Skill Accumulation in the Market and at Home

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Abstract

Learning by doing in home production is introduced into a stochastic directed search model. Workers' labor supply choices affect skill accumulation in both the home and market sectors. The optimal search behavior implies that average reemployment wages are only mildly sensitive to unemployment duration while the job finding probability is highly sensitive to duration, two facts which are documented empirically. The calibrated model is used to decompose the declining hazard out of unemployment, implying a nontrivial role for duration dependence due to skill changes. The addition of aggregate shocks leads to an asymmetric response of the unemployment rate during and after recessions, with more severe recessions resulting in stronger hysteresis in labor force participation.

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1 Introduction

Recent empirical evidence shows that reemployment and reservation wages are only mildly sensitive to duration (Fernández-Blanco and Preugschat 2016, Krueger and Mueller 2016, Schmieder et al. 2013), despite the well-documented fact that the job finding probability falls greatly over the duration of unemployment (Kaitz 1970). This paper aims to reconcile these two facts by introducing learning by doing[1] in home production into a model with unemployment risk. Workers devote their labor to market production when employed and home production otherwise. Skill accumulation in home production causes unemployed workers' outside options to improve, while skill loss in market work further increases the relative attractiveness of nonemployment. Thus, firms find it less profitable to hire the long-term unemployed, decreasing workers' job finding probabilities but increasing their reemployment wages.

There is little empirical evidence on duration's effect on starting wages[2]. Instead, the empirical literature has mainly focused on long-term earnings losses due to unemployment[3]. To understand the full effects of unemployment spells on job search and earnings outcomes, it is essential to think not only about the impact effect of entering unemployment but also the effect of the duration of the spell. Heterogeneity in home production may be empirically relevant for workers experiencing shocks to their outside options, for instance, women of childbearing age. Using data from the Current Population Survey (CPS), I show that the two empirical facts are especially pronounced for this group, providing strong suggestive evidence of the mechanism introduced here. Importantly, I establish that the results of Fernández-Blanco and Preugschat (2016), who were the first to document the motivating facts presented here, are robust to controlling for the type of job from which workers entered unemployment, as well as to using data from the Panel Study of Income Dynamics (PSID).

In this paper I propose a new channel affecting the outside options of the unemployed: skill accumulation in home production. The model is calibrated to match the declining job finding probability and implies a small decrease in starting wages, a significant improvement over models without learning by doing at home. Further, the steady state model with home

[1]Here, similar to standard models of learning by doing (Ljungqvist and Sargent 1998), the skill that a worker uses in production appreciates and the skill not in use depreciates. More recently, Jarosch (2016) and Jung and Kuhn (2016) consider skill loss in unemployment.

[2]Papers that have studied the shorter-term effects of job loss, specifically the effects of the length of unemployment duration include Addison and Portugal (1989), Gregory and Jukes (2001), and Arulampalam (2001). The latter studies the wages of British men, and finds no evidence of declines in wages due to the length of the spell, similar to the results presented here. Similar results are found by Albrecht et al. (1999) using Swedish data.

[3]For a survey of the early literature on wage scarring in the US, see papers by Fallick (1996) and Kletzer (1998). For more recent evidence, see Couch and Placzek (2010).

productivity produces reasonable unemployment and labor force participation rates. With regard to business cycles, the model implies a persistent decrease in participation in response to temporary recessions due to the evolution of outside options. To the best of my knowledge, this is the first paper to address the business cycle implications of duration dependence in the job finding probability through the lens of a structural model.

The novelty of this paper is to introduce a home production sector with "on the job" learning into a directed search model[4]. Each worker is endowed at birth with observable home and market productivities. Unemployed workers accumulate productivity at home over time, unlike the fixed technology typically assumed in the home production literature[5]. Simultaneously, the unemployed are losing their market-related skills, as is standard with on the job learning models (for instance, Ljungqvist and Sargent 1998). The driving force in the model is an evolving outside option for unemployed workers due to changes in both skills.

The intuition regarding movements in the aggregate job finding probability and reemployment wage with duration is as follows. The effect of duration on these two moments depends on two factors: the direct effect of individual skill changes with duration, and the indirect effect due to changes in the composition of unemployed workers. First, the evolution of an unemployed worker's skills influences her own optimal job search decisions. As workers spend more time in unemployment, the payoff from remaining in home production increases relative to the payoff from returning to market work. These skill changes lead workers to become pickier about the jobs they find acceptable while at the same time they are losing market-related skills. When deciding whether to accept a job, unemployed workers face a tradeoff. By moving to employment, a worker expects to gain market skills but will give up future gains in home productivity. Therefore the net utility gain that a searching worker expects from a match depends on the effects of skill changes on the value of both employment states in a complex way. Second, in the aggregate, unemployed workers who are relatively better at market production will tend to re-enter employment quickly, shifting the composition of the unemployed towards workers with relatively low market skills as duration increases. Together the direct effect, known in the literature as "true duration dependence", and the indirect effect due to compositional changes determine the decline in the job finding rate.

The model is calibrated to match key labor market facts, including the average job finding

[4]The directed search literature builds on the seminal papers of Montgomery (1991), Moen (1997), and Burdett et al. (2001). The model in this paper is most similar to Menzio and Shi (2011), with worker- rather than match-dependent productivities.

[5]See the seminal models of home production by Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991).

rate, separation rate, and the return to experience in market work estimated in US data. The calibrated model generates a large decline in the aggregate job finding probability but only a mild decrease in reemployment wages with duration. In line with the previous literature, the model implies a more important role for the composition effect in the decline of the job finding probability, although true duration dependence plays a non-trivial role.

The effects of an aggregate market productivity shock are then studied with regard to the dynamics of the job finding, participation, and unemployment rates. The directed search framework makes business cycle analysis highly tractable. Changes in the composition of worker types across labor force statuses are essential to understand the model's cyclical predictions. Starting from a steady state in which the distribution of workers across types is constant, a recession caused by a temporary decline in aggregate productivity causes a temporary fall in the job finding rate for all job seekers, an increase in the unemployment rate, and a long-term decline in the participation rate.

Specifically, when the economy experiences a negative aggregate productivity shock, lower market productivity for firms decreases the relative value of working. This relative increase in the outside option implies that the unemployed trade off higher wages for lower matching probabilities, increasing unemployment duration for all types. If the shock is large enough, workers will leave the labor force: by strictly preferring home work, some workers will discontinue their job search. In the next period, the average skills of these individuals evolve to generate a persistent decline in the participation rate even after the decline in aggregate productivity subsides. Therefore, the model gives rise to hysteresis, resulting in a persistent fall in the participation rate[6] in response to a temporary negative aggregate shock. Finally, the recession leads to a decrease in market skills relative to home skills, as workers with longer unemployment spells have more time for their skills to evolve.

The paper proceeds as follows. Section 2 describes the empirical evidence. Section 3 describes the model, beginning with the social planner's problem followed by the decentralized economy. Section 4 outlines the method for the calibration and reports the quantitative results of the model, both in and out of steady state. Section 5 concludes.

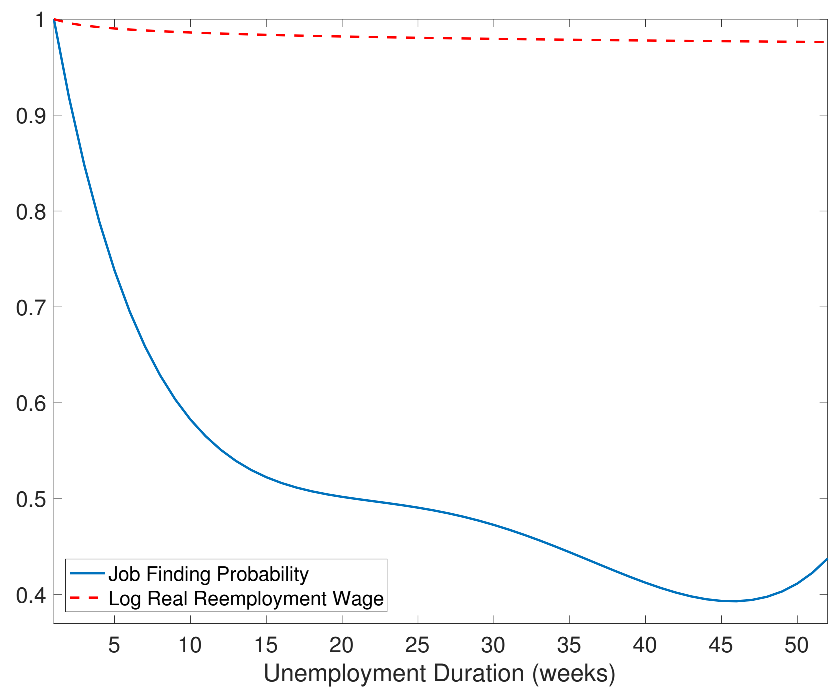
2 Empirical Evidence

This section summarizes the empirical evidence on the motivating facts discussed in the Introduction by computing reemployment wages and job finding probabilities for the unem-

[6] The model presented here has a unique equilibrium, but as with other models of human capital accumulation, may give rise to multiple steady states. Workers face a risk of death and newborn workers' skills are drawn randomly, and model simulations with a reasonable probability of death (implying a working lifetime of 30 or 40 years) suggest that the model's steady state is unique.

ployed using micro data from the CPS and PSID. The stark contrast between average job finding probabilities and reemployment wages is shown in Figure 1. The figure shows the strong negative effect of duration on the probability of transitioning from unemployment to employment, plotted as a solid line, but a much weaker effect on the reemployment wages of those workers who transition, plotted as a dashed line. Normalizing the wage and job finding probability to one at the shortest reported duration (1 week), the lines in the figure plot the predicted values over duration using the CPS estimates in column 1 of Table 2 and column 2 of Table H.13 described below.

Figure 1: Mean Job Finding Probability and Reemployment Wage by Duration



Notes: Predicted values of the mean job finding probability and log reemployment wage as functions of weekly reported unemployment duration, controlling for observables. Sample: CPS, 1994-2015, workers reporting unemployment and employment in two consecutive months, ages 18-65, with unemployment durations up to 1 year. Marginal effect of duration on the job finding probability is estimated in column (2) of Table H.13. Effect of log duration on log reemployment wage is estimated in column (1) of Table 2. Footnotes to Tables H.13 and 2 list control variables used in predictions.

In the CPS, I identify those workers who report being unemployed and actively searching for a job in month t and who report being employed in month $t+1$. The sample includes all individuals making this transition for whom both weekly unemployment duration and hourly earnings are recorded between February 1994 and December 2015. In the PSID, monthly duration is reconstructed using the monthly employment history of the head of household available in each annual interview between 1984 and 1996. Transitions are identified as observations of individuals reporting being unemployed in at least one month of the year prior to the interview who have transitioned to employment by the time of the interview. The main samples are restricted to individuals aged 18 to 65 to reduce potential issues with education and retirement, and include workers with unemployment durations up to one year. Details about the data and robustness checks are contained in Appendix C and Supplementary Appendix H, respectively.

Table 1: Linear Probability Model: Job Finding Probability on Unemployment Duration

	CPS		PSID	
	(1)	(2)	(3)	(4)
duration	-.0032***	-.0307***	-.5428***	-.4421***
	(8.45e-05)	(.0014)	(.0364)	(.0586)
duration^2		.0017***	.1165***	.0998***
		(.0001)	(.0124)	(.0201)
duration^3		-4.13e-05***	-.0107***	-.0101***
		(3.59e-06)	(.0016)	(.0026)
duration^4		3.46e-07***	.0004***	.0004***
		(3.48e-08)	(6.52 e-05)	(.0001)
R^2	.0509	.0585	.2647	.1925
N	141,916	141,916	10,803	6,154

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

In the baseline regressions, the effect of duration on the job finding probability is estimated using a linear probability model. The dependent variable of interest is a binomial variable equal to one if an unemployed worker transitioned from unemployment to employment and zero otherwise. The independent variables are unemployment duration and controls for observable heterogeneity across workers and time. Results are contained in Table 1, with robust standard errors reported in parentheses. The results indicate a strong negative correlation between duration and the job finding probability, for instance the coefficient in column (1) indicates a decline of 0.32 percentage points in the probability for each additional week spent unemployed.

An obvious concern about the estimates in columns 1 and 2 in Table 1 is that the results may be driven by unobserved heterogeneity across workers. For instance, the pool of workers with short unemployment durations may be very different from the pool of workers with long durations in a way that is unobservable to the econometrician. Since individuals participate in the CPS for only a short time[7], it is unlikely that they experience more than one unemployment spell. The longer panel structure of the PSID allows for the inclusion of these variables. Column 4 uses individual fixed effects to control for any potential unobservable heterogeneity that is fixed over time. The main insight of this regression supports those in the other columns of Table 1: even within individuals, duration has a strong negative effect on the probability of finding a job.

Table 2: Regression of log Reemployment Wage on log Duration

	CPS		PSID	
	(1)	(2)	(3)	(4)
log duration	-.0095***	-.0054	-.0197	-.0010
	(.0031)	(.0035)	(.0153)	(.0209)
dummy, > 6 mo	N	Y	Y	Y
FE	N	N	N	Y
R^2	.4117	.4117	.2150	.0112
Root MSE	.3272	.3272	.6362	
N	16,758	16,758	10,565	6,056

Notes: CPS Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E excluding those for whom the CPS allocated the hourly wage, with durations up to 52 weeks. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and total labor market experience. Column 1 reports results for the regression of workers at all durations with no long term unemployment dummy in the CPS, column 2 is the same regression with the long term dummy. Column 3 is identical to column 2 using the PSID sample, and column 4 includes individual fixed effects in the PSID sample. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Results for regressions of the reemployment wage on duration are reported in Table 2. Reemployment wages are defined as real reported hourly wages in logs, deflated using the US city average CPI. Columns 1 and 2 report results using weekly duration reported in the CPS, and columns 3 and 4 report results using monthly duration in the PSID, without and with individual fixed effects, respectively. Column 1 shows that wages decline with duration, controlling for observable characteristics, however after controlling for long term

[7]Respondents in the CPS are interviewed 8 times over a period of 16 months: interviews are conducted for 4 months consecutively, followed by 8 months of no interviews, and finally again for 4 consecutive months.

unemployment, the effect of duration on wages disappears[8].

Differently from the existing literature, the regressions shown in this section control for the industry and occupation *prior* to the unemployment spell, rather than *following* the spell. One can imagine a scenario in which workers search for jobs for which they are more and more "overqualified" as the opportunities matching their skills diminish with duration. This could lead to the pattern of flat wages because workers who are overqualified for jobs wait longer before accepting them, but are paid more than those workers at or below the qualifications who are hired after shorter unemployment spells. Instead, by controlling for the job held prior to unemployment, the regressions below show the relationship between duration and post-unemployment wages for similar workers prior to the spell.

A key robustness check in support for the mechanism introduced here is to split the sample into subgroups that may be more or less likely to experience learning by doing in the home. In particular, the table below shows regressions using the CPS data for prime-aged men, ages 25-54, and women of childbearing age, 20-40. The results in Table C.1 in Appendix C.2 show a wage that is unaffected by changes in duration for the women, but about twice as sensitive as the entire population in Column 1 of Table 2 for the men. Further suggestive evidence is discussed in Supplementary Appendix H.4.

3 Model

The economy is populated by a continuum of workers of measure 1 and a continuum of firms with a positive measure. Time is discrete and the horizon is infinite. All agents are risk neutral and discount the future at rate $\beta \in (0,1)$. There is a single consumption good produced in the economy. Workers are ex ante heterogeneous and defined by their observable productivities in the market and at home, or their "type," respectively (z,h), where $z \in Z = [\underline{z}, \overline{z}]$ and $h \in H = [\underline{h}, \overline{h}]$. The range of values the market and home productivities may take satisfy $0 < \underline{z} < \overline{z} < \infty$ and $0 < \underline{h} < \overline{h} < \infty$. Workers' skills evolve each period depending on their current employment status[9]. The stationary transition functions for these processes are given by Q_U and Q_E with $Q_i((b,c),(B,C)) = Pr((z',h') \in B \times C | (z,h) = (b,c); i)$, where $i = U, E$ indicates a current unemployed or employed worker, for given sets $B \subseteq Z$ and $C \subseteq H$ and scalars $(b,c) \in Z \times H$.

Each firm operates a constant returns to scale technology that turns 1 unit of labor from

[8] It is worth noting that the relatively small effect of duration on wages is not an artifact of all workers entering employment at the minimum wage. Of all workers reporting reemployment wages between the ages of 18 and 65 in the CPS sample, less than 10% report nominal hourly wages at or below the federal minimum wage.

[9] For an extension with human capital investment choices, see Supplementary Appendix F.

a worker of type (z, h) into Az units of output. Aggregate productivity A is common to all firms and each period lies in the set $\mathbf{A} = [\underline{A}, \overline{A}]$ where $0 < \underline{A} < \overline{A} < \infty$. This productivity is drawn from a distribution denoted by the stationary transition function $P(\cdot, A)$. Each firm maximizes its present value of profits. When a worker of type (z, h) is matched with a firm, she is employed and provides z units of effective labor inelastically. Unmatched workers are unemployed[10], and produce h units of output through home production.

3.1 Planner's Problem

Timing in each period is as follows. At the beginning of the period, nature draws the aggregate productivity A and worker-specific productivities (z, h) depending on each worker's employment status the previous period[11]. Workers face an exogenous probability of death denoted $\lambda \in (0, 1)$. I assume that workers have a bequest motive whereby they derive utility from future generations of newborn workers. The same mass λ of workers is born with productivities drawn from a given stationary distribution F_0.

The period then proceeds in the following stages: separation, production, search and matching. In the separation stage, with probability $d \in [\delta, 1]$ an employed worker separates from his match and enters unemployment, where $\delta \in (0, 1)$ is the exogenous separation probability. During the production stage, employed worker-firm pairs produce output Az using the firm's and worker's market productivities. Unemployed workers produce and consume h units of output through home production.

The labor market is defined by submarkets in which workers and vacancy-posting firms meet. The cost of posting one vacancy is a strictly positive constant k. Unemployed workers may search in one submarket each period. In this model with identical firms and worker-specific productivities, there is no motive for on-the-job search; therefore without loss of generality I assume that only unemployed workers have the opportunity to search in each period[12].

Finally, in the matching stage, the number of hires in a submarket is determined by a constant returns to scale technology $m(a, v)$ where a is the number of applicants in the submarket and v is the number of vacancies. Market tightness θ is a function of the ratio of vacancies to applicants in a given submarket. The probability that a vacancy meets a worker is $q(\theta) \equiv \frac{m(a,v)}{v}$, where $q : \mathbb{R}_+ \to [0, 1]$ is a twice continuously differentiable,

[10] In the theoretical model, I refer to all nonemployed workers as unemployed. The distinction between workers who are unemployed and out of the labor force depends on the definition of active job search and will be made in Section 4.

[11] This assumption guarantees that learning by doing occurs only after the first period of production.

[12] If market productivities were match-specific, the planner would find it optimal for employed workers to search when the productivity in the current match is sufficiently low.

strictly decreasing and convex function with $q(0) = 1$, $q'(0) < 0$, and $p'(q^{-1}(\cdot))$ concave. Similarly, the probability that a worker meets a vacancy is given by $p(\theta) = q(\theta)\theta$, where $p : \mathbb{R}_+ \to [0, 1]$ is twice continuously differentiable, strictly increasing and strictly concave with $p(0) = 0,\ p(\infty) = 1$ and $p'(0) < \infty$.

In any period, the economy is characterized by the aggregate state $\psi \equiv (A, u, e)$, with the set of possible values that ψ may take denoted by Ψ. The first element of ψ is the aggregate productivity $A \in \mathbf{A}$. The second element is a function $u : Z \times H \to [0, 1]$, describing the distribution of unemployed workers across productivities at the beginning of the production stage, where $u(z, h)$ denotes the mass of workers who are unemployed of type (z, h). Similarly, the third element is a function $e : Z \times H \to [0, 1]$, where $e(z, h)$ denotes the mass of employed workers of type (z, h) at the beginning of the production stage.

3.1.1 Formulation of the Planner's Problem

The planner's problem is to maximize aggregate consumption in the economy by choosing how to allocate workers and vacancies across submarkets. Specifically, in the separation stage the planner chooses separation probabilities for each employed worker. In the search stage, the planner chooses how many vacancies firms post in each submarket and in which submarkets unemployed workers search. Separated workers must spend one period in unemployment before searching and new hires must spend one period in employment before separating.

At the beginning of each period, the planner observes aggregate state ψ and chooses θ for each submarket and d for each worker-firm pair. Since workers of a given type are identical, the planner will choose one strategy for all workers of the same type in each employment state. Due to two-dimensional heterogeneity, the planner may find it optimal to assign the same market tightness to two types of unemployed workers. However, it is equivalent in terms of welfare to create two type-specific submarkets with the same tightness. Therefore, I assume that there is one submarket per type in each period. Denoting the skill pair $y \equiv (z, h)$ with $Y = Z \times H$, aggregate consumption conditional on θ is given by the sum of production of all workers less any vacancy costs:

$$C(\theta|\psi) \equiv \int_Y hu(y) + Aze(y) - k\theta u(y)\ dy \tag{1}$$

The planner's problem is to solve:

$$W(\psi) = \sup_{\theta \in \mathbb{R}_+} \left\{ C(\theta|\psi) + \beta \mathbb{E} \max_{d \in [\delta, 1]} \{ W(\hat{\psi}) \} \right\} \tag{2}$$

subject to the endogenous laws of motion for u and e, given by the following expressions:

$$\hat{u}(y') = \lambda f_0(y') + (1-\lambda)\left[\int_Y (1-p(\theta(y,\psi)))u(y)Q_U\big(dy,y'\big) + d(y',\hat{\psi})\int_Y e(y)Q_E\big(dy,y'\big)\right] \quad (3)$$

$$\hat{e}(y') = (1-\lambda)\left[\int_Y p(\theta(y,\psi))u(y)Q_U\big(dy,y'\big) + (1-d(y',\hat{\psi}))\int_Y e(y)Q_E\big(dy,y'\big)\right] \quad (4)$$

where $\theta(y,\psi) \equiv \theta(z,h,\psi)$ and $d(y,\psi) \equiv d(z,h,\psi)$ are the market tightness and separation probability for a given worker type y when the aggregate state is ψ. The transition functions Q_U and Q_E define the laws of motion for the level of skills next period given a worker's current levels of skills and whether she is unemployed (U) or employed (E) at the beginning of the production stage. Henceforth, for any aggregate variable, a caret denotes its value next period.

Equation (3) says that the distribution of unemployed workers of type $y' = (z', h')$ at the beginning of next period is given by a constant mass of newborn workers plus those surviving workers who were unemployed this period, did not match with a firm and drew productivities (z', h'), plus those employed workers who drew new productivities this period and separate from their matches at the beginning of next period. Equation (4) is similar and gives the mass of surviving workers who will be employed at the beginning of next period's production stage with type (z', h'). The following assumption is made for tractability.

Assumption 1. *(i) Productivities (z,h) evolve independently from A. For any $B \subseteq Y$, $C \subseteq \mathbf{A}$, $b \in Y$ and $c \in \mathbf{A}$,*

$$Pr\big(y' \in B, \hat{A} \in C \big| y = b, A = c, i\big) = Q_i(b,B)P(c,C) \quad \textit{for } i \in \{E,U\}$$

(ii) The following monotonicity conditions hold:

$$\textit{if } f: A \to \mathbb{R} \textit{ is nondecreasing, then } \int f(\hat{A})P(A,d\hat{A}) \textit{ is also nondecreasing}$$

$$\textit{if } f: Y \to \mathbb{R} \textit{ is nondecreasing, then } \int f(y')Q_i(y,dy') \textit{ is also nondecreasing for } i = U,E$$

Part (i) of Assumption 1 states that given a worker's skill pair today, the current employment state determines its evolution. Aggregate productivity is assumed to be independent of worker-specific skills to exclude the possibility of human capital externalities as in Lucas (1988). Part (ii) will be used only in the results regarding monotonicity in Theorem 1.

3.1.2 Constrained Efficiency

The above formulation of the planner's problem leads to the following theorem. All proofs are left to Appendix A.

Theorem 1. *(i) The following problem is equivalent to (2).*

$$\tilde{W}(\psi) = \int_Z \int_H W_U(z,h,A)u(z,h) + W_E(z,h,A)e(z,h)\ dh\ dz \tag{5}$$

where

$$W_U(z,h,A) = \max_{\theta\in[0,\bar{\theta}]} \Big\{ h - k\theta + \beta(1-\lambda)\Big[\mathbb{E}_U(W_U(z',h',\hat{A})|z,h,A) + p(\theta)\mathbb{E}_U\big(W_E(z',h',\hat{A}) - W_U(z',h',\hat{A})|z,h,A\big)\Big] + \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A)\Big\} \tag{6}$$

$$W_E(z,h,A) = Az + \beta(1-\lambda)\mathbb{E}_E\big(\max_{d\in[\delta,1]} \{d(h' + \beta D(z',h',\hat{A},W_U)) + (1-d)W_E(z',h',\hat{A})\}|z,h,A\big) + \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A) \tag{7}$$

where

$$D(z,h,A,W_U) = (1-\lambda)\mathbb{E}_U\big(W_U(z',h',\hat{A})|z,h,A\big) + \lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A)$$

(ii) $\tilde{W}(\psi)$ is the unique solution to (5). (iii) W_U is strictly increasing in h and weakly increasing in z and A, and W_E is strictly increasing in z and A and weakly increasing in h if Assumption 1 holds. (iv) The policy correspondences θ^* and d^* associated with (5) depend on ψ only through A: $\theta^*(z.h,\psi) = \theta^*(z,h,A)$ and $d^*(z,h,\psi) = d^*(z,h,A)$.

The expectation operator $\mathbb{E}_i$ denotes the expectation taken with respect to the transition function Q_i, $i \in \{U, E\}$ for the skill pair conditional on the current type (z,h), and aggregate state ψ, while the expectation operator $\mathbb{E}_0$ denotes the expectation taken with respect to the aggregate state and distribution F_0.

Several elements of the model complicate the analysis of the planner's problem relative to those analyzed in the previous literature. In particular, the planner's decisions in terms of market tightness and separation rates affect the endogenous distributions of worker types across employment states in a nontrivial way. Unlike models with *iid* draws of match-specific productivity (e.g. Menzio and Shi 2011), here the persistence of workers' productivities when transitioning between unemployment and employment causes the planner's choices to not only affect the level of employment, but also to dynamically affect the distribution of types across employment and unemployment. This distributional dependence interacts with the uncertainty about the aggregate productivity.

However, as Theorem 1 shows, the planner's objective of maximizing aggregate consumption is equivalent to maximizing each worker type's consumption separately. Intuitively, the law of large numbers implies that the matching and separation probabilities exactly determine the endogenous distributions of worker types next period. Since aggregate consumption is the sum of consumption of each type, it is equivalent to maximize the aggregate utilities jointly or maximize each worker type's utility separately.

3.2 Decentralized Economy

This section decentralizes the planner's problem outlined in the previous section. Within a period, timing in the decentralized economy is similar to that described for the planner, with the following changes. First, employed workers may choose to separate from their match with probability $d \in [\delta, 1]$. Second, submarkets are indexed by (x, z, h, ψ), where $x \in \mathbb{R}$ is the value in terms of the worker's lifetime utility of the match and (z, h) is the skill pair of the worker for which the vacancy is intended. Second, in the production stage, employed worker-firm pairs produce Az and employed workers consume their labor income w. In the search stage, firms choose submarkets in which to post vacancies and workers observe the distribution of offers before choosing one submarket in which to search. A firm may post a vacancy by paying a cost $k > 0$. Each vacancy in a submarket offers the same value x, and firms commit to this value as well as the type of worker they will hire if a match occurs[13].

At the beginning of the production stage, the value function for an unemployed worker of type (z, h) is

$$V_U(z,h,\psi) = \sup_x \Bigg\{ h + \beta(1-\lambda)\Bigg[(1-p(\theta(x,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{\psi})|z,h,\psi) \\ + p(\theta(x,z,h,\psi))x\Bigg] + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi)\Bigg\} \quad (8)$$

where the policy function is denoted $x(z, h, \psi)$ and the implied market tightness is denoted $\theta(x, z, h, \psi)$.

Given a wage w, the value function for an employed worker with skills (z, h) at the

[13] In this model with worker specific skills, workers do not necessarily sort across submarkets endogenously, as they do in Menzio and Shi (2011) when productivities are match specific. To guarantee that each submarket contains only one worker type, I assume firm commitment to worker skills, which are assumed to be observable.

beginning of the production stage is given by

$$V_E(z,h,\psi;w) = w + \beta(1-\lambda)\mathbb{E}_E(\max_{d\in[\delta,1]}\big\{d(h' + \beta D(z',h',\hat{\psi},V_u)) + (1-d)V_E(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad (9)$$

where the function D is defined analogously to the expression in Theorem 1. Similarly, value of a firm employing a worker whose current skills are (z,h) given her wage w and next period's separation probability d' can be written

$$J(z,h,\psi;w) = Az - w + \beta(1-\lambda)\mathbb{E}_E((1-d')J(z',h',\hat{\psi};w')|z,h,\psi) \quad (10)$$

I assume that employment contracts are complete in the sense that they specify the wage and separation probability as a function of tenure t and history of productivities $\{z^t;h^t;A^t\}$ over tenure in the match, t. As shown in Menzio and Shi (2011), this contractual environment results in bilaterally efficient contracts which maximize the sum of the firm's expected profits and the worker's expected utility. This result follows from the fact that firms must guarantee the expected value x to any worker with whom it matches, forcing the firm to internalize the optimal choices of the worker when choosing the contract. Hence the optimal choice of the separation probability is the solution to the value of the match, which is equal to the sum of (9) and (10):

$$V_M(z,h,\psi) = Az + \beta(1-\lambda)\mathbb{E}_E(\max_{d\in[\delta,1]}\big\{d(h' + D(z',h',\hat{\psi},V_u)) + (1-d)V_M(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad (11)$$

The wage is absent from (11) because it is simply a transfer from the firm to the worker, leaving the value of the match unchanged. It can be shown that the solution to (11) is $d(z,h,\psi) = \delta$ if and only if $h + D(z,h,\psi,V_u) < V_M(z,h,\psi;w')$ and $d(z,h,\psi) = 1$ otherwise.

To close the model, there is free entry into vacancy posting in every submarket, so that the firm's benefit of vacancy creation in a non-empty submarket is equal to the cost:

$$k \geq \beta(1-\lambda)q(\theta(x,z,h,\psi))(\mathbb{E}_U(V_M(z',h',\hat{\psi})|z,h,\psi) - x(z,h,\psi)) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0 \quad (12)$$

with complementary slackness. Since the timing of the model is such that matches are made at the end of the period, a firm offers lifetime utility x which is known one period before production takes place. Therefore the firm discounts the expected value of the match by $\beta(1-\lambda)$. The expectation is taken with respect to the conditional distribution of productivities while unemployed since the worker evolves once more before beginning production. This distribution depends only on the current type of the worker that the firm commits to hire, and not on the entire distribution of workers across types.

Looking at equation (12), it is clear that the present model is not equivalent to a model with one relative skill, say z/h. It is the constant cost of vacancy posting that breaks the model's homogeneity. Supplementary Appendix E extends the model to one with a proportional vacancy cost and proves that the extended model is indeed homogeneous in h. However, the intuition driving the fall in individual job finding probabilities breaks down in this case. As workers lose relative skills, their productivity falls together with the cost for firms to post vacancies targeting these workers. Thus, in general the job finding probability will not fall sharply with duration as it does in the two-skill model due to this vacancy cost-production tradeoff.

Since each firm posting a vacancy for value x commits to hire a single type, the firm knows for certain the type of worker that it will hire in any submarket. For any worker of a type different than (z, h), it is not optimal to search in submarket (x, z, h, ψ) since there is probability zero that she will be hired. Thus, the firm's decision to post a vacancy does not depend on the distribution of searching workers. Zero expected profits in equilibrium imply that firms are indifferent as to which submarket they post vacancies.

Following the literature, equilibria are restricted to those in which the market tightness satisfies complementary slackness condition (12) in every submarket. This implies that firms must be indifferent between posting vacancies in any submarket, whether or not it is active in equilibrium, so that market tightness is always pinned down by the free entry condition. I now turn to the definition of equilibrium.

Definition 1. *A block recursive equilibrium (BRE) consists of a market tightness function* $\theta : \mathbb{R} \times Z \times H \times \mathbf{A} \to \mathbb{R}_+$*, a value function for the unemployed worker* $V_U : Z \times H \times \mathbf{A} \to \mathbb{R}$*, a policy function for the unemployed worker* $x : Z \times H \times \mathbf{A} \to \mathbb{R}$*, a value function for the employed worker-firm match* $V_M : Z \times H \times \mathbf{A} \to \mathbb{R}$*, and a policy function for the match* $d : Z \times H \times \mathbf{A} \to [\delta, 1]$*, where:*

(i) $V_U(z, h, A)$ *satisfies (8)* $\forall (z, h, \psi) \in Z \times H \times \Psi$ *and* $x(z, h, A)$ *is the associated policy function.*

(ii) $V_M(z, h, A)$ *satisfies (11)* $\forall (z, h, \psi) \in Z \times H \times \Psi$ *and* $d(z, h, A)$ *is the associated policy function.*

(iii) $\theta(x, z, h, A)$ *satisfies (12)* $\forall (x, z, h, \psi) \in \mathbb{R} \times Z \times H \times \Psi$

In any BRE, agents' value and policy functions are independent of the distributions of workers across employment and unemployment as functions of their types. Given the market tightness function θ, Condition (i) ensures that unemployed workers' search strategies are optimal and condition (ii) ensures that employed worker-firm pairs' separation strategies are optimal. Condition (iii) states that the market tightness function θ is consistent with firms' incentives to create vacancies.

Given the infinite horizon programming problem faced by individuals in the decentralized economy, the analysis of equilibrium proceeds as follows. First, a lemma is stated showing that there exists a functional equation for all agents equivalent to solving equilibrium conditions (8), (11), and (12). Then, Theorem 2 shows that the functional equation from the lemma satisfies boundedness and continuity restrictions and therefore admits a unique solution. Further, by the recursive structure of the functional equation, the solutions of the problem are independent of the distributions (u, e) and satisfy Definition 1, therefore the unique decentralized equilibrium is a BRE.

Lemma 1. *An equilibrium exists if and only if it solves the following problem:*

$$\begin{aligned} V(\alpha, z, h, \psi) = \alpha\Big(& Az+\beta(1-\lambda)[\mathbb{E}_E(\max_{d\in[\delta,1]}\{(d(h'+\beta D(z', h', \hat{\psi}, V(0)))+(1-d)V(1, z', h', \hat{\psi})\})]\Big) \\ & + (1-\alpha)\max_{\theta\in[0,\overline{\theta}]}\big\{h - k\theta + \beta(1-\lambda)\big(\mathbb{E}_U(V(0, z', h', \hat{\psi})) \\ & + p(\theta)\mathbb{E}_U(V(1, z', h', \hat{\psi}) - V(0, z', h', \hat{\psi}))\big)\big\} + \beta\lambda\mathbb{E}_0(V(0, z', h', \hat{\psi})) \end{aligned} \tag{13}$$

$$\textit{where } V(0, z, h, \psi) \equiv V_U(z, h, \psi), \quad V(1, z, h, \psi) \equiv V_M(z, h, \psi)$$

$$D(z, h, \psi, V(0))) = (1-\lambda)\mathbb{E}_U\big(V(0, z', h', \hat{\psi})|z, h, \psi\big) + \lambda\mathbb{E}_0(V(0, z', h', \hat{\psi})|\psi)$$

and the period payoff function, $\alpha Az + (1-\alpha)(h - k\theta)$, *is bounded and continuous.*

Theorem 2. *(i) All equilibria are block recursive. (ii) There exists a unique BRE.*

Part (i) of Theorem 2 comes from the assumptions of directed search and complete contracts. Given a fixed aggregate productivity A, if there are two submarkets committed to hire a worker of type (z, h), the worker faces a trade off between a higher probability of matching and a higher expected value of the match. The higher is the value offered in a submarket committed to (z, h), the more applicants of type (z, h) relative to vacancies it will attract, decreasing the probability for an individual worker to find a match. Since the firm commits to hire a certain type of worker, the firm's probability of matching will depend only on one worker type rather than the distribution of searching workers across productivities. This feature of directed search is not present in random search models, in which the firm's choice depends on its expectation of the type of worker it will meet, and thus the entire distribution of searching workers across types.

The existence of type-specific submarkets acts to complete the labor market in the sense that market tightness is specific to each productivity pair and therefore provides a "price" for each type. The contracting assumption along with firm commitment allows me to restrict attention to the value of a match and pin down the lifetime value to the employed worker, x, as a function of the market tightness and the match value. Due to the two-dimensional heterogeneity of workers, without the restriction of commitment it is possible that two types of workers will find it optimal to search in the same submarket, causing the block recursive property of the equilibrium to break down. In this case, equilibria will still exist, although they will not be explored here.

3.3 Efficiency of the Decentralized Equilibrium

The following proposition states that the equilibrium described in Section 3.2 is efficient in the sense that the value and policy functions satisfying the BRE are identical to those that solve the planner's problem discussed in the previous section.

Proposition 1. *The unique BRE in the decentralized economy is efficient, that is,* $\theta(x, z, h, A) = \theta^*(z, h, A)$ *and* $d(z, h, A) = d^*(z, h, A)$.

The decentralized equilibrium is efficient because of the presence of type-specific submarkets and the assumptions of complete contracts and firm commitment to types. As discussed regarding Theorem 2, the presence of submarkets in which only one worker type (z, h) searches forces firms to internalize the externalities that are typically present in other models. Since the planner values home productivity as much as workers in the decentralized economy, it is easy to show that the planner's value of unemployment, $W_U(z, h, A)$, satisfies (8). Without complete contracts, a firm and worker would not necessarily divide the surplus optimally, and the joint value function (11) would not be solved by the value of a match to the planner $W_E(z, h, A)$. However, when the surplus is maximized by restricting the contract space, it can be shown that the value of a match in the decentralized equilibrium and the value of an employed worker to the planner both solve (11).

Since wages are not pinned down in the model due to efficiency, one must take a stand on the form of equilibrium wages. Under the assumption that wages are determined by Nash bargaining with the worker's bargaining power equal to $\gamma \in (0,1)$, wages satisfy

$$(1-\gamma)J(z,h,A;w) = \gamma\big(V_E(z,h,A;w) - h - \beta D(z,h,A,V_U)\big) \tag{14}$$

where the firm's outside option is 0 by free entry. If the worker separates, he receives his output in home production plus the discounted value of unemployment next period. Imposing the Hosios (1991) condition by setting γ equal to the elasticity of the job finding probability with respect to market tightness maintains the efficiency result of Proposition 1.

4 Quantitative Results

This section presents the quantitative results of the model. Section 4.1 discusses the calibration of model parameters. Sections 4.2 and 4.3 describe the steady state and business cycle implications of the calibrated model, respectively.

4.1 Calibration

The model is calibrated to match moments in the US data, and is solved in steady state, fixing the aggregate productivity $A = 1$. The length of a period is one month. Several parameters are chosen exogenously. First, the state spaces for individual skills z and h are discretized into grids of 9 and 12 points, respectively, with equally log-spaced state vectors where $\log(s_{i+1}) - \log(s_i) = \Delta_s$, for $i = 1, \dots N_s - 1,\ s = \{z, h\}$, with $\Delta_z = \Delta_h = .05$. The value of z_1 is normalized to 1.

The transition matrices for individual skills are defined as follows. When a worker is employed and has market skill z, with probability π_{Ez} the worker will have skill $z' = \min\{z + \Delta_z, z_{N_z}\}$ next period, and with probability $1-\pi_{Ez}$ her skill does not change: $z' = z$. Similarly, the probability that an unemployed worker's home skill increases is π_{Uh}, and the probability that an employed worker's home skill falls and an unemployed worker's market skill falls are denoted π_{Eh} and π_{Uz}, respectively. Finally, the probability of death is chosen such that the expected lifetime of a worker is 40 years and the distribution from which newborn workers' skills are drawn, F_0, is equal to the ergodic distribution of unemployed, u.

The parameters to be calibrated are summarized in Table 3 and the targets and model-implied values are shown in Table 4. The discount factor β implies an annual interest rate of 5%. Following Menzio and Shi (2011), the functional form for the probability that a worker matches with a firm is given by $p(\theta) = \min\{\theta^\gamma, 1\}$. The matching function parameter γ is set to 0.4. The remaining 7 parameters are calibrated jointly to minimize the distance between the point estimates in the simulated model and the calibration targets.

Table 3: Parameters

Parameter	Value	Description
Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^\gamma, 1\}$
Δ_h	.05	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.05	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	.82	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.31	$z' = \min\{z + \Delta_z, z_9\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	.16	$h' = \max\{h - \Delta_h, h_1\}$ with prob π_{Eh} if E, h o.w.
π_{Uh}	.69	$h' = \min\{h + \Delta_h, h_{12}\}$ with prob π_{Uh} if U, h o.w.
π_{Uz}	.45	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	2.2	Vacancy cost
δ	.023	Separation probability

The model is simulated in steady state for 50,000 workers over 1,000 periods, discarding the first 500 months. Initial productivities are drawn from the ergodic distribution u. The moments in the model, reported in Table 4, correspond to model averages over the simulations with their 95% confidence intervals in brackets. For comparability with the data, worker types with a job finding probability greater than 5% are considered unemployed. Henceforth, this threshold will be referred to as the labor force cutoff. Several robustness checks for important parameters including the choice of this threshold are discussed in Supplementary Appendix G.

The lowest home skill, h_1, is chosen such that the expected value of an unemployed worker's home productivity relative to an employed worker's market productivity in the steady state matches the estimate of the relative value of nonmarket to market activity in the model of Hall and Milgrom (2008), $\frac{\mathbb{E}_U(h)}{\mathbb{E}_E(z)} = .71$. Adding an unemployment benefit, as Hall and Milgrom do, would increase the model value closer to the data. Interestingly, the model moment is far lower than that in Hagedorn and Manovskii (2008), but including the workers out of the labor force increases the average to 0.91, close to their estimate of 0.955.

The parameters driving the accumulation and depreciation of market skills, π_{Ez} and π_{Uz}, are chosen to match the average wage increase after one year of employment and the lifetime

Table 4: Targets

Description	Target	Model [95% CI]
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.605 [.597 , .614]
Change in earnings with 1 year of market experience	2.30%	2.21% [1.67%, 2.80%]
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.56% [0.23%, 0.69%]
Lifetime earnings losses due to unemployment	−11.9%	−11.8% [-65%, -3.0%]
Annual decline, hazard out of U	44.5%	49.5% [11.1%, 60.9%]
Quarterly average EU rate	.023	.023 [0.23, 0.23]
Quarterly average UE rate	.328	.326 [.303 .348]

earnings losses due to displacement, respectively. The calibrated parameters suggest that market skill accumulation is slower than depreciation when unemployed. Wage increases are estimated by Kambourov and Manovskii (2009) as the regression coefficient representing the annual return to experience in terms of real wages for white male heads of household in the PSID between 1981 and 1992. In the model, the annual implied wage increase from experience is equal to the average annually compounded increase in wages over the cross section of employed workers. Average lifetime earnings losses are taken from estimates by Davis and von Wachter (2011) and represent the loss in the present value over 20 years of earnings of workers with at least 3 years of tenure who experienced a mass layoff relative to a counterfactual had the layoff not occurred. In the calibration, earnings losses are computed as aggregate wages over 20 years following a displacement relative to the counterfactual wages were the workers never to have entered unemployment for workers with at least 3 years of tenure. Earnings losses in the model primarily identify the skill loss during unemployment, although the speed at which skills increase upon reemployment is also an important determinant.

The probabilities dictating the evolution of home skills, π_{Uh} and π_{Eh} are chosen to match two features of the job finding probability estimated using the CPS, controlling for observable characteristics. Taking as given the choices of π_{Uz} and π_{Ez}, these moments are the percentage decline in the job finding probability between 1 and 12 months of unemployment duration and the average change in the job finding probability at one month of duration as a function of years of tenure in a worker's previous job. The former is driven by home skill accumulation during unemployment (π_{Uh}), while the latter is due to home skill loss while employed (π_{Eh}). The calibrated parameters suggest that learning at home occurs much more quickly than the same skill depreciates during employment. The decline in the job finding probability is estimated in the data in Section 2. The corresponding moment in the model is the ratio of the average job finding probability in the cross section of unemployed workers, conditional on duration.

In the data, the change in the job finding probability over tenure is the estimated marginal effect of an additional year of pre-unemployment tenure on the job finding probability in the first month of unemployment, controlling for observables[14], for workers with up to 15 years of tenure. In the model, this moment corresponds to the average change in the job finding probability in the first period of unemployment with an additional year of pre-unemployment tenure, computed in each simulated month in the cross section of newly unemployed workers with tenure between 1 and 15 years.

The employment to unemployment and unemployment to employment (EU and UE, respectively) rates in the data are computed at quarterly frequency following Shimer (2005). The UE transition rate in the model is computed as the average number of individual transitions to employment in the simulations divided by the total number of unemployed over each 3-month span, and similarly for the EU rate. In the model, the UE rate is the fraction of workers with job finding probabilities above the active search threshold who enter employment in any period. This moment is driven by the vacancy cost parameter, k. The calibrated value of this parameter suggests that the average vacancy cost is equal to 2.5 months' production in the average match, close to values used in the previous literature. The calibrated model implies that no voluntary separations occur; the model's EU rate is equal to the involuntary separation probability δ.

Table 5 summarizes some of the untargeted moments. All moments in the table are computed in the data over the period 1994-2015 to be consistent with the analysis in Section

[14]This estimate comes from the CPS's Displaced Worker Survey (DWS). See Appendix B for details.

Table 5: Untargeted Moments, Steady State

Description	Data	Model [95% CI]
% change, log reemployment wage	−1.7%	−8.6% [-11%, -5.1%]
Unemployment rate	6.0%	6.6% [5.8%, 7.4%]
Labor force participation rate	65.8%	66.2% [59.2%, 74.0%]
Initial job finding probability (1 month)	.402	.512 [.507, .520]

2. Since wages are not pinned down in the model due to efficiency, one must take a stand on the form of equilibrium wages. For comparison to the log wages in the data, wages determined by Nash Bargaining in the model are scaled such that the average reemployment wage in the model simulations is equal to the average real hourly wage reported in the CPS. The predicted decline in the average log reemployment wage[15] is 8.6% over the first year of unemployment, well above the 1.7% drop estimated in Section 2. As is shown in Figure 2 below, the calibrated model in which the home skill for all unemployed workers is constant generates a wage that falls by 19%, more than double the figure for the full model. In addition, the model-implied unemployment and participation rates are near their empirical counterparts. Finally, the level of the job finding probability after 1 month of duration in the model is larger than in data (51.2%) compared to the estimate of 40.2% conditional on observables found in Section 2.

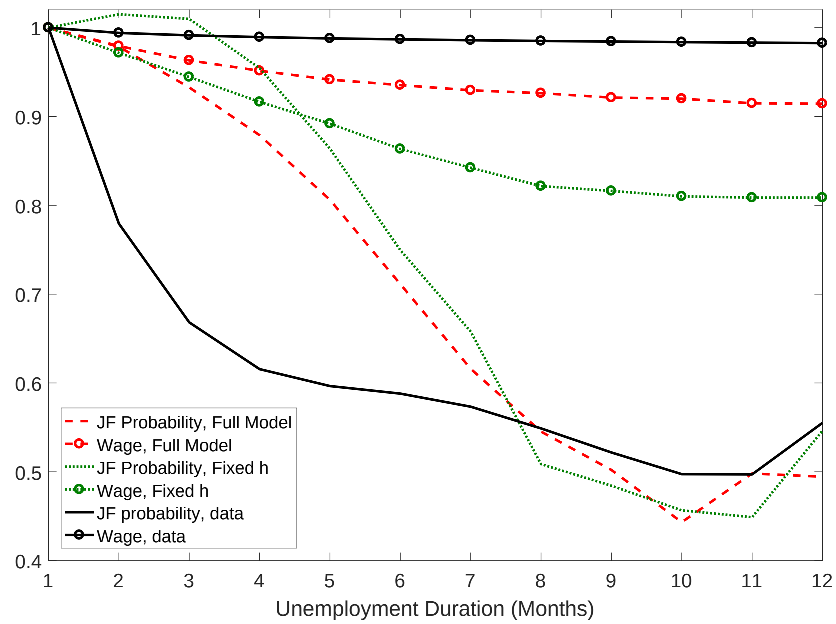
4.2 Steady State

Using the calibrated parameters described in the previous section, this section discusses the quantitative implications of the steady state model. The aggregate job finding rate and reemployment wage are drawn as dashed red lines in Figure 2. The solid black lines correspond to the predicted data controlling for observables reproduced from Figure 1. In the model, the slope of the job finding rate is targeted, which is equivalent to targeting the value of the normalized job finding probability at 12 months. For comparison, the predictions of

[15] If wages are instead equal to a constant split of the match surplus leads to a decline in the wage of 0.05%, far less than the empirical estimate.

the analogous model with a fixed home skill ("fixed h" model) are shown with green dotted lines. Model-implied moments and calibrated parameters for the fixed h model are shown in Tables D.2 and D.3. Though untargeted, the full model generates a relatively mild decline in the average reemployment wage, a large improvement over the fixed h model.

Figure 2: Normalized Job Finding Probability and Reemployment Wage

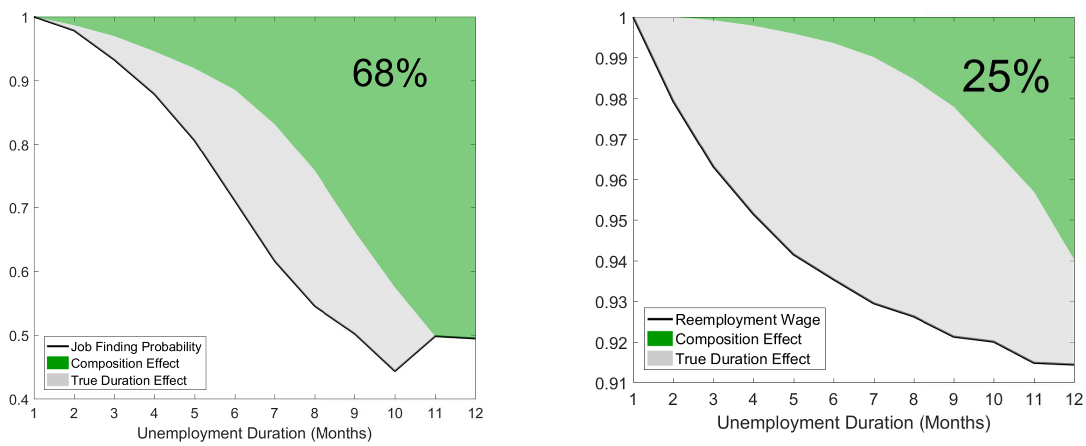


Notes: Model-implied values of the average job finding and log reemployment wages over unemployment duration, reported in months. Red dashed and green dotted lines indicate values in the benchmark and fixed h models, respectively and solid lines reproduce the predicted values from the data shown in Figure 1.

The fixed h model implies that market skills must depreciate much more quickly to match the decline in the job finding probability and earnings losses. The brief increase in the job finding probability in the first month of duration reflects the fact that at the beginning of the unemployment spell, the average worker has much to lose from remaining unemployed, leading to an increasing matching probability (and decreasing offer x). This hump-shape does not arise in the model with learning by doing in h due to the gains in consumption expected during unemployment through future increases in home skills.

The decline in the job finding probability is decomposed into the true duration and composition effects, shown in the left panel of Figure 3. The composition effect is the relatively more important factor in explaining the drop in the hazard rate: accounting for 68% of its total decline in the first year of unemployment. This result is roughly in line with recent empirical findings by Ahn and Hamilton (2016) who find that the majority of the decline is due to compositional changes. The composition effect in the model is computed as the average job finding probability holding workers' skills constant at the initial values when entering unemployment. At short durations, the true duration and composition effects represent roughly equal proportions of the total decline in the job finding probability, but at longer durations composition becomes relatively more important. This reflects the fact that those workers who remain unemployed for many months already had low job finding probabilities relative to the average worker upon entering unemployment. The true duration effect also grows over duration as workers' skills evolve in such a way that the average job finding probability falls. Results are vastly different for the average reemployment wage shown in the right panel of Figure 3, with the composition effect accounting for 25% of the total decline in wages.

Figure 3: Decomposition: Job Finding Probability and Wage



Notes: Left panel: decomposition of the model-implied average job finding probability. Right panel: decomposition of the average reemployment wage. The green shaded part of the figure indicates the proportion of the decline due to composition, and the light gray area indicates the remaining proportion of the decline, due to true duration dependence. Composition accounts for 68% of the decline in the job finding probability and 25% of the decline in the wage over 12 months of unemployment duration.

In Appendix B two additional figures corresponding to the model's steady state are shown. Paths of the policy functions for individual worker types in steady state are shown in Figure 6. Confidence intervals in the simulated data for the aggregate job finding probability and reemployment wage are shown in Figure 7.

4.3 Business Cycles

In this section aggregate uncertainty is incorporated into the calibrated model through the aggregate productivity A. In the data, aggregate productivity is chosen to match the seasonally adjusted real average output per worker in the nonfarm business sector constructed by the BLS. The process is discretized into a 5-state vector using the Rouwenhorst (1995) method, where the highest and lowest states correspond to two standard deviations above and below the mean, respectively. Normalizing the average aggregate productivity to one, the vector of aggregate productivities is given by $A = [0.9797 \;\; 0.9899 \;\; 1.0000 \;\; 1.0101 \;\; 1.0203]$.

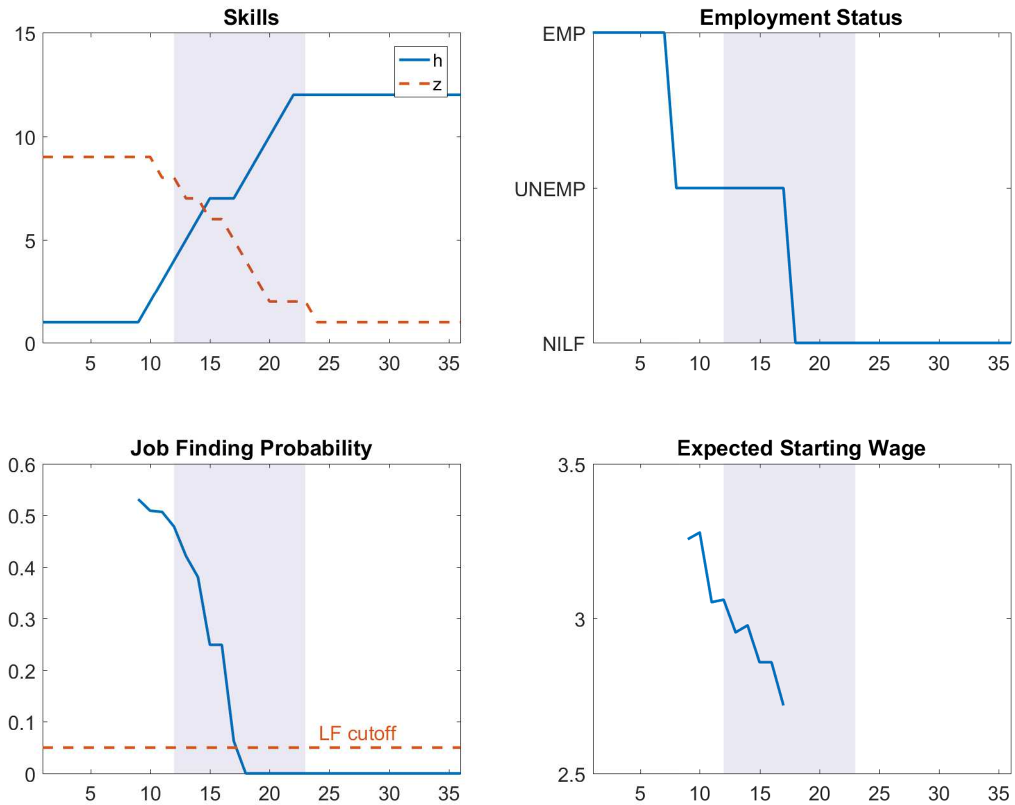
For intuition on the effects of the recession on individual outcomes, Figure 4 shows a sample path over 36 months for one worker in response to a two-standard deviation decrease in aggregate productivity that lasts for 12 months, after which aggregate productivity permanently returns to its original level. The top right panel shows the worker's employment status: at the beginning of the sample the worker separates from his job and becomes unemployed. In the top left panel, the worker's skills begin to evolve upon entry into unemployment, and are independent of the aggregate productivity by Assumption 1. When the recession arrives, this worker's expected reemployment wage falls (bottom right panel), and his job finding probability falls (bottom left panel). During the recession, the worker's home skills rise and market skills fall substantially, causing the job finding probability to fall below the labor force cutoff of 5%, after which the worker permanently drops out of the labor force, reflected in his employment status becoming not in the labor force (NILF).

In the aggregate, Figure 5 shows the responses of labor force participation, unemployment, the aggregate job finding probability, and average worker-specific productivities to the same two-standard deviation decrease in aggregate productivity. Responses are shown in percent deviations from steady state and are aggregated for those workers with job finding probabilities above the labor force cutoff.

The 12-month negative aggregate shock is indicated in Figure 5 by the shaded region. Its effect on aggregate output is depicted in panel (i). The negative shock causes all unemployed workers' job finding probabilities to fall on impact because the relative value of market work falls, making home production more attractive for all workers. In panel (ii), the participation rate falls, reflecting the fact that some workers' optimal job finding probabilities drop below the cutoff defining workers in the labor force, as seen in the individual sample path in Figure 4. This causes the unemployment rate to fall on impact as workers drop out of the labor force.

The decline in the job finding probability in panel (iv) implies longer expected unemployment durations for all workers, leading to the changes in the average worker-specific productivities of the unemployed shown in panels (v) and (vi). The impact effects in both panels reflect the effect of workers who drop out of the labor force when the recession begins. Since these workers are at the bottom (top) of the distribution of unemployed workers in terms of market (home) skills, their exit from the labor force causes the average market (home) skills to jump up (down) at the beginning of the recession. The jumps in both paths at the end of the recession reflects the re-entry to unemployment of some of these workers.

Figure 4: Business Cycle: Sample Path of an Individual Worker

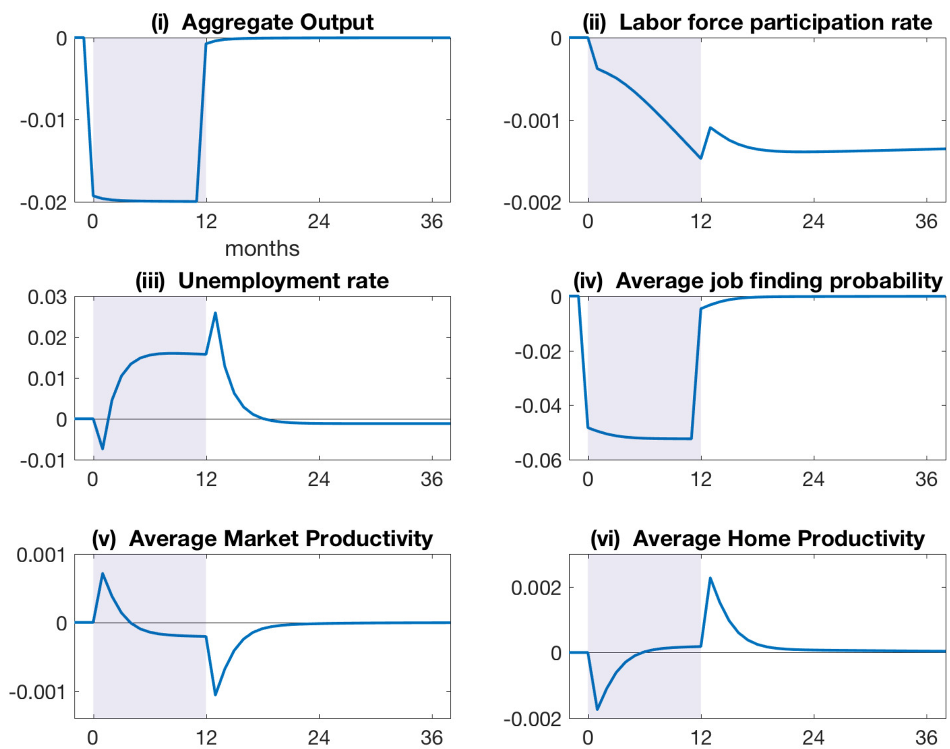


Notes: Paths of average skills, employment state, expected starting wage, and job finding probability for an individual worker over 3 years. Light and shaded areas denote normal times and times of aggregate productivity two standard deviations below its mean, respectively. The dashed line in the lower right panel denotes the 5% cutoff for the job finding probability, below which a worker is considered out of the labor force. For details on the choice of this cutoff, see Supplementary Appendix G.

During the intermediate months of the recession, average unemployment duration rises for the remaining unemployed workers, leaving more time for market (home) skills to depreciate (appreciate).

Regarding the dynamics after the recession, it is the changes in average market and home productivities that drives the hysteresis in labor force participation shown in panel (ii). Workers who choose nonparticipation continue to evolve so that they do not return to the labor force even after the recession ends. The participation rate only reverts as these workers are reborn with new productivities, taking roughly 30 years to return to its mean. The larger is the decline in aggregate productivity, the larger is the decrease in labor force participation and more persistent is its level.

Figure 5: Responses to 12 Months of Low Aggregate Productivity



Notes: Responses of aggregate variables to a 12-month decline in aggregate productivity of 2 standard deviations below the mean. Responses measured in percentage deviations from mean. Shaded region indicates periods of low aggregate productivity.

5 Conclusion

This paper develops a model with learning by doing in home and market production in order to address two facts: the insensitivity of reemployment wages to unemployment duration and high sensitivity of the job finding probability to duration. The model's key mechanism is driven by the fact that during unemployment, workers learn in home production, affecting their outside options and job search strategies. The outside option of unemployed workers tends to increase over time as home-specific skills accumulate, giving rise to an average reemployment wage that is much less elastic than the job finding probability with respect to duration. Quantitatively, the model is calibrated to target the job finding probability and generates a reemployment wage that falls little with duration, a feature that is elusive in most models with a constant unemployment benefit and market skill loss. A decomposition of the decline in the job finding probability indicates a small but significant role for changes in workers' skills during unemployment, an effect known in the literature as "true duration dependence".

I assess the effects of shocks to the aggregate market productivity to study the effects of the business cycle on skills. The model suggests that changes in workers' outside options during unemployment is an important force in generating the observed responses of aggregate labor market variables to shocks, and predicts a small but persistent change in the labor force participation rate stemming directly from these changes. Although this paper abstracts from technological change in home production, the impact of a "durable goods revolution" as studied in Greenwood et al. (2005) could be incorporated. This would not affect the model's main results, as one would expect a technology shock to home production to increase the benefits of home work, causing average duration to rise but having little effect on reemployment wages.

Future research using data on consumption or the informal sector of the economy linked to duration out of formal labor markets could shed more light on the relevance of the general mechanism of an evolving outside option. Further, the model could be extended to study the inefficiencies that arise when home production is excluded from aggregate output. Although this would not change the ability of the decentralized equilibrium to reproduce the wage and hazard patterns, a model in which households prefer nonemployment more than the planner could be used to study the policy implications regarding the prevention of long term unemployment and hysteresis in response to recessions. Finally, the value of learning in home or market work will vary over the life cycle as well as the business cycle, implying variation in skills and earnings dynamics of cohorts entering the labor market in booms and recessions.

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A Proofs

A.1 Proof of Theorem 1

(i) First, I show that solving (2) subject to (3) and (4) is equivalent to solving (5). This part of the proof shows that any solution to (2) also solves (5) and vice versa, proving the equivalence of the two formulations. To simplify notation, I will write $\mathbb{E}_U(W_U(z',h',\hat{A})) = \mathbb{E}_U(W_U(z',h',\hat{A})|z,h,A)$,
$\mathbb{E}_E(W_E(z',h',\hat{A})) = \mathbb{E}_E(W_E(z',h',\hat{A})|z,h,A)$, and $\mathbb{E}_0(W_U(z',h',\hat{A})) = \mathbb{E}_0(W_U(z',h',\hat{A})|A)$. Denote the set of solutions to (2) as $\mathcal{A}_1$ and the set of solutions to (5) as $\mathcal{A}_2$.

Suppose $(\theta^*(z,h,A), d^*(z,h,A), W_U(z,h,A), W_E(z,h,A)) \in \mathcal{A}_2$. Plugging in for the laws of motion $\hat{u}$ and $\hat{e}$,

$$
\begin{aligned}
\tilde{W}^*(\psi) = \int_Z \int_H \Bigg(&(h - k\theta^*(z,h,A))u(z,h) + Aze(z,h) \\
&+ \beta(1-\lambda)\Big[\mathbb{E}_U(W_U(z',h',\hat{A})) + p(\theta^*(z,h,A))\mathbb{E}_U\big(W_E(z',h',\hat{A}) - W_U(z',h',\hat{A})\big)\Big]u(z,h) \\
&+ \beta(1-\lambda)\mathbb{E}_E\big(d^*(z',h',\hat{A})(h' + \beta D(z',h',\hat{A},W_U)) + (1-d^*(z',h',\hat{A}))W_E(z',h',\hat{A})\big)e(z,h) \\
&+ \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A}))(u(z,h)+e(z,h))\Bigg)\, dz\, dh
\end{aligned}
$$

where

$$
\mathbb{E}_U(W_i(z',h',\hat{A})) = \mathbb{E}_{\hat{A}}\bigg(\int_Y W_i(y',\hat{A})Q_U(dy',y)\bigg), \quad i = U, E
$$

$$
\mathbb{E}_E(W_i(z',h',\hat{A})) = \mathbb{E}_{\hat{A}}\bigg(\int_Y W_i(y',\hat{A})Q_E(dy',y)\bigg), \quad i = U, E
$$

and

$$
\mathbb{E}_0(W_U(z',h',\hat{A})) = \mathbb{E}_{\hat{A}}\bigg(\int_Y W_U(y',\hat{A})f_0(y')dy'\bigg)
$$

with $y = (z,h)$ and $Y = Z \times H$.
Let

$$
C(\theta^*|\psi) = \int_Y hu(y) + Aze(y) - k\theta^*(y,A)u(y)\; dy
$$

The integrals in the equation for $\tilde{W}^*(\psi)$ may be interchanged by the independence of y and A in Assumption 1. Imposing the definitions for the laws of motion of $\hat{u}$ and $\hat{e}$, we can rewrite $\tilde{W}^*(\psi)$ as

$$\begin{aligned}\tilde{W}^*(\psi) &= C(\theta^*|\psi) + \beta E_{\hat{A}} \int_Y \Big(W_U(y', \hat{A})\hat{u}(y') + W_E(y', \hat{A})\hat{e}(y') \Big)\, dy' \\ &= C(\theta^*|\psi) + \beta E \tilde{W}^*(\hat{\psi})\end{aligned}$$

Since $\theta^*(y, A)$ solves $W_U(y, A)$ and $d^*(y, A)$ solves $W_E(y, A)$, we cannot improve upon the welfare for any one type because each type's value function was maximized separately in (5). Further, the policy functions must also maximize $\tilde{W}^*(\psi)$ since the integral of maxima is equal to the maximum of the integral when the problem of one type does not depend on the problem of any other type. Therefore the solution satisfying (5) must also satisfy (2), that is, $\mathcal{A}_2 \subseteq \mathcal{A}_1$. The proof of the converse ($\mathcal{A}_1 \subseteq \mathcal{A}_2$) is straightforward using the definitions of $\hat{u}$ and $\hat{e}$. It follows that $\mathcal{A}_2 = \mathcal{A}_1$.

(*ii*) This part of the proof shows the existence and uniqueness of the solution to (5). The optimality conditions for $W_U(z, h, A)$ imply that if $\theta^*(z, h, A) > 0$,

$$k = \beta(1-\lambda)p'(\theta^*(z, h, A))\mathbb{E}_U\big(W_E(z', h', \hat{A})) - W_U(z', h', \hat{A})\big)$$

Since the left hand side is strictly positive and $\mathbb{E}_U\big(W_E(z', h', \hat{A})) - W_U(z', h', \hat{A})\big) > 0$ whenever $\theta > 0$, $p'(\theta) \to 0$ as $\theta \to \infty$ implies that $\exists \overline{\theta} < \infty$ such that the optimality condition holds. Therefore the constraint set for θ is nonempty, compact, and continuous. Since θ and d are bounded, current period utility is bounded and continuous, $\lambda \in [0, 1]$, and $\beta \in (0, 1)$, a solution to $W_U(z, h, A)$ exists by Theorem 4.2 in Stokey, Lucas, and Prescott (1989) (henceforth, SLP). It is straightforward to show that Blackwell's sufficient conditions for a contraction hold, therefore the operator associated with $W_U(z, h, A)$ and $W_E(z, h, A)$ have unique solutions in the space of continuous bounded functions on $Z \times H \times \mathbf{A}$.

Since all variables are bounded and $\mathbb{E}_i(W_j(z', h', \hat{A}))$ for $i, j = U, E$, and $\mathbb{E}(W_U(z', h', \hat{A}))$ are bounded functions of $W_U(z, h, A)$ and $W_E(z, h, A)$, respectively, the value functions are bounded. By Theorem 4.3 in SLP, the unique solution to the operators associated with $W_U(z, h, A)$ and $W_E(z, h, A)$ coincide with the solutions to

$$\begin{aligned}W_U(z, h, A) = \max_{\theta \in [0, \overline{\theta}]} \Big\{ & h - k\theta + \beta(1-\lambda)\Big[\mathbb{E}_U(W_U(z', h', \hat{A})) \\ & + p(\theta)\mathbb{E}_U\big(W_E(z', h', \hat{A}) - W_U(z', h', \hat{A})\big)\Big] + \beta\lambda\mathbb{E}_0(W_U(z', h', \hat{A}))\Big\}\end{aligned}$$

and

$$\begin{aligned}W_E(z, h, A) = Az + \beta(1-\lambda)\mathbb{E}_E\Big(& \max_{d \in [\delta, 1]} \big\{ d(h' + \beta D(z', h', \hat{A}, W_U)) \\ & + (1-d)W_E(z', h', \hat{A})\big\}\Big) + \beta\lambda\mathbb{E}_0(W_U(z', h', \hat{A}))\end{aligned}$$

Since (5) aggregates W_U and W_E for all $(z, h) \in Z \times H$ and u and e are predetermined, the solution to (5) is unique.

(iii) This section proves the monotonicity of the value function (6) in h. Define the space of bounded, continuous functions that are nondecreasing in each of their arguments as $B(\Psi)$, and the space of bounded, continuous functions that are strictly increasing in h as $B'(\Psi) \subset B(\Psi)$. Define the operator T as

$$(Tf_U)(z,h,y) = \max_{\theta\in[0,\bar{\theta}]} \Big\{ h - k\theta + \beta(1-\lambda)\Big[\mathbb{E}_U(f_U(z',h',\hat{A})) + p(\theta)\mathbb{E}_U\big(f_E(z',h',\hat{A}) - f_U(z',h',\hat{A})\big)\Big] + \beta\lambda\mathbb{E}_0(f_U(z',h',\hat{A}))\Big\}$$

for f_U and f_E nondecreasing functions in each of their arguments. Suppose $\theta^*(z,h,y)$ achieves the maximum of the above equation, and take any $\tilde{h} > h$. Then

$$\begin{aligned}(Tf_U)(z,h,y) &= h - k\theta^*(z,h,y) + \beta(1-\lambda)\Big[\mathbb{E}_U(f_U(z',h',\hat{A})|z,h,y) \\ &\quad + p(\theta^*(z,h,y))\mathbb{E}_U\big(f_E(z',h',\hat{A}) - f_U(z',h',\hat{A})|z,h,y\big)\Big] + \beta\lambda\mathbb{E}_0(f_U(z',h',\hat{A})) \\ &< \tilde{h} - k\theta^*(z,h,y) + \beta(1-\lambda)\Big[\mathbb{E}_U(f_U(z',h',\hat{A})|z,h,y) \\ &\quad + p(\theta^*(z,h,y))\mathbb{E}_U\big(f_E(z',h',\hat{A}) - f_U(z',h',\hat{A})|z,h,y\big)\Big] + \beta\lambda\mathbb{E}_0(f_U(z',h',\hat{A})) \\ &\le \max_{\theta\in[0,\bar{\theta}]}\Big\{ h - k\theta + \beta(1-\lambda)\Big[\mathbb{E}_U(f_U(z',h',\hat{A})|z,\tilde{h},y) \\ &\quad + p(\theta)\mathbb{E}_U\big(f_E(z',h',\hat{A}) - f_U(z',h',\hat{A})|z,\tilde{h},y\big)\Big] + \beta\lambda\mathbb{E}_0(f_U(z',h',\hat{A}))\Big\} \\ &= (Tf_U)(z,\tilde{h},y)\end{aligned}$$

Where the inequality on the second line follows from the fact that Γ_U and Γ_E are monotone transition functions by Assumption 1. Since $B'(\Psi)$ is a closed subset of $B(\Psi)$ and $T(B'(\Psi)) \subset B'_z(\Psi)$, it follows from SLP Theorem 4.7 that $W_U \subset B'_z(\Psi)$. Similarly, it can be shown that $W_U(z,h,y)$ is weakly increasing in z and y, and $W_E(z,h,y)$ is strictly increasing in z and y and weakly increasing in h.

(iv) From part (ii), the policy correspondences $\theta^*(z,h,\psi)$ and $d(z,h,\psi)$ solve the maximization problems $W_U(z,h,A)$ and $W_E(z,h,A)$. Since neither the expression to be maximized nor the constraint depends on (u,e), $\theta^*(z,h,\psi)$ and $d(z,h,\psi)$ depend on ψ only through A and not on (u,e).

A.2 Proof of Lemma 1

The expectation operators in the statement of Lemma 1 are given by the following expressions.

$$\mathbb{E}_i(V(\alpha, y', \hat{\psi})) = \int_Y \int_\Psi V(\alpha, y', \hat{\psi})\ d\Gamma(\psi, d\hat{\psi})\ dQ_i(y, dy'), \quad i = U, E$$

$$\mathbb{E}(V(0, y', \hat{\psi})) = \int_Y \int_\Psi V(0, y', \hat{\psi}) f_0(y')\ d\Gamma(\psi, d\hat{\psi})\ dy'$$

for $y \in Y \equiv Z \times H$ and where Γ is the perceived law of motion for the aggregate state ψ, which must be consistent with the actual law of motion in any equilibrium.

($\Rightarrow$) Suppose an equilibrium exists. Then $(V_U, V_M, \theta^*, x^*, d^*)$ satisfy (8), (11), and (12). If $\theta^* = 0$, x^* is not pinned down by the free entry condition, but the probability that a worker meets a vacancy in that submarket is 0, therefore following the literature[16] let $x^* = 0$ when $\theta^* = 0$.

If $\theta^* > 0$, solving (12) for x^*,

$$x^* = \mathbb{E}_E(V_M(z', h', \hat{\psi})) - \frac{k}{\beta(1-\lambda)q(\theta^*)}$$

Plugging in for x from the free entry condition and noting that $\frac{p(\theta)}{q(\theta)} = \theta$, the combined value function for all agents in this equilibrium can be written

$$\begin{aligned} V(\alpha, z, h, \psi) = \alpha\Big(&Az + \beta(1-\lambda)\mathbb{E}_E(d^*(h + \beta D(z', h', \hat{\psi}, V(0))) + (1-d^*)V(1, z', h', \hat{\psi})\Big) \\ &+ (1-\alpha)\Big(h - k\theta^* + \beta(1-\lambda)\big(\mathbb{E}_U(V(0, z', h', \hat{\psi})) \\ &+ p(\theta^*)\mathbb{E}_U(V(1, z', h', \hat{\psi}) - V(0, z', h', \hat{\psi}))\big)\Big) + \beta\lambda\mathbb{E}_0(V(0, z', h', \hat{\psi})) \end{aligned} \tag{15}$$

Since x^* maximizes V_U and d^* maximizes V_M, θ^* must maximize the above expression, giving us (13). Since θ is bounded below by 0, it must be shown that $\exists\ \overline{\theta} < \infty$ such that $\theta \in [0, \overline{\theta}]\ \forall\ x, \psi$. By definition of the probability q, when $\theta \to \infty$, $q(\theta) \to 0$. Thus for large enough θ, the (binding) free entry condition is violated. It follows that $\exists\ \overline{\theta} < \infty$.

In addition, $\beta < 1$ and the per-period payoff $\alpha As + (1-\alpha)(h - k\theta)$ is continuous and bounded since all of its components are bounded. Therefore the equilibrium solves (13).

($\Leftarrow$) Take any solution to (13). For $\alpha = 0$,

$$\begin{aligned} V(0, z, h, \psi) = h + \max_{\theta \in [0, \overline{\theta}]} \{&-k\theta + \beta(1-\lambda)\big(\mathbb{E}_U(V(0, z', h', \hat{\psi})) \\ &+ p(\theta)\mathbb{E}_U(V(1, z', h', \hat{\psi}) - V(0, z', h', \hat{\psi}))\big)\} + \beta\lambda\mathbb{E}_0(V(0, z', h', \hat{\psi})) \end{aligned}$$

[16] If this were not the case, then there would exist some inactive submarkets with a positive wage in which no matches would occur.

if $x = \mathbb{E}_U(V_M(z', h', \hat{\psi})) - \frac{k}{\beta(1-\lambda)q(\theta)}$, then

$$V_U(z,h,\psi) = \sup_x \Big\{ h + \beta(1-\lambda)\Big[(1 - p(\theta(x,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{\psi})) + p(\theta(x,z,h,\psi))x\Big] + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi}))\Big\}$$

Which satisfy (8) and (12) for $\theta > 0$. Similarly, it can be shown that the value function evaluated at $\alpha = 1$ satisfies (11). Finally, for $\theta = 0$, $p(0) = 0$, and by assumption $x = 0$, therefore $V(0, z, h, \psi)$ can be written as

$$V_U(z,h,\psi) = h + \beta\big((1-\lambda)\mathbb{E}_U(V_U(z',h',\hat{\psi})) + \lambda\mathbb{E}_0(V_U(z',h',\hat{\psi}))\big)$$

which is equivalent to $V_U(z, h, \psi)$ when $x = 0$. Thus any solution to (13) is an equilibrium.

A.3 Proof of Theorem 2

Let (θ, V_U, V_M, x, d) be an equilibrium and let $V : [0,1] \times Z \times H \times \Psi$ be defined as

$$V(0,z,h,\psi) = V_U(z,h,\psi) \; \forall (z,h,\psi) \in Z \times H \times \Psi$$

$$V(1,z,h,\psi) = V_M(z,h,\psi) \; \forall (z,h,\psi) \in Z \times H \times \Psi$$

I first show the existence and uniqueness of the solution to (13). It is clear that the sets of feasible values for θ and d are nonempty, compact, and continuous. The period utility function is bounded and continuous, $\lambda \in [0,1]$ and $\beta \in (0,1)$. It immediately follows that a solution to (13) exists.

By the concavity of firms' expected profit function and of the composite function $p'(q^{-1}(\cdot))$, there is a unique choice of x for each (z, h, ψ). Let $\Omega = [0,1] \times Z \times H \times \Psi$ and $C(\Omega)$ be the space of continuous bounded functions $R : \omega \to \mathbb{R}$ for $\omega \in \Omega$ with the sup norm. Let $T : C(\Omega) \to C(\Omega)$ denote the operator associated with (13). It is easy to establish that Blackwell's sufficient conditions for a contraction are satisfied by T, thus by SLP Theorem 4.6 the mapping $TR = R$ admits a unique solution. Finally, note that since all choice and state variables are bounded, $V(\alpha, z, h, \psi)$ is bounded and by Theorem 4.3, the unique solution to $TR = R$ coincides with the solution to (13), thus the equilibrium is unique.

I now prove independence of the value and policy functions from (u, e). Following Menzio and Shi (2011), consider an arbitrary function $R \in C'(\Omega)$, denoting the set of continuous, bounded functions mapping $Z \times H \times A \to \mathbb{R}$. It is straightforward to show that $T : C'(\Omega) \to C'(\Omega)$, thus TR depends on ψ only through A. Therefore $V(\alpha, z, h, \psi) = V(\alpha, z, h, A)$ is the unique fixed point of (13). Since q is strictly decreasing and convex, $\theta(x, z, h, \psi)$ is uniquely pinned down by (12) and therefore only depends on ψ through A.

Finally, replacing ψ with A in (8) and (11) proves the independence of both value functions from (u, e). Therefore the continuation value for separating workers can be written $D(z, h, A, V_U)$. It follows that the policy functions $x(z, h, \psi)$ and $d(z, h, \psi)$ depend on ψ only through A.

A.4 Proof of Proposition 1

This proof shows that the equilibrium allocation and efficient allocation coincide, that is, $\theta(x,z,h,A) = \theta^*(z,h,A)$. Define $W'(0,z,h,A) = W_U(z,h,A)$ and $W'(1,z,h,A) = W_E(z,h,A)$ where $W_U(z,h,A)$ solves (6) and $W_E(z,h,A)$ solves (7). We can write the combined value function of the planner as

$$W'(\alpha,z,h,A) = \alpha\Bigg(Az + \beta(1-\lambda)\mathbb{E}_E\big(' \max_{d\in[\delta,1]} \big\{d(h+\beta D(z',h',\hat{A},W'(0)))$$
$$+(1-d)W'(1,z',h',\hat{A})\big\}|z,h,A\big)\Bigg)+(1-\alpha)\Bigg(\max_{\theta\in[0,\bar{\theta}]}\Bigg\{h-k\theta+\beta(1-\lambda)\Bigg[\mathbb{E}_U(W'(0,z',h',\hat{A})|z,h,A)$$
$$+ p(\theta)\mathbb{E}_U\big(W'(1,z',h',\hat{A})) - W'(0,z',h',\hat{A})|z,h,A\big)\Bigg]\Bigg) + \beta\lambda\mathbb{E}_0(W'(0,z',h',\hat{A})) \quad (16)$$

From (16) it is clear that $W'(\alpha,z,h,A)$ satisfies (13). Since V is unique, it must be the case that $V_U(z',h',\hat{A}) = W_U(z',h',\hat{A})$ and $V_M(z',h',\hat{A}) = W_E(z',h',\hat{A})$.

By definition, the allocation that solves (16) is the solution to the planner's problem, $(\theta^*(z,h,A), d^*(z,h,A))$, and the allocation that solves (13) corresponds to the decentralized equilibrium, $(\theta(x,z,h,A), d(z,h,A))$. Thus $\theta(x,z,h,A) = \theta^*(z,h,A)$ and $d(z,h,A) = d^*(z,h,A)$.

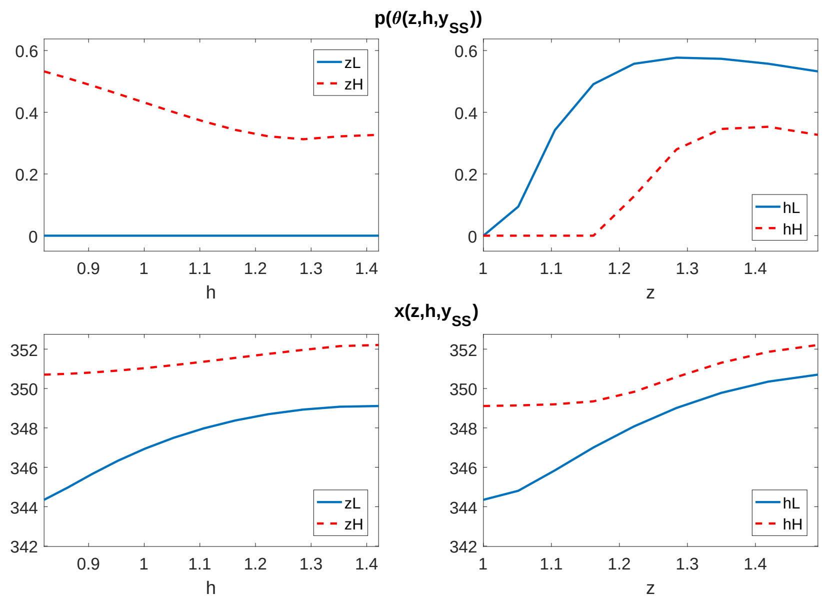
B Calibration: Details

To obtain the estimate of the job finding probability over tenure in the previous job, I identify workers who transitioned from unemployment to employment over two consecutive months of CPS interviews, and who also participated in the Displaced Worker Survey (DWS) the month they reported being unemployed between 1994 and 2010. For the sample period considered, 1994-2010, the DWS was administered 9 times. This sample includes 5,634 workers. Of these, 898 (16%) transitioned to employment in the second month.

In order to identify the depreciation of home skills during employment, I consider only those workers with unemployment spells shorter than one month at the time of the DWS. In the model, these short duration workers are those whose productivities most resemble their productivities at the time of separation. I regress a dummy variable equal to 1 if the worker transitioned from U to E and 0 if they reported being unemployed in both months on months of tenure in the previous job reported in the DWS for workers with up to 15 years of tenure, and controls for year, month, the log of the unemployment rate, gender, race, age, education, marital status, occupation, industry, and a quadratic term in total labor market experience. The estimated marginal effect of 1 additional month of pre-unemployment tenure is .03%, which is equivalent to an increase in the hazard out of unemployment of .41% for each additional year of tenure.

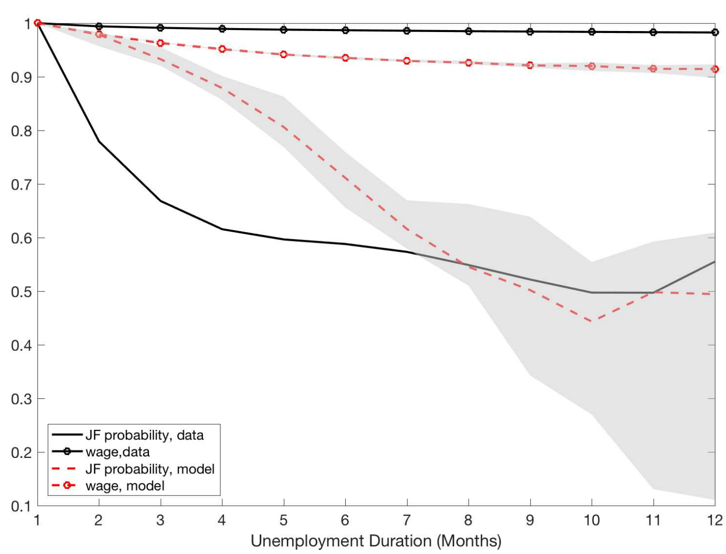
The rest of this section contains additional figures relating to the model in steady state. Figure 6 shows the policy functions for job seekers by type, both for the job finding probability and implied lifetime value, x. Figure 7 shows the aggregate job finding probability and wage in the data and model, with shaded areas indicating the 95% confidence intervals from each period of the simulated model.

Figure 6: Steady State Policy Functions: Job Finding Probability and Implied Value



Notes: Paths of individual job finding probability $p(\theta)$ and implied lifetime value of a match x in steady state holding z (first column) or h (second column) constant.

Figure 7: Steady State: Confidence Intervals for Aggregate Job Finding Probability and Reemployment Wage



Notes: Shaded bands represent 95% *confidence interval from* 50,000 *simulations used in model calibration. Dashed lines correspond to model averages and solid lines to the empirical estimates from Section 2.*

C Data

This section summarizes the data sources used in the regressions in Section 2. Robustness checks are left to Online Appendix H.

C.1 PSID

The PSID sample used here begins the first time the monthly employment histories for the year previous to the interview year are available, 1984, and ends in 1996, the year before the PSID became biannual. In this sample, job transitions are identified as workers reporting being unemployed for at least one month in the current year or the year prior to the interview and employed at the time of the interview. Durations are computed from the employment histories and therefore are recorded in months.

There are several drawbacks to using the PSID. First, duration is measured in months due to the structure of the employment histories. Therefore we may see little effect due to time aggregation bias. Second, reported wages are equal to annual income divided by annual hours, therefore it may be problematic if workers have multiple spells within a year or wage growth is strong in the first months of a new job. However, since the PSID follows individuals over many years, it may be used to control for individual fixed effects unobservable to the econometrician. Real reemployment wages are the wages reported for the year prior to the interview in which the respondent first reports being employed, deflated using the US city average CPI.

C.2 CPS

The Bureau of Labor Statistics (BLS) has conducted the Current Population Survey (CPS) on a monthly basis since 1940. Respondents participate in the CPS 8 times. Respondents in the 4th and 8th interviews, known as the outgoing rotation group (ORG), are asked additional questions about their labor income. As in Fernández-Blanco and Preugschat (2016), I consider workers whose first month of employment after an unemployment spell is in the ORG to maximize the accuracy of the measure of reemployment wages. The sample considered here begins in January 1994 and ends in December 2015. June through September 1995 are excluded due to changes in the variables required to identify individuals over time. All regressions use the monthly ORG weights.

Workers are traced over time using the household identifier, household number, and person line number. To check the accuracy of matches, I compare the age and sex of the individuals across months. Workers who report being retired, disabled, actively serving in the armed forces, or in farming and agriculture are excluded. The job finding probability is computed as those workers who transition from actively searching to employed as a fraction of active searchers. In the baseline results, "employed" includes both workers who were at work and workers who were absent the previous week. The absent employed workers make up less than 1 percent of all UE switchers and excluding these workers does not meaningfully affect the results. Real hourly wages are computed as the wages of workers who report being paid hourly deflated by the US city average CPI.

This section concludes with a table containing regression results on two key subgroups of the population: prime-aged men and women of childbearing age. The pattern of flat wages and falling job finding probabilities with duration is especially pronounced for the women, suggesting that home skills are more relevant for this group of workers.

Table C.1: Prime-Age Men and Childbearing Women, CPS

	Prime-Aged Men		Childbearing Women	
	Job Finding Probability (1)	Reemployment Wage (2)	Job Finding Probability (3)	Reemployment Wage (4)
duration	-.0376*** (.0027)		-.0271*** (.0027)	
duration^2	.0022*** (.0002)		.0014*** (.0002)	
duration^3	-5.33e-05*** (6.68e-06)		-3.26e-05*** (6.96e-06)	
duration^4	4.53e-07*** (6.43e-08)		2.64e-07*** (6.81e-08)	
log duration		-.0217*** (.0065)		-.0021 (.0057)
dummy, > 6 mo	N	N	N	N
R^2	.0638	.3051	.0654	.2872
Root MSE	.4102	.3913	.4033	.3520
N	42,994	4,392	38,910	5,141

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with duration up to 52 weeks. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 respectively show the regression of the job finding variable on duration in a linear probability model and of the linear regression of log real reemployment wage on log duration for prime-aged males (ages 25 to 54). Columns 3 and 4 are the analogous regressions for women of childbearing age (ages 20-40). $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

D Fixed Home Productivity

This section presents the calibration of the model in which all workers have the same, constant, home productivity $\overline{h}$. This model most clearly resembles that of Ljungqvist and Sargent (1998) in the context of directed search. Tables D.2 and D.3 summarize the cailbrated parameters and targeted moments used in the calibration. Figure 2 shows the job finding and reemployment wages in this model, labeled "fixed h model".

Table D.2: Parameters: Fixed Home Skill

Parameter	Value	Description
β	.9959	Discount factor
λ	.0021	Death probability
γ	.4	Job finding probability $p(\theta) = min\{\theta^{\alpha}, 1\}$
Δ_h	.05	Step in h: $\Delta_h = h_k - h_{k-1}$
Δ_z	.05	Step in z: $\Delta_z = z_j - z_{j-1}$
h_1	1.05	Lowest home skill
z_1	1	Lowest market skill
π_{Ez}	.1	$z' = \min\{z + \Delta_z, z_{N_z}\}$ with prob π_{Ez} if E, z otherwise
π_{Eh}	0	$h = \overline{h}$
π_{Uh}	0	$h = \overline{h}$
π_{Uz}	.65	$z' = \max\{z - \Delta_z, z_1\}$ with prob π_{Uz} if U, z o.w.
k	4.2	Vacancy cost
δ	.023	Separation probability

Table D.3: Targets: Fixed Home Skill

Description	Target	Model
Annual interest rate	5%	5%
Average working lifetime	40 years	40 years
Matching function elasticity w.r.t v	.4	.4
Relative value of nonmarket work	.71	.67
Lifetime earnings losses due to unemployment	−11.9%	−11.0%
Annual decline, hazard out of U	44.5%	45.3%
Quarterly average EU rate	.023	.023
Quarterly average UE rate	.328	.385

Skill Accumulation in the Market and at Home Supplementary Appendix

E Homogeneity Under Proportional Vacancy Costs

This section describes an extension of the benchmark model such that all equilibrium value and policy functions depend only on one relative skill, denoted $s = z/h$.

Define the growth rate of skill z between two consecutive periods t and $t+1$ as $g_{z,t+1} = \frac{z_{t+1}}{z_t}$, and the growth rate of h as $g_{h,t+1} = \frac{h_{t+1}}{h_t}$. The realizations of g_z and g_h are contained in the compact sets $G_z = [\frac{\underline{z}}{\overline{z}}, \frac{\overline{z}}{\underline{z}}]$ and $G_h = [\frac{\underline{h}}{\overline{h}}, \frac{\overline{h}}{\underline{h}}]$. Assume that the growth rates in the current period depend only on the current employment state such that each period, $(g_z, g_h) \sim \Pi_i$ where $\Pi_i : G_z \times G_h \to [0, 1]$ for $i = U, E$.

The laws of motion for skill growth implicitly define the Markov transition functions for their levels. Given initial skill levels $(z_0, h_0) \sim \tilde{F}_0$, the skill levels in any period t are given by $(z_t, h_t) = (g_z z_{t-1}, g_h h_{t-1})$ where $(g_z, g_h) \sim \Pi_U$ if the worker is unemployed at the beginning of period t and $(g_z, g_h) \sim \Pi_E$ if she is employed. These *iid* growth shocks implicitly define the laws of motion Q_U and Q_E for the levels of (z, h).

Timing in the extended model is identical to the benchmark, with the only difference being the cost of posting one vacancy. Here, this cost is proportional to the productivity of the worker type present[17] in a given submarket, kz, where $k > 0$ is a constant.

E.1 Homogeneity in the Planner's Problem

The planner's problem may be written as

$$W(\psi) = \sup_{\theta \in \mathbb{R}_+} \left\{ C(\theta|\psi) + \beta \mathbb{E} \max_{d \in [\delta, 1]} \{ W(\hat{\psi}) \} \right\} \tag{17}$$

subject to the laws of motion for u and e given by (3) and (4), and where aggregate consumption in a period is given by

$$C(\theta|\psi) \equiv \int_Y hu(y) + Aze(y) - kz\theta u(y) \; dy \tag{18}$$

A result similar to Theorem (1) holds in the extended model, with the planner's component value functions given by

[17] In both the social planner's problem and the decentralized economy, each submarket contains workers of only one type (z, h).

$$W_U(z,h,A) = \max_{\theta\in[0,\bar{\theta}]} \Big\{ h - kz\theta + \beta(1-\lambda)\Big[\mathbb{E}_U(W_U(z',h',\hat{A})|z,h,A)$$

$$+ p(\theta)\mathbb{E}_U\big(W_E(z',h',\hat{A}) - W_U(z',h',\hat{A})|z,h,A\big)\Big] + \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A)\Big\} \quad (19)$$

$$W_E(z,h,A) = Az + \beta(1-\lambda)\mathbb{E}_E\big(\max_{d\in[\delta,1]} \{d(h' + \beta D(z',h',\hat{A},W_U))$$

$$+ (1-d)W_E(z',h',\hat{y})\}|z,h,A\big) + \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A) \quad (20)$$

Writing the relative skill of a worker of type (z,h) as $s = \frac{z}{h}$, where $s \in S$, with $S = [\frac{\underline{z}}{\bar{h}}, \frac{\bar{z}}{\underline{h}}]$. Finally, let $\tilde{Q}_i,\ i = U,E$, denote the stationary transition function for s, and $\tilde{F}_0$ denote the exogenous distribution from which newborn workers' relative skills are drawn, and denote the associated expectation operators $\tilde{\mathbb{E}}_i,\ i = U,E,0$, respectively. The distributions of workers across employment states and relative skills are denoted $\tilde{u}$ and $\tilde{e}$. Using this notation, Assumption 1 implies that s and A are independent.

Proposition 2. *Equations (19) and (20) are homogeneous in the sense that $hw_i(s,A) \equiv hW_i(s,1,A)$, $i = \{U,E\}$, where*

$$w_U(s,A) = \max_{\theta\in[0,\bar{\theta}]} \Big\{ 1 - ks\theta + \beta(1-\lambda)\Big[\tilde{\mathbb{E}}_U(g_h' w_U(s',\hat{A})|s,A)$$

$$+ p(\theta)\tilde{\mathbb{E}}_U\big(g_h'(w_E(s',\hat{A})) - w_U(s',\hat{A}))|s,A\big)\Big] + \beta\lambda\tilde{\mathbb{E}}_0(g_h' w_U(s',\hat{A})|A)\Big\}$$

$$w_E(s,A) = As + \beta(1-\lambda)\tilde{\mathbb{E}}_E\big(g_h' \max_{d\in[\delta,1]} \{d(1 + \beta D(s',\hat{A},w_U))$$

$$+ (1-d)w_E(s',\hat{A})\}|s,A\big) + \beta\lambda\tilde{\mathbb{E}}_0(g_h' w_U(s',\hat{A})|A)$$

where

$$\tilde{D}(s,A,w_U) = (1-\lambda)\tilde{\mathbb{E}}_U\big(g_h' w_U(s',\hat{A})|s,A\big) + \lambda\tilde{\mathbb{E}}_0(g_h' w_U(s',\hat{A})|A)$$

The proof of this proposition can easily be shown by guess and verify, and is therefore omitted for brevity. Denote the optimal policies corresponding to the value functions w_U and w_E by $\theta^*(s,A)$ and $d^*(s,A)$.

E.2 Proportional Vacancy Costs in the Decentralized Economy

In the two-skill model, submarkets are indexed by (x,z,h,ψ), where $x \in \mathbb{R}$ is the value in terms of the worker's lifetime utility of the match and (z,h) is the type of worker for which the vacancy is intended. A firm may post a vacancy by paying a cost proportional to the worker's effective labor in the current period, kz, with $k > 0$. Each vacancy in a submarket offers the same value x, and firms commit to this value and the type of worker they will hire if a match occurs.

The value function for an unemployed worker of type (z, h) is

$$V_U(z,h,\psi) = \sup_x \Big\{ h + \beta(1-\lambda)\Big[(1-p(\theta(x,z,h,\psi)))\mathbb{E}_U(V_U(z',h',\hat{\psi})|z,h,\psi) + p(\theta(x,z,h,\psi))x\Big] + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi)\Big\} \quad (21)$$

where the policy function is denoted $x(z,h,\psi)$ and the implied market tightness is denoted $\theta(x,z,h,\psi)$.

The value of the employed worker can be written

$$V_E(z,h,\psi;w) = w+\beta(1-\lambda)\mathbb{E}_E(\max_{d\in[\delta,1]} \big\{d(1+D(z',h',\hat{\psi},V_u))+(1-d)V_E(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad (22)$$

where

$$D(z,h,\psi,V_U) = (1-\lambda)\mathbb{E}_U\big(V_U(z',h',\hat{\psi})|z,h,\psi\big) + \lambda\tilde{\mathbb{E}}_0(V_U(z',h',\hat{\psi})|\psi)$$

and the value for the firm given the expected separation probability d' is given by:

$$J(z,h,\psi;w) = Az - w + \beta(1-\lambda)\mathbb{E}_E((1-d')J(z',h',\hat{\psi};w')|z,h,\psi) \quad (23)$$

Combining (22) and (23) under the assumption of complete contracts gives the following match value:

$$V_M(z,h,\psi) = Az+\beta(1-\lambda)\mathbb{E}_E(\max_{d\in[\delta,1]} \big\{d(1+D(z',h',\hat{\psi},V_u))+(1-d)V_M(z',h',\hat{\psi};w')\big\}|z,h,\psi) + \beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) \quad (24)$$

where the policy function is denoted $d(z,h,\psi)$. Finally, the free entry condition may be written as:

$$kz \geq \beta(1-\lambda)q(\theta(x,z,h,\psi))(\mathbb{E}_U(V_M(z',h',\hat{\psi})|z,h,\psi) - x(z,h,\psi)) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0 \quad (25)$$

E.2.1 Homogeneity in the Decentralized Economy

Guess the following functional forms: $hV_U(s,1,A) \equiv hv_U(s,A)$, $hV_E(s,1,A) \equiv hv_E(s,A)$, where $s = \frac{z}{h}$ is the relative skill of the worker of type (z,h). The value for the unemployed worker may be written

$$hv_U(s,\psi) = \sup_x \Big\{ h + \beta(1-\lambda)\Big[(1-p(\theta(x,z,h,\psi)))\tilde{\mathbb{E}}_U(h'v_U(s',\hat{\psi})|z,h,\psi) + p(\theta(x,z,h,\psi))\tilde{x}\Big] + \beta\lambda\tilde{\mathbb{E}}_0(h'v_U(s',\hat{\psi})|\psi)\Big\}$$

Canceling terms and replacing $\frac{h'}{h}$ with g'_h,

$$v_U(s,\psi) = \sup_x \Big\{ 1 + \beta(1-\lambda)\Big[(1-p(\theta(x,s,\psi)))\tilde{\mathbb{E}}_U(g'_h v_U(s',\hat{\psi})|s,\psi) + p(\theta(\tilde{x},s,\psi))\frac{x}{h}\Big] + \beta\lambda\tilde{\mathbb{E}}_0(g'_h v_U(s',\hat{\psi})|\psi)\Big\}$$

Similarly, using the guesses for the functional forms of V_U and V_M, the value of a match is given by

$$v_M(s,\psi) = As + \beta(1-\lambda)\tilde{\mathbb{E}}_E(g'_h \max_{d\in[\delta,1]} \big\{d(1+\tilde{D}(s',\hat{\psi},v_U)) + (1-d)v_M(s',\hat{\psi};w')\big\}|s,\psi) + \beta\lambda\tilde{\mathbb{E}}_0(g'_h v_U(s',\hat{\psi})|\psi) \quad (26)$$

where

$$\tilde{D}(s,\psi,v_U) = (1-\lambda)\mathbb{E}_U\big(g'_h v_U(s',\hat{\psi})|s,\psi\big) + \lambda\mathbb{E}_0(g'_h v_U(s',\hat{\psi})|\psi)$$

Finally, the transformed free entry condition is given by

$$ks \geq \beta(1-\lambda)q(\theta(x,z,h,\psi))(\tilde{\mathbb{E}}_U(g'_h v_M(s',\hat{\psi})|s,\psi) - \frac{x(z,h,\psi)}{h}) \quad \text{and} \quad \theta(x,z,h,\psi) \geq 0$$

Solving for $x(z,h,\psi)$ when $\theta(x,s,\psi) > 0$, and plugging into the transformed value of unemployment, we are left with

$$v_U(s,\psi) = \sup_\theta \Big\{ 1 - ks\theta + \beta(1-\lambda)\Big[(1-p(\theta))\tilde{\mathbb{E}}_U(g'_h v_U(s',\hat{\psi})|s,\psi) + p(\theta)\mathbb{E}_U(g'_h v_M(s',\hat{\psi})|s,\psi)\Big] + \beta\lambda\tilde{\mathbb{E}}_0(g'_h v_U(s',\hat{\psi})|\psi)\Big\}$$

Note that the equation above can also be written as the combination of the following two equations:

$$v_U(s,\psi) = \sup_{\tilde{x}} \Big\{ 1 + \beta(1-\lambda)\Big[(1-p(\theta(\tilde{x},s,\psi)))\tilde{\mathbb{E}}_U(g'_h v_U(s',\hat{\psi})|s,\psi) + p(\theta(\tilde{x},s,\psi))\tilde{x}\Big] + \beta\lambda\tilde{\mathbb{E}}_0(g'_h v_U(s',\hat{\psi})|\psi)\Big\} \quad (27)$$

$$ks \geq \beta(1-\lambda)q(\theta(\tilde{x}, s, \psi))(\tilde{\mathbb{E}}_U(g_h' v_M(s', \hat{\psi})|s, \psi) - \tilde{x}(s, \psi)) \quad \text{and} \quad \theta(\tilde{x}, s, \psi) \geq 0 \tag{28}$$

Defining $\tilde{x} = \frac{x}{h}$, equations (27), (26), and (28) depend only on s since g_h is independent of its past realizations, verifying the guess.

F Model with Human Capital Investment

This section outlines a model which replaces the benchmark model's assumption of passive learning by doing with an active human capital investment choice. When a worker is unemployed, she chooses the fraction of time $\tau_U \in [0, 1]$ to which she devotes to actively developing her home skill h, and when she is employed, she chooses the fraction of time $\tau_E \in [0, 1]$ to which she devotes to actively developing her market skill z. The cost of this investment is foregone production time plus a quadratic adjustment cost proportional to the current skill level. For simplicity, I assume that an investment τ_U deterministically increases an unemployed worker's h at the beginning of the next period by the proportion $(1 + \tau_U)$, and similarly for the employed worker's market skill. Finally, the market skill depreciates deterministically by a proportion δ_z when the worker is unemployed; similarly, the home skill's deterministic depreciation rate when a worker is employed is denoted δ_h.

F.1 Planner's Problem

The planner's period objective function C depends on market tightness and the investment choices τ_U and τ_E and can be written as:

$$C(\theta, \tau_U, \tau_E|\psi) = \int_Z \int_H \left[((1 - \tau_U - \frac{\tau_U^2}{2})h - k\theta)u(z, h) + ((1 - \tau_E)A - \frac{\tau_E^2}{2})ze(z, h)\right] dhdz$$

The laws of motion for the distributions (u, e) are written as follows:

$$\hat{u}(z', h') = \lambda f_0(z', h') + (1 - \lambda)\Bigg[\Big(1 - p\big(\theta\big(\frac{z'}{1 - \delta_z}, \frac{h'}{1 + \tau_U}, \psi\big)\big)\Big)u\big(\frac{z'}{1 - \delta_z}, \frac{h'}{1 + \tau_U}\big) + d(z', h'\hat{\psi})e\big(\frac{z'}{1 + \tau_E}, \frac{h'}{1 - \delta_h}\big)\Bigg]$$

$$\hat{e}(z', h') = (1 - \lambda)\Bigg[p\big(\theta\big(\frac{z'}{1 - \delta_z}, \frac{h'}{1 + \tau_U}, \psi\big)\big)u\big(\frac{z'}{1 - \delta_z}, \frac{h'}{1 + \tau_U}\big) + (1 - d(z', h'\hat{\psi}))e\big(\frac{z'}{1 + \tau_E}, \frac{h'}{1 - \delta_h}\big)\Bigg]$$

Following the proof of Theorem 1, it can be shown that the planner's problem,

$$W(\psi) = \sup_{\theta \in \mathbb{R}_+, \tau_U \in [0,1], \tau_E \in [0,1]} \big\{C(\theta, \tau_U, \tau_E|\psi) + \beta \mathbb{E} \max_{d \in [\delta, 1]} \{W(\hat{\psi})\}\big\}$$

is equivalent to

$$\int_Z \int_H W_U(z,h,A)u(z,h) + W_E(z,h,A)e(z,h)\ dh\ dz$$

where

$$W_U(z,h,A) = \max_{\theta\in[0,\bar{\theta}],\tau_U\in[0,1]} \Big\{(1-\tau_U-\frac{\tau_U^2}{2})h-k\theta+\beta(1-\lambda)\Big[\mathbb{E}(W_U((1-\delta_z)z,(1+\tau_U)h,\hat{A})|z,h,A)$$
$$+p(\theta)\mathbb{E}\big(W_E((1-\delta_z)z,(1+\tau_U)h,\hat{A})-W_U((1-\delta_z)z,(1+\tau_U)h,\hat{A})|z,h,A\big)\Big]\Big\}+\beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A)$$

and

$$\begin{aligned} W_E(z,h,A) = \max_{\tau_E\in[0,1]} \big\{((1-\tau_E)A - \frac{\tau_E^2}{2})z + \beta(1-\lambda)\mathbb{E}\big(\max_{d\in[\delta,1]}\big\{d((1-\delta_h)h \\ + \beta D((1+\tau_E)z,(1-\delta_h)h,\hat{A},W_U)) + (1-d)W_E((1+\tau_E)z,(1-\delta_h)h,\hat{A})\big\}|z,h,A\big)\big\} \\ + \beta\lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A) \end{aligned}$$

where

$$\begin{aligned} D((1+\tau_E)z,(1-\delta_h)h,,A,W_U) = (1-\lambda)\mathbb{E}\big(W_U((1+\tau_E)z,(1-\delta_h)h,\hat{A})|z,h,A\big) \\ + \lambda\mathbb{E}_0(W_U(z',h',\hat{A})|A) \end{aligned}$$

The expectation operator $\mathbb{E}$ is taken with respect to the law of motion P for the aggregate productivity and is conditional on today's aggregate productivity A only, and $\mathbb{E}_0$ is the expectation The first order condition determining the choice of θ is conditional on the exogenous distribution for newborn workers' skills F_0.

$$k \geq \beta(1-\lambda)p'(\theta(z,h,A))\mathbb{E}\big(W_E((1-\delta_z)z,(1+\tau_U)h,\hat{A})-W_U((1-\delta_z)z,(1+\tau_U)h,\hat{A})|z,h,A\big)$$

The choice of investment τ_U in home skills is determined by

$$\begin{aligned} 1+\tau_U = \beta(1-\lambda)\Big[(1-p(\theta(z,h,A)))\mathbb{E}\Big(\frac{\partial W_U(z',h',\hat{A})}{\partial h'}\Big|_{z'=(1-\delta)z,\ h'=(1+\tau_U)h}|A\Big) \\ + p(\theta(z,h,A))\mathbb{E}\Big(\frac{\partial W_E(z',h',\hat{A})}{\partial h'}\Big|_{z'=(1-\delta)z,\ h'=(1+\tau_U)h}|A\Big)\Big] \end{aligned}$$

The planner's choice for the separation probability of employed workers solves the following complementary slackness conditions:

$$(1-\delta_h)h + \beta D((1+\tau_E)z,(1-\delta_h)h,\hat{A},W_U) \leq W_E((1+\tau_E)z,(1-\delta_h)h,\hat{A}) \quad \text{and } d \geq \delta$$

The choice of investment τ_E in market skills is determined by

$$A+\tau_E=\beta(1-\lambda)\Bigg[\mathbb{E}\Bigg(d((1+\tau_E)z,(1-\delta_h)h,\hat{A})\frac{\partial W_U(z',h',\hat{A})}{\partial h'}\Bigg|_{z'=(1+\tau_E)z,\ h'=(1-\delta_h)h}\Big|A\Bigg) + \mathbb{E}\Bigg(d((1+\tau_E)z,(1-\delta_h)h,\hat{A})\frac{\partial W_E(z',h',\hat{A})}{\partial h'}\Bigg|_{z'=(1+\tau_E)z,\ h'=(1-\delta_h)h}\Big|A\Bigg)\Bigg]$$

The constrained efficient policy functions in the model with human capital investment and deterministic skill depreciation are denoted $\theta^*(z,h,A)$, $\tau_U^*(z,h,A)$,$d^*(z,h,A)$, $\tau_E^*(z,h,A)$.

F.2 Decentralized Equilibrium

Timing in the decentralized equilibrium is identical to that in Section 3.2, with the stage at which agents make human capital occurring immediately before the production stage.

The value function for an unemployed worker when making her human capital investment choice is written

$$V_U(z,h,\psi)=\sup_{\tau_U\in[0,1],x}\Bigg\{(1-\tau_U-\frac{\tau_U^2}{2})h+\beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi) + \beta(1-\lambda)\Big[(1-p(\theta(x,z,h,\psi)))\mathbb{E}(V_U((1-\delta_z)z,(1+\tau_U)h,\hat{\psi})|z,h,\psi)+p(\theta(x,z,h,\psi))x\Big]\Bigg\}$$

where the policy functions are denoted $\tau_U(z,h,\psi)$ and $x(z,h,\psi)$ and the implied market tightness is denoted $\theta(x,z,h,\psi)$.

Again assuming that employment contracts are complete and that firms observe workers' investment choices, the value of the match is

$$V_M(z,h,\psi)=(1-\tau_E)Az+\beta(1-\lambda)\mathbb{E}(\max_{d\in[\delta,1]}\big\{d((1-\delta_h)h+D((1+\tau_E)z,(1-\delta_h)h,\hat{\psi},V_u)) + (1-d)V_M((1+\tau_E)z,(1-\delta_h)h,\hat{\psi};w')\big\}|z,h,\psi)+\beta\lambda\mathbb{E}_0(V_U(z',h',\hat{\psi})|\psi)$$

The policy functions for the match are denoted $\tau_e(z,h,\psi)$ and $d(z,h,\psi)$. Finally, the free entry condition is given by

$$k\geq\beta(1-\lambda)q(\theta(x,z,h,\psi))(\mathbb{E}(V_M((1-\delta_z)z,(1+\tau_U)h,\hat{\psi})|z,h,\psi)-x(z,h,\psi)) \quad \text{and} \quad \theta(x,z,h,\psi)\geq 0$$

When firms observe unemployed workers' investment decisions, it is easy to show that the decentralized equilibrium is unique and block recursive (BRE), and that the BRE is efficient, similar to the results in Theorem 2 and Proposition 1.

G Calibration: Robustness

This section presents results for the steady state model under several different parameter choices in order to evaluate the robustness of the model's mechanism. There are three main parameters of interest which may alter the predictions of the model, namely, the probability of death λ, the active search cutoff for workers in the labor force, and the elasticity of the matching function with respect to vacancies, γ. The first robustness check is to increase the probability of death, corresponding to an expected lifetime of 30 years, rather than the 40 year expected lifetime assumed in the benchmark. The second decreases the active search cutoff for unemployed workers to be in the labor force to match the 99th percentile of durations reported by the unemployed in the CPS. The final two robustness checks vary the elasticity of the matching function, first decreasing its value to match the value estimated in the CPS from 1951-2003 by Shimer (2005) of .28, and then to study the implications of increasing its value to .5. Each of these robustness checks leaves all other parameters identical to those in Table 3, and reports mean values and 95% confidence intervals from simulations of 50,000 workers over 1,000 months, discarding the first 500 months.

G.1 Changes in the Death Probability

Tables G.4 and G.5 show how the model-implied moments change with increase in the probability of death from $\lambda = .0021$ to $\lambda = .0027$. This decreases the effective discount factor, making agents more impatient since future payoffs are less likely to be realized. The tables show that a 25% decrease in the average working lifetime has a large and significant effect on the point estimate of the change in the job finding probability with tenure, and a smaller effect on the decline in the job finding probability with 12 months of unemployment. Further, it causes the average reemployment wage to fall further as a function of unemployment duration and increases the unemployment rate.

G.2 Changes in the Active Search Cutoff

This section considers different cutoffs to define workers who are in the labor force. Table G.6 shows selected percentiles of unemployment duration in the empirical distribution using the CPS and PSID samples described in Sections 2 and H. In the calibration presented in the paper, a 5% cutoff for the job finding probability was used, below which unemployed workers are considered out of the labor force. This corresponds to an expected duration of 20 months, which is equivalent to roughly 86 weeks[18], corresponding to the 93rd percentile in the CPS sample, but well beyond the 99th percentile of the PSID sample. Due to possible measurement error in the PSID sample from the annual nature of the data, the remainder of this section considers only the CPS.

Since there is a nonnegligible mass of unemployed workers with durations above the 20 month threshold, it is reasonable to decrease the minimum job finding probability to understand how this changes the results. For robustness, I use the 99th percentile from the empirical distribution of the CPS, recalibrating the model in steady state and then simulating the responses to aggregate productivity shocks.

[18] Following convention in the literature, one month is equal to roughly 4.3 weeks.

Table G.4: Targeted Moments: Higher Death Probability

Description	Target	Model [95% CI]	% Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	30 years	30 years	-25%
Matching function elasticity w.r.t v	.4	.4	0%
Relative value of nonmarket work	.71	.605 [.58 , .636]	0%
Change in earnings with 1 year of market experience	2.30%	2.19% [0.50%, 4.11%]	-0.9%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	1.00% [0.56%, 1.47%]	79%
Lifetime earnings losses due to unemployment	-11.9%	-11.9% [-62%, -3.2%]	-0.8%
Annual decline, hazard out of U	44.5%	59.6% [28.1%, 90.1%]	17%
Quarterly average EU rate	.023	.023 [0.23, 0.23]	0%
Quarterly average UE rate	.328	.327 [.271, .383]	0.3%

A monthly job finding probability of 3.6% represents the cutoff corresponding to the 99th percentile of the CPS duration distribution. Tables G.7 and G.8 show the targeted and untargeted moments of the model using this cutoff for labor force participation. A decrease in the labor force cutoff by 28% has a large effect on the point estimate of the change in the job finding probability with tenure, and a slightly smaller decrease in the reemployment wage with duration. Interestingly, this change causes the participation rate to only decrease slightly, since most workers out of the labor force are optimally choosing probability 0 of matching.

G.3 Changes in the Matching Function Elasticity

This section explores how changes in the elasticity of the matching function with respect to vacancies, denoted γ in Section 4.1, affect the results of the calibrated model. Two possibil-

Table G.5: Untargeted Moments: Higher Death Probability

Description	Data	Model	% Change from Benchmark
% change, log reemployment wage (1-12 months)	-2.2%	-9.7% [4.3%, 11.5%]	-13%
Unemployment rate	6.0%	7.5% [4.9%, 10.5%]	14%
Labor force participation rate	65.8%	66.7% [60.2%, 73.3%]	0.8%
Initial job finding probability (1 month)	.402	.513 [.474, .558]	0.2%
Relative composition effect, job finding probability		69.6%	2%
Relative composition effect, reemployment wage		26.9%	8%

Table G.6: Deciles of Unemployment Duration

Percentile	CPS	PSID
50	12 weeks	2 months
60	18 weeks	3 months
70	27 weeks	3 months
80	44 weeks	4 months
90	61 weeks	6 months
95	104 weeks	9 months
98	113 weeks	12 months
99	119 weeks	13 months

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks.
PSID: 1984-1996, annual; duration reported in months.

ities are explored in addition to the standard value: one below and one above the baseline calibration of .4. First, Shimer (2005) estimates an elasticity of 0.28 using CPS data from 1951-2003. Second, a higher value of .5 is shown to retain many of the model's predictions. Tables G.9 and G.10 show the targeted and untargeted moments of the model using the low

Table G.7: Targeted Moments: 3.6% Labor Force Cutoff

Description	Target	Model [95% CI]	% Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	40 years	0%
Matching function elasticity w.r.t v	.4	.4	0%
Relative value of nonmarket work	.71	.605 [.579 , .635]	0%
Change in earnings with 1 year of market experience	2.30%	2.23% [0.57%, 4.17%]	0.9%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.96% [0.51%, 1.31%]	71%
Lifetime earnings losses due to unemployment	−11.9%	−11.7% [-63%, -3.1%]	0.9%
Annual decline, hazard out of U	44.5%	52.7% [23.8%, 82.3%]	6%
Quarterly average EU rate	.023	.023 [0.23, 0.23]	0%
Quarterly average UE rate	.328	.327 [.272, .386]	0.3%

elasticity of the matching function, and Tables G.11 and G.12 are the corresponding tables for the high elasticity.

A decrease in the elasticity of the matching function by 30% results in a large decline in the point estimate of the change in the job finding probability with tenure, as well as its confidence interval, an increase in earnings losses due to unemployment, and a smaller effect of duration on the job finding probability. Significantly, the low elasticity leads to a large increase in the labor force participation rate, but a larger decline in reemployment wages with duration. Finally, the composition effects for both wages and the job finding probability fall by large amounts relative to the benchmark model.

Conversely, an increase in the elasticity by 25% again causes an increase in the point estimate of the change in the job finding probability with tenure, as well as its confidence interval, and larger effects with regard to the earnings losses due to unemployment and the job finding probability with duration. Regarding the untargeted moments, an increase in

Table G.8: Untargeted Moments: 3.6% Labor Force Cutoff

Description	Data	Model	% Change from Benchmark
% change, log reemployment wage (1-12 months)	−2.2%	−7.3% [1.0%, 11.4%]	15%
Unemployment rate	6.0%	6.8% [5.9%, 7.6%]	3%
Labor force participation rate	65.8%	65.9% [59.0%, 73.4%]	-0.5%
Initial job finding probability (1 month)	.402	.514 [.476, .550]	0.4%
Relative composition effect, job finding probability		71.6%	5%
Relative composition effect, reemployment wage		27.1%	8%

the elasticity causes reemployment wages to fall by much less than in the benchmark model, but also the participation rate to decrease significantly. The decomposition of reemployment wages implies a much more important role of the composition effect relative to true duration dependence.

The cyclical patterns of all of the robustness checks discussed in this section are largely identical to those of the baseline model. Specifically, the small but highly persistent decline in labor force participation, the large asymmetric response of the unemployment rate, the fall in the job finding probability, and the countercyclical behavior of the average home productivity of the unemployed. For brevity, Figures similar to 5 with the alternative parameters discussed in this section are omitted.

H Data

This section checks the robustness of the benchmark results shown in Tables 1 and 2. Additionally, it presents suggestive evidence on heterogeneity in outside options from time use data.

H.1 Job Finding Probability Using Probit Marginal Effects

Table H.13 shows the marginal effects from probit regressions corresponding to the linear probability model shown in Table 1. The last column is omitted due to the bias present in probit estimates when including fixed effects.

Table G.9: Targeted Moments: 0.28 Matching Function Elasticity

Description	Target	Model [95% CI]	% Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	40 years	0%
Matching function elasticity w.r.t v	.28	.28	0%
Relative value of nonmarket work	.71	0.60 [.587 , .613]	-.8%
Change in earnings with 1 year of market experience	2.30%	2.21% [1.35%, 3.12%]	0%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	0.27% [0.15%, 0.37%]	-52%
Lifetime earnings losses due to unemployment	−11.9%	−7.6% [-15.8%, 3.0%]	36%
Annual decline, hazard out of U	44.5%	28.5% [0%, 53.1%]	-42%
Quarterly average EU rate	.023	.023 [0.23, 0.23]	0%
Quarterly average UE rate	.328	.353 [.321, .385]	8%

H.2 Panel Study of Income Dynamics (PSID)

In Table 1, the PSID sample is restricted to heads of household age 18-65 with durations up to 12 months, though the results for "wives" are similar. Annual family weights are used in all regressions. Table H.14 shows estimated marginal effects for heads of household of all ages in Column (1), all durations in Column (2), all ages and all durations in Column (3). Table H.15 shows estimated marginal effects for the subsamples of males in Column (1), females in Column (2), workers with durations up to 6 months in Column (3), and durations above 6 months in Column (4). Over the 13-year sample period, there are 14,334 unique heads of household who respond to the survey. The total number of families responding to the survey in 1984 was 6,918 and in 1996 was 8,511. The average number of years that the head of household participates in the PSID over the sample is 9.7.

Since duration is measured in weeks in the CPS and in months in the PSID, the values

Table G.10: Untargeted Moments: 0.28 Matching Function Elasticity

Description	Data	Model	% Change from Benchmark
% change, log reemployment wage (1-12 months)	-2.2%	-13% [5.0%, 17%]	-51%
Unemployment rate	6.0%	6.3% [5.7%, 7.0%]	-5%
Labor force participation rate	65.8%	91.8% [89.5%, 93.6%]	39%
Initial job finding probability (1 month)	.402	.548 [.513, .584]	7%
Relative composition effect, job finding probability		14.7%	-78%
Relative composition effect, reemployment wage		3.48%	-86%

of the coefficients in the two surveys cannot be compared, but the results contained in Table H.14 are qualitatively similar to those using the CPS in Table 1. There continues to be a negative effect of duration on the probability of exiting unemployment as seen in the baseline CPS results in Table 1. Running the same regressions without restricting to age 18-65 gives similar results in all cases.

Tables H.16 and H.17 report the log wage regressions introduced in Table 2 with the same subsamples as in Tables H.14 and H.15, respectively. It should be noted that in the PSID, if a male adult is present in the household he is typically assigned the role of "head". Therefore, over two thirds of heads of household in the sample are males. The wage regressions below provide further evidence that the elasticity of starting wages with respect to unemployment duration is small. One can conclude that the fact that reemployment wages are less responsive to duration relative to the hazard rate of exiting unemployment is robust to the choice and timespan of the survey.

H.3 CPS: data description and robustness

Table H.18 shows similar regressions to H.14, reporting estimated marginal effects for workers of all ages in column 1, all durations in column 2, all ages and all durations in column 3. Table H.19 shows the corresponding wage regressions to Table H.16 in the unrestricted age and duration sample using the unallocated wages only in columns 1-3, and repeats the same regression including allocated wages in columns 4-6.

In the regressions of the job finding probability, the point estimate of the marginal effect of duration in column 1 of Table H.18 including workers of all durations are comparable in magnitude to the baseline results in column 2 of Table 1 focusing only on those workers

Table G.11: Targeted Moments: Matching Function Elasticity 0.5

Description	Target	Model [95% CI]	% Change from Benchmark
Annual interest rate	5%	5%	0%
Average working lifetime	40 years	40 years	0%
Matching function elasticity w.r.t v	.5	.5	25%
Relative value of nonmarket work	.71	.605 [.576 , .644]	0%
Change in earnings with 1 year of market experience	2.30%	2.00% [-0.12%, 4.53%]	-10%
Average increase in 1-month hazard out of U for each additional year of tenure	0.41%	1.56% [0.62%, 2.10%]	178%
Lifetime earnings losses due to unemployment	-11.9%	-19.3% [-67%, -2.95%]	-64%
Annual decline, hazard out of U	44.5%	81.5% [59.3%, 100%]	65%
Quarterly average EU rate	.023	.023 [0.23, 0.23]	0%
Quarterly average UE rate	.328	.308 [.255, .361]	-6%

between 18 and 65 years old. When we include workers with longer durations, the estimates of the marginal effect is only half as large as in the baseline regression. Again, including a dummy for long term unemployment causes all effects of duration on wages to disappear.

As discussed in Section 2, controls for the pre-unemployment industry and occupation are included to exclude the story of "overqualified" workers accepting worse jobs with duration, but being paid more than their new colleagues with the appropriate qualifications. Tables H.20 and H.21 show the regressions of subsamples of workers whose post-unemployment job was in a different industry, occupation, or both compared to the pre-unemployment job. Industry and occupation correspond to the "major recode." The industries considered are: Agriculture, forestry, fishing, and hunting; Mining; Construction; Manufacturing; Wholesale and retail trade; Transportation and utilities; Information; Financial activities; Professional and business services; Educational and health services; Leisure and hospitality; Other services; Public administration; Armed Forces. The occupation categories are: Man-

Table G.12: Untargeted Moments: Matching Function Elasticity 0.5

Description	Data	Model	% Change from Benchmark
% change, log reemployment wage (1-12 months)	−2.2%	−3.6% [0.6%, 6.1%]	-58%
Unemployment rate	6.0%	7.3% [6.3%, 8.7%]	11%
Labor force participation rate	65.8%	33.9% [22.7%, 44.4%]	-49%
Initial job finding probability (1 month)	.402	.509 [.444, .572]	-0.6%
Relative composition effect, job finding probability		75.6%	11%
Relative composition effect, reemployment wage		60.0%	140%

agement, business, and financial occupations; Professional and related occupations; Service occupations; Sales and related occupations; Office and administrative support occupations; Farming, fishing, and forestry occupations; Construction and extraction occupations; Installation, maintenance, and repair occupations; Production occupations; Transportation and material moving occupations; Armed Forces. Roughly 15% of the workers making a UE transition switch either major industry or occupation as defined above.

Table H.22 shows results for regressions identical to those in Table 1 for subsamples of males (columns 1-4) and females (columns 5-8). Table H.23 are the wage regressions corresponding to Table2 in the text. Results in both tables restrict the age of respondents to be between 18 and 65 as in the main results. Results in the baseline regressions are robust to splitting the sample by gender.

H.4 Empirical Support for Evolving Outside Options

Incorporating a home-specific skill into this model has implications for agents' time use over the unemployment spell. If productivity changes with duration, one expects to see individual workers adjust their allocations of time spent in different activities over the course of an unemployment spell, as long as the income and substitution effects do not perfectly offset. The American Time Use Survey (ATUS) is used to explore this possibility in the data. The survey began in 2003 and respondents are a subset of recent CPS interviewees. As above, the respondents are restricted to those who complete the ATUS survey between 18 and 65 years old with unemployment durations up to 1 year. Unless otherwise stated, time use categories are defined as in Aguiar et al. (2013).

Using the method of Krueger and Mueller (2010), duration is imputed for individuals

Table H.13: Probit: Job Finding Probability on Unemployment Duration

	CPS		PSID
	(1)	(2)	(3)
duration	-.0032***	-.0261***	-.3734***
	(.0001)	(.0014)	(.0515)
duration^2		.0014***	.0486**
		(.0001)	(.0194)
duration^3		-3.45e-05***	-.0022
		(3.99e-06)	(.0026)
duration^4		2.92e-07***	2.79 e-05
		(3.99e-08)	(.0001)
Pseudo R^2	.3702	.0555	.2564
N	141,916	141,916	10,802

Notes: CPS: January 1994-December 2015, monthly; duration reported in weeks. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. PSID: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

who are unemployed in the CPS and the ATUS as the duration reported in the CPS plus the length of time in weeks between the CPS and ATUS interviews. For those individuals employed in the CPS and unemployed in the ATUS, duration is set to half the time in weeks between the two interviews. Finally, duration for individuals who are employed in the ATUS is set to 0. This ensures that workers who are out of the labor force have recorded duration only if they reported being unemployed in the CPS and recently transitioned out of the labor force.

The data indicates that duration affects individuals' allocations of time, both in core home production (mostly cooking and cleaning) as well as childcare activities. Results are shown in Tables H.24 and H.25. Both tables show that there is a significant effect of duration on time use in home production for the population (column 1), which is strongly driven by females (column 3). Column (2) suggests that there is little effect of duration on time use in home production for males.

Regression results using minutes of core home production excluding child care as the dependent variable are shown in Table H.25. Column 1 shows results from the regression of core home production on a cubic term for duration, plus controls for observable heterogeneity (see footnote below Table for details). Columns 2 and 3 repeat the regression of column 1 for the subsamples of males and females, respectively. Though the effect is nonlinear, the length of an individual's unemployment spell changes the allocation of time in core home

Table H.14: Probit Regressions: all ages and durations, PSID

	(1)	(2)	(3)
duration	-.3691***	-.3761***	-.3753***
	(.0513)	(.0164)	(.0163)
duration^2	.0471**	.0508***	.0505***
	(.0193)	(.0036)	(.0036)
duration^3	-.0020	-.0024***	-.0024***
	(.0026)	(.0002)	(.0002)
duration^4	2.08e-05	3.08e-05***	3.05e-05***
	(.0001)	(3.33e-06)	(3.38e-06)
Pseudo R^2	.2570	.2602	.2608
N	10,855	10,938	10,991

Notes: PSID: 1984-1996, annual. Universe: heads of household unemployed in at least one month of the PSID employment history in the annual interview. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of the job finding variable on duration plus all controls for heads of household of all ages with durations up to 12 months, and for workers ages 18-65 for all durations, respectively. Column 3 shows results for heads of household of all ages and all durations. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

production, providing some indirect evidence in favor of the model mechanism. These results are robust to controlling for the number of children, age of the youngest child, and family income. Table H.24 shows the analogous regressions to those shown in Table H.25 using the sum of time spent in core home production and childcare activities.

An alternative explanation of the mechanism in this model is that the preferences for leisure activities are driving the results shown in the CPS and PSID. Results similar to those in this model would occur in a framework where workers develop a habit for leisure over the unemployment spell, implying that the outside option changes not through productivity but through preferences. Although this paper does not address it due to efficiency, the distinction between home production and leisure matters for the policy implications. In particular, if workers are developing habits for leisure which are inherently unproductive, a planner would choose a higher employment rate than workers in the decentralized economy. However, if home production contributes to output, as is assumed in this paper, worker specialization through learning by doing is optimal from the planner's perspective, leaving no room for policy.

Since for any specification of preferences, leisure has no income effect, in a model in which nonemployed workers allocate their time between home production and leisure, one would unambiguously expect an increase in leisure time with unemployment duration as the substitution effect dominates individuals' choices. The results of the ATUS indicate that the allocation of time in "home" activities changes over the unemployment spell, but the average

Table H.15: Probit Regressions: by sex and long term spell, PSID

	(1)	(2)	(3)	(4)
duration	-.2969***	-.6496***	.0430	-.0359
	(.0576)	(.1107)	(.2466)	(.0220)
duration^2	.0196	.1491***	-.1902	.0045*
	(.0219)	(.0394)	(.1412)	(.0024)
duration^3	.0014	-.0143***	.0478	-.0002*
	(.0030)	(.0051)	(.0316)	(9.64e-05)
duration^4	-.0001	.0005**	-.0034	2.46e-06*
	(.0001)	(.0002)	(.0024)	(1.14e-06)
Pseudo R^2	.2570	.3551	.2404	.3601
N	8,773	2,020	9,872	1,060

Notes: PSID: 1984-1996, annual. Universe: heads of household unemployed in at least one month of the PSID employment history in the annual interview, ages 18-65 and with duration up to 12 months. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 report results for the subsample of males and females, respectively. Column 3 reports results for the subsample of workers with durations less than 6 months, and column 4 for the subsample with durations greater than 6 months. Results reported are the estimated marginal effect of duration on the job finding probability.$*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

time spent in leisure activities does not, as shown in Table H.26. While the direction of the change in time spent doing home production depends on the specification of preferences, the data suggest that one can rule out the alternative explanation of a growing habit for leisure.

Table H.16: Wage Regressions: all ages and durations, PSID

	(1)	(2)	(3)
log duration	-.0228	-.0305**	-.0327**
	(.0155)	(.0137)	(.0138)
dummy, > 6 mo	Y	Y	Y
FE	N	N	N
R^2	.2144	.2167	.2161
Root MSE	.6382	.6361	.6381
N	10,617	10,694	10,746

Notes: PSID sample: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience, and a dummy indicating whether the most recent unemployment spell was over 6 months. Columns 1 and 2 show the regression of log real reemployment wages on log duration plus all controls for heads of household of all ages with durations up to 12 months, and for workers ages 18-65 for all durations, respectively. Column 3 shows results for heads of household of all ages and all durations. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table H.17: Wage Regressions: by sex and long term spell, PSID

	(1)	(2)	(3)	(4)
log duration	-.0198	-.0276	-.0027	-.2064
	(.0165)	(.0362)	(.0142)	(.1611)
dummy, > 6 mo	Y	Y	N	N
FE	N	N	N	N
R^2	.1941	.3039	.2096	.4989
Root MSE	.6412	.5723	.6278	.6261
N	8,593	1,972	9,738	827

Notes: PSID sample: 1984-1996, annual; duration reported in months. Universe: heads of household unemployed in at least one month of the PSID employment history with reported duration up to 12 months, ages 18-65. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Columns 1 and 2 report results for the subsample of males and females, respectively. Column 3 reports results for the subsample of workers with durations less than 6 months, and column 4 for the subsample with durations between 6 and 12 months. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table H.18: Probit Regressions: all ages and durations, CPS

	(1)	(2)	(3)
duration	-.0245***	-.0124***	-.0117***
	(.0013)	(.0005)	(.0005)
duration^2	.0014***	.0003***	.0003***
	(.0001)	(2.11e-05)	(2.01e-05)
duration^3	-3.28e-05***	-3.41e-06***	-3.23e-06***
	(3.79e-06)	(3.22e-07)	(3.08e-07)
duration^4	2.79e-07***	1.26e-08***	1.20e-08***
	(3.80e-08)	(1.53e-09)	(1.46e-09)
Pseudo R^2	.0559	.0646	.0647
N	157,169	166,814	183,544

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with duration up to 52 weeks. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of the job finding variable on duration plus all controls for respondents of all ages with durations up to 52 weeks, and for respondents ages 18-65 for all durations, respectively. Column 3 shows results for all ages and all durations. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

Table H.19: Wage Regressions: all ages and durations, CPS

	(1)	(2)	(3)	(4)	(5)	(6)
log duration	-.0075***	-.0126***	-.0109***	-.0009	-.0059**	-.0037*
	(.0029)	(.0028)	(.0026)	(.0026)	(.0024)	(.0022)
dummy, > 6 mo	N	N	N	N	N	N
R^2	.3181	.2959	.3135	.2906	.2635	.2849
Root MSE	.3487	.3572	.3495	.3705	.3786	.3717
N	18,549	17,763	19,601	26,416	25,631	28,090

Notes: Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E. Controls for observables include the aggregate unemployment dummies for the interview year and month, the log of the aggregate unemployment rate, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and total labor market experience. Columns 1-3 exclude those workers for whom the CPS allocated the hourly wage. Columns 1 and 2 show the regression of the log real reemployment wage on log duration plus all controls for respondents of all ages with durations up to 52 weeks, and for respondents ages 18-65 for all durations, respectively. Column 3 shows results for all ages and all durations. Columns 4-6 repeat the regressions including workers with allocated hourly wages. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table H.20: Probit Regressions: Workers with Changes in Industry and/or Occupation, CPS

	(1)	(2)	(3)
duration	-.0079**	-.0134***	-.0075**
	(.0033)	(.0034)	(.0038)
duration^2	.0003	.0008***	.0003
	(.0003)	(.0003)	(.0003)
duration^3	-3.85e-06	-1.86e-05*	-2.46e-06
	(9.46e-06)	(9.81e-06)	(1.09e-05)
duration^4	-7.01e-09	1.39e-07	-2.10e-08
	(9.47e-08)	(9.84e-08)	(1.09e-07)
Pseudo R^2	.0627	.0566	.0586
N	22,357	21,397	16,433

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with duration up to 52 weeks. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of the job finding variable on duration plus all controls for respondents whose industry or occupation of the job after the unemployment spell differs from the industry or occupation of the previous job, respectively. Column 3 shows results for respondents whose industry and occupation of the job after the unemployment spell differs from the industry and occupation of the previous job. Results reported are the estimated marginal effect of duration on the job finding probability. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

Table H.21: Wage Regressions: Workers with Changes in Industry and/or Occupation, CPS

	(1)	(2)	(3)
log duration	-.0026	-.0048	-.0017
	(.0035)	(.0035)	(.0040)
dummy, > 6 mo	N	N	N
FE	N	N	N
R^2	.2102	.2060	.1868
Root MSE	.3674	.3629	.3583
N	13,929	13,448	10,502

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with duration up to 52 weeks. Controls include the log of the aggregate unemployment rate, plus dummies for the interview year and month, gender, race, age, education, marital status, state, industry and occupation in the previous job, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 2 show the regression of log reemployment unallocated wages on log duration plus all controls for respondents whose industry or occupation of the job after the unemployment spell differs from the industry or occupation of the previous job, respectively. Column 3 shows results for respondents whose industry and occupation of the job after the unemployment spell differs from the industry and occupation of the previous job. $*$ denotes $p < .1$, $**$ $p < .05$, and $***$ $p < .01$.

Table H.22: Probit Regressions: Job finding probability on unemployment duration by gender

	Males				Females			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
duration	-.0042***	-.0359***	-.0577***	-.6741**	-.0038***	-.0302***	-.0397***	-.0715
	(.0002)	(.0023)	(.0076)	(.2761)	(.0001)	(.0024)	(.0076)	(.3029)
duration^2		.0021***	.0057***	.0277**		.0017***	.0033**	.0032
		(.0002)	(.0013)	(.0112)		(.0002)	(.0013)	(.0123)
duration^3		-5.35e-05***	-.0003***	-.0005**		-4.02e-05***	-.0002*	-6.42e-05
		(6.49e-06)	(8.32e-05)	(.0002)		(6.86e-06)	(8.53e-05)	(.0002)
duration^4		4.74e-07***	5.03e-06***	3.32e-06***		3.48e-07***	2.71e-06	4.85e-07
		(6.48e-08)	(1.76e-06)	(1.29e-06)		(6.89e-08)	(1.82e-06)	(1.42e-06)
Pseudo R^2	.0693	.0767	.0702	.0619	.0737	.0803	.0758	.0616
N	64,098	64,098	49,947	14,151	57,044	57,044	45,645	11,399

Notes: CPS: January 1994-December 2015, monthly. Universe: workers unemployed in at least one month of the CPS with reported duration up to 52 weeks, ages 18-65. Controls include the aggregate unemployment rate, plus dummies for the interview year and month, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and a quadratic term in total labor market experience. Columns 1 and 5 report results for the regression of workers on a linear term for duration, columns 2 and 6 is the same regression with a 4th degree polynomial of duration, columns 3 and 7 for workers with durations up to 6 months, and columns 4 and 8 for workers with durations between 6 and 12 months. Results reported are the estimated marginal effects of duration on the job finding probability. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table H.23: Regression of log reemployment wage on log unemployment duration by gender

	Males				Females			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
log duration	-.0120***	-.0073	-.0077	.1142	-.0058	-.0073	-.0056	.0686
	(.0040)	(.0050)	(.0051)	(.0507)	(.0042)	(.0050)	(.0060)	(.0447)
dummy, > 6 mo	N	Y	N	N	N	Y	N	N
R^2	.3979	.3982	.4036	.4422	.3981	.3982	.3902	.5447
Root MSE	.3395	.3394	.3393	.3391	.3292	.3394	.3319	.3012
N	8,678	8,678	7,607	1,071	8,874	8,874	7,819	1,055

Notes: Sample: January 1994-December 2015, monthly. Universe: respondents aged 18-65 who transitioned from U to E excluding those for whom the CPS allocated the hourly wage, with durations up to 52 weeks. Reemployment wage is the reported hourly wage. Duration is reported duration (prunedur). Controls for observables include the aggregate unemployment dummies for the interview year and month, the aggregate unemployment rate, gender, race, age, education, marital status, state, industry, occupation, the reason for unemployment, and total labor market experience. Columns 1 and 5 report results for the regression of workers at all durations with no long term unemployment dummy, columns 2 and 6 are the same regression with the long term dummy, columns 3 and 7 for workers with durations up to 6 months, and columns 4 and 8 for workers with durations between 6 and 12 months. * denotes $p < .1$, ** $p < .05$, and *** $p < .01$.

Table H.24: Regression: Hours per week of "core" home production plus childcare on duration

	All (1)	Men (2)	Women (3)
duration	-.8134**	-.4092	-1.149**
	(.3290)	(.3885)	(.5134)
duration^2	.0382***	.0209	.0501**
	(.0139)	(.0165)	(.0218)
duration^3	-.0005***	-.0003	-.0006**
	(.0002)	(.0002)	(.0003)
N	5,359	2,297	3,062
R^2	.1809	.1348	.1085

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, and presence of an employed partner. Column 1 reports results for all workers and columns 2 and 3 report results for the subsamples of males and females, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table H.25: Regression: minutes of "core" home production on duration

	(1)	(2)	(3)
duration	-6.002***	-3.190	-8.241**
	(2.249)	(2.816)	(3.432)
duration^2	.2664***	.1505	.3554**
	(.0957)	(.1225)	(.1446)
duration^3	-.0033***	-.0020	-.0043**
	(.0012)	(.0015)	(.0018)
N	80,545	39,314	41,231
R^2	.0997	.0240	.0797

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, presence of an employed partner, and labor force status. Column 1 reports results for the regression for all workers and columns 2 and 3 report results for the subsamples of males and females, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table H.26: Regression: Minutes leisure on duration

	(1)	(2)	(3)	(4)	(5)	(6)
duration	3.035	.3710	-5.373	9.385	-6.636	5.693
	(4.166)	(4.048)	(6.312)	(5.011)	(6.358)	(4.511)
duration^2	-.1009	-.0110	.2730	-.3942	.3130	-.2733
	(.1788)	(.1734)	(.2710)	(.2148)	(.2724)	(.1929)
duration^3	.0011	.0001	-.0036	.0047	-.0040	.0035
	(.0022)	(.0022)	(.0034)	(.0027)	(.0034)	(.0024)
N	80,545	80,545	39,314	41,231	39,314	41,231
R^2	.0842	.0586	.1002	.0726	.0697	.0489

Notes: ATUS: January 2003-December 2013, monthly. Universe: respondents with no "unclassified" time use, ages 18-65, with imputed durations up to 52 weeks. Controls for observables include dummy variables for the year and month of the interview, race, age, gender (column 1 only), state of residence, education level, presence of an employed partner, and labor force status. Columns (1) and (2) report results for all workers using definitions of leisure time including and excluding sleep, respectively. Columns (3) - (6) report results for the subsamples of males and females under each of the two definitions, respectively. * denotes $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$