Data-driven Estimation by Aggregation in Inverse problems

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Abstract

Inverse problems are becoming increasingly important in a diverse range of disciplines, including geophysics, astronomy, medicine and economics. In the econometrics literature three particular models and their extensions have been intensively discussed, namely regression models in the presence of endogenous or functional covariates and deconvolution, each of them leading naturally to an inverse problem. Roughly speaking, in all of these applications the observable signal is a transformation of the functional parameter of interest f. Consequently, inference on f usually requires an inversion of the transformation at hand and is thus called an inverse problem. Moreover, the signal and the inherent transformation are unknown in practice, although they can be estimated from the data. Our objective in this context is the construction of a fully data-driven estimation procedure of the unknown function f which is optimal in an oracle or minimax sense. In the presentation we consider the non-parametric estimation of the function f based on a dimension reduction and additional thresholding. The proposed estimation procedure relies on the choice of a dimension parameter, which in turn, crucially influences its attainable accuracy. An optimal choice, however, follows often from a classical squared-bias-variance compromise and requires a prior knowledge about certain characteristics of f and the inherent transformation, which is usually inaccessible in practice. Given a family of estimators $\{\hat{f}_m, m \in \mathcal{M}\}$ of f indexed by a dimension parameter m belonging to a pre-specified collection of models $\mathcal{M}$ the selection of a dimension parameter $\hat{m} \in \mathcal{M}$ as a minimiser of a penalised contrast criterion leads in many cases to an optimal estimator $\hat{f}_{\hat{m}}$ in an oracle or minimax sense (for regression models in the presence of functional or endogenous covariates and deconvolution with unknown error distribution see, for instance, Comte and Johannes [2012], Breunig and Johannes [2015], Asin and Johannes [2016], and Johannes and Schwarz [2013]). In this presentation we propose a fully data-driven aggregation of the estimators, $\hat{f}_{\hat{w}} = \sum_{m\in\mathcal{M}} \hat{w}_m \hat{f}_m$, which shares the optimally properties of the estimator $\hat{f}_{\hat{m}}$. The construction of the random weights $\{\hat{w}_m, m \in \mathcal{M}\}$ is inspired by the recent work of Johannes et al. [2015] where a fully data-driven Bayes estimator in an indirect sequence space model with hierarchical prior is constructed. Notably, in Johannes and Loizeau [2016] a Bayesian construction of the random weights allows to characterise the model selection estimator $\hat{f}_{\hat{m}}$ as a limit case of the data-driven aggregation strategy. As illustration we consider non-parametric regression with random design and non-parametric density estimation and we discuss its potential extension to deconvolution models as well as non-parametric functional linear and instrumental regression.

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