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# “A Contribution Margin Approach to Imperfect Competition”

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# A Contribution Margin Approach to Imperfect Competition

*Sufficient statistics for pass-through, concentration and market expansion*

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### Abstract

This paper studies symmetric oligopoly when marginal cost is not constant. In this environment, the Lerner index is not a sufficient measure of profitability: firms' relevant margin is the contribution margin, defined relative to variable cost. The paper introduces a contribution margin index and relates it to the Lerner index through a profitability factor. This factor captures the wedge between local markups and average profitability. It also governs comparative statics for cost pass-through, market expansion, entry, profits, and concentration. The framework nests the constant marginal cost benchmark and shows how cost curvature changes the interpretation of standard oligopoly statistics.

**JEL**: D21, H22, H32, L13, L51

**Keywords**: Pass-through, Cost Structure, Contribution Margin, Oligopoly.

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# 1 Introduction

Most models of imperfect competition treat marginal cost as constant. This assumption is analytically convenient, but it is not innocuous. When marginal cost varies with output, the Lerner index remains a local measure of market power, but it no longer summarizes firms' profitability. A firm may have a high markup over marginal cost and yet a low margin over variable cost, or conversely. This distinction matters for pass-through, entry, concentration, and the effects of market expansion.

This paper develops a contribution-margin approach to symmetric oligopoly when marginal cost is not constant. The central object is the contribution margin index, defined as the margin over variable cost per unit of revenue. Under constant marginal cost, this index coincides with the Lerner index. In general, however, the two objects differ. Their ratio is summarized by a profitability factor, denoted by $\lambda$, which captures the wedge between local markups and average profitability. This factor equals one under constant marginal cost, is below one when average variable cost is locally increasing, and is above one when average variable cost is locally decreasing.

Relative to the pass-through literature, the paper shifts the sufficient statistic from the Lerner margin to the contribution margin. Existing formulas usually characterize how prices respond to marginal cost shocks. This paper instead emphasizes that, when marginal cost is not constant, the same local markup may correspond to different levels of profitability depending on the shape of variable cost. The profitability factor $\lambda$ summarizes this difference. It links pass-through, profitability, market expansion, and endogenous entry within a single oligopoly framework.

The contribution of the paper is threefold. First, it shows how equilibrium profitability in a broad class of symmetric oligopoly models can be characterized by the contribution margin index rather than by the Lerner index alone. The framework treats demand and cost symmetrically and expresses equilibrium outcomes in terms of a small set of elasticities and curvature measures. This provides a tractable way to separate the role of demand curvature, cost curvature, and the mode of competition.

Second, the paper derives pass-through formulas that explicitly incorporate cost curvature. These formulas are consistent with existing sufficient statistic approaches to pass-through, but they clarify how non-constant marginal cost changes the transmission of additive and multiplicative cost shocks. Convex costs dampen pass-through, while concave costs amplify it. This effect can be interpreted by comparing the actual industry with a "ghost industry" that has the same equilibrium price, output, and marginal cost, but locally constant marginal cost. The difference between the two industries isolates the role of cost curvature.

Third, the paper uses the same objects to study market expansion, entry, and concentration. The framework shows how short-run pass-through statistics also discipline long-run comparative statics when the number of firms adjusts. In particular, market expansion or cost shocks may increase or decrease prices, profits, and concentration depending on the interaction between demand curvature, cost curvature, and entry. The analysis nests several limit cases, including price-increasing competition and negative long-run pass-through, as special cases of a unified structure.

The approach builds on a companion paper, Bontems (2026), which develops the contribution-margin approach for monopoly and monopolistic competition.[1] The present paper extends the analysis to symmetric oligopoly under a general mode of competition. This includes price and quantity competition, variety preferences, and endogenous market structure. The objective is not to replace fully structural models, but to provide a transparent benchmark in which the effects of demand curvature, cost curvature, and competition can be identified through sufficient statistics.

The rest of the paper is organized as follows. The last part of this section discusses the related literature. Section 2 presents the model, including the demand system, the cost structure, and the mode of competition. Section 3 introduces the contribution margin index and relates it to the Lerner index. Section 4 studies cost pass-through. Section 5 analyzes market size, entry, concentration, and long-run pass-through. Section 6 discusses empirical implications. Section 7 concludes.

**Related literature.** The paper is related first to the sufficient-statistic approach to pass-through and imperfect competition. Weyl and Fabinger (2013) show that pass-through is a central object for the analysis of market power, tax incidence, and welfare under imperfect competition. Their framework provides a general way to connect demand curvature, conduct, and the incidence of cost shocks. This approach builds on, and systematizes, an earlier literature on tax incidence and pass-through in oligopoly, including Delipalla and Keen (1992), Anderson et al. (2001a), and Anderson et al. (2001b). More recent contributions further connect pass-through, market power, and welfare in oligopoly environments, including Häckner and Herzing (2016), Adachi and Fabinger (2022) and Kroft et al. (2021). The present paper follows this tradition by working with general demand, general cost, and a general mode of competition. It differs by focusing on the contribution margin rather than on the Lerner index alone. This distinction is immaterial under constant marginal cost, but becomes

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[1] This companion paper has been submitted to another journal, so it is quoted anonymously here.

central when marginal cost varies with output.

The closest paper is Ritz (2024), who studies the relationship between competition and pass-through when marginal costs are not constant. Ritz shows that the standard intuition according to which more competition raises pass-through can fail under convex costs: as competition expands aggregate output, firms move along steeper marginal cost schedules, which may reduce pass-through. The present paper shares the view that cost curvature is essential, but differs in its object and scope. Ritz focuses on how cost convexity modifies the effect of competition on pass-through. This paper uses cost curvature to build a contribution-margin framework that links local market power, average profitability, cost pass-through, market expansion, and endogenous entry. In this sense, the two approaches are complementary: Ritz identifies how non-constant marginal cost changes the competition–pass-through relationship, while the present paper studies how the wedge between the Lerner index and the contribution margin affects both short-run and long-run oligopoly outcomes.

The paper also relates to recent empirical work on conduct and cost structure. In differentiated-product industries, conduct tests and welfare calculations often depend on assumptions about marginal cost. If marginal cost is incorrectly assumed to be constant, the inferred conduct parameter or the measured welfare effects of policy may be biased. Duarte et al. (2025) extend conduct tests to allow for scale economies and show, in the context of the U.S. automobile market, that cost curvature can matter for inference on competition and trade policy. Kwon et al. (2023) emphasize the role of economies of scale in the long-run evolution of corporate concentration. The present paper provides a theoretical counterpart to this empirical concern by identifying the sufficient statistics through which cost curvature affects profitability, pass-through, and concentration.

Finally, the paper contributes to the literature on endogenous market structure and market size. Bertoletti and Etro (2016) and Mrázová and Neary (2017) study entry and market outcomes under general demand conditions while maintaining constant marginal cost. Parenti et al. (2017) analyzes endogenous market structure in oligopoly environments with rich demand-side mechanisms. The present paper instead keeps the demand system symmetric and quasi-linear, but allows for general variable cost. This makes it possible to isolate the role of cost curvature in determining how market expansion affects prices, profits, entry, and concentration. The contribution is therefore complementary to demand-based explanations of market structure: it shows how the cost side alone can create a wedge between market power, profitability, and concentration.

## 2 The model

Consider that there are $n \geq 1$ single-product and symmetric firms in the industry, indexed by $i = 1...n$. The market structure is assumed oligopolistic with a general mode of competition along the lines of Weyl and Fabinger (2013) and Adachi and Fabinger (2022) that generalize the approach of Genesove and Mullin (1998) and Bresnahan (1989). I also follow the partial equilibrium tradition in assuming that all goods outside the industry are perfectly competitively supplied. In addition, there are no externalities in the economy and information is complete.

**Demand side.** Even if goods may be distinct in the consumers' eye, the demand system is assumed to be fully symmetric. There are $L$ identical consumers that share the same preferences and thus I abstract from distributional considerations. I denote the individual symmetric demand for variety $i$ as $x_i = D_i(p_1, ..., p_n)$ for all $i$, where $D_i(.)$ is twice continuously differentiable with $\partial D_i/\partial p_i < 0$ and where $p_j$ is the price of variety $j$ for any $j$. The demand addressed to firm/variety $i$ is then $q_i = Lx_i = LD_i(p_1, ..., p_n)$ for all $i$. I also denote the individual symmetric inverse demand for variety $i$ as $p_i = P_i(x_1, ..., x_n)$ for all $i$, where $P_i(.)$ is twice continuously differentiable with $\partial P_i/\partial x_i < 0$. Furthermore, I consider demand systems that satisfy the following assumption.

**Assumption 1.** *For all $i$, the own-price or own-quantity effect strictly dominates the cross-price or cross-quantity effects:*

$$\sum_j \frac{\partial D_i}{\partial p_j} < 0 \text{ and } \sum_j \frac{\partial P_i}{\partial x_j} < 0.$$

Denote the industry production/total sales as $Q = \sum_i q_i$. Moreover, denote $\varepsilon_d$ as the *elasticity of industry demand.*[2] To derive this elasticity, consider a symmetric equilibrium with price $p_i = p$, production per variety $q_i = q = Q/n$ and consumption per head and per variety as $x_i = x = Q/nL = q/L$ for all $i$. Define the industry demand as $Q = nLD(p, n)$ where $D(p, n) \equiv D_i(p, ..., p)$ for any $i$. Define also the inverse of $D(p, n)$ with respect to price as the industry inverse demand, i.e. $p = P(x, n)$ where $P(x, n) \equiv P_i(x, ..., x)$ for any $i$.

For the analysis to follow, it is convenient to define the elasticity of industry demand as a function of $x$ by:

$$\varepsilon_d(x, n) = -\frac{P(x, n)}{xP_x(x, n)} > 0$$

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[2] This is also referred to the price elasticity of market-wide demand in the literature.

where $P_x(x,n) = \sum_j \frac{\partial P_i}{\partial x_j} = \left(\sum_j \frac{\partial D_i}{\partial p_j}\right)^{-1} < 0$ is the slope of the industry inverse demand.[3] The fact that the industry inverse demand is downward sloping follows directly from the Assumption 1 made above on the demand system.

Furthermore, I consider the elasticity of the slope $P_x(x,n)$ as a measure of curvature for the industry inverse demand:

$$\rho_d(x,n) = -\frac{xP_{xx}(x,n)}{P_x(x,n)}$$

where $P_{xx}(x,n) = \sum_{j,k} \frac{\partial^2 P_i}{\partial x_j \partial x_k}$.

As is common in the literature (Seade, 1980a; Delipalla and Keen, 1992; and Kroft et al., 2021), I ignore the integer constraint on $n$ and treat it as a continuous variable for simplicity. I also impose the following assumption on all demand systems considered.

**Assumption 2.** *Holding the consumption per variety $x$ constant, adding new varieties (weakly) reduces the inverse demand for each variety, i.e. $P_n(x,n) \leq 0$. Holding price constant, adding new varieties (weakly) reduces the demand for each variety, i.e. $D_n(p,n) \leq 0$.*

This assumption holds for standard demand and inverse demand functions like the linear demand and inverse demand function, the demand system generated by additive preferences à la Dixit and Stiglitz (1977) or the Logit demand and inverse demand.[4]

When dealing with welfare, I will consider that the demand system originates from individual preferences represented by a quasi-linear utility function $U^n(x_1, ...x_n) + z$ where $z$ is the consumption of the outside good.[5] Considering a symmetric equilibrium, let us denote $U(x,n) \equiv U^n(x,...,x)$ as the utility level for a consumption $x$ per variety and with $n$ varieties. Marginal utility w.r.t. quantity is $U_x(x,n) = \sum_i \frac{\partial U^n}{\partial x_i} = nP(x,n)$.

Also, the marginal utility w.r.t. the number of varieties is given by:

$$\begin{aligned} U_n(x,n) &= \int_0^x \left[P(s,n) + nP_n(s,n)\right] ds \\ &= \frac{U(x,n)}{n} + \int_0^x nP_n(s,n)ds. \end{aligned} \tag{1}$$

using $U_x(x,n) = nP(x,n)$. I assume that preferences are characterized by love-for-variety but at a

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[3] Writing the price elasticity of industry demand as a function of $x$ stems from $P_x(x,n) = \left.\frac{1}{D_p(p,n)}\right|_{P(x,n)=p}$. By contrast, the elasticity of the industry inverse demand function $P(x,n)$ is $-xP_x(x,n)/P(x,n) = 1/\varepsilon_d(x,n)$. Starting with industry demand $D(p,n)$, the price elasticity of industry demand writes as $-pD_p(p,n)/D(p)$ and the elasticity of the industry inverse demand writes as a function of price according to $-D(p,n)/(pD_p(p,n))$.

[4] See Appendix A.

[5] The subutility function $U^n(.)$ is assumed thrice continuously differentiable, symmetric and strictly concave.

decreasing rate, i.e. $U$ is increasing concave in $n$. Observe from (1) that Assumption 2 is a sufficient condition for the latter property.

Quasi-linearity of preferences ensures that the consumers' surplus given by $LU^n - L\sum_i p_i x_i$ constitutes an adequate measure of consumers' welfare changes. Moreover, at a symmetric equilibrium, consumers' surplus writes:

$$CS(x,n) = LU(x,n) - LnxP(x,n). \tag{2}$$

Let us define

$$\mathcal{V} \equiv \frac{LU_n - pq}{p} \tag{3}$$

as a unit-free measure of the "variety effect" which captures the effect of a marginal change in the number of varieties $n$ on consumers' surplus *keeping price and consumption per variety fixed*, per unit of price.[6]

**Cost side.** I assume that all firms face the same cost function that depends only on each own output $q$. More precisely, let us denote $c(q)$ the variable cost (VC) function for any output $q$ with $c(0) = 0$. For the purpose of clarity and because the analysis that follows focus on variable profits at the equilibrium, let us assume that there are no fixed costs for the moment.[7]

Importantly, I also denote $w(q) \equiv c(q)/q$ as the average variable cost (AVC) function. In the following, I focus on all VC smooth functions that are (weakly) increasing in quantity, $c'(q) = w'(q)q + w(q) \geq 0$. I also allow for convex and concave cost functions. Indeed, concave costs represent important cases because, as suggested by Bykadorov et al. (2015), endogenous technology choice can yield situations where higher output fosters investment in marginal cost reduction which implies concave cost.[8] The empirical literature has also documented the possibility of concave costs for some manufacturing industries (Diewert and Wales, 1987; Friedlander *et al.*, 1983; and Ramey, 1991). While concave costs may appear less common than the standard convex specification, considering this case is analytically useful, as it generates sometimes paradoxical outcomes (e.g. on the impact of cost shocks) that enrich the theoretical comparisons with the benchmark case of convex costs.

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[6] Note that Kroft et al. (2021) define the variety effect as $LU_n - pq$. Here it is defined per unit of price to obtain a unit-free measure for convenience.

[7] Fixed costs will be reintroduced later when considering entry on the market.

[8] Consider the example provided by Bykadorov et al. (2015), where $c(q) \equiv \min_e e + \tilde{c}(e)q$ and $e \geq 0$ is a (continuous) technology choice that decreases $\tilde{c}(e)$ but a decreasing rate ($\tilde{c}''(e) > 0$). The FOC w.r.t $e$ is $\tilde{c}'(e) = -1/q$ which is sufficient thanks to convexity of $\tilde{c}$ and it defines the optimal tech choice $e(q)$. Totally differentiating the FOC indicates that more production fosters investment, i.e. $e'(q) > 0$. It follows that the resulting production cost $c(q)$ following the endogenous technology choice is such that $c'(q) = \tilde{c}(e(q))$ and $c''(q) = \tilde{c}'(e(q))e'(q) < 0$. Hence, $c(q)$ is concave.

Note that the AVC function $w(.)$ could also be interpreted as a price per unit to be paid to a supplier in order to be able to deliver the good to final consumers. This is particularly meaningful when the firm is a pure merchant that acts as an intermediary between some buyers and some sellers, as analyzed by e.g. Stahl (1988) and Hamilton et al. (2015). This merchant may benefit from exploiting market power not only downstream towards buyers but also upstream where $w'(q) > 0$ would correspond to an upwards sloping inverse supply from sellers.

Keeping this analogy with the merchant's problem, let us interpret the AVC function $w(.)$ *as if* it were an "inverse supply" so that the elasticity of "supply" is indexed with $s$ and defined by:[9]

$$\varepsilon_s(q) = \frac{w(q)}{qw'(q)}$$

Importantly, our assumption of increasing cost does not constrain the sign of $w'$. More precisely, the elasticity $\varepsilon_s$ is related to the elasticity $\varepsilon_c$ of VC according to $\varepsilon_c(q) = \frac{qc'(q)}{c(q)} = \frac{\varepsilon_s(q)+1}{\varepsilon_s(q)}$. An increasing cost ($\varepsilon_c \geq 0$) translates into either $\varepsilon_s(q) \geq 0$ or $\varepsilon_s(q) \leq -1$. In the limit case where the marginal cost is constant ($c'(q) = c > 0$ and thus $\varepsilon_c = 1$) then $w'(q) = 0$ and consequently $\varepsilon_s = \infty$. Let us denote this situation as *Constant AVC* (hereafter CAVC).

Consider next the case where the firm's equilibrium lies in a point where $\varepsilon_s(q) \geq 0$. This means that locally the AVC function is increasing ($w'(q) \geq 0$) and I denote this feature as *Increasing AVC* (hereafter IAVC). Clearly, IAVC is a necessary but non sufficient condition to the presence of decreasing returns to scale as long as there are fixed costs of production.

Now consider the opposite situation where at the equilibrium $\varepsilon_s(q) \leq -1$. This means that locally the AVC function is decreasing ($w'(q) \leq 0$) and I denote this feature as *Decreasing AVC* (hereafter DAVC). Note that DAVC implies that the average cost, that would include any fixed cost, is locally decreasing and thus it is a sufficient but not necessary condition to the presence of increasing returns to scale (IRS). While imperfect competition is often justified by the presence of IRS, this feature of the cost structure is compatible with either DAVC or IAVC.

Finally, I denote $\rho_s(q)$ the following measure of the curvature of the supply function (i.e. the elasticity of $w'(q)$):

$$\rho_s(q) = -\frac{qw''(q)}{w'(q)}$$

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[9] In the merchant case, the "supply" is a function $q(w)$ and it describes the quantity that the merchant's suppliers are ready to supply given the wholesale price $w$. The function $w(q)$ is the corresponding inverse supply.

Both $\varepsilon_s$ and $\rho_s$ vary with output unless the AVC function is iso-elastic (or CES) and writes as follows:[10]

$$w(q) = \beta q^{1/\varepsilon_s} \Rightarrow \rho_s = \frac{\varepsilon_s - 1}{\varepsilon_s} \leq 2.$$

The special case of Cobb-Douglas is obtained when $\varepsilon_s = -1$ and $\rho_s = 2$. Finally, note that $\varepsilon_s$ and $\rho_s$ can be related to their counterparts for the VC function $c(.)$. Indeed, it is immediate to check that $\varepsilon_c = qc'/c = (\varepsilon_s + 1)/\varepsilon_s > 0$ and $\rho_c = -qc''/c' = -(2 - \rho_s)/(1 + \varepsilon_s)$. In particular, the curvature measure $\rho_c$ of the cost function $c$ is negative if and only if $c$ is convex.

**Mode of competition.** In the analysis below, I follow the general approach developed by Weyl and Fabinger (2013) (see also Adachi and Fabinger (2022)) by not specifying a particular mode of interaction between firms. Instead, modelling the nature of competition using a general market conduct index $\theta$ allows to consider simultaneously a wide range of competition models in a very concise way, without specifying precisely the mode and the nature of competition (e.g. whether strategic variables are substitutes or complements).[11] More precisely, following Weyl and Fabinger (2013), a symmetric equilibrium is characterized by an elasticity-adjusted Lerner index $\frac{p-c'}{p}\varepsilon_d$ that is equal to $\theta$, which implies that all the first-order conditions at a symmetric equilibrium reduce to the following unique condition:

$$MP \equiv P(x,n) + \theta x P_x(x,n) - w(q) - qw'(q) = 0 \tag{4}$$

such that the *perceived* marginal profit with respect to output, and denoted $MP$, equals zero.

In general, $\theta$ depends on $q$ (or equivalently on $x$) except in some particular cases as shown below. It also depends on $n$ directly. However, *it does not depend directly on market size $L$ or on the cost side*, but only indirectly through the quantities/prices levels.[12] This would prove important when dealing with market expansion (see Section 5.3).

With substitute varieties which we assume in the following, the market conduct index $\theta$ belongs to $(0, 1)$.[13] A value for $\theta$ close to zero means that competition is very intense and thus the equilibrium price is close to marginal cost. Conversely, when $\theta$ is close to 1, then the industry almost behaves like a colluding one or a monopoly. Hence, $\theta$ measures the degree of market monopolization. Similarly, $\theta = 1$ when products are independent.

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[10] The proof that the elasticity $\varepsilon_s$ and curvature measure $\rho_s$ are constant if and only if $w(.)$ is iso-elastic is as follows.The fact that iso-elasticity of $w$ is sufficient for constant elasticity is obvious and necessity comes from setting $\frac{w(q)}{qw'(q)}$ equal to a constant $\varepsilon_s$ and integrating.

[11] Online Appendix O1 describes with details how the market conduct index $\theta$ is derived in our framework.

[12] See Online Appendix O1.

[13] With complement varieties, $\theta > 1$ (Weyl and Fabinger, 2013).

To illustrate, consider the "Generalized Cournot model" (Delipalla and Keen (1992)) with homogenous product. In that case, $\theta = \nu/n$ where $\nu$ is a constant that belongs to $[0, n]$. Cournot competition corresponds to $\nu = 1$, Bertrand competition to $\nu = 0$ and perfect collusion to $\nu = n$. Now, consider a symmetrically differentiated product oligopoly where firms compete in quantities. The inverse demand for a variety $i$ is $p_i = P_i(x_i, x_{-i})$ and

$$\theta = \frac{\partial P_i}{\partial x_i} / \sum_j \frac{\partial P_j}{\partial x_i} > 0.$$

If the inverse demand system is linear then $\theta$ is constant.

Finally, for a symmetrically differentiated product oligopoly where firms compete in prices, the demand for variety $i$ is $q_i = LD_i(p_1, ..., p_n)$ and

$$\theta = \sum_j \frac{\partial D_j}{\partial p_i} / \frac{\partial D_i}{\partial p_i} = 1 - A > 0$$

where $A = \sum_{j \neq i} d_{ij}$ where $d_{ij} = -\frac{\partial D_j}{\partial p_i} / \frac{\partial D_i}{\partial p_i} \geq 0$ is the price diversion ratio from $i$ to $j$. $A$ is the aggregate diversion ratio from any individual firm to the rest of the industry, and it represents the fraction of sales lost by a firm when it increases its price and that is captured by the competitors (Shapiro, 1995). If the demand system is linear then $A$ and thus $\theta$ are constant.[14]

As noted earlier, $\theta$ is a function of $q$ and $n$ in general. For further reference in the analysis, I denote $\varepsilon_{\theta,q} \equiv \partial \log \theta / \partial \log q$. I define similarly $\varepsilon_{\theta,n}$. Both measures reflect the sensitivity of the market conduct index w.r.t respectively $q$ and $n$. For instance, in the Generalized Cournot model, $\theta = \nu/n$ and thus $\varepsilon_{\theta,q} = 0$ and $\varepsilon_{\theta,n} = -1$.

For ease of reference, Appendix Table 1 summarizes the main notation used throughout the paper.

## 3 A contribution margin approach to the oligopoly equilibrium

**Analysis.** Bontems (2026) introduces a contribution margin approach to the monopoly's problem by showing how the firm's optimum can be described using only the elasticities of demand and "supply" ($\varepsilon_d$ and $\varepsilon_s$), as well as their curvature measures ($\rho_d$ and $\rho_s$). In this section, I extend this approach to consider the case of a symmetric oligopoly. To proceed, I first focus on a *profitability ratio* built on the comparison between price and AVC for each firm.

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[14] The market conduct index approach can also accommodate the case of monopolistic competition (see Weyl and Fabinger, 2013) and the case of competition in supply functions à la Klemperer and Meyer (1989) (see Mahoney and Weyl, 2017).

**Definition 1.** *The Contribution Margin Index, or in short the $\mathcal{C}$-index, is defined at the firm's level by:*

$$\mathcal{C}_i = \frac{p_i - w_i}{p_i}.$$

Obviously, at the symmetric equilibrium, $\mathcal{C}_i = \mathcal{C} \equiv (P(x,n) - w(q))/P(x,n)$ for all $i$. The $\mathcal{C}$-index represents the portion of total sales revenues not used to cover the aggregate variable cost of the industry and thus that contributes to covering the industry's aggregate fixed cost. Like the Lerner index ($\mathcal{L} = (p - c')/p$), the $\mathcal{C}$-index lies in the range (0,1) at the equilibrium. At the firm's level, we have $\pi = \mathcal{C}r$ where $r = pq$ is the revenue and $\pi$ the variable profit. And at the industry's level, we have $\Pi = \mathcal{C}R$ where $R = pQ$ is the industry's revenue and $\Pi$ the industry's variable profit. Hence, while the Lerner index is a usual measure of market power, the $\mathcal{C}$-index is a measure of profitability in terms of sales revenues, both at the firm's and industry's levels.

Second, let us introduce the useful notion of *effective* (industry) demand elasticity.

**Definition 2.** *The effective elasticity of industry demand is defined by:*

$$\tilde{\varepsilon}_d = \varepsilon_d/\theta$$

*that is the industry demand elasticity deflated by the intensity of competition as measured by $\theta$.*

The following Proposition indicates how the profit-maximizing oligopoly symmetric equilibrium can be described using the $\mathcal{C}$-index, instead of using the usual Lerner index.

**Proposition 1.** *At a symmetric equilibrium under imperfect competition, the $\mathcal{C}$-index obeys an inverse pseudo-elasticity rule:*

$$\mathcal{C} = \frac{p - w}{p} = \frac{1}{\varepsilon}$$

*where the pseudo-elasticity $\varepsilon$ is proportional to the effective demand elasticity:*

$$\varepsilon \equiv \lambda \tilde{\varepsilon}_d \qquad \text{with} \qquad \lambda = \frac{\varepsilon_s + 1}{\varepsilon_s + \tilde{\varepsilon}_d},$$

*and $\varepsilon \geq 1$ as well as $\tilde{\varepsilon}_d \geq 1$ are required at the equilibrium.*

*Proof.* To obtain the result, rewrite (4) as $p(1 - 1/\tilde{\varepsilon}_d) = w(1 + 1/\varepsilon_s)$ by using the definition of the effective elasticity of industry demand and of the elasticity of "supply". Then, rearranging to form $\mathcal{C}$ yields the desired formulation. An alternative formulation of the $\mathcal{C}$-index can be derived by relying on revenue and cost, i.e. $\mathcal{C} = (r(q) - c(q))/r(q) = 1/\varepsilon$ where $\varepsilon$ (and thus $\lambda$) can be written as function of the elasticities $\varepsilon_c$, $\varepsilon_r$ and $\theta$. See Appendix C for details. ■

This extends the result obtained by Bontems (2026) in the case of monopoly, to the oligopoly framework. Indeed, in the monopoly case ($n = 1$ and thus $\theta = 1$), then the effective demand elasticity is simply the monopolist's demand elasticity and the $\mathcal{C}$-index is inversely proportional to a pseudo-elasticity $\varepsilon$ which represents the demand elasticity factorized by the profitability factor $\lambda$. This profitability factor $\lambda$ is itself a function of both the demand and "supply" elasticities. More generally, in the oligopoly case ($n \geq 2$), the $\mathcal{C}$-index can be expressed in a similar way, except that the demand elasticity should be replaced with the effective demand elasticity $\tilde{\varepsilon}_d$, thus reflecting the various possibilities in terms of competition intensity in the range of oligopolies models permitted by the framework.

The equilibrium restriction on the pseudo-elasticity $\varepsilon$ also requires that $\lambda$ should be strictly positive. Furthermore, excluding the Bertrand/perfect competition case ($\theta = 0$) for the moment, the following Proposition indicates that the properties of the profitability scale factor $\lambda$ are intimately related to whether the firms operate under CAVC, IAVC or DAVC at the equilibrium.

**Corollary 1.** *Assume $\theta > 0$. If the equilibrium lies under IAVC ($\varepsilon_s > 0$) then $\lambda \in (0, 1)$. If on the contrary the equilibrium lies under DAVC ($\varepsilon_s < -1$) then $\lambda > 1$. Last, under CAVC, then $\lambda = 1$ for any equilibrium output.*

*Proof.* Note first that when the marginal cost is constant (CAVC), i.e. $\varepsilon_s = \infty$, then $\lambda$ is equal to 1 for any output level. Now assume that there is IAVC at the equilibrium, then the restriction $\varepsilon \geq 1$ implies that $\frac{\varepsilon_s+1}{\varepsilon_s+\tilde{\varepsilon}_d}\tilde{\varepsilon}_d \geq 1$ or equivalently $\tilde{\varepsilon}_d \geq 1$ as $\varepsilon_s > 0$. Thus $\lambda - 1 = \frac{1-\tilde{\varepsilon}_d}{\varepsilon_s+\tilde{\varepsilon}_d} < 0$. As $\lambda$ is also positive, we thus have $\lambda \in (0, 1)$ for the IAVC regime. Similarly, under DAVC at the equilibrium ($\varepsilon_s \leq -1$), $\varepsilon \geq 1 \Leftrightarrow \tilde{\varepsilon}_d \geq 1$ and thus $\lambda > 1$. ■

As shown in Corollary 1, the profitability factor $\lambda$ is constant precisely under CAVC. The non-constancy of marginal cost on the contrary opens up the possibility for $\lambda$ to depend on equilibrium output.

To complete the Corollary, note that under Bertrand or perfect competition, $\theta \to 0$ and then $\tilde{\varepsilon}_d \to \infty$ (if $\varepsilon_d$ is finite) and $\lambda \to 0$ (if $\varepsilon_s$ is finite). One gets $\lim_{\theta\to 0} \frac{\varepsilon_s+1}{\varepsilon_s+\tilde{\varepsilon}_d}\tilde{\varepsilon}_d = \varepsilon_s + 1$ and thus $\lim_{\theta\to 0} \mathcal{C} = \frac{p-w}{p} = \left.\frac{1}{\varepsilon_s+1}\right|_{\theta=0}$. Hence, under Bertrand or perfect competition, clearly only the regimes CAVC or IAVC are possible as $\varepsilon_s \geq 0$ is needed. If in addition marginal cost is constant, then $\varepsilon_s = \infty$ and consequently $P = w$.

**Interpreting the profitability factor $\lambda$.** The factor $\lambda$ has a simple economic interpretation. It measures the extent to which a local markup over marginal cost is informative about the firm's average profitability. At the symmetric equilibrium, the Lerner index and the contribution margin index are related by

$$\mathcal{L} = \lambda \mathcal{C},$$

or equivalently

$$\lambda = \frac{p - c'(q)}{p - w(q)}.$$

The numerator is the margin over marginal cost, while the denominator is the margin over average variable cost. Hence $\lambda$ compares a local object, the markup relevant for the first-order condition, with an average object, the contribution margin relevant for variable profits.

Under constant average variable cost, marginal cost and average variable cost coincide, so $\lambda = 1$ and the Lerner index is also a profitability index. When average variable cost is increasing, marginal cost exceeds average variable cost. The Lerner index then lies below the contribution margin index: $\lambda < 1$ and using the Lerner index understates profitability. Conversely, when average variable cost is decreasing, marginal cost lies below average variable cost. The Lerner index then lies above the contribution margin index: $\lambda > 1$ and using the Lerner index overstates profitability.

Thus $\lambda$ summarizes the wedge between local market power and average profitability created by non-constant marginal cost. This wedge is absent under constant marginal cost, but it becomes central whenever firms move along non-flat cost schedules. In the analysis below, the same factor also enters the pass-through formulas and the long-run comparative statics with endogenous entry.

It is also interesting to compare the industry with $n$ firms under scrutiny with non constant marginal cost with its counterpart for which marginal cost would be constant but equal to the *equilibrium value of* $w$ of the original industry. This particular *ghost* industry faces the same demand, has the same number of members and the same mode of competition, but differs in that its profitability factor $\lambda$ is 1 as marginal cost is constant, and hence its Lerner index and its $\mathcal{C}$-index are identical. More precisely, the original industry with non constant marginal cost produces a quantity $x^*$ given by (4) and that corresponds to a black dot in Figure 1 at the intersect between the perceived marginal revenue curve, $P(x,n) + \theta x P_x(x,n)$, and the marginal cost curve $w(q) + qw'(q)$.[15] The two black dots illustrates two possible different equilibria, one under IAVC and the other one under DAVC. By

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[15] To illustrate the different equilibrium regimes, the Figure 1 considers a classic specification with a U-shaped AVC function (based on a bipower function which in turn determines the marginal cost curve) and iso-elastic demands.

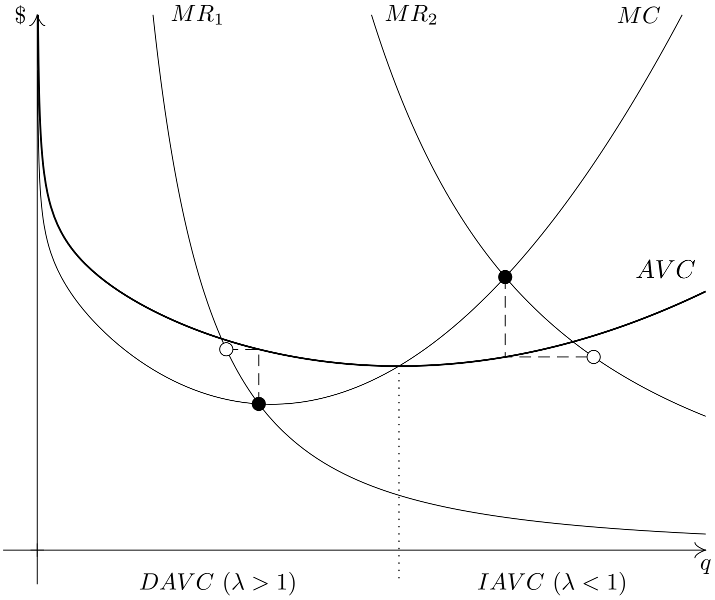

Figure 1: Equilibrium output for the industry and its ghost. $MR_i$ is the perceived marginal revenue curve for $i = 1, 2$, $MC$ is the marginal cost curve, $AVC$ is the average variable cost curve. Black dots indicate output equilibria for an industry with flexible marginal cost while white dots indicate output equilibria for the corresponding ghost industry that has constant marginal cost equal to the equilibrium value of AVC of the original industry.

contrast, the ghost industry has a constant marginal cost equal precisely to $w(q^*)$ and produces a quantity that corresponds to a white dot in Figure 1, at the intersect between the perceived marginal revenue curve and the value $w(q^*)$. Clearly, at an equilibrium under IAVC, the industry with non constant marginal cost produces less and has a larger profitability ratio than its ghost industry, because it benefits from a larger price while having the same AVC. Conversely, at an equilibrium under DAVC, the industry with non constant marginal cost produces more than the ghost industry and has a lower profitability ratio.

Intuitively, under IAVC, $\lambda \in (0, 1)$ and it represents a profitability factor that partially absorbs the importance of the effective elasticity of demand in the calculation of profitability. In other words, holding the effective demand elasticity constant, the industry benefits from the non-constancy of marginal cost (compared to its ghost) as the presence of IAVC constitutes an additional motive for a representative firm to reduce production because this allows to reach a lower average variable cost. Under DAVC, $\lambda > 1$ and the profitability of the industry with non constant marginal cost is reduced compared to the ghost industry because of the presence of countervailing incentives: on the one hand, each firm is willing to increase output to benefit from a lower AVC and thus from the presence of increasing return to scale, but on the other hand, each firm would like to reduce output to better extract consumer surplus.

Note that another type of ghost industry can be defined and will prove useful in the rest of the paper. Indeed, consider the ghost industry defined by a constant marginal cost equal to the *equilibrium marginal cost* of the original industry. Both industries produce the same output, but the ghost industry benefits (suffers) from a lower (higher) AVC under DAVC (IAVC) and hence earns more (less) in terms of profit.

**A stylized example.** To build intuition, let us consider a stylized example of imperfect competition, designed to isolate the role of cost curvature while remaining analytically tractable. Inverse demand is assumed to be linear and $n$ firms produce a homogeneous good under generalized Cournot competition, with VC functions of the form $c(q) = cq + \frac{d}{2}q^2$, where $c > 0$ is a baseline marginal cost and $d \in \mathbb{R}$ captures IAVC/DAVC. Although in this example IAVC (DAVC) actually corresponds strictly to VC convexity (concavity), I still continue in the following to refer to IAVC and DAVC regimes whenever needed. Strategic interaction is governed by parameter $\nu \in [0, n]$, which measures the degree of collusion or aggressiveness in firms' behavior. The detailed derivations of equilibrium outcomes are presented in Online Appendix O3.

A central outcome of this model is the expression for the *profitability factor*:

$$\lambda = \frac{b\,\nu}{b\,\nu + \frac{d}{2}},$$

where $b$ is the slope of the inverse demand curve. Actually, $b\nu$ represents the markup per unit sold.[16] This compact formula highlights that $\lambda$ decreases with cost curvature $d$, while the ghost industry with the same primitives but constant marginal cost equal to the equilibrium marginal cost of the original industry provides a natural benchmark with $\lambda = 1$. Under Bertrand ($\nu = 0$), then $\lambda = 0$. Moreover, $\lambda$ increases with the markup per unit $b\nu$ if and only if IAVC ($d > 0$). An increase in the per–unit markup $b\nu$ reflects weaker competitive pressure. In that case, the profitability factor $\lambda$ monotonically converges to one, regardless of whether IAVC or DAVC prevails. Hence the curvature of costs matters less when market conduct is more collusive, since $\lambda$ becomes almost insensitive to the sign or magnitude of $d$ as $b\nu$ grows.

Figure 2 illustrates how the profitability factor $\lambda$ evolves as the ratio AVC/MC deviates from the ghost industry benchmark, where it is equal to 1. Parameter $a$ represents the intercept of the linear

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[16] In Online Appendix O3, denoting $q^*$ as the unique equilibrium quantity and $p^*$ its price, the first-order condition (4) reveals that the markup $p^* - c - dq^*$ is equal to $b\nu q^*$ .

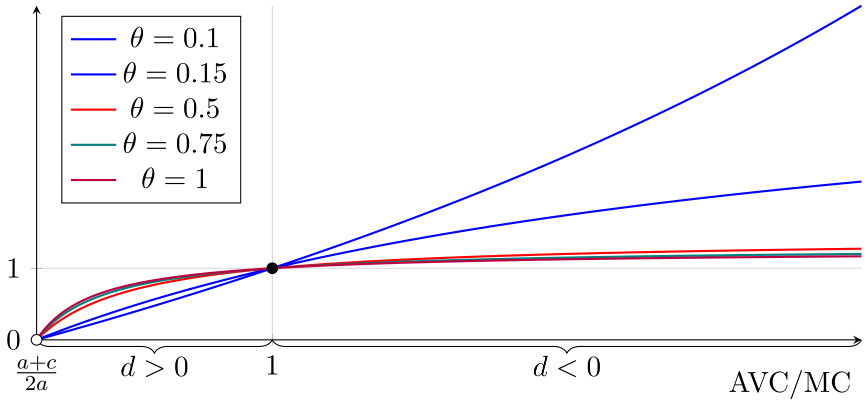

Figure 2: profitability factor $\lambda$ as a function of AVC/MC for different values of the market conduct index $\theta$.

inverse demand. Different values of $\theta = \nu/n$ shift both the slope and the curvature of the function, showing how market conduct amplifies or dampens the departure from $\lambda = 1$. Although the primitives of the model are $(a, b, c, d, \nu, n)$, expressing $\lambda$ directly as a function of AVC/MC hides the primitives from the explicit argument, except for $\theta$. This representation is nonetheless insightful: for a fixed $\theta$, any change in AVC/MC reflects an underlying variation in the primitives (typically in $d$), and the curve representing $\lambda$ immediately shows how the profitability factor reacts to such changes.

It is also instructive to look at how the cost structure through its parameters, namely $c$ as the baseline marginal cost and $d$ as the cost curvature parameter, influences market power and profitability. The Lerner index and the contribution margin ratio are given respectively by:

$$\mathcal{L} = \frac{(a-c)b\nu}{ab\nu + bnc + ad} \quad \text{and} \quad \mathcal{C} = \frac{(a-c)(b\nu + \frac{1}{2}d)}{ab\nu + bnc + ad}.$$

Clearly, the degree of market power measured through $\mathcal{L}$ is reduced whenever $c$ or $d$ raise. The profitability ratio $\mathcal{C}$ also decreases in $c$, but is non monotonic in curvature $d$. More precisely, $\mathcal{C}$ raises in curvature $d$ if and only if $\theta < \frac{c}{a} < 1$. Hence, when competition is strong enough, raising $d$ improves paradoxically profitability.

**Existence, unicity and stability.** Like Weyl and Fabinger (2013), Adachi and Fabinger (2022) and Kroft et al. (2021), I assume that the conditions for existence and uniqueness of the interior symmetric equilibrium are satisfied. In particular, following Seade (1980b), I assume that the stability condition holds in the sense that the perceived marginal profit at the symmetric equilibrium is decreasing in $q$. More precisely, given (4), the stability condition writes as follows:

$$MP_q \equiv \frac{1}{L}(1 + \theta + q\theta_q)P_x + \frac{1}{L}\theta x P_{xx} - \frac{d}{dq}\left[w(q) + qw'(q)\right] < 0$$

where $MP_q = \partial MP/\partial q$ and $\theta_q = \partial\theta/\partial q$ are the partial derivatives of respectively $MP$ and $\theta$ w.r.t $q$.

For further reference in the analysis to follow, it is convenient here to consider an *average measure of curvature* for both demand and "supply" as follows.

**Definition 3.** *The "average curvature" measure* $\rho \equiv \lambda\rho_d + (1-\lambda)\rho_s$ *is the average of curvature measures for the demand and the "supply" with weight* $\lambda$ *and* $1-\lambda$ *respectively.*

When marginal cost is constant ($\lambda = 1$), then observe that $\rho$ is simply the curvature of demand $\rho_d$. It is well known that under CAVC the stability condition puts an upper bound on $\rho_d$ whose value depends on the competition model studied, as will be clear below. When marginal cost is not constant, the new insight brought by our analysis is that the stability condition amounts to put an upper bound on the "average curvature" $\rho$ defined above. Indeed, straightforward manipulations allow to rewrite $MP_q$ as follows:[17]

$$MP_q = \frac{\theta P_x}{\lambda L}\left[\lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right) + 2 - \rho\right] \tag{5}$$

Throughout the paper, I thus assume that demand and cost functions satisfy the following condition globally.

**Assumption 3** (Stability condition)**.** $MP_q < 0$ *or equivalently,*

$$\rho < 2 + \lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right). \tag{6}$$

This general stability condition includes many specific conditions corresponding to particular cases. Observe that the term between brackets on the right hand side of the inequality depends only on $\theta$ and on $\varepsilon_{\theta,q}$, i.e. it depends on the characteristics of the mode of competition.

First, consider the case of a monopoly ($n = 1$ and thus $\theta = 1$). In that case, (6) reduces to $\rho < 2$, which represents the second-order condition for the monopolist's profit maximization problem exhibited by Bontems (2026).[18] If we further assume CAVC ($\lambda = 1$), then (6) amounts to the well known second-order condition $\rho_d < 2$.

Now consider a $n$ symmetric Cournot oligopoly ($\theta = 1/n$) with CAVC ($\lambda = 1$) and observe that (6) reduces to the condition exhibited by Seade (1980b): $\rho_d < n + 1$.[19] When marginal cost is non constant, (6) provides a direct generalization of Seade (1980b): $\rho < 2 + \lambda\left(n - 1\right)$. Finally, consider

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[17] See Appendix D.

[18] The stability condition is similar in the case of a perfectly colluding industry with $n$ members.

[19] Note that the formulation of the stability condition in Seade (1980b) is $(1 + 1/n)\hat{P}' + x\hat{P}'' < 0$ where the inverse demand is $\hat{P}(nx)$. With our notation, $P(x,n) = \hat{P}(nx)$ and thus $P_x = n\hat{P}'$ and $P_{xx} = n^2\hat{P}''$. Then the condition of Seade is equivalent to $(1 + 1/n)P_x + (1/n)xP_{xx} < 0$, which is equivalent to the condition in the text, using $P_x < 0$ and the definition of $\rho_d$.

a differentiated product oligopoly where firms compete in price and where demands are linear, then $\theta = 1 - A$ and is constant w.r.t $q$, and (6) amounts to $\rho < 2 + \lambda \frac{A}{1-A}$.

**Stability in the long run.** In the long run, the number of firms is endogenous and is driven by free exit/entry on the market. To enter the market, a fixed cost $f$ has to be spent by each entrant and the zero profit condition writes:

$$(P(x, n) - w(q))q - f = 0 \tag{7}$$

For expositional clarity, the same cost function is maintained in both the short-run and long-run analyses. This does not imply identical technologies across horizons, but rather holds technology fixed in order to isolate how entry and market structure adjust when other constraints are relaxed. Such a specification facilitates the derivation of the long-run stability condition, as changes in equilibrium outcomes can then be attributed to structural adjustments rather than shifts in production technology. The focus is therefore on comparative statics of market conduct, not on dynamic transition paths.

Condition (7) as well as the behavioral response of symmetric firms given by (4) allow to determine the long run equilibrium in terms of output per firm and number of varieties. At a long run equilibrium, the $\mathcal{C}$-index satisfies $\mathcal{C}r = f$ or equivalently $r = \varepsilon f$.

As usual, the stability condition in the long run amounts to have the following matrix $M$ as being definite negative:

$$M = \begin{pmatrix} MP_q & MP_n \\ (1-\theta)xP_x & qP_n \end{pmatrix} \tag{8}$$

where $MP_n$ denotes the marginal impact of $n$ on the perceived marginal profit holding $x$ constant:

$$MP_n = \frac{\partial}{\partial n}\left(P + \theta x P_x - w - qw'\right) = P_n + \theta x P_{xn} + \theta_n x P_x. \tag{9}$$

Note that, although $P_n \leq 0$ under Assumption 2 and $\theta_n < 0$, the sign of $MP_n$ remains ambiguous at this level of generality.[20] Nevertheless, the stability of the long run equilibrium imposes that $MP_n$ cannot be too negative, as seen below.

**Assumption 4** (Long run stability conditions)**.**

$$MP_q + qP_n < 0 \text{ and } MP_q qP_n - (1-\theta)xP_x MP_n > 0. \tag{10}$$

In particular, for the perfectly colluding oligopoly ($\theta = 1$), Assumption 2, i.e. $P_n < 0$, is necessary for the long run stability conditions (10) to hold.

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[20] The sign of $MP_n$ and its interpretation will be discussed later in Section and will be shown to be related to whether business stealing or expansion occurs at the equilibrium.

## 4 Reformulating cost pass-through

We examine in this section the incidence in terms on price, quantity and profit of some cost shock in the short run.[21] This allows to show that the expression of absolute and relative cost pass-through can be solely determined through the values of the elasticity $\lambda$ and the "average" convexity measure $\rho$ as well as the intensity of competition $\theta$ and its sensitivity to output, $\varepsilon_{\theta,q}$. For the relative cost pass-through, the pseudo elasticity $\varepsilon$ is also involved. The new expressions obtained are compared with those of the literature (in particular to Weyl and Fabinger, 2013). As will be shown in Sections 5.1 and 5.3, these cost pass-through measures are key elements to explain how market concentration and market expansion affect market outcomes.

**Cost pass-through in the short run.** Let us start with defining the absolute and relative cost pass-through in our setting. The absolute pass-through (hereafter *APT*) corresponds to the absolute impact on price of an infinitesimal additive cost shock $t$ (e.g. a specific tax/subsidy). For this, let us write the profit for any firm $i$ as $\pi_i = (p_i - w(q_i) - t)q_i$ and at the symmetric equilibrium, $APT = \left.\frac{dp}{dt}\right|_{t=0}$. Similarly, the relative pass-through (hereafter *RPT*) corresponds to the relative impact on price of a multiplicative cost shock $\tau$ (e.g. an ad-valorem tax or subsidy on cost) valued in $\tau = 1$. In this case, profit writes $\pi_i = (p_i - \tau w(q_i))q_i$ and $RPT = \left.\frac{d\log p}{d\log \tau}\right|_{\tau=1}$.[22]

**Proposition 2.** *The absolute and relative pass-through are positive and given respectively by:*

$$APT = \frac{\lambda}{\theta(2-\rho) + \lambda\left(1 - \theta + \theta\varepsilon_{\theta,q}\right)} \tag{11}$$

*and*

$$RPT = \frac{\varepsilon - \lambda}{\varepsilon} APT. \tag{12}$$

*Proof.* See Appendix E. ■

The stability condition (6) ensures that *APT* and *RPT* are positive. Note also that the relationship (12) also writes $RPT = (1-\mathcal{L})APT$ or equivalently $\mathcal{L} = 1 - RPT/APT$. This formulation is equivalent to the one proposed by Adachi and Fabinger (2022, Proposition 3).[23] Because the Lerner index belongs to $(0,1)$ under imperfect competition, then clearly $RPT < APT$. It follows that the multiplicative cost

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[21] Long-run consequences will be examined later in Section 5.2.

[22] Because it is taken in $\tau = 1$, *RPT* is also equivalent to $d\log P/d\tau$, that is the semi-elasticity of $P$ w.r.t $\tau$.

[23] Actually, Proposition 3 in Adachi and Fabinger (2022) is more general in that it shows that this relationship between the pass-through of a specific tax and the pass-through of an ad-valorem tax, measured by the tax pass-through semi-elasticity $d\log P/d\tau$, holds when both taxes coexist and are non zero and non unitary respectively. **?** considers the contribution margin approach to oligopolies in regulation contexts.

shock is less likely to be overshifted, i.e. $RPT > 1$, than the additive one, i.e. $APT > 1$.[24] Also under perfect competition, we get the well-known result that the pass-through of both shocks are the same.

The novelty here is the formulation of $APT$ which is based solely on $\lambda$ and $\rho$ as well as $\theta$ and $\varepsilon_{\theta,q}$. To better understand the role of the different components of $APT$, it is instructive to consider some specific cases of the general formula (11).

- Consider first the case of a monopoly ($n = 1$), the case of a perfectly colluding $n$-industry and the case of independent products. In all of these cases, $\theta = 1$ and thus $\varepsilon_{\theta,q} = 0$, so that (11) reduces to

  $$APT|_{\theta=1} = \frac{\lambda}{2-\rho}. \tag{13}$$

  This is the $APT$ formulation obtained by Bontems (2026) in his study of monopolistic competition and it is a straightforward extension of the well-known formulation one gets when marginal cost is constant. Indeed, under CAVC ($\lambda = 1$), $APT = 1/(2-\rho_d)$ which is the classic $APT$ formulation obtained by Bulow and Pfleiderer (1983), and there is overshifting of the additive cost shock if and only if demand is strictly log-concave at the equilibrium, i.e. $\rho_d > 1$ (Seade, 1985; Stern, 1987).[25]

  When MC is not constant, (13) reveals that there is undershifting of the cost shock if and only if $\rho < 2 - \lambda$. Conversely, there is overshifting if and only if $2 - \lambda < \rho < 2$.

  Finally, we obtain $RPT = \frac{\varepsilon-\lambda}{\varepsilon}\frac{\lambda}{2-\rho}$ which is a generalization of the formulation $\frac{\varepsilon_d-1}{\varepsilon_d(2-\rho_d)}$ proposed by Mrázová and Neary (2017) in the particular case of CAVC.

- Consider now the oligopoly case but where $\theta$ is constant w.r.t. output (Generalized Cournot model or Nash in price/quantity with differentiated products and linear direct or inverse demands). Then $\varepsilon_{\theta,q} = 0$ and the cost pass-through simplifies to:

  $$APT|_{\theta=\text{cst}} = \frac{\lambda}{\theta(2-\rho) + \lambda\,(1-\theta)}$$

  Observe that there is overshifting of the cost shock if and only if $APT|_{\theta=\text{cst}} > 1$ which yields the same condition on $\rho$ as for the monopoly above: $\rho > 2 - \lambda$. This condition generalizes the condition $\rho_d > 1$ obtained by e.g. Seade (1985) and Stern (1987) in oligopoly contexts under CAVC.

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[24] This result extends to differentiated products the result obtained by Delipalla and Keen (1992) in the context of the Generalized Cournot model.

[25] As argued by Delipalla and Keen (1992), this condition can even be traced back to Cournot (1960).

In the general case, if $\theta$ increases in $q$ then lower quantities/higher prices create more competitive market conduct and pass-through tends to be smaller (compared to the case with constant $\theta$). And conversely.

- Finally, consider now the case of price competition with homogenous product (Bertrand), then $\theta = 0$ and thus $\lambda \to 0$. Also,

$$\mathit{APT}|_{\theta=0} = \mathit{RPT}|_{\theta=0} = \frac{1}{1 + \frac{\varepsilon_d}{\varepsilon_s+1}(2-\rho_s)} = \frac{1}{1+\frac{\varepsilon_d}{\varepsilon_{c'}}} \in (0,1)$$

  where $\varepsilon_{c'} = c'/qc'' = (1+\varepsilon_s)/(2-\rho_s)$ is the inverse of the elasticity of marginal cost or equivalently the elasticity of the supply function defined by the marginal cost curve. Hence the pass-through decreases in the ratio of the elasticity of demand to that of supply.[26]

Importantly, note that, in any case, (11) provides a consistent formulation but alternative to the one proposed by Weyl and Fabinger (2013):

$$\mathit{APT} = \frac{1}{1 + \frac{\theta}{\epsilon_\theta} + \frac{\varepsilon_d - \theta}{\varepsilon_{c'}} + \frac{\theta}{\varepsilon_{ms}}}$$

which relies on the elasticity of inverse marginal surplus $ms = -xP_x$, i.e. $\varepsilon_{ms} = \frac{xP_x}{x(P_x + xP_{xx})}$, the elasticity of supply, i.e. $\varepsilon_{c'}$ and the sensitivity of $\theta$ to output as measured by Weyl and Fabinger through $\epsilon_\theta = 1/\varepsilon_{\theta,q}$.[27]

Another interesting result relates to the impact of cost curvature on the cost pass-through. For this, consider the ghost industry where each firm has a constant marginal cost equal to the equilibrium marginal cost of the original industry and where the mode of competition is the same. As argued above, the two industries produce the same output. Nevertheless, the impacts of additive and multiplicative cost shocks differ in the two industries depending on the convexity or concavity of VC. The following Proposition extends to oligopoly models a result obtained for the monopoly by Bontems (2026).

**Proposition 3.** *Whatever the mode of competition and whether the cost shock is additive or multiplicative, the non-constancy of marginal cost entails reduced (increased) cost pass-through on price or quantity (in absolute value) under VC convexity (concavity).*

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[26] The proof can be established directly like Weyl and Fabinger by considering perfect competition and the equality between demand and supply under the tax intervention (see Weyl and Fabinger (2013), Principle of Incidence under perfect competition) or indirectly from our general formulation (11) when $\theta \to 0$ and using L'Hospital rule (see Appendix E for details).

[27] To see the equivalence, note that $\varepsilon_{ms} = \frac{xP_x}{x(P_x+xP_{xx})} = \frac{1}{1-\rho_d}$ with our notations. Moreover, note that $\varepsilon_{c'} = \frac{1+\varepsilon_s}{2-\rho_s}$ and note that $\frac{\tilde{\varepsilon}_d - 1}{\varepsilon_s+1} = \frac{1-\lambda}{\lambda}$. We thus have that $1 + \frac{\theta}{\epsilon_\theta} + \frac{\varepsilon_d - \theta}{\varepsilon_{c'}} + \frac{\theta}{\varepsilon_{ms}} = 1 + \theta\varepsilon_{\theta,q} + \theta\frac{\varepsilon-1}{1+\varepsilon_s}(2-\rho_s) + \theta(1-\rho_d)$ and this further simplifies into $1 - \theta + \theta\varepsilon_{\theta,q} + \theta\frac{(2-\rho)}{\lambda}$.

*Proof.* By comparing $\mathit{APT}$ for $\lambda = 1$, that we denote $\mathit{APT}_{\text{ghost}}$, and $\mathit{APT}$ for $\lambda \neq 1$ at the equilibrium, we have the following inequality

$$\mathit{APT} = \frac{\lambda}{\theta(2-\rho) + \lambda\left(1-\theta+\theta\varepsilon_{\theta,q}\right)} < \mathit{APT}_{\text{ghost}} = \frac{1}{1+\theta-\theta\rho_d+\theta\varepsilon_{\theta,q}}$$

that is equivalent to $\lambda(2-\rho_d) < 2-\rho$ which in turn is equivalent to $(1-\lambda)(2-\rho_s) > 0$. From the definition of $c = qw$, we have $c'' = 2w' + qw'' = (2-\rho_s)w'$. As $w' > 0 \Leftrightarrow \lambda < 1$, the sign of $c''$ is also the sign of $(1-\lambda)(2-\rho_s)$. Hence the result follows. Also, because $\mathit{RPT} = (1-\mathcal{L})\mathit{APT}$ and the Lerner index is the same for both industries, the convexity/concavity of VC also determines the comparison of $\mathit{RPT}$ in the same way. Finally, because $\mathit{APT} = \left.\frac{dp}{dt}\right|_{t=0} = \frac{1}{L}P_x \left.\frac{dq}{dt}\right|_{t=0}$ and $\mathit{RPT} = \left.\frac{d\log p}{d\log\tau}\right|_{\tau=1} = -\frac{1}{\theta\tilde{\varepsilon}_d}\frac{d\log q}{d\log\tau}$, the result also extends to the absolute value of pass-through on quantity, whether we consider an additive or multiplicative cost shock. ■

Before studying the long term consequences of the cost shock, it is also instructive to look at the profit impact in the short run, because, as will be clear below, it drives the consequences in the long run in terms of entry/exit on the market.

**Proposition 4.** *The additive cost shock is profit-enhancing if and only if* $(1-\theta)\mathit{APT} > 1$ *while the multiplicative cost shock is profit-enhancing if and only if* $(1-\theta)\mathit{RPT} > 1-\mathcal{C}$.

*Proof.* See Appendix F. ■

Intuitively, in both cases, the cost shock brings two effects, a negative direct effect on profit and a positive effect in terms of reducing output which brings aggregate output closer to the collusive one and thereby enhancing profits holding the number of firms fixed. The condition for the additive cost shock is due to Weyl and Fabinger (2013). As $\theta < 1$ (substitute products), it follows that overshifting, i.e. $\mathit{APT} > 1$, is needed for the shock to be profit-increasing. The present condition for the multiplicative cost shock is new and allows to draw new insights on the comparison between both types of shocks. In particular, Proposition 4 suggests that overshifting, i.e. $\mathit{RPT} > 1$, is *not* needed for the multiplicative shock to be profit-increasing.

Moreover, using Proposition 2 and $\mathcal{C} = 1/\varepsilon$, the second condition in Proposition 4 can be rewritten as $(1-\theta)\mathit{APT} > (\varepsilon-1)/(\varepsilon-\lambda)$. Observe that under CAVC ($\lambda = 1$) then the two conditions in Proposition 4 coincide. On the contrary, when $\lambda > 1$ (DAVC) then an additive cost shock is more likely to enhance profits than a multiplicative cost shock. Conversely, under IAVC ($\lambda < 1$), a multiplicative cost shock is more likely to enhance profits than the additive shock.

**Pass-throughs in the stylized example.** In Online Appendix O3, we prove that in the stylized example, pass-throughs write as follows:

$$APT = \frac{bn}{b(n+\nu)+d}$$

and

$$RPT = (1-\mathcal{L})APT = \frac{bn}{b(n+\nu)+d}\frac{bc(n+\nu)+ad}{ab\nu+ad+bnc}.$$

Note also that, from Proposition 2, the pass-through ratios comparing the industry to the ghost one are equal and write

$$\frac{APT}{APT_{\text{ghost}}} = \frac{RPT}{RPT_{\text{ghost}}} = \frac{b(n+\nu)}{b(n+\nu)+d}.$$

Figure 3 illustrates Proposition 3, by comparing the pass-through ratios of the industry to its ghost counterpart as functions of the cost ratio AVC/MC. The common curve intersects at the benchmark point $(1,1)$, corresponding to the ghost industry where marginal and average variable costs coincide. They also meet at $\left(\frac{a+c}{2a},0\right)$, which corresponds to the limit case of highly convex costs. For AVC/MC $< 1$ (i.e. under IAVC), the ratio lies strictly below one, indicating that pass-through is reduced relative to the ghost benchmark. Conversely, when AVC/MC $> 1$ (i.e. under DAVC), the ratio exceeds one, implying an amplification of pass-through compared to the constant-MC case. An important feature of the example is that the ratio is strictly concave in AVC/MC. As a result, the cost pass-through appears more sensitive to deviations from the constant-MC benchmark when IAVC ($d>0$) prevails than when DAVC occurs ($d<0$).

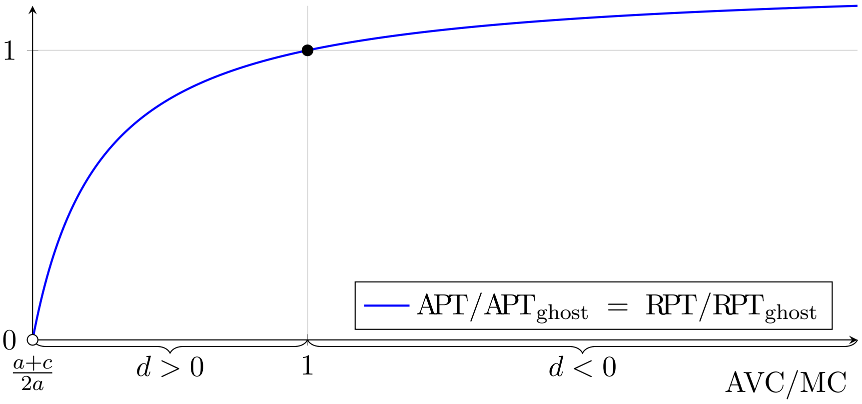

Figure 3: Pass-through ratios as a function of AVC/MC.

Last, to illustrate Proposition 4, in contrast to additive shocks which never raise profits, I show that multiplicative shocks may be profit-enhancing under strong competition provided $d$ is positive and belongs to a set $[d_-, d_+] \subset \mathbb{R}_+$. However, this possibility disappears in the ghost industry.

# 5 Comparative statics

In this section, I successively: (i) examine the comparative statics of the oligopoly model with respect to the number of firms $n$ (section 5.1), (ii) analyze the cost pass-through in the long run when the number of firms is endogenous (section 5.2) and (iii) consider the comparative statics of the oligopoly model when the number of consumers $L$ varies (section 5.3).

## 5.1 Market concentration incidence

Let us first analyze the incidence of market concentration. As we will see, a key factor in determining the evolution of the industry when $n$ changes is the relative size of the *APT*, i.e. the strength of the price response to an infinitesimal additive cost shock (e.g., a unit tax). This also provides a number of useful results for the sequel.

First, the sign of $MP_n$, i.e. which measures the marginal impact of $n$ on the perceived marginal profit, determines whether there is a business stealing or a business expansion externality in the industry. Indeed, total differentiation of (4) gives $dq/dn = -MP_n/MP_q$ and thus *business stealing, i.e.* $dq/dn < 0$*, occurs if and only if* $MP_n < 0$. From (9), recall that $MP_n$ is given by:

$$MP_n = P_n + \theta x P_{xn} + \theta_n x P_x,$$

the sign of which is ambiguous in general and hence whether there is business stealing or expansion depends on the demand and the market conduct index properties.[28]

Second, observe that *the equilibrium profit decreases in* $n$ if and only if the long run stability conditions hold. Indeed, totally differentiating $\pi = (P(x,n) - w(q))q$ yields

$$d\pi = \frac{\partial \pi}{\partial n} dn + \frac{\partial \pi}{\partial q} dq = q P_n dn + (1-\theta) x P_x dq$$

Using $dq/dn = -MP_n/MP_q$ and replacing, one obtains:

$$\frac{d\pi}{dn} = qP_n - (1-\theta) x P_x \frac{MP_n}{MP_q} = \frac{1}{MP_q}\left(MP_q q P_n - MP_n (1-\theta) x P_x\right) < 0$$

under (10). Observe that by using Proposition which shows that $MP_q = (P_x/L)/APT$, one can rewrite the long-run stability condition (10) as follows:

$$P_n - MP_n (1-\theta) APT < 0 \tag{14}$$

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[28] See Cao et al. (2021) for a theoretical and empirical study of competition between dockless bikesharing firms in China that suggests that entry of a new firm can generate business extension for an incumbent.

Finally, the impact of market concentration on price can be measured through:

$$\frac{dp}{dn} = P_x \frac{dx}{dn} + P_n = P_n - MP_n APT$$

using the total derivation of (4) which gives $dq/dn = -MP_n/MP_q$ and $MP_q = (P_x/L)/APT$.

For further reference, I gather all these results in the following Proposition.

**Proposition 5.** *When the number $n$ of competitors increases, then*

**(i)** *there is business stealing ($dq/dn < 0$) if and only if $MP_n < 0$,*

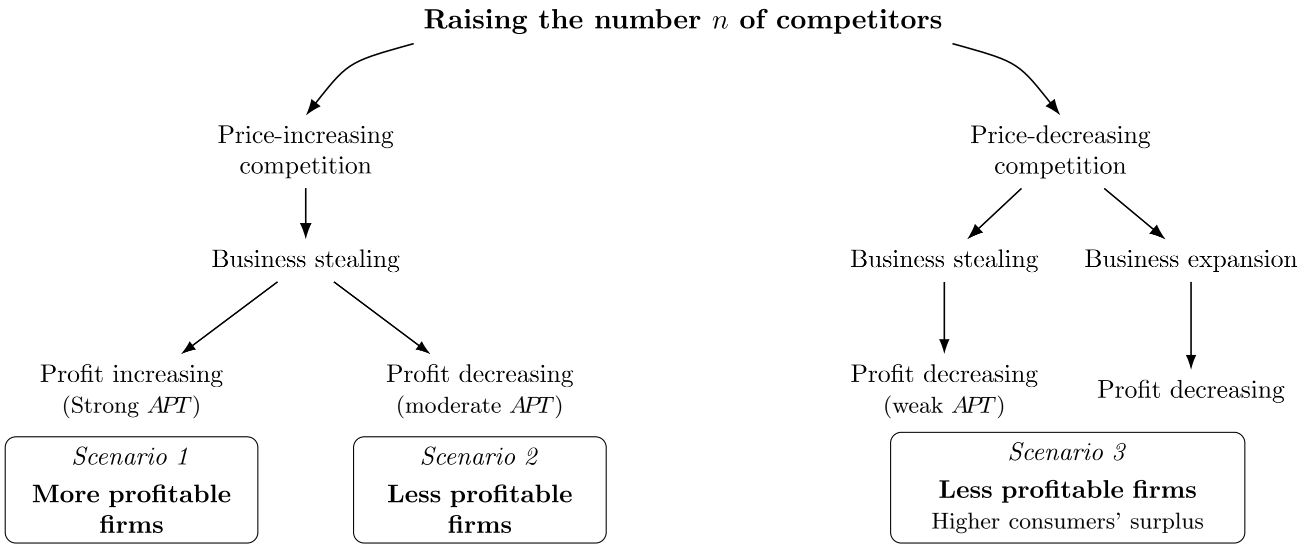
**(ii)** *the equilibrium profit decreases if and only if $P_n - MP_n(1-\theta)APT < 0$ (long-run stability condition),*

**(iii)** *the equilibrium price decreases if and only if $P_n - MP_n APT < 0$.*

The combination of the conditions expressed in Proposition 5 results in three different scenarios highlighted in Figure 4. Each scenario occurs based on only two conditions: (i) whether there is business stealing or expansion, and (ii) whether the *APT* is strong, moderate or weak. As an example, scenario 3 is characterized by the usual situation of downward price competition, which unambiguously benefits consumers. This actually occurs under two different contexts. First, observe that the presence of a *business expansion* effect, i.e. $MP_n > 0$, is a sufficient condition for the equilibrium price to be decreasing in $n$. However, despite increasing output per firm, profit is unambiguously decreasing. Profit per firm is also decreasing when there is business stealing and the *APT* is weak, i.e. $APT < P_n/MP_n$. The industry in scenario 3 is thus characterized by less profitable firms even though their size in terms of output may increase.

By contrast, under scenarios 1 and 2, competition drives up prices. More precisely, scenario 1 occurs because there is business stealing and the *APT* is strong. In this scenario, competition results in higher prices and profits, resulting in a larger industry with more profitable but smaller firms. In scenario 2, competition also increases prices, but because the *APT* is moderate, the industry evolves into a larger industry with less profitable and smaller firms. In both scenarios 1 and 2, consumers suffer from higher prices but benefit from greater variety, so that the impact of competition on consumers remains ambiguous at this stage.

Despite the fundamental insight of economics that competition usually lowers prices, the situation of price-increasing competition has already been identified as plausible both in the empirical and the-



Business stealing (expansion) means $MP_n < (>) 0$. Strong $APT$ means $APT > P_n/((1-\theta)MP_n)$. Moderate $APT$ means $P_n/MP_n < APT < P_n/((1-\theta)MP_n)$. Weak $APT$ means $APT < P_n/MP_n$.

Figure 4: Scenarios following an increase in the number $n$ of competitors.

oretical literature.[29] In particular, Chen and Riordan (2008) shows that, in a discrete choice model of product differentiation, the symmetric duopoly price can be higher than the single-product monopoly price. Specifically, because of competition for market share, duopolists have an incentive to price below the monopoly price, but in some situations, greater consumer choice may steepen the demand curve for each firm, and the latter incentive to raise the price may dominate. Similarly, Bertoletti and Etro (2016) show that, in a representative consumer model with preferences characterized by generalized linear direct utility, Nash in price or in quantity equilibria could be characterized by increasing prices in $n$, especially when the number of firms is large enough.

The present analysis covers a wide range of oligopoly models and, like in Bertoletti and Etro (2016), it treats $n$ as a continuous variable. The impact of a greater number of firms on the market price results from the confrontation of two effects. On the one hand, at constant consumption per capita and per variety, the increase in $n$ exerts a downward pressure on the price ($P_n < 0$). On the other hand, holding $n$ constant, the possible business stealing effect ($MP_n < 0$) lowers the equilibrium quantity per variety and therefore it increases the price. When the latter dominates the former, especially when $APT$ is sufficiently large, then competition drives up prices. In addition, this analysis also identifies

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[29] As early as Satterthwaite (1979) and Rosenthal (1980), it has been shown that the equilibrium price can raise following an increase in the number of firms. Empirical evidence has been found in medical services (Pauly and Satterthwaite, 1981), in the automobile retail and automobile tire markets (Bresnahan and Reiss, 1990 and 1991), in the drug market (Grabowski and Vernon, 1992; Perloff et al., 2006), and private labels in the food industry (Ward et al., 2002) and Internet access (Chen and Savage, 2011). More recently, Mangin (2022) theoretically examines the conditions for price-increasing competition in a random utility model where the number of firms is ex ante uncertain.

circumstances in which competition also increases profits in the short run.

**Market concentration incidence in the stylized example.** In the stylized model, we have $MP_n = P_n = -bLx < 0$, so that an increase in $n$ always reduces each firm's perceived marginal profit (business stealing). Moreover, the equilibrium price may increase with $n$ only when $d$ is sufficiently negative ($d < -b\nu$), while equilibrium profit is always decreasing in $n$ and the long-run stability condition is automatically satisfied under existence. Two forces move price when the number of firms $n$ increases. First, $P_n = -bLx < 0$: holding $x$ fixed, the inverse demand becomes steeper in $x$ as $n$ rises, which pushes price down. Second, firms adjust $x$ in response to the change in perceived marginal profitability; this adjustment feeds back into price through pass-through. The sign of $P_n - MP_n APT$ in Proposition 5 is exactly the balance of these two channels. When $d$ is sufficiently negative ( $d < -b\nu$ ), additive pass-through exceeds one. Then the price response to the induced quantity adjustment *overcompensates* the direct downward effect $P_n < 0$, flipping the total effect positive, producing the "price-increasing competition" paradox occuring in Scenario 2.

**Private versus social incentives to enter the market.** It is well known that whether oligopolistic competition does result in too few or too many products from a welfare point of view depends on the relative forces of two effects (Mankiw and Whinston, 1986). On the one hand, firms do not take into account the business stealing/expansion externality and on the other hand, firms usually do not fully internalize consumer surplus. Hence, each potential entrant only considers the net profit $\pi - f$ to be made while a social planner unable to control the behavior of active firms would consider the marginal impact of entry on welfare. Let us take welfare as the sum of consumers and producers surplus as follows:

$$W = LU(x,n) - nqw(q) - nf.$$

Computing the derivative of welfare with respect to $n$ yields:

$$\begin{aligned} W'(n) &= LU_n + LU_x\frac{dx}{dn} - qw - n(w + qw')\frac{dq}{dn} - f \\ &= \underbrace{p\mathcal{V}}_{\text{taste for diversity}} + \underbrace{\pi - f}_{\text{net profit}} + \underbrace{n(p - w - qw')\frac{dq}{dn}}_{\text{business stealing/expansion}} \end{aligned}$$

recalling that $U_x = np$ and that $\mathcal{V} = (LU_n - pq)/p$ denotes the variety effect. The marginal impact of an additional variety on welfare can be decomposed into three terms: (i) a taste for diversity term which is positive when $\mathcal{V} > 0$, (ii) a net profit term because a new variety means additional profit in the industry and (iii) a business stealing/expansion term that appears as long as price is above marginal cost and that is negative if and only if there is business stealing, i.e. $dq/dn < 0$. In other words, the presence of a taste for diversity and of a business stealing/expansion effect drives a wedge between the marginal entrant's evaluation of the interest of entry and the social planner's.

If the sum of the taste for diversity and business stealing/expansion terms is negative then $\pi - f > W'(n)$ and private incentives to enter exceeds the social planner's. Using $\mathcal{L} = (p - w - qw')/p = \theta/\varepsilon_d$, $dq/dn = -MP_n/MP_q$ and $MP_q = (P_x/L)/APT$, the latter condition rewrites

$$p\mathcal{V} + Q\theta MP_n APT < 0. \tag{15}$$

In particular, when the latter condition holds at the free entry equilibrium, then there is excess entry on the market from a welfare point of view.[30] Under product homogeneity, the variety effect vanishes ($\mathcal{V} = 0$) and excess entry arises if and only if there is business stealing, i.e. $MP_n < 0$ (see Amir et al. (2014) for establishing rigorously this result in the case of Cournot oligopolies and for any cost function). When products are differentiated, excess entry remains as long as the variety effect $\mathcal{V}$ is not too strongly positive. Mankiw and Whinston (1986) have shown this result for the class of direct additive preferences *à la* Dixit and Stiglitz (1977).[31]

## 5.2 Cost pass-through in the long run

Consider the pass-through of an additive cost shock $t$ in the long run. The equilibrium is characterized by the behavioral response of firms and the zero profit condition:

$$\begin{aligned} P(x,n) + \theta x P_x(x,n) - w(q) - qw'(q) - t &= 0 \qquad (16) \\ (P(x,n) - w(q) - t)q - f &= 0. \qquad (17) \end{aligned}$$

Similarly, for a multiplicative cost shock, the long run equilibrium is characterized by:

$$\begin{aligned} P(x,n) + \theta x P_x(x,n) - \tau(w(q) + qw'(q)) &= 0 \qquad (18) \\ (P(x,n) - \tau w(q))q - f &= 0. \qquad (19) \end{aligned}$$

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[30] Indeed, denoting $n^*$ the social optimum and $n^e$ the equilibrium number of firms under free entry then $\pi - f > W'(n) \Rightarrow \pi(n^*) - f > W'(n^*) = 0 = \pi(n^e) - f$. Because the long run stability condition implies that $\pi'(n) < 0$ (see Proposition 5), then $n^e > n^*$.

[31] Bertoletti and Etro (2016) also study optimal market structures in a fairly general model with respect to preferences, but where (i) marginal cost is constant and more importantly (ii) labor is the only input and labor supply is inelastic, while in the present setting, labor supply is implicitly assumed to be elastic which is a wide-spread assumption in standard oligopoly models. See also Parenti et al. (2017) for a study of imperfect competition with labor as the sole input and a fixed labor supply.

Applying the Implicit Function Theorem to both cases, I identify the circumstances under which a cost shock is procompetitive or anticompetitive. The Proposition below also indicates the circumstances under which output per firm raises or not following the cost shock.

**Proposition 6.** *In the long run,*

**(i)** *an additive cost shock is procompetitive if and only if* $(1-\theta)APT > 1$ *while a multiplicative cost shock is procompetitive if and only if* $(1-\theta)APT > (\varepsilon - 1)/(\varepsilon - \lambda)$,

**(ii)** *an additive cost shock decreases the output per firm if and only if* $P_n - MP_n < 0$ *while a multiplicative cost shock decreases output per firm if and only if* $P_n < MP_n(\varepsilon - 1)/(\varepsilon - \lambda)$,

**(iii)** *Under IAVC (*$\lambda < 1$*), multiplicative shocks are more often pro-competitive than additive shocks, and they reduce output more frequently when business stealing dominates (*$MP_n < 0$*), but less frequently when business expansion prevails (*$MP_n > 0$*).*

**(iv)** *Conversely, under DAVC (*$\lambda > 1$*), multiplicative shocks are less often pro-competitive, and they reduce output less frequently under business stealing but more frequently under business expansion.*

*Proof.* See Appendix G. ■

Consider first the additive cost shock. Recall that $(1-\theta)APT > 1$ is precisely the condition given in Proposition 4, under which an additive cost shock raises profit keeping $n$ constant. Intuitively, the same condition evaluated at the long run equilibrium reveals that the additive cost shock is procompetitive (part (i)). For this to hold, a sufficiently strong overshifting of the cost shock is needed as $\theta < 1$, and an equivalent formulation of the condition is $\rho > 2 + \lambda\varepsilon_{\theta,q}$ using the definition of *APT* contained in Proposition 2.

As indicated in part (ii), the change in output $q$ depends on the sign of $P_n - MP_n$, which describes how the difference between the price and the perceived marginal profit changes with the number of competitors, holding $x$ constant. Note that $P_n - MP_n$ equivalently represents how the *perceived inverse marginal surplus per consumer*, i.e. $-\theta x P_x$, changes in $n$ holding $x$ constant. Hence, we have:

$$P_n - MP_n = \frac{\partial}{\partial n}\left(-\theta x P_x\right) = -\frac{\theta}{n} x P_x \left(\varepsilon_{P_x,n} + \varepsilon_{\theta,n}\right), \tag{20}$$

where $\varepsilon_{P_x,n} = nP_{xn}/P_x$ is the partial elasticity of the slope of the inverse demand w.r.t $n$. It follows that $P_n - MP_n < 0$ if and only if $\varepsilon_{P_x,n} + \varepsilon_{\theta,n} < 0$. Clearly, more competitors naturally means a raising competition intensity and thus a lower market conduct index ($\varepsilon_{\theta,n} < 0$), but the elasticity value depends on the mode of competition. The demand term $\varepsilon_{P_x,n}$ results of the consumers' preferences independently of the specification for the mode of competition. Moreover, the additive cost shock is output-neutral in the long-run if and only if $\varepsilon_{P_x,n} + \varepsilon_{\theta,n} = 0$. To illustrate, assuming linear demands, this is indeed the case under Nash in quantities competition and the competition effect and the demand effect are perfectly counterbalanced. On the contrary, under Nash in prices competition, we have $P_n - MP_n < 0$, thereby indicating that the additive cost shock decreases output per firm in the long-run.

By combining the two results above, let us now evaluate the price change by computing the long run absolute pass-through denoted as $APT^{LR}$. As shown below, it can be decomposed into the sum of an intensive and extensive margin terms:[32]

$$\begin{aligned} APT^{LR} &= \left.\frac{dp}{dt}\right|_{t=0} = \underbrace{\frac{P_x}{L}\left.\frac{dq}{dt}\right|_{t=0}}_{\text{Intensive margin}} + \underbrace{P_n\left.\frac{dn}{dt}\right|_{t=0}}_{\text{Extensive margin}} \\ &= 1 + \frac{\theta(P_n - MP_n)APT}{P_n - MP_n(1-\theta)APT}. \end{aligned} \tag{21}$$

It follows that overshifting of the additive cost shock occurs in the long run if and only if output per firm decreases, i.e. iff $P_n - MP_n < 0$. Note that perfect competition ($\theta = 0$) entails that $APT^{LR} = 1$ meaning that under free entry the cost shock is fully transmitted to consumers. Under imperfect competition, this situation also occurs whenever the cost shock is output-neutral in the long-run.

The possible scenarios for the industry in the long run resulting from the above observations are depicted in Figure 5. First, when the additive shock is procompetitive, it is also characterized by overshifting and this results in an ***fragmented*** industry with more varieties/firms but where each firm is smaller in terms of output (scenario 1).[33]

Second, when the additive cost shock is anticompetitive, the final outcome depends on whether there is over or undershifting (scenarios 2 or 3 respectively). Note that in scenario 3 (***market shake-out***, i.e. fewer varieties but each in larger quantities), it is theoretically possible that the additive cost shock ends up with a *negative pass-through on price*, i.e. $APT^{LR} < 0$. This outcome would correspond to a version of Edgeworth's taxation paradox according to which taxation can reduce market price of the good.[34] What are the conditions for a negative pass-through on price?

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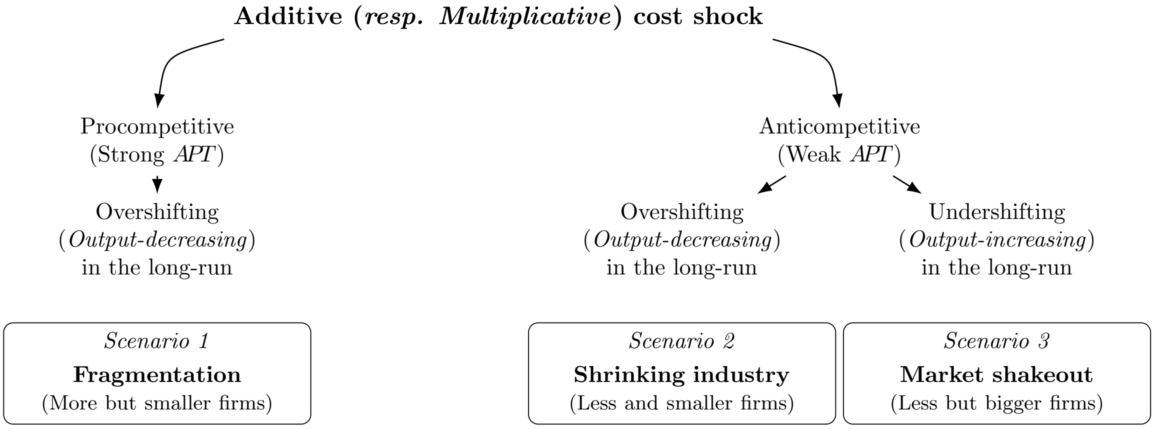
[32] See Appendix G for details of the derivation.

[33] Indeed, there are two possible cases. Either $MP_n > 0$ (business stealing), and then overshifting occurs as $P_n - MP_n < 0$. Or $MP_n < 0$ (business creation) and the stability condition $P_n < MP_n(1-\theta)APT$ implies that $P_n - MP_n < 0$ as $1 < (1-\theta)APT$ in scenario 1. Overall, cost overshifting always occurs in scenario 1.

[34] More precisely, Edgeworth's taxation paradox (1925) states that a unit tax can decrease the price of the taxed good



In the case of an **additive cost shock**, strong (weak) $APT$ means $APT > (<) 1/(1-\theta)$, and overshifting (undershifting) in the long-run means $MP_n > (<) P_n$. In the case of a ***multiplicative cost shock***, strong (weak) $APT$ means $APT > (\varepsilon - 1)/[(\varepsilon - \lambda)(1-\theta)]$, and output-decreasing (-increasing) in the long-run means $MP_n(\varepsilon - 1)/(\varepsilon - \lambda) > (<) P_n$.

Figure 5: Long-run scenarios following an additive or a multiplicative cost shock.

First, observe that business stealing ($MP_n < 0$) is necessary in scenario 3 for output per firm to increase in the long run, i.e. for $P_n - MP_n > 0$ to hold. Moreover, following a straightforward manipulation of (21) and using the stability condition, we obtain that $APT^{LR} < 0$ if and only if $0 > MP_n(1-\theta)APT > P_n > \max\left(MP_n, MP_n \frac{APT}{1+\theta}\right)$.[35] Hence, when $APT$ valued at the long run equilibrium is greater than $1+\theta$, the conditions for scenario 3 highlighted in Figure 5 already ensure that the long run pass-through is negative. When $APT < 1+\theta$, the condition $P_n > MP_n \frac{APT}{1+\theta}$ has to be added to obtain that $APT^{LR} < 0$.[36]

Let us now turn to the case of a multiplicative cost shock. Proposition 6 suggests that when marginal cost is constant (CAVC, $\lambda = 1$), there is no difference between an additive and a multiplicative cost shock in terms of the conditions describing the change in output per firm and number of firms under free entry/exit. However, if we consider IAVC ($\lambda < 1$), both conditions (i) and (ii) now involve the factor $(\varepsilon - 1)/(\varepsilon - \lambda) < 1$ and a multiplicative cost shock is more likely to be pro-competitive and to reduce output per firm than an additive cost shock. The opposite holds for DAVC. Figure 5 describes the different scenarios in a similar way as for the additive cost shock. However, the equivalence between overshifting and output reduction found for the additive cost shock does not hold for the multiplicative cost shock. This is why it is whether the shock is output-decreasing or increasing

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in the context of a multiproduct monopoly under conditions of complementarity between goods.

[35] As $\theta \in (0,1)$, we can check that $1-\theta < 1/(1+\theta)$.

[36] Note that the characteristics of scenario 3 where cost pass-through can become negative are opposite to those obtained by Ritz (2015). In that paper, Ritz considers firms that are heterogeneous in terms of constant marginal cost and shows that a unit tax can induce the entry of a new firm in the face of an incumbent monopoly and ultimately reduce the price within the resulting duopoly a la Cournot. The key conditions for this result are (i) the unitary tax can increase profit and (ii) entry reduces the price. Here, within symmetric oligopolies with endogenous entry, a cost pass-through can only become negative if the cost shock reduces profit and firm exit reduces price.

that drives the different scenarios.

Finally, the long-run relative cost pass-through, denoted $\mathit{RPT}^{LR}$, can be written as follows:[37]

$$\begin{aligned} \mathit{RPT}^{LR} &= \left.\frac{\tau}{P}\frac{dp}{d\tau}\right|_{\tau=1} \\ &= (1-\mathcal{C})\mathit{APT}^{LR} + \frac{1-\lambda}{\varepsilon}\frac{\theta P_n \mathit{APT}}{P_n - MP_n(1-\theta)\mathit{APT}} \end{aligned} \tag{22}$$

Under CAVC, (22) reduces to:

$$\mathit{RPT}^{LR} = (1-\mathcal{L})\mathit{APT}^{LR}$$

that is a *similar relationship* as for the short-run, which is given by (12). When $\lambda \neq 1$, however, $\mathit{RPT}^{LR}$ differs from $(1-\mathcal{C})\mathit{APT}^{LR}$ by an additional term whose sign is determined solely by the sign of $1-\lambda$. Hence, $\mathit{RPT}^{LR} > (1-\mathcal{C})\mathit{APT}^{LR}$ if and only if there is IAVC at the long-run equilibrium.

**Long-run pass-throughs in the stylized example.** In the stylized model, long-run equilibrium existence requires $d > -2b\nu$ and sufficiently large demand $(a - c > (b\nu + d)\sqrt{f/(b\nu + d/2)})$. A key feature is that the scale factor $\lambda = b\nu/(b\nu + d/2)$ coincides in the short and long run, as both $p-\text{MC}$ and $p-\text{AVC}$ scale linearly with firm size. Additive shocks are output-neutral and yield $\mathit{APT}^{LR} = 1$, while multiplicative shocks can be pro-competitive only under concave costs ($d < 0$), when entry and scale responses dominate the direct cost effect. The relative pass-through $\mathit{RPT}^{LR}$ is strictly positive for $d \geq 0$, but may become negative for admissible $d < 0$, generating an Edgeworth-type paradox where a higher ad valorem tax lowers the long-run equilibrium price. This paradox illustrates how cost curvature and free entry jointly shape pass-through outcomes in the long run.

### 5.3 Market expansion incidence

Let us now turn to the comparative statics of the short run and the long run equilibria w.r.t. market size $L$. As will be clear below, the contribution margin approach allows to obtain a set of sufficient statistics to describe the profit and output consequences in the short run as well as the consequences on the intensive and extensive margins in the industry in the long run.

**Short-run analysis.** To proceed, it is convenient to define the *ghost industry* here in a slightly different way than above.[38]

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[37] See Appendix G for details of the derivation.

[38] To save on notations, we nevertheless keep the same notation $\mathit{APT}_{ghost}$.

**Definition 4.** *The ghost industry is the industry with constant marginal cost equal to the equilibrium value of marginal cost and with $\theta$ being constant in output and equal to the equilibrium value of $\theta$. It follows that the APT for the ghost industry writes as:*

$$APT_{ghost} = APT|_{\lambda=1,\ \theta=cst} = \frac{1}{1+\theta-\theta\rho_d}.$$

As argued above, both industries produce the same amount. Let us denote the relative change in (variable) profit due to market expansion as $\varepsilon_{\pi,L} \equiv \frac{L}{\pi}\frac{d\pi}{dL}$ and accordingly the relative change in output as $\varepsilon_{q,L} = \frac{L}{q}\frac{dq}{dL}$. Straightforward computations lead to the results contained in the following Proposition.

**Proposition 7.** *Holding the number of firms constant,*

**(i)** *market expansion always raises output per firm according to $\varepsilon_{q,L} = APT/APT_{ghost} > 0$.*

**(ii)** *market expansion is price-decreasing if and only if $\varepsilon_{q,L} > 1$ or equivalently $APT > APT_{ghost}$.*

**(iii)** *the relative change in equilibrium variable profit resulting from changes in $L$ is given by $\varepsilon_{\pi,L} = \frac{\lambda}{\theta}\left[1-(1-\theta)\varepsilon_{q,L}\right]$.*

**(iv)** *market expansion is profit-enhancing if and only if $(1-\theta)APT < APT_{ghost}$.*

*Proof.* See Appendix H. ∎

The elasticity of output per firm w.r.t market size is governed by the comparison between the pass-through $APT$ for the oligopoly and the corresponding pass-through $APT_{ghost}$ for the ghost industry. The relative size of $\varepsilon_{q,L}$ with respect to 1 also determines whether market expansion is price-decreasing or price-increasing in the short run, and this actually depends on two potentially conflicting terms. To show this, let us compute the following quantity:

$$\frac{1}{APT} - \frac{1}{APT_{ghost}} = \frac{\theta}{\lambda}\left[\underbrace{\lambda\varepsilon_{\theta,q}}_{\text{competition intensity}} + \underbrace{(1-\lambda)(2-\rho_s)}_{\text{cost convexity/concavity}}\right]$$

which can be decomposed into a competition intensity term and a cost term whose sign depends on the convexity/concavity of $c(q)$. Consider first that $\theta$ is constant in output ($\varepsilon_{\theta,q} = 0$). The above comparison is such that $\varepsilon_{q,L} < 1$ if and only if the variable cost function $c(q)$ is convex.[39] Intuitively, with convex variable cost, market expansion raises output but at a moderate pace. Conversely, concave

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[39] Recall that $c(q)$ is convex if and only if $(1-\lambda)(2-\rho_s) > 0$.

variable costs ensure a strong output reaction to market expansion ($\varepsilon_{q,L} > 1$). Finally, under CAVC, then $\lambda = 1$ and thus $\varepsilon_{q,L} = 1$. In this case, output per firm is directly proportional to market size. Now, consider that higher quantities/lower prices create more competitive market conduct, i.e. $\varepsilon_{\theta,q} < 0$, then we have seen that this tends to make *APT* larger in Section 4 and in particular compared to $APT_{ghost}$, thereby favoring a strong output reaction market expansion ($\varepsilon_{q,L} > 1$).

Furthermore, Proposition 7 suggests that the profitability factor $\lambda$ has an interesting interpretation when the industry is perfectly colluding ($\theta = 1$) which echoes the result obtained by Bontems (2026) for the monopoly and monopolistic competition cases. Indeed, when $\theta = 1$ then $\lambda$ *represents the elasticity of (variable) profit with respect to market size at the equilibrium.* When $\theta \neq 1$ then $\lambda/\theta$ represents the partial elasticity of profit w.r.t. market size, holding $q$ constant.

Also, parts (iii) and (iv) state that $\varepsilon_{\pi,L} < 0$ if and only if $\varepsilon_{q,L} > 1/(1-\theta) > 1$ as $\theta \in (0,1)$ given the assumption of substitute products. It follows that market expansion is profit decreasing (holding constant the number of firms) whenever the induced expansion of production is large enough to cause a deleterious drop in prices. This situation is all the more plausible as the competition is strong, i.e. when $\theta$ is low.

**Market expansion in the long run.** As earlier, with free entry, the long run equilibrium is described by the zero profit condition (7) as well as the first-order condition (4). Applying the Implicit Function Theorem to this system of equations yields the following result.

**Proposition 8.** *At the long run equilibrium, market expansion*

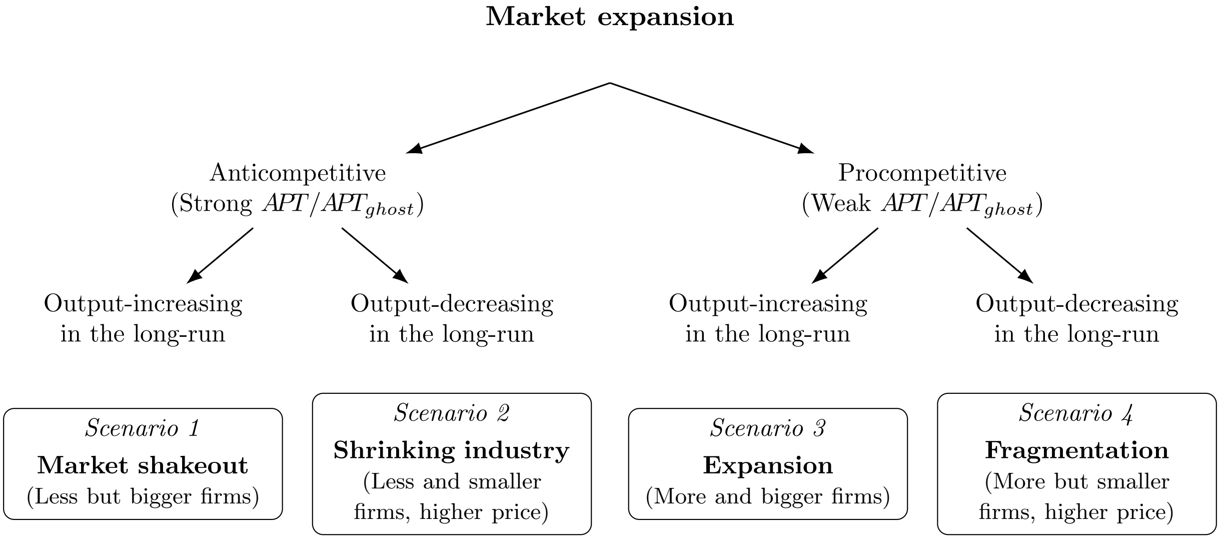
**(i)** *encourages entry if and only if* $(1-\theta)APT < APT_{ghost}$*,*

**(ii)** *raises output per firm if and only if* $P_n - MP_n APT_{ghost} < 0$*,*

**(iii)** *is price-increasing if* $P_n - MP_n APT_{ghost} > 0$*.*

*Proof.* See Appendix I. ■

Part (i) indicates that the condition under which market expansion raises profits holding $n$ constant intuitively implies that, in the long run, the number of firms increases with the size of the market. Market expansion is anticompetitive in the case of a sufficiently high value of the $APT/APT_{ghost}$ ratio, and procompetitive either. Part (ii) describes the necessary and sufficient condition for market



Strong (weak) $APT/APT_{ghost}$ means $APT/APT_{ghost} > (<) 1/(1-\theta)$. Output-decreasing (-increasing) means $MP_n < (>) P_n/APT_{ghost}$.

Figure 6: Scenarios following market expansion.

expansion to raise output per firm in the long run. Taken together, these two conditions generate four possible scenarios depicted in Figure 6.[40]

Note that under business expansion ($MP_n > 0$), necessarily the output size of firms grows with market size and either Scenario 1 (Market shakeout) or Scenario 3 (Expansion) occurs. Part (iii) indicates that whenever market expansion decreases output per firm, then the equilibrium price raises. This actually occurs in Scenario 2 (Shrinking industry) and Scenario 4 (Fragmentation). Whether market expansion decreases price in the long-run in the other scenarios depends on whether the long-run behavioral response in output is sufficiently stronger than its short-run counterpart.[41]

**Market expansion in the stylized example.** In the short-run, market expansion always raises output, decreases price if and only if DAVC prevails, and raises profit if $d$ is positive or not too negative. In the long-run, if $d > -b\nu$ then market expansion is always procompetitive and when $d$ is sufficiently negative, i.e. $d \in (-2b\nu, -b\nu)$, it is easy to find examples where it is anticompetitive. As market expansion always raises output in the long-run, it follows that only Scenarios 1 (Market shakeout) or Scenario 3 (Expansion) can occur at the long-run equilibrium, depending on whether market expansion encourages entry or not.

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[40] The impact of market expansion on welfare can also be studied in detail (see Online Appendix O2).

[41] See Appendix I.

## 6 Empirical implications and diagnostics

This section discusses how the contribution-margin framework can discipline empirical work. The objective is not to provide a complete structural estimation strategy for asymmetric industries. Rather, the results above deliver a set of reduced-form restrictions and diagnostic tests that can be applied around a symmetric industry equilibrium. These diagnostics are useful because they separate three objects that are often conflated in empirical applications: local market power, average profitability, and cost curvature.

**Testing constant marginal cost.** A first empirical implication concerns the hypothesis of locally constant marginal cost. Standard approaches to this question typically rely on flexible cost-function estimation, production-based markups, or structural supply models. Production-function approaches infer markups and marginal costs from production data, following the logic of Hall (1988), Roeger (1995), and Loecker and Warzynski (2012). Structural demand-and-supply models, starting from Berry et al. (1995), recover marginal costs from equilibrium pricing conditions. Conduct tests in the tradition of Bresnahan (1989) and Genesove and Mullin (1998) also rely on restrictions linking demand, costs, and pricing behavior. These methods are powerful, but they often require rich data and strong assumptions on demand, technology, or conduct.

The present framework delivers a complementary diagnostic. Under constant marginal cost, marginal cost and average variable cost coincide, so the Lerner index and the contribution margin index are equal. In the notation of the paper, this is equivalent to

$$\lambda = 1.$$

When $\lambda \neq 1$, the Lerner index is no longer a sufficient measure of profitability. If $\lambda < 1$, marginal cost exceeds average variable cost and the Lerner index understates the contribution margin. If $\lambda > 1$, marginal cost lies below average variable cost and the Lerner index overstates the contribution margin.

This observation gives a simple empirical test whenever prices, measures of average variable cost, and pass-through estimates are available. The contribution margin index can be measured directly as

$$\mathcal{C} = \frac{p - w}{p}.$$

The Lerner index can be recovered from the comparison between additive and multiplicative pass-through:

$$\mathcal{L} = 1 - \frac{RPT}{APT}.$$

The null hypothesis of locally constant marginal cost can then be written as

$$H_0: \quad \lambda = \frac{\mathcal{L}}{\mathcal{C}} = 1.$$

This test does not require estimating a full demand system. It requires a source of cost variation that identifies pass-through and an observable or constructible measure of average variable cost, such as variable COGS, wholesale acquisition costs, or product-level variable input costs.

The same logic also provides information about the slope of average variable cost. Conditional on the symmetric-equilibrium interpretation of the model, one can recover

$$\varepsilon_s = \frac{1-\mathcal{C}}{\mathcal{C}(1-\lambda)} = \frac{1-\mathcal{C}}{\mathcal{C}-\mathcal{L}}.$$

This object should not be interpreted as a fully structural technology parameter without additional assumptions. It is an equilibrium statistic summarizing how average variable cost varies with output near the observed equilibrium. It is therefore most useful for comparisons across products, markets, or time periods exposed to similar demand and cost shocks. This interpretation is close in spirit to recent empirical work emphasizing that cost curvature and scale economies matter for inference on markups and market power, such as Loecker et al. (2020), Duarte et al. (2025), and Kwon et al. (2023).

**Pass-through restrictions without cost data.** Even when average variable cost is not observed, the framework yields testable restrictions. Under constant marginal cost, the distinction between additive and multiplicative pass-through collapses in a specific way: the thresholds governing several short-run and long-run comparative statics coincide. When $\lambda \neq 1$, these thresholds diverge. Hence systematic differences between the implications of additive and multiplicative pass-through provide indirect evidence of non-constant marginal cost.

This diagnostic builds on the empirical pass-through literature, where cost shocks are used to infer market power, incidence, and the nature of competition. Exchange-rate shocks, commodity-price shocks, unit taxes, ad-valorem taxes, and wholesale price changes all provide potential sources of variation. The empirical content of the theory is not that pass-through is high or low in absolute terms, but that the relation between different pass-through measures reveals whether local markups and average profitability move together. In that respect, the framework complements the sufficient-statistic approach of Weyl and Fabinger (2013) and the empirical applications of pass-through to conduct and welfare analysis.

**Market expansion, entry, and concentration.** The framework also provides empirical guidance for studying market expansion and endogenous entry. The long-run results show that the effect of market size on prices, profits, entry, and concentration depends on a small number of objects: short-run pass-through, the profitability factor, the effect of entry on price, and the effect of entry on perceived marginal profit.

This structure can be used to organize empirical evidence on market-size effects. Consider a market expansion that increases the number of potential consumers. In the short run, holding the number of firms fixed, the effect on prices is governed by the same pass-through logic as a cost shock. In the long run, entry adjusts and may either reinforce or offset the short-run effect. Whether market expansion leads to fragmentation, concentration, a shrinking industry, or a market shakeout depends on how entry affects prices and marginal profitability.

These implications connect the paper to the empirical literature on market size, entry, and concentration. A large literature studies whether larger markets support more firms, lower prices, or higher product variety. The present framework adds that, when marginal cost is not constant, market-size effects also depend on how expansion moves firms along their cost schedules. This is relevant for interpreting empirical patterns in industries where scale economies, capacity utilization, or learning-by-doing are important. It also relates to recent work emphasizing the role of scale economies in the evolution of concentration, including Kwon et al. (2023).

These implications are not directly observable without additional structure. However, they can be taken to the data once pass-through is estimated from a cost shock and the signs of the entry effects are disciplined by entry variation, demand estimates, or institutional variation across markets. In particular, the model suggests comparing markets that differ in size but are otherwise similar, or studying shocks that expand demand while leaving the cost environment unchanged. The relevant empirical question is not only whether larger markets have more firms, but whether the induced entry changes prices and profitability in a way consistent with the pass-through statistics.

The same logic applies to concentration. Standard interpretations often view higher concentration as evidence of weaker competition or higher markups. With non-constant marginal cost, this inference is incomplete. A change in concentration may reflect changes in the profitability of entry generated by cost curvature, even if conduct is unchanged. The contribution-margin framework therefore suggests interpreting concentration jointly with prices, pass-through, and measures of variable profitability.

**Practical implementation.** The empirical implementation of the framework depends on the data available. With prices, quantities, and variable cost measures, one can construct the contribution margin index directly and compare it with a Lerner index recovered from pass-through. With prices and cost shocks but no cost data, one can still test whether additive and multiplicative pass-through obey the restrictions implied by constant marginal cost. With market-size variation and entry data, one can evaluate whether the observed response of entry and concentration is consistent with the long-run comparative statics.

The framework is therefore most useful as a diagnostic device. It identifies which empirical objects are needed to distinguish local market power from average profitability, and it clarifies when the Lerner index can or cannot be used as a sufficient statistic for profits. This distinction is especially important in industries with increasing returns, capacity utilization, learning-by-doing, or other mechanisms that make marginal cost vary with output.

## 7 Conclusion

This paper has studied symmetric oligopoly when marginal cost is not constant. In this environment, the Lerner index remains a local measure of market power, but it is no longer a sufficient measure of profitability. The relevant profitability object is the contribution margin index, defined relative to average variable cost. The wedge between the Lerner index and the contribution margin index summarizes how cost curvature separates local markups from average profitability.

The analysis shows that this wedge is not only an accounting correction. It also governs comparative statics. When marginal cost is non-constant, cost curvature affects the transmission of additive and multiplicative cost shocks, the interpretation of pass-through, and the long-run effects of market expansion and entry. Convex costs tend to dampen pass-through and make local markups understate variable profitability, while concave costs have the opposite effect. These mechanisms imply that the standard constant marginal cost benchmark can be misleading when firms move along non-flat cost schedules.

The contribution-margin approach also clarifies the relationship between market power, profitability, and concentration. Changes in concentration do not necessarily reflect changes in conduct or markups alone. They may also reflect changes in the profitability of entry generated by the cost structure. This distinction matters for interpreting market-size effects, entry responses, and the long-run consequences of cost shocks.

The framework is deliberately parsimonious. It focuses on symmetric oligopoly in order to obtain transparent sufficient statistics linking demand curvature, cost curvature, conduct, and entry. This makes the model a benchmark rather than a substitute for fully structural empirical analysis. The empirical diagnostics discussed above show how prices, pass-through, variable cost measures, and entry variation can be used to distinguish local market power from average profitability.

A natural next step is to extend the contribution-margin approach to asymmetric industries. Such an extension would allow firm-level differences in costs, market shares, and pass-through to interact with the wedge between Lerner margins and contribution margins. The results in this paper suggest that this wedge should remain central: whenever marginal cost is not constant, the distinction between local markups and average profitability is essential for the analysis of oligopoly.

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# Appendix

## A Examples of inverse demand functions satisfying Assumption 2

From $x = D(p,n)$ and $p = P(x,n)$, we have $D_n = -P_n/P_x$ and, as thanks to Assumption 1 $P_x < 0$, it is sufficient to check that $P_n \leq 0$.

**Linear inverse demands.** Consider the symmetric demand system based on linear inverse demands, $P_i(x_1,...,x_n) = \alpha - \beta x_i - \gamma \sum_{j \neq i} x_j$ with $\beta \geq \gamma > 0$. It follows that under symmetry, $P(x,n) = \alpha - \beta x - \gamma(n-1)x$ and $P_n(x,n) = -\gamma x < 0$.

**Inverse demands based on additive preferences.** Consider the additive and quasi linear preferences given by $z + V\left(\sum_i u(x_i)\right)$ where $z$ is the consumption of the outside good taken as the numeraire, $V(.)$ and $u(.)$ are increasing concave, so that inverse demand given by utility maximization under the budget constraint is $P_i(x_1,...,x_n) = u'(x_i)V'\left(\sum_i u(x_i)\right)$. Under symmetry, define $P(x,n) = u'(x)V'(nu(x))$ and it follows that $P_n(x,n) = u'(x)u(x)V''(nu(x)) < 0$.

**Inverse Logit demands.** Let us posit the inverse demand function for variety $i$ as a function of market shares given by:

$$\hat{P}_i(s_1,...s_n) = \frac{1}{\alpha}\left[\beta - \log \frac{s_i}{s_0}\right]$$

where $s_i = x_i/\left(z+X\right)$ is the market share of variety $i$, $X = \sum_i x_i$ is the aggregate consumption of differentiated products per head and $s_0 + \sum_i s_i = 1$ with $s_0$ being the market share of the outside good consumed in quantity $z$, i.e. $s_0 = z/(z+X)$. Under symmetry, $s_i = s = x/(z+nx)$ for all $i$ and we have

$$\hat{P}(s,n) = \frac{1}{\alpha}\left[\beta - \log \frac{s}{1-ns}\right]$$

and it follows that $\hat{P}_n(s,n) < 0$.

## B Notation

Table 1 summarizes the main notation used throughout the paper.

Table 1: Main notation

| Symbol | Meaning |
|---|---|
| $n$ | Number of firms or varieties |
| $L$ | Market size |
| $q$ | Output per firm |
| $x = q/L$ | Consumption per consumer and variety |
| $Q = nq$ | Industry output |
| $p = P(x, n)$ | Symmetric equilibrium price |
| $c(q)$ | Variable cost |
| $w(q) = c(q)/q$ | Average variable cost |
| $\varepsilon_d$ | Elasticity of industry demand |
| $\rho_d$ | Curvature of industry inverse demand |
| $\varepsilon_s$ | Elasticity associated with inverse supply $w(q)$ |
| $\rho_s$ | Curvature of inverse supply $w(q)$ |
| $\theta$ | Market conduct index |
| $\tilde{\varepsilon}_d = \varepsilon_d/\theta$ | Effective demand elasticity |
| $\mathcal{L}$ | Lerner index |
| $\mathcal{C}$ | Contribution margin index |
| $\lambda = \mathcal{L}/\mathcal{C}$ | Profitability factor |
| $\varepsilon = \lambda\tilde{\varepsilon}_d$ | Pseudo-elasticity governing the contribution margin |
| $\rho$ | Average curvature term combining demand and cost curvature |
| $APT$ | Absolute pass-through |
| $RPT$ | Relative pass-through |
| $APT^{LR}$ | Long-run absolute pass-through |
| $RPT^{LR}$ | Long-run relative pass-through |

# C Relationships between revenue, cost, demand and AVC elasticities

In this section, I connect the elasticities measures for industry demand and AVC to the ones one can define for the revenue and cost. Let us start by the demand side. From the equilibrium revenue function (per firm) $r(q) \equiv qP(q/L, n)$, straightforward manipulations allow to derive the elasticity $\varepsilon_r$ as a function of $\varepsilon_d$:

$$\varepsilon_r = \frac{qr'}{r} = \frac{\varepsilon_d - 1}{\varepsilon_d} \in (0, 1) \tag{23}$$

Now, let us define the *effective elasticity of revenue* by replacing $\varepsilon_d$ with the effective elasticity of demand $\tilde{\varepsilon}_d$ in (23):

$$\tilde{\varepsilon}_r = \frac{\tilde{\varepsilon}_d - 1}{\tilde{\varepsilon}_d} = 1 - \mathcal{L}. \tag{24}$$

Given that at the equilibrium necessarily the Lerner index belongs to $(0, 1)$ then the equilibrium value of the effective elasticity of revenue $\tilde{\varepsilon}_r$ is lower than unity.

For the cost side, I similarly obtain the following relationships between the elasticity of cost $\varepsilon_c$ and the elasticity of "supply" $\varepsilon_s$:

$$\varepsilon_c = \frac{qc'}{c} = \frac{\varepsilon_s + 1}{\varepsilon_s} > 0 \tag{25}$$

The consequence of this connection is that the contribution margin approach can be alternatively expressed in function of revenue and cost instead of demand and AVC. From (24) and (25) I respectively deduce that

$$\tilde{\varepsilon}_d = \frac{1}{1 - \tilde{\varepsilon}_r} > 1 \text{ and } \varepsilon_s = \frac{1}{\varepsilon_c - 1}.$$

Replacing in the expression of $\lambda = (\varepsilon_s + 1)/(\varepsilon_s + \tilde{\varepsilon}_d)$, I obtain

$$\lambda = \frac{\varepsilon_c \left(1 - \tilde{\varepsilon}_r\right)}{\varepsilon_c - \tilde{\varepsilon}_r}$$

As $\lambda$ is strictly positive and $\tilde{\varepsilon}_r \in (0, 1)$ at the equilibrium, this implies that $\varepsilon_c > \tilde{\varepsilon}_r$. Moreover, under DAVC, $\lambda > 1$ entails that $\varepsilon_c < 1$. Similarly under IAVC, $\lambda < 1$ entails that $\varepsilon_c > 1$. And finally, a constant marginal cost is equivalent to have $\varepsilon_c$ constant and equals to unity as well as $\lambda$. The contribution margin ratio writes

$$\mathcal{C} = \frac{p - w}{p} = \frac{r - c}{r} = \frac{1}{\varepsilon}$$

with

$$\varepsilon = \lambda \tilde{\varepsilon}_d = \frac{\varepsilon_c}{\varepsilon_c - \tilde{\varepsilon}_r} > 1.$$

# D The stability condition

We have

$$MP_q = \frac{1}{L}(1 + \theta + q\theta_q)P_x(x, n) + \frac{1}{L}\theta x P_{xx}(x, n) - 2w'(q) - qw''(q) \tag{26}$$

Dropping arguments for the sake of simplicity, and using the definition of $\rho_d$ and $\varepsilon_s$, we get

$$MP_q = \frac{1}{L}(1 + \theta + q\theta_q)P_x - \frac{1}{L}\theta\rho_d P_x - (2 - \rho_s)\, w'$$

Recall that the first-order condition for firms writes $P - w + \theta x P_x - qw' = 0$. As $P - w = P/(\lambda\tilde{\varepsilon}_d) = P/(-\lambda P/\theta x P_x) = -\theta x P_x/\lambda$ and replacing in the first-order condition, we get

$$\frac{1}{L}\theta \left(1 - \frac{1}{\lambda}\right) P_x = w' \tag{27}$$

Replacing $w'$ in (26) leads to

$$MP_q = \frac{1}{L}(1 + \theta + q\theta_q)P_x - \frac{1}{L}\theta\rho_d P_x - \frac{1}{L}\theta \left(1 - \frac{1}{\lambda}\right) P_x \left(2 - \rho_s\right)$$

and rearranging we finally get

$$MP_q = \frac{\theta P_x}{\lambda L}\left[\lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right) + 2 - \rho\right]$$

# E Cost pass-through formulas

APT derivation: The first-order conditions at the symmetric equilibrium with a specific tax $t$ all write $P + \theta(q/L)P_x(q/L) = w(q) + qw'(q) + t$ and their total differentiation subsequently taken in $t = 0$ yields $MP_q dq = dt$ and $MP_q$ takes the form given by (5) when $t = 0$. As $APT = \left.\frac{dp}{dt}\right|_{t=0} = \frac{1}{L}P_x \left.\frac{dq}{dt}\right|_{t=0} = \frac{P_x/L}{MP_q} > 0$ following the stability condition (6), we obtain the desired result.

RPT derivation: The first-order conditions at the symmetric equilibrium with an ad-valorem tax $\tau$ all write $P + \theta(q/L)P_x(q/L) = \tau w(q) + \tau q w'(q)$ and their total differentiation subsequently taken in $\tau = 1$ yields $MP_q dq = \left(w(q) + qw'(q)\right) d\tau$. We thus have:

$$\frac{d\log q}{d\tau} = \frac{w(1 + 1/\varepsilon_s)}{qMP_q} = -\tilde{\varepsilon}_d \frac{w}{P}\frac{1 + 1/\varepsilon_s}{\frac{1}{\lambda}\left[\lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right) + 2 - \rho\right]} \tag{28}$$

using $\varepsilon_s = w/qw'$, $\tilde{\varepsilon}_d = -P/\theta(q/L)P_x$ and (5). Recall that $\lambda\tilde{\varepsilon}_d = \varepsilon$ and that, from the definition of the $\mathcal{C}$-index in Proposition 1, we get $w/P = (\varepsilon - 1)/\varepsilon$. Also, from the definition of $\lambda$ in Proposition 1, we have $\varepsilon_s = (\varepsilon - 1)/(1 - \lambda)$. Replacing all these terms in (28) and rearranging, we obtain that

$$\frac{d\log q}{d\tau} = -\frac{\varepsilon - \lambda}{\left[\lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right) + 2 - \rho\right]}$$

and from

$$\left.\frac{d\log P}{d\log\tau}\right|_{\tau=1}=\frac{1}{P}\frac{dp}{d\tau}=\frac{P_x}{LP}\frac{dq}{d\tau}=-\frac{1}{\theta\tilde{\varepsilon}_d}\frac{d\log q}{d\log\tau}\tag{29}$$

we obtain the desired result:

$$\left.\frac{d\log P}{d\log\tau}\right|_{\tau=1}=\frac{\varepsilon-\lambda}{\varepsilon}\frac{\lambda}{\theta\left(2-\rho\right)+\lambda\left(1-\theta+\theta\varepsilon_{\theta,q}\right)}>0.$$

Derivation for perfect competition/Bertrand: note that for constant $\theta$, we have

$$APT=\frac{\lambda}{\theta(2-\rho)+\lambda\left(1-\theta\right)}$$

Under perfect competition/Bertrand with homogenous products, $\theta=0$ and when $\theta\to0$, then $\lambda\to0$. Also the denominator $\theta(2-\rho)+\lambda\left(1-\theta\right)$ tends towards 0. Applying L'Hospital rule, we get

$$\frac{\partial\lambda}{\partial\theta}=\frac{\partial\lambda}{\partial\tilde{\varepsilon}_d}\frac{\partial\tilde{\varepsilon}_d}{\partial\theta}=-\frac{(\varepsilon_s+1)}{(\varepsilon_s+\tilde{\varepsilon}_d)^2}\left(-\frac{\varepsilon_d}{\theta^2}\right)=\frac{(\varepsilon_s+1)\varepsilon_d}{(\theta\varepsilon_s+\varepsilon_d)^2}\underset{\theta\to0}{\longrightarrow}\frac{\varepsilon_s+1}{\varepsilon_d}$$

and

$$\begin{aligned}\frac{\partial}{\partial\theta}\left(\theta(2-\rho)+\lambda\left(1-\theta\right)\right) &= 2-\rho+\theta\left(2-\frac{\partial\lambda}{\partial\theta}\rho_d+\frac{\partial\lambda}{\partial\theta}\rho_s\right)-\lambda+\frac{\partial\lambda}{\partial\theta}\left(1-\theta\right)\\ \underset{\theta\to0}{\longrightarrow}2-\rho_s+\left.\frac{\partial\lambda}{\partial\theta}\right|_{\theta=0} &= 2-\rho_s+\frac{\varepsilon_s+1}{\varepsilon_d}\end{aligned}$$

We thus obtain that $APT\underset{\theta\to0}{\longrightarrow}\frac{\left.\frac{\partial\lambda}{\partial\theta}\right|_{\theta=0}}{\left.\frac{\partial}{\partial\theta}\left(\theta(2-\rho)+\lambda(1-\theta)\right)\right|_{\theta=0}}=\frac{\frac{\varepsilon_s+1}{\varepsilon_d}}{2-\rho_s+\frac{\varepsilon_s+1}{\varepsilon_d}}$ which simplifies into the formula in the text.

# F Proof of Proposition 4

Consider the additive cost shock $t$. In this case, equilibrium profit writes $\pi=(P(x,n)-w(q)-t)q$. Differentiating totally, we have $d\pi=(\partial\pi/\partial q)dq+(\partial\pi/\partial t)dt$ and thus

$$\left.\frac{d\pi}{dt}\right|_{t=0}=(1-\theta)xP_x\left.\frac{dq}{dt}\right|_{t=0}-q=(1-\theta)xP_x\frac{1}{MP_q}-q=q\left[(1-\theta)APT-1\right]$$

and the result follows.

Now consider the multiplicative cost shock $\tau$, then $\pi=(P(x,n)-\tau w(q))q$. Differentiating totally, we have $d\pi=(\partial\pi/\partial q)dq+(\partial\pi/\partial\tau)d\tau$ and the relative change of profit taken in $\tau=1$ is such that:

$$\begin{aligned}\left.\frac{\tau}{\pi}\frac{d\pi}{d\tau}\right|_{\tau=1} &= \frac{1}{\pi}(1-\theta)xP_x\left.\frac{dq}{d\tau}\right|_{\tau=1}-\frac{qw}{\pi}\\ &= \frac{1}{P-w}(1-\theta)xP_x\left.\left(\frac{\tau}{q}\frac{dq}{d\tau}\right)\right|_{\tau=1}-\frac{w}{P-w}\end{aligned}$$

Recall that

$$P-w=-\frac{\theta}{\lambda}xP_x$$

and

$$\frac{w}{P-w}=\varepsilon-1.$$

Replacing and rearranging, we get

$$\begin{aligned}\left.\frac{\tau}{\pi}\frac{d\pi}{d\tau}\right|_{\tau=1} &= -\frac{\lambda}{\theta}(1-\theta)\left.\left(\frac{\tau}{q}\frac{dq}{d\tau}\right)\right|_{\tau=1}-(\varepsilon-1)\\ &= (1-\theta)\varepsilon RPT-(\varepsilon-1)\end{aligned}$$

where the second line uses (29) and hence the result follows, recalling that $\mathcal{C}=1/\varepsilon$.

# G Proof of Proposition 6

Consider first the system (16)-(17) corresponding to the additive cost shock. Differentiating it totally and taking $t = 0$ yields:

$$M \begin{pmatrix} dq \\ dn \end{pmatrix} = \begin{pmatrix} 1 \\ q \end{pmatrix} dt \tag{30}$$

Note that from (11) and (5) we have that $MP_q = \frac{P_x/L}{APT}$, and thus the matrix $M$ can be rewritten as:

$$M = \begin{pmatrix} \frac{P_x/L}{APT} & MP_n \\ (1-\theta)xP_x & qP_n \end{pmatrix}.$$

Solving the linear system (30), we find that the entry effect is given by:

$$\frac{dn}{dt} = \frac{\frac{1}{APT}xP_x - (1-\theta)xP_x}{\frac{1}{APT}xP_xP_n - (1-\theta)xP_xMP_n} = \frac{1-(1-\theta)APT}{P_n - MP_n(1-\theta)APT}$$

where the denominator is negative following the stability condition (10). Indeed, using $MP_q = \frac{P_x/L}{APT}$ and rearranging, the second condition in (10) rewrites

$$P_n - MP_n(1-\theta)APT < 0.$$

It follows that the introduction of the tax is procompetitive if and only if $1-(1-\theta)APT < 0$. Also the output effect is given by:

$$\frac{dq}{dt} = \frac{qP_n - qMP_n}{\frac{1}{APT}xP_xP_n - (1-\theta)xP_xMP_n} = \frac{APT}{P_x/L}\frac{P_n - MP_n}{P_n - MP_n(1-\theta)APT}$$

and $t$ increases the output per firm if and only if $P_n - MP_n > 0$.

The long run price effect of the additive cost shock is evaluated through:

$$\begin{aligned} APT^{LR} &= \left.\frac{dp}{dt}\right|_{t=0} = \underbrace{\frac{P_x}{L}\left.\frac{dq}{dt}\right|_{t=0}}_{\text{Intensive margin}} + \underbrace{P_n\left.\frac{dn}{dt}\right|_{t=0}}_{\text{Extensive margin}} \\ &= APT\frac{P_n - MP_n}{P_n - MP_n(1-\theta)APT} + P_n\frac{1-(1-\theta)APT}{P_n - MP_n(1-\theta)APT} \\ &= 1 + \frac{\theta APT(P_n - MP_n)}{P_n - MP_n(1-\theta)APT} \end{aligned} \tag{31}$$

Now consider the multiplicative cost shock and the associated system of equations (18)-(19). Totally differentiating it and taking $\tau = 1$ yields:

$$M \begin{pmatrix} dq \\ dn \end{pmatrix} = \begin{pmatrix} w + qw' \\ qw \end{pmatrix} d\tau$$

and hence the entry effect is given by:

$$\frac{dn}{d\tau} = \frac{\frac{1}{APT}\frac{qw}{L}P_x - (1-\theta)\frac{q(w+qw')}{L}P_x}{\frac{1}{APT}\frac{q}{L}P_xP_n - (1-\theta)\frac{q}{L}P_xMP_n} \tag{32}$$

Recalling that $\varepsilon_s = w(q)/qw'(q) = (\varepsilon-1)/(1-\lambda)$, we have that $w + qw' = w(1+1/\varepsilon_s) = w\frac{\varepsilon-\lambda}{\varepsilon-1}$ and rearranging (32) yields

$$\frac{dn}{d\tau} = w\frac{1-(1-\theta)\frac{\varepsilon-\lambda}{\varepsilon-1}APT}{P_n - MP_n(1-\theta)APT}$$

Hence, the multiplicative cost shock increases the number of firms if and only if $(1-\theta)\frac{\varepsilon-\lambda}{\varepsilon-1}APT > 1$, which is exactly the condition for having a profit-enhancing cost shock in the short run (see Proposition 4).[42]

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[42] To check this, note that $\frac{\varepsilon-\lambda}{\varepsilon-1} = \frac{1-\lambda/\varepsilon}{1-\varepsilon} = \frac{1-\mathcal{L}}{1-\mathcal{C}}$ and recall that $RPT = (1-\mathcal{L})APT$.

Also the output effect is given by:

$$\frac{dq}{d\tau} = \frac{(w+qw')qP_n - qwMP_n}{\frac{1}{APT}\frac{q}{L}P_xP_n - (1-\theta)\frac{q}{L}P_xMP_n} = w\frac{APT}{P_x/L}\frac{\frac{\varepsilon-\lambda}{\varepsilon-1}P_n - MP_n}{P_n - MP_n(1-\theta)APT}$$

The multiplicative cost shock $\tau$ increases the output per firm if and only if $\frac{\varepsilon-\lambda}{\varepsilon-1}P_n - MP_n > 0$.

Finally, the long-run price effect of the multiplicative cost shock is evaluated through:

$$\begin{aligned} RPT^{LR} &= \frac{\tau}{P}\frac{dp}{d\tau}\bigg|_{\tau=1} = \frac{P_x/L}{P}\frac{dq}{d\tau}\bigg|_{\tau=1} + \frac{P_n}{P}\frac{dn}{d\tau}\bigg|_{\tau=1} \\ &= \frac{wAPT}{P}\frac{\frac{\varepsilon-\lambda}{\varepsilon-1}P_n - MP_n}{P_n - MP_n(1-\theta)APT} + \frac{wP_n}{P}\frac{1-(1-\theta)\frac{\varepsilon-\lambda}{\varepsilon-1}APT}{P_n - MP_n(1-\theta)APT} \\ &= \frac{w}{P}\left(APT\frac{\frac{\varepsilon-\lambda}{\varepsilon-1}P_n - MP_n}{P_n - MP_n(1-\theta)APT} + P_n\frac{1-(1-\theta)\frac{\varepsilon-\lambda}{\varepsilon-1}APT}{P_n - MP_n(1-\theta)APT}\right) \end{aligned}$$

Rearranging and using (31) and $\frac{w}{P} = 1 - \mathcal{C}$, we obtain that:

$$RPT^{LR} = (1-\mathcal{C})APT^{LR} + \frac{1-\lambda}{\varepsilon}\frac{\theta P_n APT}{P_n - MP_n(1-\theta)APT}$$

Part (i) and (ii) of Proposition 6 follow from the above results. Part (iii) and part (iv) are immediately obtained by noting that $\frac{\varepsilon-\lambda}{\varepsilon-1} > (<)1$ when $\lambda < (>)1$.

# H Proof of Proposition 7

Part (i): The first order condition for a symmetric equilibrium writes

$$P(q/L,n) + \theta(q/L)P_x(q/L,n) - w(q) - qw'(q) = 0 \tag{33}$$

Let us differentiate totally (33) to obtain:

$$P_x d(q/L) + (q/L)P_x\theta_q dq + \theta\left(P_x + (q/L)P_{xx}\right)d(q/L) = \left(2w' + qw''\right)dq$$

which becomes, using $d(q/L) = (1/L)dq - (q/L^2)dL$ and the definition of $MP_q$:

$$MP_q dq = (q/L^2)\left[P_x + \theta\left(P_x + (q/L)P_{xx}\right)\right]dL$$

Recalling that $MP_q = \frac{P_x/L}{APT}$ and rearranging, we get:

$$\varepsilon_{q,L} = \frac{L}{q}\frac{dq}{dL} = \left(1 + \theta - \theta\rho_d\right)APT.$$

Part (ii): it follows from computing the market size elasticity of price as:

$$\begin{aligned} \varepsilon_{p,L} &= \frac{L}{p}\left(\frac{dP(q/L,n)}{dL}\right) = \frac{LP_x}{p}\left(\frac{1}{L}\frac{dq}{dL} - \frac{q}{L^2}\right) \\ &= \frac{xP_x}{p}\left(\varepsilon_{q,L} - 1\right) \end{aligned}$$

and the desired result follows.

Part (iii): Differentiating profit totally, we obtain $d\pi = (\partial\pi/\partial q)dq + (\partial\pi/\partial L)dL$, where

$$\frac{\partial\pi}{\partial q} = xP_x + P - qw' - w \tag{34}$$

$$\frac{\partial\pi}{\partial L} = -(q^2/L^2)P_x \tag{35}$$

Substituting (33) into (34) yields

$$\frac{\partial \pi}{\partial q} = (1-\theta) x P_x$$

Hence, we have

$$\begin{aligned} \varepsilon_{\pi,L} &= (1-\theta)\frac{L}{\pi} x P_x \frac{dq}{dL} - \frac{L}{\pi}\frac{q^2}{L^2}P_x = \frac{L}{\pi}\frac{q^2 P_x}{L^2}\left[-1+(1-\theta)\frac{L}{q}\frac{dq}{dL}\right] \\ &= \frac{xP_x}{P-w}\left[-1+(1-\theta)\varepsilon_{q,L}\right] \end{aligned}$$

From the first order condition, we also have

$$P - w = qw' - \theta(q/L)P_x = \frac{q}{L}\left[\frac{qw'}{(q/L)P_x} - \theta\right]P_x = \frac{q}{L}\left[\frac{qw'}{w}\frac{w}{P}\frac{P}{(q/L)P_x} - \theta\right]P_x$$

Recall that $\varepsilon_s = w/qw' = (\varepsilon-1)/(1-\lambda)$, $w/P = (\varepsilon-1)/\varepsilon$ and that $\varepsilon_d = -P/(q/L)P_x = \theta\varepsilon/\lambda$. Hence, replacing and rearranging, we have

$$P - w = -\theta\left[\frac{1-\lambda}{\lambda} + 1\right] xP_x$$

so that finally, the relative change in profit writes:

$$\varepsilon_{\pi,L} = -\frac{1}{\theta\left[\frac{1-\lambda}{\lambda}+1\right]}\left[-1+(1-\theta)\varepsilon_{q,L}\right] = \frac{\lambda}{\theta}\left[1-(1-\theta)\varepsilon_{q,L}\right].$$

Part (iv) follows straightforwardly from parts (i) and (iii).

# I Proof of Proposition 8

Let us differentiate totally the system above to see how $q$ and $n$ change in $L$ at the long run equilibrium. We get

$$M\begin{pmatrix} dq \\ dn \end{pmatrix} = \begin{pmatrix} \frac{q}{L^2}\left(P_x + \theta(P_x + \frac{q}{L}P_{xx})\right) \\ \frac{q^2}{L^2}P_x \end{pmatrix} dL \tag{36}$$

Observe that

$$\frac{q}{L^2}\left(P_x + \theta(P_x + \frac{q}{L}P_{xx})\right) = \frac{q}{L^2}P_x\left(1+\theta-\theta\rho_d\right) = \frac{q}{L^2}\frac{P_x}{APT_{ghost}}$$

Solving the system (36), we get

$$\begin{aligned} \varepsilon_{q,L}^{LR} &= \frac{APT}{APT_{ghost}}\frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT} \\ &= \varepsilon_{q,L}\frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT} \end{aligned} \tag{37}$$

Hence, the long run elasticity of output w.r.t market size is the short run elasticity factorized by $\frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT}$. And we have

$$\begin{aligned} \varepsilon_{q,L}^{LR} &> \varepsilon_{q,L} \Leftrightarrow \frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT} > 1 \Leftrightarrow P_n - MP_n APT_{ghost} < P_n - MP_n(1-\theta)APT \\ &\Leftrightarrow -MP_n APT_{ghost} < -MP_n(1-\theta)APT \end{aligned}$$

Moreover, we get

$$\varepsilon_{n,L} = \frac{xP_x}{nP_n}\left(1-(1-\theta)\varepsilon_{q,L}\right) \tag{38}$$

Finally, the impact on price is such that:

$$\frac{dp}{dL} = P_x\frac{dx}{dL} + P_n\frac{dn}{dL} = P_x\left(\frac{1}{L}\frac{dq}{dL} - \frac{q}{L^2}\right) + P_n\frac{dn}{dL}$$

and thus in terms of elasticities,

$$\varepsilon_{p,L} = \frac{1}{\varepsilon_d}\left(1 - \varepsilon_{q,L}\right) + \frac{nP_n}{p}\varepsilon_{n,L} \tag{39}$$

Using (37) and (38) and replacing in (39), we get

$$\begin{aligned}
\varepsilon_{p,L} &= \frac{1}{\varepsilon_d}\left(1 - \frac{APT}{APT_{ghost}}\frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT}\right) + \frac{nP_n}{p}\frac{xP_x}{nP_n}\left(1 - (1-\theta)\frac{APT}{APT_{ghost}}\right) && (40)\\
&= \frac{1}{\varepsilon_d}\frac{APT}{APT_{ghost}}\left(1 - \theta - \frac{P_n - MP_n APT_{ghost}}{P_n - MP_n(1-\theta)APT}\right) && (41)
\end{aligned}$$

Hence, if $P_n - MP_n APT_{ghost} > 0$ then $\varepsilon_{p,L} > 0$. Otherwise, using (37), the sign of $\varepsilon_{p,L}$ depends only on the sign of the term in brackets in (41), i.e. $1 - \theta - \frac{\varepsilon^{LR}_{q,L}}{\varepsilon^{SR}_{q,L}}$. Hence, market expansion raises price in the long-run iff $\varepsilon^{LR}_{q,L} < (1-\theta)\varepsilon^{SR}_{q,L}$.

# Online Appendix to
# "A Contribution Margin Approach to Imperfect Competition"

## O1 Mode of competition

In this Appendix, I provide a detailed account of how the index $\theta$ of competitive intensity, centered on the focal point of overall industry profit maximization, is constructed. Let us write the profit for any industry's member $i$ as $\pi_i = (p_i - w_i(q_i))\, q_i$, keeping for the moment the possibility of asymmetry in costs and in demand system, and consider the impact of a change $d\sigma_i$ in the strategic variable $\sigma_i$ of firm $i$ on industry's aggregate profit, which can be decomposed as the sum of a pecuniary effect and a real effect as follows:

$$\frac{d}{d\sigma_i}\left(\sum_j \pi_j\right) = \underbrace{\sum_j q_j \frac{dp_j}{d\sigma_i}}_{\text{pecuniary effect}} + \underbrace{\sum_j (p_j - w_j - q_j w_j')\frac{dq_j}{d\sigma_i}}_{\text{real effect}} \tag{O1}$$

denoting $w_j = w(q_j)$. Let us define $\theta_i$ as the ratio between the real effect and the pecuniary effect:

$$\theta_i = \frac{\sum_j (p_j - w_j - q_j w_j')\frac{dq_j}{d\sigma_i}}{-\sum_j q_j \frac{dp_j}{d\sigma_i}} \tag{O2}$$

which allows to rewrite (O1) as follows:

$$\frac{d}{d\sigma_i}\left(\sum_j \pi_j\right) = (1-\theta_i)\sum_j q_j \frac{dp_j}{d\sigma_i}$$

Observe that if the strategic choice $\sigma_i$ maximizes industry's profit then this change $d\sigma_i$ in strategy would be perfectly internalized so that $\theta_i = 1$ in that case. Besides this focal point, $\theta_i$ diverges in general from unity and it can be lower or greater than 1 as will be clearer below.

When the strategic choice maximizes firm $i$'s profit for any $i$, then using the corresponding first-order condition, we can obtain an expression of the mark-up $m_i = p_i - w_i - q_i w_i'$. Indeed

$$\frac{d\pi_i}{d\sigma_i} = 0 \Leftrightarrow m_i = -q_i \frac{dp_i}{d\sigma_i} \Big/ \frac{dq_i}{d\sigma_i}.$$

Substituting in (O2) and rearranging, one can rewrite $\theta_i$ as follows:

$$\theta_i = \sum_j \left(\frac{q_j \frac{dp_j}{d\sigma_j}}{\sum_j q_j \frac{dp_j}{d\sigma_i}}\right) \frac{\frac{dq_j}{d\sigma_i}}{\frac{dq_j}{d\sigma_j}}. \tag{O3}$$

When considering a symmetric equilibrium with $p_i = p$, $q_i = q = Q/n$ and $x_i = x = q/L$ as well as $w_i(q_i) = w(q)$ for all $i$, let us denote $\theta \equiv \theta_i$ for any $i$ as the market conduct index that captures the intensity of competition in the game for profit-maximising firms. A more concise expression for $\theta$ can be obtained by denoting the own price derivative $\frac{dp_j}{d\sigma_j} = p_\sigma^o$ for all $j$ and the cross price derivative $\frac{dp_j}{d\sigma_i} = p_\sigma^c$ for all $i \neq j$ and using similar notations for the own and cross quantity derivative, i.e. $\frac{dq_j}{d\sigma_j} = q_\sigma^o$ and $\frac{dq_j}{d\sigma_i} = q_\sigma^c$. Then (O3) becomes under symmetry:

$$\theta = \frac{1 + (n-1)q_\sigma^c / q_\sigma^o}{1 + (n-1)p_\sigma^c / p_\sigma^o}. \tag{O4}$$

Consider the case of a homogenous product oligopoly. In that case, when a firm $i$ chooses its quantity, it assumes that a change $dq_i$ will induce a change $dQ = \nu dq_i$ in aggregate output (or equivalently that each other firm $j$ will change its quantity by $dq_j = \frac{\nu-1}{n-1}dq_i$ in response). Cournot competition corresponds to $\nu = 1$, Bertrand competition to $\nu = 0$ and perfect collusion to $\nu = n$.

More generally, when $\nu$ is a constant that belongs to $[0,n]$ (as in Delipalla and Keen (1992) conjectural variation model also denoted "Generalized Cournot model"), then (O4) gives $\theta = \nu/n$.[1]

Now, consider a symmetrically differentiated product oligopoly where firms compete in quantities. The inverse demand for a variety $i$ is $p_i = P_i(x_i, x_{-i})$ and from (O4), one obtains:[2]

$$\theta = \frac{\partial P_i}{\partial x_i} / \sum_j \frac{\partial P_j}{\partial x_i} > 0. \tag{O5}$$

If the inverse demand system is linear then $\theta$ is constant.

Finally, for a symmetrically differentiated product oligopoly where firms compete in prices, the demand for variety $i$ is $q_i = LD_i(p_1, ..., p_n)$ and from (O4), one gets:

$$\theta = \sum_j \frac{\partial D_j}{\partial p_i} / \frac{\partial D_i}{\partial p_i} = 1 - A > 0 \tag{O6}$$

where $A = \sum_{j \neq i} d_{ij}$ where $d_{ij} = -\frac{\partial D_j}{\partial p_i} / \frac{\partial D_i}{\partial p_i} \geq 0$ is the price diversion ratio from $i$ to $j$. $A$ is the aggregate diversion ratio from any individual firm to the rest of the industry, and it represents the fraction of sales lost by a firm when it increases its price and that is captured by the competitors (Shapiro, 1995). If the demand system is linear then $A$ and thus $\theta$ are constant.[3]

## O2 Welfare incidence of market expansion

The welfare incidence of market expansion in the long run is given by:

$$\begin{aligned} \frac{dW}{dL} &= \frac{d}{dL}\left(LU(x,n) - nqw(q) - nf\right) \\ &= U + LU_x \frac{dx}{dL} + LU_n \frac{dn}{dL} - qw(q)\frac{dn}{dL} - nc'(q)\frac{dq}{dL} - f\frac{dn}{dL}. \end{aligned}$$

Using $U_x = np$ and $\frac{dx}{dL} = \frac{1}{L}\frac{dq}{dL} - \frac{q}{L^2}$ and replacing, we get

$$\frac{dW}{dL} = U - p\frac{Q}{L} + n(p - c'(q))\frac{dq}{dL} + (LU_n - qw - f)\frac{dn}{dL}$$

Using the zero-profit condition, $(p-w)q = f$, and the definition of consumer surplus, $CS = LU - pQ$, we get

$$\begin{aligned} \frac{dW}{dL} &= \frac{CS}{L} + n(p - c'(q))\frac{dq}{dL} + (LU_n - pq)\frac{dn}{dL} \\ &= \frac{CS}{L} + np\mathcal{L}\frac{dq}{dL} + p\mathcal{V}\frac{dn}{dL} \\ &= \frac{CS}{L} + \frac{np}{L}\left(\mathcal{L}q\varepsilon_{q,L} + \mathcal{V}_0\varepsilon_{n,L}\right) \end{aligned}$$

The welfare incidence of market expansion exceeds the additional consumer surplus, $CS/L$, if and only if $\mathcal{L}q\varepsilon_{q,L} + \mathcal{V}_0\varepsilon_{n,L} > 0$.

## O3 A stylized benchmark model

This appendix provides a complete analytical characterization and comparison of equilibrium outcomes in a stylized benchmark model of a generalized Cournot oligopoly with flexible marginal costs and its

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[1] Taking $\sigma_i = q_i$ for all $i$, we have $q_\sigma^o = 1$ and $q_\sigma^c = (\nu - 1)/(n-1)$. Also, homogeneity implies $p_\sigma^o = p_\sigma^c$. Hence, replacing in (O4) leads to $\theta = \nu/n$.

[2] Taking $\sigma_i = q_i$ for all $i$, we have $q_\sigma^c = 0$ (Cournot behavior) and $q_\sigma^o = 1$. Also $p_\sigma^o = (1/L)\partial P_i/\partial x_i$ and $p_\sigma^c = (1/L)\partial P_j/\partial x_i$. Replacing in (O4) leads to (O5).

[3] The market conduct index approach can also accommodate the case of monopolistic competition (see Weyl and Fabinger, 2013) and the case of competition in supply functions à la Klemperer and Meyer (1989) (see Mahoney and Weyl, 2017).

corresponding “ghost industry”. The ghost industry replicates the same demand function, number of firms, and same conduct parameter $\theta$, but assumes a constant marginal cost equal to the realized marginal cost in the original industry. Let the inverse industry demand be linear with $P(x,n) = a - bnLx$, with $a > 0$ and $b > 0$, so that

$$\varepsilon_d(x,n) = -\frac{P(x,n)}{xP_x(x,n)} = \frac{a - bnLx}{bnLx} \text{ and } \rho_d(x,n) = -\frac{xP_{xx}(x,n)}{P_x(x,n)} = 0.$$

The variable cost function is $c(q) = cq + \frac{1}{2}dq^2$ with $c > 0$, $d \in \mathbb{R}$ and $q = Lx$. When $d > (<)0$, marginal cost is increasing (decreasing). AVC is $w(q) = c + \frac{1}{2}dq$, so that

$$\varepsilon_s(q) = \frac{w(q)}{qw'(q)} = 1 + \frac{2c}{dq} \text{ and } \rho_s(q) = -\frac{qw''(q)}{w'(q)} = 0.$$

Firms play a generalized Cournot game with conduct parameter $\theta = \nu/n$ where $\nu \in [0, n]$.

**A unique equilibrium.** Standard derivations yield the following unique equilibrium quantity per firm and price (except in the knife-edge case where $d = -b\nu$, see below):

$$q^* = \frac{a-c}{b(n+\nu)+d} \text{ and } p^* = a - bnq^* = \frac{ab\nu + ad + bnc}{b(n+\nu)+d}. \tag{O7}$$

Note that the resulting markup is:

$$p^* - c - dq^* = b\nu q^* > 0$$

so that $b\nu$ is the markup per unit. The resulting profit is

$$\pi^* = p^*q^* - cq^* - \frac{1}{2}dq^{*2} = \frac{(a-c)^2}{(b(n+\nu)+d)^2}(b\nu + \frac{1}{2}d) > 0,$$

To ensure existence, we make the following assumption.

**Assumption 1.** *Let $a > c$ and $d > \underline{d} \equiv \max\left(-b(n\frac{c}{a} + \nu), -2b\nu\right)$ for any $\nu \in [0, n]$.*

Assumption 1 suggest that $d$ positive or not too negative, together with $a > c$, ensure positive profit and quantity/price. Moreover, best-response functions are such that

$$q_i(Q_{-i}) = \frac{a - c - b\,Q_{-i}}{b(1+\nu)+d}, \qquad \frac{\partial q_i}{\partial Q_{-i}} = -\frac{b}{b(1+\nu)+d}.$$

Therefore, quantities are (i) strategic substitutes if $d > -b(1+\nu)$, (ii) strategic complements if $\underline{d} < d < -b(1+\nu)$.[4] In the knife-edge case where $d = -b(1+\nu) > \underline{d}$, the best response function becomes vertical (strategic neutrality) and all first-order conditions express as $Q_{-i} = (a-c)/b$ for all $i$. Despite the infinite slope, the intersect of these FOCs yield a unique equilibrium quantity, necessarily symmetric, such that $q^* = (a-c)/(b(n-1))$ as given by (O7). The other knife-edge case is where $d = -b\nu$ where all FOCs reduce to $Q = (a-c)/b$ such there is a unique symmetric equilibrium quantity and a continuum of asymmetric equilibria.

**Lerner, contribution margin ratio and profitability scale factor.** The Lerner index at the equilibrium is

$$\mathcal{L} = \frac{1}{\tilde{\varepsilon}_d} = \frac{(a-c)b\nu}{ab\nu + bnc + ad} \tag{O8}$$

and the equilibrium value of the contribution margin ratio is given by

$$\mathcal{C} = \frac{(a-c)(b\nu + \frac{1}{2}d)}{ab\nu + bnc + ad}. \tag{O9}$$

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[4] Quantities as strategic complements appear when $\underline{d} < -b(1+\nu)$, i.e. either $\theta \geq c/a$ and $a \leq nc$, or $\theta < c/a$ and $\nu \geq 1$.

It follows that the profitability scale factor $\lambda$ is valued at the equilibrium as:

$$\lambda = \frac{\mathcal{L}}{\mathcal{C}} = \frac{b\nu}{b\nu + \frac{1}{2}d} > 0. \tag{O10}$$

The profitability scale factor only depends on the markup per unit, $b\nu$, and on curvature $d$, and decreases unambiguously in $d$. Note that when $\nu$ is very large, which is possible when competition is weak and $n$ large, then $\lambda$ is close to 1 whatever $d$ finite. In this limiting case of a nearly perfect cartel of a large size, the curvature influence becomes negligible.

It is instructive to look at the influence of the cost structure through baseline cost parameter $c$ and curvature parameter $d$ on these outcomes. First, we can see that the exercise of market power measured through the Lerner index (O8) is reduced whenever $c$ or $d$ raise. Second, the profitability ratio given (O9) also decreases in $c$, but is non monotonic in curvature $d$. More precisely,

$$\frac{\partial \mathcal{C}}{\partial d} = \frac{bn(a-c)(c-a\theta)}{2(ab\nu + bnc + ad)^2}$$

so that $\mathcal{C}$ raises in curvature $d$ iff $\theta < \frac{c}{a} < 1$. Hence, when competition is strong enough, raising $d$ improves paradoxically profitability. This can be explained as follows: recall that $\mathcal{C} = 1 - \frac{\text{AVC}}{p^*}$ and both AVC and $p^*$ increase with $d$, but their speeds of adjustment differ. We have

$$\text{AVC} = c + \frac{d}{2}q^* = \frac{bc(n+\nu) + \frac{d}{2}(a+c)}{b(n+\nu)+d} \text{ and } p^* = \frac{ab\nu + ad + bnc}{b(n+\nu)+d}. \tag{O11}$$

Whenever $\theta < \frac{c}{a}$, competition is strong and $p^*$ reacts more strongly to $d$, which implies that the contribution margin $\mathcal{C}$ rises with $d$. Conversely, when $\theta < \frac{c}{a}$, competition is weaker and AVC adjusts faster relative to $p^*$ and $\mathcal{C}$ declines in $d$.

Let us check the stability condition which writes from (6) as:

$$\rho < 2 + \lambda\left(\frac{1}{\theta} - 1 + \varepsilon_{\theta,q}\right) \Leftrightarrow 0 < 2 + \frac{b\nu}{b\nu + \frac{1}{2}d}\left(\frac{n}{\nu} - 1 + 0\right)$$

given that $\rho = \lambda\rho_d + (1-\lambda)\rho_s = 0$ and using (O10) as well as $\theta = \nu/n$. Rearranging, we get

$$0 < 2 + \frac{b\,(n-\nu)}{b\nu + \frac{1}{2}d}$$

which always holds under Assumption 1.

**Pass-throughs in the short-run.** The absolute pass through is given by Proposition 2 and using (O10), we finally get:

$$\mathit{APT} = \frac{bn}{b(n+\nu)+d}, \tag{O12}$$

whereas the relative pass through is obtained using (O8) and (O12):

$$\mathit{RPT} = (1-\mathcal{L})\mathit{APT} = \frac{bn}{b(n+\nu)+d}\frac{bc(n+\nu)+ad}{ab\nu + ad + bnc}. \tag{O13}$$

Let us consider now the ghost industry where MC is constant and set equal to $c + dq^*$ from the original industry. The conduct parameter is also set to $\nu/n$ as before. The ghost industry produces the same quantity $q^*$ and thus charges the same price $p^*$. The Lerner index is $\mathcal{L}_{\text{ghost}} = \mathcal{L}$ as in the original industry. However, $\lambda_{\text{ghost}} = 1$ and the contribution margin ratio is now $\mathcal{C}_{\text{ghost}} = \mathcal{L}$ and it is lower than $\mathcal{C}$ if and only if $d < 0$ (decreasing AVC). The absolute and relative pass-throughs are given respectively by:

$$\mathit{APT}_{\text{ghost}} = \frac{n}{n+\nu} < \mathit{APT} \text{ iff } d < 0$$

and

$$\mathit{RPT}_{\text{ghost}} = \frac{n}{n+\nu}\frac{bc(n+\nu)+ad}{ab\nu + ad + bnc} < \mathit{RPT} \text{ iff } d < 0. \tag{O14}$$

Note also that

$$\frac{APT}{APT_{\text{ghost}}} = \frac{RPT}{RPT_{\text{ghost}}} = \frac{b(n+\nu)}{b(n+\nu)+d}.$$

We can write these ratios as a function of the ratio AVC/MC valued at the equilibrium. We have AVC given by (O11) and also

$$\text{MC} = c + dq^* = \frac{bc(n+\nu)+ad}{b(n+v)+d},$$

and it follows that

$$\frac{\text{AVC}}{\text{MC}} = \frac{c+\frac{d}{2}q^*}{c+dq^*} = \frac{bc(n+\nu)+\frac{d}{2}(a+c)}{bc(n+\nu)+ad}.$$

We deduce that:

$$\frac{APT}{APT_{\text{ghost}}} = \frac{ar - \frac{a+c}{2}}{(a-c)(r-\frac{1}{2})}$$

where $r = \frac{\text{AVC}}{\text{MC}}$. This function is increasing and strictly concave in $r$ as described in Figure 3. Note that $\lim_{d\to\infty} \frac{\text{AVC}}{\text{MC}} = \frac{a+c}{2a}$ and thus $\lim_{r\to 0} \frac{APT}{APT_{\text{ghost}}} = 0$.

**When are cost shocks profit-enhancing?** Starting from the inequality given by Proposition 4,

$$(1-\theta)APT > 1$$

we note that it does not hold under Assumption 1:

$$(1-\frac{\nu}{n})\frac{bn}{b(n+\nu)+d} > 1 \Leftrightarrow d < -2b\nu. \tag{O15}$$

Hence, an additive cost shock can never be profit-enhancing in the benchmark example. By contrast, the pro-profit condition for a multiplicative cost shock is

$$RPT > \frac{1-\mathcal{C}}{1-\theta}, \tag{O16}$$

and because $RPT = (1-\mathcal{L})APT$, (O16) rewrites as

$$(1-\theta)APT > \frac{1-\mathcal{C}}{1-\mathcal{L}}. \tag{O17}$$

Clearly, given that $(1-\theta)APT < 1$, (O17) can only hold when $\frac{1-\mathcal{C}}{1-\mathcal{L}} < 1$, that is for $\lambda < 1$ or equivalently $d > 0$ (DAVC).

Now, substituting the expressions of $\mathcal{C}$ and $RPT$ in (O16), one obtains after simplification

$$H(d) < 0$$

with $H(d)$ a convex polynomial function of degree 2 in $d$ :

$$H(d) = (a+c)\,d^2 - b\left[a\left(n-3\nu\right) - 3c(n+\nu)\right]d + 4b^2c\nu(n+\nu).$$

Hence, whenever the discriminant

$$\Delta = b^2\left[a^2n^2 - 6a^2n\nu + 9a^2\nu^2 - 6acn^2 - 4acn\nu + 2ac\nu^2 + 9c^2n^2 + 2c^2n\nu - 7c^2\nu^2\right]$$

is negative then $H(d) \geq 0$ for all $d$ and the multiplicative shock cannot be profit enhancing. On the contrary, when $\Delta > 0$, there are two real roots

$$d_{\pm} = \frac{b\left[an - 3a\nu - 3c(n+\nu)\ \pm\ \sqrt{\Delta}/\,b\right]}{2\,(a+c)},$$

that have the same sign because

$$d_{-}d_{+} = \frac{4b^2c\nu(n+\nu)}{a+c} > 0,$$

and their *common sign* is the sign of the sum

$$d_- + d_+ = \frac{b\,[\,a(n-3\nu) - 3c(n+\nu)\,]}{a+c}.$$

The multiplicative shock is thus *pro-profit* precisely for $d \in [d_-, d_+]$ when $a(n-3\nu) > 3c(n+\nu)$ so that $d_- < d_+$ are both positive. Assuming $a > 3c$,the latter condition rewrites as

$$\theta < \frac{a-3c}{3(a+c)} < 1$$

so that when competition is sufficiently strong then a multiplicative shock is profit-enhancing whenever $d \in [d_-, d_+]$. If $a < 3c$, the multiplicative shock is never profit-enhancing.

By contrast, in the ghost industry, a multiplicative shock can never be profit enhancing. Indeed, we can check that by using (O14) and using the fact that $\mathcal{C}_{\text{ghost}} = \mathcal{L}_{\text{ghost}}$ given by (O8), we finally have

$$\frac{1-\mathcal{C}_{\text{ghost}}}{1-\theta} = \frac{n+\nu}{n-\nu} RPT_{\text{ghost}} > RPT_{\text{ghost}}$$

for $n > \nu$.

**Market concentration incidence.** With $P(x,n) = a - b\,n\,L\,x$, we have

$$P_n = \frac{\partial P}{\partial n} = -b\,L\,x, \qquad P_x = \frac{\partial P}{\partial x} = -b\,n\,L, \qquad P_{xn} = \frac{\partial^2 P}{\partial x\,\partial n} = -b\,L,$$

and $\theta = \nu/n$ so that $\theta_n = -\nu/n^2 = -\theta/n$. By definition,

$$MP_n \;=\; P_n + \theta\,x\,P_{xn} + \theta_n\,x\,P_x \;=\; \underbrace{(-b\,L\,x)}_{P_n} + \theta x(-bL) - \frac{\theta}{n} x(-b\,n\,L) \;=\; -b\,L\,x = P_n.$$

Thus, in this model, $MP_n = P_n$ holds for all $(x,n)$. Using the equilibrium quantity in (O7), this gives

$$MP_n = P_n = -\frac{b\,(a-c)}{b(n+\nu)+d}. \tag{O18}$$

Hence, under Assumption 1, one has $MP_n < 0$: an increase in $n$ lowers each firm's perceived marginal profit (*business stealing*).

To check for the condition exhibited in Proposition 5 that provides a price-increasing competition test, let us compute:

$$P_n - MP_n APT = P_n\,(1 - APT) = -\frac{b(a-c)\,(b\nu + d)}{[\,b(n+\nu)+d\,]^2},$$

using (O12). Therefore, $P_n - MP_n APT > 0$ if and only if $\underline{d} \le d < -b\nu$ and the equilibrium price is *increasing* in $n$.

We check also that equilibrium profit is always decreasing in $n$. Consider the long-run stability condition (14):

$$P_n - MP_n(1-\theta)APT < 0.$$

which rewrites using (O18) as $1-(1-\theta)APT > 0$. The latter condition always hold under Assumption 1 as shown above (see (O15)).

**Long-run equilibrium and pass-throughs.** The long-run symmetric equilibrium $(q^{*LR}, n^{*LR})$ is such that the first-order condition w.r.t quantity and the zero profit condition both hold:

$$p^{*LR} - c - dq^{*LR} = \nu b q^{*LR}$$

and

$$\left(p^{*LR} - c - \frac{d}{2}q^{*LR}\right) q^{*LR} = f. \tag{O19}$$

Since MC $-$ AVC $= \frac{d}{2}q$, we have $p-$ AVC $= p-$ MC $+$ MC $-$ AVC $= (b\nu + \frac{d}{2})q$. It follows that (O19) yields

$$q^{*LR} = \sqrt{\frac{f}{b\nu + \frac{d}{2}}}. \tag{O20}$$

Therefore,

$$p^{*LR} = c + (b\nu + d)\sqrt{\frac{f}{b\nu + \frac{d}{2}}}, \tag{O21}$$

$$Q^{*LR} = \frac{a-c}{b} - \frac{b\nu + d}{b}\sqrt{\frac{f}{b\nu + \frac{d}{2}}}, \tag{O22}$$

and

$$n^{*LR} = \frac{a-c}{b}\sqrt{\frac{b\nu + \frac{d}{2}}{f}} - \frac{b\nu + d}{b}. \tag{O23}$$

**Assumption 2.** *Let* $d > -2b\nu$ *and* $a - c > (b\nu + d)\sqrt{\frac{f}{b\nu + \frac{d}{2}}}$.

The parameter restrictions in Assumption 2 ensure positive quantity, price and number of firms. A closer inspection reveals that Assumption 2 holds when $d \in (-2b\nu, -b\nu] \cup (\max(-b\nu, d_-), d_+)$ where

$$d_\pm = \frac{\frac{(a-c)^2}{2} - 2fb\nu \pm (a-c)\sqrt{\frac{(a-c)^2}{4} + 2fb\nu}}{2f}.$$

If $f$ is close to 0, then Assumption 2 reduces to $d > -2b\nu$ and $a > c$. If $\nu = 0$ (pure Bertrand), existence requires $d \in (0, d_+)$, i.e. $d$ positive. When $d$ tends to $(-2b\nu)^+$ then $q^{*LR} \to \infty$ and $n^{*LR} \to 0$ (natural monopoly tendency).

In the long-run, the expression of the profitability scale factor $\lambda^{LR}$ coincides with the one in the short-run given by (O10). Indeed, we have

$$\lambda = \frac{p - \text{MC}}{p - \text{AVC}}.$$

Here, in the short-run,

$$\lambda = \frac{b\nu q^*}{(b\nu + \frac{d}{2})q^*} = \frac{b\nu}{b\nu + \frac{d}{2}}$$

and in the long-run,

$$\lambda^{LR} = \frac{b\nu q^{*LR}}{(b\nu + \frac{d}{2})q^{*LR}} = \frac{b\nu}{b\nu + \frac{d}{2}} = \lambda.$$

This result comes from the specification of the stylized example. Indeed, this is because both $p-$ MC and $p-$ AVC are proportional to the same aggregate (here $q$) with coefficients that do not depend on $n$. More precisely, when $n$ adjusts in the long-run, it does affect the equilibrim levels (price, quantity) but not the proportionality relationship between price, MC and AVC that enter into the definition of $\lambda$.

For completeness, let us consider the ghost industry as usual defined as having a constant MC (and thus AVC) denoted $\bar{m}$ and equal to the equilibrium marginal cost of the original industry, here valued at the long run equilibrium, i.e. $\bar{m} = c + dq^{*LR}$. Clearly, at the long-run equilibrium, the ghost industry diverges significantly from the original one, as quantity per firm and number of active firms are different. The behavioral response of firms is now given by $p - \bar{m} = \nu bq$, while free entry ensures that $(p - \bar{m})q = f$. These conditions yields

$$q^{*LR}_{\text{ghost}} = \sqrt{\frac{f}{\nu b}}.$$

And it follows that $p^{*LR}_{\text{ghost}} = \bar{m} + \sqrt{\nu b f}$, $Q^{*LR}_{\text{ghost}} = \frac{a-\bar{m}}{b} - \sqrt{\frac{\nu f}{b}}$ and $n^{*LR}_{\text{ghost}} = (a-\bar{m})\sqrt{\frac{\nu}{bf}} - \nu$. The existence condition is $a - \bar{m} > \sqrt{\nu b f}$. Note also the clean scale comparison with the original industry's long-run output per firm $q^{*LR} = \sqrt{2f/(2b\nu + d)}$:

$$\frac{q^{*LR}_{\text{ghost}}}{q^{*LR}} = \sqrt{\frac{2b\nu + d}{2b\nu}} \begin{cases} > 1 & \text{if } d > 0, \\ = 1 & \text{if } d = 0, \\ < 1 & \text{if } d \in (-2b\nu, 0). \end{cases}$$

Anchoring MC at $\bar{m}$ at the original industry's long-run MC isolates the role of cost *curvature* $d$: when $d > 0$, the original MC was rising, so replacing it by a flat $\bar{m}$ relaxes marginal congestion and yields larger plants ($q^{*LR}_{\text{ghost}} > q^{*LR}$) and fewer firms; for $d < 0$ (DAVC), freezing MC at $\bar{m}$ removes the inframarginal cost advantage from further expansion, shrinking optimal plant size ($q^{*LR}_{\text{ghost}} < q^{*LR}$) and supporting more entry. Finally, if $\nu = 0$ (price-taking/Bertrand limit), the zero profit condition admits no finite solution , hence no finite-entry long-run equilibrium in the ghost industry.

Let us now consider the results contained in Proposition 6. Starting with the conditions in part (i), we first note that

$$(1-\theta)APT = \frac{n^{*LR} - \nu}{n^{*LR}} \frac{bn^{*LR}}{b(n^{*LR} + \nu) + d} = 1 - \frac{2b\nu + d}{a - c} q^{*LR} < 1$$

where the first step comes from using (O12) and the final expression from using (O20) and (O23). It follows that an additive cost shock is *always anticompetitive* in this model. By contrast, a multiplicative cost shock can be procompetitive under some condition. From Proposition **??**, the latter condition is

$$(1-\theta)APT > \frac{\varepsilon - 1}{\varepsilon - \lambda} \tag{O24}$$

and a necessary condition for this to be possible is $\frac{\varepsilon-1}{\varepsilon-\lambda} < 1$, given that $(1-\theta)APT < 1$ as shown above. Hence, a necessary condition for a multiplicative shock to be procompetitive is that $\lambda > 1$, i.e. $d < 0$. Using $\varepsilon = 1/\mathcal{C}$, and using (O9), (O10) together with (O23), (O24) rewrites

$$q^{*LR} < \tilde{q} \equiv \frac{\sqrt{d^2 + \frac{(a-c)cd}{f}} - d}{2c}$$

Now, with respect to part (ii) on output consequences from cost shocks, we have shown earlier that, in this model, $MP_n = P_n$ holds for all $(x, n)$ (see (O18)). It follows that an additive cost shock is always output-neutral in this model. By contrast, a multiplicative shock is reducing output if and only if $\frac{\varepsilon-1}{\varepsilon-\lambda} < 1$, or equivalently iff $d < 0$.

The case where (O24) holds arises only under DAVC ($d < 0$). In this region, marginal costs decrease with firm size, so free entry leads each firm to expand its scale $q^{*LR}$ as $d$ becomes more negative. While a multiplicative cost shock tends to increase prices, the concavity of costs implies that the denominator of $\frac{\varepsilon-1}{\varepsilon-\lambda} = \frac{dq^{*LR}/2+c}{dq^{*LR}+c}$ shrinks faster than $(1-\theta)APT = 1 - \frac{(2b\nu+d)}{a-c} q^{*LR}$. As a result, $\frac{\varepsilon-1}{\varepsilon-\lambda}$ falls more quickly than $(1-\theta)APT$, and the inequality (O24) is satisfied. Economically, this corresponds to a pro-competitive effect of a cost shock: entry and scale adjustment dominate the cost increase. The condition is therefore true for sufficiently negative $d$ (strong marginal economies of scale), moderate values of conduct $b\nu$ (so that $2b\nu + d$ is not too large), and not too high fixed cost $f$, ensuring that $q^{*LR}$ adjusts strongly.

Regarding the pass-throughs, let us consider first the additive cost shock in the long run. The fact that an additive shock is output neutral immediately yields

$$APT^{LR} = 1,$$

using (21). Two channels drive the price effect of entry: the *direct* impact of more firms on inverse demand ($P_n < 0$) and the *strategic* feedback via firms' perceived marginal profit $MP_n$. With linear demand $P = a - bLnx$, these terms exactly offset each other, so the marginal effect relevant for pass-through is just $P_n$. Under free entry, a unit rise in per-unit cost shifts $AC$ up by one everywhere; to keep profits at zero, once entry re-adjusts, the equilibrium price must rise one-for-one. Hence

$APT^{LR} = 1$: entry absorbs all extensive-margin adjustments, leaving the intensive margin to track the unit cost level exactly.

The long-run relative pass-through is given by:

$$RPT^{LR} = \frac{c + q^{*LR}\dfrac{d\,(3b\nu + d)}{2(2b\nu + d)}}{c + (b\nu + d)q^{*LR}} = \frac{c \;+\; \sqrt{2f}\,\dfrac{d\,(3b\nu + d)}{2\,(2b\nu + d)^{3/2}}}{c \;+\; \sqrt{2f}\,\dfrac{b\nu + d}{\sqrt{2b\nu + d}}}$$

It is interesting to describe the shape of $RPT^{LR}$ when $d$ varies. As $d$ decreases towards $-2b\nu$, then we have already shown that $q^{*LR}$ grows toward being infinite and $n^{*LR}$ tends to 0. It follows that $\lim_{d\to -2b\nu^+} RPT^{LR} = +\infty$. At $d = 0$,

$$RPT^{LR} = \frac{c}{c + \sqrt{b\nu f}} \in (0, 1),$$

which represents actually the (variable) cost share in revenue, $cq^{*LR}/p^{*LR}q^{*LR}$. When $d > 0$ then $RPT^{LR}$ is strictly positive. Nevertheless, it is easy to find examples where $RPT^{LR}$ is strictly negative (Edgeworth-type reversed pass-through in the long run) when $d < 0$. This means that an infinitesimal multiplicative cost shock reduces the long-run equilibrium price, an Edgeworth-type paradox driven by cost curvature and the zero-profit condition.

Consider $b = 1$, $\nu = 1$, $f = 1$, $c = 1$, and $a = 3$. Choose $d = -1$ (so $-2b\nu = -2 < d$). Then

$$q^{*LR} = \sqrt{\frac{f}{b\nu + \frac{d}{2}}} = \sqrt{\frac{1}{1 - \frac{1}{2}}} = \sqrt{2},$$

and the long-run feasibility constraint holds: $a - c > (b\nu + d)q^{*LR}$ since $(b\nu + d)q^{*LR} = 0 \Rightarrow a - c - (b\nu + d)q^{*LR} = 2 > 0$. The equilibrium number of firms is positive

$$n_{LR} = \frac{a - c}{b\,q^{*LR}} - \nu - \frac{d}{b} = \sqrt{2} > 0.$$

Evaluating the pass-through,

$$RPT^{LR} = \frac{c + q^{*LR}\dfrac{d(3b\nu + d)}{2(2b\nu + d)}}{c + (b\nu + d)q^{*LR}} = \frac{1 + \sqrt{2}\,\dfrac{(-1)(2)}{2(1)}}{1 + 0 \cdot \sqrt{2}} = -0.4142 < 0.$$

Hence an infinitesimal multiplicative chost shock *lowers* the long-run price. Intuitively, with decreasing AVC, the tax perturbs the zero-profit entry condition in a way that intensifies effective competition (via the scale/entry margin) and dominates the direct cost-increasing effect, delivering a negative long-run pass-through.

**Market expansion incidence.** Let us consider first the short-run impact of market expansion. We have that the elasticity of the equilibrium quantity w.r.t market size $L$ is given by:

$$\varepsilon_{q,L} = \frac{APT}{APT_{\text{ghost}}} = \frac{b(n + \nu)}{b(n + \nu) + d} \tag{O25}$$

using (O12) and Definition 4:

$$APT_{\text{ghost}} = \frac{1}{1 + \theta - \theta\rho_d}$$

with $\rho_d = 0$. Clearly, market expansion is price-decreasing (-increasing), i.e. $\varepsilon_{q,L} > (<)1$ if and only if $d < (>)0$. This pattern is driven purely by cost concavity/convexity and not by change in competition intensity as $\theta$ is constant. Following Proposition 7 and using (O10), (O25), we also have

$$\varepsilon_{\pi,L} = \frac{\lambda}{\theta}\left(1 - (1 - \theta)\varepsilon_{q,L}\right) = \frac{2b}{2b\nu + d}\,\frac{b\nu(n + \nu) + dn}{b(n + \nu) + d}$$

and market expansion raises profit iff $\varepsilon_{q,L}$ is not too large, i.e. $\varepsilon_{q,L} < 1/(1-\theta)$ which translates into:

$$d > -b\nu(1+\theta), \tag{O26}$$

that is $d$ positive or not too negative.

In the long run, Proposition 8 suggests that market expansion is procompetitive iff (O26) taken at the long-run equilibrium holds, i.e. $d > -b\nu(1+\nu/n^{*LR})$. Clearly, if $d > -b\nu$ and $d$ feasible, then this always holds. When $d \in (-2b\nu, b\nu)$, then it is easy to find examples where market expansion is anticompetitive. Indeed, take $(a,b,c,f,\nu,d) = (20,2,4,4,3,-10)$ then $q^{*LR} = \sqrt{\frac{4}{6-5}} = 2$, $n^{*LR} = \frac{20-4}{2}\sqrt{\frac{4}{6-5}} - 3 - \left(-\frac{10}{2}\right) = 6$ and $-b\nu(1+\nu/n^{*LR}) = -6\,(1+3/6) = -9$ so that $d < -b\nu(1+\nu/n^{*LR})$.

Moreover, following Proposition 8, market expansion raises output iff $1 - \mathit{APT}_{\text{ghost}} > 0$ as $P_n = -MP_n < 0$ here. This condition always hold as

$$1 - \mathit{APT}_{\text{ghost}} = 1 - \frac{1}{1+\theta} \geq 0 \text{ as } \theta \geq 0.$$

It follows that only Scenarios 1 (Market shakeout) or Scenario 3 (Expansion) occurs at the long-run equilibrium, depending on whether condition (O26) holds or not.

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