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Socially efficient discounting under ambiguity aversion

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Introduction

- Which discount rate should be used for the distant future? What is the socially efficient level of long-termism?

- Applications:
  - Nuclear wastes, pension systems, public debt,…
  - Copenhagen Consensus and Nordhaus versus Stern Review;
  - Is it socially responsible to invest in biofuel tech?

- Investment: Get 10 000 € in 2209 for each euro invested today.

$$IRR = 4.7\%$$

Copenhagen Consensus

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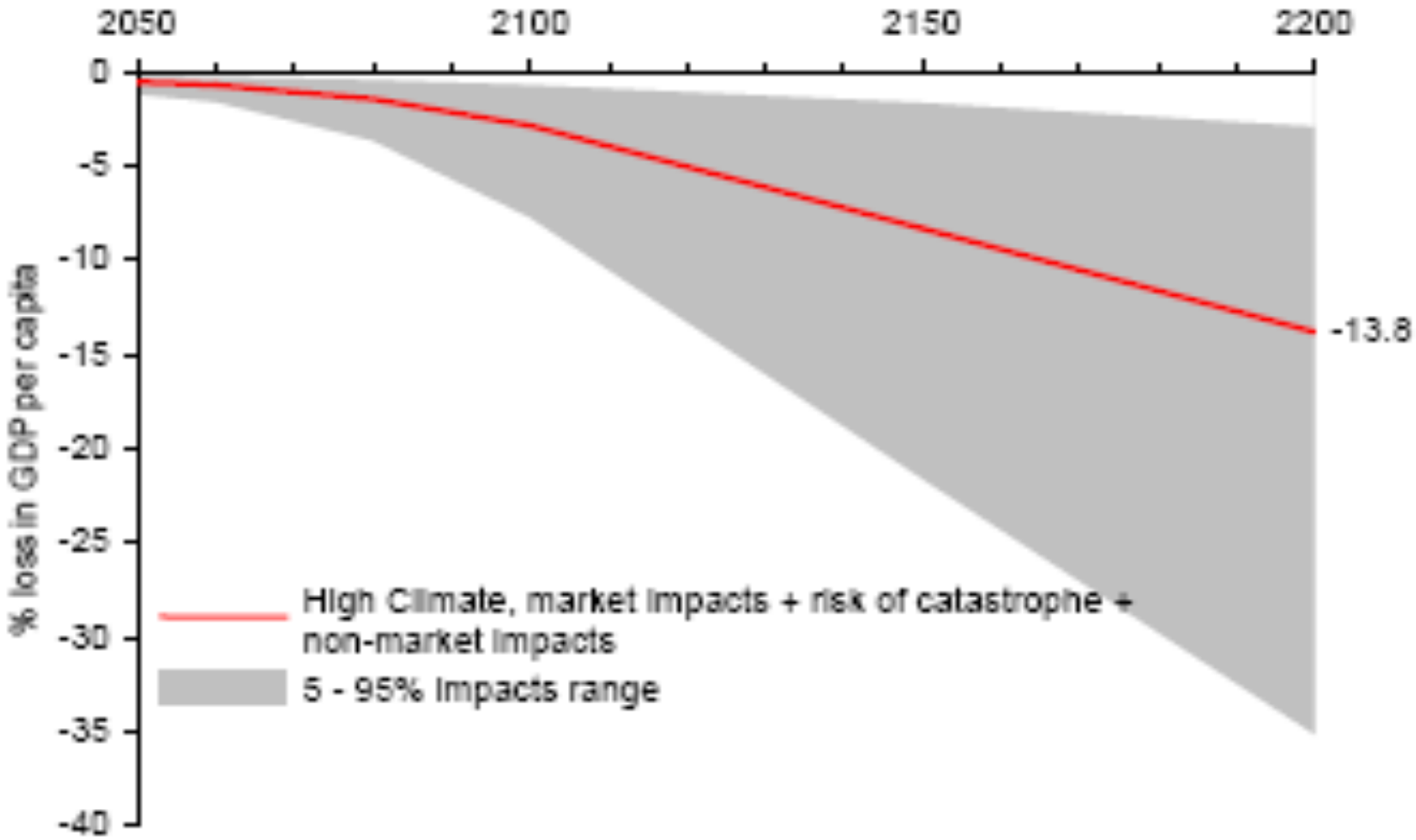
Project rating		Challenge	Opportunity
Very Good	1	Diseases	Control of HIV/AIDS
	2	Malnutrition	Providing micro nutrients
	3	Subsidies and Trade	Trade liberalisation
	4	Diseases	Control of malaria
Good	5	Malnutrition	Development of new agricultural technologies
	6	Sanitation & Water	Small-scale water technology for livelihoods
	7	Sanitation & Water	Community-managed water supply and sanitation
	8	Sanitation & Water	Research on water productivity in food production
	9	Government	Lowering the cost of starting a new business
Fair	10	Migration	Lowering barriers to migration for skilled workers
	11	Malnutrition	Improving infant and child nutrition
	12	Malnutrition	Reducing the prevalence of low birth weight
	13	Diseases	Scaled-up basic health services
Bad	14	Migration	Guest worker programmes for the unskilled
	15	Climate	Optimal carbon tax
	16	Climate	The Kyoto Protocol
	17	Climate	Value-at-risk carbon tax

Note to table: Some of the proposals were not ranked (see text below)

Stern Review



Stern versus Norhaus

- Using a discount rate of 1.4%, Stern concludes that it would be socially efficient to set a price of 85 $/tCO2, which corresponds to 25 c/liter of oil.

- Using a discount rate of 5%, Nordhaus sets a price of 8 $/ tCO2.

Abatement costs: McKinsey Study

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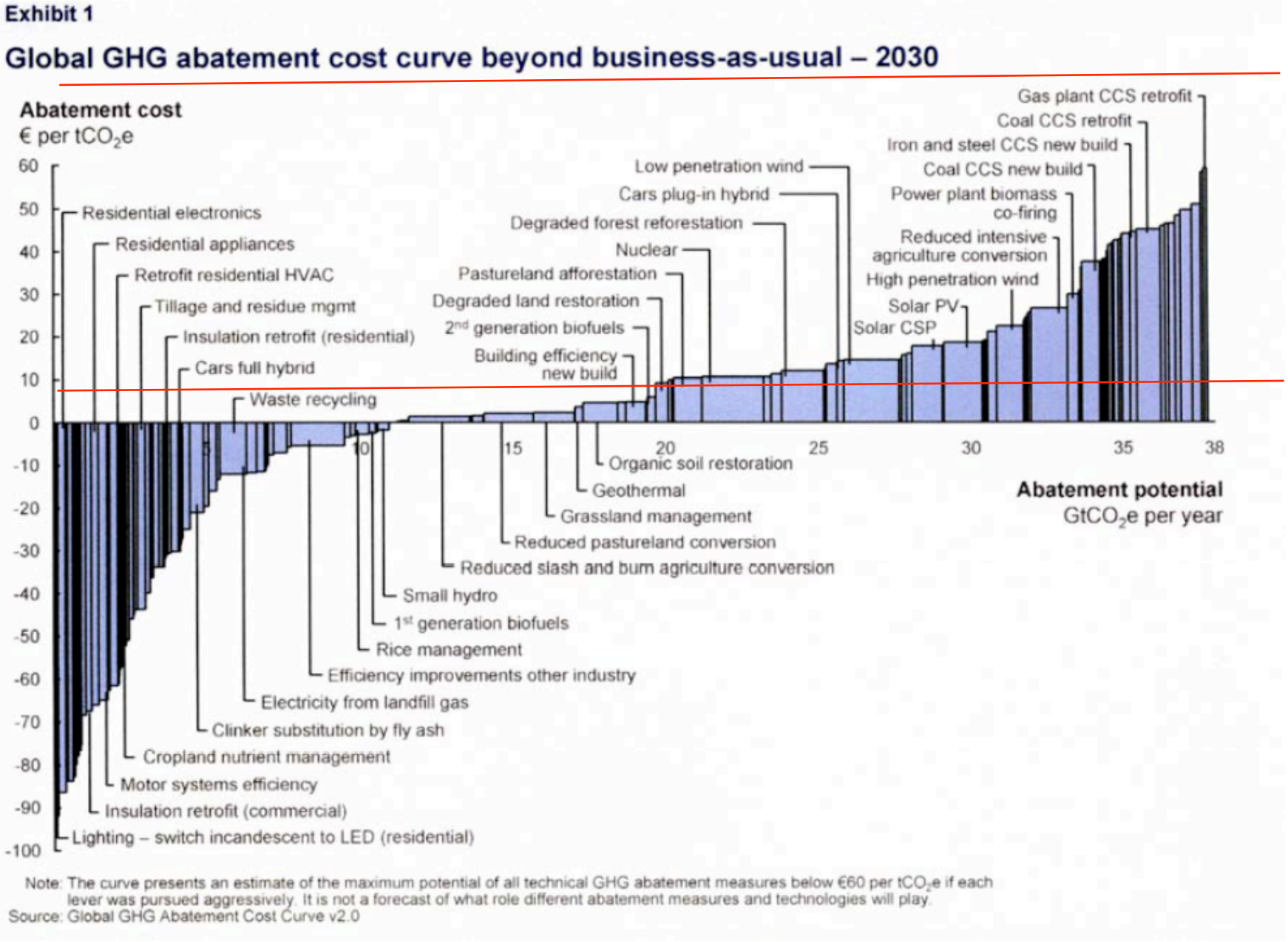


Stern:
85€/tC02

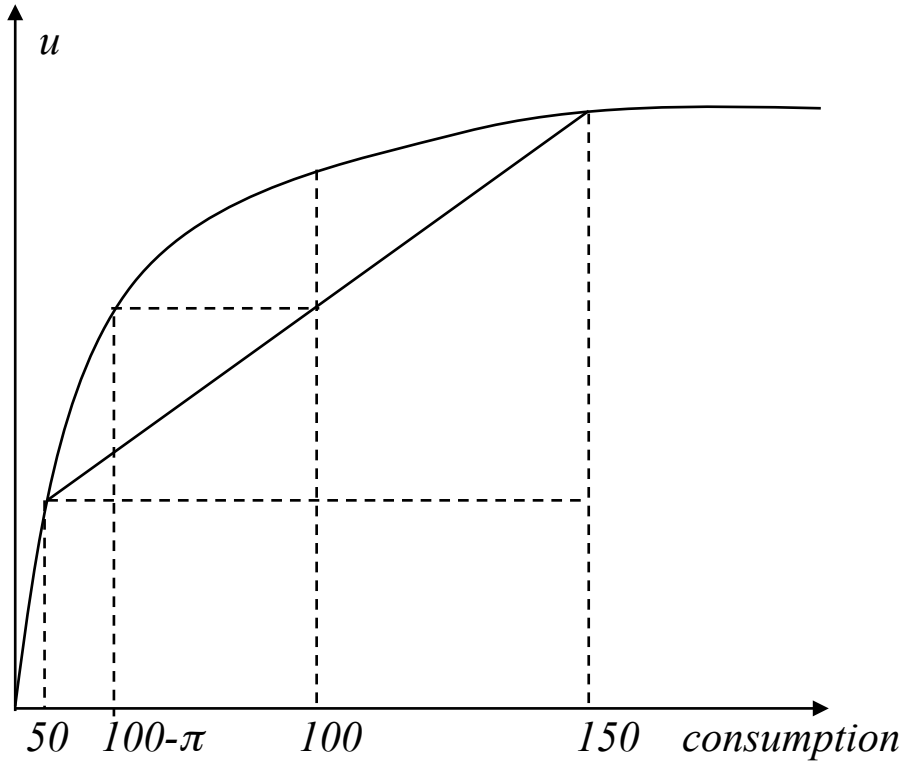
Nordhaus:
8€/tC02

Why do we discount in economics?

- The arbitrage argument. But no interest rate for the distant future.
- The impatience argument.
- The wealth effect:
  - One will be wealthier in the future: We consume 50 times more goods and services than in the early XIXth century;
  - One is averse to consumption inequalities over time.

Inequality aversion

- Consider two economies:
  - Economy A: Half of the population consumes 150, half of the population consume 50.
  - Economy B: Everyone consumes 100-π.
- What is the value of π which makes us indifferent to live in A or B, under the veil of ignorance?
- $$SWF = \frac{1}{2}u(150) + \frac{1}{2}u(50) = u(100 - \pi)$$



Estimate your own degree of inequality aversion γ

- Under the veil of ignorance, you are indifferent to live in Society A *(50, 150)* or in Society B *(100−π).*

Inequality aversion γ	*Inequality premium π*
0.5	*6.7*
1	*13.4*
4	*37.8*
10	*46.0*

The discounted utility model

- The planner wants to maximize

$$SWF = u(c_0) + e^{-\delta} u(c_1) + e^{-2\delta} u(c_2) + \ldots$$

- Consider the minimum return on an investment that transfers consumption from the present to the future.
- What is the minimum return r on this investment that makes you willing to implement it?
- Ramsey rule: $r_t = \delta + \gamma g_t$
- $\gamma=2$, $g=2\%$, $\delta=0\%$ implies $r=4\%$.

A third determinant of the discount rate

- Wealth effect: Why should we sacrifice our welfare in favour of people much wealthier than us?
- But what do we know about about future generations' achievement level?
- There is a lot of uncertainty about that.
- Effect on the socially efficient discount rate, or optimal sacrifice?
- Link with the notion of precautionary saving, and of prudence.

Extended Ramsey rule

$$r_t = \delta + \gamma\mu - 0.5\gamma^2\sigma^2$$

- Underlying assumptions:
  - Multiplying mean wealth does not affect π/w;
  - Risk on the growth rate is normally distributed without serial correlation.
- Calibration: $\gamma=2$, $\sigma=2\%$ implies precautionary effect= -0.08% !

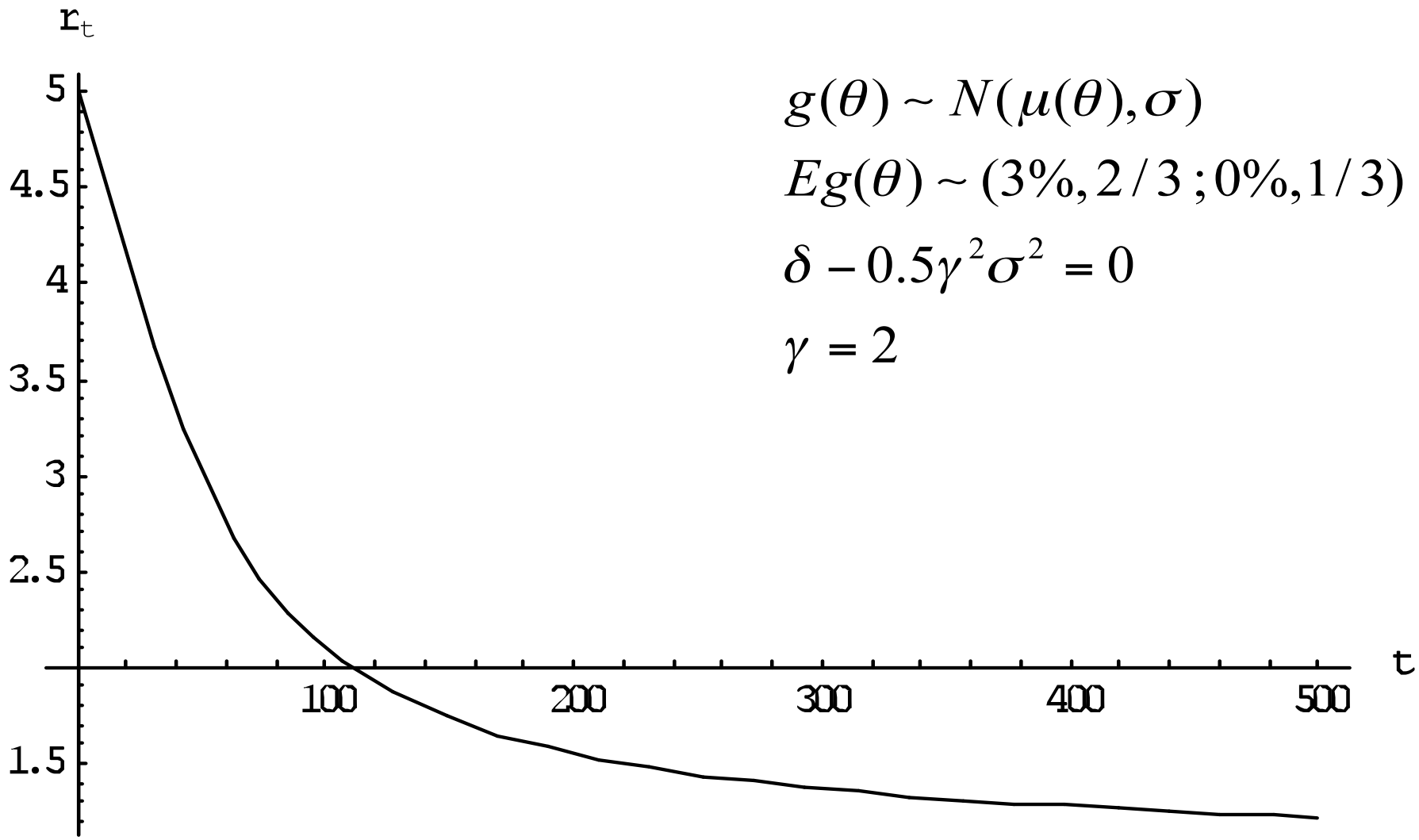
The slope of the yield curve

- Is it socially efficient to reduce the discount rate for longer time horizons?
- A potential argument:
  - more distant futures are more uncertain.
  - Under prudence, it has a negative effect on the discount rate.
  - But this is potentially counterbalanced by the fact that more distant generations are also wealthier on average.
- Comparing the degrees of riskiness of GDP per capita for different horizons.
- Serial correlations in growth rates are important.

Related literature

- The theory of the term structure of interest rates: Vasicek (1977), Cox, Ingersoll and Ross (1985),…
- Weitzman (1998, 2001), Groom, Koundouri, Panipoulou and Pantelides, (2007): risk neutral representative agent, serially correlated productivity of capital. STRONG HORIZON EFFECT
- Gollier (2002a, 2002b): risk-averse representative agent, i.i.d. growth process. WEAK HORIZON EFFECT
- Weitzman (2008) + Gollier (2007, 2008) : risk-averse representative agent, serial correlation in growth rates.

Calibration in a model of parametric uncertainty
FAT TAILS



$$g(\theta) \sim N(\mu(\theta), \sigma)$$

$$Eg(\theta) \sim (3\%, 2/3\,; 0\%, 1/3)$$

$$\delta - 0.5\gamma^2\sigma^2 = 0$$

$$\gamma = 2$$

A simple version of Ellsberg game

- An urn contains 100 balls, either black or white. Prize=100,000 €.
- In the unambiguous urn, the proportion of black balls is exactly 50%.
  - On which color do you want to bet?
  - How much are you ready to pay to play this game?
- In the ambiguous urn, the proportion of black balls is unknown.
  - On which color do you want to bet?
  - How much are you ready to pay to play this game?

Actuaries' reaction to ambiguity

- Context 1: Sure probability of 2% to pay an indemnity of $100,000. Commercial premium?

- Context 2: Unknown probability p to pay an indemnity of $100,000. Expert A says $p=1\%$, whereas expert B says $p=3\%$. Commercial premium?

Scenario	Statistics	Context 1	Context 2
Pollution	Mean premium/AV	1.35	1.88
Earthquake	Mean premium/AV	1.43	2.01

Source: Cabantous (2006)

Ambiguous growth and ambiguity aversion

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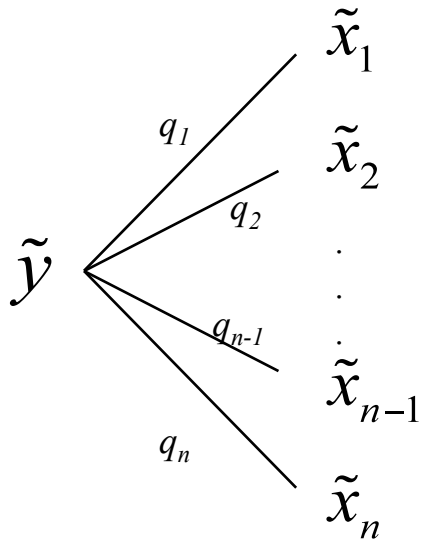
- Two new ingredients:
  - Ambiguity on the μ and σ^2 for the next 200 years.
  - People are ambiguity-averse. The following two situations are *not* equivalent:
    - The economy will grow at a rate of 2% with probability ½;
    - The economy will grow at a rate of 2% with an unknown probability with mean ½.
- This paper: Role of ambiguity and ambiguity aversion on
  - The term structure of equilibrium interest rates;
  - The term structure of the socially efficient discount rates.
- Conjecture: Ambiguity aversion should reduce the discount/interest rate.

Ambiguity

- The payoff x of the action is risky and uncertain.
- Parameter uncertainty: the distribution of x depends upon a parameter θ which can take value $\theta=1,\ldots,n$, respectively with probability $(q_1,\ldots,q_n)$.
- Distribution of x conditional to θ : $\tilde{x}_\theta$.



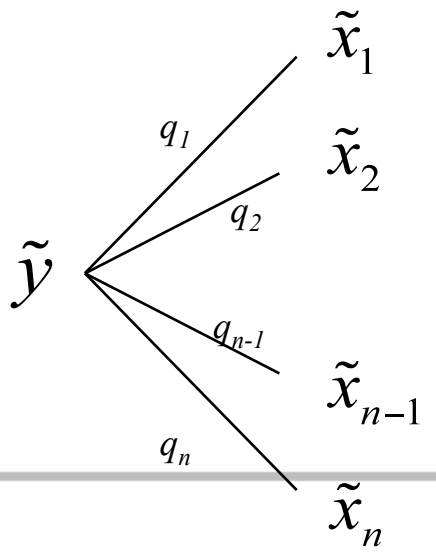
Expected utility

- The choice problem under expected utility is to maximize

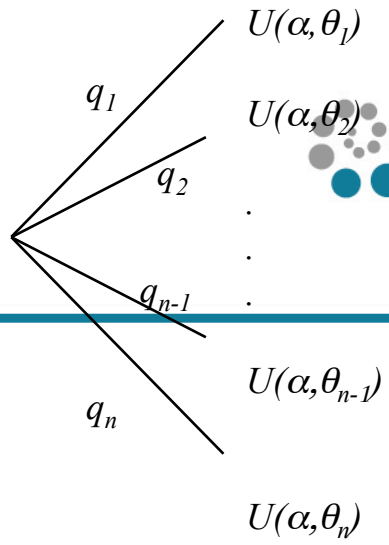
$$EU(\alpha) = \sum_{\theta=1}^{n} q_\theta U(\alpha,\theta) = \sum_{\theta=1}^{n} q_\theta Eu(w_0 + \alpha\tilde{x}_\theta) = Eu(w_0 + \alpha\tilde{y})$$

- The agent is neutral to any mean-preserving spread in the probability space.

- Indifference between the two urns.



Ambiguity aversion



- The agent is averse to any mean-preserving spread in the probability/U space.

- Klibanoff, Marinacci and Mukerji (2005): The preference functional V is a «certainty equivalent»:

$$V(f) = \phi^{-1}\left(\sum_{i=1}^{n} q_i \phi(U(f,\theta_i))\right) = \phi^{-1}\left(\sum_{i=1}^{n} q_i \phi(Eu(w_0 + f(x)))\right)$$

- The degree of concavity of function ϕ $(-\phi''/\phi')$ is a measure of ambiguity aversion (Pratt (1964)).

- Gilboa and Schmeidler (1989): the « maxmin » model is a special case with $\phi(U)=-\eta^{-1}exp(-\eta U)$, η tending to infinity.

An analytical solution: Power –power normal-normal case

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- Specification:

$$\ln c_t \,|\, \theta \sim N(\ln c_0 + \theta t, \sigma^2 t) \qquad (d \ln c_t = \theta dt + \sigma dz)$$

$$\theta \sim N(\mu, \sigma_0^2)$$

$$u(c) = c^{1-\gamma} / (1-\gamma)$$

$$\phi(V) = V^{1-\eta} / (1-\eta) \qquad \text{(when } \gamma \text{ is smaller than unity)}$$

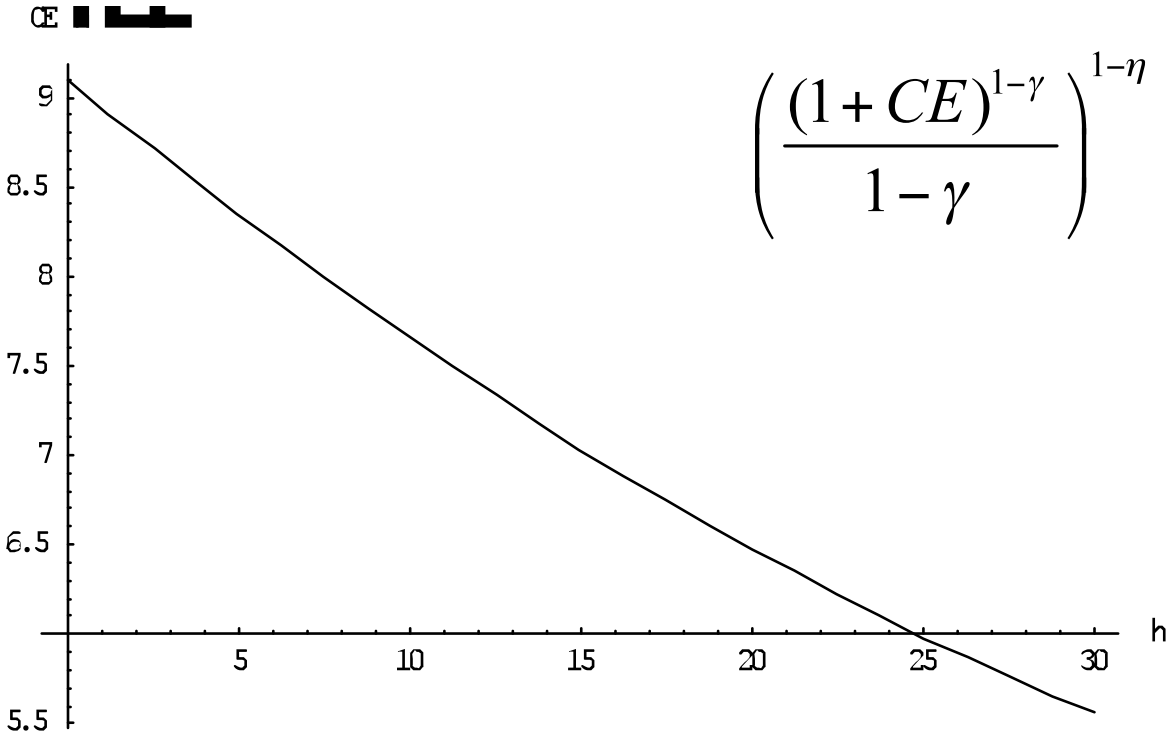
- Solution:

$$r_t = \delta + \gamma\mu - 0.5\gamma^2(\sigma^2 + \sigma_0^2 t) - 0.5\eta\left|1-\gamma^2\right|\sigma_0^2 t$$

Evaluate your own CRAA

- *Suppose that the growth rate in the next 20 years is either 20% with prob θ, or 0% with prob 1−θ. Suppose that θ is uniformly distributed on [0,1].*

- *What is the certainty equivalent (CE) growth rate?*



$$\left(\frac{(1+CE)^{1-\gamma}}{1-\gamma}\right)^{1-\eta} = \int_0^1 \left(\theta \frac{1.2^{1-\gamma}}{1-\gamma} + (1-\theta)\right)^{1-\eta} d\theta$$

$\gamma=2$

Numerical illustration

$$\ln c_t \sim N(\ln c_0 + \theta t, \sigma^2 t)$$

$$\theta \sim N(\mu, \sigma_0^2)$$

$$u(c) = c^{1-\gamma}/(1-\gamma)$$

$$\phi(V) = V^{1-\eta}/(1-\eta)$$

- Power-power, normal-normal.
  - δ=2%; γ=2, μ=2%, σ=2% implies $r_t = 5.88\% ? 3\alpha_0^2 t$ 1 + P/2=
  - σ_0=1%.

t	$\eta = 0$	$\eta = 5$	$\eta = 10$
10	5.58%	4.83%	4.08%
30	4.98%	2.73%	0.48%

An AR(1) process for log consumption with an ambiguous long-term trend

$$\ln c_{t+1} = \ln c_t + x_t$$

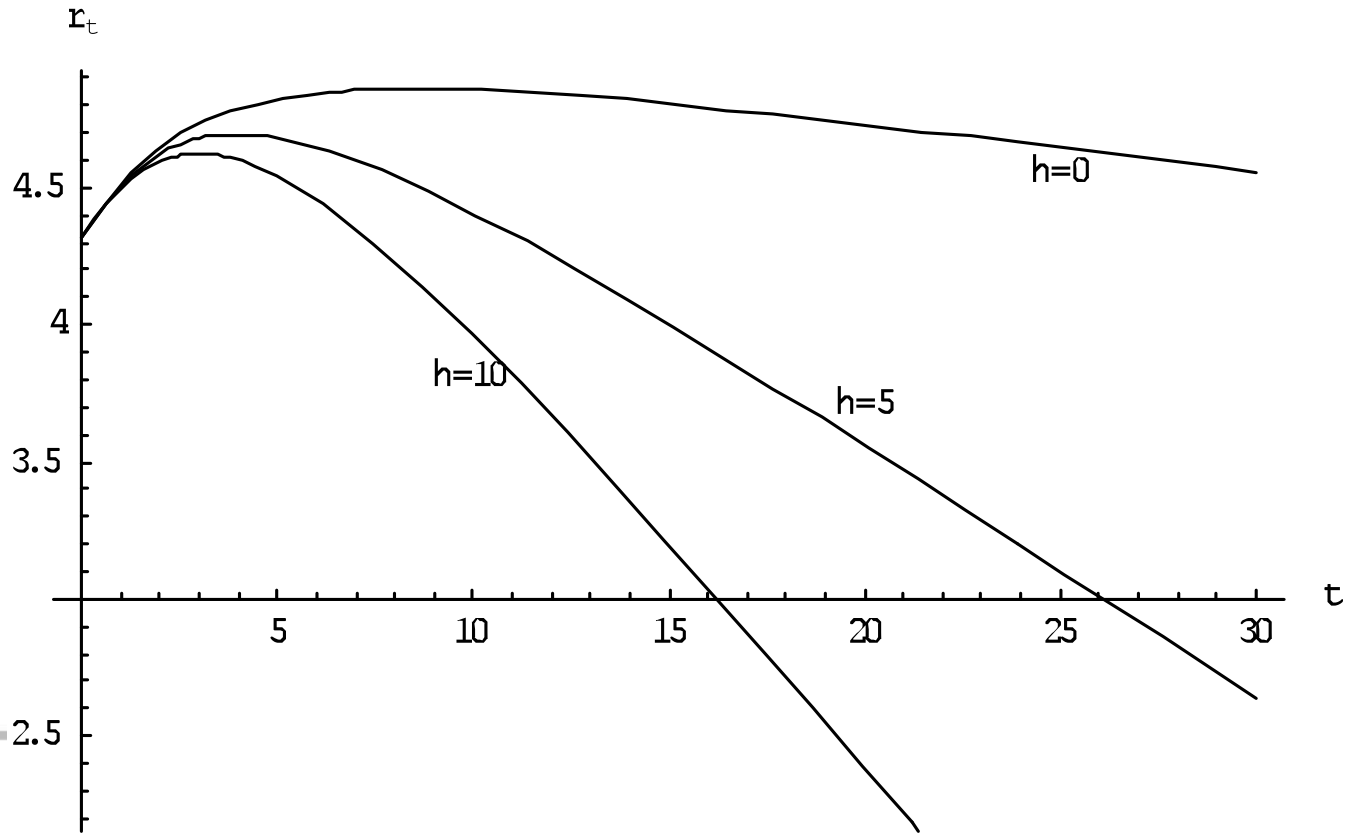
$$x_t = \psi x_{t-1} + (1-\psi)\Omega + \pi_t$$

$$\pi_t \sim N(0, \alpha^2) \quad \pi \perp \Omega$$

$$\Omega \sim N(\Omega_0, \alpha_0^2)$$

$$\Omega = 2\%,\ \Lambda = 2,\ \Omega_0 = 2\%,\ \alpha = 2\%,\ \alpha_0 = 1\%, \text{and } x_{-1} = 1\%$$

$$\psi = 0.7 \text{ year}^{-1}$$



Conclusion

- The growth process is ambiguous.
- Human beings are ambiguity-averse.
- These two ingredients raises the willingness to save, and reduces interest rates.
- Many projects in the agenda of research:
  - Recursive approach;
  - Dynamic portfolio choices;
  - Conditions for decreasing risk/uncertainty aversion;
  - Aggregation of preferences and beliefs;
  - …