Vertical Information Restraints: Pro- and Anti-Competitive Impacts of Minimum Advertised Price Restrictions

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Abstract

We consider vertical contracts where the retail market may involve search frictions. Minimum advertised price restrictions (MAP) act as a restraint on customers' information and so can increase search frictions in the retail sector. Such restraints, thereby, soften retail competition—an impact also generated by resale price maintenance (RPM). However, by accommodating (consumer or retailer) heterogeneity, MAP can allow for higher manufacturer profits than RPM. We show that they can do so through facilitating price discrimination among consumers; encouraging service provision; and facilitating manufacturer collusion. Thus, welfare effects may be positive or negative compared to RPM or to the absence of such restrictions.

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1 Introduction

When a retailer enters into a distribution agreement with a manufacturer, it is well understood that the agreement may impose 'vertical' restraints on the retailer that restrict price or non-price aspects of retailer activity.[1] Price restraints include resale price maintenance (RPM), in which the manufacturer imposes floors and/or ceilings in retail pricing. Commonly considered non-price restraints typically include exclusivity provisions or the imposition of sales territories. This paper considers a third type of vertical restraint, information restraints. To focus the discussion, minimum advertised price (MAP) policies are examined.[2] MAP policies impose a floor on the price at which retailers can advertise a product, but, crucially, not the price at which it can be sold to a consumer. That is, even if a MAP policy imposes a floor of \$10 a unit on advertised prices, nothing restricts the retailer from selling it to a consumer for \$7.

The pro- and anti-competitive impacts of MAP policies are examined through the lens of a series of closely related models. Since, superficially, MAP and RPM policies appear similar, the economic impact of these policies are compared and contrasted. Two themes emerge. First, for MAP policies to have any market impact,

[1]For a survey of common practices and the related economic issues, see Rey and Vergé (2008).

[2]MAP policies appear the most common form of vertical information restraint. As an illustration of the scope of their use, the first 40 policies found in a search for "minimum advertised price pdf" using the Google search engine on April 13, 2016 covered the following product categories (companies): Adjustable office and industrial chairs (Neutral Posture); Antennas (Winegard Company); Aquarium filtration systems (Lifegard Aquatics); Baby and toddler pillows (Smuggwugg); Batteries, flashlights, solar energy equipment (NOCO); Cameras, security cameras (Sony); Chainstays, gripping bar tape (Lizard Skins); Chairs (Allseating); Clutches, flywheels, accessories (Midway Industries); Data storage electronics (Seagate); Designer faucets, showers, and bath accessories (Sonoma Forge); Digital display mounting (Ergotron); Enzymes (Enzyme Science); Exhaust systems and mufflers (Flowmaster); Games (Cool Mini or Not); Health bars, drinks, and powders (Powercrunch); High-pressure hydraulic tools (Enerpac); Holster clips (Utiliclip); Home, mobile and automobile electronics (JVC Professional Video Products); Instruments for measuring and calibrating (Meriam Process Technologies); LCD and plasma tvs (Microtek); Mobile cameras (GoPro); Mobile telephone cases (Urban Armor Gear); Outerwear, corporate wear and imprintable apparel (Charles River Apparel); Pistols (Glock); Rock-crawling and off-road cars (GMADE); Security cameras and surveillance equipment (Samsung Techwin America); Scented pencils, stickers and paper clips (Scentoco); Shallow water anchors (JL Marine Systems); Speakers (Eminence); Stove ranges, broilers, ovens, kitcken equipment (Montague Company); Temperature-controlled food display cases (Structural Concepts); Thermal imaging infrared cameras (FLR Commercial Systems); Toy car track tape (InRoad Toys and PlayTape); Triathalon clothing and gear (Pearl Izumi); Ventilation fans (Panasonic Home & Environment Co.); Wall art, picture frames, jewelry, mugs, pillows (Glory Haus); Water bottles (Blender Bottle); Wood flooring (Kahrs International); Work boots (Thorogood Shoes). All these policies are available at www.johnasker.com/MAP.zip.

consumers need to have an informational (search) friction that advertising helps alleviate. Hence, in every setting we consider search frictions are a central feature. Without search, MAP is irrelevant to market outcomes. Second, MAP policies soften competition by obscuring prices. Thus, consumers allocate themselves to competitors somewhat randomly, being unable to sort as they would if perfectly informed as to prices. By contrast, RPM softens competition by equalizing prices. When consumers, or retailers, are heterogenous, MAP is helpful in retaining the flexibility to profitably accommodate heterogeneity. Thus, MAP will have an impact on markets when search and heterogeneity among similarly situated economic actors are important features of the environment.

We explore these themes through three interrelated models. These models are:

- The *price discrimination* model: in which it is shown that MAP policies allow manufacturers to imperfectly separate high and low search cost consumers, and better extract surplus from high value consumers with high search costs. That is, the MAP policy obscures actual prices, allowing search patterns to be leveraged as a screening device.[3]

- The *service* model: in which it is shown that by obscuring prices, MAP policies soften competition and protect retailer profits, while allowing retailers to optimize subject to their heterogenous marginal costs of retailing. This can increase the returns to retailers from providing service that expands the market (such as informative advertising). By softening competition, while retaining retailer flexibility, MAP can dominate RPM as a means for manufacturers to profitably incentivize retailers.[4]

- The *collusion* model: in which MAP raises cartel profits and stability, in markets where consumer search is important, by allowing manufacturers to more easily monitor each other's behaviour. Notably this is done without sacrificing the ability of cartel members to tailor actual transaction prices to local market conditions.[5] That is, whereas RPM may be viewed as having features of a

[3]Throughout we assume that retailers cannot price discriminate. Instead, discrimination occurs through the consumers' choice of retailer. Chen (1999) considers the effect of RPM when retailers can engage in price discrimination.

[4]Telser (1960) is typically cited for the idea that RPM promotes service (though it is also discussed in Yamey (1954), for example). More recent formalizations and developments include Matthewson and Winter (1984); Klein and Murphy (1988); and Deneckere, Marvel and Peck (1997). We follow the literature in considering pro-consumer service, although note that, as in the model of exclusion in Asker and Bar-Isaac (2014), service need not be helpful to the consumer.

[5]In formalizing this argument, we adapt the framework of Jullien and Rey (2007) to accommodate MAP.

price-fixing scheme, MAP may be viewed as analogous to a market division scheme.

In understanding the way MAP restrictions work, and in particular how they differ from the price restraint embodied in an RPM restriction, it is useful to examine a typical MAP provision. As an example, consider the January 1, 2016, MAP policy of Samsung Techwin America (a manufacturer of security cameras and surveillance equipment).[6] The MAP price is specified as a percentage of the manufacturer's suggested retail price (in this instance, 40% of MSRP). The policy explicitly applies to all advertisements in all media, including online. The MAP restriction only applies to advertised prices and not to the price at which products are actually sold. In physical stores, this means that the posted price in the store is not affected by the MAP restriction. As regards online pricing the policy states:

> Pricing listed on an internet site is considered an "advertised price" and must adhere to the MAP policy. Once the pricing is associated with an actual purchase (an internet order), the price becomes the selling price and is not bound by this MAP policy. Statements such "we will match any price", and "call for price" are acceptable.

In particular, such policies allow retailers to advise customers that they will be able to see the price when the item is in a (digital) shopping basket.

Lastly, Samsung reserves the right to punish non-compliance with termination.[7]

This paper adds to the nascent literature (notably Janssen and Shelegia (2015) and Lubensky (2014)) that examines the effects of vertical contracts when the final goods market is characterized by search frictions. In particular, in such markets it has long been understood that the law of one price need not hold, and price dispersion may arise.[8] Naturally, such frictions have implications for the contracts between manufacturer and retailer and for a manufacturer's profitability. A minimum advertised price restriction (MAP) which limits the price that retailers may advertise (with no restriction on the price they may charge) can make it more difficult for consumers to find the retailers charging the lowest prices and lead to retail price

[6]Available at www.johnasker.com/MAP.zip

[7]Interestingly, the policy also describes its purpose, which in this instance is to "to help ensure the legacy of STA as a top producer of high performance, high quality, professional security products and to protect the reputation of its name and products ...[and]... to ensure dealers, retailers and distributors have the incentive to invest resources into services for STA's customers.".

[8]This idea goes back to Stigler (1961) at least. Baye, Morgan and Scholten (2006) provide a useful overview.

dispersion (if not advertised price dispersion). As is illustrated in this paper, a manufacturer (and even consumers) may benefit.

The paper also contributes to the large and growing literature on obfuscation in search markets. (See, for example, Ellison and Wolitzky (2012), Wilson (2010), Piccione and Spiegler (2012), Chioveanu and Zhou (2013)).[9] Much of the economic force of a MAP restriction is through making firms identical from the point of view of consumers. In this way, MAP plays a role in obfuscating the actual as opposed to advertised price (indeed this is its only role in our analysis). This paper is distinct from the rest of the obfuscation literature (to our knowledge) by explicitly considering the opportunities obfuscation gives to an upstream manufacturer, as opposed to a retailer.

Only four papers, that we know of, consider MAP policies in the economic literature. Kali (1998) and Cetinkaya (2009) explore theoretical models that treat MAP as an RPM provision with an additional advertising subsidy. A very different approach is adopted in this paper. Charness and Chen (2002) conduct an experimental study of the determinants of MAP compliance. Israeli, Anderson and Coughlan (forthcoming) empirically examine detected violations of MAP, using data from a firm engaged, on behalf of manufacturers, in monitoring MAP compliance of online retailers. They also provide an informative discussion of MAP's prevalence.[10] Importantly for our study, Israeli et al find that 14-22% of authorized dealers violate MAP, as compared to 46-54% of unauthorized dealers. This suggests that MAP policies are genuine restraints on retailer behavior.

Lastly, this paper contributes to the well established literature, in policy and academic circles, on the antitrust implications of vertical contracts.[11] Where MAP provisions have been considered in this literature, the point of departure (as in this paper) has been RPM restrictions. As with many vertical restraints, the approach to MAP and RPM taken in US and European law differs, with the E.U. being the less permissive. In 2007 in the *Leegin* case federal U.S. law reversed earlier precedent, shifting RPM from a complicated version of a *per se* offense to a rule of reason regime.[12] Even prior to the *Leegin* case, US law had ruled in favor of MAP provisions, acknowledging their pro-competitive potential in preserving service

[9]More broadly, recent literature has explored retailers' policies which influence search and pricing behavior in search markets. Such practices include stochastic discounts off the list price in Gill and Thanassoulis (forthcoming), lowest price guarantees in Janssen and Parakhonyak (2013), and exploding offers in Armgstrong and Zhou (2016).

[10]The data used in Israeli et al comes from a firm offering MAP compliance monitoring of internet based sellers.

[11]See Rey and Vergé (2008) for a survey.

[12]*Leegin Creative Leather Products, Inc. v. PSKS, Inc.*, 551 U.S. 877 (2007)

incentives.[13] Post-*Leegin* cases in the U.S. have also failed to gain traction.

By contrast, in Europe, MAP provisions have tended to be viewed with more suspicion. In particular, MAP has been found to be a *de facto* form of RPM.[14] In turn, RPM has been found to impose a sufficient degree of harm to competition that there is no need to examine its effects in determining liability (that is, making it an restriction of competition by object, somewhat similar to a *per se* offense in U.S. law).[15]

Hence, jurisdictions differ in their approach to regulating MAP provisions. This paper contributes to understanding this divergence by providing the precise frameworks needed to make economic arguments supporting each approach. By doing so it helps clarify the trade-offs implicit in arriving at any policy position vis-a-vis MAP.

The rest of this paper is organized as follows. Section 2 describes the common modeling approach used in all models. Of particular importance is Subsection 2.3, which summarizes the timing and describes the equilibrium concept applied in the rest of the paper. Section 3 explores the price discrimination model. Section 4 investigates how MAP can enhance customer service. Section 5 shows how a MAP program can help coordinate a manufacturer cartel. Lastly, Section 6 offers some final remarks.

2 Model Structure

An (essentially) homogenous good is sold in a market with two retailers.[16] The retailers purchase from a manufacturer and sell to consumers. Retailers also engage

[13]See *Holabird Sports Discounters v. Tennis Tutor, Inc*, 1993 No. 92-2504, 1993 U.S. App. LEXIS 10918, (4th Cir. May 7, 1993), and at appeal 993 F.2d 228 (1993) and *Commodore Business Machines, Inc. v. Montgomery Grant, Inc,* No. 90 Civ. 7498 (LMM), 1993 U.S. Dist. LEXIS 262 (S.D.N.Y. Jan. 13, 1993). In 2003, following on from an FTC enforcement action, plaintiffs obtained a settlement in *In re Compact Disc Minimum Advertised Price Antitrust Litigation*, 216 F.R.D. 197, 199-200 (D. Me.2003) (approving settlement) alleging that MAP provisions facilitated a cartel among manufacturers (in this case, distributors of recorded music). Albert (2012) and Passo (2016) provide extensive reviews of recent U.S. policy and litigation.

[14]See Commission Decision 16 July 2003 *PO/Yamaha* (COMP/37.975), OFT Decision *Agreements between Lladro Comercial SA and UK retailers fixing the price for porcelain and stoneware figures*, CP/0809-01, 31 March 2003 and, recently, the Decision of the UK Competition and Markets Authority *Online resale price maintenance in the commercial refrigeration sector* Case CE/9856/14. All these cases apply E.U. law.

[15]See, for instance, the overview of relevant E.U. law in the Decision of the UK Competition and Markets Authority *Online resale price maintenance in the commercial refrigeration sector* Case CE/9856/14.

[16]In the collusion model differentiation exists at the retailer level.

in price advertising and demand increasing service. Consumers have search costs, which makes the service and price advertising helpful. In what follows the common structure of the modeling framework is explained in detail, together with the specific parameterizations used to consider the impact of MAP policies on i) price discrimination, ii) retailer service and iii) manufacturer collusion.

2.1 Consumers

Let there be a unit mass of potential consumers. Consumer i purchasing from Retailer j derives utility from consumption given by $v(q) + \xi_j + \varepsilon_i$.[17] In each of the models, we simplify this general structure:

1. In the price discrimination model, $\xi_j = 0$, $\varepsilon_i \in \{l, h\}$ and are i.i.d. with $\Pr(\varepsilon_i = l) = \lambda$. Moreover, consumers have unit demand; that is, $v(q \geq 1) = v$ and 0 otherwise.

2. In the service model, $\varepsilon_i = \xi_j = 0$. Here consumers do not have unit demand; instead, $v'(q) > 0$, $v''(q) < 0, 0 < v'(0) \leq \overline{v}$. This implicitly defines a demand curve $q(p)$ with standard properties for a consumer who decides to purchase at a price p.

3. In the collusion model, $\varepsilon_i = 0$, $\xi_j \sim_{i.i.d} U\,[0, 1]$; consumers have unit demand.

Minimum advertised prices only have bite if there are limits on the ability of consumers to freely compare different retailers' prices. We model these in a simple (albeit fairly standard) way. Specifically, some fraction of consumers can search costlessly, and others have sufficiently high search costs that, if observing the same advertised price at both retailers, they will choose one at random.[18] That is, they visit only one retailer and, from the point of view of these consumers, advertised prices are the only point of differentiation between retailers. We write σ_x to denote the proportion of consumers, with $\varepsilon_i = x$, that search both retailers, with the remainder searching only a single retailer. It is immediate that MAP can play no role if $\sigma_x = 1$ $\forall x$. Therefore, we assume throughout at least for some x, $\sigma_x < 1$. Indeed, to

[17] As one would expect: (i) $v(q) \geq 0$ $\forall q$; and (ii) Net utility is $v(q) + \xi_j + \varepsilon_i - p$.

[18] This is an extreme but convenient, and often-used, way to examine search frictions and consumers who are heterogeneous in their search behavior. See, for example, Varian (1980). Alternatively, an ex-ante costly decision to engage in search in the style of Burdett and Judd (1983) would allow the fraction of searchers to be endogenously determined (at the cost of additional notation for these search costs, and a somewhat more involved analysis).

highlight the role of these non-searchers, in the service and collusions models we set $\sigma_x = 0$ for all values for $\varepsilon_i = x$. That is, in these models, consumers only visit one store. However, the distinction and associated notation plays a crucial role in the price discrimination model.

2.2 Firms

The two retailers are denoted R_1 and R_2. The marginal cost of retailing for a retailer, c_j, takes on a value in $\{c_L, c_H\}$. We write $\Pr(c_j = c_H) = \alpha_j \in [0,1]$. Costs are ordered such that $0 = c_L \leq c_H$. Specific model parameters are:

1. In the price discrimination model of Section 3, $\alpha_1 = 0, \alpha_2 = 1$.
2. In the service model of Section 4, $\alpha_1 = \alpha_2 = \alpha \in (0,1)$.
3. In the collusion model of Section 5, $\alpha_1 = \alpha_2 = 0$ (or, equivalently, $c_H = c_L$).

Although the retailers may be heterogeneous, it is assumed that the manufacturer offers the retailers identical terms. Where these retailers differ in their cost realizations, we assume that they are indistinguishable to consumers, and either they are indistinguishable to the manufacturer or the manufacturer is restricted from discriminating between them.[19]

Retailers may engage in two activities supporting sales. The first is the provision of service that improves consumers' awareness of the product. This can be thought of as informative non-price advertising, or simply 'service'. We suppose that service provision plays no role in the price discrimination and collusion models; instead all consumers are aware of the product.[20] Clearly it plays a role in the service model, as we discuss in Section 4.

The second activity is price advertising. A retailer's advertised retail price is denoted p_j^a. We assume that this is flexible, and so p_j^a is set contemporaneously with the retailers' transaction price. We make the following assumption throughout the paper. It ensures that the advertised price is not simply cheap talk.

A1. Advertising cannot be fraudulent in the following sense: the advertising price can be no lower than the retail transaction price p_j, i.e. $p_j^a \geq p_j$.

[19] This may be due to legal restrictions, or simply induced by the timing assumptions below.

[20] In an extension of the price discrimination model, we allow for a "simple" service model with a discrete decision so that the service level can take on two values—'off' and 'on'. Further, we suppose that service by both retailers is necessary for consumers to be aware of the product.

Retailers buy the product from a manufacturer (M) according to a two-part tariff with linear wholesale price w and fixed fee T.[21] In the absence of vertical restrictions, retailers take the wholesale price w as given and set the retail transaction price and advertised price. If manufacturers impose point RPM, retailers are bound to charge the RPM price, $p^{RPM} = p_j^a = p_j$. Similarly, minimum and maximum RPM imposes constraints such that $p^{RPM} \leq p_j \leq p_j^a$ and $p^{RPM} \geq p_j^a \geq p_j$, respectively. If manufacturers impose MAP, retailers are bound to advertise at the MAP price or higher, $p^{MAP} \leq p_j^a$.

The manufacturer's marginal cost is equal to zero.

Thus far, it has been assumed that there is only one manufacturer. The collusion model deviates from this structure and considers two manufacturers. Each manufacturer uses a single retailer. Manufacturers do not share retailers. Given this structure, manufacturers and retailers both share the same subscript, such that R_j provides retail service to M_j.

Each model assumes complete information, with the following exceptions:

1. Retailer costs are private information to the retailer. These are realized after contracts are agreed.[22]

2. In the collusion model, w_j is only observed by M_j and R_j. ξ_j is only observed by (all) consumers and R_j.

2.3 Timing and Equilibrium

Lastly all models share a common timing structure, as follows.

1. The manufacturer sets the contract terms (T, w) and any restraints (RPM, MAP etc.).

2. Retailers accept or reject the offered contract.

3. Retailer cost realizations occur, as do realizations of ξ_j and ε_i, these are privately observed by retailers and consumers respectively.

4. Retailers each decide on a service level, denoted s_j.

[21] Since we assume that the manufacturer cannot distinguish between retailers, there are no j-subscripts on these variables to denote different retailers.

[22] This assumption only has bite in the service model, since costs are deterministic in the price discrimination and collusion model.

5. Retailers each set their price, p_j, and advertised price, p_j^a, subject to any vertical restraints.

6. Consumers sort into exploring purchases from R_1 or R_2. In particular, they choose to visit the retailer where they expect greatest surplus (as long as the expected surplus is non-negative); if indifferent, they are equally likely to visit either retailer.

7. Purchases are made and profits realized.

In the collusion model, this constitutes the stage game within a repeated game setting (described further in Section 5).

We conclude by discussing our equilibrium concept and introduce some simple and intuitive results that characterize equilibrium play in the subgame beginning at step 5 above, in which retailers set p_j and p_j^a. In essence, it ensures that the homogenous-good Bertrand structure of price competition between retailers carries over to the price advertising game considered here. Given that some consumers can visit only a single retailer, beliefs as to p_j given p_j^a need to be considered. This results in the relevant equilibrium concept being (a refinement of) perfect Bayesian equilibrium.

We introduce the following definitions.

Definition 1 *Consumers' beliefs are* ***monotone*** *if,* $\forall p_j^a > \widehat{p}_j^a$, $E(p_j|p_j^a) > E(p_j|\widehat{p}_j^a)$.

Definition 2 *A* ***monotone perfect Bayesian equilibrium*** *is a perfect Bayesian equilibrium in which consumers' beliefs are monotone.*

In principle, there is considerable flexibility in assigning off-equilibrium beliefs and, thereby, inducing perverse equilibrium behavior. However, Assumption A1 specifies that a retailer cannot advertise a price below the price that it charges imposes real constraints. For example, suppose that a retailer charging a price of \$4 or charging a price of \$6 were expected to advertise the same price. The advertised price would have to be a price at or above \$6 to conform with this restriction. Suppose that this advertised price is \$20 and that these are the only two kinds of retailer expected to advertise at \$20 in equilibrium. Then consumers expect on average some price above \$4 (if it was equally likely that retailers charge \$4 and \$6, consumers anticipate \$5 on average).[23] Suppose that the retailer charging \$4

[23] Recall that retailers are indistinguishable to consumers, so that if $p_j^a = p_k^a$ then $E(p_j|p_j^a) = E(p_k|p_k^a)$.

instead of advertising a price of \$20 advertises a price of \$4. Given the restriction that retailers cannot advertise at prices below what they charge, it must be that consumers *even if this is an off-equilibrium advertised price* anticipate paying a price below \$4; it follows that this advertised price attracts more consumers, as their expected surplus must be strictly greater, and hence would be more profitable.[24]

When there are no advertising restrictions, the simple logic in the example above has considerable bite. The following result establishes that there is no loss in supposing that retailers will advertise the price that they actually charge.[25]

Lemma 1 *Suppose that there are no advertising restrictions, then any perfect Bayesian equilibrium is equivalent to one in which each retailer sets its advertised price equal to its actual price i.e.* $p_j^a = p_j \ \forall j$.

Proof. See the appendix. ■

Note that, since perfect Bayesian equilibria allow for considerable flexibility on off-equilibrium beliefs, it is immediate that monotone perfect Bayesian where $p_j^a = p_j$ can be constructed.

In case of advertising restrictions, a retailer may not be able to set the advertised price equal to its actual price. Moreover, when the MAP price is high, the logic in the example above that underlies Lemma 1 has little bite. Returning to the example above, suppose that there was a MAP price of \$10, and consumers expected that retailers advertising a price of \$20 were equally likely to actually charge \$6 or \$4, supported by the (off-equilibrium) beliefs that any retailer advertising anything between \$10 and \$20 actually charged \$9. Here, since the MAP restriction prevents the retailer charging \$4 from advertising at a sufficiently low price, this retailer may have little to gain from advertising at a price below \$20. A focus on monotone perfect Bayesian equilibrium rules out such perverse outcomes.[26] Trivially the monotone belief assumption leads retailers to advertise at as low as a price as possible. We obtain the following result.

Lemma 2 *In a MAP regime, in all monotone perfect Bayesian equilibria, each retailer advertises its actual price unless the MAP restriction binds, and in this case it advertises the MAP price; i.e.* $p_j^a = \max\left\{p^{MAP}, p_j\right\}$.

[24] If, in equilibrium, competing retailers advertise and charge a price of \$6, but the retailer of interest has an actual price of \$4, any advertised price in the interval $[4, 6)$ is considered equivalent to advertising a price of \$4.

[25] This logic does not pin down all advertised prices. In particular, suppose that in equilibrium no retailer charges a price above \$10, then if the retailer charging \$10 is expected to advertise at \$20, it has no strict gain by advertising a price of \$10 instead.

[26] Again, flexibility regarding off-equilibrium beliefs allows such equilibria to be constructed.

Proof. See the appendix. ■

In all that follows, the focus is on monotone perfect Bayesian equilibria, and so, following the two lemmas above, it is without loss to suppose that retailers advertise their actual price, or the MAP price in the event that it binds.

3 MAP as price discrimination

MAP can facilitate price discrimination. That is, in the face of consumer preference heterogeneity, MAP can be used to sort consumers among retailers in ways that extract greater surplus than might otherwise be the case. This is because MAP by preventing all consumers from searching amongst retailers costlessly can allow for price dispersion. As noted by Salop (1977), an integrated monopolist would value such price dispersion as it both segments the market and charges higher prices to less efficient searchers. An appropriate MAP contract enables the manufacturer both to allow for such price dispersion, and to extract the surplus that this creates in the industry as a whole.

To illustrate this idea as simply as possible, we begin by abstracting from service provision and suppose that consumers have unit demand with valuations equal to either l or h. Specifically, a fraction $(1-\lambda)$ of consumers have a valuation equal to h and the remainder have a valuation equal to l. In this section we suppose that one retailer has costs equal to $c_L = 0$ and the other $c_H \geq 0$.

With a two part-tariff, it is immediate that even without the use of vertical restraints, the manufacturer can induce a retail price of h or a retailer price of l and enjoy the full surplus of doing so.[27] If industry profit is maximized by selling to all consumers at $p = l$, by setting $w = \max\{0, l - c_H\}$, the manufacturer can induce the retailer to set prices at $p = l$ and set T to extract the remaining surplus from the retailers. Conversely, if industry profit is maximized by selling to only high value consumers, all surplus can be extracted by the manufacturer (for example, by setting $p = h$ and $T = 0$). Thus, the manufacturer's profit is equal to $\max\{l, (1-\lambda)h\}$. Note that RPM does not provide an avenue through which to raise profits above this level.

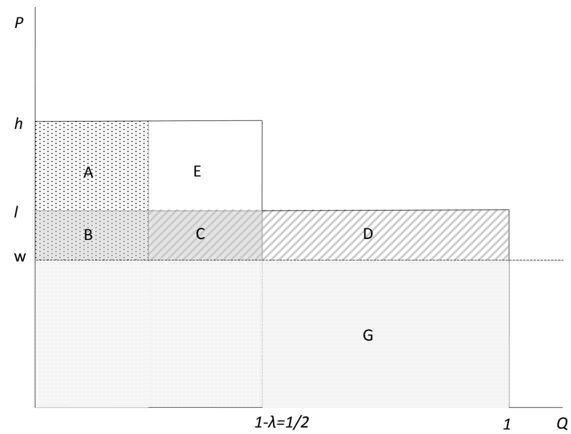
However, the addition of MAP to this environment may allow for further surplus extraction by the manufacturer. We illustrate this with a simple example.

Consider an environment in which $\sigma_l = 1$ and $\sigma_h = 0$, such that low value consumers costlessly search both retailers, and the high value consumers will only

[27] As is well understood, the use of a two-part rather than linear tariff allows the manufacturer to overcome the familiar double-marginalization problem highlighted in Spengler (1950).

visit one retailer. Further, since cost heterogeneity on the part of retailer is not crucial, assume $c_H = c_L = 0$.

Figure 1: Outcomes with MAP, a two part tarrif and no service



Consider the profit that a manufacturer that sets the MAP price at $p^{MAP} = h$ can extract. It is clear that for MAP to be strictly profitable, it must lead one retailer (denoted R_H) to charge a retail price equal to h and the other, R_L, to set the retail price to l.

Figure 1 illustrates this scenario (with $\lambda = 0.5$). Of the high value consumers, due to the identical advertised price imposed by MAP and their inability to search, half go to R_H and half to R_L. All the low value consumers (who are assumed to be able to search costlessly between retailers) go to R_L. Hence, the profit (ignoring T) made by R_H is $\frac{1}{2}(h - w)(1 - \lambda)$, the sum of rectangles A and B. Similarly, the profit made by R_L is $(l - w)(\lambda + (1 - \lambda)\frac{1}{2})$, the diagonally hatched rectangle corresponding to the sum of C and D. By setting w such that the two profits are equal, then using T, the manufacturer can capture all the surplus other than area E. Assuming that this is optimal policy for the manufacturer, it remains to show that this is an equilibrium for the retailers.[28]

Remark 1 *Let $\sigma_l = 1$, $\sigma_h = 0$, and $c_H = c_L = 0$. Suppose the manufacturer offers (w, T) and sets $p^{MAP} = h$. A monotone perfect Bayesian equilibrium in which both*

[28] Depending on parameters, the manufacturer may do better by setting $w = h$, and only selling to high value consumers. This will be the case if rectangle E is bigger than $D + G$ $(= \lambda l)$ at the optimum. Retailer equilibrium if $w = h$ is trivial.

retailers set $p^a = h$*, and* R_L *sets* $p = l$ *and* R_H *sets* $p = H$*, exists if*

$$(h-w)(1-\lambda)\frac{1}{2} \geq (l-w)\left(\lambda + \frac{1-\lambda}{2}\right) \tag{1}$$

$$(l-w)\left[\lambda + (1-\lambda)\frac{1}{2}\right] \geq (h-w)(1-\lambda)\frac{1}{2} \tag{2}$$

If this equilibrium exists, it is unique if Conditions 1 and 2 are strict inequalities. The optimal MAP policy sets w^**, such that Condition 2 holds with equality, and* T^* *such that* $(h-w)(1-\lambda)\frac{1}{2} - T^* = 0$*.*

Proof. See the appendix. ■

It is easy to find parameterizations such that this equilibrium exists and this discriminatory pricing policy is optimal for the manufacturer, as shown in Example 1, below.

Example 1 *Let* $\sigma_l = 1$*,* $\sigma_h = 0$*,* $c_H = c_L = 0$*,* $h = 2$*,* $l = 1$ *and* $\lambda = 0.5$*. The conditions in Remark 1 are satisfied when* $(w, T) = (\frac{1}{2}, \frac{3}{8})$ *and* $p^{MAP} = 2$*. Further, the manufacturers profit from this pricing scheme is* 1.25*. The profit from a two part tariff, absent MAP, is 1. Thus, the MAP policy, in this example, leads to strictly greater profit for the manufacturer.*

All price discrimination operates by (usually imperfectly) homogenizing consumers so that targeted prices extract more surplus.[29] The MAP policies explored here are no exception. By introducing an information friction, the manufacturer is able to isolate at least some of the high value consumers and extract surplus from them via the high price charged by R_H. By contrast, RPM does little to segregate consumers and so serves only to solve a possible double margin problem (which arises if T is restricted to 0). This illustrates the sense in which MAP has elements analogous to the much more invidious market division schemes that invite criminal sanction under antitrust laws, while RPM is similarly analogous to price fixing.[30] Merely agreeing on a common price does not help retailers, in this setting, to extract

[29]Sometimes the emphasis is tilted toward better targeting the prices (in the extreme, Type 1 price discrimination), while other times the emphasis is on homogenizing the consumers (bundling, and Type 3 price discrimination, being obvious examples).

[30]To be clear, we do not argue that MAP and RPM should invite the same sanctions as market division and price fixing. Indeed, as much of this paper shows, both often serve to make markets more efficient and more consumer friendly.

more surplus. However, if retailers could allocate high value consumers to R_H and low value consumers to R_L, as might be done in some ideal market division scheme, then they would be able to extract all available gains from trade. MAP generates an imperfect implementation along these lines, albeit ultimately to the benefit of the upstream manufacturer.[31]

Remark 2 *Much as in the classic price discrimination literature (Adams and Yellen (1976), Schmalensee (82, 84), McAfee et al (1989)) negative correlation between valuation and search costs is not a necessary condition for this policy to work. If we adjust Example 1 such that $\sigma_l = \sigma_h = \sigma$, it can be shown that profits from optimal MAP-based pricing is given by $1 + \frac{\sigma(1-\sigma)}{3\sigma+1}$ which will strictly dominate the non-MAP profit, for $\sigma \in (0,1)$.*

We finish the section with a general proposition characterizing the MAP-based price discrimination scheme, and its optimality, for this environment.

Proposition 1 *The optimal MAP-based discriminatory price scheme by the manufacturer sets*

$$T^*(w) = \min \left\{ \begin{array}{c} (h - w - c_H)(1-\sigma_H)\frac{1-\lambda}{2}, \\ (l-w)\left(\sigma_L\lambda + \sigma_H(1-\lambda) + (1-\sigma_L)\frac{\lambda}{2} + (1-\sigma_H)\frac{1-\lambda}{2}\right) \end{array} \right\}.$$

and

$$w^* = \arg\max_w \quad w\left(1 - (1-\sigma_L)\frac{\lambda}{2}\right) + 2T^*(w)$$

s.t.

$$\begin{aligned} \left(\sigma_L\lambda + \sigma_H(1-\lambda) + (1-\sigma_L)\frac{\lambda}{2} + (1-\sigma_H)\frac{1-\lambda}{2}\right)(l-w) &\geq (1-\lambda)\frac{1+\sigma_H}{2}(h-w) \\ \left(\sigma_L\lambda + \sigma_H(1-\lambda) + (1-\sigma_L)\frac{\lambda}{2} + (1-\sigma_H)\frac{1-\lambda}{2}\right)(l-w-c_H) &\leq (h-w-c_H)(1-\sigma_H)\frac{1-\lambda}{2} \\ w &\leq \min\{l, h-c_H\}. \end{aligned}$$

Retailers play a (almost unique) monotone perfect Bayesian equilibrium in which R_L sets $p = l$ and R_H sets $p = h$.[32] The solution of this program constitutes the optimal pricing policy for the manufacturer for a positive measure of parameter values.

[31] Interestingly, contingent on IC constraints being satisfied, as more retailers are used in a market, surplus extraction via MAP will approach perfect (type 1) price discrimination in this setting.

[32] As in Remark 1, the equilibrium may not be unique if $c_H = 0$ so that the firms may play different roles in price setting.

Proof. The proof extends to the intuition developed in Remark 1. It is immediate that the discriminatory MAP scheme should involve the retail prices set at l and h. Since R_H has higher marginal costs of production than R_L, more industry profit is generated when the R_L sets a price of l (and sells a greater quantity) and R_H sets a price of h. As argued below, the manufacturer can ensure that this is the equilibrium outcome.

The fixed fee is set to extract as much surplus as possible subject to both the low cost retailer and high cost retailer being willing to take up the contract; that is, it is equal to the minimum of their (gross of fixed) fee profits.

The manufacturer chooses the input price w^* to maximize its profits.

The penultimate two inequalities at the end of the statement of the proposition correspond to the incentive constraints that guarantee that the two retailers do not wish to deviate from their equilibrium pricing strategies. The first ensures that the low cost retailer prefers charging a price of l than a price of h, and the second that the high cost retailer prefers charging a price of h to undercutting the low cost retailer and attracting all searchers. The final inequality ensures that both retailers prefer to make positive sales. ■

Note that in this section, given the simplicity of the set-up (where there are only two types of consumers, each with unit-demand), the level of the price restriction (as long as it is sufficiently high) plays no role. Instead, it could be considered as analogous to a ban on advertising. In richer environments with downward-sloping demand, the level of the advertising restriction can be used to affect the double marginalization concern raised by Spengler (1950) in addition to allowing for the kind of discrimination described in this section. As will become apparent, the level of the advertising restriction plays a substantive role in the variants of the model that we consider in Sections 4 and 5.

3.1 Price discrimination and simple service

We extend the model in Section 3, to allow for service in a simple way. Here, we suppose that service is either 'on' or 'off'.[33] Moreover, both retailers need to provide service for demand to be non-zero.[34] Due to the 'on'/'off' nature of service, retailers incur a fixed cost of service of F. It is noteworthy that this service consideration implies that MAP may be optimal for the manufacturer even when the manufacturer

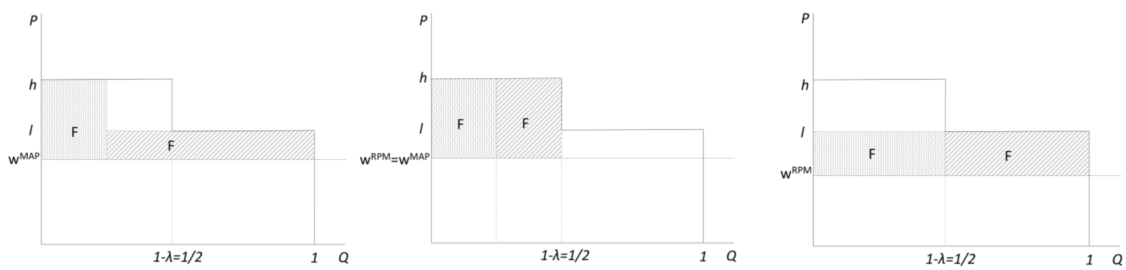
[33] This could correspond, for example, to a fixed cost to participate in the market.

[34] This is an (extreme) way of capturing the free-riding effect explored in Matthewson and Winter (1984) and many subsequent papers.

is restricted to linear tariffs.[35]

In this model of discrete service provision, where both retailers must be active, the market breaks down in the absence of vertical restrictions. This is due to the Bertrand competition between the retailers – at least one retailer will earn zero profits, before any fixed costs are deducted, and hence, anticipating this outcome, the high cost retailer has no incentive to provide service at cost $F > 0$. As a result, there is no demand for the product, and the product is not offered to the market.

Figure 2: Service with MAP and RPM



(a) Service with MAP (b) Service with RPM when $p^{RPM}=h$ (c) Service with RPM when $p^{RPM}=l$

Notes $c_L=c_H=0$, $\sigma_l=1$ and $\sigma_h=0$.

Panel (a) of Figure 2 illustrates how a price discrimination-based MAP program can be used to generate service. Building on the model illustrated in Figure 1, Panel (a) shows that, rather than capturing surplus using the lump-sum part of a two part tariff, the manufacturer may leave surplus with the retailers and the retailers can use this to fund service. That is, when the wholesale unit price is w^{MAP}, the rectangles with area F will be sufficient to provide service. The profit to the manufacturer in Panel (a) is equal to w. Note that all customers get served and some consumer surplus is generated for those high value consumers that find the low price retailer.

Panel (b) of Figure 2 illustrates the outcome when $p^{RPM} = h$. Given a cost of service F, the optimal wholesale unit price is equal to w^{MAP}. This is easy to see, once it is noted that the rectangles F in Panel (b) must be the same size as the rectangle F in Panel (a).[36] The key difference between Panel (b) and Panel (a) is that, while

[35]This is immediate, if for example it happens to be that $F = T^*(w^*)$ where $T^*(w^*)$ is as characterized in Proposition 1; but, of course, is true much more generally.

Instead, if $F = 0$ and the manufacturer is restricted to linear tariffs, it can be shown that the manufacturer would never gain through the use of a MAP restriction.

[36]It is also immediate from this observation that, in this model, if service can be provided under MAP, there exists an RPM policy that will also induce service.

service is provided in both instances, in Panel (b) a quantity distortion exists. This is, by virtue of MAP allowing more effective price discrimination, more consumers are served under MAP than RPM. Further, some consumer surplus is generated under MAP, but not when using RPM. Finally, the profit of the manufacturer using RPM is half that obtained when using MAP (w is the same, but half the consumers are served). Thus, MAP dominates RPM for both consumers and the manufacturer, despite the fact that service is provided under both policies.

Panel (c) shows what happens when the optimal RPM policy is $p^{RPM} = l$.[37] Note that $w^{RPM} < w^{MAP}$, since, under RPM, the retailers are sharing the entire market at the low price: this means consumers have more surplus, and to give the retailer sufficient rents to fund service, the manufacturer has to drop the wholesale unit price. Thus, consumers are better off under RPM, service is provided under both RPM and MAP, and no distortions exist. It still remains that, since $w^{MAP} > w^{RPM}$, the MAP policy is the more profitable way for the manufacturer to provide service.

The dominance, from the point of view of the manufacturer, of MAP policies as a way to incentivize service is not surprising, as it has been already established that, in this setting, MAP is a superior method of surplus extraction. What is perhaps more notable, is that when gains from trade are 'low', consumers also prefer MAP policies to RPM, as MAP does not generate a quantity restriction.[38]

We end by providing a parametrized example in which the possibility results and intuitions provided in this section are easily verified.

Example 2 *Let $\sigma_l = 1$, $\sigma_h = 0$, $c_H = c_L = 0$, $h = 1 + \epsilon$, $l = \epsilon$ and $\lambda = 0.5$. Let $F = \frac{3}{8}$. For $\epsilon < \frac{1}{2}$, neither RPM nor MAP will provide sufficient surplus to retailers to induce them to provide service and allow the manufacturer to make profits. For $\epsilon \in \left(\frac{1}{2}, 1\right)$, the optimal RPM price is h, $w = h - 4F$ and service is provided, but only the high value consumers are served and a deadweight loss exists. By contrast, with a MAP policy in which $p^{MAP} = h$ and $w = h - 4F$, all consumers are served, and no deadweight loss occurs. Further, half the high value consumers receive some consumer surplus. For $\epsilon > 1$, there is no deadweight loss under either MAP or RPM, service is provided and RPM generates the greater consumer surplus ($p^{RPM} = l$). For any $\epsilon > \frac{1}{2}$ the manufacturer's profits from MAP will be strictly higher than that using RPM.*

[37] This will occur after some point as h and l are increased by the same additive constant.

[38] That is, when $p^{RPM} = h$.

4 MAP and service provision

In this section, we develop the idea that a service component might lead a MAP arrangement to be optimal, independently of the usefulness of MAP as a price discrimination device (as in Section 3.1).

In this section, we focus on retailer cost heterogeneity, and suppose that consumers are homogeneous. This shuts down price discrimination. To further homogenize consumers, and to highlight the role of search costs, we set $\sigma_L = \sigma_H = 0$. That is, all consumers have high costs of search and so visit only one store. The model is enriched, such that that consumers have downward-sloping, rather than unit, demand. This means that retailers with different marginal costs have different optimal retail prices.

On the retailer side, we enrich the scope of service. Specifically, retailer j sets a service level $s_j \in [0, 1]$ at cost $I(s_j)$, which is continuously differentiable and increasing and $I'(s)$ is sufficiently high that equilibrium investments are strictly less than 1. Thus, service is now a continuous choice variable for each retailer. It acts to increase consumer awareness of the product.[39] A consumer that is aware of the product can purchase from either retailer, as in Matthewson and Winter (1984). An investment of s_j will expose a measure of consumers of size s_j to the product. The probability that a consumer is exposed to the investment of one retailer is independent of whether it is exposed to the other, so the total measure of consumers that are aware of the product is equal to $\mathcal{S} = 1 - (1 - s_1)(1 - s_2)$.

Finally, we suppose that each retailer's marginal cost of retailing is independently drawn from an ex ante known distribution. This marginal cost takes the value c_H with probability α, and is $c_L = 0$ otherwise. Retailer costs remain private information; that is, retailers do not know each others' realizations.[40] Note that that retailers' cost are realized only after entering into contracts with the manufacturer. Hence, the two-part tariff extracts surplus by setting the fixed fee equal to a retailer's expected net profit.[41]

This model shows how MAP can lead to pro-competitive service when the price discrimination channel outlined above is not operative. That is, the focus is shifted from MAP's efficacy in leveraging consumer heterogeneity, to its efficacy in environments dominated by retailer heterogeneity.

[39] As pointed out in Asker and Bar-Isaac (2014) service need not be pro-competitive, and (at least in their model) can lead to exclusion when retailers are gatekeepers to a market. All the arguments made in Asker and Bar-Isaac (2014) can be transferred to environments with MAP.

[40] At a technical level, this provides ex post price dispersion, which is needed for RPM and MAP to have a meaningful role.

[41] This is implied by the timing in Section 2.

4.1 No vertical restrictions

Assuming $p_1 \leq v'(0)$ so that there are positive sales, profit for R_1 conditional on a realization of $\mathcal{S}$, the measure of aware consumers, is given by

$$\pi(p_1, p_2) = \begin{cases} \mathcal{S}q(p_1)(p_1 - w - c_1) & \text{if } p_1 < p_2 \\ \frac{1}{2}\mathcal{S}q(p_1)(p_1 - w - c_1) & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \tag{3}$$

Profit for R_2 is similarly defined.

In considering the equilibrium of the pricing subgame (stage 5 in the timing in Section 2.3), note that this is a discrete analog to Spulber (1995) which points out that Bertrand with privately known costs mirrors the equilibrium in a first price auction with risk aversion. Equilibrium of the pricing subgame then follows Proposition 2 in Spulber (1995) which is, itself, an adaptation of Theorem 2 in Maskin and Riley (1984). Equilibrium pricing is characterized as follows.

Proposition 2 *There exists a unique symmetric equilibrium pricing strategy such that, for $j \in \{1, 2\}$:*

1. if $c_j = c_H$, $p_H = w + c_H$
2. $\overline{p} = \min\{p_H, p_L^M(w)\}$ where p_L^M is the monopoly price a low cost retailer would charge; i.e. $p_L^M = \arg\max(p - w)q(p)$
3. $\underline{p}$ is implicitly defined by $q(\underline{p})\left(\underline{p} - w\right) = \alpha(\overline{p} - w)q(\overline{p})$
4. if $c_j = 0$, price (p_L) is drawn from the distribution $F_L(p) = \frac{(p-w)q(p) - \alpha(\overline{p}-w)q(\overline{p})}{(1-\alpha)(p-w)q(p)}$ with the support $[\underline{p}, \overline{p})$.

Proof. See the appendix. ■

Next, we proceed by backward induction to consider the equilibrium of the investment subgame. Attention is restricted to symmetric equilibria where a retailer with a low cost realization chooses an interior investment $s_L > 0$. Note that given the pricing equilibrium in Proposition 2; it is immediate that a high cost retailer earns no (net of investment) profits and, therefore, that there is no investment at a high cost realization; that is, $s_H = 0$.

In choosing s_L, each retailer solves the following problem (illustrated here for Retailer 1):

$$\max_{s_1} E_{s_2}(\mathcal{S})\tilde{\pi}(c_L) - I(s_1) \tag{4}$$

where $E(\mathcal{S}) = 1 - (1 - s_1)(1 - \alpha s_H - (1 - \alpha) s_L) = 1 - (1 - s_1)(1 - (1 - \alpha) s_L)$ and $\tilde{\pi}(c_L) = \alpha(\overline{p} - w)q(\overline{p})$ (note, by construction, in the mixed strategy equilibrium profits adjusted by the probability of winning is constant over p). This yields the following best response function in the investment subgame that arises from the first order condition:

$$I'(s_1(c_L)) = (1 - (1 - \alpha) s_L)\alpha(\overline{p} - w)q(\overline{p}). \tag{5}$$

By imposing symmetry, the equilibrium advertising levels are implicitly defined as follows:[42]

$$I'(s_L) = (1 - (1 - \alpha) s_L)\alpha(\overline{p} - w)q(\overline{p}). \tag{6}$$

Finally, we can turn to the problem of the manufacturer. The manufacturer sets T equal to a retailer's expected profits so that

$$T = (1 - \alpha) \left[(1 - (1 - s_L)(1 - (1 - \alpha) s_L))\alpha(\overline{p} - w)q(\overline{p}) - I(s_L)\right],$$

where this expression follows on noting that only the low cost retailer earns profits and expected size of market and expected profits per customer follow from the expressions that appear below (4). The manufacturer chooses w to maximize:

$$2T + w\left[2\alpha(1 - \alpha)s_L \int_{\underline{p}}^{\overline{p}} q(p)f_L(p)dp + (1 - \alpha)^2(1 - (1 - s_L)^2) \int_{\underline{p}}^{\overline{p}} q(p)2f_L(p)(1 - F_L(p))dp\right],$$

where the term in square brackets corresponds to expected sales. To understand this term, first note that when both retailers have high costs then there is no investment in advertising and so there are no sales; with probability $2\alpha(1 - \alpha)$ there is just one low cost retailer, a mass of s_L consumers aware of the product and each consumer purchases $\int_{\underline{p}}^{\overline{p}} q(p)f_L(p)dp$ in expectation; finally the second term inside the square brackets describes the event in which both retailers get low cost realizations. When both retailers are low cost, consumers aware of the product will purchase from the lower of two price drawn independently from $F_L(.)$; thus, the final integral reflects the expected quantity corresponding to a price distributed as the second order static of $F_L(.)$.

4.2 RPM

From the perspective of the manufacturer, resale price maintenance can strictly improve on the outcome with no restraints. The channels that allow this are standard.

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[42]Imposing specific functional forms allows for a closed form solution, for example when $I(s) = \frac{s^2}{2}$ then $s_L = \frac{\alpha(\overline{p}-w)q(\overline{p})}{1+(1-\alpha)\alpha(\overline{p}-w)q(\overline{p})}$.

First, minimum RPM can soften price competition between retailers, and hence, by allowing the retailers higher expected profits, can induce greater investment. For example, consider the case where α is close to 0, so that both retailers are almost certainly low cost retailers. Retailer (net) profits, and hence advertising investments, would be negligible. In this case, a minimum RPM provision that guaranteed retailers some net profits and induce some investment would clearly help induce service. Second, retail prices (for retailers with low cost realizations) may be too high due to a standard double mark-up problem; maximum RPM can help solve this. Here, the focus is on the case of a binding minimum RPM, set at a level denoted by P, reflecting the need for service.[43]

First, note that if minimum RPM is binding for both high and low cost retailers, the retail price (regardless of the retailers' cost realizations) is given by P. Instead, if the restriction is binding only for retailers with low cost realizations, then the pricing equilibrium is as follows:

Proposition 3 *Suppose that the minimum RPM price is $P < w + c_H$ (so that it does not bind given a high cost realization) but binds given the low cost realization, there exists a unique symmetric equilibrium pricing strategy such that for $j \in \{1, 2\}$:*

1. if $c_j = c_H$, $p_H = w + c_H$
2. $\overline{p}_{\min} = \min\{c_H + w, p_L^M(w)\}$
3. for $p < \overline{p}_{\min}$, $F_{\min P}(p)$ is characterized by $\alpha(\overline{p}_{\min} - w)q(\overline{p}_{\min}) = q(p)(p-w)(\alpha + (1-\alpha)(1 - F_{\min P}(p))$
4. $\underline{p}_{\min}$ is implicitly defined by $q(P)(P-w)\left[\alpha + (1-\alpha)\frac{F_{\min P}(\underline{p}_{\min})}{2} + (1-\alpha)(1 - F_{\min P}(\underline{p}_{\min}))\right] = q(\underline{p}_{\min})(\underline{p}_{\min} - w)(\alpha + (1-\alpha)(1 - F_{\min P}(\underline{p}_{\min}))$
5. if $c_j = 0$, and $\underline{p}_{\min} < \overline{p}_{\min}$ then price (p_L) is drawn from the distribution $F_{\min P}(p)$ with support $P \cup (\underline{p}_{\min}, \overline{p}_{\min})$ and an atom at P
6. if $c_j = 0$, and $\underline{p}_{\min} \geq \overline{p}_{\min}$ then $p_L = P$ with probability 1.

Proof. See the appendix. ■

[43] While the double margin issues are not the primary focus, it should be noted that the problem is limited by $p_H = w + c_H$.

The characterization of the pricing equilibrium suggests that in case the optimal minimum RPM induces price dispersion, it cannot aid the manufacturer through inducing service provision.[44]

Corollary 1 *Suppose that the minimum RPM price is $P < w + c_H$ (so that it does not bind given a high cost realization) and that a low cost retailer plays a mixed strategy in pricing, then the level of investment in service is identical to the level of investment in the absence of RPM or any other restrictions—that is the solution to (4).*

Proof. This is immediate on noting that a retailer with a high cost realization makes no net profits and that $\overline{p}_{\min} = \overline{p}$ so that a retailer's profit per aware consumer is identical to the case of no restrictions, so that the low cost retailer faces the identical maximization problem (4) in its choice of investment in service. ■

It follows that RPM can only play a role in inducing service provision in one of two cases (i) either it binds for both low and high cost realizations; that is, if $P > w + c_H$; or (ii) it leads a firm with a low cost realization to choose P with certainty (which requires $\underline{p}_{\min} > \overline{p}_{\min}$ as defined in Proposition 3). We provide a complete characterization of the manufacturer's problem with RPM, analogous to our analysis of the case with no restrictions, above, in Appendix B. Here, it is worth noting that in the former case, where both high and low cost firms set a price equal to $P > w + c_H$, demand for a retailer irrespective of its costs is given by $\frac{q(P)}{2}$. Service for low cost and high cost realization can be easily implicitly characterized through the first order conditions as follows:

$$I'(s_L(P)) = (1 - \alpha s_H - (1-\alpha) s_L) \frac{q(P)}{2} (P - w), \text{ and} \tag{7}$$

$$I'(s_H(P)) = (1 - \alpha s_H - (1-\alpha) s_L) \frac{q(P)}{2} (P - w - c_H). \tag{8}$$

Note that these equations must be satisfied simultaneously; in comparison, in the case of no restrictions solving for a high cost retailer's investment is trivial which allows for a simple characterization of a low cost retailer's investment in (5). In particular, investment by the high cost and low cost retailers are strategic substitutes.[45]

[44] Interestingly, minimum RPM (and not only maximum RPM) can play a role in affecting "double mark-up" concerns by changing the price distribution and so may still dominate no restriction from the manufacturer's perspective even if it has no impact on service provision.

[45] For specific parameterizations unique closed form solutions are assured. For example, for quadratic investment costs $I(s) = \frac{s^2}{2}$, $s_L(P) = \frac{(P-w)q(P)}{2+(P-w-\alpha c)q(P)}$ and $s_H(P) = \frac{(P-w-c)q(P)}{2+(P-w-\alpha c)q(P)}$.

It is straightforward to find instances where the manufacturer might benefit from imposing such a price restriction; in particular, this will be the case for α close to 0 or 1.

Summarizing the discussion above, we obtain Proposition 4.

Proposition 4 *Minimum RPM set at P can strictly improve the manufacturer's profits relative to the case of no restrictions in the case of smooth advertising investments. It can only do so through raising retailers' advertising investments when it leads retailers to choose pure pricing strategies.*

Proof. See the appendix. ∎

4.3 MAP

MAP can have the benefit of the minimum RPM scheme described above in dulling competition between retailers and allowing them higher profits, thereby encouraging investment in advertising. However, MAP has the benefit that it allows some retailers to drop actual price below the advertised minimum. The reason that they may want to do this is that their monopoly price may be below the MAP price. Moreover, this can be in the interest of the manufacturer as it increases the sales per customer for customers who are aware relative to the minimum RPM policy, and can increase the low cost retailer's level of investment and the overall number of customers aware of the product.

Specifically, MAP at the MAP price P plays a role that might be different from RPM when the high cost retailer charges this price, and the low cost retailer charges a different (lower) price.[46] It is clear that the low cost retailer would charge $p^m(w)$ which maximizes $\frac{q(p^m(w))}{2}(p^m(w) - w)$.

Under MAP, half of the consumers who are aware of the good purchase from either retailer (irrespective of its costs), since the retailers advertise the same price. These consumers then respond to the price that they observe at the retailer. Consequently, service for low cost and high cost realization can be easily implicitly characterized

[46] At a sufficiently high MAP price, both a high cost retailer and a low cost retailer would charge a lower price than the MAP price. However, since a high cost retailer's price can be predicted—it is simply its monopoly price (given w)—the manufacturer could set the MAP price at this level. Consequently, it is without loss of generality to suppose that the MAP price is always binding for a high cost retailer in equilibrium. For expositional simplicity we assume $P \leq p_H^M(w)$.

through the first order conditions as follows:[47]

$$I'(s_L(P)) = (1-\alpha s_H - (1-\alpha)\, s_L)\frac{q(p^m(w))}{2}(p^m(w)-w), \text{ and} \tag{9}$$

$$I'(s_H(P)) = (1-\alpha s_H - (1-\alpha)\, s_L)\frac{q(P)}{2}(P-w-c_H). \tag{10}$$

These differ from the equations that characterize investment under RPM at the same price P, in (7) and (8) since the low cost retailer sells at a price $p^m(w)$ rather than P and sells $q(p^m(w))$ to each consumer who arrives at the retailer rather than $q(P)$. Given that retailers' investments are strategic substitutes, this affects the equilibrium investments of both retailers. Thus, although greater investment from a low cost retailer might be anticipated as a result of the imposition of MAP, a consequence would be a force that dampens the high cost retailer. As for the case of no restrictions, the manufacturer optimally T equal to a retailer's expected profits and chooses w to maximize expected profits. A formal treatment can be found in Appendix B.

We argue that the different economics of MAP as compared to RPM, by allowing the low cost retailer to charge a lower price than the MAP price, lead to higher industry profits and more effective investments. We do so both by example and in the Proposition below.

Proposition 5 *When investment in service take any value in* $[0,1]$*, MAP can earn* M *strictly higher profits than minimum RPM. When investment costs are sufficiently convex or if the firm is unlikely to be low cost (*α *is high enough), then MAP can never earn less than minimum RPM.*

Proof. Suppose that the optimal min RPM involves a two-part tariff (T, w) and the level of the price minimum at P. Rather than considering the optimal MAP scheme, suppose that MAP is imposed at P and with a wholesale price of w, it is immediate that a retailer with a high cost realization would choose a retail price equal to P. However, it is possible that the monopoly price for a retailer with a low cost realization is below P. In this case, under MAP, a retailer with a low cost realization would charge this monopoly price $p_L^m(w) = \arg\max(p-w)q(p) < P$.

This potentially affects M's profits in several ways:

First, through sales directly, by increasing the sales per aware consumer (since a low cost retailer charges a lower price). Secondly, it affects sales by changing retailers'

[47]For example, with quadratic investment costs $I(s) = \frac{s^2}{2}$, a unique closed form solution is assured. Specifically, $s_L(P) = \frac{(P-w)q(P)}{2+(P-w-\alpha c)q(P)}$ and $s_H(P) = \frac{(P-w-c)q(P)}{2+(P-w-\alpha c)q(P)}$.

investments: Holding fixed the service level of a high cost retailer, it is clear that a lower cost retailer would invest more. However, the higher investment of a low cost retailer reduces a high cost retailer's gains from investment, as discussed below Equation (10). Overall, the combined effect is ambiguous. With sufficiently convex costs (or high α), the effects on the low cost retailer's investment is negligible leading to a positive overall effect on profits.

Finally, there is an effect through a (potentially) higher fixed fee T^{MAP} which might also reflect different service levels and so, in principle, the sign of the effect might be ambiguous. Again, when the investment cost function is sufficiently convex or α high enough, the impact on service levels is small, so that this effect is also positive. Thus, overall M's profits rise.

Finally, to easily observe that it indeed possible to have $p_L^m(w) < P$ and, thereby, prove the first statement of the proposition, consider a case where α is close to 1 and $c_H > p_L^m(0)$. ■

It is worth noting that MAP may under-perform RPM from the perspective of the manufacturer. This is (trivially) the case when all consumers can search costlessly ($\sigma_l = \sigma_h = 1$) and MAP serves no useful function.

That said, the earlier Example 2 and Example 3, below, which provides a numerical counterpoint to Proposition 5, make it clear that settings do exist in which, by allowing for greater investment in efficient service and simultaneously allowing more sales to realized, MAP can be pro-competitive (whether judged on a consumer surplus or total surplus criterion).

Example 3 *Let $q(p) = 1 - p$, $\alpha = 0.9$ and $c_H = 0.4$. Table 1 shows the characteristics of the optimal polices in the no restrictions, RPM and MAP cases. MAP generates the highest consumer and total surplus, and the highest expected manufacturer and retailer profits.*[48]

5 MAP and collusion

Vertical restrictions, and RPM in particular, have long been regarded as potentially collusive mechanisms. That is, the binding of retailers, by the manufacturer, to a common price, can look, at first glance, like a price fixing agreement.[49] In the modern

[48] Matlab code for generating the results in Table 1 can be found at www.johnasker.com/MAPservice.zip

[49] See Overstreet (1983) for a comprehensive historical survey.

Table 1: Results for example 3: $\alpha = 0.9$, $c_H = 0.4$, $q(p) = 1 - p$

	No restrictions	RPM	MAP
Manufacturer expected profits	0.0057	0.0077	0.0084
Retail expected profits	0.0020	0.0039	0.0041
Low cost R profits	0.0203	0.0109	0.0125
High cost R profits	0	0.0031	0.0032
P	-	0.6723	0.7000
w	0.0805	0	0
s_L	0.1836	0.1048	0.1140
s_H	0	0.0424	0.0428
Consumer Surplus	0.0061	0.0051	0.0065
Total Surplus	0.0159	0.0205	0.0231

era, the concern that RPM facilitates collusion continues to shape policy, supported by research, most notably the central contribution of Jullien and Rey (2007).[50] Jullien and Rey (2007) demonstrate that RPM facilitates collusion by making deviations more transparent, at the cost of reducing responsiveness to local idiosyncratic demand fluctuations.[51,52] In the analysis below, it is shown that MAP can similarly soften competition, again by inducing demand patterns akin to a market division scheme, without reducing responsiveness to local market demand fluctuations. This results in MAP facilitating a more profitable and stable cartel.

The idea can been seen heuristically in the following simple example. Two manufacturers sell to a market via dedicated (non-shared) retailers. Consider a market in which there is always a high value consumer who would pay h. In addition, with probability μ, there is an additional consumer who has a low valuation l. This additional consumer is local to one of the two manufacturers' retailers (with equal

[50] Jullien and Rey (2007) provide a formal foundation for ideas that have existed, albeit in only heuristic form, since at least Telser (1960) and Yamey (1954).

[51] Julien and Rey (2007) note other elements of the tradeoff, notably that RPM can make punishment more severe.

[52] In a recent paper Sugaya and Wolitzky (2016) highlight that even costless transparency might not always be beneficial for sustaining a cartel; although in many standard settings it is.

probability). That is with probability $1 - \mu$ there is only a high value consumer, and with probability $(\mu/2)$ there is also a low value consumer available to manufacturer 1 and with probability $(\mu/2)$ a low value consumer available to manufacturer 2. Suppose that $2l > h$ so that if a low value consumer is available, the industry would earn higher profits by selling to both consumers than to the high value consumer alone.[53]

Jullien and Rey (2007) argue that RPM allows the manufacturers to more easily monitor compliance with a collusive scheme at the cost of less responsiveness to demand (or cost shocks). In the simple environment outlined above, RPM does not help manufacturers collude (with this simple unit demand specification, the manufacturers can agree on a common (high) retail price by setting $w_1 = w_2 = h$), and observing retail prices allows manufacturers to monitor their compliance with this scheme. MAP accommodates heterogeneity in this case by allowing for responses to local demand shocks but without intensifying competition (since, for a retailer, there is always a 50% chance of selling to the high value consumer and if there is a low cost consumer then the relevant retailer will want to sell to him and does so). Hence, here, MAP can increase the scope for collusion relative to no vertical arrangements or, indeed, relative to RPM.

In what follows we embed this logic in a formal model that builds on the Jullien and Rey (2007) framework. The modeling objective is to allow price advertising to have an explicit role in the market, while retaining tractability and the economic forces illustrated in Jullien and Rey (2007). The resulting framework is one in which RPM facilitates a more stable cartel (in the sense of admitting collusion for a wider range of discount rates than a cartel without restraints), at the cost of flexibility. MAP, by contrast, can facilitate a yet more stable cartel, and does not sacrifice flexibility, leading to higher cartel profits.

5.1 Model details

All model details are in Section 2. The essential elements are provided here for convenience. There are two manufacturers, each of whom distributes to the local market via a dedicated (non-shared) retailer. In this section, we suppose that consumers have unit demand and value retailer R_j's good at $v + \xi_j$, where $\xi_j \sim U[0, 1]$ is a manufacturer-retailer specific demand shock reflecting idiosyncratic local market factors. It follows that consumers will choose to visit retailer R_j if

$$v + \xi_j - E\left(p_j | p_j^a\right) \geq 0 \tag{11}$$

[53] Note that this also implies that each manufacturer would rather sell with certainty at a price l than sell at a price of h with probability (1/2).

and

$$v + \xi_j - E\left(p_j | p_j^a\right) > v - \xi_k - E\left(p_k | p_k^a\right) \tag{12}$$

The first inequality reflects that the consumer must expect non-negative surplus, and the second that the consumer expects greater surplus from visiting R_j than R_k. If the first inequality holds and the second condition holds with equality, then we suppose that the consumer purchases from whichever retailer offers a higher ξ. To keep exposition simple, $\sigma = 0$ so consumers visit at most only one store (search costs are high).

Furthermore, we suppose that ξ_j is not observed by any manufacturer nor by the rival retailer; it is only privately observed by R_j after contracting with the manufacturer and before setting the retail price. Each manufacturer sells to its dedicated retailer using a two-part tariff $\{w_j, T_j\}$ (which are not publicly observed).[54] Lastly, $v > 1$, which ensures that w_j is always set such that all ξ_j realizations are served. Advertised prices are observed by all—including rival manufacturers.

5.2 Collusion with no vertical restrictions

With no vertical restraints, deriving the equilibrium of this game is somewhat involved. Here we sketch the elements, and provide a characterization that is fleshed out in the appendix. Many of the details follow the underlying logic in Jullien and Rey (2007), albeit applied to a different demand system. Once characterized, collusion in the absence of vertical restraints provides a benchmark against which to compare the much simpler RPM- and MAP-facilitated schemes.

We begin by presenting a preliminary result that will prove useful. Specifically, we begin by examining the pricing game between retailers, supposing (as will turn out to be the case in equilibrium) that manufacturers choose the same input price; that is, supposing that $w_1 = w_2 = w$. In this case, the pricing game between retailers is a minor variant on the standard IPV symmetric first price sealed bid auction in which each retailer solves:

$$p_j = \arg \max_{w \leq p \leq v + \xi_j} \; (p - w) \Pr\left(\xi_j - p > \xi_k - p_k\right) \tag{13}$$

Equilibrium pricing is stated in Proposition 6 and derived following the steps in, for instance, Krishna (2002).

[54] These assumptions reflect those in Julien and Rey (2007). The assumptions on ξ_j give a reason to give some pricing discretion to the retailer. The assumptions on $\{w_j, T_j\}$ make the cartel's monitoring of compliance non-trivial (and keeps IR constraints relatively simple).

Proposition 6 *Assume that $w_j = w_k = w$. In a monotone perfect Bayesian equilibrium, $p_j^a = p_j = w + \frac{\xi_j}{2} - \frac{w-v}{2}\mathbb{1}_{v<w}$.*

Proof. See the appendix. ■

Proposition 6 means that if $w = 0$, as would arise in the static version of the game, then retailers price using the rule $p_j = \frac{\xi_j}{2}$. That is, prices fluctuate by half the actual fluctuations in local demand. The retailers' expected profit, which is then transferred to the manufacturer through the fixed fee, T_j, is equal to the expectation of $\frac{\xi_j^2}{2}$, which is $\frac{1}{6}$.[55]

Instead, the manufacturer-optimal scheme in this setting involves both manufacturers setting $w = v$ and $T = \frac{1}{6}$. That is, the manufacturers set w as high as possible without excluding any consumers,[56] and then retailers compete accounting for idiosyncratic market conditions.[57] The manufacturers extract the remaining expected surplus from retailers using the lump sum component of the two part tariff.

In a collusive outcome, this scheme is enforced using a variant of a grim trigger strategy, in which manufacturers set $w_t = v$ if all past advertised pricing has been in the interval $[v, v + \frac{1}{2}]$, and $w_t = 0$ otherwise. This results in an expected per period collusive profit, for each manufacturer, of $\frac{v}{2} + \frac{1}{6}$. The characterization is complicated by the fact that determining the optimal deviation is not immediate. In particular, if a manufacturer charges a slightly lower wholesale price in some period t, because of the local demand ξ_j demand variations, this may lead to only a marginal gain but will have only a marginal probability of detection. Alternatively, a manufacturer might prefer a more drastic deviation that leads to a substantive gain albeit at the cost of a higher probability of detection. These two forms of deviation can, in principle, lead to two locally optimal deviations, which then need to be compared. Full details appear in the appendix.[58] The following proposition provides the lower bound on the discount factor, δ, required for this collusive scheme to be supportable in equilibrium.

Proposition 7 *A collusive equilibrium in which manufacturers set $w_t = v$ if $p_j^a \in [v, v+\frac{1}{2}]$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable if $\delta > \frac{6v-2}{9v-2}$. In such an equilibrium, the per period profit earned by each manufacturer, π_{NR}^c, is $\frac{v}{2} + \frac{1}{6}$.*

Proof. See the appendix. ■

[55] That is, the retailer's expected profit is $\int_0^1 \frac{\xi_j}{2} \Pr(\xi_j > \xi_k) d\xi_j$.

[56] This follows from the assumption that $v > 1$.

[57] We assume retailers are not parties to the cartel agreement and always play statically optimal equilibrium strategies.

[58] Jullien and Rey's central proof is aimed at eliminating exactly the same types of deviations in their setting.

5.3 Collusion with RPM

When RPM is available, the manufacturers can set the price at which the retailers sell to customers. As discussed in Section 2, this will also be the advertised price. The value of RPM is that it removes any uncertainty on the part of the cartel members as to whether there has been a deviation or not, albeit at the expense of losing some flexibility in adjusting to local market conditions (that is, adjusting for ξ_j realizations). Thus, collusion can now take the form: Set $w_t = p^{RPM} = v$ if the same advertised pricing has been observed in all past periods, and $w_t = 0$ otherwise.[59] Proposition 8 describes this much simpler, collusive environment.

Proposition 8 *A collusive equilibrium in which manufacturers set $w_t = p_t^{RPM} = v$ as long as $p_{j,t}^a = v$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable if $\delta \geq \underline{\delta}^{RPM} = \frac{3v}{6v-1}$. In such an equilibrium, the per period profit earned by each manufacturer, π_{RPM}^c, is $\frac{v}{2}$.*

Proof. The proof is standard. It involves solving for δ such that $\frac{\pi_{RPM}^c}{1-\delta} \geq \pi_{RPM}^D + \frac{\delta}{1-\delta}\pi^p$, where $\pi_{RPM}^c = \frac{v}{2}$, $\pi_{RPM}^D = v$ and $\pi^p = \frac{1}{6}$. ■

5.4 Collusion with MAP

Collusion, facilitated by MAP, involves coordinating on the advertised price, but allowing the transaction price (as usual under MAP) to only be constrained to be less than or equal to that advertised price. The value of this to the colluding manufacturers is that it obfuscates pricing in the market for consumers, effectively introducing a form of market division, while retaining flexibility on the part of consumers to adjust prices to local idiosyncratic conditions; that is it is a scheme that partially accommodates the realized demand heterogeneity. In this environment, the optimal collusive strategy is to set $p_{j,t}^a = p_{k,t}^a = v + 1$, resulting in the market being split equally. Retailers, having retained the flexibility to adjust prices downward, now are free from competition and can extract all consumer surplus by setting $p_{j,t} = v + \xi_{j,t}$ (in contrast, with no restraints there is competition so that $p_{j,t} = v + \frac{\xi_{j,t}}{2}$). Thus the MAP provision, in this environment allows the retailer to extract all available surplus from consumers they service (assuming a rationing rule in which consumers

[59] An alternative punishment phase may involve using RPM and setting $w_t = p^{RPM} = 0$. This is a more extreme form of punishment. With this punishment stage equilibrium, $\pi^p = 0$ in the notation of the proof of Proposition 8. Thus, in Proposition 8, the threshold would adjust to $\delta > \frac{1}{2}$.

purchase the good for which they have the highest value, given their indifference between goods after expected pricing is taken into account).[60] Proposition 9 formalizes this intuition.

The proposition supposes that if the cartel agreement breaks down, the manufacturers revert to play the static equilibrium policies.[61]

Proposition 9 *A collusive equilibrium in which manufacturers enforce $p_{j,t}^{MAP} = v+1$ as long as $p_{j,t}^{a} = v+1$ for all j in all past periods, and $w_t = 0$ without MAP otherwise, is supportable if $\delta \geq \underline{\delta}^{MAP} = \frac{1}{2}$. In such an equilibrium, the per period profit earned by each manufacturer, π_{MAP}^{c}, is $\frac{v}{2} + \frac{1}{3}$.*

Proof. The proof is standard, requiring solving for δ such that $\frac{\pi_{MAP}^{C}}{1-\delta} \geq \pi_{MAP}^{D} + \frac{\delta}{1-\delta}\pi^{p}$, where $\pi_{MAP}^{c} = \frac{v}{2} + \frac{1}{3}$, $\pi_{MAP}^{D} = v + \frac{1}{2}$ and $\pi^{p} = \frac{1}{6}$. ■

5.5 Vertical restraints can facilitate collusion

A comparison of Propositions 7, 8 and 9 makes it clear that the collusive profits with MAP dominate those without restrictions, which in turn dominate those from employing RPM. That is, $\pi_{MAP}^{c} > \pi_{NR}^{c} > \pi_{RPM}^{c}$.

Further, employing a vertical restraint (whether MAP or RPM) for the cartel can increase stability, in the sense that the force of future retribution for a deviation coupled with the prospect of losing collusive gains becomes a stronger incentive relative to the gains from deviating. This increase in the strength of dynamic incentives is captured by comparing the lower bound of the range of discount factors under which collusion can be supported. It can be easily verified that for v high enough (a sufficient condition is that $v > \frac{2+2\sqrt{2}}{3}$ so that $\frac{3v}{6v-1} < \frac{6v-2}{9v-2}$) a cartel can be more easily sustained with RPM. MAP, by contrast, increases stability relative to no restraints for a wider range (or, indeed, RPM). This follows as it is easier to monitor a manufacturer deviation from the cartel arrangement, when this arrangement involves a vertical restraint. When v is low then an RPM arrangement may be sacrificing too much in terms of profits, just as in Jullien and Rey (2007), and the flexibility to

[60] In any other rationing rule, the surplus extraction is inefficient, in that by retarding competition, consumers are not directed toward those products that maximize gains from trade. In the case where consumers who are indifferent are equally likely to visit either firm, the corresponding expressions for those that appear in Proposition 9 are $\pi_{MAP}^{C} = \frac{v}{2} + \frac{1}{4}$ and $\underline{\delta}^{MAP} = \frac{6v+3}{12v+4}$.

[61] An alternative punishment phase may still involve using MAP and setting $w = p^{MAP} = 0$. This is a more extreme form of punishment. With this punishment stage equilibrium, $\pi^{p} = 0$. In this case, Proposition 9 would instead require $\delta > \frac{1}{2}$.

respond to local demand conditions is relatively valuable. MAP, on the other hand, maintains flexibility and so does not demand the same trade-off.

Finally, as mentioned above, $\underline{\delta}^{MAP} < \underline{\delta}^{RPM}$ suggesting that MAP policies may help in stabilizing cartel agreements relative to RPM in addition to providing higher cartel profits.[62] This follows since, although in both arrangements it is equally easy to monitor deviations from the cartel agreements, the MAP arrangement allows for a higher cartel profit since it accommodates the heterogeneity associated with local demand variation.

Thus MAP policies can allow a manufacturers' cartel to attain greater surplus extraction than a cartel that does not use MAP or RPM, and strengthen the dynamic incentives that give the cartel stability, such that it is at least as stable as RPM. As such, in this setting, MAP appears a more effective cartel facilitation device that RPM.

6 Discussion

This paper has discussed both pro- and anti-competitive features of MAP policies. MAP policies are distinct from RPM in that they can serve as a more effective way to incentivize service, and as a more effective way to facilitate collusion. As such it suggests that, to the extent that controversy surrounds the appropriate policy treatment of vertical price restraints, the same, or greater, controversy should surround vertical information restraints, of which MAP is a prominent example. At the very least, carefully considering the nature and impact of information constraints in markets where search is a central feature seems warranted.

This paper leaves at least three areas of enquiry open. First, aside from the collusion model, the frameworks presented here do not consider the impact of competition at the manufacturer level. Given that information restraints will likely change the cross-price elasticities between competitors, even absent collusion, this seems a rich area for further investigation.

Second, the models presented here limit the role of advertising to pure price advertising. This is analytically helpful in creating a clear mapping between the advertised price and the transaction price, but suppresses aspects that may be important in a richer model. An obvious issue in the price discrimination and collusion models is that unit demand means that prices do not influence quantities to the extent they might in a real market. A more delicate, and interesting, issue is that advertised

[62]As already noted above, in Footnotes 59 and 61, if MAP or RPM is used in the punishment stage, then $\underline{\delta}^{MAP} = \underline{\delta}^{RPM}$.

prices may signal quality—an issue that is claimed to be a concern in some MAP policy statements.

Lastly, the extent to which MAP, like other vertical restraints, can generate exclusion of a rival is not explicitly investigated, although it is conjectured that, as elsewhere, this effect can be generated.

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Appendix A: Proofs

Lemma 1 *Suppose that there are no advertising restrictions, then any perfect Bayesian equilibrium is equivalent to one in which each retailer sets its advertised price equal to its actual price i.e.* $p_j^a = p_j\ \forall j$.

Proof. Suppose, toward a contradiction, that there is an equilibrium where this is not the case. Then the equilibrium must involve (at least) two retailers, who charge different prices, choosing the same advertised price a. Further, this must happen with non-zero probability.

Consider all retailers who advertise at a. Of this set, let $p_{\min}(a)$ denote the price of the retailer with the lowest actual price. Note that since there is more than one retailer who advertises at a, it must be the case that consumers anticipate a non-zero probability of an actual price strictly greater than $p_{\min}(a)$, and hence expect surplus smaller than that generated by receiving $p_{\min}(a)$ with certainty.

The restriction that a retailer cannot charge a price higher than its advertised price implies that, if the retailer charging $p_{\min}(a)$ advertised its actual price $p_{\min}(a)$, consumers could not put any probability on the retailer charging a higher price than $p_{\min}(a)$ (and might even put some probability on a lower price). That is they anticipate an actual price that is equal to, or lower than $p_{\min}(a)$. Expected surplus is similarly strictly increased.

Putting together the last observations in each of the two paragraphs above, it is immediate that setting the advertised price at $p_{\min}(a)$ would attract strictly greater demand as compared to choosing the advertised price a. This may be because demand is downward sloping, or through competition with another retailer. This generates the required contradiction.

Lastly, note that the argument above may support multiple equilibria. For instance, consider a market with two retailers. If equilibrium prices are uniquely set at p_1 and p_2 ,such that $p_1 < p_2$, then the argument above can be used to construct equilibria in which $p_1^a \in [p_1, p_2)$ and $p_2^a \geq p_2$. Given no economically meaningful distinction exists between all these equilibria, they are considered *equivalent* to an equilibrium in which $p_j^a = p_j$ for all retailers j. In this example, the essential point is that $p_1^a < p_2 \leq p_2^a$. ■

Lemma 2 I*n a MAP regime, in all monotone perfect Bayesian equilibria, each retailer advertises its actual price unless the MAP restriction binds, and in this case it advertises the MAP price; i.e.* $p_j^a = \max\left\{p^{MAP}, p_j\right\}$.

Proof. The proof follows that of Lemma 1, but appeals to monotone beliefs rather than Bayesian updating (that is, monotone beliefs take the place of the requirement that advertised prices be at least as great as actual prices). ■

Remark 1 *Let $\sigma_l = 1$, $\sigma_h = 0$, and $c_H = c_L = 0$. Suppose the manufacturer offers (w, T) and sets $p^{MAP} = h$. A monotone perfect Bayesian equilibrium in which both retailers set $p^a = h$, and R_L sets $p = l$ and R_H sets $p = H$, exists if*

$$(h-w)(1-\lambda)\frac{1}{2} \geq (l-w)\left(\lambda + \frac{1-\lambda}{2}\right) \tag{14}$$

$$(l-w)\left[\lambda + (1-\lambda)\frac{1}{2}\right] \geq (h-w)(1-\lambda)\frac{1}{2} \tag{15}$$

If this equilibrium exists, it is unique if Conditions 1 and 2 are strict inequalities. The optimal MAP policy sets w^*, such that Condition 2 holds with equality, and T^* such that $(h-w)(1-\lambda)\frac{1}{2} - T^* = 0$.

Proof. Condition 1 corresponds to R_H preferring to charge h rather than l (in Figure 1, that $A+B \geq B+C$). Condition 2 corresponds to R_L preferring to charge l rather than h (in Figure 1, that is, $C+D \geq A+B$). Given that either $p = h$ or $p = l$ dominate all other retail prices, these are conditions for best responses. Coupled with Lemma 2, this establishes existence.

Next, note that if Condition 1 binds, then Condition 2 is slack, since $\left[\lambda + (1-\lambda)\frac{1}{2}\right] > \frac{1}{2}$. Conversely, by the same reasoning, if Condition 2 binds then Condition 1 is slack. Note it is impossible for both conditions to bind.

If Condition 1 binds and 2 is slack, then another equilibrium exists in which both retailers set $p = l$. Having Condition 1 bind, implies that retailers are indifferent between $p = l$ and $p = h$ when the other retailer is pricing l. Similarly, if Condition 2 binds and 1 is slack, then another equilibrium exists in which both retailers set $p = h$.

Uniqueness applies, up the identity of the retailers. If both conditions hold as strict inequalities, then the best response sets become singletons.

To see optimality, note that, if the surplus of the retailers are not equal, then the lump-sum payment, T, will leave some surplus with one of the retailers. Hence, full extraction requires that Condition 2 bind. As already noted, this means that Condition 1 is satisfied. ■

Proposition 2 *There exists a unique symmetric equilibrium pricing strategy such that, for $j \in \{1, 2\}$:*

1. *if $c_j = c_H$, $p_H = w + c_H$*

2. $\overline{p} = \min\{p_H, p_L^M(w)\}$ *where* p_L^M *is the monopoly price a low cost retailer would charge; i.e.* $p_L^M = \arg\max(p - w)q(p)$

3. $\underline{p}$ *is implicitly defined by* $q(\underline{p})\left(\underline{p} - w\right) = \alpha(\overline{p} - w)q(\overline{p})$

4. *if* $c_j = 0$, *price* (p_L) *is drawn from the distribution* $F_L(p) = \frac{(p-w)q(p)-\alpha(\overline{p}-w)q(\overline{p})}{(1-\alpha)(p-w)q(p)}$ *with the support* $[\underline{p}, \overline{p})$.

Proof. By standard Bertrand reasoning, if $c_j = c_H$ then $\pi(p_H) = 0$. This implies $p_H = c_H + w$.

For $c_j = 0$, the equilibrium price distribution resembles that in Stahl (1989). The maximal price is the minimum of the monopoly price given w and $c_j = 0$, denoted $p_L^M(w)$, and $c_H + w - \varepsilon$ such that $\varepsilon \longrightarrow 0$. That is, if $p_L^M(w) \geq c_H + w$ then the maximal price of the price distribution will be arbitrarily close to $c_H + w$. Hence, if $p_L^M(w) \geq c_H + w$, $\overline{p} = c_H + w$. Otherwise $\overline{p} = p_L^M(w)$.

Since a mixed strategy requires that $\alpha(\overline{p} - w)q(\overline{p}) = [\alpha + (1-\alpha)(1 - F_L(p))](p - w)q(p)$ for all p in the support, it follows that $F_L(p) = \frac{(p-w)q(p)-\alpha(\overline{p}-w)q(\overline{p})}{(1-\alpha)(p-w)q(p)}$.

The lower bound of the support is found by setting $F_L(p) = 0$. ■

Proposition 3 *Suppose that the minimum RPM price is* $P < w + c_H$ *(so that it does not bind given a high cost realization) but binds given the low cost realization, there exists a unique symmetric equilibrium pricing strategy such that for* $j \in \{1, 2\}$:

1. *if* $c_j = c_H$, $p_H = w + c_H$

2. $\overline{p}_{\min} = \min\{c_H + w, p_L^M(w)\}$

3. *for* $p < \overline{p}_{\min}$, $F_{\min P}(p)$ *is characterized by* $\alpha(\overline{p}_{\min} - w)q(\overline{p}_{\min}) = q(p)(p - w)(\alpha + (1-\alpha)(1 - F_{\min P}(p))$

4. $\underline{p}_{\min}$ *is implicitly defined by* $q(P)(P - w)\Big[\alpha + (1-\alpha)\frac{F_{\min P}(\underline{p}_{\min})}{2} + (1-\alpha)(1 - F_{\min P}(\underline{p}_{\min}))\Big] = q(\underline{p}_{\min})(\underline{p}_{\min} - w)(\alpha + (1-\alpha)(1 - F_{\min P}(\underline{p}_{\min}))$

5. *if* $c_j = 0$, *and* $\underline{p}_{\min} < \overline{p}_{\min}$ *then price* (p_L) *is drawn from the distribution* $F_{\min P}(p)$ *with support* $P \cup (\underline{p}_{\min}, \overline{p}_{\min})$ *and an atom at* P

6. *if* $c_j = 0$, *and* $\underline{p}_{\min} \geq \overline{p}_{\min}$ *then* $p_L = P$ *with probability* 1.

Proof. The logic is similar to the proof of Proposition 2.

That there must be a mass point at P follows from the fact that it binds. Bertrand reasoning determines the behavior of retailers with high cost realization.

Indifference (so that profits are the same at all prices that a low cost retailer chooses in the pricing equilibrium) determines both properties 3 and 5. ■

Proposition 6 *Assume that* $w_j = w_k = w$*. In a monotone perfect Bayesian equilibrium,* $p_j = w + \frac{\xi_j}{2} - \frac{w-v}{2}\mathbb{1}_{v<w}$.

Proof. The proof follows the proof of equilibrium in a IPV symmetric first price seal bid auction. See Krishna (2002) p.16ff for omitted details, easily adapted to this setting. The retailers solve

$$p_j(\xi_j) = \arg \max_{w \leq p \leq v+\xi_j} \quad (p-w)\Pr(\xi_j - p > \xi_k - p_k) \tag{16}$$

Let $u_k(\xi_k) = \xi_k - p_k(\xi_k)$ and denote the equilibrium strategy to be $\beta_k(\xi) = u(\xi_j)$, and $\beta^{-1}(u_k) = \xi_k$. Assume that this strategy is monotone and increasing. To begin assume that $w \leq v$. Recall that ξ is uniformly distributed on $[0,1]$. The retailers' problem is then:

$$p_j = \arg\max_p \quad (p-w)\,\beta^{-1}(\xi_j - p) \tag{17}$$

First order conditions yield

$$\beta^{-1}(\xi_j - p) - (p-w)\,\frac{\partial \beta^{-1}(\xi_j - p)}{\partial(\xi_j - p)} = 0 \tag{18}$$

Imposing the equilibrium condition that $\beta(\xi_j) = u(\xi)$, yields

$$\begin{aligned}
\xi_j - (p(\xi_j) - w)\,\frac{1}{1-p'(\xi)} &= 0 && (19)\\
p(\xi) + p'(\xi)\xi &= \xi + w && (20)\\
\frac{\partial}{\partial \xi}(p(\xi)\xi) &= \xi + w && (21)\\
p(\xi)\xi &= \int (\xi + w)d\xi && (22)
\end{aligned}$$

Noting that $u(0) = 0$, allows the integral to be evaluated, yielding

$$p(\xi) = \frac{\xi}{2} + w \tag{23}$$

It remains to deal with the case where $w \geq v$. In this case, some measure of ξ are excluded. The measure of included realizations is now $U[w-v,1]$. This implies that the boundary condition is now $u(w-v)=0$, yielding the pricing part of the proposition. That $p_j^a = p_j$ follows from Lemma 1. ■

Proposition 7 *A collusive equilibrium in which manufacturers set $w_t = v$ if $p_j^a \in [v, v+\frac{1}{2}]$ for all j in all past periods, and $w_t = 0$ otherwise, is supportable if $\delta > \frac{6v-2}{9v-2}$. In such an equilibrium, the per period profit earned by each manufacturer, π_{NR}^c, is $\frac{v}{2}+\frac{1}{6}$.*

Proof. There are three possible deviations. The first is $w \in [0, v-1]$, denoted D1. The second is $w \in \left(v-1, v-\frac{1}{2}\right)$, denoted D2. The third is $w \in \left[v-\frac{1}{2}, v\right)$, denoted D3.

Given these three types of deviation, there are three conditions that are necessary for collusion to be able to be sustained. These are:

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D1} + \Pr(p_j < v|w^{D1})\frac{\delta}{1-\delta}\pi^p + \left(1-\Pr(p_j < v|w^{D1})\right)\frac{\delta}{1-\delta}\pi_{NR}^c, \quad (24)$$

and

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D2} + \Pr(p_j < v|w^{D2})\frac{\delta}{1-\delta}\pi^p + \left(1-\Pr(p_j < v|w^{D2})\right)\frac{\delta}{1-\delta}\pi_{NR}^c; \quad (25)$$

and

$$\frac{\pi_{NR}^c}{1-\delta} \geq \pi^{D3} + \Pr(p_j < v|w^{D3})\frac{\delta}{1-\delta}\pi^p + \left(1-\Pr(p_j < v|w^{D3})\right)\frac{\delta}{1-\delta}\pi_{NR}^c; \quad (26)$$

where π_{NR}^c denotes the per-period collusive profit, and following discussion in the text $\pi_{NR}^c = \frac{v}{2}+\frac{1}{6}$; and in case the collusion breaks down, manufacturers earn the one-shot profit which is denoted by $\pi^p = \frac{1}{6}$.

Note that it is immediate that deviating to $w > v$ cannot be optimal.

The proof proceeds by establishing properties of the optimal deviation of each type, first by characterizing the retailers pricing when $w_j < w_k = v$ and then examining the manufacturers problem in setting w_j for each type of deviation. Finally, a δ that is sufficient for none of the deviations to be attractive is derived.

First, consider the pricing of a retailer with a wholesale unit price of w_j facing a rival that prices in line with the cartel rule, such that $p_k = v + \frac{\xi_k}{2}$.[63] The retailer's pricing problem is

$$p_j^*(\xi_j) = \arg\max_p \quad (p-w_j)\Pr(\xi_j - p > \xi_k - v - \frac{\xi_k}{2}) \quad (27)$$

[63]Recall that retailers are not part of any cartel agreement and compete in a static game.

Given that ξ_k is uniformly distributed on $[0,1]$, this amounts to maximizing

$$(p-w)\left(2\xi_j+2v-2p\right) \tag{28}$$

in the region where $(2\xi_j+2v-2p)\in[0,1]$ (outside of this region, either there is no chance of winning, or the retailer wins for certain and is merely forgoing revenue by dropping price). Ignoring this constraint results in the pricing rule $p^*(\xi_j):=\frac{v+w_j}{2}+\frac{\xi_j}{2}$. If $2\xi_j+2v-2p^*>1$, then the pricing rule is $p=v-\frac{1}{2}+\xi_j$.[64] Further, if $2\xi_j+2v-2p^*(\xi_j)>1$, then the same inequality holds for all $\xi>\xi_j$. This results in the following retailer pricing rule when $w_j<w_k=v$: $p_j=\max\left\{\frac{v+w_j}{2}+\frac{\xi_j}{2},v-\frac{1}{2}+\xi_j\right\}$.

Next we turn to consider the manufacturer's strategy. The first style of deviation ($D1$), involves choosing the optimal deviation in the interval $w\in[0,v-1]$. In this interval, by inspection of the retailer's pricing rule derived above, the probability of detection is equal to 0.5 regardless of w. Hence, the optimal deviation can be derived by maximizing the combined retailer-manufacturer-pair deviation payoff (since the manufacturer can extract the retailer's profit through the fixed fee). It is clear that this is exactly what the retailer will do when $w_j=0$. Hence, the profit maximizing $D1$ deviation arises when the retailer sets prices such that $p_j=v-\frac{1}{2}+\xi_j$. From the manufacturers point of view, this retailer pricing policy will arise for any $w\in[0,v-1]$ and so can be implemented in a variety of ways, all resulting in the same deviation profit of v (the lump-sum component of the manufacturer's two part tariff will be used to extract remaining expected profits from the retailer). Thus the D1 deviation yields $\pi^{D1}=v$ and $\Pr(p_j<v|w^{D1})=\frac{1}{2}$.

Given this, we can solve for the minimum δ such that a $D1$ deviation is not attractive. The $D1$ condition requires that

$$\left(\frac{v}{2}+\frac{1}{6}\right)\frac{1}{1-\delta}\geq v+\frac{1}{2}\frac{\delta}{1-\delta}\frac{1}{6}+\frac{1}{2}\frac{\delta}{1-\delta}\left(\frac{v}{2}+\frac{1}{6}\right) \tag{29}$$

implying that when $\delta\geq\frac{6v-2}{9v-2}$ a $D1$ deviation is not attractive.

The second style of deviation ($D2$), leaves the probability of detection unchanged at 0.5. To see this note that for all $w\in\left(v-1,v-\frac{1}{2}\right)$, $p_j=\max\left\{\frac{v+w_j}{2}+\frac{\xi_j}{2},v-\frac{1}{2}+\xi_j\right\}=v$ when $\xi=\frac{1}{2}$ and that p is monotonic in ξ and so the deviation is detected for $\xi\leq\frac{1}{2}$ and undetected otherwise. Also, recall that when $w=0$ the retailer chooses $p_j=v-\frac{1}{2}+\xi_j$. Hence, by setting $w\in\left(v-1,v-\frac{1}{2}\right)$, the manufacturer diminishes stage profits, with no compensating return in terms of adjusting the probability

[64] Recall, $v>1$, making this event relevant. In this event, the pricing rule is derived by raising p until the chance of winning is equal to 1.

around detection (that is, leaving the continuation value unchanged). Hence, a $D2$ deviation must always be dominated by a $D1$ deviation.

The third style of deviation ($D3$), involves choosing the optimal deviation in the interval $w \in \left[v - \frac{1}{2}, v\right)$. In this interval, a change in w will affect the probability of detection; specifically, in this range, $\Pr(p_j < v|w) = \Pr(\frac{v+w}{2} + \frac{\xi_j}{2} < v) = \Pr(\xi_j < v - w) = v - w$. Trivially, $\frac{d\Pr(p_j<v|w_j)}{dw_j} = -1$. Given that $w + 1 - v$ is the value of ξ_j such that $\frac{v+w}{2} + \frac{\xi_j}{2} = v - \frac{1}{2} + \xi_j$, the deviation profit can be written as:

$$\pi^{D3} = \int_0^{w_j+1-v} \left(\frac{v + w_j}{2} + \frac{x}{2}\right)(v + x - w_j)\,dx + \int_{w_j+1-v}^1 \left(v + \frac{1}{2} + x\right)dx. \quad (30)$$

It will be useful to note that $\frac{\partial \pi^{D3}}{\partial w_j} = -1 - w_j\,(w_j + 1 - v)$ and that equation (30) also describes π^{D2}.

The optimal $D3$ deviation is the solution to

$$\max_{w_j} \quad D3(w_j) \equiv \max_{w_j} \quad \pi^{D3} + \Pr(p_j < v|w_j)\frac{\delta}{1-\delta}\pi^p + (\Pr(1 - p_j < v|w_j))\frac{\delta}{1-\delta}\pi^c_{NR} \quad (31)$$

which, taking the derivative with respect to w_j, yields

$$\begin{aligned} \frac{\partial D3(w_j)}{\partial w_j} &= \frac{\partial \pi^{D3}}{\partial w_j} - \frac{\partial \Pr(p_j < v|w_j)}{\partial w_j}\frac{\delta}{1-\delta}\left(\pi^c_{NR} - \pi^p\right) && (32) \\ &= -1 - w_j\,(w_j + 1 - v) + \frac{\delta}{1-\delta}\frac{v}{2} && (33) \end{aligned}$$

where the last equality follows on substituting for the two derivatives, as calculated above. Note that the second derivative is $\frac{\partial^2 D3(w_j)}{\partial w_j^2} = v - 1 - 2w_j < 0$ in the range $w \in [v - \frac{1}{2}, v)$.

Necessary conditions for the existence of an optimal D3 deviation (i.e. an interior solution in $\left[v - \frac{1}{2}, v\right)$) are that $\left.\frac{\partial D3(w_j)}{\partial w_j}\right|_{w_j=v-\frac{1}{2}} \geq 0$ and $\left.\frac{\partial D3(w_j)}{\partial w_j}\right|_{w_j=v} < 0$.

From equation (33), $\left.\frac{\partial D3(w_j)}{\partial w_j}\right|_{w_j=v-\frac{1}{2}} \geq 0$ implies that $\delta \geq \frac{2v+3}{4v+3}$. Similarly, $\left.\frac{\partial D3(w_j)}{\partial w_j}\right|_{w_j=v} < 0$ implies $\delta < \frac{2v+2}{3v+2}$. Hence, for a $D3$ deviation to exist it must be that $\delta \in \left[\frac{2v+3}{4v+3}, \frac{2v+2}{3v+2}\right)$.

It remains to establish that a $D1$ deviation will always dominate a $D3$ deviation. It is established above that the optimal $D1$ deviation can be implemented by setting $w = v - 1$. In the region $w \in \left(v - 1, v - \frac{1}{2}\right)$ (a $D2$ deviation), deviation profits,

π^{D2}, decrease. Hence, adapting equation (33), and noting that $\frac{\partial \pi^{D3}}{\partial w_j} = \frac{\partial \pi^{D2}}{\partial w_j} = -1 - w_j\left(w_j + 1 - v\right)$ for a $D3$ deviation to dominate a $D1$ deviation it must be that

$$\int_{v-1}^{w^{D3}} -1 - w\left(w + 1 - v\right) dw + \int_{v-\frac{1}{2}}^{w^{D3}} \frac{\delta}{1-\delta}\frac{v}{2} dw > 0 \tag{34}$$

where w^{D3} is the optimal w for a $D3$ deviation. Note that, on the assumption that equation (34) is true,

$$\begin{aligned}
& \int_{v-1}^{w^{D3}} -1 - w\left(w + 1 - v\right) dw + \int_{v-\frac{1}{2}}^{w^{D3}} \frac{\delta}{1-\delta}\frac{v}{2} dw && (35)\\
\leq \; & \int_{v-1}^{v-\frac{1}{2}} -1 - w\left(w + 1 - v\right) dw + \int_{v-\frac{1}{2}}^{w^{D3}} -\frac{v}{2} - \frac{3}{4} dw + \int_{v-\frac{1}{2}}^{w^{D3}} \frac{\delta}{1-\delta}\frac{v}{2} dw && (36)\\
\leq \; & \int_{v-1}^{v-\frac{1}{2}} -1 - w\left(w + 1 - v\right) dw + \int_{v-\frac{1}{2}}^{v} -\frac{v}{2} - \frac{3}{4} dw + \int_{v-\frac{1}{2}}^{v} \frac{\delta}{1-\delta}\frac{v}{2} dw && (37)
\end{aligned}$$

Where the first inequality arises from noting that $\frac{\partial^2 D3(w)}{\partial w^2} < 0$ and substituting in $\left.\frac{\partial D3(w)}{\partial w}\right|_{w=v-\frac{1}{2}}$. The second inequality comes from noting that the second and third integrals are linear in w, and so their difference, under the maintained assumption, is increasing in the limits of integration. Setting, $\delta = \frac{2v+2}{3v+2}$ (to maximize the third integral) and evaluating yields

$$\left(-\frac{5}{12} - \frac{3v}{8}\right) + \left(-\frac{v}{4} - \frac{3}{8}\right) + \left(\frac{v}{2} + \frac{1}{2}\right) < 0, \tag{38}$$

which, by contradiction, establishes that $D1$ is the more profitable deviation. ■

Appendix B: Additional material

Section 4.2: The manufacturer's problem with RPM

As described in the text, RPM can only play a role in inducing service provision in one of two cases (i) either it binds for both low and high cost realizations; that is, if $P > w + c_H$; or (ii) it leads a firm with a low cost realization to choose P with certainty (which requires $\underline{p}_{\min} > \overline{p}_{\min}$ as defined in Proposition 3). In the latter case, it is immediate that a firm with a high cost realization makes no investment, and the level of the investment of a firm with a high cost realization satisfies:

$$I'(s_L(P)) = \left(\alpha + \frac{(1-\alpha)(1-s_L(P))}{2}\right)(P-w)q(P); \tag{39}$$

the manufacturer sets T to maximize a retailer's expected profits

$$T = (1-\alpha)\left[\left(\alpha s_L + \frac{(1-\alpha)(1-(1-s_L)^2)}{2}\right)(P-w)q(P) - I(s_L)\right], \tag{40}$$

and w to maximize expected profits

$$2T + wq(P)\left[2\alpha(1-\alpha)s_L + (1-\alpha)^2(1-(1-s_L)^2)\right] + w\alpha^2 q(w+c_H). \tag{41}$$

In the former case, where both high and low cost firms set a price equal to $P > w + c_H$, demand for a retailer irrespective of its costs is given by $\frac{q(P)}{2}$. Service for low cost and high cost realization can be easily implicitly characterized through the first order conditions described in the main text in (8) and (7). Finally, we can turn to the problem of the manufacturer who sets T equal to a retailer's expected profits so

$$\begin{aligned} T &= \alpha\left[(P-w-c)\frac{q(P)}{2}(1-(1-s_H)(1-\alpha s_H-(1-\alpha)s_L)) - I(s_H)\right] \qquad (42)\\ &\quad +(1-\alpha)\left[(P-w)\frac{q(P)}{2}(1-(1-s_L)(1-\alpha s_H-(1-\alpha)s_L) - I(s_L)\right], \end{aligned}$$

and chooses w and $P > w + c_H$ to maximize

$$2T + wq(P)\left[\alpha^2(1-(1-s_H)^2) + 2\alpha(1-\alpha)(1-(1-s_L)(1-s_H)) + (1-\alpha)^2(1-(1-s_L)^2)\right]. \tag{43}$$

Section 4.3: The manufacturer's problem with MAP

The manufacturer set T as follows

$$\begin{aligned} T &= \alpha\left[(P-w-c)\frac{q(P)}{2}(1-(1-s_H)(1-\alpha s_H-(1-\alpha)s_L))-I(s_H)\right] \\ &\quad +(1-\alpha)\left[(p^m(w)-w)\frac{q(p^m(w))}{2}(1-(1-s_L)(1-\alpha s_H-(1-\alpha)s_L)-I(s_L)\right], \end{aligned} \tag{44}$$

and chooses w and $P > w + c_H$ to maximize expected profits—the sum of the fixed fee T and expected revenue from the per unit fee:

$$2T + w\left[\begin{array}{c} \alpha^2(1-(1-s_H)^2)q(P)+(1-\alpha)^2(1-(1-s_L)^2)q(p^m(w)) \\ +\alpha(1-\alpha)(1-(1-s_L)(1-s_H))(q(P)+q(p^m(w))) \end{array}\right]. \tag{45}$$