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One-Sided Enforcement in a Model with Persistent Adverse Selection

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ABSTRACT. We study a repeated buyer-seller relationship with persistent adverse selection and one-sided enforcement, where a prepaid seller can breach by taking the money and running. The optimal stationary contract depends on enforcement strength and the discount factor. Three regimes arise. With a strong legal system, penalties deter breach and the optimal static contract can be repeated. With a weak system, the penalty caps transfers, forcing bunching among efficient (low-cost) types. With a very weak system, compliance relies on relational rents, causing large downward distortions. Strengthening public enforcement relaxes both incentive and enforcement constraints, reducing allocative inefficiency.

KEYWORDS. Adverse selection, Limited enforcement, Relational contracts, Contract breach.

JEL CLASSIFICATION. D82, D86, K12, O17.

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1. Introduction

In many economic transactions, particularly in procurement, construction, and international trade, buyers must commit funds before goods or services are delivered. This temporal gap creates a fundamental enforcement problem: the seller may accept the prepayment but fail to deliver, opting instead to "take the money and run." This risk is particularly acute in developing economies where legal institutions are frail, and the cost of recovering damages often exceeds the value of the transaction.

Empirical evidence highlights the severity of this friction. Antràs and Foley (2015) document that in environments with weak contractual enforcement, exporters often demand cash-in-advance terms, shifting the risk entirely to the importer. Conversely, in the Kenyan rose export sector, Macchiavello and Morjaria (2015) show that when formal contracts are unenforceable, trade relies heavily on the value of future relationships, with reliability increasing as the relationship matures. Similarly, in the construction industry, statutory bonds and adjudication mechanisms are often required to prevent contractors from absconding with funds, yet these mechanisms are imperfect substitutes for robust legal enforcement.[1]

In this paper, we develop a model of repeated interaction between a buyer (principal) and a seller (agent) with persistent private information regarding production costs. We analyze how the strength of the legal system, *modeled as the magnitude of an enforceable penalty for breach*, shapes the optimal contract. We depart from the assumption of perfect commitment found in standard dynamic mechanism design (Baron and Besanko (1984)) by introducing a one-sided enforcement constraint: the seller can breach the contract after receiving payment but before delivery, paying a penalty Π and terminating the relationship.

Our analysis reveals that the optimal stationary contract depends critically on the magnitude of Π relative to the seller's information rents and the value of future trade. We identify three contractual regimes:

(i) *Strong Enforcement ($\Pi \geq \Pi^{os}$):* When the penalty for breach is sufficiently large, the legal system provides a complete deterrent. The enforcement constraint is slack,

[1] See, for example, *Civil Mining & Construction Pty Ltd v Isaac Regional Council* (2014), where adjudication provided interim cash flow during a dispute, effectively preventing a total breakdown of the trade relationship.

and the optimal contract is the infinite repetition of the optimal static contract derived by Baron and Myerson (1982). Here, statutory penalties alone suffice to support second-best efficiency.

(ii) *Weak Enforcement* ($\Pi^* \leq \Pi < \Pi^{os}$)*:* When penalties are moderate, they cannot support the high transfers required to elicit truthful revelation from efficient types in the standard second-best contract. To prevent the seller from taking the payment and paying the fine, the principal must cap the transfer to the most efficient type at exactly Π. Because incentive compatibility links transfers across types, this cap forces the principal to pool the most efficient types (bunching) and distort output downwards for less efficient types. In this regime, the contract is enforced by the penalty, but the *structure* of the contract is distorted by the legal system's limitations.

(iii) *Very Weak Enforcement* ($\Pi < \Pi^*$)*:* In environments where the legal penalty is negligible, the monetary fine is insufficient to deter breach. Enforcement must rely on the "shadow of the future" (the discount factor δ). The seller complies only to preserve the value of future rents. The optimal contract involves bunching and severe output distortions to generate sufficient continuation value to prevent breach. This aligns with the findings of Berglöf and Claessens (2006), who argue that in transition economies, private ordering must substitute for public enforcement, often at the cost of restricted trade volumes or exclusion of new market entrants.

Our results provide a theoretical foundation for policy interventions in weak institutional environments. We show that improvements in the legal system (increasing Π) and mechanisms that enhance the visibility of reputation (effectively increasing δ) are complementary. By increasing the penalty for breach, policymakers do not just punish deviance; they relax the constraints on screening, allowing for more sophisticated contracts that reduce information rents and increase aggregate welfare, even though distributional effects may be non-monotonic.

LITERATURE REVIEW. The paper is related to two actively developing areas of incentive theory: dynamic mechanism design and relational contracts. In dynamic adverse selection models, the agent commits to the contract. This literature generally assumes that the agent's type is constant over time.[2] Under such full commitment and costless enforcement, Baron and Besanko (1984) show that the optimal contract is stationary and it can be implemented by the optimal static contract. By assuming that the parties can commit to the contract but not to continuing the relationship, our model weakens the full commitment assumption in long-run contracting with fixed types. Laffont and Tirole (1993) study dynamic contracting with short-term contracts. The optimal contract involves some pooling where the "ratchet effect" leads to bonuses to the efficient agent, but it creates incentives for the take-the-money-and-run strategy. We show that the enforcement constraint alone may involve some pooling.

In contrast, the relational contracts literature mostly focuses on self-enforcing incentive contracts with independent types (Levin (2003), Wolitzky (2010), and Halac (2012)). In his important contribution, Levin (2003) considers an enforcement constraint which balances the incentives to benefit from the current contracting terms with the present value of the relationship. The optimal contract is stationary, and as in our paper has partial pooling for efficient types and exhibits more distortion than the second-best solution. Malcomson (2016) considers a variant of Levin's model with persistent types. In his model bunching arises because of the ratchet effect.

Martimort et al. (2017b) analyze a similar environment with two-sided enforcement and the restriction to stationary contracts, where both the principal and agent can breach. They show that the relevant constraint depends on the *aggregate* penalty of both parties. While they identify the existence of strong and weak enforcement regimes involving bunching, their aggregation of penalties masks the specific strategic role of the agent's "take-the-money-and-run" option when legal penalties are negligible. The current paper complements that work by isolating the one-sided enforcement problem. This allows us to identify a third regime (*very weak enforcement*), where legal penalties are insufficient to deter breach, forcing the contract to rely entirely on relational rents (the discount factor) and necessitating positive rents for the least efficient type.

Martimort et al. (2017a) present a dynamic adverse selection model with two-sided enforcement. Type is drawn from a binary distribution and it is fixed. The optimal contract is non-stationary: transfers are increasing for the most efficient type in order to

[2] Baron and Besanko (1984), Laffont and Tirole (1996), and Battaglini (2005) extend this literature and consider types that are correlated over time. The general result with correlated types is that the optimal long-term contract is non-stationary and has a memory.

prevent opportunistic behavior by the seller in the initial rounds. Penalties for breach of contract for the buyer and seller are aggregated and it is only the aggregated penalty that matters. Thus if there is only an agent's enforcement problem, as in our paper, then the contract is enforced at no cost.

ORGANIZATION OF THE PAPER. Section 2 presents the model and discusses the enforcement constraint. In Section 3, we present the principal's problem and formulate the main result. In Section 4, we solve the model using the reduction to two simple programs. Section 5 introduces some possible extensions and discusses assumptions of the model. Proofs are relegated to the Appendix.

2. MODEL

• PREFERENCES AND INFORMATION. We consider a long-term relationship between a buyer (the principal) and a seller (the agent). In each period, the buyer purchases a (non-durable) good q from the seller and pays a transfer t.[3] The seller and the buyer have per-period utility functions given respectively by

$$V(q,t) = S(q) - t, \qquad U(q,t,\theta) = t - \theta q,$$

where θ is the seller's marginal cost: the agent's type. Assume that the agent's type is drawn once and for all at the beginning of the relationship. The agent privately learns his cost parameter θ which is drawn from the atomless distribution $F(\cdot)$ on the interval $\Theta = [\underline{\theta}, \overline{\theta}]$ with density $f(\cdot)$. The distribution $F(\cdot)$ is common knowledge. The gross surplus function $S(\cdot)$ is increasing and strictly concave $(S'(\cdot) > 0 > S''(\cdot))$ and satisfies the Inada conditions $S'(0) = +\infty, S(0) = 0$. These assumptions ensure that the first-best surplus is always positive (i.e., $\max_{q \geq 0} S(q) - \overline{\theta} q \geq 0$).

The time horizon is infinite, discrete, and the parties have a common discount factor $\delta \in [0,1]$.

Output is observable each period. At the beginning of the relationship the buyer offers a contract to the seller which runs for all periods. The seller, after learning the parameter θ, accepts or rejects this contract. Up to this point the framework is identical to Baron and Besanko (1984). The main differences with Baron and Besanko (1984) and Battaglini (2005) are the focus on stationary mechanisms and imperfect commitment.

[3] The good q can be interpreted as quantity, or quality in the case of a single unit.

At each period, the agent, after receiving payment from the buyer, may decide not to deliver the good, in which case he pays for the breach of contract a (financial) penalty Π. This penalty is exogenously specified and is part of the contractual environment.

• TIMING. Without loss of generality we consider a direct mechanism $\{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ (see, for example, Baron and Besanko (1984)). The contracting game unfolds as follows:

(i) The agent learns the value of θ, which is his private information;

(ii) The principal offers a contract $\mathcal{C} = \{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ that runs for all periods of the relationship;

(iii) The agent accepts or rejects the contract, and if he accepts he reports his type $\hat{\theta}$, which may be different from the true type θ;

(iv) In each period τ, the agent, after receiving the per-period payment $t(\hat{\theta})$, decides to stay in the relationship or to walk away. In the latter case the agent pays Π for breach of contract and the relationship ends.[4]

By restricting attention to stationary mechanisms we avoid dependence of mechanisms on the history of play.[5]

• INCENTIVE COMPATIBILITY AND PARTICIPATION CONSTRAINTS. Because of full commitment, the Revelation Principle applies to our setting. We thus focus on direct, stationary mechanisms of the form $\{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta}\in\Theta}$. We denote the seller's per period-rent as

$$U(\theta) = t(\theta) - \theta q(\theta).$$

His (ex post) participation constraint can be written as

$$U(\theta) \geq 0 \qquad \forall\theta \in \Theta. \tag{2.1}$$

LEMMA 2.1 (Incentive compatibility and monotonicity). *Suppose the mechanism is truthful. Then q is non-increasing and a.e. differentiable with $\dot{q}(\theta) \leq 0$, and at any point where q and t are differentiable,*

$$\dot{t}(\theta) = \theta\,\dot{q}(\theta) \quad \textit{a.e. on}\ \Theta. \tag{2.2}$$

[4]Here the penalty Π is included in the formal description of the contract. Importantly, this amount is enforceable by courts. Thus, even if a large penalty is stipulated in the contract but cannot be enforced, we assume that the legal system is weak and the actual amount of Π is considerably smaller than the one designated in the formal contract.

[5]We comment on the stationarity assumption in Section 5.

This lemma can equivalently be expressed as saying that U is absolutely continuous with $\dot{U}(\theta) = -q(\theta)$ a.e., and U is convex.

DEFINITION 2.2 (Admissible contracts). A pair (t, q) is *admissible* if: (i) q is piecewise continuous and of bounded variation with $\dot{q}(\theta) \leq 0$ a.e.; (ii) t is absolutely continuous and (2.2) holds a.e.; and (iii) (2.1) is satisfied.

REMARK 2.3 (Continuity of the optimal schedule). Although admissibility allows for discontinuities, under Assumption 3.2 every *optimal* stationary quantity schedule is in fact continuous (and weakly decreasing); see Proposition 6.1 in Appendix 6. Thus, the structural statements in the main text (e.g., a single bunching interval) can be interpreted for continuous schedules without loss.

• ENFORCEMENT CONSTRAINT. Accounting for the agent's option to breach after receiving the period-τ payment,

$$\frac{1}{1-\delta} U(\theta) \;\geq\; \max_{\hat{\theta} \in \Theta,\, \tau \geq 0} \left\{ \sum_{s=0}^{\tau-1} \delta^s \big(t(\hat{\theta}) - \theta q(\hat{\theta})\big) \;+\; \delta^\tau \big(t(\hat{\theta}) - \Pi\big) \right\}. \tag{2.3}$$

LEMMA 2.4 (Reduction of enforcement). *An admissible (t, q) is enforceable iff*

$$t(\theta) - \theta q(\theta) \;\geq\; (1-\delta)\big(t(\underline{\theta}) - \Pi\big) \qquad \forall \theta \in \Theta. \tag{2.4}$$

When an agent engages in non-compliant behavior, his optimal strategy involves mimicking the most efficient type, securing the payment intended for such a type, followed by breach of contract. This strategy is reminiscent of the well-known *"take-the-money-and-run"* strategy that is found in models with spot contracting.[6] As shown in those models, deviations become a lesser concern when rewards for efficient types are moderated. Specifically, when the penalty for breaching the contract, Π, is moderate, the right-hand side of (2.4) is positive and the enforcement constraint significantly limits the set of implementable allocations. Reducing the payment $t(\underline{\theta})$ facilitates enforcement for the most efficient type. However, by incentive compatibility such distortion also requires the reduction of payments for all the least efficient types, potentially hampering production.

[6]See, for instance, Laffont and Tirole (1993) (Chapter 9).

Conversely, when the penalty for breach Π is sufficiently large, the right-hand side of (2.4) is negative and the enforcement constraint is implied by the participation constraint for the least efficient agent. This case essentially converges with the conventional framework of the optimal static contract.

3. Optimal Contract

The principal's problem is

$$(P): \quad \max_{\{(t,q) \text{ admissible}\}} \int_{\underline{\theta}}^{\bar{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

subject to (2.1) and (2.4).

We extend the classical second-best problem to include the enforcement constraint (2.4). The results of Baron and Besanko (1984) and Laffont and Tirole (1993) in a setting without an enforcement constraint (2.4) (or under the assumption of a very large penalty) involve the perpetual implementation of the optimal static contract $(q^{os}(\theta), t^{os}(\theta))$ (Baron and Myerson (1982)). Here $t^{os}(\theta) = \theta q^{os}(\theta) + \int_{\theta}^{\bar{\theta}} q^{os}(x) dx$, and $q^{os}(\theta)$ is determined by

$$S'\left(q^{os}(\theta)\right) = \theta + \frac{F(\theta)}{f(\theta)} \quad \forall \theta \in \Theta.$$

This equation represents the second-best optimality criterion: the buyer's marginal benefit equals the seller's *virtual cost* $\theta + \frac{F(\theta)}{f(\theta)}$. This virtual cost, exceeding the actual cost, reduces the information rent left to the agent by decreasing output. The following definition generalizes the optimal static contract.

DEFINITION 3.1. For any $r \geq 0$, define the generalized static output $q^{(r)}(\theta)$ as the solution to

$$S'\left(q^{(r)}(\theta)\right) = \theta + \frac{r + F(\theta)}{f(\theta)} \quad \forall \theta \in \Theta.$$

Note that $q^{os}(\theta) = q^{(0)}(\theta)$. When $r > 0$, the resulting output is lower than that stipulated by the optimal static contract. The following assumption is similar to the standard monotone hazard rate assumption, which guarantees that $q^{os}(\theta)$ is decreasing.

ASSUMPTION 3.2. (i) $\frac{r+F(\theta)}{f(\theta)}$ is an increasing function of θ for all $r \geq 0$.
(ii) The density function $f(\theta)$ is differentiable.

Under this assumption, the generalized static output is decreasing. Examples of distributions satisfying this assumption include any (weakly) decreasing hazard rate, such as uniform and exponential distributions.[7]

Next, we define the output which plays an important role in characterizing the optimal contract.

DEFINITION 3.3. For any $r \geq 0$, define the output $q_r(\theta)$ by

$$q_r(\theta) = \begin{cases} q^{(r)}(\theta_r) \text{ if } \theta < \theta_r, \\ q^{(r)}(\theta) \quad \text{if } \theta_r \leq \theta \leq \bar{\theta}, \end{cases} \tag{3.1}$$

where $\theta_r \in [\underline{\theta}, \overline{\theta}]$ is the (minimal) solution of

$$r = \frac{F^2(\theta_r)}{\theta_r f(\theta_r) - F(\theta_r)}, \tag{3.2}$$

if it exists, otherwise $\theta_r = \overline{\theta}$.

When $\underline{\theta} < \theta_r < \bar{\theta}$, the output exhibits bunching for types smaller than θ_r. For types greater than θ_r, the output aligns with the generalized static output (see Figure 1). When $r = 0$, the threshold θ_r is $\underline{\theta}$ and the corresponding output $q_r(\theta)$ is the optimal static contract. If no solution to (3.2) exists, then the output $q_r(\theta)$ consists of complete bunching at $q^{(r)}(\overline{\theta})$.

The transfers required to implement the output $q_r(\theta)$ are determined by

$$t_r(\theta) = \theta q_r(\theta) + \int_{\theta}^{\bar{\theta}} q_r(x)\,dx + \left(t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta})\right). \tag{3.3}$$

Let $r^* = \frac{1-\delta}{\delta}$.[8] The following lemma establishes the existence of the output $q_r(\theta)$ for all $r \in [0, r^*]$.

LEMMA 3.4. *For any $r \in [0, r^*]$, either the solution $\theta_r \leq \bar{\theta}$ to (3.2) exists or there exists a unique $\hat{r} \in [0, r^*]$ such that $r(\bar{\theta}) = \hat{r}$.*

[7] Relaxing this assumption does not change the main findings, though it does introduce additional notation. If $\frac{r+F(\theta)}{f(\theta)}$ increases over certain intervals, the optimal contract could involve bunching for intermediate types, similar to Guesnerie and Laffont (1984).

[8] It will be shown that r^* represents the maximal value for which the optimal output is defined.

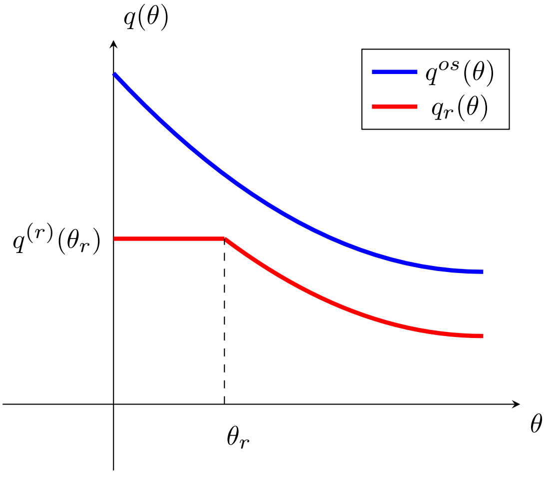

FIGURE 1. Optimal static contracts

In the left panel of Figure 2, equation (3.2) defines the locus of thresholds θ_r and r that determine the output $q_r(\theta)$. The function r as a function of θ_r increases and intersects the line r^* at $\theta_{r^*} < \overline{\theta}$. Thus, r^* determines the output $q_{r^*}(\theta)$ with non-trivial bunching for $\theta \leq \theta_{r^*}$ and separation for $\theta > \theta_{r^*}$. Conversely, if $\theta_{r^*} = \overline{\theta}$, then $q_{r^*}(\theta) = q^{(r^*)}\left(\overline{\theta}\right)$ for all θ.

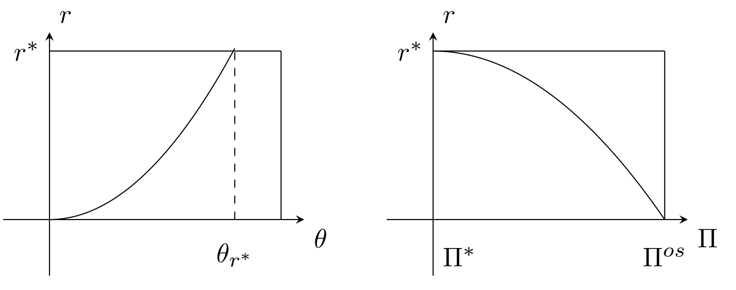

FIGURE 2. r as a function of θ_r and Π

In the right panel of Figure 2, each $r \in [0, r^*]$ corresponds to a penalty $\Pi \in [\Pi^*, \Pi^{os}]$ according to

$$\Pi = \underline{\theta} q_r(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} q_r(\theta)\, d\theta. \tag{3.4}$$

The function $r(\Pi)$ is decreasing. Furthermore, as indicated by (3.3), the right-hand side of (3.4) equals $t_r(\underline{\theta})$ provided that the rent of the least efficient type, $t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta})$, is zero. If this rent is positive, then the penalty Π is less than the transfer to the most efficient type.

Using (3.4), define the penalties Π^{os} and Π^* corresponding to $r = 0$ and $r = r^*$ respectively. The penalty Π^{os} defines the threshold beyond which the enforcement constraint is implied by the agent's participation constraint and, therefore, the corresponding output is the optimal static contract.

The optimal contract for the agent's enforcement problem can now be described.

THEOREM 3.5. *(i)* ***Strong enforcement.*** *For $\Pi \geq \Pi^{os}$, the optimal contract involves the infinite repetition of the optimal static contract.*

(ii) ***Weak enforcement.*** *For $\Pi^* \leq \Pi \leq \Pi^{os}$, the optimal contract is $\big(t_r(\theta), q_r(\theta)\big)$, where the parameter r is defined by*

$$t_r(\underline{\theta}) = \Pi. \tag{3.5}$$

Here, the least efficient type has zero rent, i.e., $t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta}) = 0$.

(iii) ***Very weak enforcement.*** *For $\Pi \leq \Pi^*$, the optimal contract is $\big(t(\theta), q_{r^*}(\theta)\big)$. The transfers $t(\theta)$ are given by*

$$t(\theta) = t_{r^*}(\theta) + \frac{1-\delta}{\delta}\left(\Pi^* - \Pi\right).$$

If $\Pi < \Pi^*$, then the least efficient type earns a positive rent,

$$t(\bar{\theta}) - \bar{\theta} q_{r^*}(\bar{\theta}) = \frac{1-\delta}{\delta}\left(\Pi^* - \Pi\right) > 0.$$

Theorem 3.5 partitions enforcement environments according to two thresholds, Π^{os} and Π^*. When $\Pi \geq \Pi^{os}$, the enforcement constraint is slack and the optimal stationary contract coincides with the repeated optimal static contract. When $\Pi^* \leq \Pi < \Pi^{os}$, the transfer cap binds at the top, forcing bunching among efficient types and generating downward distortions relative to q^{os}. When $\Pi < \Pi^*$, public enforcement is too weak to deter breach at the top, so compliance must be supported by continuation values, and the least efficient type earns a strictly positive rent.

WHY BUNCHING ARISES UNDER WEAK ENFORCEMENT. For $\Pi^* \leq \Pi < \Pi^{os}$, the enforcement cap binds at the top, so that $t(\underline{\theta}) = \Pi$. Reducing $t(\underline{\theta})$ lowers the gain from a "take-the-money-and-run" deviation, but incentive compatibility links transfers across types. As a result, types slightly less efficient than $\underline{\theta}$ are attracted to the allocation intended for the most efficient type. To mitigate these countervailing incentives, the principal optimally pools the most efficient types rather than raising $t(\underline{\theta})$, which would violate enforceability. This logic leads to bunching at the top of the type distribution.

For types above the bunching cutoff θ_r, the optimal allocation coincides with the generalized static rule. Relative to the standard virtual cost $\theta + \frac{F(\theta)}{f(\theta)}$, enforcement introduces an additional wedge $\frac{r}{f(\theta)}$, which can be interpreted as a *virtual enforcement cost.* This term disciplines information rents so that the induced transfers remain compatible with the enforcement constraint.

RENT DECOMPOSITION AND DEVIATION LOGIC. Under strong and weak enforcement ($\Pi \geq \Pi^*$), the least efficient type earns zero rent, $U(\overline{\theta}) = 0$, and the agent's rent is purely informational:

$$U(\theta) = \int_{\theta}^{\overline{\theta}} q_r(x)\, dx.$$

In the weak-enforcement region, a tighter cap (lower Π) compresses transfers at the top and therefore compresses information rents, weakening the incentive to imitate the top contract and then breach.

By contrast, when $\Pi < \Pi^*$, the maximal transfer exceeds the penalty, so any type can profitably breach unless additional rents are provided. To deter such deviations, the principal must leave a positive continuation payoff to the least efficient type, yielding the decomposition

$$U(\theta) = \underbrace{\frac{1-\delta}{\delta}(\Pi^* - \Pi)}_{\text{enforcement rent}} + \underbrace{\int_{\theta}^{\overline{\theta}} q_{r^*}(x)\, dx}_{\text{information rent}} .$$

The first term is a compliance premium required to sustain self-enforcement when public enforcement is weak; the second term is the standard information rent.

OPTIMALITY OF TOP POOLING. The optimality of bunching among efficient types follows from a standard positive-variation argument. Suppose that, for $\theta \leq \theta_r$, output strictly decreases, with $q_r(\underline{\theta}) > q_r(\theta_r)$. Consider any positive variation $dq_r(\theta)$ on $[\underline{\theta}, \theta_r]$ such that $dq_r(\underline{\theta}) = dq_r(\theta_r) = 0$ and the modified schedule remains decreasing. This variation preserves incentive compatibility and leaves $t(\underline{\theta})$ unchanged, so the enforcement constraint continues to bind. Since the variation moves the allocation closer to the optimal static contract, it strictly raises surplus. Hence, in the optimum, output must be flat over $[\underline{\theta}, \theta_r]$, implying $q_r(\underline{\theta}) = q_r(\theta_r)$.

CONNECTION TO RELATIONAL CONTRACTING. This logic connects to Levin (2003). Limited enforceability generates additional distortions relative to the second best and can induce pooling. In our model, public enforcement is still operative when $\Pi^* \leq \Pi < \Pi^{os}$, but its limited magnitude caps transfers and triggers top pooling; when $\Pi < \Pi^*$, compliance is sustained entirely by relational incentives through the continuation value.

4. PROOF OF THEOREM: REDUCTION TO TWO PROGRAMS

Rewriting constraints (2.1) and (2.4) respectively yields

$$t(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta, \tag{4.1}$$

$$t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta. \tag{4.2}$$

Constraint (4.2) includes, on the right-hand side, the transfer for the most efficient type $t(\underline{\theta})$, representing the seller's net gain in the event of breach. This right-hand side becomes non-positive when the net gain falls below the penalty, making the enforcement constraint ineffective. In this case, the problem becomes a second-best problem with a cap Π on transfers. If the net gain from any deviation is less than the penalty, the seller's overall benefit from breaching the contract is non-positive. Therefore, the seller has no incentive to breach as long as ex post rents are non-negative, which is assured if (4.1) holds.

When the seller's net gain from breach exceeds the penalty, the participation constraint follows from the enforcement constraint. The seller can breach, cover the penalty with the gain, and leave the relationship. To prevent breach, the contract must ensure that the seller obtains enough rent from continuation. This leads to two regimes of constraints:

$$(A): \begin{cases} t(\underline{\theta}) - \Pi \geq 0, \\ t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta, \end{cases}$$

and

$$(B): \begin{cases} t(\underline{\theta}) - \Pi \leq 0, \\ t(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta. \end{cases}$$

We split the problem (P) into two problems (P^A) and (P^B), obtained from (P) by replacing (4.1) and (4.2) by systems (A) and (B) respectively.

An optimal solution of (P) is necessarily optimal for either (P^A) or (P^B). Conversely, among the solutions to (P^A) and (P^B), the one that provides the highest payoff to the principal is the optimal solution of (P).

Program (P^A)*:* Consider first (P^A). To eliminate $t(\underline{\theta})$ from the right-hand side of (4.2), introduce the adjusted transfer $y(\theta)$: [9]

$$y(\theta) = t(\theta) - (1-\delta)\left(t(\underline{\theta}) - \Pi\right).$$

Then system (A) becomes

$$\begin{cases} y(\underline{\theta}) - \Pi \geq 0, \\ y(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta. \end{cases}$$

Note that the second constraint must bind at $\overline{\theta}$:

$$y(\overline{\theta}) - \overline{\theta} q(\overline{\theta}) = 0. \tag{4.3}$$

Indeed, if $y(\overline{\theta}) - \overline{\theta} q(\overline{\theta}) > 0$, consider the contract $\left(q(\theta)+\varepsilon, y(\theta)\right)$, for ε such that $y(\overline{\theta}) - \overline{\theta}(q(\overline{\theta})+\varepsilon) = 0$. This change does not affect feasibility and strictly increases the principal's payoff.

Problem (P^A) can thus be written as

$$\max_{\{y(\cdot),q(\cdot)\}\text{ admissible}} \int_{\underline{\theta}}^{\overline{\theta}} \left(S(q(\theta)) - y(\theta)\right) f(\theta) d\theta - \frac{1-\delta}{\delta}\left(y(\underline{\theta}) - \Pi\right)$$

subject to (4.3) and

$$y(\underline{\theta}) - \Pi \geq 0. \tag{4.4}$$

Problem (P^A) depends on Π, which enters both the objective and constraint (4.4). We treat (P^A) as an optimal control problem with boundary constraints (4.3)–(4.4) and a scrap value $-\frac{1-\delta}{\delta}\left(y(\underline{\theta}) - \Pi\right)$.

LEMMA 4.1. *(i) When* $\Pi \in [\Pi^*, \Pi^{os}]$*, the optimal output for* (P^A) *is* $q_r(\theta)$ *with* r *determined by*

$$\theta_r q^{(r)}(\theta_r) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}(\theta)\, d\theta = \Pi. \tag{4.5}$$

[9]Notice that $y(\underline{\theta}) = \delta t(\underline{\theta}) + (1-\delta)\Pi$ and $t(\underline{\theta}) = \frac{y(\underline{\theta}) - (1-\delta)\Pi}{\delta}$. Thus $t(\underline{\theta}) = \Pi$ if and only if $y(\underline{\theta}) = \Pi$.

Furthermore, constraint (4.4) *is binding:* $y(\underline{\theta}) = \Pi$.

(ii) For $\Pi < \Pi^*$*, the optimal output is* $q_{r^*}(\theta)$ *and* (4.4) *is slack:* $y(\underline{\theta}) > \Pi$.

In Case 1, $t(\underline{\theta}) - \Pi \geq 0$ is binding. Enforcement is not an issue and the optimal contract does not depend on δ. Case 2, of very weak enforcement, arises when $\Pi < \Pi^*$. In this case, the output is fixed at $q_{r^*}(\theta)$. Efficiency cannot be further compromised in favor of enforcement. Given the minimal penalty, enforcement relies on the discount factor, reflected in $r^* = \frac{1-\delta}{\delta}$.

Program (P^B)*:* Now consider

$$(P^B): \quad \max_{\{t(\cdot),q(\cdot)\} \text{ admissible}} \int_{\underline{\theta}}^{\bar{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

subject to (4.1) and

$$\Pi - t(\underline{\theta}) \geq 0. \tag{4.6}$$

Problem (P^B) is a second-best problem with a cap on transfers. It does not depend on δ. There are two differences between (P^B) and (P^A): there is no scrap value in the objective of (P^B), and constraint (4.6) is the reverse of (4.4). The optimal static contract describes the maximum payoff in the presence of the enforcement constraint. Therefore, when $\Pi \geq \Pi^{os}$ the optimal enforcement contract is simply the optimal static contract.

When $\Pi \leq \Pi^{os}$, constraint $\Pi - t(\underline{\theta}) \geq 0$ is binding. Thus, for $\Pi \in [\Pi^*, \Pi^{os}]$ the optimal outputs and transfers are identical in (P^A) and (P^B).

LEMMA 4.2. *(i) If* $\Pi \leq \Pi^{os}$*, the optimal output is* $q_r(\theta)$*, where* r *is defined by* (4.5)*; in addition,* (4.6) *is binding:* $t(\underline{\theta}) = \Pi$.

(ii) If $\Pi > \Pi^{os}$*, the optimal output is the static second-best contract, and* (4.6) *is strict:* $t(\underline{\theta}) < \Pi$.

COMPARISON BETWEEN (P^A) AND (P^B): Intuitively, for small Π, the value derived from (P^A) exceeds that of (P^B), and the reverse holds as Π becomes large. Specifically, for $\Pi \geq \Pi^{os}$, (P^B) has the static second-best contract as its solution. For (P^A), the constraint on $t(\underline{\theta})$ distorts transfers away from the optimal static contract. The seller's gain in the event of breach, $t(\underline{\theta})$, must at least match the large penalty, leading to an upward distortion of $t^{os}(\underline{\theta})$. Consequently, for $\Pi > \Pi^{os}$, the value of (P^B) exceeds that of (P^A) and the optimal contract is the static second-best.

For $\Pi \in [\Pi^*, \Pi^{os}]$, both (P^A) and (P^B) share the same necessary and sufficient conditions and, therefore, the same solutions. Given that $y(\underline{\theta}) - \Pi$ is binding, the scrap value in the objective of (P^A) is zero. Hence, both programs have the same value.

For any $\Pi < \Pi^*$, consider the optimal contract $\big(t(\theta), q(\theta)\big)$ for (P^B). By construction, $t(\underline{\theta}) = \Pi$, so this contract is feasible for (P^A) and attains the same objective value as in (P^B). Therefore the solution to (P^A) is also optimal for (P).

PROPOSITION 4.3. *(i) For $\Pi < \Pi^*$, the solution to (P^A) specified in Lemma 4.1 is optimal for (P).*

(ii) For $\Pi \in [\Pi^*, \Pi^{os}]$, both programs have the same value and yield the same optimal contract.

(iii) For $\Pi > \Pi^{os}$, the solution to (P^B) specified in Lemma 4.2 is optimal for (P).

5. DISCUSSION

We have characterized the optimal stationary contract when the agent can commit ex ante but cannot commit not to default ex post. The optimal allocation is pinned down by the interaction between public enforcement (the breach penalty Π) and private enforcement (the continuation value governed by δ).

The model delivers regime-dependent comparative statics that map naturally into observable contract terms in procurement, construction, and trade (e.g., cash-in-advance, progress payments, retention, performance bonds, and relationship-based enforcement). Two primitives have empirical counterparts: public enforcement Π (court effectiveness, recoverability of damages, enforceable penalties) and private enforcement δ (relationship value driven by repeat trade, reputational capital, switching costs, or platform ratings).

DECOMPOSITION OF RENTS. As discussed after Theorem 3.5, incentive compatibility implies the envelope condition $\dot{U}(\theta) = -q(\theta)$ a.e., hence

$$U(\theta) = U(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(x)\, dx. \tag{5.1}$$

The regime-dependent term is $U(\bar{\theta})$.

- **Strong and Weak Enforcement ($\Pi \geq \Pi^*$).** In these regimes the least efficient type earns zero rent, $U(\bar{\theta}) = 0$. Thus the agent's rent is purely informational:

$$U(\theta) = \int_{\theta}^{\bar{\theta}} q_r(x)\,dx,$$

  with $r = 0$ under strong enforcement ($\Pi \geq \Pi^{os}$) and $r = r(\Pi) \in (0, r^*(\delta)]$ under weak enforcement ($\Pi^* \leq \Pi \leq \Pi^{os}$). A higher penalty Π relaxes the transfer cap and reduces distortions (lower r), thereby increasing information rents.

- **Very Weak Enforcement ($\Pi < \Pi^*$).** Here public enforcement is insufficient to deter breach at the top; compliance must be supported by relational incentives. The optimal quantity schedule is $q_{r^*}(\theta)$ with $r^*(\delta) = \frac{1-\delta}{\delta}$, and the principal must leave a strictly positive *enforcement rent* to the least efficient type:

$$U(\bar{\theta}) = \frac{1-\delta}{\delta}\big(\Pi^* - \Pi\big) > 0.$$

  Using (5.1), the agent's rent decomposes into

$$U(\theta) = \underbrace{\frac{1-\delta}{\delta}\big(\Pi^* - \Pi\big)}_{\text{Enforcement rent}} + \underbrace{\int_{\theta}^{\bar{\theta}} q_{r^*}(x)\,dx}_{\text{Information rent}}.$$

  Holding δ fixed, raising Π reduces the enforcement rent one-for-one while leaving q_{r^*} unchanged; thus the agent is strictly worse off as Π increases within this regime, until Π reaches Π^*.

This yields the following comparative statics.[10]

COROLLARY 5.1 (Institutional improvements and surplus). *Consider the optimal stationary contract at* (Π, δ).

(i) ***(Output and bunching.)*** *For* $\Pi \geq \Pi^{os}$, $q = q^{os}$ *(no bunching) and is independent of* δ. *For* $\Pi^*(\delta) \leq \Pi \leq \Pi^{os}$, *the optimal contract is* (t_r, q_r) *with* $r = r(\Pi)$ *decreasing in* Π, *so bunching weakly decreases with* Π. *For* $\Pi < \Pi^*(\delta)$, *the optimal quantity is* $q_{r^*(\delta)}$, *independent of* Π, *and* $r^*(\delta) = \frac{1-\delta}{\delta}$ *decreases in* δ, *implying less distortion as* δ *rises. Moreover,* $\Pi^*(\delta)$ *is increasing in* δ.

(ii) ***(Principal payoff.)*** *The principal's expected payoff is weakly increasing in* Π *and weakly increasing in* δ.

[10] Recall that Π^* depends on δ through $r^* = \frac{1-\delta}{\delta}$. We write $\Pi^* = \Pi^*(\delta)$ emphasizing this dependence.

(iii) ***(Agent payoff.)*** *Holding δ fixed, the agent's expected payoff is non-monotone in Π: it strictly decreases with Π on $\Pi < \Pi^*(\delta)$ (reduced enforcement rents), and weakly increases with Π on $\Pi^*(\delta) \le \Pi \le \Pi^{os}$ (higher information rents), and is constant for $\Pi \ge \Pi^{os}$. Holding Π fixed, the effect of δ on the agent's expected payoff is* locally zero *in the Strong and Weak regimes (i.e., for changes in δ that do not move the economy across the threshold $\Pi^*(\delta)$), and is generically ambiguous in the Very Weak regime.*[11]

Parts (1)–(2) follow because increasing Π or δ relaxes the enforcement constraint, expanding the feasible set and allowing the principal to do weakly better. For the agent, (5.1) implies that changes in $U(\bar{\theta})$ shift all rents pointwise. In the very weak regime, $q = q_{r^*}$ is independent of Π while $U(\bar{\theta}) = \frac{1-\delta}{\delta}(\Pi^*(\delta) - \Pi)$, so increasing Π reduces $U(\bar{\theta})$ and hence $U(\theta)$ for all types. In the weak regime, $U(\bar{\theta}) = 0$ and increasing Π lowers r, increasing q_r pointwise and therefore increasing information rents $\int_\theta^{\bar{\theta}} q_r(x)\,dx$.

Finally, our analysis focuses on stationary contracts. Future research might relax this assumption. Yet stationarity is a robust benchmark: when enforcement is costless, Baron and Besanko (1984) obtain a stationary optimum, and stationary terms are also prevalent in practice (e.g., franchising royalties; Lafontaine and Shaw (1999)).

6. Appendix

Proof of Lemma 2.4. Multiply both sides of (2.3) by $(1-\delta)$ and write $U(\hat{\theta};\theta) := t(\hat{\theta}) - \theta q(\hat{\theta})$. For any fixed report $\hat{\theta}$ and any integer $\tau \ge 0$,

$$(1-\delta)\sum_{s=0}^{\tau-1} \delta^s\, U(\hat{\theta};\theta) + (1-\delta)\delta^\tau\big(t(\hat{\theta}) - \Pi\big) = (1-x)\,U(\hat{\theta};\theta) + (1-\delta)x\big(t(\hat{\theta}) - \Pi\big),$$

where $x := \delta^\tau \in [0,1]$. For fixed $\hat{\theta}$ this expression is affine in x, hence its maximum over $x \in [0,1]$ is attained at an endpoint $x \in \{0,1\}$ and is worth:

$$\max\Big\{U(\hat{\theta};\theta),\ (1-\delta)\big(t(\hat{\theta}) - \Pi\big)\Big\}.$$

Taking now the maximum over $\hat{\theta} \in \Theta$ yields

$$(1-\delta)\cdot \text{RHS of (2.3)} = \max\Big\{\underbrace{\max_{\hat{\theta}} U(\hat{\theta};\theta)}_{=U(\theta)\text{ by IC}},\ \underbrace{(1-\delta)\max_{\hat{\theta}} t(\hat{\theta})}_{=(1-\delta)\,t(\underline{\theta})} - (1-\delta)\Pi\Big\}.$$

[11] Higher δ improves efficiency but reduces enforcement rents.

The equality $\max_{\hat{\theta}} U(\hat{\theta};\theta) = U(\theta)$ uses truthfulness (Lemma 2.1). Since q is non-increasing and $\dot{t} = \theta\dot{q} \leq 0$ a.e., t is weakly decreasing in θ, so $\max_{\hat{\theta}} t(\hat{\theta}) = t(\underline{\theta})$. Therefore, (2.3) is equivalent to

$$U(\theta) \geq \max\Big\{U(\theta),\ (1-\delta)\big(t(\underline{\theta}) - \Pi\big)\Big\} \quad \forall\theta,$$

which is in turn equivalent to (2.4). □

PROOF OF LEMMA 3.4. Note first that (3.2) yields $r(\underline{\theta}) = 0$. Second, the denominator $\theta f(\theta) - F(\theta)$ is positive at $\theta = \underline{\theta}$. Consider an interval $[\underline{\theta}, \varepsilon]$ such that this denominator is positive for all $\theta \in [\underline{\theta}, \varepsilon]$. Differentiating (3.2) with respect to θ gives $r'(\theta) > 0$ for all $\theta \in [\underline{\theta}, \varepsilon]$. Indeed,

$$r'(\theta_r)\left(\theta_r - \frac{F(\theta_r)}{f(\theta_r)}\right) = 2F(\theta_r) f(\theta_r) - r\theta_r f'(\theta_r) = 2F(\theta_r) f(\theta_r) - \frac{F^2(\theta_r) + rF(\theta_r)}{f(\theta_r)} f'(\theta_r).$$

The numerator on the right-hand side is

$$2F(\theta_r) f^2(\theta_r) - F^2(\theta_r) f'(\theta_r) - rF(\theta_r) f'(\theta_r) = F(\theta_r)\Big(2f^2(\theta_r) - F(\theta_r) f'(\theta_r) - rf'(\theta_r)\Big) > 0,$$

where the last inequality follows from Assumption 3.2. Proceeding by extending the interval step by step, either we reach some θ' such that $\theta' f(\theta') - F(\theta') = 0$, or for all $\theta \in [\underline{\theta}, \overline{\theta}]$ we have $\theta f(\theta) - F(\theta) > 0$. In the first case, for all $r \geq 0$ there exists a unique $\theta_r \in [\underline{\theta}, \overline{\theta}]$ such that $r = \frac{F^2(\theta_r)}{\theta_r f(\theta_r) - F(\theta_r)}$. In the second case, we can assume that $\theta_r = \overline{\theta}$ for all $r \geq \widehat{r} = r(\overline{\theta})$. □

PROOF OF LEMMA 4.1. *Optimality conditions for problem* (P^A). We explicitly incorporate monotonicity by introducing $z(\theta) = \dot{q}(\theta)$. Problem (P^A) is then formulated as an optimal control problem with state variables $y(\theta)$ and $q(\theta)$ and control $z(\theta)$. The co-state variables corresponding to (6.2) and (6.3) are denoted by $\lambda_1(\theta)$ and $\lambda_2(\theta)$ respectively.

The Hamiltonian is

$$H(y, q, z, \lambda_1, \lambda_2, \theta) = (S(q) - y) f(\theta) + \lambda_1 \theta z + \lambda_2 z,$$

which is concave in (y, q, z) for all θ. Let $\big(y(\theta), q(\theta), z(\theta)\big)$ be an admissible triplet with continuous, a.e. differentiable $y(\theta), q(\theta)$ and piecewise continuous $z(\theta)$. Then $\big(y(\theta), q(\theta), z(\theta)\big)$ is optimal if and only if there exist continuous, piecewise differentiable co-state variables $(\lambda_1(\theta), \lambda_2(\theta))$ such that the following conditions (see Seierstad and Sydsaeter (1986), pp. 85, 396) are satisfied:

$$z(\theta) \in \arg\max_z H\big(y(\theta), q(\theta), z, \lambda_1(\theta), \lambda_2(\theta), \theta\big) \quad \forall\theta, \tag{6.1}$$

$$\dot{y}(\theta) = \theta z(\theta), \tag{6.2}$$

$$\dot{q}(\theta) = z(\theta) \leq 0, \tag{6.3}$$

$$\dot{\lambda}_1(\theta) = f(\theta) \quad \text{a.e.}, \tag{6.4}$$

$$\dot{\lambda}_2(\theta) = -S'\big(q(\theta)\big) f(\theta) \quad \text{a.e.}, \tag{6.5}$$

$$\lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta(1-\delta), \quad \lambda_2(\underline{\theta}) = 0, \quad \beta \geq 0 \; (= 0 \text{ if } y(\underline{\theta}) - \Pi > 0), \tag{6.6}$$

$$\lambda_1(\bar{\theta}) = \gamma, \quad \lambda_2(\bar{\theta}) = -\gamma\bar{\theta}, \quad \gamma \geq 0. \tag{6.7}$$

Denote

$$r = \lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta(1-\delta) = \frac{(1-\delta)(1-\beta\delta)}{\delta}. \tag{6.8}$$

Conditions (6.2), (6.3) and (6.4)–(6.6) imply

$$\lambda_1(\theta) = r + F(\theta), \tag{6.9}$$

and

$$\lambda_2(\theta) = -\int_{\underline{\theta}}^{\theta} S'\big(q(u)\big) f(u)\, du. \tag{6.10}$$

Define $\psi(\theta) = \lambda_1(\theta)\theta + \lambda_2(\theta)$. From (6.9)–(6.10) we get

$$\psi(\theta) = \big(r + F(\theta)\big)\theta - \int_{\underline{\theta}}^{\theta} S'\big(q(u)\big) f(u)\, du.$$

Optimality condition (6.1) yields

$$\psi(\theta) z(\theta) = 0, \qquad \psi(\theta) \geq 0 \quad \forall\theta. \tag{6.11}$$

Derivation of r^*. Note that

$$\psi(\underline{\theta}) = r\,\underline{\theta} \geq 0. \tag{6.12}$$

Thus $r \geq 0$. From (6.8), the minimal value $r = 0$ corresponds to $\beta = 1/\delta > 0$, and the maximal value $r^* = \frac{1-\delta}{\delta}$ corresponds to $\beta = 0$.

The form of optimal $q(\theta)$. If $\psi(\theta) > 0$ over a non-degenerate interval, then $z(\theta) = 0$ on this interval, rendering both state variables $q(\theta)$ and $y(\theta)$ constant due to (6.2) and (6.3). Conversely, if $\psi(\theta) = 0$ over a non-degenerate interval Θ' it follows that $\dot{\psi}(\theta) = 0$, leading to

$$S'\big(q(\theta)\big) = \theta + \frac{F(\theta) + r}{f(\theta)} \quad \text{for all } \theta \in \Theta'.$$

This condition implies that $q(\theta) = q^{(r)}(\theta)$ for all $\theta \in \Theta'$.

We now show that there can be at most one bunching interval. Suppose, to the contrary, that there are two disjoint bunching intervals, denoted $\Theta_1 = [\underline{\theta}, \theta_1)$ and $\Theta_3 = (\theta_2, \overline{\theta}]$.[12]

Within the intermediate interval $\Theta_2 = (\theta_1, \theta_2)$, we have $\psi(\theta) = 0$, so $q(\theta) = q^{(r)}(\theta)$ for all $\theta \in \Theta_2$. For all $\theta \in \Theta_1$, $q(\theta) = q^{(r)}(\theta_1) = q_1$, and for all $\theta \in \Theta_3$, $q(\theta) = q^{(r)}(\theta_2) = q_3$. If Θ_3 were nonempty, then $q_3 > q^{(r)}(\theta)$ for all $\theta \in \Theta_3$, and hence $S'\big(q_3\big) < S'\big(q^{(r)}(\theta)\big)$. Thus,

$$\dot{\psi}(\theta) = r + F(\theta) + f(\theta)\theta - S'(q_3)f(\theta) > r + F(\theta) + f(\theta)\theta - S'\big(q^{(r)}(\theta)\big)f(\theta) = 0$$

for all $\theta \in \Theta_3$. Since $\psi(\theta_2) = 0$ and, by the boundary condition, $\psi(\overline{\theta}) = 0$, this is a contradiction. Thus there may be only one bunching interval $[\underline{\theta}, \theta_1]$.

Derivation of (3.2). The first equation to determine θ_1 and r is $\psi(\theta_1) = 0$:

$$0 = \big(r + F(\theta_1)\big)\theta_1 - \int_{\underline{\theta}}^{\theta_1} S'\big(q(u)\big)f(u)\,du. \tag{6.13}$$

Since $q(u) = q_1 = q^{(r)}(\theta_1)$ for $u \in [\underline{\theta}, \theta_1]$, and $S'(q_1) = \theta_1 + \frac{r+F(\theta_1)}{f(\theta_1)}$, we obtain

$$r\theta_1 = \big(r + F(\theta_1)\big)\frac{F(\theta_1)}{f(\theta_1)}, \tag{6.14}$$

which is equivalent to (3.2).

Construction of the optimal contract. We have established that the optimal output has the form in Figure 1 with $\theta_1 = \theta_r$.

[12] Note that θ_1 may coincide with $\underline{\theta}$.

For all $\Pi \geq \Pi^{os}$ we have $r = 0$. Hence (4.5) has only the trivial solution $\theta_r = \underline{\theta}$ and the optimal output is the static second-best contract. In this case, $\beta = \frac{1}{\delta} > 0$, so (4.4) binds and $y(\underline{\theta}) = \Pi^{os}$.

For all $\Pi \in [\Pi^*, \Pi^{os}]$, constraint (4.4) binds so $y(\underline{\theta}) = \Pi$ and $t(\underline{\theta}) = \Pi$. Define r via

$$\theta_r q^{(r)}(\theta_r) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}(\theta) d\theta = \Pi.$$

Differentiating with respect to r gives $\Pi'(r) < 0$, so r is uniquely determined for $\Pi \in [\Pi^*, \Pi^{os}]$.

For $\Pi < \Pi^*$, constraint (4.4) is slack and the solution is the same as at Π^*. □

PROOF OF LEMMA 4.2. Modulo the change of variables, the optimality conditions (6.1)–(6.5) are the same as for (P^A). The transversality condition (6.6) is replaced by

$$\lambda_1(\underline{\theta}) = \beta'(1-\delta), \quad \lambda_2(\underline{\theta}) = 0, \quad \beta' \geq 0 \ (= 0 \text{ if } \Pi - t(\underline{\theta}) > 0).$$

In this case the parameter $r = \beta'(1-\delta)$ is unbounded. If $r = 0$, then the optimal output is the static second-best contract and the corresponding transfers are $t_0(\theta)$ defined by (3.3) with zero rent for $\overline{\theta}$. If $r > 0$, then the optimal output is $q_r(\theta)$ and transfers are given by (3.3) with zero rent for $\overline{\theta}$. □

PROPOSITION 6.1 (Continuity). *Under Assumption 3.2, every optimal stationary solution* (t, q) *has a quantity schedule* $q : \Theta \to \mathbb{R}_+$ *that is continuous and weakly decreasing.*

PROOF. Let (t, q) be an optimal stationary admissible contract.

Step 1 (Weak decrease). By Lemma 2.1, truthfulness implies that q is weakly decreasing (and a.e. differentiable with $\dot{q}(\theta) \leq 0$ a.e.).

Step 2 (Continuity). We prove that q has no jumps. Consider Program (P^A) (the only case in which the pointwise constraint is potentially binding over a nontrivial set of types). Recall the adjusted transfer

$$y(\theta) = t(\theta) - (1-\delta)\big(t(\underline{\theta}) - \Pi\big),$$

so that the per-type feasibility constraint in (P^A) is

$$y(\theta) - \theta q(\theta) \geq 0 \qquad \forall \theta \in \Theta. \tag{6.15}$$

Define the associated (adjusted) rent

$$\widetilde{U}(\theta) := y(\theta) - \theta q(\theta).$$

Since y differs from t by a constant, Lemma 2.1 implies that $\widetilde{U}$ is absolutely continuous and satisfies the envelope condition

$$\dot{\widetilde{U}}(\theta) = -q(\theta) \qquad \text{a.e. on } \Theta. \tag{6.16}$$

Moreover, (6.15) is a *linear* (hence concave) pointwise mixed constraint in $(\widetilde{U}, q)$, while the objective integrand in (P^A),

$$(S(q(\theta)) - y(\theta))f(\theta) = (S(q(\theta)) - \theta q(\theta) - \widetilde{U}(\theta))f(\theta),$$

is concave in q (because S is strictly concave) and linear in $\widetilde{U}$. The remaining term in the objective of (P^A) is a linear endpoint (scrap-value) term in $y(\underline{\theta})$ and therefore does not affect the local concavity argument.

Thus (P^A) is a concave optimal control problem with a pointwise concave (mixed) constraint of exactly the form studied in Simons (2025). In particular, Simons (2025) prove that, under concavity of the objective and pointwise concavity of the mixed constraint, optimal solutions cannot exhibit jumps in the quantity schedule. Therefore, the optimal $q(\cdot)$ is continuous on Θ. □

■

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