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Abstract

Degrowth is often advocated as a response to the climate crisis, but its consistency with growth theory remains unclear. We develop an endogenous growth model of directed technical change and climate in which the economy relies on both a polluting fossil resource and a clean renewable alternative. We fully characterize the social optimum and show that accounting for climate damages may justify an initial phase of degrowth. Such a phase is more likely when fossil resources are abundant and fossil-oriented research is relatively inefficient. In all cases, long-run optimal growth remains positive.

Keywords: degrowth; directed technical change; endogenous growth; social optimum; climate change.

JEL classification: O33; O44; Q32; Q54; Q55

1 Introduction

The issue of degrowth has gained increasing attention in public debate, often presented as a potential response to the climate crisis.[1] It is explicitly mentioned in the latest IPCC report as part of the mitigation pathways (Riahi *et al.*, 2022). While a growing body of research explores degrowth-related issues, few contributions rely on formal economic analysis, and even fewer on modeling approaches (Savin and van den Bergh, 2024). Yet it seems crucial to examine what insights economic theory – and in particular, growth theory – can offer on this topic.[2]

As early as the 1970s, theory showed that resource scarcity could lead to negative optimal growth if technological progress fails to offset time preference *(e.g.*, Stiglitz, 1974). This

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[1]For an analysis of different factors of degrowth, see *e.g.*, Sasaki (2021) (declining population), or Bosi *et al.* (2023) (low accumulation of human capital).

[2]While Aghion *et al.* (2025) show that an economy can endogenously converge to zero material growth when sustaining positive welfare growth through quality improvements, our question is different: we examine the conditions under which a social planner who internalizes the climate externality would optimally choose a negative growth rate of aggregate output.

aligns with Solow's view that societal preferences may lead to eventual extinction despite technological feasibility (Solow, 1974). These insights have since been extended to endogenous growth frameworks (*e.g.*, Grimaud and Rouge, 2003; Groth and Schou, 2006; Groth, 2007).

Directed technical change (DTC) models – such as the benchmark developed by Acemoglu *et al.* (2012) and surveyed by Hémous and Olsen (2021) – provide a natural framework to jointly address the climate consequences of emissions and the endogenous direction of technological progress. In this setting, the possibility of green growth hinges on the availability of a renewable resource and an adequate orientation of innovation. We adopt this now-standard DTC framework to revisit the question of optimal degrowth. We compute the social optimum (Section 2), and analyze the resulting growth path, identifying the conditions and timing under which negative optimal growth may arise (Section 3).

2 The social planner problem

2.1 The model

The model builds on Grimaud and Rouge (2025), which is close to Acemoglu *et al.* (2012). At each time t[3], the economy produces a flow $\tilde{Y}(t)$ of final good from a fossil-based input $Y_f(t)$ and a renewable-based input $Y_r(t)$ according to the following technology:

$$\tilde{Y}(t) = \left[Y_r(t)^{\frac{(\varepsilon-1)}{\varepsilon}} + Y_f(t)^{\frac{(\varepsilon-1)}{\varepsilon}}\right]^{\frac{\varepsilon}{(\varepsilon-1)}}, \tag{1}$$

where $\varepsilon \in (0, +\infty)$. The production of each input Y_j, $j = r, f$, requires a natural resource (either R or F), a continuum of sector-specific intermediate goods, x_{ji}, and an associated stock of knowledge, A_{ji}, $i \in [0, 1]$:

$$Y_r(t) = R^{1-\alpha} \int_0^1 A_{ri}(t)^{1-\alpha} x_{ri}(t)^{\alpha} di \quad \text{and} \quad Y_f(t) = F(t)^{1-\alpha} \int_0^1 A_{fi}(t)^{1-\alpha} x_{fi}(t)^{\alpha} di, \tag{2}$$

with $\alpha \in (0, 1)$. The available flow R of clean renewable resource is considered constant (*e.g.* average flow of solar or wind energy). The fossil resource $F(t)$ is costlessly extracted from a limited stock $S(t)$ of initial size S_0:

$$\dot{S}(t) = -F(t), \quad \text{with } S(0) \equiv S_0. \tag{3}$$

At each time t, a quantity $L_{ji}(t)$ of labor is used in the R&D subsector i of sector j to improve the quality of the intermediate good $x_{ji}(t)$. Knowledge accumulation in this subsector writes:

$$\dot{A}_{ji}(t) = \eta_j L_{ji}(t) A_{ji}(t), \tag{4}$$

[3]Time is continuous. Hereafter, we will denote by $g_G(t)$ the growth rate $\dot{G}(t)/G(t)$ of any variable $G(t)$ at date t. We sometimes omit the time index when this is clear from the context.

with $A_{ji}(0) = A_{jk}(0)$, $\forall i,k \in [0,1]$. We define $A_j(t) \equiv \int_0^1 A_{ji}(t)di$ as the aggregate stock of sector j-specific knowledge. As shown later (Section 3), parameter $\eta_j > 0$, $j = r, f$, which characterizes the efficiency of the research sectors, is crucial in determining whether an optimal degrowth phase is possible, which highlights the importance of using a DTC model to analyze this optimality question.

The economy is endowed with a constant labor flow, normalized to one, allocated to the two competing R&D sectors:

$$\int_0^1 L_{ri}(t)\, di + \int_0^1 L_{fi}(t)\, di = 1, \ \forall t \geq 0\,. \tag{5}$$

The intermediate good $x_{ji}(t)$ associated with the knowledge stock $A_{ji}(t)$ is produced according to the following linear technology, supposed to be identical in all sectors:

$$x_{ji}(t) = \frac{1}{\psi} y_{ji}(t)\,, \tag{6}$$

with $\psi > 0$ and where $y_{ji}(t)$ is an amount of final good.

Using the non-renewable resource releases carbon emissions that accumulate into the atmospheric carbon stock $Z(t)$, of initial state Z_0. For simplicity, we approximate this emission flow to resource extraction, which implies:

$$\dot{Z}(t) = F(t) - \theta Z(t)\,, \quad \text{with } Z(0) \equiv Z_0, \tag{7}$$

where $\theta > 0$ denotes the exponential rate of natural carbon removal. As in Nordhaus (2013), carbon accumulation impacts the economy through the depreciation of the final output $\tilde{Y}(t)$ by a "penalty weight" $[1 - D(Z)]$, where $D(Z) = 1 - e^{-\delta Z}$ denotes the damage function, with $\delta \geq 0$.[4] The resulting expression of the output, net of the degradation due to climate change, is thus:

$$Y(t) = \tilde{Y}(t)[1 - D(Z(t))] = \tilde{Y}(t)e^{-\delta Z(t)}\,. \tag{8}$$

Denoting ρ the social discount rate, the utility function is:

$$U = \int_0^{+\infty} \ln\big(C(t)\big)e^{-\rho t}\, dt, \tag{9}$$

Last, the final net output can be either used for consumption or for the production of intermediate goods:

$$Y(t) = C(t) + \int_0^1 y_{ri}(t)di + \int_0^1 y_{fi}(t)di, \ \forall t \geq 0\,. \tag{10}$$

[4]We use the same damage function as in Golosov *et al.* (2014). Note that the forthcoming discussion on the sign of the optimal growth rate of the economy does not depend on this specific form, which is used for computational convenience only. Indeed, log-differentiating (8) with respect to time implies $\dot{g}_Y = \dot{g}_{\tilde{Y}} - \frac{D'(Z)}{[1-D(Z)]}\dot{Z}$, which displays two terms that can be of opposite sign when both the output and the atmospheric carbon stock increase over time, thus giving rise to a possible negative growth rate. Last, directly introducing the carbon stock into the utility function, as in Grimaud and Rouge (2025) or Aghion *et al.* (2025), in addition to the production function, would not change the main result of this paper.

2.2 The optimal solution

The social planner maximizes the intertemporal utility function (9) subject to the dynamic constraints (3), (4), and (7), to the allocation constraints (5) and (10), and given the technologies (1), (2) and (6). We set aside the non-negativity constraints on the control variables, and we prove ex-post the existence of an interior solution, that is, a solution in which both research sectors are simultaneously active ($L_r, L_f > 0$). Detailed computation and full characterization of the social optimum are provided in Appendix A. The main results are presented in the following Proposition.

Proposition 1. *If $\rho < \eta_f$, an interior optimal solution exists and is characterized as follows.*

- ***Fossil resource:***

$$\begin{aligned} F(t) &= \frac{e^{\rho B S_0} - 1}{B(e^{\rho t} + e^{\rho B S_0} - 1)}, & (11)\\ S(t) &= \frac{1}{\rho B} \ln \left[1 + (e^{\rho B S_0} - 1) e^{-\rho t} \right], & (12)\\ g_F(t) &= \frac{-\rho}{1 + (e^{\rho B S_0} - 1) e^{-\rho t}}, & (13) \end{aligned}$$

where $B \equiv -\bar{\lambda}_Z \left(\frac{\eta_r + \eta_f}{\eta_r} \right) > 0$ *and* $\bar{\lambda}_Z = \frac{-\delta}{(\rho+\theta)(1-\alpha)} < 0$.

- ***Research efforts:***

$$L_r(t) = \frac{\eta_f + g_F(t)}{\eta_r + \eta_f} \quad \textit{and} \quad L_f(t) = \frac{\eta_r - g_F(t)}{\eta_r + \eta_f}. \tag{14}$$

- ***Knowledge stocks:***

$$\frac{A_r(t)}{A_f(t)} = \frac{F(t)}{R} \left(\frac{\eta_f}{\eta_r} \right)^{\frac{\varepsilon(1-\alpha)+\alpha}{(1-\alpha)(\varepsilon-1)}} \quad \textit{and} \quad g_{A_j}(t) = \eta_j L_j(t), \quad j = r, f. \tag{15}$$

- ***Intermediate goods, consumption, output:***

$$\begin{aligned} x_r(t) &= \frac{\eta_f \alpha Y(t)}{\psi(\eta_r + \eta_f)} \quad \textit{and} \quad x_f(t) = \frac{\eta_r \alpha Y(t)}{\psi(\eta_r + \eta_f)}, & (16)\\ C(t) &= (1-\alpha) Y(t), & (17)\\ g_Y(t) &= g_C(t) = g_{x_j}(t) = \eta_r L_r(t) - \frac{\delta \dot{Z}(t)}{(1-\alpha)}, \quad j = r, f. & (18) \end{aligned}$$

Given (14), L_f is always positive, while a necessary condition for L_r to remain positive for all t is $\rho < \eta_f$. Hence, the rate of time preference must be low enough compared with the productivity in the fossil-oriented R&D sector to guarantee an interior solution. We also observe that, in the absence of climate damage ($\delta = 0$), the social cost of carbon, referred to as $\bar{\lambda}_Z$ (*cf.* Appendix A), is nil and the growth rate of carbon emissions, g_F, is constant and equal to $-\rho$. Moreover, the research efforts, L_r and L_f, are constant too. Hence, climate change leads to postponing resource extraction ($g_F(t)$ is increased for all t) and has a direct impact on labor in R&D by increasing $L_r(t)$ and decreasing $L_f(t)$ for all t.

3 Occurrence of socially optimal (de)growth

In addition to technical change, the dynamics of the economy are driven by two laws of motion: i) from (3), the fossil resource stock is progressively depleted; ii) from (7), the carbon stock can either increase or decrease according to the emissions intensity. To understand the resulting overall dynamics, we need to construct a phase diagram in the (S, Z) plane (see Appendix B for technical details).

The isocline curve $\left\{\dot{Z}(t) = 0\right\}$ is such that:

$$Z = \frac{1 - e^{-\rho B S}}{\theta B} \equiv h(S)\,, \tag{19}$$

with $h(0) = 0$, $\lim_{S\to+\infty} h(S) = \frac{1}{\theta B}$, $h'(S) > 0$ and $h''(S) < 0$. The associated phase diagram is depicted in Figure 1. We observe that if S_0 is small relatively to Z_0 (case (S_0', Z_0') in Figure 1), then Z is always decreasing over time. In this case, the optimal growth rate of the economy, *i.e.* g_Y as defined by (18), is always positive. If S_0 is large as compared to Z_0 (case (S_0'', Z_0'')), Z grows over time in the short term, and then declines. The trajectory of Z is thus inverted U-shaped. In this case, $g_Y(t)$ is not necessarily always positive.

Next, we can express g_Y as the following function of S and Z:

$$g_Y(t) = \frac{\eta_r\,(\eta_f - \rho)}{\eta_r + \eta_f} - \frac{\delta\theta^2}{(\rho+\theta)(1-\alpha)} h(S(t)) + \frac{\delta\theta}{(1-\alpha)} Z(t)\,, \tag{20}$$

which implies:

$$g_Y \gtreqless 0 \quad \Leftrightarrow \quad Z \gtreqless \left(\frac{\theta}{\rho+\theta}\right) h(S) - \frac{(1-\alpha)\eta_r(\eta_f - \rho)}{\delta\theta(\eta_r + \eta_f)} \equiv k(S)\,. \tag{21}$$

Note that, as $\eta_f - \rho > 0$ (existence condition of an interior solution), then $k(S) < h(S)$ for all S. Since $k'(S) > 0$, $k''(S) < 0$ and $k(0) < 0$, this means that a degrowth regime can arise only if $\lim_{S\to+\infty} k(S) = \frac{(1-\alpha)\eta_r[\theta-(\eta_f-\rho)]}{\delta\theta(\eta_r+\eta_f)} > 0$, *i.e.*, if $\eta_f < \rho + \theta$. Figure 2 illustrates the resulting three possible growth regimes for $Y(t)$, which depends on the values of the carbon and fossil resource stocks, and their respective dynamics. Proposition 2 summarizes these findings.

Proposition 2. *If $\rho < \eta_f < \rho + \theta$ and Z_0 is relatively low compared to S_0, that is, if $Z_0 < k(S_0)$, the economy experiences an initial optimal phase of degrowth ($g_Y(t) < 0$) and eventually*

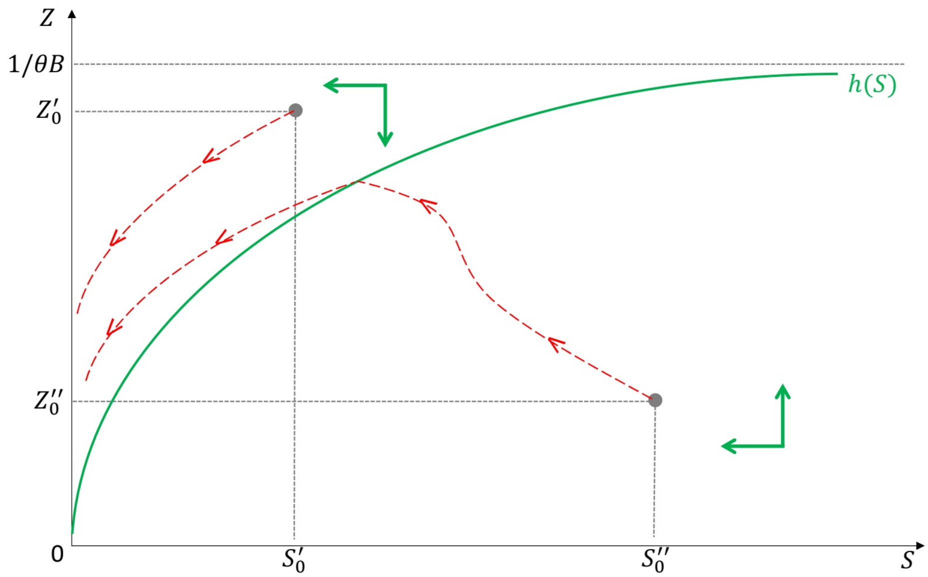

Figure 1: Joint dynamics of $S(t)$ and $Z(t)$

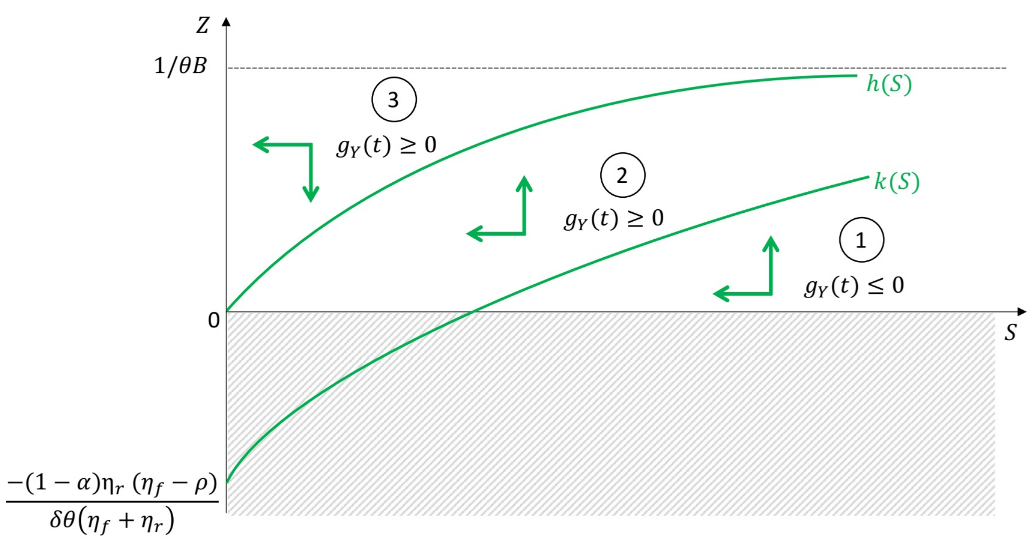

Figure 2: The different growth regimes of $Y(t)$

converges to a regime of positive growth along which the stock of carbon first increases, and then asymptotically decreases toward zero.

Proposition 2 indicates that if the efficiency rate of the fossil-oriented research sector, η_f, is large enough (higher than the social discount rate ρ), but not too large (smaller than the "environmentally" augmented discount rate $\rho + \theta$), and if Z is relatively low compared to S, the economy experiences an initial phase of decline (area 1 in Figure 2). Considering the expression of g_Y as given by (20), this result can be explained by the following factors. Here, the carbon stock is initially relatively low, so its natural decay, θZ, is weak. Additionally, as F continuously decreases over time, the initial carbon emissions are the highest. Hence, the increase in the carbon stock is high during this first phase and the exponential impact of carbon accumulation on production is high. In this case, knowledge accumulation is not sufficient to warrant positive growth: Equation (18) indeed shows that optimal growth is negative when the research effort in the renewable-oriented sector, L_r, is too low compared to carbon accumulation, $\dot{Z}$.

Subsequently, as the fossil resource stock diminishes and the carbon stock increases, the economy transitions to a new regime in which growth becomes positive (area 2 in Figure 2). Here, the carbon stock continues to grow. However, the accumulation of knowledge has reached a threshold where it offsets this negative effect, as well as that of fossil resource depletion, on growth. Furthermore, in this regime, carbon accumulation is more strongly restrained by natural decay than in the previous regime, since the carbon stock is higher and natural depreciation is proportional to its level. Finally, the economy reaches a final regime (area 3 in Figure 2), in which the carbon stock decreases, the resource continues to be depleted, and output growth remains positive.

Equation (18), together with Equations (13) and (14), shows that an increase in the environmental damage parameter δ affects the economy's optimal growth rate through two opposing channels. The first is a research-reallocation effect, captured by the term involving L_r. A higher δ slows fossil resource extraction ($g_F < 0$), which allows the fossil-related knowledge stock to grow at a lower optimal rate so as to keep the augmented ratio $RA_r(t)/F(t)A_f(t)$ constant (see Equation (15)). The social planner therefore reallocates research effort away from fossil-oriented innovation toward renewable-oriented research. Faster accumulation of renewable knowledge thus mitigates the negative impact of carbon accumulation and can foster output growth. The second effect is negative and operates through carbon accumulation: when $\dot{Z} > 0$, stronger climate damage directly reduces the socially optimal growth rate. Numerical simulations in Appendix C illustrate cases in which the research-reallocation effect dominates the production-reduction effect.

The efficiency of fossil-oriented research, η_f, operates through the aforementioned research-allocation effect: a higher η_f induces a reallocation of research toward renewables, making short-run optimal degrowth less likely. Appendix C illustrates this and shows that, conversely, higher renewable research efficiency η_r increases the likelihood of short-run decline.

4 Conclusion

By using the benchmark framework of directed technical change with climate (Acemoglu *et al.*, 2012; Hémous and Olsen, 2021), we have characterized the socially optimal trajectory of an economy that relies on both renewable energy and fossil fuels, the latter being detrimental to production. This allows us to examine how accounting for the climate impact of economic activity affects the sign of the optimal growth rate, and to offer novel insights beyond standard results.

We have shown that an initial phase of output decline can be socially optimal, particularly when climate damages to production are substantial, and the atmospheric carbon stock remains low relative to the available fossil resource. However, the more efficient fossil-oriented research is, the less likely this degrowth phase becomes. In all cases, this optimal phase of negative growth is temporary, and the long-run optimal growth rate remains – or eventually becomes – positive.

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Appendix

Appendix A: Proof of Proposition 1

Optimal conditions

The Lagrangian in current value associated with the social planner program writes:

$$\begin{aligned}\mathcal{L}(t) &= \ln C(t) + \int_0^1 \lambda_{A_r i}(t)\eta_r L_{ri}(t)A_{ri}(t)di + \int_0^1 \lambda_{A_f i}(t)\eta_f L_{fi}(t)A_{fi}(t)di \\ &\quad -\lambda_S(t)F(t) + \lambda_Z(t)\left[F(t) - \theta Z(t)\right] + \nu(t)\left[1 - \int_0^1 L_{ri}(t)di - \int_0^1 L_{fi}(t)di\right] \\ &\quad +\xi(t)\left[Y(t) - C(t) - \psi \int_0^1 x_{ri}(t)di - \psi \int_0^1 x_{fi}(t)di\right],\end{aligned}$$

where $\lambda_{A_{ji}}(t)$, $\lambda_S(t)$ and $\lambda_Z(t)$ are the costate variables associated to constraints (4), (7), and (3), respectively. $\nu(t)$ and $\xi(t)$ are the Lagrange multipliers associated to constraints (5) and (10). Maximizing this Lagrangian leads to a well-known result in the literature, which is symmetry across sectors i. To avoid some computational burdens, we directly incorporate this result into the first-order conditions of the program, which thus take the following forms:

$$C(t)^{-1} = \xi(t) \tag{22}$$

$$\alpha e^{-\delta Z(t)}\tilde{Y}(t)^{1/\varepsilon}Y_j(t)^{(\varepsilon-1)/\varepsilon}x_j(t)^{-1} = \psi, \text{ for } j = r, f \tag{23}$$

$$\lambda_{A_j}(t)\eta_j A_j(t) = \nu(t), \text{ for } j = r, f \tag{24}$$

$$(1-\alpha)\xi(t)e^{-\delta Z(t)}\tilde{Y}(t)^{1/\varepsilon}Y_f(t)^{(\varepsilon-1)/\varepsilon}F(t)^{-1} = \lambda_S(t) - \lambda_Z(t)\,. \tag{25}$$

The dynamic optimality conditions are:

$$\dot{\lambda}_S(t) = \rho\lambda_S(t) \tag{26}$$

$$\dot{\lambda}_Z(t) = (\rho+\theta)\,\lambda_Z(t) + \delta\xi(t)e^{-\delta Z(t)}\tilde{Y}(t) \tag{27}$$

$$\dot{\lambda}_{A_j}(t) = (\rho - \eta_j L_j(t))\,\lambda_{A_j}(t) - (1-\alpha)\xi(t)e^{-\delta Z(t)}\tilde{Y}(t)^{1/\varepsilon}Y_j(t)^{(\varepsilon-1)/\varepsilon}A_j(t)^{-1}. \tag{28}$$

By defining $O_j(t) \equiv \lambda_{A_j}(t)A_j(t)$ and by using (4), Condition (28) becomes:

$$\dot{O}_j(t) = \rho O_j(t) - (1-\alpha)\xi(t)e^{-\delta Z(t)}\tilde{Y}(t)^{1/\varepsilon}Y_j(t)^{(\varepsilon-1)/\varepsilon}\,. \tag{29}$$

Last, the transversality conditions are :

$$\lim_{t\to+\infty}\lambda_S(t)S(t)e^{-\rho t} = \lim_{t\to+\infty}\lambda_Z(t)Z(t)e^{-\rho t} = \lim_{t\to+\infty}O_j(t)e^{-\rho t} = 0\,. \tag{30}$$

Output, intermediate goods and R&D effort

By using Equations (1) and (23), we get:

$$\alpha Y(t) = \psi\left[x_r(t) + x_f(t)\right]\,. \tag{31}$$

Then, (10) and (31) give:

$$C(t) = (1-\alpha)Y(t) = (1-\alpha)e^{-\delta Z(t)}\tilde{Y}(t)\,. \tag{32}$$

Moreover, (24) implies $\eta_f O_f = \eta_r O_r$ and thus $g_{O_f} = g_{O_r}$. Using (29), we have $O_f(t)/O_r(t) = [Y_r(t)/Y_f(t)]^{(\varepsilon-1)/\varepsilon}$. Hence, given (23), we get:

$$\frac{x_r(t)}{x_f(t)} = \left[\frac{Y_r(t)}{Y_f(t)}\right]^{(\varepsilon-1)/\varepsilon} = \frac{\eta_f}{\eta_r}. \tag{33}$$

Using (31), we finally obtain:

$$x_f(t) = \frac{\alpha Y(t)}{\psi(1+\eta_f/\eta_r)} \quad \text{and} \quad x_r(t) = \frac{\alpha Y(t)}{\psi(1+\eta_r/\eta_f)} \tag{34}$$

which, together with (32), implies $g_{x_f}(t) = g_{x_r}(t) = g_Y(t) = g_C(t) = g_{\tilde{Y}}(t) - \delta\dot{Z}(t)$.

Time differentiating $\tilde{Y}(t)$ yields $d\tilde{Y}(t)/dt = \tilde{Y}(t)^{1/\varepsilon}\left[g_{Y_f}(t)Y_f(t)^{(\varepsilon-1)/\varepsilon} + g_{Y_r}(t)Y_r t)^{(\varepsilon-1)/\varepsilon}\right]$. Using (33), it comes:

$$g_{\tilde{Y}}(t) = g_{Y_f}(t) = g_{Y_r}(t)\,. \tag{35}$$

From (2), we get $g_{Y_r} = (1-\alpha)g_{A_r} + \alpha g_{x_r}$. From (34) and (35), we also know that $g_{Y_r} = g_{\tilde{Y}} = g_{x_r} + \delta\dot{Z}$. This implies $g_{x_r} = g_{A_r} - \delta\dot{Z}/(1-\alpha) = \eta_r L_r - \delta\dot{Z}/(1-\alpha)$.

By using (2), (10), and (35), we obtain: $g_F = g_{A_r} - g_{A_f} = \eta_r L_r - \eta_f L_f$. We deduce:

$$L_r(t) = \frac{\eta_f + g_F(t)}{\eta_r + \eta_f} \quad \text{and} \quad L_f(t) = \frac{\eta_r - g_F(t)}{\eta_r + \eta_f}\,. \tag{36}$$

Last, the optimal growth rate of the output is derived from (2), (4), (8), and (35): $g_Y = g_{\tilde{Y}} - \delta\dot{Z} = g_{Y_r} - \delta\dot{Z} = (1-\alpha)g_{A_r} + \alpha g_{x_r} - \delta\dot{Z} = (1-\alpha)\eta_r L_r + \alpha g_{x_r} - \delta\dot{Z}$. As $g_Y = g_{x_r}$ from (34), it comes immediately:

$$g_Y(t) = \eta_r L_r(t) - \frac{\delta\dot{Z}(t)}{(1-\alpha)}\,. \tag{37}$$

Dynamics of carbon accumulation

Thanks to (22) and (32), (27) can be rewritten as $\dot{\lambda}_Z(t) = (\rho+\theta)\lambda_Z(t) + \delta Y(t)/C(t) = (\rho+\theta)\lambda_Z(t) + \delta(1-\alpha)^{-1}$. This implies $\lambda_Z(t) = e^{(\rho+\theta)t}\left[\lambda_Z(0) + \delta(1-\alpha)^{-1}\int_0^t e^{-(\rho+\theta)s}ds\right]$. Moreover, the solution of (7) is given by $Z(t) = e^{-\theta t}\left[Z(0) + \int_0^t F(s)e^{\theta s}ds\right]$. Replacing Z and λ_Z by these last two expressions in (30) yields $\lambda_Z(0) = -\delta(1-\alpha)^{-1}\int_0^\infty e^{-(\rho+\theta)s}ds$. We finally obtain:

$$\lambda_Z(t) = \bar{\lambda}_Z = \frac{-\delta}{(\rho+\theta)(1-\alpha)}\,, \tag{38}$$

which reads as the social cost of carbon: the value, discounted to date 0, of the social cost of one unit of carbon emitted at date t expressed in utility terms.[5]

[5] To see this, let us assume that, at a given point in time $\bar{t}$, resource extraction increases by $dF(\bar{t})$. Additional released carbon emissions accumulate into the atmosphere, but they are gradually dissipated at rate θ at each

Dynamics of fossil resource use

By using (22) and (32), Equation (25) becomes $\left[\tilde{Y}(t)/Y_f(t)\right]^{(1-\varepsilon)/\varepsilon} = \left[\lambda_S(t) - \bar{\lambda}_Z\right] F(t)$. Since, from (35), $g_{\tilde{Y}} = g_{Y_j}$, $j = r, f$, time differentiating this equation gives:

$$g_F(t) = \frac{-\rho\lambda_S(t)}{\lambda_S(t) - \bar{\lambda}_Z}, \tag{39}$$

where $\lambda_S(t) = \lambda_S(0)e^{\rho t}$ from (26). Moreover, by plugging (23), (32), and (34) into (25), we get:[6]

$$F(t) = \left(\frac{\eta_r}{\eta_r + \eta_f}\right)\left(\frac{1}{\lambda_S(0)e^{\rho t} - \bar{\lambda}_Z}\right),$$

where $\lambda_S(0)$ is such that $\int_0^{+\infty} F(t)dt = S_0$. From this, we deduce:

$$\lambda_S(0) = \frac{-\bar{\lambda}_Z}{e^{\rho BS_0} - 1}, \text{ with } B \equiv -\bar{\lambda}_Z\left(\frac{\eta_r + \eta_f}{\eta_r}\right) = \frac{\delta(\eta_r + \eta_f)}{(\rho + \theta)(1 - \alpha)\eta_r} > 0\,. \tag{40}$$

Hence, we have:

$$F(t) = \frac{e^{\rho BS_0} - 1}{B(e^{\rho t} + e^{\rho BS_0} - 1)}\,, \tag{41}$$

which implies:

$$g_F(t) = \frac{-\rho}{1 + (e^{\rho BS_0} - 1)e^{-\rho t}}. \tag{42}$$

Note that $g_F(t) < 0$ for all t. We also deduce $g_F(0) = -\rho e^{-\rho BS_0}$ and $\lim_{t\to+\infty} g_F(t) = -\rho$. The fossil resource is thus depleted asymptotically.

Appendix B: Dynamics of the system in the (S, Z) plane

To build a phase diagram in the (S, Z) plane, we first combine (11) and (12) to rewrite the fossil resource extraction under its open-form expression:

$$F(t) = \frac{1 - e^{-\rho BS(t)}}{B}\,. \tag{43}$$

Then, given (7), the isocline curve $\left\{\dot{Z}(t) = 0\right\}$ is such that $Z = \frac{1-e^{-\rho BS}}{\theta B} \equiv h(S)$, with $h(0) = 0$, $\lim\limits_{S\to+\infty} h(S) = \frac{1}{\theta B}$, $h'(S) > 0$ and $h''(S) < 0$. The associated phase diagram is depicted in Figure 1.

time $t \geq \bar{t}$, which results in an increase in the atmospheric carbon stock by $dZ(t) = dF(\bar{t})e^{-\theta(t-\bar{t})}$. Given (1) and (8), this rise in the carbon concentration reduces the final output at any time $t \geq \bar{t}$, and then the consumption, by an amount $dC(t) = dY(t) = -\delta Y(t)dZ(t)$, which entails a diminution of the utility of consumption by $-\delta u_C(t)Y(t)dF(\bar{t})e^{-\theta(t-\bar{t})}$. Given that $u(C) = \ln C$, and from (32), this instantaneous utility loss equals $\frac{-\delta}{(1-\alpha)}dF(\bar{t})e^{-\theta(t-\bar{t})}$ which, in cumulative discounted value, yields $\int_{\bar{t}}^{\infty} \frac{-\delta}{(1-\alpha)}dF(\bar{t})e^{-(\rho+\theta)(t-\bar{t})}dt = \frac{-\delta}{(1-\alpha)(\rho+\theta)}dF(\bar{t})$.

[6] We use the following general result on indefinite integrals of exponential forms: $\int \frac{e^{at}}{b+ce^{at}}dt = \frac{1}{ac}\ln(b+ce^{at})$.

Next, using (12) and (13), we can write:

$$g_F(t) = -\rho\left[1 - BF(t)\right] . \tag{44}$$

Replacing $\dot{Z}$, L_r and g_F into (18) by their expressions coming from (7), (14) and (44), respectively, the optimal growth rate of the economy can be rewritten as $g_Y = \frac{\eta_r(\eta_f-\rho)}{\eta_r+\eta_f} - \left(\rho\bar{\lambda}_Z + \frac{\delta}{1-\alpha}\right) F + \left(\frac{\delta\theta}{1-\alpha}\right) Z$. Last, replacing F by $\theta h(S)$ from Equation (43), expanding the expression of $\bar{\lambda}_Z$ and rearranging, we get:

$$g_Y(t) = \frac{\eta_r\left(\eta_f-\rho\right)}{\eta_r+\eta_f} - \frac{\delta\theta^2}{(\rho+\theta)(1-\alpha)} h(S(t)) + \frac{\delta\theta}{(1-\alpha)} Z(t) \,. \tag{45}$$

Appendix C: Sensitivity analysis

We study the effects of some key parameters on the set of all pairs (S_0, Z_0) such that the socially optimal growth of output is initially negative, as illustrated by area 1 in Figure 2. Let us denote by $\Omega(S)$ this area, that is, the surface below the positive part of the $k(S)$-curve for any $S > \hat{S}$, with $\hat{S}$ being such that $k(\hat{S}) = 0$: $\Omega(S) \equiv \int_{\hat{S}}^{S} k(x)dx$.

By rewriting the function k as $k(S) = \frac{1}{(\rho+\theta)B}\left[\frac{(\rho+\theta-\eta_f)}{\theta} - e^{-\rho BS}\right]$, with $B \equiv \frac{\delta(\eta_r+\eta_f)}{\eta_r(1-\alpha)(\rho+\theta)}$, and by defining $\hat{S}$ such that $e^{\rho B\hat{S}} = \frac{\theta}{\rho+\theta-\eta_f}$, we can express $\Omega(S)$, after computation, as:

$$\begin{aligned}\Omega(S) &= \frac{1}{(\rho+\theta)B}\left[\left(S-\hat{S}\right)e^{-\rho B\hat{S}} + \frac{e^{-\rho BS} - e^{-\rho B\hat{S}}}{\rho B}\right] \\ &= \frac{1}{\rho(\rho+\theta)B^2}\left\{e^{-\rho BS} + \left(\frac{\rho+\theta-\eta_f}{\theta}\right)\left[\rho BS - 1 - \ln\left(\frac{\theta}{\rho+\theta-\eta_f}\right)\right]\right\} .\end{aligned} \tag{46}$$

Given the complexity of this expression, it is not possible to sign the partial derivative of Ω with respect to the various parameters. We will instead run simulations to identify the most relevant results. We focus on three parameters: the R&D efficiency parameter in the carbon-free and carbon-based sectors, η_r and η_f, and the marginal rate of climate damage, δ.[7] We then proceed as follows:

- Step 1: We draw the $\Omega(S)$-curves for any $S > \hat{S}$ and for two different values of the parameter of interest, all the others being given (see Figure 3);

- Step 2: We confirm the result identified at step 1 by depicting the area Ω as a function of this parameter of interest, and for restricted values of S (see Figure 4).

From these simulations, we can observe that the set of pairs (S_0, Z_0) such that $g_Y(t) < 0$ is expanding with η_r – meaning a higher risk of degrowth in the short run – and is contracting with η_f and δ.

[7] These simulations are produced by using the following calibration: $\varepsilon = 0.4$, $\alpha = 0.2$, $\psi = 1$, $\rho = 0.05$, $\theta = 0.05$. Moreover, we consider the following baseline values for the parameters of interest: $\eta_r = 0.1$, $\eta_f = 0.06$, $\delta = 1$. The objective here is not to provide a complete and realistic calibration of our model, but to numerically illustrate our main theoretical results.

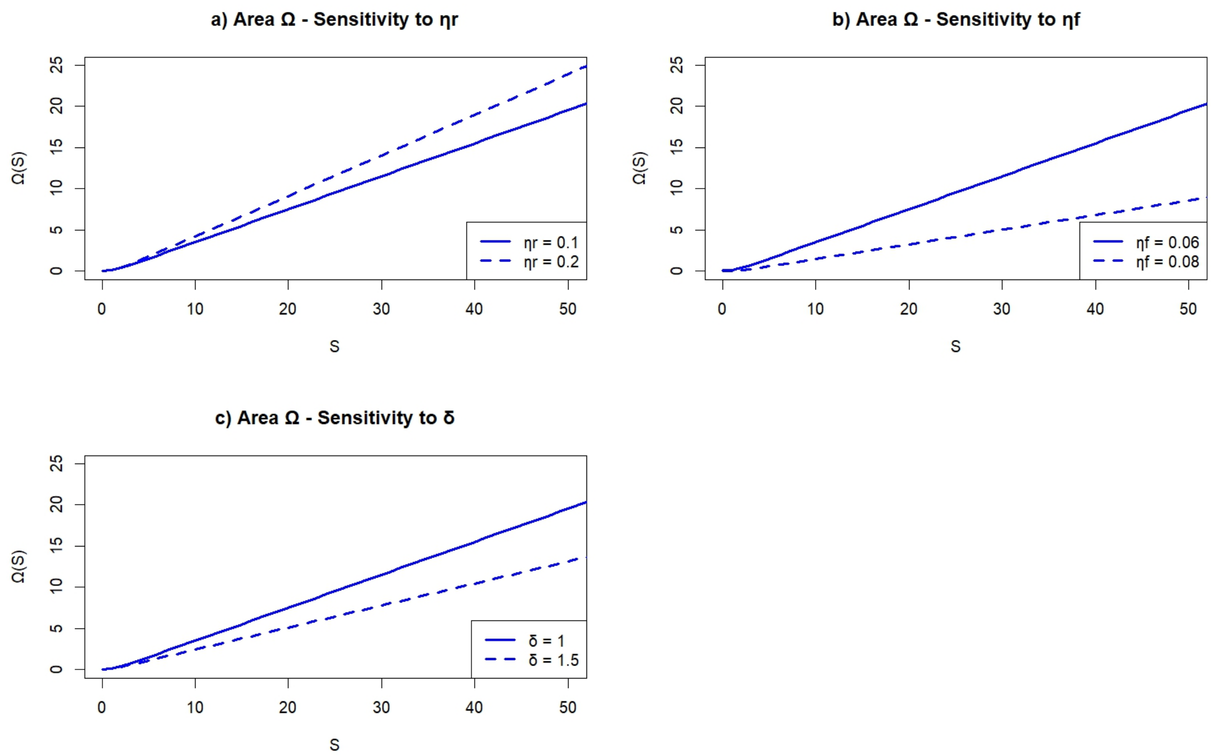

Figure 3: Sensitivity analysis of the area $\Omega(S)$ – Step 1

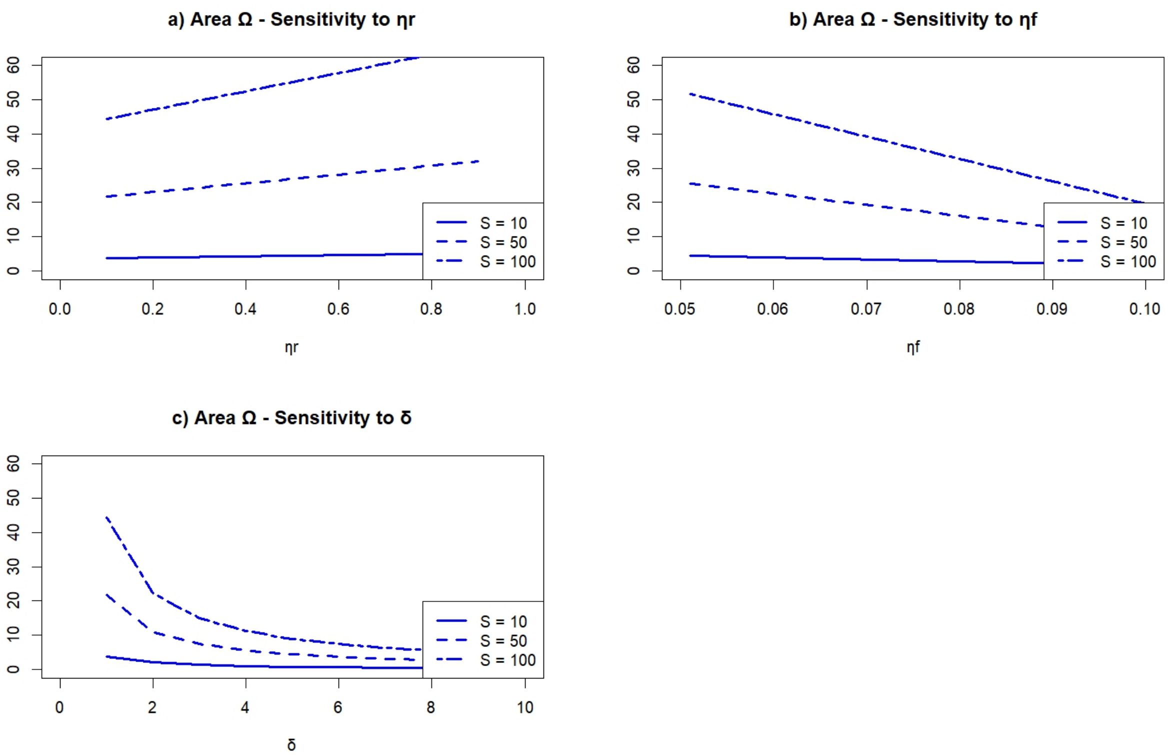

Figure 4: Sensitivity analysis of the area $\Omega(S)$ – Step 2