Welfare Effects of R&D Support Policies*

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Abstract

We conduct a welfare analysis of R&D subsidies and tax credits using a model of innovation policy incorporating externalities, limited R&D participation and financial market imperfections. We estimate the model using R&D project level data from Finland. The optimal R&D tax credit rate (0.24) is lower than the average R&D subsidy rate (0.36). The intensive, not the extensive margin of R&D is important for policy. Tax credits and subsidies increase R&D investments and spillovers compared to laissez-faire but to levels below the first best. R&D support policies don't improve welfare.

KEY WORDS: R&D subsidies, R&D tax credits, extensive and intensive margin, welfare, counterfactual, economic growth.

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1 Introduction

A large body of evidence suggests that enhanced productivity through innovation is the main driver of economic growth. Economic theory, starting with Nelson (1959) and Arrow (1962), suggests that market failures provide a motivation for government intervention regarding private R&D investments. In line with these results, an increasing number of countries resort to various financial support policies such as R&D subsidies and tax credits to encourage private sector R&D: e.g., OECD countries spend in excess of $50 Billion on such support annually.[1] The existing theoretical and empirical literature however is not well-suited for giving guidance as to the extent and allocation of such support. For example, growth models assume that all firms invest in R&D when data shows that not to be the case; empirical research, too, mostly fails to differentiate between the effects of support at the extensive and intensive margins of R&D; and there are few if any empirical studies contrasting R&D subsidies and R&D tax credits. Most importantly, the vast empirical literature on the effects of R&D subsidies and tax credits does not address the ultimate question: are these R&D support policies welfare enhancing or not?[2]

This paper develops and applies a framework that allows us to first, contrast the impact of support at the extensive and intensive margins; second, to compare the impacts of R&D subsidies to those of R&D tax credits; and third, to empirically benchmark R&D subsidies and R&D tax credits against policies of no government support, and first and second best.

The two well known market failures motivating public support to private R&D are appropriability problems and financial market imperfections. Government innovation policy officials often add to this list the objective to entice non-R&D-performing firms to start investing in R&D, suggesting a

[1] We arrive at this figure by multiplying Business Enterprise R&D (BERD) measured in 2010 PPP US$ by the percentage of BERD financed by government (OECD Main Science and Technology Indicators www-site, accessed Sept 16th 2015).

[2] See surveys by Garciá-Quevedo (2004), Cerulli (2010), and Zúñica-Vicente et al. (2014). Recent important contributions include Bronzini and Iachini (2014), Einiö (2014), Hünermund and Czarnitzki (2016), and Howell (2017) on R&D subsidies, and Dechezleprêtre et al. (2016) on R&D tax credits.

market failure related to the extensive margin of R&D investments that is not captured by growth models guiding policy making. We build a model of an innovation policy that incorporates all these three rationals for public support to private R&D. We use revealed preference to identify the structural parameters by estimating four key decisions: the firms' project level R&D investments yield information on parameters governing the marginal profitability of R&D; the decision to invest in R&D allows us to identify the fixed costs of R&D; the decision of a firm to apply for subsidies is informative about the costs of application; and finally, the government agency's decision of what fraction of R&D costs to reimburse allows us to identify the parameters of the government utility function. We thus identify the parameters of the government's utility function from its own decisions.

In our welfare analysis, we keep the government utility function, identified from the estimation of the subsidy regime, constant while varying the R&D policy regimes. We first displace R&D subsidies with an optimally calculated R&D tax credit. This counterfactual is informative of the different effects that the two main government financial support policies used throughout the world have on private R&D. To provide benchmarks for these support policies, we consider a laissez-faire regime with no government support, and the first and second best regimes where the government can directly determine the level of private R&D investments (subject to the firms' zero-profit condition in the second best regime).

We take the model to R&D project-level data from Finland where the R&D to GDP ratio is among the highest in the world. In the mid 1980s a government agency (Tekes) was established to provide R&D subsidies to firms, and other forms of public financial support to R&D (e.g., R&D tax credits) were abolished. Our data cover the period 2000-2008. Finland's R&D subsidy regime is comparable, for example, to those of Belgium, Germany, the Netherlands and to the US SBIR programs, and is highly regarded.[3] The

[3]A recent evaluation of Tekes (van der Veen et al. 2012, pp. 29) concluded that "Tekes richly deserves its international reputation as a leading technology and innovation agency". Yet, this evaluation and other similar evaluations of Tekes and subsidy programs of other countries, cannot answer the question of whether tax payers' money is well spent or not. A contribution of our paper is to provide a tool for such a welfare analysis.

R&D tax credit regime we model is motivated by those used, for example, by Belgium, the Netherlands, Norway, and the UK.

In terms of theory, our model shows how both the calculation of optimal R&D subsidies and of optimal R&D tax credits becomes become much more complex when the extensive margin of R&D investment is introduced. We also demonstrate that the effect of financial market imperfections on the level of optimal support delicately depends on the margin at which the support operates.

Empirically, it turns out that there are only small differences across policy regimes in the R&D participation rate which is slightly above 50%. In other words, almost half of the Finnish firms do not invest in R&D, nor should they - their R&D ideas are neither privately nor socially profitable. Subsequently, the two R&D support policies have on average almost no impact at the extensive margin.[4] Conditional on investing, there are however large differences in the level of R&D: the R&D support policies increase R&D investments by more than 40%, and in the first best regime R&D investments are on average more than 100% higher than in laissez-faire.

The main difference between the two support regimes is that R&D subsidies tailor support to particular projects while reaching only a small fraction of R&D performing firms. Reflecting this, our counterfactual shows that in the R&D subsidy regime, the supported projects become clearly larger than average. In contrast, R&D tax credits are available to all R&D investing firms, but support does not vary according with project characteristics. In terms of fiscal cost, tax credits are 9% more expensive than R&D subsidies (ignoring administrative costs).

While differences in spillovers (i.e., welfare externalities of a firm's R&D such as consumer surplus and technological spillovers) across regimes are of the same order of magnitude as differences in the R&D investments, differences in profits are only a few percentage points. It turns out that profits are the main element of welfare, and we find virtually no differences in welfare between laissez-faire and the public support regimes, and only a couple of

[4]There are subtle differences at the extensive margin across the different policy regimes but the effects mostly cancel out in the aggregate numbers.

percentage points between the first best and laissez-faire. An explanation for spillovers being low relative to profits is that a significant fraction of spillovers generated by the Finnish R&D are likely to be flowing outside Finland, and should be ignored by a Finnish agency.

We differ from the vast majority of papers studying the effects of public support to private R&D in that we build a model to derive the estimation equations. One of our four main estimation equations is a familiar-looking R&D equation albeit with a different interpretation, as our model acknowledges the heterogeneity in firms' innovation emphasized by Akcigit and Kerr (2016). According to our data, this heterogeneity appears to be well-understood by innovation policy makers, and we use the large variation in government subsidy decisions - Figure 1 displays the distribution of the fraction of R&D cost covered by the government - that most papers ignore.[5] Our approach also potentially leads to the "treatment parameter" of the existing literature to be heterogeneous as a function of firm characteristics, though this turns out not to be the case in our data.

FIGURE 1 HERE

We believe to be the first to build and estimate a model of innovation policy where firms do not fully appropriate the returns to their R&D and where financial market imperfections and fixed cost of R&D lead to some firms not investing in R&D. This provides a basis for a welfare evaluation of R&D support policies, which has hitherto proven elusive. While the empirical literature on the effects of R&D support policies is vast (see footnote 2), it has focused on estimating the (causal) effect of a policy on some other outcome variable (e.g., on private R&D) than welfare. Nor do the existing models of innovation provide a solid foundation for a welfare analysis: for example, while useful for us as a starting point, the model in Takalo, Tanayama and Toivanen (2013a, TTT henceforth) assumes frictionless financial markets and

[5] A large fraction of the literature uses a dummy variable for a firm obtaining public support for R&D (González et al. 2005, and Arqué-Castells and Mohnen 2015 are among the few exceptions). Even fewer seek to "illuminate how planners make treatment decisions" (Manski 2001, pp. 106; see also Manski 2004) in which Manski sees "potentially enormous payoffs".

- in clear violation of any data - exhibits an equilibrium where all firms invest in R&D. It is also challenging to compare the merits of R&D subsidies and R&D tax credits without integrating them in a unified framework.

Methodologically, our paper is close to the macro-oriented literature on optimal R&D policy (e.g., Acemoglu et al. 2013, and Akcigit, Hanley, and Stantcheva 2017). We differ from this literature both in terms of data and modeling. Our data are more dis-aggregated, in particular when it comes to government support and R&D investment decisions which we observe and model at the project level, taking into account both the intensive and the extensive margin. We offer a richer model of the R&D subsidy process, i.e., who applies, who gets and how large subsidies, and what the investment effects of these subsidies are, but in a partial equilibrium context.

Another close paper to ours is Bloom, Schankerman, and van Reenen (2013) who share with us the interest on R&D spillovers and the estimation of social returns to R&D. Our approach to identifying spillovers and social returns complements theirs. Our result on the intensive margin being more important than the extensive margin is reminiscent of the results of Garcia-Macia, Hsieh and Klenow (2016) who find that productivity improvements by incumbents are more prevalent than those by entrants.

Our precursors in the small literature estimating structural models of innovation include TTT (2013a) and González, Jaumandreu, and Pazó (2005) who focus on R&D subsidies, Peters et al. (2017) who use a dynamic empirical model to uncover the fixed and sunk costs of R&D, and Doraszelski and Jaumandreu (2013) who study R&D and productivity. Also relevant are Xu (2008), who estimates an industry equilibrium model with R&D spillovers, Arqué-Castells and Mohnen (2015) who study the impact of fixed and sunk costs of R&D on the effectiveness of R&D subsidies, and Boller, Moxnes and Ulltveit-Moe (2015) who study the link between R&D, imports and exports.

We proceed by first discussing the Finnish institutional environment for R&D in the next section where we also present our data. We turn to our model in section 3. Section 4 is devoted to explaining how we estimate our model. Estimation results are presented in section 5 and section 6 contains the counterfactual experiment. Section 7 concludes.

2 Institutional Environment and Data

2.1 Institutional Environment

As pointed out by Trajtenberg (2001), Finland rapidly transformed from a resource- to an innovation and knowledge-based economy at the end of the millennium. The R&D/GDP ratio in Finland doubled over the two decades and overtook that of the US, though in the last couple of years it has decreased slightly (see Appendix A). The bulk of Finnish R&D is conducted by the private sector; its share has been slowly increasing.

The Finnish innovation policy hinges on direct R&D subsidies. In particular, during the period of our data (2000-2008) there were no R&D tax credits. Tekes, where our subsidy data comes from, is the main public organization providing funding for private investments in innovation. It provides both grants and loans. Some other public funding organizations such as Finnish Industry Investment, Finnvera, and Sitra also provide some limited finance for innovation, but their funding is not focused on R&D investments and does not generally consist of subsidies.

Tekes' objectives. Tekes' mission is to promote "*the development of industry and services by means of technology and innovations. This helps to renew industries, increase value added and productivity, improve the quality of working life, as well as boost exports and generate employment and well being*" (Tekes 2008 and 2011). In 2012 Tekes funding was circa 600M€, up from circa 400M€ in 2004 (see Appendix A). A large majority of this funding goes to firms, the rest to universities and other research institutes. In its funding decisions, Tekes emphasizes small and medium sized enterprises (SMEs), especially those seeking growth in global markets. However, large companies may also obtain funding from Tekes.

According to Tekes, its funding decisions are based on "*the novelty of the project, market distance, and the size of the company*" (Tekes 2011). After receiving an application, a team of Tekes' experts reviews and grades it in several dimensions, of which we use two: technological challenge and market risk.[6] The screening stage includes a thorough interview with the applicant's

[6]To acquaint ourselves with Tekes' decision making, one of us spent 11 months in

representatives. The expert team then makes a funding proposal for a funding committee which decides the subsidy rate. The maximum financing share may reach, depending on the applicant and the project, 70% of the project costs. Tekes can give firms that satisfy the European Union (EU) criteria for SMEs a 10 percentage point higher maximum financing share than for large companies.

2.2 Data

Our data comes from two main sources: from Tekes, we obtained detailed data on all R&D subsidy applications between 1/2000 and 12/2008. These data include the applied amount of funding, Tekes' internal screening outcomes and final funding decisions, the realized project expenses and reimbursements by Tekes. We matched these data to the R&D survey and balance-sheet data from Statistics Finland. After matching this information with firm characteristics, we end up with 25 505 firm-year observations for 8 363 firms.[7] In addition to Tekes and Statistics Finland, we obtain cost-of-borrowing data for Finland from the European Central Bank Statistical Data Warehouse (see Table 1). In contrast to TTT (2013a), our data cover a longer time period and contain information on the actual (instead of planned) R&D expenditure and reimbursements at the project level for successful applicants, and information on firm level R&D for all firms.

We show descriptive statistics in Table 1. The average age of non-applicant (applicant) firms in our data is 17 (13) years; the average number of employees is 107 (176), and the average sales per employee, normalized to year 2005 in 100 000€, is 0.27 (0.21). Of the non-applicant (applicant) firms in our data, 70% (73%) are SMEs, 17% (20%) are located in the re-

Tekes. It became clear that technological challenge and market (commercial) risk are the two most important grading dimensions. As in TTT (2013a), we estimate ancillary grading equations; see Appendix B.

[7] We follow TTT (2013a) and randomly choose one application for those firms with more than one application in a given year. In essence, we are assuming that each firm receives only one R&D idea per year. Relaxing this assumption provides a challenging task for future research. We also pool the different funding tools of Tekes as in TTT (2013a), and conduct a related robustness exercise in Appendix B. We explain how we trim the estimation sample and provide some further descriptive statistics of our data in Appendix B.

gions eligible for EU regional aid, and 55% (84%) invested in R&D in the preceding year. All these differences between applicants and non-applicants are statistically significant. As the figures of Table 1 also imply, on average some 60% firms invest in R&D and only some 20% of the firms apply for subsidies.

Table 1 also displays descriptive statistics for accepted and rejected applicants; here the differences are not statistically significant. For those firms that obtain a subsidy, the average subsidy rate is 0.36 with a large standard deviation. The average project level R&D investment over the (max. 3 year) lifetime of a project is 393 000€. As explained in detail in Appendix B, we convert the original Likert scale 0-5 grades of both technological challenge (*tech*: ranging from 0 = "no technological challenge" to 5 = "international state-of-the-art") and market risk *(risk*: ranging from 0 = "no identifiable risk" to 5 = "unbearable risk") to 1-3 by combining grades 0 and 1, and grades 3, 4, and 5 because of very few observations at the tales. Using the augmented grades, the average technological challenge is 2.1 (1.9 on the original scale) and the average market risk 2.3 (2.4).

TABLE 1 HERE

3 The Model

3.1 Overview

In this section we present our model which is builds on TTT (2013a,b). A key generalization is that we model financial market imperfections.[8] An extensive literature (see surveys by Hall and Lerner 2010, and Kerr and Nanda 2015) suggests that imperfections are a pervasive feature of innovation finance.

[8]There are other important differences: the model we estimated in TTT (2013a) was based on stronger functional form assumptions and besides assuming perfect capital markets did not model the extensive margin of R&D either. The model in TTT (2013b), which has not been estimated, added the fixed costs of R&D. Neither of our earlier papers modeled R&D tax credits.

There is a firm run by an entrepreneur with an R&D project, a public agency allocating R&D subsidies, and a competitive private sector financier of R&D. Henceforth, we refer to the public agency as the "agency" and to the private sector financier as the "investor", and treat the entrepreneur's decisions as if they would be made by the firm. All three agents are risk neutral and for brevity there is no time preference. The firm's R&D project involves both a variable and a fixed cost. The firm has no funds of its own. The process of obtaining outside funding is hampered by both moral hazard and incomplete information problems.

Moral hazard. As in Holmström and Tirole (1997), the firm's ability to borrow is constrained by a dual moral hazard problem. The firm has access to different R&D projects and is tempted to choose a less productive project with a higher non-verifiable return. The investor can solve the firm's moral hazard problem through a monitoring technology that is costly. The investor thus has an incentive to shirk. As is standard, the firm's project choice and the investor's monitoring decision are non-verifiable to third parties. Investment level, project success, and payments from the firm to the investor are verifiable.

Incomplete information. The type of the firm and the investor are common knowledge, but the type of the agency (i.e., part of the agency's payoff function) is unknown to all agents when the firm contemplates whether to apply for a subsidy or not. Thus, incomplete information means that the firm faces uncertainty about the agency's valuation of its project when making a subsidy application decision. The agency's valuation of the project equals the social value of the project under the assumption of a welfare-maximizing agency.

Compared to standard corporate finance models, our assumption may sound unorthodox.[9] Assuming common knowledge about the firm's type may ignore some interesting features of R&D subsidy programs.[10] The ad-

[9]But note that there is a growing corporate finance literature building on the analogous asumption that a lender knows the borrower's creditworthiness better than the borrower herself (see, e.g., Inderst and Mueller, 2006) .

[10]For example, by using the more familiar informational assumption, Takalo and Tanayama (2010) show how a subsidy decision by the agency acts as a signal about the

vantage of our assumption is that it ensures, in line with data, equilibrium outcomes with rejected applications without the need to model complexities arising from signaling games. Furthermore, it seems reasonable to us to assume that a firm does not exactly know the agency's objective function. The other features of our incomplete information assumption are less controversial. We assume that the agency learns its valuation of the firm's project after receiving and screening an application and w.l.o.g. also that its type becomes common knowledge thereafter.[11]

Agency behavior. The agency's decision is an ex ante commitment to reimburse a fraction of the project's variable costs ex post; this we call the subsidy rate. We assume that the agency can extend funding neither to fixed costs nor to external financing costs.[12] In line with the institutional environment, the agency's subsidy rate decision is subject both to a maximum constraint that is strictly less than unity, and to a minimum constraint of zero, which binds if there is no application or the application is rejected.

We assume that the agency cannot dictate the firm's investment level. For simplicity we also assume that the agency's budget constraint does not bind, but allow for a costs of public funds. We show that the agency will reject applications in equilibrium.

Timing of events. In period zero, nature draws the agents' types. In period one, the firm decides whether or not to apply for a subsidy. If the firm applies, in period two the agency evaluates the proposed project, learns its valuation of the project, and decides the subsidy rate. The agency's type becomes common knowledge. In period three, the firm and the investor sign a financing contract which stipulate the size of the project, how the project is financed, and how the profit is shared between them. The investor makes a

firm's type for private sector financiers, and Lach, Neeman, and Schankerman (2015) study the possibilities to design subsidy programs so as to screen appropriate applicants.

[11] In a more dynamic model it would be natural to assume that the firm learns the agency's type over time. This is an interesting avenue for further research.

[12] In our setting the agency (Tekes) has rules on eligible expenses and regularly does not accept all types of costs included by applications. In particular, the costs of raising external finance are non-eligible. It is also arguably more difficult to get a reimbursement from the agency for fixed costs than for variable costs that are easy to allocate for subsidized projects.

monitoring decision. In period four the firm chooses the project and invests.[13] If the firm has been granted a subsidy in period two, it will be reimbursed. In period five, the project returns are realized, and divided according to the financing contract.

Next we present the model. To obtain our econometric model, we use more specific functional forms than would be necessary from a purely theoretical point of view. As parts of the model are similar to TTT (2013a,b), we relegate some derivations to Appendix C.

3.2 R&D Technology

A firm needs to incur a variable cost $R > 1$ and a fixed cost $F \geq 0$ to undertake an innovation project in period four. Investing in the project yields, in period five, a verifiable financial return equaling either zero, or

$$\pi = A\left(\frac{R^{1-\gamma}-1}{1-\gamma}\right) \tag{1}$$

in case of success. In equation (1), $A \geq 0$ is a constant shifting the conditional returns, and $\gamma \geq 0$ is a measure of the concavity of the conditional profit function.[14]

To formalize the moral hazard problem, we assume that the firm can privately choose between two projects. A "good" project succeeds with probability $P \in (0,1)$, but provides no private benefit. A "bad" project succeeds with a zero probability but involves a non-transferable private benefit $b > 0$ per unit of investment.[15] If the firm does not launch the project, the returns are zero.

3.3 The Financing Contract

Since the firm has no liquid funds of its own and since the public agency at maximum reimburses a fraction of the investment ex post, the firm must

[13]Note that the investor can commit to monitoring before the firm makes the project choice as in Winton (1993) and Holmström and Tirole (1997). This assumption of means we need not consider mixed strategy equilibria.

[14]When $\gamma \to 1$, a logarithmic conditional return function emerges. This is the reason to have -1 in the numerator. Also for this reason, $R > 1$ in equilibrium whenever the firm goes ahead with project. When $\gamma \to 0$, π becomes linear in R.

[15]It would be straightforward to extend the model to allow the bad project to succeed with a positive probability lower than P.

raise external funding from an outside investor in period three. The investor can flexibly raise funds at a constant rate $r \geq 1$ independent of the project.

A financing contract between the firm and its investor stipulates that the returns from a successful project in period five are split according to

$$\pi = \pi^I + \pi^E, \tag{2}$$

where π^I and π^E denote the investor's and the firm's (the superscript E stands for the entrepreneur running the firm) share of project returns. Neither party is paid anything if the project fails. In our setting this return sharing rule is optimal, and accommodates both equity and debt contracts.

The investor has access to a monitoring technology that allows her to prevent the firm from choosing the bad project at a monitoring cost $c > 0$ per unit of investment. We assume that the firm's private benefit b from the bad project is large enough to make the bad project privately attractive to the her unless the investor monitors, i.e., $b \geq P\pi$.[16]

We assume that the investor behaves competitively in the sense that a project financing deal, if any, yields zero profits to the investor. Consequently, we may seek an optimal financing contract that maximizes the firm's payoff.[17] An optimal financing contract solves the program

$$\max_{\{\pi^E \geq 0, \pi^I \geq 0, R \geq 0\}} \Pi^E = P\pi^E \tag{3}$$

subject to equation (2),

$$\Pi^E \geq 0, \tag{4}$$

[16] If private benefits and monitoring costs are included in the welfare calculus, we should also ensure that monitoring and choosing the good project are socially desirable. We could impose an upper bound for b and then assume that $c < b$ so as to satisfy that welfare criterion, but assume for simplicity that the bad project involves low enough social externalities to render it inferior to the good project from a welfare point of view.

[17] In essence, this is equivalent to assuming a financial sector with free entry of identical investors. When there are no externalities among investors, the investors would offer the same contract as the one that is offered by a single investor that maximizes the firm's payoff subject to the investor's zero profit condition.

$$\Pi^I = P\pi^I - (r+c)(R+F) + sR \geq 0, \tag{5}$$

and

$$P\pi^I - c(R+F) \geq 0. \tag{6}$$

Equations (4) and (5) are the firm's and the investor's participation constraints. The latter shows that if the firm's investment is successful, the investor receives π^I. The investor needs to fund the whole investment $R+F$ and needs to cover the opportunity cost of her funds r and the costs of monitoring c. Since subsidies are paid ex post, the investor gets the subsidy, if any, granted to the firm by the agency. This is shown by the last term of equation (5), where $s \in [0, \overline{s}]$, $\overline{s} < 1$, is the subsidy rate. Thus the investor needs to get at least $\pi^I = [(r+c)(R+F) - sR]/P$ to participate.[18]

Equation (6) is the investor's incentive constraint. On the left-hand side is the investor's payoff to monitoring. Equation (6) thus implies that the investor needs to get at least $\pi^I \geq c(R+F)/P$ to invest in monitoring. Whenever the investor is monitoring, the bad project is eliminated from the firm's choice set, rendering the firm's incentive constraint irrelevant. Comparing equations (5) and (6) shows that that the investor's incentive constraint (6) is slack since $r \geq 1$, and $s \leq \overline{s} < 1$. In essence, monitoring is part of the investor's participation decision and, thus, the relevant constraint is equation (5).

Solving the program of equations (3)-(5) yields (see Appendix C) the firm's optimal R&D investment rule as

$$R^*(s) = \mathbb{I}_{[0,\infty)}\left(\Pi^E\left(R^{**}(s), s\right)\right) R^{**}(s), \tag{7}$$

[18] In other words, we assume that the financing contract is written contingent on the agency's subsidy decision. If the financing contract were not written contingent on subsidies, the firm's cost of outside funding would be higher but all other features of the model would be unchanged (see Appendix C where we use this alternative assumption in the case of R&D tax credits). Since evidence (see Demeulemeester and Hottenrott, 2015) suggests that subsidies lower firms' cost of outside funding, we choose the more realistic assumption.

where the firm's optimal variable R&D investment and equilibrium participation constraints are given by

$$R^{**}(s) := \arg\max_{R\geq 0} \Pi^E(R,s) = \left(\frac{\alpha}{\rho - s}\right)^{\frac{1}{\gamma}}, \tag{8}$$

and

$$\Pi^E\left(R^{**}(s),s\right) = \frac{\alpha}{1-\gamma}\left[\gamma\left(\frac{\alpha}{\rho-s}\right)^{\frac{1-\gamma}{\gamma}} - 1\right] - \rho F \geq 0, \tag{9}$$

respectively, and where $\mathbb{I}_{[0,\infty)}(\cdot)$ is an indicator function taking value one if equation (9) holds and zero otherwise. In equations (8) and (9), $\alpha := AP$ is a constant shifting the expected profitability of the R&D project, $\rho := r+c > 1$ denotes the investor's marginal cost, and $\rho - s$ captures the marginal cost of R&D to the firm.

3.4 Public Funding

The agency's decision on how much to subsidize the firm's investment is made in period two. The agency's utility from an R&D project is given by

$$U\left(R(s),s\right) = vR\left(s\right) + \Pi^E\left(R(s),s\right) + \Pi^I\left(R(s),s\right) - gsR\left(s\right) \tag{10}$$

where $g > 1$ is the constant opportunity cost of the public funds (same for all projects). As the second and third right-hand side terms of equation (10) show, the firm's and investor's profits enter the agency's objective function. The first and last terms on the right-hand side captures the effects of the firm's R&D on the agency beyond the firm's and investor's payoffs.

Equation (10) is our measure of welfare. Our approach rests on the idea that identifying the parameters governing equation (10) allows us to meaningfully compare counterfactual policies to the current policy from the government's point of view without necessarily taking a stand on whether the government is a benevolent social planner or not. In particular, we may think of v as being the spillover rate per unit of R&D, in which case vR gives the total spillovers generated by the project. The spillover rate v can reflect standard positive welfare externalities of R&D investments such as

consumer surplus and technological spillovers, but also private benefits from funding the project to the agency's civil servants. The parameter v can also be negative e.g. due to duplication of R&D costs, business stealing effects, or negative environmental externalities. Spillovers (vR) are assumed linear in the investment level R, as is common in the literatures on economic growth and R&D spillovers. Referring to our incomplete information assumption discussed in section 3.1, we assume that v is known to the agency when it makes the subsidy decision but is unknown to the firm when it contemplates applying (i.e., v determines the type of the agency).

The agency chooses $s \in [0, \overline{s}]$ to maximize its objective function (10) subject to the firm's participation constraint (9) and investment rule (7), and to the agency's participation constraint

$$U\left(R^*(s^*), s^*\right) \geq 0. \tag{11}$$

Equation (11) implies that the agency's total benefits from the project should be non-negative when it grants an optimal positive subsidy rate ($s^* > 0$); otherwise the agency rejects the application and $s^* = 0$.

To ensure that the agency's problem is well behaving we impose

Assumption 1. $\gamma < \frac{g}{g-1}$.

Assumption 1 is a necessary condition for the existence of an interior solution for the agency's problem.[19]

The agency's program is solved for in Appendix C. It turns out that, depending on the parameter values, the agency's optimal subsidy rate s^* is 0, $\tilde{s}$, $s^{**}(v)$, or $\bar{s}$ where

$$s^{**}(v) := \arg\max_{s \in \mathbb{R}} U(R^{**}(s), s) = \frac{v - \rho\gamma(g-1)}{g - \gamma(g-1)} \tag{12}$$

[19] If Assumption 1 failed to hold, the agency would either award a minimum subsidy rate of zero or the maximum subsidy rate of $\overline{s}$, depending on the parameter values. Assumption 1 can be relaxed but at a substantial cost. In our empirical application, we use $g = 1.2$; then Assumption 1 implies that $\gamma < 6$.

is the solution for the agency's unconstrained problem, and

$$\tilde{s} := \rho - \alpha^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{\alpha + \rho F(1-\gamma)} \right]^{\frac{\gamma}{1-\gamma}}. \tag{13}$$

is the subsidy rate at which the firm's participation constraint (9) holds as an equality. Note from Assumption 1 and equation (12) that the denominator of $s^{**}(v)$ is positive and, thus, $s^{**}(v)$ is strictly increasing.

To characterize the agency's optimal decision rule, it is useful to divide the parameter space into two dimensions, F and v. In Appendix C we show that there exist two threshold values of F, denoted $\tilde{F}$ and $\bar{F}$, and satisfying $\tilde{F} < \bar{F}$. If $F > \bar{F}$, fixed costs are so high that they prevent investment even with a maximum subsidy rate $\bar{s}$.

In contrast, if $F \leq \tilde{F}$ the firm's participation constraint never binds. In that case, equation (12) suggests that the minimum constraint of zero on the subsidy rate binds for sufficiently low spillover rates, i.e., for $v \leq \underline{v} := \rho\gamma(g-1)$, $s^*(v) = 0$. Similarly, the maximum constraint of $\overline{s}$ binds for high enough spillover rates, implying that $s^*(v) = \bar{s}$ for $v \geq \overline{v}$ where $\overline{v} := \underline{v} + \bar{s}[g - \gamma(g-1)]$. When $v \in (\underline{v}, \bar{v})$, the agency grants the optimal unconstrained subsidy rate $s^{**}(v)$.[20]

Finally, if $F \in \left(\tilde{F}, \bar{F}\right]$, the firm will invest only if it receives a subsidy. In Appendix C we show that in this case the agency's optimal decision rule is $s^*(v) = 0$ if $v < v^0$, $s^*(v) = \tilde{s}$, if $v \in [v^0, \tilde{v})$, $s^*(v) = s^{**}(v)$ if $v \in [\tilde{v}, \bar{v})$, and $s^*(v) = \bar{s}$ if $v \geq \bar{v}$, where v^0 and $\tilde{v}$ (with $v^0 \leq \tilde{v} \leq \overline{v}$) denote the values of v that satisfy $U(R^{**}(\tilde{s}), \tilde{s})$=0 and $s^{**}(\tilde{v}) = \tilde{s}$, respectively. This implies that for a project yielding high agency benefits but low profits, the agency may increase the subsidy rate so high as to satisfy the firm's participation constraint, but does so only if its own participation constraint is also satisfied.

3.5 Firm's Application Decision

In period one, the firm has to decide whether or not to apply for a subsidy.

[20] $g > 1$ and Assumption 1 yield $0 < \underline{v} < \overline{v}$.

If the firm does not apply, its profits in period five are

$$\Pi_0^E = max\left\{0, \Pi^E\left(R^{**}\left(0\right), 0\right)\right\}, \tag{14}$$

where the subscript 0 indicates that the firm does not apply for a subsidy. Equation (14) reflects the firm's option to invest without a subsidy.

The firm's expected profits in case it applies for a subsidy are given by

$$\Pi_1^E = \mathbb{E}_v\left[max\left\{0, \Pi^E\left(R^{**}\left(s^*\right), s^*\right)\right\}\right] - K \tag{15}$$

where subscript 1 indicates that the firm applies for a subsidy, $K > 0$ is the fixed cost of applying for subsidies, and $\mathbb{E}_v$ denotes the expectation operator over the agency types. The max operator captures the possibility of the subsidy being so low that it is unprofitable to invest.

Equation (15) shows how the firm, when contemplating an application, must take expectation over all possible types of the agency, and then calculate all possible subsidy rates resulting from those agency types. The firm can then calculate the expected costs of external financing and its expected investment levels resulting from those subsidy rates, and, ultimately, its expected profits resulting from those investments and subsidy rates. We assume that the agency type is drawn from a known type space V according to a distribution with probability density function $\phi(v)$ and cumulative density function $\Phi(v)$.

The firm applies for a subsidy only if the application constraint $\Delta\Pi^E := \Pi_1^E - \Pi_0^E \geq 0$ holds. The firm's application decision $d \in \{0, 1\}$ can then be expressed as an indicator function $\mathbb{I}_{[0,\infty)}\left(\Delta\Pi^E\right)$. We describe $\Delta\Pi^E$ in more detail in Appendix C.

3.6 Equilibria

Recall from section 3.2 that the firm can choose between two projects in period four. Let us denote the firm's project choice by $h \in \{B, G\}$ where "B" and "G" represent a bad and a good project. Let $m \in \{0, 1\}$ denote the investor's decision to monitor in period three (1 = monitor; 0 = don't).

A strategy for the firm consists of an indicator function $\mathbb{I}_{[0,\infty)} : \mathbb{R} \to \{0, 1\}$

that describes the application decision, d, in period one, and of functions $f_h : \{0,1\}^2 \times [0,\bar{s}] \times [0,\infty) \to \{B,G\}$ and $f_R : \{0,1\}^2 \times [0,\bar{s}] \times [0,\infty) \to [0,\infty)$ that describe, in period four, a project choice, h, and the size of the R&D investment, R, as functions of the application decision, the agency's subsidy rate decision, and the monitoring decision, and profit share required by the investor. A strategy for the investor consists of two functions $f_{\pi^I} : \{0,1\} \times [0,\bar{s}] \to [0,\infty)$ and $f_m : \{0,1\} \times [0,\bar{s}] \to \{0,1\}$ that describe, in period three, the required profit share π^I and monitoring decision m as functions of the firm's application decision and the agency's subsidy rate. A strategy for the agency is a function $f_s : V \times \{0,1\} \to [0,\bar{s}]$ mapping the agency's type v and the firm's application decision d into a subsidy rate s in period two.

We focus on perfect Bayesian equilibria (PBE) satisfying the following five criteria: 1) the firm rationally assigns a probability $\phi(v)$ to type $v \in V$; 2) the firm's optimal strategy is $d^* = \mathbb{I}_{[0,\infty)}\left(\Delta\Pi^E\right)$, $R^*(s)$ as given by equation (7), $h^* = G$ if $m = 1$ and $h^* = B$ if $m = 0$; 3) the competitive investor's optimal strategy is $\pi^{I*}(s)$ as given by equation (23) and $m^* = 1$; 4) if $d = 1$, the agency's optimal strategy is $s^*(v) \in \{0, \tilde{s}, s^{**}(v), \bar{s}\}$ where $s^{**}(v)$ and $\tilde{s}$ are given by equations (12) and (13), respectively, and if $d = 0$, $s^*(v) = 0$ for all v; and 5) if a rejection of an application is optimal for the agency, $s^*(v) = 0$. [21] In our model the firm's and investors' posterior beliefs concerning the agency's type v after observing a subsidy decision are inconsequential, so there is no need to model the updating of beliefs.

We obtain the following proposition:

Proposition 1. *For given F and v, there is a unique PBE with the following properties:*

i) $\pi^{I*}(s) = \left[\rho\left(R^*(s) + F\right) - sR^*(s)\right]/P$, $m^* = 1$, and $h^* = G$.

ii) Suppose $F \leq \tilde{F}$. Then $R^*(s) = R^{**}(s)$ for all d and v. If equation (30) (see the proof in Appendix C) does not hold, $d^* = 0$ and $s^*(v) = 0$.

[21] These criteria are standard save for the latter part of the fourth ($s^*(v) = 0$ for all v if $d = 0$) and the fifth criteria. These simplifying criteria are motivated by the practice of R&D subsidy programs. As mentioned in section 3.1, we assume that an agency cannot give a positive subsidy rate if it receives no application or wishes to reject an application.

Otherwise $d^* = 1$, $s^*(v) = 0$ for $v \leq \underline{v}$, $s^*(v) = s^{**}(v)$ for $v \in (\underline{v}, \overline{v})$, and $s^*(v) = \overline{s}$ for $v \geq \overline{v}$.

iii) Suppose $F \in \left(\tilde{F}, \overline{F}\right]$. If equation (31) (see the proof in Appendix C) does not hold, $d^* = 0$, $s^*(v) = 0$, and $R^*(0) = 0$. Otherwise, $d^* = 1$, $s^*(v) = 0$ for $v < v^0$, $s^*(v) = \tilde{s}$ for $v \in [v^0, \tilde{v})$ $s^*(v) = s^{**}(v)$ for $v \in [\tilde{v}, \overline{v},)$ and $s^*(v) = \overline{s}$ for $v \geq \overline{v}$. For $v < v^0$, $R^*(0) = 0$. For $v \geq v^0$, $R^*(s) = R^{**}(s)$.

iv) Suppose $F > \overline{F}$. Then for all v, $d^* = 0$, $s^*(v) = 0$, and $R^*(0) = 0$.

Proof: in Appendix C.

Part i) of Proposition 1 follows directly from our definition of a PBE, summarizing the interaction between the investor and the firm. Parts ii-iv) focus on the interaction between the firm and the agency, characterizing the conditions under which it is optimal to launch a project.

While part i) of Proposition 1 is trivial, its implications are not. In principle the equilibrium outcome where a project is not launched ($R^*(s) = 0$) could be supported by multiple combinations of monitoring and project choice decisions and by a continuum of profit sharing rules. Our definition of a PBE, resulting in part i) of Proposition 1, solves this indeterminacy by putting restrictions on the out-of-equilibrium play between the investor and the firm: the investor is always willing to participate and monitor a project by requiring a competitive rate of return that covers the cost of monitoring. As a result the firm then goes ahead with the good project, if any. While in theory it would be easy to relax these restrictions on out-of-equilibrium play, they imply that the firm's decision to not launch a project is a result of prohibitively high, but uniquely specified, cost of external funding.

Finally, note from equation (12) that the effect of ρ on the optimal unconstrained subsidy rate s^{**} is negative. Since ρ is a sum of r, the market rate of return reflecting the economy-wide financial conditions and c, the cost of monitoring capturing the firm or project specific financial frictions, this suggests that room for using various adverse financial market conditions to

motivate R&D subsidy policies may be more limited than what is commonly thought (see also the discussion in TTT 2013b).

4 Econometric Implementation

In this section we describe how to estimate the agents' four key decisions in the theoretical model: the firm's decision whether to launch an R&D project and the optimal R&D investment levels conditional on starting a project, the decision to apply for a subsidy, and the agency's subsidy rate decision.[22]

We discuss the identification of each equation separately. Our model allows for two key features of the unobservables: first, spillovers and profits are allowed to be correlated; second, the set of firms that apply for subsidies is allowed to systematically differ from other firms. We collect the formal assumptions on the unobservables at the end of this section. All estimation equations are defined at the project level except for the R&D participation decision which is at the firm level. We use the following generic notation where possible: $\mathbf{X}_{it}^{l}$ denotes a vector of observable firm and project characteristics, and β^{l} denotes the associated vector of parameters to be estimated. Subscript i denotes a project (and a firm), subscript t denotes the year of the firm's subsidy application decision, and superscript $l \in \{F, K, R, v\}$ refers to the variable of the interest in an estimation equation. We specify that $\mathbf{X}_{it}^{l}$ contain the following: a 3^{rd} order polynomial in firm (log) age, (log) number of employees and sales/employee; a dummy for a calendar year, an industry, an R&D investment in the previous year, and the eligibility for EU regional aid. All explanatory variables are lagged by one year.[23]

R&D investment. Let us define the constant shifting the expected

[22] While our estimation procedure builds on TTT (2013a), the introduction of the fixed cost of R&D leads to a much more involved estimation procedure.

[23] We bootstrap the whole estimation procedure to obtain standard errors for the R&D investment, fixed cost of R&D and application cost parameters.

profitability of an R&D project (see equations (8) and (9)) as

$$\alpha_{it} := e^{\gamma_{it}\left(\mathbf{X}_{it}^{R}\beta^{R}+\varepsilon_{it}\right)}, \tag{16}$$

where ε_{it} is a random shock affecting the expected profitability of the R&D project i in year t. This profitability shock is observed by all three agents of the model but unobserved by the econometrician. Besides parameters included in β^R, our model allows the estimation the structural parameter γ_{it} which, as mentioned, is a measure of the concavity of the conditional profit function.

Substituting equation (16) into equation (8), and taking the natural logarithms of both sides of the resulting equation yields

$$\ln R_{it}(s_{it}) = \mathbf{X}_{it}^{R}\beta^{R} - \frac{1}{\gamma_{it}}\ln\left(\rho_t - s_{it}\right) + \varepsilon_{it}. \tag{17}$$

Equation (17) is our estimation equation for R&D investment, conditional on the firm starting a project. There is a linear, firm-specific coefficient of $\ln\left(\rho_t - s_{it}\right)$, and a standard, additively linear error term ε_{it}. At this final stage of the model, s_{it} is known (and independent of ε_{it}, see below). We approximate the cost external funds, ρ_t, by the annual average cost of borrowing of the Finnish non-financial corporations; hence it is not indexed by i. We specify $\gamma_{it} = \Phi(\mathbf{X}_{it}^{\gamma}\beta^{\gamma})$ where $\Phi\left(\cdot\right)$ is the standard normal cumulative distribution function. With $\mathbf{X}_{it}^{R}$, ρ_t, and s_{it} being observed, estimating equation (17) yields $\hat{\gamma_{it}}$ and $\hat{\beta}^{R}$. Because we estimate equation (17) by ML, we also identify the variance of ε_{it}.

There is a sample selection problem as we only observe the project level R&D investments of those firms that apply for and receive a subsidy. We estimate equation (17) with standard sample selection methods. For identification, we exploit the agency's goal of prioritizing SMEs in its subsidy allocation decisions (see section 2). In particular, the maximum subsidy is 10 percentage point higher for SMEs. The criteria for qualifying as an SME is decided at the EU level and can hence be viewed exogenous to the Finnish

environment. This non-linearity of the agency decision rule in firm size means that we assume that an SME is more likely to apply for a subsidy, but the SME status should have no impact on the firm's R&D investment level.

In the first stage of the sample selection model, the dependent variable is a dummy taking value 1 if a firm was granted a subsidy in year t. In the second stage, we estimate the realized R&D investments of subsidized projects as specified by equation (17), from which the SME dummy is excluded. The sample for the first stage consists of all firm-year observations, that for the second stage of those firms that obtain a positive subsidy and whose actual R&D decision we thus observe.

R&D participation. The firm's decision of whether to launch an R&D project is given by equation (9). The fixed cost of an R&D project is assumed to take the form

$$F_{it} := e^{\mathbf{X}_{it}^F \beta^F + \zeta_{it}}. \tag{18}$$

Substituting equation (18) into equation (9) and using some algebra, we can express the firm's participation constraint as an indicator function:

$$\mathbb{I}_{[0,\infty)} \left(\ln \frac{\hat{\alpha_{it}}}{1-\hat{\gamma_i}} \left[\hat{\gamma_{it}} \left(\frac{\hat{\alpha_{it}}}{\rho_t - s_{it}} \right)^{\frac{1-\hat{\gamma_{it}}}{\hat{\gamma_{it}}}} - 1 \right] - \ln \rho_t + \mathbf{X}_{it}^F \beta^F + \zeta_{it} \right), \tag{19}$$

where $\hat{\alpha}_{it} := \exp \hat{\gamma}_{it}(\mathbf{X}_{it}^R \hat{\beta}^R + \varepsilon_{it})$. Since $\hat{\gamma_{it}}$ and $\hat{\beta}^R$ are obtained from the estimation of equation (17), and since ρ_t, s_{it}, and $\mathbf{X}_{it}^F$ are observed, the vector of parameters to be estimated from equation (19) is β^F. We have identifying variation because the fixed cost is independent of the subsidy rate, but the subsidy rate affects the expected discounted profits gross of fixed cost. Estimation is done using simulated maximum likelihood (see Appendix B).

Agency decision. The agency's constrained optimal subsidy rate, $\hat{\hat{s}}$, can directly be obtained by plugging equation (18) together with the observed ρ_t and the parameters $\hat{\alpha}_{it}$, $\hat{\gamma_{it}}$, and $\hat{\beta}^F$ into equation (13).

To derive an estimable equation for the agency's unconstrained optimal

subsidy rate (12) we specify that

$$v_{it} := \mathbf{X}_{it}^{v}\beta^{v} + \eta_{it}, \tag{20}$$

where η_{it} is a random shock to spillovers from project i in year t. It is observed by the agency when it evaluates an application in stage 2, but it is unobserved by the econometrician and by the firm in stage 1 (for the investor, v_{it} is irrelevant). Substitution of equation (20) into equation (12) then gives

$$s_{it}^{**}[g - \hat{\gamma_{it}}(g-1)] = \mathbf{X}_{it}^{v}\beta^{v} - \rho_t\hat{\gamma_{it}}(g-1) + \eta_{it}, \tag{21}$$

To estimate equation (21), we assume that the shadow cost of public funds, g, is constant and takes value 1.2, and we only use those observations of the agency decisions where $s_{it} > \hat{\tilde{s}}_{it}$ because, according to our model, the agency decision is only then based on the interior solution. The agency decision rule is estimated by a generalized two-limit Tobit. This estimation provides us $\hat{\beta}^{v}$ (recall that ρ_t are $\hat{\gamma_{it}}$ are also known at this stage). The vector of observable firm and project characteristics $\mathbf{X}_{it}^{v}$ includes the SME dummy to accommodate the agency's priorities, and the agency's grades for each project. The estimation of the agency's grading of projects follows TTT (2013a) and is explained in Appendix B.

Note that our model allows spillovers and profits to be correlated: equations (10), (17), and (20) show how spillovers generated by project i, $v_{it}R_{it}$, are a function of both η_{it} and ε_{it}. The key identifying assumption is that while spillovers and profits are correlated,the shock to the *spillover rate* v_{it} (i.e., spillovers per euro of R&D) and the shock to profitability of R&D (ε_{it}) are uncorrelated. As a result, the agency decision rule is not subject to selection on unobservables.

Subsidy application. To be able to estimate the firm's subsidy application decision, characterized in section 3.5, we need to specify an empirical counterpart to the firm's application costs, K_{it}. We hence define that

$$K_{it} := e^{\mathbf{X}_{it}^{K}\beta^{K} + \mu_{it}}, \tag{22}$$

where μ_{it} is a random shock to the application costs, observed by the firm but unobserved by the econometrician (the observability of μ_{it} by the agency and the investor is irrelevant).

The firm's application decision is also estimated by simulated maximum likelihood. For each simulation draw, we numerically integrate the expected discounted profits from applying for subsidies (equation (15)) with equation (22) substituted for the costs of applying. We use all the parameters estimated in the prior stages of the estimation process, i.e., the parameters of the R&D investment function, the fixed cost of R&D, and the agency decision rule. To calculate the expected benefits from applying for a subsidy, we also need to take into account the way the agency grades each application it receives (see Appendix B). The exclusion restrictions are based on the agency decision rule being a function of the SME status of a firm, and the R&D investment being function of the subsidy rate, whereas neither the SME status nor the subsidy rate should affect the application cost.

Statistical assumptions. The unobservables (ε_{it}, ζ_{it}, η_{it}, μ_{it}) of the main estimation equations are assumed to be normally distributed with mean zero, and variances that we estimate, and uncorrelated with observed applicant characteristics. All this is assumed to be common knowledge.

We also assume that a) $\mu_{it} = \xi\varepsilon_{it} + \mu_{0it}$, where μ_{0it} is a random shock whose variance is normalized to unity; b) $\eta_{it}, \zeta_{it} \perp \varepsilon_{it}$; c) $\eta_{it}, \zeta_{it} \perp \mu_{0it}$ and d) $\eta_{it} \perp \zeta_{it}$. As assumption a) shows, the application cost shock, μ_{it}, and the shock to the expected profitability of R&D investments, ε_{it}, can be correlated with each other. This allows for the possibility that firms with higher profitability shocks have systematically different application costs than otherwise similar firms. The economic interpretation of assumption b) is discussed above: spillovers are correlated with the profitability shock ε_{it}, but the shock to the spillover rate η_{it} is uncorrelated with ε_{it}.

Assumptions a)-d) mean that the spillover rate shock η_{it} and the shock to fixed cost of R&D ζ_{it} are uncorrelated with the application cost shock μ_{it} and with each other. This rules out a selection problem for the subsidy rate equation (21), makes the subsidy rate s_{it} independent of the profitability shock ε_{it}, and renders the observability of μ_{it} inconsequential for the agency. Note that

assumptions b) and c) also imply that $\varepsilon_{it} \perp \mu_{0i}$. However, the assumptions introduce the selection problem for the R&D investment equation (17) that is discussed above. Under these assumptions, we can identify all the structural parameters of our model, including those governing the distribution of the shocks.

5 Estimation Results

We first discuss the estimated coefficients, and then turn to their implications.[24] Preliminary estimations of the R&D equation using $\gamma_{it} = \Phi(\mathbf{X}_{it}^{\gamma}\beta^{\gamma})$ suggested that $\gamma_{it} = 1$ for all i and t. This implies that firms' profits appear to be logarithmic in R&D as sometimes *assumed* in the literature (including TTT 2013a). We impose the constraint $\hat{\gamma_{it}} = 1$ since this yields considerable computational savings. The coefficient estimates from all main estimation equations are collected into Table 2.

R&D investment. Column 1 of Table 2 displays the estimated coefficients of the (log) R&D equation. These coefficients measure how firm characteristics affect the intensive margin of R&D, i.e., marginal profitability of R&D. Echoing the findings in Garcia-Macia et al. (2016), we find that firm age, size, and sales per employee all have an impact on R&D. Exporters invest more, as do firms who invested in the previous period. Firm location does not affect R&D investment. Column 1 of Table 2 also reveals that the estimated standard error of ε_{it} is 1.5, giving us insights into the distribution of shocks to the expected profitability of R&D project ideas. Year dummies suggest that Finnish firms invested more in the early and late 2000s than in the base year 2005, and that there is significant heterogeneity in marginal profitability of R&D across industries.

TABLE 2 HERE

Fixed cost of R&D. Column 2 of Table 2 suggests that the fixed cost of R&D is also a function of firm age, firm size and sales per employee. Each of these firm characteristics appear to work to the opposite direction at the extensive (fixed cost) and at the intensive margin (R&D investment level):

[24] Results of the estimation of the grading equations are in Appendix B.

the different polynomial terms of firm age, size, and sales per employee have the same signs in Columns 1 and 2. Export status does not affect the fixed cost. In line with the literature (Arqué-Castells and Mohnen 2015, Peters et al. 2017), having invested in R&D in the previous year greatly reduces the fixed costs. The omitted results suggest that fixed costs are higher in the first two years of our data and vary over industries.

Subsidy rate equation. Column 3 shows the estimated coefficients of the agency decision rule. Firm age, size and sales per employee all affect the optimal subsidy rate with again a similar sign-pattern. This suggests that Tekes evaluates those firms with higher private profitability of R&D to also have a higher spillover *rate*. Exporters obtain larger subsidies, but neither past R&D nor location of the applicant affects the subsidy rate. As explained, the subsidy rules allow SMEs to obtain up to 10 percentage points higher subsidy rates: our results suggest that SMEs receive on average 5 percentage points higher subsidy rates. Tekes internal grading variables only appear to play a minor role. According to the unreported coefficients, the awarded subsidy rates were lower in the early years of the millennium. We find no statistically significant differences across industries.

Application cost. In column 4 we find the familiar pattern of coefficient signs for firm age, size and sales per employee, but now the statistical significance levels are lower. Exporters face a higher application cost, as do firms that invested in R&D in the previous year. We find, as in TTT (2013a), that the shocks to application costs are positively correlated with the profitability shock, though the parameter is not statistically significant. A positive correlation implies that higher quality projects in terms of expected discounted profits have higher application cost.

Implications of the estimated coefficients. Using the estimated parameters we can simulate the fixed costs of R&D and subsidy application costs; see Table 3. The simulated mean fixed R&D cost is 112 000€ but the median is less than 19 000€, suggesting that the mean is driven by the long right tail. There is indeed a large amount of variation: the lowest decile of firms have fixed cost lower than 3 000€, and those in the highest quartile higher than 100 000€. The mean application cost may seem high at 84 000€,

but is similarly explained by the long right tail. In line with this, we find quite modest application costs at the lower end of the application cost distribution: in a given simulation round, 10% of firms have application costs that are lower than 6 000€, and 25% lower than 9 000€, but the median is already 26 000€. Long right tails of the fixed and application cost distributions are natural consequence of our data and model. Recall from section 2.2. that only some 60% of the firms invest in R&D and 20% of apply for subsidies. The main mechanisms in our model that explain why a firm does not invest in R&D or, in case it invests, why it does not apply for subsidies, are the fixed costs of R&D and costs of applying.

TABLE 3 HERE

6 Counterfactual Analysis

6.1 Policies

We use our model and empirical results to simulate four counterfactual policies: i) an optimal R&D tax credit policy; ii) a laissez-faire scenario without government interventions in firms' R&D investments; iii) the first-best policy where the social planner can force the firms to invest the desired amount in each project; and iv) the second-best (Ramsey) policy where the social planner is constrained by the firm's zero profit condition.

Optimal R&D tax credits. To analyze an optimal R&D tax credit, we make two modifications to our basic model: first, we set the subsidy rate s to zero. Second, we introduce a corporate tax rate $\tau \in [0,1]$, and a R&D tax credit rate $\tilde{\tau}_R \in [0,1]$. We assume that the R&D tax credit means that a firm investing R euros in R&D is reimbursed for $\tilde{\tau}_R R$ euros. It is more convenient to work with an "adjusted" tax credit rate $\tau_R := \tilde{\tau}_R/(1-\tau)$.

The way we model the tax credit policy is similar to the tax credit regime in Belgium, the Netherlands, and Norway, and the UK: in case the firm makes a loss, it is compensated directly by the same amount it would have saved

in taxes had it made a profit. To facilitate the comparison of the tax credit policy with the subsidy policy, we assume that all variable R&D costs but no fixed costs are subject to the tax credit. For brevity, we also assume that all firms that invest in R&D claim the R&D tax credit.[25]

Using these assumptions we show in Appendix C that the firm's optimal R&D investment rule with an R&D tax credit is equivalent to the one given by equations (7)-(9) with τ_R replacing s. Note that this implies that, as in basic text book models of corporate taxation and investment, the corporate tax rate τ has no effect on the R&D investment in our model.[26]

While subsidies and tax credits have identical impacts on the firms' objective functions, they crucially differ from the agency's point of view: unlike the subsidy, the tax credit is not project specific, and it is a treatment decision rule that usually is not conditioned on covariates (Manski 2001, 2004). The benefit is that access to treatment is not hindered by application costs.

To determine the optimal level of τ_R, we replace s by τ_R in the project specific agency objective function (10) and aggregate the resulting objective function over all projects (firms). We perform a grid search over the region $\tau_R \in [0, 1]$ with a step size of 0.01, and choose the value that maximizes agency welfare. We simulate the relevant shocks (i.e., all but the shock to application costs, ν_0) 100 times from their estimated distributions.

We find that the socially optimal τ_R is 0.33 (with a bootstrapped standard error of 0.02), which is slightly less than the mean subsidy rate of the successful applicants (0.36). In calculating the optimal tax credit the agency needs to take into account that some projects should not be subsidized at all. The agency however also takes into account that shocks to application costs and R&D profitability are positively correlated, i.e., that an average firm is likely to have a more profitable R&D project than an average firm that applies for subsidies. Our result suggests that the former consideration outweighs the latter, and hence the optimal (adjusted) R&D tax credit rate is lower than the mean optimal subsidy rate.

[25] In practice, eligible firms may fail to claim R&D tax credits, see e.g. Verhoeven, van Stel, and Timmermans, (2012) and Busom, Corchuelo, and Martínez-Ros (2014).

[26] It is well known that this neutrality of corporate taxation is sensitive to a number of issues. See Mukherjee et al. (2017) for a recent study of corporate taxation and innovation.

Laissez-faire, first and second best. In our laissez-faire scenario, there are neither R&D subsidies nor tax credits. In the first best scenario the (perfectly informed) agency chooses for each project the level of R&D investment. The agency thereby internalizes the spillovers and all the costs. We assume that R&D is financed at the same cost as private funding is provided. As the first best investment level may lead to negative profits for a firm, we also consider the second best policy where the agency chooses the optimal level of each R&D investment subject to the firms' zero profit constraint.

It is possible that laissez-faire generates higher welfare than the R&D subsidy policy, because the agency chooses the optimal subsidy rate for a project only conditional on receiving an application. Each application creates application costs, and the agency does not take into account the effects of its policy on the number and costs of applications. In other words, we assume that the agency's policy is discretionary without a possibility to commit to a subsidy rate rule.

6.2 Results

We compare our five different policy regimes (subsidies, tax credits, the first and the second best, and laissez-faire) in various dimensions. The reported means are calculated over all firms and simulation draws (see Appendix D for details), unless otherwise indicated. We report percentiles of firm-specific means.

Probability to invest in R&D. In Table 4 we report the firms' propensity to conduct R&D in various policy regimes. Under laissez-faire, 53% of firms invests in R&D in a given year. A quarter of the firms invest less than 13% of the time, the median investment probability over all firms is 72%, and one quarter of the firms invest at least 83% of the time. Neither subsidies nor tax credits induce a higher R&D participation rate than laissez-faire. These results are in line with Dechezlepêtre et al. (2016) and Peters et al. (2017) who find little effects of R&D tax credits at the extensive margin. The first best policy increases R&D participation only one percentage point from laissez-faire. Note that the differences across the regimes are somewhat larger than suggested by Table 4: for example, the first best includes (excludes) some projects generating positive (negative) welfare but negative (positive) profits which are excluded from (included in) the laissez-

faire outcome.

TABLE 4 HERE

R&D investment. Table 5 shows that, in contrast to the extensive margin, there are large differences across policy regimes at the intensive margin, again in line with Dechezlepêtre et al. (2016) and Peters et al. (2017). The mean R&D investment under laissez-faire, conditional on investing (left panel), is roughly 190 000€ per project over all simulation rounds. The mean investment under the first and the second best policies is more than two times higher. We report the unconditional means in the right panel: these allow us to compare the R&D investments generated in the economy by different policies taking both the extensive and intensive margins of R&D investments into account. Given that there are only small differences across policies in the probability to invest in R&D, the rankings and ratios in the right panel are close to those in the left panel.

R&D tax credit and subsidy policies induce clearly higher average R&D investments than laissez-faire but fall short of first and second best. The R&D tax credit generates a marginally higher mean investment than the subsidy regime (280 000€ versus 270 000€) since the tax incentive is given to all firms investing in R&D, whereas subsidies are only granted to those who successfully apply for them. However, the mean R&D investment of successful applicants (last row, lower panel) is substantially higher than investments under either laissez-faire or R&D tax credits, emphasizing the effectiveness of the ability to tailor the subsidy to each project.

The medians are clearly lower than the means, indicating that the R&D distribution is skewed to the right. To give an idea of the differences in the distribution of R&D, we plot the distribution from one simulation round of the counterfactual analysis across policy regimes in Figure 2. R&D support policies and first and second-best shift the R&D distribution to the right. Some differences are however not clearly visible from Figure 2: e.g., the difference in the size of R&D investments between the subsidy and laissez-faire regimes is increasing with the project size.[27]

[27] The mean 50th percentile for the subsidy regime over all simulation rounds is 69 000€ and that for laissez-faire 55 000€, a difference of 25%. The difference at the 90th percentile is 36%. The differences between laissez-faire and first and second best are even more strongly increasing in the percentile. In contrast, for the R&D tax credit the difference is 41-44% irrespective of where along the distribution

TABLE 5 & FIGURE 2 HERE

Profits. The counterfactual profit estimates are displayed in Table 6. Profit differences across policy regimes are much smaller than those in R&D investment because, as suggested by Table 4, almost half of the firms are not investing in R&D in any of the regimes and are hence unaffected by the policies. The mean expected discounted profits are almost identical under laissez-faire and the two R&D support policies. Profits in the first and second best regimes are lower, though not by much, because firms no longer invest at the profit-maximizing R&D levels.

TABLE 6 HERE

Spillovers. In Table 7 we report on spillovers. Because spillovers are the product of spillovers per euro of R&D times the amount of R&D, the ranking of the regimes in terms of spillovers and the ratio to laissez-faire follow the ranking of regimes in terms of R&D investments. Spillovers are much lower than firm profits in all regimes, ranging from 68 000€ under laissez-faire to 175 000€ under first best. The subsidy and R&D tax incentive regimes produce almost identical average spillovers. While both R&D support policies increase spillovers almost 50% compared to laissez-faire, they are less than 2/3 of the spillovers generated by the first and second best regimes.

TABLE 7 HERE

Welfare. The ultimate measure of different R&D support policies is their impact on welfare. Our welfare analysis compares counterfactual outcomes to what the Finnish government obtains through the current policy, as measured by our revealed preference approach to identify parameters of equation (10). We find (see Table 8) that all regimes are very close in terms of welfare. Although the first and second best policies substantially increase R&D investments and spillovers from laissez-faire, they lead to lower profits. Since spillovers only constitute a fraction of

of R&D investment one measures it.

profits, the welfare improvement in the first and second best regimes compared to laissez-faire is small (2%). This does not leave much room for any policy to increase welfare. Thus, while results in Tables 5-7 show how the two R&D support policies increase R&D investments, spillovers, and profits, results in Table 8 suggests that they do not improve welfare once the shadow costs of public funds are taken into account.

Note that our estimations of the welfare of the R&D support policies do not capture some relevant considerations. On the one hand, our welfare estimates are likely to be upward biased: although we take into account the firms' application costs, we ignore the agency's administrative costs. On the other hand, global welfare is likely to be understated because, e.g., a large part of consumer surplus and technological spillovers generated by the Finnish R&D projects is captured abroad but that part should not be included in the agency's objective function. The fact that we ignore firm's international R&D location decisions may also lead us to underestimate the benefits of support policies. In the case of R&D tax credits, we assume that all eligible firms use the R&D tax credit although evidence suggests that this is not the case. This leads to an upward bias in both benefits and costs of the R&D tax credit policy.

TABLE 8 HERE

Policy parameters. We have collected parameters of direct policy interest into Table 9. Across all simulations, on average 24% of firms apply for a subsidy and the average subsidy rate, conditional on getting a subsidy, is 39%. Both figures are very close to those in the data. As mentioned, we find that the optimal "adjusted" tax credit rate τ_R is 0.33. By using a corporate tax rate of $\tau = 0.26$ (which was the corporate tax credit rate in Finland in 2005-2011), this transforms to an optimal R&D tax credit rate of $\tilde{\tau}_R = \tau_R(1-\tau) = 0.24$. Turning to the fiscal costs of R&D support policies, we find that the mean subsidy, conditional on applying for one, is 81 000€ whereas the mean tax credit conditional on getting one (i.e., conditional on investing in R&D) is 92 000€. When we calculate these across all simulation draws (i.e. irrespective of whether a firm invests in R&D or applies for subsidies), the averages are 55 000€ for the subsidy regime and 60 000€ for the optimal tax

credit regime. The tax credit regime would thus be 9% (≈60/55 - 1) more expensive in terms of fiscal expenditure. One might have expected a larger difference given that all R&D investments get the tax credit, but only some 20% the subsidy. The explanation is that while the average subsidy rate is only somewhat higher than the tax credit, some (large) projects get high subsidy rates.

Robustness. We re-estimate our model and recalculate our counterfactual outcomes, first, using only data on subsidies instead of subsidies and subsidized loans, and second, excluding the three largest firms in the estimation sample. Appendix D contains details of these two robustness tests. We find that our results on R&D participation and welfare comparison of the regimes relative to laissez-faire are unchanged from Tables 4 and 8. The estimated levels of R&D investment and by extension, profits, spillovers and welfare, are somewhat lower when using only subsidies, and somewhat higher when excluding the three largest firms by employment. The former effect expected as we make the support regime less generous and thereby less attractive to the firms. The latter effect suggests that the three largest firms do not have particularly large and profitable R&D projects.

TABLE 9 HERE

7 Conclusions

Government support to private R&D has a solid basis in economic theory, and is widely used in numerous countries. A large empirical literature applies the tools of the treatment effect literature on both R&D tax credits and R&D subsidies, mostly but not exclusively studying the treatment effect of support on the level of private R&D (exceptions include Demeulemeester and Hottenrott 2015, Hünermund and Czarnitzki 2016, and Dechezleprêtre et al. 2016 who study effects on cost of debt, firm growth and patenting, respectively). Notwithstanding the insights of this literature, there are limits to how informative its results are for optimal policy design. The ultimate objective of policy evaluation should be welfare effects, yet this question is rarely addressed regarding R&D support policies.

This paper is an attempt to study of the welfare effects of innovation policies. Extending our earlier work we build and estimate a model of an innovation policy,

incorporating the main policy motivations, and conduct a counterfactual analysis of different R&D support policies. In a departure from most existing work, we utilize the variation in government R&D subsidy rate decisions to identify the parameters of the government's utility function. In our counterfactual exercise, we keep these parameters constant.

Our model yields theoretical results that concern both the regularly cited policy motivations and the interpretation of the R&D investment equation. Contrary to conventional wisdom, the effect of financial market imperfections on the level of optimal support delicately depends on the margin at which the support operates. At the intensive margin, an increase in financial market imperfections leads to a decrease in the optimal level of support, while at the extensive margin the conventional view of a positive relation is observed. As to the R&D investment equation, our model shows how the treatment parameter that is often the center of interest - the coefficient of the subsidy variable - is actually a function of the R&D production technology.

Complementing the findings of Garcia-Macia et al. (2016), we find that larger and more productive firms invest more. The firms that invest more at the intensive margin also have higher fixed costs of R&D. The agency takes firm characteristics into account in deciding the optimal subsidy rate and grants SMEs a higher subsidy rate. Costs of applying for subsidies are heterogeneous and greatly affect the effectiveness of a R&D subsidy policy.

In the counterfactual policy analysis the optimal R&D tax credit is 24%, which is lower than the average subsidy rate in our data (36%). R&D tax credits and R&D subsidies yield significantly higher R&D investment than laissez-faire, but do not increase R&D participation. First and second best R&D levels are twice as large as under laissez-faire. The same ranking applies to spillovers, but profits are roughly constant over policies. Profits are an order of magnitude larger than spillovers; an explanation for this is that the Finnish agency only takes into account spillovers to Finland, and these are most likely a small fraction of total spillovers. Because profits dominate in welfare calculus, differences in welfare are minor despite the large differences in R&D investment and spillovers: first and second best yield only 2% more welfare than laissez-faire. Given this space for welfare improvements, it is not surprising that the R&D tax credit and subsidy policies fail to improve welfare despite increasing R&D and spillovers by more than 40%.

References

Acemoglu, D., U. Akcigit, N. Bloom and W. Kerr, 2013, Innovation, reallocation and growth. NBER Working Paper No. 18993.

Akcigit, U., D. Hanley, and S. Stantcheva, 2017, Optimal taxation and R&D policies. NBER Working Paper No. 22908.

Akcigit, U., and W. Kerr, 2016, Growth through heterogenous innovations. *Journal of Political Economy*, forthcoming.

Arqué-Castells, P. and P. Mohnen, 2015, Sunk costs, extensive R&D subsidies and permanent inducement effects. *Journal of Industrial Economics*, LXIII, pp. 458-494.

Arrow, K.J., 1962, Economic welfare and the allocation of resources for invention. In Nelson, R. R. (ed.), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, pp. 609-625. Princeton, N.J.: Princeton University Press.

Bloom, N., M. Schankerman, and J. Van Reenen, 2013, Identifying technology spillovers and product market rivalry. *Econometrica,* 81, pp. 1347 - 1393.

Boller, E. A., A. Moxnes and K. Ulltveit-Moe, 2015, R&D international sourcing, and the joint impact on firm performance. *American Economic Review,* 105, 12, pp. 3704-3739.

Bronzini, R. and E. Iachini, 2014, Are incentives for R&D effective? Evidence from a regression discontinuity approach. *American Economic Journal: Economic Policy*, 6, pp. 100-134.

Busom, I., B. Corchuelo, and E. Martínez-Ros, 2012, Tax incentives....or subsidies for R&D. *Small Business Economics*, 43, pp. 571-596.

Cerulli, G., 2010, Modelling and measuring the effect of public subsidies on business R&D: a critical review of the economic literature. *Economic Record,* 86, pp. 421-449.

Dechezleprêtre, A., E. Einiö, R. Martin, K.-T. Nguyen and J. van Reenen, 2016, Do tax incentives for research increase firm innovation? An RD design for R&D. NBER Working Paper No. 22405.

Demeulemeester, S, and Hottenrott, H., 2015, R&D subsidies and firms' cost of debt, KU Leuven, Faculty of Economics and Business, Working Papers MSI_1511.

Doraszelski, U. and J. Jaumandreu, 2013, R&D and productivity: estimating endogenous productivity. *Review of Economic Studies*, 80, pp. 1338 - 1383.

Einiö, E., 2014, R&D subsidies and company performance: evidence from geographic variation in government funding based on the ERDF population-density rule, *Review of Economics and Statistics*, 96, pp. 710-728.

Garcia-Macia, D., C.-T. Hsieh, and P. Klenow, 2016, How destructive is innovation? NBER Working Papers No. 22953.

Garciá-Quevedo, J., 2004, Do public subsidies complement business R&D? A meta-analysis of the econometric evidence. *KYKLOS,* 57, pp. 87–102.

González, X., J. Jaumandreu and C. Pazó, 2005, Barriers to innovation and subsidy effectiveness. *RAND Journal of Economics,* 36, pp. 930-950.

Hall, B. and J. Lerner, 2010, The financing of R&D and innovation. In Hall, B. and N. Rosenberg (eds) *Handbook of the Economics of Innovation*, pp 609-639. Netherlands: Elsevier.

Holmström, B. and J. Tirole, 1997, Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics,* 112, pp. 663-982.

Howell, S., 2017, Financing innovation: evidence from R&D grants. *American Economic Review,* 107, pp. 1136-1164

Hünermund, P. and D. Czarnitzki, 2016, Estimating the local average treatment effect of R&D subsidies in a Pan-European program. ZEW Discussion Paper No. 16-039.

Inderst, R. and H. Mueller, 2006, Informed lending and security design. *Journal of Finance*, 61, pp. 2137–2162.

Kerr, W. and R. Nanda, 2015, Financing innovation. *Annual Review of Financial Economics*, 7, pp. 445-462

Lach, S., Z. Neeman and M. Schankerman, 2015, Government funding for R&D start-ups. Mimeo.

Manski, C., 2001, Designing programs for heterogeneous populations: the value of covariate information. *American Economic Review, Papers and Proceedings*, 91, pp. 103 - 106.

Manski, C., 2004, Statistical treatment rules for heterogeneous populations. *Econometrica*, 72, pp. 1221 - 1246.

McFadden, D., 1989, A method of simulated moments for estimation of discrete response models without numerical integration. *Econometrica*, 57, pp. 995-1026.

Meh, C. and K. Moran, 2010, The role of bank capital in the propagation of shocks. *Journal of Economic Dynamics and Control*, 34, pp. 555-576,

Mukherjee, A., Singh, M., and A. Zaldokas, 2017, Do corporate taxes hinder innovation? *Journal of Financial Economics*, 124, pp. 195-221.

Mohnen, P. and B. Lokshin, 2010, What does it take for an R&D tax incentive policy to be effective? In Ghosal, V. (ed), Reforming Rules and Regulations, pp. 33-58. Cambridge, MA: MIT Press.

Nelson, R.R., (1959), The simple economics of basic scientific research. *Journal of Political Economy*, 49, pp. 297-306.

OECD, 2011, OECD Science, Technology and Industry Scoreboard 2011. OECD Publishing.

Peters, B., M. Roberts, V. A. Vuong, and H. Fryges, 2017, Estimating the dynamic R&D choice: an analysis of costs and long-run benefits. *RAND Journal of Economics*, 48, pp. 409-437.

Stern, S., 1997, Simulation-based estimation. *Journal of Economic Literature*, 35, pp. 2006-2039.

Takalo, T. and T. Tanayama, 2010, Adverse selection and financing of innovations: is there need for R&D subsidies? *Journal of Technology Transfer,* 35, pp. 16-41.

Takalo, T., T. Tanayama, and O. Toivanen, 2013a, Estimating the benefits of targeted R&D subsidies. *Review of Economics and Statistics,* 95, pp. 255-272.

Takalo, T., T. Tanayama, and O. Toivanen, 2013b, Market failures and the additionality effects of public support to private R&D: theory and empirical implications. *International Journal of Industrial Organization,* 31, pp. 634-642.

Tekes, 2008, Annual report.

Tekes, 2011, Annual report.

Trajtenberg, M., 2001, Innovation in Israel 1968-1997: a comparative analysis using patent data. *Research Policy*, 30, pp. 363-389.

van der Veen, G., E. Arnold, P. Boekholt, J. Deuten, A. Horvarth, P. Stern and J. Stroyan, 2012, Evaluation of Tekes, Final Report. Publications of the Ministry of Employment and the Economy, Innovation 22/2012.

Verhoeven, W., A. van Stel, and N. Timmermans, 2012, Evaluatie WBSO 2006-2010: effecten, doelgroepbereik en uitvoering. https://www.rijksoverheid.nl/

documenten/rapporten/2012/04/02/hoofdrapport-evaluatie-wbso-2006-2010

Winton, A., 1993, Limitation of liability and the ownership structure of the firm. *Journal of Finance,* 48, pp. 487-512,

Xu, D., 2008, A structural empirical model of R&D, firm heterogeneity and industry evolution. Mimeo.

Zúñica-Vicente, J.A., C. Alonso-Borrego, F. Forcadell, and J. Galan, 2014, Assessing the effect of public subsidies on firm R&D investment: a survey. *Journal of Economic Surveys,* 28, pp. 36-67.

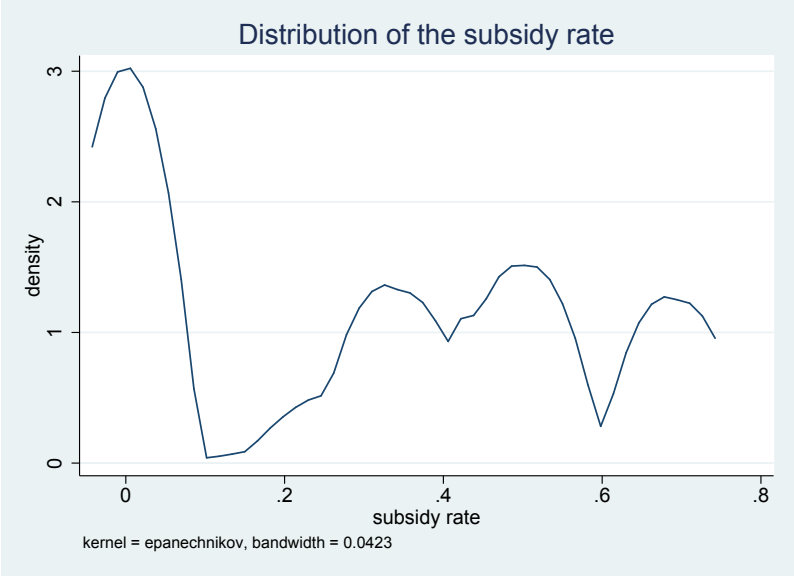


Figure 1. Distribution of the subsidy rate

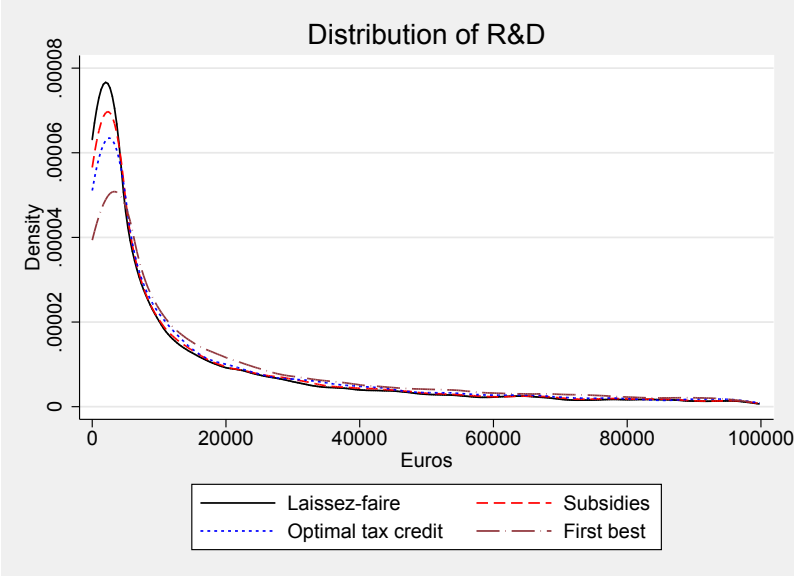


Figure 2. Distribution of R&D across policy regimes.

Table 1. Descriptive statistics

	Non-applicants			Applicants			Rejected applicants			Successful applicants		
	mean	s.d.	p50	mean	s.d.	p50	mean	s.d.	p50	mean	s.d.	p50
subsidy rate	-	-	-	0.30	0.27	0.35	0	-	-	0.36	0.25	0.35
R&D, realized	-	-	-	392 902	824 671	151 965	17 511	148 268	0.00	468 932	867 083	201 531
tech				2.07	0.79	2	1.69	0.78	1	2.13	0.77	2
risk				2.33	0.81	2	2.29	0.92	3	2.34	0.79	2
prev applicant	0.15	0.35	0	0.23	0.42	0	0.23	0.42	0	0.24	0.42	0
$1[R\&D]_{t-1}$	0.55	0.50	1.00	0.84	0.37	1.00	0.82	0.38	1.00	0.84	0.37	1.00
SME	0.70	0.46	1.00	0.73	0.44	1.00	0.72	0.45	1.00	0.74	0.44	1.00
age	16.74	15.65	13.00	12.62	13.21	9.00	13.02	13.26	9.00	12.52	13.20	9.00
#empl.	106.60	262.49	33.00	176.45	612.94	18.00	164.40	494.87	23.90	156.57	510.80	17
sales/empl.	0.27	0.39	0.14	0.21	0.36	0.11	0.23	0.41	0.11	0.21	0.35	0.11
region	0.17	0.38	0.00	0.20	0.40	0.00	0.20	0.40	0.00	0.20	0.40	0.00
interest	0.06	0.01	0.06	0.06	0.01	0.06	0.06	0.01	0.06	0.06	0.01	0.06
#Obs.	19 718			5 787			940			4 847		

NOTES: *subsidy rate* is the fraction of R&D paid by the government. *R&D* is the actual R&D investment in the project, measured in 2005 euros.

tech is the technological challenge of a project as evaluated by the agency, on a 1-4 Likert scale. 1 = no or small risk; 2 = risk; 3 = high risk; 4 = very high or unbearable risk.

risk is the marketing risk of a project as evaluated by the agency, on a 1-3 Likert scale. 1 = no or small risk; 2 = risk; 3 = high risk to unbearable risk.

The number of observations for *tech* (*risk*) is 2825 (2852) for all applicants, 407 (406) for unsuccessful applicants, and 2418 (2446) for successful applicants.

prev applicant takes value 1 if the firm applied for a subsidy in year $t-1$and 0 otherwise. $1[R\&D]_{t-1}$ takes value 1 if the firm invested in R&D in year $t-1$and 0 otherwise.

SME takes value 1 if the firm in year t is an SME according to the EU - guidelines, and zero otherwise. *age* is the age of the firm in year t in years;

region takes value 1 if the firm is located in a region eligible for EU regional aid and 0 otherwise.*interest* is the ECB interest rate for non-financial corporations.

Loans other than revolving loans and overdrafts, credit card debt. See http://sdw.ecb.europa.eu/browse.do?node=9613587

All differences between non-applicants and applicants significant at 5% level.

Only the differences in interest is significant between successful and rejected applicants. Observations at firm-year level.

Table 2. Coefficient estimates

	R&D investment	R&D	subsidy rate	application
$\ln age$	1.5116	2.7450***	0.0699*	0.4686
	(1.0678)	(0.8405)	(0.0360)	(0.5067)
$\ln age2$	-0.8381	-1.3669***	-0.0272*	-0.2760
	(0.5313)	(0.4142)	(0.0158)	(0.2232)
$\ln age3$	0.1157*	0.1870***	0.0032	0.0377
	(0.0710)	(0.0567)	(0.0021)	(0.0293)
$\ln emp$	0.6373***	0.5180***	0.0224**	0.6146***
	(0.1488)	(0.1588)	(0.0094)	(0.2163)
$\ln emp2$	-0.1410	-0.0586	-0.0103***	-0.0897
	(0.0950)	(0.0775)	(0.0027)	(0.0615)
$\ln emp3$	0.0168	0.0029	0.0009***	0.0092
	(0.0116)	(0.0104)	(0.0002)	(0.0057)
$sales/emp$	-3.9275**	-5.2901***	-0.1318**	-2.3490**
	(1.957)	(1.5108)	(0.0599)	(1.1597)
$sales/emp2$	7.1329***	8.9839***	0.1673	5.5431***
	(2.2319)	(2.2119)	(0.1028)	(1.5866)
$sales/emp3$	-2.7292***	-3.3343***	-0.0481	-2.2962***
	(0.6897)	(0.8225)	(0.0411)	(0.6154)
$exporter$	0.2910**	-0.3180	0.0145*	0.4501*
	(0.1506)	(0.2935)	(0.0078)	(0.2456)
$region$	0.0133	0.0855	0.0024	-0.1906
	(0.1116)	(0.3480)	(0.0071)	(0.3223)
RD_{t-1}	1.5687**	-3.4478*	-0.0052	2.5080
	(0.6703)	(1.9627)	(0.0081)	(1.5893)
#Obs.	3 530	25 172	1 615	25 172

NOTES: standard errors (in parentheses) are bootstrapped (201 rounds) for the $R\&D$ investment, RD participation and application cost equations, and asymptotic and robust for the subsidy rate equation..

*** p<0.01, ** p<0.05, * p<0.1.

Table 2. Coefficient estimates

	R&D investment	R&D	subsidy rate	application
SME			0.0444***	
			(0.0110)	
risk			0.0061*	
			(0.0035)	
tech			0.0073*	
			(0.0040)	
prev applicant				-0.4279***
				0.2426
Constant	3.4505*	11.3482***	1.2326	3.6039
	(3.5939)	(3.4306)	(9.0591)	(4.6099)
σ_ε	1.4965***			
	(0.8219)			
σ_η			0.0993***	
			(0.0019)	
ξ				1.3524
				(4.5920)
#Obs.	3 530	25 172	1 615	25 172
Year dummies	YES	YES	YES	YES
Industry dummies	YES	YES	YES	YES

NOTES: standard errors (in parentheses) are bootstrapped (201 rounds) for the *R&D* investment, *RD* participation and application cost equations, and asymptotic and robust for the subsidy rate equation.

*** p<0.01, ** p<0.05, * p<0.1

Table 3. Fixed cost of R&D and cost of application

	mean	s.d.	p10	p25	median	p75
F	402101	3272480	737	1310	3858	148690
K	45726	38328	5320	8457	17420	63027

NOTES: The cost figures are from the counterfactual simulations.

Percentiles are calculated over firm averages.

Table 4. R&D participation

Regime	Mean	s.d.	p25	median	p75	ratio
Laissez-faire	0.53	0.35	0.13	0.72	0.83	1.00
1^{st} best	0.54	0.35	0.14	0.74	0.85	1.02
2^{nd} best	0.53	0.35	0.13	0.72	0.83	1.00
τ_R	0.54	0.35	0.13	0.73	0.84	1.02
s	0.54	0.35	0.13	0.73	0.84	1.02

NOTES: the figures are calculated over all simulation rounds and firms.

ratio = mean for the regime in question divided by the laissez-faire mean.

Table 5 R&D investment

Simulation rounds conditional on $R > 0$							All simulation rounds					
Regime	Mean	s.d.	p25	median	p75	ratio	Mean	s.d.	p25	median	p75	ratio
Laissez-faire	192988	641245	44114	85724	176517	1.00	125453	606540	9302	38034	101084	1.00
1^{st} best	478807	1609925	112019	214185	415239	2.48	291540	1294571	24275	97776	256166	2.32
2^{nd} best	428582	1368817	111464	213234	409252	2.22	278535	1274400	22412	94507	248976	2.22
τ_R	278115	928760	63320	123513	254498	1.44	182246	879938	13641	55453	147155	1.45
s	268435	929107	53317	113636	245797	1.39	177487	873228	10421	50829	148085	1.41
$s \vert s > 0 \,\&\, R\&D > 0$	437374	1063471	58751	195134	481516	2.27						

NOTES: the figures are calculated over over simulation rounds and firms with $R > 0$ (left panel), or all simulation rounds and firms (right panel).

ratio = ratio between the mean for the regime in question and the laissez-faire mean.

$s \vert s > 0 \,\&\, R\&D > 0$ shows the avg. R&D investment from the subsidy regime, conditional on a firm receiving a strictly positive subsidy.

Table 6. Profit

Regime	Mean	s.d.	p25	median	p75	ratio
Laissez-faire	1829289	10999334	80441	429784	1280492	1.00
1^{st} best	1755193	10770476	68918	392743	1202067	0.96
2^{nd} best	1766540	10769543	74657	402695	1214608	0.97
τ_R	1878900	11238477	84347	444939	1321620	1.03
s	1859831	11214498	80705	431689	1299060	1.02

NOTES: The figures are calculated over all simulation rounds and firms.

ratio = ratio between the mean for the regime in question and the laissez-faire mean.

Table 7. Spillovers

Regime	Mean	s.d.	p25	median	p75	ratio
Laissez-faire	68388	316449	5624	23120	60568	1.00
1^{st} best	175686	772275	14966	60832	158188	2.57
2^{nd} best	162800	735990	13757	58254	153963	2.38
τ_R	99376	459200	8233	33707	88229	1.45
s	100314	476422	6272	30873	89940	1.47

NOTES: the figures are over all simulation rounds and firms.

ratio = ratio between the mean for the regime in question and the laissez-faire mean.

Table 8. Welfare

Regime	Mean	s.d.	p25	median	p75	ratio
Laissez-faire	1897677	11313050	86287	452573	1342183	1.00
1^{st} best	1930879	11444876	89895	466206	1375898	1.02
2^{nd} best	1929340	11444712	88958	465711	1373001	1.02
τ_R	1906106	11346032	87216	456121	1350427	1.00
s	1894054	11344328	85699	447637	1330920	1.00

NOTES: The figures are calculated over all simulation rounds and firms.

ratio = ratio between the mean for the regime in question and the laissez-faire mean.

Table 9. Counterfactual estimates

variable	mean
Pr[$apply$]	0.24
$subsidy\ rate \vert s > 0$	0.39
τ_R	0.33
government cost, $s \vert s > 0$ & $R\&D > 0$	80563
government cost, $\tau_R \vert R\&D > 0$	91778
government cost, s	55076
government cost, τ_R	60141

NOTES: the figures are calculated over all simulation rounds and firms unless stated otherwise.

Pr[$apply$] is the average probability to apply for a subsidy. $subsidy\ rate \vert s > 0$ is the average subsidy rate conditional on it being strictly positive. τ_R is the optimal tax credit.

government cost, $s \vert s > 0$ & $R\&D > 0$ is the avg. euro-cost to the government from those projects it subsidizes.

government cost, s is the average cost of subsidies over all firms and simulation rounds.

government cost, $\tau_R \vert R\&D > 0$ and government cost, τ_R are defined similarly for R&D tax credits.

Appendix

Appendix A: figures

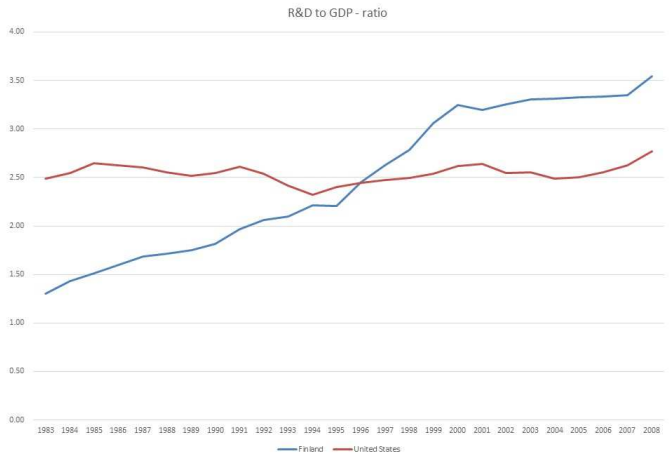
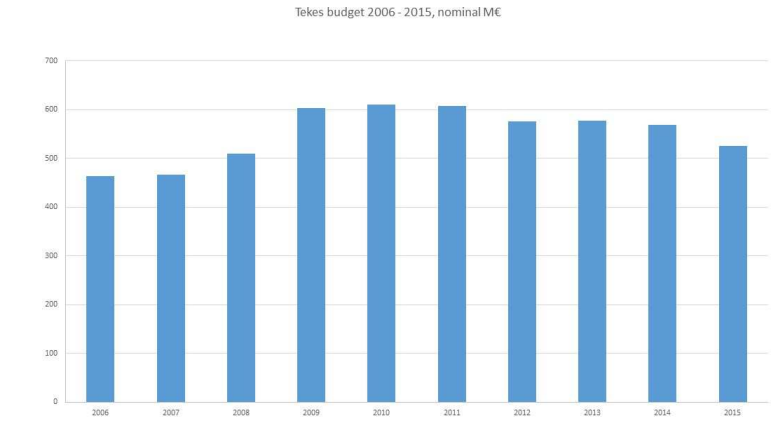


Figure A1. R&D/GDP-ratio, Finland and the US. Source: OECD Main Science and Technology Indicators.



Source: https://www.tekes.fi/globalassets/global/tekes/.../tekesin_organisaatio.pptx

Figure A2. Tekes budget 2006 - 2015.

Appendix B: further descriptive statistics and estimation details

Estimation sample

We first drop those observations where sales are negative (8 observations). We then exclude those firms for which we don't observe age at any point (8 453 obs.); we further drop those firm-year observations for whom we don't observe employment in the year in question, or in either of the adjacent years (307 obs.): in case employment is observed in adjacent years but not in the year in question, we substitute primarily the employment level in the previous, and secondarily the employment level in the following year. We exclude from the estimations outliers as follows: we first exclude all observations in the top 0.01% of the size (#employees) distribution (405 obs.); second, we drop any remaining observations in the top 0.01% of the age distribution (197 obs.). We then drop all those 18 158 firm-year observations for which we don't observe the R&D expenditure; these come from firms not included in the R&D survey of Statistics Finland. Finally, we drop those 7 910 firm-year observation for which we don't observe the firm's R&D-status in the previous year.

According to the Statistics Finland www-site,[28] statistics on research and development are based on the European Union's Regulations (Decision No 1608/2003/EC of the European Parliament and of the Council and Commission Implementing Regulation No 995/2012) . The inquiry includes enterprises in different fields having reported R&D activities in the previous inquiry, enterprises having received product development funding from the Finnish Funding Agency for Technology and Innovation Tekes and the Finnish Innovation Fund Sitra, and all enterprises with more than 100 employees and a sample of enterprises with 10 to 99 employees. We experimented with using weights that correct for the sampling frame. As these had no material impact on the estimations but increased the computation time significantly, we do not use weights in the reported estimations.

Number of observations per firm

Table B1 shows the distribution of the number of observations per firm in our estimation sample.

[28] See http://tilastokeskus.fi/keruu/yrtk/index_en.html, accessed June 17, 2017).

Table B1. Distribution of #obs / firm

#obs	#firm-year obs.	%	Cum. %
1	2 902	11.38	11.38
2	3 456	13.55	24.93
3	3 357	13.16	38.09
4	2 848	11.17	49.26
5	2 780	10.9	60.16
6	2 238	8.77	68.93
7	2 170	8.51	77.44
8	1 704	6.68	84.12
9	4 050	15.88	100
Total	25 505	100	

Descriptive statistics on number of applications

Table B2 reports the distribution of the number of applications by firm across our estimation sample. Table B3 shows the distribution of the number of applications per in a given year.

Table B2. Distribution of #applications / firm

#applications	#firms	%	Cum. %
0	5 209	62.29	
1	1 799	21.51	83.80
2	726	6.68	2.48
3	313	3.74	96.22
4	146	1.75	97.97
5	79	0.94	98.91
6	45	0.54	99.45
7	26	0.31	99.76
8	14	0.17	99.93
9	6	0.07	100
Total #firms	8 363	100	

Table B3. Distribution of #applications/ year

year	#applications
2000	589
2001	660
2002	570
2003	652
2004	649
2005	655
2006	698
2007	675
2008	639
Total # applications	5 787

Agency's grading and grading equations

Upon receiving an application the agency grades it in two dimensions, technological challenge and market risk, by using a 5-point Likert scale. The agency has six grades but uses only five of them in practice. A loose translation of the six grades of technological challenge is 0 = "no technological challenge", 1 = "technological novelty only for the applicant", 2 = "technological novelty for the network or the region", 3 = "national state-of-the-art", 4 = "demanding international level", and 5 = "international state-of-the-art". For market risk, it is 0 = "no identifiable risk", 1 = "small risk", 2 = "considerable risk", 3 = "big risk", 4 = "very big risk", and 5 = "unbearable risk". As explained in the main text, we group some grades as follows: grades 0 and 1 on the one hand, and grades 3, 4 and 5 on the other hand. Table B4 displays the original and the augmented grades' distribution.

Using the process described in TTT (2013a, see in particular equation (9)), we estimate the two grading rules by using ordered probits. The dependent variables are the grades, and the explanatory variables are firm characteristics. The unobservables of the two grading equations are assumed to be normally distributed and uncorrelated with each other, and with the four unobservables (ε_{it}, ζ_{it}, η_{it}, μ_{0it}) of the main equations. This estimation provides us with two vectors of parameters that are used to generate a firm's prediction on how the agency would grade its application in the two grading dimensions, if the firm applied for a subsidy. Estimation is by maximum likelihood. The results are presented in Table B5. We use the thus generated probabilities for calculating the expected discounted profits from applying for a subsidy (see below for more detail).

Table B4. Distribution of agency grades

	tech		risk	
grade	original	augmented	original	augmented
0	0.67		0.84	
1	27.08	27.75	18.02	18.86
2	37.59	37.59	31.35	31.35
3	33.10	34.65	47.34	47.34
4	1.56		2.38	
5	0		0.07	
#Obs.	2 825		2 852	

NOTES: numbers given are the %

of observations with a particular grade.

Table B5. Tekes grading rule estimation and first state of the R&D sample selection model

	tech	risk
$\ln age$	0.6669**	0.2599
	(0.3078)	(0.2977)
$\ln age2$	-0.2878**	-0.1165
	(0.1376)	(0.1336)
$\ln age3$	0.0399**	0.0134
	(0.0189)	(0.0184)
$\ln emp$	0.1049	-0.0706
	(0.0700)	(0.0672)
$\ln emp2$	-0.0167	-0.0002
	(0.0210)	(0.0200)
$\ln emp3$	0.0014	0.0002
	(0.0018)	(0.0017)
$sales/emp$	-1.2446**	-2.3677***
	(0.5101)	(0.4887)
$sales/emp2$	2.2645**	2.3539***
	(0.8842)	(0.8352)
$sales/emp3$	-0.8063**	-0.5704*
	(0.3504)	(0.3314)
$exporter$	0.1647**	0.0244
	(0.0645)	(0.0636)
$region$	0.0102	-0.0053
	(0.0572)	(0.0561)
RD_{t-1}	0.2411***	0.1425**
	(0.0670)	(0.0670)
Observations	2 800	2 826
Year dummies	YES	YES
Industry dummies	YES	YES

NOTES: asymptotic robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Simulation for estimation

We use the simulation estimator for discrete choice introduced by McFadden (1989); see also Stern (1997). We simulate the profitability shock of the firm (ε_{it}) both for the R&D participation and the subsidy application decisions. We use 40 simulation rounds and draw the shocks using Halton sequences. The draws are the same for all estimation equations.

Expected profits from applying for subsidies

To estimate the firm's application decision, we need to deal with both agency grading and the stochastic component of agency utility, η_{it}. These are all unknown to the firm contemplating application. Our assumption is that the firm knows the probabilities of obtaining particular grades for *tech* and *risk*, and the distribution of η_{it}. We therefore calculate for each firm and each simulation draw the expected discounted profits from obtaining a particular grade combination, integrating over the distribution of η_{it}. These profits are then weighted by the probability of getting a particular grade combination; we obtain these probabilities from the ancillary (ordered probit) grading equations. For numerical integration we use Simpson's method. The integration is repeated separately for each simulation round and each iteration.

Bootstrap

We bootstrap the whole estimation process and the generation of the optimal tax credit. To speed up computation, we limit the number of Newton-Raphson iterations to 5 for the R&D investment, R&D participation and application equations, while using the estimated coefficients as starting values. We restrict the number of iterations to 150 for the agency decision rule. We further restrict the number of simulation rounds for the calculation of the optimal tax credit to 50 (100 in the estimation), and restrict the support of the grid search to be [20,50] (in the estimation [0,100]). The grid step is kept at 1 (percentage point). For the calculation of the optimal tax credit, we restrict the number of simulation rounds to 50 (we use 100 rounds in the estimation).

Appendix C: details and proofs of the theoretical model

Derivation of the firm's R&D investment rule of section 3.3

As we are seeking a contract which maximizes the firm's payoff and the financial market is competitive, the constraint (5) may be written as an equality. As a result, the investor's share of project return can be written by using $\rho := r + c$ as

$$\pi^I = \frac{\rho\,(R+F) - sR}{P}. \tag{23}$$

By using equation (2) we can rewrite the firm's objective function from equation (3) as $\Pi^E = P\left(\pi - \pi^I\right)$. Substitution of equations (1) and (23) for π and π^I by using $\alpha := AP$ then gives

$$\Pi^E(R,s) = \alpha\left(\frac{R^{1-\gamma}-1}{1-\gamma}\right) - (\rho - s)\,R - \rho F. \tag{24}$$

This equation specifying the firm's objective function shows how $\rho - s$ captures the marginal cost of R&D to the firm. Maximizing equation (24) with respect to R gives equation (8) of the main text.

Next, after substituting equation (8) for (24), we can rewrite the constraint (4) as equation (9) of the main text. The firm's optimal R&D investment decision rule given by equations (7)-(9) then follows.

Derivation of the agency's subsidy decision rule of section 3.4

The agency seeks to grant optimal subsidies given its objective function (10). As the investor's participation constraint (5) is binding, we can write the agency's problem in stage two of the game as

$$\max_{s\in[0,\overline{s}]} U\left(R^*(s),s\right) = vR^*\left(s\right) + \Pi^E\left(R^*\left(s\right),s\right) - gsR^*\left(s\right), \tag{25}$$

subject to equations (8), (9), and (11).

To characterize the optimal agency decision, we first ignore all constraints to the agency's problem (25). Using the envelope theorem and equations (8) and (24), we can write the first-order condition for the agency's unconstrained problem (25) as equation (12) of the main text.

If the firm's participation constraint (9) is not satisfied at $s = s^{**}$, the agency needs to decide between a higher subsidy rate and no support. Letting equation (9) hold as an equality and solving for s gives equation (13) of the main text. This is the optimal subsidy rate if the firm's participation constraint (9) is not satisfied at $s = s^{**}$ if it also satisfies the agency's participation constraint (11); as the agency's objective function is concave in s (see the proof of Proposition 1), it is sub-optimal to give any larger subsidy.

When $s = 0$, equation (9) holds if

$$F \leq \tilde{F} := \frac{\alpha}{\rho(1-\gamma)} \left[\gamma \left(\frac{\alpha}{\rho} \right)^{\frac{1-\gamma}{\gamma}} - 1 \right]. \tag{26}$$

In words, if equation (26) holds, the firm's participation constraint never binds, i.e., fixed costs are so small that they affect neither the agency's nor the firm's decisions. Similarly, letting $s = \bar{s}$ in equation (9) implies that if

$$F > \bar{F} := \frac{\alpha}{\rho(1-\gamma)} \left[\gamma \left(\frac{\alpha}{\rho - \bar{s}} \right)^{\frac{1-\gamma}{\gamma}} - 1 \right], \tag{27}$$

fixed costs prevent investment even with a maximum subsidy rate $\bar{s}$. It is immediate from equations (26) and (27) that $\tilde{F} < \bar{F}$.

If $F \in \left(\tilde{F}, \bar{F} \right]$, the firm will invest only if it receives a subsidy. Now awarding $\tilde{s}$ as given by equation (13) is an option to the agency. Since $\tilde{s}$ is independent of v but $s^{**}(v)$ is strictly increasing in v, there exists a unique value of v, denoted by $\tilde{v}$, such that $s^{**}(\tilde{v}) = \tilde{s}$. Equations (12) and (13) then yield

$$\tilde{v} := \rho g - \left\{ \alpha^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{\alpha + \rho F(1-\gamma)} \right]^{\frac{\gamma}{1-\gamma}} \right\} [g - \gamma(g-1)]. \tag{28}$$

Because $s^{**}(v)$ is strictly increasing, the firm's participation constraint remains irrelevant for the agency for sufficiently high spillover rates, $v \geq \tilde{v}$. Thus, only if $v < \tilde{v}$, the agency may award subsidy $\tilde{s}$ that just satisfies the firm's participation constraint. This requires that the agency's participation constraint (11) holds at $s = \tilde{s}$. Since the investor's participation constraint (5) is binding and since the firm's participation constraint (9) is also biding at $s = \tilde{s}$ by definition, we observe from equation (10) that $U(R^{**}(\tilde{s}), \tilde{s}) = vR^{**}(\tilde{s}) - g\tilde{s}R^{**}(\tilde{s})$. As a result, $U(R^{**}(\tilde{s}), \tilde{s}) \geq 0$ if $v - g\tilde{s} \geq 0$. Inserting $\tilde{s}$ from equation (13) into $v - g\tilde{s} \geq 0$ yields $v \geq v^0$ where

$$v^0 := g \left\{ \rho - \alpha^{\frac{1}{1-\gamma}} \left[\frac{\gamma}{\alpha + \rho F(1-\gamma)} \right]^{\frac{\gamma}{1-\gamma}} \right\}. \tag{29}$$

Using equations (28) and (29) we can show that $v^0 \leq \tilde{v}$, with the inequality being strict for $\gamma > 0$. As a result, $s^*(v) = \tilde{s}$ constitutes the optimal agency decision for $v \in \left[v^0, \tilde{v} \right)$. If $v < v^0$, the agency's and firm's participation constraints cannot be satisfied for any positive subsidy rate, rendering a zero subsidy rate optimal. Since $\tilde{v} \leq \bar{v}$, we can summarize the agency's optimal decision rule for $F \in \left(\tilde{F}, \bar{F} \right]$ as follows: $s^*(v) = 0$ if $v < v^0$, $s^*(v) = \tilde{s}$, if $v \in \left[v^0, \tilde{v} \right)$, $s^*(v) = s^{**}(v)$ if $v \in [\tilde{v}, \bar{v})$, and $s^*(v) = \bar{s}$ if $v \geq \bar{v}$.

Finally, note the following complication to the optimal subsidy rule, ignored in the main text for brevity: from equations (26) and (27) we observe that it is possible that $\tilde{F} \leq 0$ or that $\bar{F} \leq 0$. If $\bar{F} \leq 0$, there are no R&D investments in the economy. If $\tilde{F} \leq 0 < \bar{F}$, the firm will invest only if it receives a subsidy and $F \leq \bar{F}$ and will not invest otherwise, i.e., the optimal policy is characterized as in the case of $F \in \left(\tilde{F}, \bar{F} \right]$.

Characterization of the firm's application decision of section 3.5

In the main text we write the application constraint simply as $\Delta\Pi^E = \Pi_1^E - \Pi_0^E \geq 0$. Let us now characterize $\Delta\Pi^E$ in mode detail. Recall that although the firm does not know the agency's type v, it knows that v is drawn from V according to the pdf $\phi(v)$ and cdf $\Phi(v)$, and it can calculate the agency's decision rule as a function of the type.

If condition (26) holds, the firm will launch the project even without a subsidy. As established in section 3.4, the firm knows that in this case $s^*(v) = 0$ if $v \leq \underline{v}$, $s^*(v) = s^{**}(v)$ if $v \in (\underline{v}, \bar{v})$, and $s^*(v) = \bar{s}$ if $v \geq \bar{v}$. When condition (26) holds we thus have

$$\Pi_1^E = \mathbb{E}_v \Pi^E \left(R^{**}(s^*(v)), s^*(v)\right) = \Phi(\underline{v})\, \Pi^E \left(R^{**}(0), 0\right)$$

$$+ \int_{\underline{v}}^{\overline{v}} \Pi^E \left(R^{**}(s^{**}(v)), s^{**}(v)\right) \phi(v)\, dv + (1 - \Phi(\overline{v}))\, \Pi^E \left(R^{**}(\overline{s}), \overline{s}\right).$$

Also, $\Pi_0^E = \Pi^E \left(R^{**}(0), 0\right)$ under condition (26). As a result the application constraint $\Delta\Pi^E \geq 0$ can be written as

$$\int_{\underline{v}}^{\overline{v}} \Pi^E \left(R^{**}(s^{**}(v)), s^{**}(v)\right) \phi(v)\, dv + (1 - \Phi(\overline{v}))\, \Pi^E \left(R^{**}(\overline{s}), \overline{s}\right) \tag{30}$$

$$- (1 - \Phi(\underline{v}))\, \Pi^E \left(R^{**}(0), 0\right) \geq K,$$

and the firm's application decision as $d = \mathbb{I}_{[0,\infty)} \left(\Delta\Pi^E \left| F \leq \tilde{F}\right.\right)$ that equals 1 if condition (30) holds.

If $F \in \left(\tilde{F}, \bar{F}\right]$, the firm will not launch the project without a subsidy (equation (14) becomes $\Pi_0^E = 0$). Again, the firm can calculate the agency's decision for each agency type. As shown in section 3.4, the firm knows that if $v \geq \tilde{v}$, the firm's participation constraint is irrelevant for the agency's decision, and that if $v < \tilde{v}$, the firm will either receive no subsidy in which case it will not invest or it will receive subsidy $\tilde{s}$ that just satisfies the firm's participation constraint, which by definition also leads to the zero profits. The application constraint $\Delta\Pi^E \geq 0$ simplifies now to

$$\int_{\tilde{v}}^{\overline{v}} \Pi^E \left(R^{**}(s^{**}(v), s^{**}(v)\right) \phi(v)\, dv + (1 - \Phi(\overline{v}))\, \Pi^E \left(\overline{v}\right) \geq K, \tag{31}$$

and the firm's application decision is $d = \mathbb{I}_{[0,\infty)} \left(\Delta\Pi^E \left| F \in \left(\tilde{F}, \bar{F}\right]\right.\right)$ that equals 1 if condition (31) holds.

If condition (27) holds, the firm will not invest even if it received the maximum subsidy rate $\bar{s}$. Therefore $\Delta\Pi^E = -K$ and $d = \mathbb{I}_{[0,\infty)} \left(\Delta\Pi_d^E \left| F > \bar{F}\right.\right) = 0$.

Proof of Proposition 1 of section 3.6

Part i). This follows directly from our definition of a PBE, which in turn follows directly from our analysis in section 3.3 where we establish that equation (23) satis-

fies a competitive investor's participation constraint and that if the investor participates, she will always monitor. Therefore in any PBE we must have $m^* = 1$ and $\pi^{I*}(s) = [\rho(R^*(s) + F) - sR^*(s)]/P$. In section 3.3 we further establish that if $m^* = 1$, then $h^* = \text{G}$.

Part ii). When $F \leq \tilde{F}$, condition (9) does not bind. The firm is able to raise external funding in period three and invest in R&D in period four even without a subsidy, i.e., equation (7) implies $R^*(s) = R^{**}(s)$ for all for all v and d. The firm's best-reply function $R^{**}(s)$ as given by equation (8) is well-behaving since the second derivative of the firm's objective function (24) is negative:

$$\frac{\partial^2 \Pi^E}{\partial R^2} = -\gamma\alpha R^{-\gamma-1} < 0. \tag{32}$$

In stage two, the agency solves the program (25) conditional on its v and $d = 1$, and anticipating that $R^*(s) = R^{**}(s)$. We want to prove that for each $v \in V$, there is a unique optimal subsidy rate $s^*(v)$. Since $U(R^{**}(s), s)$ is continuous and we have linear constraints of minimum and maximum subsidies it suffice to show that $U(R^{**}(s), s)$ is concave when evaluated at the interior solution, $s = s^{**}$, i.e., we want to show that $d^2U(R^{**}(s), s)/ds^2\,|_{s=s^{**}} < 0$.

From equation (8) we get that

$$R' := \frac{dR^{**}}{ds} = \frac{\alpha^{\frac{1}{\gamma}}(\rho - s)^{-\frac{1}{\gamma}-1}}{\gamma} = \frac{R^{**}}{\gamma(\rho - s)} > 0 \tag{33}$$

and that

$$R'' := \frac{d^2R^{**}}{ds^2} = \frac{(1+\gamma)\alpha^{\frac{1}{\gamma}}(\rho - s)^{-\frac{1}{\gamma}-2}}{\gamma^2} = \frac{(1+\gamma)R^{**}}{[\gamma(\rho - s)]^2} > 0. \tag{34}$$

Then, we differentiate $U(R^{**}(s), s) = vR^{**}(s) + \Pi^E(R^{**}(s), s) - gsR^{**}(s)$ twice with respect to s. Suppressing all function arguments for brevity, the first differentiation of U with respect to s gives

$$\frac{dU}{ds} = vR' + \frac{\partial \Pi^E}{\partial R}R' + \frac{\partial \Pi^E}{\partial s} - gR^{**} - gsR',$$

and the second differentiation yields

$$\frac{d^2U}{ds^2} = vR'' + \frac{\partial^2 \Pi^E}{\partial^2 R}(R')^2 + \frac{\partial \Pi^E}{\partial R}R'' + \frac{\partial^2 \Pi^E}{\partial s^2} + 2\frac{\partial^2 \Pi^E}{\partial R \partial s}R' - 2gR' - gsR''. \tag{35}$$

Now, $\partial \Pi^E/\partial R = 0$ by the envelope theorem, and from equation (24) we get that $\partial^2 \Pi^E/\partial s^2 = 0$ and $\partial^2 \Pi^E/\partial R \partial s = 1$. By using these insights, equation (35) simplifies to

$$\frac{d^2U}{ds^2} = (v - gs)R'' + \frac{\partial^2 \Pi^E}{\partial^2 R}(R')^2 + (1-g)2R'.$$

Inserting equations (32)-(34) into the right-hand side of the above equation gives

$$\frac{d^2U}{ds^2} = \frac{R}{\gamma(\rho - s)}\left[\frac{(1+\gamma)(v-gs)}{\gamma(\rho-s)} + 2(1-g) - \frac{\alpha R^{-\gamma}}{\rho - s}\right].$$

After using equation (8) to substitute $\rho - s$ for $\alpha R^{-\gamma}$ in the above equation we get

$$\frac{d^2U}{ds^2} = \frac{R}{\gamma(\rho - s)}\left\{\frac{(1+\gamma)(v-gs)}{\gamma(\rho-s)} + 1 - 2g\right\}.$$

Then, substituting s^{**} from equation (12) for s in the term in the square brackets shows that the term is negative when $g - \gamma(g-1) > 0$. This holds under Assumption 1. This suffices to prove that $d^2U(R^{**}(s), s)/ds^2 \mid_{s=s^{**}} < 0$. Consequently, equation (12) characterizes the unique type-contingent maximum for the agency's unconstrained decision problem.

Because $U(R^{**}(s), s)$ is continuous, constraints of minimum and maximum subsidies are linear, and the optimal unconstrained subsidy $s^{**}(v)$ is increasing in v (see equation (12)), the optimal subsidy rate is given by $s^*(v) = 0$ for $v \leq \underline{v}$, $s^*(v) = s^{**}(v)$ for $v \in (\underline{v}, \bar{v})$, and $s^*(v) = \bar{s}$ for $s \geq \bar{s}$. This is the optimal subsidy allocation rule given $d = 1$. If the agency does not receive an application ($d = 0$), $s^*(v) = 0$ for all v by assumption. Thus, the agency's optimal subsidy allocation rule in stage two is a function $s^* : V \times \{0,1\} \to \{0, s^{**}, \bar{s}\}$, i.e., conditional on v and d, the action of the agency in stage two is unique.

In period one the firm decides whether to apply or not given $\phi(v)$, $s^*(v)$, and $\pi^{I*}(s^*)$. Since in a PBE the firm's choice must maximize the profits and the firm's beliefs must be consistent with the agency's strategy, $d^* = 1$ only if condition (30) holds and $d^* = 0$ otherwise. Clearly, the agency's best response to $d^* = 1$ is $s^*(v) \in \{0, s^{**}(v), \bar{s}\}$, and $d^* = 0$ implies $s^*(v) = 0$ for all v. Thus, together with part i) of the proof, we have found a PBE that satisfies the five equilibrium criteria defined in section 3.6. Since the utility maximizing action in each stage of the game is unique for each $v \in V$, the equilibrium is also unique.

Part iii). When $F \in \left(\tilde{F}, \bar{F}\right]$, the firm will be able to raise funding in period three and invest in period four only if it gets a subsidy rate which is at least $\tilde{s}$ as given by equation (13). Conditional on $s^{**}(v) \geq \tilde{s}$, the proof follows step i) above and is omitted. We may thus focus on the range of parameter values where $s^{**}(v) < \tilde{s}$. For $v < \tilde{v}$, the firm is not able to invest if $s = s^{**}$ since the cost of finance $\pi^{I*}(s^{**})$ would be prohibitively high. Therefore, $s = \tilde{s}$ might constitute an optimal agency decision for $v < \tilde{v}$. But this requires that the agency's participation constraint (11) holds. As shown in section 1.4, the agency's participation constraint holds if $v \geq v^0$, and that $v^0 \leq \tilde{v}$, with the latter

inequality being strict for $\gamma > 0$. As a result, $s^*(v) = \tilde{s}$ constitutes the optimal agency decision for $v \in \left[v^0, \tilde{v}\right)$.

For $v < v^0$, the agency's participation constraint is no longer satisfied for $s = \tilde{s}$. Because for $v < v^0 \leq \tilde{v}$, $s^{**}(v) < \tilde{s}$ and because the agency's payoff $U(R^{**}(\tilde{s}), \tilde{s})$ is decreasing for $s \geq s^{**}(v)$, the agency is not willing to participate for any $s \geq \tilde{s}$ either. The agency might be willing to participate if $s \in [0, \tilde{s})$ but that would result in prohibitively high cost of finance and thus in $R^*(s) = 0$. As a result, $U(R^*(s), s) = 0$. Our fifth criterion for PBE stipulates that in this case $s^*(v) = 0$.

In sum, we have shown that when $F \in \left(\tilde{F}, \bar{F}\right]$ and $d = 1$, $s^*(v) = 0$ for $v < v^0$, $s^*(v) = \tilde{s}$ for $v \in \left[v^0, \tilde{v}\right]$, $s^*(v) = s^{**}(v)$ for $v \in (\tilde{v}, \bar{v})$, and $s^*(v) = \bar{s}$ for $v \geq \bar{v}$. If the agency does not receive an application ($d = 0$), $s^*(v) = 0$ for all v. Therefore, the agency's optimal subsidy rate decision in period two is a function $s^* : V \times \{0, 1\} \to \{0, \tilde{s}, s^{**}, \bar{s}\}$.

In period one the firm decides whether to apply or not given $\phi(v)$, $s^*(v)$, and $\pi^{I*}(s^*)$. Since in a PBE the firm's choice must maximize the profits and the firm's beliefs must be consistent with the agency's strategy, $d^* = 1$ only if condition (31) holds and $d^* = 0$ otherwise. Clearly, the agency's best response to $d^* = 1$ is $s^*(v) \in \{0, \tilde{s}, s^{**}(v), \bar{s}\}$ and to $d^* = 0$, $s^*(v) = 0$ for all v so, together with part i), we have found a PBE. Since the utility maximizing action in each stage of the game is unique for each $v \in V$, the equilibrium is also unique.

Part iv) When $F > \bar{F}$, the agency will reject any application since it knows that the firm would not be able to raise funding and invest even if it received a maximum feasible subsidy rate $\bar{s}$. In theory, when $F > \bar{F}$, all feasible subsidy levels $s \in [0, \bar{s}]$ amount to a rejection of an application. However, our fifth criterion for PBE stipulates that in this case $s^*(v) = 0$ for all v. Since condition (27) is independent of v, the firm knows when $F > \bar{F}$. Hence the firm does not apply for a subsidy it will not receive for sure, i.e. $d^* = 0$. But $F > \bar{F}$ implies by construction that market funding without a subsidy becomes so expensive that the firm cannot profitably raise funding and invest, i.e., $R^*(0) = 0$.■

Derivation of the firm's optimal R&D investment rule with an R&D tax credit of section 6.1

Recall from the main text that we modify our theoretical model of section 3 by setting the subsidy rate, s, to zero and introducing a corporate tax rate $\tau \in [0, 1]$ and a R&D tax credit rate $\tilde{\tau}_R \in [0, 1]$, which the firm obtains whether or not the project succeeds.

To derive the investor's profit function, we assume that the investor has a large number of projects whose success probabilities are independently and identically distributed. Then, by the law of large numbers, fraction P of these projects are successful and fraction $1 - P$ of the projects fail. The investor's net profits after paying corporate taxes are given by

$$\Pi^I = (1 - \tau)\left\{P\left[\pi^I - \rho(R + F)\right] - (1 - P)\left[\rho(R + F)\right]\right\},$$

which simplifies to

$$\Pi^I = (1-\tau)\left[P\pi^I - \rho\left(R+F\right)\right]. \tag{36}$$

As equation (36) shows, we assume for simplicity that monitoring expenses, too, are tax-deductible.[29] To maintain the consistency of the theoretical framework developed in section 3, we continue to assume that monitoring costs are non-verifiable. An interpretation is that the investor's total cost of supplying funding to the firms are verifiable to third parties (e.g., tax authorities) but allocation of that cost between monitoring and other expenses such costs of raising funding remains non-verifiable.

An optimal financing contract solves the program

$$\max_{\{\pi^E \geq 0, \pi^I \geq 0, R \geq 0\}} \Pi^E = (1-\tau) P\pi^E + \tilde{\tau}_R R \tag{37}$$

subject to the return sharing rule (equation (2)), the firm's and the investor's participation constraints, $\Pi^E \geq 0$ and $\Pi^I \geq 0$, respectively, and the investor's incentive constraint which is unchanged from equation (6), because the corporate taxes cancel out. Note from equations (36) and (37) that we assume that the financing contract is not made contingent on the R&D tax credit rate.[30]

As in section 3.3, equations (6) and (36) show that the investor's participation constraint rather than her incentive constraint is binding. As a result, a competitive investor's return share is given by

$$\pi^I = \frac{\rho\left(R+F\right)}{P}. \tag{38}$$

After substitution of equations (1), (2), and (38) for equation (37), the problem of seeking an optimal financing contract boils down to

$$\max_{\{R \geq 0\}} \Pi^E = (1-\tau)\left[\alpha\left(\frac{R^{1-\gamma}-1}{1-\gamma}\right) - \left(\rho - \tau_R\right)R - \rho F,\right] \tag{39}$$

subject to the firm's participation constraint $\Pi^E \geq 0$. In equation (39), $\tau_R = \tilde{\tau}_R/\left(1-\tau\right)$ denotes the "adjusted" tax credit rate. The firm's objective function Π^E in equation (39)

[29] If monitoring costs are non-monetary, as in Holmström and Tirole (1997), and cannot hence be deducted from taxes, then the firm's cost of outside funding becomes a function of corporate tax rate. This would substantially complicate the analysis since corporate taxation would no longer be neutral with respect to R&D investments. For similar reasons, quantitative models based on Holmström-Tirole type financial frictions (e.g., Meh and Moran, 2010) typically assume that monitoring costs are monetary expenses.

[30] Assuming that the financing contract would be contingent on the tax credit rate would essentially yield the same results, as is evident from comparison of the equations of this appendix with the corresponding ones in section 3. The only difference would be that the firm's cost of outside funding would be lower. See also footnote 18 in section 3.

corresponds to equation (24) save for s being replaced by τ_R. It is thus clear that the optimal R&D investment decision rule with an R&D tax credit must be identical to the one given by equations (7)-(9) with τ_R replacing s.

Appendix D: counterfactual

Execution

For the counterfactual, we draw shocks (ε_{it}, ζ_{it} η_{it}, μ_{it}) from their estimated (joint) distribution. We replace those draws in the top 1% with the value at the 99^{th}%. We also remove from the calculations the top 0.02% of observations with the highest simulated mean R&D investments. We use 100 simulation rounds.

Robustness

In Tables D1 and D2 we present results from our counterfactual when 1) we estimate the model ignoring (soft) loans Tekes gives and only use subsidies as our measure of s_{it} and 2) excluding the largest 3 firms in the estimation sample. The loans Tekes are soft in two senses: first, the interest rate a firm has to pay is subsidized; second, in case the project fails, the firm may not need to pay the (whole) loan back. We report the means of the same objects reported in the main text.

Table D1. Counterfactual results from the robustness tests

Only Tekes subsidies	R&D participation	R&D	ratio (R&D)	R&D | s >0	profit	spillovers	welfare	ratio (welfare)
Laissez-faire	0.53	80800	1.00	120105	1546144	72172	1618315	1.00
1^{st} best	0.54	205708	2.55	314038	1419034	206978	1626012	1.02
2^{nd} best	0.53	196598	2.43	292649	1471657	183838	1655495	1.02
τ_R	0.54	128222	1.59	188643	1588200	113336	1613252	1.00
s	0.54	121924	1.51	175870	1569582	108790	1607180	1.00
$s\|s>0$				303432				
Excluding largest 3 firms	**R&D participation**	**R&D**	**ratio (R&D)**	**R&D | s >0**	**profit**	**spillovers**	**welfare**	**ratio (welfare)**
Laissez-faire	0.53	162388	1.00	224598	3347821	123176	3470997	1.00
1^{st} best	0.54	368631	2.27	531086	3157554	327794	3485348	1.02
2^{nd} best	0.53	358015	2.20	498942	3234610	293121	3527731	1.02
τ_R	0.54	235802	1.45	323951	3414795	177681	3464192	1.00
s	0.53	237432	1.46	325580	3407749	186293	3465189	1.00
$s\|s>0$				581216				

NOTES: the reported numbers are the means over all firms and simulation rounds for R&D participation, R&D investment, R&D conditional on a positive subsidy rate, profit, spillovers and welfare. Ratio (R&D) is the ratio between average R&D in the regime in question and under laissez-faire; ratio (welfare) is the ratio between welfare in the regime in question and under laissez-faire.

Table D2. Counterfactual estimates

variable	Only Tekes subsidies	Excluding largest 3 firms
$\Pr[apply]$	0.23	0.23
$subsidy\ rate \vert s > 0$	0.41	0.39
τ_R	0.39	0.33
government cost, $s \vert R\&D > 0$	59327	107378
government cost, $\tau_R \vert R\&D > 0$	73571	106903
government cost, s	43385	79450
government cost, τ_R	50007	77815

NOTES: the figures are calculated over all simulation rounds and firms..