Asset Markets with Heterogeneous Information

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Abstract

This paper studies competitive equilibria of economies where assets are heterogeneous and traders have heterogeneous information about them. Markets are defined by a price and a procedure for clearing trades and any asset can in principle be traded in any market. Buyers can use their information to impose acceptance rules which specify which assets they are willing to trade in each market. The set of markets where trade takes place is derived endogenously. The model can be applied to find conditions under which these economies feature fire-sales, contagion and flights to quality.

Keywords: Asymmetric information, competitive equilibrium, fire sales, expertise

JEL codes: D82, D41, G14

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1 Introduction

I study competitive asset markets where traders have different information about the assets being traded. Sellers own a portfolio of assets of heterogeneous quality and there are potential gains from trade in selling them to a group of buyers. Since Akerlof (1970), one special case has been studied in great detail: where sellers are informed and buyers are uninformed. Instead, I allow for different buyers to have different information about each of the assets.

The leading example in the paper is a financial market where different traders have different ability to assess the quality of the assets in a portfolio. However, the same information patterns naturally arise in many settings. Venture capital funds may have heterogeneous ability to judge the business plans of start-ups; some sports teams may have different information than others about the talent levels of the available free agents; some house buyers may be differentially informed about house construction quality or neighborhood trends. This paper proposes answers to some basic questions that arise in such environments: which assets will trade, who will buy each one at what price and what allocation will result.

Wilson (1980) and Hellwig (1987) first showed that in simple trading environments with asymmetric information the predictions are sensitive to the exact way that competition is modeled: the order of decisions, who proposes prices, etc. Faced with this difficulty, one approach has been to study these problems as games where all of these features are spelled out completely (Rothschild and Stiglitz 1976, Wilson 1977, Miyazaki 1977, Stiglitz and Weiss 1981, Arnold and Riley 2009). An alternative approach, which I pursue in this paper, has been to attempt to abstract from the details of how trading takes place and adapt the notion of Walrasian competitive equilibrium (Gale 1992, 1996, Dubey and Geanakoplos 2002, Bisin and Gottardi 2006) or competitive search equilibrium (Guerrieri et al. 2010, Guerrieri and Shimer 2012, Chang 2011) to settings with asymmetric information.

Existing definitions of competitive equilibrium for asymmetric information environments start by defining a set of markets with prespecified prices and allow any asset to be traded in any market; traders' decision problem is then to choose supply or demand in each market. Markets may be assumed to be exclusive (each trader may only trade in one market) or non-exclusive (each trader can trade in several markets at once). This paper studies the case where markets are non-exclusive.

Just defining markets with different prices is not enough to handle environments with many differently-informed buyers. The reason is that when different assets trade in the same market, some buyers may have enough information to tell them apart while other buyers do not. Analyzing this possibility requires developing a new notion of competitive equilibrium where traders can act on this differential information in a way that's not reducible to just choosing quantities. In the equilibrium deifnition below, buyers can act on their information by imposing acceptance rules. These specify which assets the buyer is willing to buy in each market. Each buyer's acceptance rules must be consistent with his own information; if a buyer's information is not sufficient to tell two assets apart, his acceptance rule cannot discriminate beween them.

Allowing different buyers to impose their own acceptance rules in a given market can give rise to situations where there is more than one possible way to clear the market. This indeterminacy can be resolved by defining the set of all possible market-clearing algorithms and allowing traders to direct their trades to markets that use the algorithm they prefer. Thus, the set of markets is defined as the set of all price-alogrithm pairs. Equilibrium is defined in terms of quantities and acceptance rules for each market.

I focus on a basic case where there are two qualities of assets, good and bad, in known proportions. Each seller owns a representative portfolio of these assets, and the source of gains from trade is that a fraction of sellers are impatient. Sellers know the quality of each asset they own but buyers cannot observe it directly. Instead, they each observe an imperfect binary signal about each asset. I first chracterize the equilibrium in a case with "false positives" only: buyers may observe good signals from bad assets but not the other way around. I then study the opposite information structure, with "false negatives" but no false positives. In both cases, buyers can be ranked by their expertise, i.e. their probability of making mistakes.

For the false positives case, the equilibrium can be characterized quite simply. All assets trade at the same price; sellers of good assets can sell as many units as they choose at that price but sellers of bad assets face rationing. Bad assets that are more likely to be mistaken for good assets face less rationing than easily detectable ones, and some assets cannot be traded at all. Only buyers who observe sufficiently informative signals choose to trade, while the rest stay out of the market.

One question of applied interest has to do with what happens if the number of impatient sellers increases. Will there be fire-sale effects, with prices falling with the number of impatient sellers? Uhlig (2010) has shown that in a pure asymmetric information case with equally uninformed buyers prices should go up with the number of impatient sellers because these are the only ones that sell good assets. With differentially informed buyers, there are countervailing effects. When more assets are sold the set of active buyers must expand, so the marginal buyer has worse information; as a result, the net effect depends on the joint distribution of buyers' wealth and information quality in a way that is easily characterized. The price will drop if the density of wealth conditional on the marginal buyer's information quality is low, so that a large fall in the cutoff level of information quality is needed to absorb an increase in supply.

The model also implies that as long as two asset classes share at least part of the pool of investors, there will be contagion between them. If an increase in the number of distressed sellers in one assets class leads to a drop in prices, investors will shift towards that asset class; the set of active buyers in the other asset class must expand to make up for this so the quality of information of the marginal buyer will fall, requirng lower prices too.

For the false negatives case, different good assets trade at different prices, which depend on how many buyers are able to realize that the asset is good. Thus more transparent assets command a premium. In this case, an increase in the number of impatient sellers leads to flight-to-quality effects, where the premium for the most transparent assets increases.

2 The Economy

Dates and assets

There are two periods, $t = 1$ and $t = 2$. Consumption at time t is denoted c_t.

Assets are indexed by $i \in [0, 1]$. Asset i will produce $q(i) = \mathbb{I}(i \geq \lambda)$ goods at $t = 2$ for some $\lambda \in (0, 1)$. This means that a fraction $1 - \lambda$ of assets (those with indices $i \geq \lambda$) are good assets and will pay a dividend of 1 at $t = 2$ and a fraction λ (those with indices $i < \lambda$) are bad and will pay nothing.

Agents, preferences and endowments

Agents are divided into buyers and sellers. Buyers are indexed by $b \in [0, 1]$. Preferences for buyers are

$$u(c_1, c_2) = c_1 + c_2$$

and their consumption is constrained to be nonnegative. Buyer b has an endowment of $w(b)$ goods at $t = 1$, where w is a continuous, strictly positive function. Let

$$W(b) \equiv \int_b^1 w\left(\tilde{b}\right) d\tilde{b}$$

be the total endowment of buyers whose indices are at least b.

Sellers are indexed by $s \in [0,1]$. Preferences for sellers are

$$u(c_1, c_2, s) = c_1 + \beta(s) c_2$$

with

$$\beta(s) = \mathbb{I}(s \geq \mu)$$

I refer to buyers of types $s < \mu$ as "impatient" or "distressed". Their impatience is the source of gains from trade. Each seller is endowed with a portfolio containing one unit of each asset.

I will assume that

$$W(0) \geq \mu(1-\lambda) \tag{1}$$

i.e. that the total endowment of all buyers is at least as large as the total dividends of the assets owned by distressed sellers.

Linearity in preferences is assumed for simplicity. For sellers, it means that the decision of what to do with one asset does not depend on what the seller does with any other asset. For buyers, it means that if they choose to trade they will be at a corner solution where they spend all their endowment.

Information

Each seller knows the index i, and therefore the dividend $q(i)$, of each asset he owns. Buyers do not observe i. Instead, buyer b observes a signal $x(i,b)$ whenever he analyzes asset i. If $x(\cdot,b)$ were invertible, i.e. if $x(i,b) \neq x(i',b)$ whenever $q(i) \neq q(i')$, then buyer b would be perfectly informed about asset qualitites. The interesting case arises when this is not the case for at least some buyers, who can therefore not tell apart some assets of different qualities.

I will focus on two possible cases, illustrated in Figure 1.[1] In the false positives case, buyer b observes

$$x(i,b) = \mathbb{I}(i \geq b\lambda) \tag{2}$$

When an asset is good, every buyer observes $x(i,b) = 1$. When an asset i is a bad, those buyers of types $b \leq \frac{i}{\lambda}$ will observe $x(i,b) = 1$, so they cannot distinguish it from a good asset; instead, buyers with $b > \frac{i}{\lambda}$ will observe $x(i,b) = 0$ and conclude that the asset is bad.

[1] In Appendix C I characterize the equilibrium for two further cases: one with both false positives and false negatives and one with false positives but non-nested information sets.

A buyer's type b can therefore be thought of as an index of expertise: higher values of b means that there is a smaller subset of bad assets that the buyer might misidentify as good assets. Furthermore, expertise is nested: if type b can identify that asset i is bad, then so can all types $b' > b$.

Conversely, in the false negatives case, buyer b observes

$$x(i,b) = \mathbb{I}\left(i \geq 1 - b(1-\lambda)\right) \tag{3}$$

When an asset is bad all buyers observe $x(i,b) = 0$ but when it is good only those buyers with $b \geq \frac{1-i}{1-\lambda}$ observe $x(i,b) = 1$ and realize it is good. Again, b can be thought of as an index of expertise.

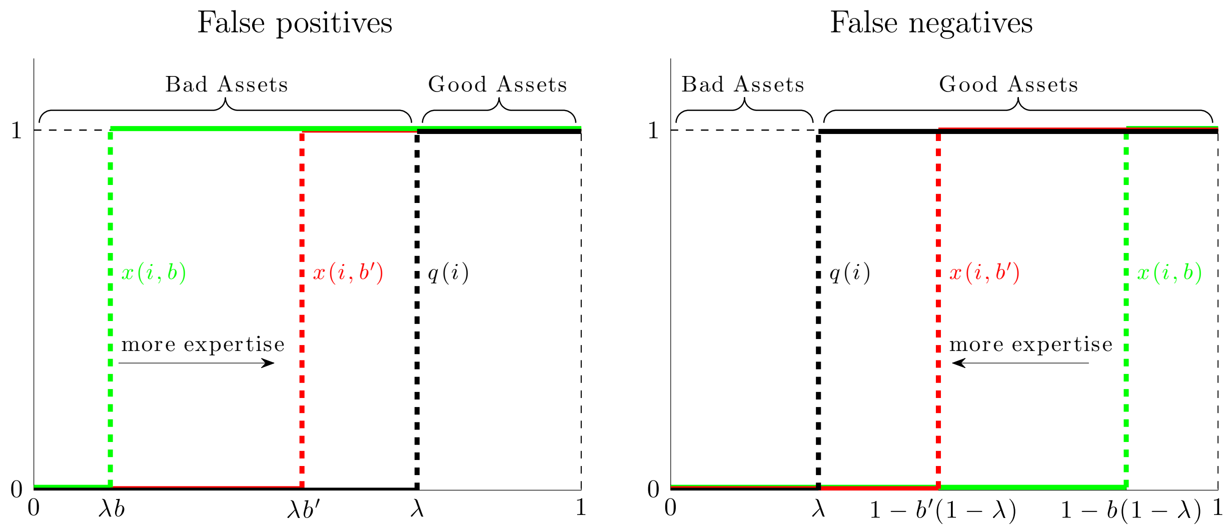

Figure 1: Information of buyers in the two examples

3 Equilibrium

Markets

There is no market for trading $t = 1$ goods against $t = 2$ goods. If there was such a market, which can be interpreted as a market for uncollateralized borrowing, then impatient sellers would borrow up to the point where $c_2 = 0$ and the gains from trade would be exhausted. Instead, the only way to achieve some sort of intertemporal trade is to trade $t = 1$ goods for assets. These assets will in turn produce $t = 2$ goods.

There are many markets, open simultaneously, where agents can exchange goods for assets. Each market m is defined by two features. The first is a price $p(m)$ of assets in terms of goods. The second is what I call a clearing algorithm. Clearing algorithms (described in more detail below) are rules that dictate how assets that are brought to a market will be allocated, depending on the actions of buyers and sellers.

Markets do not specify which assets will be traded in them so, in principle, any asset can be traded in any market. However, markets need not clear: assets that are offered for sale in market m may remain totally or partially unsold.

Markets are assumed to be non-exclusive. In particular, sellers are allowed to offer the same asset for sale in as many markets as they want. Whether exclusivity or non-exclusivity is a more appropriate assumption depends on the application. Non-exclusivity seems more appropriate in contexts where it is hard for market participants to observe each others' endowments or trades, so that sellers cannot credibly demonstrate that they are abiding by a commitment to exclusive dealing. For instance, it seems a better fit for the problem of hedge funds trading securities than for a publicly traded corporation issuing debt, where exclusivity can be more easily enforced.

The set of all markets is denoted by M.

Seller's problem

Sellers must choose how much to supply of each asset in each market. Formally, each seller chooses a function $\sigma : [0,1] \times M \to [0,1]$, where $\sigma(i,m)$ represents the number of i assets that the seller supplies in market m.

From the point of view of the sellers, markets are characterized by their prices $p(m)$ and a rationing function η.

Definition 1. A rationing function η assigns a measure $\eta(\cdot;i)$ on M to each possible asset i.

If $M_0 \subseteq M$ is a set of markets, $\eta(M_0;i)$ is the number of assets of index i that the seller will end up selling if he supplies one unit of asset i to each market $m \in M_0$. For instance, if $\eta(m;i) = \alpha$, this means that a seller who supplies one unit of asset i in market m will end up selling α units in that market. Implicit in this formulation is the idea that assets are perfectly divisible, so there is exact pro-rata rationing rather than a probability of selling an indivisible unit. η is an endogenous object, which results from clearing algorithms and from equilibrium supply and demand. Each seller simply takes it as given.

Seller s solves the following problem:

$$\max_{c_1, c_2, \sigma} u(c_1, c_2, s) \tag{4}$$

$$s.t.$$

$$c_1 = \int_{[0,1]} \left[\int_M p(m)\, \sigma(i,m)\, d\eta(m;i) \right] di \tag{5}$$

$$c_2 = \int_{[0,1]} q(i) \left[1 - \int_M \sigma(i,m)\, d\eta(m;i) \right] di \tag{6}$$

$$\int_M \sigma(i,m)\, d\eta(m;i) \leq 1 \quad \forall i \tag{7}$$

$$0 \leq \sigma(i,m) \leq 1 \quad \forall i, m \tag{8}$$

$$c_1 \geq 0 \quad c_2 \geq 0 \tag{9}$$

Constraint (5) computes how many goods the seller gets at $t = 1$ as a result of his sales. For each asset i, he supplies $\sigma(i,m)$ in market m and receives $p(m)$ for each unit he sells. Integrating across markets using measure $\eta(\cdot; i)$ and adding across all assets i results in (5). Constraint (6) computes how many goods the seller gets at $t = 2$ as a result of the assets which he does not sell. For each asset i his unsold assets are equal to his endowment of 1 minus what he sold in all markets, and each yields $q(i)$ goods.

Constraint (7) just says that the total sales of any given asset (added across all markets) are constrained by the seller's endowment. However, the constraint is on actual sales and not on attempted sales, i.e. the following constraint is *not* imposed:

$$\sum_{m \in M_0} \sigma(i,m) \leq 1 \quad \forall M_0 \subseteq M \text{ countable}, \forall i \tag{10}$$

Imposing (7) rather than (10) reflects the non-exclusivity assumption.

Constraint (8) says that supply is nonnegative and that a buyer can at most attempt to sell his entire endowment of each asset in any given market. This is important when $\eta(m;i) < 1$. It rules out a strategy of offering, say, 4 units for sale when he only owns 1 because he knows that due to rationing, only $\frac{1}{4}$ of the units are actually sold. The fact that the upper bound on supply is 1 is not essential. Nothing would change if instead one imposed $\sigma(i,m) \leq K$ for a large K. What matters is that if there is rationing in a market, sellers cannot fully undo it by offering unbounded amounts of assets for sale.

The choice of $\sigma(i,m)$ for any single market m such that $\eta(m;i) = 0$ has no effect on the utility obtained by the seller. The interpretation of this is that if he is not going to be able to sell, it doesn't matter whether or not he tries. Formally, this means that program (4) has multiple solutions. I am going to assume that when this is the case, the solution has to be robust to small positive $\eta(m;i)$, meaning that the seller must attempt to sell an asset in all the markets where if he could he would want to and must not attempt to sell an asset in any market where if he could he would not want to.

Definition 2. A solution to program (4) is *robust* if for every $\{i_0, m_0\}$ such that $\eta(m_0; i_0) = 0$ there exists a sequence of strictly positive real numbers $\{z_n\}_{n=1}^{\infty}$ and a sequence of consumption and selling decisions c_1^n, c_2^n, σ^n such that, defining

$$\eta^n(M_0; i) = \eta(M_0; i) + z_n \mathbb{I}(m_0 \in M_0)\,\mathbb{I}(i = i_0)$$

1. c_1^n, c_2^n, σ^n solve program

$$\max_{c_1, c_2, \sigma} u(c_1, c_2, s) \tag{11}$$
$$s.t.$$
$$c_1 = \int_{[0,1]} \left[\int_M p(m)\,\sigma(i,m)\,d\eta^n(m;i) \right] di$$
$$c_2 = \int_{[0,1]} q(i) \left[1 - \int_M \sigma(i,m)\,d\eta^n(m;i) \right] di$$
$$0 \le \sigma(i,m) \le 1 \quad \forall i, m$$
$$\int_M \sigma(i,m)\,d\eta^n(m;i) \le 1 \quad \forall i$$
$$c_1 \ge 0 \quad c_2 \ge 0$$

2. $z_n \to 0$

3. $c_1^n \to c_1$, $c_2^n \to c_2$ and $\sigma^n(i,m) \to \sigma(i,m)$ for all i, m.

Lemma 1. *Every robust solution to program (4) satisfies*

$$\sigma\left(i,m\right)=\begin{cases}1 & \textit{if}\ p\left(m\right)>p^{R}\left(i\right)\\ 0 & \textit{if}\ p\left(m\right)<p^{R}\left(i\right)\end{cases}$$

for some $p^{R}\left(i\right)$*.*

Lemma 1 implies that sellers will use a simple cutoff rule for deciding what markets to try to sell their assets in. For each asset i they will choose a reservation price $p^{R}\left(i\right)$. They will try to sell their entire endowment of i assets in every market where $p\left(m\right)>p^{R}\left(i\right)$ and will not attempt to sell i assets in any market where $p\left(m\right)<p^{R}\left(i\right)$. The Lemma does not exactly specify what sellers do in markets where $p\left(m\right)=p^{R}\left(i\right)$. They may for instance choose to attempt to sell their assets in some but not others.

Imposing rubustness in seller's decisions will rule out self-fulfilling equilibria where sellers don't supply assets at certain prices because there are no buyers and buyers do not try to buy at those prices because there are no sellers. In a robust solution, sellers will always supply their assets in markets where the price is attractive, even if they know they won't be able to sell them.

Buyer's problem

When buyers place orders in a market, they can specify both the quantity of assets that they demand and what subset of assets they are willing to accept. An example of an order will be "I offer to buy 5 assets as long as the indices i of those assets satisfy $i\geq0.4$". I formalize the idea that buyers can be selective by defining acceptance rules:

Definition 3. An *acceptance rule* is a function $\chi:[0,1]\rightarrow\{0,1\}$.

$\chi\left(i\right)=1$ means that a buyer is willing to accept asset i and $\chi\left(i\right)=0$ means he is not. Buyers cannot just impose any selection rule that they want, such as accepting only good assets. They are not necessarily able to tell different assets apart from each other since they do not observe i but just the imperfect signal $x\left(i,b\right)$. Feasible acceptance rules are those that only discriminate between assets that buyers can actually tell apart.

Definition 4. An acceptance rule χ is *feasible for buyer* b if it is measurable with respect to buyer b's information set, i.e. if

$$\chi\left(i\right)=\chi\left(i'\right)\quad\text{whenever}\quad x\left(i,b\right)=x\left(i',b\right)$$

In general, since different buyers observe different signals, the set of feasible acceptance rules will be different for each of them and in equilibrium they will end up imposing different acceptance rules. I denote the set of possible acceptance rules by X and the set of acceptance rules that are feasible for buyer b by X_b.

Buyers must choose a market m to buy from, the number of units δ to demand in it and the acceptance rule χ that they will impose.[2] From their point of view, markets are characterized by their prices $p(m)$ and an allocation function A.

Definition 5. An *allocation function* A assigns a measure $A(\cdot;\chi,m)$ on $[0,1]$ to each acceptance rule-market pair $(\chi,m) \in X \times M$.

If $I_0 \subseteq [0,1]$, $A(I_0,\chi,m)$ represents the amount of assets $i \in I_0$ that a buyer will obtain if he demands one unit in market m and imposes acceptance rule χ. A is an endogenous object, which results from the clearing algorithms and equilibrium supply and demand. Each buyer simply takes it as given.

Buyer b solves the following problem:

$$\max_{c_1,c_2,\delta,m,\chi} u(c_1,c_2) \tag{12}$$

$$s.t.$$

$$c_1 = w(b) - p(m)\,\delta A([0,1];\chi,m) \tag{13}$$

$$c_2 = \delta \int_{[0,1]} q(i)\, dA(i;\chi,m) \tag{14}$$

$$\chi \in X_b \tag{15}$$

$$c_1 \geq 0 \quad c_2 \geq 0 \quad \delta \geq 0 \tag{16}$$

Constraint (13) says that $t = 1$ consumption is equal to the buyer's endowment minus what he spends on buying assets. Upon demanding δ assets in market m and imposing acceptance rule χ he obtains $\delta A([0,1];\chi,m)$ assets and pays $p(m)$ for each of them. Constraint (14) computes the total amount of $t = 2$ goods that the buyer will obtain. This is given by adding up the dividends from the assets he acquires using measure $A(\cdot;\chi,m)$. Constraint (15) restricts the buyer to using a feasible acceptance rule.

[2] One could allow buyers to demand assets from more than one market, or to use more than one acceptance rule. Given the linearity in the environment, restricting them to a single market and acceptance rule is without loss of generality.

Clearing algorithms

Each market is defined by a price $p(m)$ and a clearing algorithm. A clearing algorithm is a rule that determines what trades take place as a function of what trades are proposed by buyers and sellers.

Consider the example in Table 1.

i	$q(i)$	$\chi(i)$ of buyer b_1	$\chi(i)$ of buyer b_2	Supply
Black	0	0	0	1.5
Red	0	1	0	1.5
Green	1	1	1	1.5
		$\delta_{b_1}=1$	$\delta_{b_2}=1$	

Table 1: Example of supplies and demands in a market

There are three types of assets. Black and Red assets are bad while Green assets are good. There are two buyers, with types b_1 and b_2. Type b_1 cannot tell apart Red and Green so he must either accept both of them or reject both of them; assume he is willing to accept both of them but rejects Black assets, which he can tell apart. Type b_2 can distinguish the worthless Black and Red assets from the good Green assets so he can impose that he will only accept Green assets. Each of the buyers demands a single unit. The total supply from all sellers is 1.5 units of each asset.

One possible clearing algorithm would say: "let b_1 choose first and take a represenative sample of the assets he is willing to accept; then b_2 can do the same". This would result in the following allocation and rationing functions:

$$A(i;\chi,m)=\begin{cases}\begin{array}{ccc}\text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black}\\ 0.5 & 0 & \text{if } i=\text{Red}\\ 0.5 & 1 & \text{if } i=\text{Green}\end{array}\end{cases};\quad \eta(m;i)=\begin{cases}0 & \text{if } i=\text{Black}\\ \frac{1}{3} & \text{if } i=\text{Red}\\ 1 & \text{if } i=\text{Green}\end{cases} \tag{17}$$

Type b_1 picks randomly from the sample excluding the rejected Black assets. Since there are equal amounts of Red and Green assets and the total exceeds his demand, he gets a measure 0.5 of each. After that, type b_2 gets to pick. He only accepts Green assets and there is one unit left, which is exactly what he wants. From the sellers' point of view, all Green assets are sold but only $\frac{1}{3}$ of Red assets and no Black assets are sold.

Another possible clearing algorithm would say "let b_2 choose first and take a representative sample of the assets he is willing to accept; then b_1 can do the same". This results in:

$$A\left(i;\chi,m\right)=\begin{cases} \text{if } \chi=\{0,1,1\} & \text{if } \chi=\{0,0,1\} & \\ 0 & 0 & \text{if } i=\text{Black} \\ 0.75 & 0 & \text{if } i=\text{Red} \\ 0.25 & 1 & \text{if } i=\text{Green} \end{cases} ;\quad \eta\left(m;i\right)=\begin{cases} 0 & \text{if } i=\text{Black} \\ \frac{1}{2} & \text{if } i=\text{Red} \\ \frac{5}{6} & \text{if } i=\text{Green} \end{cases} \tag{18}$$

After b_2 picks one unit of Green assets, there are only 0.5 units left, and there are still 1.5 units of Red assets. A respresentative sample from this remainder will give type b_1 a total of 0.75 units of Red assets and 0.25 units of Green assets.

Clearly different algorithms result in different allocations and it is necessary to determine which algorithm will be used. The equilibrium definition below assumes that there exist separate markets for each possible clearing algorithm and traders can choose which of these markets they wish to trade in. To make this statement precise, I need to describe the set of possible clearing algorithms.

Definition 6. A *clearing algorithm* is a total order on X.

Clearing algorithms are rules for ordering acceptance rules. If acceptance rule χ precedes acceptance rule χ' under algorithm ω, this is denoted by $\chi <_\omega \chi'$. Let Ω be the set of all clearing algorithms, i.e. the set of all total orders on X.

Once buyers' trades have been ordered, they are executed sequentially. When a buyer's trade is executed, the buyer picks a representative sample of the acceptable assets, if any, that remain on sale in the market. The assumption that buyers always pick representative samples is meant to capture the idea that all assets that a buyer accepts look alike to him. It is then natural to assume that they obtain a random sample among them or, if the law of large numbers applies, an exact representative sample.

In the example from Table 1, the first clearing algorithm dictates $\{0,1,1\} <_\omega \{0,0,1\}$ while the second clearing algorithm dictates $\{0,0,1\} <_\omega \{0,1,1\}$.[3]

Allocation and rationing functions result from applying each market's clearing algorithm to the demand and supply in that market. See Appendix B for details.

[3]The examples don't specify when trades with other possible acceptance rules would clear. A complete description of the clearing algorithm would specify that too.

Definition of equilibrium

The set of markets $M = \mathbb{R}^+ \times \Omega$ is the set of all possible pairs of a positive price and a clearing algorithm. An equilibrium consists of:

1. Consumption and supply for each seller: $c_{1,s}, c_{2,s}, \sigma_s$
2. Consumption, asset demand, choice of market and acceptance rules for each buyer: $c_{1,b}, c_{2,b}, \delta_b, m_b, \chi_b$
3. An allocation function A
4. A rationing function η

such that

1. $c_{1,s}, c_{2,s}, \sigma_s$ are a robust solution to program (4) for each seller s, taking η as given
2. $c_{1,b}, c_{2,b}, \delta_b, m_b, \chi_b$ solve program (12) for each buyer b, taking A as given
3. A and η follow from applying each market's clearing algorithm

4 False Positives Case

For this information structure, there is an essentially unique equilibrium. Define less-restrictive-first algorithms (LRF) as follows:

Definition 7. ω is a *less-restrictive-first* algorithm if it orders acceptance rules of the form $\chi_g = \mathbb{I}(i \geq g)$ according to $\chi_g <_\omega \chi_{g'}$ if $g < g'$.

Given the information structure, feasible acceptance rules must take the form of a simple cutoff rule. LRF algorithms order them from least restrictive (lower cutoff for acceptance) to most restricitve (higher cutoff for acceptance).

Let p^* and b^* de defined by the solution to:

$$\int_{b^*}^{1} \frac{1}{\lambda(1-b) + \mu(1-\lambda)} \frac{w(b)}{p^*} db = 1 \tag{19}$$

$$p^* = \frac{\mu(1-\lambda)}{\lambda(1-b^*) + \mu(1-\lambda)} \tag{20}$$

Lemma 2. *There is a unique solution to equations (19) (20), with $b^* \in (0,1)$ and $p^* \in (0,1)$.*

Let market m^* be a market where the price is p^* and the algorithm is LRF.

Proposition 1. *There exists a competitve equilibrium where:*

1. *The reservation price for asset i for seller s is:*

$$p_s^R(i) = \begin{cases} p^* & \text{if } i \geq \lambda \text{ and } s < \mu \\ 1 & \text{if } i \geq \lambda \text{ and } s \geq \mu \\ 0 & \text{if } i < \lambda \end{cases} \tag{21}$$

2. *Buyers with $b \geq b^*$ demand $\frac{w(b)}{p^*}$ assets in market m^*, imposing $\chi(i) = \mathbb{I}(i \geq \lambda b)$ and other buyers don't demand assets*

3. *The allocation function gives rule $\mathbb{I}(i \geq \lambda b)$ in market m^* the following density over assets:*

$$a(i, \mathbb{I}(i \geq \lambda b), m^*) = \frac{\mathbb{I}(i \in [\lambda b, \lambda)) + \mu \mathbb{I}(i \geq \lambda)}{\lambda(1-b) + \mu(1-\lambda)} \tag{22}$$

4. *Rationing in market m^* is*

$$\eta(m^*; i) = \begin{cases} 1 & \text{if } i \geq \lambda \\ \int_{b^*}^{\frac{i}{\lambda}} \frac{1}{\lambda(1-b)+\mu(1-\lambda)} \frac{w(b)}{p^*} db & \text{if } i \in [\lambda b^*, \lambda) \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

See Appendix A for a full statement of all the equilibrium objects. The equilibrium works as follows. There is a single market m^*, with $p(m^*) = p^*$, where all trades take place. In this market, distressed sellers supply all their assets while non-distressed sellers only supply bad assets. Total supply is therefore μ of each good asset and 1 of each bad asset.

Supply decisions in markets $m \neq m^*$ have no effect on sellers' utility since $\eta(m; i) = 0$, so they are determined in equilibrium by the robustness requirement. By Lemma 1, this involves a reservation price for each asset for each seller. For good assets, it turns out that fraction that they are able to sell in market m^* is 1. Therefore distressed sellers' reservation price for them is p^*: they supply them in all markets where the price is above p^* (where if they could, they would rather sell them) but don't supply them in any markets where the price is below p^* (since they are able to sell them for sure at p^*, they don't want to sell them at a lower price). For bad assets, the fraction that can be sold in market m^* is strictly below 1. Therefore the reservation price for all sellers is 0: all sellers supply them in all markets.

The clearing algorithm in market m^* is LRF. As in example (17), being preceded by less-restrictive trades is not a problem for buyers because these trades don't change the relative proportions of acceptable assets in the residual supply faced by a more-restrictive buyer. Instead, as in example (18), any buyer faces more adverse selection if higher-b buyers have cleared before him. Therefore buyers self-select into trading in a LRF market, where all buyers end up receiving a representative sample of the overall supply of assets they are willing to accept. Informally, one could think that a lower-b buyer would rather trade in a market where the price is $p+\varepsilon$ but he is guaranteed to be first in line than in a market where the price is p but higher-b buyers clear their trades before him.

Sellers, for their part, are indifferent regarding what algorithm is used to clear trades: they just care about the price and fraction of assets they will be able to sell. Therefore they supply the same assets in all markets that have the same price.[4]

Buying from markets with prices other than p^* is not optimal for buyers. At prices lower than p^*, the supply includes only bad assets, so buyers prefer to stay away, whereas at prices above p^*, the supply of assets is exactly the same as at p^* but the price is higher.

This does not settle the question of whether a buyer chooses to buy at all. Buyers who choose to buy from market m^* can reject some of the bad assets that are on sale there, but not all of them. Consider a buyer of type b. The sample of assets he accepts includes all the good assets that are supplied, of which there are $\mu(1-\lambda)$, as well as all bad assets with indices $i \in (b\lambda, \lambda]$, which total $\lambda(1-b)$. Therefore the terms of trade (in terms of $t=2$ goods per $t=1$ good spent) for buyer b are

$$\tau(b) = \frac{1}{p^*}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)} \tag{24}$$

$\tau(b)$ is increasing in b because the higher-b buyers can reject more of the bad assets and therefore draw from a better sample overall. Condition (20) implies that the terms of trade for type b^* are $\tau(b^*)=1$, which leave him indifferent between buying or not. Buyers with $b>b^*$ get $\tau(b)>1$, so they spend all their endowment buying assets and buyers with $b<b^*$

[4] All markets besides m^* have zero demand, so no matter what the clearing algorithm, a buyer in those markets would receive a representative sample of the assets he accepts, just as in market m^*. This means that buyers are indifferent between buying in market m^* or in other markets where the price is also p^*, but sticking to m^* is one of the optimal choices. Conversely, imposing robustness in sellers' solution does not settle what sellers do about markets with price p^* and other clearing algorithms. Supplying the same assets they supply in m^* is one of the optimal choices.

would get $\tau(b) < 1$, so they prefer not to buy at all.

The fraction of assets i that can be sold in market m^* is given by the ratio of the total allocation of that asset across of buyers to the supply of that asset. For good assets, the supply is $\mu(1-\lambda)$ and buyer b (with $b \geq b^*$) obtains $\frac{w(b)}{p}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$ units. Adding across buyers and imposing that all good assets get sold results in (19).

Figure 2 illustrates how the equilibrium p^* and b^* are determined. The indifference condition (20) defines an upward-sloping relationship between p^* and b^*: at higher prices, terms of trade are lower so the marginal buyer needs to be more expert. The market-clearing condition (19) defines a downward-sloping relationship between p^* and b^*: at higher prices, more wealth is needed to buy the entire supply of good assets, which requires the participation of lower-b buyers.

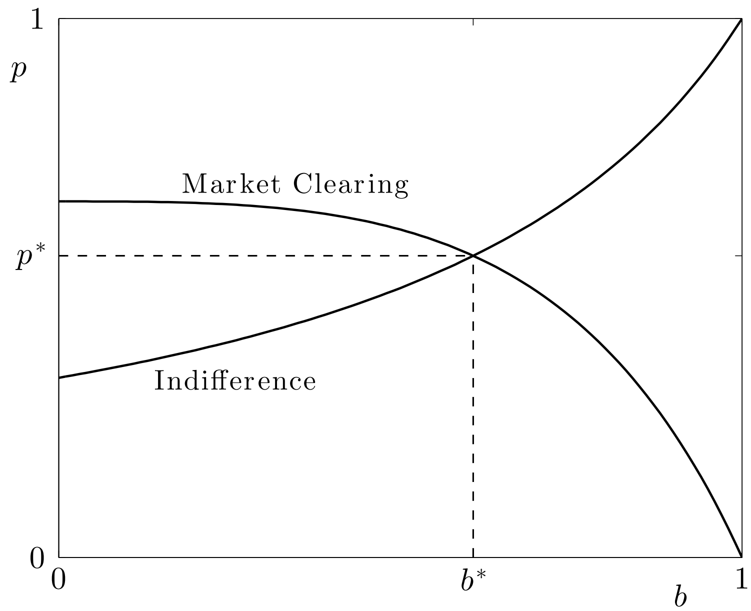

Figure 2: Equilibrium conditions. The example uses $\lambda = 0.5$, $\mu = 0.5$ and $w(b) = 0.7b^2$.

For assets $i \in (\lambda b^*, \lambda]$, the supply is 1 and buyer b obtains $\frac{w(b)}{p}\frac{1}{\lambda(1-b)+\mu(1-\lambda)}$ as long as $b \in [b^*, \frac{\lambda}{i})$; lower types demand nothing and higher types reject asset i. This implies the rationing function (23), as illustrated in Figure 3. Notice that $\eta(m^*, i)$ is continuous in i. Bad assets with indices just below λ fool almost all buyers into thinking they are likely to be good and therefore sellers are able to sell a high fraction of them; assets with indices just above λb^* fool very few buyers and only a low fraction are sold. Assets with $i < \lambda b^*$ are rejected by all buyers who choose to trade and cannot be sold at all.

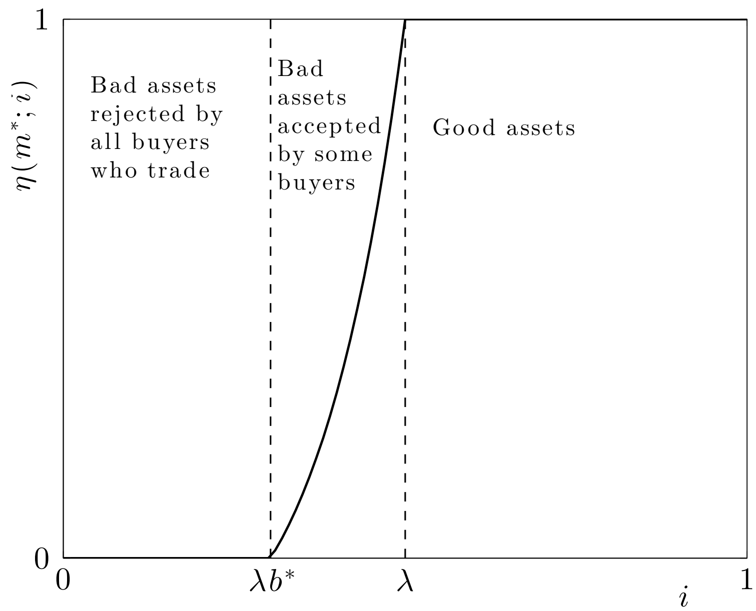

Figure 3: Rationing function. The example uses $\lambda = 0.5$, $\mu = 0.5$ and $w(b) = 0.7b^2$.

The equilibrium described above is (essentially) unique.

Proposition 2. *In any equilibrium, the price and allocations are those of the equilibrium described by equations (19)-(23).*

The proof (in Appendix A) proceeds in several steps. I first show (Lemmas 3 and 4) that the logic of the example in Table 1 generalizes: given the acceptance rules that buyers use, as the rounds of a clearing algorithm advance, the pool of remaining assets always weakly worsens; therefore (Lemma 5) given any acceptance rule, buyers obtain the best possible terms of trade if their trades clear in the first round. This allows an easy characterization of an upper bound on the terms of trade that buyer b can obtain in any market m (Lemma 6): they can never do better than what would result from imposing $\chi(i) = \mathbb{I}(i \geq \lambda b)$ and clearing in the first round. Using this result, I show that in equilibrium it must be that all trades take place at the same price (Lemma 7): if there were more than one price, say p_H and p_L where trades take place, I can always find a market where $p \in (p_L, p_H)$ where any buyer can obtain better terms of trade than the upper bound on what he can obtain by buying at p_H.[5] I then show (Lemma 8) that in any equilibrium where there is trade at price p it must be that all distressed sellers are able to sell all the good assets at price p: otherwise buyers would be able to obtain better terms of trade at prices below p. The combination of a single price, the condition that all good assets can be sold and buyer optimization implies that equations (19) and (20) must hold.

Other equilibria are possible, but they all lead to the same allocation. They only differ in terms of in which of the markets where $p(m) = p^*$ trades take place. Trades could, for instance, all take place in a market where the acceptance rule $\chi(i) = \mathbb{I}(i \in (0.3, 0.5))$ takes precedence over all others and after that the rule is less-restrictive-first. Since $\chi(i) = \mathbb{I}(i \in (0.3, 0.5))$ is not feasible for any buyers, this would make no difference for allocations. Trades could also take place in more than one market. For instance, buyers $b \in [b^*, b^* + \Delta]$ could trade in an LRF market while buyers $b \in (b^* + \Delta, 1]$ trade in a market that is LRF but only for rules of the form $\chi(i) = \mathbb{I}(i \geq g)$ with $g > (b^* + \Delta)\lambda$. What is common to all equilibria is that all trades take place at price p^* and that no buyer trades after a more-informed buyer in the same market.

Notice that the equilibrium nests the special case with no information differentials among buyers, which obtains when $w(b)$ is degenerate at $b = 0$. In that case $p = \frac{\mu(1-\lambda)}{\lambda+\mu(1-\lambda)}$ and all assets are sold.

Pooling

The equilibrium described above features a form of pooling, in that different assets trade in the same market at the same price. This is in contrast to Gale (1992, 1996), Guerrieri et al. (2010), Guerrieri and Shimer (2012) and Chang (2011). All of these models use a similar many-market construct: rather than letting the price clear markets, all possible prices coexist and markets clear by rationing. However, unlike the current model, they all feature separating equilibria.

The reason for this difference is the assumption of non-exclusivity. All the above-mentioned models assume that markets are exclusive. Under this assumption, a seller's decision to offer an asset for sale in market m entails a commitment to keep it in case in equilibrium it is not sold in that market. This commitment is used as a signal of quality,

[5] The single-price result is special to the case where the dividend paid by bad assets is exactly zero. If that dividend were $q_L > 0$, then it is easy to show that there would also be trade in at least one market with $p(m) = q_L$. Indeed, even if we maintain the assumption that the dividend of bad assets is zero, there are equilibria where as well as trading at p^*, traders trade bad assets at a price of zero. Of course, these trades don't matter for allocations.

which sustains separation. Instead, if markets are non-exclusive, the decision to offer an asset for sale in market m says nothing about a seller's willingness to keep the asset because nothing prevents him from offering it for sale in market m' as well. The pooling outcome is a consequence of this.

Wilson (1980) proposes a game-theoretic analysis of Akerlof's model, distinguishing between a "buyers' equilibrium" and a "sellers' equilibrium". In the first, buyers propose prices and sellers are not constrained to attempt to sell at a single price, resulting in pooling. In the second, sellers propose prices and are committed to the price they announce, which allows for separation.

The fact that pooling follows from non-exclusivity is not reliant on heterogeneously informed buyers. It would still hold in the special case where all buyers are equally uninformed (if the distribution of endowments were degenerate at $b = 0$). What heterogeneity among buyers does is to allow different buyers to draw different proportions of assets from the same pool.

Changes in information

The model can be used to examine what happens if the quality of information changes.

Definition 8. For two otherwise identical economies where the endowment functions are w and $\tilde{w}$ respectively, with $W(0) = \tilde{W}(0)$, the economy with endowment $\tilde{w}$ has better information if $\tilde{W}(b) \geq W(b)$ for all b.

Given the way the model is parametrized, the information of each buyer is fixed; better information is represented by a shift in wealth towards higher-expertise buyers in a FOSD sense, holding total wealth constant. This is isomorphic to assuming that the wealth of each buyer is held fixed but their expertise shifts up. Conversely, a deterioration of information can equivalently be the result of the highest-expertise buyers losing their wealth or of all buyers unergoing a drop in their level of expertise.

Proposition 3. *p^* and b^* increase with better information.*

Proposition 3 provides a meaningful way to think about the effects of changes in the degree of informational asymmetry. If buyers become less expert (or, equivalently, if the more expert buyers lose wealth), the marginal buyer will become less expert and prices will drop. Increases in the informational asymmetry could be the result of traders realizing the

the models that they relied on to value securities are not as accurate as they thought or, as Dang et al. (2009) argue, they could be result of negative shocks themselves.

Figure 4 shows the result graphically. A worsening of information leads to a downwad shift in the market clearing condition (19) and hence p^* and b^* must fall.

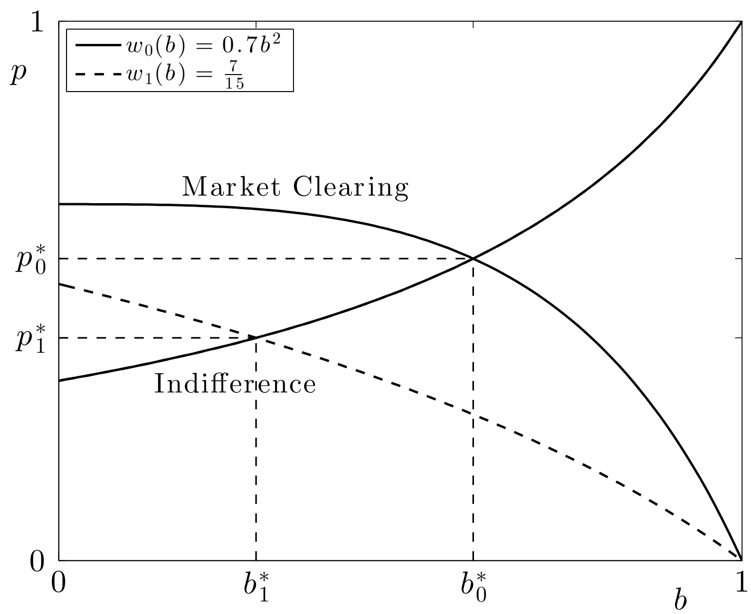

Figure 4: A worsening of information. The example uses $\lambda = 0.5$ and $\mu = 0.5$.

Fire Sales

The term “fire sales” is sometimes used to refer to situations where traders’ urgency for funds leads them to sell assets at prices that are far below their usual price. Fire sales have been documented in many different markets, including used aircraft (Pulvino 1998), real estate (Campbell et al. 2011), equities (Coval and Stafford 2007), corporate bonds (Ellul et al. 2011) and convertible bonds (Mitchell et al. 2007).

Examples of traders with an urgent need of funds include hedge funds facing margin calls, banks facing runs on their deposits, etc. This distress could itself be the result about bad news about the value of the assets, in which case a drop in price is no puzzle. The question is whether the need to sell itself makes the price drop, an effect that is at the heart of a sizable literature (see Shleifer and Vishny (2011) for a recent survey).

In the context of the current model, one can ask whether an increase in μ (the fraction of sellers who are distressed) leads to a decrease in p^*. If so, then the model has the potential to explain fire sales.

Proposition 4.

1. *p^* is decreasing in μ if and only if*

$$w(b^*) < \left[\lambda + \frac{\mu(1-\lambda)}{1-b^*}\right]\mu(1-\lambda)\int_{b^*}^{1}\frac{w(b)}{[\lambda(1-b)+\mu(1-\lambda)]^2}db \tag{25}$$

2. *If $w(b)$ is a constant, then $\frac{dp^*}{d\mu} = 0$.*

In general, there are two opposing effects when more sellers become distressed, as illustrated in Figure 5. On the one hand, since $p^* < 1$, distressed sellers are the only ones who are willing to sell good assets. Other things being equal, more distressed sellers should improve the pool of assets being sold and thus lead to higher, not lower, prices. This is reflected in an upward shift in the indifference condition (20). This is the effect emphasized by Uhlig (2010), who concludes that an equally-uninformed-buyers model cannot be the entire explanation for fire-sale patterns. Indeed, with equally uninformed buyers the market clearing condition (19) is fixed and vertical, so an upward shift in the indifference condition (20) implies higher p^*.[6]

However, more distressed sellers mean that more assets are being offered for sale. This is reflected in a downward shift in the market clearing condition (19). Given that the more expert buyers exhaust their wealth, an increased supply makes it is necessary to resort to less expert buyers. These less expert buyers are aware that they are less clever at filtering out the bad assets so, other things being equal, they will make up for this by only entering the market if prices are lower. The net effect on p^* depends on which of these two shifts is greater. In Figure 5 the second effect dominates and p^* falls.

Proposition 4 shows that which effect dominates (locally) depends on the density of wealth at the equilibrium cutoff level of expertise. If $w(b^*)$ is high, this means that a large

[6] Models with equally uninformed buyers and exclusive markets also don't produce fire sales. As long as a version of (1) holds, assets in the separating equilibria of Gale (1996) and others are always fairly priced, so prices do not move with μ.

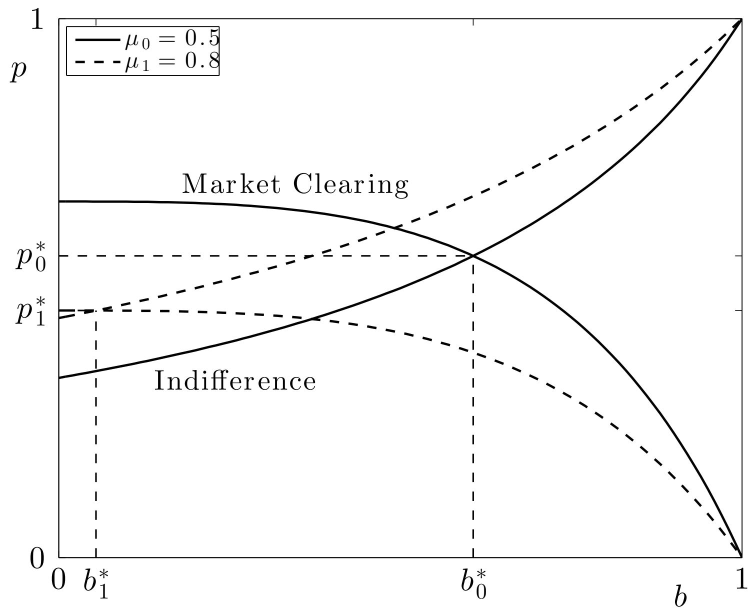

Figure 5: An increase in μ. The example uses $\lambda = 0.5$ and $w(b) = 0.7b^2$.

amount of wealth would enter the market if the cutoff level of expertise was lowered slightly. In this case, the direct selection effect dominates and prices rise, meaning there are no fire sales. Instead when $w(b)$ is low, cutoff level of expertise needs to fall a lot in order to attract sufficient wealth to buy the extra units supplied. In this case, the changing-threshold effect dominates and prices fall. Interestingly, for the special case where wealth is uniformly distributed across all levels of expertise, the price is the same for any μ, so both effects cancel out.

The model has both differences and similarities with other theories of fire-sales in the existing literature. One class of theories (Shleifer and Vishny 1992, 1997, Kiyotaki and Moore 1997) emphasizes that the marginal buyer of an asset can be a second-best user with diminishing marginal product. If first-best users need to sell more units, asset prices will fall along the marginal product curve of second-best users. This effect could operate along an intensive margin (a single second-best user absorbing higher quantity) or an extensive margin (moving from a second-best user to a third-best user and so on). This mechanism is probably better suited to explain fire sales for real assets that can be given alternative uses than for financial assets. The holder of a financial asset does not need to use his expertise and/or complementary assets in order to extract value from it, so the idea of a second-best user does not naturally fit fire-sales in financial markets. However, the current model illustrates that expertise may be relevant in the trade itself, and moving along a gradient of expertise can induce to fire-sale effects.

A second class of theories (Fostel and Geanakoplos 2008, Geanakoplos 2009) derives a diminishing-marginal-valuation schedule among potential buyers as a consequence of differences of opinion about the true value of the asset combined with borrowing constraints, even though actual payoffs from holding the asset are the same for all traders. The current setup, instead, is based on standard common-prior beliefs and the differences among buyers are in the quality of their information. Besides this basic difference, the two setups have much in common. First, changes in the identity of the marginal buyer are the key driver of changes in prices. Second, borrowing constraints are the reason why the natural buyers have limits on the positions they can take. A maintained assumption in the current model is that the high-b buyers cannot borrow to increase the volume of assets they buy. Otherwise, $b = 1$ buyers would drive up the price all the way to 1 and reject all bad assets. One possible interpretation is that $w(b)$ represents the total resources available to buyer b after they have exhausted their borrowing capacity.[7]

A third class of theories (Allen and Gale 1994, 1998, Acharya and Yorulmazer 2008) relies on the notion of cash-in-the-market pricing. There is a given amount of purchasing power available, so if more units are to be sold, the price must fall. But this class of models typically leaves unanswered the question of why buyers with deep pockets (for instance, rich individuals) stay out of the market. The current model provides an explanation for buyers staying out of the market: even though there are good deals available for those who have expertise, those who do not have expertise are rationally worried that they are not able to select the deals among all the assets on offer. In other words, given their expertise, buying from this market does not provide excess returns, even though it does for experts.

A fourth class of theories, building on Grossman and Stiglitz (1980) and Kyle (1985) is, like the current model, based on limited information. In those models, increases in the supply of the asset as a result of "noise traders" play a similar role to increases in μ. Crucially, however, the assumption is that the aggregate net supply from noise traders is unobserved. They lead to falls in prices (over and beyond what is needed to persuade traders to hold extra units of a risky asset) because uninformed traders face a signal extraction problem: they are rationally unsure whether there has been an increase in supply or more-informed traders have received bad news. Those models have the implication that fire sales would not take place if traders were aware of the supply shocks. In the current model, instead, fire

[7]Using the assets as collateral would not undo borrowing constraints because lower-b buyers (natural lenders) would not have the ability to distinguish good from bad collateral.

sales can take place even though the parameter μ is commonly known.

The different theories of fire sales discussed above could have different normative implications. Suppose for instance that at some cost one could introduce regulation that prevented μ from shifting up: would that be desirable? If fire sales are the result of a second-best-user effect, then one should measure the output differential between first best and second best users, multiply that times the probability of a fire sale and compare it to the cost of regulation. Instead, in the current model, the assumptions on sellers' preferences (with $\beta(s)$ only taking the values 0 and 1) imply that the equilibrium allocations are Pareto Efficient: all distressed sellers sell all their good assets so gains from trade are exhausted. If that were the true model, the regulation would be undesirable. This is not meant to argue that differential information is necessarily the right explanation in every instance, just that knowing the underlying reason for fire sales, i.e. what makes the demand for assets downward sloping, can be important.

Contagion

Consider the following extension of the model to a case where there are two asset classes, A and B. For instance, A assets could be high-yield corporate bonds and B assets emerging market sovereign bonds. Each asset class contains a fraction of bad assets, λ_A and λ_B respectively. They are held by separate groups of sellers, of whom fractions μ_A and μ_B respectively are distressed. There is a common pool of buyers, with types $(b_A, b_B) \in [0,1] \times [0,1]$ and endowments $w(b_A, b_B)$. Buyers know whether an asset belongs to asset class A or B; for asset i in asset class $z \in \{A, B\}$, they observe signal $x(i, b_z) = \mathbb{I}(i \geq b_z \lambda_z)$. In this formulation, b_z represents the expertise of the buyer in asset class z.

The equilibrium is characterized by a generalization of equations (19) and (20). As in the single-asset-class case, only one market for each asset class is active and all good assets held by distressed sellers are sold. Buyers have three options: buying A assets, buying B assets or not buying at all. Their decision will depend on (b_A, b_B), as illustrated in Figure 6. The terms of trade that buyers can obtain are still given by (24) so buyers will be indifferent between buying A and B assets when

$$\frac{1}{p_A^*} \frac{\mu_A (1 - \lambda_A)}{\lambda_A (1 - b_A) + \mu_A (1 - \lambda_A)} = \frac{1}{p_B^*} \frac{\mu_B (1 - \lambda_B)}{\lambda_B (1 - b_B) + \mu_B (1 - \lambda_B)} \tag{26}$$

This condition is represented by the dotted line in Figure 6. Buyers Southeast of this line are more expert in A assets while buyers Northwest of it are more expert in B assets. Buyers with sufficiently low b_A and b_B would have $\tau < 1$ for both asset classes and stay out of the market.

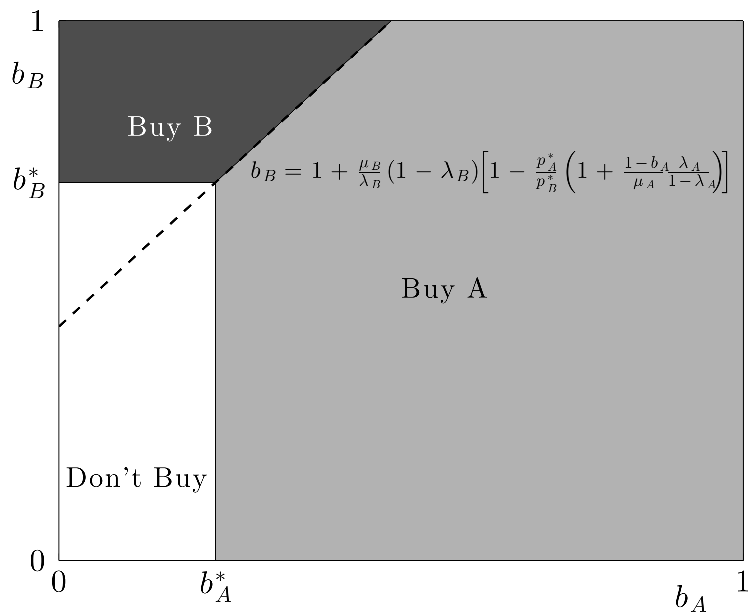

Figure 6: Buyer's decisions as a function of (b_A, b_B). The example uses $p_A = 0.7$, $p_B = 0.9$, $\mu_A = 0.2$, $\mu_B = 0.3$, $\lambda_A = 0.1$ and $\lambda_B = 0.1$.

The equilibrium p_A^*, p_B^*, b_A^* and b_B^* satisfy:

$$\frac{1}{p_A^*}\frac{\mu_A(1-\lambda_A)}{\lambda_A(1-b_A^*)+\mu_A(1-\lambda_A)} = 1 \tag{27}$$

$$\int_{b_A^*}^{1}\left(\int_{0}^{\bar{b}_B\left(b_A,p_A^*,p_B^*\right)} \frac{1}{\lambda_A(1-b_A)+\mu_A(1-\lambda_A)}\frac{w(b_A,b_B)}{p_A^*}db_B\right)db_A = 1 \tag{28}$$

(and symmetrically for asset class B), where

$$\bar{b}_B(b_A,p_A^*,p_B^*) \equiv \min\left\{1+\mu_B\frac{1-\lambda_B}{\lambda_B}\left[1-\frac{p_A^*}{p_B^*}\left(1+\frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right)\right],1\right\} \tag{29}$$

is derived from rearranging (26). Condition (27) is the analogue of equation (20). Buyers with A-type b_A^* are exactly indifferent between buying A assets and not buying. Condition (28) is the analogue of equation (19). Integrating all the good assets bought by buyers in the light shaded area of Figure 6 must equal the endowment of good A assets from distressed sellers.

As in the single-asset-class case, an increase in μ_A could raise or lower p_A^*, depending on the density of wealth of marginal buyers.[8] A different question is what happens to prices of A assets when μ_B increases.

Proposition 5. *Suppose p_B^* is decreasing in μ_B. Then p_A^* is decreasing in μ_B.*

If an increase in μ_B results in a fall in p_B^*, then buying B assets becomes more attractive for all buyers. Other things being equal, marginal buyers who are indifferent between buying A and B assets will shift towards buying B assets. These marginal buyers exist as long as there is positive density of wealth along the dotted line in Figure 6, i.e. as long as there are active investors whose expertise in both asset classes is sufficiently even that they are willing to shift from one asset class to the other. In order to restore equilibrium to the A market, p_A^* must fall. Thus, there will be contagion from distress of the owners of one asset class to the prices of the other asset class through the equilibrium decisions of the common pool of potential buyers.[9] Calvo (1999) makes a related argument applied to the Russian crisis of 1998, in a model based on a signal-extraction problem in the style of Grossman and Stiglitz (1980).

5 False Negatives Case

I describe the equilibrium informally, relegating a formal statement, together with the proof that it is unique, to Appendix A. Thanks to Lemma 1, each seller's decision can be summarized in terms of a reservation price $p^R(i)$ for each asset. As in the false-positives case, $p^R(i) = 0$ for bad assets for all sellers and $p^R(i) = 1$ for good assets for non-distressed sellers. Unlike the false-positive case, the $p^R(i)$ for distressed sellers is different for different good assets. Distressed seller's preferences imply that $p^R(i)$ must be such that they are able to sell asset i for sure by supplying it in all markets with $p(m) \geq p^R(i)$ (otherwise they would do better by supplying it at lower prices as well); hence finding $p^R(i)$ is equivalent to finding the lowest price at which asset i trades. Unlike the false-positives case, where all trades of

[8] Here marginal buyers include both those indifferent between buying A assets and not buying and those indifferent between buying A assets and B assets, i.e. everyone on the frontier of the lightly shaded region of Figure 6.

[9] As with the single-asset-class fire sales, the key to the effect lies in a shift in the marginal buyer; similar effects would arise if buyers were heterogeneous in their intrinsic valuation for the asset rather than their information.

a given asset take place at the same price, in this case some fraction of asset i could trade at prices above $p^R(i)$ as well. Therefore to characterize the equilibrium one must find both $p^R(i)$ and any other prices at which asset i trades.

For each $i \in [\lambda, 1]$, $p^R(i)$ for for distressed sellers falls into one of three possible classes: a "cash-in-the-market" price, a "bunching" price or a "nonselective" price.

Cash-in-the-market price

The basic way to determine $p^R(i)$ is by a form of cash-in-the-market pricing. Define $\hat{b}(i)$ as

$$\hat{b}(i) \equiv \frac{1-i}{1-\lambda} \tag{30}$$

$\hat{b}(i)$ is the lowest buyer type that observes $x(i,b) = 1$, i.e. the least expert buyer who realizes that asset i is good. The cash-in-the-market price $p^C(i)$ for asset i is the price such that buyer $\hat{b}(i)$ can afford to buy enough units so that all units held by distressed sellers are sold. Hence

$$p^C(i) = \frac{1}{r(i)(1-\lambda)} w\left(\hat{b}(i)\right) \tag{31}$$

where $r(i)$ is the number of units held by distressed sellers that they were not able to sell at prices above $p^C(i)$ and the term $(1-\lambda)$ is the result of a change of measure: $d\hat{b}(i) = -\frac{1}{1-\lambda} di$.[10]

As long as $p^C(i)$ defines a function that is strictly increasing and sufficiently high (in a sense made precise below), the logic of cash-in-the-market pricing works as follows. Each asset $i \in [\lambda, 1]$ will be supplied by distressed sellers in all markets where $p(m) \geq p^C(i)$ and in no market with a lower price, while all bad assets are supplied in all markets. Each buyer will attempt to buy assets in the cheapest market where he can find assets for which he observes $x(i,b) = 1$, i.e. where he can detect good assets, and will impose the acceptance rule $\chi(i) = \mathbb{I}(i \geq 1 - b(1-\lambda))$. Consider a market where $p = p^C(i)$. In it there will be assets in the range $[\lambda, i]$ on sale, but no assets in the range $(i, 1]$, since those can be sold at higher prices. Buyer $b = \hat{b}(i)$ will be able to see good assets in this market but buyers $b < \hat{b}(i)$ will not. Indeed, if $p^C(i)$ is strictly increasing, this is the cheapest market where buyer $\hat{b}(i)$ can detect good assets so he will spend his entire endowment in this market.

[10] $r(i) \equiv \mu\left[1 - \eta\left(\left\{m : p(m) > p^C(i)\right\}, i\right)\right]$, so $r(i)$ could be equal to or lower than μ (the total endowment of distressed sellers) depending on whether $\eta\left(\left\{m : p(m) > p^C(i)\right\}, i\right) > 0$, i.e. on whether it is possible to sell some units of asset i at prices above $p^C(i)$. This will be the case if nonselective pricing applies to some assets with indices above i.

Equation (31) implies that this will exhaust the remaining supply of asset i, confirming the conjecture that i will not be on sale at prices below $p^C(i)$. Notice that, because a single type of buyer demands assets at each price, it does not matter what clearing algorithm is used.

There are two reasons why assets might not actually trade at the prices described by expression (31). First, $p^C(i)$ need not be monotonic. Second, it could be so low that it makes it attractive for buyers to buy at price $p^C(i)$ and impose $\chi(i) = 1$ (i.e. accept all assets). These considerations lead to bunching and nonselective pricing respectively.

Bunching

Since the endowment function w could have any shape, the function p^C could have any shape too and need not be increasing in i. If it happens to be decreasing over some range, then the cash-in-the-market pricing logic described above breaks down. Suppose for some i, i' with $\lambda < i < i'$ it were the case that $p^C(i') < p^C(i)$. Buyer $\hat{b}(i)$ can identify both i and i' as good assets, so if asset i' is on sale at price $p^C(i')$, he would prefer to buy in that market. Therefore there would be no buyer for asset i. By this logic, if all good assets held by distressed sellers are to be sold, their reservation price must be (weakly) monotonically increasing in i, so that easier-to-recognize good assets trade at a higher price than harder-to-recognize ones.

Imposing monotonicity results in a form of bunching. An "ironing" procedure similar to that in Mussa and Rosen (1978) results in a weakly monotone function that restores a version of the cash-in-the-market logic. Each buyer spends his entire endowment buying from the cheapest market where he can detect good assets and distressed sellers can sell all their good assets. In markets where there is bunching, the clearing algorithm used lets lower-b buyers, who impose more restrictive acceptance rules, trade before higher-b buyers. This ensures that there are enough good assets remaining for all buyers.

Condition (1) implies that the total amount of wealth is more than enough to buy all good assets from distressed sellers at a price of 1. This implies that there is necessarily a range of assets at the top that are bunched at a price of 1. In other words, the most transparent assets will be sold at no discount.

Nonselective pricing

Suppose distressed sellers offer asset $i > \lambda$ at price p. This implies that all assets in the interval $[\lambda, i)$ from distressed sellers will also be on sale in all markets where the price is p. Any buyer can decide to be "nonselective" and demand assets in a market where the price is p and the algorithm is LRF, imposing $\chi(i) = 1$ (i.e. accept everything). He will obtain a representative sample from a pool of $\mu(i-\lambda)$ good assets mixed with λ bad assets and therefore obtain a fraction of

$$p^N(i) \equiv \frac{\mu(i-\lambda)}{\mu(i-\lambda)+\lambda} \tag{32}$$

good assets. If $p < p^N(i)$, this would be better than not trading. Since assumption (1) implies that some buyers do not trade, this must mean that no asset i is offered at a price below $p^N(i)$. Therefore p^N provides a lower bound on reservation prices.

When this lower bound is operative, trade will take place in markets where both selective and nonselective buyers are active. In the market where the price is $p^N(i)$, nonselective buyers will buy just enough assets (distributed pro-rata among the assets offered) so that buyer $\hat{b}(i)$ can afford to buy all the remaining i assets. Buyer $\hat{b}(i)$ can afford to buy $\frac{w(\hat{b}(i))}{p^N(i)(1-\lambda)}$ units so nonselective buyers buy the remainining units. The fact that these buyers are nonselective implies that their purchases will include the same number of units of all assets in $[\lambda, i)$ as of asset i. Therefore the amount of asset i bought by buyer $\hat{b}(i)$ is equal to the amount of any asset $j \in [\lambda, i)$ that remains unsold. Hence, since w is assumed to be continuous:

$$r(i) = \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)} \tag{33}$$

Equation (33) implies that in any single market, nonselective buyers buy a zero measure of assets; they only buy a positive measure of assets once one aggregates over range of markets. This has a simple interpretation. Suppose asset i is such that $p^C(i) = p^N(i)$. This means that buyer $\hat{b}(i)$ can afford to buy exactly $r(i)$ assets at price $p^N(i)$. Since w is continuous, buyers $\hat{b}(j)$ for j close to i have to be very close to being able to afford the same amount. Hence a small purchase from nonselective buyers is enough for selective buyers to exhaust the supply.

By the same logic of the false-positives case, the clearing algorithm in these markets will dictate that nonselective buyers clear first.

Example

Figure (7) shows an example of how $p^R(i)$ is constructed. The left panel shows $w\left(\hat{b}(i)\right)$, which in the example is relatively high for high values of i, i.e. low-b buyers who can only recognize high-i assets have most of the wealth. The top bunching region ends at $i = 0.91$. For $i \in [0.83, 0.91]$, $p^C(i)$ is lower than 1 and increasing, so cash-in-the-market pricing prevails. For $i \in [0.74, 0.83]$, $p^C(i)$ would fall below $p^N(i)$ if $r(i)$ were constant at μ, so nonselective pricing prevails. Nonselective buyers buy a respresentative sample of all assets on offer at each price, so $r(i)$ falls from $r(0.83) = \mu = 0.5$ to $r(0.74) = 0.16$. Between $i = 0.36$ and $i = 0.74$ the cash-in-the market price is nonmonotonic because buyers near $\hat{b}(0.36)$ have more wealth than buyers near $\hat{b}(0.74)$. Hence there is a bunching region, with assets $i \in [0.36, 0.74]$ all trading at the same price. Finally, assets in $i \in [0.3, 0.36]$ have cash-in-the-market pricing.

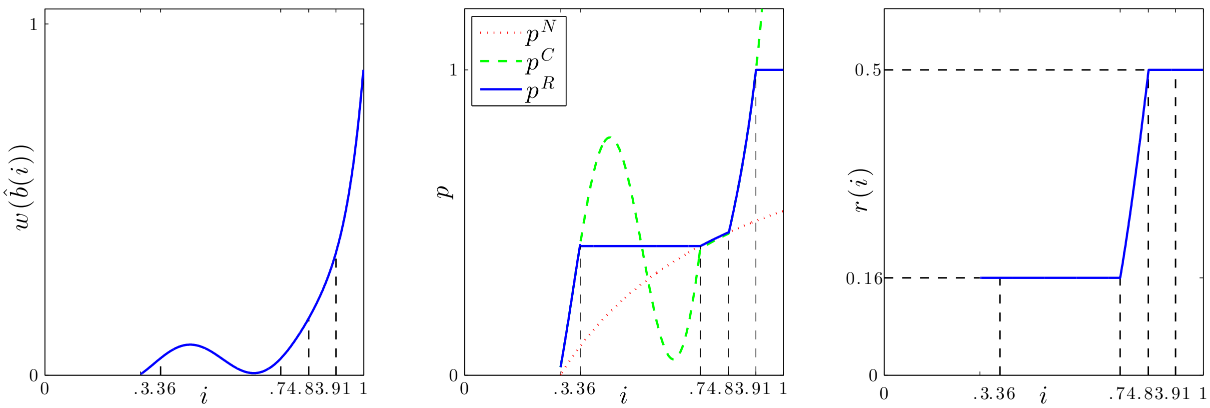

Figure 7: Construction of the reservation price p^R. The example uses $\lambda = 0.3$, $\mu = 0.5$ and $w(b) = 0.08e^{-b} + 0.8e^{-10b} + 0.05\sin(10b)$

Having determined p^R, the rest of the equilibrium is straightforward. Buyers for whom $p^R(1 - b(1 - \lambda)) < 1$ can detect good assets in markets where the price is below 1, so they spend their entire endowment in them; the rest are indifferent between not buying, buying (selectively) from the market where $p = 1$ and buying nonselectively from a market in a nonselective region if there is one.

Flights to Quality

What happens if μ increases? As in the false-positives case, the answer depends on parameters. Suppose first that parameters were such that there was no region of nonselective pricing, so that $r(i) = \mu$ for all i. In this case, equation (31) implies that the cash-in-the-market price falls when there is an increase in μ. This is precisely the logic of cash-in-the-market pricing, as in Allen and Gale (1994). If all assets are bought by selective buyers, their price is determined by the ratio of buyers' wealth to quantity supplied; an increase in the fraction of assets that are held by distressed sellers implies an increase in quantity supplied and necessitates a fall in prices. On the other hand, equation (32) says that nonselective prices are increasing in μ, since more distressed sellers improve the overall mix of assets that a nonselective buyer will encounter. Overall, the effect could be mixed, with prices for some assets increasing while others decrease.

Figure (8) shows the effect of an increase in μ from 0.5 to 0.7 in the example from Figure (7). For high values of i such that there are no nonselective-pricing regions to the right, then prices fall when μ increases. However, the lower bound from nonselective pricing is higher and nonselective buyers end up buying larger amounts, so for lower values of i prices rise when μ increases.

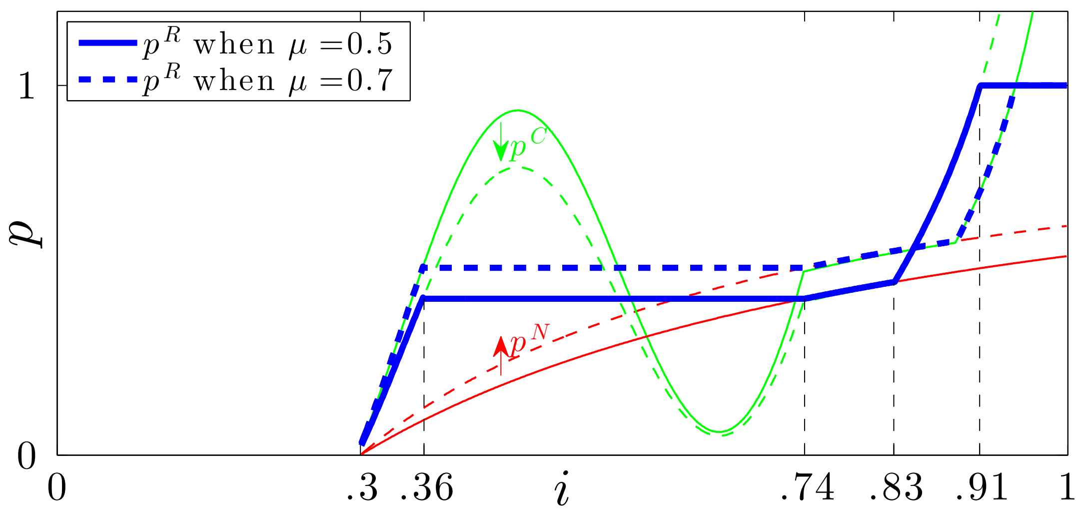

Figure 8: An increase in μ in the False-Negatives case

Some features of the example in Figure (8) turn out to hold in general.

Proposition 6.

1. *Let i_1 be the lowest i such that $p^R(i) = 1$. $p^R(i)$ is weakly decreasing in μ for any $i > i_1$ and, if $i_i > \lambda$, strictly decreasing in μ in a neighborhood of i_1.*

2. *$p^R(i)$ is increasing in μ for any i in a nonselective pricing interval.*

Part 1 of Proposition 6 can be interpreted as a model of a "flight to quality". For any given μ, there is a range of assets that are sufficiently transparent that their price is 1. But even within this range some assets are more transparent than others. If μ increases, this creates a difference between the most transparent assets, for which it is still the case that the buyers who can identify them as good have enough wealth to afford them at $p = 1$ and the slightly-less-transparent ones, for which the buyers who can identify them as good cannot afford the increased supply at the original price. This could be an explanation of why the premium for the most transparently good assets (such as US Treasuries) over high-quality assets that may require some expertise to assess (such as high-grade corporate bonds), increases in times of market stress.

6 Final Remarks

This paper provides a theory of competitive equilibrium for economies with heterogeneous assets and heterogeneous information, as well as existence and uniqueness results and a characterization for some simple cases. This information pattern is a feature of many markets beyond the market for financial assets. For instance, Kurlat and Stroebel (2013) find evidence consistent with it in residential real estate.

Consistent with the simple environment, the types of trades allowed in the model are very limited: only exchanges of goods for assets. However, the same type of equilibrium notion could be applied to study the trade of more complicated, multidimensional objects such as insurance contracts. This could be useful, for instance to ask what happens when there are competitive insurance companies with different abilities to assess risks.

Moreover, the equilibrium is not equivalent to the outcome of a mechanism design problem. In this environment the information of different buyers is very highly correlated, and hence the logic of Crémer and McLean (1988) applies. A mechanism designer could obtain all buyers' information and implement allocations other than the equilibrium outcome. The equilibrium concept implicitly restricts the outcomes to those that result from buyers, in some order, choosing representative samples of acceptable assets.

Due to the assumptions on seller's preferences, with $\beta(s)$ only taking the values 0 and 1, the equilibrium allocations are Pareto Efficient: all distressed sellers sell all their good assets so gains from trade are exhausted. This would no longer be true if $\beta(s)$ took intermediate values: in that case there would be some sellers with $\beta(s) < 1$ that inefficiently kept some good assets. In Kurlat (2013) I analyze that case and show that in general expertise is socially valuable, i.e. the total surplus increases if $w(b)$ shifts towards higher values. However, the social value may be greater or smaller than the private value, so there could be both over-and under-investment in acquiring expertise.

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Appendix A: Detailed Statements of the Equilibria and Proofs

Proof of Lemma 1

Assume the contrary. This implies that there are two markets, m and m' with $p(m') > p(m)$ such that, for some i, the seller chooses $\sigma(i,m) > 0$ and $\sigma(i,m') < 1$. There are four possible cases:

1. $\eta(m;i) > 0$ and $\eta(m';i) > 0$. Then the seller can increase his utility by choosing supply $\tilde{\sigma}$ with $\tilde{\sigma}(i,m') = \sigma(i,m') + \varepsilon$ and $\tilde{\sigma}(i,m) = \sigma(i,m) - \varepsilon\frac{\eta(m';i)}{\eta(m;i)}$ for some positive ε.

2. $\eta(m;i) > 0$ and $\eta(m';i) = 0$. Consider a sequence such that $\eta^n(m';i) > 0$. By the argument in part 1, for any n the solution to program (11) must have either $\sigma(i,m) = 0$ or $\sigma(i,m') = 1$ (or both). Therefore either the condition that $\sigma^n(i,m) \to \sigma(i,m)$ or the condition that $\sigma^n(i,m') \to \sigma(i,m')$ in a robust solution is violated.

3. $\eta(m;i) = 0$ and $\eta(m';i) > 0$. Consider a sequence such that $\eta^n(m;i) > 0$. By the argument in part 1, for any n the solution to program (11) must have either $\sigma(i,m) = 0$ or $\sigma(i,m') = 1$ (or both). Therefore either the condition that $\sigma^n(i,m) \to \sigma(i,m)$ or the condition that $\sigma^n(i,m') \to \sigma(i,m')$ in a robust solution is violated.

4. $\eta(m;i) = \eta(m';i) = 0$. Consider a sequence such that $\eta^n(m';i) > 0$ and suppose that there is a sequence of solutions to program (11) which satisfies $\sigma^n(i,m') \to \sigma(i,m') < 1$. This implies that for any sequence such that $\eta^n(m,i) > 0$ and for any n, the solution to program (11) must have $\sigma^n(i,m) = 0$. Therefore the condition that $\sigma^n(i,m) \to \sigma(i,m) > 0$ in a robust solution is violated.

Proof of Lemma 2

Replacing (20) into (19):

$$\int_{b^*}^{1} \frac{w(b)}{\mu(1-\lambda)} \frac{\lambda(1-b^*) + \mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} db = 1 \tag{34}$$

The left-hand-side of (34) is decreasing in b^* so any solution must be unique. Setting $b^* = 1$ the left-hand-side is equal to 0 while setting $b^* = 0$

$$\int_0^1 \frac{w(b)}{\mu(1-\lambda)} \frac{\lambda + \mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} db > \int_0^1 \frac{w(b)}{\mu(1-\lambda)} db \geq 1$$

where the second inequality follows from the assumption (1). Hence, by continuity a solution with $b^* \in (0,1)$ exists.

Equilibrium in the False Positives case

Let market m^* be the market defined by price p^* and an LRF algorithm.

1. Seller decisions

   (a) For $s < \mu$:

$$\sigma_s(i,m) = \begin{cases} 1 \quad \text{if} \begin{cases} i < \lambda \\ \text{or} \\ p(m) \geq p^* \end{cases} \\ 0 \qquad \text{otherwise} \end{cases} \tag{35}$$

$$c_{1s} = p^* \int_0^1 \eta(m^*; i)\, di$$

$$c_{2s} = 0$$

   (b) For $s \geq \mu$:

$$\sigma_s(i,m) = \begin{cases} 1 \quad \text{if} \begin{cases} i < \lambda \\ \text{or} \\ p(m) \geq 1 \end{cases} \\ 0 \qquad \text{otherwise} \end{cases} \tag{36}$$

$$c_{1s} = p^* \int_0^\lambda \eta(m^*; i)\, di$$

$$c_{2s} = 1 - \lambda$$

where $\eta(m^*; i)$ is given by (40) below.

2. Buyer decisions

   (a) For $b < b^*$:

$$\begin{aligned} \delta_b &= 0 \\ m_b &= m^* \\ \chi_b(i) &= \mathbb{I}(i \geq \lambda b) \\ c_{1b} &= w(b) \\ c_{2b} &= 0 \end{aligned}$$

   (b) For $b \geq b^*$:

$$\begin{aligned} \delta_b &= \frac{w(b)}{p} \\ m_b &= m^* \\ \chi_b(i) &= \mathbb{I}(i \geq \lambda b) \\ c_{1b} &= 0 \\ c_{2b} &= \frac{w(b)}{p^*} \frac{\mu(1-\lambda)}{\lambda(1-b) + \mu(1-\lambda)} \end{aligned}$$

3. Allocation function

   (a) For market m^* and $\chi(i) = \mathbb{I}(i \geq g)$ for some $g \in [0, \lambda]$:

$$a(i; \chi, m^*) = \frac{\mathbb{I}(i \in [g, \lambda)) + \mu \mathbb{I}(i \geq \lambda)}{[\lambda - g] + \mu[1-\lambda]} \tag{37}$$

   (b) For market m^* and any other acceptance rule:

$$a(i; \chi, m^*) = \begin{cases} \frac{\chi(i)[1-\eta(m^*;i)]}{\int_0^\lambda \chi(i)[1-\eta(m^*;i)]di} & \text{if } \chi(i) \notin X_0 \text{ and } \int_0^\lambda \chi(i)[1-\eta(m^*;i)]\,di > 0 \\ \frac{\chi(i)[1-\eta(m^*;i)]}{\sum_i \chi(i)[1-\eta(m^*;i)]} & \text{if } \chi(i) \notin X_0,\ \int_0^\lambda \chi(i)[1-\eta(m^*;i)]\,di = 0 \text{ but } \sum_i \chi(i)[1-\eta(m^*;i)] > 0 \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

   where $\eta(m^*; i)$ is given by (40) below.

(c) For any other market:

$$a(i;\chi,m)=\begin{cases}\frac{\chi(i)S(i;m)}{\int\chi(i)S(i;m)di} & \text{if } \int\chi(i)\,S(i;m)\,di>0\\ \frac{\chi(i)S(i;m)}{\sum_i\chi(i)S(i;m)} & \text{if } \int\chi(i)\,S(i;m)\,di=0 \text{ but } \sum_i\chi(i)\,S(i;m)>0\\ 0 & \text{otherwise}\end{cases}\tag{39}$$

where

$$S(i;m)=\begin{cases}1 & \text{if } \begin{cases} i<\lambda\\ \text{or}\\ p(m)\geq 1\end{cases}\\ \mu & \text{if } i\geq\lambda,\ p(m)\in[p^*,1)\\ 0 & \text{if } i\geq\lambda,\ p(m)<p^*\end{cases}$$

4. Rationing function

$$\eta(M_0;i)=\begin{cases}1 & \text{if } m^*\in M_0 \text{ and } i\geq\lambda\\ \int_{b^*}^{\frac{i}{\lambda}}\frac{1}{\lambda(1-b)+\mu(1-\lambda)}\frac{w(b)}{p^*}db & \text{if } m^*\in M_0 \text{ and } i\in[\lambda b^*,\lambda)\\ 0 & \text{otherwise}\end{cases}\tag{40}$$

Proof of Proposition 1

1. Seller optimization.

   The rationing function (40) implies that sellers will be able to sell all assets $i\in[\lambda,1]$ and a fraction $\eta(m^*;i)<1$ of assets $i\in[\lambda b^*,\lambda)$ in market m^*, and nothing else. A necessary and sufficient condition for a solution to program (4) is that in market m^* distressed sellers supply the maximum possible amount of assets $i\in[\lambda b^*,1]$ and non-distressed sellers supply the maximum possible amount of assets $i\in[\lambda b^*,\lambda)$ and no assets $i\in[\lambda,1]$. The decisions (36) and (35) about other assets in market m^* and about all assets in markets $m\neq m^*$ are consistent with a robust solution. Consumption then follows from the budget constraint.

2. Buyer optimization.

   Choosing any feasible acceptance rule other than $\chi(i)=x(i,b)$ in market m^* would, according to (37) and (38), result in a lower fraction of good assets, so choosing $\chi(i)=\mathbb{I}(i\geq\lambda b)$ is optimal.

Define the terms of trade that a buyer obtains in market m with acceptance rule χ as

$$\tau(m,\chi) \equiv \begin{cases} \frac{\int q(i)dA(i;\chi,m)}{p(m)A([0,1];\chi,m)} & \text{if } A([0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

Let

$$\tau^{\max}(b) \equiv \max_{m\in M,\chi\in X_b} \tau(m,\chi)$$

be the best terms of trade that buyer b can achieve, and let $M^{\max}(b)$ be the set of markets where buyer b can obtain terms of trade $\tau^{\max}$ with a feasible acceptance rule.

Necessary and sufficient condition for buyer optimization are that buyers for whom $\tau^{\max}(b) < 1$ choose zero demand, buyers for whom $\tau^{\max}(b) > 1$ spend their entire endowment in a market $m \in M^{\max}(b)$ and buyers for whom $\tau^{\max}(b) = 1$ choose a market $m \in M^{\max}(b)$. Using equation (37), a buyer b that uses acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ in market m obtains terms of trade

$$\tau(m,\chi) = \begin{cases} \frac{1}{p(m)} \frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)} & \text{if } p(m) \geq p^* \\ 0 & \text{otherwise} \end{cases}$$

so for all buyers

$$\tau^{\max}(b) = \frac{1}{p^*} \frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$$

and the maximum is attained in any market where the price is p^*, including m^*. Together with condition (20), this implies that types $b < b^*$ have $\tau^{\max}(b) < 1$ so zero demand is optimal for them while buyers with types $b \geq b^*$ have $\tau^{\max}(b) \geq 1$, so spending their entire endowment in market m^* is optimal for them too. Consumption then follows from the budget constraint.

3. Allocation function.

   In all markets except m^* demand is zero, so for any clearing algorithm the residual supply faced by any buyer equals the original supply. For these cases, (39) follows directly from (65).

   For market m^*, the LRF algorithm implies that a buyer who imposes $\chi(i) = \mathbb{I}(i \geq g)$ will face a residual supply of acceptable assets that is proportional to the original supply. Therefore the measure of assets he will obtain is the same as if he traded first. Therefore (37) follows from (65).

For market m^* and rules that are not of the form $\chi(i) = \mathbb{I}(i \geq g)$ with $g \leq \lambda$ (which nobody uses in equilibrium), their trades clear after all active buyers, so the supply they face only includes assets $i < \lambda$ and is given by $S^{\chi}(i;m) = 1 - \eta(m^*; i)$. Therefore (38) follows from (65).

4. Rationing function.

   (40) follows from (19) and direct application of formula (67)

Proof of Proposition 2

I first establish a series of preliminary results.

Lemma 3. *In any equilibrium, buyers who demand positive amounts of assets only use acceptance rules of the form $\chi(i) = \mathbb{I}(i \geq g)$ for $g \leq \lambda$.*

Proof. Given their information, any buyer b has only three feasible acceptance rules:

$$\begin{aligned}
\chi_b(i) &= \mathbb{I}(i \geq \lambda b) \\
\chi_0(i) &= 1 \\
\chi_{-b}(i) &= \mathbb{I}(i < \lambda b)
\end{aligned}$$

Rule χ_{-b} will never be associated to positive demand in equilibrium, because it implies accepting only assets that are known to be bad. The other two rules are of the required form. $\square$

Lemma 4. *Consider an arbitrary market m and an acceptance rule $\chi = \mathbb{I}(i \geq g)$ for some $g \leq \lambda$. Denote the residual supply after acceptance rule χ trades by $S^{\chi+}$. Then in any equilibrium, if $i_L \in [\lambda b, \lambda)$ and $i_H \in [\lambda, 1]$*

$$\frac{S^{\chi+}(i_H; m)}{S^{\chi}(i_H; m)} \leq \frac{S^{\chi+}(i_L; m)}{S^{\chi}(i_L; m)} \tag{42}$$

Proof. Residual supply after rule χ trades is

$$S^{k+}\left(i_H;m\right)=S^k\left(i_H;m\right)-\frac{S^k\left(i_H;m\right)}{\int_g^1 S^k\left(i;m\right)di}D\left(\chi,m\right)$$

so

$$\frac{S^{k+}\left(i_H;m\right)}{S^k\left(i_H;m\right)}=1-\frac{1}{\int_g^1 S^k\left(i;m\right)di}D\left(\chi,m\right) \tag{43}$$

Similarly:

$$\frac{S^{k+}\left(i_L;m\right)}{S^k\left(i_L;m\right)}=1-\frac{\mathbb{I}\left(i_L\geq g\right)}{\int_g^1 S^k\left(i;m\right)di}D\left(\chi,m\right) \tag{44}$$

Subtracting (44) from (43):

$$\frac{S^{k+}\left(i_H;m\right)}{S^k\left(i_H;m\right)}-\frac{S^{k+}\left(i_L;m\right)}{S^k\left(i_L;m\right)}=\frac{\mathbb{I}\left(i_L\geq g\right)-1}{\int_g^1 S^k\left(i;m\right)di}D\left(\chi,m\right) \tag{45}$$

The right hand side of (45) is nonpositive, which implies (42). □

Lemma 4 states that as the clearing algorithm progresses, good assets leave the pool at a (weakly) greater rate than bad assets. This implies that, other things being equal, buyers prefer to trade in markets where their trades will clear sooner rather than later. This helps establish an upper bound on the terms of trade a buyer might obtain in a given market: the best terms of trade that a buyer can obtain in a market are those that result if his trades clear in the first round so that he obtains a representative sample of the acceptable assets supplied to that market.

Lemma 5. *Let $\tau\left(m,\chi\right)$ be defined by (41). Then in equilibrium, for any m,χ:*

$$\tau\left(m,\chi\right)\leq\frac{1}{p\left(m\right)}\frac{\int q\left(i\right)\chi\left(i\right)S\left(i;m\right)di}{\int\chi\left(i\right)S\left(i;m\right)di} \tag{46}$$

Proof. Using (41) and (65):

$$\tau\left(m,\chi\right)=\frac{1}{p\left(m\right)}\frac{\int_\lambda^1\chi\left(i\right)S^\chi\left(i;m\right)di}{\int_0^1\chi\left(i\right)S^\chi\left(i;m\right)di}$$

Lemma 4 implies that the term

$$\frac{\int_\lambda^1\chi\left(i\right)S^\chi\left(i;m\right)di}{\int_0^1\chi\left(i\right)S^\chi\left(i;m\right)di}$$

is weakly lower when rules other than χ clear before χ. Therefore $\tau(m,\chi)$ is bounded above by the terms of trade that would result if χ clears first, which implies inequality (46). $\square$

Lemma 5 places an upper bound on the terms of trade than can be obtained in a market given an acceptance rule. It is also possible to compute an upper bound for a given buyer who can choose among all his feasible acceptance rules.

Lemma 6. *Let*

$$\tau(m,b) \equiv \max_{\chi \in X_b} \tau(m,\chi)$$

In equilibrium, for any m, b:

$$\tau(m,b) \leq \frac{1}{p(m)} \frac{\int_{\lambda}^{1} S(i;m)\, di}{\int_{\lambda b}^{1} S(i;m)\, di}$$

Proof. For any χ, Lemma 5 implies

$$\tau(m,\chi) \leq \frac{1}{p(m)} \frac{\int q(i)\,\chi(i)\, S(i;m)\, di}{\int \chi(i)\, S(i;m)\, di} \tag{47}$$

The acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ satisfies $\chi \in \arg\max_{\chi \in X_b} \frac{\int q(i)\chi(i)S(i;m)di}{\int \chi(i)S(i;m)di}$ so taking the maximum in X_b on both sides of (47) implies the result. $\square$

Knowing the upper bound on the terms of trade a buyer can obtain in a given market m is useful because if one can find a market m' where a buyer can obtain better terms of trade than the upper bound on $\tau(m,b)$, this implies that buyer b will not buy from market m. Using this fact, the following result establishes that in equilibrium all trades take place at the same price.

Lemma 7. *In equilibrium there is trade at only one positive price*

Proof. Assume the contrary, suppose there is trade in markets m_H and m_L where $p(m_H) > p(m_L) > 0$. If buyers are willing to buy in market m_L, then it means that distressed sellers are willing to sell some good assets at a price $p(m_L)$. Lemma 1 implies that in a robust solution to problem (4), for any m such that $p(m) > p(m_L)$, then $\sigma_s(i;m) = 1$ whenever $s < \mu$ and $i \geq \lambda$, i.e. all distressed sellers supply the maximum amount of any asset $i \geq \lambda$ (and hence $S(i;m) = \mu$) in all markets where $p > p_L$.

Fix any b and take a market m where $p(m) \in (p_L, p_H)$ and the clearing algorithm clears rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ first. In such a market

$$\begin{aligned}
\tau(m, \chi(i) = \mathbb{I}(i \geq \lambda b)) &= \frac{1}{p(m)} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m)\, di + \mu(1-\lambda)} \\
&> \frac{1}{p(m_H)} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m_H)\, di + \mu(1-\lambda)} \\
&\geq \tau(m_H, b) \text{ for any } m' \text{ such that } p(m') = p_H
\end{aligned}$$

The first step follows from from applying (65) directly; the second from $p(m) < p(m_H)$ and the fact that, by Lemma 1, $S(i;m_H) \geq S(i;m)$ and the last step follows from Lemma 6. Therefore buyer b will not buy from any market where the price is p_H. Since this applies to all b, there can be no trade at p_H. □

Next I show that in any equilibrium where there is trade at price p^*, sellers are able to sell all their good assets.

Lemma 8. *Let $M_p = \{m \in M : p(m) = p\}$. In any equilibrium where there is trade at price p^*, $\eta(M_{p^*}; i) = 1$ for all $i \geq \lambda$.*

Proof. Assume the contrary. Since no feasible acceptance rule for any buyer distinguishes between different good assets, $\eta(M_{p^*}; i) < 1$ for some $i \geq \lambda$ implies $\eta(M_{p^*}; i) < 1$ for all $i \geq \lambda$. By Lemma 7, there is no trade at any other price, which means that a fraction of good assets held by distressed sellers remains unsold. Therefore in a robust solution to program (4), distressed sellers will supply $\sigma(i,m) = 1$ for all m for $i \geq \lambda$, which implies $S(i;m) = \mu$ for any $i \geq \lambda$. Now take any buyer b and consider any two markets: m, where $p(m) = p^*$ and m', where $p(m') < p^*$ and acceptance rule $\chi(i) = \mathbb{I}(i \geq \lambda b)$ clears in the first round. The terms of trade buyer b can obtain in market m' are

$$\begin{aligned}
\tau(m', \chi(i) = \mathbb{I}(i \geq \lambda b)) &= \frac{1}{p(m')} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m')\, di + \mu(1-\lambda)} \\
&> \frac{1}{p^*} \frac{\mu(1-\lambda)}{\int_{\lambda b}^{\lambda} S(i;m)\, di + \mu(1-\lambda)} \\
&\geq \tau(m, b)
\end{aligned}$$

where the inequalities follow in the same way as in the proof of Lemma 7. This implies that buyer b prefers to buy from market m' rather than from market m. Since this is true for all

b and any m where $p(m) = p^*$, it contradicts the assumption that there is trade at p^*. $\square$

Using Lemmas (4)-(8), Proposition 2 follows by the following argument.

Proof. Let p^* and b^* be defined by equations (19) and (20). Suppose there is an equilibrium where trade takes place at price $p_H > p^*$. For any market m where $p = p_H$, supply satisfies $S(i;m) = 1$ for $i < \lambda$ and $S(i;m) \leq \mu$ for $i \geq \lambda$. Therefore

$$\tau(m,b) \leq \frac{1}{p_H}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$$

so the lowest b that may be willing to buy is b_H, defined by

$$\frac{1}{p_H}\frac{\mu(1-\lambda)}{\lambda(1-b_H)+\mu(1-\lambda)} = 1$$

Equation (20) implies $b_H > b^*$. The maximum measure of good assets that buyer b can get is

$$\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}\frac{w(b)}{p_H}$$

which implies that for any $i \geq \lambda$,

$$\eta(M_{p_H};i) \leq \int_{b_H}^{1} \frac{1}{\lambda(1-b)+\mu(1-\lambda)}\frac{w(b)}{p_H}db$$

but because $p_H > p^*$ and $b_H > b^*$, equation (19) implies that $\eta(M_{p_H};i) < 1$, which contradicts Lemma (8).

Suppose now that there is an equilibrium where trade takes place at $p_L < p^*$. For any market m where $p(m) \in (p_L, 1)$, supply satisfies $S(i;m) \leq 1$ for $i < \lambda$ and $S(i;m) = \mu$ for $i \geq \lambda$. This is true in particular for markets where acceptance rule $\chi(i) = x(i,b)$ is cleared in the first round, so for any $p \in (p_L, 1)$, buyer b can find a market where the terms of trade are

$$\tau(m, \chi(i) = \mathbb{I}(i \geq \lambda b)) \geq \frac{1}{p}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$$

Therefore, in order for trade to only take place at p_L, it must be that all buyers with $b > b_L$ obtain terms of trade of at least

$$\frac{1}{p_L}\frac{\mu(1-\lambda)}{\lambda(1-b)+\mu(1-\lambda)}$$

in markets where $p = p_L$, where b_L is defined by

$$\frac{1}{p_L} \frac{\mu (1-\lambda)}{\lambda (1-b_L) + \mu (1-\lambda)} = 1$$

This would require a total of

$$\int_{b_L}^{1} \frac{\mu (1-\lambda)}{\lambda (1-b) + \mu (1-\lambda)} \frac{w (b)}{p_L} db$$

good assets, but because $p_L < p^*$ and $b_L < b^*$, equation (19) implies this is more than $\mu (1-\lambda)$, which is the total supply of good assets, so not all buyers can obtained the required terms of trade.

This means that in any equilibrium, all trades take place at $p = p^*$. The rest of the equilibrium objects follow immediately. $\square$

Proof of Proposition 3

Define $b(x) \equiv W^{-1}(W(0) - x)$. Better information implies that $b(x)$ shifts up. Using that $\frac{db}{dx} = \frac{1}{w(b(x))}$, conditions (19) and (20) can be rewritten as:

$$\int_{b(x^*)}^{1} \frac{1}{\lambda (1-b(x)) + \mu (1-\lambda)} \frac{1}{p^*} dx = 1 \tag{48}$$

$$p^* = \frac{\mu (1-\lambda)}{\lambda (1-b(x^*)) + \mu (1-\lambda)} \tag{49}$$

If $b(x)$ shifts up, then either a higher p^* or a higher $b(x^*)$ is needed to restore equation (48); since equation (49) implies a positive relatioship between p^* and $b(x^*)$, this implies that they must both increase.

Proof of Proposition 4

From equations (19) and (20), one can compute

$$\frac{dp^*}{d\mu} = \frac{\lambda (1-\lambda)}{\lambda (1-\lambda) \mu + [\lambda (1-b^*) + \mu (1-\lambda)] w (b^*)} \left[\begin{array}{c} \frac{1}{\lambda(1-b^*)+\mu(1-\lambda)} w (b^*) (1-b^*) \\ -\mu \int_{b^*}^{1} \frac{(1-\lambda)}{[\lambda(1-b)+\mu(1-\lambda)]^2} w (b) \, db \end{array} \right]$$

Part 1 follows from setting $\frac{dp^*}{d\mu} < 0$ and rearranging; part 2 follows from replacing $w(b) = w, \forall b$ and simplifying.

Proof of Proposition 5

From (29),

$$\frac{\partial \bar{b}_B}{\partial \mu_B} = \begin{cases} \frac{1-\lambda_B}{\lambda_B}\left[1 - \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right)\right] & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \leq 0 \tag{50}$$

$$\frac{\partial \bar{b}_B}{\partial p_B^*} = \begin{cases} \mu_B \frac{1-\lambda_B}{\lambda_B}\frac{p_A^*}{\left(p_B^*\right)^2}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \geq 0 \tag{51}$$

$$\frac{\partial \bar{b}_B}{\partial p_A^*} = \begin{cases} -\mu_B \frac{1-\lambda_B}{\lambda_B}\frac{1}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) & \text{if } \frac{p_A^*}{p_B^*}\left(1 + \frac{1-b_A}{\mu_A}\frac{\lambda_A}{1-\lambda_A}\right) \geq 1 \\ 0 & \text{otherwise} \end{cases} \leq 0 \tag{52}$$

Equations (50) and (51) imply that, other things being equal, if μ_B rises and p_B^* falls, $\bar{b}_B$ falls. Using (52), this implies that for (28) to hold, either p_A^* must fall, b_A^* must fall, or both. Since equation (27) implies that b_A^* and p_A^* must move in the same direction, both fall.

Equilibrium in the False Negatives case

I first describe the procedure for determining what range of assets has each kind of pricing and then state the equilibrium in full.

Define

$$E(i,p,r) \equiv \max_{j\in[\lambda,i]} \int_{\hat{b}(i)}^{\hat{b}(j)} w(b)\, db - p \cdot r \cdot (i-j) \tag{53}$$

For a given asset i, price p and remaining supply r, $E(i,p,r)$ measures the maximum over $j < i$ of difference between the endowment of all buyers who can recognize that i is good but cannot recognize that j is good and what it would cost to buy r units of all assets in $[j,i]$ at price p. An asset can only be priced by cash-in-the-market if $E\left(i, p^C(i), r(i)\right) = 0$. A strictly positive value would mean that there exists a range of buyers $\left[\hat{b}(i), \hat{b}(j)\right]$ for some $j < i$, all of whom can identify some asset in the range $[j,i]$ as a good asset (but not any asset lower than j) and whose collective endowment exceeds what is necessary to buy all assets in

$[j, i]$ for a price $p^C(i)$. Since these buyers will want to spend their entire endowment buying assets, it must be that some asset in the range $[j, i]$ must be priced above $p^C(i)$; monotonicity then implies that the price of asset i exceeds $p^C(i)$.

Suppose one knows that $\bar{i}$ is the upper limit of one type of region. Then the following procedure finds the lower end of the region, the type of region immediately below and the prices within the region.

1. For a cash-in-the market region, the lower end is

$$\sup\left\{i < \bar{i} : p^N(i) > p^C(i) \text{ or } E\left(i, p^C(i), r(i)\right) > 0\right\} \tag{54}$$

   and the region to the left is a nonselective region or a bunching region respectively, depending on which of the two conditions is met. Within the region, $p^R(i) = p^C(i)$ and $r(i) = r(\bar{i})$.

2. For a bunching region, the lower end is

$$\max\left\{i < \bar{i} : E\left(i, p^R(\bar{i}), r(\bar{i})\right) = 0\right\} \tag{55}$$

   and the region to the left is always a cash-in-the-market region. Within the region $p^R(i) = p^R(\bar{i})$ and $r(i) = r(\bar{i})$.

3. For a nonselective region, the lower end is

$$\sup\left\{i < \bar{i} : \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)} > r(j) \text{ for some } j \in (i, \bar{i}) \text{ or } E\left(i, p^N(i), r(i)\right) > 0\right\} \tag{56}$$

   and the region to the left is a cash-in-the-market region or a bunching region respectively, depending on which of the two conditions is met. Within the region, $p^R(i) = p^N(i)$ and $r(i) = \frac{w\left(\hat{b}(i)\right)}{p^N(i)(1-\lambda)}$.

The first region is a bunching region with $\bar{i} = 1$, $p(\bar{i}) = 1$ and $r(\bar{i}) = \mu$. When one of the sets defined by (54), (55) or (56) is empty, then the region extends up to $i = \lambda$ and p^R has been completely defined.

Definition 9. ω is a *nonselective-then-more-restrictive-first* (NMR) algorithm if it orders acceptance rules of the form $\chi_g = \mathbb{I}(i \geq g)$ according to $\chi_0 <_\omega \chi_g$ for all $g > 0$ and $\chi_g <_\omega \chi_{g'}$ for $g > g' > 0$.

For any $i \in [\lambda, 1]$, let $m(i)$ denote the market where the price is $p^R(i)$ (where $p^R(i)$ is found by the procedure above) and the clearing algorithm is NMR. Note that because of bunching, $m(i)$ could mean the same market for different i. For any $I_0 \subseteq [0,1]$, let the set of markets $M(I_0)$ be $M(I_0) = \{m(i) : i \in I_0\}$. The set of active markets is $M([\lambda, 1])$

The equilibrium is given by:

1. Seller decisions

   (a) For $s < \mu$:

$$\sigma_s(i,m) = \begin{cases} 1 & \text{if } \begin{cases} i < \lambda \\ \text{or} \\ p(m) \geq p^R(i) \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{57}$$

$$c_{1s} = \int_\lambda^1 \left[\int_{M([i,1])} p(m)\, d\eta(m; i) \right] di + \lambda \int_{M([\lambda,1])} p(m)\, d\eta(m; 0)$$

$$c_{2s} = 0$$

   (b) For $s \geq \mu$:

$$\sigma_s(i,m) = \begin{cases} 1 & \text{if } \begin{cases} i < \lambda \\ \text{or} \\ p(m) \geq 1 \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{58}$$

$$c_{1s} = \lambda \int_{M([\lambda,1])} p(m)\, d\eta(m; 0)$$

$$c_{2s} = 1 - \lambda$$

   where $\eta(m^*; i)$ is given by (64) below. Selling decisions follow the reservation price defined in the text. Consumption results from selling decisions using the rationing function η defined below. For distressed sellers, the first term in $c_{1,s}$ is the proceeds of trying to sell asset a good asset i in markets $M([i,1])$ and the integrating across i. The second term is the proceeds of trying to sell bad assets in markets $M([\lambda,1])$. It incorporates the fact that in equilibrium all bad assets sell at the same rate so $\eta(\cdot; i) = \eta(\cdot; 0)$ for all $i \in [0, \lambda)$.

2. Buyer decisions.

Let i_1 be the lowest i such that $p^R(i) = 1$. Define b_N by

$$\int_{b_N}^{\hat{b}(i_1)} w(b)\, db \equiv \mu (1 - i_1) \tag{59}$$

Define the function $\tilde{b}(i)$ as the solution to the following differential equation

$$\tilde{b}'(i) \equiv -\frac{1}{w\left(\tilde{b}(i)\right)} p^R(i) \left[\frac{\lambda}{\mu} + (i - \lambda)\right] r'(i) \tag{60}$$

with boundary condition $\tilde{b}(1) \equiv b_N$. Finally, let $b_0 \equiv \tilde{b}(\lambda)$ and define $\tilde{i}(b)$ for $b \in [b_0, b_N]$ by

$$\tilde{i}(b) \equiv \min\left\{i : \tilde{b}(i) = b\right\}$$

(a) For $b \geq b_N$:

$$\begin{aligned}
\delta_b &= \frac{w(b)}{p^R(1 - b(1 - \lambda))} \\
m_b &= m(1 - b(1 - \lambda)) \\
\chi_b(i) &= \mathbb{I}(i \geq 1 - b(1 - \lambda)) \\
c_{1b} &= 0 \\
c_{2b} &= \frac{w(b)}{p^R(1 - b(1 - \lambda))}
\end{aligned}$$

(b) For $b \in [b_0, b_N)$:

$$\begin{aligned}
\delta_b &= \frac{w(b)}{p^R\left(\tilde{i}(b)\right)} \\
m_b &= m\left(\tilde{i}(b)\right) \\
\chi_b(i) &= 1 \\
c_{1b} &= 0 \\
c_{2b} &= w(b)
\end{aligned}$$

(c) For $b < b_0$

$$\begin{aligned} \delta_b &= 0 \\ m_b &= m(1) \\ \chi_b(i) &= 1 \\ c_{1b} &= w(b) \\ c_{2b} &= 0 \end{aligned}$$

Buyers $b \geq b_N$ spend their entire endowment buying assets in market $m(1 - b(1-\lambda))$, i.e. in the market for the lowest i for which they observe a good signal, and use the selective acceptance rule $\mathbb{I}(i \geq 1 - b(1-\lambda))$, which only accepts good assets. Buyers $b \in [b_0, b_N)$ are nonselective. The function $\tilde{i}(b)$ assigns each one to a market. It is constructed as follows. In market $m(i)$, nonselective buyers bring down the unsold remainder by $r'(i)$, which requires buying $r'(i)(i-\lambda)$ good assets and $r'(i)\frac{\lambda}{\mu}$ bad assets. These cost $p^R(i)\left[\frac{\lambda}{\mu} + i - \lambda\right] r'(i)$. If buyer $\tilde{b}(i)$ is the nonselective buyer that buys in market $m(i)$ then the total nonselective wealth available in this market is $-w\left(\tilde{b}(i)\right)\tilde{b}'(i)$, so market clearing implies (60). Inverting this function results in buyer b choosing market $m\left(\tilde{i}(b)\right)$. Buyers $b < b_0$ don't buy anything. Note that since they are indifferent between buying and not buying, many other patterns of demand among nonselective buyers are possible.

3. Allocation function

(a) For markets $m \notin M([\lambda, 1])$ or markets $m(i) \in M([\lambda, 1])$ where i falls in either a cash-in-the-market or a nonselective range:

$$a(i;\chi,m) = \begin{cases} \frac{\chi(i)S(i;m)}{\int \chi(i)S(i;m)di} & \text{if } \int \chi(i)\, S(i;m)\, di > 0 \\ \frac{\chi(i)S(i;m)}{\sum_i \chi(i)S(i;m)} & \text{if } \int \chi(i)\, S(i;m)\, di \text{ but } \sum_i \chi(i)\, S(i;m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{61}$$

where

$$S(i;m)=\begin{cases}1 & \text{if } \begin{cases} i<\lambda \\ \text{or} \\ p(m)\geq 1\end{cases} \\ \mu & \text{if } i\geq\lambda,\ p(m)\in\left[p^R(i),1\right) \\ 0 & \text{if } i\geq\lambda,\ p(m)<p^R(i)\end{cases}$$

(b) For markets $m(i)$ where i falls in a bunching range $[i_L,i_H]$ and χ is of the form $\chi(i)=\mathbb{I}(i\geq g)$:

$$a(i;\chi,m)=\begin{cases}\frac{\chi(i)S^g(i;m)}{\int\chi(i)S^g(i;m)di} & \text{if } \int\chi(i)S^g(i;m)\,di>0 \\ \frac{\chi(i)S^g(i;m)}{\sum_i\chi(i)S^g(i;m)} & \text{if } \int\chi(i)S^g(i;m)\,di \text{ but } \sum_i\chi(i)S^g(i;m)>0 \\ 0 & \text{otherwise}\end{cases} \tag{62}$$

where $S^g(i;m)$ is the solution to the differential equation

$$\frac{dS^g(i;m)}{dg}=\begin{cases}\frac{w\left(\frac{1-g}{1-\lambda}\right)}{p(m)}\frac{S^g(i;m)\mathbb{I}[g\leq i\leq i_H]}{\int_g^{i_H}S^g(j;m)dj} & \text{if } g\in[i_L,i_H] \\ 0 & \text{otherwise}\end{cases} \tag{63}$$

with terminal condition

$$S^1(i;m)=\begin{cases}1 & \text{if } i<\lambda \\ \mu & \text{if } i\in\left[\lambda,i^H\right] \\ 0 & \text{otherwise}\end{cases}$$

Except for bunching markets, all buyers will draw assets from a sample that is proportional to the original supply, since their trades are never executed after another selective buyer. This results in (61). In bunching markets, buyer b imposes rule $\chi(i)=\mathbb{I}(i\geq g)$ with $g=1-b(1-\lambda)$; therefore when he trades the supply of asset i falls in proportion to this buyer's demand $\frac{w\left(\frac{1-g}{1-\lambda}\right)}{p(m)}$ times the ratio between the supply of asset i and all the assets acceptable by buyer b (as long as buyer b accepts asset i). This results in the differential equation (63), which describes how the supply of each asset falls as clearing algorithm progresses.

4. Rationing function

$$\eta(M([j,1]);i)=\begin{cases}1-\frac{r(j)}{\mu} & \text{if } j>i \\ 1 & \text{if } j\leq i\end{cases} \tag{64}$$

and $\eta(M_0; i) = 0$ if $M_0 \cap M([\lambda, 1]) = \emptyset$.

The rationing function simply says that a seller who offers an asset $i < j$ at every market with $p(m) \in \left[p^R(j), 1\right]$ will be able to sell a fraction $1 - \frac{r(j)}{\mu}$ (so that the unsold assets from the μ distressed sellers are $r(j)$), and $\frac{r(i)}{\mu}$ can be sold in market $m(i)$.

Proposition 7. *Equations (57)-(64) describe an equilibrium*

Proof.

1. Seller optimization.

   Since $r(j) > 0$, the rationing function (64) implies that in order to sell all of a seller's holdings of asset i, the reservation price must be $p^R(i)$. Therefore supply decisions are optimal, and consumption follows from the budget constraint.

2. Buyer optimization.

   For $b \in [b_N, 1]$, each buyer chooses the lowest-price market m where there is an i such that $x(i, b) = 1$ and $S(i; m) > 0$; since $p(m) \leq 1$, this is optimal. For $b \in [b_0, b_N)$, buyers only place weight on markets where nonselective pricing prevails; (32) implies that they are indifferent between trading or not and since the lowest-price market where there is an i such that $x(i, b) = 1$ and $S(i; m) > 0$ has $p(m) = 1$, there is no other market in which they would prefer to trade. For $b < b_0$, the same logic implies that not trading is also optimal. Consumption of all buyers follows from the budget constraint.

3. Allocation function.

   For markets $m \notin M([\lambda, 1])$ demand is zero, so for any clearing algorithm the residual supply faced by any buyer equals the original supply. For these cases, (61) follows directly from (65).

   For markets $m(i) \in M([\lambda, 1])$ where i falls in either a cash-in-the-market or a non-selective range, the NMR algorithm implies that all buyers face a residual supply proportional to the original supply, so (61) follows from (65).

   For markets $m(i)$ where i falls in a bunching range $[i_L, i_H]$ and χ is of the form $\chi(i) = \mathbb{I}(i \geq g)$, then the differential equation (63) follows from (66). Then (62) follows from applying the NMR algorithm.

4. Rationing function.

   Equation (64) follows from applying formula (67).

□

Proposition 8. *In any equilibrium, the prices and allocations are those of the equilibrium described by equations (57)-(64).*

I first establish some preliminary results. Recall that by Lemma 1, in any equilibrium there must be a price $p^R(i)$ for each asset such that distressed buyers supply asset i in all markets with $p(m) > p^R(i)$ and in no markets where $p(m) < p^R(i)$.

Lemma 9. *In any equilibrium* $p^R(i)$ *is nondecreasing in* i *over the range* $[\lambda, 1]$.

Proof. Assume the contrary. There there exist assets i, i' with $i' > i > \lambda$ such that $p^R(i') < p^R(i)$. For this to be consistent with seller optimization, it must be that

$$\eta(M_0; i') < \eta(M_0; i) = 1$$

where $M_0 = \left\{m : p(m) \geq p^R(i)\right\}$. But buyer optimization and the signal structure (3) requires that buyers only use rules of the form $\chi(i) = \mathbb{I}(i \geq g)$. This implies that for any $M_0 \subseteq M$,

$$\eta(M_0; i') \geq \eta(M_0; i)$$

□

Lemma 10. *In any equilibrium,* $\tau(m, \chi(i) = 1) \leq 1$ *for all* m*, where* τ *is defined by (41).*

Proof. Assume the contrary. Since the acceptance rule $\chi(i) = 1$ is feasible for all buyers, this implies that all buyers will want to spend their entire wealth in some market. Condition (1) implies that this is inconsistent with equilibrium. □

The proof of Proposition (8) is as follows:

Proof. Assume that there exists an equilibrium where the reservation prices for distressed seller are $\tilde{p}^R(i) \neq p^R(i)$. Consider first the case where $\tilde{p}^R(i) > p^R(i)$ for at least one $i \in [\lambda, 1]$ and let i_0 be the highest i where this is the case.

1. If i_0 is in a cash-in-the market region, (31) implies that at price $\tilde{p}^R(i_0)$ buyer $\hat{b}(i_0)$ cannot afford to buy all the units that distressed sellers own; buyers $b > \hat{b}(i)$ can find good assets at lower prices because by Lemma (9), $\tilde{p}^R(i)$ is monotonic so they do not buy at price $\tilde{p}^R(i)$; buyers $b < \hat{b}(i_0)$ do not observe any good signals at price $\tilde{p}^R(i)$ because i_0 is the highest i where $\tilde{p}^R(i) > p^R(i)$ so no assets for which they observe good signals are on sale at $\tilde{p}^R(i)$; they could buy nonselectively but the fact that i_0 is in a cash-in-the-market region means that they would prefer not to buy. Therefore some units owned by distressed sellers will remain unsold at price $\tilde{p}^R(i)$. Given distressed seller's preferences, this implies that reservation price $\tilde{p}^R(i)$ cannot be optimal.

2. If i_0 is in a bunching region, then $\tilde{p}^R(i) > p^R(i)$ violates monotonicity.

3. If i_0 is in a nonselective region, then $\tilde{p}^R(i) > p^R(i)$ implies that nonselective buyers would prefer not to buy, so some units owned by distressed sellers will remain unsold, contradicting optimality.

Consider instead the case where $\tilde{p}^R(i) < p^R(i)$ for at least one $i \in [\lambda, 1]$ and let i_0 be the largest i where this is the case.

1. Suppose i_0 is in a cash-in-the market region and consider the decision of buyers of type $\hat{b}(i_0)$. Since $\tau\left(m(i_0), \chi(i) = \mathbb{I}\left(i \geq 1 - \hat{b}(i)(1-\lambda)\right)\right) > 1$, optimality requires that they spend their entire wealth. Monotonicity (Lemma 9) implies that they cannot find assets for which they observe good signals at prices below $\tilde{p}^R(i_0)$ and Lemma 10 implies that they do not want to buy nonselectively, so they must spend all their wealth buying at price $\tilde{p}^R(i_0)$. But (31) implies that there distressed sellers do not own enough units to exhaust buyers' wealth at that price, so it cannot be part of an equilibrium.

2. If i_0 is in a bunching region $[i_L, i_H]$, condition (55) together with the monotonicty condition (Lemma 9) imply that the distressed sellers' endowment of assets $i \in [i_L, i_H]$ is not enough to exhaust the wealth of buyers $b \in \left[\hat{b}(i_H), \hat{b}(i_L)\right]$, so it cannot be part of an equilibrium.

3. If i_0 is in a nonselective region, (32) implies that $\tau(m(i_0), \chi(i) = 1) > 1$, which contradicts Lemma 10.

Therefore it must be the case that in an equilibrium reservation prices are those defined in the text. The rest of the equilibrium objects follow immediately.

$\square$

Proof of Proposition 6

1. By definition, $p^R(i) = 1$ for all $i > i_1$; since $p^R(i)$ is never above 1, it must be decreasing in μ. (55) and (53) imply that if $i_1 > \lambda$, then when μ increases, cash-in-the-market pricing will apply in a neighborhood of i_1. The result then follows because (31) implies $p^C(i)$ is decreasing in $r(i)$ and $r(i) = \mu$ in a neighborhood of i_1.

2. This follows directly from (32).

Appendix B: Applying Clearing Algorithms

This appendix details how a given pattern of choices by buyers and sellers results, through the application of the relevant clearing algorithms, in an allocation function A and a rationing function η.

First define aggregate supply of asset i in market m as:

$$S(i;m) = \int_{s\in[0,1]} \sigma_s(i;m)\,ds$$

Aggregate demand is a measure D on $X \times M$. $D(X_0, M_0)$ is the total amount of assets demanded by buyers who impose acceptance rules $\chi \in X_0 \subseteq X$ in markets $m \in M_0 \subseteq M$. Letting

$$B(X_0, M_0) = \{b : \chi_b \in X_0, m_b \in M_0\}$$

then

$$D(X_0, M_0) = \int_{b\in B(X_0,M_0)} \delta_b db$$

Allocation functions are derived from aggregate supply and demand as follows. Consider a market m and let $S^\chi(i,m)$ denote the residual supply that is faced by a buyer who imposes rule χ in market m. Let

$$a(i;\chi,m) = \begin{cases} \frac{\chi(i)S^\chi(i;m)}{\int \chi(i)S^\chi(i;m)di} & \text{if } \int \chi(i)\, S^\chi(i;m)\, di > 0 \\ \frac{\chi(i)S^\chi(i;m)}{\sum_i \chi(i)S^\chi(i;m)} & \text{if } \int \chi(i)\, S^\chi(i;m)\, di = 0 \text{ but } \sum_i \chi(i)\, S^\chi(i;m) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{65}$$

As long as rule χ accepts some assets that remain in positive supply by the time the clearing algorithm reaches rule χ, then $a(i,\chi,m)$ defines either a density with respect to the Lebesgue measure or a discrete measure, which describes the assets received by rule χ in market m. In either case, these assets constitute a representative sample of the χ-acceptable assets that remain. If no χ-acceptable assets remain, then the demand associated with rule χ is left unsatisfied.[11]

The residual supply faced by a buyer who imposes acceptance rule χ is computed by subtracting from the original supply all the units that were allocated to all acceptance rules that precede rule χ under the algorithm ω_m used in market m:

$$S^{\chi}(i;m) = S(i;m) - \int\limits_{\tilde{\chi} <_{\omega_m} \chi} a(i;\tilde{\chi},m)\, dD(\tilde{\chi},m) \tag{66}$$

The rationing function is then:

$$\eta(M_0;i) = \int\limits_{\substack{m \in M_0 \\ \chi \in X}} \frac{a(i;\chi,m)}{S(i;m)} dD(\chi,m) \tag{67}$$

Equation (67) says that the number of units of asset i that can be sold in markets M_0 (per unit supplied) is computed in the following way. For each acceptance rule χ and market m, $\frac{a(i,\chi,m)}{S(i;m)}$ is the ratio between how much the buyers who impose χ get per unit of demand and how much the sellers offered. Adding up over markets and acceptance rules using the demand measure yields how many units sellers are able to sell. For instance $\eta(m;\text{Green})$ in equation (18) results from the following calculation:

$$\begin{aligned}\eta(m;\text{Green}) &= \frac{a(\text{Green};\{0,1,1\};m)}{S(\text{Green};m)} D(\{0,1,1\};m) + \frac{a(\text{Green};\{0,0,1\};m)}{S(\text{Green};m)} D(\{0,0,1\};m) \\ &= \frac{0.25}{1.5} \times 1 + \frac{1}{1.5} \times 1 \\ &= \frac{5}{6}\end{aligned}$$

[11] If demand exceeds residual supply, i.e. if $D(\chi,m) > \int \chi(i)\, S^{\chi}(i;m)\, di > 0$, then $a(i;\chi,m) = \frac{\chi(i) S^{\chi}(i;m)}{D(\chi,m)}$. In equilibrium, this issue never arises.

Appendix C: Two Other Examples

This Appendix describes (without formally stating all the equilibrium objects) what the equilibrium looks like under two other possible information structures.

False Positives and False Negatives Together

Suppose

$$x(i,b) = \mathbb{I}(i \geq b)$$

so buyers with $b < \lambda$ observe some "false positives" while buyers with $b > \lambda$ observe some "false negatives". Suppose further that for $b > \lambda$, $w(b)$ is strictly increasing and differentiable.

The key equilibrium object is a function $p(i)$, defined for $i \in [\lambda, 1]$, which denotes the lowest price at which asset i is offered for sale. As in the false negatives case, false-negative buyer b will want to buy in the cheapest market where he can observe good assets. If $p(i)$ is increasing (which will be true in equilibrium because $w(b)$ is increasing), then this means a market where the price is $p(b)$. The same reasoning that leads to equation (31) implies

$$p(i) = \frac{w(i)}{r(i)} \tag{68}$$

where $r(i)$ is the number of i assets held by distressed sellers that they were unable to sell at prices above $p(i)$.

For false-positive buyers, if they buy in a market where the price is $p(i)$, they will be drawing bad assets in the range $[b, \lambda)$ (coming from all sellers) and good assets in the range $[\lambda, i]$ (only from distressed sellers). They obtain terms of trade

$$\tau(b, i, p(i)) = \frac{1}{p(i)} \frac{\mu(i-\lambda)}{\mu(i-\lambda) + \lambda - b} \tag{69}$$

In choosing what market to buy from, they face a tradeoff beween better selection and lower prices. The marginal rate of substitution is

$$MRS(b,i,p) \equiv \frac{\frac{\partial \tau(b,i,p)}{\partial i}}{\frac{\partial \tau(b,i,p)}{\partial p}} = -\frac{p(i)}{(i-\lambda)} \frac{\lambda - b}{\mu(i-\lambda) + \lambda - b}$$

which is increasing in b. Hence there is single crossing and false-positive buyers sort into different markets: higher-b buyers (which, among false-positive buyers, means more expert) prefer lower prices and worse selection compared to lower-b buyers. False-positive buyers' sorting can be summarized by a function $\tilde{b}(i)$ that says which buyer buys in the market where the price is $p(i)$.

Therefore the equilibrium is given by the functions $p(i)$, $\tilde{b}(i)$ and $r(i)$. These must satisfy the following conditions. The first is a first order condition for buyer $\tilde{b}(i)$ to find it optimal to buy from market $p(i)$. Using (69):

$$p'(i) = \frac{p(i)}{i-\lambda} \frac{\lambda - \tilde{b}(i)}{\mu(i-\lambda) + \lambda - \tilde{b}(i)} \tag{70}$$

The second condition is a market-clearing condition. The purchases of each false-positive buyer are spread over many assets but add up to determine the unsold remainder $r(i)$. In market $p(i)$, false positive buyers buys $r'(i)(i-\lambda)$ good assets and $r'(i)\frac{\lambda-\tilde{b}(i)}{\mu}$ bad assets. This requires spending a total of $p(i)\left[\frac{\lambda-\tilde{b}(i)}{\mu} + i - \lambda\right] r'(i)$. These buyers have a total wealth of $-w\left(\tilde{b}(i)\right)\tilde{b}'(i)$. Hence, in equilibrium it must be that

$$\tilde{b}'(i) = -\frac{1}{w\left(\tilde{b}(i)\right)} p(i) \left[\frac{\lambda - \tilde{b}(i)}{\mu} + (i-\lambda)\right] r'(i) \tag{71}$$

Finally, (68) can be rewritten in differential form as

$$r'(i) = \frac{\frac{\partial w(i)}{\partial i} p(i) - p'(i) w(i)}{[p(i)]^2} \tag{72}$$

Equations (70)-(72) constitute a system of differential equations. The terminal conditions are:

$$\frac{1}{p(i^*)} \frac{\mu(i^*-\lambda)}{\mu(i^*-\lambda) + \lambda - \tilde{b}(i^*)} = 1 \tag{73}$$

$$\tilde{b}(\lambda) = \lambda \tag{74}$$

$$r(i^*) = \mu \tag{75}$$

$$p(i^*) = \frac{w(i^*)}{\mu} \tag{76}$$

Equation (73) is an indifference condition. For some cutoff asset i^*, buyer $\tilde{b}(i^*)$ is indifferent between buying and not buying. As in the false positives case, buyers with $b < \tilde{b}(i^*)$ find that τ is below 1 in all markets and do not trade. Equation (74) says that all false-positive buyers with $b \in \left[\tilde{b}(i^*), \lambda\right]$ buy assets $i \in [\lambda, i^*]$, which requires that buyer $b = \lambda$ buy asset λ. Equation (75) says that, since no false-positive buyers are present in markets where prices are above $p(i^*)$, distressed buyers cannot sell any asset below i^* in those markets, so the unsold remainder of asset i^* equals their entire endowment μ. Finally, equation (76) is equation (68) evaluated at i^*.

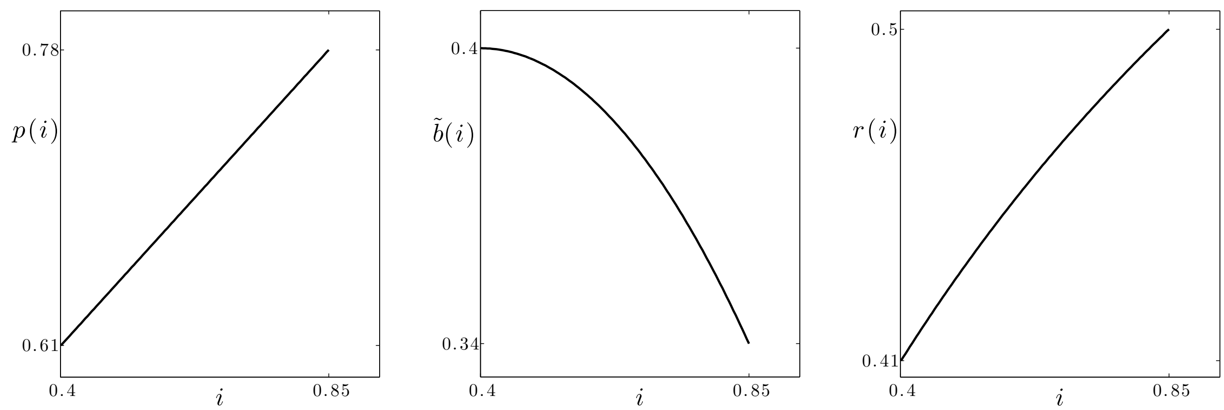

Figure 9: Solution to the differential equations. The example uses $\lambda = 0.4$, $\mu = 0.5$ and $w(b) = \max\{0.05(b+1), 0.1\}$.

Figures 9 and 10 illustrate the equilibrium. Figure 9 shows the solution to the system of differential equations. The cutoff asset for this example is $i^* = 0.85$. The left panel shows the price function for assets in the range $[\lambda, i^*]$. Buyer $\tilde{b}(i^*) = 0.34$ buys asset i^*; buyers below this cutoff do not buy at all and buyers between 0.34 and $\lambda = 0.4$ spend all their wealth buying assets. Their collective purchases bring down the unsold remainder from $r(i^*) = \mu = 0.5$ up to $r(\lambda) = 0.41$.

Figure 10 shows what markets buyers buy from, what assets they obtain and what are the resulting terms of trade. The first panel shows which market each buyer chooses to buy from. Buyer λ, who gets a perfect signal, buys good assets at the lowest possible price. Away from λ in either direction, buyer choose higher-priced markets. To the left of $b = \lambda$, false positive buyers buy in higher-priced markets because they need to mix in more good assets

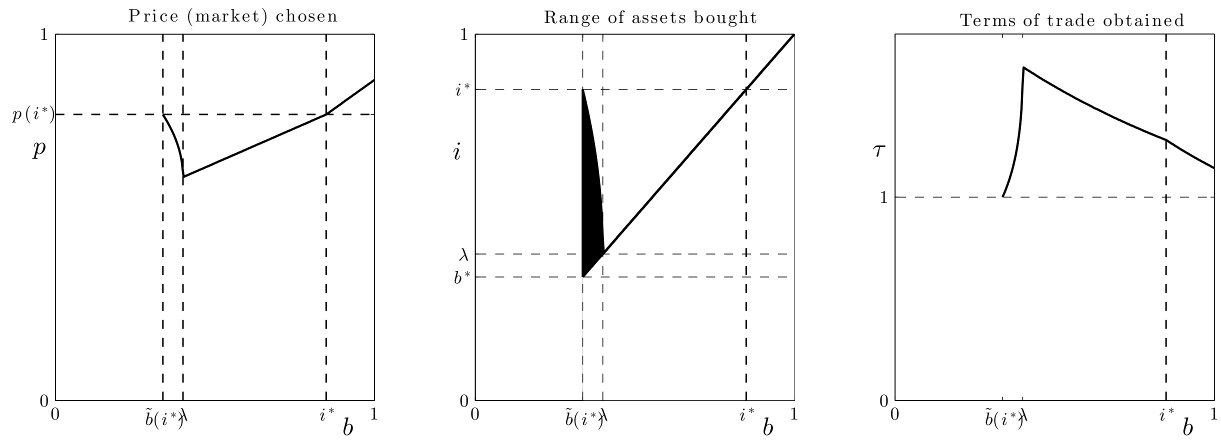

Figure 10: Characterization of the equilibrium. The example uses $\lambda = 0.4$, $\mu = 0.5$ and $w(b) = \max\{0.05(b+1), 0.1\}$.

with the bad assets that they are unable to filter out. To the right of $b = \lambda$, less expert false-negative buyers need to buy at higher prices because they cannot detect the good assets that are on sale at $p(\lambda)$. Those with $b \in (\lambda, i^*)$ buy in markets where a false-positive buyer is also present, whereas those with $i > i^*$ buy in markets with no other buyers. Notice however that even in markets where more than one type of buyer is present, the assets they accept overlap on a zero-measure set, so the allocation they obtain does not depend on the order in which they clear. Hence, the clearing algorithm that is used is indeterminate.

The selection of assets that each buyer obtains is shown in the second panel. By choosing a higher-priced market, lower-b false-positive buyers find higher-i good assets on sale, which makes up for the fact that the lower bound on the assets the accept is lower. False-negative buyers, on the other hand, each buy a single asset type: $i = b$, the lowest i that they can tell is a good asset.

The third panel shows the terms of trade that result. Buyer $b = \lambda$ gets the best terms of trade and they fall off in either direction. For this example, τ is above 1 for all false-negative buyers (because $p(i) < 1$), so they all strictly choose to trade, but τ reaches 1 for $b = 0.34$, so buyers below this point choose not to trade. This need not be the case in general; it depends on the function $w(b)$.

Non-Nested Signals

Suppose

$$x(i,b) = \mathbb{I}\left(i \in [0, \max\{\lambda b + \Delta - \lambda, 0\}] \cup [\lambda b, \lambda b + \Delta] \cup [\lambda, 1]\right)$$

for some $\Delta < \lambda$, as illustrated in Figure 11. All buyers observe false positives for Δ assets, but the set of bad assets for which they observe $x(i,b) = 1$ are not nested. For $b \leq \lambda - \Delta$, buyers oberve false positives for an interval of length Δ starting at $i = \lambda b$. For $b > \lambda - \Delta$, buyers observe false positives for $i \in [\lambda b, \lambda]$ and for $i \in [0, \lambda b + \Delta - \lambda]$.

Further suppose $w(b) = w$ is a constant. As a result, all buyers are symmetric (because they observe the same fraction of false positives and have the same wealth) and all bad assets are symmetric (because they give false positive signals to the same fraction of buyers).

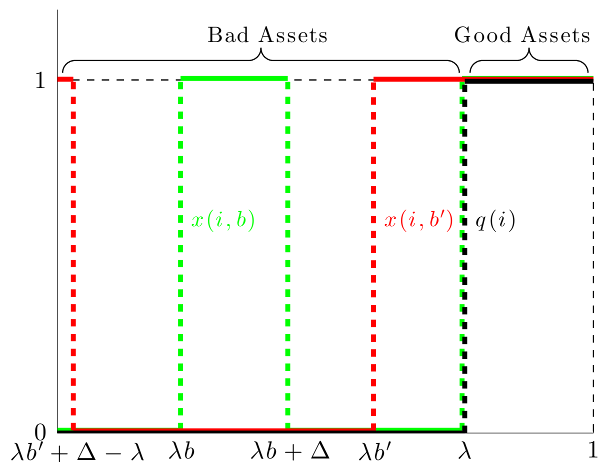

Figure 11: Information of buyers with non-nested signals

The equilibrium for this case is as follows. All assets trade at the same price p^* but in different markets since all buyers direct their demand to a market where they are guaranteed to trade first. In any market at price p^* the supply includes a measure $\mu(1-\lambda)$ of good assets from distressed sellers and a measure λ of bad assets from all sellers, of which any buyer accepts a measure Δ. Assumption (1) implies that buyers do not spend all their endowment, so they must be indifferent between buying and not buying, which implies

$$p^* = \frac{\mu(1-\lambda)}{\mu(1-\lambda) + \Delta}$$

Adding across all markets with $p = p^*$, all good assets are sold, as well as a fraction $\frac{\Delta}{\lambda}$ of bad assets.