Just starting out: Learning and equilibrium in a new market*

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Abstract

We document the evolution of the new market for frequency response within the UK electricity system over a six-year period. Firms competed in price while facing considerable initial uncertainty about demand and rival behavior. We show that over time prices stabilized, converging to a rest point that is consistent with equilibrium play. We draw on models of fictitious play and adaptive learning to analyze how this convergence occurs and show that these models predict behavior better than an equilibrium model prior to convergence.

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1 Introduction

What do competing agents or firms do when their environment changes? Answering this question is necessary for making predictions about market evolution following policy changes or changes to market institutions. The approach to analyzing changes used in empirical work is typically based on computing counterfactual equilibria. However, convergence to equilibrium after a perturbation may not be swift or indeed certain, and the adjustment mechanism may well be integral in determining which among alternative possible equilibria the market converges to. Understanding how firms adjust and the ensuing learning process is thus central to the analysis of environmental changes.

This paper offers a case study of a newly deregulated market, the frequency response (FR) market in the UK. Initially, firms faced tremendous uncertainty both about the determinants of demand and about what their rivals would do. Following Brandenburger (1996), we refer to a firm's uncertainty about the behavior of its rivals as strategic uncertainty.[1] We explore how demand and strategic uncertainty manifest themselves in the behavior of firms from "day one," tracing their behavior over the next six years.

Broadly speaking, FR is a product required by the system operator to keep the electricity system running smoothly. Historically, electricity generating firms had been obligated to provide FR to the system operator at a fixed price. Deregulation created a market in which firms are allowed to bid for providing FR, thus setting the stage for price competition. An attractive feature of this market is that the demand for FR and the set of market participants were, at least in the first three and a half years, relatively stable, so that changes in bids can be plausibly attributed to learning rather than changes in the environment.

The first part of the paper documents bidding behavior over time. We distinguish three phases in the evolution of the FR market. The early phase of the FR market is characterized by heterogeneous bidding behavior and frequent and sizeable adjustments of bids. Some firms appear to experiment with their bids. Other firms appear to "follow the leader." Yet other firms do not change their bids at all for many months. The price of FR exhibits a noticeable upward trend during the early phase that culminates in a "price bubble." During the middle phase of the FR market, this trend reverses itself. Competition between firms drives the

[1]Brandenburger (1996) draws a distinction between strategic uncertainty, i.e., uncertainty about the actions and beliefs of other players, and structural uncertainty, i.e., uncertainty about the primitives of the game.

highest bids down, leading to a dramatic reduction in the range of bids. Adjustments of bids are less frequent and smaller than in the early phase. By the time the FR market enters its late phase, it appears to have reached a "rest point." This rest point is consistent with a complete information Nash equilibrium, and we show that thereafter bids stay close to their equilibrium levels despite periodically occurring smaller changes in the environment. The industrial organization literature routinely assumes that equilibrium reasserts itself, so finding that it does in a particular example is reassuring (and to the best of our knowledge ours is the first paper to empirically analyze the convergence process). On the other hand, the FR market can only be considered to have converged to a rest point after three and a half to four years of monthly strategic interaction.

The second part of the paper analyzes in more detail how this convergence occurs through the "lens" of alternative learning models. To do so, we first estimate the demand and cost primitives under a relatively weak rationality assumption that we view as appropriate for the late phase of the FR market. This enables us to estimate profits for any vector of bids. Assuming actual bids are determined by perceptions of likely profits, we can then analyze how the realizations of its rivals' bids and demand impact a firm's perceptions of the profitability of alternative strategies. To structure our analysis of strategic uncertainty we use fictitious play models in which firms form their beliefs based on past observed rival behavior (Brown 1951). To structure our analysis of firms' perceptions about demand we use adaptive learning models in which these perceptions are grounded in a statistical analysis of the data they have available to them when they form their bids (Sargent 1993, Evans and Honkapohja 2001, Evans and Honkapohja 2013). We judge alternative parameterizations of our learning models by comparing both their "one-step-ahead" and "multi-period" bid predictions to the actual bids.

The heterogeneous behavior and experimentation by some firms in the early phase of the FR market is hard to rationalize with these models, so we focus our analysis of learning models on the last two phases. During the middle phase of the FR market, the best-fitting models are those in which firms more heavily weight recent rival behavior in forming beliefs about their rivals' bids and adaptively learn about the price elasticity of demand. In this phase the predictions from the learning models are noticeably better than those from a complete information Nash equilibrium where all agents know the demand parameters. Moreover, the learning models make predictions which lead to what seems to be the "rest point" that we observe in the last phase of the FR market. With some caveats that we point out below, our work is therefore broadly supportive of these learning models — models that have previously only been tested in lab experiments.

In contrast, during the late phase of the FR market the equilibrium model fits the data about as well as the best learning models. Since there are a series of changes in the environment that have been largely absent in the earlier phases, the performance of the equilibrium model during this phase is notable. Of course, by the late phase firms had been able to acquire quite a bit of information about rival behavior as well as demand.

Related literature. Our paper is closely related to a large body of work in micro, macro and experimental economics. Going back to Cournot (1838), there has been work on the theory of learning in normal-form and, more recently, extensive-form games. This literature mainly aims to derive conditions on the underlying game under which the canonical models of belief-based learning (including fictitious play) and reinforcement learning imply convergence to equilibrium (Milgrom and Roberts 1991, Fudenberg and Kreps 1993, Börgers and Sarin 1997, Hart and Mas-Colell 2000). Belief-based learning starts with the premise that players keep track of the history of play and form beliefs about what their rivals will do in the future based on their past play. Reinforcement learning assumes that strategies are "reinforced" by their past payoffs and that the propensity to choose a strategy depends in some way on its stock of reinforcement.

Experimental economists have pushed this theoretical literature further by using lab experiments to determine which learning models best describe how people actually learn (Erev and Roth 1998). On the one hand, this has resulted in the development of more general models such as experience-weighted attraction learning (Camerer and Ho 1999) and models with sophisticated learners who try to influence how other players learn (Camerer, Ho and Chong (2002); see also Mohlin, Ostling and Wang (2014) for an imitation dynamic consistent with data from a Swedish gambling game). On the other hand, there is a growing consensus that telling apart belief-based learning from reinforcement learning is difficult in practice (Salmon 2001).

A second, distinct, theoretical literature considers behavior when agents have only partial knowledge of the environment in which they operate. There is a long literature in applied mathematics and statistics analyzing bandit problems, in which forward-looking agents trade off "exploration" versus "exploitation" (Robbins 1952). Easley and Kiefer (1988) study under what conditions optimizing agents learn the true parameters governing the data generating process. Economists have also contributed to this literature by analyzing what happens when multiple agents compete in a partially known environment, noting informational free-riding incentives (Bolton and Harris 1999, Keller, Rady and Cripps 2005) and incentives to "signal jam" (Riordan 1985, Mirman, Samuelson and Urbano 1993).

Macroeconomists largely think about learning in terms of expectation formation. The influential idea of adaptive learning (Sargent 1993, Evans and Honkapohja 2001, Evans and Honkapohja 2013) posits that agents proceed like an econometrician and use the available data to estimate a model of the economy and a rule for forming expectations. The central question is whether the economy reaches a rational-expectations equilibrium. Large shocks can have persistent effects through changing the agents' "data sets" (Venkateswaran, Veldkamp and Kozlowski 2015). There is also a corresponding experimental literature on expectation formation (Fehr and Tyran 2008, Anufriev and Hommes 2012).

We combine models for beliefs about rival behavior with models for learning about the underlying structural parameters and provide empirical evidence on how well they fit the data. There is existing theoretical work on how firms learn about demand (Rothschild 1974, Bergemann and Välimäki 1996, Bergemann and Välimäki 2006, Bernhardt and Taub 2015), but little empirical work. What empirical work there is in the industrial organization and marketing literatures is largely about how consumers experiment to learn their demand for experience goods (Erdem and Keane 1996, Ackerberg 2003, Dickstein 2013) or how firms learn about their cost (Griliches 1957, Porter 1995, Benkard 2000, Conley and Udry 2010, Zhang 2010, Covert 2013, Newberry 2016).

There has been some empirical work assessing whether behavior in new markets converges to some notion of equilibrium, but no structured analysis of how convergence occurs. Joskow, Schmalensee and Bailey (1998) study the emissions rights market that was created by the 1990 Clean Air Act Amendments, concluding that the market "had become reasonably efficient" (p. 669) within four years. Sweeting (2007) examines the electricity spot market in England and Wales between 1995 and 2000, and finds evidence of tacit collusion between the two largest generators. Hortaçsu and Puller (2008) look at the electricity spot market in Texas from 2001 to 2003, following a restructuring that introduced a uniform-price auction. They find that firms with large stakes made bids that were close to optimal, while small players deviated significantly. Luco, Hortaçsu, Puller and Zhu (2017) further investigate the impact of this heterogeneity in strategic ability on market efficiency.

Structure of paper. In Sections 2 and 3 we describe the FR market, our data, and offer some descriptive evidence on how this market evolved over time. Section 4 outlines our strategy for estimating the demand and cost primitives. In Section 5 we consider how well different learning models fit the data, before concluding in Section 6. Additional derivations and information on the construction of the data are contained in the data appendix. The online appendix presents several robustness checks and extensions.

2 The FR market

The UK electricity market is a network of generators and distributors, connected by a transmission grid. This grid is owned and operated by a company called National Grid plc (NG). NG is responsible for the transmission of electricity from the generators to the distributors, as well as the balancing of supply and demand in real time. Figure 1 summarizes the UK electricity market.

The unit of exchange in this market is a given amount of power supplied for a half-hour (measured in megawatt hours (MWh)). About 98% of electricity is sold through bilateral forward contracts between generators and distributors. These contracts can be formed months or even years in advance. There are also shorter term contracts (both day ahead and day of) which are often traded on power exchanges. One hour prior to the settlement period, both generators and distributors must submit their contracted positions to NG, along with bids and offers indicating the terms under which they are willing to be repositioned. NG then acts to equate supply and demand over the settlement period by accepting bids and offers in something akin to a multi-unit discriminatory auction. This process is called the balancing mechanism (BM), and it accounts for the remaining 2% of electricity sales. The generators bidding in the BM are called BM units. A power station typically consists of multiple BM units, and multiple stations may be owned by the same firm. The BM units belonging to the same station tend to be identical.

Frequency response. NG is obligated by government regulation to maintain a system frequency within a one-percent band of 50Hz (Hertz, the number of cycles per second). System frequency is determined in real time by imbalances between the supply and demand of electricity. The higher demand is relative to supply, the lower the system frequency is,

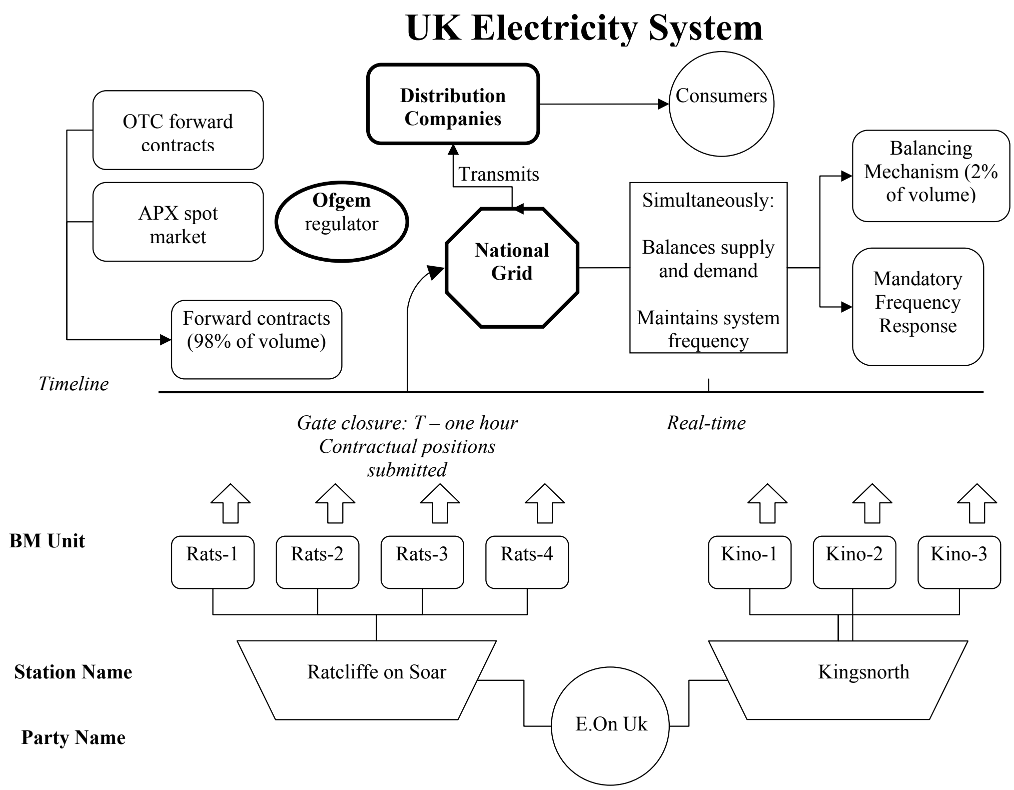

Figure 1: Overview of the UK electricity market.

and vice versa. Imbalances occur due to shocks that cannot be corrected in advance through the BM. To balance supply and demand in real time, NG instructs one or more BM units into FR mode. Once in this mode, NG can rapidly adjust the electricity production of the BM unit using so-called governor controls.

NG is required by government regulation to hold a certain amount of FR capacity at all times.[2] This response requirement is based on risk-response curves that assess the likelihood

[2]There are in fact three types of FR. Primary response is additional energy from a BM unit that is available ten seconds after an event and can be sustained for a further twenty seconds. Secondary response is additional energy that is available within thirty seconds for up to thirty minutes. High response is a reduction of energy within thirty seconds. These responses are technologically constrained and correspond to dilating the steam valve (primary), increasing the supply of fuel (secondary), and decreasing the supply of fuel (high). For historical reasons, BM units are instructed into FR mode in the combinations primary-high and primary-secondary-high. To simplify the presentation and analysis, we aggregate the three types of FR (see the data appendix for details).

and magnitude of possible shocks given the total amount of electricity demanded. As the total amount of electricity demanded evolves, NG instructs BM units in and out of FR mode to satisfy its response requirement. To the best of our knowledge, the response requirement remained unchanged over the sample period.[3]

FR services are thus a second product, distinct from electricity, that BM units can sell to NG, and the FR market is distinct from the main market (comprised of the BM and bilateral forward contracts). Providing FR is costly: a BM unit in FR mode incurs additional wear and tear as it may have to make rapid, small adjustments to its electricity production in response to supply and demand shocks. It also runs less efficiently, with a degraded heat rate. The BM unit is compensated by NG by a holding payment and an energy response payment. The holding payment is per unit of FR capacity and paid for the time that the BM unit spends in FR mode, regardless of whether or not it actually makes adjustments to its electricity production. We explain below in more detail how the holding payment is calculated. The energy response payment compensates the BM unit for the adjustments to its electricity production that NG calls for to maintain system frequency (when actually needed).[4] The energy response payment is considered by industry insiders to be a relatively small source of profit, and is thus ignored in what follows.[5]

Deregulation. Our interest in FR stems from a change in the way the holding payment is determined. This change occurred with the enactment of an amendment to the Connection and Use of System Code called CAP047 and "went live" on November 1, 2005. Pre CAP047, providing FR was mandatory, and the holding payment was at an administered price which had been fairly constant over time (see Figure 2).[6] CAP047 replaced the mandatory provision

[3]We have checked the publicly available minutes of all meetings of the Balancing Services Standing Group (comprising representatives of the generators and NG) and found no discussion of a change in the response requirement.

[4]If the BM unit produces more energy than it was initially contracted to in the BM, NG pays it 125% of the current market price per additional unit of energy; if the BM unit produces less energy, it pays NG 75% of the current market price.

[5]FR services are paid for by NG, who in turn charges both generators and distribution companies and ultimately consumers through the basic service charge, see `https://www.ofgem.gov.uk/electricity/transmission-networks/charging` for additional details.

[6]According to an OFGEM report (`https://www.ofgem.gov.uk/publications-and-updates/potential-income-adjusting-events-under-ngets-200506-system-operator-incentive-scheme`) the volatility during the period from October 11 to October 30, 2005 was due to a dispatch error resulting from a new FR provider being dispatched on indicative contract prices, but settled at higher final contract prices, which had not been updated in NG's dispatch file. Adjusting for this event, NG advised that the average holding payment price in October 2005 would have been £1.75/MWh.

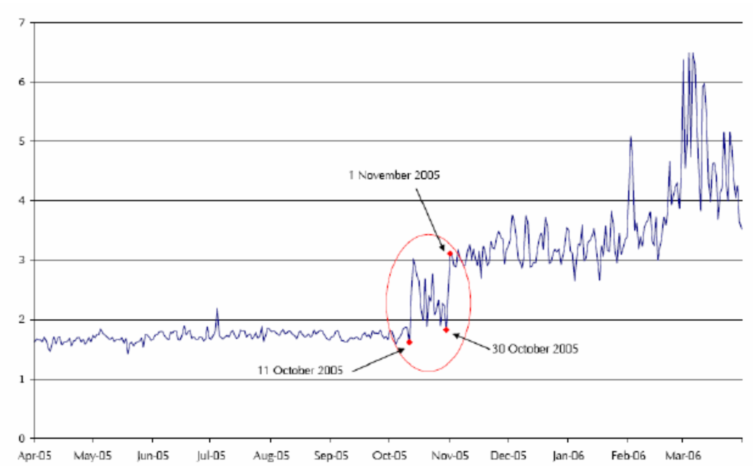

Figure 2: Holding payment in £/MWh for high response by day pre and post CAP047. Source: National Grid.

of FR with a market.

In this market, a BM unit tenders a (scalar) bid each month for providing FR. The bid for the next month is submitted before the 20th of the current month, well in advance of electricity production, and consists of a price per unit of FR capacity (measured in £/MWh). Its bid commits the BM unit to offer FR at a fixed price over the next month. If called upon by NG, the BM unit is paid a holding payment equal to its bid times the quantity of MWh it provides (i.e., it gets "paid-as-it-bids"). The quantity is the product of its FR capacity at its current operating position when instructed into FR mode (measured in megawatt, MW) and the time spent in FR mode (measured in hours, h). The BM unit's FR capacity, in turn, is a function of its current operating position and the current system deviation, laid out in a contract with NG that generally does not change over the sample period.[7]

NG can call upon any BM unit at any time, and often does not choose the lowest bidders to provide FR. Instead, it simultaneously accepts bids in the BM and instructs BM units into FR mode to equate supply and demand and maintain the mandated amount of FR capacity in the most cost-effective way. In practice, the cost minimization problem that

[7]This contract takes the form of a 5×3 matrix for each type of FR (see footnote 2) that specifies the quantity delivered at five deload points (operating positions) and three system deviations (0.2Hz, 0.5Hz, and 0.8Hz away from 50Hz). At other deload points and deviations, the quantity is determined by linear interpolation. The matrices are proprietary information, but selected entries are published by NG in the capability data (see the data appendix). For over 80% of the BM units, the observed entries do not change over the sample period.

jointly governs the FR market and the BM is solved in real time by a proprietary linear program running on a supercomputer. NG may not choose the lowest bids for at least three reasons. First, BM units differ in the precision of their governor controls, and NG may prefer to call upon more expensive but more precise BM units. The precision of a BM unit is thus a source of product differentiation. Second, because the FR capacity of a BM unit depends on its operating position, NG may prefer to call upon a BM unit operating in the middle of its range, with plenty of FR capacity, rather than a BM unit operating at the extremes of its range.[8] Third, transmission constraints may affect the ability of some BM units to provide FR and therefore make them ineligible.

The market for FR was proposed by RWE Npower Renewables Ltd., one of the largest firms in the UK electricity market. This proposal was opposed by NG, who argued that since its demand for FR is regulated and inelastic, firms would be able to exploit their market power and the price of FR would rise. The government regulator dismissed these concerns and on November 1, 2005 introduced CAP047. Figure 2 shows that NG had every reason to worry about CAP047, as the holding payment doubled within the year.

From the pre-CAP047 period, firms had an understanding of the response requirement NG is obligated to satisfy and the relative desirability of their BM units, as well as the cost of providing FR. However, firms were uncertain about the demand for their FR services because they did not know how their rivals would bid in the newly established market. In addition to this strategic uncertainty, the firms faced demand uncertainty in that they did not know how price sensitive NG was.

Our goal is to understand how firms learned to bid in the presence of this uncertainty, and how this contributed to the evolution of the holding payment in Figure 2. The setup of the FR market provides firms with ample opportunity to learn. NG regularly publishes the submitted bids and the FR quantities it allocated to the various BM units. By the time a firm prepares its bids for the next month, it knows its rivals' bids up to and including the current month and their FR quantities up to and including the previous month.

[8]Indeed, NG may first alter the operating position of the BM unit by taking over part of its obligations in the BM before instructing the BM unit into FR mode. As a result, a BM unit does not have to withhold generating capacity from the main market in order to participate in the FR market. Our data shows that BM units can — and do — contract out all of their capacity in the forward market while still actively participating in the FR market. We thank Frank Wolak for pointing out to us that in many other countries the FR market is run separately from the BM. As a result, a BM unit has to withhold generating capacity to participate in the FR market. Because of the resulting opportunity cost, the holding payment is an order of magnitude larger than in the UK.

Data. Our analysis focuses on the first six years of the operation of the FR market from November 2005 to October 2011. We collected most of our data from two public sources. Our data on the FR market comes from NG. For the post-CAP047 period we have the bids submitted by each BM unit at a monthly level and the quantities provided of each type of FR (in MWh, see footnote 2) by each BM unit at a daily level. The combination of bid and quantity data allows us to calculate the holding payment received by each BM unit.

Our data on the BM comes from Elexon Ltd. Elexon is contracted by the government regulator to manage measurement and financial settlement in the BM. For every BM unit we have data on the bids and acceptances in the BM every half-hour. In combination with data on the contracted position that the BM unit submits to NG one hour prior to the settlement period, this allows us to assess the operating position of the BM unit.

Finally, we collected data on ownership and characteristics of power stations and fuel prices from various sources. See the data appendix for further details on data sources as well as sample and variable construction.

Market participants. There are 130 BM units grouped into 61 power stations owned by 29 firms. The FR market is mildly concentrated with a ten-firm-concentration ratio of just over 80% and an HHI of 76.5. Table 1 summarizes revenue in the FR market for the ten largest firms over the first six years of the market's existence.

The largest firm, Drax, had over 20% of the FR market and earned about £100,000,000 over the sample period, or about £1,400,000 per month. Drax is a single-station firm, while the next two largest firms, E.ON and RWE, are multi-station firms. Anecdotally, Drax's disproportionate share is attributable to having a relatively new plant, with accurate governor controls, making it attractive for providing FR. The smallest firm, Seabank, still makes around £200,000 per month. This suggests that the FR market was big enough that firms may have been willing to devote time to actively managing their bidding strategy, at least when the profitability of the market became apparent. Indeed, in 2006 Drax hired a trader to specifically deal with the FR market.[9] Within a year, Drax's revenue from the FR market increased more than threefold.

[9]Source: private discussion with Ian Foy, Head of Energy Management at Drax.

Table 1: Firms with the largest frequency response revenues

Rank	Firm name	Num Units Owned	Total Revenue	Revenue Share (%)	Cumulative Share (%)
1	Drax Power Ltd.	6	99.4	23.8	23.8
2	E.ON UK plc	20	67	16	39.9
3	RWE plc	23	48.4	11.6	51.6
4	Eggborough Power Ltd	4	29.8	7.1	58.7
5	Keadby Generation Ltd	9	24.2	5.8	64.5
6	Barking Power Ltd	2	17.8	4.2	68.8
7	SSE Generation Ltd	4	15.2	3.6	72.5
8	Jade Power Generation Ltd	4	15	3.6	76.1
9	Centrica plc	8	14.7	3.5	79.6
10	Seabank Power Ltd	2	14	3.3	83

Inflation-adjusted revenue in millions of british pounds (base period is October 2011). There is information on 72 months in the data. The number of units owned is the maximum ever owned by that firm during the sample period.

Supply and demand of FR. The demand for and supply of FR are relatively stable over most of the sample period, although there are some changes to the market environment towards the end of the sample. We argue this using a sequence of figures, each with dashed vertical lines separating the three phases we distinguish below. Starting with the demand for FR, the left panel of Figure 3 plots the monthly quantity of FR. Though this series is clearly volatile, it is no more volatile at the beginning than at the end of the period we study (but, as we show in Section 3, the bids are). The right panel of Figure 3 shows some evidence of modest seasonality.

In addition to the mandatory frequency response (MFR) that is the focus of this paper, NG uses long-term contracts with BM units to procure FR services. This is known as firm frequency response (FFR). Figure 4 plots the monthly quantity of FFR and, for comparison purposes, that of MFR (see also the left panel of Figure 3). The quantity of FFR remains relatively stable over our sample period up until July 2010, when it almost doubles and thereafter remains stable at the new level.

The vast majority of FFR is provided by pumped-storage BM units, who provide negligible amounts of MFR. However, Drax — the largest firm in the MFR market — signed an FFR contract from July to September 2007 and again from May to September 2010. This may have been a short-lived attempt by NG to curtail the market power of Drax.

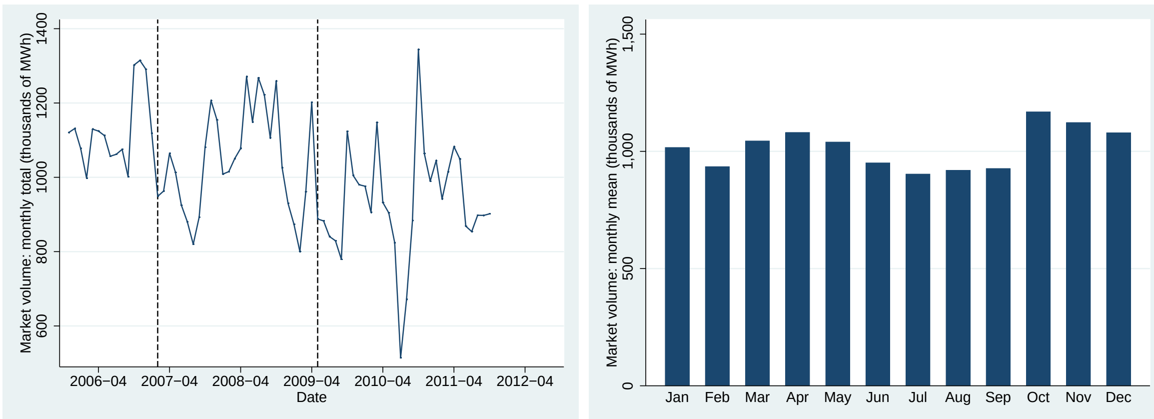

Figure 3: MFR quantity by month (left panel) and on average by month-of-year.

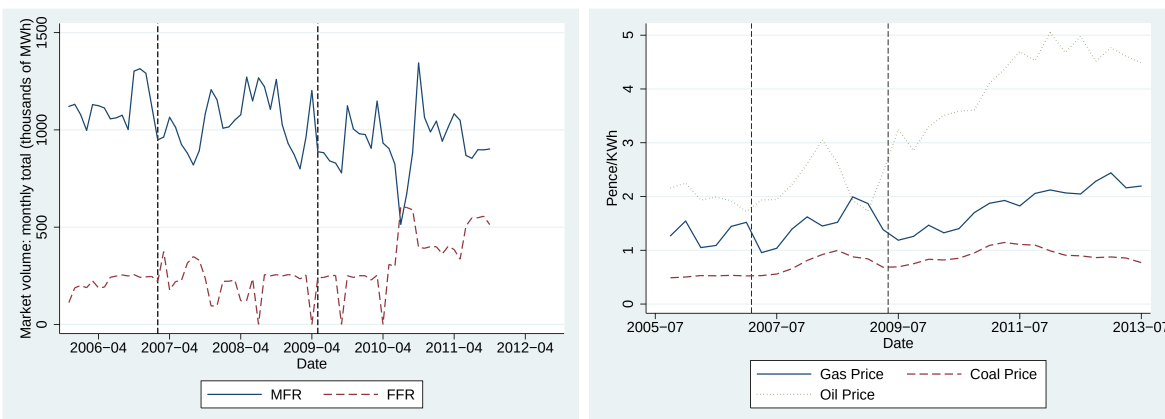

Figure 4: MFR and FFR quantities by month (left panel) and fuel prices by quarter (right panel).

Turning from the demand to the supply of FR, the right panel of Figure 4 plots quarterly fuel prices paid by power stations in the UK. Fuel prices may matter for the FR market in that they change the "merit order" in the main market. For example, when gas is relatively expensive, gas-powered BM units may be part-loaded and therefore available for FR, whereas coal-powered BM units may be operating at full capacity and thus require repositioning in the BM in preparation for providing FR. Though there are some upward trends in oil and — to a lesser extent — gas prices, they are largely confined to the end of the sample period.

Finally, a BM unit can opt out of the FR market by submitting an unreasonably high bid. The left panel of Figure 5 plots the number of "active" power stations over time, where

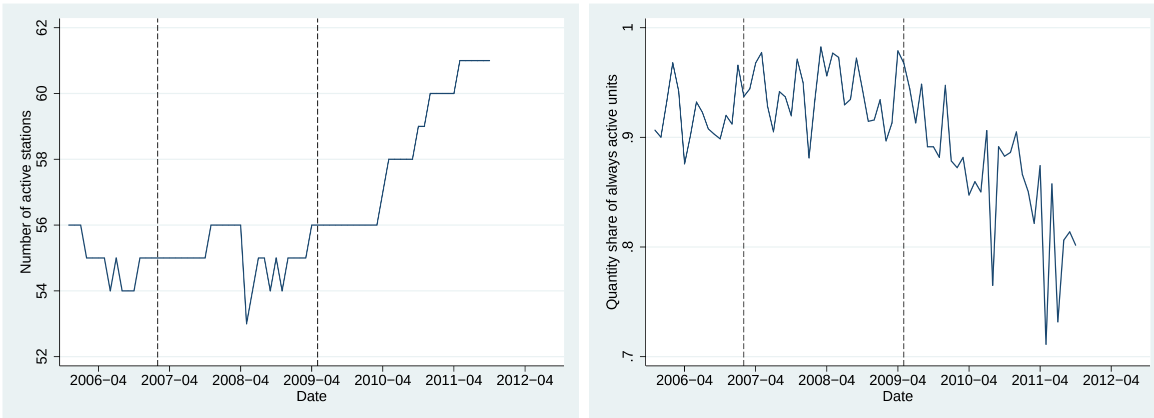

Figure 5: Number of active power stations by month (left panel) and market share of always-active power stations (right panel).

we define a station as active if one of its BM units submits a competitive bid of less than or equal to £23/MWh (see the data appendix for details). The number of active stations fluctuates a bit, ranging from 53 to 61 over the sample period. In the first four years of the FR market, the fluctuations are relatively small and none of the stations who become active or inactive is particularly large. The right panel of Figure 5 shows that the share of stations that are always active is steady at around 95% but that there are some larger fluctuations in the last two years of the FR market.

In sum, until the middle of 2009, the physical environment and demand and supply conditions are stable. After that date, FFR plays a larger role and the number of active power stations rises, as do oil and gas prices. Thus, at least prior to the middle of 2009 any volatility in bids is unlikely to be caused by changes in demand or supply conditions.

3 Evolution of the FR market

Our discussion divides the evolution of the FR market into three phases that differ noticeably in bidding behavior. Figure 6 shows the average monthly price of FR, computed as the quantity-weighted average bids, with vertical lines separating the three phases. For comparison purposes, Figure 6 also shows the unweighted average bids.

During the early phase from November 2005 to February 2007, the price exhibits a noticeable upward trend, moving from an initial price of £3.1/MWh to a final price of £7.2/MWh. The

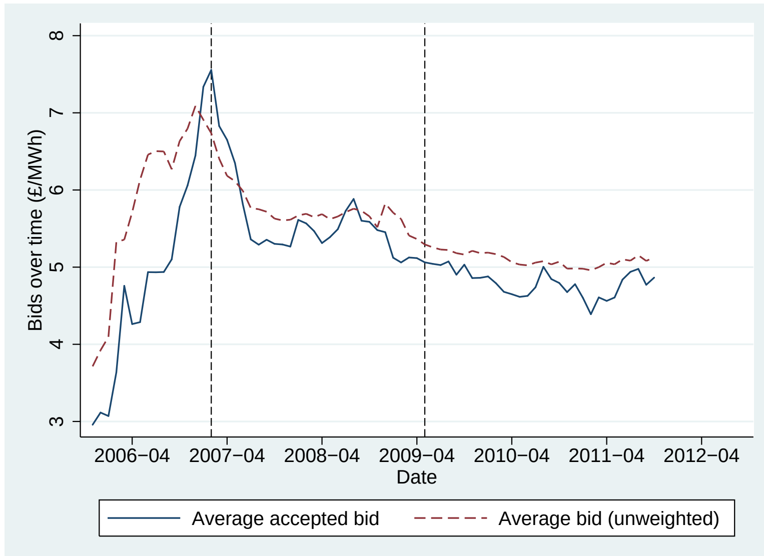

Figure 6: Quantity-weighted and unweighted FR price by month. Weights are based in month t.

upward trend culminates in a "price bubble." During the middle phase from March 2007 to May 2009, this trend reverses itself and the price falls back down to £4.8/MWh. From June 2009 to the end of our sample in October 2011 there is no obvious trend at all. While there are fluctuations during this late phase, they are smaller, and the price stays in the range of £4.3/MWh to £5.1/MWh. The sharper movements in one direction are relatively (to the earlier phases) quickly "corrected" by movements in the opposite direction.

The movements in the price of FR in the earlier phases in Figure 6 occurred despite the relative stability of the demand and supply conditions (see Section 2) and are too persistent to be driven by seasonality in the demand for FR. Though there are some changes in FFR and an upward trend in the number of active power stations as well as in the oil and gas prices, most of this occurs towards the end of the sample when the price of FR has become quite stable. We therefore look for an alternative explanation for the changes in bidding behavior over time, in particular learning. Since none of the participants in the FR market had any experience bidding into it, it seems unlikely that they had strong priors about how their rivals would bid, or how their allocation of FR would vary with their bid conditional on how their rivals would bid. We begin with a summary of how bidding behavior changed from one phase to the next. After providing the overview, we look more closely at the role of individual power stations.

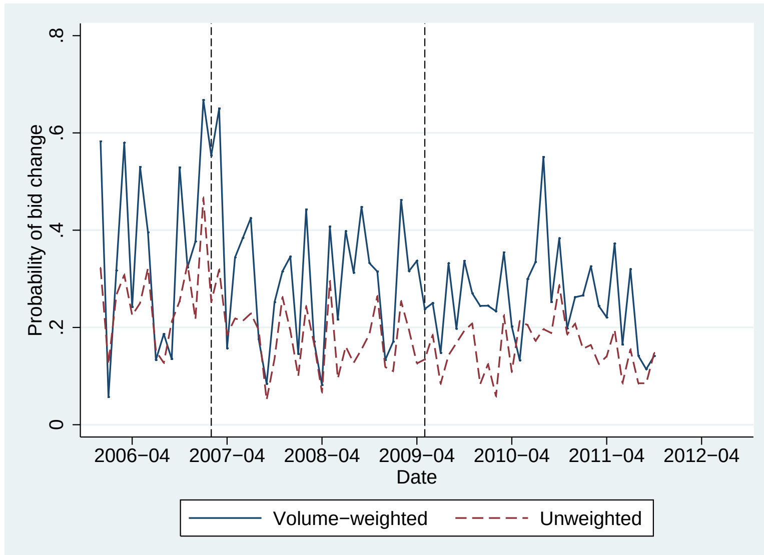

Figure 7: Quantity-weighted and unweighted probability of a bid change between month t and $t-1$. Weights are based in month $t-1$.

Early or rising-price phase (November 2005 – February 2007). In the early or rising-price phase, firms change the bids of their BM units more often and by larger amounts (in absolute value) than in the middle and late phases. On average, the bids of 4 out of 10 BM units change each month by between £1/MWh and £3/MWh (conditional on changing). This is illustrated in Figures 7 and 8.

In addition to changing their bids more often and by noticeably larger amounts, firms tender very different bids in the early phase. Figure 9 shows that the range of bids as measured by the variance of bids across BM units is an order of magnitude larger than in the middle and late phases.

Comparing the left and right panels of Figure 10 shows that most of the variance stems from differences in bids between firms (across-firm variance, right panel) rather than from differences between BM units within firm (within-firm variance, left panel). What within-firm variance there is, is highest in the early phase and then declines, suggesting that firms initially experimented by submitting different bids for their BM units, and that such experimentation became less prevalent over time.

Figure 11 shows the monthly bids of the eight largest power stations by revenue in the FR market. The top left panel provides a more detailed look at the early phase. In line with the

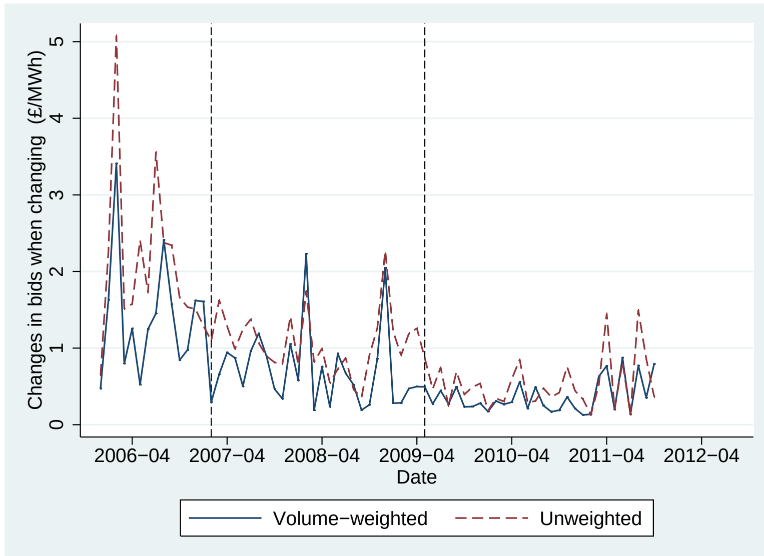

Figure 8: Quantity-weighted and unweighted absolute value of bid change conditional on changing between month t and $t-1$. Weights are based on month $t-1$ and are zero if the BM unit's bid did not change.

wide range of bids documented in Figure 9 and the right panel of Figure 10, the levels and trends of the bids are quite different across stations. Barking, Peterhead, and Seabank bid very high early on — pricing themselves out of the market — and then drift back down into contention. The remaining stations start low and then gradually ramp up. The big increase in bids by Drax during late 2006 and early 2007 leads to the "price bubble" in Figure 6.

Middle or falling-price phase (March 2007 – May 2009). In the middle or falling-price phase, firms change the bids of their BM units less often and by much smaller amounts (in absolute value) than in the early phase. As Figures 7 and 8 illustrate on average the bids of 3 out of 10 BM units change each month by around £1/MWh (conditional on changing). Figure 9 shows that the range of bids is much narrower than in the early phase.

The top right panel of Figure 11 provides more detail. The "price bubble" bursts when Seabank and Barking sharply decrease their bids and steal significant market share from Drax. Drax follows Seabank and Barking down, and this inaugurates intense competition and the noticeable downward trend in the price of FR in Figure 6. Experiments with increased bids are not successful. Drax, for example, increased its bid at the end of 2007 for exactly two months, giving its rivals an opportunity to see its increased bid and follow suit. When

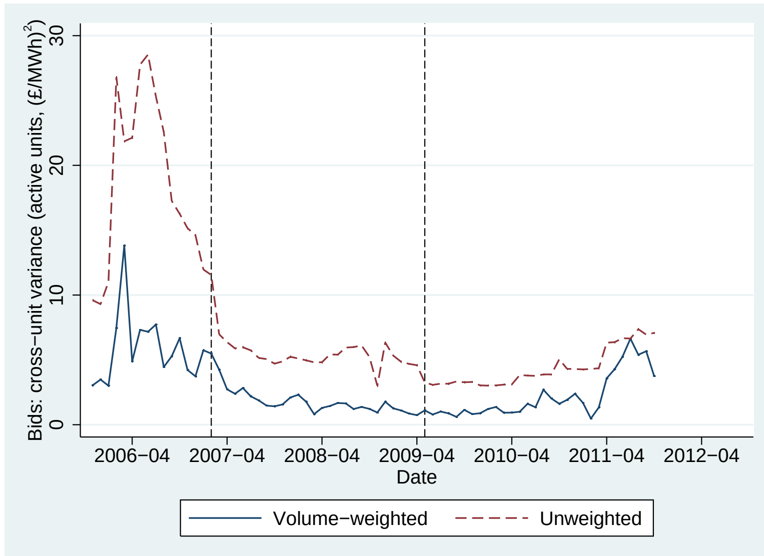

Figure 9: Quantity-weighted and unweighted variance in bids across BM units by month. Weights are based in month t.

no one did, Drax decreased its bid.

The dominant trend in the top right panel of Figure 11 is for the bids of the different power stations to move toward one another. Stations that entered the middle phase with relatively high bids decreased their bids while stations that entered this phase with relatively low bids maintained those bids. This intense competition generated the marked decrease in the variance of bids in Figure 9.

Late or stable-price phase (June 2009 – October 2011). In the late or stable-price phase, firms change the bids for their BM units as often as in the middle or falling-price phase, but by much smaller amounts (in absolute value). As Figures 7 and 8 illustrate, on average, the bids of 3 out of 10 BM units change each month by around £0.5/MWh (conditional on changing). As noted, the range of bids is again much narrower than in either of the earlier phases. The bottom panel of Figure 11 provides more detail. While bids at some power stations continue to fall (Rats and Cottam), others are more erratic or rise (Drax and Eggborough), and others are almost completely flat (Peterhead). Overall, however, the bids of the different stations are noticeably closer to one another in this phase. By the time the FR market has entered its late phase, it appears to have reached a "rest point" that is periodically perturbed by small changes in the environment.

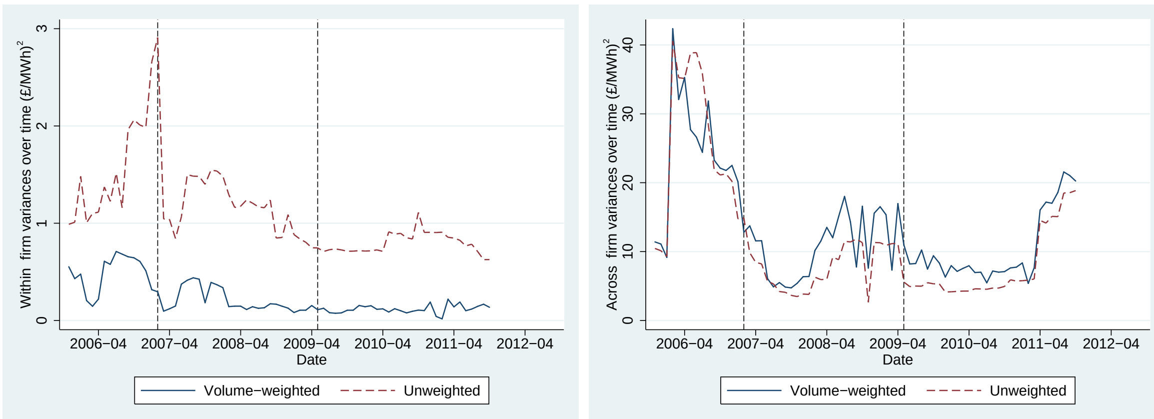

Figure 10: Quantity-weighted and unweighted variance in bids within a firm (left panel) and across firms (right panel). The right panel shows quantity-weighted variance across firms in the quantity-weighted mean firm bids and the unweighted variance across firms in the unweighted mean firm bids. Note the difference in the y-axes between panels.

Summary. The early phase of the FR market is characterized by heterogeneous bidding behavior and frequent and sizable adjustments of bids. During the middle and late phases, bids grow closer and the frequency and size of adjustments to bids falls.

In the early or rising-price phase firms had no prior experience with bidding in the FR market. Those firms who viewed the market as potentially profitable may have taken the opportunity to experiment with their bids. This view is consistent with a comment by Ian Foy, head of energy management at Drax, who stated: "The initial rush by market participants to test the waters having no history to rely upon; to some extent it was guess work, follow the price of others and try to figure out whether you have a competitive edge." Different firms pursued different strategies, with at least some firms responding to rivals' experiments. As a result, a model able to explain bidding behavior in the early phase is likely to have to allow firms to consider the gains from alternative experiments in a competitive environment; a task beyond the scope of this paper.

We view the middle or falling-price phase as a period of firms learning about how best to maximize current profit. We thus treat the middle phase as dominated by firms bidding to "exploit" perceived opportunities rather than to experiment. Section 5 analyzes this phase with the aid of learning models.

Finally, we view the late or stable-price phase as firms having reached an understanding of the behavior of competitors, the resulting allocation of FR, and the likely impact of changes

Figure 11: Quantity-weighted average bids of the largest power stations by month. November 2005 – February 2007 (top left panel), March 2007 – May 2009 (top right panel), and June 2009 – October 2011 (bottom panel). Stations ranked by revenue in the FR market during early and middle phases. Bids are censored above at £10/MWh to improve visual presentation.

in the physical environment. As a result, firms are able to adjust quickly to the changes that occurred in the late phase.

4 Demand and cost estimation

In this section we model and estimate the demand and cost primitives. These serve as an input to the learning models we use in Section 5 to analyze the evolution of bids in the middle and late phases of the FR market.

4.1 Demand

We estimate a generously parameterized logit model at the BM unit-month level to approximate the market shares that are being generated by the proprietary linear program that NG solves in real time to satisfy its response requirement by instructing BM units into FR mode. We model demand at the monthly level because bids are tendered monthly.[10] We focus on the $J = 72$ BM units owned by the ten largest firms in Table 1.[11] Together these "inside goods" account for just over 80% of revenue in the FR market. The share of the remaining BM units becomes the share of the "outside good."

In addition to parsimoniously parameterizing own- and cross-price elasticities when there are this many goods, an advantage of using a logit model for market shares is that it avoids having to model market size. As the right panel of Figure 3 shows, the monthly quantity of FR is seasonal. A disadvantage of using a logit model is that it cannot account for a BM unit receiving a zero market share in a month. There are many zeros since BM units may be unavailable for FR as they undergo maintenance or may submit a non-competitive bid for some other reason. We deal with these zeros by combining our logit model with a probit model that predicts whether a BM unit receives a positive market share. We say that the BM unit is "eligible" if it receives a positive market share.

[10] As noted by a referee, this implies that we estimate an average of the demand functions for shorter (in our case half-hourly) periods, similar to many other studies of demand. We do not explicitly account for the variance in the demand functions across the shorter periods.

[11] Due to non-competitive or missing bids, we subsume 10 of the 82 BM units listed in that table into the outside good.

Model. Let i index firms, j BM units, and t months. In month $t-1$ firm i submits a bid $b_{j,t}$ for BM unit j in month t. Let $\mathcal{J}_i$ denote the indices of the BM units that are owned by firm i and $b_{i,t} = (b_{j,t})_{j \in \mathcal{J}_i}$ the bids for these BM units. We adopt the usual convention to denote the bids for all BM units in month t by $b_t = (b_{i,t}, b_{-i,t})$.

Let $s_{j,t}$ denote the market share of BM unit j in month t and $s_{0,t} = 1 - \sum_j s_{j,t}$ the market share of the outside good. Let $e_{j,t} = 1(s_{j,t} > 0)$ be the indicator for BM unit j being eligible for providing FR services — and thus having a positive market share — in month t. Accounting for eligibility, we specify a logit model for the market share of BM unit j in month t as

$$s_{j,t} = \frac{e_{j,t} \exp\left(\alpha \ln b_{j,t} + \beta x_{j,t} + \gamma_j + \mu_t + \xi_{j,t}\right)}{1 + \sum_k e_{k,t} \exp\left(\alpha \ln b_{k,t} + \beta x_{k,t} + \gamma_k + \mu_t + \xi_{k,t}\right)}, \tag{1}$$

where γ_j and μ_t are BM-unit and month fixed effects and $x_{j,t}$ and $\xi_{j,t}$ are observable and unobservable (to the econometrician) characteristics of BM unit j in month t.

The month fixed effect μ_t subsumes any time-varying characteristics of the outside good. The BM-unit fixed effect γ_j captures the time-invariant preferences of NG for a BM unit due to, for example, the precision of its governor controls or transmission constraints. In addition to its bid $b_{j,t}$, BM unit j has time-varying observed characteristics, $x_{j,t}$, and a time-varying unobserved characteristic, $\xi_{j,t}$, in month t which are meant to capture the main time-varying forces that influence demand in the FR market. The observable characteristics $x_{j,t}$ include two controls for the operating position of the BM unit, namely the fraction of the month the BM unit is fully loaded and the fraction of the month it is part-loaded. As discussed in Section 2, NG uses long-term contracts to procure FFR services that may be a substitute for MFR services. To capture this, $x_{j,t}$ also includes a dummy for whether BM unit j is under contract with NG in month t and provides positive FFR volume. Finally, we allow the unobservable characteristics $\xi_{j,t}$ to follow an $AR(1)$ process with

$$\xi_{j,t} = \rho \xi_{j,t-1} + \nu_{j,t},$$

where the innovation $\nu_{j,t}$ is assumed to be iid across BM units and months and mean independent of current and past bids $(b_{j,\tau})_{\tau \leq t}$ and observable characteristics $(x_{j,\tau})_{\tau \leq t}$. This setup allows a firm to condition its current bid on past unobservable (to the econometrician) characteristics but not on the current innovation, in line with the fact that the bid for the current month is submitted before the 20th of the previous month.

Our probit model for BM unit j being eligible for providing FR services in month t is

$$e_{j,t} = 1(\breve{\beta} x_{j,t} + \breve{\gamma}_j + \breve{\mu}_t + \eta_{j,t} > 0),$$

where $\breve{\gamma}_j$ and $\breve{\mu}_t$ are BM-unit and month fixed effects, $x_{j,t}$ are the same observable characteristics of BM unit j in month t as in equation (1), and $\eta_{j,t} \sim N(0, 1)$ is a standard normally distributed disturbance that is iid across BM units and months and, similar to $\nu_{j,t}$, mean independent of current and past bids and observable characteristics.[12] It follows that

$$\Pr(e_{j,t} = 1 | x_{j,t}) = 1 - \Phi\left(-\breve{\beta} x_{j,t} - \breve{\gamma}_j - \breve{\mu}_t\right) = \Phi\left(\breve{\beta} x_{j,t} + \breve{\gamma}_j + \breve{\mu}_t\right), \tag{2}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF). We estimate equation (2) by maximum likelihood (ML).

Equation (1) implies

$$\ln s_{j,t} - \ln s_{0,t} \equiv \delta_{j,t} = \alpha \ln b_{j,t} + \beta x_{j,t} + \gamma_j + \mu_t + \xi_{j,t} \tag{3}$$

as long as $e_{j,t} = 1$. We can estimate equation (3) by ordinary least squares (OLS) if $\rho = 0$ and $\nu_{j,t}$ is independent of $\eta_{j,t}$.

However, if $\rho \neq 0$ so that $\xi_{j,t}$ is correlated with $\xi_{j,t-1}$, then OLS is likely biased because $\xi_{j,t-1}$ is known to the firm when it chooses $b_{j,t}$ and likely influences estimates of $\xi_{j,t}$. This induces a correlation between $\xi_{j,t}$ and $b_{j,t}$. To correct for this, we quasi-first-difference equation (3) to obtain

$$\delta_{j,t} - \rho\delta_{j,t-1} = \alpha(\ln b_{j,t} - \rho \ln b_{j,t-1}) + \beta(x_{j,t} - \rho x_{j,t-1}) + \widetilde{\gamma}_j + \widetilde{\mu}_t + \nu_{j,t}, \tag{4}$$

where $\widetilde{\gamma}_j = (1 - \rho)\gamma_j$ and $\widetilde{\mu}_t = \mu_t - \rho\mu_{t-1}$. As long as $e_{j,t} = e_{j,t-1} = 1$ and $\nu_{j,t}$ is independent

[12] Because our demand and cost estimates depend on whether or not a BM unit is eligible as captured by $e_{j,t}$ in the data but not on how we model eligibility, we keep the eligibility model as simple as possible. In particular, our probit model neglects the fact that eligibility is persistent. In the online appendix, we include lagged eligibility $e_{j,t-1}$ and show that it is statistically significant. Also our probit model assumes that the probability of having a positive market share is not affected by the bid itself. In the online appendix, we include the log bid $\ln b_{j,t}$ in a number of ways and show that although it is statistically significant, it is economically small: in our preferred specification, a £1/MWh increase in bid (corresponding to 18% of the mean and 36% of the standard deviation of bids) decreases the probability of being eligible by -0.021 on a baseline of 0.75, or by about 2.8%. While the specification of the eligibility model impacts the analysis of learning in Section 5, we expect the impact to be small.

Table 2: Summary Statistics (top 10 firms only)

	Mean	Std. Dev.	Min	Max
Share	0.011	0.016	0.000	0.131
Eligibility	0.752	0.432	0.000	1.000
Bid	5.453	2.759	1.515	21.003
Fully loaded	0.133	0.236	0.000	0.997
Part loaded	0.551	0.373	0.000	1.000
Missing operating position	0.115	0.319	0.000	1.000
Positive FFR volume	0.007	0.085	0.000	1.000
Number of observations	5175			

Summary statistics on the frequency response market. An observation is a bmunit-month, and the sample is restricted to units owned by the top 10 biggest firms (ranked by revenue over the sample period). Eligibility is an indicator for a bmunit receiving positive share. Fully loaded is the fraction of time the unit's final physical notification is that it is fully loaded (i.e. operating at or close to capacity). Part loaded is the corresponding fraction when it is operating below capacity. FFR volume is the quantity of FR provided through firm frequency response contracts (i.e. outside of this market).

of $\eta_{j,t}$, we can estimate equation (4) by non-linear least squares (NLLS).[13] We maintain this independence assumption for ease of presentation since allowing for correlation has little effect on our conclusions (see the online appendix).

Data. Table 2 summarizes the data used in the estimation. Over the first six years of the operation of the FR market, we have 5175 observations at the BM unit-month level. Market shares are small with an average of 1%, although there is considerable heterogeneity and the maximum over months and BM units is 13%. In about 25% of observations, the market share is zero. Bids are £5.5/MWh on average. Some data on operating position is missing, and where it is, we include a dummy for missing operating position in $x_{j,t}$ and interact it with the controls for being fully loaded and part-loaded.

Results: estimates. The first column of Table 3 shows OLS estimates from equation (3) and the second column NLLS estimates from equation (4). The number of observations differs because we require $s_{j,t} > 0$ for OLS and $s_{j,t} > 0$ and $s_{j,t-1} > 0$ for NLLS.

[13] Due to the BM-unit fixed effects equation (4) is estimated using a "within" estimator. The transformation used in estimation has the average of both $\nu_{j,t}$ and $\ln b_{j,t}$ on the right-hand side of the estimation equation. Correlation between these terms is a possible source of bias in the parameter estimates. The econometrics literature shows that this bias in a linear (balanced) panel model is of the order ρ/T (Nickell 1981), and since we observe a BM unit for a median of $T = 72$ months we ignore it.

Table 3: Demand System Estimates

	Market Share		Eligibility
	OLS	NLLS	ML
Log bid	-1.648	-1.614	
	(0.132)	(0.119)	
Fully loaded	1.666	1.949	2.501
	(0.220)	(0.182)	(0.355)
Part loaded	2.111	2.234	2.168
	(0.156)	(0.139)	(0.335)
Positive FFR volume	-0.794	-0.587	-0.500
	(0.200)	(0.245)	(0.461)
Unit and Month FE	yes	yes	yes
ρ	–	0.41	–
s.e. ρ	–	0.03	–
Estimated R^2 (of shares)	0.49	0.67	–
N	3831	3509	5175

In the first two columns, the dependent variable is the log ratio of the share to the outside good share. In the last column it is an indicator for eligibility. The second market share specification allows for an AR(1) process in the error term, and we estimate the quasi-first-differenced equation by non-linear least squares (we provide an estimate of the autocorrelation coefficient ρ and the standard error of that estimate). The R^2 measure reported is for the fit of predicted versus actual shares (again omitting zero-share observations). Standard errors are clustered by bmunit.

The estimates are remarkably similar across specifications. Because market shares are small, the coefficient on log bid closely approximates the price elasticity of demand. It is negative and significantly less than -1, as one would expect. The coefficients on fully loaded and part-loaded in $x_{j,t}$ are positive and significant. This makes sense because a BM unit can provide FR only if it is currently operating. The coefficient on part-loaded is larger than that on fully loaded in line with our expectation that NG prefers to call upon a BM unit in the middle of its operating range. The coefficient on positive FFR volume in $x_{j,t}$ is negative and significant, indicating that a BM unit has a smaller share of the MFR market if it is already under contract with NG, also as expected. Finally, the NLLS estimates from equation (4) in the second column of Table 3 provide evidence of persistence in the unobservable characteristics $\xi_{j,t}$ as the $AR(1)$ coefficient ρ is positive and significant.[14]

[14] We conducted a number of robustness checks. Adding squared terms in the log bid to equation (4) does very little for fit, with the R^2 increasing from 0.6305 to 0.6309. Perhaps the most notable change occurred when we instrument for $\ln b_{j,t}$ with its lag $\ln b_{j,t-1}$. The estimate for α decreases from -1.614 in the middle column of Table 3 to -1.801. While the estimate for α decreases, the estimate for β remains virtually unchanged. We find the same when we additionally instrument with the second lag $\ln b_{j,t-2}$. These changes

The third column of Table 3 shows ML estimates from equation (2). They echo our logit model for market shares. In particular, the coefficients on fully loaded and part-loaded are positive and significant, indicating that a BM unit is more likely to be eligible for providing FR services if it is up and running.

Results: goodness of fit. To assess goodness of fit, we predict the market share of BM unit j in month t conditional on $s_{j,t} > 0$. To do so, we sample independently and uniformly from the empirical distribution of residuals $\hat{\xi}_{j,t}$ for the OLS specification in equation (3) and from the empirical distribution of residuals $\hat{\nu}_{j,t}$ for the NLLS specification in equation (4).[15] In both cases we repeatedly sample to integrate out over the empirical distribution of residuals. The logit model fits the data reasonably well. Comparing the realized with our predicted market shares from equation (3) and equation (4), we obtain an R^2 of 0.49 and 0.67. This reinforces the importance of persistence in the unobservable characteristics $\xi_{j,t}$ and prompts us to take the NLLS estimates from equation (4) in the second column of Table 3 as our leading estimates.[16]

Figure 12 shows that the fit is good even for the largest power stations, whose market shares change quite dramatically from one month to the next. The fact that predicted market shares closely track actual market shares over time makes it clear that the good fit is not solely a consequence of having BM-unit fixed effects.

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are not large enough to affect our conclusions.

[15]In the latter case, we proceed as follows: We first obtain the residuals $\hat{\nu}_{j,t}$ along with the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$ from equation (4). We then rewrite equation (3) as $\delta_{j,t} - \alpha \ln b_{j,t} - \beta x_{j,t} = \gamma_j + \mu_t + \xi_{j,t}$, substitute in $\hat{\alpha}$ and $\hat{\beta}$, and estimate by OLS. This yields the residuals $\hat{\xi}_{j,t}$ along with the estimated BM-unit and month fixed effects $\hat{\gamma}_j$ and $\hat{\mu}_t$. We simulate $\xi_{j,t}$ by substituting $\hat{\xi}_{j,t-1}$ and a draw from the empirical distribution of residuals $\hat{\nu}_{j,t}$ into the law of motion $\xi_{j,t} = \rho\xi_{j,t-1} + \nu_{j,t}$. If BM unit j has a zero share in month $t-1$ so $\xi_{j,t-1}$ is missing, then we go back to the first month $\tau_1 < t-1$ such that $s_{j,\tau_1} > 0$ and we go forward to the first month $\tau_2 > t-1$ such that $s_{j,\tau_2} > 0$. We assume that $\nu_{j,l} = \nu$ for all $l = \tau_1, \ldots, \tau_2$ and solve the equations $\xi_{j,t-1} = \rho^{t-1-\tau_1}\xi_{j,\tau_1} + \nu \sum_{l=0}^{t-1-\tau_1-1} \rho^l$ and $\xi_{j,\tau_2} = \rho^{\tau_2-t+1}\xi_{j,t-1} + \nu \sum_{l=0}^{\tau_2-t} \rho^l$ for ν and $\xi_{j,t-1}$. If missing for a stretch at the beginning so that τ_1 is not defined, then we use the second equation alone with $\nu = 0$; if missing for a stretch at the end so that τ_2 is not defined, then we use the first equation alone with $\nu = 0$.

[16]As we pointed out in Section 2, the demand for and supply of FR are relatively stable over most of the sample period. As noted by a referee, this may seem surprising in light of the advent of wind and solar power. To investigate further, we computed the standard deviation of the residuals $\hat{\nu}_{j,t}$ across BM units over time. There was no discernible increase in volatility over the sample period.

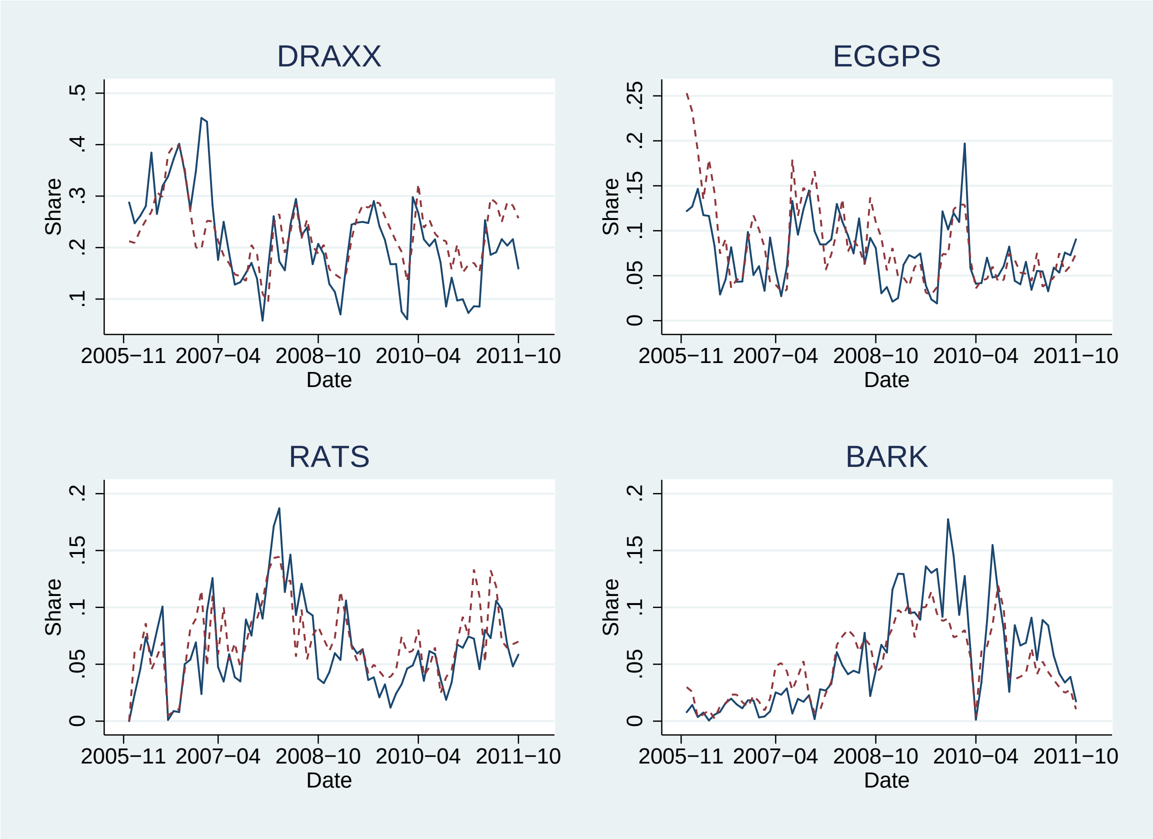

Figure 12: Goodness of fit. Realized (blue, solid) and predicted (red, dashed) market share by month for the four largest power stations Drax (top left panel), Eggborough (top right panel), Ratcliffe (bottom left panel), and Barking (bottom right panel).

4.2 Cost

Since the firms we are modeling have been providing FR for a long time, we assume that they know their own cost (but not necessarily those of their rivals). The cost of providing FR is not known to us, however, and we have to estimate it as an input to the learning models in Section 5.

The main source of cost is the additional wear and tear that a BM unit incurs while in FR mode, which we expect to be relatively stable over time. Let c_j denote the constant marginal cost of BM unit j for providing FR. The realized profit of firm i in month t is

$$\pi_{i,t} = \sum_{j \in \mathcal{J}_i} (b_{j,t} - c_j) M_t s_j(b_t, x_t, \xi_t, e_t; \theta_0), \tag{5}$$

where M_t is market size in month t. The market share of BM unit j in month t depends on the bids b_t, characteristics x_t and ξ_t, and eligibilities e_t, of all BM units, as well as on the true parameters θ_0 of the demand system. In contrast to market share, market size M_t is independent of bids b_t because the response requirement NG is obligated to satisfy is exogenously determined by government regulation as a function of the demand for electricity.

We estimate the marginal cost $c_i = (c_j)_{j\in\mathcal{J}_i}$ for the BM units that are owned by firm i from the bidding behavior of the firm in the late or stable-price phase of the FR market from June 2009 to October 2011. We maintain that a firm's bidding behavior stems from the firm "doing its best" in the sense of choosing its bid to maximize its expected profit conditional on the information available to it. More formally, the bids $b_{i,t}$ of firm i in month $t \geq 44$ maximize the firms' perception of expected profit conditional on the information it has at its disposal at the time the bid is submitted:

$$\max_{b_{i,t}} \mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t} \left[\sum_{j\in\mathcal{J}_i} (b_{j,t} - c_j)\, M_t s_j(b_t, x_t, \xi_t, e_t; \theta_t) \middle| \Omega_{i,t-1} \right], \tag{6}$$

where, in a slight abuse of notation, we use $\Omega_{i,t-1}$ to denote both the firm's perceptions and the information used to form these perceptions. The notation in equation (6) is designed to stress the two main sources of uncertainty that a firm faces: (i) strategic uncertainty about its rivals' bids $b_{-i,t}$ and (ii) demand uncertainty generated by the realizations of ξ_t and e_t and the fact that the parameters θ_t of the demand model may not be known (so to the firm the demand parameters are a random variable). Using the information available to it, the firm forms perceptions about $b_{-i,t}$, ξ_t, e_t, and θ_t, and these perceptions may change over time as more information becomes available.[17] These perceptions underlie the expectation operator $\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}[\cdot|\Omega_{i,t-1}]$ in equation (6). How perceptions are formed is the central question for the learning models that we turn to in Section 5, but for now we remain agnostic.

Equation (6) implies that the firm believes its current bids do not impact future profit, and because of this rules out most models of experimentation. It is therefore not an appropriate characterization of the bidding behavior in the early phase of the FR market. It also rules out collusive equilibria, since in that case firms act to maximize a different objective function. We come back to the possibility of collusion below.

[17]We make the simplifying assumption that the firm has perfect foresight about market size M_t and the characteristics x_t to avoid modeling their perceptions about these objects. Our estimates do not depend on this assumption in any way because our approach is robust to M_t and x_t being unknown to the firm. In contrast, our analysis of learning and equilibrium in Section 5 relies on this assumption.

Equation (6) implies that the bids $b_{i,t}$ of firm i in month $t \geq 44$ solve the first-order conditions

$$\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}\left[M_t s_k(b_t, x_t, \xi_t, e_t; \theta_t) + \sum_{j\in\mathcal{J}_i} (b_{j,t} - c_j)\, M_t \frac{\partial s_j(b_t, x_t, \xi_t, e_t; \theta_t)}{\partial b_{k,t}} \middle| \Omega_{i,t-1}\right] = 0, \quad \forall k \in \mathcal{J}_i. \tag{7}$$

Since we have not specified how the firm forms its perceptions, the system of first-order conditions in equation (7) does not provide the restrictions needed for estimating marginal cost. To derive an estimator for c_i, we use a relatively weak rationality assumption that restricts perceptions in a way that we view as appropriate for the late phase.

By the time the FR market enters the late phase, a firm has had ample opportunity to observe how its rivals bid as well as the resulting allocation of market shares. There are changes in the environment during the late period, and these do cause changes in bids, but we assume that the firm's bids are by now free of bias. That is, we find our estimate of c_i by first substituting the realized market size M_t and market shares $s_{i,t} = (s_{j,t})_{j\in\mathcal{J}_i}$ for the BM units that are owned by firm i as well as our estimate $\hat{\alpha}$ from Table 3 into equation (7) and then setting the time-average of the first-order conditions for months $t \geq 44$ to zero, or

$$\frac{1}{29}\sum_{t=44}^{T=72}\left[M_t s_{k,t} + \sum_{j\in\mathcal{J}_i}(b_{j,t} - c_j)\, M_t \left(1(k=j) - s_{k,t}\right)\frac{\hat{\alpha} s_{j,t}}{b_{k,t}}\right] = 0, \quad \forall k \in \mathcal{J}_i, \tag{8}$$

where we have substituted out for the derivatives in equation (7) using the properties of the logit and $1(\cdot)$ is the indicator function. We estimate c_i by solving this system of $|\mathcal{J}_i|$ equations for the $|\mathcal{J}_i|$ unknowns. This is straightforward because the equations are linear in the unknowns.

Given the behavioral assumption in equation (6), sufficient conditions for our estimation procedure to yield a consistent estimate of c_i as the time horizon $T \to \infty$ are that (i) our estimate of θ is consistent for that parameter, (ii) the firm's perceptions about $b_{-i,t}$, ξ_t, e_t, and θ_t lead to an unbiased estimate of the time-averaged first-order conditions in equation (8) as $T \to \infty$, and (iii) values of $\tilde{c}_i$ different from the true marginal cost c_i lead to values of the time-averaged first-order conditions that are bounded away from zero as $T \to \infty$.[18]

The behavioral assumption in equation (6) is standard: the econometrician has to understand the incentives faced by the firm in order to use the implications of the firm's actions in

[18] Formally, let $y_{i,t} \equiv (M_t, b_{i,t}, s_{i,t})$ and define $h^k(c_i, \alpha, y_{i,t}) \equiv M_t s_{k,t} + \sum_{j\in\mathcal{J}_i}(b_{j,t} - c_j)\, M_t \left(1(k=j) - s_{k,t}\right)\frac{\alpha s_{j,t}}{b_{k,t}}$, and $h(c_i, \alpha, y_{i,t}) \equiv \left[h^1(c_i, \alpha, y_{i,t}), \ldots, h^{J_i}(c_i, \alpha, y_{i,t})\right]'$. Let

estimation. The consistency condition (i) and the identification condition (iii) are also standard (identification is hopeless if the objective function cannot asymptotically distinguish the true marginal cost from alternative values). But notice that we have not specified how a firm forms its perceptions. Condition (ii) does not require us to assume that the market is in a "rational expectations" equilibrium or that the environment necessarily reaches some sort of rest point (neither the actual nor the perceived distribution of bids have to be stationary). It does, however, require that the average of the firm's perceptions of its first-order conditions converges to the true average over time. Although we think of condition (ii) as a weak rationality condition (as in Fershtman and Pakes 2012), we do not know of a characterization of the necessary conditions that a learning model has to satisfy in order to generate it.[19] In the appendix we show that a sufficient condition for such convergence is that the subjective probability distribution underlying the firm's perceptions converges weakly to the objective probability distribution (uniformly over information sets).

Results: estimates. The average of the marginal cost c_j that we estimate for the $J = 72$ BM units owned by the ten largest firms is £1.40/MWh, with a standard deviation of £0.66/MWh across BM units.[20] The estimates are reasonably precise, with an average standard error of £0.04/MWh. By comparison, pre CAP047 the "cost reflective" administered price was around £1.7/MWh.[21] Since we expect some markup to be built into the administered price, the marginal costs we recover are in the right ballpark.

$h^T_{i,t}(c_i) = h(c_i, \hat{\alpha}_T, y_{i,t})$ and $h^e_{i,t}(c_i) = \mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}[h(c_i, \alpha, y_{i,t})|\Omega_{i,t-1}]$. We require that

$$\|T^{-1}\sum_{t=44}^{T}\left(h^T_{i,t}(c_i) - h^e_{i,t}(c_i)\right)\| = o_p(1) \quad \text{and} \quad \sup_{\|\tilde{c}_i - c_i\| \geq \epsilon} \|T^{-1}\sum_{t=44}^{T} h^T_{i,t}(\tilde{c}_i)\|^{-1} = O_p(1), \quad \forall \epsilon > 0,$$

where $\|\cdot\|$ is the Euclidean norm, $o_p(1)$ indicates convergence in probability to zero, and $O_p(1)$ indicates stochastically bounded. The proof of consistency follows from Theorem 3.1 in Pakes and Pollard (1989).

[19] In general, learning need not occur. Easley and Kiefer (1988) show that a single optimizing agent who controls a stochastic process may fail to learn the parameters of that process, depending on his incentive to "explore" versus "exploit." In games, fictitious play does not necessarily guarantee that players have correct perceptions of each other's play even if play is continued indefinitely (see the rock, paper, scissors example in Shapley 1964). The available convergence results typically rely on stringent assumptions on the underlying game (see Fudenberg and Levine 1998).

[20] Because one BM unit has zero share during the late phase, we impute its marginal cost with that of the other BM unit in the same power station.

[21] We have two sources: Figure 2 and a document prepared just prior to CAP047 by NG for Ofgem, the government regulator. (`www.ofgem.gov.uk/ofgem-publications/62273/8407-21104ngc.pdf`). It states in paragraph 5.3 that the holding payment is "of the order of £5/MWh" for the bundle of primary, secondary, and high response, implying an average of £1.67/MWh per type of FR.

Table 4: Cost estimates for the top 8 stations (by total revenue)

Station	# Units	Fuel	Vintage	Mean	Std. Dev. (within station)
Barking	2	CCGT	1994	1.20	.01
Connah's Quay	4	CCGT	1996	1.04	.03
Cottam	4	Coal	1969	1.35	.04
Drax	6	Coal	1974	1.06	.04
Eggborough	4	Coal	1968	1.53	.06
Peterhead	1	CCGT	2000	1.54	0
Ratcliffe	4	Coal	1968	1.33	.06
Seabank	2	CCGT	1998	1.59	.01

Summary statistics on the unit-specific cost estimates derived from solving the firm first order condition arising from the demand system, reported as the within-station average cost and standard deviation in costs.

Table 4 shows the average marginal cost for the BM units belonging to the eight largest power stations. It is quite reasonable and varies between £1.04/MWh and £1.59/MWh across stations. The standard deviation of marginal cost within a station is very small, on the same order as the standard error of the estimates. Most of the variation in marginal cost is therefore across stations. This is in line with the fact that the BM units belonging to the same station tend to be identical.

Table 5 shows the result of regressing marginal cost on the characteristics of the BM units. As expected, a (typically smaller) BM unit using dual fuel or oil has lower cost than a BM unit using other fuel types. Moreover, although not statistically significant, the estimates suggest that a BM unit of later vintage has lower cost.

Results: residuals. Using our estimates, we evaluate realized values of the profit derivative $M_t s_{k,t} + \sum_{j \in \mathcal{J}_i} (b_{j,t} - c_j) M_t (1(k=j) - s_{k,t}) \frac{\hat{\alpha} s_{j,t}}{b_{k,t}}$ in equation (8) for each BM unit and month. For simplicity, we call this value a "residual." The average of this residual over the late phase of the FR market is zero for all BM units by construction. Figure 13 shows the time series of the average residual across BM units. It contrasts the early and middle phases in the left panel with the late phase of the FR market in the right panel (for visual clarity, we scale the vertical axis differently in the two panels).

The average residual starts well above zero in the early phase before falling below zero in the middle phase. The standard deviation falls throughout, consistent with our earlier discussion

Table 5: Projecting costs onto unit characteristics

	Cost estimate
Unit vintage	-0.015
	(0.017)
Dual Fuel	-0.819
	(0.466)
Large Coal	-0.463
	(0.429)
Medium Coal	-0.683
	(0.544)
Oil	-0.967
	(0.397)
R^2	0.13
N	71

The dependent variable is the cost estimate c_j. The omitted fuel type is combined cycle gas turbines (CCGT). One observation is dropped because of missing vintage data.

of convergence. In the late phase, the average residual is above zero in some months and below zero in others and the standard deviation does not exhibit a trend. Interestingly, even after the substantial increase in FFR volume that occurs in July 2010 (see Figure 4; July 2010 is marked with a dotted line in the right panel of Figure 13) and changes in participation during this phase (see Figure 5), the standard deviations of the residual are still an order of magnitude smaller than in the earlier phases of the FR market.

We also examined whether the residuals are autocorrelated. The first three columns of Table 6 display the coefficients from separate regressions of the residual on its lagged value for each of the three phases of the FR market, including BM-unit fixed effects in all regressions. In the last three columns we further restrict attention to observations in which the BM unit's bid changed between months.

We find significant autocorrelation in all regressions but the last. Assuming our specification and cost estimates are correct, this indicates that some firms are making mistakes that are not corrected in the subsequent month. This may reflect persistent differences between a firm's perceptions of its expected profits and reality, an interpretation that makes particular sense in the early and middle phases of the FR market where firms had little experience and behaved quite differently. Relatedly it is striking how the R^2 falls over the three phases of the FR market, indicating that the lagged value explains progressively less of the variation in

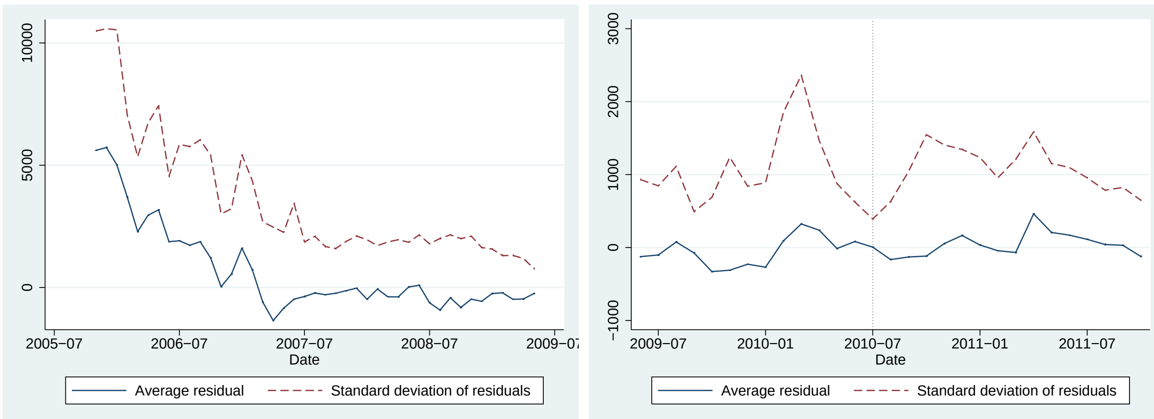

Figure 13: Average and standard deviation of residuals during the early and middle phases (left panel) and late phase (right panel). In the right panel, the dotted line indicates July 2010, when FFR volume nearly doubles.

Table 6: Autocorrelation in residuals

	Early	Middle	Late	Early	Middle	Late
		All			Bid changes only	
Lagged residual	0.542	0.343	0.445	0.389	0.126	0.029
	(0.087)	(0.050)	(0.063)	(0.059)	(0.042)	(0.080)
R^2	0.63	0.48	0.20	0.74	0.38	0.08
N	1080	1931	2088	355	449	401

The dependent variable is the residual in the FOC at the estimated costs. Controls are the lagged residual and unit fixed effects. The regressions with bid changes only include only observations in which the unit's bid was different from its bid in the previous period. Standard errors are clustered by unit.

the residual. In the third phase, we still find significant autocorrelation using all observations (third column), but that autocorrelation essentially disappears when we restrict attention to observations in which the BM unit's bid changed between months (last column). Our interpretation of this is that by the third phase firms have reasonably accurate perceptions, but do not always update their bid. When they do choose to update their bid, they do so in a way that accounts for the information contained in the lagged residual.

Robustness check: fuel price. As noted in Section 2, there is an upward trend in oil and — to a lesser extent — gas prices towards the end of the sample period. While the major cost of providing FR is additional wear and tear, a BM unit in FR mode may run with a degraded heat rate. Hence, the cost of providing FR may be tied to the fuel price. As a robustness check, we model the marginal cost of BM unit j in month t as $c_{j,t} = c_j + \mu f_{j,t}$, where c_j is a BM-unit fixed effect, $f_{j,t}$ is the fuel price that the BM unit faces, and μ is a parameter to be estimated. In the online appendix we provide details on how we accommodate the additional parameter in the estimation. We find that μ is negative and significant but economically small. The negative sign appears inconsistent with the intuition that a BM unit in FR mode consumes more fuel. We therefore re-estimated cost allowing for a time trend (with and without the fuel price) and found that the impact of the fuel price is indistinguishable from a downward time trend. So there may be something changing cost over time in our data, but it is not necessarily related to the fuel price. To ensure that the downward time trend does not affect our results, we re-ran the analysis of the learning models in Section 5 with the alternative specification of cost. As documented in the online appendix, our conclusions remain unchanged.

Robustness check: repositioning in the BM. One might worry that equation (5) does not reflect the full set of incentives a firm faces, as it does not account for the profit that accrues to a BM unit as it is repositioned in the BM in preparation for providing FR. In the online appendix, we incorporate these incentives. Using additional data on the BM, we first model and estimate demand for repositioning. Extending equation (8), we then simultaneously estimate the marginal cost of providing FR and the markup on repositioning. The estimated markup is very small and not statistically different from zero, and the marginal cost of providing FR does not change materially from that reported earlier in this section.

The markup partly reflects the amount of attention paid to the profit from repositioning when deciding on the FR bid. Our estimate may thus be explained by the fact that FR bids and bids in the BM are made by different people within the firm, and those deciding on FR bids may not pay attention to the BM. This is consistent with our conversations with Ian Foy, who told us that people in the industry do not think of repositioning incentives when deciding on FR bids.[22]

Robustness check: collusion. We finally turn to the possibility that firms collude and that equation (5) is therefore misspecified. In the online appendix, we try three different

[22] A possible explanation may be that the compensation of different employees is tied to different markets. We thank an anonymous referee for pointing this out.

Table 7: Profit Statistics

Rank	Firm name	Months Changed	Matched Direction (%)	Total Profit	Ex-Post Lost Profit (%)	Ex-Post Bid Diff.	Ex-Post Bid Diff. (%)
1	Drax Power Ltd.	23	80	68	2.0	1.1	20
2	E.ON UK plc	52	67	44	2.0	0.8	14
3	RWE plc	15	86	25	3.1	0.9	23
4	Eggborough Power Ltd	18	58	18	1.4	0.8	17
5	Keadby Generation Ltd	17	80	15	5.1	1.0	19
6	Barking Power Ltd	57	49	12	6.2	0.7	15
7	Jade Power Generation Ltd	15	54	10	4.7	1.2	24
8	SSE Generation Ltd	17	62	9	0.5	0.4	8
9	Seabank Power Ltd	9	89	9	8.2	1.1	20
10	Centrica plc	42	73	9	2.6	0.9	18

Months changed indicates the total number of months in which a firm changed the bid of one or more of their units. Matched direction indicates the share-weighted percent of bid changes that are in the same direction as our estimated ex-post-optimal bid. Ex-post lost profit indicates the share-weighted percent increase in profit by taking the estimated ex-post-optimal bid instead of the actual bid. Ex-post bid difference indicates the share-weighted average absolute difference between the ex-post-optimal and actual bids, in absolute and relative terms.

ways of examining the data for evidence of collusion. The first is to examine whether bid changes across BM units owned by different firms are correlated in either timing or direction. The pairwise correlations are symmetrically distributed around zero (whereas the within-firm correlations are virtually all positive). This is incompatible with most models of coordinated pricing, though it is possible that an unknown subset of firms collude and that there is correlation within that subset. Our second approach is more direct. We assume particular collusive arrangements and infer cost given the assumed conduct. Our estimate of marginal cost becomes negative, which we take as evidence against these arrangements. Finally, at the suggestion of a referee, we ask how much weight the firms can put on their rivals' profits when optimizing their bids while ensuring that the observed bids are consistent with non-negative cost. We find that the maximum weight is relatively small.

Timing of bid changes and ex-post profitability. One of the striking empirical regularities of the data is that some firms take a far more active approach to bidding than others. This is documented in Table 7, where we count the number of months in which a firm updated the bid of *any* of its BM units. E.ON, Barking, and Centrica change their bids in more than half of the months, but the remainder of the top 10 firms (notably including Drax) make changes far less frequently (column labelled "months changed"). The frequency of bid changes is not significantly correlated with firms' realized profits over the full sample period: the three most active firms are ranked second, sixth and tenth in terms of profits (column labelled "total profit").

To get a sense of how costly this infrequent adjustment may be, we use our demand and cost estimates to compute a firm's ex-post optimal bid, i.e., the bid that would have been optimal had the firm known $b_{-i,t}$, ξ_t, and e_t when choosing its bid. The ex-post optimal bid yields an upper bound to what the firm can earn as it requires information that the firm does not have. Moreover since it can be computed without committing to a particular model of how the firm forms perceptions, it allows us to offer statistics that are more "model-free" than those in the learning analysis below. We also define the ex-post lost profit as the difference between profit at the ex-post optimal bid $b^*_{i,t}$ and profit at the observed bid $b_{i,t}$.

Somewhat surprisingly, the ex-post lost profit does not allow us to explain the timing of bid changes. While we expect firms to be more likely to adjust their bids in months where the ex-post lost profit is large, we find no statistically significant support for this in any of a number of probit regressions that explore different plausible specifications.[23]

The ex-post optimal bid helps to explain the direction in which firms adjust their bids, conditional on adjusting. The column labelled "matched direction" in Table 7 indicates the percentage of times that such adjustments are in the direction of the ex-post optimal bid (share-weighted across the BM units within a firm). It is well above 50% for many firms. For the most active firms we match the direction less often, consistent with our account of firms "exploring" during the early phase of the FR market. The percentage of matches is at most weakly correlated with firm size as measured by realized profit over the sample period: the four highest percentages are ranked ninth, third, first, and fifth in terms of firm size. This seems different than Hortaçsu and Puller's (2008) finding that firms with large stakes made bids that were closer to optimal in the electricity spot market in Texas.

To measure how much money the firms have "left on the table" we look at the ex-post lost profit over the full sample period as a percentage of realized profit in the column labelled

[23]We note that this makes it unlikely that switching costs are the root cause for the infrequent adjustments to bids, as most models with switching costs predict bid changes exactly at times when the perceived gains to adjustment are largest. Despite our skepticism of these models, we provide alternative cost estimates in the online appendix that are consistent with a very simple model of switching costs or inattention; these line up closely with our leading estimates.

“ex-post lost profit (%)” of Table 7. The magnitudes are generally small. For example, RWE plc rarely updated its bids and slowly tracked the market upwards. As a result, we estimate that they lost £768,000 or 3.06% over six years. While this is enough money that we may expect RWE plc to pay more attention and update its bids more frequently, it is perhaps not enough to justify hiring a full-time employee to study the FR market and optimize bidding. Contrast these small ex-post profit differences with the average absolute difference between a firm's (share-weighted) average bid and the ex-post optimal bid, shown in the last two columns labelled “ex-post bid difference” and “ex-post bid difference (%)” in absolute and relative terms. These magnitudes are much larger, with most firms placing bids that are around 15% to 20% away from their ex-post optimal bids.

This suggests a possible reason why we have been unable to explain the timing of bid changes: once one allows for the uncertainty which the ex-post optimal bid abstracts from, it may not be obvious to a firm that adjusting its bid increases its profit in any substantial way.[24] Yet, as Akerlof and Yellen (1985) have noted — and as our study seems to illustrate — even small departures from perfectly rational behavior may lead to aggregate behavior that is quite different from equilibrium. To further investigate this disequilibrium bidding behavior, we use learning models.

5 Learning and equilibrium

In this section we consider how well different learning models fit the data. We noted in Section 3 that realistically accounting for the bidding behavior in the early phase of the FR market requires an explanation for the heterogeneity in the way firms learn and a model that allows for experimentation. These are topics we do not tackle here. Instead, we consider models of the bidding behavior in the middle and late phase of the FR market. The middle phase is characterized by a convergence of bids in a relatively stable environment whereas there are several environmental changes in the late phase.

Our learning models capture the two main sources of uncertainty that a firm faces, namely (1) strategic uncertainty about its rivals' bids $b_{-i,t}$, and (2) demand uncertainty generated

[24] The fact that profit is not overly sensitive to a deviation from the ex-post optimal bid may have lengthened the time it took the FR market to reach a rest point. The lack of sensitivity may have implied a combination of rational inattention (Sims 2003) and satisficing behavior (Simon 1955) that may have been optimal along the observed path of play given firms' various human resource and institutional constraints.

both by the realizations of ξ_t and e_t and by the fact that the parameters θ of the logit model may not be known. As noted in Section 1, the literature traditionally uses different types of models for how a firm forms perceptions about rivals' bids and for how the firm forms perceptions about demand, and so do we. Our learning models combine fictitious play as a model for learning about rival's bids with adaptive learning about demand.

Recall that prior to CAP047 providing FR was mandatory, so at the start of our study firms already had quite a bit of experience with demand and cost. We therefore assume throughout that firms know the marginal cost c of all BM units, the $AR(1)$ process generating ξ_t, the objective probability distribution of e_t, and the BM-unit fixed effects $\gamma = (\gamma_j)_{j=1,\dots,J}$. The latter capture the time-invariant preferences of NG for the different BM units. However, there is reason to think firms had to learn about other aspects of demand. In particular, since prior to CAP047 the holding payment was at an administered price which had been fairly constant over time, firms may not have been able to assess the sensitivity of NG to the bids they submit, a sensitivity captured by the parameter α in our model. They may also have been uncertain about the parameter β to the extent that the time-varying characteristics $x_{j,t}$ mattered differently for NG post CAP047. Finally, firms may have been uncertain about the month fixed effects $\mu = (\mu_t)_{t=1,\dots,72}$ that subsume any time-varying characteristics of the outside good.

As a baseline we also compute a complete information Nash equilibrium. This is the leading model in the empirical industrial organization literature, so how it compares to our learning models is of some interest.

5.1 Complete information Nash equilibrium

The assumptions underlying a complete information Nash equilibrium are more restrictive than those we needed to estimate cost. Formally, the bids b_t^* of all BM units satisfy the system of J equations

$$\begin{aligned}\mathrm{E}_{\xi_t,e_t}\Bigg[& M_t s_k(b_t^*, x_t, \xi_t, e_t; \theta_0) \\ & + \sum_{j\in\mathcal{J}_i} \left(b_{j,t}^* - c_j\right) M_t \frac{\partial s_j(b_t^*, x_t, \xi_t, e_t; \theta_0)}{\partial b_{k,t}} \Bigg| M_t, x_t, \xi_{t-1}, \theta_0, c\Bigg] = 0, \quad \forall k = 1, \dots, J. \qquad (9)\end{aligned}$$

The notation $\mathrm{E}_{\xi_t,e_t}[\cdot|\cdot]$ in equation (9), in contrast to $\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}[\cdot|\cdot]$ in equation (6), is meant to indicate that the expectation is computed with respect to the objective probability distribution of ξ_t and e_t conditional on the information available to firm i in month $t-1$ when it submits its bids $b_{i,t}$ for month t. We assume that the firm has perfect foresight about market size M_t and the observable characteristics x_t. In addition, we condition the expectation operator on the unobservable characteristics ξ_{t-1} with the implicit understanding that $\xi_{j,t}$ follows the $AR(1)$ process $\xi_{j,t} = \rho\xi_{j,t-1} + \nu_{j,t}$ with ρ known. Finally, we abstract from uncertainty about the parameters of the demand system for now and condition the expectation operator on θ_0. Later on we extend the complete information Nash equilibrium to accommodate adaptive learning about demand.

Note that the expectation operator in equation (9) does not condition on $b_{-i,t}$. While the best response of firm i depends on its perceptions of its rivals' bids, in a complete information Nash equilibrium these perceptions are consistent with actual play. A complete information Nash equilibrium is thus obtained by solving the above J equations in the J unknowns b_t^*.

We are not aware of a result in the extant literature that shows that there is a unique complete information Nash equilibrium in our setting. However, we have checked extensively for multiplicity by trying different starting points for the equation solver and never found more than one complete information Nash equilibrium.

To make equation (9) practical we replace ξ_{t-1}, θ_0, and c by our estimates of those objects and evaluate the expectation operator using Monte Carlo integration. To this end we generate a random sample $\left(\xi_t^{(s)}, e_t^{(s)}\right)_{s=1,\dots,S}$ with $S = 50,000$ and replace the expectation operator by the corresponding sample average. To obtain $\xi_{j,t}^{(s)}$, we sample independently and uniformly from the empirical distribution of residuals $\hat{\nu}_{j,t}$ from the NLLS specification in equation (4) (for further details see footnote 15). To obtain $e_{j,t}^{(s)}$, we sample independently from a Bernoulli distribution with success probability $\Pr(e_{j,t} = 1|x_{j,t})$ as specified in equation (2).

5.2 Learning models

We now turn to the learning models we use for firms' perceptions about rivals' bids and demand.

Perceptions about rivals' bids: fictitious play. Belief-based learning starts with the premise that players keep track of the history of play and form beliefs about what their rivals will do in the future based on their past play. We consider fictitious play as a leading example of belief-based learning.[25] In particular, we assume that in month $t-1$ when firm i chooses its bids $b_{i,t}$ it believes that its rivals' bids $b_{-i,t}$ are sampled from the empirical distribution of their past play. To account for correlation in the bids of firm i's rivals, we sample an entire vector of rivals' bids. Since firm i may believe that its rivals' bids $b_{-i,t}$ are likely to be more similar to more recent observations, we allow for geometrically declining sampling weights. In particular, we assign sampling weight $\delta^{t-\tau-1}$ to rivals' bids $b_{-i,\tau}$ in month $\tau \leq t-1$, where $\delta \in [0,1]$ is a decay parameter, and then normalize so the sampling weights sum to one. For month $\tau = t-1, t-2, t-3, \ldots$ the sampling weight is therefore proportional to $1, \delta, \delta^2, \ldots$.[26]

We let $F(\delta)$ denote our model of fictitious play with decay parameter δ. Note that $\delta = 1$ indicates no decay and $\delta = 0$ full decay. Under $F(0)$ firm i believes that its rivals' bids $b_{-i,t}$ in month t are equal to its rivals' bids $b_{-i,t-1}$ in month $t-1$ with certainty. Adaptive best response thus arises as a special case of fictitious play with $\delta = 0$. In the other extreme, under $F(1)$ all past observations are weighted equally.[27]

Perceptions about demand: adaptive learning. To account for uncertainty about demand in addition to uncertainty about rivals' bids, we allow a firm to adaptively learn about the parameters θ of the logit model. Developed in the macroeconomics literature, adaptive learning stipulates that agents learn about parameters in the same way as econometricians do, by using the available data to estimate them. As more data becomes available over time, agents update their estimates.[28]

We focus on a simple form of adaptive learning that assumes that firms use the regression procedure described in Section 4.1 and the data available to them to estimate the parameters

[25] For a discussion of fictitious play and its variants see Fudenberg and Levine (1998).

[26] In nine instances the bid $b_{j,t}$ of BM unit j in month t is missing. To facilitate sampling, we impute it by going back in time to the first month $\tau < t$ such that $b_{j,\tau}$ is not missing.

[27] The decay parameter δ implicitly also determines the variance of a firm's beliefs about its rivals' bids. In particular, the variance vanishes as δ approaches zero. While this is a feature of the fictitious play models used in the theory and experimental literatures, a fruitful avenue for future research may be to consider other learning models that disentangle the variance of a firm's beliefs from the weight the firm assigns to the more distant past.

[28] For a detailed treatment of adaptive learning see Evans and Honkapohja (2001).

they are uncertain about. In month $t-1$, when firms prepare their bids for month t, firms only have data for month $\tau \leq t-2$ at their disposal (NG does not publish quantities for a month until the very end of the month). We refer to the estimates obtained in this manner as the "sequential" estimates for month t and distinguish them from the "full-sample" estimates obtained in Section 4.1. We assume throughout that firms ignore any uncertainty in the sequential estimates so that their perceptions put point mass on those estimates.

We noted above that there is reason to think that firms may have been uncertain about the price sensitivity parameter α, the coefficient β on the time-varying characteristics $x_{j,t}$, and about the month fixed effects μ. Accordingly, we distinguish four combinations of parameters that firms may have been uncertain about, namely: α, α and β, α and μ, and α, β, and μ. Our notation for and implementation of adaptive learning for these four cases plus the baseline case that abstracts from demand uncertainty follows:

1. $A(\alpha)$: Using data for month $\tau \leq t-2$ and fixing β, ρ, $\widetilde{\gamma}$, and $(\widetilde{\mu}_\tau)_{\tau \leq t-2}$ at the full-sample estimates in Table 3, estimate equation (4) by OLS to obtain the sequential estimate $\hat{\alpha}^{(t)}$ for month t.

2. $A(\alpha, \beta)$: Using data for month $\tau \leq t-2$ and fixing ρ, $\widetilde{\gamma}$, and $(\widetilde{\mu}_\tau)_{\tau \leq t-2}$ at the full-sample estimates, estimate equation (4) by OLS to get the sequential estimates $\hat{\alpha}^{(t)}$ and $\hat{\beta}^{(t)}$ for month t.

3. $A(\alpha, \mu)$: Using data for month $\tau \leq t-2$ and fixing β, ρ, and $\widetilde{\gamma}$ at the full-sample estimates, estimate equation (4) by OLS to obtain the sequential estimates $\hat{\alpha}^{(t)}$ and $\left(\widehat{\widetilde{\mu}}_\tau^{(t)}\right)_{\tau \leq t-2}$ for month t. Using data for month $\tau \leq t-2$ and fixing β and γ at the full-sample estimates and α at the sequential estimate for month t, estimate equation (3) by OLS to obtain the sequential estimates $\left(\hat{\mu}_\tau^{(t)}\right)_{\tau \leq t-2}$ (for more details see footnote 15). Extrapolate $\hat{\mu}_t^{(t)} = \hat{\mu}_{t-2}^{(t)}$ to obtain the sequential estimate $\hat{\mu}_t^{(t)}$ for month t.

4. $A(\alpha, \beta, \mu)$: Using data for month $\tau \leq t-2$ and fixing ρ and $\widetilde{\gamma}$ at the full-sample estimates, estimate equation (4) by OLS to obtain the sequential estimates $\hat{\alpha}^{(t)}$, $\hat{\beta}^{(t)}$, and $\left(\widehat{\widetilde{\mu}}_\tau^{(t)}\right)_{\tau \leq t-2}$ for month t. Using data for month $\tau \leq t-2$ and fixing γ at the full-sample estimate and α and β at the sequential estimates for month t, estimate equation (3) by OLS to obtain the sequential estimates $\left(\hat{\mu}_\tau^{(t)}\right)_{\tau \leq t-2}$. Extrapolate $\hat{\mu}_t^{(t)} = \hat{\mu}_{t-2}^{(t)}$ to obtain the sequential estimate $\hat{\mu}_t^{(t)}$ for month t.

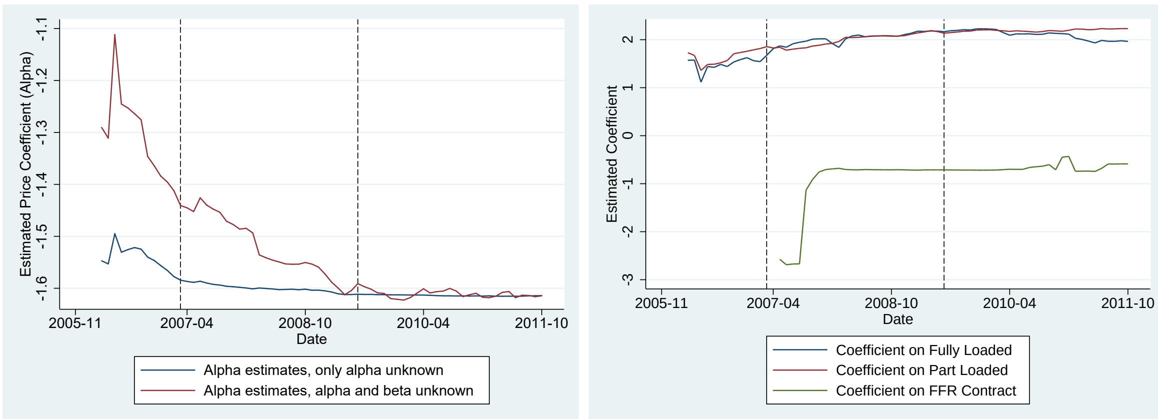

Figure 14: Sequential estimates $\hat{\alpha}^{(t)}$ under $A(\alpha)$ and $A(\alpha,\beta)$ (left panel) and $\hat{\beta}^{(t)}$ under $A(\alpha,\beta)$ (right panel) by month.

5. $A(\emptyset)$: Fix all parameters θ at the full-sample estimates in Table 3 to abstract from demand uncertainty.

Figure 14 illustrates the sequential estimates. The left panel shows $\hat{\alpha}^{(t)}$ obtained alternatively under $A(\alpha)$ and $A(\alpha,\beta)$ and the right panel shows $\hat{\beta}^{(t)}$ obtained under $A(\alpha,\beta)$. Over time the sequential estimates by construction approach the full-sample estimates obtained in Section 4.1. The sequential estimates for the price sensitivity parameter α start out small in absolute value and gradually decrease. Until the late phase of the FR market, $\hat{\alpha}^{(t)}$ is considerably smaller under $A(\alpha)$ than under $A(\alpha,\beta)$. The sequential estimate produced by $A(\alpha)$ is within 2% of the full sample estimate by the start of the middle phase. The sequential estimate $\hat{\beta}^{(t)}$ on positive FFR volume is far from its full-sample estimate until month 25 (because there is little FFR volume in the early part of the data), but stabilizes thereafter. The sequential estimates $\hat{\beta}^{(t)}$ on part-loaded and fully loaded are much less volatile and gradually trend towards their full-sample estimates.

Predictions. We combine fictitious play with adaptive learning to make predictions. We measure fit by comparing our predictions to the observed bids. We make two kinds of predictions. The first is one-period predictions: for month t during the middle and late phases of the FR market, we take the data available to the firms at the time they bid (which includes bids $(b_r)_{r\leq t-1}$ and market shares $(s_r)_{r\leq t-2}$) and predict their bids. This corresponds to the thought experiment of predicting the next move of a player in a game and is analogous to the one-step-ahead predictions used to assess predictive accuracy in the experimental and computational literatures (Erev and Roth 1998, Fershtman and Pakes 2012).

However, industrial organization analysts are often asked to predict how prices evolve for many periods following a policy change or a change in market institutions. We therefore also consider multi-period predictions in which we sequentially predict bids and market shares and then overwrite the observed data with our predictions as we go. To see how these differ, consider the following example: if the strategic model specifies that firms best respond to last period's bids, and we predict in period t that BM unit 1 bids 7 but it actually bids 11, then in the one-period prediction exercise we predict that in period $t+1$ the other firms best respond to the actual bid of 11, but in the multi-period prediction exercise they best respond to the predicted bid of 7. This allows the possibility that our multi-period predictions increasingly diverge from the observed data. Indeed, if the model is correct, then the variance of the prediction error should grow with the distance between the base period and the period for which we are predicting.

A fictitious play model $F(\delta)$ with $\delta \in [0,1]$ and an adaptive learning model $A(y)$ with $y \in \{\alpha, (\alpha,\beta), (\alpha,\beta,\mu), \emptyset\}$ together with available data D_t determine a predicted bid $b_{i,t}^{(\delta,y)}$ for firm i in month t. That bid $b_{i,t}^{(\delta,y)}$ solves the system of $\mathcal{J}_i$ equations

$$\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}\Bigg[M_t s_k(b_{i,t}^{(\delta,y)}, b_{-i,t}, x_t, \xi_t, e_t, \theta_t) \\ + \sum_{j\in\mathcal{J}_i}\left(b_{j,t}^{(\delta,y)} - c_j\right) M_t \frac{\partial s_j(b_{i,t}^{(\delta,y)}, b_{-i,t}, x_t, \xi_t, e_t, \theta_t)}{\partial b_{k,t}}\Bigg| F(\delta), A(y), D_t\Bigg] = 0, \quad \forall k \in \mathcal{J}_i. \tag{10}$$

The notation $\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}[\cdot|\cdot]$ indicates that the expectation operator is with respect to the subjective probability distributions of $b_{-i,t}$ and θ_t induced by $F(\delta)$ and $A(y)$. To make equation (10) practical we replace ξ_{t-1} and c in D_t by our estimates. In the single-period prediction exercises, we estimate θ_t for the adaptive learning model $A(y)$ as detailed above using the observed data for month $\tau \leq t-2$. We evaluate the remaining random variables in the expectation operator by drawing samples $\left(b_{-i,t}^{(s)}, \xi_t^{(s)}, e_t^{(s)}\right)_{s=1,\ldots,S}$ with $S = 50,000$, proceeding as in Section 5.1 for sampling $\xi_{j,t}^{(s)}$ and $e_{j,t}^{(s)}$. We sample $b_{-i,t}$ from the data $(b_{-i,\tau})_{\tau\leq t-1}$ according to $F(\delta)$.

The multi-period prediction exercises are identical except for the data D_t: for any model $(F(\delta), A(y))$ we replace the observed bids and market shares during the middle and late phases with our predictions $b_t^{(\delta,y)}$, thus changing what firms observe.[29] This has two effects. First, when sampling $b_{-i,t}$ according to $F(\delta)$, we now sometimes sample our predicted bids. Second, when estimating θ_t, the dataset used for estimation now contains our predicted bids and market shares. These effects generate differences between the single- and multi-period predictions. For reasons that will become clear below, for the multi-period prediction exercises we only consider the adaptive learning models $A(\emptyset)$ and $A(\alpha)$.

5.3 Measures of fit

We consider two different measures of fit. First, we ask how well alternative learning models and the complete information Nash equilibrium predict the overall cost of the FR market to NG. Second, we compare the models to each other using the traditional measure of model performance, namely the mean square prediction error. In this section, we define our measures of fit and in Section 5.4 we describe how we conduct formal statistical tests. Throughout we focus on the $J = 72$ BM units owned by the ten largest firms.

NG's costs. We measure how well the model $(F(\delta), A(y))$ predicts NG's costs as

$$CE^{(\delta,y)} = \frac{1}{T}\sum_t CE_t^{(\delta,y)} = \frac{1}{T}\sum_t \left| \sum_j \left(b_{j,t}^{(\delta,y)} - b_{j,t}\right) \frac{s_{j,t}}{\sum_j s_{j,t}} \right|.$$

$CE^{(\delta,y)}$ is the average across months of the absolute value of the prediction error $CE_t^{(\delta,y)}$. The actual price of FR paid by NG in month t is $\sum_j b_{j,t}\frac{s_{j,t}}{\sum_j s_{j,t}}$, while the price predicted by the model is $\sum_j b_{j,t}^{(\delta,y)}\frac{s_{j,t}}{\sum_j s_{j,t}}$. Replacing $b_{j,t}^{(\delta,y)}$ by $b_{j,t}^*$, we use CE^* and CE_t^* to denote the analogous measures for the complete information Nash equilibrium in Section 5.1.

Model performance. We measure the performance of model $(F(\delta), A(y))$ as

$$MSE^{(\delta,y)} = \frac{\sum_t \sum_j \left(b_{j,t}^{(\delta,y)} - b_{j,t}\right)^2 s_{j,t}}{\sum_t \sum_j s_{j,t}},$$

[29]To predict market shares we use the estimated parameter vector $\hat{\theta}$ regardless of the adaptive learning model $A(y)$, as it is our best guess for that parameter.

Table 9: Middle Phase: Prediction Error in NG's Monthly Cost ($CE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.33	0.28	1.29	0.50	0.93	0.50	0.31
F(0.5)	0.33	0.27	1.30	0.51	0.94	0.48	0.28
F(1)	0.51	0.44	1.03	0.36	0.67	0.60	0.39
Eq.	0.53	0.42	1.87	0.63	1.29	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

or the weighted average of the square prediction error $\left(b_{j,t}^{(\delta,y)} - b_{j,t}\right)^2$, where the weight $\frac{s_{j,t}}{\sum_t \sum_j s_{j,t}}$ is the share of BM unit j in month t in the total share of the inside goods across months. We again use MSE^* to denote the corresponding measure for the complete information Nash equilibrium.

5.4 Results

We begin with discussing how well different learning models fit the data from the middle phase of the FR market and then move on to the late phase.

Middle or falling-price phase. Table 9 provides the average absolute prediction error $CE^{(\delta,y)}$ and CE^* for the middle phase and Table 10 provides the mean square prediction error $MSE^{(\delta,y)}$ and MSE^*. The entries in the table are arranged to reflect our two-way classification of learning models: the first three rows specify the fictitious play model used to form perceptions of rivals' bids, followed by the complete information Nash equilibrium, and the columns specify the adaptive learning model used to form perceptions of the demand parameters. The five columns on the left pertain to single-period predictions and the two columns on the right to multi-period predictions.

Starting with the single-period predictions, the 2×2 sub-matrix in the top left corner of Tables 9 and 10 have the smallest values. Regardless of whether we are concerned with $CE^{(\delta,y)}$ or $MSE^{(\delta,y)}$, the preferred model for perceptions about rivals' bids is either $F(0)$ (full decay beyond the immediate past, or adaptive best response) or $F(0.5)$ (intermediate decay).

Table 10: Middle Phase: Mean Squared Error of Bid Predictions ($MSE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	1.26	1.22	3.65	1.74	2.63	1.38	1.21
F(0.5)	1.26	1.22	3.73	1.77	2.68	1.34	1.18
F(1)	1.49	1.39	2.69	1.42	1.94	1.55	1.32
Eq.	1.45	1.32	6.35	1.94	3.86	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

The preferred model for perceptions about demand is either $A(\emptyset)$ (no demand uncertainty) or $A(\alpha)$ (uncertainty about the price sensitivity parameter α). Both measures of fit noticeably deteriorate under the adaptive learning models $A(\alpha,\beta)$, $A(\alpha,\mu)$ and $A(\alpha,\beta,\mu)$ that presume that firms are uncertain about additional demand parameters.

When we searched for a point estimate of the decay parameter δ in the fictitious play model, we obtained the smallest mean square prediction error at $\delta = 0.3$. However, there was very little difference in the mean square prediction error when δ took on values between 0 and 0.6. So we conclude only that the data from the middle phase of the FR market appears to favor a fictitious play model in which firms rely disproportionately on more recent observations to form beliefs about rivals' bids.

The single-period predictions in the five columns on the left of Tables 9 and 10 seem to give only a slight edge to $A(\alpha)$ over $A(\emptyset)$ (as is suggested by Figure 14). This edge, however, is accentuated in the multi-period predictions in the two columns on the right. The multi-period predictions compound differences between the sequential estimates $\hat{\alpha}^{(t)}$ of the price sensitivity parameter α that underlie $A(\alpha)$ and the full-sample estimate that underlies $A(\emptyset)$. More surprising is how well the multi-period predictions using $A(\alpha)$ do. They have essentially the same mean square prediction error as the single-period predictions using $A(\alpha)$ and do just up to 10% worse in terms of the average absolute prediction error. In contrast, the multi-period predictions using $A(\emptyset)$ do 5% to 10% worse than the single-period predictions using $A(\emptyset)$ in terms of the mean square prediction error and 50% worse in terms of the average absolute prediction error.

Perhaps the most striking feature of Tables 9 and 10 is that the fictitious play models $F(0)$

Table 11: Middle Phase: Differences Between the Learning Model with $(\delta, y) = (0, \emptyset)$ and Nash Equilibrium

	Dependent Variable			
	$CE_t^{(0,\emptyset)} - CE_t^*$		$\left(b_{j,t}^{(0,\emptyset)} - b_{j,t}\right)^2 - \left(b_{j,t}^* - b_{j,t}\right)^2$	
Constant	-0.194	-0.368	-0.192	-0.631
	(0.032)	(0.055)	(0.088)	(0.225)
Time		0.013		0.031
		(0.003)		(0.012)
N	27	27	1470	1470

An individual observation for the CE_t regressions is a month that is formed from aggregating across BM units with weights as specified in section 5.3. For the third and fourth column, an individual observation is a BM unit in a given month; these regressions are estimated via share-weighted least squares and the standard errors are clustered at the month level. The regressors are a constant and a linear time trend; time=1 for the first month of the phase.

and $F(0.5)$ outperform the complete information Nash equilibrium. This is more pronounced for the average absolute prediction error, but clearly noticeable for both measures of fit. Table 11 further illustrates just how different the predictions from the learning and equilibrium models are. In this table, we specifically compare $F(0)$ and $A(\emptyset)$, i.e. adaptive best response with the full sample estimate, with the complete information Nash equilibrium. To do this, we regress of $CE_t^{(0,\emptyset)} - CE_t^*$ first on a constant (column 1) and then a constant and time trend (column 2). Columns 3 and 4 show the corresponding regressions when the dependent variable is instead $\left(b_{j,t}^{(0,\emptyset)} - b_{j,t}\right)^2 - \left(b_{j,t}^* - b_{j,t}\right)^2$, the difference in squared prediction error. In all regressions, we share-weight the observations to give greater importance to precise predictions for BM units with larger market shares.

We find statistically and economically significant differences between the learning model and equilibrium predictions, both with and without a time trend. Moreover, the regressions that include a time trend indicate that the difference starts out large and declines over the middle phase of the FR market, with both the initial difference and the subsequent decrease clearly significant. We find similar statistically and economically significant differences between the predictions of the other leading learning models $(F(0.5), A(\emptyset))$, $(F(0), A(\alpha))$, and $(F(0.5), A(\alpha))$ and the complete information Nash equilibrium.

Figure 15 shows the time path of the price of FR paid by NG as predicted by alternative learning models and the complete information Nash equilibrium as well as the time path of

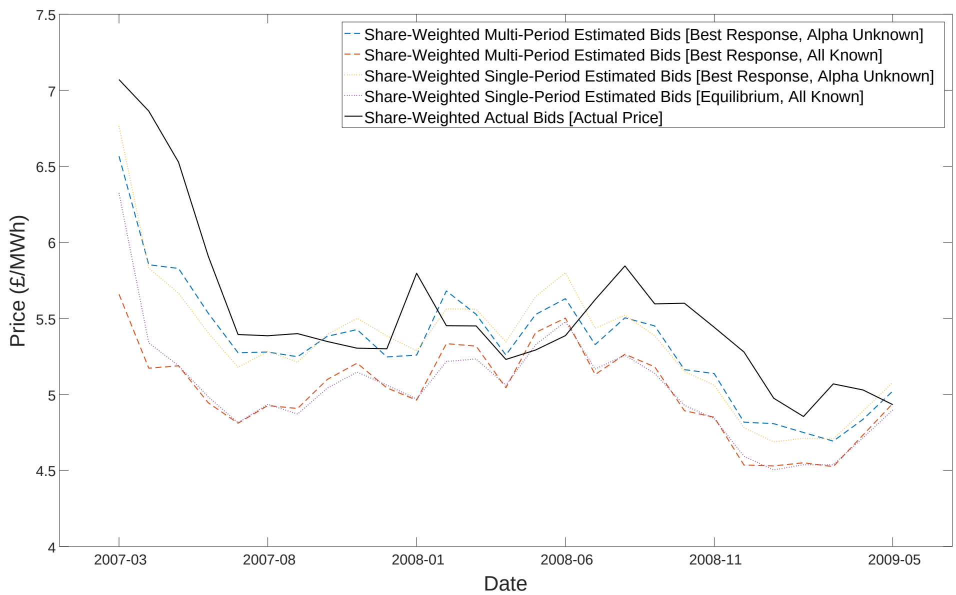

Figure 15: Predicted and actual FR price by month. Share-weighted average computed from bids predicted by fictitious play $F(0)$ with adaptive learning $A(\alpha)$ in both single and multi-period simulations, complete information Nash equilibrium without demand uncertainty $A(\emptyset)$, and actual bids. Middle phase.

the actual price of FR (green line). Modulus multiplicity issues, only changes in the demand and cost primitives can cause changes in the complete information Nash equilibrium. As the turquoise line in Figure 15 shows, these changes alone do not allow the complete information Nash equilibrium to vary sufficiently to track the actual price of FR with much accuracy.

The learning models allow changes in the perceptions about rivals' bids and demand to vary over time and to impact the predictions for bids. The purple line in Figure 15 shows single-period predictions from the model $(F(0), A(\alpha))$ and demonstrates that allowing for these changes in perceptions enables the bids to change over time in ways that mimic the changes in the actual price of FR.[30] Even the multi-period predictions from the learning models do considerably better than the single-period prediction of the complete information Nash equilibrium: the blue line shows multi-period predictions from the model $(F(0), A(\alpha))$ and starts out just above the purple line that shows single-period predictions from the same model. However, this relies on adaptive learning about demand. The red line shows the

[30]The time path of single-period predictions from the model $(F(0.5), A(\alpha))$ is similar and thus omitted.

Table 13: Late Phase: Prediction Error in NG's Monthly Costs ($CE^{(\delta,y)}$)*

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.15	0.15	0.23	0.18	0.17	0.15	0.27
F(0.5)	0.15	0.15	0.24	0.19	0.17	0.15	0.27
F(1)	0.24	0.24	0.33	0.27	0.25	0.17	0.28
Eq.	0.17	0.17	0.28	0.21	0.19	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

Table 14: Late Phase: Mean Squared Error of Bid Predictions ($MSE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.31	0.31	0.40	0.37	0.39	0.30	0.38
F(0.5)	0.32	0.32	0.40	0.38	0.39	0.30	0.38
F(1)	0.39	0.39	0.52	0.47	0.50	0.32	0.41
Eq.	0.31	0.31	0.44	0.38	0.38	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

multi-period predictions from the model $(F(0), A(\emptyset))$. The fact that the difference between the red and the blue line is so notable indicates that allowing for learning about the price sensitivity parameter α is important for the multi-period predictions to come close to the actual price of FR.

Late or stable-price phase. Tables 13 and 14 provide the average absolute prediction error $CE^{(\delta,y)}$ and CE^{*} and the mean square prediction error $MSE^{(\delta,y)}$ and MSE^{*} for the late phase of the FR market; they are analog to Tables 9 and 10 for the middle phase. Both measures of fit improve in the late phase for the alternative learning models and for the complete information Nash equilibrium. In particular, the mean square prediction error is a third or less of its value in the middle phase.

Recall that we estimate the marginal cost c by setting the time-average of the first-order conditions over the late phase to zero (see equation (8)). It may therefore not be a surprise that the predictions from the complete information Nash equilibrium fit the data from the late phase well. However, this does not imply that the bids predicted by the learning models are either close to the data or close to the predictions from the equilibrium model.

It is difficult to determine the preferred models for perceptions about rivals' bids and demand in the late phase from our measures of fit. Tables 13 and 14 appear to favor the fictitious play models $F(0)$ and $F(0.5)$ over $F(1)$. The baseline model $A(\emptyset)$ that abstracts from demand uncertainty and the adaptive learning models $A(\alpha)$, $A(\alpha, \beta)$, and $A(\alpha, \mu)$ perform similarly according to both measures of fit. Part of the explanation may be that the sequential estimates of the various demand parameters are identical to their full-sample estimates by the end of the late phase.

The most striking conclusion from Tables 13 and 14 is that in the late phase the difference between the predictions from the leading learning models $(F(0), A(\emptyset))$, $(F(0.5), A(\emptyset))$, $(F(0), A(\alpha))$, and $(F(0.5), A(\alpha))$ and the predictions from the equilibrium model are a fraction of what they were in the middle phase. Table 15, which is the analog of Table 11 for the middle phase, reports a precisely estimated average difference in average absolute prediction error of about 0.02 (compared to about 0.19 in the middle phase), and a precisely estimated average difference in mean square prediction error of about 0.002 (compared to about 0.19 in the middle phase).

The learning models continue to do slightly better than the equilibrium model, at least in terms of average absolute prediction error. One can see why in Figure 16, which is the analog of Figure 15 for the middle phase, and shows the predicted time paths of the cost of FR paid by NG from alternative models. In the late phase, the equilibrium predictions are slightly above the single- and multi-period predictions from the learning models which, in turn, are slightly above the data. However, these differences are much smaller than those in the middle phase, and the learning and equilibrium models move in very similar ways; indeed they seem to mimic each other.

Our findings support the presumption that the learning models are "well behaved" in the sense that they seem to converge to a complete information Nash equilibrium or something close to it. Convergence is not generally guaranteed under fictitious play (see Shapley (1964) and Fudenberg and Levine (1998)), so it is encouraging that we find empirical support for it.

Table 15: Late Phase: Differences Between the Learning Model with $(\delta, y) = (0, \emptyset)$ and Nash Equilibrium

	Dependent Variable			
	$CE_t^{(0,\emptyset)} - CE_t^*$		$\left(b_{j,t}^{(0,\emptyset)} - b_{j,t}\right)^2 - \left(b_{j,t}^* - b_{j,t}\right)^2$	
Constant	-0.023	-0.007	-0.002	0.026
	(0.007)	(0.014)	(0.001)	(0.012)
Time		-0.002		-0.002
		(0.001)		(0.001)
N	29	29	1529	1529

An individual observation for the CE_t regressions is a month that is formed from aggregating across BM units with weights as specified in section 5.3. For the third and fourth column, an individual observation is a BM unit in a given month; these regressions are estimated via share-weighted least squares and the standard errors are clustered at the month level. The regressors are a constant and a linear time trend; time=1 for the first month of the phase.

To the extent that learning models are used in counterfactual analysis (see Lee and Pakes (2009) and Wollman (2016)), this suggests that counterfactual outcomes may be close to equilibrium outcomes that could plausibly be sustained over time. On the other hand, we also see that there is still some room left between our best predictions and the actual bids.

While the learning and equilibrium models converge to about the same place, there remains the question whether some of these models are better able to adjust to the environmental changes that occurred during the late phase. Here we have less to say. When we regress the actual bids on BM-unit fixed effects and the predicted bids from each model separately, we obtain highly significant positive coefficients on the predicted bids. However, the coefficients obtained from the different models are virtually identical, in line with the fact that the different models are close to indistinguishable according to mean square prediction error.

Efficiency consequences. An interesting question is whether the out-of-equilibrium behavior in the middle phase of the FR market had important efficiency consequences, or merely amounted to a transfer from NG to firms.[31] As FR demand is perfectly inelastic, efficiency hinges on the social cost of meeting that demand. To further assess this, we first compute the average cost per MWh of FR provided by the $J = 72$ BM units owned by the top ten firms as $\sum_t \sum_j \hat{c}_j s_{j,t} / \sum_t \sum_j s_{j,t}$. We then re-compute this cost had the FR mar-

[31] We thank a referee for suggesting that we investigate this topic.

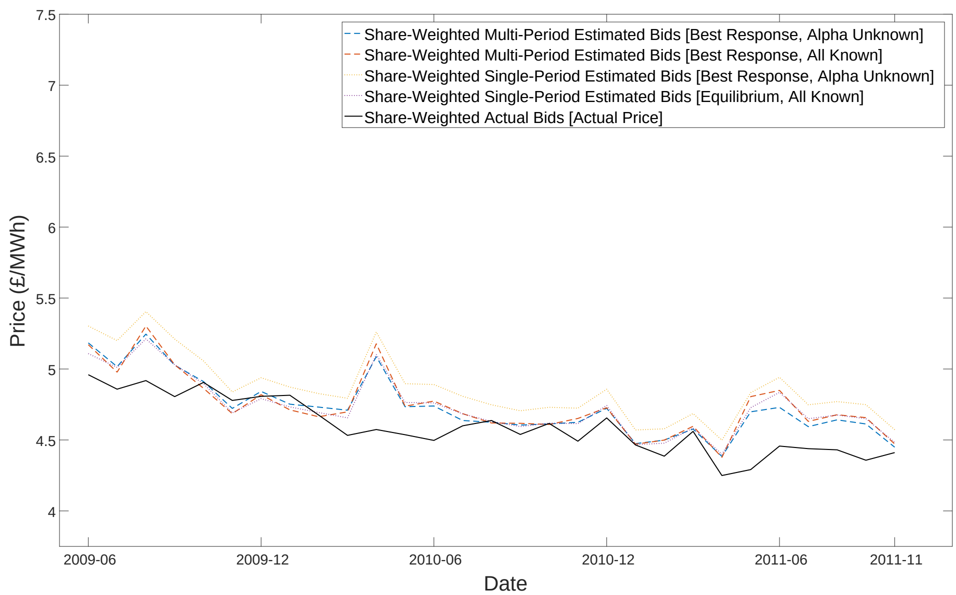

Figure 16: Predicted and actual FR price by month. Share-weighted average computed from bids predicted by fictitious play $F(0)$ with adaptive learning $A(\alpha)$ in both single and multi-period simulations, complete information Nash equilibrium without demand uncertainty $A(\emptyset)$, and actual bids. Late phase.

ket counterfactually been in equilibrium as $\sum_t \sum_j \hat{c}_j s^*_{j,t} / \sum_t \sum_j s^*_{j,t}$, where $s^*_{j,t}$ is the market share of BM unit j in month t implied by the complete information Nash equilibrium. The average cost is £1.318/MWh in the data compared to £1.3133/MWh in the counterfactual, implying relatively minor efficiency consequences. This is both because the cost differences across units are not that big (see Table 4) and because the changes in shares are not large.

Summary. Different models may be appropriate for different periods in an industry's evolution. In periods shortly after a major change in the environment, it may be better to rely on models that allow for learning than to rely on equilibrium for either one's understanding of — or one's predictions for — behavior. This seems true of the middle phase of the FR market for single-period predictions and to an even larger extent for multi-period predictions. During this time, the data appear to favor fictitious play models in which firms rely disproportionately on more recent observations to form beliefs about rivals' bids in combination with adaptive learning models that accommodate uncertainty about some aspects of demand. On the other hand, in the late phase we find that the complete information Nash equilibrium seems to fit about as well as the best learning models.

6 Conclusion

We have analyzed the evolution of the FR market in the UK following its deregulation. We find that the market seems to have converged to a rest point after about three and a half years, or 42 periods, of interaction. Subsequent changes in the environment seem to cause much smaller changes in bids than the bid changes that were observed in the prior "learning" period. The rest point seems consistent with a complete information Nash equilibrium in that cost estimates derived from a necessary equilibrium condition are plausible and in that learning models gravitate towards it.

There is, however, substantial heterogeneity in how the major market participants approach this rest point. Early on some firms experiment, while others are more cautious and make infrequent adjustments. During the middle phase, firm behavior is more predictable: bid predictions from fictitious play models in which firms best respond to recent rival behavior are able to explain a substantial share of the variance in bids. The fit is further improved by allowing for adaptive learning about the price sensitivity parameter. These models match the observed bids better than the bids predicted by the complete information Nash equilibrium as the latter approach the rest point much more rapidly than the actual data does, while the learning models approach the rest point more slowly and, as a result, mimic the actual data more closely. In the final phase, once the rest point is reached and firms had quite a bit of data on the play of their competitors, the equilibrium predictions fit about as well as the best fitting learning models and seem to adjust about as rapidly as do the learning models to changes in the environment.

Our empirical "case study" supports the idea that in stable environments play will generally converge to a Nash equilibrium. This is consistent with much of the theory of learning in games, and the corresponding idea that equilibrium play is a good prediction for the long-run outcome of a game with strategic players. But for predicting short-run play following a change in the environment we find that models of fictitious play and adaptive learning outperform the complete information Nash equilibrium. This lends empirical support to the use of learning models, which to the best of our knowledge have thus far only been tested in the lab. One area where learning model may be particularly helpful is in simulating counterfactual outcomes, a type of analysis increasingly used by regulatory authorities.

Finally, we emphasize that there are gaps in the fit of both learning and equilibrium models over the entire sample period which we study. None of these models explains the heterogeneity in firms' activity levels. Somewhat surprisingly, the cost of infrequent adjustment to bids in terms of lost profits seems small, which may itself be part of the explanation. Still the gaps in the fit suggests that other forms of analysis might also be helpful in predicting market evolution.

References

Ackerberg, Daniel A., "Advertising, Learning, and Consumer Choice in Experience Good Markets: An Empirical Examination," *International Economic Review*, 2003, *44* (3), 1007–1040.

Akerlof, George and Janet Yellen, "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?," *The American Economic Review*, 1985, *75* (4), 708–720.

Anufriev, Mikhail and Cars Hommes, "Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments," *American Economic Journal: Microeconomics*, 2012, *4* (4), 35–64.

Benkard, C. Lanier, "Learning and Forgetting: The Dynamics of Aircraft Production," *American Economic Review*, 2000, *90* (4), 1034–1054.

Bergemann, Dirk and Juuso Välimäki, "Learning and Strategic Pricing," *Econometrica*, 1996, *64* (5), 1125–1149.

___ **and** ___ , "Dynamic Price Competition," *Journal of Economic Theory*, 2006, *127* (1), 232–263.

Bernhardt, Dan and Bart Taub, "Learning About Common and Private Values in Oligopoly," *Rand Journal of Economics*, 2015, *46* (1), 66–85.

Bolton, Patrick and Christopher Harris, "Strategic Experimentation," *Econometrica*, 1999, *67* (2), 349–374.

Borenstein, Severin, James B. Bushnell, and Frank A. Wolak, "Measuring Market Inefficiencies in California's Restructured Wholesale Electricity Market," *American Economic Review*, 2002, *92* (5), 1376–1405.

Börgers, Tilman and Rajiv Sarin, "Learning Through Reinforcement and Replicator Dynamics," *Journal of Economic Theory*, 1997, *77* (1), 1–14.

Brandenburger, Adam, "Strategic and Structural Uncertainty in Games," in Richard J. Zeckhauser, Ralph L. Keeney, and James K. Sebenius, eds., *Wise Choices: Games, Decisions, and Negotiations*, Brighton: Harvard Business School Press, 1996, pp. 221–232.

Brown, George W., "Iterative Solutions of Games by Fictitious Play," in Tjalling C. Koopmans, ed., *Activity Analysis of Production and Allocation*, New York: Wiley, 1951.

Camerer, Colin and Teck-Hua Ho, "Experienced-Weighted Attraction Learning in Normal Form Games," *Econometrica*, 1999, *67* (4), 827–874.

___ , ___ , and Juin-Kuan Chong, "Sophisticated Experience-Weighted Attraction Learning and Strategic Teaching in Repeated Games," *Journal of Economic Theory*, 2002, *104* (1), 837–874.

Conley, Timothy and Christopher Udry, "Learning About a New Technology: Pineapple in Ghana," *American Economic Review*, 2010, *100* (1), 35–69.

Cournot, A. Augustine, *Recherches sur les principes mathmatiques de la thorie des richesses*, Paris: Hachette, 1838.

Covert, Thomas, "Learning About Hydraulic Fracturing in the North Dakota Bakken Shale," Working Paper, Harvard University 2013.

Dickstein, Michael, "Efficient Provision of Experience Goods: Evidence from Antidepressant Choice," Working Paper, Stanford University 2013.

Easley, David and Nicholas M. Kiefer, "Controlling a Stochastic Process with Unknown Parameters," *Econometrica*, 1988, *56* (5), 1045–64.

Erdem, Tülin and Michael P. Keane, "Decision-Making under Uncertainty: Capturing Dynamic Brand Choice Processes in Turbulent Consumer Goods Markets," *Marketing Science*, 1996, *15* (1), 1–20.

Erev, Ido and Alvin E. Roth, "Predicting How People Play Games: Reinforcement Learning in Experimental Games with Unique, Mixed Strategy Equilibria," *American Economic Review*, 1998, *88* (4), 848–881.

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton: Princeton University Press, 2001.

____ **and** ____ , "Learning as a Rational Foundation for Macroeconomics and Finance," in Roman Frydman and Edmund S. Phelps, eds., *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton: Princeton University Press, 2013.

Fehr, Ernst and Jean-Robert Tyran, "Limited Rationality and Strategic Interactions: The Impact of the Strategic Environment on Nominal Inertia," *Econometrica*, 2008, *76* (2), 353–394.

Fershtman, Chaim and Ariel Pakes, "Dynamic Games with Asymmetric Information: A Framework for Empirical Work," *Quarterly Journal of Economics*, 2012, *127* (4), 1611–1661.

Fudenberg, Drew and David Kreps, "Learning Mixed Equilibria," *Games and Economic Behavior*, 1993, *5* (3), 320–67.

____ **and David Levine**, *The Theory of Learning in Games*, Cambridge: MIT Press, 1998.

Griliches, Zvi, "Hybrid Corn: An Exploration in the Economics of Technological Change," *Econometrica*, 1957, *25* (4), 501–522.

Hart, Sergiu and Andreu Mas-Colell, "A Simple Adaptive Procedure Leading to Correlated Equilibrium," *Econometrica*, 2000, *68* (5), 1127–1150.

Heckman, James, "Sample Selection Bias as a Specification Error," *Econometrica*, 1979, *47* (1), 153–161.

Hortaçsu, Ali and Steven L. Puller, "Understanding Strategic Bidding in Multi-unit Auctions: A Case Study of the Texas Electricity Spot Market," *Rand Journal of Economics*, 2008, *39* (1), 86–114.

Joskow, Paul L., Richard Schmalensee, and Elizabeth M. Bailey, "The Market for Sulfur Dioxide Emissions," *American Economic Review*, 1998, *88* (4), 669–685.

Keller, Godfrey, Sven Rady, and Martin Cripps, "Strategic Experimentation with Exponential Bandits," *Econometrica*, 2005, *73* (1), 39–68.

Lee, Robin S. and Ariel Pakes, "Multiple Equilibria and Selection by Learning in an Applied Setting," *Economics Letters*, 2009, *104* (1), 13–16.

Luco, Ferando, Ali Hortaçsu, Steven L. Puller, and Dongni Zhu, "Does Strategic Ability Affect Efficiency? Evidence from Electricity Markets," Working Paper, University of Chicago 2017.

Milgrom, Paul and John Roberts, "Adaptive and Sophisticated Learning in Normal-Form Games," *Games and Economic Behavior*, 1991, *3* (1), 82–100.

Mirman, Leonard J., Larry Samuelson, and Amparo Urbano, "Duopoly Signal Jamming," *Economic Theory*, 1993, *3* (1), 129–149.

Mohlin, Erik, Robert Ostling, and Joseph Tao-yi Wang, "Learning by Imitation in Games: Theory, Field, and Laboratory," Economics Series Working Papers 734, University of Oxford, Department of Economics November 2014.

Newberry, Peter, "An Empirical Study of Observational Learning," *Rand Journal of Economics*, 2016, *47* (2), 394–432.

Nickell, Stephen, "Biases in Dynamic Models with Fixed Effects," *Econometrica*, 1981, *49* (6), 1417–1426.

Pakes, Ariel and David Pollard, "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 1989, *57* (5), 1027–1057.

Porter, Robert H., "The Role of Information in U.S. Offshore Oil and Gas Lease Auctions," *Econometrica*, 1995, *63* (1), 1–27.

Riordan, Michael H., "Imperfect Information and Dynamic Conjectural Variations," *Rand Journal of Economics*, 1985, *16* (1), 41–50.

Robbins, Herbert, "Some Aspects of the Sequential Design of Experiments," *Bulletin of the American Mathematical Society*, 1952, *58* (5), 527–535.

Rothschild, Michael, "A Two-Armed Bandit Theory of Market Pricing," *Journal of Economic Theory*, 1974, *9* (2), 185–202.

Salmon, Timothy C., "An Evaluation of Econometric Models of Adaptive Learning," *Econometrica*, 2001, *69* (6), 1597–1628.

Sargent, Thomas J., *Bounded Rationality in Macroeconomics*, Oxford: Oxford University Press, 1993.

Shapley, Lloyd S., "Some Topics in Two-Person Games," in Dresher Melvin, Lloyd S. Shapley, and Albert W. Tucker, eds., *Advances in Game Theory*, Princeton: Princeton University Press, 1964.

Simon, Herbert A., "A Behavioral Model of Rational Choice," *Quarterly Journal of Economics*, 1955, *69* (1), 99–118.

Sims, Christopher A., "Implications of Rational Inattention," *Journal of Monetary Economics*, 2003, *50* (3), 665–690.

Sweeting, Andrew, "Market Power in the England and Wales Wholesale Electricity Market 1995-2000," *Economic Journal*, 2007, *117* (520), 654–685.

Venkateswaran, Venky, Laura Veldkamp, and Julian Kozlowski, "The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation," Working Paper No. 21719, NBER 2015.

Wolak, Frank A., "Identification and Estimation of Cost Functions Using Observed Bid Data: An Application to Electricity," in Mathias Dewatripont, Lars Peter Hansen, and Stephen J. Turnovsky, eds., *Advances in Econometrics: Theory and Applications*, Cambridge: Cambridge University Press, 2003, pp. 133–169.

——, "Quantifying the Supply-Side Benefits from Forward Contracting in Wholesale Electricity Markets," *Journal of Applied Econometrics*, 2007, *22* (7), 1179–1209.

Wollman, Thomas, "Trucks without Bailouts: Equilibrium Product Characteristics for Commercial Vehicles," Working Paper, University of Chicago 2016.

Zhang, Juanjuan, "The Sound of Silence: Observational Learning in the U.S. Kidney Market," *Marketing Science*, 2010, *29* (2), 315–335.

A Appendix

A.1 Belief convergence

We show that if our estimate of θ is consistent for that parameter and if the firm's subjective probability distribution converges weakly to the objective probability distribution (uniformly across information sets), then

$$\left\| T^{-1} \sum_{t=44}^{T} \left(h_{i,t}^{T}(c_i) - h_{i,t}^{e}(c_i) \right) \right\| = o_p(1),$$

where the notation follows footnote 18. This is one of the sufficient conditions for obtaining consistent estimates of c_i (the other one being an identification condition).

Let firm i have a subjective probability measure $P^{i,t}(b_{-i,t}, \xi_t, e_t, \theta_t \big| \Omega_{i,t-1})$ underlying $\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}\left[\cdot|\Omega_{i,t-1}\right]$, so that $\mathcal{E}_{b_{-i,t},\xi_t,e_t,\theta_t}\left[f(u)|\Omega_{i,t-1}\right] = \int f(u) dP^{i,t}(u|\Omega_{i,t-1})$. Let α_0 denote the true price parameter. By the triangle inequality

$$\begin{aligned} \left\| T^{-1} \sum_{t=44}^{T} \left(h_{i,t}^{T}(c_i) - h_{i,t}^{e}(c_i) \right) \right\| \leq & \left\| T^{-1} \sum_{t=44}^{T} \left(h(c_i, \hat{\alpha}_T, y_{i,t}) - h(c_i, \alpha_0, y_{i,t}) \right) \right\| \\ & + \left\| T^{-1} \sum_{t=44}^{T} \left(h(c_i, \alpha_0, y_{i,t}) - h_{i,t}^{0}(c_i) \right) \right\| \\ & + \left\| T^{-1} \sum_{t=44}^{T} \left(h_{i,t}^{0}(c_i) - h_{i,t}^{e}(c_i) \right) \right\|, \end{aligned}$$

where $h_{i,t}^{0}(c_i) = E[h(c_i, \alpha, y_{i,t})|\Omega_{i,t-1}]$ and $E[\cdot|\Omega_{i,t-1}]$ denotes the expectation with respect to the objective probability measure (which puts point mass on $\alpha = \alpha_0$). By assumption, $\text{plim}_{T\to\infty} \hat{\alpha}_T = \alpha_0$, so the first term converges to zero by the continuous mapping theorem. The second term converges in probability to zero by a WLLN, since each term in the summation is a mean zero random variable, independent of the previous term because of the conditioning on $\Omega_{i,t-1}$. The third term converges in probability to zero since $P^{i,t}(\cdot|\Omega_{i,t-1})$ weakly converges to the objective probability measure, uniformly in $\Omega_{i,t-1}$, h is continuous and bounded, and so $h_{i,t}^{e}(c_i) \equiv \int h(c_i, \alpha, y_{i,t}) dP^{i,t}(\alpha, y_{i,t}|\Omega_{i,t-1}) \to_p E[h(c_i, \alpha, y_{i,t})|\Omega_{i,t-1}] = h_{i,t}^{0}(c_i)$. Convergence of the sequence of individual terms implies convergence of the sequence of averages. Then, since the right-hand side converges in probability to zero, so does the left-hand side.

A.2 Data appendix

Data sources. Since a redesign on November 1, 2013, the data website of NG is available at `http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmission-operational-data/Data-explorer/Outcome-Energy-Services/`. The data on the FR market is available under the tab "Frequency Response — FFR & Mandatory." We downloaded our data from a previous version of the NG data website. In those cases detailed below where the data is no longer available on the NG data website, it is available from the authors on request. NG used to publish Seven Year Statements detailing their projections of energy supply and demand and upcoming challenges. These used to be available at `http://www.nationalgrid.com/uk/Electricity/SYS/archive/`.

- **Bids**: We obtained FFR bid data directly from the NG data website. The relevant file is labeled "Prices." Currently, a version is available that starts in January 2007 and is updated every month. From the old version of the data website, we downloaded one file for the period from November 2005 to January 2010, and another file for January 2007 to July 2013. These files contain monthly bids (in £/MWh) by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. The combined data period from the two files is November 2005 to July 2013.

- **Capabilities**: We obtained FR capabilities data directly from the NG data website. The relevant file is labeled "Capabilities." Currently, a version is available that starts in January 2006 and is updated every month. From the old version of the data website, we downloaded one file for the period from November 2005 to January 2010, and another file for January 2006 to August 2013. The former file reports that November and December 2005 are not available, so only the latter file is relevant, since it contains all the data that is available. The file contains monthly response capabilities by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. For the market segment primary, response capabilities in MWh are given at 0.2Hz, 0.5Hz, and 0.8Hz, while for the market segments secondary and high, only response capabilities at 0.2Hz and 0.5Hz are listed. In each case, the column on the right represents the maximum over the operating range. These values are constant over the sample period for more than 80% of BM units. The data period is January 2006 to August 2013.

- **Quantities**: We obtained FR quantity data directly from the NG data website. Unfortunately, the new data website no longer provides historic quantities, and only a file that holds quantities from August 2013 is available. We downloaded monthly quantity files for November 2005 thru June 2013. Each of these files contains one month of daily holding quantities in MWh by every BM unit with mandatory FR provision requirements separately for the market segments primary, secondary, and high. The combined data period of these monthly files is November 2005 to June 2013.

- **Main market**: Elexon publishes all messages submitted to the Balancing Mechanism Reporting System on a given day at `http://www.bmreports.com/`. An example for a daily file is `http://www.bmreports.com/tibcodata/tib_messages.2003-01-01.gz`. Each file collects the messages submitted as part of the BM on a given day. These messages contain information on final physical notification (FPN), maximum export limit (MEL), bid-offer data (BOD), or bid-offer acceptance level (BOAL) for typically a half-hour interval.

- **Electricity demanded**: We take information on electricity demanded from NG at `http://www.nationalgrid.com/uk/Electricity/Data/Demand+Data/`. The data is stored in a sequence of excel spreadsheets, each of which has the quantity demanded on a given day.

- **Firm frequency response**: We obtain information on FFR from the reports published at `http://www2.nationalgrid.com/UK/Industry-information/Electricity-transmissio` `Report-explorer/Services-Reports/`. The data is stored in a sequence of excel spreadsheets published monthly, each of which has FFR volumes by day.

- **Fuel type**: We take fuel type information from appendix F1 of the Seven-Year Statement prepared by NG in 2011 at `http://nationalgrid.com/NR/rdonlyres/3B1B4AE4-2368-4B6E-8DA4-539A67EAD41F/47211/NETSSYS2011AppendixF1.xls`. The sheet "F-2," corresponding to table F.2, provides fuel type for every BM unit listed under the column "Plant type." For an additional eleven stations, we take information on fuel type from Variable Pitch at `http://www.variablepitch.co.uk/grid/`.

- **Fuel prices**: The UK Department of Energy and Climate Change publishes quarterly and annual prices of fuels purchased by generators and of gas at UK delivery points. A file titled "Average prices of fuels purchased by the major UK power producers and of gas at UK delivery points (QEP 3.2.1)" is available at `https://www.gov.uk/government/statistical-data-sets/prices-of-fuels-purchased-by-major-power-producers`. The sheet "Quarterly" contains the quarterly price of coal, oil, and gas, measured in pence per kilowatt hour (KWh), in columns D, F, and G.

- **Vintage**: We take fuel type information from appendix F1 of the Seven-Year Statement prepared by NG in 2011 at `http://nationalgrid.com/NR/rdonlyres/3B1B4AE4-2368-4B6E-8 47211/NETSSYS2011AppendixF1.xls`. The sheet "F-2," corresponding to table F.2, provides vintages for most BM units under the column "Commissioning Year." The cell is empty for almost all hydro power stations, so we take this information from the website of the British Hydropower Association at `http://www.british-hydro.org/`. For an additional eleven power stations we take this information from Wikipedia (5), from press releases prepared by the respective operator (5), and the website `www.scottish-places.info` (1). We are missing vintage for FAWN-1, which is connected with the Esso refinery in Fawley.

- **Ownership**: After registration on `https://www.elexonportal.co.uk/`, information on the registered party is contained in the file `reg_bm_units.csv` available under "Operational Data" → "Registration Information" → "Registered BM units" or under `https://www.elexonportal.co.uk/REGISTEREDBMUNITS`. It is based on registration data at the Central Registration Agency and under "Party Name" lists the registered party. We downloaded a version of this file on December 29, 2009, and July 15, 2013, but there were no conflicts.

Sample and variable construction. The unit of observation is BM unit by month. We consider the time period November 2005 to October 2011. We include BM units in the analysis if they provided positive FR quantity in at least one of these months. We aggregate quantities for the three market segments primary, secondary, and high (see footnote 2) by summing daily quantities across segments and days. For BM unit j in month t we thus obtain FR quantity as

$$q_{j,t} = \sum_{k=P,S,H} \sum_{\tau \in t} q_{k,\tau,j,t},$$

where k indexes market segments and τ days, and we abuse notation to denote as $\tau \in t$ the days in month t. The FR bids are constructed as quantity-weighted averages of segment-specific bids, where the weights are constant and given by the overall quantities of the three segments over the sample period:

$$b_{j,t} = \left(\sum_{k=P,S,H} Q_k b_{k,j,t} \right) / Q,$$

where $Q_k = \sum_j \sum_t \sum_{\tau \in t} q_{k,\tau,j,t}$ and $Q = Q_P + Q_S + Q_H$.

Because a bid above £23/MWh is only accepted 12 times in our dataset of over 9000 observations, we label such a bid non-competitive; we otherwise label the bid competitive. One reason to opt out of the FR market by submitting a non-competitive bid is that the BM unit undergoes maintenance that month. Modeling maintenance and other reasons a BM unit opts out of the FR market is beyond the scope of this paper, and throughout we simply drop the corresponding observations. We also drop observations if the bid is missing.

Online appendix for "Just starting out: Learning and equilibrium in a new market"

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A.1 Selection

Selection on observables: persistence in eligibility. We extend the probit model in equation (2) to include lagged eligibility $e_{j,t-1}$:

$$\Pr(e_{j,t}=1|e_{j,t-1},x_{j,t}) = 1-\Phi\left(-\breve{\alpha}e_{j,t-1}-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t\right) = \Phi\left(\breve{\alpha}e_{j,t-1}+\breve{\beta}x_{j,t}+\breve{\gamma}_j+\breve{\mu}_t\right). \tag{11}$$

The first column of Table 17 shows ML estimates for this model. There is statistically significant and economically meaningful evidence of persistence in eligibility. Footnote 12 in the main text explains why we decided not to model this in the main paper.

Selection on observables: bid. To investigate selection on observables, we extend the probit model in equation (2) to include the log bid $\ln b_{j,t}$:

$$\Pr(e_{j,t}=1|b_{j,t},x_{j,t}) = 1-\Phi\left(-\breve{\alpha}\ln b_{j,t}-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t\right) = \Phi\left(\breve{\alpha}\ln b_{j,t}+\breve{\beta}x_{j,t}+\breve{\gamma}_j+\breve{\mu}_t\right). \tag{12}$$

The second and third columns of Table 17 show ML estimates. In the third column, we allow the bid to enter more flexibly through a series of dummies for $b_{j,t}$ being in each decile of the distribution of bids. The coefficient on log bid $\ln b_{j,t}$ is statistically significant, as are half of the decile coefficients in the flexible specification. However, as noted in footnote 12 in the main text, the impact of the log bid $\ln b_{j,t}$ is economically small.

Selection on unobservables. To examine selection on unobservables, we revert to the probit model in equation (2). We allow for correlation between $\nu_{j,t}$ and $\eta_{j,t}$ (and hence $\xi_{j,t}$ and $\eta_{j,t}$) and assume that they are iid across BM units and months and jointly normal distributed as

$$\begin{pmatrix}\nu_{j,t}\\ \eta_{j,t}\end{pmatrix} \sim N\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma^2 & \lambda\sigma\\ \lambda\sigma & 1\end{pmatrix}\right).$$

It follows that

$$
\begin{aligned}
&\mathrm{E}\left(\nu_{j,t}|e_{j,t}=e_{j,t-1}=1,x_{j,t}\right)\\
&=\mathrm{E}\left(\nu_{j,t}|\eta_{j,t}>-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t,\eta_{j,t-1}>-\breve{\beta}x_{j,t-1}-\breve{\gamma}_j-\breve{\mu}_{t-1},x_{j,t}\right)\\
&=\mathrm{E}\left(\nu_{j,t}|\eta_{j,t}>-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t,x_{j,t}\right)\\
&=\lambda\sigma\frac{\phi\left(-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t\right)}{1-\Phi\left(-\breve{\beta}x_{j,t}-\breve{\gamma}_j-\breve{\mu}_t\right)}=\lambda\sigma\frac{\phi\left(\breve{\beta}x_{j,t}+\breve{\gamma}_j+\breve{\mu}_t\right)}{\Phi\left(\breve{\beta}x_{j,t}+\breve{\gamma}_j+\breve{\mu}_t\right)},
\end{aligned}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal probability density function (PDF) and CDF. Hence, $\mathrm{E}\left(\nu_{j,t}|e_{j,t}=e_{j,t-1}=1,x_{j,t}\right)\neq 0$ as long as $\lambda\neq 0$ and there is correlation between $\nu_{j,t}$ and $\eta_{j,t}$.

Estimating equation (4) requires adding an inverse Mills ratio selection correction (Heckman 1979). Table 18 shows the resulting NLLS estimates. The coefficient on the inverse Mills ratio is significant but the remaining coefficients are very similar to our leading estimates in Table 3.

Table 17: Determinants of positive volume

	Indicator for positive share		
	(1)	(2)	(3)
Positive FFR volume	-0.009	-0.581	-0.527
	(0.233)	(0.481)	(0.451)
Lagged Eligibility	1.708		
	(0.074)		
Log bid		-0.526	
		(0.203)	
Fully loaded		2.604	2.591
		(0.365)	(0.349)
Part loaded		2.277	2.436
		(0.344)	(0.300)
Bid decile 2			-0.003
			(0.360)
Bid decile 3			0.442
			(0.314)
Bid decile 4			-0.430
			(0.360)
Bid decile 5			-0.699
			(0.326)
Bid decile 6			-0.959
			(0.335)
Bid decile 7			-0.729
			(0.356)
Bid decile 8			-0.693
			(0.341)
Bid decile 9			-0.443
			(0.317)
Bid decile 10			-0.866
			(0.320)
Unit and Month FE	yes	yes	yes
Flexible bid controls	no	no	yes
N	5099	5175	5175

An observation is a unit-month, and the dependent variable is an indicator for a unit having positive volume. Inactive units are omitted. The regressors are the bid (either in logs, or with indicators for the bid being in decile bins) , the fraction of time the unit's final physical notification is that it is fully loaded (i.e. operating at capacity) and part loaded (i.e. operating below capacity); and whether the unit is under a firm frequency response contract. Standard errors are clustered by bmunit.

Table 18: Demand System Estimates

	Log share ratio QFD
Log bid	-1.649
	(0.117)
Fully loaded	1.580
	(0.226)
Part loaded	1.927
	(0.185)
Positive FFR volume	-0.573
	(0.246)
Millsratio	-0.517
	(0.182)
Unit and Month FE	yes
ρ	0.40
s.e. ρ	0.03
N	3509

The dependent variable is the log ratio of the unit share to the outside good share (an observation is a unit-month), coded as missing where the share is zero and omitted in estimation. The specification allows for an AR(1) process in the error term, and we estimate the quasi-first-differenced equation by non-linear least squares (we provide an estimate of the autocorrelation coefficient ρ and the standard error of that estimate).

A.2 Fuel price

We model the marginal cost $c_{j,t}$ of BM unit j in month t as $c_{j,t} = c_j + \mu f_{j,t}$, where c_j is a BM-unit fixed, $f_{j,t}$ is the fuel price that the BM unit faces, and μ is a parameter.

Estimation. To estimate the $J+1$ parameters $c = (c_j)_{j=1,\dots,J}$ and μ, we replace equation (8) by

$$\frac{1}{29}\sum_{t=44}^{T=72}\left[M_t s_{k,t} + \sum_{j\in\mathcal{J}_i}(b_{j,t} - c_{j,t})\,M_t\,(1(k=j) - s_{k,t})\,\frac{\hat{\alpha}s_{j,t}}{b_{k,t}}\right] = 0, \quad \forall k = 1,\dots,J,$$

and we add the equation

$$\frac{1}{29J}\sum_{k=1}^{K}\sum_{t=44}^{T=72}\left[\left[M_t s_{k,t} + \sum_{j\in\mathcal{J}_i}(b_{j,t} - c_{j,t})\,M_t\,(1(k=j) - s_{k,t})\,\frac{\hat{\alpha}s_{j,t}}{b_{k,t}}\right] f_{k,t-1}\right] = 0, \tag{13}$$

where $f_{k,t-1}$ is the fuel price relevant for BM unit k in month $t-1$. These $J+1$ equations are linear in the $J+1$ unknowns.

Results. We estimate μ to be -0.0137 with a standard error of 0.0040. This is economically small: on average across BM units, marginal cost decreases from £1.44/MWh to £1.35/MWh over the final phase of the FR market.

To probe this estimate, we re-specify $c_{j,t} = c_j + \lambda t$, where t is a time trend that is common across BM units. To estimate, we replace equation (13) by

$$\frac{1}{29J}\sum_{k=1}^{K}\sum_{t=44}^{T=72}\left[\left[M_t s_{k,t} + \sum_{j\in\mathcal{J}_i}(b_{j,t} - c_{j,t})\,M_t\,(1(k=j) - s_{k,t})\,\frac{\hat{\alpha}s_{j,t}}{b_{k,t}}\right] t\right] = 0. \tag{14}$$

We estimate λ to be -0.00295 with a standard error of 0.0012. We finally re-specify $c_{j,t} = c_j + \mu f_{j,t} + \lambda t$. To estimate, we use equations (13) and (14). We estimate μ to be -0.01928 wit a standard error of 0.0104 and λ to be 0.001550 with a standard error of 0.0030. Hence, neither coefficient is statistically significant. We conclude that the impact of fuel price is indistinguishable from a downward time trend in cost.

Table 20: Middle Phase: Prediction Error in NG's Monthly Cost with Variable Marginal Costs ($CE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.30	0.26	1.35	0.56	0.98	0.44	0.29
F(0.5)	0.30	0.27	1.37	0.57	0.99	0.42	0.28
F(1)	0.48	0.40	1.10	0.42	0.73	0.55	0.35
Eq.	0.53	0.42	1.87	0.63	1.29	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

Table 21: Middle Phase: Mean Squared Error of Bid Predictions with Variable Marginal Costs($MSE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	1.23	1.20	3.91	1.84	2.79	1.31	1.19
F(0.5)	1.23	1.20	4.00	1.88	2.84	1.28	1.17
F(1)	1.43	1.34	2.89	1.48	2.04	1.47	1.27
Eq.	1.45	1.32	6.35	1.94	3.86	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

Learning models. As a robustness check, we re-ran our analysis of learning models in Section 5 under the assumption that the marginal cost $c_{j,t}$ of BM unit j in month t is $c_{j,t} = c_j + \mu f_{j,t}$, as specified and estimated using the procedure outlined above.

Tables 20, 21, 22, and 23 correspond to 9, 10, 13, and 14 in the main text. The broad conclusions are robust to allowing for time-varying marginal cost: the best fitting models remain those with $A(\emptyset)$ or $A(\alpha)$ and $F(0)$ or $F(0.5)$, and these fit substantially better in the middle phase of the FR market and only slightly better in the late phase.

Table 22: Late Phase: Prediction Error in NG's Monthly Costs with Variable Marginal Costs ($CE^{(\delta,y)}$)*

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.15	0.15	0.24	0.19	0.18	0.16	0.27
F(0.5)	0.16	0.16	0.25	0.19	0.18	0.16	0.27
F(1)	0.24	0.24	0.34	0.27	0.26	0.17	0.29
Eq.	0.17	0.17	0.28	0.21	0.19	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

Table 23: Late Phase: Mean Squared Error of Bid Predictions with Variable Marginal Costs ($MSE^{(\delta,y)}$) *

	Single Period Prediction					Multi-Period Prediction	
	$A(\emptyset)$	$A(\alpha)$	$A(\alpha,\mu)$	$A(\alpha,\beta)$	$A(\alpha,\beta,\mu)$	$A(\emptyset)$	$A(\alpha)$
F(0)	0.32	0.32	0.41	0.38	0.39	0.31	0.40
F(0.5)	0.32	0.32	0.42	0.38	0.40	0.31	0.39
F(1)	0.40	0.40	0.54	0.48	0.50	0.33	0.42
Eq.	0.31	0.31	0.44	0.38	0.38	-	-

* $F(x)$ denotes fictitious play with $\delta = x$. $A(x)$ denotes adaptive learning where all parameters except x are known and x is sequentially estimated. Single period predictions take the data from period $t-1$ and predict bids in period t. Multi period predictions take the bid data from the period before each phase, and make a series of bid predictions for the entire phase, taking the path of all other covariates as known.

A.3 Repositioning in the BM

We account for the profit that accrues to a BM unit as it is repositioned in the BM in preparation for providing FR. The BM is a multi-unit discriminatory auction that is held every half-hour. Prior to this auction, a BM unit submits its contracted position to NG along with its bid. A bid in the BM is essentially a supply curve that is centered at the BM unit's contracted position. This supply curve is described by price-quantity pairs through which the BM unit can offer to increase its energy production in up to five increments above its contracted position. If NG accepts an offer, the BM unit is paid by NG accordingly. The supply curve is further described by up to five price-quantity pairs through which the BM unit can bid to decrease its energy production below its contracted position. If NG accepts a bid, the BM unit pays NG accordingly.

The BM in other countries has been studied in great detail by Borenstein, Bushnell and Wolak (2002), Wolak (2003, 2007), Sweeting (2007), and Hortaçsu and Puller (2008). In line with our focus on the FR market, we work with a much simpler model of the BM that is designed to merely give us a sense of the profit that accrues to a BM unit as it is repositioned in the BM and how that profit changes with its bid for providing FR. We proceed in two steps. First, we estimate a demand model for repositioning. To account for the interdependency between the BM and the FR market, we include the bid for providing FR in the demand model. Second, to obtain profit, we estimate the markup in the BM jointly with the cost of providing FR.

Data. For every BM unit we have data on bids and offers (up to ten price-quantity pairs), contracted position, and actual position every half-hour. The quantity of upward repositioning $q^+_{j,\tau}$ of BM unit j in half-hour τ effected through the BM is therefore the larger of zero and the difference between actual and contracted position; the quantity of downward repositioning $q^-_{j,\tau}$ is the larger of zero and the difference between contracted and actual position. Market size $M^+_\tau = \sum_j q^+_{j,\tau}$ and $M^-_\tau = \sum_j q^-_{j,\tau}$ is the total amount of upward, respectively, downward repositioning in half-hour τ.

We face two problems with the data. First, if BM unit j is not repositioned up or down in the BM in half-hour τ, then $q^+_{j,\tau} = 0$, respectively, $q^-_{j,\tau} = 0$. This happens quite frequently, and we account for it in our demand model. Second, the bids and offers can take on extreme values. This sometimes happens even though the BM unit is repositioned so that $q^+_{j,\tau} > 0$

or $q^-_{j,\tau} > 0$. Hence, taken at face value, the bids and offers imply an implausibly huge profit. We deal with this by directly estimating the markup rather than marginal cost in the BM.

The only place in which the offers are used in what follows is to construct a grid of 24 prices for upward repositioning as follows: pooling across all BM units and half-hours, we consider the distribution of offers and take the 4th through 96th percentiles. We proceed analogously to fix a grid of 24 prices for downward repositioning.

Demand. As with the FR market, the "inside goods" are the $J = 72$ BM units owned by the ten largest firms in Table 1 and the "outside good" encompasses the remaining BM units. To simplify the exposition, we focus on the demand for upward repositioning. The demand for downward repositioning is analogous.

Let $s^+_{j,\tau}$ denote the market share of upward repositioning of BM unit j in half-hour τ and $s^+_{0,\tau} = 1 - \sum_j s^+_{j,\tau}$ the market share of the outside good. Let $e^+_{j,\tau} = 1(s^+_{j,\tau} > 0)$ be the indicator for BM unit j being eligible for repositioning in the BM — and thus having a positive market share — in half-hour τ. Accounting for eligibility, we use a logit model for the market share of BM unit j in half-hour τ with

$$s^+_{j,\tau} = \frac{e^+_{j,\tau} \exp\left(\alpha^+ \ln b_{j,t} + \beta^+ x^+_{j,\tau} + \gamma^+_j + \xi^+_{j,\tau}\right)}{1 + \sum_k e^+_{k,\tau} \exp\left(\alpha^+ \ln b_{k,t} + \beta^+ x^+_{k,\tau} + \gamma^+_k + \xi^+_{k,\tau}\right)}. \tag{15}$$

γ^+_j is a BM-unit fixed effect. $b_{j,t}$ is the bid for providing FR of BM unit j in the month t to which half-hour τ belongs. $x^+_{j,\tau}$ are controls that parsimoniously represent the supply curves that the BM units bid in the BM. We include in $x^+_{j,\tau}$ the hypothetical market share of BM unit j in half-hour τ at each of the 24 prices in the grid for upward repositing.[1] Finally, $\xi^+_{j,\tau}$ is a disturbance that, we assume, is mean independent of $b_{j,t}$ and $x^+_{j,\tau}$. This rules out that a firm conditions its bid in the BM on $\xi^+_{j,\tau}$.

We use a probit model for BM unit j being eligible for repositioning in the BM in half-hour τ with

$$e^+_{j,\tau} = 1(\breve{\alpha}^+ \ln b_{j,t} + \breve{\beta}^+ \breve{x}^+_{j,\tau} + \breve{\gamma}^+_j + \eta^+_{j,\tau} > 0).$$

$\breve{\gamma}^+_j$ is a BM-unit fixed effect. $b_{j,t}$ is the bid for providing FR of BM unit j in the month t to

[1] From its supply curve we can infer a hypothetical quantity of upward repositioning for BM unit j in half-hour τ at any given price. We compute the hypothetical market share of BM unit j in half-hour τ from the hypothetical quantities of all BM units, irrespective of whether they are part of the inside or outside goods.

which half-hour τ belongs. $\breve{x}^+_{j,\tau}$ contains additional half-hour-of-day (same for each day), day-of-week (same for each week), week-of-year (same for each year), and year fixed effects and controls that parsimoniously represent the supply curves that the BM units bid in the BM. We include in $\breve{x}^+_{j,\tau}$ the lowest offer of BM unit j in half-hour τ along with the corresponding quantity. Next we compute the distribution of lowest offers of all BM units (irrespective of whether they are part of the inside or outside goods) in half-hour τ. We include in $\breve{x}^+_{j,\tau}$ ten dummies for the decile in which the lowest offer of BM unit j in half-hour τ falls. We proceed similarly for the quantity corresponding to the lowest offer and include in $\breve{x}^+_{j,\tau}$ another ten dummies for the decile in which the quantity corresponding to the lowest offer of BM unit j in half-hour τ falls. Finally, $\eta^+_{j,\tau} \sim N(0,1)$ is a standard normally distributed disturbance that, we assume, is mean independent of $b_{j,t}$ and $\breve{x}^+_{j,\tau}$ and independent across BM units and half-hours.

It follows that

$$\Pr(e^+_{j,\tau} = 1 | b_{j,t}, \breve{x}^+_{j,\tau}) = 1 - \Phi\left(-\breve{\alpha}^+ \ln b_{j,t} - \breve{\beta}^+ \breve{x}^+_{j,\tau} - \breve{\gamma}^+_j\right) = \Phi\left(\breve{\alpha}^+ \ln b_{j,t} + \breve{\beta}^+ \breve{x}^+_{j,\tau} + \breve{\gamma}^+_j\right), \tag{16}$$

where $\Phi(\cdot)$ is the standard normal CDF. We estimate equation (16) by ML. Moreover, equation (15) implies

$$\ln s^+_{j,\tau} - \ln s^+_{0,\tau} \equiv \delta^+_{j,\tau} = \alpha^+ \ln b_{j,t} + \beta^+ x^+_{j,\tau} + \gamma^+_j + \xi^+_{j,\tau}$$

as long as $e^+_{j,\tau} = 1$. We assume $\xi^+_{j,\tau}$ and $\eta^+_{j,\tau}$ are independent of each other and estimate by OLS.

Results. Tables 24 and 25 show our estimates for the logit model in equation (15) and the probit model in equation (16). In the first and third columns, we exclude the controls $\breve{x}^+_{j,\tau}$ and $x^+_{j,\tau}$; in the second and fourth columns, we include them. The number of observations differs because we require $s_{j,t} > 0$ for OLS.

The coefficient on log FR bid $\ln b_{j,t}$ is significantly different from zero and negative in the logit model in equation (15) and the probit model in equation (16), both for upward and downward repositioning. This indicates that a BM unit that submits a low FR bid is more likely to be repositioned in the BM and also by larger amounts, presumably so that it can provide FR services. However, the impact is economically small. For example, in the logit

Table 24: Repositioning Share Analysis

	Log relative share of repositioning(if positive)			
	Upward repositions		Downward repositions	
logFRbid	-0.086	-0.108	-0.076	-0.100
N	260482	260482	885659	885659
R^2	0.57	0.58	0.53	0.54

In the first pair of regressions, the dependent variable is the log share ratio of upward repositioning volume; in the next two columns, it is the corresponding log share ratio of downward repositioning volume. Controls for the share of volume that a uniform auction would assign this unit based on its offers (upward) and bids (downward) at a set of 24 increasing prices are included in the second and fourth columns, although their coefficients are omitted. Unit fixed effects are included in all specifications and standard errors are clustered by half-hour periods (an observation is a unit-half-hour).

Table 25: Repositioning Probit Analysis

	Probability of Repositioning			
	Upward repositions		Downward repositions	
logFRbid	-0.04915	-0.03726	-0.19963	-0.09871
Closest bid/offer price		-0.00000		0.00000
Closest bid/offer quantity		-0.00009		0.00156
N	1511766	1511766	1511765	1508785

Estimates from a 20% random sample of observations. In the first pair of regressions, the dependent variable is the indicator variable of whether a unit get repositioned upward; in the next two columns, it is the corresponding indicator for downward repositions. Controls for the bid/offer closest to the current contracted position are shown; dummies for the percentile of the bid and offer (relative to contemporaenous offers) are included in the second and fourth columns but suppressed. Month-of-year, day-of-week and hour-of-day dummies fixed effects are included in all specifications.

model in equation (15), the elasticity of market share with respect to FR bid is on the order of -0.1, compared to around -1.6 in the FR market.

Markup and profit. To simplify the exposition, we again focus on upward repositioning. Conditional on eligibility (or in realization), the market share of BM j in half-hour τ is $s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+)$, as defined on the right-hand side of equation (15). We use the shorthands $x_\tau^+ = \left(x_{j,\tau}^+\right)_{j=1,\ldots,J}$, $\xi_\tau^+ = \left(\xi_{j,\tau}^+\right)_{j=1,\ldots,J}$, and $e_\tau^+ = \left(e_{j,\tau}^+\right)_{j=1,\ldots,J}$. θ^+ denotes the parameters of the logit model in equation (15). Unconditionally (or in expectation), the market share of BM j in half-hour τ is

$$s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+) = \sum_{e_\tau^+ \in \{0,1\}^J} s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) w^+(b_t, \breve{x}_\tau^+, e_\tau^+; \breve{\theta}^+),$$

where

$$w^+(b_t, \breve{x}_\tau^+, e_\tau^+; \breve{\theta}^+) \equiv \prod_{l=1,\ldots,J} \Phi\left(\breve{\alpha}^+ \ln b_{l,t} + \breve{\beta}^+ \breve{x}_{l,\tau}^+ + \breve{\gamma}_l^+\right)^{e_{l,\tau}^+} \left(1 - \Phi\left(\breve{\alpha}^+ \ln b_{l,t} + \breve{\beta}^+ \breve{x}_{l,\tau}^+ + \breve{\gamma}_l^+\right)\right)^{1-e_{l,\tau}^+} \tag{17}$$

and the summation is over all 2^J possible values of e_τ^+. $\breve{\theta}^+$ denotes the parameters of the probit model in equation (16).

We assume that the profit that accrues to BM unit j as it is repositioned in the BM over the course of month t (again unconditionally or in expectation) can be written as

$$\mu_j \sum_{\tau \in t} \left(M_\tau^+ s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+) + M_\tau^- s_j^-(b_t, x_\tau^-, \xi_\tau^-, \breve{x}_\tau^-; \theta^-, \breve{\theta}^-)\right),$$

where we abuse notation to denote as $\tau \in t$ the half-hours in month t. μ_j is a common markup for upward and downward repositioning. If NG accepts an offer to increase energy production, then the BM unit is paid by NG according to its offer but bears the cost of the additional fuel. If NG accepts a bid to decrease energy production, then the BM unit pays NG according to its bid but saves on fuel cost. Because bids and offers are under the control of the firm owning the BM unit, we expect the markup to be nonnegative.

Recalling that $\mathcal{J}_i$ denotes the indices of the BM units that are owned by firm i, the profit of firm i in the BM over the course of month t (again unconditionally or in expectation) is

$$\sum_{j \in \mathcal{J}_i} \mu_j \sum_{\tau \in t} \left(M_\tau^+ s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+) + M_\tau^- s_j^-(b_t, x_\tau^-, \xi_\tau^-, \breve{x}_\tau^-; \theta^-, \breve{\theta}^-)\right).$$

We are interested in how this profit changes with the bid for providing FR. Recall that the bid for the current month is submitted before the 20th of the previous month while bidding in the BM takes place during the current month. We simplify and assume that in preparing its bid for providing FR a firm ignores $\frac{\partial x_\tau^+}{\partial b_{j,t}}$, $\frac{\partial \breve{x}_\tau^+}{\partial b_{j,t}}$, $\frac{\partial x_\tau^-}{\partial b_{j,t}}$, and $\frac{\partial \breve{x}_\tau^-}{\partial b_{j,t}}$ for all $\tau \in t$. In essence, this says that the firm ignores that through its bid for providing FR it can influence the competitive landscape for the subsequent bidding in the BM. Under some conditions the envelope theorem ensures that this assumption is satisfied with respect to the bids and offers for the BM units that are owned by the firm. We emphasize, however, that this assumption has bite with respect to the bids and offers for the BM units that are owned by the firm's rivals.

It remains to compute $\frac{\partial s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+)}{\partial b_{k,t}}$ and $\frac{\partial s_j^-(b_t, x_\tau^-, \xi^-\tau, \breve{x}_\tau^-; \theta^-, \breve{\theta}^-)}{\partial b_{k,t}}$. We have

$$\frac{\partial s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+)}{\partial b_{j,t}} = \sum_{e_\tau^+ \in \{0,1\}^J} \Bigg(s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) \left(1 - s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+)\right) \frac{\alpha^+}{b_{j,t}}$$
$$+ s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) \frac{\breve{\alpha}^+ \phi\left(\breve{\alpha}^+ \ln b_{j,t} + \breve{\beta}^+ \breve{x}_{j,\tau}^+ + \breve{\gamma}_j^+\right)}{b_{j,t}\left(\Phi\left(\breve{\alpha}^+ \ln b_{j,t} + \breve{\beta}^+ \breve{x}_{j,\tau}^+ + \breve{\gamma}_j^+\right) + e_{j,\tau}^+ - 1\right)} \Bigg) w^+(b_t, \breve{x}_\tau^+, e_\tau^+; \breve{\theta}^+)$$

for $k = j$ and

$$\frac{\partial s_j^+(b_t, x_\tau^+, \xi_\tau^+, \breve{x}_\tau^+; \theta^+, \breve{\theta}^+)}{\partial b_{k,t}} = \sum_{e_\tau^+ \in \{0,1\}^J} \Bigg(- s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) s_k^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) \frac{\alpha^+}{b_{k,t}}$$
$$+ s_j^+(b_t, x_\tau^+, \xi_\tau^+, e_\tau^+; \theta^+) \frac{\breve{\alpha}^+ \phi\left(\breve{\alpha}^+ \ln b_{k,t} + \breve{\beta}^+ \breve{x}_{k,\tau}^+ + \breve{\gamma}_k^+\right)}{b_{k,t}\left(\Phi\left(\breve{\alpha}^+ \ln b_{k,t} + \breve{\beta}^+ \breve{x}_{k,\tau}^+ + \breve{\gamma}_k^+\right) + e_{k,\tau}^+ - 1\right)} \Bigg) w^+(b_t, \breve{x}_\tau^+, e_\tau^+; \breve{\theta}^+)$$

for $k \neq j$. Note that these derivatives are themselves expectations over eligibility e_τ^+ using probability weights $w^+(b_t, \breve{x}_\tau^+, e_\tau^+; \breve{\theta}^+)$.

To jointly estimate the marginal cost of providing FR and the markup on repositioning operations, we adjust the estimation equation (8) as follows: When we substitute in realizations and parameter estimates, then the bids $b_{i,t}$ of firm i in month $t \geq 44$ during the late phase satisfy the system of equations[2]

$$
\begin{aligned}
&\frac{1}{29}\sum_{t=44}^{72}\Bigg[\Bigg(M_t s_k(b_t,x_t,\xi_t,e_t;\theta)+\sum_{j\in\mathcal{J}_i}(b_{j,t}-c_j)\,M_t s_j(b_t,x_t,\xi_t,e_t;\theta)\,(1(k=j)-s_k(b_t,x_t,\xi_t,e_t;\theta))\,\frac{\alpha}{b_{k,t}}\\
&\quad+\sum_{j\in\mathcal{J}_i}\mu_j\sum_{\tau\in t}\Bigg(M_\tau^+\Bigg(s_j^+(b_t,x_\tau^+,\xi_\tau^+,e_\tau^+;\theta^+)\left(1(k=j)-s_k^+(b_t,x_\tau^+,\xi_\tau^+,e_\tau^+;\theta^+)\right)\frac{\alpha^+}{b_{k,t}}\\
&\qquad+s_j^+(b_t,x_\tau^+,\xi_\tau^+,e_\tau^+;\theta^+)\frac{\breve{\alpha}^+\phi\left(\breve{\alpha}^+\ln b_{k,t}+\breve{\beta}^+\breve{x}_{k,\tau}^+ +\breve{\gamma}_k^+\right)}{b_{k,t}\left(\Phi\left(\breve{\alpha}^+\ln b_{k,t}+\breve{\beta}^+\breve{x}_{k,\tau}^+ +\breve{\gamma}_k^+\right)+e_{k,\tau}^+-1\right)}\Bigg)\\
&\qquad+M_\tau^-\Bigg(s_j^-(b_t,x_\tau^-,\xi_\tau^-,e_\tau^-;\theta^-)\left(1(k=j)-s_k^-(b_t,x_\tau^-,\xi_\tau^-,e_\tau^-;\theta^-)\right)\frac{\alpha^-}{b_{k,t}}\\
&+s_j^-(b_t,x_\tau^-,\xi_\tau^-,e_\tau^-;\theta^-)\frac{\breve{\alpha}^-\phi\left(\breve{\alpha}^-\ln b_{k,t}+\breve{\beta}^-\breve{x}_{k,\tau}^- +\breve{\gamma}_k^-\right)}{b_{k,t}\left(\Phi\left(\breve{\alpha}^-\ln b_{k,t}+\breve{\beta}^-\breve{x}_{k,\tau}^- +\breve{\gamma}_k^-\right)+e_{k,\tau}^--1\right)}\Bigg)\Bigg)\Bigg)\otimes(1,f_{k,t-1})\Bigg]=0,\quad\forall k\in\mathcal{J}_i,
\end{aligned}
$$

where $\otimes$ denotes the Kronecker product, 1 the constant, and $f_{k,t-1}$ the fuel price relevant for BM unit k in month $t-1$. We omit distinguishing between parameters and estimates to simplify the notation.

These $2|\mathcal{J}_i|$ equations not only require that the first-order conditions are on average correct in the late phase but also that they are uncorrelated with the lagged fuel price that is known to the firm at the time it prepares its current FR bid. To facilitate the estimation, we assume the markup is common across BM units and firms and solve the resulting overdetermined system of linear equations by OLS.

Results. Accounting for repositioning incentives has a relatively small impact on the estimated marginal cost of providing FR: as Table 26 shows, the average across BM units falls from £1.41/MWh to £1.36/MWh. The estimated markup is not significantly different from zero.

[2] We make the simplifying assumption that the firm has perfect foresight about $M_t^+=(M_\tau^+)_{\tau\in t}$, $M_t^-=(M_\tau^-)_{\tau\in t}$, $x_t^+=(x_\tau^+)_{\tau\in t}$, $\breve{x}_t^+=(\breve{x}_\tau^+)_{\tau\in t}$, $x_t^-=(x_\tau^-)_{\tau\in t}$, and $\breve{x}_t^-=(\breve{x}_\tau^-)_{\tau\in t}$.

Table 26: Cost Estimates with Repositioning Incentives

	Without repositioning	With repositioning
Average marginal cost	1.40	1.36
Main market markup	–	-0.0014
s.e. markup	–	0.0041

Cost and markup estimates, with and without accounting for repositioning. Estimation is by generalized method of moments. The first column estimates are the same as those discussed in the main text.

A.4 Collusion

We try three different ways of examining the data for evidence of collusion. The first is to look for coordination in the timing and direction of bid changes across BM units. To capture timing, we define a dummy for BM unit j changing its bid between months $t-1$ and t and, to capture direction, another dummy for the BM unit increasing its bid. We compute all pairwise correlations between BM units in the dummy for a BM unit changing its bid and in the dummy for a BM unit increasing its bid (conditional on both BM units in the pair changing their bids). In Figure 17 we plot the distribution of correlation coefficients separately for BM units owned by the same firm ("within firm", left panels) and for BM units owned by different firms ("across firms", right panels). Note that we expect some across-firm correlation in both the timing and the direction of bid changes due to common shocks to demand.

The within-firm correlations for the timing and direction of bid changes in the left panels are positive and substantial. This reinforces our contention that decisions are centralized at the level of the firm rather than made at the level of the BM unit. The right panels show correlations pretty much evenly distributed around zero, consistent with independent decision making across firms. While we cannot rule out collusion in the earlier phases — and indeed we believe Drax attempted to establish a tacitly collusive arrangement in the middle phase — the lack of significant bid correlation is suggestive evidence against this.

Our second approach is more direct. We assume particular collusive arrangements and infer cost given the assumed conduct. Specifically, we re-solve equation (8) for the cost c_i that is consistent with observed play during the late phase of the FR market under the assumption that the top 10 firms colluded and maximized the combined profits of all their BM units. This yields an estimated average cost of £-9.8/MWh for the BM units, which is clearly implausible. The estimates are negative because demand is relatively inelastic, and

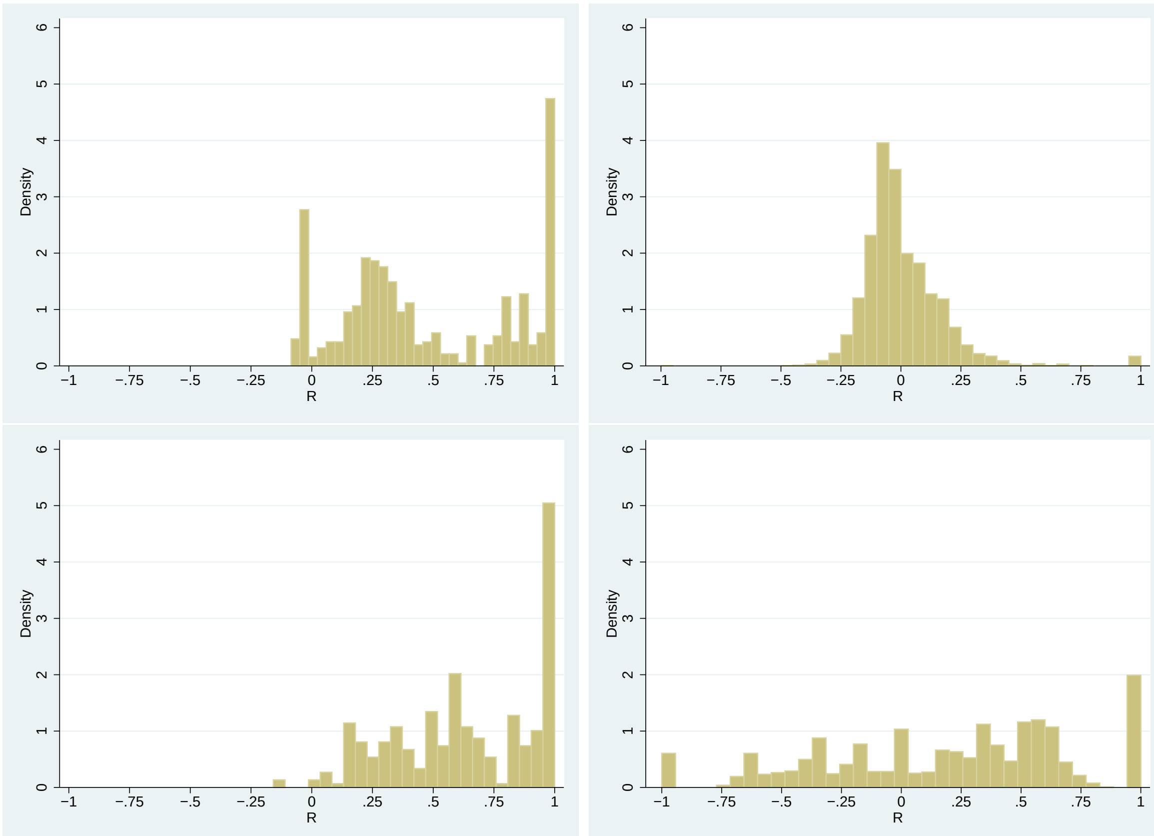

Figure 17: Top left is within-firm correlation in bid changes; top right is across-firm correlation in bid changes; bottom left is within-firm correlation in direction of change (conditional on both changing); bottom right is across-firm correlation in directions.

so rationalizing the bids in the face of increased market power requires low cost. When we repeat the exercise assuming that only the top 3 firms collude, the implied average cost is £-0.25/MWh, still negative.

Finally, we ask at how much weight μ we can have a firm put on its rivals' profits while still ensuring that the observed bids b_t for months $t \geq 44$ are consistent with non-negative cost $c = (c_j)_{j=1,\ldots,J}$. We formalize this question as the program

$$\max_{\mu,c} \mu$$

subject to

$$
\frac{1}{29}\sum_{t=44}^{T=72}\Bigg[M_t s_{j,t} + \sum_{k\in\mathcal{J}_i}\left(b_{k,t}-c_k\right)M_t\left(1(j=k)-s_{j,t}\right)\frac{\hat{\alpha}s_{k,t}}{b_{j,t}}
$$
$$
+\mu\sum_{k\in\mathcal{J}_{-i}}\left(b_{k,t}-c_k\right)M_t\left(1(j=k)-s_{j,t}\right)\frac{\hat{\alpha}s_{k,t}}{b_{j,t}}\Bigg] = 0,\quad \forall j\in\mathcal{J},
$$

and

$$
\mu \geq 0,
$$
$$
c_j \geq 0,\quad \forall j = 1,\dots,J.
$$

Note that μ pertains to BM units $k \in \mathcal{J}_{-i}$ that are *not* owned by firm i.

Conditional on μ the program is linear in c. We thus start with $\mu = 0$ and solve for c. Then we successively increase μ and re-solve for c. The implied cost estimates for various values of μ are shown in Figure 18. The horizontal axis is μ and the vertical axis is the share of BM units. The blue line shows the share of BM units with negative cost and the orange line is the share for which the 95% confidence interval is entirely negative. By $\mu = 0.28$, over 5% of BM units have negative cost with 95% confidence. Consistent with the first two approaches, we see little evidence of collusion.

A.5 Switching costs and inattention

Suppose there is a fixed costs to adjusting bids or that a firm is occasionally inattentive to the FR market for exogenous reasons. Suppose further that firms are myopic. Then the first-order conditions in equation (7) hold in those months in which the firm actually adjusted its bids. This allows us to re-estimate cost by simply restricting the sample. These resulting estimates do not differ significantly from our baseline estimates, as Figure 19 illustrates.

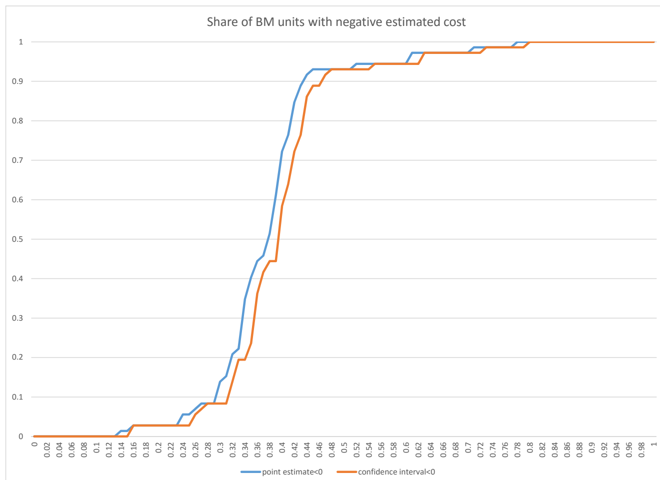

Figure 18: Share of BM units with negative estimated cost.

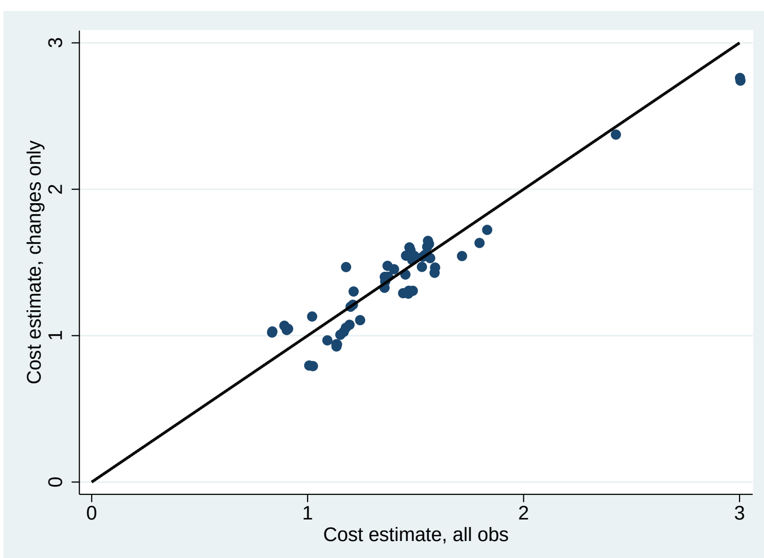

Figure 19: Comparison of cost estimates using all months versus only months in which the firm adjusted its bids.