v}]$  satisfying*

$$\frac{\mu \lambda}{\mu \lambda + r} = \frac{k}{O^S}.$$

*Trade begins with a burst or after a silent period. After trade begins, it proceeds smoothly until the end.*

As a last step, it remains to determine prices in the unique candidate RSE. To ease the exposition, I define the following objects.

**Definition 5.** *Let  $T_{\underline{v}}^{sm}$  be time at which trade stops in equilibrium in a sellers' market. That is,  $T_{\underline{v}}^{sm}$  satisfies*

$$\frac{\mu_{T_{\underline{v}}^{sm}} \lambda}{\mu_{T_{\underline{v}}^{sm}} \lambda + r} = \frac{\underline{v}}{O^S}.$$

*Moreover, define pointwise the function  $\underline{\Omega}^{sm}(\mu)$  as*

$$\underline{\Omega}^{sm}(\mu) := \left( \frac{\mu_{T_{\underline{v}}^{sm}}}{\mu} \right)^{r/\lambda} \left( \frac{1 - \mu}{1 - \mu_{T_{\underline{v}}^{sm}}} \right)^{(\lambda+r)/\lambda}. \quad (44)$$

Note that  $T_{\underline{v}}^{sm} < +\infty$  if and only if  $\underline{v} > 0$ . That is, the market clears in finite time if and only if  $\underline{v} > 0$ . When  $\underline{v} > 0$ , along the equilibrium path the seller knows that at the finite time  $T_{\underline{v}}^{sm}$  only the buyer of type  $\underline{v}$  has still to trade. So, there is no asymmetric information anymore and the seller can charge the buyer of type  $\underline{v}$  his willingness to pay. Therefore, he exercises the option of immediate trade (Equilibrium Condition 5) by setting

$$P(\underline{v}, \mu_{T_{\underline{v}}^{sm}}) = \underline{v} > 0. \quad (45)$$

From, (39), (44), and (45) we get

$$P(k, \mu) = S(k, \mu) = \frac{\mu \lambda O^S}{\lambda + r} + \underline{\Omega}^{sm}(\mu) \left( \underline{v} - \frac{\lambda \mu_{T_{\underline{v}}^{sm}} O^S}{\lambda + r} \right).$$

In contrast, when  $\underline{v} = 0$ , the seller cannot exercise the option for immediate trade at a finite time by charging a non-negative price. Thus, he cannot do anything better than setting

$$P(k, \mu) = \frac{\mu \lambda O^S}{\lambda + r}.$$

in equilibrium. The next result follows.

**Lemma 6** (Price Dynamics in a Sellers' Market – IPV). *Suppose private values are independent. In a sellers' market, in the unique candidate RSE, the price offer function satisfies:*

(a) If  $\underline{v} = 0$ ,

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r};$$

(b) If  $\underline{v} > 0$ ,

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \underline{\Omega}^{sm}(\mu) \left( \underline{v} - \frac{\lambda\mu T_{\underline{v}}^{sm} O^S}{\lambda + r} \right).$$

A verification argument shows that the unique candidate RSE is indeed an equilibrium.

## 5.2 Equilibrium Characterization in a Sellers' Market – Interdependent Values

In this section and in Section 6.2, I assume that  $\underline{v} = 0$ . This is so because with interdependent values the continuous-time limit of the discrete-time analog of the model may exhibit dynamics that do not satisfy my equilibrium restrictions. In particular, these dynamics may neither be regular nor generated by stationary strategies (see [Deneckere and Liang \(2006\)](#)). In contrast, when  $\underline{v} = 0$  [Fuchs and Skrzypacz \(2013b\)](#) show that equilibria become regular and stationary.<sup>27</sup>

Suppose that  $(K_t, \mu_t) = (k, \mu)$  for some  $t$  in the interior of a smooth-trade time interval. The seller's expected payoff satisfies the HJB equation

$$rS(k, \mu) = \sup_{\dot{K} \in (-\infty, 0]} \left\{ \mu\lambda [\underline{Q}^S(k) - S(k, \mu)] + [P(k, \mu) - S(k, \mu)] \frac{f(k)}{F(k)} (-\dot{K}) + S_1(k, \mu)\dot{K} + S_2(k, \mu)\dot{\mu} \right\}. \quad (46)$$

The interpretation of (46) is analogous to that of equation (32), but with an important difference. Now, the sellers' expected payoff upon arrival of the event,  $\underline{Q}^S(k)$ , depends on the buyer type. This is the source of interdependency between the two players' payoffs.

Again, as the right-hand side of (46) is linear in  $\dot{K}$ , the sum of the coefficients on  $\dot{K}$  must be non-negative on the interior of a smooth-trade time interval. The seller finds it optimal to set  $\dot{K} = 0$  (i.e., not to trade at all) if the coefficients on  $\dot{K}$  add up to something strictly positive. Lemma 16 in Appendix B.2 uses the necessary conditions for the buyers' problem

<sup>27</sup>In the spirit of [Daley and Green \(2017\)](#), one can show that singular dynamics do not necessarily arise in my setup even when  $\underline{v} > 0$ . In fact, for equilibrium to alternate between bursts of trade and silent periods, during a silent period the sellers' belief must be exactly such that the Coasian desire to speed up trade is absent. With the addition of learning, the seller's belief cannot remain constant at such a belief over any time interval. As a result, singular dynamics need not arise in equilibrium. I thank Stephan Lauermaun for pointing this out to me. Such an extension of the model is in progress.

to show that  $\dot{K} = 0$  cannot occur after parties begin to trade. Thus, for now suppose that  $\dot{K} < 0$ ; in this case, the coefficients on  $\dot{K}$  must add up to zero, which means that the seller must be indifferent between speeds of trade.

Setting the coefficients on  $\dot{K}$  to zero in (46) yields the PDEs

$$S_1(k, \mu) = \left[ P(k, \mu) - S(k, \mu) \right] \frac{f(k)}{F(k)}, \quad (47)$$

$$S_2(k, \mu)\dot{\mu} = (\mu\lambda + r)S(k, \mu) - \mu\lambda \underline{O}^S(k), \quad (48)$$

which describe the seller's best response problem in the interior of a smooth-trade time interval of any candidate equilibrium when  $\dot{K} < 0$ .

The next lemma, established in Appendix B.1, characterizes smooth-trade prices and their properties in a sellers' market.

**Lemma 7** (Smooth Trade Prices in a Sellers' Market – IV). *Suppose valuations are interdependent. In a sellers' market, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} < 0$ :*

(i) *Prices are determined by the seller's indifference between speeds of trade.*

(ii) *The price offer function satisfies the PDE*

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - \mu\lambda \left( O^S(k) + \underline{O}^S(k) \right). \quad (49)$$

(iii) *The PDE in (49) has general solution*

$$P(k, \mu) = \frac{\mu\lambda \left( O^S(k) + \underline{O}^S(k) \right)}{\lambda + r} + \mathcal{K} \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}},$$

where  $\mathcal{K}$  is the constant of integration. Thus, the price offer function depends on the buyer type that trades at any given instant but is independent of the distribution of buyer types.

By Lemma 16 in Appendix B.2, trade proceeds smoothly after it begins. So there are only two candidate RSEs. One in which all types trade at the same instant, and one in which trade is efficient. Lemma 18 in Appendix B.2 shows that the seller has always a profitable deviation from any price schedule that sustains instantaneous trade with all buyer types. Thus, there exists a unique candidate equilibrium.

To determine prices in the unique candidate equilibrium, note that, as  $\underline{v} = 0$ , the seller cannot exercise the option for immediate trade at a finite time by charging a non-negative price. Thus, he cannot do anything better than setting

$$P(k, \mu) = \frac{\mu\lambda \left( O^S(k) + \underline{O}^S(k) \right)}{\lambda + r}.$$

As a last step, it remains to determine how the seller screens out types in equilibrium. Now, the total derivative of the equilibrium condition  $B^k(k, \mu) = k - P(k, \mu)$  with respect to time is

$$B_1^k(k, \mu)\dot{K} + B_2^k(k, \mu)\dot{\mu} = P_1(k, \mu)\dot{K} + P_2(k, \mu)\dot{\mu} \quad (50)$$

That is, as now the price offer function depends on the buyer type that trades at any given instant, the term  $P_1(k, \mu)\dot{K}$  does not disappear from the right-hand side of (50). Replacing (41) and (50) into the HJB equation in (40) yields

$$P_1(k, \mu)\dot{k} + P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - (\mu\lambda + r)k. \quad (51)$$

Together, the equilibrium necessary conditions (49) and (51) yield

$$k = \frac{\mu\lambda O^S(k)}{\mu\lambda + r} + \frac{\mu\lambda(O^S(k) - P_1(k, \mu)\dot{k})}{\mu\lambda + r}. \quad (52)$$

Now, the second term on the right hand side of (52) represent the inefficiency arising with interdependent valuations.

A verification argument shows that the unique candidate RSE is indeed an equilibrium.

### 5.3 Bargaining Dynamics in a Sellers' Market

Before discussing bargaining dynamics in a sellers' market, I summarize in the next theorem the equilibrium characterization.

**Theorem 2** (Bargaining Dynamics in a Sellers' Market). *There exists a unique RSE of the bargaining game in a sellers' market. In this RSE:*

(a) *Under independent private values (IPV):*

(i) *Trade is efficient.*

(ii) *Trade begins with a burst or after a silent period. Next, it proceeds smoothly until the end.*

(iii) (a) *If  $\underline{v} = 0$ , the market does not clear in finite time and*

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r}.$$

(b) *If  $\underline{v} > 0$ , the market clears at the finite time  $t = T_{\underline{v}}^{sm}$  and*

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \underline{\Omega}^{sm}(\mu) \left( \underline{v} - \frac{\lambda\mu T_{\underline{v}}^{sm} O^S}{\lambda + r} \right).$$

*In both case (a) and case (b), the price offered strictly decreases over time.*

(iv) *The market clears in finite time if and only if market clearing prices are strictly positive. In this case, prices are higher than the competitive price.*

(b) *Under interdependent values (IV):*

(i) *There is inefficient delay in equilibrium.*

(ii) *Trade begins with a burst or after a silent period. Next, it proceeds smoothly until the end.*

(iii) *The price offer function is*

$$P(k, \mu) = \frac{\mu\lambda(O^S(k) + \underline{O}^S(k))}{\lambda + r}.$$

*Thus, the price offered decreases over time. The seller's expected payoff is*

$$S(k, \mu) = \frac{\mu\lambda\underline{O}^S(k)}{\lambda + r}.$$

(iv) *The market does not clear in finite time.*

In a sellers' market, the timing of agreements is efficient when valuations are independent. In contrast, transactions occur inefficiently late when the seller's payoff upon the arrival of the event depends on the buyer type.

If private values are independent and the seller cannot clear the market in finite time at a positive price, the seller is unable to exercise market power. In this case,  $S(k, \mu)$  has the property that at any point in the game (for any state  $(k, \mu)$ ) the expected payoff of the seller is equal to his payoff from waiting for the (uncertain) arrival of the event. Moreover, the price schedule coincides with the competitive one. In particular, as  $P(k, \mu) = S(k, \mu)$ , for any  $(k, \mu)$ , the price coincide with the expected present value from waiting for the (uncertain) arrival of the event. This is a kind of no-ex post regret property: upon the price being accepted by some buyer type, the seller does not regret not slowing down the trade. This case suggests an interpretation of the Coase Conjecture analogous to that of [Fuchs and Skrzypacz \(2010\)](#): although the conjecture does not hold in terms of the price dropping immediately to zero, the Coasian dynamics force down the seller's profit to his outside option (his "marginal cost"). There are, however, two important differences. First, this dynamic result occurs in my setting without interdependent valuations. Second, it does not occur at the expenses of efficiency.

In a sellers' market, there are two ways the seller can exercise market power and price above his marginal cost. The first possibility occurs with independent value, when the seller can credibly commit to clear the market in finite time at a positive price. In this case, the intuition for why prices are higher than the competitive price is analogous to that in [Section 4](#) and market power is not at odds with efficiency. The second way the seller exercises market power is when valuation are interdependent. The result holds even though the seller cannot

exercise the option for immediate trade. With interdependent valuations,  $S(k, \mu)$  still has the property that for any state  $(k, \mu)$ , the expected payoff of the seller is equal to his payoff from waiting for the (uncertain) arrival of the event. However, prices are now higher than the competitive price – the competitive price now being equal to

$$\frac{\mu \lambda \underline{O}^S(k)}{\lambda + r}.$$

Moreover, for any  $(k, \mu)$ , the price is now also higher than the expected present value the seller would have earned from type  $k$  if he waited for (uncertain) the arrival, this value being

$$\frac{\mu \lambda O^S(k)}{\lambda + r}.$$

Therefore, prices do not have a kind of no-ex post regret property, but rather exhibit a form of “ex post satisfaction.” In this case, however, market power comes at the expenses of efficiency. These price dynamics sharply contrast the findings with independent private values, and the results in [Fuchs and Skrzypacz \(2010\)](#).

## 6 Bargaining in a General Market

In this section, I characterize bargaining dynamics in a general market, where post-arrival payoffs are non-trivial for both parties. As in Section 5, I split the analysis into two parts, considering first the case of independent private values, and then the case of interdependent valuations. Without loss of insight, this section only considers the case where  $O^B(v)/v$  is strictly decreasing, so that negative selection occurs in equilibrium.

The analysis builds on the results developed in Sections 4 and 5. The conceptual steps to construct an equilibrium are the same. In the main text I characterize equilibrium dynamics when  $\dot{K} < 0$  and present the main result. Appendix C shows that there can neither be a silent period after trade begins, nor a burst of trade. Appendix C also shows that instantaneous trade with all buyer types never occurs in equilibrium. A verification argument shows that the unique candidate is indeed an equilibrium.

### 6.1 Equilibrium Characterization in a General Market – Independent Private Values (IPV)

Suppose private values are independent, and let  $O(k) := O^S + O^B(k)$ . From the analysis of the seller’s problem in Section 5.1, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} < 0$ , the price offer function satisfies the PDE (see Lemma 4)

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - \mu\lambda O^S. \tag{53}$$

Moreover, from the analysis of the buyers' problem in Section 4.1 (see (28)),

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - (\mu\lambda + r)k + \mu\lambda O^B(k). \quad (54)$$

Together, the equilibrium necessary conditions (53) and (54) yield

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^S(k)},$$

which is the efficiency condition. To ease the exposition, I define the following objects.

**Definition 6.** Let  $T_{\underline{v}}^{gm}$  be time at which trade stops in equilibrium under negative selection in a general market. That is,  $T_{\underline{v}}^{gm}$  satisfies

$$\frac{\mu_{T_{\underline{v}}^{gm}}\lambda}{\mu_{T_{\underline{v}}^{gm}}\lambda + r} = \frac{\underline{v}}{O^B(\underline{v})}.$$

Moreover, define pointwise the function  $\underline{\Omega}^{gm}(\mu)$  as

$$\underline{\Omega}^{gm}(\mu) := \left( \frac{\mu_{T_{\underline{v}}^{gm}}}{\mu} \right)^{r/\lambda} \left( \frac{1 - \mu}{1 - \mu_{T_{\underline{v}}^{gm}}} \right)^{(\lambda+r)/\lambda}. \quad (55)$$

Note that  $T_{\underline{v}}^{gm} < +\infty$  if and only if  $\underline{v} > 0$ . That is, the market clears in finite time if and only if  $\underline{v} > 0$ . When  $\underline{v} > 0$ , along the equilibrium path the seller knows that at the finite time  $T_{\underline{v}}^{gm}$  only the buyer of type  $\underline{v}$  has still to trade. So, there is no asymmetric information anymore and the seller can charge the buyer of type  $\underline{v}$  his willingness to pay. Therefore, he exercises the option of immediate trade (Equilibrium Condition 5) by setting

$$P(\underline{v}, \mu_{T_{\underline{v}}^{gm}}) = \underline{v} > 0. \quad (56)$$

Now, (56) can be used as the terminal condition to provide an expression for the equilibrium price schedule. From (39), (55), and (56), we get

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \underline{\Omega}^{gm}(\mu) \left( \underline{v} - \frac{\lambda\mu_{T_{\underline{v}}^{gm}} O(\underline{v})}{\lambda + r} \right).$$

In contrast, when  $\underline{v} = 0$ , the seller cannot exercise the option for immediate trade at a finite time by charging a non-negative price. Thus, he cannot do better than setting

$$P(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r}.$$

in equilibrium.

## 6.2 Equilibrium Characterization in a General Market – Interdependent Values (IV)

Suppose private values are independent, and let  $O(k) := O^S(k) + O^B(k)$ . From the analysis of the seller's problem in Section 5.2, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} < 0$ , the price offer function satisfies the PDE (see Lemma 7)

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - \mu\lambda\left(O^S(k) + \underline{O}^S(k)\right). \quad (57)$$

Moreover, from the analysis of the buyers' problem in Section 4.1 (see (28)),

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu) - (\mu\lambda + r)k + \mu\lambda O^B(k). \quad (58)$$

Together, the equilibrium necessary conditions (57) and (58) yield

$$k = \frac{\mu\lambda O(k)}{\mu\lambda + r} + \frac{\mu\lambda\left(\underline{O}^S(k) - P_1(k, \mu)\dot{k}\right)}{\mu\lambda + r}. \quad (59)$$

Now, the second term on the right hand side of (59) represent the inefficiency arising with interdependent valuations.

## 6.3 Bargaining Dynamics in a General Market

I summarize with the next theorem the equilibrium characterization in a general market. The intuition behind the results is similar to that discussed in Sections 4 and 5. It is thus omitted. An important difference, however, is that now equilibrium prices may not evolve monotonically over time. If outside opportunities are relatively more attractive to the buyer than to the seller, as time elapses without the arrival of the event, uncertainty unravels in the seller's favor. Then, the seller begins to gently increase prices in order to exploit the buyer's cost of waiting for now less and less likely outside opportunities.

**Theorem 3** (Bargaining Dynamics in a General Market). *There exists a unique RSE of the bargaining game in a general market. In this RSE:*

(a) *Under independent private values (IPV):*

(i) *Trade is efficient.*

(ii) *Trade begins with a burst or after a silent period. Next, it proceeds smoothly until the end.*

(iii) (a) *If  $\underline{v} = 0$ , the market does not clear in finite time and*

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r}.$$

Thus, the price offered strictly decreases over time.

(b) If  $\underline{v} > 0$ , the market clears at the finite time  $t = T_{\underline{v}}^{gm}$  and

$$P(k, \mu) = S(k, \mu) = \frac{\mu\lambda O^S}{\lambda + r} + \underline{\Omega}^{gm}(\mu) \left( \underline{v} - \frac{\lambda\mu T_{\underline{v}}^{gm} O^B(\underline{v})}{\lambda + r} \right).$$

The price offer uncton may be non-monotone over time.

(iv) The market clears in finite time if and only if market clearing prices are strictly positive. In this case, prices are higher than the competitive price.

(b) Under interdependent values (IV):

(i) There is inefficient delay in equilibrium.

(ii) Trade begins with a burst or after a silent period. Next, it proceeds smoothly until the end.

(iii) The price offer function is

$$P(k, \mu) = \frac{\mu\lambda(O^S(k) + \underline{O}^S(k))}{\lambda + r}.$$

Thus, the price offered decreases over time. The seller's expected payoff is

$$S(k, \mu) = \frac{\mu\lambda \underline{O}^S(k)}{\lambda + r}.$$

(iv) The market does not clear in finite time.

## 7 Extensions and Discussion

### 7.1 The Bargaining Game without Learning

Assume that  $\mu^0 = 1$ , but that the bargaining game remains otherwise unchanged. In this case, (superior) outside options arrive stochastically, but are certain to exist. Thus, learning about the existence of outside opportunities plays no role. The next two subsections show that, absent learning, the two players either agree immediately or never do so. The result holds both in a sellers' and in a buyers' market.<sup>28</sup> The rich bargaining dynamics that emerge in my setting are due to the interaction between (exogenous) learning about the bargaining environment and (endogenous) learning about parties' private information. Without this interaction, bargaining dynamics and equilibrium outcomes resemble those of the standard Coase Conjecture framework.

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<sup>28</sup>An exception is when valuations are interdependent.

### 7.1.1 Bargaining without Learning in a Buyers' Market

**Proposition 2** (Bargaining without Learning in a Buyers' Market). *Suppose  $\mu^0 = 1$  and consider any stationary perfect Bayesian equilibrium of the discrete-time game. In the limit where the period length goes to zero:*

(a) *If  $\frac{\lambda O^B(v)}{\lambda+r} > v$  for all  $v \in [\underline{v}, \bar{v}]$ :*

(i) *No trade occurs, with all buyer types waiting for the superior outside opportunity to arrive;*

(ii) *The seller's payoff is zero, and the payoff of the buyer of type  $v$  is  $\frac{\lambda}{\lambda+r} O^B(v)$ .*

(b) *If there exists some  $v^* \in (\underline{v}, \bar{v}]$ , such that  $\frac{\lambda}{\lambda+r} O^B(v) > v$  for all  $v < v^*$  and  $\frac{\lambda O^B(v)}{\lambda+r} \leq v$  for all  $v \geq v^*$ :*

(i) *All buyer types  $v < v^*$  do not trade and wait for the superior outside opportunity to arrive;*

(ii) *The seller's opening price converges to zero and all buyer types  $v \geq v^*$  trade immediately;*

(iii) *The seller's payoff converges to zero; the payoff of the buyer of type  $v \geq v^*$  converges to  $v$ , and the expected payoff of the buyer of type  $v < v^*$  is  $\frac{\lambda O^B(v)}{\lambda+r}$ .*

*In both case (a) and case (b) the market outcome is efficient.*

### 7.1.2 Bargaining without Learning in a Sellers' Market

**Proposition 3** (Bargaining without Learning in a Sellers' Market). *Suppose private values are independent and that  $\mu^0 = 1$ . Consider any stationary perfect Bayesian equilibrium of the discrete-time game. In the limit where the period length goes to zero:*

(a) *If  $\frac{\lambda O^S}{\lambda+r} > v$  for all  $v \in [\underline{v}, \bar{v}]$ :*

(i) *No trade occurs, with the seller waiting for the superior outside opportunity to arrive;*

(ii) *The seller's payoff is  $\frac{\lambda O^S}{\lambda+r}$  and the buyer's payoff is zero.*

(b) *If there exists some  $v^* \in (\underline{v}, \bar{v}]$ , such that  $\frac{\lambda O^S}{\lambda+r} > v$  for all  $v < v^*$  and  $\frac{\lambda O^S}{\lambda+r} \leq v$  for all  $v \geq v^*$ :*

(i) *The seller's price offer is  $p = \frac{\lambda O^S}{\lambda+r}$ ;*

(ii) *All buyer types  $v < v^*$  do not trade and all buyer types  $v \geq v^*$  trade immediately;*

(iii) *The seller's payoff is  $\frac{\lambda O^S}{\lambda+r}$ ; the payoff of the buyer of type  $v \geq v^*$  is  $v - \frac{\lambda O^S}{\lambda+r}$  and the expected payoff of the buyer of type  $v < v^*$  is zero.*

In both case (a) and case (b) the market outcome is efficient.

Thus, the rich bargaining dynamics that emerge in a buyers' market are due to the interaction between (exogenous) learning about the bargaining environment and (endogenous) learning about parties' private information.

## 7.2 The Case of Complete Information

Suppose that the buyer's valuation  $v \in [\underline{v}, \bar{v}]$  for the good is common knowledge between the seller and the buyer. If the event does not occur before trade takes place, then:

- (a) Price discrimination is perfect and the seller, who has all the bargaining power, extracts all the surplus from the transaction. As a result, the transaction occurs at time  $t(v)$ , the efficient trading time with the buyer of type  $v$  (see part (iii) of Proposition 1). Trade takes place at price  $p$  equal to the buyer's willingness to pay at time  $t(v)$ . This is given by the difference between the buyer's valuation  $v$  and the present discounted value of his outside option at time  $t(v)$ , which is  $\mu_{t(v)}\lambda O^B(v)/(\lambda + r)$ . That is,

$$p = v - \frac{\mu_{t(v)}\lambda O^B(v)}{\lambda + r}.$$

- (b) The seller implements the complete information outcome by setting any price schedule which is greater than the buyer's willingness to pay,  $v - \mu_t\lambda O^B(v)/(\lambda + r)$ , at all times  $t \neq t(v)$ , and equal to  $v - \mu_{t(v)}\lambda O^B(v)/(\lambda + r)$  at time  $t = t(v)$ . In such case, the buyer of type  $v$  is willing to pay price  $p_{t(v)}$  at time  $t(v)$ , as this leaves him indifferent between trading and waiting for the arrival of the outside option, but refuses to trade at any other time.

The next proposition characterizes the complete information outcome. Parts (i) and (ii) follow from the previous discussion. Part (iii) holds by straightforward calculations, which are in Appendix D.1.

**Proposition 4** (Complete Information Outcome). *Suppose the buyer type  $v \in [\underline{v}, \bar{v}]$  is common knowledge, and that the event does not occur before trade takes place. Then:*

- (i) Trade is efficient.
- (ii) The seller extracts all the surplus from trade by implementing any price schedule  $p: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ ,  $t \mapsto p_t$ , such that

$$p_t \begin{cases} = v - \frac{\mu_t\lambda O^B(v)}{\lambda + r} & \text{if } t = t(v) \\ > v - \frac{\mu_t\lambda O^B(v)}{\lambda + r} & \text{if } t \neq t(v) \end{cases},$$

where  $t(v)$  is the efficient trading time for the buyer of type  $v$ .

(iii) The seller's and the buyer's payoffs, denoted by  $S_C(v)$  and  $B_C(v)$ , are:

$$S_C(v) = \begin{cases} v - \frac{\mu^0 \lambda O^B(v)}{\lambda + r} & \text{if } v \geq \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r} \\ v - \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)} & \text{if } v < \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r} \end{cases},$$

and

$$B_C(v) = \begin{cases} \frac{\mu^0 \lambda O^B(v)}{\lambda + r} & \text{if } v \geq \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r} \\ \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)} & \text{if } v < \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r} \end{cases}.$$

The next corollary, which follows by simple calculations collected in Appendix D.2, contains comparative statics for the complete information outcome. The results have a natural interpretation.

**Corollary 1** (Comparative Statics). *Fix a buyer type  $v \in [\underline{v}, \bar{v}]$ . In the complete information outcome:*

(i) *If  $v \geq \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r}$ , then*

(a)  *$S_C(v)$  is decreasing in  $O^B(v)$  and independent of  $O^S(v)$ ;*

(b)  *$B_C(v)$  is increasing in  $O^B(v)$  and independent of  $O^S(v)$ .*

(ii) *If  $v < \frac{\mu^0 \lambda O(v)}{\mu^0 \lambda + r}$ , then*

(a)  *$S_C(v)$  is increasing in  $O^S(v)$ , decreasing in  $O^B(v)$  when  $O^S(v) > v$ , increasing in  $O^B(v)$  when  $O^S(v) < v$ , and independent of  $O^B(v)$  when  $O^S(v) = v$ ;*

(b)  *$B_C(v)$  is decreasing in  $O^S(v)$ , increasing in  $O^B(v)$  when  $O^S(v) > v$ , decreasing in  $O^B(v)$  when  $O^S(v) < v$ , and independent of  $O^B(v)$  when  $O^S(v) = v$ .*

### 7.3 Burst of Trade after Trade Begins

In my model, trade may begin with a burst, but proceeds smoothly afterwards. What drives slow screening of buyer types in equilibrium is the assumption that  $O^S(v)/v$  is strictly monotone. If  $O^S(v)/v$  monotone but not strictly so, bursts of trade may occur after trade begins. As long as  $O^S(v)/v$  is constant (and monotone) over finitely many subintervals of  $[\underline{v}, \bar{v}]$ , regular stationary equilibria allow to capture bargaining dynamics where bursts of trade alternate with periods of smooth trade. This alternative specification of the benchmark model may be appropriate in some applications.

### 7.4 More General Learning Processes

I assume that learning occurs via conclusive news. While this is a natural modeling choice in my setting, one may consider more gradual learning processes. For instance, news may arrive

via a non-conclusive Poisson process (see [Keller and Rady \(2010\)](#)). I can nest this case in my model by assuming that the event corresponds to the first jump of the Poisson process and using  $O^S(v)$  and  $O^B(v)$  to capture the two parties' payoffs in the continuation game that follows the jump. Another option is to assume that news about the bargaining environment are revealed via a Brownian diffusion process (see [Bolton and Harris \(1999\)](#)). Although this is an interesting theoretical extension of my model, I expect the main insights of the analysis to remain valid in such alternative setup.

## 7.5 Optimal Sales Mechanisms under Commitment

In section 3.1 I characterize the efficient benchmark, but I do not discuss its implementation. When valuations are independent, however, the analysis shows that the “freestyle” bargaining implements the efficient outcome. With interdependent valuations, this is no longer true. In this case, it is natural to inquire whether an efficient mechanism exists and, if so, if it can be implemented in prices. If the answer to either question is negative, then the bargaining outcome is necessarily inefficient.

In a companion work in progress – [Lomys \(2017\)](#) – I adopt a mechanism design approach to study the same trading environment as the one of this paper. In particular, I consider the design of profit maximizing mechanisms when the seller has full commitment power and the design of efficient trade mechanisms. A preliminary investigation suggests that the bargaining outcome is not second-best efficient when valuations are interdependent.<sup>29</sup>

## 8 Concluding Remarks

Parties to a negotiation often have reason to inquire whether the current counterparty represent the best available trading opportunity or, in contrast, a more satisfactory use of their resources exists. Superior opportunities not only take time to arise, but are also of unknown existence. Uncertainty, however, unravels over time if parties engage in some form of market experimentation. These features are common in many markets – durable goods, labor, housing, and financial markets, just to name a few – and, arguably, the trade-offs they give rise to are a defining feature of many bargaining relationships. In this paper, I develop a framework to understand bargaining dynamics when parties learn whether superior outside opportunities are available during their negotiations. I show that the resulting tension between an immediate agreement and the option value of waiting to learn is of first-order importance in shaping bargaining relationships. It affects the timing of agreements, the dynamics of prices, surplus division, and the seller's ability to exercise market power.

Trade no longer takes place immediately with the informed party capturing all the rents. In contrast, the seller screens out buyers over time by charging different prices to different types.

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<sup>29</sup>See [Deneckere and Liang \(2006\)](#) for a result in this spirit. However, while theirs is a static mechanism design problem, mine is dynamic.

While learning accounts for delay, inefficiently timed agreements only occur if valuations are interdependent. Although other explanations have been proposed for the observed delay in bargaining, I believe mine to be a very natural one. It shows that delay is to be expected in markets with search and learning. My results, however, question the view that long disputes result in inefficient bargaining outcomes: absent additional frictions in the protocol (e.g., interdependent valuations), the Coasean force towards efficiency remains overwhelming when parties engage in market experimentation during their negotiations.

I also show that the seller may exercise substantial market power. In particular, market power is present when the seller is able to clear the market in finite time at positive prices, and when the seller has the option to wait and learn whether superior outside opportunities are available to him. In these cases, prices are (often) higher than the competitive price and the seller's payoff is larger than what he would get if he were awaiting for the possible arrival of a superior opportunity or unable to screen using prices (or both).

My model is flexible enough to serve as a stepping stone for future research. I discuss several extensions in Section 7. I plan to build on this setup to explore further bargaining and trade relationships when the economic environment is non-stationary because of learning.

## A Proofs for Section 4: Bargaining in a Buyers' Market

### A.1 Lemmas for Equilibrium Construction in a Buyers' Market

**Lemma 8** (No Silent Periods after Trade Begins in a Buyers' Market – Negative Selection). *Suppose  $O^B(v)/v$  is strictly decreasing. In any candidate RSE, if  $K_t < \bar{v}$ , then  $\dot{K}_s < 0$  for all  $s \geq t$ .*

**Lemma 9** (No Instantaneous Trade in a Buyers' Market – Negative Selection). *Suppose  $O^B(v)/v$  is strictly decreasing. In any candidate RSE, instantaneous trade with all buyer types never occurs.*

**Proof.** Suppose  $\underline{v} = 0$ . At time  $t = t_\mu$  the willingness to pay of the buyer of type  $v$  is

$$v - \frac{\mu\lambda O^B(v)}{\lambda + r}. \quad (60)$$

As (60) is strictly negative for a positive-measure subset of buyer types at any  $t_\mu \in \mathbb{R}_+$ , instantaneous trade with all buyer types occurs only if the seller sets a negative price. Since the seller can always secure himself a payoff of zero by not trading, this can never be an equilibrium.

Now suppose  $\underline{v} > 0$ . Let  $\underline{T}$  be the time satisfying

$$\underline{v} - \frac{\mu_{\underline{T}}\lambda O^B(\underline{v})}{\lambda + r} = 0.$$

Instantaneous trade with all buyer types at  $t < \underline{T}$  requires a negative price. Again, as the seller can always secure himself a payoff of zero by not trading, this can never be an equilibrium. It remains to rule out instantaneous trade with all buyer types at any  $t \geq \underline{T}$ . ■

### A.2 Bargaining in a Buyers' Market: Equilibrium Characterization under Positive Selection

Now suppose that  $O^B(v)/v$  is strictly increasing, so that positive selection occurs in equilibrium. The analysis mimics that under negative selection. The main difference arises when writing the seller's necessary conditions for optimality in the interior of a smooth-trade time interval: The seller now starts screening buyer types from the top of the distribution, and not from its bottom.

Suppose  $(K_t, \mu_t) = (k, \mu)$  for some  $t$  in the interior of a smooth-trade time interval. In

this case, the seller's expected payoff satisfies the HJB equation

$$rS(k, \mu) = \sup_{\dot{K} \in [0, +\infty)} \left\{ -\mu\lambda S(k, \mu) + [P(k, \mu) - S(k, \mu)] \frac{f(k)}{1 - F(k)} (-\dot{K}) + S_1(k, \mu)\dot{K} + S_2(k, \mu)\dot{\mu} \right\}. \quad (61)$$

The linearity in  $\dot{K}$  of the right-hand side of (61) implies that the sum of the coefficients on  $\dot{K}$  must be non-positive on the interior of a smooth-trade time interval. In fact, if the coefficients on  $\dot{K}$  in (61) added up to something positive, the seller would maximize his payoff by trading as fast as possible, that is by setting  $\dot{K} = +\infty$ , which is incompatible with smooth trade. Now, note that the seller finds it optimal to set  $\dot{K} = 0$  (i.e., not to trade at all) if the coefficients on  $\dot{K}$  add up to something strictly negative. The next lemma uses the necessary conditions for the buyers' problem, which are the same as for the case of negative selection, to show that  $\dot{K} = 0$  cannot occur after parties begin to trade.

**Lemma 10** (No Silent Periods after Trade Begins in a Buyers' Market – Positive Selection). *Suppose  $\mathcal{O}^B(v)/v$  is strictly increasing. In any candidate RSE, if  $K_t > \underline{v}$ , then  $\dot{K}_s > 0$  for all  $s \geq t$ .*

Thus, for now suppose that  $\dot{K} > 0$ ; in this case, the coefficients on  $\dot{K}$  must add up to zero, which means that the seller must be indifferent between speeds of trade. Setting the coefficients on  $\dot{K}$  to zero in (61) yields the PDEs

$$S_1(k, \mu) = [P(k, \mu) - S(k, \mu)] \frac{f(k)}{1 - F(k)}, \quad (62)$$

$$S_2(k, \mu)\dot{\mu} = (\mu\lambda + r)S(k, \mu), \quad (63)$$

which describe the seller's best response problem on the interior of a smooth-trade time interval of any candidate equilibrium. The PDE in (63) has general solution

$$S(k, \mu) = \mathcal{K}_S \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (64)$$

where  $\mathcal{K}_S$  is the constant of integration. By (64),

$$S_1(k, \mu) = 0. \quad (65)$$

Now, as  $f(k)/(1 - F(k)) > 0$  by assumption, (62) and (65) yield

$$P(k, \mu) = S(k, \mu).$$

The next lemma follows.

**Lemma 11** (Smooth Trade Prices in a Buyers' Market – Positive Selection). *Suppose  $O^B(v)/v$  is strictly increasing. In a buyers' market, on the interior of a smooth-trade time interval of any candidate RSE, if  $\dot{K} > 0$ :*

(i) *Prices are determined by the seller's indifference between speeds of trade.*

(ii) *The price offer function satisfies the PDE*

$$P_2(k, \mu)\dot{\mu} = (\mu\lambda + r)P(k, \mu), \quad (66)$$

where  $P_2(k, \mu)$  denotes the partial derivative of  $P(k, \mu)$  with respect to its second argument, and

$$P(k, \mu) = S(k, \mu). \quad (67)$$

(iii) *Moreover,*

$$P(k, \mu) = S(k, \mu) = \mathcal{K}_P \frac{(1 - \mu)^{(\lambda+r)/\lambda}}{\mu^{r/\lambda}}, \quad (68)$$

where  $\mathcal{K}$  is the constant of integration. Thus, the price offer function is independent of the distribution of buyer types and of the buyer type that trades at any given instant.

Together, the equilibrium necessary conditions (66) and (28) yield

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^B(k)},$$

which is the efficiency condition.

By Lemma 10, trade proceeds smoothly after it begins. So we are left with only two candidate equilibria. One in which all types trade at the same instant, and one in which trade is efficient. The next lemma shows that the seller has always a profitable deviation from any price schedule that sustains instantaneous trade with all buyer types. Thus, this case can never be part of an equilibrium.

**Lemma 12** (No Instantaneous Trade in a Buyers' Market – Positive Selection). *Suppose  $O^B(v)/v$  is strictly increasing. In any candidate RSE, instantaneous trade with all buyer types never occurs.*

The next result follows.

**Lemma 13** (Efficient Trade in a Buyers' Market – Positive Selection). *In a buyers' market, there exists a unique candidate RSE. In this RSE, trade is efficient. At time  $t = 0$ , trade occurs with all buyer types  $k \in [\underline{v}, \bar{v}]$  for which  $\mu^0\lambda O^B(k) \leq (\mu^0\lambda + r)k$ . At time  $t_\mu > 0$  trade occurs with the buyer type  $k \in [\underline{v}, \bar{v}]$  satisfying*

$$\frac{\mu\lambda}{\mu\lambda + r} = \frac{k}{O^B(k)}.$$

Trade begins with a burst or after a silent period. After trade begins, it proceeds smoothly until the end.

As a last step, it remains to determine the exact price schedule in the unique candidate RSE. Let  $T_{\bar{v}}$  be time at which trade stops in equilibrium (see Definition 4), and note that  $T_{\bar{v}} < +\infty$ . That is, the market clears in finite time. Along the equilibrium path the seller knows that at the finite time  $T_{\bar{v}}$  only the buyer of type  $\bar{v}$  has still to trade. So, the asymmetric information disappears and the seller can charge the buyer of type  $\bar{v}$  his willingness to pay. Therefore, he exercises the option of immediate trade at time  $T^S = T_{\bar{v}}$  (Equilibrium Condition 5) by setting

$$P(\bar{v}, \mu_{T_{\bar{v}}}) = \bar{v} - \frac{\lambda \mu_{T_{\bar{v}}} O(\bar{v})}{\lambda + r} > 0. \quad (69)$$

Now, (69) can be used as the terminal condition to provide an expression for the equilibrium price schedule. From (68), (30), and (69) we get

$$P(k, \mu) = S(k, \mu) = \bar{\Omega}(\mu) \left( \underline{v} - \frac{\lambda \mu_{T_{\underline{v}}} O(\bar{v})}{\lambda + r} \right).$$

The next result follows.

**Lemma 14** (Price Dynamics in a Buyers' Market – Positive Selection). *Suppose  $O^B(v)/v$  is strictly increasing. In a buyers' market, in the unique candidate RSE, the price offer function satisfies*

$$P(k, \mu) = S(k, \mu) = \bar{\Omega}(\mu) \left( \bar{v} - \frac{\lambda \mu_{T_{\bar{v}}} O(\bar{v})}{\lambda + r} \right).$$

Theorem 1 in the main text summarizes the equilibrium characterization under positive selection.

## B Proofs for Section 5: Bargaining in a Sellers' Market

### B.1 Proof of Lemma 7

By the fundamental theorem of calculus,

$$S(k, \mu) = \int_{\underline{v}}^k S_1(v, \mu) dv + S(\underline{v}, \mu). \quad (70)$$

Replace (47) in (70) to obtain

$$S(k, \mu) = \int_{\underline{v}}^k \left[ P(v, \mu) - S(v, \mu) \right] \frac{f(v)}{F(v)} dv + S(\underline{v}, \mu), \quad (71)$$

and so, by the Leibniz integral rule,

$$S_2(k, \mu) = \int_{\underline{v}}^k \left[ P_2(v, \mu) - S_2(v, \mu) \right] \frac{f(v)}{F(v)} dv + S_2(\underline{v}, \mu). \quad (72)$$

In turn, replacing (71) and (72) in (48) yields

$$\begin{aligned} & \left[ \int_{\underline{v}}^k \left[ P_2(v, \mu) - S_2(v, \mu) \right] \frac{f(v)}{F(v)} dv + S_2(\underline{v}, \mu) \right] \dot{\mu} \\ &= (\mu\lambda + r) \left[ \int_{\underline{v}}^k \left[ P(v, \mu) - S(v, \mu) \right] \frac{f(v)}{F(v)} dv + S(\underline{v}, \mu) \right] - \mu\lambda \underline{O}^S(k). \end{aligned} \quad (73)$$

Note also that (48) reads as

$$S_2(k, \mu) = \frac{(\mu\lambda + r)S(k, \mu)}{\dot{\mu}} - \frac{\mu\lambda \underline{O}^S(k)}{\dot{\mu}}. \quad (74)$$

Now plug (74) into (73) to obtain

$$\begin{aligned} & \left[ \int_{\underline{v}}^k \left[ P_2(v, \mu) - \frac{(\mu\lambda + r)S(k, \mu) + \mu\lambda \underline{O}^S(k)}{\dot{\mu}} \right] \frac{f(v)}{F(v)} dv + S_2(\underline{v}, \mu) \right] \dot{\mu} \\ &= (\mu\lambda + r) \left[ \int_{\underline{v}}^k \left[ P(v, \mu) - S(v, \mu) \right] \frac{f(v)}{F(v)} dv + S(\underline{v}, \mu) \right]. \end{aligned} \quad (75)$$

Simplifying (75) yields

$$\begin{aligned} & \left[ \int_{\underline{v}}^k P_2(v, \mu) \frac{f(v)}{F(v)} dv \right] \dot{\mu} + \int_{\underline{v}}^k \mu\lambda \underline{O}^S(k) \frac{f(v)}{F(v)} dv + S_2(\underline{v}, \mu) \dot{\mu} \\ &= (\mu\lambda + r) \left[ \int_{\underline{v}}^k P(v, \mu) \frac{f(v)}{F(v)} dv + S(\underline{v}, \mu) \right] - \underline{O}^S(k). \end{aligned} \quad (76)$$

Observe that

$$\frac{\partial \underline{O}^S(k)}{\partial k} = \frac{\partial}{\partial k} \left[ \int_{\underline{v}}^k O^S(v) \frac{f(v)}{F(v)} dv \right] = O^S(k) \frac{f(k)}{F(k)}. \quad (77)$$

Moreover,

$$\begin{aligned}
\frac{\partial \underline{O}^S(k)}{\partial k} &= \frac{\partial}{\partial k} \left[ \int_{\underline{v}}^k \underline{O}^S(v) \frac{f(v)}{F(v)} dv \right] \\
&= \frac{\partial}{\partial k} \left[ \int_{\underline{v}}^k \left[ \int_{\underline{v}}^v O^S(s) \frac{f(s)}{F(s)} ds \right] \frac{f(v)}{F(v)} dv \right] \\
&= \left[ \int_{\underline{v}}^k O^S(s) \frac{f(s)}{F(s)} ds \right] \frac{f(k)}{F(k)} \\
&= \underline{O}^S(k) \frac{f(k)}{F(k)}.
\end{aligned} \tag{78}$$

Finally, differentiating both sides of (76) with respect to  $k$ , using (77) and (78), and simplifying yields

$$P_2(k, \mu) \dot{\mu} = (\mu\lambda + r)P(k, \mu) - \mu\lambda \left( O^S(k) + \underline{O}^S(k) \right), \tag{79}$$

which establishes (49) in Lemma 7. Part (iii) of Lemma 7 follows. ■

## B.2 Lemmas for Equilibrium Construction in a Sellers' Market

**Lemma 15** (No Silent Periods after Trade Begins in a Sellers' Market – IPV). *Under IPV, in any candidate RSE, if  $K_t < \bar{v}$ , then,  $\dot{K}_s < 0$  for all  $s \geq t$ .*

**Lemma 16** (No Silent Periods after Trade Begins in a Sellers' Market – IV). *Under IV, in any candidate RSE, if  $K_t < \bar{v}$ , then  $\dot{K}_s < 0$  for all  $s \geq t$ .*

**Lemma 17** (No Instantaneous Trade in a Sellers' Market – IPV). *In any candidate RSE, instantaneous trade with all buyer types never occurs.*

**Proof.** Instantaneous trade with all buyer types occurs only if the seller sets a price equal to  $\underline{v}$ . Since the seller can always secure himself a payoff higher than  $\underline{v}$  by setting a price equal to  $\mu\lambda O^S/(\lambda + r)$ , instantaneous trade with all buyer types can never occur in equilibrium. ■

**Lemma 18** (No Instantaneous Trade in a Sellers' Market – IV). *In any candidate RSE, instantaneous trade with all buyer types never occurs.*

**Proof.** Instantaneous trade with all buyer types occurs only if the seller sets a price equal to  $\underline{v}$ . Since the seller can always secure himself a payoff higher than  $\underline{v}$  by setting a price equal to  $\mu\lambda O^S(k)/(\lambda + r)$ , instantaneous trade with all buyer types can never occur in equilibrium. ■

## C Proofs for Section 6: Bargaining in a General Market

### C.1 Lemmas for Equilibrium Construction in a Sellers' Market

**Lemma 19** (No Silent Periods after Trade Begins in a General Market – IPV). *Under IPV, in any candidate RSE, if  $K_t < \bar{v}$ , then,  $\dot{K}_s < 0$  for all  $s \geq t$ .*

**Lemma 20** (No Silent Periods after Trade Begins in a General Market – IV). *Under IV, in any candidate RSE, if  $K_t < \bar{v}$ , then  $\dot{K}_s < 0$  for all  $s \geq t$ .*

**Proof.** Suppose  $\underline{v} = 0$ . At time  $t = t_\mu$  the willingness to pay of the buyer of type  $v$  is

$$v - \frac{\mu\lambda O^B(v)}{\lambda + r}. \quad (80)$$

As (80) is strictly negative for a positive-measure subset of buyer types at any  $t_\mu \in \mathbb{R}_+$ , instantaneous trade with all buyer types occurs only if the seller sets a negative price. Since the seller can always secure himself a strictly positive payoff by setting a price equal to  $\mu\lambda O^S(k)/(\lambda + r)$ , instantaneous trade with all buyer types can never occur in equilibrium.

Now suppose  $\underline{v} > 0$ . Let  $\underline{T}$  be the time satisfying

$$\underline{v} - \frac{\mu_{\underline{T}}\lambda O^B(\underline{v})}{\lambda + r} = 0.$$

Instantaneous trade with all buyer types at  $t < \underline{T}$  requires a negative price. Again, as the seller can always secure himself a strictly positive payoff by setting a price equal to  $\mu\lambda O^S(k)/(\lambda + r)$ , instantaneous trade with all buyer types can never occur in equilibrium. It remains to rule out instantaneous trade with all buyer types at any  $t \geq \underline{T}$ . ■

**Lemma 21** (No Instantaneous Trade in a General Market – IPV). *In any candidate RSE, instantaneous trade with all buyer types never occurs.*

**Lemma 22** (No Instantaneous Trade in a General Market – IV). *In any candidate RSE, instantaneous trade with all buyer types never occurs.*

**Proof.** At time  $t = t_\mu$  the willingness to pay of the buyer of type  $v$  is

$$v - \frac{\mu\lambda O^B(v)}{\lambda + r}. \quad (81)$$

As (81) is strictly negative for a positive-measure subset of buyer types at any  $t_\mu \in \mathbb{R}_+$ , instantaneous trade with all buyer types occurs only if the seller sets a negative price. Since the seller can always secure himself a strictly positive payoff by setting a price equal to  $\mu\lambda O^S(k)/(\lambda + r)$ , instantaneous trade with all buyer types can never occur in equilibrium. ■

## D Proofs for Section 7: Extensions and Discussion

### D.1 Proof of Proposition 4

If  $v \geq \mu^0 \lambda O(v) / (\mu^0 \lambda + r)$ , trade occurs at time  $t = 0$  (equivalently, belief  $\mu^0$ ). The seller's payoff is

$$S_C(v) = p_0 = v - \frac{\mu_0 \lambda O^B(v)}{\lambda + r},$$

and the buyer's payoff is

$$B_C(v) = v - p_0 = \frac{\mu_0 \lambda O^B(v)}{\lambda + r}.$$

If  $v < \mu^0 \lambda O(v) / (\mu^0 \lambda + r)$ , trade occurs at belief

$$\mu(v) = \frac{rv}{\lambda(O(v) - v)}.$$

Let  $t_{\mu(v)}$  be the time at which  $\mu_t = \mu(v)$ . Thus, the seller's payoff is

$$S_C(v) = p_{t_{\mu(v)}} = v - \frac{\mu(v) \lambda O^B(v)}{\lambda + r} = v - \frac{rv}{\lambda(O(v) - v)} \frac{\lambda O^B(v)}{\lambda + r} = v - \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)},$$

and the buyer's payoff is

$$B_C(v) = v - p_{t_{\mu(v)}} = \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)}. \quad \blacksquare$$

### D.2 Proof of Corollary 1

Part (i) is immediate. The only non-obvious statement in part (ii) is the dependence of  $S_C(v)$  and  $B_C(v)$  on  $O^B(v)$ . This is so because  $O^B(v)$  appears in both the nominator and the denominator of  $S_C(v)$  and  $B_C(v)$ . So, suppose that  $v < \mu^0 \lambda O(v) / (\mu^0 \lambda + r)$ . Then

$$\frac{\partial S_C(v)}{\partial O^B(v)} = \frac{\partial}{\partial O^B(v)} \left[ v - \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)} \right] = -\frac{r \lambda v (O^S(v) - v)}{O(v) - v},$$

and

$$\frac{\partial B_C(v)}{\partial O^B(v)} = \frac{\partial}{\partial O^B(v)} \left[ \frac{r \lambda v O^B(v)}{\lambda(\lambda + r)(O(v) - v)} \right] = \frac{r \lambda v (O^S(v) - v)}{O(v) - v}.$$

By observing that

$$\frac{\partial S_C(v)}{\partial O^B(v)} \geq 0 \iff O^S(v) \leq v,$$

and

$$\frac{\partial B_C(v)}{\partial O^B(v)} \geq 0 \iff O^S(v) \geq v,$$

the desired result follows.  $\blacksquare$

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