Optimal Water Portfolios and Non-discretionary Consumption

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Abstract

Non-discretionary water consumption is an important component of the aggregate demand for water and in the determination of optimal water supply portfolios. To capture these features an intertemporal optimization model is developed to generate an optimal water portfolio, which weights alternative sources of water according to the uncertainty of inflows, supply costs and stochastic habit formation. The model is calibrated to the city of Taoyuan, Taiwan, which, despite experiencing rainfalls at three times the world average, is nonetheless subject to water shortage problems and a good example of the widespread nature of the water crisis. We show that habit has a significant effect on the optimal water supply portfolio. In the case of purely discretionary water consumption the optimal water supply portfolio consists primarily of reservoir water. Allowing for habit formation results in greater reliance on manufactured water, but less so when habit formation is sensitive to the shocks in water supply. This has important implications for policies that target water consumption behavior. In addition to reducing water demand, these policies enable the realization of long run water supply portfolios that are more cost-effective as they rely less on the costly, riskfree sources of water.

KEYWORDS: Habit Persistence, Water Supply, Gamma Distribution, Water Consumption

1 Introduction

Approximately one fifth of the world's population faces physical water scarcity and another one quarter suffers inappropriate infrastructure to access natural water sources (Watkins, 2006). With persistently increasing water demand surpassing population growth and limited water supply capacity, managing water scarcity remains a fundamental challenge for public water utilities. While augmenting existing water supply may partially meet growing demands such investments frequently involve significant costs. An alternative to increasing supply is to modify consumption patterns so current availability meets present and future needs more efficiently (Brent, Cook and Olsen, 2015). To this end it is necessary to understand the factors that determine water consumption patterns and in particular those factors that determine habit formation (Gregory and DiLeo, 2003).

Non-discretionary, or habit level consumption, is particularly important in the context of household water consumption and is a contributing factor to the low price elasticity of water demand that is commonly observed (Garcia-Valiñas et al., 2014). Importantly, the level of non-discretionary water consumption has been found to be time varying (Martìnez-Espiñeira and Nauges, 2004) and weather dependent (Gaudin et al., 2001). A fundamental question that is addressed in this paper concerns the role of habit formation, how it is affected by the variability in weather patterns over time, and how these risks, in turn, impact upon the optimal consumption and portfolio decisions of water users. Building on the work by Leroux and Martin (2016) and Howe (1986) an intertemporal optimization model with stochastic habit formation is specified to derive an optimal water portfolio that weights alternative sources of water flows according to the uncertainty surrounding rainfall and reservoir inflows, supply costs, factors determining water consumption behaviour, as well as the risk attitude of water managers towards water shortages. The model represents an infinite horizon problem where the water manager determines the optimal paths for water consumption and the allocation of alternative sources of water within the portfolio, including reservoirs and harvested stormwater as well as manufactured water from desalination.

The proposed framework is applied to construct a water portfolio for the city of Taoyuan, which is located in the northern end of the Taiwan strait. Despite the city experiencing rainfalls at three times the world average it is nonetheless subject to water shortage problems. In 2006 water managers in Taoyuan considered building a desalination plant to augment the city's water supply, but eventually dismissed the idea due to the comparatively higher costs. Since, and as recently as 2015, water use is being rationed periodically in Taoyuan. The model is calibrated to monthly reservoir inflows and rainfall data from 1987 to 2014, collected from the Central Weather Bureau and the Water Resources Agency in Taiwan, to generate an optimal water portfolio consisting of water from reservoirs, harvested stormwater and manufactured water. The key results of the paper show that habit persistence and stochastic habit formation have a significant effect on the optimal water supply portfolio, especially on the shares of reservoir and manufactured water. The contribution of manufactured water to the total water stock is highest when deterministic habit formation is assumed, as the portfolio needs to ensure that habit level consumption can be met regardless of shocks to rainfall and inflows. Allowing for stochastic habit formation with habitual outdoor water use increasing in dryer years and water saving behaviour being encouraged by lower reservoir inflows, translates into a lower reliance on manufactured water but not eliminating the need for it altogether as is the case when the habit level consumption is ignored. The optimal level of consumption assuming a base case of stochastic habit formation close to the observed level, but the cost of the portfolio is relatively higher due to the positive share of manufactured water in the portfolio.

The rest of the paper proceeds as follows. The portfolio model of water is presented in Section 2 with optimal solutions for water consumption and the portfolio shares for the alternative water sources also given. The framework is applied in Section 3 by calibrating the model to derive an optimal water portfolio for Taoyuan. Concluding comments are given in Section 4 with key derivations presented in the Appendix A.

2 A Portfolio Model of Water

The formal model is now specified and derived. An important ingredient of the model is an allowance for the role of habit in water consumption and how it is influenced by changes in weather patterns that impact upon reservoir inflows and rainfall patterns. The number of water assets specified in the model is restricted to three, consisting of two assets that are stochastic as they are affected by weather variations and a third asset that is interpreted as "risk-free" as it is assumed to be independent of variations in rainfall and climatic trends. Extensions of the trivariate model of water assets to N assets is conceptually straight forward although making the derivations more complicated.

2.1 Specification

The social planner is assumed to choose water consumption (x) where the stock of water (W) comes from three alternative sources: reservoirs (r), harvested rainwater (h) and manufactured water (m). Following Leroux and Martin (2016) the stock of water is adjusted for supply costs. Reservoirs and harvested rainwater are assumed to be risky assets as water flows from these sources are subject to variations in weather patterns, whereas manufactured water is treated as risk-free from the point of view that water flows from this source are perfectly reliable.

Letting θ_r and θ_h represent respectively the cost-adjusted shares of reservoirs and harvested water in the water portfolio, with the share from manufactured water determined by the constraint $\theta_m = 1 - \theta_r - \theta_h$, the optimisation problem consists of solving

$$\max_{x,\theta_r,\theta_h} E \int_0^\infty \left[e^{(\xi-\delta)t} \frac{(x-y)^{1-\gamma}}{1-\gamma} \right] dt, \tag{1}$$

subject to the following constraints

$$\begin{aligned} dS_i &= \mu_i dt + \sigma_i dz_i, \qquad \{i = r, h, m\}, & (2) \\ dy_t &= (bx - ay)\, dt + \beta_r dz_r + \beta_h dz_h, & (3) \\ dW &= \left(a_r\theta_r + a_h\theta_h + \frac{\mu_m}{S_m} + \frac{c_m}{p}\lambda_m\mu_m\right) W dt - x_t dt + \\ & \left(\frac{\sigma_r}{S_r} + \frac{c_r}{p}\lambda_r\sigma_r\right)\theta_r W dz_r + \left(\frac{\sigma_h}{S_h} + \frac{c_h}{p}\lambda_h\sigma_h\right)\theta_h W dz_h. & (4) \end{aligned}$$

The objective function in (1) represents the discounted net present value of utility (u_t), which is a function of the difference between water consumption (x_t) and habit (y_t). The shape of the utility function is determined by the risk aversion parameter γ, with water utility managers exhibiting risk aversion when $\gamma > 0$, whereby $\gamma = 1$ corresponds to logarithmic preferences and $\gamma > 1$ represents relatively high risk aversion.

The Arrow-Pratt measure of relative risk aversion (RRA) is

$$\text{RRA} = \frac{-x u_{xx}(x, y)}{u_x(x, y)} = \gamma \frac{x}{x - y}, \tag{5}$$

where u_x and u_{xx} are respectively the first and second derivatives of the utility function with respect to water consumption. Relative risk aversion is time-varying according to movements in consumption and habit with the property that the water manager becomes more risk averse as the current level of water consumption approaches the habit or subsistence level. The surplus ratio is high when water consumption exceeds the habit level of consumption, implying that individuals are able to consume more to receive higher utility resulting in the social planner being less risk averse. In the special case where all water consumption is discretionary $(y = 0)$, relative risk aversion is no longer time-varying reducing to the parameter γ. The discount parameter is δ, while ξ represents the rate of population growth, which are both assumed to be constant and satisfying the property $\delta > \xi$. Following Muellbauer (1988) and Rozen (2010) and findings in the empirical literature on non-discretionary water consumption (Garcia-Valiñas et al., 2014), it is assumed that the habit level of water consumption stays below consumption to ensure there is no dis-utility obtained from consumption.

Equation (2) gives the water flows for the alternative sources of water in the portfolio with S_r representing the storage capacity of reservoirs, S_h representing the storage capacity for harvested rainwater and S_m representing the stock of manufactured water. The parameter μ_i is the average inflow per annum of the i^{th} water source with σ_i controlling the degree of uncertainty associated with water flows caused by the Brownian motion z_i, which are assumed to be distributed as $N(0, dt)$. As the flow of manufactured water is assumed to be risk free, $\sigma_m = 0$. The uncertainty associated with reservoir and harvested stormwater inflows is assumed to be correlated with parameter ρ, such that $dz_r dz_h = \rho dt$.

The importance of habit behaviour in water consumption and its dynamic characteristics are well documented in empirical studies of water demand. Being vulnerable to drought and water shortages is identified as one of the key drivers for the decision to retro-fit water saving technologies by Taiwanese water consumers (Lam, 2006). Garcia-Valiñas et al. (2014) find that the adoption of water-saving behaviour and investments reduce the non-discretionary level of consumption (y), especially when these changes affect indoor water use. In the current framework habit behaviour is assumed to be stochastic with changes over time determined by (3). This form of the stochastic differential equation is motivated by previous work of Sundaresan (1989), Constantinides (1990), Abel (1990), Campbell and Cochrane (1994), Kiley (2010) and Nakagawa (2012). The parameter b measures the relationship between contemporaneous water consumption and habit with larger values of $b > 0$ causing current consumption to accelerate habit formation. The parameter $a > 0$ controls the persistence of habit with larger values of a resulting in faster mean reversion to the equilibrium level of habit, with a habit/consumption ratio in equilibrium of b/a. In the extreme case of $a \to \infty$, there is instantaneous adjustment to the equilibrium level of habit.

Random variations to habit are assumed to be determined by the uncertainty in reservoir and harvested stormwater inflows which are controlled respectively by the parameters β_r and β_h. The parameter β_h is expected to be negative with a negative shock to rainfall increasing the habitual level of water consumption as outdoor activities, including watering the garden are required more frequently and at greater intensity. In contrast, the parameter β_r is expected to be positive as decreases in the water stocks of reservoirs encourage consumers to be more frugal in household consumption by limiting their use of water and encouraging investment in water saving household devices and appliances. In the special case where $\beta_r = \beta_h = 0$, changes in habit formation are deterministic.

The water constraint equation in (4) is based on Leroux and Martin (2016) and derived in Appendix A, where

$$a_r = \frac{\mu_r}{S_r} - \frac{\mu_m}{S_m} + \frac{c_r}{p}\lambda_r\mu_r + \frac{{\sigma_r}^2}{S_r}\frac{c_r}{p}\lambda_r - \frac{c_m}{p}\lambda_m\mu_m, \tag{6}$$

$$a_h = \frac{\mu_h}{S_h} - \frac{\mu_m}{S_m} + \frac{c_h}{p}\lambda_h\mu_h + \frac{{\sigma_h}^2}{S_h}\frac{c_h}{p}\lambda_h - \frac{c_m}{p}\lambda_m\mu_m, \tag{7}$$

represent the cost-adjusted excess inflows from reservoirs and harvested rainwater relative to the inflows from manufactured water. The term

$$\lambda_i = \frac{d}{dS_i}\left(\frac{p}{c_i}\right), \qquad \{i = r, h, m\}, \tag{8}$$

captures the change in the relative price (p) to cost (c_i) ratio arising from changes in the i^{th} stock of water.

2.2 Solution

The optimal solution of (1) to (4) is based on solving the following dynamic programming problem (see, Kamien and Schwartz, 1981, p.248)

$$
\begin{aligned}
(\delta-\xi)V = \max_{x,\theta_r,\theta_h} \Bigg\{ & \frac{(x-y)^{1-\gamma}}{1-\gamma} \\
& + \left[\left(a_r\theta_r + a_h\theta_h + \frac{\mu_m}{S_m} + \mu_m\lambda_m\frac{c_m}{p}\right)W - x\right]V_W \\
& + \left[\frac{1}{2}\left(\frac{\sigma_r}{S_r} + \sigma_r\lambda_r\frac{c_r}{p}\right)^2\theta_r^2 + \frac{1}{2}\left(\frac{\sigma_h}{S_h} + \sigma_h\lambda_h\frac{c_h}{p}\right)^2\theta_h^2\right. \\
& \left. + \left(\frac{1}{S_r} + \lambda_r\frac{c_r}{p}\right)\left(\frac{1}{S_h} + \lambda_h\frac{c_h}{p}\right)\rho\sigma_r\sigma_h\theta_r\theta_h W^2 V_{WW}\right] \\
& + (bx - ay)V_y + \left(\frac{1}{2}\beta_r^2 + \frac{1}{2}\beta_h^2 + \beta_r\beta_h\rho\right)V_{yy} \\
& + \left[\left(\frac{\sigma_r}{S_r} + \frac{c_r}{p}\lambda_r\sigma_r\right)(\beta_r + \beta_h\rho)\theta_r\right. \\
& \left. + \left(\frac{\sigma_h}{S_h} + \frac{c_h}{p}\lambda_h\sigma_h\right)(\beta_h + \beta_r\rho)\theta_h\right]WV_{Wy}\Bigg\},
\end{aligned} \tag{9}
$$

where V is the time-invariant indirect utility function corresponding to the maximum feasible value gained from solving (9), which is a function of the water stock W and the level of habit y. The derivatives V_W and V_{WW} denote the first and second derivatives of the value function with regards to the water stock, the derivatives V_y and V_{yy} represent the first and second order derivatives of the value function with respect to habit and V_{Wy} is the cross derivative with respect to the change in the water stock and changes in habit consumption.

Maximising the right-hand side of (9) with respect to x and rearranging gives the following expression for the optimal level of water consumption

$$
x = (V_W - bV_y)^{-\frac{1}{\gamma}} + y. \tag{10}
$$

Now maximising the right-hand side of (9) with respect to the shares for reservoir (θ_r)

and harvested rainwater (θ_h) gives the optimal water portfolio shares as

$$\begin{aligned}\left[\theta_r\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)^2+\left(\frac{1}{S_r}+\lambda_r\frac{c_r}{p}\right)\left(\frac{1}{S_h}+\lambda_h\frac{c_h}{p}\right)\sigma_{rh}\theta_h\right]W^2V_{WW}\\ +a_rWV_W+\left(\frac{\sigma_r}{S_r}+\frac{c_r}{p}\lambda_r\sigma_r\right)(\beta_r+\beta_h\rho)\,WV_{Wy}=0,\end{aligned} \tag{11}$$

and

$$\begin{aligned}\left[\theta_h\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)^2+\left(\frac{1}{S_r}+\lambda_r\frac{c_r}{p}\right)\left(\frac{1}{S_h}+\lambda_h\frac{c_h}{p}\right)\sigma_{rh}\theta_r\right]W^2V_{WW}\\ +a_hWV_W+\left(\frac{\sigma_h}{S_h}+\frac{c_h}{p}\lambda_h\sigma_h\right)(\beta_h+\beta_r\rho)\,WV_{Wy}=0.\end{aligned} \tag{12}$$

Rearranging (11) and (12) to solve for θ_r and θ_h, gives

$$\begin{aligned}\theta_r &= \left(\frac{\rho a_h}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)}-\frac{a_r}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)^2}\right)\times\\ &\quad \frac{1}{(1-\rho^2)}\frac{V_W}{WV_{WW}}-\frac{\beta_r}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)}\frac{V_{Wy}}{WV_{WW}},\end{aligned} \tag{13}$$

and

$$\begin{aligned}\theta_h &= \left(\frac{\rho a_r}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)}-\frac{a_h}{\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)^2}\right)\times\\ &\quad \frac{1}{(1-\rho^2)}\frac{V_W}{WV_{WW}}-\frac{\beta_h}{\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)}\frac{V_{Wy}}{WV_{WW}}.\end{aligned} \tag{14}$$

Upon substituting the optimality expressions for consumption in (10), and for the reservoir and harvested stormwater shares in (13) and (14) respectively, into the right hand-side of (9) yields the nonlinear second order differential equation

$$\begin{aligned}(\delta-\xi)V &= \frac{(x-y)^{1-\gamma}}{1-\gamma}+\left[\left(a_r\theta_r+a_h\theta_h+\frac{\mu_m}{S_m}+\mu_m\lambda_m\frac{c_m}{p}\right)W-x\right]V_W \\ &+\left[\frac{1}{2}\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)^2\theta_r^2+\frac{1}{2}\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)^2\theta_h^2\right. \\ &\left.+\left(\frac{1}{S_r}+\lambda_r\frac{c_r}{p}\right)\left(\frac{1}{S_h}+\lambda_h\frac{c_h}{p}\right)\sigma_{rh}\theta_r\theta_h\right]W^2V_{WW} \\ &+(bx-ay)V_y+\left(\frac{1}{2}\beta_r^2+\frac{1}{2}\beta_h^2+\beta_r\beta_h\rho\right)V_{yy} \\ &+\left[\left(\frac{\sigma_r}{S_r}+\frac{c_r}{p}\lambda_r\sigma_r\right)(\beta_r+\beta_h\rho)\theta_r\right. \\ &\left.+\left(\frac{\sigma_h}{S_h}+\frac{c_h}{p}\lambda_h\sigma_h\right)(\beta_h+\beta_r\rho)\theta_h\right]WV_{Wy}. \end{aligned} \tag{15}$$

A closed-form solution to (15) is given by

$$V=A\,(W+By)^{1-\gamma}. \tag{16}$$

Upon substituting the following derivatives in (15)

$$\begin{aligned} V_W &= A(1-\gamma)(W+By)^{-\gamma} \\ V_{WW} &= -A\gamma(1-\gamma)(W+By)^{-\gamma-1} \\ V_y &= AB(1-\gamma)\,(W+By)^{-\gamma} \\ V_{yy} &= -AB^2(1-\gamma)\gamma\,(W+By)^{-\gamma-1} \\ V_{Wy} &= -AB\gamma(1-\gamma)\,(W+By)^{-\gamma-1}, \end{aligned} \tag{17}$$

and rearranging shows that the solutions for A and B in (16) are respectively

$$\begin{aligned} A &= \left[\frac{\delta-\xi-(1-\gamma)\left(\frac{\mu_m}{S_m}+\mu_m\lambda_m\frac{c_m}{p}\right)}{\gamma}-\frac{1-\gamma}{2\gamma^2(1-\rho^2)}\left(\frac{a_r^2}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)^2}-\right.\right. \\ &\left.\left.\frac{2a_ra_h\rho}{\left(\frac{\sigma_r}{S_r}+\sigma_r\lambda_r\frac{c_r}{p}\right)\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)}+\frac{a_h^2}{\left(\frac{\sigma_h}{S_h}+\sigma_h\lambda_h\frac{c_h}{p}\right)^2}\right)\right]^{-\gamma}\frac{(1-bB)^{\gamma-1}}{1-\gamma}, \end{aligned} \tag{18}$$

and

$$B = -\left[\frac{\mu_m}{S_m} + \mu_m\lambda_m\frac{c_m}{p} + a - b + \frac{\beta_r}{\left(\frac{\sigma_r}{S_r} + \sigma_r\lambda_r\frac{c_r}{p}\right)}a_r + \frac{\beta_h}{\left(\frac{\sigma_h}{S_h} + \sigma_h\lambda_h\frac{c_h}{p}\right)}a_h\right]^{-1}. \tag{19}$$

The expression for B in (19) represents a risk adjusted return on weather-dependent sources of water relative to the risk-free rate corresponding to manufactured water with habit formation entering the expression through the conditional mean parameters a and b, as well as the risk-behavioural parameters β_r and β_h. The combined term By is interpreted as the discounted sum of current and future habit levels thereby providing an overall measure of the stock of water devoted to habit. Given the negative sign of B this implies that $W + By$ represents the excess water stock available to discretionary consumption net of the total water stock used for habit consumption.

The expression for A in (18) is a function of all of the parameters in the model. For values of the relative risk aversion parameter of $\gamma < 1$, then $A > 0$, whereas for higher levels of relative risk aversion with $\gamma > 1$, then $A < 0$. In the special case where the habit parameter $b = 0$, the conditional mean of habit formation is independent of consumption, resulting in A reducing to the expression given in Leroux and Martin (2016) corresponding to the case where all water consumption is discretionary (see also Kamien and Schwartz, 1981).

To derive the optimal level of water consumption in terms of the parameters of the model, V_W and V_y from (17) are substituted into (10) to give

$$x = (A(1-\gamma))^{-\frac{1}{\gamma}}(1 - bB)^{-\frac{1}{\gamma}}(W + By) + y. \tag{20}$$

Dicretionary consumption of water at each point in time is proportional to $W + By$, the excess water stock available for consumption net of the total water stock devoted for habit consumption. The effects on current water consumption from an change to habit is

$$\frac{dx}{dy} = (A(1-\gamma))^{-\frac{1}{\gamma}} (1-bB)^{-\frac{1}{\gamma}} B + 1. \tag{21}$$

From (18), as assuming the relative risk aversion parameter satisfies $\gamma > 1$, then $A(1-\gamma) > 0$ so a one standard deviation positive shock to habit causes an increase in water consumption, but by a smaller increase than the size of the shock, provided that the condition $-1 < (A(1-\gamma))^{-\frac{1}{\gamma}} (1-bB)^{-\frac{1}{\gamma}} B < 0$ is satisfied. Alternatively, if $(A(1-\gamma))^{-\frac{1}{\gamma}} (1-bB)^{-\frac{1}{\gamma}} B < -1$ holds, the positive shock to habit actually results in a fall in water consumption.

The effect of the stochastic habit parameters β_r and β_h on optimal consumption is

$$\frac{dx}{d\beta_i} = \left(\frac{b(W+By)}{\gamma\,(A(1-\gamma))^{\frac{1}{\gamma}} (1-bB)^{1+\frac{1}{\gamma}}} + \frac{y}{(A(1-\gamma))^{\frac{1}{\gamma}} (1-bB)^{\frac{1}{\gamma}}} \right) \frac{dB}{d\beta_i}, \tag{22}$$

which is positive for $\gamma > 1$, as $B < 0$ and $dB/d\beta_i > 0$ from (19). Hence an increase (decrease) in β_i results in optimal consumption levels being more (less) sensitive to shocks in water flows.

To derive the final expressions for the optimal water shares in terms of the parameters of the model, consider combining the derivatives in (17) as follows

$$\begin{aligned}
\frac{{V_{Wy}}^2}{V_{WW}} &= -AB^2(1-\gamma)(W+By)^{-\gamma-1} \\
\frac{V_{Wy}}{V_{WW}} &= B \\
\frac{V_W}{V_{WW}} &= -\frac{W+By}{\gamma} \\
\frac{{V_W}^2}{V_{WW}} &= -A\frac{1-\gamma}{\gamma}(W+By)^{-\gamma+1} \\
\frac{V_{Wy}V_W}{V_{WW}} &= AB(1-\gamma)(W+By)^{-\gamma}.
\end{aligned} \tag{23}$$

Using these expressions in (13) and (14) gives

$$\theta_r = \frac{k_r a_r - k a_h}{\gamma} + (k_r a_r - k a_h)\frac{By}{\gamma W} - \frac{\beta_r}{\left(\frac{\sigma_r}{S_r} + \sigma_r \lambda_r \frac{c_r}{p}\right)} \frac{B}{W}, \tag{24}$$

and

$$\theta_h = \frac{k_h a_h - k a_r}{\gamma} + (k_h a_h - k a_r)\frac{By}{\gamma W} - \frac{\beta_h}{\left(\dfrac{\sigma_h}{S_h} + \sigma_h \lambda_h \dfrac{c_h}{p}\right)}\frac{B}{W}, \tag{25}$$

where

$$k_r = \frac{1}{(1-\rho^2)\left(\dfrac{\sigma_r}{S_r} + \sigma_r \lambda_r \dfrac{c_r}{p}\right)^2}, \quad k_h = \frac{1}{(1-\rho^2)\left(\dfrac{\sigma_h}{S_h} + \sigma_h \lambda_h \dfrac{c_h}{p}\right)^2}, \quad k = \rho\sqrt{k_r k_h}. \tag{26}$$

An important special case of the water portfolio shares in (24) and (25) is where there is extreme risk aversion with $\gamma \to \infty$, resulting in the optimal solutions for reservoir and harvested stormwater reducing to

$$\theta_r = -\sqrt{k_r\,(1-\rho^2)}\beta_r \frac{By}{W}, \qquad \theta_h = -\sqrt{k_h\,(1-\rho^2)}\beta_h \frac{By}{W}. \tag{27}$$

This result contrasts with solutions based on models where water consumption is entirely discretionary where the effect of extreme risk aversion results in an optimal solution consisting of all water being sourced from desalination, $\theta_m = 1$. In fact, this latter result is achieved by assuming that there is no deterministic habit consumption by setting $y = 0$ in (27) resulting in the optimal portfolio shares from reservoirs and harvested stormwater reducing to $\theta_r = \theta_h = 0$.

3 A Water Portfolio for Taoyuan

The water portfolio model is now applied to Taoyuan which is a city located in the northern end of the Taiwan strait. Taoyuan is of particular interest because of its geographic location and prolonged struggle with water crises, which is representative of many cities in Taiwan (Lam, 2006) and illustrates that critical water shortages are not limited to cities located in arid and semi-arid climates. Given its abundant rainfall with levels at three times the world average it would seem to be unusual that this sub-tropical city should be subjected to water shortage problems. However, per capita consumption in the region is only one fifth of the world average which is due to its topographical features and high population density. With a population of two million and proximity to the capital Taipei, Taoyuan is characterised by fast population growth and is a leading industrial and technological city with over one third of the world's top five hundred technology companies have manufacturing sites in Taoyuan. Even though Taoyuan was once referred to as the 'Thousand-Pond Township', highlighting its historically plentiful water supply, nonetheless in 2002 the city was left without water for twenty-two days. To address the city's water shortage problems in 2006 the construction of a desalination plant was proposed, but abandoned due to the high cost of manufactured water and the low price of water in Taoyuan at the time. In 2015 water rationing was introduced, limiting water use to five days a week.

3.1 Choice of Model Parameter Values

3.1.1 Flow Parameters

The average inflow and rainfall parameters μ_r and μ_h in equation (2) and the corresponding uncertainty parameters σ_r and σ_h, are estimated by using monthly reservoir inflows data obtained from the Water Resources Authority for the Shihmen Reservoir and monthly rainfall data across seven weather stations collected from the Central Weather Bureau in Taiwan between 1987 and 2014. The data on reservoir inflows are presented in Figure 1 and for rainfall in Figure 2. Inspection of the figures shows that reservoir inflows and rainfall exhibit high volatility with relatively high levels of inflows and rainfall followed by much lower levels of inflows and rainfall. Descriptive statistics presented in Table 1, show that inflows and rainfall peak between April and September corresponding to the monsoon season in the region. In other months, reservoir inflows and rainfall are significantly lower. Based on annual data, the average total inflows in a year are $1548GL$ with a standard deviation of $560GL$, while the average accumulated rainfall is $2030mm$ with a standard deviation of $465mm$.

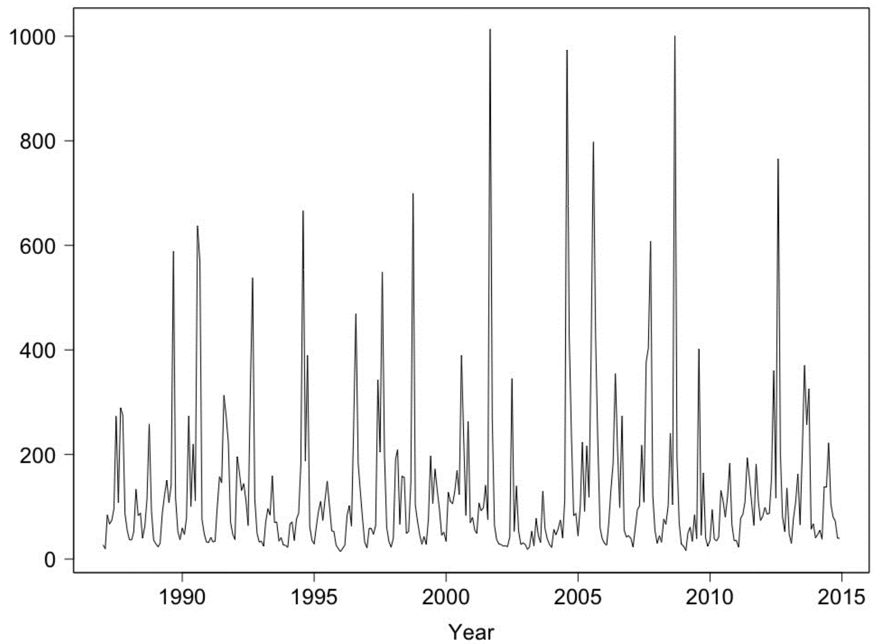

Figure 1: Monthly reservoir inflows for the Shihmen reservoir measured in GL.

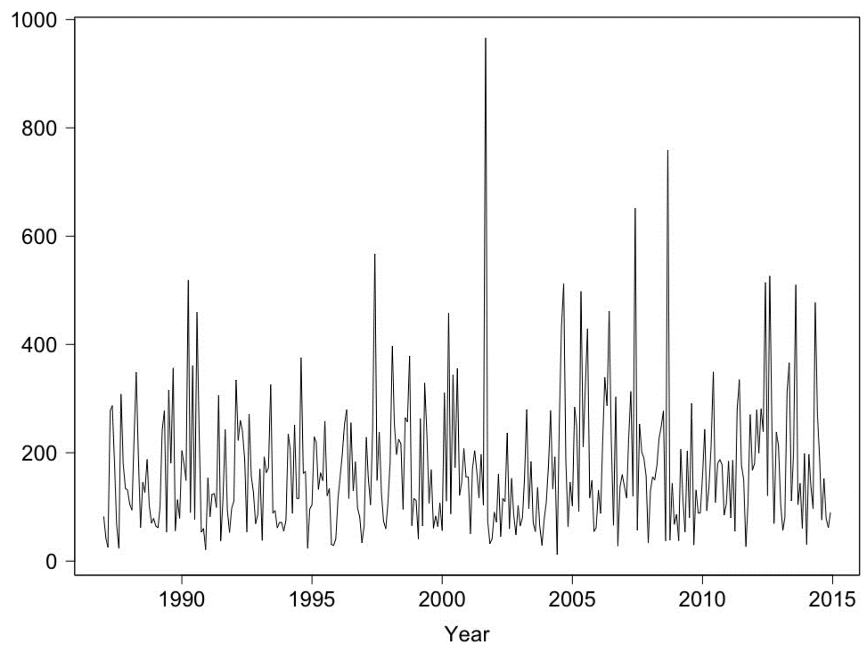

Figure 2: Monthly rainfall across 7 weather stations in Taoyuan measured in mm.

Table 1:
Descriptive statistics on monthly reservoir inflows and rainfall for Taoyuan, January 1987 to December 2014.

Month	Inflows (GL)				Rainfall (mm)			
	Mean	SD	Min	Max	Mean	SD	Min	Max
January	37	16	14	82	117	44	31	204
February	58	48	16	196	155	104	38	397
March	73	51	24	223	177	61	26	278
April	84	49	25	273	208	121	46	519
May	100	48	24	217	224	109	54	498
June	148	87	40	360	250	158	12	652
July	155	90	39	399	159	84	38	320
August	300	269	31	974	218	152	23	527
September	290	257	34	1014	231	208	27	966
October	190	161	41	699	114	71	28	379
November	68	46	26	264	92	63	23	271
December	46	25	20	136	98	49	21	213
Total (p.a.)	1548	560	558	2740	2030	465	3101	1279

To estimate the mean parameters μ_r and μ_h and the uncertainty parameters σ_r and σ_h, a gamma distribution is specified

$$g(z;\alpha;\beta) = \frac{1}{\Gamma[\alpha]\beta}\left(\frac{z}{\beta}\right)^{\alpha-1} \exp\left(-\frac{z}{\beta}\right), \tag{28}$$

where z represents either inflows or rainfall. This choice of distribution has been used recently by Leroux and Martin (2016) and has been advocated by Wilks (1990) and Groisman et al. (1999) as it provides flexibility in capturing different inflow and rainfall patterns. The parameters α and β respectively represent the shape parameter and the scale parameter, which are related to the mean and uncertainty parameters in the water flow equations as $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$. The α, β parameters of the gamma distribution are estimated by maximum likelihood methods with the results given in Table 2. All computations are performed using the R software.

All of the estimates of α in (28) are greater than 1, suggesting the shapes of the distribution are skewed. There are large differences within the estimates of the scale parameter β, which implies large variation in rainfall for specific months. For the

Table 2:
Maximum likelihood estimates of the gamma distribution parameters in (28) for the monthly reservoir inflows and rainfall for Taoyuan, January 1987 to December 2014.

Month	Inflows (GL)		Rainfall (mm)	
	α	β	α	β
January	7.101	5.191	6.364	18.32
February	2.060	27.924	2.195	70.754
March	2.925	24.817	6.143	28.855
April	3.611	23.292	2.946	70.466
May	4.059	24.746	4.145	53.976
June	3.411	43.338	2.161	115.576
July	3.006	51.453	3.384	46.925
August	1.280	234.272	1.906	114.154
September	1.574	183.931	1.804	128.282
October	1.863	101.907	3.202	35.531
November	3.691	18.496	2.744	33.458
December	4.926	9.414	3.778	26.036
Total	7.131	217.057	19.318	105.596

same shape parameter, a higher scale parameter implies a flatter curve, suggesting that reservoir inflows and rainfall are much higher for particular months than other periods during the year. Some examples of the distributions of inflows over the year are given in Figure 3 for January, April, July and October. Corresponding distributions for rainfall are given in Figure 4.

3.1.2 Calibration Parameters

The water portfolio model is calibrated to a period prior to the Taoyuan government's decision to build a desalination plant at the end of 2006. Prior to 2006, the Shihmen reservoir has been the dominant source of water for Taoyuan. The reservoir was established in 1963 and has a storage capacity of $S_r = 251.88GL$. The unit cost of supplying water from the Shihmen reservoir is $\$0.167/KL$, which is composed of an operational cost of $\$0.05/KL$ (WRA, 2014) and an assumed per unit fixed cost of $\$0.117/KL$.[1] From Table 3, the average annual return and volatility for reservoir

[1] All prices and costs are expressed in US dollars by using a $US\$/NT\$$ exchange rate of 0.031.

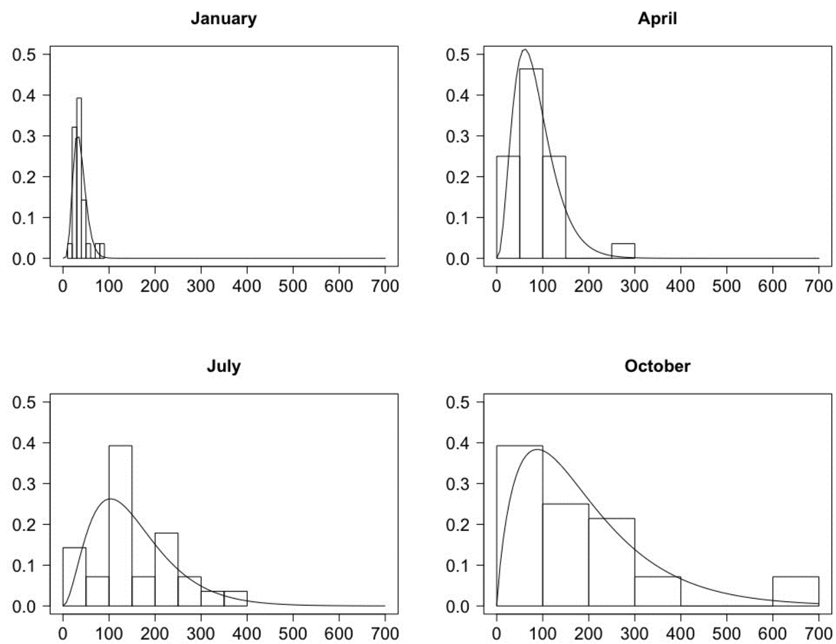

Figure 3: Estimated kernel density estimates of reservoir inflows (measured in GL) for selected months, January 1987 to December 2014.

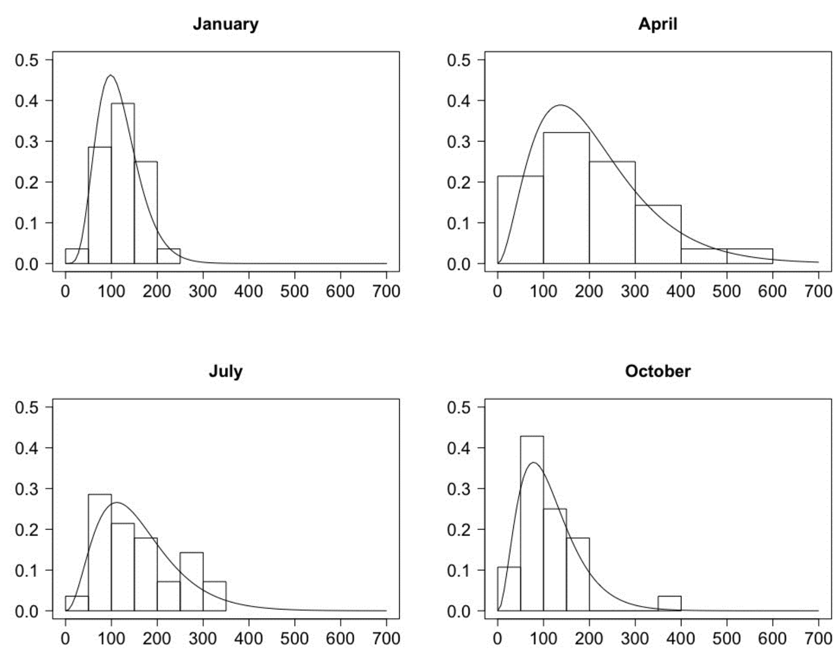

Figure 4: Estimated kernel density estimates of rainfall (measured in mm) for selected months, January 1987 to December 2014.

inflows $\mu_r = 7.131 \times 217.057 = 1548GL$ and $\sigma_r = (7.131 \times 217.057^2)^{0.5} = 580GL$ respectively. Given the storage capacity of the Shihmen reservoir, these flows imply that the reservoir is fully replenished several times per year.

In the midst of the water crisis, Taoyuan has considered alternative water sources such as harvesting rainwater. Taiwan has abundant rainfall and rainwater capturing devices. These tend to have low running costs, provided that usage is limited to low risk activities such as gardening and toilet flushing, constituting 27% of total water consumption (WRA, 2008). The existing rainwater harvesting schemes in Taoyuan are limited to a small number of participating schools in the Sustainable Campus Program. In order to construct an optimal water portfolio that provides insights into the possible implementation of the current supply portfolio, the model is calibrated to a stormwater harvesting project in Taipei, the capital of Taiwan. The model is calibrated using the largest pilot stormwater harvesting project in Taiwan at the Taipei Zoo, where large rainwater water tanks have been installed for toilet flushing and other non-contact activities. The stormwater storage capacity is $S_h = 0.0002GL$ for a catchment area of 0.22 hectares. The annualised mean flow from harvested rainwater is calculated for each month as the product of the rainfall in mm and the catchment area. In the case of October for example, the annualised average rainfall is computed as $\mu_h = 3.202 \times 35.531 \times 12 \times 0.22 \times 10^{-5} = 0.003GL$ and the corresponding volatility is computed as $\sigma_h = (3.202 \times 35.531^2 \times 12^2)^{0.5} \times 0.22 \times 10^{-5} = 0.002GL$. The cost of water supply through harvesting rainwater is $\$0.33/KL$ (WRA, 2014).

In 2006, the Taoyuan City government proposed a plan to establish a desalination plant. The plant was designed to supply $30ML$ of water per day, suggesting a water stock of $S_m = 10.95GL$, which represents about 7% of total water supply at the time of the announcement. The construction cost (K_m) was estimated at \$ $3.74m$ such that the unit supply cost (c_m) of desalination was \$ $0.93/KL$ (WRA, 2006). As it is assumed that there are no uncertainties associated with water flows generated from manufactured wated, the volatility for this source of water is fixed at $\sigma_m = 0$.

The price of water (p) is \$0.471/KL, which corresponds to the wholesale price by the Taiwan Water Corporation in 2013 (TWC, 2014). The population growth (ξ) is 2% according to the General Statistical Analysis Report published by the National Statistics, Taiwan (NS, 2014). A discount rate of 3% is chosen to represent the long term planning horizon of water investment projects.

Observed household water consumption in Taoyuan is $243GL$, of which 40% or $97.3GL$, is assumed to be non-discretionary (CWSD, 2014). With a population of just over 2.1 million this translates into non-discretionary consumption of $130L$ per person per day. While there are no empirical studies of non-discretionary water consumption in Taiwan, this amount is above levels in Germany and Spain, lower than levels in France and Portugal, and well within the range of values observed across the world (Garcia-Valiñas et al., 2014). The habit function parameters are calibrated in accordance with a deterministic habit-consumption ratio, where $b = a\frac{y}{x} = 0.6$ when $a = 1.5$.

3.1.3 Risk Aversion Parameter

The risk aversion parameter γ used to calibrate the model is chosen by identifying the implied value of γ of the water managers under a particular scenario assuming that water allocations are determined optimally. Formally this is obtained by substituting the optimal water shares for reservoirs and harvested stormwater from (24) and (25) respectively into the adding-up constraint $\theta_r + \theta_h + \theta_m = 1$, to give

$$\begin{aligned} 1 &= (k_r a_r - k a_h)\frac{1}{\gamma} + \left(\frac{k_r a_r - k a_h}{\gamma} - \frac{\beta_r}{\left(\frac{\sigma_r}{S_r} + \sigma_r \lambda_r \frac{c_r}{p}\right)} \right) \frac{By}{W} \\ &\quad + (k_h a_h - k a_r)\frac{1}{\gamma} + \left(\frac{k_h a_h - k a_r}{\gamma} - \frac{\beta_h}{\left(\frac{\sigma_h}{S_h} + \sigma_h \lambda_h \frac{c_h}{p}\right)} \right) \frac{By}{W} + \theta_m. \end{aligned}$$

Table 3:

Parameter values used to calibrate the water asset model.

Parameter	Value	Unit	Description
S_r	251.880	GL	Total reservoir capacity
S_h	0.200	ML	Total rainwater harvesting capacity
S_m	10.950	GL	Desalination plant capacity
μ_r	1548.000	GL	Mean reservoir inflow (p.a.)
μ_h	0.004	GL	Mean rainfall (p.a.)
μ_m	0.000	GL	Mean manufactured water flow (p.a.)
σ_r	580.000	GL	Standard deviation of reservoir inflow
σ_h	0.001	GL	Standard deviation of rainfall
σ_m	0.000	GL	Standard deviation of manufactured water
ρ	0.642		Correlation coefficient
x	243.150	GL	Observed consumption (p.a.)
y	97.260	GL	Non-discretionary consumption (p.a.)
a	1.500		Mean reversion parameter
b	0.600		Habit formation parameter
ξ	0.020		Population growth rate (p.a.)
δ	0.030		Discount rate (p.a.)
γ	1.137		Implied risk aversion parameter
c_r	0.167	$/KL	Operating costs for reservoir
c_h	0.333	$/KL	Operating costs for rainfall harvesting
c_m	0.934	$/KL	Operating costs for desalination plant
p	0.471	$/KL	Retail price of water
	0.031	US$/NT$	Currency conversion rate into USD

Upon rearranging for γ yields an expression for the implied risk aversion parameter in terms of the parameters of the model according to

$$\gamma = \frac{(k_r a_r - k a_h + k_h a_h - k a_r)\left(1 + \frac{By}{W}\right)}{1 - \theta_m + \left(\frac{\beta_r}{y\left(\frac{\sigma_r}{S_r} + \sigma_r \lambda_r \frac{c_r}{p}\right)} + \frac{\beta_h}{y\left(\frac{\sigma_h}{S_h} + \sigma_h \lambda_h \frac{c_h}{p}\right)}\right)\frac{By}{W}}. \tag{29}$$

In the special case of deterministic habit with $\beta_r = \beta_h = 0$, the implied risk aversion parameter in (29) reduces to

$$\gamma = \frac{(k_r a_r - k a_h + k_h a_h - k a_r)\left(1 + \frac{By}{W}\right)}{1 - \theta_m}. \tag{30}$$

A further simplification occurs where there is no habit so $y = 0$, resulting in (30) reducing to

$$\gamma = \frac{(k_r a_r - k a_h + k_h a_h - k a_r)}{1 - \theta_m}, \tag{31}$$

which is the expression for the implied γ parameter used in Leroux and Martin (2016). A comparison of (30) and (31) shows that as $B < 0$, the introduction of deterministic habit consumption of water into the model reduces the implied risk aversion parameter. Given the parameter values presented in Table 3 and setting $\theta_m = 0.09$, the implied risk aversion parameter is set at[2]

$$\gamma = 1.137. \tag{32}$$

3.2 Optimal Water Portfolio

The optimal water portfolio is calculated for the base case of stochastic habit formation and contrasted with the optimal portfolios under deterministic habit formation and the extreme case of where all water consumption is non-discretionary.

[2]The choice of θ_m corresponds to the share of manufactured water that would have resulted from the construction of the desalination plant as planned.

The base case of stochastic habit formation assumes that a reduction in rainfall by one standard deviation, increases habit level consumption by 0.1 of a standard deviation, $\beta_h = -0.1$, as a result of the increased outdoor watering needs of the moderately abundant green spaces in Taoyuan in a low rainfall year. The base case also assumes a downward shock to reservoir inflows by one standard deviation results in a decrease in habit level consumption of half a standard deviation, $\beta_r = 0.5$. This is a behavioural effect that arises from the perception of greater vulnerability to water shortages when storage levels are low, which encourages the adoption of water saving behaviour and technologies (Lam, 2006). Both parameters values are subject to a sensitivity analysis in the following section.

Table 4 shows that the optimal water portfolio under stochastic habit formation consists of 88% of the total water stock being sourced from the Shihmen reservoir, 3% from rainwater harvesting schemes and 9% from desalination. The optimal water portfolio is similar to the portfolio that was envisaged in 2006 when the construction of a desalination plant was proposed. Moreover, from Table 4 the optimal consumption of water is $242GL$ per year which is also comparable to the observed level of $243GL$ in 2006 (CWSD, 2014). The total cost of supplying water from the optimal stochastic habit portfolio is US \$$122m$.

The water portfolio in the case of deterministic habit formation is obtained by calibrating the model with the habit parameters in (3) restricted to $\beta_r = \beta_h = 0$, with the level of water consumption from habit equal to $y = 97.3GL$. Table 4 shows that the deterministic habit level of consumption increases the optimal share of manufactured water from 9% to 21% in the stochastic case, while the share of harvested rainwater reduces slightly from 3% to 2%. The greater reliance on the risk free water source arises because no allowance is made for the possibility that consumers adjust their habit level consumption when lower dam levels are observed. Optimal consumption is also lower at $195GL$ per year. The annual supply costs of water from this portfolio are now US

Table 4:

Optimal levels of water consumption (x) (in GL), and portfolio shares for reservoirs (θ_r), harvested rainwater (θ_h) and manufactured water (θ_m) for alternative models of consumption. Calculations based on an implied risk aversion parameter of $\gamma = 1.137$.

Habit Model				Shares			Cons.	Costs
	β_r	β_h	y	θ_r	θ_h	θ_m	x	$(US\ \$m)$
Stochastic:	0.5	−0.1	97.3	0.88	0.03	0.09	242	122
Deterministic:	0.0	0.0	97.3	0.77	0.02	0.21	194	169
None:	0.0	0.0	0.0	0.97	0.03	0.00	206	86

$\$169m$ and much larger than the optimal portfolio based on stochastic habit as a result of a greater reliance on manufactured water which is relatively more costly.

The implications of ignoring non-discretionary consumption is obtained by calibrating the model with the restrictions $\beta_r = \beta_h = 0$, together with a habit level of water of $y = 0$. From Table 4 the optimal portfolio now comprises primarily of reservoir water (97%), a small amount of harvested rainfall water (3%) and no manufactured water. This portfolio is closest to the observed portfolio in 2006, which is characterised by having a much lower cost of supply equal to $US\ \$86m$ per year as a result of the greater reliance on the cheaper source of reservoir water.

The analysis shows that allowing for habit level consumption increases the need for including water from a risk free or weather independent source. This is the case especially for deterministic habit formation where the non-discretionary level of consumption has a strong tendency to revert to its mean. Stochastic habit formation is allowed for, whereby it is assumed that consumers align their non-discretionary consumption with the availability of reservoir water, yielding a portfolio characterised by a lower reliance on manufactured water.

3.3 Sensitivity Analysis

Two sensitivity analyses are conducted to identify the impact of the stochastic habit formation parameters from rainfall and reservoir inflow shocks. Table 5 shows that for a given $\beta_h = -0.1$, increasing the sensitivity of the habit level of consumption to rainfall shocks (β_h) decreases the optimal share of the safe water source (θ_m), while increasing the share allocated to reservoir water. For each of these scenarios there is at most marginal changes in the share allocated to harvested stormwater in the optimal water portfolio. In line with the theoretical predictions of the model as identified in (22), the increase in β_h leads to increases in the optimal consumption of water in this case, where shocks to reservoir inflows are on average positive, $dz_r > 0$.

The effects from increasing the sensitivity of non-discretionary water consumption to rainfall shocks (β_h) are given in Table 6. In contrast to the sensitivity results presented in Table 5 there is now an increasing reliance on manufactured water in the optimal water portfolio, largely at the expense of reservoir water. If negative rainfall shocks translate into relative higher outdoor demands then greater reliance on manufactured water is needed to service this increased demand. In the extreme scenario given in Table 6 where $\beta_r = 0.5$ and $\beta_h = -0.8$, the share allocated to manufactured water is one-third with nearly all of the remaining water sourced from the reservoir. This change in the water portfolio from reservoir water to manufactured water progressively results in higher costs given the relatively higher costs of water from the latter source that the former source.

The sensitivity results from both tables show that the optimal share of harvested water remains relatively constant at around 3% across most scenarios, suggesting that the hedging opportunities that exist between the two weather dependent sources are robust to the assumptions about habit formation.

The cost implications of the different optimal portfolios with varying sensitivity to reservoir inflow shocks and rainfall shocks are that as individuals become more respon-

Table 5:

Optimal levels of portfolio shares for reservoirs (θ_r), harvested rainwater (θ_h) and manufactured water (θ_m), and optimal water consumption (x) (in GL), for alternative values of the harvested stormwater stochastic habit parameter β_r with $\beta_h = -0.1$. Calculations based on an implied risk aversion parameter of $\gamma = 1.137$.

Habit Parameters		Shares			Cons.	Costs
β_r	β_h	θ_r	θ_h	θ_m	x	$(US\ \$m)$
0.00	-0.10	0.72	0.02	0.26	179.61	189.31
0.10	-0.10	0.79	0.02	0.19	200.42	161.22
0.20	-0.10	0.83	0.03	0.14	215.23	145.11
0.30	-0.10	0.85	0.03	0.12	226.29	134.65
0.40	-0.10	0.87	0.03	0.10	234.88	127.33
0.50	-0.10	0.88	0.03	0.09	241.74	121.90
0.60	-0.10	0.90	0.03	0.07	247.34	117.72
0.70	-0.10	0.90	0.03	0.07	252.00	114.41
0.80	-0.10	0.91	0.03	0.06	255.94	111.71

sive to reservoir inflow shocks, the cost of constructing an optimal supply portfolio decreases while the opposite holds the more orthogonal habit level consumption is to rainfall shocks. This finding suggests that policies that aim to align demand-side behaviour and investment in water saving technologies with water availability could prove cost effective by enabling less costly water supply portfolios.

4 Conclusions

An optimal water portfolio is derived where the water authorities choose an optimal time path for current and future water consumption that maximises the discounted net present value of utility from water consumption in the presence of non-discretionary consumption of water. The water assets consist of inflows from reservoirs and harvested stormwater which are treated as risky in terms of the reliability of their inflows, as well as manufactured water from desalination for example, that generates inflows that are considered to be risk-free. The model allows for uncertainty in weather patterns, supply

Table 6:

Optimal levels of portfolio shares for reservoirs (θ_r), harvested rainwater (θ_h) and manufactured water (θ_m), and optimal water consumption (x) (in GL), for alternative values of the harvested stormwater stochastic habit parameter β_h with $\beta_r = 0.5$. Calculations based on an implied risk aversion parameter of $\gamma = 1.137$.

Habit Parameters		Shares			Cons.	Costs
β_r	β_h	θ_r	θ_h	θ_m	x	(US \$$m$)
0.50	0.00	0.89	0.03	0.08	245.71	119.01
0.50	-0.10	0.88	0.03	0.09	241.74	121.90
0.50	-0.20	0.88	0.03	0.09	237.14	125.38
0.50	-0.30	0.87	0.03	0.10	231.75	129.65
0.50	-0.40	0.85	0.03	0.12	225.32	135.06
0.50	-0.50	0.84	0.02	0.14	217.47	142.20
0.50	-0.60	0.81	0.02	0.17	207.62	152.26
0.50	-0.70	0.78	0.02	0.20	194.74	168.30
0.50	-0.80	0.69	0.01	0.30	176.82	203.85

costs, as well as water managers being risk averse to water shortages. An important part of the model is that this non-discretionary component of water consumption is assumed to be stochastic as it is affected by shocks to reservoir inflows and harvested rainwater.

The model is applied to the case of Taoyuan in Taiwan, which has experienced severe water restrictions despite its sub-tropical climate and higher than average rainfall. Allowance for non-discretionary water consumption and stochastic habit formation is made, whereby negative rainfall shocks increase non-discretionary consumption due to the increased demand for outdoor irrigation, while negative shocks to reservoir inflows decrease habit level of water consumption as a result of water saving behaviours being adopted. Under these assumptions, the optimal portfolio involves reservoir water being supplemented with some harvested rainfall water as well as manufactured water. The optimal consumption level from this portfolio is similar to the observed level, but the costs of the portfolio are higher due to the positive share of high-cost manufactured water. Overall, the portfolio that is optimal under stochastic habit formation resembles the portfolio that would have resulted from the construction of the desalination plant that was proposed for Taoyuan in 2006. The share of manufactured water and the cost of the optimal portfolio increase when the habit level of water consumption is deterministic and the possibility of it being sensitive to rainfall and reservoir shocks is ignored. In contrast, ignoring habit level consumption altogether yields an optimal portfolio that is close to the observed portfolio and relatively cheaper as it does not include any contribution from manufactured water.

The importance of habit level consumption has been well established in the empirical literature suggesting that pricing policies may be less effective in the presence of non-discretionary water consumption. Allowing for non-discretionary water consumption in the determination of an optimal water portfolio, introduces the additional constraint that the portfolio needs to satisfy the non-discretionary level to be met regardless of weather conditions. This type of behaviour implies a greater portfolio share of manufactured water, which increases the total cost of supplying a given stock of water from this portfolio. If however, stochastic habit formation is allowed for and habit level consumption is assumed to adjust to some extent to the availability of reservoir water, the optimal portfolio includes a greater share of reservoir water, delivering water at a lower average cost. Policies that lead to outdoor water demand being less sensitive to rainfall, combined with policies designed to raise the sensitivity of habitual water consumption to reservoir water availability, facilitate the investment in water supply portfolios that are subject to greater supply variability and more cost-effective.

A Appendix: Water Constraint Derivation

The following derivations generate the expression for the change in the water stock presented in (4). From Leroux and Martin (2016) the cost-adjusted stock of water is defined as

$$\frac{W}{p} = \sum \frac{N_i S_i}{c_i}. \tag{33}$$

Differentiating both sides yields an expression for the change in the total water stock

$$\begin{aligned} dW &= \sum \frac{p}{c_i} dN_i S_i + \sum \frac{p}{c_i} N_i dS_i + \sum \frac{p}{c_i} dN_i dS_i \\ &+ \sum d\frac{p}{c_i} N_i S_i + \sum d\frac{p}{c_i} dN_i S_i + \sum d\frac{p}{c_i} N_i dS_i + \sum d\frac{p}{c_i} dN_i dS_i. \end{aligned} \tag{34}$$

Defining the consumption-investment trade-off as

$$\sum \frac{p}{c_i} dN_i S_i + \sum \frac{p}{c_i} dN_i dS_i + \sum d\frac{p}{c_i} dN_i S_i + \sum d\frac{p}{c_i} dN_i dS_i = -x(t)dt, \tag{35}$$

the expression for the change in the water stock becomes

$$dW = \sum \frac{p}{c_i} N_i dS_i + \sum d\frac{p}{c_i} N_i S_i + \sum d\frac{p}{c_i} N_i dS_i - x(t)dt. \tag{36}$$

The second term on the right-hand side of (36) is rewritten as

$$\sum d\frac{p}{c_i} N_i S_i = \sum \frac{d}{dS_i}\left(\frac{p}{c_i}\right) N_i S_i dS_i = \sum \lambda_i N_i S_i dS_i, \tag{37}$$

where

$$\lambda_i = \frac{d}{dS_i}\left(\frac{p}{c_i}\right).$$

As the cost adjusted share is defined as $\theta_i = \frac{pN_iS_i}{c_iW}$, (37) becomes

$$\sum \lambda_i N_i S_i dS_i = \sum \theta_i \lambda_i \frac{c_i}{p} W dS_i. \tag{38}$$

Now consider rewriting the last term of the right-hand side of (36) as

$$\sum d\frac{p}{c_i} N_i dS_i = \sum \frac{d}{dS_i}\left(\frac{p}{c_i}\right) N_i dS_i^2 = \sum \lambda_i N_i \sigma_i^2 dt, \tag{39}$$

where the last step uses Ito's lemma applied to the expression for dS_i from the stochastic differential equation in (2). Again using the definition of the cost adjusted share $\theta_i = \frac{pN_iS_i}{c_iW}$, (39) becomes

$$\sum \lambda_i N_i \sigma_i^2 dt = \sum \theta_i \lambda_i \sigma_i^2 \frac{W}{S_i} \frac{c_i}{p} dt. \tag{40}$$

Using (38) and (40) in (36) yields the following expression for the change in the water stock

$$dW = \sum \frac{p}{c_i} N_i dS_i + \sum \theta_i \lambda_i \frac{c_i}{p} W dS_i + \sum \theta_i \lambda_i \sigma_i^2 \frac{W}{S_i} \frac{c_i}{p} dt - x(t)dt \tag{41}$$

which is further rewritten as

$$dW = \sum \theta_i \frac{W}{S_i} dS_i + \sum \theta_i \lambda_i \frac{c_i}{p} W dS_i + \sum \theta_i \lambda_i \frac{c_i}{p} \sigma_i^2 \frac{W}{S_i} dt - x(t)dt, \tag{42}$$

as $\theta_i = \frac{pN_iS_i}{c_iW}$.

Using equation (2) to substitute for dS_i and rearranging gives

$$\begin{aligned} dW &= \sum \mu_i \theta_i \frac{W}{S_i} dt + \sum \mu_i \theta_i \lambda_i \frac{c_i}{p} W dt + \sum \theta_i \lambda_i \frac{c_i}{p} \sigma_i^2 \frac{W}{S_i} dt - x(t)dt \\ &\quad + \sum \sigma_i \theta_i \frac{W}{S_i} dz_i + \sum \sigma_i \theta_i \lambda_i \frac{c_i}{p} W dz_i, \end{aligned} \tag{43}$$

or in long form

$$\begin{aligned} dW &= \left(\mu_r \theta_r \frac{W}{S_r} + \mu_h \theta_h \frac{W}{S_h} + \mu_m \theta_m \frac{W}{S_m} \right) dt \\ &\quad + \left(\mu_r \theta_r \lambda_r \frac{c_r}{p} W + \mu_h \theta_h \lambda_h \frac{c_h}{p} W + \mu_m \theta_m \lambda_m \frac{c_m}{p} W \right) dt \\ &\quad + \left(\theta_r \lambda_r \frac{c_r}{p} \sigma_r^2 \frac{W}{S_r} + \theta_h \lambda_h \frac{c_h}{p} \sigma_h^2 \frac{W}{S_h} + \theta_m \lambda_m \frac{c_m}{p} \sigma_m^2 \frac{W}{S_m} \right) dt \\ &\quad - x(t)dt \\ &\quad + \left(\sigma_r \theta_r \frac{W}{S_r} dz_i + \sigma_h \theta_h \frac{W}{S_h} dz_h + \sigma_m \theta_m \frac{W}{S_m} dz_m \right) \\ &\quad + \left(\sigma_r \theta_r \lambda_r \frac{c_r}{p} W dz_r + \sigma_h \theta_h \lambda_h \frac{c_h}{p} W dz_h + \sigma_m \theta_m \lambda_m \frac{c_m}{p} W dz_m \right). \end{aligned} \tag{44}$$

As manufactured water is considered to be risk-free, $\sigma_m = 0$ thereby reducing the expression for dW in (44) to

$$
\begin{aligned}
dW &= \left(\left(\frac{\mu_r}{S_r} + \mu_r \lambda_r \frac{c_r}{p} + \frac{\sigma_r^2}{S_r} \lambda_r \frac{c_r}{p} \right) \theta_r + \left(\frac{\mu_h}{S_h} + \mu_h \lambda_h \frac{c_h}{p} + \frac{\sigma_h^2}{S_h} \lambda_h \frac{c_h}{p} \right) \theta_h \right. \\
&\quad \left. + \left(\frac{\mu_m}{S_m} + \mu_m \lambda_m \frac{c_m}{p} \right) \theta_m \right) W dt - px(t) dt \\
&\quad + \left(\frac{\sigma_r}{S_r} + \sigma_r \lambda_r \frac{c_r}{p} \right) \theta_r W dz_r + \left(\frac{\sigma_h}{S_h} + \sigma_h \lambda_h \frac{c_h}{p} \right) \theta_h W dz_h. \qquad (45)
\end{aligned}
$$

Also, applying the adding up restriction $\theta_m = 1 - \theta_r - \theta_h$ the expression for dW simplifies to

$$
\begin{aligned}
dW &= \left(a_r \theta_r + a_h \theta_h + \frac{\mu_m}{S_m} + \mu_m \lambda_m \frac{c_m}{p} \right) W dt - x(t) dt \\
&\quad + \left(\frac{\sigma_r}{S_r} + \sigma_r \lambda_r \frac{c_r}{p} \right) \theta_r W dz_r + \left(\frac{\sigma_h}{S_h} + \sigma_h \lambda_h \frac{c_h}{p} \right) \theta_h W dz_h, \qquad (46)
\end{aligned}
$$

where

$$
a_r = \frac{\mu_r}{S_r} - \frac{\mu_m}{S_m} + \mu_r \lambda_r \frac{c_r}{p} + \frac{\sigma_r^2}{S_r} \lambda_r \frac{c_r}{p} - \mu_m \lambda_m \frac{c_m}{p}, \qquad (47)
$$

$$
a_h = \frac{\mu_h}{S_h} - \frac{\mu_m}{S_m} + \mu_h \lambda_h \frac{c_h}{p} + \frac{\sigma_h^2}{S_h} \lambda_h \frac{c_h}{p} - \mu_m \lambda_m \frac{c_m}{p}, \qquad (48)
$$

represent the cost-adjusted excess flows of reservoir water and harvested stormwater relative to manufactured water. Equation (46) is the expression used for the change in the water stock given in (4).

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