Approximation of the Solution of the Backward Stochastic Differential Equation

We consider the problem of estimation of the solution of the backward stochastic differential equation. Suppose that we observe a diffusion process X satisfying following forward SDE

$$\mathrm{d}X_t = S\left(\vartheta, t, X_t\right) \mathrm{d}t + \sigma\left(t, X_t\right)\, \mathrm{d}W_t, \quad X_0,\ 0 \leq t \leq T,$$

where the drift coefficient depends on the unknown parameter $\vartheta \in \Theta = (\alpha, \beta)$. For two given functions $f\left(t, x, y, z\right)$ and $\Phi\left(x\right)$, the question is to construct a pair of processes $\left(Y_t, Z_t, 0 \leq t \leq T\right)$ which is a solution of the backward SDE

$$\mathrm{d}Y_t = -f\left(t, X_t, Y_t, Z_t\right)\, \mathrm{d}t + Z_t\, \mathrm{d}W_t, \qquad Y_0, \quad 0 \leq t \leq T,$$

with the final condition $Y_T = \Phi\left(X_T\right)$. This problem was introduced as forward-backward stochastic differential equations (FBSDE) in El Karoui N., Peng S. and Quenez M. (1997) Backward stochastic differential equations in finance, *Math. Finance*, 7, 1-71. For example, let X be the dynamics of some basic securities and consider a contingent claim $\Phi\left(X_T\right)$ at maturity T. Then the value of the replicating (hedging) portfolio is given by Y and Z corresponds to the portfolio process.

Note that the solution $\left(Y_t, Z_t, 0 \leq t \leq T\right)$ is Markovian in the sense that it is given by a function of time t and a state process X_t.

We propose an approximation $\left(\hat{Y}_t, \hat{Z}_t, 0 \leq t \leq T\right)$ of the solution of the BSDE based on the one-step MLE of the unknown parameter.