# Searching For Information and the Diffusion of Knowledge

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Abstract

We study a dynamic learning model in which heterogeneously connected Bayesian players choose between two activities: learning from one's own experience (*work*) or learning from the experience of others (*search*). Players who work produce an inflow of information which is local and dispersed around the society. Players who search aggregate the information produced by others and facilitate its diffusion, thereby transforming what inherently is a private good into information that everyone can access more easily. The structure of social connections affects the interaction between equilibrium information production and its social diffusion in ways that are complex and subtle. We show that increasing the connectivity of the society can lead to a strict decrease in the quality of social information. We link these inefficiencies to frictions in peer-to-peer communications. Moreover, we find that the socially optimal allocation of learning activities can differ dramatically from the equilibrium one. Under certain conditions, the planner would flip the equilibrium allocation, forcing highly connected players to work, and moderately connected ones to search. We conclude with an application that studies how resilient a society is to external manipulation of public opinion through changes in the meeting technology.

**JEL Classification Numbers:** D83 , D85, D62.

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# 1. Introduction

This paper studies learning in large connected societies. The novelty of our approach is to explicitly capture the interactions between the *creation* of information and its social *diffusion*. When a society is effective at diffusing information, this reduces individual incentives to create new information. Analogously, when those who create information are few and peripheral, this hinders the diffusion of information. Our goal is to study the effects of these interactions on learning and their dependence on the structure of social connections. We identify a novel externality that we call the *noise-amplification* effect. This is an equilibrium mechanism by which noise reverberates and amplifies throughout the society, a phenomenon akin to the broken telephone game. We show the implications of this externality on different aspects of social learning: Do societies allocate learning tasks among differently connected players in ways that promote learning? Are more connected societies necessarily better informed? Are they more resilient to the external manipulation of public opinions?

We introduce a dynamic model of learning in societies in which players are heterogeneously connected to each other. A population of Bayesian players choose how to allocate time between two activities: learning directly from information sources (*work*) and learning indirectly from others (*search*). Players who work produce an inflow of information, which is initially local, available only to the player who produces it. Search, instead, is a frictional for two natural reasons. First, in order learn from others a player needs to meet them and this takes time. The rate at which a player can meet others is heterogeneous and determined by her *type*, representing how connected she is to the rest of society. Second, searcher-to-searcher communication is potentially subject to frictions. Namely, we allow for some information to be lost in these exchanges. A distinguishing feature of our model is that it captures the important social role played by those who search for information. While not producing new information, searchers aggregate the information produced by others and facilitate its diffusion. By doing so, they transform what inherently is a private good, information produced by a worker for herself, into a *more public* one, information that can be accessed more easily by everyone else. This increases the value of search and attracts in it a group of marginally connected players. Their diffusion ability, however, is no better than their ability to create information. This introduces a distortion that reverberates through the rest of the searching population, thereby causing its amplification.

Our main contribution is to identify novel inefficiencies in social learning, generated by the interaction between information production and its diffusion. First, we highlight the critical role played by those who search for information in the aggregation and diffusion of information. From a collective point of view, a society needs searchers to achieve some degree of informational efficiency. And yet, we show that the structure of social connections can affect efficiency in ways that are complex and subtle. To explore this, we study the equilibrium consequences of increasing the connectivity of a society. We show that increasing connectivity can lead to a strict decrease in the quality of social information. Our results reveal how this inefficiency is directly linked to communication frictions. Second, we analyze how players of different connectivity levels choose their learning activity in equilibrium, and compare this to the planner's solution where each type is allocated to an activity to maximize social welfare. This allows us to study more generally the inefficiencies associated with social learning. We show that allocation of activities in the planner's solution can be in direct contrast with the equilibrium one, especially when communication frictions are severe. In such cases, the planner's solution requires players with high connectivity to be the producers of information in the society, whereas in equilibrium, this role is necessarily taken on by players with low connectivity. Third, we apply our model to study how resilient a society is to external manipulations of public opinions and how this depends on the connectivity of the society. An important implication of our analysis is that societies that are very effective in aggregating and diffusing information are also particularly prone to manipulations.

This paper combines ideas and modeling tools from literatures such as multi-armed bandits, networks and search theory. In order to study *both* the equilibrium production of information and its social diffusion, we introduce heterogeneity across players in a parsimonious way. We focus on a single dimension of heterogeneity, how connected each player is to these society. The basic assumption we make is that the connectivity of a player determines the *rate* at which she meets others, not *who* she meets. This affects the frequency with which she encounters opportunities to receive, as well as to transfer information to others. While this formulation abstracts from other interesting ways in which the network structure can affect learning dynamics, we argue that it captures its most prominent feature, namely heterogeneity in the *number* of connections. Second, we allow for frictions in searcher-to-searcher communication. These frictions are modeled as a garbling, a "depreciation" of the quality of a signal as it travels among searchers. These frictions create a gap between *first-hand* information, the information created by players who work, and *second-hand* information, the information collected by anybody else. When these frictions are absent, our model reduces to the special case in which players perfectly observe each other's posterior beliefs upon meeting.

The stationary equilibrium of this game is unique and has a remarkably simple structure. Specifically, the allocation into activities is fully characterized by a threshold connectivity level, below which players work and, above which players search. Intuitively, more connected players meet others at a higher rate. These players, instead of working, they prefer to learn from others, even when this entails receiving information of lower quality on average. Due to their higher connectivity level, searchers are also easier to meet. Therefore, the information they carry is made *more accessible* to everybody else in the society, further attracting players away from work. Frictions in peer-to-peer communication drive a wedge between a searcher's ability to *aggregate* information and her ability to *diffuse* it to others. In equilibrium, there exists a group of moderately connected players who decide to search but, by the act of doing so, impose a negative externality on the rest of the society. Due to communication frictions, these players' diffusion abilities as searchers are inferior to their diffusion abilities as workers. This introduces an inefficiency that goes beyond the fact that meeting these players is now less informative. Indeed, it impacts *all* social meetings. These moderately connected players are effectively responsible for injecting extra-noise in the society. As it travels from one searcher to another, this noise accumulates and amplifies through the structure of social connections. We call this effect the *noise-amplification* externality.

We study whether this externality is fueled or, rather, dampened when the society becomes more connected. The net effect of increasing connectivity ultimately arises from the conflict of two opposing forces. A highly connected society provides more opportunities for players to learn from others and them players to aggregate information at a faster rate. But at the same time, it can tilt incentives away from producing information, thereby enlarging the pool of searchers and, thus, increasing the noise-amplification externality. We model increased connectivity with a class of stochastic transformations of the type-distribution. In particular, we focus on sequences of stochastic transformations satisfying single-crossing property and a version of the monotone likelihood ratio property. We show that the equilibrium quality of social information is *quasi-convex*. This property implies that, along any sequence of increasingly connected societies, the equilibrium quality of social information undergoes two phases. During the first one, it declines because increased connectivity comes at the cost of amplifying these social distortions. In the second phase, instead, increased diffusion ability overcomes noise, and social encounters become more informative. What determines the relative importance of these two phases is the communication technology.

To better understand these inefficiencies, we study how the planner allocates types into learning activities to maximize welfare. We show that the socially optimal allocation can diverge substantially from the equilibrium one. In particular, players with a higher connectivity do not necessarily spend more time searching, a feature that is necessarily true in equilibrium. Depending on the constraints the social planner faces, the socially optimal allocation can differ from equilibrium in two distinct ways: *reversal*, a situation in which highly connected players work, whereas lower types search, in stark contrast with equilibrium; or *time-switching*, a situation in which a region of players is constantly switched back and forth between work and search as a function of the actual information they carry. Both deviations highlight how the planner's allocation can differ from equilibrium in a qualitative sense. These deviations are caused by the interplay of connectivity and frictions in communication.

When these frictions are severe, a searcher's contribution to social information is curtailed. Although she accumulates information at a high rate, her diffusion ability are poor. From a social perspective, she free-rides more than she can diffuse information. By forcing a highly connected searcher to work, the planner trades-off her individual gains with the fact that every signal she produces as a worker will be easier to find by those who search. This is because her type determines *both* the rate at which she meets others and the rate at which others meet her.

Finally, we apply our framework to study how resilient a society is to external manipulations of public opinions and how this depends on the connectivity of the society. We imagine altering the meeting technology in a way that consistently exposes a small group of players to biased information.[1] We evaluate the overall impact of this manipulation on the distribution of posterior beliefs of the society. To do so, we construct a measure of *influence* for players in our society. It combines all the different forces that are at play in equilibrium. Our analysis shows that searchers become more influential as the share of the population producing information declines and connectivity of the society becomes concentrated on searchers. This result is line with the role of the amplification mechanism described above. An important implication of our analysis is that that societies that are very effective in aggregating and diffusing information also correspond to those that are highly susceptible to manipulations. As the influence of each type increases in this society –possibly due to the communication channel becoming more efficient or the society becoming more connected– it becomes easier to shift public opinion by manipulating the learning process for an increasingly small share of agents in the population.

The rest of the paper is structured as follows. Section 2 gives a comprehensive account of the related literature. Our model is introduced in Section 3 and we discuss its main assumptions in Section 3.4. We proceed by characterizing the equilibrium in Section 4 and derive our main results in Section 5. Section 6 is dedicated to normative solutions and efficiency benchmarks. In Section 7, we introduce our measure of influence and study the resilience of the society to external manipulations. Finally, Section 8 provides a discussion of our results in relation to possible extensions, while Section 9 concludes.

[1]Recently, a number of controlled large-scale social media experiments have shown the power of altering the news-feed in affecting users' beliefs and behavior. This corresponds to tweaking the probability that a given content will be shown to (in the language of our model, "will be found by") a given user. Aral (2012) study the impact of manipulation on the decision to vote, Muchnik et al. (2013) study the likelihood of informational cascades and Bakshy et al. (2012) study product adoption decisions.

## 2. Related Literature

This paper borrows ideas and tools from several distinct literatures. The fundamental trade-off between producing information and learning from others is a classic feature of strategic bandit problems, as in Bolton and Harris (1999), Keller et al. (2005), and Rosenberg et al. (2007), among many others. In bandit problems, players learn via costly experimentation or the observation of other players' experimentation. Pulling the *safe* arm effectively consists in free-riding on the information produced by others. However, experimentation is public, as these models do not accommodate heterogeneity in connections. Therefore, the *diffusion problem* is trivial. Relative to bandit problems, we reduce players' strategic interactions to their minimal components. In particular, we work with a continuum of players in a stationary environment. Moreover, the absence of a "safe" allows us to abstract from the classic experimentation-exploration trade-off, which is of interest in that literature. This makes the player's problem simple, allowing us to enrich social interactions in novel directions that are of fundamental importance for our questions. Recently, Che and Horner (2015) and Frick and Ishii (2016) have analyzed bandit-like environments with a continuum of players to study optimal information design and technological adoption.

A fundamentally new feature in our paper is the introduction of heterogeneity in players connections. Empirical evidence shows how vastly different people are when it comes to how connected they are to the rest of the society (Newman (2010)). Of course, this heterogeneity affects players' ability to learn from others as well as the influence they exert on others (Ballester et al. (2006)). One of our motivations is to understand how the structure of social connections affects equilibrium outcomes. From this point of view, we borrowed a lot from the social networks literature (Jackson (2008) and Golub and Sadler (2016)). In particular, our paper somewhat relates to the games studied by Bramoullé and Kranton (2007) and Bramoullé et al. (2014) and to the questions in Acemoglu et al. (2010). We substantially deviate from most of this literature as we model connections probabilistically in the context of a search framework. We do so by tweaking the standard search theory set-up to conveniently account for degree-heterogeneity. In our model, players are characterized by a *type* that determines the rate at which they meet other players. These meetings are random and their nature is a function of the type-distribution itself. With this, we are able to capture the idea that more connected players are easier to meet. The idea of learning from others by sampling opinions from the society is a feature that comes from the word-of-mouth learning literature, e.g. Banerjee (1993), Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2004). In a different context, it is also a feature of Duffie et al. (2009) and Duffie et al. (2014) and, to some extent, of Callander (2011). Caplin and Leahy (1998) and Caplin and Leahy (2000) also use search tools to model learning in an economy with a continuum of agents. Our paper differs substantially from these ones as we explicitly account for heterogeneity in the rate at which players meet. Farboodi et al. (2016) have independently developed a similar meeting technology to model this heterogeneity, although they have applied it to a markedly distinct environment. Moreover, unlike these authors, we are explicitly interested in the study of equilibrium outcomes as the underlying society becomes *increasingly connected*. To this purpose, the tools we develop are orthogonal to the literature above. Our main Theorem reduces to an integral aggregation of the single-crossing property. This problem closely relates to a tradition in economics that studies comparative statics under uncertainty. Seminal examples are Milgrom and Shannon (1994), Athey (2001), Athey (2002) and, specifically, Quah and Strulovici (2012) from which our definition of "regular" sequence is inspired.

A second key innovation in our paper is to explicitly model frictions in the communication technology. The idea is that communications between players take place through a communication channel with *finite capacity* that inevitably distorts the message. This idea is common in the information theory literature, especially in computer science. In economics, finite capacity channels have been used to model rational inattentive agents, Sims (2003), Steiner et al. (2016) and Jung et al. (2016). We abstract away from the problem of strategic information transmission that has been amply studied in the communication literature, see for example, Milgrom (1981), Grossman (1981), Jovanovic (1982), Crawford and Sobel (1982), Okuno-Fujiwara et al. (1990) and more recently by Kamenica and Gentzkow (2011). Frictions in communication are an implicit feature also in most of the herding literature, Banerjee (1992) and Bikhchandani et al. (1992), Acemoglu et al. (2011), but also Gale and Kariv (2003). In this paper, we create a flexible environment in which to interact these communication frictions together with changes in the social structure of the underlying society.

Finally, our paper also relates to some recent work done in growth theory. In particular, Perla and Tonetti (2014) and Lucas and Moll (2014) study growth models in which firms, by search among other firms, can "update" and improve their own technologies. Jovanovic (2015) studies a dynamic learning problem in which an agent chooses between production or investment in information. Fogli and Veldkamp (2014) analyze, both theoretically and empirically, the dual aspects of diffusion: encouraging the spread of good versus bad behavior. In political science, Larson and Lewis (2016) model an information diffusion process that accounts for the fact that people may trust some of their contacts more than others. In such context, higher network density potentially impedes the wide reach of information to diverse communities. Relatedly, Grossman et al. (2014) study empirically how the access to information communication technology affects who gets heard and what gets communicated to politicians.

# 3. Model

In this section, we introduce the model. We begin by describing players' characteristics and objectives, and the learning activities available to them. In Section 3.2, we define and solve the players' dynamic choice problem. In Section 3.3, we model information exchanges and introduce the communication technology. We postpone the discussion of our model to Section 3.4, in which we provide intuitions and motivation for our main modeling assumptions.

## 3.1. Types and Meetings

Time runs continuously and uncertainty is characterized by a persistent and unknown binary state of nature $\theta \in \{-1, 1\}$. A continuum of Bayesian and forward looking players enter and leave the economy at a fixed Poisson rate $\delta > 0$. We index their age with $t \geq 0$ and denote $\tau(t) := \delta e^{-\delta t}$ its distribution. Players discount the future at common rate $r > 0$ and, when entering the game, have a common prior belief $p_0 := \Pr(\theta = 1)$.[2] Each player is born with a *type* $x \in \mathbb{R}_+$, distributed according to density $f$ with support $X$. Denote $\mathcal{F}$ the set of all such densities. We refer to the distribution of types $f \in \mathcal{F}$ as a *society*.[3]

Each player is endowed with a type-dependent search technology, allowing her to actively search in order to meet others in the society. In particular, a player's type describes how *connected* she is to the rest of the society. Her type measures of how easily she can meet others and, potentially, learn from them. Specifically, a player's type $x$ denotes the rate at which meetings take place. The nature of these meetings is random. Their distribution is given by the conditional density function $h(y) := yf(y)/\int_X zf(z)dz$, independent of the player's type $x$. That is, while a player's type $x$ determines the extent to which she meets others, it does not affect the conditional likelihood with which she meets one type versus another. Notice that, *ceteris paribus*, higher types are also those that are more likely to be met by others.

Players want to learn about the state of nature $\theta \in \Theta$ as quickly and accurately as possible. For simplicity, we assume a player's flow utility to be given by $u(p(x, t)) := \max\{p(x, t), 1 - p(x, t)\}$, with $p(x, t)$ denoting the player's posterior belief at time $t$. To learn about $\theta$, each player continuously chooses between two *activities*: working and searching. These activities provide them with private signals, informative about the state $\theta$. Signals that originate from the *work* activity are exogenous, as they do not depend on the activities chosen by other players. Moreover, they are type-independent; thus, everyone has equal access to this

[2] Neither of these two assumptions is necessary for most of our results. When dealing with heterogeneous prior, however, it is important that such heterogeneity is common knowledge.

[3] Throughout, for any measurable function $q \in \mathbb{R}^X$, we write $\mathbb{E}_f(q(x)) := \int_X q(x)f(x)dx$.

technology. Specifically, when a player chooses to work in a $dt$-interval, she receives a signal $\pi_w$, distributed normally with mean $\eta_w \theta dt$ and variance $dt$. These signals are conditionally independent across both time and players.[4]

Signals that originate from the *search* activity are, instead, equilibrium objects. These information structures depend on which activities the other players have chosen, the meeting technology, the amount of information each player has accumulated, and on the potential frictions in communication, which will be introduced in Section 3.3. We broadly refer to the information players can receive from others as *social information*. When searching, type $x$ meets other players at rate $x$ according to the conditional density $h$. From each one of them, she extracts some information. The player receives an endogenous signal from each player she randomly meets. Clearly, the nature of these signals will differ, depending on the type of the player she meets, as well of the age, activity and particular experience etc. Yet, what matters for her decision to work or search in the $dt$-interval is the information she *expects* to receive, which can be represented as a signal that aggregates all the characteristics listed above. We denote this equilibrium object as $\pi_s(x)$ and posit that it is normally distributed with mean $x\eta_s\theta dt$ and variance $dt$.[5] Hence, the characteristics of the signal $\pi_s(x)$ depend on the searcher's type $x$ only to the extent that it scales the mean-to-variance ratio $\eta_s$. This reflects the idea that one's type only affects the rate at which meetings take place, but since the conditional meeting density $h$ is type-independent, it does not affect the nature of these meetings. Also, the signal does not depend on time $t$. This is true in a stationary equilibrium of this dynamic model, and our analysis will mostly focus on equilibria that are stationary. In a stationary equilibrium, even if players do learn as they grow old, the society-wide distribution of posteriors is a stationary object. This implies that the information structure that characterizes the search activity is time-independent.

In this simple model, the moments $\eta_w$ and $\eta_s$ fully characterize the information processes associated with the two activities. They provide a measure of how informative each activity is. In particular, $\eta_s$ can be interpreted as the "per-meeting" expected quality of social information, a measure of the informativeness of a random meeting in the society. It will represent one of the main objects of interest in our model. Before we discuss how $\eta_s$ is determined in equilibrium, we discuss how a player, given a triple $(x, \eta_x, \eta_s)$, chooses between the two learning activities.

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[4]We normalize the variance of signals to 1, letting $\eta_w$ capture the mean-to-standard deviation ratio. This normalization is without loss of generality with respect to the problem solved by each player.

[5]The distributions of the signals associated with working and searching that we just posited can be shown to be the continuous-time analogs of the distributions in a discrete-time version of our model.

## 3.2. Optimal Learning Activities

Players continuously allocate time between work and search to most effectively learn about the uncertain state $\theta$. Let $v(p_t)$ be the value of the player's problem with posterior $p_t$ at age $t$, and $\alpha_t$ denote the instantaneous probability that a player searches at age $t$. Her dynamic problem can be expressed recursively as follows:

$$v(p_t) = \max_{\alpha_t \in [0,1]} (r+\delta)u(p_t)dt + e^{-(r+\delta)dt}\mathbb{E}\big(v(p_{t+dt})|\alpha_t\big), \tag{1}$$

where the expectation is taken with respect to the future posterior beliefs $p_{t+dt}$ given the choice of $\alpha_t$. The choice of $\alpha$ only affects future information. Since, we focus on a stationary equilibrium and since players are strategically small, the choice of a player's activity does not affect any of the aggregate variables. In this set-up, her objective is equivalent to maximizing the variance of her posterior beliefs. In Lemma A3, relegated to the Appendix, we show that the recursive equation above can be written as

$$v(p_t) = \max_{\alpha_t \in [0,1]} u(p_t) + \frac{2}{r+\delta} p_t^2 (1-p_t)^2 v''(p_t) Q(\alpha_t).$$

The term $Q(\alpha_t) := ((1-\alpha_t)\eta_w)^2 + (\alpha_t x \eta_s)^2$ captures how the chosen activity affects the variance of posterior beliefs. In the Proof of the next result, we will also show that $v$ is convex, a consequence of the fact that the player is information-loving. She wants to learn as fast as possible. The variance of her future posterior depends on $p_t$. In particular, the more extreme $p_t$ is, the smaller the variance. However, the choice of the activity does not depend on $p_t$ as the next result establishes.

**Lemma 1.** *Given $\eta_w$ and $\eta_s$, there exists a unique threshold type $x^\star = \frac{\eta_w}{\eta_s}$, such that all types above $x^\star$ search $(\alpha_t = 1)$ and all types below $x^\star$ work $(\alpha_t = 0)$.*

In particular, players never switch between activities during their lives in a stationary equilibrium in which $\eta_s$ is time-independent. There is a unique threshold type $x^\star$, whose connectivity increases with $\eta_w$ and decreases with $\eta_s$. We denote the map

$$\eta_s(x) = \frac{\eta_w}{x} \tag{2}$$

the *individual rationality* condition (IR). Given any number $\bar{\eta}_s$, each and every type above $x$ finds optimal to search (exclusive) if and only if $\bar{\eta}_s = \eta_s(x)$.

## 3.3. Information Exchanges and Communication Technology

When a searcher meets a type $x$, a transfer of information occurs *from* $x$ *to* the searcher. The amount of information that is successfully transferred depends naturally on two factors:

how much information type $x$ has accumulated up to that point, and on how efficiently he can communicate it to the searcher. The natural upper bound is that a player can never transfer more information than she possesses. Similarly, the lower bound is the one in which no information can be transferred. Let $\Gamma(x,t)$ denote the *stock of information* a player of type $x$ has gathered up to time $t$. From Lemma 1, we know that players do not switch between activities. It is therefore particularly simple to model the stochastic process $\Gamma(x,t)$, which takes the form of a Brownian motion, potentially with endogenous drift-to-variance ratio. We have that:

$$\Gamma(x,t) := \begin{cases} \eta_w t\theta + B(t) \sim \mathcal{N}(\eta_w t\theta, t) & \text{if } x \text{ works,} \\ \eta_s x t\theta + B(t) \sim \mathcal{N}(\eta_s x t\theta, t) & \text{if } x \text{ searches,} \end{cases} \tag{3}$$

where $B(t)$ is the standard Weiner process. Intuitively, when $\Gamma(x,t)$ is positive (resp. negative), the player has accumulated evidence in favor of hypothesis $\theta = 1$ (resp. $\theta = -1$). We show in Lemma A4 that the stock of information $\Gamma(x,t)$ and the posterior belief $p(x,t)$ are in a one-to-one relationship. Therefore, we can think of observing the stock of information $\Gamma(x,t)$ as observing $p(x,t)$.

A communication technology allowing for instantaneous transfer of all the information a player has ever received in her life is certainly an extraordinary efficient one. More realistically, communication technologies introduce distortions in the information that can be transferred between any two players. We want our model to be flexible with respect to these deviations from the efficient benchmark, that has been amply studied in the literature. To do so, it is convenient to normalize $\Gamma(x,t)$ as $\pi(x,t) := t^{-\frac{1}{2}}\Gamma(x,t)$, which consequently becomes normally distributed with some mean - activity and time dependent - and unit variance.[6] This normalization concentrates all the heterogeneity coming from the age, $t$, and type of player, $x$, to differences in means. We think of the communication from a *searcher* of type $x$ and any other searcher to happen through a communication technology with possibly finite capacity.[7] Specifically, the communication technology accounts for potential loss in information when moving from the input signal, $\pi(x,t)$, to the output signal, $\tilde{\pi}(x,t)$, which can be interpreted as a garbled version of it. A communication technology is defined as follows:

**Definition 1.** *A* ***communication technology*** *is a map*

$$g \in \mathcal{G} := \{g \in C(\mathbb{R}_+) \mid \ g \text{ non-decreasing and } g(y) \leq y\}.$$

*Let $(\mathcal{G}, \geq)$ be the poset of communication technologies. When $g' \geq g$, we write that $g'$ is* more informative *than $g$.*

[6]This normalization is without loss of generality: since both $x$ and $t$ are observable, and activities are persistent, the normalized signals induce the same update of $p_t$, which is ultimately the state variable that matters for the player.

[7]Sims (2003) popularized the notion of a finite capacity channel in economics. See Steiner et al. (2016) and Jung et al. (2016) for recent applications.

| | Input | | Output |
|---|---|---|---|
| If $x$ works | $\pi(x,t) \sim \mathcal{N}\Big(\sqrt{t}\theta\eta_w, 1\Big)$ | $\to$ | $\tilde{\pi}(x,t) \sim \mathcal{N}\Big(\sqrt{t}\theta\eta_w, 1\Big)$ |
| If $x$ searches | $\pi(x,t) \sim \mathcal{N}\Big(x\sqrt{t}\theta\eta_s, 1\Big)$ | $\rightsquigarrow$ | $\tilde{\pi}(x,t) \sim \mathcal{N}\Big(g(x\sqrt{t})\theta\eta_s, 1\Big)$ |

Table 1: Communication Frictions

In the Definition above, we make two natural assumptions on the communication technology. First, $g$ is non-decreasing. This captures the idea that more input necessarily produces (weakly) more output. Second, $g(y) \leq y$. That is, no player can ever transfer more signals than those she has at a given point in time. Frictions in communication are implemented as illustrated in Table 1. The basic assumption we make is that, due to these frictions, it could be harder to transfer signals *intermediated* by searchers. This introduces an inherent difference between *first-hand* information, the one coming directly from the source, i.e. a worker, and *second-hand* information, the one coming from a searcher, someone who has herself learned indirectly from either a worker or another searcher. We postpone further discussions on the communication technology to Section 3.4.

Searchers receive a signal from $\tilde{\pi}(x,t)$ upon meeting type $x$ at age $t$. Yet, meetings are random, taking place continuously. In order to assess the value of the search activity, a searcher needs to evaluate the amount of information she can *expect* to receive whenever she engages in the search activity for $dt$ amount of time. From Section 3.1, we know this is captured by $\eta_s$. To close the model, we specify how $\eta_s$ is linked to the distribution of types $f$ and the signals $\tilde{\pi}(x,t)$ that can be acquired from each player in the society:

$$\mathbb{E}(\pi_s|\eta_s) = \mathbb{E}\Big(\int_X \Big(\int_0^\infty \tilde{\pi}(x,t)\ \tau(t)\ dt\Big) h(x)dx\Big). \tag{4}$$

The condition in Equation 4 directly links the informativeness of the search activity, $\eta_s$, to the "amount" of information that is expected to be communicated in a random meeting taking place in this society.We refer to the condition in Equation 4 as *Bayes consistency.*

Equation 4 makes explicit that the information process characterizing the search activity revolves around a fixed-point argument. In order to determine the behavior of a given player - which activity she will choose, and how much information she will gather consequently - one needs to pin down $\eta_s$. However, $\eta_s$ itself depends on the amount of information that can be communicated by a type $x$ at age $t$, which is given by $\tilde{\pi}(x,t)$, also depending on $\eta_s$. We characterize and solve this fixed-point problem in Section 4.

## 3.4. Discussion of the Model

Before moving to the analysis of our model, it is useful to discuss our main assumptions, their motivation and robustness of our results with respect to them.

*The Meeting Technology.* A principal feature of our model is that there is heterogeneity in how easily players can learn from others. We assumed that highly connected players meet others more easily. This has two implications: first, highly connected players meet more people and, therefore, are better at extracting information *from* the rest of the society. Second, highly connected players, precisely because of their relative *size*, attract other players more frequently and are therefore more likely to be met by others, independently of their activity. This is the fundamental difference between $x$ and $h(x)$, we have introduced in Section 3.1. To keep the model tractable, we assumed that the connectivity of an agent $x$ only affects the *rate* at which meetings take place when she searches, not their nature. A society $f \in \mathcal{F}$ is not, generically speaking, a *network*.[8] Hence, by reducing all the heterogeneity in the society to heterogeneity in connectivity, we abstract from other potentially interesting features of a network, such as the heterogeneity in people's neighborhoods. Instead, we are able to capture only the first-order level of heterogeneity that characterizes real-world networks, namely degree heterogeneity, which posits that different people may have different levels of "access" to the network, because they have more or less connections. From this perspective, $f$ is a convenient way to introduce a fundamental level of heterogeneity in the model without explicitly having to account for a full-scale network. This assumption makes our model particularly tractable and it also allows us to study *global* and general shifts in the distribution of connections $f$, rather than *local* and particular changes, such as the deletion of one particular node or the other.

*Information Exchanges and Communication Frictions.* The second key component in our analysis is the way in which information is exchanged from one player to the other. In this paper, we abstract away from the *motives* that lead players to share information available to them with other players. The problem of strategic information transmission has been amply studied in the literature and goes beyond the scope of this paper.[9] In the context of our model, this problem is immaterial because players are too small to have an effect on the level of social information, and therefore they are indifferent between transferring all, some or no information. The novelty of our approach is to introduce frictions in the communication technology that transfer information from one player to the other. Frictions only apply to information that is accumulated via the *search* activity, thus introducing a wedge between

[8]It is indeed a very particular type of network: an infinite, complete and weighted graph on $X$, where the weights are proportional to the *type* of a node.

[9]See for example Crawford and Sobel (1982), Grossman (1981), Milgrom (1981), Jovanovic (1982) and Kamenica and Gentzkow (2011).

*first-hand* and *second-hand* information. We posit that workers, having literally produced the information themselves, are able to perfectly relay it to others without loss. Searchers, instead, having received information from others, may not be as effective as workers at relaying this knowledge to others. Of course, this is a reductive assumption, but it makes the main tensions in this model more transparent and straightforward. In Section 8, we discuss an extension of our model in which the communication technology is frictional for all players, independently of their activity. In Appendix B, we show that most of our results go through, at the cost of a stronger requirement - concavity - on $g \in \mathcal{G}$.[10]

*Stationarity and Payoffs.* In Section 8, we discuss the dynamic version of our model. There, $\eta_s(t)$ becomes a time-dependent equilibrium object and players possibly switch from work to search during the course of their life. In Appendix C, we show that dynamic equilibria, although difficult to characterize, have information paths $\eta_s(t)$ that necessarily converge to the stationary equilibrium. We otherwise focus attention on stationary equilibria. In a stationary environment, we think of players becoming aware of, or interested in, a given *issue* $\theta$ about which they have no particular previous knowledge. At a random future time $\tau(t)$, this player will have to take an irreversible guess which will determine her material payoff given the state $\theta$ is realized. Since the time at which her choice becomes payoff relevant is random, the optimal strategy for her is to continuously update her guess, as new information comes along. The flow utility $u(p_t)$ represents the value of such guessing problem, that is $u(p_t) := \max_{b_t \in \Theta} \mathbb{E}_{p_t}\big(\mathbb{1}(b_t = \theta)\big) = \max\{p_t, 1 - p_t\}$.[11] We interpret the random time $\tau(t)$ at which the player has to cast her final guess as the time at which issue $\theta$ becomes subjectively obsolete for the player. A new issue $\theta'$ will become of interest for her and she leaves the game. While we focus on stationary equilibria mostly for analytical tractability, there are many environments in which it is a natural solution concept. Consider a population where each agent sequentially faces different issues or problems about which she has to form an opinion. How much time each player spends on an issue is stochastically determined by $\tau(t)$ and depends, among many other things, on how important this particular issue is to the player or her awareness of other issues. When a player decides to learn about this issue from other players, she is able to acquire information mostly from those that are also currently interested on that specific issue. This is natural in social networks that are issue-specific. The same is true even in a general-interest social network, such as Facebook or Twitter, as

[10] A concave $g \in \mathcal{G}$ captures a natural idea. Under a "finite capacity" communication channel, the higher the number of signals that need to be transfered, the harder it is to transfer of an additional one. See Section 8 for a more detailed discussion.

[11] The results we present in this paper do not hinge on the particular choice of $u(p_t)$. Any flow utility $u$ generating a convex value $v$ will do. This simply requires players to be information lovers. That is, a player's flow payoff increases, in expectation, with information. Our choice of $u$ is motivated by the fact that it provides us with a simple analytical solution for the second-order partial differential equation describing the player's control problem.

long as active players only post information about the issues they are currently interested in, while older posts become obsolete or inaccessible. From this perspective, a stationary solution concept can be interpreted as applying to those environments in which the share of the population interested in a specific issue is evolving over time and yet its size is relatively stable.

# 4. Equilibrium

In this Section, we establish existence and uniqueness of a stationary equilibrium for the model introduced in Section 3. We show how to reduce the equilibrium to a relatively simple fixed-point map, thus condensing all the complexity introduced in the previous sections into one simple equation, from which most of our results will be later derived. To begin, we formally define what a stationary equilibrium is for this game.

**Definition 2.** *A* **Stationary Equilibrium** *is a pair $(x^\star, \eta_s)$, composed by a threshold type $x^\star$ and a social information quality $\eta_s$, that satisfies the following conditions:*

*(IR) Given $\eta_s$, $x^\star$ satisfies Individual Rationality as by Equation 2. In particular, type $x$ searches if and only if $x \geq x^\star$.*

*(BC) Given $x^\star$, $\eta_s$ is Bayes Consistent. That is, $\eta_s$ is a fixed-point of Equation 4.*

In the definition above, the first requirement is that no player, at any point in her life, wants to deviate from the activity she is engaged in. For this to be true, type $x^\star$ has to be indifferent between work and search. All types with higher connectivity will search and, vice versa, all types with lower connectivity will work. The second requirement, instead, can be seen as a particular kind of *market clearing* for information. As in a production economy, players cannot "consume" more than the economy is producing. Similarly, Bayes consistency requires that information shall not be created from nowhere or erased with no reason. Information must be produced and diffused according to the "rules of the game" that we have outlined in the Section 3.

The fixed point implicit in Bayes consistency has a unique solution (Lemma A5), which is given by the following expression:

$$\eta_s = \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}(z)h(z)dz}, \tag{5}$$

where $\tilde{g}(z) := \mathbb{E}_\tau\big(g(z\sqrt{t})\big) : X \to \mathbb{R}_+$ and $c := \mathbb{E}_\tau(\sqrt{t})$. Function $\tilde{g}(z)$ represents the expected informational contribution across all possible ages $t$ of a given type $z$, as it is filtered through the communication technology $g$. To rule out explosive dynamics, we need to guarantee $\eta_s \in \mathbb{R}_+$. A sufficient condition is given by the following assumption.

**Assumption 1.** *We assume $f \in \mathcal{F}$ satisfies $\mathbb{E}_f(x^2) \leq \mathbb{E}_f(x)$ and that $\mathbb{E}(\sqrt{t}) \leq 1$.*

Assumption 1 imposes restrictions on how thick the upper tail of $f$ is and on how long players are expected to remain in the game.[12] If this condition fails to apply, the society is able to multiply information unboundedly, precluding the possibility for a stationary environment. We will maintain Assumption 1 throughout the paper. In equilibrium, according to Definition 2, both individual rationality and Bayes consistency (Equations 2 and 5 respectively) are satisfied simultaneously. This provides us with a single fixed-point equation that fully characterizes the equilibrium:

$$x = \frac{1}{c} + \int_x m(z)h(z)dz, \tag{6}$$

where we denoted $m(z) := \frac{1}{c}\big(cx - \tilde{g}(z)\big)$. The following result establishes existence and uniqueness of our stationary equilibrium. Figure 1 provides a graphical representation of the interactions between individual rationality and Bayes consistency.

**Proposition 1.** *Fix a society $f \in \mathcal{F}$ and a communication technology $g \in \mathcal{G}$. A stationary equilibrium exists and is unique.*

In equilibrium, we observe rich and non-trivial interactions among activities, types and the level of $\eta_s$. Work and search cannot be reduced to complements or substitutes of each other. For example, substitution is important when the worker population shrinks to a low level. In such a circumstance, there is very little information injected in the system and, therefore, little information to be searched for. When the society is not particularly rich of information, the search activity becomes less attractive, especially for types whose connectivity is not particularly high. This creates incentives for these players to switch to work, therefore re-balancing the inflow of "new" information in the society. The tension we just described, which seems to suggest that when players switch to work, the search activity becomes more profitable, does not apply uniformly across the population. Generically, that is for almost all $g \in \mathcal{G}$, the quality of social information $\eta_s$ is non-monotone in the threshold type $x^\star$ (Figure 1). Past a critical value $\hat{x}$, at which social information quality is maximal, $\eta_s$ starts declining as the working population becomes larger. This technical observation has the power to uncover the unique role that searchers play in equilibrium. While searchers do not create "new" information, they nevertheless $(i)$ aggregate information produced by others and $(ii)$ enable it to be diffused to different parts of the society. Searchers transform what inherently is a *private good* - i.e. information produced by a worker for her own self - into a *public good*, information that everybody else can access more easily. Due to their higher

[12] The requirement on $f$ can be relaxed by strengthening the requirement on $\tau$. Equilibrium exists uniquely as long as $\mathbb{E}_\tau(\sqrt{t})\mathbb{E}_f(x^2) \leq \mathbb{E}_f(x)$.

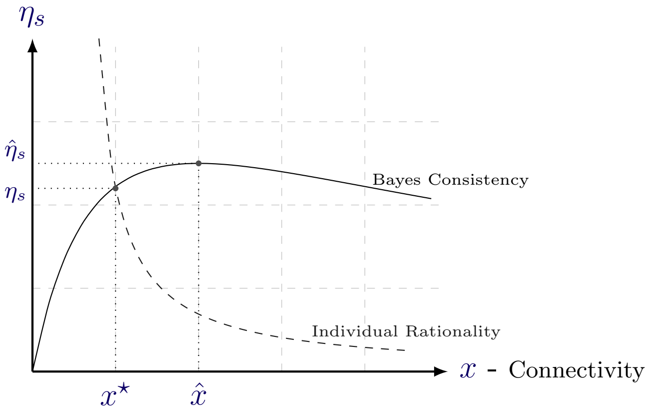


FIGURE 1: A graphical representation of the stationary equilibrium.

connectivity, searchers meet (and are met by) people more often than others. Searchers are better able to aggregate information and making it available to be shared with others, modulo the distortions introduced by the communication channel. For highly connected types, choosing the search activity might not only be beneficial on an individual level, but it can also be beneficial for the society as a whole. With no one searching, information is scattered around the society, in "private goods" which are of use solely for the players who have produced them. Searchers bring these goods together and make them more accessible to the general public. By doing so, they can increase the value of search and, in principle, the welfare of the society. And yet, the now higher value of search can attract workers away from their activity, thus possibly reducing the value of search. The unique balance among these rich interactions is a feature of the equilibrium of this game, which we analyze in the next section.

# 5. Results

Our model is characterized by two principal components: the distribution of types $f \in \mathcal{F}$, describing how connected the society is; and the communication technology $g \in \mathcal{G}$, describing potential frictions in information exchanges. In this section, we study how $f$ and $g$ affect the equilibrium of this game and how these two components interact with each other.

## 5.1. Equilibrium and the Communication Technology

We start by uncovering the role of the communication technology $g$ in the determination of the equilibrium. In Definition 1, we have introduced a natural order on $\mathcal{G}$. We said that $g'$ is more informative than $g$ if $g' \geq g$. When this is the case, uniformly less information can be transferred under $g$ than under $g'$. The partially ordered set of communication technologies has two obvious extrema: $\max(\mathcal{G}, \geq) = \mathrm{id}_{\mathbb{R}_+}$, the identity function on $\mathbb{R}_+$, and $\min(\mathcal{G}, \geq) = 0$. When the communication technology is maximally informative, it is *as if* players could observe each other's posterior belief upon meeting. There is no loss entailed in peer-to-peer information exchanges. Instead, when $g = 0$, second-hand information is fully depreciated and searchers can learn only through workers. In this case, searchers serve no social role. They only free-ride on the information produced by workers.

**Proposition 2.** *Fix a society $f \in \mathcal{F}$. As the communication technology $g$ becomes* more informative*, the mass of workers shrinks* and *the equilibrium information quality $\eta_s$ increases.*

As the communication technology improves, we observe players shifting away from work towards search. This effect is intuitive. A better communication technology implies that, *ceteris paribus*, a player is able to extract more information for any given meeting. This implies that, the marginal type under $g$, who was not connected enough to be a searcher under $g$, could strictly prefer to search under $g' \geq g$. This leads to a decrease in $x^\star$ and, therefore, in the share of those who produce information. However, even though information production is reduced, the equilibrium information quality increases because, under the new communication technology, information gets aggregated and diffused with higher efficiency.

It is no surprise that improvements in the communication technology are unambiguously beneficial for the society. In fact, not only $\eta_s$ increases in equilibrium, but it is also the case that $\eta_s(x)$ increases conditional on any $x \in X$, as depicted in Figure 2. This implies that, under the superior communication technology, any level of $\eta_s$ can be achieved with a strictly smaller set of workers. When $g$ is maximally informative, a particular feature of the equilibrium $(x^\star, \eta_s)$ is that $x^\star = \hat{x}$, that is, the quality of social information is maximized. Under such $g$, players can effectively access each other's posterior beliefs with no friction and learn instantaneously all the information a player has ever collected in her life. This is a noiseless society in which no signal is ever lost.

**Proposition 3** (Observing Posteriors Beliefs)**.** *Let $(x^\star, \eta_s)$ be the equilibrium under $g \in \mathcal{G}$ and $f \in \mathcal{F}$. The quality of social information $\eta_s$ is maximal if and only if $g$ is maximally informative.*

Proposition 3 highlights how, in the presence of frictions in the communication technology, players might fail to choose the activity that maximizes the quality of social information $\eta_s$. Under any frictional $g$, there exists a region $[x^\star, \hat{x})$ of players with intermediate connectivity who choose to search, even when they would have contributed more to $\eta_s$ as workers. A frictional communication technology, indeed, introduces a wedge between an player's own incentives and her social role in the determination of $\eta_s$. When searching, a player aggregates

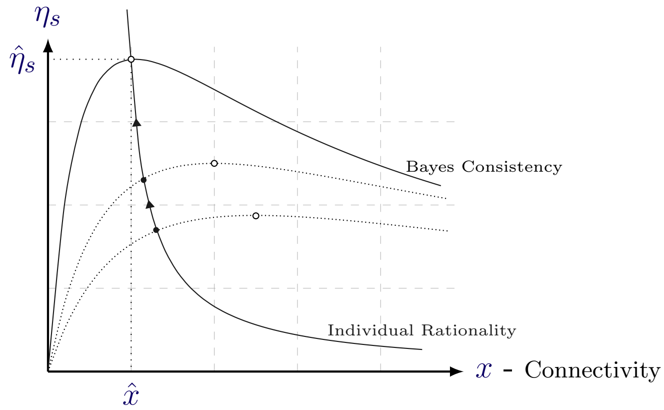


FIGURE 2: Equilibrium and Communication Technology

information at a rate proportional to her type $x$. However, her ability to diffuse information is depressed due to the friction imposed by the communication technology $g$. While players' individual incentives are entirely determined by the former, their social role is also linked to the latter. A discrepancy between the two creates a region characterized by players who are better than workers at aggregating information, but worse than them at its diffusion. Of course, this situation has implications for the efficiency of these equilibria and, more specifically, on how a social planner would redistribute players into activities, as we will see in Section 6. For the rest of this section, however, we focus on the effects that this region produce on equilibrium outcomes.

## 5.2. Searching and the Amplification of Noise

The previous discussion highlighted the role of communication frictions in creating a region in the type-space that is populated by searchers whose individual decisions are detrimental for $\eta_s$, the quality of social information. In this subsection, we single out the details of the implications of such decisions. In particular, we show how these distortions are amplified through the activity of searchers, even those who do not belong to the compromised region.

When the communication technology $g \in \mathcal{G}$ is frictional, we know from Proposition 3 that Bayes consistency implies $\eta_s(x)$ is strictly increasing in $x$, for all $x \in [x^\star, \hat{x})$ (Figure 1). Using the definition of $\eta_s$ in Equation 5 and rearranging, it is easy to see that,

$$\frac{d\eta_s(x)}{dx} > 0 \quad \Rightarrow \quad 1 - \int_x \tilde{g}(z)h(z)dz > \tilde{g}(x)H(x) \quad \Rightarrow \quad \tilde{g}(x)\eta_s < c\eta_w \quad \text{for all } x \in [x^\star, \hat{x}).$$

Fix any type $\bar{x} \in X$. We can think of $c\eta_w$ (resp. $\tilde{g}(\bar{x})\eta_s$) as the extent to which type $\bar{x}$ is expected to contribute to $\eta_s$ in her lifetime, if she works (resp. searches). Type $\bar{x}$'s contribution when she works is exogenous, it does not depend on her type and is not affected by frictions. It is only a function of the distribution of ages $\tau$. On the contrary, type $\bar{x}$'s contribution when she searches is endogenous, it is type dependent and affected by frictions. When $\bar{x} \in [x^\star, \hat{x})$, type $\bar{x}$'s decision to search effectively reduces $\eta_s$. Equivalently, she introduces *noise* in the society. When someone meets such a player, the signal she receives is *noisier* than it should have been.

The distortions for which type $\bar{x}$ is responsible, however, go well beyond the simple fact that meeting her is now less profitable. The noise that type $\bar{x}$, with her decision to search, seeded in the society is collected by all searchers who meet her and, therefore, spread around the society. It becomes part of everyone's information set. In this sense, a problem that was created locally, that is, in the region $[x^\star, \hat{x})$, becomes a *global* phenomenon affecting all searchers. This *noise-amplification mechanism* reduces the informativeness, not only of those social meetings that involve players from the compromised region, but of *all* social encounters. Types in the region $[x^\star, \hat{x})$ seed extra-noise in the society.

Next, we formalize this idea of amplification mechanism. To do so, we compute a relative measure of social information elasticity. To begin, we fix a type $\bar{x} \in X$ and an activity $w$ or $s$. We compute the elasticity of $\eta_s$ with respect to a marginal increase in type $\bar{x}$'s diffusion abilities:

$$\varepsilon_w(\bar{x}) := \frac{d\eta_s/dc_{\bar{x}}}{\eta_s/c_{\bar{x}}} \qquad \text{and} \qquad \varepsilon_s(\bar{x}) := \frac{d\eta_s/d\tilde{g}(\bar{x})}{\eta_s/\tilde{g}(\bar{x})}$$

These elasticities capture how much $\eta_s$ is affected when type $\bar{x}$ becomes more efficient at diffusing information. We model such an experiment by simulating a marginal increase - affecting specifically type $\bar{x}$ - in either $c$ or $\tilde{g}(\bar{x})$, according to which activity type $\bar{x}$ plays in equilibrium. The relative difference between these two elasticities, namely $\varepsilon_s(\bar{x}) - \varepsilon_w(\bar{x})$, provides us with a measure of the total effect that type $\bar{x}$ produces on $\eta_s$ when she decides to transit *from* work *to* search. In the next result, we compute and decompose such measure.

**Proposition 4.** *The* relative elasticity *of* search *versus* work *for a given type $\bar{x} \in X$ can be decomposed as:*

$$E(\bar{x}) := \varepsilon_s(\bar{x}) - \varepsilon_w(\bar{x}) = \underbrace{\Big(\tilde{g}(\bar{x})\eta_s - c\eta_w\Big)}_{\substack{\text{social}\\ \text{contribution}}} \underbrace{\frac{\bar{x}}{H(x^\star)}}_{\substack{\text{amplification}\\ \text{effect}}} \kappa(\bar{x}), \tag{7}$$

*where $\kappa(\bar{x}) = \frac{f(\bar{x})}{c\eta_w \mathbb{E}_f(z)}$ only depends on primitives.*

Proposition 4 provides a decomposition of the negative externality that players in the compromised region $[x^\star, \hat{x})$ are exerting on $\eta_s$. For these players, we have established that $\tilde{g}(\bar{x})\eta_s < c\eta_w$. Therefore, the first term represents the extra-noise that their activity seeds in the system. This wedge, we said, is entirely due to frictions implied by $g$. Yet, these

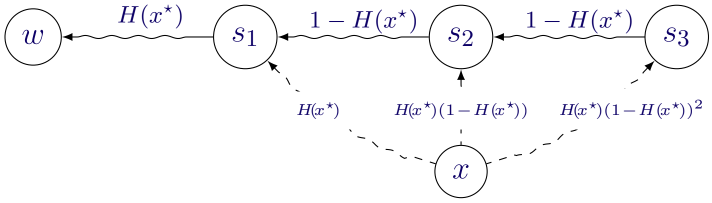


FIGURE 3: The amplification effect.

distortions are not meant to stay local. They are amplified by the rest of the searching population, as captured by the second term in Equation 7. This term has two factors:

*Centrality.* The amplification effect is increasing in $\bar{x}$. The value of $\bar{x}$ measures how attractive, or "central," this player is. The higher $\bar{x}$, the higher the rate at which other searchers will meet her, the stronger the social distortions that her individual decision creates.

*Expected Path Length.* The amplification effect is decreasing in $H(x^\star)$. In our model, all signals originate from some workers. These signals are then spread around the society by the activity of searchers. The term $\frac{1}{H(x^\star)}$ captures exactly the expected number of searchers each signal encounters before reaching a player $x \in X$. To see this, consider Figure 3. Player $x$, a searcher, meets another searcher called $s$. Although $x$ can observes $s$'s type and age, she cannot reconstruct the path followed by the information she is about to relay. However, for each signal player $s$ has collected, player $x$ can speculate on the path it traveled. For example, if $s = s_1$, the signal that $s$ is carrying comes directly from a worker $w$. Such path has length $k = 1$. This event has probability $H(x^\star)$, namely the probability player $s$ meets a worker. If $s = s_2$, instead, the signal went through another searcher before reaching $s_2$. This path has length $k = 2$ and probability $H(x^\star)\big(1 - H(x^\star)\big)$. Player $x$ can compute the probability of each $k$ and, thus, the expected length of such paths:

$$\mathbb{E}\Big(\text{path of length } k \;\Big|\; x^\star\Big) = \sum_{k=1}^{\infty} kH(x^\star)\big(1 - H(x^\star)\big)^{k-1} = \frac{1}{H(x^\star)}$$

The smaller $H(x^\star)$, the higher the expected path length that each signal travels in this society, the higher the probability the signal ever went through the compromised region $[x^\star, \hat{x})$.[13]

The combination of the strength of attraction of type $\bar{x}$ and the amplification power of the

[13] This is reminiscent of what, in the United States, is sometimes referred to as the *telephone game* effect. The telephone game consists of having a group of people arranged in a line with a message being whispered by one player to her immediate neighbor, until it reaches the last player, who then announces the message to the group. Errors typically accumulate and amplify in the re-tellings, so that the statement announced by the last player differs significantly from the one uttered by the first. A conceptually similar force is at play in our model.

society determine the overall impact of the negative externality that type $\bar{x}$ induces on the whole society.

## 5.3. The Pitfalls of Increasingly Connected Societies

We now turn the analysis to the effects of the social structure on equilibrium outcomes. More precisely, we are interested in understanding how the equilibrium and, in particular, the ability to produce and diffuse information, is affected when the society becomes *more connected*. Our discussion in Section 5.1 shows how a frictional communication technology is bound to introduce a wedge between an agent's individual incentives and her social role. This implicitly suggests that increasing connectivity in a society could have ambiguous effects on the quality of social information $\eta_s$. We begin this Section by making this intuition explicit. We show how different changes in the structure of the society - always increasing aggregate connectivity - can systematically produce opposite effects on the quality of social information. This illustrates how the ambiguity mentioned above is an endemic feature of this problem. In our Theorem 1, we put forward a positive resolution to this ambiguity. We show that, by organizing the changes in the social structure in a coherent and reasonable manner, the tension between negative and positive effects gains some order, thus providing us with a clean and general illustration of the equilibrium effects of increasing social connectivity.

To begin, let us introduce a partial order $\trianglerighteq$ on $\mathcal{F}$ capturing the idea that, if $f' \trianglerighteq f$, society $f'$ is *more connected* than society $f$. Since $f$ is a probability density function, it seems natural to use stochastic orderings to track how the distribution $f$ is changing. The use of stochastic ordering is particularly convenient in our model because it allows us to abstract away from *local* changes to the social structure, like adding a connection between two particular players, and focus on *global* changes. We capture the idea of $f'$ being more connected than $f$ via first-order stochastic dominance.

**Definition 3.** *Let $(\mathcal{F}, \trianglerighteq)$ be the poset of societies endowed with the first order stochastic order. We say that $f'$ is* **more connected** *than $f$ whenever $f' \trianglerighteq f$.*

Now consider any society $f \in \mathcal{F}$ with a *frictional* communication technology $g \in \mathcal{G}$. By Proposition 3, we know that $x^\star < \hat{x}$. Players whose type falls between $x^\star$ and $\hat{x}$ are contributing negatively to the equilibrium quality of information society. They find individually optimal to be searcher, but due to frictions in the communication technology, they end up relaying less information than they would, if they had worked. In the next Observation, we construct two examples of first-order stochastic shifts that, in the one case, exacerbate the *influence* of these trouble-types and, in the other, alleviate it. Denote $\eta_s(f)$ the equilibrium quality of social information under a given society $f$.

**Observation 1.** *Fix a society $f \in \mathcal{F}$ and any* frictional *communication technology $g \in \mathcal{G}$. There exists two societies $f', f'' \in \mathcal{F}$, both* more connected *than $f$, such that, at their respective equilibrium, $\eta_s(f') < \eta_s(f) < \eta_s(f'')$.*

Observation 1 highlights how increasing connectivity can produce opposite effects on the quality of social information, according to which types have increased their social influence. The way examples in Observation 1 are constructed is particularly instructive on the more general tensions that characterize the transitions from one society to a more connected one. To illustrate this clearly, fix $f$ and let $(x^\star, \eta_s)$ be the respective stationary equilibrium. When the communication technology $g \in \mathcal{G}$ is frictional, we know from the previous section that $\eta_w c > \eta_s \tilde{g}(x)$ for all $x \in [x^\star, \hat{x})$ and $\eta_w c < \eta_s \tilde{g}(x)$ for all $x \in (\hat{x}, \infty)$. To better understand the effects of increasing connectivity, it is particularly useful to consider the following decomposition of $\eta_s$ (see Equation A.2):

$$\eta_s = \eta_w c + \int_{x^\star}^{\hat{x}} \underbrace{\left(\eta_s \tilde{g}(z) - \eta_w c\right)}_{\text{negative}} h(z)dz + \int_{\hat{x}}^{\infty} \underbrace{\left(\eta_s \tilde{g}(z) - \eta_w c\right)}_{\text{positive}} h(z)dz$$

From a social perspective, players in $x \in [x^\star, \hat{x})$ creates a negative externality on the equilibrium allocation. Searchers in this region could have contributed to $\eta_s$ more effectively if only they had worked. The effects on $\eta_s$ of increasing connectivity crucially depend on which types see their relative "weight" increased, whether it is $[x^\star, \hat{x})$ or $(\hat{x}, \infty)$. This provides intuition on how the examples in Observation 1 can be constructed. If $f$ increases in a first-order stochastic sense so does $h$. If the implied change is such that under the new conditional density $h'$, mass has been shifted from $[0, x^\star]$ to $[x^\star, \hat{x})$, the equilibrium adjustment of $\eta_s$ will be negative. Vice versa, if mass has been shifted from $[0, x^\star]$ to $[\hat{x}, \infty)$, bypassing the trouble region $[x^\star, \hat{x})$, the equilibrium adjustment of $\eta_s$ will be positive.[14]

This discussion highlights, once again, how the potential negative effects associated with increasing connectivity are closely tied to the frictions in the communication channel. As shown in Proposition 3, both $x^\star$ and $\hat{x}$ are changing as the communication channel becomes more efficient and, in particular, one converges to the other, therefore making the share of players $[x^\star, \hat{x}]$ increasingly small. We summarize the discussion above in the next result.

**Proposition 5.** *Fix a society $f \in \mathcal{F}$ and a communication technology $g \in \mathcal{G}$. The following are equivalent:*

*(i)* $\eta_s(f') > \eta_s(f)$ *for all* $f' \in \mathcal{F}$ *such that* $f' \trianglerighteq f$.

[14]Formally, this shows that $\eta_s(f', x^\star(f)) < \eta_s(f, x^\star(f)) < \eta_s(f'', x^\star(f))$. However, since the equation that defines individual rationality is strictly decreasing in $x$, this will imply that $\eta_s(f', x^\star(f')) < \eta_s(f, x^\star(f)) < \eta_s(f'', x^\star(f''))$, as we wished to show.

*(ii)* $g \in \mathcal{G}$ *is maximally informative.*

The quality of social information unambiguously improves irrespectively of the shift if and only if players can observe each other's posteriors, thereby perfectly transferring all the information they ever accumulated in their life. When frictions are present, instead, there always exist a more connected society in which $\eta_s$ has decreased.

We have focused on $\eta_s$ while looking at comparative statics. Note that a decrease in $\eta_s$ doesn't directly imply a decline in social welfare. Social welfare in equilibrium is determined by the interaction between the new distribution of connectivities $f'$ and the new quality of social information $\eta_s(f')$. When $\eta_s(f')$ goes down, conditional on one's type, expected utility unequivocally goes down. Nonetheless, it is always the case that higher types do better. Thus, it is possible that the increase in connectivity for the society is such that the overall welfare effect is positive. However, it is important to emphasize that an increase in welfare in *never* guaranteed whenever there are frictions in the communication technology. Particularly, it is always possible to construct examples where a decrease in $\eta_s$ is accompanied by a decline in social welfare.

The result of Proposition 5 hinges on manipulating in specific ways the distribution of connectivities, while respecting the stochastic order $\trianglerighteq$. The ambiguous effect on $\eta_s$ is produced by the fact that the set of first-order stochastic shifts is large and weakly structured. In the real world, increases in connectivity often occur in more *regular* ways. A natural requirement could be that if a type $x$ is less likely under $f'$ than under $f$, the same should apply for all others $z < x$, something that is not guaranteed under $\trianglerighteq$. Similarly, if $f$ belongs to a known parametric family of distributions, it's unclear whether one can replicate the manipulations in Observation 1 within the class of such distributions. For these reasons, we now introduce more structure to our problem. Specifically, we define a natural class of stochastic shifts that has the merit of introducing order in the way the negative and positive components highlighted in Observation 1 affect social information. Since the structure of connectivity in the society influences the equilibrium only through the meeting technology $h$, we directly put structure on $h$.[15]

**Definition 4.** *A sequence* $(h^n(z))_{n\in N} \subset \mathcal{H}$ *is* ***increasing*** *if for all* $n$, $h^n_\Delta := h^{n+1} - h^n$ *crosses* $0$ *only once. The sequence is* ***regular*** *if, whenever positive,* $h^{n+1}_\Delta / h^n_\Delta$ *is non-decreasing in* $z$*.*

A sequence is *increasing* if concentration of connectivity moves from low types to high types in a monotone way. For example, if a player with connectivity $x$ becomes more prevalent in

[15] One can always consider the meeting technology $h$, instead of the distribution of connectivity $f$, as the primitive of our model. Starting with $f$ is more natural for introducing the model.

the new society $h^n$ relative to the old one $h^{n-1}$, all types $x'$ with connectivity higher than $x$ weakly become become more prevalent as well. Alternatively, if it the case that type $x$ loses prevalence, it must be that all lower types becomes less prevalent as well.

*Regularity* of a sequence, instead, imposes a condition on the way $h$ increases along the sequence. The condition closely resembles the Monotone Likelihood Ratio, but applies to changes in $h$ along the sequence not to $h$ directly, and thus, accounts for cases where the ratio can take negative values.[16] One natural implication of this property is that, for $n'' > n' > n$, letting $\bar{z}$ and $\underline{z}$ being respectively defined as $h^{n''}(\bar{z}) = h^{n'}(\bar{z})$ and $h^{n'}(\underline{z}) = h^n(\underline{z})$, we have $\bar{z} \geq \underline{z}$, something that is not guaranteed by having an increasing sequence alone. When this is not the case, i.e. when $\bar{z} < \underline{z}$, all types $z \in [\bar{z}, \underline{z}]$ would be less prevalent' in $h_{n'}$ relative to $h_n$, but more prevalent in $h_{n''}$ relative to $h_{n'}$. Definition 4 rules out these anomalies, by imposing a form of regularity along the sequence. An alternative way to interpret the condition is to focus on a specific type $x$ and see how $h^n(x)$ changes along the sequence with $n$. Regularity guarantees that the sequence can be divided into at most two parts, the first part where $h^n(x)$ is increasing and then the subsequent part where it is decreasing.

**Theorem 1.** *Fix a communication technology $g \in \mathcal{G}$ and a regular sequence $(h^n(z))_{n\in N} \subset \mathcal{H}$ of increasingly connected societies. Let $(\eta_s^n)_{n\in N}$ be the corresponding sequence of equilibrium social information qualities. The sequence $(\eta_s^n)_{n\in N}$ is* quasi-convex *in $n$.*

Formally, $\eta_s(h)$ is quasi-convex along any increasing and regular sequence. That is, the equilibrium evolution of the quality of social information $\eta_s$ has two distinct phases. In the first one, $\eta_s$ *decreases* and the quality of social information deteriorates. In this phase, the increase in connectivity comes at the cost of amplifying the negative social role that the new searchers are exerting. Due to their increased connectivity, these players are more attracted to search. Yet, they don't internalize the social cost that their choice imposes on the rest of the society. In the second phase, $\eta_s$ starts increasing. As the society becomes more connected, so do highly connected types. At some point the increased ability at diffusing information overcomes the negative impact of the additional noise that the marginal searchers are introducing. What determines the relative importance of these two phases is, once again, the communication technology $g$ and, in particular, how severe communication frictions are. To pair this result with Proposition 5, when $g$ is maximally efficient, the decreasing phase disappears along any increasing and regular sequence.

Figure 4 illustrates the result graphically. Theorem 1 sheds light on the relationship between how connected a society is and the quality of the information that it is able to produce and

[16]This condition is a version of the Monotone Signed Ratio property introduced by Quah and Strulovici (2012). It is a regularity condition necessary for successfully aggregating the single crossing property.

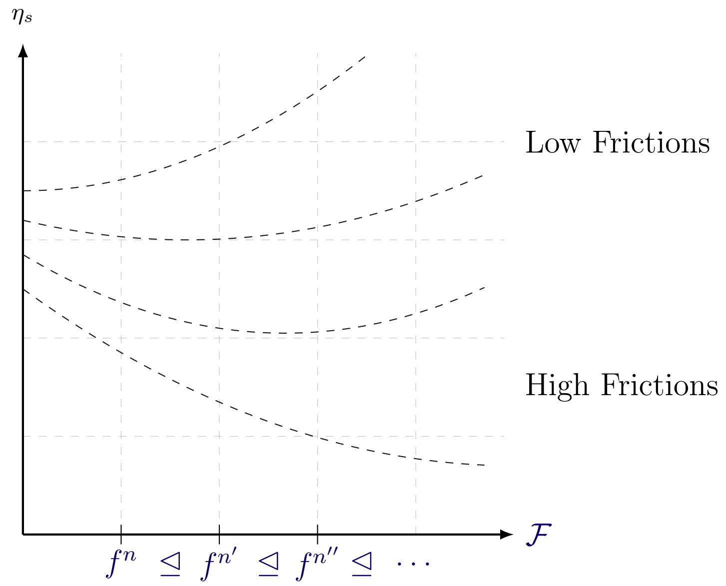


Figure 4: Social information quality as the society becomes increasingly connected.

diffuse in equilibrium. Our result arises from the clash between two opposing forces. On the one hand increased connectivity improves the speed at which information is aggregated and diffused in a society. This contributes positively to the overall information quality. This is reflected in the fact that searchers, conditional on meeting another searcher, always prefer to meet a more connected type rather than a less connected one. However, this comparative static is only true *conditional* on meeting another searcher. Frictions in the communication technology imply the existence of cases where a searcher prefers to meet a less connected worker to a more connected searcher. These correspond to instances where the searcher has accumulated more information than a worker, but is actually able to transfer less information due to the frictions in communication. Moreover, increased connectivity also provides incentives for some players to quit their working activity, and to switch to search. This effect decreases the size of the working population, and thus the amount of "original" information that is injected into the system. The now larger searching population propagates noise at a higher magnitude, because the average path length connecting the signal fetched by a worker to its "final" user is now longer and thus signals are garbled more often.

We conclude this section with the analysis of the two extreme cases of no frictions and maximal frictions. In the first case, our model captures interactions in which players can observe each other's posteriors. In the second case, our model converges to a pure exploitation problem. Searchers benefit from their higher connectivity to learn from workers more effectively. However, they serve no social role as the information they collect cannot be re-used by anyone else. Consistently, in these two extreme case, we obtain two opposite and extreme results.

**Corollary 1.** *Consider any regular sequence $(f^n(z))_{n\in N} \subset \mathcal{F}$ of increasingly connected societies. The equilibrium quality of social information $\eta_s$ is monotonically increasing (in $n$) if $g = \max(\mathcal{G}, \geq)$ and monotonically decreasing if $g = \min(\mathcal{G}, \geq)$.*

# 6. Normative Solutions

We now turn to the normative part of our analysis. This section analyzes the efficiency of the equilibrium we characterized in the previous sections. In particular, we will ask how *should* activities be allocated as to maximize social welfare. We will consider two distinct definitions of welfare and planner. We will conclude that equilibrium allocations are generically inefficient, in the sense that they are efficient only in the extreme case in which players observe each others posteriors. Due to the richness of our type structure, however, the equilibrium allocation and the optimal one may not simply diverge at a quantitative level, but also at qualitative one.

In the first problem, the planner allocates players into activities as a function of their type and maximizes the *ex ante* welfare of a generation of newborns. We show that in the optimal allocation, the planner may entirely reverse the order of the society by allocating lower types to search and higher type to work (non-monotone allocations). In the second problem, the planner maximizes *ad interim* welfare of the society by optimally allocating players into activities as a function of both their type and the information they have accumulated up to any given point. We show that, even if we are still in a stationary environment, a non-empty region of players is constantly swapped between activities by the planner, as a function of the amount of information they have acquired.

## 6.1. The Optimal Time-Independent Allocation of Labor

In this section, the planner can allocate players into activities at the beginning of their life and maximizes the present discounted value of a generation of newborn players. Formally, the planner chooses an allocation function $\alpha \in \mathcal{A} := \{\alpha : X \to [0,1]\}$. The planner is not bound to respect individual incentives, as described in Lemma 1. Apart from this, the society functions according to the rules spelled out in Section 3.3. In particular, the planner does not affect how meetings take place and how information is collected, exchanged and, possibly, compromised due to frictions in peer-to-peer communication. More precisely, the planner is constrained by the fact that the quality of social information $\eta_s$ still needs to be

Bayes consistent. Equation 5, however, takes a more general form in this case:

$$\eta_s(\alpha) := \eta_w \, \frac{c \int_X (1-\alpha(z))h(z)dz}{1 - \int_X \alpha(z)\tilde{g}(z)h(z)dz}. \tag{8}$$

The consistency condition above differs from Equation 5 as the social planner is not bound to respect individual rationality and therefore she can choose allocations that are no longer characterized by a unique threshold-type.[17] The planner's problem can be expressed in the following way.

$$\begin{aligned} W^{\mathrm{SP}} \;=\; & \max_{\alpha \in \mathcal{A}} \quad \int_X \Big( \big(1-\alpha(z)\big) v_w(\eta_w) + \alpha(z) v_s(z,\eta_s) \Big) \, f(z)dzdt, \\ & \text{sub to} \quad \eta_s = \eta_s(\alpha) \text{ as in Equation 8.} \end{aligned} \tag{9}$$

In Lemma A9, we show that the planner's trade-off relative to the allocation of type $z$ can be described as follows:

$$\underbrace{v_s(z,\eta_s) - v_w(\eta_w)}_{\text{net individual gain}} \geq \underbrace{z\big(\eta_w c - \eta_s \tilde{g}(z)\big)K}_{\text{net social loss}}. \tag{10}$$

The left-hand side of the above inequality represents the marginal *individual* gain of having type $z$ searching rather than working. The right-hand side, instead, represents the marginal *social* loss stemming from allocating type $z$ to search rather than work. If type $z$ searches, throughout her life she will contribute to $\eta_s$ at rate $z\tilde{g}(z)$. The first term $z$ captures the idea that the more connected type $z$ is, the more frequently she will be met by others (independently of her activity), and therefore the more important her contribution to the equilibrium $\eta_s$ is. The other term $\tilde{g}(z)$, instead, captures her life-long contribution to the social information $\eta_s$, a function of how quickly she can gather information if she searches and of how severe the communication frictions are. The life-long contribution as a worker, $c := \mathbb{E}(\sqrt{t})$, is not filtered through the function $g$ and does not depend on the type, since neither enter the worker's problem.

**Proposition 6.** *Generically, the equilibrium allocation is* ex ante *inefficient. In particular, it is efficient if and only if the communication technology is maximally informative.*

The word generically, again, captures the idea that, when $g = \max\{\mathcal{G}, \geq\}$, that is when there are no frictions whatsoever in peer-to-peer communications, then the equilibrium allocation coincides with the planner's. The result in Proposition 6 demonstrates that, whenever there are frictions in the communication technology, we should expect the equilibrium allocation

[17] Specifically, Equation 8 reduces to Equation 5 if there exists some threshold type $x \in X$ such that $\alpha(z) \in \{0,1\}$, with $\alpha(z) = 1$ if and only if $z \geq x$.

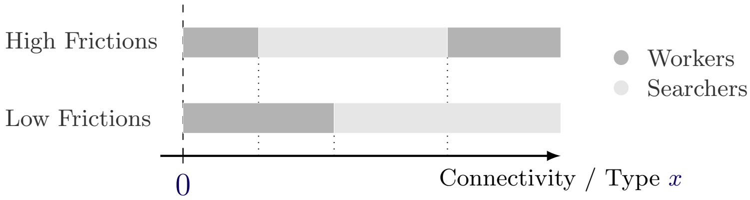


FIGURE 5: Planner's allocation for different $g \in \mathcal{G}$

be inefficient, and in particular to fall short in the supply of the "public good" that is produced by this economy, namely information. This result is in line with the literature in public good provision games. When there are free-riding problems, the equilibrium usually underprovides the public good.[18] However, our model produces outcomes that go beyond a mere quantitative deviation from the optimal amount of information that the society should produce. Indeed, when the communication frictions are particularly severe, the equilibrium allocation is inefficient also in a qualitative sense. The social planner could find optimal to entirely reverse the order of the society. We know that equilibrium allocations are characterized by a simple threshold-type structure. This is no longer the case of the socially optimal allocation. For example, the planner can allocate very low *and* very high type to work, while leaving all intermediate types searching (see Figure 5).

**Corollary 2.** *The optimal allocation can fail to have a monotone threshold type structure.*

The efficient social allocation can be qualitatively different from the equilibrium one. For example, it could feature three regions: low types working, intermediate types searching and high type working. The intuition is the following. Consider a highly connected type $z$ such that $c\eta_w > \tilde{g}(z)\eta_s$. This situation is bound to happen when $g$ is particularly concave, namely frictions are particularly severe. In such case, this player is contributing less than she could to the total amount of information in the society. However, since $z$ is highly connected, she will be met extremely often by others in the society. This effect is captured by the multiplicative term $z$ appearing on the right hand side of Equation 10. Therefore, the negative social contribution of player $z$ is amplified by the fact that she is going to be met a lot by others in the society. For this reason, the marginal social loss she generates by searching can offset her individual gain and the planner would rather want to have her work.

## 6.2. Optimal Time-Dependent Allocation of Labor

In this section, we consider a more demanding definition of a planner. Not only she can allocate people into activities based on their type, but she can also condition based on which

[18]This is also the case in Keller et al. (2005), for example.

information they have at any given point in their life. Formally, the planner selects an allocation function $\alpha \in \mathcal{A}' := \{\alpha : X \times \mathbb{R} \to [0,1]\}$, a map that is adapted to the filtration it induces. The number $\alpha(x,t)$ denotes the instantaneous probability that type $x$ will be assigned to search when she is of age $t$. Through the choice of $\alpha$, the planner chooses what players do at every instant of their life, as a function of their type and their experience. The planner maximizes total welfare subject to Bayes consistency:

$$\eta_s(\alpha) := \eta_w \frac{\int_{X\times\mathbb{R}_+} \big(1-\beta(z,t)\big)\sqrt{t}h(z)\tau(t)dzdt}{1-\int_{X\times\mathbb{R}_+} g(\beta(z)z\sqrt{t})h(z)\tau(t)dzdt}. \tag{11}$$

where $\beta(t) := \int_0^t \alpha(t)dt = \frac{1}{t}\int_0^t \beta(t)dt$ directly derives from $\alpha$. The value of $\beta(t) \in [0,1]$ captures the proportion of time player $x$ spent on the search activity until age $t$. The planner's problem can be expressed in the following way.

$$\begin{aligned} W^{\mathrm{SP}} \quad = \quad & \max_{\alpha\in\mathcal{A}'} \quad \mathbb{E}\Big(\int_X\int_{\mathbb{R}_+} u(p(x,t))\ \tau(t)dt\ f(z)dz\ \Big|\ \alpha\Big), \\ & \text{sub to} \quad \eta_s = \eta_s(\alpha) \text{ as in Equation 11.} \end{aligned} \tag{12}$$

To begin, notice that the set of values that the planner can achieve under $\mathcal{A}$, the set of allocation functions from previous section, is also achievable under $\mathcal{A}'$. Indeed, these allocations depend, not only on type, but also on time. This immediately suggests that for all frictional communication technologies, the equilibrium allocation is inefficient also in this stronger, *ad interim*, sense. In the next proposition we show that not only this is the case, but the allocation becomes efficient even in the *ad interim* sense when the communication technology is maximally informative.

**Proposition 7.** *Generically, the equilibrium allocation is* ad interim *inefficient. In particular, it is efficient if and only if the communication technology is maximally informative.*

This result stresses once again how special the extreme case of $g = \max\{\mathcal{G}, \geq\}$ is. When players are able to observe each other posteriors, the equilibrium goes from being inefficient in the weak *ex ante* sense, to being efficient in the stronger *ad interim* sense. When $g$ is not maximal, the equilibrium allocation is inefficient not only from a quantitative point of view but also from a qualitative one. Whenever there are communication frictions, the planner would like to modify the equilibrium allocation in two different dimensions: types and time.

**Corollary 3.** *The planner solution is characterized in the following way. There exists thresholds $0 < x_1 \leq x_2 \leq \infty$ such that for $x \in X$,*

- *If $x < x_1$, the player is allocated to work, independently of time.*
- *If $x_1 < x < x_2$, the player is switched between work and search as a function her posterior beliefs.*

– *If $x_2 < x$, the player is allocated to search, independently of time*

The planner trades off two principal forces. On the one hand, she wants to allocate players to those activities that maximize their current individual gains. To do so, she chooses the activity that induces the highest posterior variance. However, the variance itself depends on $p(x,t)$. The idea is that when $p(x,t)$ is very close to 1 (or equivalently 0) the gains from learning more are small especially if compared with those of the other activity. Vice versa when $p(x,t)$ is very close to $\frac{1}{2}$ gains from learning are very high. From a social point of view, instead, the planner internalizes the net effect that every player induces on $\eta_s$. There exists a group of types for which it is individually optimal to search but contribute to $\eta_s$ negatively in relative terms. The planner finds optimal to switch these types back and forth between work and search as a function of how informed they are. The idea is that, when a player is poorly informed, individual gains from learning are high. When instead she is very informed, these gains are negligible. In the first case, this player would be allocated to search, so to learn quick, and would be switch back to work when she build a sufficient stock of information.

# 7. Social Influence and Public Opinion

In this section, we use our model to assess how resilient a society is against external manipulations of the information process for a small share of players. In particular, we study how this resilience depends on the structure of social connections, especially as the society becomes more connected. We think of "manipulations" as tweaks in the meeting technology $h$. These tweaks are such that the targeted group of players is consistently exposed to biased information. These manipulations happen under a regime of unawareness, that is, the society commonly believes that $h$ is unbiased. Although somewhat non-standard, we believe this assumption to be extremely descriptive of how these kinds of manipulations could happen in the real world. Ultimately, we are interested in understanding how difficult it is to influence public opinion by manipulating the information of a relatively small group of players, which group is optimal to target and, finally, how this depend on the level of connectivity of such society.

In this application, we assume that the meeting algorithm is manipulated for a mass $\delta$ of players in the society, with $\delta \in (0,1)$. The manipulation is implemented in a simple way. The manipulator selects a *target* type $\bar{x} \in X$. All types in the $\delta$-neighborhood of this type $\bar{x}$ will have their meeting technology manipulated in the following way.[19] When the searching for $dt$ period of time, these players receive an aggregate signal, $\pi_s(x)$, distributed normally

[19]The $\delta$-neighbor of $\bar{x}$ is defined as the centered interval that solves $F([\bar{x}-\epsilon, \bar{x}+\epsilon]) = \delta$.

with mean $(\eta_s+b)x\theta dt$ and variance $dt$. The parameter $b \in \mathbb{R}$ is the *size* of bias and assumed to be exogenous. Without loss of generality, we will assume $b = 1$. We can interpret this as a filtering of the signals that would otherwise be available to player $x$. That is, the meeting technology is tweaked in such a way that signals that are biased in one direction are more likely to be observed by the player. Notice that the more connected a type is, the stronger is going to be the effect of the bias, as $b$ is multiplied by $x$. Our goal is to calculate how biasing the meeting technology for these players affects overall public opinion.

Our model offers natural tools to evaluate the effects of these manipulations. In Section 3.3, we showed that the evolution of posterior beliefs for players who learn from others can be described in terms of the following Brownian motion:

$$\Gamma(x,t) = \eta_s xt\theta + B(t) \sim \mathcal{N}(\eta_s xt\theta, t)$$

This implies that, values for $\eta_w$ and $\eta_s$, associated with working and searching, respectively, together with the distribution of types $f$, pin down the entire distribution of posterior beliefs for the society. In fact, we can capture how manipulations in the meeting technology affect public opinion by studying their impact on $\eta_s$. Then we can combine changes in $\eta_s$ with their effects on the evolution of beliefs for the society, as specified in the equation above. As a simple example of this, we already know that $\eta_s$ does not affect beliefs of the working population. Therefore, manipulations of $h$ are bound to be less disruptive the higher is the share of players working.

Let $\tilde{\eta}_s(\bar{x})$ be the altered $\eta_s$ that follows from manipulating $h$ for all players in the $\delta$-neighbor of $\bar{x}$. Denote $\tilde{\Gamma}(x,t) := \tilde{\eta}_s xt\theta + B(t)$ the corresponding altered information process for a type $x$ who searches. Therefore, the impact on public opinions that follows from a manipulation of the meeting algorithm for a $\delta$-neighbor of $\bar{x} \in X$ can be defined as the expected aggregate deviation between the manipulated $\tilde{\Gamma}(x,t)$ and the original $\Gamma(x,t)$:

$$I(\bar{x},\delta) := \mathbb{E}\left( \int_{X\times\mathbb{R}_+} \big(\tilde{\Gamma}(x,t) - \Gamma(x,t)\big) f(z)\tau(t)dzdt \;\Big|\; \theta \right)$$

This discrepancy can be interpreted as the average distortion in beliefs that the manipulation has induced. Clearly, $I(\bar{x},\delta)$ depends on how types in the $\delta$-neighbor of player $\bar{x}$ can affect the opinion of other players. Therefore, it is natural to think of $I(\bar{x},\delta)$ as a measure of their influence on the rest of the society. Following this logic, we can transform $I(\bar{x},\delta)$ into a measure of *influence* for any specific type $x$.

**Definition 5.** *The* **social influence** *of type $x \in X$ is defined as $\iota(x) := \lim_{\delta\to 0} \frac{1}{\delta} I(x,\delta)$.*

The study of public opinion manipulations led us to the definition of a measure of influence. This is a natural outcome. In fact, how susceptible a society is to external manipulation is a function of the relative influence of its components. The stronger the influence a single player exerts on the whole society, the easier its manipulation. The next result fully characterizes our measure of influence.

**Proposition 8.** *Fix $f \in \mathcal{F}$ and $g \in \mathcal{G}$ and let $(x^\star, \eta_s)$ be stationary equilibrium. The social influence exerted by a searcher of type $x$ is given by*

$$\iota(x) = \frac{x\tilde{g}(x)\eta_s}{c\eta_w}\frac{1 - H(x^\star)}{H(x^\star)}.$$

Proposition 8 shows how the different forces in our model jointly contribute to determining the influence of any given type. The equilibrium $(x^\star, \eta_s)$, the meeting function $h$, the communication frictions $g$, a player's type, all these ingredients affect $\iota(x)$ in non trivial ways. To begin with, workers' influence is zero, as altering their meeting algorithm does not affect the information they receive. Other players do not exert influence on them. Therefore, targeting workers to manipulate public opinion would result in no manipulation whatsoever. Searchers instead are manipulable, as their information process depends on $h$. First, the higher their type, the stronger their influence. This is because, they attract more players to them and, thus, their impact on everybody's opinion is particularly important. Second, the higher their type, the higher is $\frac{\tilde{g}(x)}{c}$. This is because they also collect information at a faster rate, relative to workers, as they also meet others at a higher rate. In a sense, they make use of the tweaked meeting function $h$ more than others. Finally, and perhaps most importantly, the influence of a players also depend on equilibrium variables. In particular, $\iota(x)$ depends on two important equilibrium objects:

*The Relative Speed of Learning.* The ratio $\frac{\eta_s}{\eta_w}$ can be thought of as the relative speed at which searchers learn as compared to workers. The higher $\eta_s$ the faster opinion spread out the stronger is the influence of any given searcher.

*Amplification effect.* As discussed in Section 5.2, information in our model can be thought of being exchanged from player-to-player, in chains that can reach arbitrary length. These chains necessarily start with a worker, the player who first seeded the information in the society. Their length is important for equilibrium purposes. The idea is that the longer the chain, the more its original signal was distorted under a potentially frictional communication technology $g$ distorted the original signal. Therefore, the average length of a chain affects the expected quality of social information. It is natural, then, to find in our measure of influence the term $\frac{1}{H(x^\star)}$, the mean of a geometric distribution with parameter $H(x^\star)$. In particular, when $x^\star$ decreases, the expected length of a chain increases and so does the influence of player $x$. This intuition is clear. When $x^\star$ decreases, more players are searching over a total stock of information that became smaller. In such a context, manipulating the information for a given player has amplified effects on the whole society. The biased information rebounds among searchers many more times than it used and therefore it has the ability to influence more players in the society.

An important implication of this analysis is that any change in $g$ or $f$ leading to an increase in $\eta_s$, also make the society less resilient to external manipulations. This observation is captured in the next result.

**Corollary 4.** *Let $f, f' \in \mathcal{F}$ be such that $f'$ is more connected than $f$. The increase in connectivity either decreases the quality of social information $\eta_s$ or makes the society less resilient to external manipulation.*

We observe that societies that are highly effective at aggregating and diffusing information also happen to be societies that are particularly susceptible to manipulations. These societies are less reliant on work, precisely because they are efficient at the diffusion of information. However, this situation creates a weak spot. As the influence of each type increases in this society, it becomes easier to shift public opinion by manipulating the learning process for an increasingly smaller share of players in the population.

# 8. Discussion

In this section, we discuss in more depth some of our assumptions, their generalization and extension.

## 8.1. Communication Technology with Finite Capacity

In our model, the communication technology $g$ has the power to impose frictions in the peer-to-peer communications among searchers. In particular, it does not apply to workers. The idea behind this assumption is that workers, by virtue of their activity, are perfectly able to communicate the information they themselves have produced. This captures the idea that it ought to be more informative to receive information directly from its original source as opposed to receiving it from someone who herself got it for someone else (who in turn may have collected it from someone else, and so on). We do so as this assumption makes the main tensions introduced by communication frictions in this model more transparent and straightforward. Still, most of our results extend to the more general case in which the communication frictions apply *equally* to everyone independently of their activity. In particular, in Appendix B we discuss the variation of our model in which we apply the communication technology to all players in the economy. For our results to go through, we impose more stringent requirements on $g$ (Definition 1). In particular, we replace continuity of $g$ with a *concavity* assumption. Concavity captures the idea of a *finite capacity* communication channel. This idea goes back to Shannon (1948) and it is now a standard tool in information theory. In recent years, finite capacity communication channels have been used and discussed also in economics as a modeling tool for rational inattention (see the work of Sims (2003), Steiner et al. (2016) and Jung et al. (2016), among others). The idea is that the more *bits* of information one collects the more laborious their communication is going to be. Consider the problem of choosing between $y$ signals of high precisions $\eta_w$ versus receiving $y' > y$ signals of lower precision $\eta_s$. Suppose that these numbers are such that when $g = \mathrm{id}_{\mathbb{R}_+}$, a Bayesian agent would prefer to receive $y'$-many signals of lower precision. Under a communication channel with finite capacity, $g(y) < y$, some of these signals are going to be lost. There always exists a $g$ concave enough such that a Bayesian player would strictly prefer reeving fewer signals $g(y)$ of lower precision $\eta_w$, rather than more signals $g(y') > g(y)$ of higher precision $\eta_s$.

A concave $g$ that applies equally to all players is bound to induce the same sort of distortions like the ones we analyzed in this paper. In expectation, those who search in equilibrium collect signals at a higher intensity than anybody who works. Therefore, their stock of information will be hit more severely by the concavity of $g$. This introduces the same sort of "primacy of the original source," a wedge between first and second-hand information, similar to the one discussed in Section 3. When $g$ is increasing and concave, more informed players are still more informative to meet. However, the more signals a player has collected, the stronger the implied distortion. In our model $c := \mathbb{E}(\sqrt{t})$ represents a measure of the expected number of signals a worker has collected upon meeting. When the distortions apply to workers, as well as searchers, our model will change in that $c = \int g(\sqrt{t})\tau(t)dt =: \tilde{g}(1)$. The trade-off in the social contribution of a "marginal" searcher $x$ becomes $\eta_w\tilde{g}(1) - \eta_s\tilde{g}(x)$, which is deemed to be negative for types $x \in [x^\star, \hat{x}]$.[20] Interestingly, in this more general specification of our model, concavity of $g$ is not strictly speaking necessary for Theorem 1, but rather for uniqueness in Proposition 1, on which the Theorem builds.

## 8.2. Stationary and Non-Stationary Equilibria

Stationarity of the dynamic environment is one of the working assumption in our model. In our game, players enter and leave at stochastic times. The outflow of "knowledge" connected with players' departure guarantees that the amount of information in the economy is bounded and, in fact, stable on aggregate. Undoubtedly, the focus on stationary equilibria makes the model tractable and the comparison of equilibria as $f$ shifts stochastically a feasible

[20]Where these two threshold $x^\star$ and $\hat{x}$ are now computed using $c = \int g(\sqrt{t})\tau(t)dt =: \tilde{g}(1)$.

exercise. In Appendix C, we discuss the dynamic equilibria of this model. There is an initial time $t = 0$ at which a unit mass of players is born and shares a common prior. As time enfolds, new players enter the economy, while older players may leave. Our analysis leads to three conclusions. First, the equilibrium takes the form of a non-linear second-order differential equation, the solution of which determines the "information path" for $\eta_s(t)$, now a time-dependent equilibrium object. The existence and uniqueness of such equilibria can be investigated invoking Picard-Lindelöf theorem and transforming the equilibrium condition in a system of first-order ODEs, whenever possible. Second, we show that any information path of a given dynamic equilibrium necessarily converges to the stationary equilibrium defined in Section 4. Third, we show that players will monotonically transit *from* work *to* search, as the $\eta_s(t)$ increases. These dynamics are intuitive. At time zero, players share a common prior and therefore cannot learn anything from each other. The time they spend working builds a stock of information, which makes $\eta_s(t)$ increase. Highly connected types will then find optimal to switch to search.

# 9. Conclusion

In this paper, we introduced a simple and versatile model of frictional learning to study the equilibrium production of information and its diffusion in a large connected society. Players in our model face a basic tension between learning by doing (*work*) and learning from others (*search*). Learning is *frictional* as we explicitly allow for both search and communication frictions. Under a frictional communication technology, we assumed that any peer-to-peer information exchange could entail some loss of information. In the unique stationary equilibrium of our game, all existing information is originally produced by some worker. Players who search do not contribute to the production of "new" information. Nevertheless, these players exert a critical social role in enabling information that would otherwise remain local to be aggregated and diffused to different parts of the society.

Our main contribution is to formally identify the inefficiencies that a highly connected society is capable of setting in motion. We show that, generically, the equilibrium of our game is characterized by *excessive searching*. In this situation, there exists a region of players, which are only moderately connected, who decide to search, even when, from a social perspective, their diffusion abilities do not compensate for the implicit loss in information that they could have produced. These players are effectively responsible for injecting extra noise into the society. This noise is then collected by other searchers and, as it travels from one searcher to another, it amplifies throughout the whole society. We formalize these effects in three ways. First, we show that a more connected society may not be a society with better information. Second, we show how the equilibrium allocation of activities can dramatically differ from the one a benevolent social planner would choose. Finally, we show how increasing connectivity can make the society less resilient to external manipulation of public opinion.

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# A. Proofs.

**Lemma A1.** *Signals $\pi_s$ are independent across time and types.*

*Proof.* We show that independence holds along any sequence of discrete-time models that converges to the continuous-time model. Fix any $\Delta > 0$. For any player $x \in X$, let $M_1(x,t) \subset X$ denote the set of players she has met before period $t$. Denote $M_2(x,t)$ the set of players that were before $t$ met by those players in $M_1(x,t)$, and so on. Define $M(x,t) := \cup_{i\geq 1} M_i(x,t)$. It is enough to show that, for any $x', x'' \in X$, the sets $M(x',t)$ and $M(x'',t)$ are almost surely disjoint. In such case, the information of player $x'$ is independent from that of player $x''$. When time is finite, the cardinality of $M_i(x,t)$ is finite for all $i \in \mathbb{N}$. Therefore, $M(x,t)$ is countably infinite, for all $x$ and $t$, and so is $M(x',t) \cap M(x'',t)$. Since the probability measure underlying the meeting density $h$ is atom-less, the measure of any countable set is zero. Therefore, we conclude that $M(x',t)$ and $M(x'',t)$ are almost surely disjointed. $\square$

**Lemma A2.** *Following a choice of $\alpha_t$, posterior belief $p_t$ evolves according to:*

$$dp_t \sim \mathcal{N}\Big(0, 4dt\big(p_t(1-p_t)\big)^2\big((1-\alpha_t)^2\eta_w^2 + \alpha_t^2 x^2 \eta_s^2\big)\Big).$$

*Proof.* Fix a time $t$ and a posterior belief $p_t$. To begin with, suppose we want to compute $dp_t$ following a generic signal $ds \sim \mathcal{N}(\mu\theta dt, \sigma^2 dt)$ for some $\mu$ and $\sigma$. From Bayes' rule,

$$p_{t+dt} = \frac{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2}}{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} + (1-p_t)e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}}.$$

Therefore,

$$\begin{aligned} dp_t &= p_{t+dt} - p_t \\ &= \frac{p_t(1-p_t)\left(e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} - e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}\right)}{p_t e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} + (1-p_t)e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\underline{\theta}dt\right)^2}} \end{aligned}$$

Taking the squares and using the fact that $ds_t^2 = \sigma^2 dt$, the exponential terms can can be simplified. For example,

$$e^{-\frac{1}{2\sigma^2 dt}\left(ds_t - \mu\bar{\theta}dt\right)^2} = e^{-\frac{1}{2}} e^{\bar{\theta}\frac{\mu}{\sigma^2}ds_t - \frac{1}{2}\frac{\mu^2}{\sigma^2}dt}.$$

Moreover, notice that, by a Taylor expansion,

$$e^{\bar{\theta}\frac{\mu}{\sigma^2}ds_t - \frac{1}{2}\frac{\mu^2}{\sigma^2}dt} = 1 + \bar{\theta}\frac{\mu}{\sigma^2}ds_t$$

where we neglected all terms of order $dt^{\frac{3}{2}}$ and higher. Putting all this together in the expression for $dp_t$:

$$
\begin{aligned}
dp(x,t) &= \frac{2p_t(1-p_t)\frac{\mu}{\sigma^2}ds_t}{1+(2p_t-1)\frac{\mu}{\sigma^2}ds_t} \\
&= 2p_t(1-p_t)\frac{\mu}{\sigma^2}ds_t\big(1-(2p_t-1)\frac{\mu}{\sigma^2}ds_t\big) \\
&= 2p_t(1-p_t)\big(\frac{\mu}{\sigma^2}ds_t-(2p_t-1)\frac{\mu^2}{\sigma^2}dt\big)
\end{aligned}
$$

where we used the approximation $(1+x)^{-1} \approx 1-x$. Inside the expression in the last line, there is the random variable $\frac{\mu}{\sigma^2}ds_t - (2p_t-1)\frac{\mu^2}{\sigma^2}dt$. Unconditional on $\theta$, its expectation is zero and its variance $\frac{\mu^2}{\sigma^2}dt$. It inherits its distribution from $ds_t$. Therefore,

$$dp_t \sim \mathcal{N}\Big(0, \frac{4\mu^2 dt}{\sigma^2}\big(p_t(1-p_t)\big)^2\Big).$$

Now suppose that in the interval $[t, t+dt)$, a player of type $x$ chooses $\alpha_t$. The evolution of $p_t$ can be written as $dp_t = (1-\alpha_t)dp_{t,w} + \alpha_t dp_{t,S}$, where $dp_{t,w}$ and $dp_{t,s}$ are the evolutions of $dp_t$ in case the signal received is $d\pi_w$ and $d\pi_s$, respectively. Notice that since $d\pi_w$ or $d\pi_s$ are independent, so are $dp_{t,w}$ and $dp_{t,s}$. Therefore, using the result proven above:

$$dp_t = (1-\alpha_t)dp_{t,w} + \alpha_t dp_{t,s} \sim \mathcal{N}\Big(0, 4dt\big(p_t(1-p_t)\big)^2\big((1-\alpha_t)^2\eta_w^2 + \alpha_t^2\eta_s^2x^2\big)\Big).$$

Rearranging gives us the result. □

**Lemma A3.** *The HBJ equation of the agent's problem is:*

$$v(p_t) = \max_{\alpha_t \in [0,1]} u(p_t) + \frac{2}{\delta + r}p_t^2(1-p_t)^2 v''(p)Q(\alpha_t),$$

*where* $Q(\alpha_t) = (1-\alpha_t)^2\eta_w^2 + \alpha_t^2 x^2 \eta_s^2$.

*Proof.* We can approximate $v$ with a second-order Taylor expansion:

$$\mathbb{E}(v(p_{t+dt})|\alpha_t) \approx \mathbb{E}\Big(v(p_t) + v'(p_t)dp_t + \frac{1}{2}v''(p_t)(dp_t)^2 \;\Big|\; \alpha_t\Big)$$

where $dp_t$ is a random variable that depends on $\alpha_t$. By Lemma A2, we know the distribution of $dp_t$ and we can write

$$\mathbb{E}(v(p_{t+dt})|\alpha_t) \approx v(p_t) + v''(p)2p_t^2(1-p_t)^2Q(\alpha_t)dt,$$

since $\mathbb{E}(dp_t) = 0$, by Lemma A2. Therefore, plugging this back into Equation 1 gives:

$$v(p_t) = (r+\delta)u(p_t)dt + \big(1-(r+\delta)dt\big)\Big(v(p_t) + v''(p)2p_t^2(1-p_t)^2Q(\alpha_t)dt\Big)$$

where we used the approximation $e^{-(r+\delta)t} \approx 1-(r+\delta)dt$. Rearranging and ignoring terms $dt^2$, gives us the result. □

**Proof of Lemma 1**: From Lemmas A2 and A3, we have that

$$v(p_t) = \max_{\alpha_t\in[0,1]} u(p_t)+\frac{2}{r+\delta}p_t^2(1-p_t)^2 v''(p_t)Q(\alpha_t) = u(p_t)+\frac{2}{r+\delta}p_t^2(1-p_t)^2 v''(p_t) \max_{\alpha_t\in[0,1]} Q(\alpha_t),$$

where $Q(\alpha_t) := (1-\alpha_t)^2\eta_w^2 + \alpha_t^2 x^2 \eta_s$. If $v'' > 0$, the problem is maximized with $\alpha^\star = 1$, if $x^2\eta_s > \eta_w^2$, and with $\alpha_t^\star = 0$, otherwise. Rearranging gives $x^\star = \eta_w/\eta_s$. We are therefore left to show that $v$ is convex. To do so, we solve the ODE.

Rewrite the ODE can be written as $\bar{v}(p) = p + Kp^2(1-p)^2\bar{v}''(p)$, for $p \geq \frac{1}{2}$ and $\underline{v}(p) = 1-p+Kp^2(1-p)^2\underline{v}''(p)$ otherwise, where we set $K := \frac{2Q(\alpha_t^\star)}{r+\delta}$ and drop all $t$ subscripts. Let $\zeta := \frac{\sqrt{1+\frac{4}{K}}}{2}$ and $a := \frac{1}{2} - \zeta < 0$ and $b := \frac{1}{2} + \zeta > 0$. It can be verified by substitution that equations

$$\begin{aligned} \bar{v}(p) = &\quad p + c_1 p^a(1-p)^b + c_2 p^b (1-p)^a \\ \underline{v}(p) = &\ 1-p+\tilde{c}_1 p^a (1-p)^b + \tilde{c}_2 p^b (1-p)^a \end{aligned} \tag{A.1}$$

are generic solutions of their respective ODE. To pin down the values of $c_1$, $c_2$, $\tilde{c}_1$, and $\tilde{c}_2$, we invoke three properties that $v$ must posses: (1) symmetry around $p = \frac{1}{2}$, (2) smooth pasting at $\frac{1}{2}$, and (3) and boundaries conditions.

(1) The problem faced by the agent is symmetric in the sense that the flow payoff she optimally respond to beliefs symmetric around $\frac{1}{2}$, e.g. $\bar{p} = \frac{1}{2} - \varepsilon$ or $\underline{p} = \frac{1}{2} + \varepsilon$, when $\varepsilon \in [0, 1/2]$, are the same. Thus, also the corresponding values $\underline{v}(\bar{p})$ and $\bar{v}(\underline{p})$ need to match. We require that for all such $\varepsilon$, $\bar{v}(\underline{p}) = \underline{v}(\bar{p})$. Notice that $1 - \bar{p} = \underline{p}$. Thus, symmetry implies that

$$\underline{p}^a \bar{p}^b (c_1 - \tilde{c}_2) = \underline{p}^b \bar{p}^a (c_2 - \tilde{c}_1)$$

which is true for all $\varepsilon \in [0, 1/2]$ if and only if $c_1 = \tilde{c}_2$ and $c_2 = \tilde{c}_1$.

(2) Next, we impose smooth pasting at $p = \frac{1}{2}$. This requires that $\bar{v}'(p^\star) = \underline{v}'(p^\star)$. Computing the derivatives and evaluating them at $p$ gives

$$\bar{v}'(p^\star) = \ 1 + c_1\big(a - p^\star\big) + c_2\big(b - p^\star\big) \ = \ 1 + c_1(-\zeta) + c_2\zeta$$

$$\underline{v}'(p^\star) = -1 + \tilde{c}_1\big(a - p^\star\big) + \tilde{c}_2\big(b - p^\star\big) = -1 + \tilde{c}_1(-\zeta) + \tilde{c}_2\zeta$$

hence

$$2 + \zeta(\tilde{c}_1 + c_2) = \zeta(c_1 + \tilde{c}_2).$$

(3) Finally, at the boundaries $p \in \{0, 1\}$, the agent is certain that the state is either $1$ or $-1$. Thus, the value of the problem must necessarily be equal to 1. Let $\bar{v}(1) = 1$. Then, since $a < 0$,

$$\bar{v}(1) = 1 + c_1 1^a 0^b + c_2 1^b 0^a = 1 + c_1 0 + c_2 \infty = 1.$$

Thus, $c_2 = 0$ is the only constant that can guarantee $\bar{v}(1) = 1$. A similar reasoning at $p = 0$ gives us $\tilde{c}_1 = 0$.

Putting these three conditions together we get the system

$$\begin{cases} c_1 = \tilde{c}_2 \text{ and } c_2 = \tilde{c}_1 \\ 2 + \zeta(\tilde{c}_1 + c_2) = \zeta(c_1 + \tilde{c}_2) \\ \tilde{c}_1 = c_2 = 0 \end{cases} \quad \Rightarrow \quad \begin{matrix} c_1 = \tilde{c}_2 = \frac{1}{\zeta} = 2\frac{\tilde{\mu}\sqrt{x}}{\sqrt{\tilde{\mu}^2 x + 2(r+\delta)}} \\ \tilde{c}_1 = c_2 = 0 \end{matrix}$$

To conclude, the value function is:

$$v(p) = \begin{cases} p + cp^a(1-p)^b & \text{if } p \geq \frac{1}{2} \\ 1 - p + cp^b(1-p)^a & \text{else.} \end{cases}$$

where $c > 0$, $a < 0$ and $b > 0$. For $p > \frac{1}{2}$, its second derivative is

$$v''(p) = cp^a(1-p)^b\Big(\frac{a(a-1)}{p^2} + \frac{b(b-1)}{(1-p)^2}\Big) = -cp^a(1-p)^b ab\Big(\frac{1}{p^2} + \frac{1}{(1-p)^2}\Big) > 0$$

where we used $a - 1 = -b$ and $b - 1 = -1$ and $ab < 0$. In a specular way, one can show that $v''(p) > 0$ for $p \leq \frac{1}{2}$. □

**Lemma A4.** *Fix a player of type $x$ and a time $t$. Let $\pi(x,t)$ be the stock of information and $p(x,t)$ the one for posterior beliefs. There exists a one-to map $\xi : \mathbb{R} \to \mathbb{R}$, independent of $x$ and $t$, such that $p(x,t) = \xi(\pi(x,t))$.*

*Proof.* Define the log-likelihood ratio of posterior beliefs as follow:

$$z(x,t) := \ln \frac{p(x,t)}{1 - p(x,t)} = \ln \frac{p_0 \phi(\pi(x,t)|\theta = 1)}{(1 - p_0)\phi(\pi(x,t)|\theta = -1)} = z(x,0) + \ln \frac{\phi(\pi(x,t)|\theta = 1)}{\phi(\pi(x,t)|\theta = -1)}$$

where $\phi(\pi(x,t)|\theta)$ is the probability density of finding $\pi(x,t)$ at the given level, conditional on the state being $\theta$. Notice that,

$$z(x,t) = K + 2\eta_a \pi(x,t)$$

where $K$ is a constant and $\eta_a = \eta_w$ if $x$ is a worker and $\eta_s$ otherwise. We conclude that, the process for $p(x,t)$ is a one-to-one transformation of $z(x,t)$, which, in turn, is a linear one transformation of $\pi(x,t)$. □

**Lemma A5.** *Bayes consistency $\mathbb{E}(\pi_s) = \eta_s \theta$ implies a unique positive solution given by*

$$\eta_s = \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}(z)h(z)dz} \geq 0.$$

*Proof.* From Equation 4, we have that

$$\eta_s = \eta_w H(x) \int \sqrt{t}\tau(t)dt + \eta_s \int_x \Big(\int g(z\sqrt{t})\tau(t)dt\Big) h(z)dz. \tag{A.2}$$

Denote $\tilde{g}(z) = \mathbb{E}_\tau(g(z\sqrt{t}))$ and $c = \mathbb{E}_\tau(\sqrt{t})$ and rearranging we get the result. We are left to show that $\eta_s \geq 0$ for all $x$. Notice that $1 - \int_x \tilde{g}(z)h(z)dz$ is increasing in $x$. Moreover, for any $g \in \mathcal{G}$ such that $g \leq \mathrm{id}_{\mathbb{R}_+}$, we have

$$\tilde{g}(z) := \int g(z\sqrt{t})\tau(t)dt \leq \int z\sqrt{t}\,\tau(t)dt.$$

Therefore for all $x \in X$ and $g \in \mathcal{G}$, we have

$$1 - \int_x \tilde{g}(z)h(z)dz \geq 1 - \int_0 \Big( \int z\sqrt{t}\,\tau(t)dt \Big) h(z)dz.$$

Therefore, in order to ensure $\eta_s \geq 0$, it is enough to show that the right hand side of this equation is positive. However, notice that

$$\int_0 \Big( \int z\sqrt{t}\,\tau(t)dt \Big) h(z)dz = \mathbb{E}_h(z)\mathbb{E}_\tau(\sqrt{t}).$$

By definition of $f$, we have that $\mathbb{E}_h(z) := \mathbb{E}_f(z^2)/\mathbb{E}_f(z)$. Finally, Assumption 1 ensures that $\mathbb{E}_h(z)\mathbb{E}_\tau(\sqrt{t}) \leq 1$.

**Proof of Proposition 1.** (*Existence*) Equation (6) can be rewritten as

$$\Phi(x) := cxH(x) + \int_x \tilde{g}(z)h(z)dz = 1. \tag{A.3}$$

First we show that the $\Phi(x)$ crosses 1 at least once. Notice that at $x = 0$, $\int_0 \tilde{g}(z)h(z)dz \leq 1$, as shown in the proof of Lemma A5. Therefore $\Phi(0) \leq 1$. Vice versa, $\lim_{x\to\infty} Q(x) = \infty$. Next, we show that $\Phi$ is continuous, as being the sum and products of continuous functions. First, notice that $H$ is absolutely continuous, as it admits a density $h$. Second, for any sequence $(x_n)$ such that $x_n \to x$, the sequence $\int_{x_n} \tilde{g}(z)h(z)dz$ is a positive and non-increasing. Every monotonic and bounded sequence admits a limit point, from which we conclude also $\int_x \tilde{g}(z)h(z)dz$ is continuous. Continuity of $\Phi$, via a straightforward application of Bolzano's Theorem, guarantees the existence of a crossing point $\Phi(x) = 1$.

*(Uniqueness)*. To show the fixed-point is unique, we show that $\Phi$ is strictly increasing. Fix $x' > x$. We have

$$\begin{aligned}\Phi(x') - \Phi(x) &= c\big[x'H(x') - xH(x)\big] - \textstyle\int_x^{x'} \tilde{g}(z)h(z)dz \\ &\geq c\Big(\big[x'H(x') - xH(x)\big] - \textstyle\int_x^{x'} zh(z)dz\Big).\end{aligned}$$

Therefore, $\Phi(x') - \Phi(x) > 0$, as we wished to show. We conclude that the equilibrium is unique. □

**Proof of Proposition 2.** Fix any $g, g' \in \mathcal{G}$ with $g' \geq g$. We need to show that equilibrium $\eta_s$ under $g$ is lower than under the equilibrium $\eta_s$ under $g'$. To show this, we prove a stronger claim, which we will later use in the main text. Namely, let $\eta_s(x, g)$ be defined as in Equation

5, were we make explicit the dependence on $x$ and $g$. We show next that $\eta_s(x,g) \leq \eta_s(x,g')$, for all $x \in X$. We have

$$\eta_s(x,g) = \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}(z)h(z)dz} \leq \eta_w \frac{cH(x)}{1 - \int_x \tilde{g}'(z)h(z)dz} = \eta_s(x,g'),$$

since, for all $x$,

$$\tilde{g}'(x) := \int g'(x\sqrt{t})\tau(t)dt \geq \int g(x\sqrt{t})\tau(t)dt =: \tilde{g}(x).$$

Since the equation that defines individual rationality, $\eta_s = \eta_w/x$, is strictly decreasing in $x$, this proves our claim. $\square$

**Proof of Proposition 3.** First, notice that $\eta_s(x)$ as defined by Equation 5, has a maximum at

$$\frac{d\eta_s(x)}{dx} = 0 \quad \Rightarrow \quad 1 - \int_x \tilde{g}(z)h(z)dz = \tilde{g}(x)H(x).$$

From Equation A.3, the equilibrium $x^\star$ is pinned down by:

$$1 - \int_{x^\star} \tilde{g}(z)h(z)dz = cx^\star H(x^\star).$$

Notice that $\tilde{g}(x) \leq cx$, where the inequality is strict for all $g < \mathrm{id}_X$. Putting all together we conclude that, for all $g < \mathrm{id}_X$, at the respective equilibrium $x^\star$, $\frac{d\eta_s(x)}{dx} > 0$. Vice versa, when $g = \mathrm{id}_X$, $\frac{d\eta_s(x)}{dx} = 0$. $\square$

**Proof of Proposition 4** Let $(x^\star, \eta_s)$ be the equilibrium. Fix $\bar{x} \in X$ as suppose $\bar{x} < x^\star$. The information elasticity for this type can be computed as follows:

$$\varepsilon_w(\bar{x}) := \frac{d\eta_s/dc_{\bar{x}}}{\eta_s/c_{\bar{x}}} = \frac{\eta_w h(\bar{x})}{1 - \int_{x^\star} \tilde{g}(z)h(z)dz} \frac{c_{\bar{x}}}{\eta_s} = \frac{h(\bar{x})}{H(x^\star)} \frac{\eta_w c_{\bar{x}}}{\eta_w c}.$$

Since $\bar{x}$ is a worker $c_{\bar{x}} = c$. Moreover, by definition of $h$, we have $h(\bar{x}) = \bar{x}f(\bar{x})/\mathbb{E}_f(z)$. Therefore,

$$\varepsilon_w(\bar{x}) = c\eta_w \frac{\bar{x}}{H(x^\star)} \frac{f(\bar{x})}{c\eta_w \mathbb{E}_f(z)}.$$

Now suppose $\bar{x} \geq x^\star$. The information elasticity for this type can be computed as follows:

$$\varepsilon_s(\bar{x}) := \frac{d\eta_s/d\tilde{g}(\bar{x})}{\eta_s/\tilde{g}(\bar{x})} = \frac{c\eta_w H(x^\star)}{\left(1 - \int_{x^\star} \tilde{g}(z)h(z)dz\right)^2} h(\bar{x}) \frac{\tilde{g}(\bar{x})}{\eta_s} = \tilde{g}(\bar{x})\eta_s \frac{h(\bar{x})}{H(x^\star)} \frac{1}{c\eta_w}.$$

Using the definition of $h$,

$$\varepsilon_s(\bar{x}) = \tilde{g}(\bar{x})\eta_s \frac{\bar{x}}{H(x^\star)} \frac{f(\bar{x})}{c\eta_w \mathbb{E}_f(z)}.$$

From there, we can compute relative difference $\varepsilon_s(\bar{x}) - \varepsilon_w(\bar{x})$ and rearrange. $\square$

**Proof of Proposition 5.** $(ii) \Rightarrow (i)$. Let $g(y) = y$ for all $y \in \mathbb{R}_+$ and consider any $f' \trianglerighteq f$. It is straightforward to check that the latter implies $h' \trianglerighteq h$. The fixed-point map of Equation 6 can be rewritten as

$$x = \frac{1}{c} + \int_x (x - z)h(z)dz.$$

Moreover,

$$\int_x (x-z)h(z)dz = \int_X (x-z)\mathbb{1}_{\{z>x\}}(z)h(z)dz > \int_X (x-z)\mathbb{1}_{\{z>x\}}(z)h'(z)dz = \int_x (x-z)h'(z)dz,$$

by definition of FOSD and the fact that $(x - z)\mathbb{1}_{\{z>x\}}$ is non-increasing. Thus, letting $x^\star$ be the equilibrium under $f$,

$$x^\star = \frac{1}{c} + \int_{x^\star} (x^\star - z)h(z)dz > \frac{1}{c} + \int_{x^\star} (x^\star - z)h'(z)dz.$$

Notice that $\frac{d}{dx} \int_x (x - z)h(z) = 1 - H(x) > 0$. Therefore, the equilibrium is re-established at $f'$ by decreasing $x^\star$, therefore increasing $\eta_s$ as we wished to prove.

$(i) \Rightarrow (ii)$. This direction is equivalent to $\neg(ii) \Rightarrow \neg(i)$ which, however is the content of Observation 1. □

**Proof of Theorem 1.** For clarity, we divide the proof of these result in four Lemmas. To being, fix an increasing uniform sequence $(h_n)_{n\in N}$, fix $n' > n$ in $N$ and some $x \in X$. Denote:

$$h_\Delta(z, n) := h_{n'}(z) - h_n(z) \qquad \text{and} \qquad D(z) := \frac{h_\Delta(z, n')}{\int_x h_\Delta(z, n')dz} - \frac{h_\Delta(z, n)}{\int_x h_\Delta(z, n)dz}.$$

Notice that, since $h_{n'} \trianglerighteq h_n$, the function $h_\Delta(z, n)$ is single-crossing (SC)[21] in $z$ and integrate to 0. In fact,

$$h_\Delta(z, n) := h_{n'}(z) - h_n(z) = (\gamma(z) - 1)h_n(z),$$

where $\gamma(z) = h_{n'}(z)/h_n(z)$ is positive, non-decreasing and crosses 1, by definition of MLR. Since $h_n(z) > 0$, we have that $h_\Delta(z, n) \geq 0$ implies $h_\Delta(z', n) \geq 0$ for all $z' \geq z$. Moreover, $\int_0^\infty h_\Delta(z, n)dz = 1 - 1 = 0$. For this reason, together with the fact that $h_\Delta(z, n)$ is SC, we have that $\int_x h_\Delta(z, n)dz \geq 0$. Also, notice that $\int_x^\infty D(z)dz = 0$.

We begin by showing that $D(z)$ inherits the single-crossing property from $h_\Delta(z, n')$ and $h_\Delta(z, n)$.[22]

**Lemma A6.** *$D(z)$ is single-crossing in $z$ in the interval $[x, \infty)$.*

*Proof.* Since the SC property is preserved under scalar transformation, we prove that the function

$$D'(z) := h_\Delta(z, n') - \beta h_\Delta(z, n) \qquad \text{with} \quad \beta := \frac{\int_x h_\Delta(z, n')dz}{\int_x h_\Delta(z, n)dz} \geq 0$$

[21] A function $f : \mathbb{R} \to \mathbb{R}$ is single-crossing if $f(z) \geq 0$ implies $f(z') \geq 0$ for all $z' \geq z$.

[22] This is done in a very much similar spirit of Athey (2002) and, more closely, of Quah and Strulovici (2012).

is single-crossing in $z$ in the interval $[x,\infty)$. Let $D'(z) \geq 0$. We want to show that $D'(z) \geq 0$ for all $z' \geq z$. Note that, since $h_n$ and $h_{n'}$ belong to a uniform sequence, we have that $h_\Delta(z,n') \geq 0$ implies that $h_\Delta(z,n) \geq 0$. That is, $h_\Delta(z,n)$ crosses zero before $h_\Delta(z,n)$. Therefore, we have to consider only three cases: (1) when $h_\Delta(z,n'), h_\Delta(z,n) \geq 0$, (2) when $h_\Delta(z,n') \leq 0 \leq h_\Delta(z,n)$ and (3) $h_\Delta(z,n'), h_\Delta(z,n) \leq 0$.

(1) Suppose $h_\Delta(z,n'), h_\Delta(z,n) \geq 0$. Since $D'(z) \geq 0$, we have $h_\Delta(z,n') \geq \beta h_\Delta(z,n)$. Therefore,
$$\beta \leq \frac{h_\Delta(z,n')}{h_\Delta(z,n)} = \frac{h_{n''}(z) - h_{n'}(z)}{h_{n'}(z) - h_n(z)}.$$
which is increasing since the sequence is uniform (Definition 4). We conclude that for any $z' \geq z$, $D'(z') \geq 0$.

(2) Suppose $h_\Delta(z,n') < 0 \leq h_\Delta(z,n)$. This implies $D(z) < 0$, a contradiction.

(3) Finally, suppose $h_\Delta(z,n'), h_\Delta(z,n) \leq 0$. We will show that this is incompatible with $D'(z) \geq 0$. By way of contradiction, suppose that $D'(z) \geq 0$. Note that at the right-most boundary of this region, we have that $h_\Delta(z_0,n) = 0$ (since $h_\Delta(z,n)$ crosses zero first). Therefore, at $z_0$, we have that $D'(z_0) = h_\Delta(z_0,n') \leq 0$. By continuity, there must be a $z^\star \in [z, z_0]$, such that $D'(z^\star) = 0$. This implies that $\frac{h_\Delta(z^\star,n')}{h_\Delta(z^\star,n)} = \beta$. Since, the sequence is uniform, we have $\frac{h_\Delta(z,n')}{h_\Delta(z,n)} \leq \beta$. Now we use the definition of $\beta$.
$$\beta := \frac{\int_x h_\Delta(z,n')dz}{\int_x h_\Delta(z,n)dz} = \frac{\int^x h_\Delta(z,n')dz}{\int^x h_\Delta(z,n)dz} = \frac{\int^x h_\Delta(z,n')\frac{h_\Delta(z,n)}{h_\Delta(z,n)}dz}{\int^x h_\Delta(z,n)dz} < \frac{\int^x h_\Delta(z,n)\beta dz}{\int^x h_\Delta(z,n)dz} = \beta$$
This gives us the contradiction. In the second equality, we used that for any $n$ and $x$, $\int h_\Delta(z,n)dz = 0$. For the inequality, we used the fact that $x < z^\star$. This is automatically the case since we are trying to show $\mathcal{SC}$ of $D'$ on $[x,\infty)$.

This shows that $D'(z) \geq 0$ only in case (1), where we showed $D'(z') \geq 0$ for all $z' \geq z$. Therefore $D'$ is $\mathcal{SC}$ in the interval $[x,\infty)$ and so is $D$. □

Now that we have established the SCP for $D(z)$, we move to a second instrumental result, which builds on Lemma A6.

**Lemma A7.** *We have that* $\int_x m(z)D(z)dz \leq 0$.

*Proof.* By definition of $D(z)$, notice that $\int_x^\infty D(z)dz = 0$. Since $D(z)$ is $\mathcal{SC}$ in the interval $[x,\infty)$ (Lemma A6), we have that $\int_x^y D(z)dz \leq 0$ for any $y < \infty$. Integrating by parts:
$$\begin{aligned}\int_x m(z)D(z)dz &= m(z)\int_x^z D(y)dy\Big|_{z=x}^{z=\infty} - \int_x \Big(m'(z)\int_x^z D(y)dy\Big)dz \\ &= -\int_x \Big(m'(z)\int_x^z D(y)dy\Big)dz \leq 0.\end{aligned}$$

The second equality comes from the fact that $\int_x^\infty D(z)dz = \int_x^x D(z)dz = 0$. The inequality comes from $\int_x^y D(z)dz \leq 0$ and the fact that $m'(z) \leq 0$. To confirm the latter, recall that $m(z) := c^{-1}\big(cx - \tilde{g}(z)\big)$. Therefore, $m'(z) = -c^{-1}\int \sqrt{t}g'(z\sqrt{t})\tau(t)dt \leq 0$, by the fact that $g' \geq 0$ is increasing. (Definition 1) □

Finally, the last and most important of these instrumental results.

**Lemma A8.** *Fix $x \in X$ arbitrarily and consider an $\trianglerighteq$-increasing uniform sequence in $\mathcal{H}$. The functional $L : N \to \mathbb{R}$, defined as*

$$L(n) = \int_x m(z)h_n(z)dz,$$

*is quasi-concave in $n \in N$.*

*Proof.* To show this, it is enough to prove that, for $n'' \geq n' \geq n$,

$$L(n'') - L(n') \geq 0 \quad \Rightarrow \quad L(n') - L(n) \geq 0.$$

Notice that

$$0 \leq L(n'') - L(n') = \int_x m(z)\big(h_{n''}(z) - h_{n'}(z)\big)dz = \int_x m(z)h_\Delta(z, n')dz$$

Therefore, we need to show that, for any $n' \geq n$

$$\int_x m(z)h_\Delta(z, n')dz \geq 0 \quad \Rightarrow \quad \int_x m(z)h_\Delta(z, n)dz \geq 0.$$

Fix $n' \geq n$ and assume $\int_x m(z)h_\Delta(z, n')dz \geq 0$. As argued above, $\int_x h_\Delta(z, n')dz \geq 0$ (for any $n'$). Therefore,

$$\frac{\int_x m(z)h_\Delta(z, n')dz}{\int_x h_\Delta(z, n')dz} \geq 0.$$

Moreover,

$$\frac{\int_x m(z)h_\Delta(z, n')dz}{\int_x h_\Delta(z, n')dz} - \frac{\int_x m(z)h_\Delta(z, n)dz}{\int_x h_\Delta(z, n)dz} = \int_x m(z)D(z)dz \leq 0.$$

Thus,

$$0 \leq \frac{\int_x m(z)h_\Delta(z, n')dz}{\int_x h_\Delta(z, n')dz} \leq \frac{\int_x m(z)h_\Delta(z, n)dz}{\int_x h_\Delta(z, n)dz} \quad \Rightarrow \quad \int_x m(z)h_\Delta(z, n)dz \geq 0,$$

concluding the proof of Lemma A8. □

With this last result, we can finally provide the proof for Theorem 1.

Let $(h_n)_n$ be a $\trianglerighteq$-increasing uniform sequence in $\mathcal{H}$ and let $h_{n''} \trianglerighteq h_{n'} \trianglerighteq h_n$. Call $x_n$, $x_{n'}$ and $x_{n''}$ the fixed points of Equation (6) for $h_n$, $h_{n'}$ and $h_{n''}$, respectively. To show quasi-concavity we need to show that $x_{n'} \geq \min\{x_n, x_{n''}\}$. That is, we need to show: (Case 1) if

$x_{n'} \leq x_{n''}$, then $x_n \leq x_{n'}$, and (Case 2) if $x_n \geq x_{n'}$ then $x_{n'} \geq x_{n''}$. To begin, notice that, by Proposition 1, we know that the self-map in Equation (6) has a unique fixed point. Since $c > 0$, it must be the case that the function $\frac{1}{c} + L(x, h)$ crosses the function $x$ from *above*. Indeed,

$$\frac{1}{c} + L(0, n) = \frac{1}{c} + \frac{1}{c}\int_0^\infty (0 - \tilde{g}(z))h_n(z)dz = \frac{1}{c}\Big(1 - \int_0^\infty \tilde{g}(z)h_n(z)dz\Big) > 0.$$

We discuss Case 1 and Case 2 separately.

*Case 1.* Let $x_{n'} \leq x_{n''}$. Then, by the argument just made, it must be that $\frac{1}{c} + L(x_{n'}, n'') \geq \frac{1}{c} + L(x_{n'}, n') = x_{n'}$, otherwise we would contradict $x_{n'} \leq x_{n''}$. This implies that $L(x_{n'}, n'') \geq L(x_{n'}, n')$. By Lemma A8, we know $L$ is quasi-concave in $h$. That is, $L(x_{n'}, n'') \geq L(x_{n'}, n')$ implies that $L(x_{n'}, n') \geq L(x_{n'}, n)$. Again, by the single crossing argument above, this implies that $x_n \leq x_{n'}$.

*Case 2.* This case mimics the previous one. Let $x_n \geq x_{n'}$. We know that this implies $L(x_{n'}, n) \geq L(x_{n'}, n')$. By Lemma A8, we get that $L(x_{n'}, n') \geq L(x_{n'}, n'')$ and conclude that $x_{n'} \geq x_{n''}$.

The two cases above showed that $x_{n'} \geq \min\{x_n, x_{n''}\}$. Since $n'' \geq n' \geq n$ were arbitrary, we conclude that the fixed point $x$ of Equation 6 is quasi-concave in $n$. By the equation that defines individual rationality this implies that $\eta_s$ is quasi-convex, concluding the proof of Theorem 1. □

**Lemma A9.** *Fix a type $z \in X$. The derivative of $W^{SP_1}(\alpha)$ with respect to a marginal increase in $\alpha(z)$ is:*

$$W^{SP_1}_{\alpha(z)}(\alpha) = f(z)\Big(v_s(z, \eta_s) - v_w(\eta_w) + z\Big(\eta_s\tilde{g}(z) - \eta_w c\Big)K\Big),$$

*where $K \geq 0$ is a positive constant.*

*Proof.* We compute the derivative of $W^{\mathrm{SP}_1}$ with respect to a marginal increase in $\alpha(z)$, the probability that type $z$ is allocated to search. In computing this derivative, we need to compute

$$\frac{dv_s(z, \eta_s)}{d\alpha(z)} = \frac{dv_s(z, \eta_s)}{d\eta_s}\frac{d\eta_s}{d\alpha(z)}.$$

We know that the first term is positive for all $z$. The second term instead is

$$\frac{d\eta_s}{d\alpha(z)} = zf(z)C\Big(\tilde{g}(z)\eta_s - c\eta_w\Big).$$

where we used the definition of $h$, of $\eta_s$ and we denoted $C := \big(\mathbb{E}_f(z)\big(1 - \int_X \alpha(z)\tilde{g}(z)h(z)dz\big)\big)^{-1}$, a positive constant. Putting all together, we get

$$W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha) = f(z)\Big(v_s(z, \eta_s) - v_w(\eta_w) + z\Big(\eta_s\tilde{g}(z) - \eta_w c\Big)K\Big),$$

where $K = C\int_X \alpha(y)\frac{dv_s(y, \eta_s)}{d\eta_s}f(y)dy \geq 0$. □

**Proof of Corollary 1.** The first part follows from Proposition 5. We are left to show that for any regular sequence $(f^n(z))_{n\in N} \subset \mathcal{F}$, $\eta_s$ is monotonically decreasing if $g = \min(\mathcal{G}, \geq)$. When the communication technology is completely uninformative, we we have that $\eta_s = \eta_w c H(x^\star)$, a strictly increasing function of $x^\star$. If $f' \trianglerighteq_{MLR} f$, the respective $h'$ and $h$ are also ranked by $\trianglerighteq_{MLR}$, i.e. $h' \trianglerighteq_{MLR} h$. Moreover, since $\trianglerighteq_{MLR} \subset \trianglerighteq$, $H'(x) \leq H(x)$, for all $x \in X$. Thus, fixing $x^\star$, $\eta_s$ is decreasing in the shift. Since $\eta_s = \eta_w/x$ is strictly decreasing in $x$, we conclude that $x^\star$ is increasing in the shift, and therefore $\eta_s$ is decreasing.

**Proof of Proposition 6.** (*If part*). Let $g = \max\{\mathcal{G}, \geq\}$. As by Lemma A9, the sign of $W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha)$ is positive, meaning that the planner wants a type $z$ to search, if and only if $v_s(z, \eta_s) - v_w(\eta_w) \geq z\big(\eta_w c - \eta_s \tilde{g}(z)\big)K$. In the particular case when $g$ is maximal, we have that $\tilde{g}(z) = cz$. Therefore, the inequality becomes

$$v_s(z, \eta_s) - v_w(\eta_w) \geq zc\big(\eta_w - z\eta_s\big)K.$$

Now let $(x^\star, \eta_s)$ be the equilibrium under such $g$. Notice that by definition of $x^\star$, we have $\eta_w = x^\star\eta_s$ and $v_s(x^\star, \eta_s) = v_w(\eta_w)$. For all types above $x^\star$, the LHS of the inequality is strictly positive, while the RHS is strictly negative. For all types below $x^\star$, the RHS of the inequality is strictly negative, while the RHS is strictly positive. Therefore, $W^{\mathrm{SP}_1}_{\alpha(z)}(\alpha) \geq 0$ if and only if $x \geq x^\star$, showing that the allocation is indeed efficient.

(*Only if part*). Now take any $g < \max\{\mathcal{G}, \geq\}$. By continuity, we have that $\tilde{g}(z) < \mathbb{E}(\sqrt{t})z$. Now consider the equilibrium $(x^\star, \eta_s)$. We have that

$$\eta_w \mathbb{E}(\sqrt{t}) - \eta_s \tilde{g}(x^\star) > c\big(\eta_w - x^\star\eta_s\big) = 0 = v_s(x^\star, \eta_s) - v_w(\eta_w).$$

The planner would strictly prefer this type to work. This constitutes a deviation from the equilibrium allocation, thereby proving that it cannot be efficient. □

**Proof of Corollary 2.** Consider the extreme case $g(y) := 0$. In this case, meeting other searchers is completely unproductive. We have that $\tilde{g}(z) = 0$ for all $z \in X$ and $\eta_s$ can be written as

$$\eta_s(\alpha) = \eta_w c \int_X (1 - \alpha(z))h(z)dz.$$

The planner's incentive can be represented with the following inequality

$$v_s(z, \eta_s) - v_w(\eta_w) \geq zc\eta_w K.$$

The left hand side is increasing and strictly concave, starting at $-v_w(\eta_w)$ when $z = 0$. The right hand side is linear and increasing. For appropriate values of $\eta_w$ and $c$, there are exactly two solutions for this equation, $x_1$ and $x_2$ with $x_1 \leq x_2$, such player $z$ searches if and only if $z \in (x_1, x_2)$. □

**Proof of Proposition 7.** (*Only if part*). When $g$ is not maximally informative, the equilibrium allocation is inefficient in the *ex ante* sense (Proposition 6) and, *a fortiori*, is inefficient in the *ad interim* sense.

(*If part*). Suppose $g$ is maximally informative, i.e. $g = \mathrm{id}_{\mathbb{R}_+}$, and let $(x^\star, \eta_s)$ be the equilibrium. Consider a type $x$ of arbitrary age $t$ and suppose there exists a profitable deviation from the equilibrium plan. We consider a *simple* deviation that consists in switching type $x$'s activity for a $dt$ interval and then reverting back to the equilibrium allocation forever. Let $x$ be a searcher. Switching $x$ to work for a $dt$ interval generates two effects. First, $dp_t$, namely the instantaneousness change in type $x$'s posterior beliefs, has lower variance. On an individual basis, type $x$ is worse off. Second, her social contribution is affected. In the $dt$ interval she accumulated information at rate $\eta_w dt$ rather than $x\eta_s dt$. Since $x$ searches in equilibrium, $x\eta_s dt > \eta_w dt$. Since $g = \mathrm{id}_{\mathbb{R}_+}$, this necessarily implies her social contribution is diminished. The deviation considered reduces both $\eta_s$ *and* type $x$'s present discounted value and therefore cannot improve social welfare. □

**Proof of Corollary 3.** When $g = \max\{\mathcal{G}, \geq\}$, Proposition 7 shows that $x_1 = x_2 = x^\star$ and there is nothing to prove. Let $g < \max\{\mathcal{G}, \geq\}$. There are three distinct cases to consider:

*Case 1.* Suppose there exists no such $x_1 > 0$. This implies that the social planner finds optimal to allocate $x = 0$ to search at some particular $p_t$. However, both type $x$ and the society lose from this deviation. As a searcher, type $x$'s contribution to the society is null and so is her personal gain. As a worker, these are both strictly positive. A contradiction. Therefore there must exists $x_1 > 0$. By monotonicity, all types below $x_1$ will be allocated in a similar way.

*Case 2.* Next, suppose $x_2 < \infty$. For any type $x > x_2$, the cost of reverting back to work is higher. In terms of social contribution, the most extreme case is when $g = 0$. In such case, $x$ contributes exactly as $x_2$. Yet, the individual gains for $x$ dominate those of $x_2$, while the implied social loss is the same. Therefore, $x$ is allocated to search independently of time.

*Case 3.* Now consider a type $x_1 < x < x_2$. By Case 1 and 2, this type individually gains from search, at all $p_t$, but she contributes negatively to society due to the interaction between her type and the communication technology $g$. When $p_t$ converges to 0 or 1, her individual gain for engaging in search relative to work goes to zero, whereas her social contribution does not. Therefore, the planner would want this type to $p_t$ to work. Vice versa, when $p_t$ goes to $\frac{1}{2}$, her individual gain are maximized, the planner would want this type to search. □

**Proof of Proposition 8.** Fix a target type $\bar{x}$ a bias $b$ and a basin of $\delta$. The altered $\tilde{\eta}_s$ can be computed in ways similar to Lemma A5. We have that

$$\tilde{\eta}_s = c\eta_w H(x^\star) + \tilde{\eta}_s \int_{x^\star} \tilde{g}(z)h(z)dz + b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz = \frac{c\eta_w H(x^\star) + b\int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z)h(z)dz}{1 - \int_{x^\star} \tilde{g}(z)h(z)dz}$$

where $\alpha(x) = \tilde{g}(x)$ if $x \geq x^\star$ and $\alpha(x) = c$ otherwise. The influence exerted by the

$\delta$-neighbor of $\bar{x}$ is then

$$I(\bar{x},\delta) = \int_{x^\star} z(\tilde{\eta}_s - \eta_s) f(z) dz$$

$$= \mathbb{E}_f(x)(1 - H(x^\star)) \frac{b \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z) h(z) dz}{1 - \int_{x^\star} \tilde{g}(z) h(z) dz}$$

$$= \mathbb{E}_f(x) \frac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} b \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z) h(z) dz.$$

where we used that $h(z) = zf(z)/\mathbb{E}_f(x)$. Finally, setting $b = 1$,

$$\iota(x) := \lim_{\delta \to 0} \frac{I(x,\delta)}{\delta}$$

$$= \mathbb{E}_f(x) \frac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} \lim_{\delta \to 0} \frac{1}{\delta} \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} \alpha(z) h(z) dz$$

$$= \frac{\mathbb{E}_f(x)}{\mathbb{E}_f(x)} \frac{(1-H(x^\star))\eta_s}{cH(x^\star)\eta_w} \lim_{\delta \to 0} \frac{1}{\delta} \int_{\bar{x}-\epsilon_\delta}^{\bar{x}+\epsilon_\delta} z\alpha(z) f(z) dz$$

$$= \frac{\eta_s}{c\eta_w} \frac{1-H(x^\star)}{H(x^\star)} x\alpha(x)$$

$$= \frac{x\alpha(x)\eta_s}{c\eta_w} \frac{1-H(x^\star)}{H(x^\star)},$$

since $F([\bar{x} - \epsilon_\delta, \bar{x} + \epsilon_\delta]) = \delta$. □

**Proof of Corollary 4.** Take any two societies $f, f' \in \mathcal{F}$ with $f' \trianglerighteq f$. If $\eta_s(f') < \eta_s(f)$ there is nothing to prove. Suppose $\eta_s(f') > \eta_s(f)$. In such case, we have that $x^\star$ declined. Since $f' \trianglerighteq f$ implies $h' \trianglerighteq h$, we must have $H(x^\star)$ has also decreased under this transformation. From Proposition 8 we know that the influence of all searchers has strictly increased. The manipulative strategy can therefore produce higher distortions for any fixed $\delta$, or the same amount of distortion for a strictly smaller $\delta$. □

# B. Finite Capacity Communication Technologies

In this Appendix, we extend our model to a communication technology $g$ that equally applies to both workers and searchers.

**Definition B1.** *A* communication technology with finite capacity *is a* concave *and* non-decreasing *self-map $g$ on $\mathbb{R}_+$ with $g(y) \leq y$. Let $\mathcal{G}$ denote the set of such functions.*

Relative to the model presented in the paper, the coefficient $c$, which was defined as $c := \mathbb{E}_\tau(\sqrt{(t)})$ in Section 3, becomes a $\tilde{g}(1) := \mathbb{E}_\tau(\sqrt{(t)})$, an object that directly depends on the communication channel. Following Lemma A5, it is straightforward to show that Bayes consistency implies

$$\eta_s = \eta_w \frac{\tilde{g}(1)H(x)}{1 - \int_x \tilde{g}(z)h(z)dz} \geq 0. \tag{B.1}$$

The fixed-point map of 6, becomes

$$x = \frac{1}{\tilde{g}(1)} + \int_x m(z)h(z)dz, \tag{B.2}$$

where we denoted $m(z) := \frac{1}{\tilde{g}(1)}\big(\tilde{g}(1)x - \tilde{g}(z)\big)$. The following result establishes existence and uniqueness of our stationary equilibrium when $g$ is a finite capacity channel.

**Assumption B2.** *Let $\delta$ be high enough so that $\mathbb{E}_{h,\tau}(x\sqrt{t} \mid x \geq 1) \leq 1$.*

**Lemma B10.** *Fix a society $f \in \mathcal{F}$ and a communication technology with finite capacity $g \in \mathcal{G}$. A stationary equilibrium exists and is unique.*

*Proof.* (*Existence*) Notice that Equation B.2 can be rewritten as

$$x\tilde{g}(1)H(x) + \int_x \tilde{g}(z)h(z)dz = 1.$$

First we show that the left hand side crosses 1 at least once. Notice that at $x = 0$, $\int_0 \tilde{g}(z)h(z)dz \leq 1$, by Assumption 1. When $x \to \infty$ the right hand side grows unboundedly. Since the right hand side is continuous in $x$, this proves that there exists at least one stationary equilibrium in this game. Next we show that $x \geq 1$.

*(Uniqueness).* To show that the left hand side crosses 1 exactly once, we start by computing the derivative of Equation B.1:

$$\frac{d\eta_s}{dx} = K\Big(1 - \int_x \tilde{g}(z)h(z)dz - \tilde{g}(x)H(x)\Big),$$

where $K > 0$. It is enough to show that $1 - \int_x \tilde{g}(z)h(z)dz \geq \tilde{g}(x)H(x)$ at any equilibrium point to ensure uniqueness.

To begin, suppose the equilibrium point is $x \geq 1$. Notice that, at the equilibrium, it must be that

$$1 - \int_x \tilde{g}(z)h(z)dz = \tilde{g}(1)H(x).$$

Therefore, $1 - \int_x \tilde{g}(z)h(z)dz \geq \tilde{g}(x)H(x)$ at the equilibrium if $\tilde{g}(x) \leq x\tilde{g}(1)$. This is true if $g(x\sqrt{t}) \leq xg(\sqrt{t})$, which is guaranteed by concavity of $g$ (Definition B1), together with the fact that $x \geq 1$.[23]

[23] To see this, fix $x \geq 1$ and $t$ arbitrarily. Let $\alpha := \frac{1}{x} < 1$ and notice that by concavity of $g$,

$$g(\alpha x\sqrt{t} + (1-\alpha)0) \geq \alpha g(x\sqrt{t}) + (1-\alpha)g(0),$$

and, since $g(0) = 0$, $g(\alpha x\sqrt{t}) \geq \alpha g(x\sqrt{t})$. Rearranging gives us $g(x\sqrt{t}) \leq xg(\sqrt{t})$.

To complete the proof, we analyze the $x < 1$ and conclude that there cannot be an equilibrium of this kind. Notice that at $x = 1$, $1 - \int_1 \tilde{g}(z)h(z)dz > \tilde{g}(1)H(1)$. Indeed,

$$\begin{aligned}
1 - \int_1 \tilde{g}(z)h(z)dz - H(1)\tilde{g}(1) &> 1 - (1 - H(1))\mathbb{E}(\sqrt{xt}|x > 1) - H(1)\mathbb{E}(\sqrt{t}) \\
&> 1 - (1 - H(1))\mathbb{E}(\sqrt{xt}|x > 1) - H(1)\mathbb{E}(\sqrt{xt}|x > 1) \\
&> 1 - \mathbb{E}(\sqrt{xt}|x > 1) \\
&\geq 0
\end{aligned}$$

where the last inequality follows from Assumption B2. Since the derivative is single-crossing, $1 - \int_x \tilde{g}(z)h(z)dz > \tilde{g}(x)H(x)$ for any $x < 1$. Moreover, concavity of $g$ in the interval $x \in [0, 1]$ implies $xg(1) < g(x)$. Therefore, $1 - \int_x \tilde{g}(z)h(z)dz > x\tilde{g}(1)H(x)$ and therefore $x < 1$ cannot be an equilibrium. □

After ensuring the existence and uniqueness of the stationary equilibrium, Theorem 1 follows immediately since the fixed-point problem of Equation B.2 has the same property than the one of Equation 6, in particular, function $m$ is decreasing in $z$. Similarly, the proofs of Proposition 2, 3 and 5 can be readily applied to the new problem of Equation B.2 to reach the same conclusions.

# C. Non-Stationary Equilibria and Convergence

In this section, we discuss the non-stationary version of our model. For concreteness, we will assume there is a unit mass of players alive a time $t = 0$ and that $\delta = 0$. Players are infinitely lived is and there is no inflow of newborn player in the society. Moreover, we restrict attention to a simpler class of communication technologies, linear functions $g(y) = \kappa g(y)$, $0 \leq \kappa \leq 1$. Let $\eta_s : \mathbb{R}_+ \to \mathbb{R}$ be a time-dependent precision for social information. We refer to $\eta_s(t)$ as a *social information path*. It describes the precision of the instantaneous signal coming from search. Due to the continuum of players, we can think of $\eta_s$ as being deterministic. We will denote $E_s(t) := \int_0^t \eta_s(t')dt'$ and, therefore, $E'_s(t) = \eta_s(t)$. At time $t$, every players has two channel from which she can receive information, work and search:

$$\pi_w \sim \mathcal{N}\Big(\eta_w \theta dt, dt\Big) \qquad \pi_s(x) \sim \mathcal{N}\Big(xE'_s(t)\theta dt, dt\Big)$$

Players allocate their time optimally. Since they are of measure zero, they cannot affect the path of $\eta_s$. Their problem is in fact very similar to the of Section 3.2. A player at time $t$ faces a decision problem that consists in choosing the signal, between $\pi_w$ and $\pi(x)$, that maximizes the variance of her posterior beliefs. As in Lemma 1, her problem is solved by the following stopping function:

**Lemma C11.** *Fix a strictly increasing social information path $E'_s(t)$. There exists a stopping rule $\zeta : X \to \mathbb{R}_+$, such that, for all $x \in X$, player $x$ searches at time $t$ if and only if $t \geq \zeta(x)$. Moreover, $\zeta(x)$ is strictly decreasing in $x$.*

*Proof.* Fix a path $E_s(t)$. From Lemma 1, we know that at time $t$ the *indifferent type* at is pinned down by Equation $\eta_w = x^\star(t)E_s'(t)$. Define $\zeta(x)$ the inverse function of $x^\star(t) = \eta_w/E_s'(t)$. Since $E_s'(t)$ is strictly increasing, $\zeta$ is well-defined. $\square$

When $E_s'(t)$ is increasing, $\zeta$ is decreasing. This means that players unravel away from work, starting from highly connected types down to less connected types. In such case, players necessarily switch from *work to search*. The stock of information that each players collect is

$$\Gamma(x,t) := \begin{cases} \eta_w t\theta + B(t) & \text{if } t < \zeta(x), \\ \Big(\eta_w\zeta(x) + x\big(E(t) - E(\zeta(x))\big)\Big)\theta + B(t) & \text{else.} \end{cases} \tag{C.1}$$

We normalize to 1 the variance of the signal that type $(x,t)$ relay onto others. Its conditional expectation becomes, $\mathbb{E}\big(\pi(x,t)|\theta\big) = \eta_w\sqrt{t}\theta$ if if $t < \zeta(x)$, and $\mathbb{E}\big(\pi(x,t)|\theta\big) = \eta_w\frac{\zeta(x)}{\sqrt{t}}\theta + \frac{\kappa x}{\sqrt{t}}\big(E(t) - E(\zeta(x)\big)\theta$ otherwise.

**Definition C2.** *A Dynamic Equilibrium is a* stopping rule $\zeta : X \to \mathbb{R}_+$ *and* social information path $E_s : \mathbb{R}_+ \to \mathbb{R}_+$ *such that*

1. *Given the social information path $E_s$, the stopping rule is adapted to $E_s$ as by Lemma C11. That is, for all types $x$ and times $t$, player $x$ searches at $t$ if and only if $t \geq \zeta(x)$.*
2. *Given the stopping rule $\zeta$, $E_s$ us Bayes consistent, that is*

$$E_s'(t) = \int_X \mathbb{E}(\pi(z,t)|E_s)h(z)dz \qquad \forall t \geq 0.$$

In the equilibrium definition, we impose the obvious consistency requirement, which replace its stationary counterpart of Equation 4. By replacing the values for $\mathbb{E}(\pi(z,t)|E_s)$ in the Bayes consistency requirement and rearranging, we get

$$\sqrt{t}E_s'(t) = \eta_w tH(x^\star(t)) + \int_{x^\star(t)} \Big(\eta_w\zeta(z) + \kappa x\big(E_s(t) - E_s(\zeta(z))\big)\Big)h(z)dz. \tag{C.2}$$

This condition can be expressed as a second-order non-linear ODE, as we show in the next result.

**Lemma C12.** *A pair $(\zeta, E_s)$ is a Dynamic Equilibrium if and only if $E_s$ is a solution to the ODE*

$$tE_s''(t) = \eta_w\sqrt{t}H\Big(\frac{\eta_w}{E_s'(t)}\Big) + E_s'(t)\Big(\sqrt{t}\int_{\frac{\eta_w}{E_s'(t)}} zh(z)dz - \frac{1}{2}\Big)$$

*and $\zeta$ is adapted to $E_s$.*

*Proof.* We begin by deriving Equation C.2. Since by definition of $\zeta$ we have $\zeta(x^\star(t)) = t$, the derivative simplifies a lot. In particular, notice that

$$
\begin{aligned}
\tfrac{d}{dt}\sqrt{t}E_s'(t) &= \eta_w H(x^\star(t)) + x_t^\star(t)h(x^\star(t))\Big(\eta_w t - \eta_w\zeta(x^\star(t)) + \kappa x^\star(t)\big(E_s(t) - E_s(\zeta(x^\star(t)))\big)\Big) \\
&\quad +\kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz \\
&= \eta_w H(x^\star(t)) + x_t^\star(t)h(x^\star(t))\Big(\eta_w t - \eta_w t + \kappa x^\star(t)\big(E_s(t) - E_s(t)\big)\Big) + \kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz \\
&= \eta_w H(x^\star(t)) + \kappa \textstyle\int_{x^\star(t)} zE_s'(t)h(z)dz.
\end{aligned}
$$

Therefore,

$$\frac{1}{2\sqrt{t}}E_s'(t) + \sqrt{t}E_s''(t) = \eta_w H(x^\star(t)) + \kappa E_s'(t)\int_{x^\star(t)} zh(z)dz$$

and rearranging, with $x^\star(t) = \eta_w/E_s'(t)$,

$$tE_s''(t) = \eta_w\sqrt{t}H\Big(\frac{\eta_w}{E_s'(t)}\Big) + E_s'(t)\Big(\kappa\sqrt{t}\int_{\frac{\eta_w}{E_s'(t)}} zh(z)dz - \frac{1}{2}\Big)$$

which concludes the proof. □

## C.1. Convergence Towards the Stationarity Equilibrium

The dynamic model introduced in the previous section differs from the on introduced in Section 3 because players are infinitely lived. In this section, we perform the same exercise of the previous one, but in a dynamic model in which players at different ages can coexist. Although equilibria cannot be readily expressed as we did in Lemma C12, we can conclude they has nice features. In particular we show that when $g(x) = x$, $\eta_s(t)$ is necessarily a strictly increasing path that converges to the stationary equilibrium of Section 4. To avoid confusion, we will refer to time with variable $t$ and to age with variable $m$. Notice that $m \leq t$. As a consequence of Lemma C11, we still have that the activity choice for a player of type $x$ only depends on $t$, not $m$. In particular, such choice is determined by $\tau(x)$ the stopping rule adapted to $E_s$. The stock of information is given by

$$\Gamma(x,m,t) := \begin{cases} \eta_w m\theta + B(m) & \text{if } t < \tau(x), \\ \big(\eta_w(\tau(x) - m) + x\big(E(t) - E(\tau(x))\big)\theta + B(m) & \text{if } m > t - \tau(x). \\ \big(x\big(E(t) - E(m)\big)\theta + B(m) & \text{if } m < t - \tau(x). \end{cases}$$

Similarly, we can define the normalized $\mathbb{E}\big(\pi(z,m,t)\big|E_s\big)$, as we did in the previous section. The consistency conditions now takes into account the fact that a type can be a multiple different ages at the same time. In particular, at time $t$, the probability that type $x$ is of age $m \leq t$ is given by the truncated exponential distribution $\tau(m|t)$. Therefore, Bayes consistency becomes:

$$E_s'(t) = \int_X \Big(\int_0^t \mathbb{E}\big(\pi(z,m,t)\big|E_s\big)\tau(m|t)dm\Big)h(z)dz \qquad \forall t \geq 0. \tag{C.3}$$

The equilibrium is then defined as follows:

**Definition C3.** *A Dynamic Equilibrium with overlapping generations is composed by a* stopping rule $\zeta : X \to \mathbb{R}_+$ *and a* social information path $E_s : \mathbb{R}_+ \to \mathbb{R}_+$ *such that*

1. *Given the social information path $E_s$, the stopping rule is adapted to $E_s$ as by Lemma C11. That is, for all types $x$ and times $t$, player $x$ searches at $t$ if and only if $t \geq \zeta(x)$.*
2. *Given the stopping rule $\zeta$, $E_s$ is a solution to Equation C.3.*

In the next result we argue that, when an equilibrium exists, its information path $\eta_s(t)$ it has two important property. continuous solution will satisfy the property of being monotonically increasing and converging to the stationary equilibrium analyzed in Section 4. For that, we prove first two other results about $\eta_s(t)$.

**Lemma C13.** *Let $(x^\star, \eta_s^\star)$ be the unique stationary equilibrium. Suppose $(\tau, E_s)$ is a dynamic equilibrium with overlapping generations and that $g(y) = y$. Then $\eta_s(t)$ is strictly increasing.*

*Proof.* Suppose not. Let $\bar{t}$ be the first time at which $\eta'(t) \leq 0$. Fix a type $z$ and an age $m$. We want to show $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) \geq \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. This compares the social contribution of two identical players that where born $dt$-apart from each others. If $\bar{t} < \zeta(z)$, then both players have worked all their lives, which are of length $m$. Therefore they accumulated the same amount of information in expectation, or $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) = \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. If, instead, $\bar{t} \geq \zeta(z)$, the younger player, the one who is of age $m$ at time $\bar{t} + dt$, has collected more information in expectation, $\mathbb{E}(\pi(z, m, \bar{t} + dt)|\eta_s) > \mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$. This is because, by assumption on $\bar{t}$, in the interval $[0, \bar{t})$ the information path $\eta_s(t)$ was strictly increasing. Therefore, keeping $\tau(m|\bar{t})$ constant, the integral in Equation C.3 is strictly increasing. The change in the distribution only reinforce this effect. In fact, the distribution of ages $\tau(m|\bar{t})$ is first-order stochastically dominated by $\tau(m|\bar{t} + dt)$. Since $\mathbb{E}(\pi(z, m, \bar{t})|\eta_s)$ is trivially increasing in $m$ (older players have more information in expectation), we conclude that $\eta_s(\bar{t}) < \eta_s(\bar{t} + dt)$ and therefore $\eta_s'(\bar{t}) > 0$. A contradiction. $\square$

**Lemma C14.** *Let $(x^\star, \eta_s^\star)$ be the unique stationary. Suppose $(\tau, E_s)$ is a non-stationary equilibrium with overlapping generations and that $g(y) = y$. Then $\eta_s(t)$ is bounded above by $\eta_s^*$*

*Proof.* Suppose not. Then, by continuity of $\eta_s(t)$, there must be a $\bar{t}$ such that $\eta_s(\bar{t}) = \eta_s^\star$. By Claim C13, we know that $\eta_s(t) < \eta^\star$ for all $t \in [0, \bar{t})$. As before, fix any type $x$ and any age $m$. Under the dynamic equilibrium, this player cannot have accumulated more information than the same player of the same age under the stationary equilibrium. If $(z, m)$ works at $\bar{t}$, she has been working since $\bar{t} - m$ and has the expected amount of information under the two regimes. If she ever searched, then by $\eta_s(t) < \eta^\star$, she must have strictly less information under the dynamic equilibrium. Therefore, keeping $\tau(m|\bar{t})$ constant, at $\bar{t}$ the integral in Equation C.3 is strictly smaller than $\eta_s^\star$, or $\eta_s(\bar{t}) < \eta_s^\star$, a contradiction. $\square$

**Lemma C15.** *Let* $(x^\star, \eta_s^\star)$ *be the unique stationary equilibrium. Suppose* $(\tau, E_s)$ *is a non-stationary equilibrium with overlapping generations and that* $g(y) = y$*. Then* $\lim \eta_s(t) = \eta^\star$*.*

*Proof.* We know that $\eta_s(t)$ is an monotone increasing (Claim C13) and bounded (Claim C14) sequence. As such, it necessarily converges to some real limit point $\lim \eta_s(t) \leq \eta^\star$. Suppose $\lim \eta_s(t) < \eta^\star$. In such case, $\lim \eta_s(t)$, together with the implied threshold $x$, must satisfies Definition 2. A contradiction on the uniqueness result of Proposition 1. □