Currency Manipulation*

Tarek A. Hassan† Thomas M. Mertens‡ Tony Zhang§

December 31, 2015

Preliminary and Incomplete

Abstract

We propose a novel, risk-based transmission mechanism for the effects of currency manipulation: policies that systematically induce a country's currency to appreciate in bad times, lower its risk premium in international markets and, as a result, lower the country's risk-free interest rate and increase domestic capital accumulation and wages. Currency manipulations by large countries also have external effects on foreign interest rates and capital accumulation. Applying this logic to policies that lower the variance of the bilateral exchange rate relative to some target country ("currency pegs"), we find that a small economy pegging its currency to a large economy increases domestic capital accumulation and wages. The size of this effect increases with the size of the target economy, offering a potential explanation why the vast majority of currency pegs in the data are to the US dollar, the currency of the largest economy in the world. A large economy (such as China) pegging to a larger economy (such as the US) diverts capital accumulation from the target country to itself, increasing domestic wages, while decreasing wages in the target country.

JEL classification: F3, G0
Keywords: fixed exchange rate, currency manipulation, exchange rate peg

*We thank Mark Aguiar, Adrien Auclert, Oleg Itskhoki, Loukas Karabarbounis, Matteo Maggiori, Brent Neiman, Vania Stavrakeva, and Ivan Werning for helpful comments. Hassan is grateful to the Fama-Miller Center at the University of Chicago for providing financial support. The views expressed here are solely those of the authors and do not necessarily represent those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

†**University of Chicago**, NBER, and CEPR; Postal Address: 5807 S Woodlawn Avenue, Chicago IL 60637, USA; E-mail: tarek.hassan@chicagobooth.edu.

‡**Federal Reserve Bank of San Francisco**; Postal Address: 101 Market Street Mail Stop 1130, San Francisco, CA 94105, USA; E-mail: thomas.mertens@sf.frb.org.

§**University of Chicago**; Postal Address: 5807 S Woodlawn Avenue, Chicago IL 60637, USA; E-mail: tony.wtz@gmail.com.

1 Introduction

Differences in interest rates across developed economies are large and persistent; some countries have lower interest rates than others for decades rather than years. These long-lasting differences in interest rates correlate with differences in capital-output ratios across countries, and account for the majority of excess returns on the carry trade, which is a trading strategy where international investors borrow in low interest rate currencies, such as the Japanese Yen, and lend in high interest rate currencies, such as the New Zealand dollar (Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2015).

A growing literature studying these "unconditional" differences in currency returns argues that they may be attributable to heterogeneity in the stochastic properties of exchange rates: currencies with low interest rates pay lower returns because they tend to appreciate in bad times and depreciate in good times, providing a hedge to international investors and making them a safer investment (Lustig and Verdelhan, 2007; Menkhoff, Sarno, Schmeling, and Schrimpf, 2013). This literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates, ranging from differences in country size (Martin, 2012; Hassan, 2013) and financial development (Maggiori, 2013) to differential resilience to disaster risk (Farhi and Gabaix, 2015). The common theme across these papers is that whatever makes countries different from each other results in differential sensitivities of their exchange rates to various shocks, such that some currencies tend to appreciate systematically in states of the world when the shadow price of traded goods is high. Currencies with this property then pay lower expected returns and have lower risk-free interest rates.

In this paper, we argue that this risk-based view of currency returns provides a novel way of thinking about the effects of currency manipulation: interventions in currency markets that change the stochastic properties of exchange rates should also change the expected returns on currencies and other assets. In particular, policies that induce a country's currency to appreciate in bad times should lower domestic interest rates, lower the cost of capital for the production of non-traded goods, and, as a result, increase capital accumulation. Moreover, if these interventions are large enough, that is, if the country manipulating its exchange rate is large relative to the world, its policies will affect the interest rates and capital accumulation in other countries, potentially diverting capital accumulation from other countries to itself. Policies that change the variances and covariances of exchange rates should thus, via their effect on interest rates and asset returns, affect the allocation of capital across countries.

After making this argument in its most general form, we illustrate the implications of this view with an application to currency pegs. Table 1 shows that 88% of countries (representing 47% of world GDP) manipulate their exchange rates by pegging their currency relative to some target country (Reinhart and Rogoff, 2004). Such currency pegs specify a target currency (two thirds of them the US dollar) and set an upper bound for the volatility of the real or nominal exchange rate relative to that target country. A "hard" peg may set this volatility to zero, while a "soft" peg may officially or unofficially specify a band of allowable fluctuations around some mean. The common feature of all of these policies is that they manipulate the variances and covariances of exchange rates by changing the states of the world and the extent to which they appreciate and depreciate, without necessarily manipulating the level of the exchange rate.

Table 1: 2010 Exchange Rate Arrangements based on Reinhart and Rogoff (2004, 2011)

	% of Countries	% of GDP
Panel A	*Exchange rate arrangement*	
Floating	3%	34%
Pegged	88%	47%
soft	47%	32%
hard	41%	15%
Currency union	9%	19%
Panel B	*Target currencies of pegs*	
Dollar	67%	80%
Euro	27%	19%

Notes: Classification of exchange rate regimes as of 2010 according to Reinhart and Rogoff (2004, 2011). All data are available on Carmen Reinhart's website at `www.carmenreinhart.com/data/browse-by-topic/topics/11`.

We analyze the effects of such pegs on interest rates, capital accumulation, and wages within a generic model of exchange rate determination. Households consume a bundle of a freely traded good and a country-specific nontraded good. The nontraded good is produced using capital and labor as inputs. In equilibrium, the real exchange rate fluctuates in response to country-specific (supply) shocks to the productivity in the production of nontraded goods across countries, (demand) shocks to preferences, and (monetary) shocks to the inflation rates of national currencies.

As a stand-in for the various potential sources of heterogeneity in the stochastic properties of countries' exchange rates studied in the literature outlined above, we add heterogeneity in country size to this canonical setup as in Hassan (2013). That is, we assume that all shocks are common within countries and some countries account for a larger share of world GDP than others. This heterogeneity in country size generates differences in the stochastic properties of countries' exchange rates, where the currencies of larger countries tend to appreciate in "bad" times: households react to supply, demand, and monetary shocks by shipping traded goods across countries in an effort to share risk across borders. However, shocks that affect larger countries are harder to diversify internationally. For example, when a country has a low per capita output of nontraded goods, its consumption bundle becomes relatively more expensive and its real exchange rate appreciates. To compensate for the shortfall of nontraded goods, the country imports additional traded goods from the rest of the world. However, a low output of nontraded goods in a large country simultaneously triggers a rise in the world market price of traded goods, while a low output of nontraded goods in a small country does not. As a consequence, currencies of large countries tend to appreciate when the world market price of traded goods is high, offering a hedge to world-wide consumption risk. Because of these hedging properties, the currencies of large countries pay lower expected returns and have lower risk-free interest rates. Lower interest rates in turn lower the cost of capital in these countries, prompting them to install higher capital-output ratios and pay higher wages in equilibrium.

Within this economic environment we study the positive effects of a class of policies that lower the variance of one "pegging" country's real exchange rate relative to a "target" country's currency, while leaving the mean of the real exchange rate unaffected. We largely focus our discussion on real pegs, that is, policies that manipulate the real rather than the nominal exchange rate, although all of our main results generalize to nominal pegs. We also focus almost exclusively on the positive predictions of our model, mainly because these predictions are invariant to whether real exchange rates move as the result of supply, demand, or monetary shocks and thus appear highly robust. By contrast, the welfare effects of currency pegs depend on many details of the model, such as the degree of market completeness and the relative importance of monetary shocks.

To sustain the peg, the pegging country's government alters the state-contingent plan of imports and exports of traded goods. In particular, when the target country appreciates, it matches this appreciation by reducing traded goods consumption and thus raising domestic marginal utility. Similarly, when the pegging country suffers a shock that increases domestic marginal utility that would ordinarily result in an appreciation, it imports additional traded goods to lower domestic marginal utility. In our model, the pegging country's government implements these policies by using a set of state-contingent taxes on Arrow-Debreu securities, and a lump sum transfer financed by using an independent source of wealth ("currency reserves"). More generally, we might also imagine such policies being implemented by offering to exchange foreign currencies for domestic currency at a pre-determined rate or other kinds of interventions in currency markets.

We first consider the case in which the pegging country is small and thus only affects its own price of consumption. A small country that imposes a hard peg on a larger country inherits the stochastic properties of the larger country's exchange rate: the pegged exchange rate now tends to appreciate when marginal utility of traded consumption is high in world markets, making the pegging country's curency a better hedge against consumption risk, lowering its risk-free interest rate and expected return on its currency. Similarly, investments in the pegging country's capital stock now become more valuable, increasing its capital-output ratio and raising wages within the country.

To sustain the peg, the pegging country ships additional traded goods to the rest of the world when the target country appreciates. If the target country is large, these states again tend to be states when the shadow price of traded goods is high. As a result, pegging to a larger target country generates an insurance premium, making it cheaper to peg to larger countries. If households are sufficiently risk averse and the target country is sufficiently large, this insurance premium may be so large that the currency peg generates positive revenues, that is, it accumulates rather than depletes reserves.

This revenue-generating effect of currency pegs to larger countries, however, diminishes when the pegging country itself becomes larger. The reason is that the peg exaggerates the spikes in the pegging country's own demand for traded goods, increasing its price impact: in states of the world in which the pegging country has high marginal utility and would ordinarily appreciate relative to the target country, it must import even more traded goods than it would have in the absence of the peg to prevent appreciation. When the pegging country is large enough to affect the equilibrium shadow price of traded goods, the peg thus induces an unfavorable change in the state-contingent prices of traded goods. The larger the pegging country, the more reserves are required to maintain the peg.

Our model also allows us to solve for the effects of the peg on the target country: a country that becomes the target of a peg imposed by a country that is large enough to affect world prices (or the target of multiple pegs imposed by a non-zero measure of small countries) experiences a rise in its risk-free interest rate, a decrease in its capital-output ratio, and a decrease in wages. The reason is that, to sustain its peg, the pegging country supplies additional traded goods to the world market whenever the target country appreciates. This activity dampens the impact of the target country's shocks on the shadow price of traded goods, reducing their spill-over to the world market. The lower this impact, the lower is the co-movement between the shadow price of traded goods and the target country's exchange rate. The currency of a large country, that is, the target of a peg thus becomes a less attractive hedge for international investors, raising its risk-free interest rate.

In various robustness checks we show that this broad set of conclusions arises regardless of whether variation in exchange rates are driven primarily by supply, demand, or monetary shocks; regardless of whether the peg is real or nominal; and regardless of whether financial markets are complete or segmented within countries.

We also examine the welfare effects of currency pegs for a special case of our model, where markets are complete and exchange rates vary exclusively as a result of supply shocks. In this simpler model, currency pegs are never welfare increasing for the pegging country because any gains in revenues from the peg or from the increase in capital accumulation are outweighed by the adverse effect of an increase in the volatility of consumption. Conversely, becoming the target of a peg reduces one's variance of consumption, resulting in a net welfare increase, despite the detrimental effects on the target country's capital stock. However, these welfare results depend strongly on the details of the model.

Taken together, we believe our results provide a novel way of thinking about currency manipulation in a world in which risk-premia affect the level of interest rates. First, by manipulating exchange rates, policymakers may be able to manipulate the allocation of capital across countries. Second, although currency pegs do not appear optimal under standard welfare measures, our model shows that policymakers might have a motive to peg if their objective is to increase wages, increase capital accumulation, or raise revenue. For example, we might think of political reasons why policymakers might have an interest in raising wages or in externalities that may make it optimal to increase capital accumulation. Third, whatever the motive for pegging, pegs to larger countries appear to be cheaper to implement and more impactful on all dimensions than pegs to smaller countries, offering a potential explanation for the fact that almost all pegs in the data are imposed on the euro or dollar. Fourth, our model speaks to the external effects of pegs on the target country, providing a meaningful notion of what it means to be at the center of the world's monetary system: countries that peg to a common target divert capital accumulation from the target while dampening the effects of shocks emanating from the target on the world economy.[1]

This latter point also offers an interesting perspective on the large public debate over the Chinese exchange rate regime: U.S. policymakers have often voiced concern that China may be undervaluing its exchange rate and that this undervaluation may be bad for U.S. workers and good for Chinese workers. The official Chinese response to these allegations has been that China is merely reducing the volatility of the dollar - RMB exchange rate and not systematically distorting its level. The implication of our analysis is that even if this assertion is accurate, the mere fact that China is pegging its currency to the dollar may divert capital accumulation from the U.S. to China, a policy that is likely to be bad for U.S. workers.

A large literature studies the effects of monetary stabilization and exchange rate pegs in the presence of nominal frictions.[2] Most closely related are Kollmann (2002) and

[1] In this sense, our paper also relates to a growing literature that argues for a special role of the US dollar in world financial markets. See for example Gourinchas and Rey (2007), Lustig et al. (2011), Maggiori (2013), and Miranda-Agrippino and Rey (2015).

[2] One strand of the literature analyzes optimal monetary policy in small open economies with fixed exchange rates (Kollmann, 2002; Parrado and Velasco, 2002; Gali and Monacelli, 2005), while another

Bergin and Corsetti (2015), where currency pegs affect markups and the level of capital accumulation through their effects on nominal rigidities. We add to this literature in three ways. First, we study a novel effect of currency pegs on risk premia that operates even in a frictionless economy in which money is neutral. Second, we are able to study how the effects of currency pegs vary with the choice of the target country. Third, we are able to study the external effects of the currency peg on the target country.

More broadly, our paper also relates to a large literature on capital controls.[3] Similar to the work by Costinot, Lorenzoni, and Werning (2014), who argue that capital controls may be thought of as a manipulation of intertemporal prices, we show that currency pegs, and other policies altering the stochastic properties of exchange rates, may be thought of as a manipulation of state-contingent prices. The key difference between the two concepts is that capital controls affect allocations through market power and rents, while currency manipulation affects allocations through risk premia even when the country manipulating its exchange rate has no effect on world market prices. In addition, our work shows that, in contrast to capita controls, currency pegs cannot be rationalized as optimal policies within a frictionless neoclassical model.

Finally, as mentioned above, our paper relates to a growing empirical literature that argues that "unconditional" differences in currency returns may be attributable to heterogeneity in the stochastic properties of exchange rates.[4] The theoretical side of this literature has explored various potential drivers of heterogeneity of the stochastic properties of countries' exchange rates.[5] We add to this literature by showing that this class of model implies that exchange rate manipulation may transmit itself through its effect on currency risk premia.

deals with the choice of the exchange rate regime in the presence of nominal rigidities (Helpman and Razin, 1987; Bacchetta and van Wincoop, 2000; Devereux and Engel, 2003; Corsetti, Dedola, and Leduc, 2010; Schmitt-Grohé and Uribe, 2012; Bergin and Corsetti, 2015).

[3]See for example Calvo and Mendoza (2000), Jeanne and Korinek (2010), Bianchi (2011), Farhi and Werning (2012), Schmitt-Grohé and Uribe (2012), Farhi and Werning (2013), and Korinek (2013).

[4]See for example Lustig and Verdelhan (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Lustig et al. (2011), Menkhoff, Sarno, Schmeling, and Schrimpf (2012), David, Henriksen, and Simonovska (2014), and Verdelhan (2015). Also see the evidence in some of the aforementioned papers, including Hassan (2013), Ready, Roussanov, and Ward (2013), Tran (2013), and Richmond (2015).

[5] These include differences in country size (Govillot et al., 2010; Martin, 2012; Hassan, 2013), the size and volatility of shocks affecting the nontraded sector (Tran, 2013), financial development (Maggiori, 2013), factor endowments (Ready et al., 2013; Powers, 2015), trade centrality (Richmond, 2015), and resilience to disaster risk (Farhi and Gabaix, 2015).

2 Reduced Form Model of Exchange Rates

We begin by deriving the main insights of our analysis in their most general form. Consider a class of models in which the utility of a representative household in each country n depends on its consumption of a final good that consists of a country-specific nontraded good and a freely traded good. In this class of models, we may write the price of the final good in country n in reduced form as

$$p^n = a\lambda_T - bx^n, \tag{1}$$

where p^n is the log of the number of traded goods required to purchase one unit of the final good in country n, λ_T is the log shadow price traded goods in the world market, b is a constant greater than zero, and $x \sim N(0, \sigma_x^2)$ is a normally distributed shock to the log price of consumption in country n. We may think of this shock interchangeably as the effect of a country-specific supply, demand, or monetary shock; in other words, it is a stand-in for any factor that affects the price of consumption in one country more than in others. The higher x, the lower is the price of domestic consumption.

If households can share risk in world markets by shipping traded goods between countries, these country-specific shocks will also be reflected in the equilibrium shadow price of traded goods in the world. Thus, if many countries have adverse shocks, the shadow price of traded goods will be high in the world, and vice versa. In the model we derive below, this relationship is linear with

$$\lambda_T = -\sum_n w^n x^n, \tag{2}$$

where the weights $w^n \geq 0$ may differ across countries.

The real exchange rate between two countries f and h is the relative price between their respective final goods. We can write the log real exchange rate as

$$s^{f,h} = p^f - p^h.$$

The risk-based view of differences in currency returns applies some elementary asset pricing to this expression. Using the Euler equation of an international investor, one can show that the log expected return to borrowing in country h and to lending in country f is

$$r^f + \mathbb{E}s^{f,h} - r^h = cov\left(\lambda_T, p^h - p^f\right) = b\left(w^h - w^f\right)\sigma_x^2, \tag{3}$$

where r^n is the risk-free interest rate in country n. [6] This statement means that a currency that tends to appreciate when the shadow price of traded goods is high pays a lower expected return and, if $\mathbb{E}s^{f,h} = 0$, also has a lower risk-free interest rate. Currencies that appreciate in bad times thus provide a hedge against world-wide consumption risk and must pay lower returns in equilibrium. These "systemic" currencies are the currencies of countries that have a relatively large w^n, that is, the currencies of countries whose shocks spill over to world markets more than the shocks of other countries.

This line of argument (equations (1)-(3)) is the main ingredient of any risk-based model of unconditional differences in interest rates across countries, where different approaches model differences in w^n as the result of heterogeneity in country size, the volatility of shocks to the non-traded sector, trade specialization, financial development, factor endowments, etc.

We make a simple point relative to this literature: if there is merit to this risk-based view of currency returns, policies that alter the covariance between a country's exchange rate and the shadow price of traded goods can alter interest rates, currency returns, and the allocation of capital across countries. In particular, a country that adopts policies that increase the price of domestic consumption in states of the world where λ_T is high, can lower its risk-free interest rate relative to all other countries in the world. As an example, consider a "pegging" country (p) that levies a state-contingent tax on domestic consumption of traded goods that is proportional to the realization of x^t in some target country t, such that

$$p^p = a\lambda_T - bx^p - cx^t.$$

Note that this state-contingent tax is zero on average across states $\left(E\left(x^t\right) = 0\right)$, such that it affects only the stochastic properties but not the level of country p's log exchange rate. If the target country's shock affects the world price of traded goods, that is if $w^t > 0$,

[6] See Appendix A for a formal proof.

this policy increases the covariance between p^p and λ_T and, as a result, lowers country p's interest rate relative to all other countries in the world. The larger w^t, the larger is this effect. In this sense, currency manipulations that hone in on the shocks affecting the most systemic countries in the world are most impactful.

In addition, and this will become clear when we move to our fully specified model, the state-contingent tax also impacts country p's state-contingent plan of shipping traded goods to and from the world. Specifically, a tax that increases the price of consumption when x^t is low induces shipments of traded goods from country p to the rest of the world in those states. If the country that manipulates its exchange rate is itself large in the sense that its actions affect the equilibrium shadow price of traded goods, its policy reduces the the target country's weight w^t in (2). That is, it dampens the extent to which the target country's shock spills over to other countries. As a consequence, the covariance between p^t and λ_T falls, increasing the interest rate in the target country. A state-contingent tax of the form above thus *raises* the interest rate in the target country, while lowering it in the country that manipulates its exchange rate.

If interest rates play a role in allocating capital across countries (as it is the case in our fully specified model), manipulations of the stochastic properties of exchange rates can thus divert capital accumulation from the target country of the manipulation to the country that conducts the manipulation, and, more broadly, alter the equilibrium allocation of capital across countries.

The remainder of this paper fleshes out this simple argument in the context of a canonical model of exchange rate determination.

3 A Model of Currency Pegs

In this section, we set up our fully specified model in which the allocation of capital across countries and the stochastic properties of real exchange rates are jointly determined as a function of supply and demand shocks. The model generalizes the framework in Hassan (2013) and Hassan, Mertens, and Zhang (2015) by allowing for currency manipulation. The model nests the canonical real business cycle model of exchange rate determination of Backus and Smith (1993), augmented with preference shifters as in Pavlova and Rigobon

(2007), as well as a simplified version of the incomplete-markets model with monetary shocks by Alvarez, Atkeson, and Kehoe (2002). The purpose of including multiple types of shocks is simply to show the generality of our argument — all results go through with just one type of shocks. Within this canonical model of exchange rate determination, one country, labelled the "pegging" country, deviates from the competitive equilibrium by imposing a hard or soft peg on its real or nominal exchange rate with respect to a "target" country.

3.1 Setup

There are two discrete time periods, $t = 1, 2$. There exists a unit measure of households $i \in [0, 1]$, partitioned into three subsets Θ^n of measure θ^n. Each subset represents the constituent households of a country. We label these countries $n = \{p, t, o\}$ for the "pegging", "target", and "outside" country, respectively. Households make an investment decision in the first period. All consumption occurs in the second period.

Households exhibit constant relative risk aversion according to

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(\exp(\chi^n)C_2(i))^{1-\gamma}\right], \tag{4}$$

where $C_2(i)$ is the consumption index for household i, χ^n is a common shock to the preferences of households in country n,

$$\chi^n \sim N\left(-\frac{1}{2}\sigma_\chi^2, \sigma_\chi^2\right),$$

and $\gamma > 0$ is the coefficient of relative risk aversion. The consumption index is defined as

$$C_2(i) = C_{T,2}(i)^\tau \; C_{N,2}(i)^{1-\tau},$$

where $C_{N,2}$ is consumption of the country-specific nontraded good, $C_{T,2}$ is consumption of the traded good, and $\tau \in (0, 1)$.

At the start of the first period, each household receives a deterministic endowment of traded goods $\left(Y_{T,1}^n\right)$ and one unit of capital. Traded goods can be stored for consumption in the second period and are freely shipped internationally. Capital goods can be freely shipped only in the first period when they are invested for use in the production of nontraded goods in the second period.

Households also produce their country specific nontraded good using a Cobb-Douglas production technology that employs capital and labor. Households purchase capital in international markets in the first period. Each household supplies one unit of labor inelastically. The per capita output of nontraded goods is

$$Y_{N,2}^n = \exp(\eta^n)\,(K^n)^\nu$$

where $0 < \nu < 1$ is the capital share in production, K^n is the *per capita* stock of capital in country n and η^n is a country-specific productivity shock to the production of nontraded goods realized at the start of the second period,

$$\eta^n \sim N\left(-\frac{1}{2}\sigma_N^2, \sigma_N^2\right).$$

Throughout we use the tradable consumption good as the numéraire, such that all prices and returns are accounted for in the same units.

In the first period, a fixed proportion ϕ of households within each country trades a complete set of state-contingent securities in international markets. Label these households as "active". The remaining $1 - \phi$ fraction of households within each country are excluded from trading state-contingent securities. Label these households as "inactive". Inactive households cede the claims to their endowments, their wages, and firm profits to active households in return for a nominal bond. Each active household thus receives a fraction $\frac{1}{\phi}$ of per-capita second period wages and firm profits.

The nominal bond given to inactive households pays off one unit of the country's nominal consumer price index. We write this payment to inactive households as $P_2^n e^{-\mu^n}$, where μ^n is a country-specific inflation shock to the price of one unit of the traded good in terms of the currency of country n,

$$\mu^n \sim N\left(-\frac{1}{2}\tilde{\sigma}, \tilde{\sigma}\right).$$

To simplify notation, let ω represent the realization of productivity, preference and inflation shocks and let $g(\omega)$ be the associated multivariate density. All households take prices as given. Active households maximize their utility (4) subject to the constraint

$$\begin{aligned}&\int Q(\omega)\left(P_2^n(\omega)C_2^n(\omega)+\frac{1-\phi}{\phi}P_2^n(\omega)e^{-\mu^n}\right)d\omega\\ \leq&\frac{1}{\phi}\left[Y_{T,1}^n+q_1-q_1K^n+\int Q(\omega)P_{N,2}^n(\omega)\exp(\eta^n)\left(K^n\right)^{\nu}d\omega+\kappa^n\right]\end{aligned}$$

where $Q(\omega)$ is the price of a security that pays one unit of the traded good if state ω occurs, P_2^n denotes the number of traded goods required to buy one unit of the country-specific consumption index in country n, and $\frac{1-\phi}{\phi}$ is the number of inactive households per active household in each country. q_1 is the world-market price of a unit of capital in the first period. To simplify the derivation, we also assume that active households receive a country-specific transfer, κ^n, before trading begins, which equalizes household wealth across countries.

Inactive households also maximize (4), but are subject to the constraint

$$P_2^n\hat{C}_2(i)\leq P_2^n(\omega)e^{-\mu^n}, \tag{5}$$

where we use hats to denote the consumption of inactive households.

3.2 Currency Pegs

The pegging country's government has the ability to levy a state-contingent consumption tax. It also has access to an independent supply of traded goods (currency reserves) that it can use to finance its taxation scheme. The government's objective is to decrease fluctuations of its country's log real exchange rate with the target country by a fraction $\zeta\in(0,1]$ relative to the freely-floating regime without distorting the conditional mean of the log real exchange rate. As a result, it chooses a taxation scheme such that

$$var\left(s^{t,p}\right)=(1-\zeta)^2var\left(s^{t,p*}\right) \tag{P1}$$

and

$$\mathbb{E}\left[s^{t,p}|\{K^n\}\right] = \mathbb{E}_1\left[s^{t,p*}|\{K^n\}\right]. \tag{P2}$$

where asterisks denote values under a free floating exchange rate regime, and where we refer to $\zeta = 1$ as a "hard" peg.

We also consider pegs of the *nominal* exchange rate that decrease the variance of the log nominal exchange rate between the pegging and target countries, $var\left(\tilde{s}^{t,p}\right) = (1-\zeta)^2 var\left(\tilde{s}^{t,p*}\right)$, while keeping the conditional mean of the log nominal exchange rate unchanged, $\mathbb{E}\left[\tilde{s}^{t,p}|\{K^n\}\right] = \mathbb{E}\left[\tilde{s}^{t,p*}|\{K^n\}\right]$.

The government achieves this policy through a combination of a state contingent tax on consumption goods delivered in the country, $Z(\omega)$ and a lump sum transfer, $\bar{Z}$. Formally, households in the pegging country face the budget constraint

$$\begin{aligned}&\int Z(\omega)Q(\omega)\left(P_2^p(\omega)C_2^p(\omega) + \frac{1-\phi}{\phi}P_2^p(\omega)e^{-\mu^n}\right)d\omega\\ \leq&\frac{1}{\phi}\left[Y_{T,1}^p + q_1 - q_1K^p + \int Z(\omega)Q(\omega)P_{N,2}^p(\omega)\exp(\eta^p)\left(K^p\right)^\nu d\omega + \kappa^p + \bar{Z}\right].\end{aligned}$$

3.3 Equilibrium

The market clearing conditions for traded, nontraded, and capital goods are

$$\int_i C_{T,2}(i,\omega)di = \sum_n \theta^n Y_{T,1}^n(\omega), \tag{6}$$

$$\int_{i\in\theta^n} C_{N,2}(i,\omega)di = \theta^n Y_{N,2}^n(\omega), \tag{7}$$

and

$$\sum_n \theta^n K^n = \sum_n \theta^n = 1 \tag{8}$$

The economy is in an equilibrium when all households maximize utility subject to their budget constraints, firms maximize profits, and goods markets clear.

3.4 Solving the Model

Although financial markets are incomplete, the model's solution remains tractable. Appendix B shows the solution to the inactive household's problem. Their behavior is relevant only for understanding how monetary shocks affect the equilibrium, which we discuss in detail later. Active households in the target and outside countries maximize utility subject to their budget constraints [7]. Because all active households within a given country are identical, we can write their consumption bundle as $\left(C^n_{T,2}, C^n_{N,2}\right)$ and henceforth drop the household index i .

The first-order conditions with respect to $C^n_{T,2}$ equate the shadow price of tradable consumption across active households in the target and outside countries

$$\tau \exp\left((1-\gamma)\chi^n\right)\left(C^n_2(\omega)\right)^{-\gamma}\left(C^n_{T,2}(\omega)\right)^{-1} = \Lambda_{T,2}(\omega) \tag{9}$$

and the first-order conditions with respect to $C^n_{N,2}$ define the shadow prices of non-traded goods within each country

$$(1-\tau)\exp\left((1-\gamma)\chi^n\right)\left(C^n_2(\omega)\right)^{-\gamma}\left(C^n_{N,2}(\omega)\right)^{-1} = \Lambda^n_{N,2}(\omega). \tag{10}$$

We derive the Euler equation by taking first-order conditions with respect to K^n_N and using the fact that competitive markets imply $P^n_{N,2}(\omega) = \Lambda^n_{N,2}(\omega)/\Lambda_{T,2}(\omega)$.

$$K^n_N = \frac{\nu}{\Lambda_{T,1} q_1}\mathbb{E}\left[\Lambda^n_{N,2} Y^n_{N,2}\right] \tag{11}$$

The Euler equation defines the level of capital accumulation in country n as a function of first-period prices, $\Lambda_{T,1}$, the stochastic properties of the shadow price of non-traded consumption, $\Lambda^n_{N,2}$, and $Y^n_{N,2}$. $\Lambda_{T,1} = \mathbb{E}\left[\Lambda_{T,2}(\omega)\right]$ can be interpreted as the shadow price of a traded good in the first period, prior to the realization of shocks.

In addition, it is useful to keep track of the (redundant) first-order condition with respect to the consumption index C^n_2, because it pins down the marginal utility of con-

[7] See Appendix C for a formal setup and proof.

sumption of active households in each country

$$\exp\left((1-\gamma)\chi^n\right)\left(C_2^n(\omega)\right)^{-\gamma} = \Lambda_2^n(\omega). \tag{12}$$

By definition, the real exchange rate between two countries h and f equals the ratio of these shadow prices,

$$S^{f,h}(\omega) = \Lambda_2^f(\omega)/\Lambda_2^h(\omega).$$

Solving the problem of active households in the pegging country, we find that active households in the pegging country equate their marginal utility of traded consumption with the after tax shadow price of traded goods [8]

$$\tau \exp\left((1-\gamma)\chi^p\right)\left(C_2^p(\omega)\right)^{-\gamma}\left(C_{T,2}^p(\omega)\right)^{-1} = Z(\omega)\Lambda_{T,2}(\omega) \tag{13}$$

The first order condition with respect to nontraded consumption still defines the shadow price of nontraded goods in the pegging country

$$(1-\tau)\exp\left((1-\gamma)\chi^p\right)\left(C_2^p(\omega)\right)^{-\gamma}\left(C_{N,2}^p(\omega)\right)^{-1} = \Lambda_{N,2}^p, \tag{14}$$

The state-contingent tax that implements the exchange rate peg appears as a consumption wedge in the first-order condition of traded consumption.

The first order condition with respect to capital accumulation in the pegging country still simplifies to

$$K^p = \frac{\nu}{\Lambda_{T,1}q_1}\mathbb{E}\left[\Lambda_{N,2}^p Y_{N,2}^p\right]$$

Importantly, this Euler equation holds in all countries, even the pegging country. To see this, note that the pegging government's intervention alters the firm's problem in the pegging country in two offsetting ways: the tax alters the domestic price of all Arrow-Debreu securities, and thus the state-contingent valuation of nontraded output as well as the state-contingent valuation of nontraded consumption. The two effects cancel such that (11) holds in all countries.

Equation (6) defines the resource constraint for traded goods. Equation (7) defines

[8]See Appendix D for a formal setup and proof

the (three) resource constraints for non-traded goods in each country, and equation (8) defines the resource constraint for capital goods. Equations (9), (10), (11), (13), (14) define the three first-order conditions with respect to tradable consumption, the three first-order conditions with respect to non-tradable consumption and the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. These 14 equations implicitly define the following 14 endogenous variables: $\left\{C^n_{N,2}, C^n_{T,2}, K^n, \Lambda^n_{N,2}\right\}_{n\in\{p,t,o\}}, \Lambda_{T,2}$ and q_1.

4 Results

To study the model in closed form, we log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$) and all firms have a capital stock that is fixed at the deterministic steady state level. That is, we study the *incentives* to accumulate different levels of capital across countries, while holding the capital stock fixed. We thus ignore the feedback effect of differential capital accumulation on the size of risk premia. Doing so allows us to simplify the exposition of the solution.

We begin by characterizing the state contingent taxes that the pegging country can implement to impose a real or nominal exchange rate peg. Throughout, lowercase variables refer to natural logs.

Lemma 1

A tax on all assets paying off consumption goods in the pegging country of the form

$$z(\omega) = \zeta\frac{1-\tau}{\tau\left(\tau+\phi(1-\tau)\right)}\left(y^p_N - y^t_N\right) + \zeta\frac{(1-\tau)(1-\phi)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(\mu^p - \mu^t\right) + \zeta\frac{\gamma-1}{\tau+\phi(1-\tau)}\left(\chi^t - \chi^p\right)$$

implements a real exchange rate peg of strength ζ.

A tax on all assets paying off consumption goods in the pegging country p of the form

$$\tilde{z}(\omega) = z(\omega) + \zeta\frac{\gamma\tau+\phi(1-\tau)}{\tau+\phi(1-\tau)}\left(\mu^p - \mu^t\right)$$

implements a nominal exchange rate peg of strength ζ.

The cost of the peg, κ^p_{Cost}, is the sum of the change in the cost of purchasing state-contingent claims to tradable consumption

$$\kappa^p_{Cost} = \int Q(\omega) C^p_T d\omega - \int Q^*(\omega) C^{p*}_T d\omega.$$

Proof. See Appendix E. ■

4.1 Real Business Cycle Model

We first analyze the case where all households are active, $\phi = 1$, and the variance of the preference shock is zero ($\sigma_{\chi,n} = 0$). In this case, all endogenous variables are determined exclusively by shocks to the production of nontraded goods and our model coincides with the canonical real business cycle model of exchange rate determination (Backus and Smith, 1993).

4.1.1 The freely floating regime

In the absence of currency manipulation ($\zeta = 0$), equilibrium consumption of traded goods in an arbitrary country n (recall that lowercase variables denote logs) is given by

$$c^{n*}_T = \frac{(1-\tau)(\gamma-1)}{(1-\tau)+\gamma\tau} \left(\bar{y}_N - y^n_N\right),$$

where $\bar{y}_N = \sum_n \theta^n y^n_N$ is the average log per-capita output of nontraded goods across countries. The expression shows that households use the traded good to insure themselves against shocks to the output of nontraded goods. If $\gamma > 1$, households receive additional tradables whenever they have a lower-than-average output of non-traded goods, and vice versa.[9]

[9]Condition $\gamma > 1$ ensures that the cross-partial of marginal utility from tradable consumption with respect to the nontraded good is negative, that is, the relative price of a country's nontraded good falls when its supply increases. As most empirical applications of international asset pricing models find a relative risk aversion significantly larger than one, and an elasticity of substitution around one, we follow the literature in assuming that this condition is met, but refer to it whenever it is relevant (see Coeurdacier (2009) for a detailed discussion).

This risk-sharing behavior generates a shadow price of traded goods of the form given in (2),

$$\lambda_T^* = -(\gamma - 1)(1 - \tau) \sum_n \theta^n y_N^n, \tag{15}$$

where each country's weight is proportional to its size: shocks to the productivity of larger countries affect a larger measure of households and thus tend to spill over to the rest of the world in the form of higher shadow prices of traded goods. When $\gamma > 1$, the shadow price of traded goods falls with the average output of nontraded goods across countries. Thus, λ_T tends to be low in "good" states of the world when countries experience positive productivity shocks in their non-traded sectors.

The real exchange rate between two arbitrary countries f and h is

$$s^{f,h*} = \lambda^{f*} - \lambda^{h*} = \frac{\gamma(1-\tau)}{(1-\tau) + \gamma\tau} \left(y_N^h - y_N^f \right)$$

The currency of the country with lower per-capita output of non-traded goods appreciates because its final consumption bundle is expensive relative to that in other countries.

Inspecting λ_T^* and $s^{f,h*}$ shows that currencies of larger countries tend to appreciate when the shadow price of traded goods is high whenever a country suffers a low productivity shock and its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is (note that $s^{f,h*}$ is independent of θ). However, a shock to a larger country has a larger impact on the rest of the world. For example, in states of the world in which the US (the largest economy in the world) draws a low productivity shock, it imports a large share of the world's traded goods, raising the shadow price of traded goods for every country. As a result, the US dollar tends to appreciate when λ_T is high, producing a positive covariance between the US real exchange rate and λ_T. It immediately follows from the first equality in (3) that larger countries have a lower risk-free rate.

$$r^{f*} + \mathbb{E}s^{f,h*} - r^{h*} = cov\left(\lambda_T^*, p^{h*} - p^{f*}\right) = \frac{(\gamma-1)\gamma(1-\tau)^2}{1 + (\gamma - 1)\tau} \left(\theta^h - \theta^f\right) \sigma_N^2$$

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, we can rearrange the Euler equation for capital accumulation (11) and obtain a form similar to (3): take logs of both sides of the equation, substitute $\lambda^n_{N,2} = p^n_{N,2} + y^n_{N,2}$, and take differences across countries to obtain

$$k^{f*}_N - k^{h*}_N = \frac{1}{2} var\left(p^f_N + y^f_N\right) - \frac{1}{2} var\left(p^h_N + y^h_N\right) + cov\left(p^f_N + y^f_N - p^h_N - y^h_N, \lambda_T\right) \quad (16)$$

where we can interpret $p^f_N + y^f_N$ as the value of non-traded output in terms of traded goods, or as the payoff of a unit of stock in the non-tradable sector of country f. Ignoring the two variance terms on the right hand side for the moment, this expression suggests that countries whose output increases in value when λ_T is high should accumulate more capital per capita. The solution of the model yields

$$p^{f*}_N + y^{f*}_N = \frac{(1-\tau)(\gamma-1)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y^f_N\right).$$

It shows that differences in the payoff of stocks behave in the same way as exchange rates: when country f suffers a low productivity shock, its currency appreciates and the value of its firm's output in terms of traded goods increases. If country f is large, the same adverse productivity shock also raises λ_T, inducing a positive covariance between λ_T and the value of the firm's output.

Larger countries thus not only have lower interest rates but also have incentives to accumulate higher capital-output ratios. Solving for the variances and covariances in (16) yields

$$k^{f*}_N - k^{h*}_N = \frac{(\gamma-1)^3(1-\tau)^2\tau}{1+(\gamma-1)\tau}\left(\theta^f - \theta^h\right)\sigma^2_N.$$

It is efficient to accumulate more capital in the larger country because a larger capital stock in larger country represents a good hedge against global consumption risk. Households around the world are worried about states of the world in which the large country receives a low output from its non-tradable sector, because larger countries transmit these shocks to the rest of the world through a higher shadow price of tradable consumption. Although households cannot affect the realization of productivity shocks, they can partially insure themselves against low output in the non-tradable sector of large countries by accumulating more capital in these countries. This raises expected output in the non-tradable sector and dampens the negative effects of a low productivity shock.

4.1.2 Internal effects of a peg

We have described allocations in an economy with freely floating exchange rates. All else equal, larger countries have lower risk-free rates and higher capital investment per capita. Now, we analyze how a country can influence these allocations with an exchange rate peg. We start by analyzing the effect of the exchange rate peg on allocations and prices in the pegging country alone. Afterwards, we analyze its impact on prices and quantities in the rest of the world.

The exchange rate peg makes the price level in the pegging country behave more in line with the price level in the target country.

$$\lambda^p = \lambda^{p*} + (1-\theta^p)\zeta\frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}\left(y^p_{N,2} - y^t_{N,2}\right)$$

Similar to the intervention considered in (1), the real exchange rate peg increases the effect of the target country's shock, while also decreasing the weight of its own shock. The same is true for the value of the firm's output in the pegging country

$$p^p_N + y^p_N = (p^{p*}_N + y^{p*}_N) + \zeta\frac{(1-\tau)\left(\theta^p + (\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\left(y^p_N - y^t_N\right).$$

If the target country is sufficiently large relative to the pegging country, the exchange rate peg thus increases the covariance of both λ^p and $p^p_N + y^p_N$ with λ_T, lowering the country's interest rate and increasing its capital accumulation.

Proposition 1

In the real business cycle model of exchange rate determination with $\gamma > 1$, a country that imposes a hard real exchange rate peg on a target country larger than itself lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.

Proof. When the smaller country imposes a hard real exchange rate peg, the interest rate differential becomes

$$r^p + \mathbb{E}s^{p,t} - r^t = cov\left(\lambda_T, p^t - p^p\right) = \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \frac{(1-\tau)^2\gamma\left(\theta^t - \theta^p\right)(\gamma-1)\tau}{\tau\left(1+(\gamma-1)\tau\right)}\sigma^2_N.$$

See Appendix G for the corresponding proof for capital accumulation. ■

Aside from these effects on interest rates and capital accumulation, maintaining the currency peg affects the pegging government's resources (currency reserves). From (15), we already know that the cost of the peg is simply the cost of altering the state-contingent purchases of traded goods in world markets. The cost of the peg thus depends on the change in the pegging country's equilibrium consumption of traded goods. We can write this change as

$$c_T^p - c_T^{p*} = \zeta \frac{(1-\tau)(1-\theta^p)}{\tau\left((1-\tau)+\gamma\tau\right)} \left(y_N^t - y_N^p\right).$$

When the target country receives a relatively bad productivity shock ($y_N^t < y_N^p$), its price of consumption increases. To mirror this increase, the pegging country reduces its consumption of traded goods relative to the freely floating regime, ships additional traded goods to the rest of the world, and thus raises its own marginal utility. Conversely, when the pegging country receives a relatively bad shock, its price of consumption would ordinarily increase. To offset this increase and prevent its currency from appreciating, it imports even more traded goods than it would have ordinarily.

The peg thus induces the pegging country to sell additional traded goods in response to adverse productivity shocks in the target country, and to buy additional traded goods in response to adverse productivity shocks at home. If the target country is larger than the pegging country, traded goods are more expensive in the states in which it sells than in the states in which it buys. In this case, the peg induces the pegging country to provide insurance to the world market, pocketing an insurance premium.

Proposition 2

In the real business cycle model of exchange rate determination with $\gamma > 1$, if the pegging country is small, $\theta^p = 0$, then the cost of the peg decreases with the size of the target country. Additionally, the cost of the peg is negative if and only if

$$\theta^t > \frac{\zeta + (\gamma-1)\tau}{(\gamma-1)^2\tau^2}.$$

Proof. Combining equations (18) with the expression for the cost of the peg in Lemma

1 allows us to derive the following expression for the (log-linear) cost of the peg

$$\begin{aligned}\log\left(\kappa^{p}_{Cost}\right) &= \log\mathbb{E}\left[\exp\left(\lambda_{T,2}+c^{p}_{T,2}-\lambda_{T,1}\right)\right]-\log\mathbb{E}\left[\exp\left(\lambda^{*}_{T,2}+c^{p*}_{T,2}-\lambda^{*}_{T,1}\right)\right]\\ &= \frac{1}{2}var\left(\lambda_{T,2}+c^{p}_{T,2}-\lambda_{T,1}\right)-\frac{1}{2}var\left(\lambda^{*}_{T,2}+c^{p*}_{T,2}-\lambda^{*}_{T,1}\right)\\ &= \frac{\left[(\zeta+(\gamma-1)\tau)-\tau^{2}(1-\gamma)^{2}\theta^{t}\right](1-\tau)^{2}\zeta\sigma^{2}_{N}}{\tau^{2}\left(1+(\gamma-1)\tau\right)^{2}}\end{aligned}$$

where we derive second line by assuming prices and quantities are log-normally distributed. Recall, $\lambda_{T,1} = \log\left(\mathbb{E}\left[\Lambda_{T,2}\right]\right) = \mathbb{E}\lambda_{T,2} + \frac{1}{2}var\left(\lambda_{T,2}\right)$ is the shadow price of a traded good in the first period. The expression derived above is decreasing in the size of the target country, and becomes negative if and only if the target country is large enough. ■

If the target country is sufficiently large relative to the pegging country and risk aversion is sufficiently high, this insurance premium can be so large that the peg generates revenues for the government, accumulating rather than depleting currency reserves.

When the pegging country itself is large ($\theta^{p} > 0$), its purchases and sales of traded goods also affect the equilibrium shadow price of traded goods, λ_{T}, increasing the cost of the peg. The reason is that pegging effectively increases the volatility of shipments of traded goods to the rest of the world. In states where the pegging country has a bad productivity shock, it imports more traded goods than it ordinarily would have. In states where the target country has bad productivity shock, it exports more. The more price impact the pegging country has, the more costly it therefore is to maintain the peg.

4.1.3 External effects of a peg

When the pegging country is large ($\theta^{p} > 0$), the exchange rate peg affects traded consumption and prices in the rest of the world. The shadow price of traded goods is given as

$$\lambda_{T,2} = -(1-\tau)(\gamma-1)\bar{y}_{N} + \frac{\theta^{p}(1-\tau)}{\tau}\zeta\left(y^{t}_{N,2}-y^{p}_{N,2}\right).$$

The second term on the right hand side shows that if the pegging country is large, the peg dampens the effect of the target country's shocks on the shadow price of tradables, reducing the extent to which it spills over to the rest of the world and making it *less* systemic. As a result, the currency peg decreases the covariance between the target country's real exchange rate and $\lambda_{T,2}$, increasing the target country's interest rate and lowering capital accumulation.

Proposition 3

In the real business cycle model with $\gamma > 1$, a country that becomes the target of a peg imposed by a large country experiences a rise in its risk-free interest rate, a fall in capital accumulation, and a fall in average wages relative to all other countries.

Proof. The interest rate differential between the target and outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta \frac{\theta^p (1-\tau)^2 \gamma}{\tau \left(1 + (\gamma - 1)\tau\right)} \sigma_N^2.$$

See Appendix H for the corresponding proof for capital accumulation. ■

In this sense, a currency peg can divert capital from the target country to the pegging country even though it has no effect on the level of the real exchange rate. This finding is particularly interesting because it sheds new light on recent public controversies, for example between Chinese and U.S. officials, which usually focuses on the idea that an under-valuation of the Chinese real exchange rate favors Chinese workers at the expense of U.S. workers. By contrast our results suggest, that even a currency peg that manipulates the variance but not the level of the real exchange rate can have this effect.

4.1.4 Welfare & the Rationale for Pegging

Finally, we may use the real business cycle version of our model to perform a simple welfare calculation. So far we have assumed that a currency peg has two objectives, to reduce the variance of the log real exchange rate (P1) while not distorting its level (P2). For the purposes of this calculation, we now drop the objective (P2) and debate the cost of the peg to households in the pegging country. That is, households in the pegging country bear the costs of imposing the exchange rate peg by shifting the level of their tradable consumption in all states of the world.

Changes in the expected utility of households in the pegging country are a result of changes in the expected level and variance of consumption. We have already seen that a peg to a larger country can increase the level of consumption by increasing capital accumulation and by generating revenues (a negative cost of the peg). However, the peg also increases the variance of consumption, which reduces expected utility. In equilibrium, the latter effect dominates and the exchange rate peg decreases the welfare of households within the pegging country. Similarly, the currency peg decreases the volatility of consumption in the target country because it dampens the effect of the target country's shock on the shadow price of traded goods.

Proposition 4

In the real business cycle model of exchange rate determination with $\gamma > 1$,

1. *a country that imposes an exchange rate peg decreases the welfare of its households.*
2. *a country that becomes the target of an exchange rate peg imposed by a smaller country with positive mass will see the volatility of its households' consumption decrease. Expected utility in the target country increases as a result of the peg.*

Proof. See Appendix I. ■

In contrast to the positive results outlined above, these welfare results do not generalize easily to our full model with incomplete markets, inflation, and preference shocks and should therefore be interpreted with some caution.

Moreover, although our model does not deliver a clear welfare-based motive for pegging, it may nevertheless rationalize currency pegs, and in particular the pegs to the US observed in the data, if policymakers have an interest in increasing capital accumulation, increasing wages, or generating revenues from a peg. For example, a peg to the largest economy in the world may be optimal if policymakers in a pegging country maximize a function of the form

$$EU^n + \mu_1 K^n - \mu_2 \kappa^p,$$

where μ_1 and μ_2 are constants that may reflect the political influence of workers, externalities from capital accumulation, or a motive for generating revenues in a way that avoids direct taxation of households or firms.

4.2 Incomplete Markets and Preference Shocks

In the previous section, we established the impact of an exchange rate peg on the equilibrium allocation in a conventional real business cycle model with complete markets. Although the real business cycle model is an important benchmark, it has a number of well-known empirical shortcomings. First, it predicts a perfectly negative correlation between appreciations of the real exchange rate and aggregate consumption growth — a currency appreciates when the country's aggregate consumption falls (Backus and Smith, 1993). Second, the model predicts that consumption should be more correlated across countries than output, whereas the opposite is true in the data (Backus, Kehoe, and Kydland, 1994). Third, real exchange rates and terms of trade seem much too volatile to be rationalized exclusively by real (productivity) shocks alone (Chari, Kehoe, and McGrattan, 2002). As a result, many authors have argued for incomplete market models that allow for an effect of monetary shocks on equilibrium real exchange rates, or models with demand shocks.

In this subsection, we analyze the effects of exchange rate manipulation in our full model, featuring inflation shocks, market segmentation, and preference shocks, and show that the intuition and all positive results from the previous section carry over to this more general model. To simplify the discussion, we derive all results for the case where productivity is deterministic, $\sigma_N = 0$.

The punchline is that both inflation and preference shocks generate a relationship between exchange rates and the shadow price of traded goods identical to the structure in (1) and (2): inflation shocks affect exchange rates by shifting away resources within a given country from inactive households, who are excluded from financial markets (and thus are irrelevant for prices in international markets), to active households whose marginal utilities price assets in international markets. Inactive households hold nominal bonds denominated in their national currencies and are thus vulnerable to inflation shocks. A positive inflation shock to the price of traded goods in terms of the domestic currency acts as an "inflation tax" on inactive households. The higher the inflation shock, the less their nominal bonds are worth and the less these households are able to consume. Since inflation shocks have no bearing on the real resources available for consumption, this reduction of inactive household's wealth shifts resources towards the country's active households such that they receive more traded and nontraded goods, depreciating the domestic price of consumption in both real and nominal terms. At the same time, risk-sharing compels the active households to ship some of the additional traded goods to active households in other countries, transmitting part of the inflation shock to active households in other countries via the shadow price of traded goods.

Similarly, preference shocks move exchange rates by shifting the level of utility derived from each unit of consumption. A high preference shock reduces the marginal utility of households' consumption, and thus also depreciates the country's currency in real and nominal terms. Again, risk-sharing with households in other countries then compels domestic households to ship traded goods to the rest of the world, transmitting part of the shock to other countries.

Solving our model with inflation and preference shocks yields

$$\begin{aligned}\lambda^p = & -\frac{(1-\phi)\gamma^2\tau}{\phi\left(\phi(1-\tau)+\gamma\tau\right)}\bar{\mu} - \frac{(1-\phi)(1-\tau)\gamma}{\phi(1-\tau)+\gamma\tau}\mu^p - \frac{\gamma\tau(\gamma-1)}{\gamma\tau+(1-\tau)\phi}\bar{\chi} - \frac{(1-\tau)(\gamma-1)\phi}{\gamma\tau+(1-\tau)\phi}\chi^p \\ & + (1-\theta^p)\zeta\frac{\gamma(1-\tau)(1-\phi)}{\phi(1-\tau)+\gamma\tau}\left(\mu^p-\mu^t\right) + (1-\theta^p)\,\zeta\frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau+(1-\tau)\phi}\left(\chi^p-\chi^t\right)\end{aligned}$$

and

$$\begin{aligned}\lambda_T = & -\gamma\left(\frac{1-\phi}{\phi}\right)\bar{\mu} - (\gamma-1)\bar{\chi} \\ & + \zeta\frac{\theta^p(1-\tau)}{\gamma\tau}\left(\gamma(1-\phi)\left(\mu^t-\mu^p\right) + \phi(\gamma-1)\left(\chi^t-\chi^p\right)\right),\end{aligned}$$

where $\bar{\mu} = \sum_n \theta^n \mu^n$ and $\bar{\chi} = \sum_n \theta^n \chi^n$ are the weighted sums of inflation and preference shocks in all countries, respectively. The first lines in both expressions show the price of country p's consumption index and the shadow price of traded goods in the freely floating regime. Note that, as with productivity shocks, a high realization of both inflation and preference shocks depreciates the price of domestic consumption and lowers λ_T in proportion to the size of the country. The second line in both expressions shows that, again, a currency peg makes the pegging country's price of consumption behave more like the target country's price, and that the peg lowers the weight of the target country's shock in λ_T, while simultaneously increasing the weight of the pegging country's shock.

This change in the size of spill-overs in the shadow price of traded goods results from the fact that active households in the pegging country ship additional traded goods to the rest of the world whenever the target country appreciates, and import additional traded goods whenever shocks raise the price of their own consumption relative to that in the target country.

$$c^{p}_{T,2} - c^{p*}_{T,2} = \zeta \Xi^{p}_{T} \left(\gamma(1-\phi)\left(\mu^{t} - \mu^{p}\right) + \phi(\gamma - 1)\left(\chi^{t} - \chi^{p}\right)\right), \qquad (17)$$

where Ξ^{p}_{T} is a positive constant derived in the Appendix J.

It follows directly that all of our positive predictions about the effects of currency pegs carry over to our full model.

Proposition 5

In the full model with market segmentation, inflation shocks, preference shocks, and productivity shocks with $\gamma > 1$,

1. *a smaller country that imposes a hard real exchange rate peg on a sufficiently large target country lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.*

2. *when the pegging country is small, the cost of the peg decreases with the size of the target country and increases with the size of the pegging country.*

3. *if a country becomes the target of a peg imposed by a large country ($\theta^{p} > 0$), its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.*

Proof. See Appendix K ■

In addition to reinforcing the main insights from our analysis of the real business cycle case, the full model also improves the quantitative implications of the model along the three dimensions outlined above. The the combination of market segmentation, inflation shocks, and preference shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth, lowers the correlation of aggregate consumption across countries, and increases the volatility of real and nominal exchange rates (Alvarez et al., 2002; Pavlova and Rigobon, 2007; Kollmann, 2012).

4.3 Nominal Pegs

Up until now, we have characterized the internal and external effects of a real exchange rate peg. In practice, nominal exchange rate pegs are often easier to implement. In fact, most exchange rate pegs in the data appear to be nominal.

The log nominal exchange rate between two countries is equal to the ratio of their nominal price indices

$$\tilde{s}^{f,h} = p_2^f - \mu^f - \left(p_2^h - \mu^h\right).$$

When markets are complete ($\phi = 1$), inflation shocks impact real allocations only through their impact on the pegging country's tradable consumption. Lemma 1 shows that the state-contingent tax that is used to implement a nominal exchange rate peg, is identical to that used to impose a real exchange rate peg plus random noise induced by inflation shocks. Naturally, the nominal peg then leads to allocations and risk premia identical to those under the real peg plus an additional noise component. As long as the volatility of the inflation shock is low relative to productivity and preference shocks (in the data, real and nominal exchange rates are highly correlated), all positive results then continue to hold.

If $\phi < 1$, real and nominal exchange rate pegs both respond to differences in inflation shocks between the pegging and target countries. The only difference is the magnitude of this adjustment. A nominal peg forces households to change tradable consumption to offset the differences in real price levels as well as the differences in inflationary shocks. Hence, we might suspect that a nominal exchange rate peg is just a "stronger" version of the real exchange rate peg, and that there is some equivalence between a nominal exchange rate peg of strength $\tilde{\zeta}$ and a real exchange rate peg of strength ζ. The following proposition shows that this is indeed the case.

Proposition 6

Suppose the variance of real shocks and preference shocks are zero ($\sigma_{N,n}, \sigma_{\chi,n} = 0$)*. A tax* *on Arrow-Debreu securities purchased by residents of country p of the form*

$$z(\omega) = \tilde{\zeta} \frac{\gamma - (\gamma - 1)(1 - \tau)\phi}{\gamma\tau\left(\tau(1 - \phi) + \phi\right)}$$

implements a nominal exchange rate peg of strength $\tilde{\zeta}$*. This is equivalent to implementing a real exchange rate peg of strength*

$$\zeta = \tilde{\zeta} \frac{\gamma - (\gamma - 1)(1 - \tau)\phi}{\gamma(1 - \tau)(1 - \phi)}.$$

5 Conclusion

This paper solves an international asset pricing model that endogenizes the stochastic properties of exchange rates, international asset prices, and the level of capital accumulation across countries. It explores the effects of exchange rate pegs on the economies of the target country and of countries outside the peg. We are able to characterize the impact of the peg on the consumption of households in each country, the exchange rates between countries and the spreads on bonds and stocks in the world. Additionally, we solve for the impact of exchange rate pegs on differences in capital investment between countries.

References

Alvarez, F., A. Atkeson, and P. J. Kehoe (2002). Money, interest rates, and exchange rates with endogenously segmented markets. *Journal of Political Economy 110*(1), 73–112.

Bacchetta, P. and E. van Wincoop (2000). Does exchange-rate stability increase trade and welfare. *American Economic Review 90*(5), 1093–1109.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994). Determinants of the trade balance and the terms of trade: The j curve? *American Economic Review 84*(1), 84–103.

Backus, D. K. and G. W. Smith (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics 35*, 297–316.

Bergin, P. R. and G. Corsetti (2015). International competitiveness and monetary policy. *Unpublished Manuscript*.

Bianchi, J. (2011, December). Overborrowing and systemic externalities in the business cycle. *American Economic Review 101*(7), 3400–3426.

Calvo, G. and E. Mendoza (2000). Rational contagion and the globalization of securities markets. *Journal of International Economics 51*(1), 79–113.

Campbell, J. Y., K. Serfaty-De Medeiros, and L. M. Viceira (2010). Global currency hedging. *The Journal of Finance 65*(1), 87–121.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002). Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies 69*(3), 533–563.

Coeurdacier, N. (2009). Do trade costs in goods market lead to home bias in equities? *Journal of International Economics 77*(1), 86–100.

Corsetti, G., L. Dedola, and S. Leduc (2010). Optimal monetary policy in open economies. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics* (1 ed.), Volume 3, Chapter 16, pp. 861–933. Elsevier.

Costinot, A., G. Lorenzoni, and I. Werning (2014). A theory of capital controls as dynamic terms-of-trade manipulation. *Journal of Political Economy 122*(1), 77–128.

David, J. M., E. Henriksen, and I. Simonovska (2014). The risky capital of emerging markets. NBER Working Paper Series 20769, National Bureau of Economic Research.

Devereux, M. B. and C. Engel (2003). Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility. *Review of Economic Studies*.

Farhi, E. and X. Gabaix (2015). Rare disasters and exchange rates. *Quarterly Journal of Economics forthcoming*.

Farhi, E. and I. Werning (2012). Dealing with the trilemma: Optimal capital controls with fixed exchange rates. *mimeo, Massachusetts Institute of Technology*.

Farhi, E. and I. Werning (2013). Dilemma not trilemma? capital controls and exchange rates with volatile capital flows. *mimeo, Massachusetts Institute of Technology*.

Gali, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies 72*, 707–734.

Gourinchas, P.-O. and H. Rey (2007). From world banker to world venture capitalist: U.s. external adjustment and the exorbitant privilege. In *G7 Current Account Imbalances: Sustainability and Adjustment*, NBER Chapters, pp. 11–66. National Bureau of Economic Research, Inc.

Govillot, N., H. Rey, and P. Gourinchas (2010). Us external returns and the exorbitant duty. 2010 Meeting Papers 371, Society for Economic Dynamics.

Hassan, T. A. (2013). Country size, currency unions, and international asset returns. *The Journal of Finance 68*(6), 2269–2308.

Hassan, T. A. and R. Mano (2015). Forward and spot exchange rates in a multi-currency world. *mimeo University of Chicago*.

Hassan, T. A., T. M. Mertens, and T. Zhang (2015). Not so disconnected: Exchange rates and the captial stock. *mimeo University of Chicago*.

Helpman, E. and A. Razin (1987). Exchange rate management: Intertemporal tradeoffs. *American Economic Review 77*(1), 107–123.

Jeanne, O. and A. Korinek (2010). Excessive volatility in capital flows: A pigouvian taxation approach. *American Economic Review 100*(2), 403–407.

Kollmann, R. (2002). Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics*.

Kollmann, R. (2012). Limited asset market participation and the consumption-real exchange rate anomaly. *Canadian Journal of Economics*.

Korinek, A. (2013). Capital controls and currency wars. *mimeo University of Maryland*.

Lustig, H., N. Roussanov, and A. Verdelhan (2011). Common risk factors in currency markets. *Review of Financial Studies 24*(11), 3731–3777.

Lustig, H. and A. Verdelhan (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review 97*(1), 89–117.

Maggiori, M. (2013). Financial intermediation, international risk sharing, and reserve currencies. Mimeo.

Martin, I. (2012). The forward premium puzzle in a two-country world. Mimeo.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf (2012). Carry trades and global foreign exchange volatility. *Journal of Finance 67*(2), 681–718.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf (2013). Information flows in dark markets: Dissecting customer currency trades. Bis working papers, Bank for International Settlements.

Miranda-Agrippino, S. and H. Rey (2015). World asset markets and the global financial cycle. NBER Working Paper Series 21722, National Bureau of Economic Research, Inc.

Parrado, E. and A. Velasco (2002). Optimal interest rate policy in a small open economy. NBER Working Paper Series 8721, National Bureau of Economic Research, Inc.

Pavlova, A. and R. Rigobon (2007). Asset prices and exchange rates. *Review of Financial Studies 20*(4), 1139–1180.

Powers, T. Y. (2015). The commodity currency puzzle. *mimeo Harvard University*.

Ready, R., N. Roussanov, and C. Ward (2013). Commodity trade and the carry trade: a tale of two countries. NBER Working Papers 19371, National Bureau of Economic Research, Inc.

Reinhart, C. M. and K. Rogoff (2011). From financial crash to debt crisis. *American Economic Review 101*, 1676–1706.

Reinhart, C. M. and K. S. Rogoff (2004). The modern history of exchange rate arrangements: a reinterpretation. *The Quarterly Journal of economics 119*(1), 1–48.

Richmond, R. (2015). Trade network centrality and the currency carry trade. *Mimeo UCLA*.

Schmitt-Grohé, S. and M. Uribe (2012). Pegs and pain. *mimeo Columbia University*.

Tran, N.-K. (2013). Growth risk of nontraded industries and asset pricing. *Mimeo Olin Business School*.

Verdelhan, A. (2015). The share of systematic risk in bilateral exchange rates. *mimeo MIT Sloan*.

A Differences in Log Asset Returns

Because markets are competitive, all prices outside of the pegging country must coincide with ratios of shadow prices. In particular, we can solve for the state-contingent price of an Arrow-Debreu security that pays one unit of the traded good in state ω in the target or outside country as

$$Q(\omega) = \frac{\Lambda_{T,2}(\omega)}{\Lambda_{T,1}} g(\omega) \tag{18}$$

where $\Lambda_{T,2}(\omega)$ is the shadow price of a traded good in state ω and $\Lambda_{T,1} = \mathbb{E}\left[\Lambda_{T,2}(\omega)\right]$ is the shadow price of a traded good in the first period, prior to the realization of shocks.

Consider an asset that pays off $X(\omega)$ units of the traded good. The value of this asset

$$V_X = \mathbb{E}\left[\frac{\Lambda_{T,2}(\omega)}{\Lambda_{T,1}} X(\omega)\right]$$

If we assume asset returns and marginal utilities are log-normally distributed and take logs of both sides of the previous equation

$$v_X = \mathbb{E}\left[\lambda_{T,2} - \lambda_{T,1} + x\right] + \frac{1}{2} var\left(\lambda_{T,2}\right) + \frac{1}{2}\left(x\right) + cov\left(\lambda_{T,2}, x\right)$$

because $\Lambda_{T,1}$ is deterministic and known in the first period. Hence, the log expected return on an asset with payoff X is

$$\begin{aligned} \log ER[X] = \log \mathbb{E}\left[\frac{X}{V_X}\right] &= \mathbb{E}\left[x\right] + \frac{1}{2} var\left(x\right) - v_X \\ &= \lambda_{T,1} - \mathbb{E}\left[\lambda_{T,2}\right] - \frac{1}{2} var\left(\lambda_{T,2}\right) - cov\left(\lambda_{T,2}, x\right) \end{aligned}$$

Taking the difference between the log expected returns of two different assets with payoffs X and Z yields

$$\log ER[X] - \log ER[Z] = cov\left(\lambda_{T,2}, z - x\right) \tag{19}$$

The risk free bond in country n pays P^n units of the traded good in the second period. We plug the bond payments P^h and P^f into equation (19) to derive equation (3).

B Equilibrium Consumption of Inactive Households

Inactive households in country n maximize utility, defined in equation (4), in each state of the world by splitting their wealth $\exp(-\mu_t^n)P_t^n$ optimally between traded and non-traded goods. Their optimization problem can be written as maximizing (4) in each state subject to (5)

$$\max_{\hat{C}_{T,2}(i),\hat{C}_{N,2}(i)} \frac{1}{1-\gamma}\left(e^{\chi^n}\hat{C}_{T,2}\left(i\right)^{\tau}\hat{C}_{N,2}\left(i\right)^{1-\tau}\right)^{1-\gamma}$$
$$\text{s.t. } \hat{C}_{T,2}\left(i\right)+P_{N,2}\hat{C}_{N,2}\left(i\right)=\exp(-\mu_t^n)P_t^n$$

We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $\hat{C}_{T,2}\left(i\right)$ and $\hat{C}_{N,2}\left(i\right)$.

Inactive households consume an optimal mix of traded and non-traded goods given by

$$\hat{C}_{T,2}^n=\exp(-\mu^n)(\tau P_2^n),\ \hat{C}_{N,2}^n=\exp(-\mu^n)\left(\frac{(1-\tau)P_2^n}{P_{N,2}^n}\right)$$

where $\hat{C}_T^n$ and $\hat{C}_N^n$ are the consumption of traded and non-traded goods by inactive agents in country n, respectively.

C Active Household's Problem in Target and Outside Countries

We denote the Lagrange multiplier on the households' budget constraint in country n by Ξ^n. Then, the first-order conditions with respect to traded and non-traded consumption are

$$U_{C_T^n}(i)g(\omega)=\Xi^nQ(\omega)$$
$$U_{C_N^n}(i)g(\omega)=\Xi^nQ(\omega)P_N^n(\omega)$$

The transfers κ^n equalize the marginal utility of wealth across countries. The value of endowing any household an additional unit of the traded good in the first period is the same. Hence, $\Xi^n = \Lambda_{T,1}$ for all n. Replacing Ξ^n with $\Lambda_{T,1}$ in the previous equations and using the definition of the price of a state contingent claim yields equation (9). We define $U_{C_N^n}(i) = \Lambda_{N,2}^n$ to be the shadow price of non-traded consumption. The price of non-traded goods is the ratio of the shadow prices of traded and non-traded consumption

$$\Lambda_{N,2}^n(\omega) = \Xi^n Q(\omega) P_{N,2}^n(\omega) \Rightarrow P_N^n = \frac{\Lambda_{N,2}^n}{\Lambda_{T,2}}$$

We use these definitions to derive equation (10).

The first order condition with respect to capital accumulation is

$$\Xi^n q_1 = \Xi^n \int Q(\omega) P_N^n(\omega) e^{\eta^n} \nu \left(K_N^n\right)^{\nu-1} d\omega$$

which simplifies to

$$K_N^n = \frac{\nu}{\Xi^n q_1^n} \mathbb{E}\left[\Xi^n \frac{\Lambda_{T,2}}{\Lambda_{T,1}} P_N^n Y_N^n\right]$$

We use the identity $\Xi^n = \Lambda_{T,1}$ and the definition of $P_{N,2}^n$ to derive the Euler equation (11).

D Equilibrium Consumption of Active Households in the Pegging Country

The first order conditions of the household's problem in the pegging country are

$$U_{C_T^p}(i) g(\omega) = \Xi^p Q(\omega) Z(\omega)$$
$$U_{C_N^p}(i) g(\omega) = \Xi^p Q(\omega) Z(\omega) P_N^p(\omega)$$

We assume the transfer κ^p equalized the marginal utility of wealth prior to the government's decision to impose an exchange rate peg. We consider two cases for the government transfer $\bar{Z}$. We assume that the government in the pegging country has reserves, which it can use to fund the exchange rate peg. In this section, we assume the transfer $\bar{Z}$ equalizes $\Xi^p = \Lambda_{T,1}$. We use this definition to calculate the cost of the peg, which is the amount of traded goods the government must expend from it's set of reserves. When we describe the welfare effects of the exchange rate peg, we consider a second case where the government does not fully fund the exchange rate peg. $\bar{Z}$ simply rebates (lump sum) the tax revenues from the state contingent tax back to the households. These results are derived in a later.

We derive first order conditions when the government uses reserves to pay for the exchange rate peg. By setting $\Xi^p = \Lambda_{T,1}$, we abstract from the wealth effects from imposing the exchange rate peg. When $\Xi^p = \Lambda_{T,1}$ the first-order condition with respect to traded goods simplifies to (13). Again, we define $\Lambda^p_{N,2}$ to be the marginal utility of non-traded consumption. Hence,

$$\Lambda^p_{N,2}(\omega) = Z(\omega)\Xi^p Q(\omega) P^p_{N,2}(\omega) \Rightarrow P^p_N = \frac{\Lambda^p_{N,2}}{Z(\omega)\Lambda_{T,2}}$$

We use this definition to simplify and derive equation 14.

The first order condition with respect to capital accumulation in the pegging country is

$$\Xi^p q_1 = \Xi^p \int Z(\omega) Q(\omega) P^p_N(\omega) e^{\eta^p} \nu \left(K^p_N\right)^{\nu-1} d\omega$$

Using the definition of $Q(\omega)$ and $P^p_N(\omega)$, this simplifies to

$$K^p_N = \frac{\nu}{q_1 \Xi^p} \mathbb{E}\left[\Xi^p Z \frac{\Lambda_{T,2}}{\Lambda_{T,1}} \frac{\Lambda^p_{N,2}}{Z\Lambda_{T,2}} Y^p_N\right]$$

where we can further simplify using $\Xi^p = \Lambda_{T,1}$

E Proof of Lemma 1

First, we solve for the form of the log-linear tax that implements the exchange rate peg. We assume a state contingent tax of the form

$$Z(\omega) = \left(\frac{Y^t_{N,2}}{Y^p_{N,2}}\right)^{a_1} \left(\frac{\chi^t}{\chi^p}\right)^{a_2} \left(\frac{\mu^t}{\mu^p}\right)^{a_3}$$

and solve for the volatility of the real and nominal exchange rate as a function of the tax $Z(\omega)$. We then search coefficients a_1, a_2, a_3 that decrease the volatility of the exchange rate between the pegging and target country such that (P1) holds.

To derive the cost of the peg, we first re-write the households' budget constraint to identify the components of the lump sum transfer, $\bar{Z}$. $\bar{Z}$ contains a lump-sum rebate of state contingent taxes, κ^p_{Tax} and the transfer from government reserves, κ^p_{Cost}. Conditional on already purchasing a quantity of capital, the household in the pegging country faces the following budget constraint

$$\begin{aligned}&\int (1+X(\omega))\, Q(\omega) \left(P^p_2(\omega)C^p_2(\omega) + \frac{1-\phi}{\phi}P^p_2(\omega)e^{-\mu^n}\right) d\omega \\ \leq &\frac{1}{\phi}\left[Y^p_{T,1} + \int (1+X(\omega))\, Q(\omega)P^p_{N,2}(\omega)\exp(\eta^p)\,(K^p)^\nu\, d\omega + \kappa^p + \kappa^p_{Tax} + \kappa^p_{Cost}\right].\end{aligned}$$

Where we have re-written the exchange rate peg as $Z(\omega) = 1 + X(\omega)$.

This lump-sum rebate of tax revenues is

$$\begin{aligned}\kappa^p_{Tax} = &\int X(\omega)Q(\omega)\left(P^p_2(\omega)C^p_2(\omega) + \frac{1-\phi}{\phi}P^p_2(\omega)e^{-\mu^n}\right) d\omega \\ &- \frac{1}{\phi}\int X(\omega)Q(\omega)P^p_{N,2}(\omega)\exp(\eta^p)\,(K^p)^\nu\, d\omega\end{aligned}$$

Subtracting the lump-sum rebate from both sides of the budget constraint and multiplying by ϕ yields

$$\begin{aligned}&\int Q(\omega)\left(\phi P^p_2(\omega)C^p_2(\omega) + (1-\phi)P^p_2(\omega)e^{-\mu^n}\right) d\omega \\ \leq &Y^p_{T,1} + \int Q(\omega)P^p_{N,2}(\omega)\exp(\eta^p)\,(K^p)^\nu\, d\omega + \kappa^p + \kappa^p_{Cost}\end{aligned}$$

Substituting in for the market clearing condition for non-traded goods yields

$$\int Q(\omega)\left[\phi C^p_T(\omega) + (1-\phi)\hat{C}^p_T(\omega)\right] d\omega \leq Y_{T,1} + \kappa^p + \kappa^p_{Cost}$$

We solve for lump-sum transfer that decentralizes the Social Planner's problem, κ^n, by repeating this exercise for an economy without an exchange rate peg. We find that the lump-sum transfer needed to equalize wealth across all households is

$$\kappa^n = \int Q^*(\omega) \left[\phi C_T^{p*}(\omega) + (1-\phi)\hat{C}_T^{p*}(\omega)\right] d\omega - Y_{T,1}$$

We plug in this expression for κ^p into the previous equation and solve for the cost of the peg.

F Log-linearized System of Equations

Equation (6) defines the resource constraint for traded goods. Equation (7) defines the (three) resource constraints for non-traded goods in each country, and equation (8) defines the resource constraint for capital goods. Equations (9), (10), (13), (14) define the three first order conditions with respect to tradable consumption and the three first order conditions with respect to non-tradable consumption. Finally, equation (11) defines the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. We log-linearize the model around the deterministic solution — the point at which the variances of all shocks are zero ($\sigma_{\chi,n}, \sigma_{N,n}, \tilde{\sigma} = 0$) and all firms have a capital stock that is fixed at the deterministic steady state level.

The log-linear first order conditions for the target and outside countries are

$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = \lambda_T$$
$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = \lambda_N^n$$

The log-linear first order conditions for the pegging country are

$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = \lambda_T + z$$
$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = \lambda_N^n + z$$

where z is the log-linear expression for the tax given by Lemma 1.

We log-linearize the Euler equation for capital accumulation for each country, given by equation (11)

$$\log(q_1) + \lambda_{T,1} + k_N^n = \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2} var\left(\lambda_N^n + y_N^n\right)$$

where we can treat $\log(q_1) + \lambda_{T,1}$ as a single quantity that tells us the cost of a unit of capital in terms of utility.

Finally, the log-linear resource constraints are

$$\phi c_N^n + (1-\phi)\left(-\mu^n - \tau\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right) = \eta^n + \nu k_N^n = y_N^n$$

$$\sum_{n=p,t,o} \theta^n \left[\phi c_T^n + (1-\phi)\left(-\mu^n - (1-\tau)\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right)\right] = \sum_{n=p,t,o} \theta^n y_{T,1}^n = 1$$

$$\sum_{n=p,t,o} \theta^n k_N^n = 1$$

This set of fourteen equations allows us to solve for the following fourteen unknowns $\{k_N^n, c_N^n, c_T^n, \lambda_N^n\}_{n=p,t,o}$, λ_T and $\log(q_1) + \lambda_{T,1}$. We can write these endogenous variables in terms of the following nine state variables $\{y_N^n, \mu^n, \chi^n\}$ for $n = p, t, o$. Note that agents only care about the total output of non-traded goods, $y_N^n = \eta^n + \nu k_N^n$, in each country in the second period.

G Proof of Proposition 1

In the real business cycle model of exchange rate determination with $\gamma > 1$, a smaller country that imposes a hard real exchange rate peg on a sufficiently large target country lowers its risk-free rate, increases capital accumulation, and increases the average wage in its country relative to all other countries.

The interest rate differential between the pegging and target country is

$$\begin{aligned} r^p + \mathbb{E}s^{p,t} - r^t &= cov\left(\lambda_T, p^t - p^p\right) \\ &= \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \zeta \frac{(1-\tau)^2\gamma\left(2\theta^p(1-\zeta) + \left(\theta^t - \theta^p\right)(\gamma-1)\tau\right)}{\tau\left(1 + (\gamma-1)\tau\right)}\sigma_N^2 \end{aligned}$$

When the smaller country imposes a hard real exchange rate peg, this expression simplifies to

$$r^p + \mathbb{E}s^{p,t} - r^t = cov\left(\lambda_T, p^t - p^p\right) = \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \frac{(1-\tau)^2\gamma\left(\theta^t - \theta^p\right)(\gamma-1)\tau}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

which implies the exchange rate peg decreases the risk free rate in the pegging country relative to the risk free rate in the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

From equation (16), we calculate the differential incentives to accumulate capital when the pegging country imposes a hard peg in the real business cycle economy

$$k_N^p - k_N^t = k_N^{p*} - k_N^{t*} + \frac{(\gamma-1)^3(1-\tau)^2\tau\left(\theta^t - \theta^p\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2$$

The last term of the right hand side expression shows that incentives to accumulate capital in the pegging country increase relative to the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

Because firms are competitive, wages are given by the marginal product of labor. $w^n = (1-\nu)\exp\left(\eta^n\right)\left(K_N^n\right)^\nu$. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg increases wages in the pegging country relative to all other countries.

H Proof of Proposition 3

If a country becomes the target of a peg imposed by a large country, its risk-free interest rate rises relative to the rest of the world, capital accumulation falls, and average wages fall relative to all other countries.

The interest rate differential between the target and outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = cov\left(\lambda_T, p^o - p^t\right) = \left(r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^p(1-\tau)^2\gamma}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

which implies the exchange rate peg increases the risk free rate in the target country relative to the risk free rate in the outside country.

Equation (16) tells us the differential incentives to accumulate capital when the pegging country imposes a hard peg in the real business cycle economy

$$k_N^t - k_N^o = k_N^{t*} - k_N^{o*} - \frac{\theta^p(\gamma-1)^2(1-\tau)^2}{(1+(\gamma-1)\tau)^2}\zeta\sigma_N^2$$

The last term of the right hand side expression shows that incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg decreases wages in the target country relative to all other countries.

I Proof of Welfare Results

The volatility of log consumption in the pegging country is

$$\begin{aligned} var\left(c^p\right) &= var\left(\tau c_T^p + (1-\tau)c_N^p\right) \\ &= var\left(c^{p*}\right) + \zeta\frac{2(1-\theta^p)(1-\tau)^2\left((1-\theta^p)\zeta - 1 + \left(\theta^t - \theta^p\right)(\gamma-1)\tau\right)}{(1+(\gamma-1)\tau)^2}\sigma_N^2 \end{aligned}$$

which shows that the volatility of consumption in the pegging country increases with the size of the target country. If

$$\theta^t > \theta^p + \frac{1-(1-\theta^p)\,\zeta}{(\gamma-1)\tau}$$

then the exchange rate peg increases the volatility of consumption in the pegging country. A corollary of this result is that a small country ($\theta^p = 0$) that imposes a hard exchange rate peg always increases the volatility of consumption of its households, and always decreases the expected utility of its households.

We investigate changes in expected utility by examining $(1-\gamma)U(i)$. As $(1-\gamma)U(i)$

increases, utility decreases. For household i in the pegging country, we calculate

$$\frac{d}{d\zeta}\log\left[(1-\gamma)U(i)\right] = \frac{d}{d\zeta}\left[(1-\gamma)\mathbb{E}\left(c^p\right) + \frac{(1-\gamma)^2}{2}var\left(c^p\right)\right]$$
$$= \frac{(\gamma-1)(1-\tau)^2\left((1-\theta^p)\,\zeta + \theta^p\left(1+\theta^t-\theta^p\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2 < 0$$

Hence

$$\frac{dU(i)}{d\zeta}\frac{1}{U(i)} > 0$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} < 0$ because $U(i) = \frac{1}{1-\gamma}\left(C_2^p\right)^{1-\gamma} < 0$. Hence, imposing an exchange rate peg decreases utility in the pegging country.

The volatility of log consumption in the target country is

$$var\left(c^t\right) = var\left(c^{t*}\right) - \zeta\frac{2\theta^p(1-\tau)^2\left(1-\theta^p\zeta + \left(\theta^t-\theta^p\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2$$

it is clear that $var\left(c^t\right)$ decreases when the pegging country imposes an exchange rate peg if the pegging country is smaller than the target country, $\theta^t > \theta^p$.

Again, we examine the quantity $(1-\gamma)U(i)$. For household i in the target country, we calculate

$$\frac{d}{d\zeta}\log\left[(1-\gamma)U(i)\right] = -\frac{\theta^p(\gamma-1)^2(1-\tau)^2\left(2\left(1-\theta^p\right) + \theta^p\left(1+\theta^t-\theta^p\right)\left(1+(\gamma-1)\tau\right)\right)}{\left(1-\theta^p\right)\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2 < 0$$

If we multiply both sides of the inequality by $U(i)$, we show $\frac{dU(i)}{d\zeta} > 0$ because $U(i) = \frac{1}{1-\gamma}\left(C_2^t\right)^{1-\gamma} < 0$. Hence, expected utility in the target country increases when the pegging country imposes an exchange rate peg.

J Expressions for Constants in the Incomplete Markets Model

The following constants are used to define the consumption of traded goods in the pegging country and the value of non-traded output in the incomplete markets model, respectively.

$$\Xi_T^p = \frac{(1-\theta^p)\left(\tau+(1-\tau)\phi\right)^2 + (1-\tau)(1-\phi)\left(\gamma\tau+(1-\tau)\phi\right)\left(1+(1-\tau)\right)(1-\tau)\phi}{\gamma\tau\left(\phi+(1-\tau)\phi\right)\left(\gamma\tau+\phi(1-\tau)\right)\left(\tau+(1-\tau)\phi\right)}$$

$$\Xi_N^p = \frac{(1-\tau)\phi\left(\theta^p\left(\tau(\gamma-\phi)+\phi\right)+(1-\theta^p)(\gamma-1)\tau\right)}{\gamma\tau(\tau(1-\phi)+\tau)(\gamma\tau+(1-\tau)\phi)}$$

K Proof of Proposition 5

We derive results for the quantities of interest in an economy that is affected by productivity shocks, inflation shocks and preference shocks. We will first prove results for the internal effects of a real exchange rate peg.

The interest rate differential between the pegging and target country when the pegging country imposes a hard exchange rate peg is

$$\begin{aligned} r^p + \mathbb{E}s^{p,t} - r^t =& cov\left(\lambda_T, p^t - p^p\right) = \\ & \left(r^{p*} + \mathbb{E}s^{p,t*} - r^{t*}\right) - \frac{(1-\tau)^2(\gamma-\phi)^2\gamma\left(\theta^t-\theta^p\right)}{\phi\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2 - \\ & \frac{(1-\tau)(1-\phi)^2\gamma^2\left(\theta^t-\theta^p\right)}{\phi\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 - \frac{(\gamma-1)^2(1-\tau)\phi\left(\theta^t-\theta^p\right)}{\gamma\tau+(1-\tau)\phi}\sigma_\chi^2 \end{aligned}$$

which implies the exchange rate peg decreases the risk free rate in the pegging country relative to the risk free rate in the target country as long as the target country is larger than the pegging country, $\theta^t > \theta^p$.

We show that the relative incentives to accumulate capital in the pegging country increase with the size of the target country. Hence, there exists a country size θ_{min} such that a hard exchange rate peg on any country larger than θ_{min} will increase the incentives to accumulate capital in the pegging country.

$$\frac{d}{d\theta^t}\left[k_N^p - k_N^t - \left(k_N^{p*} - k_N^{t*}\right)\right] = \frac{\zeta(\gamma-1)(1-\tau)^2\tau(\gamma-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2 + \frac{\zeta(\gamma-1)\gamma(1-\tau)\tau(\gamma-\phi)(1-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 + \frac{\zeta(\gamma-1)^3(1-\tau)\tau(\gamma-\phi)\phi^2}{\gamma\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2$$

Because firms are competitive, wages are given by the marginal product of labor. Hence, an appropriate exchange rate peg increases wages in the pegging country relative to all other countries.

The interest rate differential between the target country and the outside country is

$$r^t + \mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \mathbb{E}s^{t,o*} - r^{o*}\right) + \frac{\zeta\theta^p\gamma(1-\tau)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\sigma_N^2 + \frac{\zeta\theta^p\gamma(1-\tau)^2(1-\phi)^2}{\tau\left(\gamma\tau+\phi(1-\tau)\right)}\tilde{\sigma}^2 + \frac{\theta^p\zeta(\gamma-1)^2(1-\tau)^2\phi^2}{\gamma\tau\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2$$

which implies the exchange rate peg increases the risk free rate in the target country relative to the risk free rate in the outside country.

The differential incentives to accumulate capital in the target country relative to the outside country is given by

$$k_N^t - k_N^o = k_N^{t*} - k_N^{o*} - \frac{\theta^p\zeta(1-\tau)^2(\gamma-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_N^2 - \frac{\theta^p\gamma\zeta(1-\tau)(\gamma-\phi)(1-\phi)^2}{\left(\gamma\tau+(1-\tau)\phi\right)^2}\tilde{\sigma}^2 - \frac{\theta^p(\gamma-1)^2\zeta(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\gamma\tau+(1-\tau)\phi\right)^2}\sigma_\chi^2$$

Incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate peg decreases wages in the target country relative to all other countries.

Finally, we turn to the cost of the peg. When the pegging country is small, the change in the cost of a hard exchange rate peg as the target country gets larger is

$$\frac{d\log\left(\kappa^{p}_{Cost}\right)}{d\theta^{t}} = -\frac{\gamma(\gamma-1)(1-\tau)(1-\phi)^2(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\tilde{\sigma}^2 - \frac{(\gamma-1)(1-\tau)^2(\gamma-\phi)(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\phi(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma^2_N - \frac{(\gamma-1)^3(1-\tau)\phi(\tau(1-\phi)(\gamma(1-\tau)+\tau)+\phi)}{\gamma(\tau(1-\phi)+\phi)(\gamma\tau-\tau\phi+\phi)^2}\sigma^2_\chi < 0$$

Hence, it is cheaper to peg to a larger country.

L Results with Endogenous Capital Accumulation

We derive results for the quantities of interest in an economy that is affected by productivity shocks, inflation shocks and preference shocks and where we allow for capital to adjust endogenously. Allowing for endogenous capital accumulation only changes the expected levels of consumption, but not the covariance of consumption across countries.

We can derive results for differences in capital accumulation we can rearrange the Euler equation for capital accumulation (11) and obtain a form similar to (3): take logs of both sides of the equation, substitute $\lambda^{n}_{N,2} = p^{n}_{N,2} + y^{n}_{N,2}$, and take differences across countries to obtain