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"Gaming the Giants: How Startups Shape Innovation to Spark Acquisition Wars"

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Gaming the Giants: How Startups Shape Innovation to Spark Acquisition Wars*

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Abstract

We study a startup's choice of its "direction of innovation," how well the technology fits alternative acquirers, and the effects on acquisition outcomes and market dominance. Two horizontally differentiated firms bid to acquire the innovation and then compete in the product market. Firms differ in initial quality stock and in "absorption capabilities," how effectively the acquired innovation is integrated into their stock. The innovator designs the innovation to intensify bidding by putting firms on a more equal footing, thereby favoring the initially lower-quality firm. As a result, "increasing dominance" is less likely than under exogenous fit. The winner of the innovation is driven primarily by relative absorption capabilities rather than initial quality: the firm with higher absorption capability is more likely to win. The equilibrium innovation direction minimizes industry profit and consumer surplus. In a two-period model, decreasing dominance becomes more likely when the low-quality firm has stronger absorption capabilities.

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1 Introduction

There has been a significant increase in acquisitions of innovations by firms with strong market power, for example, in big tech and pharmaceutical industries, often in adjacent (complementary) markets (Jin et al., 2023; Eisfeld, 2023). Acquisitions have overtaken IPOs as the primary exit route for startups (Alezra and Berquier, 2024). The possibility of being acquired can significantly shape a startup innovator's incentives and behavior before any contractual agreement.

In this paper, we study how an innovator chooses the relative fit of innovation to potential acquirers' technology stacks (*the direction of innovation*), and how this choice affects the acquisition outcome, and subsequent downstream market competition. In particular, we analyze when dominant firms are likely to remain dominant by acquiring new innovations (*increasing dominance*). We account for initial asymmetries in the acquirers' technology stacks as well as in their *capabilities* to integrate the innovation into their stack.

Our two main contributions are the following. First, we show that increasing dominance is less likely when the innovator's choice for the direction of innovation is endogenous. Second, we find that the main determinant of which firm acquires the innovation is the asymmetry in acquirers' capabilities to absorb the innovation, rather than the asymmetry in their initial quality stocks.

Teece et al. (1997) defines dynamic capabilities as "the key role of strategic management in appropriately adapting, integrating, and reconfiguring internal and external organizational skills, resources, and functional competences to match the requirements of a changing environment". The management literature and industry observers have recognized the importance of firms' dynamic capabilities that enable them to achieve long-run competitive success, and sometimes it can be more important than companies large stock of valuable technology assets or resources. We introduce this concept to study competition for acquiring innovations.[1]

Some recent examples of successes and failures illustrate the importance of firm capabilities to absorb an innovation. Microsoft acquired Skype (video conference app) in 2011 and replaced it by its own Teams application. Similarly, it acquired Wunderlist (To-Do App) in 2016 and replaced it after one year with its own To-Do App, since Wunderlist's API was running on Amazon Web Services and it proved be too costly to port to Azure (Microsoft Cloud Services) (TheVerge, March 21, 2018). On the other hand, Amazon successfully acquired and integrated robotics companies, Kiva Systems and Rightbot, which have improved significantly Amazon's operational efficiency and logistics (Allgor et al., 2023).

The direction of innovation can take different forms in practice. For instance, it can capture interoperability decisions of innovators when making the innovation more compatible (optimizing for one potential acquirer against the other).[2] The innovation direction can also be more comple-

[1]For ease of exposition, we will refer to a firm's absorption capabilities to integrate an innovation instead of dynamic capabilities.

[2]The cloud services of Microsoft Azure and Amazon Web Services (AWS) are known to be incompatible, and it is very costly to move services from one architecture to another (Biglaiser et al., 2024). An innovator therefore needs to decide on which cloud architecture to build its application on.

mentarity to one acquirer's technology stack vs another. Google acquired Waze map application by outbidding Facebook and Apple (Guardian, May 2013, Marketing Dive, 2013). Waze's social and traffic data features could arguably be a better complement to Facebook or Apple than Google, since Facebook and Apple could have accessed to Waze's real-time location data and social network effects. Google in contrast had already that data from its own map application.

For a successful acquisition of innovation, both the direction of innovation and the acquirer's capability of integrating it are important. For example, in 2019 Intel acquired Habana (AI-focused startup) at $ 2 billion to improve its competitive position against Nvidia (SDxCentral, December 2019). Despite the good fit, the acquisition was not successful as Intel failed to integrate Habana (Ctech, February 2025).[3]

To study endogenous direction of innovation and its impact on the downstream market we model two horizontally differentiated firms that compete to acquire a startup and then compete in the downstream market. The downstream firms are asymmetric in two dimensions: initial quality and capability to integrate the innovation. Prior to the acquisition stage, the startup chooses the distribution of relative fit of innovation (the direction of innovation). We assume that the value of innovation is significant: either firm can win the innovation depending on its fit. After observing the realized fit, downstream firms make competing bids to acquire the startup, and the innovator sells it to the firm with the highest bid. After acquisition downstream firms compete in prices. Using this framework we study the equilibrium choice of the innovator and the resulting probability of each firm winning the innovation.

The innovator's objective is to maximize the selling price for its innovation. Given the two sources of asymmetry between the firms, the innovator's choice aims at putting the downstream firms on a more equal footing (equalizing their willingness-to-pays for innovation). To counterbalance the initial asymmetry in quality the innovator chooses the direction of innovation closer to the lower-quality firm. The magnitude of this effect depends on both initial quality asymmetry and the asymmetry in capabilities. The stronger the initial quality asymmetry, the closer the fit of innovation to the initially lower-quality firm. The fit moves even closer to the weaker firm when the higher quality firm is also more capable in integrating the innovation. On the other hand, this effect is partially mitigated when the higher quality firm is less capable.

The innovator's choice also determines the likelihood of each firm winning the innovation. We show that for any level of uncertainty on the innovation fit–even when uncertainty is nearly zero–both firms have a positive probability of acquiring the startup in equilibrium, and this likelihood depends on the relative capabilities. When firms have equal capabilities to integrate the innovation, they are equally likely to acquire the startup despite the initial quality asymmetry. Otherwise, the firm with the higher capability to integrate the innovation is more likely to acquire the startup.

The winner of the innovation is the market leader. This means that market leadership flips (the lagger becomes dominant) if the lower-quality firm wins the innovation. The endogenous nature of innovation makes decreasing dominance more likely to arise in equilibrium. To see this, suppose the

[3] Intel was one of the four investors in Habana and so had a significant influence on Habana's technology.

fit of innovation was exogenously random and consider the simplest case of symmetric absorption capabilities. The initially stronger firm would then be more likely to win the innovation than in the case where the innovator chooses the direction of innovation. With endogenous innovation the market always becomes more concentrated post-acquisition.

We find that the innovator's equilibrium choice minimizes both the industry profit and consumer surplus among all possible directions of innovation. The innovator sacrifices some industry profit to equalize the maximum willingness to pays across the firms. The endogenous choice of the innovator results in the lowest quality for the winner among all possible quality levels where it wins.

Finally, we analyze a two-period version of the model where in each period a different innovator chooses its direction of innovation. Dynamics make decreasing dominance more likely when the initially lower-quality firm is better able to absorb innovation. The opposite is true otherwise. The lower quality firm is more (less) likely to win the innovation in the first period compared to the one-period game if it is better (worse) at absorbing the innovation.

Related Literature: Anti-trust authorities have been concerned that dominant incumbents acquire potential future rivals and kill their products to remain dominant, so called "killer acquisitions" (Cunningham et al., 2021; Letina et al., 2024; Fumagalli et al., 2022). In our model each firm's acquisition strategy reflects both offensive aspects (improving own quality stock) and a defensive aspect (preventing rival from improving its quality stock), while the innovator tries to increase competition between the firms via choosing the innovation's direction. When the incumbent's quality advantage is not too large, the equilibrium direction of innovation benefits both firms directly, and so there is no killer acquisition.[4] Another concern has been the reduced incentives to innovate in some directions due to the increase in incumbents' market power when consumers expect the innovator to be acquired, so called "killer zone" (Kamepalli et al., 2020). We argue that, when some competition remains, an innovator can counter-balance the incumbent's market power by distorting innovation toward the weaker rival.

We contribute to the literature studying when increasing dominance prevails in the market (Gilbert and Newbery, 1982, 1994; Reinganum, 1983; Fudenberg et al., 1983; Segal and Whinston, 2007; Denicolo and Polo, 2021). Denicolo and Polo (2021) address the issue of increasing dominance through start-up acquisition. Innovation is treated as being exogenous in these previous studies, while we endogenize the innovator's choice of the relative value of innovation for each firm and document how this is necessary to have decreasing dominance.

Our work is also related to the literature on "merger for buyout" (Rasmusen, 1988). There is small amount of work studying the innovator's choice for the type (or direction) of innovation anticipating the possibility of acquisition by an incumbent,(Bryan and Hovenkamp, 2020; Dijk et al., 2024; Callander and Matouschek, 2022; Motta and Shelegia, 2024). In contrast to Bryan and Hovenkamp (2020), in our setup the weaker firm has a positive likelihood to acquire the innovation and

[4]If the initial quality advantage is very large, the innovator will indeed induce a killer acquisition by choosing a direction of innovation that brings no value to the incumbent, who is the acquirer of the innovation.

the innovator (via its choice of the direction of innovation) can influence the probability of being acquired by each firm and their willingness-to-pay for innovation. As a result, the innovator biases the direction of innovation against the initially stronger firm, whereas in Bryan and Hovenkamp (2020) the innovator always favors the initially stronger firm as it is always the acquirer of the innovation.[5] We differ from Dijk et al. (2024) and Motta and Shelegia (2024) since we assume that the innovator does not become a rival in the downstream market and so it only cares about the price at which it sells its innovation to downstream firms.

Our acquisition auction provides an example of auctions with externalities, as in Jehiel and Moldovanu (2000) , as well as bidder asymmetries, following Myerson (1981) and Maskin and Riley (2000). The literature on asymmetric auctions typically emphasizes the choice of auction format, often involving handicapping certain bidders (see, for example, McAfee and McMillan (1989) and Laffont and Tirole (1991)). In contrast, we hold the auction format fixed and instead endogenize the externality by allowing the innovator to design the product in a way that reduces asymmetries across bidders.

Section 2 presents our benchmark model. Section 3 provides an equilibrium analysis. Section 4 provides a demand model to study welfare implications. Section 5 provides extensions. Section 6 studies the two-period version of our model. Section 7 concludes. All formal proofs are in the Appendix.

2 Model

We consider two differentiated firms A and B with different base qualities, Q_A and Q_B respectively, such that A has an initial quality advantage of $\Delta_0 = Q_A - Q_B > 0$. We refer to B as the initially weaker firm.

There is an innovator I who chooses its direction of innovation $\theta \in [0, 1]$ determining the probability distribution $g(\alpha; \theta)$ of a parameter α, which measures how well the innovation fits product B relative to product A. The choice of θ can be interpreted as an interoperability decision of an application developer when it chooses which cloud architecture to build its application on. It can also be complementarity of the innovation to potential acquirers' quality stack, like in the example of Waze-Google, see the Introduction.

We assume a uniform distribution $g(\alpha; \theta) = \frac{1}{2\sigma}$ on interval $[\theta - \sigma, \theta + \sigma]$ where $\theta \in [\sigma, 1 - \sigma]$. The parameter σ captures the uncertainty about the realized match. We assume that σ is not too large so that $\alpha \in (0, 1)$ and often focus on the limit case where the uncertainty goes to zero.

The value of innovation is $q > 0$, which is assumed to be exogenous. Firms can differ in their *capability* to incorporate the innovation for a given level of relative innovation fit. We capture this difference by introducing an absorption capability $\lambda_j \geq 0$ for firm $j = A, B$, such that λ_j measures how well firm j's product quality is improved with the innovation. Firms' capabilities to integrate

[5] In addition, we consider significant innovation, which allows the initially weaker firm to leapfrog the leader if it acquires the innovation.

the technology into their quality stock is built on their characteristics from past investments and acquisitions of technologies. We assume that this capability is exogenous to individual innovation acquisition and difficult to adjust in the short run. Microsoft's inability to integrate Skype and Wunderlist (discussed in the Introduction) could arguable be examples of our absorption capability parameter, λ_j.

The innovator has no bargaining power and the only option to generate some profits is to induce competition between the downstream firms.[6] We assume that the innovator uses a second-price auction to sell its innovation once the location of innovation α is realized. The firm with the highest bid wins the innovation and pays to the innovator the bid of the losing firm.[7]

Let Δ^j denote the (ex-post) quality differential between firm A and firm B when firm j wins the innovation. Note that Δ^j is negative when B has a higher quality level than A. If firm A has the innovation, its quality advantage becomes

$$\Delta^A = \Delta_0 + (1-\alpha)\lambda_A q. \tag{1}$$

If firm B has the innovation, the quality difference between A and B becomes

$$\Delta^B = \Delta_0 - \alpha\lambda_B q. \tag{2}$$

Thus, how the firm's quality changes if it acquires the innovation is independent of its base quality.

To guarantee that firm B has a chance to win the innovation (at least for large α), we assume that the value of the innovation for firm B is sufficiently high compared to the initial quality advantage of firm A:

Assumption 1 $\lambda_B q > 2\Delta_0$.

Downstream profits: Let Δ be the quality differential between firm A and firm B. Given (1) and (2), Δ ranges from $\Delta_0 - \lambda_B q$ (when B wins and innovation is the best match for B, $\alpha = 1$) to $\Delta_0 + \lambda_A q$ (when A wins and innovation is the best match for A, $\alpha = 0$). We assume that the gross downstream equilibrium profits (not including payments to the innovator) are only functions of Δ and

Assumption 2 *i. Profit of firm A, $\Pi_A(\Delta)$, is increasing and convex,*

ii. Profit of firm B, $\Pi_B(\Delta)$, is decreasing and convex,

iii. Profits are symmetric: $\Pi_A(\Delta) = \Pi_B(-\Delta)$.

We provide micro-foundations under which these assumptions hold in Section 4. Next we define total industry profit,

$$\Pi^T(\Delta) = \Pi_A(\Delta) + \Pi_B(\Delta) \tag{3}$$

[6]We discuss how our qualitative results change if the innovator has more bargaining power in Section 5.4

[7]We focus on startup acquisitions and do not consider licensing of innovation. In our setup, the innovator would choose exclusive sale of innovation if it had a choice of giving it to both firms or to only one.

Assumption 2 implies that $\Pi^T(\Delta)$ is a convex, U-shaped function which is symmetric around $\Delta = 0$. The innovator's payoff is a transfer from the winning firm to the innovator, so included in the industry profit.

Timing: First, the innovator chooses its direction of innovation θ, which generates an α according to the distribution function $g(\alpha; \theta)$. Second, the firms bid in a second-price auction to acquire the innovation. Firms realize their profits.

We look for the Subgame Perfect Nash Equilibrium of the game assuming an interior solution for the direction of innovation (which is guaranteed by Assumption 3 below).

3 Equilibrium analysis

3.1 Acquisition of the innovation:

We analyze the second stage of the game given the realization of α. The firms bid simultaneously to acquire the innovation. Firm j's maximum willingness-to-pay to acquire the innovation (WTP_j) is the difference between its (gross) profits if it wins the innovation less its profit if its rival wins:

$$WTP_j(\alpha) = \Pi_j(\Delta^j) - \Pi_j(\Delta^{-j}). \tag{4}$$

Given that Δ^j is a function of innovation fit, α, from (1) and (2), WTP_j is a function of α. There are two effects that influence WTP_j. One is the *direct* effect of having the innovation on the own profit, $\Pi_j(\Delta^j)$, and the other is the *indirect* effect on the profit when the rival wins the innovation, $\Pi_j(\Delta^{-j})$. Both effects depend on the value of the innovation q, the fit of the innovation (the draw of α), and the absorption capabilities, λ_A and λ_B. The direct effect increases in the firm's own absorption capability λ_j: when the firm is better able to incorporate the innovation, this increases the value of acquiring the innovation. The indirect effect rises in the rival's absorption capability λ_{-j}: when the rival better incorporates the innovation, this decreases the firm's profit in case the rival acquires the innovation. Consequently, both firms' WTPs are increasing in λ_A and λ_B.

In equilibrium, the firm with the highest WTP wins the innovation and pays its rival's WTP (losing bid). We next present two properties of the model that are important to prove that the WTPs cross only once:

Property 1. The difference in WTPs is equal to the difference in total industry profit when each firm wins the innovation:

$$WTP_A(\alpha) - WTP_B(\alpha) = \Pi^T(\Delta^A) - \Pi^T(\Delta^B).$$

Property 2. The total industry profit is a function of A's quality differential, it is symmetric and U-shaped around $\Delta = 0$.

Property 1. implies that the WTPs are equalized when $\Pi^T(\Delta^A) = \Pi^T(\Delta^B)$. This only holds when $\Delta^A = -\Delta^B$ due to the symmetry property of the total industry profit function, Property 2.. This fact is important in the construction of the equilibrium. We next characterize a cutoff on α that determines the single crossing of the WTPs and when each firm wins the innovation.

Proposition 1 *There exists a unique cutoff $\widehat{\alpha} \in (0,1)$ at which $WTP_A(\widehat{\alpha}) = WTP_B(\widehat{\alpha})$:*

$$\widehat{\alpha} = \frac{2\Delta_0 + \lambda_A q}{q(\lambda_A + \lambda_B)},$$

and $\Delta^A(\widehat{\alpha}) = -\Delta^B(\widehat{\alpha})$. For $\alpha < \widehat{\alpha}$, $WTP_A(\alpha) > WTP_B(\alpha)$ and firm A wins the innovation. For $\alpha > \widehat{\alpha}$, $WTP_A(\alpha) < WTP_B(\alpha)$ and firm B wins the innovation.

At $\alpha = \widehat{\alpha}$, the leader quality edge is given by

$$\Delta^A(\widehat{\alpha}) = -\Delta^B(\widehat{\alpha}) = \frac{q\lambda_B\lambda_A + (\lambda_B - \lambda_A)\,\Delta_0}{\lambda_A + \lambda_B}. \tag{5}$$

In other words, at $\alpha = \widehat{\alpha}$, the fit of innovation neutralizes the initial quality asymmetry and leads to the same ex-post quality advantage for the winner of the innovation regardless of A's initial quality advantage, Δ_0. Below we show that the cutoff $\widehat{\alpha}$ is important for the innovator's equilibrium choice; its comparative follow directly implied from Proposition 1:

Corollary 1 *The location of innovation $\widehat{\alpha}$, which equalizes the WTPs, increases in initial asymmetry Δ_0 and the absorption capability of firm A, λ_A, while it decreases in the quality of innovation q and the absorption capability of firm B, λ_B. Finally, it is independent of the profit functions.*

The fact that $\widehat{\alpha}$ is independent of the profit function is due to two aspects of our model. First, recall that the difference between the WTPs corresponds to the difference between the total industry profit when firm A wins innovation versus when firm B wins, Property 1. Second, the point where this difference is zero determines $\widehat{\alpha}$ and this is independent of profit function because total industry profit is U-shaped and symmetric around $\Delta = 0$, Property 2. The comparative statics of λ_A follows from Assumption 1. We discuss the main forces behind the comparative statics of Δ_0 and q after presenting the innovator's equilibrium choice.

3.2 The innovator's equilibrium choice of θ

The innovator's payoff is equal to the lowest WTP (bid) (due to the nature of a second-price auction). Hence, the innovator's payoff is the expected returns from selling the innovation, which is equal to the probability that A wins ($\alpha \leq \widehat{\alpha}$) times the expected willingness-to-pay of the losing firm, WTP_B, plus the probability that B wins ($\alpha \geq \widehat{\alpha}$) times the expected willingness-to-pay of the losing firm, WTP_A:

$$V(\theta) = \int_0^{\widehat{\alpha}} WTP_B(\alpha)g(\alpha;\theta)d\alpha + \int_{\widehat{\alpha}}^1 WTP_A(\alpha)g(\alpha;\theta)d\alpha.$$

As a higher α implies a better fit of the innovation to firm B relative to firm A, one would expect that the WTP of firm A declines in α, while the WTP of firm B increases in α. This is the case when the direct effect of α on the winning profit outweighs its indirect effect on the losing profit. In the Appendix, Lemma 2 demonstrates that this holds when there is a sufficiently small gap in the relative absorption capabilities of two firms, $|\lambda_B - \lambda_A|$ is not too large. We make the following assumption to focus on this case.

Assumption 3 *$WTP_A(\alpha)$ is decreasing in α and $WTP_B(\alpha)$ is increasing in α.*

The assumption implies that $V(\theta)$ is increasing in θ on the range $\theta \leq \widehat{\alpha} - \sigma$ and decreasing in θ on the range $\theta \geq \widehat{\alpha} + \sigma$ (see the proof of Proposition 2). For $\theta \in [\widehat{\alpha} - \sigma, \widehat{\alpha} + \sigma]$ the payoff function of the innovator becomes

$$V(\theta) = \frac{1}{2\sigma}\left(\int_{\theta-\sigma}^{\widehat{\alpha}} WTP_B(\alpha)d\alpha + \int_{\widehat{\alpha}}^{\theta+\sigma} WTP_A(\alpha)d\alpha\right). \tag{6}$$

The innovator chooses θ^* that maximizes $V(\theta)$, which corresponds to maximizing the expected minimum WTP across the two firms. The following characterizes θ^*, assuming an interior solution.

Proposition 2 *The innovator's equilibrium choice is $\theta^* \in [\widehat{\alpha} - \sigma, \widehat{\alpha} + \sigma]$, where $\widehat{\alpha}$ is given by Proposition 1, and*

$$WTP_A(\theta^* + \sigma) - WTP_B(\theta^* - \sigma) = 0. \tag{7}$$

Increasing θ shifts the draw of innovation α towards firm B and so raises the likelihood that $\alpha > \widehat{\alpha}$, in which case firm B wins. This generates extra revenue of $WTP_A(\theta + \sigma)$ for the innovator because this shifts the upper part of the distribution above $\theta + \sigma$. It also reduces the likelihood that firm A wins, which reduces the revenue of the innovator by $WTP_B(\theta - \sigma)$. The innovator's expected profit is therefore maximized when these two effects are equal.

This incentive is similar to handicapping in asymmetric auctions (Maskin and Riley, 2000). The innovator chooses the direction of innovation moving the fit closer to the initially weaker firm, B, so as to equate the WTPs, and thus maximize its payoff.

The following corollary complements Proposition 2:

Corollary 2 *When there is no uncertainty, $\sigma \to 0$, $\theta^* = \widehat{\alpha}$. Otherwise,*

- *If absorption capabilities are the same, $\lambda_B = \lambda_A$, $\theta^* = \widehat{\alpha}$,*
- *If B has higher absorption capability, $\lambda_B > \lambda_A$, $\theta^* > \widehat{\alpha}$,*
- *If A has higher absorption capability, $\lambda_A > \lambda_B$, $\theta^* < \widehat{\alpha}$.*

We now illustrate the innovator profit and choice of fit with a focus on the relative absorption capabilities. Figure 1 illustrates WTP_A as a decreasing function of α (blue line), WTP_B as an increasing function of α (red line), and the innovator's choice of fit $\theta^* = \widehat{\alpha}$ when $\lambda_A = \lambda_B$. When

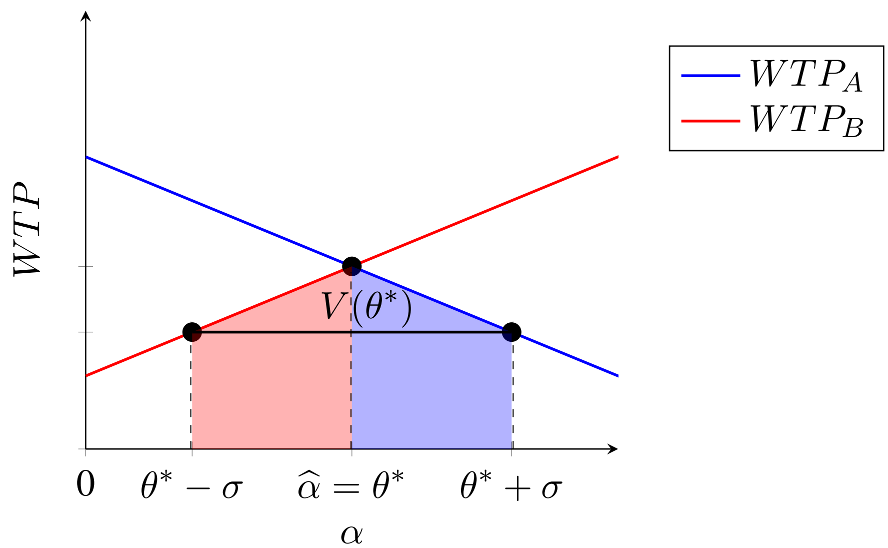
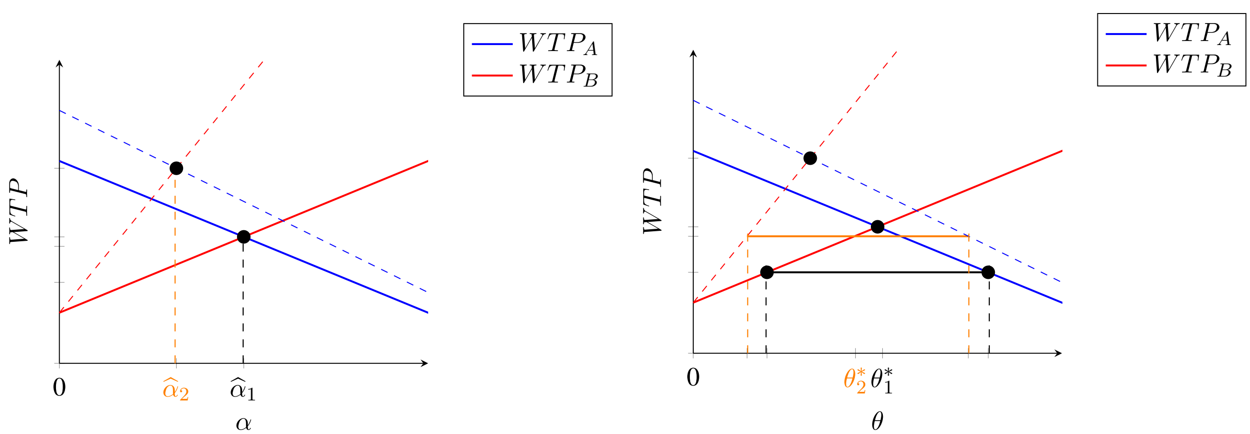


Figure 1: Equilibrium location of innovation, θ^*, when $\lambda_A = \lambda_B$

the draw of the fit α is lower than $\widehat{\alpha}$, firm A wins the innovation (Proposition 1), in which case the innovator pays the losing bid, WTP_B. The red shaded area in the figure gives the expected payoff of the innovator when A wins. If α is higher than $\widehat{\alpha}$, firm B wins the innovation, in which case the innovator pays the losing bid, WTP_A. The blue shaded area in the figure gives the expected payoff of the innovator when B wins. The sum of the red and blue areas correspond to the innovator's total expected payoff, $V(\theta^*)$. The innovator chooses the fit θ^* that maximizes its total expected payoff, and so at the innovator's choice we have $WTP_A(\theta^* + \sigma) = WTP_B(\theta^* - \sigma)$. When $\lambda_A = \lambda_B$, both firms WTPs have the same slope in absolute values (see the proof of Lemma 2 in the Appendix). Hence, the point where WTPs intersect corresponds to the point of fit that maximizes the innovator's payoff, $V(\theta)$, that is, we have $\theta^* = \widehat{\alpha}$ when $\lambda_A = \lambda_B$, with the obvious implication that each firm is equally likely to win.

The effect of a change in absorption capability is illustrated in Figures 2a and 2b. Increasing λ_B above λ_A increases firm B's WTP via raising the direct effect of having the innovation since the change makes the innovation more valuable to firm B. Increasing λ_B also affects firm A's profit via the indirect effect, since A's profit is falling in B's absorption capability in case B wins the innovation. Thus, increasing λ_B shifts both WTPs (the blue and the red lines) upwards in the figures, but it shifts the WTP of firm B more than the WTP of firm A: the direct effect dominates the indirect effect (this is implied by the convexity and symmetry of each firm's profit function in firm A's market share, which we prove in the Appendix). As a result, the intersection point of the two WTPs moves left (decreases from $\widehat{\alpha_1}$ to $\widehat{\alpha_2}$), see Figure 2a. Moreover, increasing λ_B also increases the slopes of both WTPs by amplifying the effect of the innovation location on WTPs. Firm B's direct effect reaction is steeper than firm A's indirect effect reaction, and so the change increases the slope of WTP_B more than the slope of WTP_A. For positive σ, this leads to a decrease in the equilibrium location of innovation from θ_1^* to θ_2^* to equalize the WTPs at the limits of the interval, as illustrated in Figure 2b. Thus, a higher absorption capability of firm B induces the innovator to shift the innovation's location toward firm A relative to the case with equal absorption rates. The mechanism mirrors that underlying the innovator's tendency to move away from the initially stronger firm A. However, in this case the direction of adjustment is reversed: the innovator now reduces the bias by locating the innovation closer to firm A than it would if λ_B were smaller.

By symmetric arguments, increasing λ_A above λ_B shifts both WTPs, the blue and the red lines, upwards in the figure, and it shifts the WTP of firm A more than the WTP of firm B. As a result, the intersection of the two WTPs, $\widehat{\alpha}$, moves right. Moreover, increasing λ_A also increases the slope of WTP_A more than the slope of WTP_B. For positive σ, this leads to an increase of θ^* to equalize WTPs. As a result, making firm A stronger in terms of absorption capability induces the innovator to move its location of innovation towards firm B compared to the case under the equal absorption rates. This increases the bias the innovator induces by moving away from the initially stronger firm A.



(a) Comparative statistics of $\widehat{\alpha}$.

(b) Comparative statistics of θ^*.

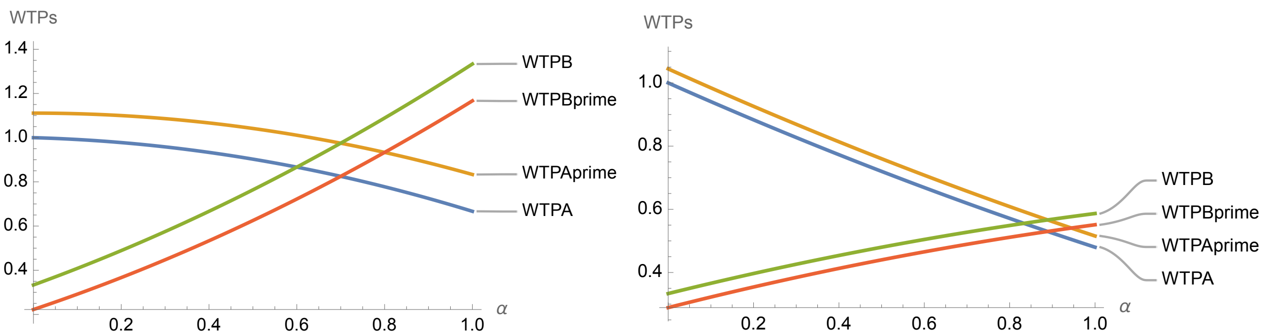
Figure 2: When λ_B increases starting from $\lambda_A = \lambda_B$.

Comparative statics of innovation location: From Corollary 2, when there is nearly no uncertainty, the innovator chooses $\theta^* = \widehat{\alpha}$. Hence, the comparative statics of $\widehat{\alpha}$ from Corollary 1 apply for θ^*. Thus, we have

Corollary 3 *When there is no uncertainty, $\sigma \to 0$, the equilibrium location of innovation $\theta^* = \widehat{\alpha}$ increases in the initial quality differential Δ_0, decreases in the value of innovation q, and is independent of the firms profit functions.*

We now explain the economic mechanism using the firms' willingness to pay for the innovation. To illustrate comparative statics, in Figures 3 and 4, we use the linear Hotelling model for downstream competition between the firms.[8]

First, consider the effects of changing the initial quality advantage of firm A, Δ_0. As Δ_0 increases, the innovator chooses the innovation fit to compensate and favor the initially weaker firm to induce stiffer competition. One can see this in Figure 3: WTP_A shifts up and WTP_B shifts down with an increase in Δ_0, because for firm A the value of winning the innovation is larger than the benefits of denying it to the rival. The fit of innovation, where $WTPs$ are equalized $\widehat{\alpha}$ increases in Δ_0. Thus, in the limit with nearly no uncertainty, the innovator's choice of θ^* increases, that is, the direction of innovation gets closer to firm B.



(a) When $\lambda_B = 1.5 > \lambda_A = 1$, the effects of increasing Δ_0 from 0.5 to 1.

(b) When $\lambda_B = 0.8 < \lambda_A = 1$, the effects of increasing Δ_0 from 0.5 to 0.7.

Figure 3: Comparative statics with respect to Δ_0 when $q = 2$.

As the size of the innovation, q, grows each firm's gross profit (before paying the transfer to the innovator (12) increases in q if it wins the innovation and decreases in q if the rival wins. As a result, both WTPs increase in q, see Figure 4. The initial asymmetry, Δ_0, matters less as q grows. This induces the innovator to choose an innovation *towards* more equal fitness to maximize competition. Intuitively, the innovator maximizes its payoff by making the firms more symmetric competitors. In the limit case, when q is very large, the innovator chooses the innovation's fit to be $\lambda_A/(\lambda_A + \lambda_B)$ and is equal to a half with equal absorption capabilities; otherwise the fit is better for the *weaker* firm, that is, the firm with the lower absorption capability.

3.3 Acquisition of startup and market dominance

We next investigate each firm's likelihood of winning the innovation in the limit where there is nearly no uncertainty, $\sigma \to 0$, and how this probability is affected by their absorption capabilities, λ_A and λ_B. We then discuss the main forces that generate increasing and decreasing dominance

[8] A mass of consumers are uniformly distributed between firm A, which is located at 0, and firm B, which is located at 1. We assume that the linear transportation cost is $\mu = 1$.

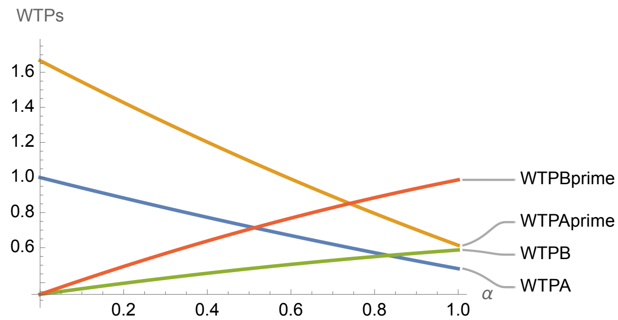


Figure 4: Comparative statics with respect to q when $\lambda_B = 0.8 < \lambda_A = 1$, $\Delta_0 = 0.5$, increasing q from 2 to 3.

in the market, and provide comparative statics with respect to the other parameters of the model, $\{q, \Delta_0\}$.

The probability that B wins the innovation is

$$Pr(B\ wins) = Pr(\alpha \geq \widehat{\alpha}) = \frac{\theta^* + \sigma - \widehat{\alpha}}{2\sigma}.$$

We show in the Appendix that in the limit this probability corresponds to

$$\lim_{\sigma \to 0} Pr(B\ wins) = \frac{1}{2} + \frac{1}{2}\frac{d\theta^*}{d\sigma} = \frac{1}{1 - \frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})}}. \tag{8}$$

From the above equation, we obtain the following:

Proposition 3 *When there is nearly no uncertainty, either firm can win the innovation with a positive probability. If $\lambda_B = \lambda_A$, both firms are equally likely to win the innovation (despite initial asymmetry). Firm B is more likely to win if and only if $\lambda_B > \lambda_A$.*

Using Figures 1 and 2 we can explain Proposition 3. When the firms have equal absorption capabilities, $\lambda_A = \lambda_B$, the innovator chooses $\theta^* = \widehat{\alpha}$ and each firm is equally likely to win. When λ_B increases, both firms WTPs for the innovation rise, but the rate of change (slope) is higher for firm B than firm A, since the direct effect is larger than the indirect effect. This results in θ^* being larger than the new $\widehat{\alpha}$. Thus, firm B is more likely to win the innovation when $\lambda_B > \lambda_A$. The innovator receives WTP_B when A wins the innovation (on the left side of $\widehat{\alpha}$) and WTP_A when B wins the innovation (on the right side of $\widehat{\alpha}$). For low levels of uncertainty, the innovator's expected payoff at $\widehat{\alpha}$ conditional on B winning the innovation is greater than its payoff conditional on A winning the innovation

$$\frac{1}{2}\int_{\widehat{\alpha}}^{\widehat{\alpha}+\sigma} WTP_A(\alpha)d(\alpha) > \frac{1}{2}\int_{\widehat{\alpha}-\sigma}^{\widehat{\alpha}} WTP_B(\alpha)d(\alpha),$$

since $|WTP'_A(\widehat{\alpha})| < WTP'_B(\widehat{\alpha})$ when $\lambda_B > \lambda_A$. Therefore, the innovator benefits from increasing the probability that firm B wins the innovation.

Given that the ratio $-\frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})}$ falls when λ_B goes above λ_A, the innovator puts more weight on WTP_A and induces firm B to win more often. To illustrate the innovator's incentives, suppose that it chooses the location of innovation at new $\widehat{\alpha}$ (or $\widehat{\alpha}_2$ in Figure 1). Moving the innovation location towards firm B by $\epsilon > 0$ increases the likelihood of B winning the innovation, say by probability Δp, and increases the innovator's payoff by $\Delta p WTP_A(\widehat{\alpha} + \sigma)$. This decreases the likelihood of A winning the innovation by Δp and so lowers the innovator's payoff by $\Delta p WTP_B(\widehat{\alpha} - \sigma)$. The net gain of the innovator from this change is

$$\Delta p \left(WTP_A(\widehat{\alpha} + \sigma) - WTP_B(\widehat{\alpha} - \sigma)\right) \approx WTP'_A(\widehat{\alpha}) + WTP'_B(\widehat{\alpha}).$$

Since $WTP'_B > |WTP'_A(\widehat{\alpha})|$ as $\lambda_B > \lambda_A$, the innovator's payoff increases by this change. By symmetric arguments, firm A is more likely to win the innovation when $\lambda_A > \lambda_B$.

Increasing vs decreasing dominance: Firm A initially offers a higher-quality product than firm B. If Firm A obtains the innovation, this increases its dominance.[9] Alternatively, if firm B gets the innovation, firm B leapfrogs firm A and becomes the dominant firm. To understand the reason why there can be decreasing dominance, first consider the case of symmetric absorption capabilities, $\lambda_A = \lambda_B$. As we discussed earlier, the innovator chooses the fit of the innovation biasing towards the initially weaker firm B in order to neutralize the initial asymmetry as this leads to the highest expected payoff for the innovator. This corresponds to the point where firms' WTPs for innovation are equal: the winning firm has the same quality advantage whether it is initially stronger (firm A) or not (firm B) (see Proposition 2). Hence, even if initially firm A has a higher quality product than firm B, the innovator's choice of fit neutralizes the initial asymmetry by moving the location of innovation closer to the initially weaker firm, firm B, such that if firm B wins the innovation it will obtain a larger quality than firm A (decreasing dominance). As we have shown in Proposition 2, when $\lambda_B > \lambda_A$, the innovator's choice of fit moves towards firm A compared to the case of equal absorption capabilities, reducing the initial bias in the location of innovation. Intuitively, now firm B is in a more competitive position than when the absorption capabilities are the same and thus the innovator does not compensate its choice of fit towards firm B as much. Symmetrically, when $\lambda_B < \lambda_A$, the innovator's choice of fit moves further away from firm A compared to the case of equal absorption capabilities, increasing the initial bias in the location of innovation.

Endogenous direction of innovation is crucial for decreasing dominance: The mechanism behind the decreasing dominance is that the direction of innovation is a choice variable and is not exogenously determined. This distinguishes our paper from the most models in the literature, where the fit of innovation is assumed to be exogenous. Now, we provide a simple example

[9] This is true for standard demand specifications where the market share of the firm increases in its quality advantage. For example, we show this in a discrete choice demand and covered market in Section 4.

to compare the winner and the consequences for market dominance when the innovation is either exogenously or endogenously determined.

First, suppose the firms have the same absorption capabilities, $\lambda_A = \lambda_B$. In this case, $\widehat{\alpha}$ is greater than 0.5 and is increasing in A's initial advantage over B. If innovation is generated as an **exogenous** uniform draw on $[0, 1]$, then A is more likely to win than B and B wins with probability $1 - \widehat{\alpha}$. When innovation is **endogenous**, the innovator chooses an innovation that is a relatively better fit for B than A. Furthermore, B's chance of winning the innovation is 1/2 $> 1 - \widehat{\alpha}$. Thus, endogenizing innovation increases B's chance of wining and reduces the chance of increasing dominance for firm A. As a consequence, if analysts assume that innovation is exogenously chosen then they falsely expect that dominance will occur more often than it actually does if innovation is endogenous. Now, suppose that λ_B increases above λ_A. From Figure 2 and the resulting discussion of it, this will reduce $\widehat{\alpha}$ and the choice of θ^* will also fall, but not as much as $\widehat{\alpha}$. This increases the probability that B wins above 1/2 and this makes increasing dominance less likely than if innovation were exogenous. This also can be seen from Proposition 3 when there is little uncertainty.

It is important to note that the the market becomes more asymmetric regardless of which firm wins the innovation, since the ex-post quality difference between the firms is always greater than the initial quality advantage of firm A: $|\Delta^{j*}| > \Delta_0$. Interestingly, the innovator chooses a better fit for the initially weaker firm, firm B, making it a stronger rival than the firm that had better quality originally, firm A.

Comparative statics of dominance: We next study how the initial quality advantage of firm A and the size of the innovation affects the likelihood that initially weaker firm (B) wins the innovation when there is nearly no uncertainty. This comparative static involving the initial quality advantage is particularly important since it sheds light on the firms' incentives to acquire the innovation in the two-period model by illustrating how the first-period quality advantage of firm A affects the probability firm B wins the innovation.

Proposition 4 *When there is nearly no uncertainty, $\sigma \to 0$, the probability that the initially weaker firm (B) wins innovation decreases in the initial quality advantage of A, Δ_0. B's likelihood to win decreases in the value of innovation q if and only if $\lambda_B > \lambda_A$.*

When the initially weaker firm (B) becomes less likely to win the innovation, increasing dominance becomes more likely. This happens when the initial quality advantage of firm A is greater. This is due to two effects. First, higher Δ_0 increases the direct effect of acquiring the innovation for firm A and decreases the direct effect of acquiring the innovation for firm B. This means WTP_A shifts upwards and WTP_B shifts downwards due to the direct effect, as illustrated in Figure 3. Second, higher Δ_0 also induces the innovator to choose the fit of innovation closer to firm B (from Proposition 1), and so lowering the profits of firm A if firm B owns the innovation. Both direct effect and indirect effects lead to an increase in WTP_A, since A wants to keep the innovation away from B. However, the direct effect implies a reduction of WTP_B, which is partially compensated by the indirect effect (via moving the innovation fit towards B). As a result, the likelihood that B wins the innovation decreases in Δ_0.

Increasing dominance becomes more likely when the value of innovation, q, increases if $\lambda_B > \lambda_A$. Higher-valued innovation increases the direct quality benefit of owning it for each firm. Thus, the direct effect implies upward shift for both WTPs, as shown in Figure 4. The magnitude of this direct effect depends on the absorption capability of the firm. When λ_j increases, the direct effect for firm j increases. On the other hand, higher q reduces the importance of the initial quality advantage of firm A and so moves the location of innovation closer to firm A. The relative slope of the WTPs with respect to the location of innovation ($-\frac{WTP'_A}{WTP'_B}$) is crucial in determining how much the location of innovation gets closer to firm A when q increases. We show in the Appendix that this relative slope increases in q if and only if $\lambda_B > \lambda_A$, given that higher q increases the market share of the firm who owns the innovation. As a result, the likelihood that B wins the innovation decreases in q if and only if $\lambda_B > \lambda_A$.

3.4 Equilibrium Profits

The "net" profit of the winner of the innovation is the gross profit minus its payment to the innovator: $\Pi_j(\Delta^{j*}) - P_j$. The net profit of the losing firm is the same as its gross profit: $\Pi_{-j}(\Delta^{j*})$, as the loser does not pay its bid. The following shows that as the uncertainty goes to zero, each firm is indifferent between losing and winning the innovation since they have equal WTPs:

Corollary 4 *When there is nearly no uncertainty, $\sigma \to 0$, at equilibrium the firms have equal net profits (after payment to the innovator):*

$$V^*(\Delta_0) = \Pi_A(\Delta^{B*}) = \Pi_B(\Delta^{A*}). \tag{9}$$

We next study the effects of the innovator's equilibrium choice on the industry profits. We showed in Proposition 1 that the total industry profit is a function of the ex-post quality differential between firm A and firm B, Δ^{j*}: it is decreasing when $\Delta^{j*} < 0$ and increasing when $\Delta^{j*} > 0$, see Property 2. If we increase α starting from $\alpha = 0$, Δ^j decreases. As long as $\alpha < \widehat{\alpha}$, firm A acquires the innovation and $\Delta^{A*} > 0$, so total profit is decreasing in α. When α goes beyond $\widehat{\alpha}$, firm B acquires the innovation and $\Delta^{B*} < 0$, so total profit is increasing in α. As a result, we prove

Corollary 5 *When uncertainty σ goes to zero, the total industry profit is minimized at the equilibrium choice of the innovator, $\theta^* = \widehat{\alpha}$.*

Comparative Statics of Net Profits We next show how the net profits change in the parameters of our model.

Proposition 5 *When there is no uncertainty, $\sigma \to 0$, both downstream firms' net profits*

i. increase (decrease) in firm A's quality advantage Δ_0 if and only if $\lambda_A > \lambda_B$ ($\lambda_B > \lambda_A$).

ii. decrease in the value of innovation q.

As the initial advantage of firm A, Δ_0, grows, the gross profit of firm A increases due to direct effect of Δ_0, regardless of the winner of the innovation. As we showed in Proposition 1, increasing Δ_0 increases the fit of the innovation for firm B. This in turn implies a reduction in firm A's profit. The net effect of Δ_0 on A's profit when B wins the innovation is positive if and only if A has better absorption capability than B. As a result, A's net profit increases if and only if $\lambda_A > \lambda_B$. In equilibrium, the firms have the same net profits, and so B's net profit increases in Δ_0 if and only if $\lambda_A > \lambda_B$.

Increasing the value of the innovation raises the gross profit of both firms of having the innovation. In addition, the benefit of denying the rival's access to the innovation also increases. The combination of these effects lead to an increase in WTP for both firms, which is more than the direct gains from having the innovation. Thus, for a given fit innovation, both firms lose when q increases.

Comparative Statics of Gross Profits Finally, we present comparative statics of the equilibrium quality differential between firm A and B when firm j wins the innovation, Δ^{j*}, and gross profits:

Proposition 6 *In the limit with no uncertainty, $\sigma \to 0$,*

i. When quality differential Δ_0 increases:

a. If $\lambda_B > \lambda_A$, the winning firm's gross profit increases and the losing firm's gross profit decreases.

b. If $\lambda_A > \lambda_B$, the winning firm's gross profit decreases and the losing firm's gross profit increases.

ii. The gross profit of the winning-firm increases in the size of the innovation q.

First consider how the initial advantage of firm A, Δ_0, affects its ex-post advantage if it is the winner:

$$\frac{d\Delta^A}{d\Delta_0} = 1 - \frac{d\alpha}{d\Delta_0}\lambda_A q. \tag{10}$$

Given the definition of $\Delta^A = \Delta_0 + (1-\alpha)\lambda_A$ (equation 1), there are two effects. First, the ex-post relative quality of A increases in Δ_0 one for one (direct effect). Second, the innovator compensates by changing the direction of innovation (the indirect effect). This second effect is composed of how the innovation's location is moved away from A, $\frac{d\alpha}{d\Delta_0}$, A's absorption capacity, λ_A, and the value of the innovation, q. These two effects have opposite signs and which one is larger depends on the relative absorption capabilities of the two firms. We show in the Appendix, that when there is no uncertainty, the location of innovation changes by $\frac{d\alpha}{d\Delta_0} = \frac{2}{(\lambda_A+\lambda_B)q}$. Consider the case where firm A wins the innovation. If the weaker firm has a higher absorption rate, $\lambda_B > \lambda_A$, the innovator does not need to increase α so much to compensate for B's initial weakness. As a result, the direct effect dominates the indirect effect, and so the ex-post quality of A increases in Δ_0. When $\lambda_A > \lambda_B$, the innovator compensates for B's initial weakness more as A is better in both dimensions (initial quality and absorption capacity). As a result, the indirect effect dominates the direct effect. It is straightforward that the winner's gross profit increases in the size of innovation q.

4 Downstream demand and welfare

To analyze consumer surplus and welfare, we must specify downstream demand. Following the demand specification of retail market equilibrium analysis we then characterize consumer surplus and welfare.

4.1 Downstream demand specification

The consumer buying the product of firm j at price p_j obtains net utility

$$Q_j - p_j - \mu\epsilon_j,$$

where ϵ_j is random taste shock for firm j and parameter μ is the measure of taste variance, which captures horizontal differentiation between the firms. We assume that the market is covered.

Define consumer type x by

$$x = \frac{1 + \epsilon_A - \epsilon_B}{2},$$

such that higher x means that the consumer has a stronger preference for firm B than firm A (similar to the location of the consumer in the Hotelling model). We assume that x is distributed on the interval $[0, 1]$ with a probability density function $f(x)$ and a cumulative distribution function $F(x)$ satisfying the following:

Assumption 4 *(i) $f(x)$ is symmetric and log-concave. (ii) The inverse hazard rate $\frac{F(x)}{f(x)}$ is convex.*[10]

Part (i) implies that the inverse hazard rate $\frac{F(x)}{f(x)}$ is increasing and that $F(1/2) = 1/2$: firm A dominates firm B if A sells to all types below $\hat{x} > 1/2$.

The following ensures an interior equilibrium of the downstream competition game:

Assumption 5 $\max\{\Delta_0 + \lambda_A q, \lambda_B q - \Delta_0\} < 2\mu(\frac{1}{2} + lim_{x\to 1}\frac{1}{f(x)})$.

[10]Assumption 4 holds for the uniform distribution and for the symmetric triangle distribution:

$$f(x) = \left\{ \begin{array}{ll} 4x, & \text{if } 0 \leq x \leq 1/2 \\ 4(1-x), & \text{if } 1/2 \leq x \leq 1. \end{array} \right\}.$$

Retail equilibrium analysis: Assuming that both firms are active, the location of the marginal consumer is:

$$x = \frac{1}{2} + \frac{\Delta + p_B - p_A}{2\mu}.$$

Firm A chooses its price p_A to maximize its profit: $\Pi_A = p_A F(x)$ and firm B chooses its price p_B to maximize its profit: $\Pi_B = p_B(1 - F(x))$. Equilibrium prices are the solution to the first-order conditions:

$$p_A = \frac{2\mu F(x)}{f(x)}, \qquad p_B = \frac{2\mu(1 - F(x))}{f(x)}.$$

Given that the location of innovation is at α, we first illustrate the equilibrium properties of firms' market shares when firm j owns the innovation in which case the quality differential between firm A and firm B is Δ^j, see (1) and (2).

Lemma 1 *For any $\Delta \in [\Delta_0 - \lambda_B q, \Delta_0 + \lambda_A q]$ the equilibrium location of the marginal consumer is unique $x(\Delta) \in (0, 1)$:*

$$x(\Delta) = \frac{1}{2} + \frac{1 - 2F(x(\Delta))}{f(x(\Delta))} + \frac{\Delta}{2\mu}, \tag{11}$$

which is larger than $\frac{1}{2}$ iff $\Delta > 0$. Furthermore, the retail market gross profits are:

$$\Pi_A(\Delta) = \frac{2\mu(F(x(\Delta))^2}{f(x(\Delta))}, \qquad \Pi_B(\Delta) \quad = \quad \frac{2\mu(1 - F(x(\Delta)))^2}{f(x(\Delta))}, \tag{12}$$

and they satisfy Assumption 2.

Firm A's demand $F(x(\Delta))$ is increasing in Δ, while firm B's demand $1 - F(x(\Delta))$ is decreasing. When $\alpha = 1$, if B acquires the innovation, its market share is larger than A's market share in case when A obtains the innovation: $1 - F(x(\Delta_0 - \lambda_B q)) > F(x(\Delta_0))$; thus, it is feasible for B to win the innovation. For any α, A's market share is larger if A acquires the innovation than if B acquires it, $F(x(\Delta^B)) < F(x(\Delta^A))$.

A direct implication of Proposition 1 is that the relative fit of innovation that equates the firms' WTPs, $\widehat{\alpha}$, is independent of differentiation between firms, μ. Thus, when there is no uncertainty, the equilibrium choice of the innovator is independent of μ (Corollary 3). We next show the effect of μ on the net and gross profits:

Proposition 7 *When there is no uncertainty, $\sigma \to 0$, increasing the level of product differentiation, μ, increases the net profits of the firms. Moreover, it increases both firms' markup, but reduces the market share of the winner, so it has an ambiguous effect on the gross profit of the winner and increases the gross profit of the losing firm.*

When the firms are more differentiated, they can raise their markups. This positive effect is present both for the winner of the innovation and losing firm. The innovator's endogenous choice for the direction of innovation makes firms symmetric despite the initial quality asymmetry: the winning firm has the same market share whether it is A or B. More differentiation would then imply that the winning firm's market share decreases. As a result, the net effect of μ on the winner's profit is unclear, but the losing firm's gross profit increases due to both positive markup effect and positive market share effect.

4.2 Total welfare and consumer surplus

We next analyze the impact of the innovator's choice on total welfare, that is, the sum of the total industry profit and consumer surplus. Let $T(\Delta)$ denote the total transportation costs of consumers when firm j has the innovation:

$$T(\Delta) \equiv \mu \int_0^{x(\Delta)} xf\left(x\right)dx + \mu \int_{x(\Delta)}^{1} \left(1-x\right)f\left(x\right)dx. \tag{13}$$

For a given draw α, welfare if firm A has the innovation and welfare if firm B has the innovation are respectively

$$W^A\left(\alpha\right) = Q_0^B + F(x(\Delta^A))\Delta^A - T\left(\Delta^A\right), \tag{14}$$

$$W^B\left(\alpha\right) = Q_0^B + F(x(\Delta^B))\Delta_0 + \left(1-F(x(\Delta^B))\right)\alpha\lambda_B q - T\left(\Delta^B\right). \tag{15}$$

where Q_0^B refers to the initial quality of firm B's product. When A wins the innovation, all consumers enjoy utility from quality Q_0^B and only those buying from firm A, measure of $F(x(\Delta^A))$, enjoy the utility from additional utility, $\Delta^A = \Delta_0 + (1-\alpha)\lambda_A q$. The total welfare is the sum of these utilities minus the total transportation costs when A wins the innovation. When B wins the innovation, all consumers enjoy utility from quality Q_0^B, only those buying from firm A, measure of $F(x(\Delta^B))$, enjoy the additional utility Δ_0 and only those buying from firm B, measure of $1-F(x(\Delta^B))$, enjoy the additional utility $\alpha\lambda_B q$. The total welfare is the sum of these utilities minus the total transportation costs when B wins the innovation.

For a given location of innovation, θ, the ex-ante expected welfare is the welfare if firm A wins the innovation, which happens when $\alpha \leq \widehat{\alpha}$, plus the welfare if firm B wins the innovation, which happens when $\alpha > \widehat{\alpha}$. Taking the limit when uncertainty over α vanishes we define;

$$W^T(\theta) = \lim_{\sigma\to 0}\frac{1}{2\sigma}\int_{\theta-\sigma}^{\theta+\sigma}\left(W^A\left(\alpha\right)\mathbb{1}\left(\alpha\leq\widehat{\alpha}\right) + W^B\left(\alpha\right)\mathbb{1}\left(\alpha>\widehat{\alpha}\right)\right)d\alpha$$

This welfare measure can then be written for $\theta \neq \widehat{\alpha}$ as[11]

$$W^T(\theta) = W^A\left(\theta\right)\mathbb{1}\left(\theta<\widehat{\alpha}\right) + W^B\left(\theta\right)\mathbb{1}\left(\theta>\widehat{\alpha}\right)$$

[11] When $\theta = \widehat{\alpha}$, the probability of each firm winning the innovation is given by (8). In that case, welfare lies between $W^A(\widehat{\alpha})$ and $W^B(\widehat{\alpha})$.

We next write consumer surplus as total welfare minus the firms' profits. For any $\alpha < \widehat{\alpha}$, A wins the innovation and the consumer surplus is

$$\begin{aligned} CS^{A}(\alpha) &= W^{A}(\alpha) - \Pi^{T}(\Delta^{A}) \\ &= Q_0^B + F\left(x(\Delta^A)\right)\Delta^A - \Pi^T\left(\Delta^A\right) - T\left(\Delta^A\right). \end{aligned}$$

For any $\alpha > \widehat{\alpha}$ B wins the innovation and the consumer surplus is

$$\begin{aligned} CS^{B}(\alpha) &= W^{B}(\alpha) - \Pi^{T}(\Delta^{B}) \\ &= Q_0^B + \alpha\lambda_B q + F\left(x(\Delta^B)\right)\Delta^B - \Pi^T\left(\Delta^B\right) - T\left(\Delta^B\right) \end{aligned}$$

As in welfare, we define $CS(\theta)$ as the limit of consumer surplus when uncertainty σ goes to 0, which coincides with $CS^A(\theta)$ when θ is below $\widehat{\alpha}$ and with $CS^B(\theta)$ when θ is strictly above $\widehat{\alpha}$.

Proposition 8 *When there is nearly no uncertainty, consumer surplus $CS(\theta)$ is U-shaped with a minimum at $\widehat{\alpha}$, and an upward jump at $\widehat{\alpha}$ that is:*

$$\frac{\partial CS(\theta)}{\partial\theta} < 0 \;\; \textit{if} \;\; \theta < \widehat{\alpha}, \quad \frac{\partial CS(\theta)}{\partial\theta} > 0 \;\; \textit{if} \;\; \theta > \widehat{\alpha}, \quad \textit{and} \quad CS(\widehat{\alpha}^{+}) - CS(\widehat{\alpha}^{-}) = \Delta_0.$$

Hence, consumer surplus is maximal at $\theta = 0$ or $\theta = 1$.

Starting from $\alpha = 0$ if we increase α in the region where firm A wins the innovation, $\alpha < \widehat{\alpha}$, this lowers the quality of firm A without affecting firm B's quality. Firm A lowers its price, but less than the reduction in quality. As a result, firm A's net utility offering is lower. As a response, firm B raises its price and so lowers its utility offering. Thus, all consumers are worse off by raising α within the region $\alpha < \widehat{\alpha}$. By a symmetric argument, raising α when $\alpha > \widehat{\alpha}$ increases the quality of firm B without affecting firm A's quality. Firm B raises its price, but less than the increase in quality. As a result, firm B's net utility offering is higher. As a response, firm A lowers its price and so raises its utility offering. There is an upward discontinuity at $\widehat{\alpha}$ which is due to the initial quality asymmetry Δ_0 and the fact that the innovator equalizes the firms' willingness-to-pays at its equilibrium choice. The innovator chooses the quality differential of the winner which is independent of the winner's identity. To compensate the initial quality advantage of firm A, this requires a larger increase in quality of firm B when it wins. Hence, the discontinuous jump in consumer surplus at $\widehat{\alpha}$ is equal to Δ_0. We next present welfare consequences of the innovator's choice:

Corollary 6 *When there is nearly no uncertainty, welfare $W^T(\theta)$ is U-shaped with a minimum at $\widehat{\alpha}$, and an upward jump at $\widehat{\alpha}$. Hence, welfare is maximal at $\theta = 0$ or $\theta = 1$.*

Given that both total industry profit and consumer surplus are U-shaped functions of θ and minimized at $\theta = \widehat{\alpha}$, the same properties hold for welfare with an upward jump at $\widehat{\alpha}$ due to the jump of consumer surplus.

5 Extensions

5.1 Preemptive (killer) acquisition

The literature typically defines “killer acquisition” as one where an incumbent acquires an innovation and shelves it to prevent the emergence of a future competitor (Cunningham et al., 2021; Motta and Shelegia, 2024; Letina et al., 2024). In our model, the motivation of the acquisition is to prevent a rival from benefiting from the innovation, “preemptive acquisition”. This is similar to a killer acquisition since in both cases an initially strong firm’s motivation is to harm a rival, even if the strong firm does not directly benefit from innovation.

Assumption 1 was made to avoid a corner solution for the direction of innovation. From above, it is immediate that it is not possible that the innovator chooses $\theta = 0$ when the uncertainty is small (i.e., $\widehat{\alpha} > 0$ for all initial quality differentials and absorption capacities). Nevertheless, if Assumption 1 is violated, A’s willingness to pay is larger than B’s willingness to pay for any realization of α. In this case A always wins the auction and the objective of the innovator is to maximize B’s willingness to pay which is increasing in α. Hence the direction of innovation is $\theta = 1$. Notice that this means that A does not benefit from the innovation and only buys it to keep it away from firm B. Hence this situation is similar to one of a ”killer acquisition”.

5.2 Early acquisition

We assumed that the innovator sells after the choice of innovation θ. Alternatively the startup could be sold before (ex-ante) an investment is made by the innovator. If this is the case, the acquirer would choose the direction that best suits the firm. That is, $\theta = 0$ for firm A and $\theta = 1$ for B. Moreover, as we saw above, if the selling occurs through an auction, the winner is the firm that generates the highest industry profits. Thus, firm B wins the auction if the quality differential $\lambda_B q - \Delta_0$ is larger than $\Delta_0 + \lambda_A q$. This is never the case if Assumption 1 is violated. When Assumption 1 is satisfied, we obtain that firm B wins the ex-ante auction whenever

$$(\lambda_B - \lambda_A)q > 2\Delta_0.$$

In terms of welfare, B winning the auction is efficient if $W_B(1) > W_A(0)$.

Proposition 9 *There exists $\overline{\lambda}_B$ such that $W_B(1) > W_A(0)$ if and only if $\lambda_B > \overline{\lambda}_B$. Moreover $(\overline{\lambda}_B - \lambda_A)q < 2\Delta_0$.*

Using the same logic as for the ex-post auction, the switching point where B wins occurs when the quality differential between the winner and the loser of the auction is independent of the winner’s identity. Equalizing quality differentials requires boosting the quality of B so as to compensate for its initial quality disadvantage. As a consequence, welfare is larger when B wins by precisely the quality disadvantage of B, that is Δ_0. Hence, both in the context of ex-ante auction and in equilibrium of the game with ex-post auctions, the weaker firm B wins less often than it should from a welfare perspective.

5.3 Uncovered market

In the benchmark model we assume that the market is covered and so the profits depend only on the quality differential between the firms. We now discuss how our qualitative results change when the market is uncovered.

If the total industry profit is increasing in quality level of each firm, Proposition 1 is valid with a different level of $\widehat{\alpha}$, which will be implicitly defined as the point where the total industry profit is independent of the winner of the innovation (the WTPs of the firms will be equal). In addition, if we assume WTP_A decreasing in α and WTP_B increasing in α (Assumption 3 holds), at $\widehat{\alpha}$ the total industry profit is minimized, the equilibrium choice of the innovator is within the interval around $\widehat{\alpha}$ so that either firm has a positive probability of winning the innovation (Propositions 2 and 3 are valid).

5.4 Bargaining power of the innovator

In the benchmark, the innovator sells its innovation via a second-price auction and so its payoff is the willingness-to-pay of the firm that loses the auction. Now, we consider an alternative selling mechanism that gives more bargaining power to the innovator.

Suppose the innovator sells the innovation to the firm with the highest willingness to pay after the realization of the innovation fit, α. We assume that with probability $\gamma > 0$ the innovator extracts the highest willingness-to-pay for the innovation and with complementary probability $1 - \gamma$, the innovator gets the second highest willingness-to-pay. Parameter γ captures the bargaining power of the innovator. When $\gamma = 0$, we have our model so the innovator chooses its location of innovation maximizing the expected value of the lowest WTP.

When $\gamma > 0$, the innovator's payoff is the weighted sum of the willingness-to-pays of firm A and firm B, where γ is the weight in front of the largest WTP. Recall that WTP_A is decreasing in α, WTP_B is increasing in α, and they intersect at $\widehat{\alpha}$. For a sufficiently small γ, like in our model, the payoff of the innovator has the same shape as in our model. Thus, for low levels of uncertainty, the innovator's choice will be close to $\widehat{\alpha}$.

As before, the innovator's choice will equate its payoff at the boundaries of α distribution. Differently the innovator's payoff is the weighted sum of the WTPs, where γ is the weight in front of the largest WTP. Following the steps as in Proposition 3, one can show that the probability of each firm winning the innovation will not be affected when the firms have equal absorption capabilities.

Proposition 10 *In the model where the innovator receives the weighted sum of the WTPs, if γ is small enough, for low levels of uncertainty (σ), we have:*

- *The innovator's choice will be close to $\widehat{\alpha}$.*

- *Compared with our model ($\gamma = 0$), firm B is more likely to win the innovation if $\lambda_B > \lambda_A$ and less likely to win the innovation if $\lambda_A > \lambda_B$.*

6 Dynamic Model

We now extend the model to two periods, $t = 1, 2$. The initial quality differential is Δ_0. In each period t there is a new innovator that generates an innovation of quality q. For conciseness we assume that the absorption capacity of each firm is constant over time, at λ_A and λ_B.

We assume that there is some technological spillover between the firms from period 1 to period 2. When firm i is the industry leader in period 1, this improves the quality of the follower at the end of the first period, since the follower learns some of the knowledge that the leader has. As a result, the quality differential of the leader firm is reduced by factor $\beta \in (0, 1)$ at the beginning of period 2. We assume that the equilibrium choice of the innovation remains interior in both periods. This is the case in the second period if β is sufficiently small relative to q.

Firms discount period 2 payoffs by $\delta \in (0, 1)$ at the start of period 1. The stage game in each period is the same as the static game described in Section 2. As before, we assume interior solution to the downstream competition (Assumption 5) and maintain Assumption 3 for each period. We solve for the equilibrium when the uncertainty goes to zero in both periods: $\sigma_2 \to 0$, then $\sigma_1 \to 0$.

We denote by $\Delta_1 \in \{\Delta^A, \Delta^B\}$ the quality differential between firms A and B after the auction in period 1–hence at the start of period 2 the quality differential is $\beta\Delta_1$. The equilibrium direction of innovation in the second period is

$$\theta_2^* = \widehat{\alpha}_2 = \frac{2\beta\Delta_1 + \lambda_A q}{(\lambda_A + \lambda_B)\, q}$$

if there is an interior solution for $\widehat{\alpha}_2$.

Consider the expected profit of firm j evaluated at the beginning of period 2, as a function of the quality differential. Recall from Corollary (4) that as the uncertainty goes to zero, both firms obtain the same profit in the one-period game. We thus compute this profit as the profit from losing the auction in period 2, for each firm,$V^*(\beta\Delta_1)$. From Proposition 5 it follows that the net profits in period 2 are increasing in Δ_1 if $\lambda_A > \lambda_B$. Symmetrically, the net profits in period 2 are decreasing in Δ_1 if $\lambda_B > \lambda_A$. The net profits in period 2 are constant in Δ_1 if $\lambda_B = \lambda_A$. We next show the firms' second period preferences over the identity of the winner of the innovation in period 1.

Corollary 7 *The second period profit of each firm is larger if the firm with larger absorption capacity (λ_i) wins the first-period innovation.*

First period equilibrium: An immediate consequence of Proposition 5 is that when $\lambda_B = \lambda_A$, the second period equilibrium of the two-period game coincides with that of the static game, as the second-period profit does not depend on the winner of the first period.

Now consider cases where the firms have different absorption capabilities. The retail competition game is the same as in the one-period model. For the auction game, we need to derive the WTP to win the auction. If firm A wins the auction it will get the profit $\Pi_A(\Delta_1^A)$ in the first period and the expected profit $V^*(\beta\Delta_1^A)$ in period 2, see (32). Losing the auction would yield profit $\Pi_A(\Delta_1^B)$ in the first period and the expected profit $V^*(\beta\Delta_1^B)$ in period 2. Thus, firm A's WTP to win in period 1 is

$$WTP_A(\alpha_1) = \Pi_A(\Delta_1^A) - \Pi_A(\Delta_1^B) + \delta\left(V^*(\beta\Delta_1^A) - V^*(\beta\Delta_1^B)\right)$$

Analogously, firm B's WTP to win in period 1 is

$$WTP_B(\alpha_1) = \Pi_B(\Delta_1^B) - \Pi_B(\Delta_1^A) + \delta\left(V^*(\beta\Delta_1^B) - V^*(\beta\Delta_1^A))\right)$$

Given these WTPs we can replicate the analysis of the one-period game to obtain:

Proposition 11 *For δ and β not too large, there exists a unique value $\widehat{\alpha}_1$ such that $WTP_A(\widehat{\alpha}_1) = WTP_B(\widehat{\alpha}_1)$. The innovator chooses a direction of innovation, θ_1^*, that converges to $\widehat{\alpha}_1$ when σ_2 and σ_1 go to zero.*

We now study how dynamic consideration affects the direction of innovation and the evolution of quality over time. Recall that we assume that $WTP_A(\widehat{\alpha}_1)$ is decreasing and $WTP_B(\widehat{\alpha}_1)$ is increasing. We established that this is the case when absorption capabilities are not too far apart and δ is not too large. To understand how the first period direction of innovation compares between the dynamic and the static model, we need to understand in which direction the WTP change when accounting for dynamics.

We next show how the static $\widehat{\alpha}$ compares to the dynamic $\widehat{\alpha}_1$:

Proposition 12 *The equilibrium direction of innovation in the first period of the dynamic game is lower than the equilibrium direction of innovation of the static game, $\widehat{\alpha}_1 < \widehat{\alpha}$, if and only if firm B has higher absorption capability than firm A in period 1, $\lambda_B > \lambda_A$.*

As we discussed in the previous subsection the second period net profits decrease in Δ_1 when $\lambda_B > \lambda_A$. This implies that firm A has a lower WTP in period 1 than the static model, whereas firm B has a higher WTP in period 1 than the static model. Figure 5 illustrates the comparison between dynamic and static $WTPs$ for λ_B greater than λ_A. As a consequence, the cutoff point where WTPs are equalized is lower in the dynamic model than in the static model (the dashed blue vertical line is to the left of the dashed red vertical line).

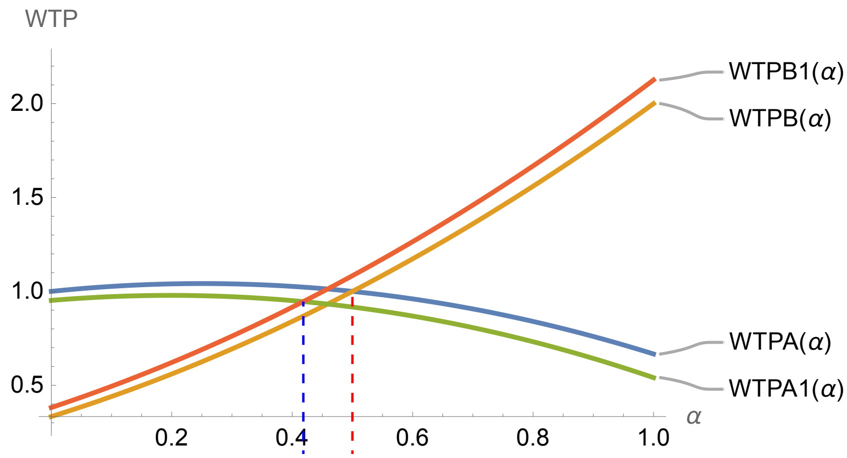


Figure 5: WTPs for innovation in the static and dynamic model for uniformly distributed $x \sim [0, 1]$, and parameter values $\beta = 0.45, q = 2, \Delta_0 = 0.5, \mu = 1, \lambda_A = 1, \lambda_B = 2, \delta = 1$.

By symmetric arguments, the net profits of the second period increase in Δ_1 when $\lambda_B < \lambda_A$. This implies that firm A has a higher WTP in period 1 than the static model, whereas firm B has a lower WTP in period 1 than the static model.

Dynamics and Market Dominance: We now study how dynamics affect the likelihood that firm B wins the innovation. We do this by comparing the probability that B wins the innovation in period 1 in the dynamic model with the probability that it wins the innovation in the static model. This comparison involves changes in the slope of the WTPs as documented in (8). Thus, in this analysis we focus on the case of $F(x)$ being a uniform distribution over $[0, 1]$. We still maintain Assumptions (5) and make the following assumptions:

Assumption 6 *Interior downstream market shares:*
(i) $\max\{\beta(\lambda_B q - \Delta_0) + \lambda_B q, \beta(\Delta_0 + \lambda_A q) + \lambda_A q\} < 3\mu$.
Interior period 2 innovation location $0 < \alpha_2 < 1$*:*
(ii) $\lambda_B q > 2\beta(\Delta_0 + \lambda_A q)$, *(iii)* $\lambda_A q > 2\beta(\lambda_B q - \Delta_0)$.

These assumptions extend the previous assumption to ensure that for small σ, both firms have positive market shares in both periods and the choice of the direction of innovation is in $(0, 1)$. The next proposition extends our previous analysis of dominance to the dynamic model.

Proposition 13 *If $\lambda_B = \lambda_A$, in both the dynamic and static models each firm has $1/2$ chance of winning the innovation in each period. If $\lambda_B > \lambda_A$, firm B wins the first period auction more often than in the case of the one-period game, while B is less likely to win if $\lambda_B < \lambda_A$.*

To sum up, we show that dynamics makes increasing dominance less likely when the initially weaker firm has stronger capacity to absorb innovation. This illustrates that the main asymmetry that matters for the dynamics of competition is not the initial asymmetry in the stock of quality, but it is the asymmetry in the firms' capabilities to absorb innovation.

7 Conclusion

We study the incentives of an innovator startup when it chooses its direction of innovation (the average relative fitness of innovation to competing firms) before the firms bid to acquire the innovation. We focus on drastic innovations which can enable leapfrogging if acquired by the initially lower-quality firm. To maximize the sales price, the innovator puts the firms on equal footing. This means choosing the location of innovation closer to the initially weaker firm to compensate for the initial quality asymmetry between the firms. This results in the weaker firm becoming the market leader if it wins the innovation.

The asymmetry in firms' capabilities to integrate the technology into their quality stock determines the probability of each firm winning the innovation and so whether increasing or decreasing dominance prevails. When the initially lower-quality firm is better able to absorb innovation, it is more likely to win the innovation and so we have decreasing dominance. This results is a consequence of allowing for endogenous relative fitness of innovation. The key asymmetry that matters for increasing dominance is the asymmetry in the firms' capabilities to absorb innovation. The innovator's choice of the relative fitness mitigates the initial asymmetry in the quality stock. Dynamics make decreasing dominance more likely when the initially lower-quality firm is better able to absorb innovation.

Our analysis shows that policies aimed at restricting acquisitions by dominant firms can have ambiguous effects on consumers. Such policies are likely to reduce the dominant firm's willingness to pay (WTP), thereby shifting the equilibrium direction of innovation toward that firm. This shift increases consumer surplus when the dominant firm acquires the innovation, but reduces it when the lagging firm becomes the acquirer. The overall effect depends on the relative likelihood of each firm securing the innovation, which in turn is determined by the relative slopes of their WTP functions.

Appendices

A Benchmark Analysis: Static model

Proof of Proposition 1 Using profit expressions, (12), we firstly write the difference in the firms' willingness-to-pay for a given draw of the innovation location, α,

$$WTP_A(\alpha) - WTP_B(\alpha) = \Pi_A(\Delta^A) - \Pi_A(\Delta^B) - \Pi_B(\Delta^B) + \Pi_B(\Delta^A)$$

Observe that this reduces to comparing total industry profit:

$$WTP_A(\alpha) - WTP_B(\alpha) = \Pi^T(\Delta^A) - \Pi^T(\Delta^B).$$

Assumption 2 implies that $\Pi^T(\Delta)$ is a convex, U-shaped and symmetric function. For α small enough so that $\Delta^B > 0$ we have $\Delta^B < \Delta^A$ implying that $\Pi^T(\Delta^A) > \Pi^T(\Delta^B)$ because $\Pi^T(\Delta)$ is increasing in Δ. Then as α gets larger, Δ^B becomes negative while Δ^A remains positive. On this range $\Pi^T(\Delta^A) - \Pi^T(\Delta^B)$ is decreasing in α. Moreover, by 1, $\Pi^T(\Delta^A) - \Pi^T(\Delta^B)$ is negative when $\alpha = 1$. Hence there exists a unique value of α that equalizes the industry profit. By symmetry, this is obtained when $\Delta^A = -\Delta^B$ which gives the value of $\widehat{\alpha}$.

Lemma 2 *If the differential in absorption capability $|\lambda_A - \lambda_B|$ is not too large, the willingness-to-pay of firm A is decreasing in α and the willingness-to-pay of firm B is increasing in α.*

Proof. The willingness-to-pay of firm A is from (4) and (12):

$$WTP_A(\alpha) = \Pi_A(\Delta^A) - \Pi_A(\Delta^B),$$

We then have

$$WTP_A'(\alpha) = -\lambda_A q \Pi_A'(\Delta^A) + \lambda_B q \Pi_A'(\Delta^B).$$

Hence, if $|\lambda_A - \lambda_B|$ is small, $WTP_A'(\alpha) < 0$ because $\Delta^A > \Delta^B$ and $\Pi_A(\Delta)$ is convex. Following similar steps, we show that $WTP_B'(\alpha) > 0$. In this case we have

$$WTP_B'(\alpha) = -\lambda_B q \Pi_B'(\Delta^B) + \lambda_A q \Pi_B'(\Delta^A).$$

Given that $\Pi_B(\Delta)$ is convex, it follows that $WTP_B'(\alpha) > 0$ if $|\lambda_A - \lambda_B|$ is small. ■

Proof of Proposition 2 and of Corollary 2 We prove the proposition in two steps:

Claim 1: The innovator's optimal choice $\theta^* \in [\widehat{\alpha} - \sigma, \widehat{\alpha} + \sigma]$ is such that

$$WTP_A(\theta^* + \sigma) = WTP_B(\theta^* - \sigma). \tag{16}$$

Proof of Claim 1:

- If $\theta \leq \widehat{\alpha} - \sigma$, $V(\theta)$ is increasing in θ, since then $\alpha < \widehat{\alpha}$, so A wins the innovation and $V(\theta) = \int_{\theta-\sigma}^{\theta+\sigma} WTP_B(\alpha)d\alpha$ and $WTP'_B(\alpha) > 0$ by Assumption 3. In that case, the innovator prefers to increase θ. Thus, the optimal choice of the innovator cannot be below $\widehat{\alpha} - \sigma$: $\theta^* \geq \widehat{\alpha} - \sigma$.
- Symmetrically, if $\theta \geq \widehat{\alpha} + \sigma$, $V(\theta)$ is decreasing, since then $\alpha > \widehat{\alpha}$, so B wins the innovation and $V(\theta) = \int_{\theta-\sigma}^{\theta+\sigma} WTP_A(\alpha)d\alpha$ and $WTP'_A(\alpha) < 0$ by Assumption 3. In that case, the innovator prefers to decrease θ. Thus, the optimal choice of the innovator cannot be above $\widehat{\alpha} + \sigma$: $\theta^* \leq \widehat{\alpha} + \sigma$.
- For $\theta \in [\widehat{\alpha} - \sigma, \widehat{\alpha} + \sigma]$, the payoff function of the innovator is

  $$V(\theta) = \frac{1}{2\sigma}\left(\int_{\theta-\sigma}^{\widehat{\alpha}} WTP_B(\alpha)d\alpha + \int_{\widehat{\alpha}}^{\theta+\sigma} WTP_A(\alpha)d\alpha\right).$$

  The innovator chooses θ maximizing $V(\theta)$. The first-ordercondition characterizes the equilibrium choice of the innovator, $\theta^* \in [\widehat{\alpha} - \sigma, \widehat{\alpha} + \sigma]$:

  $$WTP_A(\theta^* + \sigma) = WTP_B(\theta^* - \sigma)$$

Our next claim is

Claim 2: $\theta^* = \widehat{\alpha}$ if $\lambda_B = \lambda_A$, $\theta^* > \widehat{\alpha}$ if $\lambda_B > \lambda_A$, and $\theta^* < \widehat{\alpha}$ if $\lambda_B < \lambda_A$.

Proof of Claim 2: Consider $\widehat{\theta}$ such that $\Delta^{A*}(\widehat{\theta} + \sigma) + \Delta^{B*}(\widehat{\theta} - \sigma) = 0$. Using definitions (1) and (2) we have

$$\widehat{\theta} = \frac{2\Delta_0 + \lambda_A q}{q(\lambda_A + \lambda_B)} + \frac{\sigma(\lambda_B - \lambda_A)}{\lambda_A + \lambda_B} = \widehat{\alpha} + \frac{\sigma(\lambda_B - \lambda_A)}{\lambda_A + \lambda_B}. \tag{17}$$

Now consider

$$\begin{aligned}\Delta^{A*}(\widehat{\theta} - \sigma) + \Delta^{B*}(\widehat{\theta} + \sigma) &= 2\Delta_0 + (1 - \widehat{\theta} + \sigma)\lambda_A q - (\widehat{\theta} + \sigma)\lambda_B q, \\ &= 2\Delta_0 + \lambda_A q - \widehat{\theta}(\lambda_A + \lambda_B)q + \sigma(\lambda_A - \lambda_B)q\end{aligned}$$

Replacing the equality of $\widehat{\theta}$ from (17) gives that

$$\Delta^{A*}(\widehat{\theta} - \sigma) + \Delta^{B*}(\widehat{\theta} + \sigma) = 2\sigma q(\lambda_A - \lambda_B). \tag{18}$$

Suppose $\lambda_A = \lambda_B$. From (18) we have $\Delta^{A*}(\widehat{\theta} - \sigma) + \Delta^{B*}(\widehat{\theta} + \sigma) = 0$ The symmetry of the profit functions, Assumption 2, implies that $\Pi_A(\Delta^{A*}(\widehat{\theta} + \sigma)) = \Pi_B(\Delta^{B*}(\widehat{\theta} - \sigma))$ and $\Pi_A(\Delta^{B*}(\widehat{\theta} + \sigma)) = \Pi_B(\Delta^{A*}(\widehat{\theta} - \sigma))$. Using (4), we write

$$\begin{aligned} WTP_A(\theta + \sigma) &= \Pi_A(\Delta^{A*}(\theta + \sigma)) - \Pi_A(\Delta^{B*}(\theta + \sigma)), \\ WTP_B(\theta - \sigma) &= \Pi_B(\Delta^{B*}(\theta - \sigma)) - \Pi_B(\Delta^{A*}(\theta - \sigma)). \end{aligned}$$

and show that $WTP_A(\widehat{\theta} + \sigma) = WTP_B(\widehat{\theta} - \sigma)$, that is, $\widehat{\theta} = \theta^* = \widehat{\alpha}$.

Suppose $\lambda_B > \lambda_A$. From (18) we have $-\Delta^{A*}(\widehat{\theta} - \sigma) > \Delta^{B*}(\widehat{\theta} + \sigma)$. Using the monotonicity of Π_A and the symmetry of the profits, we have

$$\Pi_A(\Delta^{B*}(\widehat{\theta} + \sigma)) < \Pi_A(-\Delta^{A*}(\widehat{\theta} - \sigma)) = \Pi_B(\Delta^{A*}(\widehat{\theta} - \sigma)), \tag{19}$$

which implies that $WTP_A(\widehat{\theta} + \sigma) > WTP_B(\widehat{\theta} - \sigma)$, and so $\theta^* > \widehat{\alpha}$. By symmetric arguments $\theta^* < \widehat{\alpha}$ if $\lambda_A > \lambda_B$.

Proof of Proposition 3 We write the probability that firm B wins the innovation as

$$Pr(B\, wins) = Pr(\alpha \geq \widehat{\alpha}) = \frac{\theta^* + \sigma - \widehat{\alpha}}{2\sigma}.$$

Using L'Hôpital's Rule we calculate the limit of this probability when σ goes to zero, so θ^* goes to $\widehat{\alpha}$:

$$\lim_{\sigma \to 0} \Pr(B\, wins) = \frac{1}{2}\frac{d\theta^*}{d\sigma} + \frac{1}{2}. \tag{20}$$

We next use the equilibrium condition for θ^* and take the total derivative of both sides to characterize $\frac{d\theta^*}{d\sigma}$:

$$\begin{aligned} WTP_A(\theta^* + \sigma) &= WTP_B(\theta^* - \sigma) \\ WTP'_A(\widehat{\alpha})\left(\frac{d\theta^*}{d\sigma} + 1\right) &= WTP'_B(\widehat{\alpha})\left(\frac{d\theta^*}{d\sigma} - 1\right) \\ \frac{d\theta^*}{d\sigma} &= \left(\frac{WTP'_B(\widehat{\alpha}) + WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha}) - WTP'_A(\widehat{\alpha})}\right). \end{aligned}$$

Replacing the latter derivative into (20) gives

$$\begin{aligned} \lim_{\sigma \to 0} \Pr(B\, wins) &= \frac{1}{2}\left(1 + \frac{WTP'_B(\widehat{\alpha}) + WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha}) - WTP'_A(\widehat{\alpha})}\right) \\ &= \frac{WTP'_B(\widehat{\alpha})}{WTP'_B(\widehat{\alpha}) - WTP'_A(\widehat{\alpha})} \end{aligned}$$

Hence, we obtain $\lim_{\sigma\to 0}\Pr(B\ wins) > \frac{1}{2}$ if and only if $WTP'_B(\widehat{\alpha}) > -WTP'_A(\widehat{\alpha})$ or

$$-\lambda_B q\Pi'_B(\Delta^B(\widehat{\alpha})) + \lambda_A q\Pi'_B(\Delta^A(\widehat{\alpha})) > \lambda_A q\Pi'_A(\Delta^A(\widehat{\alpha})) - \lambda_B q\Pi'_A(\Delta^B(\widehat{\alpha})),$$

Using the symmetry of the profit functions and that $\Delta^B(\widehat{\alpha}) = -\Delta^A(\widehat{\alpha})$, we have $\Pi'_B(\Delta^B(\widehat{\alpha})) = -\Pi'_A(\Delta^A(\widehat{\alpha}))$ and $\Pi'_B(\Delta^A(\widehat{\alpha})) = -\Pi'_A(\Delta^B(\widehat{\alpha}))$. Thus, the latter inequality becomes

$$(\lambda_B - \lambda_A)\left(\Pi'_A(\Delta^A(\widehat{\alpha})) + \Pi'_A(\Delta^B(\widehat{\alpha}))\right) > 0,$$

which holds if and only if $\lambda_B > \lambda_A$.

Proof of Proposition 4 As we show in Proposition 3, when there is nearly no uncertainty, the probability that firm B wins the innovation depends on the ratio of how WTP_A changes in $\widehat{\alpha}$ to how WTP_B changes in $\widehat{\alpha}$:

$$\begin{aligned}\lim_{\sigma\to 0}\Pr(B\ wins) &= \frac{WTP'_B(\widehat{\alpha})}{WTP'_B(\widehat{\alpha}) - WTP'_A(\widehat{\alpha})} \\ &= \frac{1}{1 - \frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})}}\end{aligned}$$

Hence, the likelihood of B wins decreases in $-\frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})}$. We therefore study how this ration changes in Δ_0.

We have (from the proof of Lemma 2) that

$$\begin{aligned}WTP'_A(\widehat{\alpha}) &= -\lambda_A q\Pi'_A(\Delta^A(\widehat{\alpha})) + \lambda_B q\Pi'_A(\Delta^B(\widehat{\alpha})) \\ WTP'_B(\widehat{\alpha}) &= -\lambda_B q\Pi'_B(\Delta^B(\widehat{\alpha})) + \lambda_A q\Pi'_B(\Delta^A(\widehat{\alpha}))\end{aligned} \tag{21}$$

When $\sigma \to 0$, $\alpha \to \widehat{\alpha} == \frac{2\Delta_0 + \lambda_A q}{q(\lambda_A + \lambda_B)}$ from Proposition 1, the ex-post qualities are

$$\Delta^A(\widehat{\alpha}) = \Delta_0 + (1-\widehat{\alpha})\lambda_A q, \qquad \Delta^B(\widehat{\alpha}) = \Delta_0 - \widehat{\alpha}\lambda_B q.$$

For conciseness we omit the argument $\widehat{\alpha}$ in $\Delta^A(\widehat{\alpha})$ in this part of the proof. Using $\Delta^B(\widehat{\alpha}) = -\Delta^A(\widehat{\alpha})$, and $\Pi'_B(\Delta) = -\Pi'_A(-\Delta)$ (by symmetry), we obtain

$$-\frac{WTP'_A}{WTP'_B} = -\frac{-\lambda_A\Pi'_A\left(\Delta^A\right) + \lambda_B\Pi'_A\left(-\Delta^A\right)}{-\lambda_B\Pi'_B\left(-\Delta^A\right) + \lambda_A\Pi'_B\left(\Delta^A\right)} = -\frac{-\lambda_A\Pi'_A\left(\Delta^A\right) + \lambda_B\Pi'_A\left(-\Delta^A\right)}{\lambda_B\Pi'_A\left(\Delta^A\right) - \lambda_A\Pi'_A\left(-\Delta^A\right)}.$$

We next derive how the ratio changes in the quality differential of firm A:

$$\frac{d}{d\Delta^A}\left(-\frac{WTP'_A}{WTP'_B}\right)=\frac{d}{d\Delta^A}\left(-\frac{-\lambda_A\Pi'_A\left(\Delta^A\right)+\lambda_B\Pi'_A\left(-\Delta^A\right)}{\lambda_B\Pi'_A\left(\Delta^A\right)-\lambda_A\Pi'_A\left(-\Delta^A\right)}\right),$$

$$=-\frac{\begin{array}{c}\left(-\lambda_A\Pi''_A\left(\Delta^A\right)-\lambda_B\Pi''_A\left(-\Delta^A\right)\right)\left(\lambda_B\Pi'_A\left(\Delta^A\right)-\lambda_A\Pi'_A\left(-\Delta^A\right)\right)\\-\left(-\lambda_A\Pi'_A\left(\Delta^A\right)+\lambda_B\Pi'_A\left(-\Delta^A\right)\right)\left(\lambda_B\Pi''_A\left(\Delta^A\right)+\lambda_A\Pi''_A\left(-\Delta^A\right)\right)\end{array}}{\left(\lambda_B\Pi'_A\left(\Delta^A\right)-\lambda_A\Pi'_A\left(-\Delta^A\right)\right)^2},$$

$$=\frac{\left(\lambda_B^2-\lambda_A^2\right)\left(\Pi''_A\left(\Delta^A\right)\Pi'_A\left(-\Delta^A\right)+\Pi''_A\left(-\Delta^A\right)\Pi'_A\left(\Delta^A\right)\right)}{\left(\lambda_B\Pi'_A\left(\Delta^A\right)-\lambda_A\Pi'_A\left(-\Delta^A\right)\right)^2},$$

which has the sign of $\lambda_B-\lambda_A$ because Π_A is increasing convex.

In particular, we have

$$\frac{d\Delta^{A*}}{d\Delta_0}=\frac{(\lambda_B-\lambda_A)\Delta_0}{\lambda_A+\lambda_B},$$

and using the last two derivatives we show

$$\frac{d}{d\Delta_0}\left(-\frac{WTP'_A}{WTP'_B}\right)>0.$$

Hence, we conclude that $\lim_{\sigma\to 0}\Pr(B\ wins)$ decreases in Δ_0.

Now we study how $\lim_{\sigma\to 0}\Pr(B\ wins)$ changes in q. Following similar steps we show that

$$\frac{d}{dq}\left(-\frac{WTP'_A}{WTP'_B}\right)>0 \text{ if } \lambda_B>\lambda_A,$$

since $\frac{d\Delta^{A*}}{dq}=\frac{\lambda_B\lambda_A}{\lambda_A+\lambda_B}$. We conclude that $\lim_{\sigma\to 0}\Pr(B\ wins)$ is strictly decreasing in q if $\lambda_B>\lambda_A$ and strictly increasing in q if $\lambda_B<\lambda_A$.

Proof of Corollary 4 When there is nearly no uncertainty, $\sigma\to 0$, the innovator chooses θ^* to equalize the WTPs of both firms: $\theta^*=\widehat{\alpha}$. Each firm bids its WTP, and so in equilibrium each firm is indifferent between winning and losing the auction. We thereby compute the profit from losing the auction for each firm: $\Pi_A(\Delta^{B*})$ and $\Pi_B(\Delta^{A*})$. We have $\Pi_B(\Delta^{A*})=\Pi_A(-\Delta^{B*})$, and $-\Delta^{B*}=\Delta^{A*}$ by Proposition 1 and Corollary 2.

Proof of Proposition 5 Recall that when σ goes to zero we have

$$\Delta^{A*}=-\Delta^{B*}=\frac{q\lambda_B\lambda_A+(\lambda_B-\lambda_A)\Delta_0}{\lambda_A+\lambda_B}. \tag{22}$$

The net profit is then $\Pi_A(\Delta^{B*})$, increasing in Δ^{B*}.

Then we have

$$\frac{d\Delta^{B*}}{dq}=-\frac{\lambda_A\lambda_B}{\lambda_A+\lambda_B}.$$

and

$$\frac{d\Delta^{B*}}{d\Delta_0} = \frac{\lambda_A - \lambda_B}{\lambda_A + \lambda_B}.$$

Proof of Proposition 6 We first consider the comparative statics with respect to Δ_0. Firm A's quality differential is $\Delta^A = \Delta_0 + (1-\alpha)\lambda_A q$ if it wins the innovation and $\Delta^B = \Delta_0 - \alpha\lambda_B q$ if B is the winner. Differentiation of Δ^j with respect to Δ_0 gives

$$\frac{d\Delta^j}{d\Delta_0} = 1 - \frac{d\alpha}{d\Delta_0}\lambda_j q. \tag{23}$$

There are two ways in which a change in the initial advantage of firm A, Δ_0, affects the ex-post advantage of the firm that wins the innovation. First, there is the direct effect. Second, and more interestingly, there is the indirect effect of how initial advantage changes the innovator's choice of fit. The indirect effect is composed of how the innovator's choice is affected, the winner's absorption capacity and the size of the innovation.

We consider comparative statics in the limit of no uncertainty, so $\theta^* \to \widehat{\alpha}$ and

$$lim_{\sigma\to 0}\frac{d\alpha}{d\Delta_0} = \frac{d\widehat{\alpha}}{d\Delta_0} = \frac{2}{(\lambda_A + \lambda_B)q} > 0.$$

where we use Propositions 1 and 2. Thus, increasing Δ_0 increases the innovator's choice of fit for the initially weaker firm. Thus, the indirect effect runs counter to the direct effect. The sign of how A's initial advantage affects its ex-post quality advantage depends on the relative sizes of the direct and indirect effects. Substituting into 23 we have:

$$lim_{\sigma\to 0}\frac{d\Delta^j}{d\Delta_0} = 1 - \frac{2\lambda_j}{\lambda_A + \lambda_B} = \frac{\lambda_{-j} - \lambda_j}{\lambda_A + \lambda_B}.$$

Furthermore, the gross profit of firm A (before paying the transfer to the innovator) increases in Δ_0 if and only if $\frac{d\Delta^j}{d\Delta_0} > 0$. If A wins the innovation, the gross profit of firm A increases in Δ_0 if and only if $\lambda_B > \lambda_A$. If B wins the innovation, the profit of firm A decreases in Δ_0 if and only if $\lambda_B > \lambda_A$. Symmetrically, when A is the winner, the profit of firm B decreases in Δ_0 if and only if $\lambda_B > \lambda_A$. When B is the winner, its gross profit increases in Δ_0 if and only if $\lambda_B > \lambda_A$.

Now consider the comparative statics with respect to the value of innovation, q. In the limit of no uncertainty we have $\lim_{\sigma\to 0}\theta^* = \widehat{\alpha}$, which is given by Proposition 1. The cutoff where the WTPs are equal, $\widehat{\alpha}$, is decreasing in q:

$$\frac{d\widehat{\alpha}}{dq} = -\frac{2\Delta_0}{q^2(\lambda_A + \lambda_B)} < 0.$$

Hence, we obtain

$$\begin{aligned}\frac{d\Delta^A}{dq} &= -\frac{d\widehat{\alpha}}{dq}\lambda_A q + (1-\widehat{\alpha})\lambda_A = \frac{\lambda_A\lambda_B}{\lambda_A+\lambda_B} > 0,\\ \frac{d\Delta^B}{dq} &= -\frac{d\widehat{\alpha}}{dq}\lambda_B q - \widehat{\alpha}\lambda_B = -\frac{\lambda_A\lambda_B}{\lambda_A+\lambda_B} < 0.\end{aligned}$$

When firm A wins the innovation, the gross profit of firm A increases in q and the gross profit of firm B decreases in q. Symmetrically, when firm B wins the innovation, the gross profit of firm A decreases in q and the gross profit of firm B increases in q.

Proof of Lemma 1 In our model Δ ranges from $\Delta_0 - \lambda_B q$ when B wins and $\alpha = 1$ to $\Delta_0 + \lambda_A q$ when A wins and $\alpha = 0$. Under Assumption 4 (the monotone hazard rate property of $F(x)$) and 5, there exists a unique $x(\Delta) \in [0,1]$ which is the solution to (11). We have $x(\Delta)$ increasing. We also have $1 - x(\Delta_0 - \lambda_B q) > x(\Delta_0)$, by Assumption 1, $0 < x(\Delta) < 1$ by Assumption 5, $x(\Delta^B) < x(\Delta^A)$ as $\Delta^A > \Delta^B$ and $\frac{1}{2} < x(\Delta^A)$ since $\Delta^A > 0$ given that $\Delta_0 > 0$.

For given Δ, the location of the marginal consumer $x(\Delta)$ is given by (11), which, by symmetry of F, implies that $1 - x(\Delta) = x(-\Delta)$. Then we have

$$\Pi_B(\Delta) = 2\mu\frac{(1-F(x(\Delta))^2}{f(x(\Delta))} = 2\mu\frac{F(1-x(\Delta))^2}{f(x(\Delta))} = 2\mu\frac{(F(x(-\Delta))^2}{f(x(-\Delta))} = \Pi_A(-\Delta).$$

We now show convexity of the profit functions $\Pi_j(\Delta)$. We have

$$x'(\Delta) = \frac{\frac{1}{2\mu}}{1-\left(\frac{1-2F(x(\Delta))}{f(x(\Delta))}\right)'} > 0,$$

since $\left(\frac{1-2F(x)}{f(x)}\right)' < 0$ due to the log-concavity of $F(x)$ and the Monotone Hazard Rate property, Assumption 4. We now define function $\Gamma(x)$ as

$$\Gamma(x) = \left[\frac{F^2(x)}{f(x)}\right]' \frac{1}{1-\left(\frac{1-2F(x)}{f(x)}\right)'}. \tag{24}$$

We then have

$$\Pi_A'(\Delta) = \Gamma(x(\Delta)) \text{ and } \Pi_B'(\Delta) = -\Gamma(x(-\Delta)).$$

where the second term follows from the symmetry property of profits. We have:

$$\Gamma(x) = \frac{2Ff^2 - F^2 f'}{f^2} \frac{1}{1 - \left(\frac{-2f^2-(1-2F)f'}{f^2}\right)},$$
$$= \frac{2Ff^2 - F^2 f'}{3f^2 + (1-2F)f'} = \frac{F}{-\frac{f^2-f'}{2f^2-Ff'} + 2}.$$

Observe that $\Gamma(x) > 0$ due to the Monotone Hazard Rate property and log-concavity of $F(x)$. Furthermore, $\Gamma'(x) > 0$ if $\frac{f^2-f'}{2f^2-Ff'}$ is non-decreasing. Our final claim is that this is the case given that the inverse hazard rate $H(x) = \frac{F(x)}{f(x)}$ is increasing and convex, and the symmetry property of $f(x)$, by Assumption 4.

$$\left(\frac{f^2 - f'}{2f^2 - Ff'}\right)' = \frac{(2ff' - f'')(2f^2 - Ff') - (f^2 - f')(4ff' - (ff' + Ff''))}{(2f^2 - Ff')^2},$$
$$= \frac{f\left(f^2 f' - 2ff'' - 2(f')^2 F + fFf'' + 3(f')^2\right)}{(2f^2 - Ff')^2},$$
$$= \frac{f\left(f^2 f' + (1-F)(2(f')^2 - ff'') + (f')^2 - ff''\right)}{(2f^2 - Ff')^2}, \quad (25)$$

The symmetry property of $f(x)$ implies that $\Lambda(x) = \frac{1-F(x)}{f(x)} = H(1-x)$. The inverse hazard rate $H(x)$ being convex implies that $\Lambda''(x) > 0$. We calculate

$$\Lambda'(x) = -1 - (1-F)\frac{f'}{f^2}.$$

and

$$\Lambda''(x) = \frac{(ff' - (1-F)f'')f + 2(1-F)(f')^2}{f^3}.$$

thus, we prove that $\Lambda''(x) > 0$ implies $f'f^2 - (1-F)f''f + 2(1-F)(f')^2 > 0$, which in turn implies that $\left(\frac{f^2-f'}{2f^2-Ff'}\right)' > 0$ because f log-concavity implies $(f')^2 - ff'' > 0$, see (25), and hence we conclude that $\Gamma'(x) > 0$. As x is increasing in Δ, this shows that profit functions are convex.

Proof of Proposition 7 Consider how firm A's net profit changes in μ when σ goes to zero, that is, the same as the change in firm A's gross profit when B wins the innovation. Using 12 we derive A's gross profit at Δ^{B*}:

$$\frac{d\Pi_A^*(\Delta^{B*})}{d\mu} = 2\frac{[F(x^{B*})]^2}{f(x^{B*})} + 2\mu\left(\frac{[F(x^{B*})]^2}{f(x^{B*})}\right)' \frac{dx^{B*}}{d\mu},$$

where

$$x^{j*} = \frac{1}{2} + \frac{1-2F(x^{j*})}{f(x^{j*})} + \frac{\Delta^{j*}}{2\mu}.$$

and so

$$\frac{dx^{B*}}{d\mu} = -\frac{\frac{\Delta^{B*}}{2\mu^2}}{1 - \left(\frac{1-2F(x^{B*})}{f(x^{B*})}\right)'} > 0.$$

given that $\Delta^{B*} = -\Delta^{A*} < 0$ and Δ^{B*} is constant in μ ($\widehat{\alpha}$ is independent of μ from Proposition 1). Using the definition of $\Gamma(x)$ from (24) and $\Gamma(x) > 0$ as shown in the proof of Lemma 2, we show that Firm A's net profit increases in μ:

$$\frac{d\Pi_A^*(\Delta^{B*})}{d\mu} = 2\frac{[F(x^{B*})]^2}{f(x^{B*})} - \Gamma(x^{B*})\frac{\Delta^{B*}}{\mu} > 0.$$

where the first term captures the positive markup effect and the second term captures the positive effect of μ via increasing firm A's market share when B wins the innovation. We show in Corollary 4 that firm B's net profit is equal to firm A's net profit in equilibrium. Hence, we have the same comparative statics for firm B's net profit as A's net profit.

The market share of firm A, $F(x^{A*})$, decreases in differentiation if A wins the innovation since

$$\frac{dx^{A*}}{d\mu} = -\frac{\frac{\Delta^{A*}}{2\mu^2}}{1 - \left(\frac{1-2F(x^{A*})}{f(x^{A*})}\right)'} < 0.$$

Increasing μ affects firm A's gross profit when A wins the innovation in two ways going opposite directions. On one hand, increasing differentiation increases each firm's markup. On the other hand, the market share of firm A decreases in μ if A wins the innovation. Formally,

$$\frac{d\Pi_A^*(\Delta^{A*})}{d\mu} = 2\frac{\left(F(x^{A*})\right)^2}{f(x^{A*})} - \Gamma(x^{A*})\frac{\Delta^{A*}}{\mu},$$

where the first term captures the positive markup effect and the second term captures the negative effect of μ via lowering firm A's market share. Thus, the net effect of μ on firm A's gross profit is unclear. When A wins the innovation, Firm B's gross profit increases in μ since increasing μ increases its markup as well as its market share. Symmetrically, when B wins the innovation, the net effect of μ on Firm B's gross profit is unclear since increasing μ increases its markup but decreases its market share, $1 - x^{B*}$ decreases in μ. When B wins the innovation, Firm A's gross profit increases in μ since increasing μ increases its markup as well as its market share.

Proof of Proposition 8 Suppose $\theta < \widehat{\alpha}$. Using the marginal consumer at equilibrium prices, (11), we have at $\alpha = \theta$:

$$x^A = \frac{1}{2} + \frac{1 - 2F\left(x^A\right)}{f\left(x^A\right)} + \frac{\Delta^0 + (1-\theta)\,\lambda_A q}{2\mu}$$

Thus, $x^A(\theta)$ is an implicit function. Next we define

$$L\left(x\right) =: x - \frac{1 - 2F\left(x\right)}{f\left(x\right)} \quad \text{and} \quad l\left(x\right) =: L'\left(x\right) = 1 - \left(\frac{1-2F(x)}{f(x)}\right)'. \tag{26}$$

Note that $l(x) > 0$ by the Monotone Hazard Rate Property. Using these definitions we write the derivative of the marginal consumer with respect to θ (we drop the argument of x^A to ease the notation):

$$\frac{dx^A}{d\theta} = -\frac{\lambda_A q}{2\mu}\frac{1}{l\left(x^A\right)}.$$

Thereby, we have the derivative of consumer surplus with respect to θ:

$$\frac{dCS^A\left(\theta\right)}{d\theta} = -\lambda_A q F\left(x^A\right) + \left(-\frac{\lambda_A q}{2\mu l\left(x^A\right)}\right)\left[f\left(x^A\right)\left(\Delta^0 + (1-\theta)\,\lambda_A q\right) - T'\left(x^A\right) - \Pi^{T\prime}\left(x^A\right)\right]$$

Using the definition $T(x^j)$, (13), we obtain

$$T'\left(x^j\right) = -\mu f\left(x^j\right)\left(1 - 2x^j\right). \tag{27}$$

Moreover, from the equilibrium condition (11) when $\alpha = \theta$ we have

$$-\mu f\left(x^A\right)\left(1 - 2x^A\right) = 2\mu(1 - 2F\left(x^A\right)) + f\left(x^A\right)\left(\Delta^0 + (1-\theta)\lambda_A q\right),$$

Plugging these equations in the consumer surplus derivative gives

$$\frac{dCS^A\left(\theta\right)}{d\theta} = -\lambda_A q F\left(x^A\right) + \left(1 - 2F\left(x^A\right)\right)\frac{\lambda_A q}{l\left(x^A\right)} + \Pi^{T\prime}\left(x^A\right)\left(\frac{\lambda_A q}{2\mu l\left(x^A\right)}\right).$$

Now consider the total industry profit derivative

$$\frac{1}{2\mu}\Pi^{T\prime}\left(x\right) = \frac{d}{dx}\left(\frac{F^2(x)}{f(x)} + \frac{(1-F(x))^2}{f(x)}\right)$$

Thus, we obtain

$$\begin{aligned}\frac{1}{2\mu}\Pi^{T\prime}\left(x\right) &= 2F\left(x\right) - \frac{F^2\left(x\right)f'\left(x\right)}{f^2\left(x\right)} - 2(1 - F\left(x\right)) - \frac{(1-F\left(x\right))^2 f'\left(x\right)}{f^2\left(x\right)} \\ &= 2(2F(x) - 1) - \frac{\left(1 - 2F\left(x\right)\right)f'(x)}{f^2(x)} - \frac{2F^2\left(x\right)f'(x)}{f^2(x)}.\end{aligned} \tag{28}$$

Then we have, using $l\left(x\right)=3+\frac{(1-2F(x))f'(x)}{f^{2}(x)}$,

$$\begin{aligned}\frac{dCS^{A}\left(\theta\right)}{d\theta} &= \frac{\lambda_{A}q}{l\left(x^{A}\right)}1-2F\left(x^{A}\right)-\left(3+\frac{\left(1-2F\left(x^{A}\right)\right)f'\left(x^{A}\right)}{f^{2}\left(x^{A}\right)}\right)F\left(x^{A}\right)+2(2F(x^{A})-1)\\ &\quad -\frac{\left(1-2F\left(x^{A}\right)\right)f'(x^{A})}{f^{2}(x^{A})}-\frac{2F^{2}\left(x^{A}\right)f'(x^{A})}{f^{2}(x^{A})},\\ &= \frac{\lambda_{A}q}{l\left(x^{A}(\theta)\right)}\left(-F\left(x^{A}\right)-1-\left(1-F\left(x^{A}\right)\right)\frac{f'\left(x^{A}\right)}{f\left(x^{A}\right)^{2}}\right),\\ &= \frac{\lambda_{A}q}{l\left(x^{A}\right)}\left(-F\left(x^{A}\right)+\frac{d}{dx^{A}}\left(\frac{1-F\left(x^{A}\right)}{f\left(x^{A}\right)}\right)\right)<0,\end{aligned}$$

where the last inequality is due to the Monotone Hazard Rate Property. Hence, we prove that

$$\frac{\partial CS^{A}\left(\theta\right)}{\partial\theta}<0 \qquad \text{if} \qquad \theta<\widehat{\alpha}.$$

The proof is symmetric for $\theta>\widehat{\alpha}$. Using the marginal consumer at equilibrium prices, (11), we have

$$x^{B}=\frac{1}{2}+\frac{1-2F\left(x^{B}\right)}{f\left(x^{B}\right)}+\frac{\Delta^{0}-\theta\lambda_{B}q}{2\mu}$$

Using the definitions of $L\left(x\right)$ and $l\left(x\right)$ from (26), we write the derivatives of the marginal type with respect to θ as

$$\frac{\partial CS^{B}(\theta)}{\partial\theta}=-2\mu\left(1-2F\left(x^{B}\right)\right)\frac{dx^{B}}{d\theta}+\left(1-F\left(x^{B}\right)\right)\lambda_{B}q-\Pi^{T\prime}\left(x^{B}\right)\frac{dx^{B}}{d\theta}$$

$$\begin{aligned}\frac{\partial CS^{B}(\theta)}{\partial\theta} &= -2\mu\left(\left(2F\left(x^{B}\right)-1\right)+\frac{2F\left(x^{B}\right)-1}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)-\frac{2F^{2}\left(x^{B}\right)}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)\right)\frac{dx^{B}}{d\theta}\\ &\quad +\left(1-F\left(x^{B}\right)\right)\lambda_{B}q\end{aligned}$$

$$\begin{aligned}\frac{\partial CS^{B}(\theta)}{\partial\theta} &= \left(\left(2F\left(x^{B}\right)-1\right)+\frac{2F\left(x^{B}\right)-1}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)-\frac{2F^{2}\left(x^{B}\right)}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)\right)\left(\frac{\lambda_{B}q}{l\left(x^{B}\right)}\right)\\ &\quad +\left(1-F\left(x^{B}\right)\right)\lambda_{B}q\end{aligned}$$

$$\frac{\partial CS^{B}(\theta)}{\partial\theta}=\left(\left(2F\left(x^{B}\right)-1\right)+\frac{2F\left(x^{B}\right)-1}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)-\frac{2F^{2}\left(x^{B}\right)}{f^{2}\left(x^{B}\right)}f'\left(x^{B}\right)+\left(1-F\left(x^{B}\right)\right)l\left(x^{B}\right)\right)\left(\frac{\lambda_{B}q}{l\left(x^{B}\right)}\right)$$

$$\frac{\partial CS^B(\theta)}{\partial\theta} = \left(1 - F\left(x^B\right) + \frac{d}{dx^B}\frac{F\left(x^B\right)}{f\left(x^B\right)}\right)\left(\frac{\lambda_B q}{l\left(x^B\right)}\right) > 0.$$

Proof of Corollary 6 If $\theta \leq \widehat{\alpha}$, firm A wins the innovation and the expected welfare is

$$W^A(\theta) = Q_B + F\left(x^{A*}\right)\left(\Delta_0 + (1-\theta)\lambda_A q\right) - T\left(x^{A*}\right).$$

If $\theta > \widehat{\alpha}$, firm B wins the innovation and the expected welfare is

$$W^B(\theta) = Q_B + F\left(x^{B*}\right)\Delta_0 + \left(1 - F\left(x^{B*}\right)\right)\theta\lambda_B q - T\left(x^{B*}\right)$$

1. First we determine the jump of welfare at $\widehat{\alpha}$, which is given by

$$W^B(\widehat{\alpha}) - W^A(\widehat{\alpha}) = \left(1 - 2F\left(x^A\right)\right)\Delta_0 + F\left(x^A\right)\left(\widehat{\alpha}\lambda_B q - (1-\widehat{\alpha})\lambda_A q\right).$$

since $1 - F(x^{B*}) = F(x^{A*})$ at θ^* and when there is nearly no uncertainty $\theta^* = \widehat{\alpha}$. Using $\widehat{\alpha} = \frac{2\Delta_0 + \lambda_A q}{q(\lambda_A + \lambda_B)}$, we then get $W^B(\widehat{\alpha}) - W^A(\widehat{\alpha}) = \Delta_0$

2. For $\theta < \widehat{\alpha}$, the slope of W_A is

$$\frac{dW^A(\theta)}{d\theta} = f\left(x^{A*}\right)\left(\Delta_0 + (1-\theta)\lambda_A q\right)\frac{dx^{A*}}{d\theta} - F\left(x^{A*}\right)\lambda_A q - T'\left(x^{A*}\right)\frac{dx^{A*}}{d\theta}$$

Using $T'\left(x^{A*}\right) = -\mu f\left(x^{A*}\right)\left(1 - 2x^{A*}\right)$ and

$$\mu\left(1 - 2x^{A*}\right) = -\frac{2\mu\left(1 - 2F\left(x^{A*}\right)\right)}{f\left(x^{A*}\right)} - \left(\Delta_0 + (1-\theta)\lambda_A q\right),$$

we have

$$\begin{aligned}\frac{dW^A(\theta)}{d\theta} &= f\left(x^{A*}\right)\left(\Delta_0 + (1-\theta)\lambda_A q\right)\frac{dx^{A*}}{d\theta} - F\left(x^{A*}\right)\lambda_A q + \mu f\left(x^{A*}\right)\left(1 - 2x^{A*}\right)\frac{dx^{A*}}{d\theta} \\ &= f\left(x^{A*}\right)\left(\mu\left(1 - 2x^{A*}\right) + \Delta_0 + (1-\theta)\lambda_A q\right)\frac{dx^{A*}}{d\theta} - F\left(x^{A*}\right)\lambda_A q \\ &= -2\mu\left(1 - 2F\left(x^{A*}\right)\right)\frac{dx^{A*}}{d\theta} - F\left(x^{A*}\right)\lambda_A q < 0\end{aligned}$$

3. For $\theta > \widehat{\alpha}$ Slope of W^B is

$$\frac{dW^B(\theta)}{d\theta} = f\left(x^{B*}\right)\left(\Delta_0 - \theta\lambda_B q\right)\frac{dx^{B*}}{d\theta} + \left(1 - F\left(x^{B*}\right)\right)\lambda_B q + \mu f\left(x^{B*}\right)\left(1 - 2x^{B*}\right)\frac{dx^{B*}}{d\theta}$$

where $\mu\left(1-2x^{B*}\right)=-\frac{2\mu\left(1-2F\left(x^{B*}\right)\right)}{f(x^{B*})}-(\Delta_0-\theta\lambda_B q)$. We then have

$$\begin{aligned}\frac{dW^B(\theta)}{d\theta}&=f\left(x^{B*}\right)\left(\Delta_0-\theta\lambda_B q+\mu\left(1-2x^{B*}\right)\right)\frac{dx^{B*}}{d\theta}+\left(1-F\left(x^{B*}\right)\right)\lambda_B q\\&=-2\mu\left(1-2F\left(x^{B*}\right)\right)\frac{dx^{B*}}{d\theta}+\left(1-F\left(x^{B*}\right)\right)\lambda_B q>0\end{aligned}$$

Proof of Proposition 9: Denoting x_1^B and x_0^A the marginal consumer when B wins with $\alpha=1$ and A wins with $\alpha=0$ respectively, we have

$$W_B(1)-W_A(0)=\left(F(x_1^B)-F(x_0^A)\right)\Delta_0+\left(1-F(x_1^B)\right)\lambda_B q-F(x_0^A)\lambda_A q-T(x_1^B)+T(x_0^A)$$

First notice that $W_B(1)$ is increasing in λ_B while $W_A(0)$ is independent of λ_B, which establishes the existence of $\overline{\lambda}_B$. Moreover, evaluating the difference at the switching point $\lambda_B q-\Delta_0=\lambda_A q+\Delta_0$, we have $x_1^B=1-x_0^A$ which implies that the welfare differential is

$$\begin{aligned}W_B(1)-W_A(0)&=\left(1-2F(x_0^A)\right)\Delta_0+F(x_0^A)\left(\lambda_B q-\lambda_A q\right)\\&=\Delta_0,\end{aligned}$$

which is positive. Hence $\overline{\lambda}_B q<\lambda_A q+2\Delta_0$.

Proof of Proposition 10 The innovator's payoff is the weighted sum of the WTPs:

$$\Pi_I(\alpha)=\begin{cases}\gamma WTP_A(\alpha)+(1-\gamma)WTP_B(\alpha) & \text{if}\quad \alpha<\widehat{\alpha},\\ \gamma WTP_B(\alpha)+(1-\gamma)WTP_A(\alpha) & \text{if}\quad \alpha>\widehat{\alpha}.\end{cases}$$

Recall from Lemma 2, $WTP_A(\alpha)$ is decreasing and $WTP_B(\alpha)$ is increasing when $|\lambda_A-\lambda_B|$ is small. Given γ is small enough, $\Pi_I(\alpha)$ is increasing when $\alpha<\widehat{\alpha}$ and decreasing when $\alpha>\widehat{\alpha}$ with a kink at $\widehat{\alpha}$. Thus, the innovator's choice is the same as the baseline model.

Now consider the probability of B winning the innovation. Define $R\equiv-\frac{WTP_A'(\alpha)}{WTP_B'(\alpha)}$. In the baseline model (Proposition 8) the probability of B winning the innovation is negatively related to R. Suppose $\lambda_B>\lambda_A$ then $R<1$ (B is more likely to win the innovation than A). Similar to Proposition 8, the probability of B winning the innovation is negatively related to $\tilde{R}$, which is defined by

$$\tilde{R}\equiv-\frac{\gamma WTP_B'(\alpha)+(1-\gamma)WTP_A'(\alpha)}{\gamma WTP_A'(\alpha)+(1-\gamma)WTP_B'(\alpha)}=\frac{-\gamma+(1-\gamma)R}{-\gamma R+1-\gamma}\tag{29}$$

Observe that $\tilde{R}<R$ if and only if $R<1$. This is the case when $\lambda_B>\lambda_A$, and so the probability of B winning the innovation is larger with $\gamma>0$. By similar arguments, the probability of B winning the innovation is lower with $\gamma>0$ when $\lambda_B<\lambda_A$.

B Dynamic model

We denote by Δ_1 the quality differential between firms A and B after the auction in period 1–hence at the start of period 2–which takes on two values depending on who wins the auction in period 1:

$$\begin{aligned} \Delta_1^A &= \Delta_0 + (1-\alpha_1)\,\lambda_A q \quad \text{if firm } A \text{ wins the period 1 auction,} \\ \Delta_1^B &= \Delta_0 - \alpha_1 \lambda_B q \quad \text{if firm } B \text{ wins the period 1 auction.} \end{aligned}$$

Similarly, we denote by Δ_2 the quality difference between firm A and firm B at the end of period 2, which takes on two values depending on the winner of the auction in period 2:

$$\begin{aligned} \Delta_2^A &= \beta\Delta_1 + (1-\alpha_2)\,\lambda_A q \quad \text{if firm } A \text{ wins the period 2 auction} \\ \Delta_2^B &= \beta\Delta_1 - \alpha_2 \lambda_B q \quad \text{if firm } B \text{ wins the period 2 auction} \end{aligned}$$

where β captures how much quality differential of the market leader is carried from period 1 to period 2. Given the choice of the innovator, the market share of firm A in period t when firm j wins the auction is the implicit solution to

$$x_t^{j*} = \frac{1}{2} + \frac{1 - 2F(x_t^{j*})}{f(x_t^{j*})} + \frac{\Delta_t^j}{2\mu}. \tag{30}$$

The gross profit of firm A and firm B in period t when firm j wins the auction are respectively:

$$\Pi_A(\Delta_t^j) = 2\mu \frac{\left(F(x_t^{j*})\right)^2}{f(x_t^{j*})} \qquad \text{and} \qquad \Pi_B(\Delta_t^j) = 2\mu \frac{\left(1 - F(x_t^{j*})\right)^2}{f(x_t^{j*})}.$$

B.1 Second period equilibrium

From the static analysis, when $\sigma_2 \to 0$, the equilibrium level of innovation location is

$$\theta_2^* = \widehat{\alpha}_2 = \frac{2\beta\Delta_1 + \lambda_A q}{(\lambda_A + \lambda_B)\, q}$$

if there is an interior solution for $\widehat{\alpha}_2$. In this case, the quality difference between the firms is

$$\hat{\Delta}_2^A(\Delta_1) = -\hat{\Delta}_2^B(\Delta_1) = \beta\Delta_1 \frac{\lambda_B - \lambda_A}{\lambda_A + \lambda_B} + \frac{\lambda_A \lambda_B q}{\lambda_A + \lambda_B}. \tag{31}$$

We focus on the interior solution in the second period, which is the case if β is sufficiently small relative to q.[12] Consider the expected profit of firm j evaluated at the beginning of period 2, as a

[12] If A wins the auction in period 1 and $\beta\Delta_1^A \geq \frac{\lambda_B q}{2}$, then the direction of innovation in period 2 is the best fit for firm B (i.e., $\theta_2^* = \alpha_2 = 1$) and A wins again in period 2. If B wins the auction in period 1 and $\beta\Delta_1^B \leq -\frac{\lambda_A q}{2}$, then the direction of innovation in period 2 is the best fit for firm A (i.e., $\theta_2^* = \alpha_2 = 0$) and B wins again.

function of the quality differential. The innovator chooses $\theta^* = \widehat{\alpha}_2$ to equalize the WTPs of both firms, and so in equilibrium each firm is indifferent between winning and losing the auction. Recall from Corollary (4) that as the uncertainty goes to zero, both firms obtains the same profit in the one-period game. We can thus compute this profit as the profit from losing the auction for either firm:

$$V_2^*(\Delta_1) = \Pi_A(x_2^{B*}) = \Pi_B(x_2^{A*}), \tag{32}$$

where x_2^{A*} and x_2^{B*} are firm A's market shares if A wins and loses the period 2 auction, respectively. From Proposition 5 it follows that the net profits in period 2 are increasing in Δ_1 if $\lambda_A > \lambda_B$. Symmetrically, the net profits in period 2 are decreasing in Δ_1 if $\lambda_B > \lambda_A$. And the net profits in period 2 are constant in Δ_1 if $\lambda_B = \lambda_A$.

We now analyze the firms second period preferences over which firm should win the period 1 innovation.

Corollary 8 *The second period profit of each firm is larger if the firm with larger absorption capacity (λ_i) wins the first-period innovation.*

Proof of Propositions 11 and 12 Proposition 11 follows from continuity. Consider Proposition 12. When $\lambda_B = \lambda_A$, the dynamic equilibrium is the same as the static equilibrium. Starting from $\lambda_B = \lambda_A$, raising λ_B, so when $\lambda_B > \lambda_A$, we have $V_2^*(\Delta_1^A) - V_2^*(\Delta_1^B) < 0$ by Proposition 5. This shifts curve WTP_{A1} down and WTP_{B1} up, implying that the crossing point shifts toward lower $\widehat{\alpha}$. The reverse holds for $\lambda_B < \lambda_A$.

Proof of Proposition 13 Recall that the probability of B winning the innovation is inversely related to $-\frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})}$, see (8). For simplicity, without loss of generality, we normalize $\lambda_A = 1$.

In the static model for uniform $F(x)$ this ratio is

$$-\frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})} = \frac{-3\mu\left(\lambda_B - 1\right) + \Delta_0\left(\lambda_B - 1\right) + q\lambda_B}{3\mu\left(\lambda_B - 1\right) + \Delta_0\left(\lambda_B - 1\right) + q\lambda_B} \tag{33}$$

while in the dynamic model the first period ratio of the WTPs is

$$-\frac{WTP'_A(\widehat{\alpha_1})}{WTP'_B(\widehat{\alpha_1})} = \frac{\begin{array}{c}-3\mu\left(1 + \beta\delta\left(\lambda_B - 1\right) + \lambda_B\right)\left(\lambda_B^2 - 1\right) + \Delta_0\left(\lambda_B - 1\right)\left(\beta^2\delta\left(\lambda_B - 1\right)^2 + \left(\lambda_B + 1\right)^2\right)\\ +q\lambda_B\left(\beta\delta\left(\lambda_B - 1\right)^2 + \beta^2\delta\left(\lambda_B - 1\right)^2 + \left(\lambda_B + 1\right)^2\right)\end{array}}{\begin{array}{c}3\mu\left(1 - \beta\delta\left(\lambda_B - 1\right) + \lambda_B\right)\left(\lambda_B^2 - 1\right) + \Delta_0\left(\lambda_B - 1\right)\left(\beta^2\delta\left(\lambda_B - 1\right)^2 + \left(\lambda_B + 1\right)^2\right)\\ +q\lambda_B\left(\beta\delta\left(\lambda_B - 1\right)^2 + \beta^2\delta\left(\lambda_B - 1\right)^2 + \left(\lambda_B + 1\right)^2\right)\end{array}} \tag{34}$$

Define $D = \lambda_B - 1$ and $S = 1 + \lambda_B$. Using these definitions we rewrite (33) and (34) respectively

$$-\frac{WTP'_A(\widehat{\alpha})}{WTP'_B(\widehat{\alpha})} = \frac{-3\mu D + \Delta_0 D + q\lambda_B}{3\mu D + \Delta_0 D + q\lambda_B} \tag{35}$$

$$-\frac{WTP_A'(\widehat{\alpha_1})}{WTP_B'(\widehat{\alpha_1})} = \frac{-3\mu\left(S+\beta\delta D\right)DS+\Delta_0 D\left(\beta^2\delta D^2+S^2\right)+q\lambda_B\left(\beta\delta D^2+\beta^2\delta D^2+S^2\right)}{3\mu\left(S-\beta\delta D\right)DS+\Delta_0 D\left(\beta^2\delta D^2+S^2\right)+q\lambda_B\left(\beta\delta D^2+\beta^2\delta D^2+S^2\right)},$$

$$= \frac{-3\mu D+\Delta_0 D+q\lambda_B+\frac{\beta\delta}{S^2}D^2\left(-3\mu S+\Delta_0 D\beta+q\lambda_B\left(1+\beta\right)\right)}{3\mu D+\Delta_0 D+q\lambda_B+\frac{\beta\delta}{S^2}D^2\left(-3\mu S+\Delta_0 D\beta+q\lambda_B\left(1+\beta\right)\right)}.$$

First note that when $\lambda_B = \lambda_A = 1$, $D = 0$. Thus, both ratios are equal to 1, which implies that each firm has equal probability of winning the innovation in both dynamic and static models.

We next take the difference between the dynamic and the static ratios:

$$\frac{\frac{\beta\delta}{S^2}D^2\left(-3\mu S+\Delta_0 D\beta+q\lambda_B\left(1+\beta\right)\right)6\mu D}{\left(3\mu D+\Delta_0 D+q\lambda_B+\frac{\beta\delta}{S^2}D^2\left(-3\mu S+\Delta_0 D\beta+q\lambda_B\left(1+\beta\right)\right)\right)\left(3\mu D+\Delta_0 D+q\lambda_B\right)}$$

The denominator is positive and we have

$$-3\mu S+\Delta_0 D\beta+q\lambda_B\left(1+\beta\right)<0,$$

because as $D=\lambda_B-1<S=\lambda_B+1$ and $\lambda_B<S$,

$$-3\mu+\Delta_0\frac{D}{S}\beta+q\frac{\lambda_B}{S}\left(1+\beta\right)<-3\mu+\Delta_0\beta+q\left(1+\beta\right),$$

which is negative by Assumption 6*(i)*. So the numerator has the opposite sign of $D = \lambda_B - 1$. Hence, the ratio of the WTPs is smaller in the dynamic model than the static when $\lambda_B > 1 = \lambda_A$. This implies that the probability of B winning is greater in the dynamic model than in the static model when $\lambda_B > \lambda_A$.

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