Carbon Tax in a Production Network:

Propagation and Sectoral Incidence

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This reflects the opinions of the authors and do not necessarily express the views of the Banque de France; the European Central Bank or the Eurosystem

Motivation

Growing interest for climate change and transition policies:

- In policy debate:
  Evaluation of climate change impacts, transition costs, etc.
- In central banks more specifically:
  Development of climate stress-tests scenarios, "greenflation"

This paper's focus:

- Transition risk (vs. physical risk)
- Economic policy shock: carbon tax based on GHG emissions
- Time horizon: 5 to 10 years
- Quantification of structural impact

Method:

- Sectoral level modelling
- Multiple countries

Link with climate stress-test scenarios:

- Can be coupled with micro- and macroprudential tools
- Combined to aggregate macro models, useful for calibration

Literature

Carbon price and carbon tax

- Shadow price of carbon (Quinet report, 2019)
- Impact of carbon tax (DNB, Hebbink *et al.*, 2018, Bundesbank's Emuse, 2022)
- Carbon tax and double dividend (Freire-Gonzalez & Ho, 2019)

Production networks

- Baqaee & Farhi (2019a, 2019b)
- Fiscal multiplier with multiple sectors: Bouakez *et al.* (2023)
- Productivity shocks in multi-country multi-sector model: Johnson (2014)

Climate stress-tests

- NGFS reports
- BoE stress-test scenario
- Numerous large scale models (Imaclim, E3ME, ...)

Our paper

- Parsimonious and transparent
- General equilibrium set-up

Results preview

Model

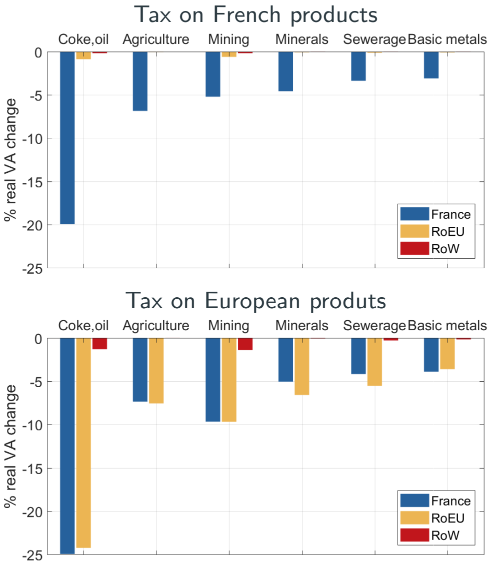
- Static, general equilibrium
- Sectoral production network
- Allowing for substitution

Geographical coverage

- France, rest of European Union, rest of the World

Policy experiment

- Implementation of a carbon tax
- In France or the whole EU
- Tax size: carbon price of 100 €/tCO_2-e
- Real GDP impact in France: -1.2% to -1.5%
- Upstream sectors more impacted



Model: Main Ingredients

Production:

- Sectoral production network
- Perfect competition within sector
- One representative producer in each sector in each country
- Output used for intermediate and final consumption

Households:

- One representative household per country
- Consumes a basket of goods
- Perfect risk-sharing across countries

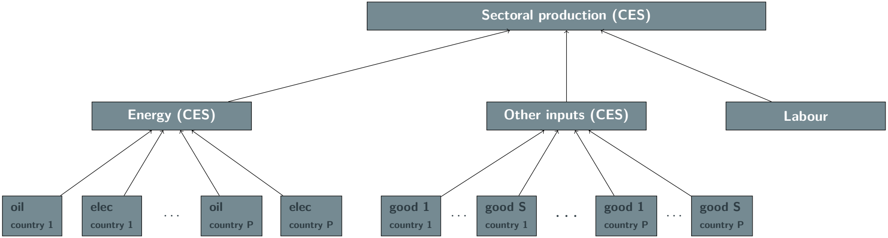
Taxes:

- On final purchase of oil & coke by households (κ)
- On intermediate purchase of oil & coke by producers (ζ)
- On production (τ)
- All proceeds redistributed to households

Model: Production

In each sector:

- Perfect competition
- Intermediate consumptions imported from the whole world
- Nested CES, with substitution across:
  - Energy types
  - Other (non-energy) inputs
  - Energy, non-energy inputs and labour
- Country-sector-specific input shares



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Model: Households and Market Clearing

Representative household in each country:

- Consumes a basket of goods from the whole world
- Supplies labour inelastically
- Receives tax proceeds as a lump-sum transfer based on her residence country

Perfect risk sharing:

- Households can trade internationally a complete set of Arrow-Debreu securities

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Market Clearing

Labour market:

- Perfect labour mobility across sectors within a country
- No cross-country labour mobility
- Wage adjusts to balance labour demand and labour supply in each country

Goods markets:

- Production used for intermediate consumption and final consumption
- Price adjusts to balance demand and supply for each good

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Model: Taxes

Household side:

Final consumption tax: rate κ_{jA} specific to each country

- Applied to **final** consumption of oil & coke
- Reflects GHG emitted by households of that country

Model: Taxes

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Production side:

Challenge: two different types of emissions linked to production

- GHG emissions due to **energy use** (e.g. burning oil, coke)
  Can be substituted by (renewable) electricity
- GHG emissions inherent to **production process** (e.g. agriculture, cement)
  No easy substitute, would need substancial innovation

$\Rightarrow$ Model two different taxes on the production side:

Intermediate consumption tax

Production tax

Model: Taxes

Intermediate consumption tax: rate ζ_{ji} specific to each country-sector

- Reflects emission intensity due to **energy use** (CO_2 emissions)
- Applied to **intermediate** consumption of oil & coke only
  $\Rightarrow$ favours substitution across inputs to pay less tax

Model: Taxes

Intermediate consumption tax: rate ζ_{ji} specific to each country-sector

- Reflects emission intensity due to **energy use** (CO_2 emissions)
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Production tax: rate τ_i specific to each country-sector

- Reflects emission intensity inherent to sectoral **production process** (e.g. methan for agriculture)
- Applied to production, passed to both final and intermediate buyers
  $\Rightarrow$ **influences both consumers' and producers'** decisions
- Tax rate fixed exogenously
  $\Rightarrow$ favours substitution in downstream sectors only
  producer cannot reduce his own tax rate

All proceeds redistributed to the household of the taxing country

Calibration

Input & consumption shares, aggregate labour

- World Input Output Database (2014)
- Aggregated into 3 blocks: France, rest of EU, rest of the World
- 55 sectors per country
- Energy: 2 sectors
  oil/coal vs. electricity/gas/steam/air conditioning

Elasticities

From literature (Baqaee & Farhi, 2017, Atalay, 2017)

	Energy types	Other inputs	Labour, energy and other inputs	Final consumption goods
Elasticity across	0.9	0.4	0.8	0.9

Households relative risk-aversion: $\varphi = 2$

Taxes

- Eurostat sectoral emissions data (2016)
- Proportional to emission intensity per € produced or per € of oil consumed
- Tax proceeds cover emissions cost, given price of tCO_2e

more

Policy experiment: Carbon Tax

Tax applied to goods produced in:

1. France (*scenario 1*)
2. France and rest of the EU (*scenario 2*)

Price of the ton of CO_2 equivalent set to 100 euros
$\Rightarrow$ Total tax receipts equal to 1.5% of VA for France, 2.1% for RoEU

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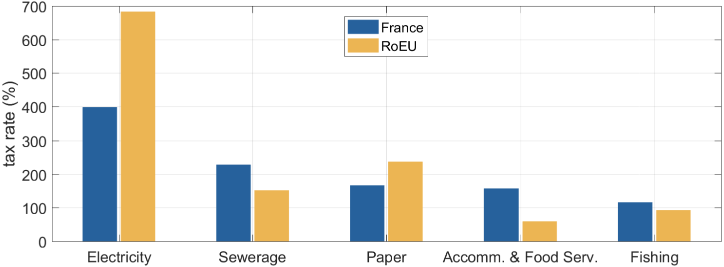
Oil intermediate consumption tax:

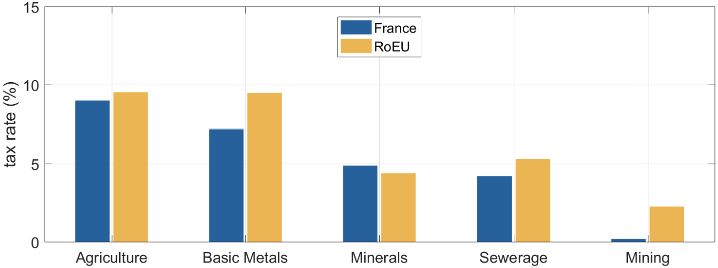
- Proportional to CO_2 emission intensity per € of oil consumed



Policy experiment: Carbon Tax

Production tax:

- Proportional to non-CO_2 emissions intensity per € produced
- Special cases for *Basic Metals* and *Minerals (excl. metals)*:



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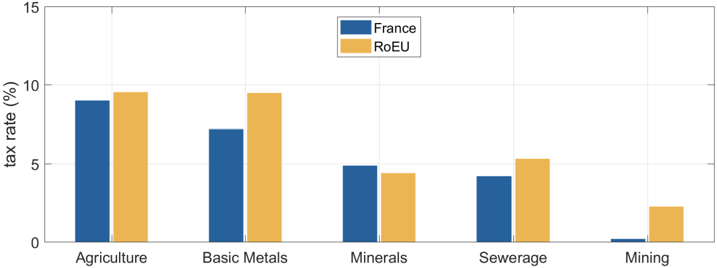


Oil final consumption tax:

- France: 42%
- RoEU: 28%

Aggregate Results: Real value added

% deviation from SS	France	Rest of the EU	Rest of the world
Tax in France (sc. 1)			
Real VA	-1.21	-0.05	-0.01
Real consumption	-0.56	0.01	0.01
Tax in whole EU (sc. 2)			
Real VA	-1.53	-1.88	-0.13
Real consumption	-0.64	-0.67	0.08

- Weak transmission across countries
- Larger impact in RoEU than France in sc. 2 (nuclear energy)
- Impact via:
  - Input prices ⇒ substitution in downstream intermediate consumptions
  - Final goods prices ⇒ substitution in final consumption basket
  - Lower real wages and lower final demand

Aggregate Results: Bilateral Real Exchange Rates

% change (decrease=appreciation)	*with respect to:*		
In:	France	RoEU	RoW
Tax in France (sc. 1)			
France	–	-1.14	-1.15
RoEU	1.15	–	-0.02
RoW	1.17	0.02	–
Tax in whole EU (sc. 2)			
France	–	0.05	-1.45
RoEU	-0.05	–	-1.50
RoW	1.47	1.52	–

Tax in France:

- Relative after-tax price levels increase in France
  $\Rightarrow$ Slight appreciation of the France RER
  But not enough to prevent consumption decrease

Tax in whole EU:

- Stable RER between France and RoEU

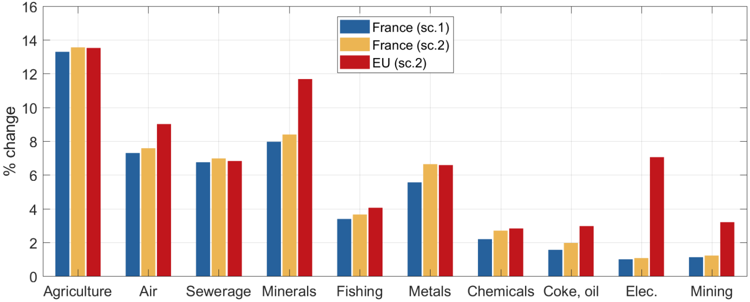
Aggregate Results: Real Bilateral Trade

% change *Exports From:*	*To:* France	 RoEU	 RoW	 Total
Tax in France (sc. 1)				
France	–	-1.21	-0.98	-1.10
RoEU	-1.63	–	-0.02	-0.24
RoW	-3.37	-0.10	–	-0.49
Total	-2.31	-0.27	-0.12	-0.43
Tax in whole EU (sc. 2)				
France	–	-2.89	-1.27	-2.12
RoEU	-3.31	–	-1.35	-1.61
RoW	-3.89	-3.60	–	-3.63
Total	-3.56	-3.50	-1.34	-2.44

Tax in France:

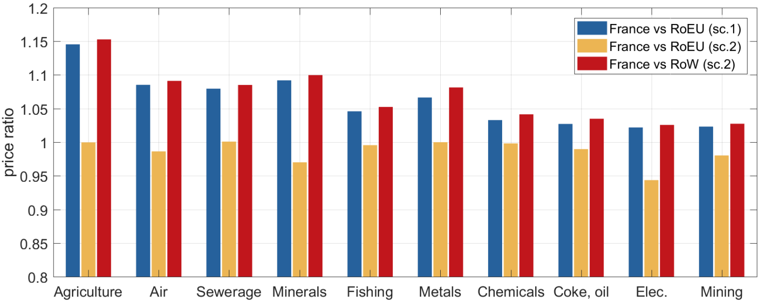
- Exports consistent with RER movements:
  RER appreciation in Fr $\Rightarrow$ Fr exports $\downarrow$
- Fr wages $\downarrow$ + financial transfers $\Rightarrow$ Fr final demand $\downarrow$ + home bias $\Rightarrow$ Fr imports $\downarrow$
- Recessive effect on total trade

Sectoral Results: Real After-tax Prices



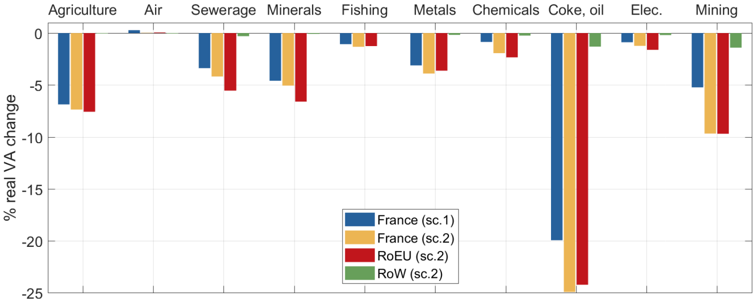
- Higher tax on RoEU *Electricity*
- Little transmission: similar prices in France when RoEU implements tax

Sectoral Results: Price Competitiveness



- French agricultural and manufacture products more expensive than RoEU and RoW ones
- Less true for services
- Price competitiveness w.r.t. RoEU restored when tax also implemented in RoEU

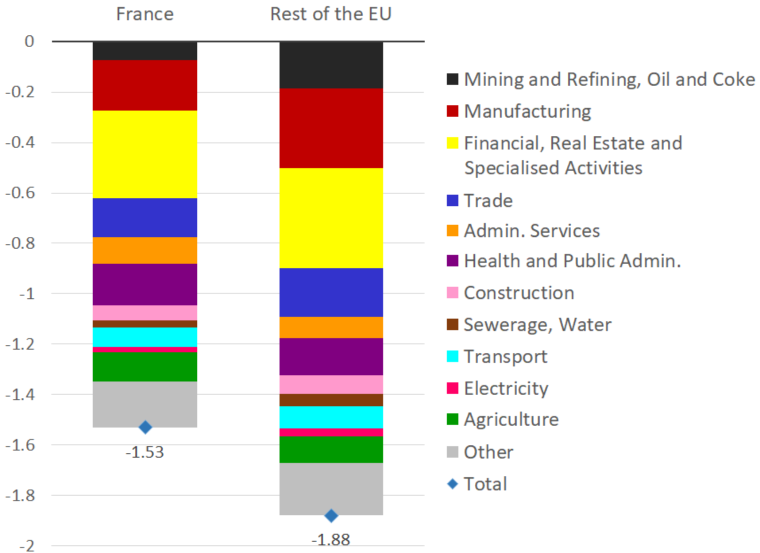
Sectoral Results: Real VA



- Related to price changes
- V. large impact on *Coke, oil* because of additional intermediate and final consumption tax
- Impact transmitted to upstream sectors even in RoW: *Mining*

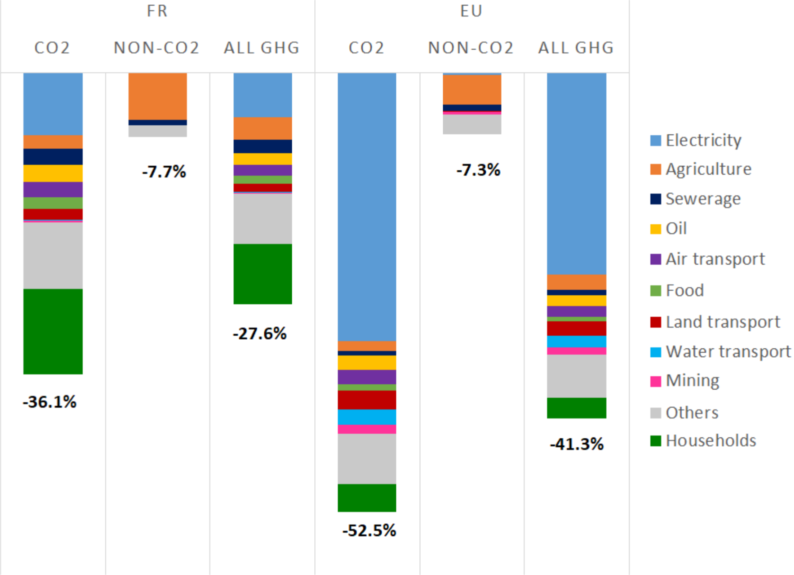
Recessionary impact

Sectoral contributions to decrease in real value added (%), tax in whole EU



GHG Emissions

Sectoral contributions to decrease in GHG emissions (%), tax in whole EU



The role of the production network: sectors

Aggregating all non-energy sectors:

- A 3-sectors multi-country model
- In each country: Oil, Electricity, the rest of the economy

	Full model		Simplified model	
% change	France	Rest of EU	France	Rest of EU
Real value added	-1.53	-1.88	-1.65	-2.04
GHG emissions	-36.1	-41.3	-23.0	-38.5

Qualitatively similar but overall more pessimistic:

- about 8% larger value added decline,
- 7% smaller emissions drop
- Less leeway to adjust inputs and consumption choices, less precise pricing of polluting activities

The role of the production network: countries

Disaggregating the Rest of EU into more separate countries:

- Progressively separating individual countries from the Rest of EU block

% change	FRA	DEU	ITA	ESP	GBR	NLD	POL	PRT	RoEU	RoW	EU\FRA
Baseline	-1.53	-	-	-	-	-	-	-	-1.88	-0.13	-1.88
+ 4 countries	-1.53	-1.81	-1.78	-1.74	-1.45	-	-	-	-2.12	-0.13	-1.84
+ 7 countries	-1.52	-1.77	-1.76	-1.74	-1.43	-2.22	-4.15	-2.24	-1.75	-0.13	-1.81

Results similar to the Baseline:

- impact on French value added unchanged
- EU excl. France marginally better
- Countries using more polluting energy sources or with heavy industry fare worse (Poland, Germany, Netherland)

Downstreamness and tax propagation

Downstreamness index from Antràs & Chor (2013)

Figure: Uniform tax rate τ in whole EU

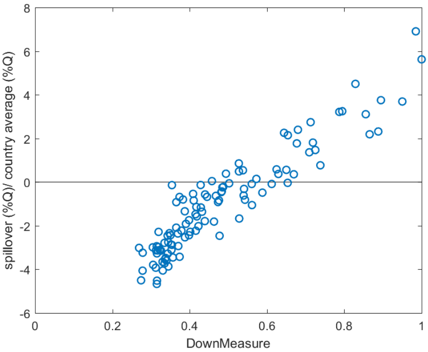
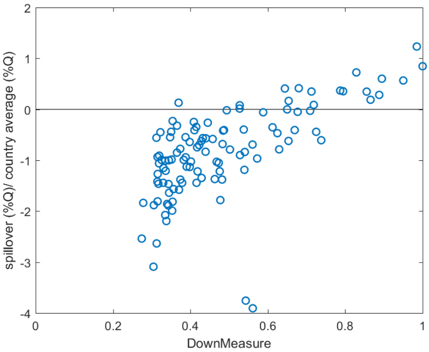


Figure: Complete carbon tax scenario



spillovers more negative in upstream sectors:

- corr(Downstreamness, Spillovers (% Q)) =0.93

somewhat blurred with heterogeneous tax rates (complete carbon tax scenario):

- corr(Downstreamness, Spillovers (% Q)) =0.42

Robustness

Calibration

Varying elasticities on the production side: more

- Substitution across energy types
- Substitution across non-energy intermediate inputs
- Substitution across energy, other inputs and labour

$\Rightarrow$ Substitution across energies is key, otherwise fairly robust

Varying household preferences parameters: more

- Substitution across final consumption goods
- Relative risk aversion

$\Rightarrow$ Fairly robust

Varying the tax level: more

- Carbon price

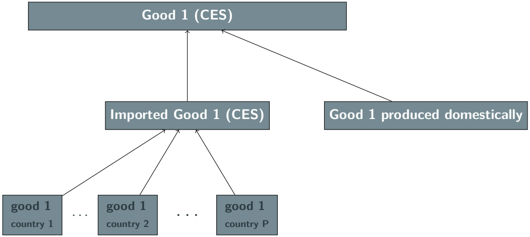
$\Rightarrow$ Close to linear

Financial Markets

Importance of perfect risk sharing more

Extensions

Armington trade aggregator to be able to specify trade elasticities



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- Results broadly similar

Using Exiobase data

- Higher sectoral granularity
- Energy sectors separating gas and electricity types (per source of energy)

Conclusion

Evaluate the medium-run impact of a carbon tax with some key ingredients:

- Sectoral production network
- Multiple countries

$\Rightarrow$ Aggregate impact on real value added: -1.5% for France, -1.9% for the RoEU

Caveats:

- No technological innovation
- No infra-sectoral heterogeneity in taxation

Further uses:

- Inserted in BdF/ACPR's framework for climate scenarios for pilot exercises in 2021 and 2023 (cf. Allen et al., 2020 and Allen et al., 2023)

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Thank you!

Appendix

Appendix: Production

Representative producer's program in each sector i:

$$\max_{L_i, Z_{ij}} \pi_i = P_i(1-\tau_i)Q_i - wL_i - \sum_{j=1}^{N} P_j(1+\zeta_{ji})Z_{ji}$$

where: $$Q_i = \left(\mu_i^{\frac{1}{\theta}} L_i^{\frac{\theta-1}{\theta}} + \alpha_{Ei}^{\frac{1}{\theta}} E_i^{\frac{\theta-1}{\theta}} + \alpha_{Ii}^{\frac{1}{\theta}} I_i^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

$$E_i = \left(\sum_{j=1}^{N_E} \left(\frac{\alpha_{ji}}{\alpha_{Ei}} \right)^{\frac{1}{\sigma}} Z_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

$$I_i = \left(\sum_{j=N_E+1}^{N} \left(\frac{\alpha_{ji}}{\alpha_{Ii}} \right)^{\frac{1}{\epsilon}} Z_{ji}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Parameters defined such that: $\alpha_{Ei} = \sum_{j=1}^{N_E} \alpha_{ji}$;
$\alpha_{Ii} = \sum_{j=N_E+1}^{N} \alpha_{ji}$;
$\alpha_{Ei} + \alpha_{Ii} + \mu_i = 1$;

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Appendix: Producers' FOC

$$\text{FOC in sector } i: \frac{L_i}{Q_i} = \mu_i \left(\frac{P_i(1-\tau_i)}{w}\right)^{\theta}$$

$$\frac{E_i}{Q_i} = \alpha_{Ei} \left(\frac{P_i(1-\tau_i)}{P_{Ei}}\right)^{\theta}$$

$$\frac{I_i}{Q_i} = \alpha_{Ii} \left(\frac{P_i(1-\tau_i)}{P_{Ii}}\right)^{\theta}$$

$$\frac{Z_{ji}}{E_i} = \frac{\alpha_{ji}}{\alpha_{Ei}} \left(\frac{P_{Ei}}{P_j(1+\zeta_{ji})}\right)^{\sigma} \forall j \leq N_E$$

$$\frac{Z_{ji}}{I_i} = \frac{\alpha_{ji}}{\alpha_{Ii}} \left(\frac{P_{Ii}}{P_j(1+\zeta_{ji})}\right)^{\epsilon} \forall j \geq N_E + 1$$

$$\text{Price index definitions: } P_{Ei} = \left(\sum_{j=1}^{N_E} \frac{\alpha_{ji}}{\alpha_{Ei}} (P_j(1+\zeta_{ji}))^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

$$P_{Ii} = \left(\sum_{j=N_E+1}^{N} \frac{\alpha_{ji}}{\alpha_{Ii}} (P_j(1+\zeta_{ji}))^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$$

$$P_i(1-\tau_i) = \left(\mu_i w^{1-\theta} + \alpha_{Ei} P_{Ei}^{1-\theta} + \alpha_{Ii} P_{Ii}^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

back

Appendix: Households' program

$$\max_{\{C_{Ajt}\},\{b_A(\omega_{t+1})\}} E_t \sum_{\tau=1}^{\infty} \beta^{\tau} \frac{C_{At+\tau}^{1-\varphi}}{1-\varphi} \tag{1}$$

s.t.

$$C_{At} = \left(\sum_{j=1}^{N} \gamma_{Aj}^{\frac{1}{\rho}} C_{Ajt}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{2}$$

$$w_{At} L_{At} + T_{At} + b_A(\omega_t) = P_{At} C_{At} + \int q(\omega_{t+1}) b_A(\omega_{t+1}) d\omega_{t+1} \tag{3}$$

where $P_A = \left(\sum_{k=1}^{N} \gamma_{kA} \left[P_k (1+\kappa_{kA}) \right]^{1-\rho} \right)^{\frac{1}{1-\rho}}$

back

Appendix: Households' FOC

- Consumption ratios in country A:

$$\forall j \in \{1, \ldots, N\}:$$
$$\frac{C_{jA}}{C_A} = \gamma_{jA} \left(\frac{P_j(1+\kappa_{jA})}{P_A} \right)^{-\rho}$$

- Risk-sharing condition:

$$\forall B \in \mathcal{C} \setminus A, \qquad \frac{C_B}{C_A} = \nu_{AB} \left(\frac{P_A}{P_B} \right)^{\frac{1}{\varphi}}$$

where $\{\nu_{AB}\}_{B \in \mathcal{C}}$: relative aggregate consumption sizes across countries in the initial steady state.

back

Appendix: Market Clearing Conditions

Labour market:

- Perfect labour mobility across sectors within a country
- No cross-country labour mobility
- Wage adjusts to balance labour demand and labour supply in each country

$$\forall A \in \mathcal{C}, \qquad L_A = \sum_{j \in \mathcal{S}_A} L_j$$

Goods markets:

- Production used for intermediate consumption and final consumption
- Price adjusts to balance demand and supply for each good

$$\forall i \in \{1, \ldots, N\}, \qquad Q_i = \sum_{j=1}^{N} Z_{ij} + \sum_{A \in \mathcal{C}} C_{iA}$$

back

Appendix: Taxes Calibration

Rates proportional to emission intensity per € produced or per € of oil consumed

Tax proceeds cover emissions cost, given price of tCO_2e

Easy for final consumption tax:

$$\sum_{i=1}^{N} \kappa_{iA} P_i C_{iA} = P_{\mathsf{CO_2}} GHG_A^H$$

implies

$$\kappa_{iA} = \begin{cases} 0 \text{ if } i \notin \text{oil refining} \\ \kappa_A = \frac{P_{\mathsf{CO_2}} GHG_A^H}{\sum_{i \in \text{oil}} P_i C_{iA}} \text{ if } i \in \text{oil refining} \end{cases}$$

back

Appendix: Taxes Calibration

Issue for production side:
Distinguish between emissions due to energy use and to production process

Approximate solution:
Separate between CO_2 and non-CO_2 GHG:

- $GHG_i = CO_{2i} + \widetilde{GHG}_i$
- Works for most sectors (agriculture, sewerage)
- Exceptions for cement, steel $\Rightarrow$ all GHG attributed to production process (impose $CO_{2i} = 0$)

Production tax

$$\tau_i Q_i P_i = P_{\mathrm{CO_2}} \widetilde{GHG}_i \quad \Rightarrow \quad \tau_i = P_{\mathrm{CO_2}} \frac{\widetilde{GHG}_i}{Q_i P_i}$$

Intermediate consumption tax

$$\zeta_{ji} = \begin{cases} 0 \text{ if } j \notin \text{oil refining} \\ \zeta_i = \frac{P_{CO_2} CO_{2i}}{\sum_{j \in oil} P_j Z_{ji}} \text{ if } j \in \text{oil refining} \end{cases}$$

back

Appendix: Financial Transfers & Trade Balance

Households' budget contraint and consumption:

- Budget constraint relaxed in taxed countries
- Consumption increases less than labour income and tax transfers in taxed countries
- Consumption increases more in non-taxed countries (between 0.03 and 0.2 ppt more)
- Due to risk-sharing: non-taxed countries receive a financial transfer from taxed countries

Trade balance mirrors this:

- Taxed countries import less than they export
- Non-taxed countries import more than they export
- Balanced by financial transfers

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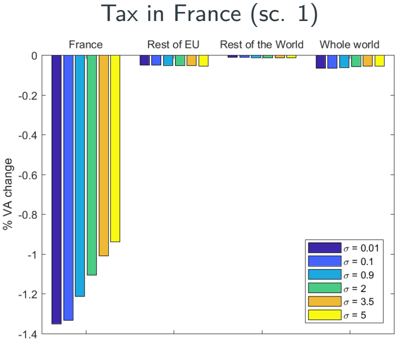
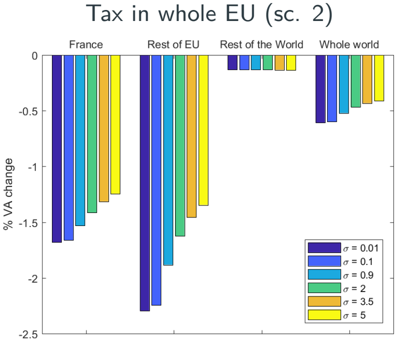
Robustness: Substitution Across Energy Types





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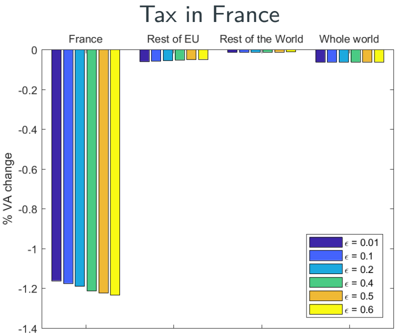
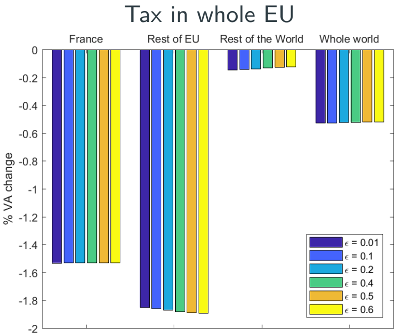
Robustness: Substitution Across Intermediate Inputs





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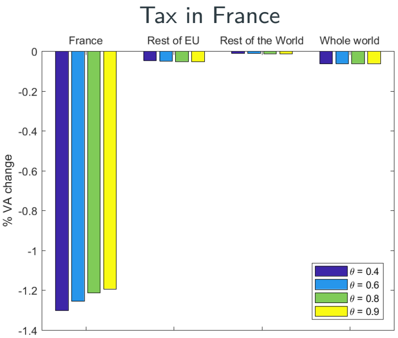
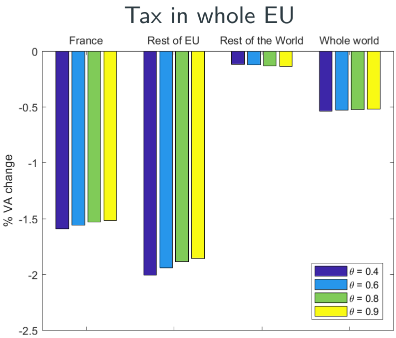
Robustness: Substitution Across Labour, Intermediate Inputs and Energy





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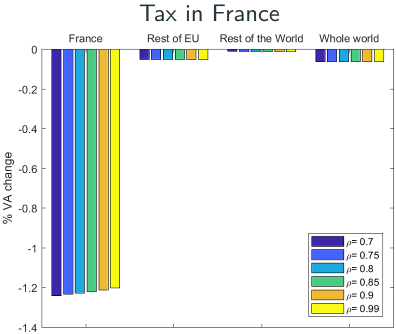
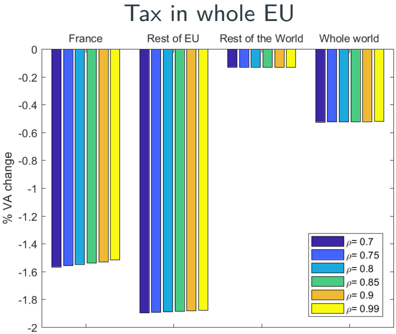
Robustness: Substitution Across Final Consumption Goods





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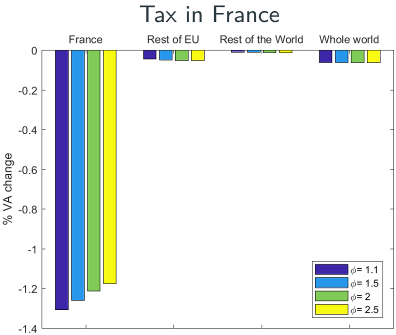
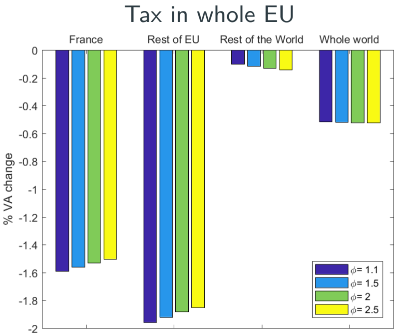
Robustness: Relative Risk Aversion





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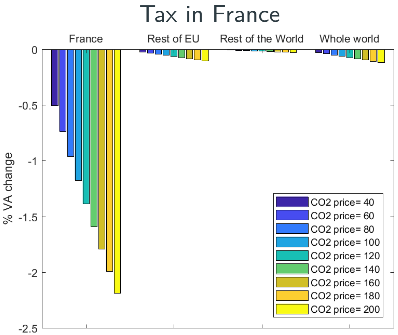
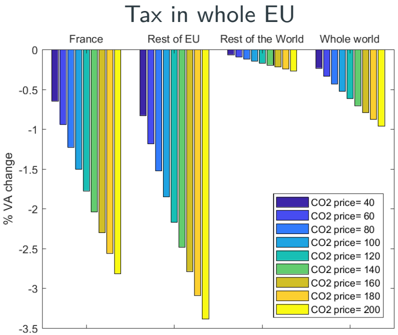
Robustness: Carbon Price





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Appendix: Production with Armington aggregator

Representative producer's program in each sector i in each country B:

$$\max_{L_{Bi}, Z_{Cij}} \pi_{Bi} = P_{Bi}(1-\tau_{Bi})Q_{Bi} - w_B L_{Bi} - \sum_C \sum_{j=1}^{N} P_{Cj}(1+\zeta_{BCji}) Z_{BCji}$$

where: $$Q_{Bi} = A_{Bi}\left(\mu_{Bi}^{\frac{1}{\theta}} L_{Bi}^{\frac{\theta-1}{\theta}} + \alpha_{BEi}^{\frac{1}{\theta}} E_{Bi}^{\frac{\theta-1}{\theta}} + \alpha_{BIi}^{\frac{1}{\theta}} I_{Bi}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$$

$$E_{Bi} = \left(\sum_{j=1}^{N_E}\left(\frac{\alpha_{Bji}}{\alpha_{BEi}}\right)^{\frac{1}{\sigma}} Z_{Bji}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$I_{Bi} = \left(\sum_{j=N_E+1}^{N}\left(\frac{\alpha_{Bji}}{\alpha_{BIi}}\right)^{\frac{1}{\epsilon}} Z_{Bji}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$Z_{Bji} = \left(\left(\frac{\alpha_{BBji}}{\alpha_{Bji}}\right)^{\frac{1}{\eta_E}} Z_{BBji}^{\frac{\eta_E-1}{\eta_E}} + \left(\frac{\alpha_{BMji}}{\alpha_{Bji}}\right)^{\frac{1}{\eta_E}}\left(\sum_{C\neq B}\left(\frac{\alpha_{BCji}}{\alpha_{BMji}}\right)^{\frac{1}{\xi_E}} Z_{BCji}^{\frac{\xi_E(\eta_E-1)}{(\xi_E-1)\eta_E}}\right)\right)^{\frac{\eta_E}{\eta_E-1}},$$

$\forall j \in \{1, \ldots, N_E\}$

$$Z_{Bji} = \left(\left(\frac{\alpha_{BBji}}{\alpha_{Bji}}\right)^{\frac{1}{\eta_I}} Z_{BBji}^{\frac{\eta_I-1}{\eta_I}} + \left(\frac{\alpha_{BMji}}{\alpha_{Bji}}\right)^{\frac{1}{\eta_I}}\left(\sum_{C\neq B}\left(\frac{\alpha_{BCji}}{\alpha_{BMji}}\right)^{\frac{1}{\xi_I}} Z_{BCji}^{\frac{\xi_I(\eta_I-1)}{(\xi_I-1)\eta_I}}\right)\right)^{\frac{\eta_I}{\eta_I-1}},$$

$\forall j \in \{N_E+1, \ldots, N\}$

Appendix: Production with Armington aggregator

Parameters defined such that:

$$\alpha_{BMji} = \sum_{C \neq B} \alpha_{BCji}$$

$$\alpha_{Bji} = \alpha_{BMji} + \alpha_{BBji}$$

$$\alpha_{Ei} = \sum_{j=1}^{N_E} \alpha_{ji}$$

$$\alpha_{Ii} = \sum_{j=N_E+1}^{N} \alpha_{ji}$$

$$1 = \alpha_{Ei} + \alpha_{Ii} + \mu_i$$

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