WORKING PAPERS

N° 1702

January 2026

"How do upstream competition and supply shocks affect investment decisions?"

Benoit Chevalier-Roignant and Stéphane Villeneuve

Toulouse School of Economics

How do upstream competition and supply shocks affect investment decisions?

Benoît **Chevalier-Roignant** (emlyon)

Stéphane **Villeneuve** (Toulouse School of Economics)

February 2, 2026

Abstract

We study the effect of upstream competition and supply shocks on a buyer's investment decisions, under demand uncertainty. Imperfect upstream competition leads to double marginalization. This effect is mitigated if the supplier pool is larger (when production costs are linear or in case of diseconomies of scale): The resulting lower equilibrium input price ultimately benefits the buyer and makes it more likely to invest sooner. A supply shock—that shrinks the supplier base—may increase the market power of the remaining suppliers and exacerbate double marginalization. Such a shock may arise either exogenously (due to a sudden external event) or endogenously (when profitability upstream is reduced). An exogenous shock, which leads to higher input prices and lower order quantities, reduces the profitability of the buyer, which is then less inclined to invest if more suppliers are affected by it. When the shock arises endogenously, the buyer may be better off and invest sooner if it subsidizes its supplier base as a way to maintain more competition upstream.

Keywords: Supply shock, supply chain, real options.

Acknowledgement. The authors thank Vlad Babich, Opher Baron, Onur Boyabatli, Jiri Chod, Andrea Gamba, Elise Gourier, Haresh Gurnani, Arnd Huchzermeier, Dan Iancu, Panos Kouvelis, Nicos Koussis, Florian Lücker, Nikolay Osadchiy, Richard Ruble, Nicola Secomandi, ManMohan Sodhi, Jean Tirole, Lenos Trigeorgis, Jan Van Mieghem, David Wuttke, and Alex Yang for useful feedback on previous versions of this work. This work has been presented at the

Supply Chain Finance & Risk Management Workshop at Washington University, Saint Louis, Missouri, and the International Real Options Conference. Stéphane Villeneuve acknowledges funding from FDR-SCOR *chaire Marché des risques et création de valeur* and ANR under grant ANR-17-EUR-0010 (Investissements d'Avenir program).

1 Introduction

There are instances where demand materializes, but investments are delayed, not due to the uncertainty inherent to the end market, but due to inefficiencies in how the supplier market is organized. Within the agribusiness, the overall adoption of alternative proteins is affected by such upstream inefficiencies: While the market for alternative proteins is expected to grow an annual 3.7% compound rate from 2022 to 2027, producers face challenges "securing enough high-quality raw materials at competitive prices" and mitigating "shortages [...] caused by extreme weather conditions and soil degradations as well as more recently by COVID-19 and war-related supply chain disruptions."[1] More generally, global supply chains face new challenges due to climate change and other major developments such as rising protectionism, the weaponization of trade, regional conflicts, sanctions, and the erosion of global institutions. Understanding the functioning of these supply chains and anticipating changes that affect them are essential, also from an investment perspective.

We propose a stylized modeling framework to study these issues expanding the classical real options theory (e.g., Dixit and Pindyck 1994, Trigeorgis 1996) by microfounding firm profits that reflect imperfect upstream competition and by considering the impact of supply shocks—besides end demand uncertainty—on a firm's investment decisions. Our framework contributes to the literature at the interface of finance, operations, and risk management (iFORM) and explains how supply uncertainties lead to delayed investments. We stress two key mechanisms at play. First, reduced *upstream competition* can lead to increased input prices for buyers and eventually to higher prices for finished goods, which depresses the demand of the end customers and consequently affects the entire supply chain. As a consequence, downstream companies may face lower profitability and reconsider their investment decisions altogether. These effects are expected to be particularly severe for inputs that are already in limited supply. For instance, the current fertilizer shortage has strongly affected the alternative protein market.[2] Second, firms may be subject to *supply shocks*—that may arise exogenously due to operational contingencies, natural hazards (e.g. lockdowns during Covid 19, 2021 Suez canal obstruction, Panama canal drought), terrorism (e.g., Houthi attacks on commercial vessels in Nov. 2023) and political instability (Kleindorfer and Saad 2005) or endogenously if

[1] www.ey.com/en_gl/insights/strategy/how-alternative-proteins-are-reshaping-meat-industries
[2] www.ey.com/en_gl/insights/strategy/how-alternative-proteins-are-reshaping-meat-industries

certain suppliers face financial distress and become unreliable. As firms face challenges in adjusting their sourcing strategy after supply shocks (Cohen et al. 2018), the upstream market becomes less competitive and the remaining suppliers exploit the new circumstances to wield more market power.

Our framework helps us articulate key, novel managerial insights based on a set of stylized models, all solved analytically (with proofs provided in an e-companion).

We first study how the degree of imperfect *upstream competition* affects the equilibrium profit of a monopsonic buyer, assuming uncertainty about end demand. If suppliers and buyers make pricing or quantity decisions in a decentralized supply chain, they pursue their own interests to achieve higher margins, typically leading to double marginalization (Spengler 1950, Tirole 1988). In our setting, the effects brought about by more upstream competition (understood as an increase in the number of suppliers) critically depend on the degree of (dis)economies the suppliers can achieve. In the cases where the suppliers (i) have linear production costs or (ii) face scale diseconomies, more upstream competition improves the buyer's profitability (see Propositions 1 and 3). Intuitively, as upstream competition intensifies, the effect of double marginalization is mitigated as suppliers collectively lose market power, to the buyer's benefit. In contrast, if (iii) economies of scale can be achieved, one supplier monopolizes the input market (see Proposition 2). If (ii) there are diseconomies of scale, the buyer may actually prefer to source from a large set of suppliers, despite reduced but still significant double marginalization, than to vertically integrate (see Proposition 4). Essentially, despite the markups charged by the suppliers onto the buyer, dispatching production among a large set of suppliers makes it possible for the buyer to avoid the diseconomies of scale that would arise if the buyer were vertically integrated. For (i) linear costs or (ii) scale diseconomies, more upstream competition makes the buyer invest earlier (see Propositions 5 and 6), while the buyer is effectively indifferent about the (potential) size of the supplier base if there are (iii) economies of scale, investing above a cutoff demand level corresponding to a setting with a monopolistic supplier.

If the input market is not already monopolized (i.e., if there are no economies of scale), a *supply shock* may exacerbate double marginalization, as the suppliers who remain after the event wield more market power and charge a higher markup. We consider and model two cases: A supply shock can either result from some exogenous event affecting a subset of one's supplier base (e.g., a disease affecting crops) or be the outcome of an endogenous decision by some suppliers to temporarily cease operations should the changing economic environment invalidate current operations (e.g., farmers deciding to stop operations in view of current low produce prices and high fixed costs). The buyer develops rational expectations about these shocks, internalizes the fragility of supply, and revises its investment strategy accordingly. If more suppliers may be affected by the exogenous supply shock, the value of operating in this market decreases for the buyer (see Proposition 7). The buyer will decide based on whether the shock has or will arrive and how many suppliers have been or are likely to be impacted (see Proposition 8). The buyer will invest if the end demand exceeds a cutoff level (see Proposition 9), sufficiently large to compensate for the additional supply risk. If more suppliers may be affected by the exogenous disruption, the buyer will increase this cutoff level (see Proposition 10). Again, a supply shock can have endogenous roots when a supplier faces a profitability challenge. In this case, regardless of any fair trade considerations, the downstream firm may have an incentive to financially support weaker suppliers (see Proposition 11): It does so under certain circumstances but not at all times—to sustain more competition upstream and to mitigate double marginalization. This potential intervention generates value and makes the buyer invest earlier (see Proposition 12).

2 Literature review

The optimal time at which to make (partly) irreversible decisions is at the core of real options theory (Dixit and Pindyck 1994, Trigeorgis 1996). Following Chevalier-Roignant et al. (2011, Sect. 3.2) and Trigeorgis and Tsekrekos (2018, Theme D), supply-chain interactions are, however, often ignored in this literature. Exceptions include Moon et al. (2011) (who determine the times at which to sell and buy in a supply chain subject to uncertain revenues and costs), Billette De Villemeur et al. (2014) (who study the timing decision of the buyer who purchases a key equipment from a supplier at its investment time), and Chevalier-Roignant et al. (2025) (who study the mechanism through which a supplier and a buyer reach a time at which to invest concomitantly). Contributing to this literature, our paper provides a microfoundation to the equilibrium profits across the supply chain and studies the impact of more intense horizontal competition at the suppliers' echelon and of supply shocks on a buyer's investment decision.

Our manuscript also relates to two streams of the literature on operations management:

Supplier-buyer relationships. Spengler (1950) introduced a simple model of supply chain relationship with "double marginalization," which has become seminal. Greenhut and Ohta (1979), Salinger (1988), Corbett and Karmarkar (2001), and Huang et al. (2016) (to name a few) have generalized this supply chain model, modeling successive Cournot oligopolies with deterministic demand, while Huberts et al. (2025) consider double marginalization arising in an investment context with a financier wielding market power. De Wolf and Smeers (1997) and DeMiguel and Xu (2009) consider variants in which the inverse demand function in the downstream market is subject to randomness and in which firms taking the role of Stackelberg leader or follower compete over quantities. In a related spirit, Gurnani and Gerchak (2007) consider the downstream firm as a Stackelberg leader, while multiple component suppliers act as Stackelberg followers and compete among themselves in a Cournot fashion. We consider a Cournot-Nash game among suppliers nested in a two-echelon supply chain game (with suppliers as leaders and a monopsonic buyer as follower). We leverage tractable demand and cost specifications and study the impact of a change in the size of the supplier base on the buyer's equilibrium profit, an impact which critically depends on the degree of (dis)economies of scale of the suppliers' production technology. We allow for uncertainty in demand in the output market and focus on the impact of changes in the supplier chain configurations (e.g., in terms of numbers of suppliers and supply shock) on a buyer's investment timing decision. We also briefly discuss the buyer's incentive to vertically integrate, in that respect modestly contributing to the rich literature on vertical integration (e.g., Salinger 1988, Corbett and Karmarkar 2001, Fang et al. 2023, Jullien et al. 2023).

Supply shocks. Supply shocks have dramatically affected global trade and have become topical in scholarly research of late (see the reviews by Gurnani et al. 2013 and Lücker et al. 2024). This research theme has been understood and studied from various perspectives (Sodhi et al. 2012). For example, the literature has studied the incentive to diversify one's supplier base in the context of a newsvendor problem subject to "yield risk," i.e., when the buyer is likely to experience a default by at least one supplier during lead time. This issue has been investigated in the context of a two-tier supply chain (Dada et al. 2007, Babich et al. 2007, Swinney and Netessine 2009, Federgruen and Yang 2009, Tang et al. 2014) or multiple-tier supply networks (Osadchiy et al. 2016, Ang et al.

2017, Bimpikis et al. 2018, 2019, Birge et al. 2023). Multisourcing helps diversify away upstream (e.g., Tomlin 2006, Allon and Van Mieghem 2010) or downstream risk (e.g., Chod et al. 2019). Our baseline model differs from a newsvendor problem, specifically (i) the product prices adjust to ensure that the input and output markets clear (as in Wadecki et al. 2013, Bimpikis et al. 2019) and (ii) lead times are assumed away, the suppliers being able to satisfy current orders but possibly disappearing from the input market in subsequent periods, which is another form of supply shock, relevant in a dynamic setup. Our model views multisourcing differently, as a way to allow more competition upstream and mitigate double marginalization. Supplier shocks in our case lead to more market power being wielded at an earlier echelon, with consequences across the supply chain. Yang et al. (2015) and Huang et al. (2016) also look at the effect of firm defaults (due to exogenous factors) on supply chains, with Huang et al. (2016) and Yang et al. (2015) focusing on defaults upstream and downstream, respectively. As in Wadecki et al. (2013) and Tang et al. (2014), we discuss the incentive of a buyer to subsidize suppliers to mitigate supply shocks.

3 Does a larger supplier base make buyers better off?

We consider an imperfect upstream market in which the price set by suppliers for a homogeneous nonstorable good depends on the purchase order of a monoposonic buyer, with monopoly market power downstream. This buyer's purchase order reflects the current level of demand in the output market, which may change over time due to demand uncertainty. This modeling framework is motivated by the current state of the agribusiness, for instance (i) the US poultry sector, where an integrator like Tyson Foods wields quasi-monopsony control over numerous small-scale contract growers raising perishable chickens, (ii) coffee cooperatives in Ethiopia that supply a dominant multinational, Nestlé, or (iii) the UK dairy industry, where hundreds of independent farms provide raw milk to a limited number of purchasers, which are either major supermarket chains (Tesco, Sainsbury's, Morrisons, etc.) or processors (Arla Foods, Müller, etc.).

3.1 Baseline supply chain model

As in Babich et al. (2007), Demirel et al. (2018) and Bimpikis et al. (2019), the suppliers compete against each other. Here, the suppliers infer the downstream demand and collectively converge (in a Cournot-equilibrium fashion) to a market-clearing price (see Spengler, 1950 or pp. 174-175 in Tirole, 1988).

Upstream demand. The monopsonic downstream firm faces a produce price $P(q, y) \geq 0$, which decreases with respect to the supply quantity q and depends on a state of demand $y > 0$ observable at each time $t \geq 0$. For instance, we may consider—as Chod and Rudi (2006, Section 3)—isoelastic demand, with

$$q \mapsto P(q, y) = yq^{-\delta} \text{ for } \delta \in (0, 1), \tag{1a}$$

where y is the realization of the (multiplicative) random demand shock and the constant δ is the reciprocal of the price elasticity of demand $|\frac{\mathrm{d}Q}{Q}/\frac{\mathrm{d}P}{P}|$. Up to a renormalization, we assume that the buyer needs q units of input to produce q units of output. The buyer buys each unit at a price $w > 0$, determined endogenously by the suppliers.

Production cost upstream. Suppliers face symmetric costs driven by a monotone increasing cost function $C(\cdot)$, which may be nonlinear in the output to account for (dis)economies of scale. For instance, the function

$$C(q) = \frac{c}{v} q^v \text{ where } c > 0 \text{ and } v > 0, \tag{1b}$$

which increases from 0 to ∞, fits these specifications. If $v = 1$, it is linear and c corresponds to a unit production cost. Because $C''(q) = c(v-1)q^{v-1}$, the marginal cost decreases (resp., increases) with the output $q \in (0, \infty)$ and each supplier benefits from economies of scale (resp., faces scale diseconomies) if $0 < v < 1$ (resp., $v > 1$). Diseconomies of scale are a common feature across various agricultural commodities (Alizamir et al. 2019), often explaining the relative small sizes of farms depending on nature of the crops.

Equilibrium conditions. Many markets are not vertically integrated, which may be well justified (cf. Section 3.2). We consider n symmetric suppliers (e.g., farmers) deciding on their output levels.

To determine the Cournot-Nash equilibrium price $\bar{w}_n \geq 0$, we first build the buyer's demand for an arbitrary input price w. Rationally, the buyer selects an output level $\bar{q}(y,w)$ that maximizes its profit $\pi(q;y,w) := qP(q,y) - wq$.[3] For suitable model specifications, the buyer's optimal output is determined from a first-order condition:

$$\pi_q\left(y, \bar{q}(y,w)\right) = 0. \tag{2}$$

From the suppliers' perspective, $w \mapsto \bar{q}(y,w)$ can be interpreted as the demand function. Under standard specifications, the order quantity $\bar{q}(y,w)$ in eq. (2) increases with demand (as $\bar{q}_y = -\pi_{qy}/\pi_{qq} \geq 0$) and decreases with the input price.

To determine a symmetric Cournot-Nash equilibrium upstream, we use the inverse demand function $Q \mapsto \bar{q}(y,\cdot)^{-1}(Q)$, which maps the total demand by the buyer Q to a price. We then compute the supplier i's *best-reply function*:

$$z \in \mathbb{R}_+ \mapsto \mathrm{R}(z) := \arg\max_{q_i \geq 0} \Big\{ \underbrace{q_i \, \underbrace{\bar{q}(y,\cdot)^{-1}(q_i + z)}_{\substack{\text{Inverse demand} \\ \text{function=input price}}}}_{\substack{\text{individual supplier's} \\ \text{revenues}}} - \underbrace{C(q_i)}_{\substack{\text{supplier's} \\ \text{cost}}} \Big\} \in \mathbb{R}_+, \tag{3}$$

where the term z is understood as the aggregate output of rivals, $z = \sum_{j \neq i} q_j$.

We consider a symmetric equilibrium for which each upstream firm i supplies an amount $q_i = \bar{q}_n(y) \geq 0$. In equilibrium, this amount solves the fixed-point equation

$$\bar{q}_n(y) = \mathrm{R}\Big((n-1)\bar{q}_n(y)\Big), \tag{4}$$

with the total output given by

$$\bar{Q}_n(y) := n\bar{q}_n(y), \tag{5}$$

[3]We use π_q, π_y, and π_{qq} to denote the partial derivatives.

and prices in the upstream and downstream markets are

$$\bar{w}_n(y) := \bar{q}(y,\cdot)^{-1}\left(\bar{Q}_n(y)\right), \tag{6}$$

$$\text{and } \bar{P}_n(y) := P\big(\bar{Q}_n(y), y\big), \tag{7}$$

respectively. Clearly, these prices are not decoupled, with changes in the end demand (driven by y) also affecting the equilibrium price $\bar{w}_n(y)$ in the upstream market. The buyer's equilibrium profit is given by

$$\bar{\pi}_n(y) := \pi\Big(\bar{Q}_n(y); y, \bar{w}_n(y)\Big), \quad y > 0, \tag{8}$$

for $\bar{Q}_n(\cdot)$ and $\bar{w}_n(\cdot)$ given in eqs. (5) and (6) respectively, while a supplier's profit reads

$$\pi_n(y) := \bar{w}_n(y)\bar{q}_n(y) - C\big(\bar{q}_n(y)\big). \tag{9}$$

The proposition below specifies equilibrium firm profits:[4]

Proposition 1 (Supply-chain equilibrium with n suppliers) *For the specifications in eq.* (1) *with $v \geq 1$, there exists a symmetric Cournot-Nash equilibrium upstream, with each supplier providing the quantity*

$$\bar{q}_n(y) = n^{-\frac{\delta}{\delta+v-1}}\left(\tfrac{1-\delta}{c}y\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}}. \tag{10}$$

In this equilibrium, the input and output prices in eqs. (6) *and* (7) *satisfy $\bar{P}_n > \bar{w}_n(y)$, while the buyer's and suppliers' profits in eqs.* (8) *and* (9) *are given by*

$$\begin{aligned}
\bar{\pi}_n(y) &= a_n y^{\epsilon}, \qquad a_n := \delta\left(n^{v-1}\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} \geq 0,\\
\pi_n(y) &= \nu_n y^{\epsilon}, \qquad \nu_n := n^{-\frac{\delta}{\delta+v-1}v}\left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}}\tfrac{1-\delta}{v}\left[v-1+\tfrac{\delta}{n}\right],
\end{aligned}$$

respectively, with $\epsilon := v/[\delta+v-1]$.

This proposition helps us derive numerous managerial insights, some of which are discussed in the following section. Proposition 1 embeds the linear-cost benchmark obtained by setting $v = 1$.

[4]Proposition 1 presumes one buyer and multiple suppliers. Our e-companion also considers a setup with two successive Cournot oligopolies.

In this case, each supplier charges an input price $\bar{w}_n(y) = \frac{c}{1-\delta/n}$ and the buyer charges to the end customer a unit output price $\bar{P}_n(y) = \frac{1}{1-\delta}\frac{c}{1-\delta/n}$. Classically, the vertical relationship features double marginalization, with $\bar{P}_n(y) > \bar{w}_n(y) > c$. The supplier's and buyer's profits are given by

$$\pi_n(y) = \frac{\delta(1-\delta)}{n^2}\left(\frac{1-\delta}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta}} y^{\frac{1}{\delta}} \quad \text{and} \quad \bar{\pi}_n(y) = \delta\left(\frac{1-\delta}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta}} y^{\frac{1}{\delta}},$$

respectively. Proposition 1 goes beyond the linear-cost benchmark. If $v > 1$ in eq. (1), the suppliers face <u>diseconomies of scale</u> as common for farms. Following Proposition 1, suppliers collectively receives a fixed share of the downstream equilibrium price. The profit functions are proportional to y^{ϵ}. Furthermore, if $v \geq 1$, the buyer and suppliers are risk seeking with respect to the level of demand in the output market: They can expand (resp., contract) production if demand builds up (resp., shrinks).

Proposition 1 disregards the strategic interactions that take place in the case where suppliers face <u>economies of scale</u> (with $1-\delta < v < 1$). This case is more involved. In particular, under such circumstances, symmetric suppliers may end up playing an asymmetric equilibrium among themselves:

Proposition 2 (Economies of scale) *In case of economies of scale ($1-\delta < v < 1$), for n suppliers and one buyer, there is no symmetric pure-strategy Cournot-Nash equilibrium. The n asymmetric strategy profiles $\{\bar{q}_1(y), 0, \ldots, 0\}$, $\{0, \bar{q}_1(y), \ldots, 0\}$,... and $\{0, \ldots, 0, \bar{q}_1(y)\}$, for $\bar{q}_1(y)$ given in eq. (10), are pure-strategy Cournot-Nash equilibria. For each of these equilibria, the input and output markets clear respectively at the prices $\bar{w}_1(y)$ given in eq. (6) and $\bar{P}_1(y)$ given in eq. (7), while the buyer's profit is $\bar{\pi}_1(y) = a_1 y^{\epsilon}$, given in Proposition 1.*

3.2 Benefits of a large supply base?

For the <u>linear-cost benchmark</u> (obtained by setting $v = 1$), more upstream competition ($n \geq 1$) leads to lower equilibrium input and output prices, $\frac{c}{1-\delta/n}$ and $\frac{1}{1-\delta}\frac{c}{1-\delta/n}$ respectively. However, this development causes the buyer to buy and sell more, with its order quantity/output $\bar{Q}_n(y) = \left(\frac{1-\delta}{c}y\,(1-\delta/n)\right)^{1/\delta}$ increasing with n. The buyer earns more ($\bar{\pi}_1 \leq \bar{\pi}_2 \leq \ldots$), while each supplier earns less ($\pi_1 \geq \pi_2 \geq \ldots$). This result suggests a "shift in market power" to the benefit of the buyer, at the expense of the suppliers. The buyer's purchase order increases with end demand (as $\partial\bar{Q}_n/\partial y > 0$), but decreases with the suppliers' unit cost (as $\partial\bar{Q}_n/\partial c < 0$).[5] The double-marginalization effect vanishes if competition in the input market becomes perfect, with the input price now corresponding to the marginal cost ($\bar{w}_n \to c$ as $n \to \infty$) and the suppliers making no profit ($\pi_n \to 0$).

When considering nonlinear production costs, two effects must be acknowledged. First, as in the linear-cost benchmark, each supplier has a natural tendency to respond strategically to more intense rivalry (indexed by $n \geq 1$) by reducing output. Second, if the production costs are nonlinear, an output reduction affects marginal costs, an effect that feeds back into the equilibrium price-setting mechanism. Specifically, a lower output leads to a lower (resp., larger) marginal production cost when a supplier faces diseconomies of scale (resp., benefits from scale economies). So, if suppliers benefit from scale economies ($1 - \delta < v < 1$), the (monopsonic) buyer faces a tradeoff: Spreading production among a larger set of suppliers leads to a larger marginal cost. Again, according to our Proposition 2, in this case, the production is not equally split among suppliers, with either supplying the buyer as if it were a monopolistic supplier, while the other supplier leaves out the game altogether. If suppliers face diseconomies of scale, then there is no tradeoff for the buyer: Because of competitive pressures, the suppliers reduce their outputs, each being less subject to scale diseconomies. Proposition 3 summarizes the net effect on the buyer's profit ($m = 1$):

Proposition 3 (Effect of supplier base size) *In case of linear cost ($v = 1$) or of scale diseconomies ($v > 1$), a buyer that can source from a larger set of suppliers orders and sells more items (i.e., $\bar{Q}_{n+1} \geq \bar{Q}_n$), items which it purchases at a lower input price (i.e., $\bar{w}_{n+1} \leq \bar{w}_n$). This buyer is better off (with $a_{n+1} \geq a_n$) despite offering a lower price to the end customers (with $\bar{P}_{n+1} \leq \bar{P}_n$). In case of scale economies ($1 - \delta < v < 1$), a larger set of potential suppliers ($n + 1 > n \geq 1$) does not affect the equilibrium conditions for the buyer.*

We focus on the cases where suppliers have linear costs ($v = 1$) or where they face scale problems ($v > 1$). Following Proposition 3, the buyer can then source its input at a lower price $\bar{w}_n(y)$ if the supplier base is more competitive, as indexed by n. In the case with diseconomies of scale ($v > 1$),

[5] For the setup with m buyers of Proposition 13 in the appendix, we also have $\bar{\pi}_{m,1} \leq \bar{\pi}_{m,2} \leq \ldots$, again implying that buyers are better off if the supplier base is larger.

sourcing from a larger set of suppliers has two benefits. First, suppliers compete with one another, which tends to depress the equilibrium price upstream (due to strategic substitution among the suppliers' strategic choices). However, each of them also produces less, so the marginal production cost decreases, which has a positive feedback effect on the equilibrium input price. Because the input cost decreases for the buyer, it produces more ($\bar{Q}_{n+1} \geq \bar{Q}_n$) and earns more ($a_{n+1} \geq a_n$). Another consequence is that end customers benefit from more upstream competition, with $\bar{P}_{n+1} \leq \bar{P}_n$. This last result is consistent with the general property (see Tirole 1988, p. 67) that a monopoly price (here, set by the buyer and charged to the end customers) increases in the marginal cost (here, the equilibrium input price).

3.3 Vertical integration

Proposition 4 briefly discusses the benefit of upstream competition (despite double marginalization) compared to being vertically integrated:

Proposition 4 (Vertical integration) *We take the specifications in eq.* (1). *For* linear costs *($v = 1$), the buyer's profit given vertical integration $\Pi(y) := \max_{q \geq 0}\{qP(y,q) - C(q)\}$ exceeds the profit the buyer would achieve given upstream competition, $\bar{\pi}_n$ in Proposition 1, independently of the number n of suppliers. However, in the case with* diseconomies of scale *($v > 1$), there exists a unique finite integer $\tilde{n} > 1$ such that $\Pi(y) \geq \bar{\pi}_n(y)$ for $1 \leq n \leq \tilde{n}$ and $\Pi(y) < \bar{\pi}_n(y)$ otherwise.*

A particular supply chain setup, covered by Proposition 4, is when the buyer purchases from a single supplier ($n = 1$). In this case, we recover the classical double marginalization result (Spengler 1950): After both the supplier and the buyer charged a markup onto the next echelon of the chain, the end price ends up larger than the price the centralized supply chain would optimally set. This inefficiency arises because the upstream firms do not take into account the externality exerted on the upstream firm by changing the wholesale price.

Under linear costs ($v = 1$), spreading production over a larger set of suppliers mitigates double marginalization, but a certain degree always remains. However, if $v > 1$ the buyer faces diseconomies of scale when it is vertically integrated, an additional cost that can offset the benefit of avoiding double marginalization, especially if the suppliers are numerous ($n > \tilde{n}$) and collectively wield limited market power.

4 Buyer's investment if there are no supply shocks

We first study the effect of upstream competition on a buyer's investment decision.

Buyer's long-term value (after investment). As Chod and Rudi (2006), we assume that the demand shock in eq. (1a) is lognormal:

$$Y_0 = y > 0 \quad \text{and} \quad \mathrm{d}Y_t = \mu Y_t \,\mathrm{d}t + \sigma Y_t \,\mathrm{d}W_t, \quad \text{with } \sigma > 0. \tag{11}$$

These dynamics model demand shifts due to changes in consumer tastes and the arrival of substitute products over time (see, e.g., Li and Kouvelis 1999).

We assume the buyer's discount rate, $r > 0$, to be constant over time (Dixit and Pindyck 1994, Li and Kouvelis 1999) and let $\mathbb{E}^y = \mathbb{E}\big[\,\cdot\,\big|Y_0 = y\big]$ denote the conditional expectation. If there are n firms that supply at all times ("reliable suppliers"), the buyer's present value (PV) is given by

$$\bar{u}_n(y) = \mathbb{E}^y \int_0^\infty e^{-rt}\bar{\pi}_n(Y_t)\mathrm{d}t, \tag{12}$$

where $\bar{\pi}_n(\cdot)$ denotes the buyer's equilibrium profit in eq. (8). Proposition 5 expresses this PV in closed form for the specifications of eq. (1).[6] To state the result, we introduce γ_+ the positive solution of equation $\mathcal{Q}(x) = 0$, where

$$\mathcal{Q}(x) := \frac{1}{2}\sigma^2 x(x-1) + \mu x - r. \tag{13}$$

A classical assumption for linear payoffs is $r > \mu$ (e.g. Dixit and Pindyck 1994), which is not sufficient here. More restrictively, we assume $\epsilon < \gamma_+$ throughout the manuscript, which is equivalent to $\mathcal{Q}(\epsilon) < 0$. If this is not satisfied, then the PV explodes ($\bar{u}_n(y) \to \infty$) because the profits grow exponentially but at a rate too strong compared to the discount rate.

[6] Proposition 5 can be generalized to accommodate for m buyers based on the equilibrium profit expression of Proposition 13 in the e-companion.

Buyer's optimal investment time. Given the PV in eq. (12), we study the buyer's propensity to invest by solving the real-options problem:

$$\bar{\psi}_n(y) := \sup_{\tau} \mathbb{E}^y \Big[e^{-r\tau} \big\{ \bar{u}_n(Y_\tau) - I \big\} \Big], \tag{14}$$

where the investment time τ is selected by the buyer and the parameter $I \geq 0$ is a known investment cost. Proposition 5 solves this problem:

Proposition 5 (Buyer's real-options problem) *For the specifications in eq.* (1)*, the PV in eq.* (12) *is*

$$\bar{u}_n(y) = \alpha_n y^\epsilon, \quad \textit{where} \quad \alpha_n := -\frac{a_n}{\mathcal{Q}(\epsilon)}, \tag{15}$$

for a_n given in Proposition 1. The buyer's investment problem in eq. (14) *has a solution:*

$$\bar{\psi}_n(y) = \begin{cases} \big[\alpha_n \bar{y}_n^\epsilon - I\big] \left(\frac{y}{\bar{y}_n}\right)^{\gamma_+}, & y < \bar{y}_n := \left(\frac{\gamma_+}{\gamma_+ - \epsilon} \frac{I}{\alpha_n}\right)^{\frac{1}{\epsilon}}, \\ \alpha_n y^\epsilon - I, & y \geq \bar{y}_n. \end{cases}$$

Its optimal investment time is $\bar{\tau}_n := \inf \big\{ t \geq 0 \big| Y_t \geq \bar{y}_n \big\}$.

In line with our earlier results about the impact of upstream competition on the buyer's profit in Proposition 3, we find that the buyer is better off if upstream competition is more intense in case of (i) linear costs ($v = 1$) or (ii) if these suppliers face diseconomies of scale ($v > 1$). If (iii) there are economies of scale to be achieved ($1 - \delta < v < 1$), having the ability to source from more suppliers effectively is of no value to the buyer because the market is monopolized (see Proposition 2). In all cases, the buyer invests if the price exceeds a level (obtained by smooth fit), higher than the NPV and Marshallian thresholds (see Dixit and Pindyck 1994, Ch. 5). The buyer thus requires extra profitability from its project before undertaking investment.

The next proposition summarizes a main managerial insight on the impact of upstream competition on the buyer's investment decision:

Proposition 6 (Impact of upstream competition on investment) *Assume $v \geq 1$. We have $\alpha_n \leq \alpha_{n+1}$ and $\bar{u}_n \leq \bar{u}_{n+1}$. Furthermore, the buyer's value function increases with the intensity of* *upstream competition ($\bar{\psi}_n \leq \bar{\psi}_{n+1}$ for $n \geq 1$), while the optimal investment time decreases ($\bar{\tau}_n \geq \bar{\tau}_{n+1}$). Moreover, the value of delay flexibility decreases with the intensity of upstream competition ($\bar{\psi}_n - (\bar{u}_n - I) \geq \bar{\psi}_{n+1} - (\bar{u}_{n+1} - I)$).*

From Proposition 3, we know that, in case of linear costs and diseconomies of scale ($v \geq 1$), more upstream competition makes the buyer earn more. Proposition 6 establishes that, under these circumstances, the buyer is also more prone to invest (earlier) because it can extract more value from its operations.

5 Buyer's investment under exogenous supply shock

Supply shocks may be caused by exogenous events, including natural disasters, the COVID-19 pandemic, strikes crippling economies, nuclear incidents (Fukushima), terrorist attacks, or embargoes (e.g., Iran, Russia) (Kleindorfer and Saad 2005, Babich et al. 2007, Lücker et al. 2024). In particular, the agribusiness is subject to such supply shocks. For instance, in 2023/2024, Brazil, the world's largest Arabica coffee producer and exporter, experienced dry weather conditions and frost events in Mina Gerais, which caused a production shortfall, while Vietnam and Indonesia were afflicted by prolonged dry weather conditions and excessive rainfalls, respectively.[7] These supply shocks may have temporary effects (e.g., droughts, floods, frosts), while others (e.g., coffee leaf rust, coffee wilt disease, coffee berry disease) have a long-term impact causing some suppliers to drop out.

If the buyer has n suppliers when demand is y, it receives a profit $\bar{\pi}_n(y) = a_n y^\epsilon$ given in Proposition 1. If the state of upstream competition remains unchanged with n *reliable* suppliers, the buyer's present value of eq. (12) is given by $\bar{u}_n(y) = \alpha_n y^\epsilon$ in eq. (15). However, supply chains may be subject to major shocks, a stylized fact that challenges our previous assumption (used in Proposition 5) that the supplier base remains constant over time. In contrast to the literature, which often considers a shock occurring between the times of ordering and receiving goods, we consider repeated relationships with some suppliers disappearing permanently at some future time. Specifically, at time 0, the buyer has identified a set of n potential suppliers it can source from. Yet, when a sudden shock arises at a random time, some suppliers disappear altogether, while a subset

[7] openknowledge.fao.org/server/api/core/bitstreams/8135b05e-a013-4080-b8f6-a6ac5b02230a/content

survives. In line with Proposition 3, the shock that occurs at that time leads to an upward jump in the equilibrium input price (because fewer upstream rivals ultimately harm the buyer), a sudden drop in the order quantity and sales volume, as well as a significant drop in the buyer's profit. End customers are also affected as the equilibrium output price increases abruptly.[8] In our model, exogenous supply shocks are assumed to be independent of demand uncertainty. For simplicity, we model one shock, not a sequence. Furthermore, as we consider nonstorable goods, the buyer cannot stock up inventory to mitigate supply shocks. At any rate, inventory buildup can help mitigate the effect of supply shocks in the short to mid term at best, but not in the long term.

Present value. The present value received if the buyer invests at time 0,

$$\tilde{u}_n(y) := \mathbb{E}\left[\int_0^T e^{-rs}\bar{\pi}_n(Y_s)\,\mathrm{d}s + e^{-rT}\bar{u}_N(Y_T)\right], \tag{16}$$

embeds the effect of a supply shock at time T, after which the buyer can only source from N suppliers. This shock occurs at a time T believed by the buyer to be exponentially distributed with parameter λ (e.g., Kouvelis and Xu 2021), while the random variable N takes values in the set $\{0,\ldots,n-1\}$. We recall the definition of the parameter α_n in eq. (15) and set $\alpha_0 = 0$ by convention. The proposition below expresses the PV of eq. (16) in closed form and provides comparative statics on the impact of the supply shock characteristics on this value. To state these comparative statics, we use the notation $N_2 \preceq N_1$ to mean that the random variable N_2 is stochastically dominated by N_1 in the first-order sense. In our context, $N_2 \preceq N_1$ implies that more suppliers are likely to disappear following the supply shock at time T if we consider the random variable N_2 instead of N_1. For instance, the random variable N_2 may reflect a situation where the supplier base is concentrated around 'patient zero' of a disease affecting crops permanently, while N_1 captures a more geographically diversified supplier base. We have:

Proposition 7 (Present value for exogenous supply disruption) *Take $v \geq 1$. At time 0,*

[8] openknowledge.fao.org/server/api/core/bitstreams/8135b05e-a013-4080-b8f6-a6ac5b02230a/content

prior to the shock, the present value of the buyer in eq. (16) *is given by*

$$\tilde{u}_n(y) = \eta_n y^\epsilon, \ \textit{where } \eta_n = \eta_n^{\lambda,N} := \alpha_n + \frac{\lambda}{\lambda - \mathcal{Q}(\epsilon)}\big\{\mathbb{E}\alpha_N - \alpha_n\big\}. \tag{17}$$

A supply shock destroys value with $\eta_n \leq \alpha_n$. Furthermore, if the disruption is likely to occur sooner or affect more suppliers, then value is also destroyed (with $\lambda_2 \geq \lambda_1$ and $N_2 \preceq N_1$ implying $\eta_n^{\lambda_2,N} \leq \eta_n^{\lambda_1,N}$ and $\eta_n^{\lambda,N_2} \leq \eta_n^{\lambda,N_1}$, respectively).

Under rational expectations, the buyer anticipates the supply shock. When it happens, the remaining suppliers will exercise greater collective market power. As a consequence, the equilibrium price upstream will increase, at the buyer's expense. A likely supply shock then leads to a downward adjustment of the buyer's present value, from $\eta_n y^\epsilon$ in eq. (15) in the case with n reliable suppliers to $\eta_n y^\epsilon$ in case of a supplier base prone to downward change. The factor $\eta_n^{\lambda,N}$ in eq. (17) depends on the arrival of a supply shock, through the parameter λ, as well as on the impact of this disruption on the supplier base through the distribution of N. In the absence of supply shock (i.e., $\lambda = 0$), the factor $\eta_n^{0,N}$ in eq. (17), which drives the buyer's present value, simplifies to the factor α_n obtained in eq. (15). As the factor $\eta_n^{\lambda,N}$ decreases with respect to λ, a higher arrival rate for the disruption depresses the present value of a buyer with rational expectations. The sensitivity of the supplier base to the disruptive event at time T is another metric the buyer may want to consider. If the buyer expects more suppliers to be affected by the disruption (in the sense that $N_2 \preceq N_1$), then its present value will again be negatively affected.

The investment problem $\sup_\tau \mathbb{E}e^{-r\tau}\big\{\tilde{u}_n(Y_\tau) - I\big\}$, for $\tilde{u}_n(\cdot)$ in eq. (17) has a closed-form solution similar in form to $\bar{\psi}_n(y)$ in Proposition 5. Yet, this investment problem is *not* time consistent because the investment decision is made solely in view of shifts in demand, ignoring the dynamics of the supply shock; It essentially presupposes that the shock can only occur after the buyer's investment decision. A time-consistent formulation of the problem (which can be solved using dynamic programming) involves three state processes: (i) demand $(Y_t)_t$ in eq. (11) as before, (ii) the state $(H_t)_t$ of the supplier base, which takes the value 1 if the shock took place and 0 otherwise, and (iii) the number $(N_t)_t$ of suppliers, which drops from the initial number of identified suppliers n to N following the shock.

The buyer may face two situations. If the shock already happened ($H_0 = h = 1$), the buyer observes which suppliers survived, i.e., the realization of the random variable N, $k \in \{0, \dots, n-1\}$, and receives the present value $\bar{u}_k(\cdot)$ in eq. (12) upon investing. Alternatively, if no shock took place ($H_0 = h = 0$), the buyer forms rational expectations and anticipates a future shock, thus receiving the present value of eq. (16). The time-consistent formulation of the buyer's real-options problem is[9]

$$\tilde{\psi}(y, h, n) := \sup_{\tau \geq 0} \mathbb{E}\left[e^{-r\tau}\left\{\tilde{u}_n(Y_\tau)(1 - H_\tau) + \bar{u}_N(Y_\tau)H_\tau - I\right\}|Y_0 = y, H_0 = h, N_0 = n\right], \tag{18}$$

for the 'state of disruption' $h \in \{0, 1\}$ and the number of surviving suppliers N. We denote by $j(\cdot)$ the distribution of the random variable N over the set $\{0, \dots, n-1\}$. We first rewrite the investment problem in eq. (18) in a more classical form:

Proposition 8 (Problem under exogenous supply shock) *If the shock took place ($h = 1$) and k suppliers have survived, the buyer's investment problem in eq.* (18) *reduces to* $\tilde{\psi}(\cdot, 1, k) \equiv \bar{\psi}_k(\cdot)$ *solved in Proposition 5 for* $k = 1, \dots, n-1$. *If the disruption did not yet take place ($h = 0$), the value function takes the form*

$$\tilde{\psi}(y, 0, n) = \mathbb{E}R_N(y) + \sup_{\tau \in \mathcal{T}} \mathbb{E}e^{-(r+\lambda)\tau}\big\{\eta_n Y_\tau^\epsilon - I - R_N(Y_\tau)\big\}, \tag{19}$$

for η_n *defined by eq.* (17) *and* $R_k(y) := \mathbb{E}e^{-(r+\lambda)T}\bar{\psi}_k(Y_T)\mathrm{d}t \geq 0$.

Proposition 8 rewrites the real-options problem in eq. (18), which depends on three state variables, into a usual optimal stopping problem with one state variable y. Compared to the problem in eq. (14), we consider a new problem (the second RHS term in eq. (19)) with (i) a higher discount rate $r + \lambda$ accounting for the risk of arrival of a supply shock and (ii) an extra term accounting for the consequence of that arrival on the buyer's supplier base.[10] We want to determine the solution of that new investment problem:

[9] In eq. (18), the buyer's investment τ is assumed to be adapted to an augmented filtration that accounts for the additional source of uncertainty with respect to the supplier base.

[10] $R_k(y)$ can be interpreted as the value of a compound option, specifically the value of a European option written on an American call, with an exponentially distributed maturity date T for the European option.

Proposition 9 (Buyer's real-options problem under exogenous disruption) *If the shock did not yet occur ($h = 0$), it is optimal for the buyer to wait for demand to exceed a threshold $\tilde{y}$, with its value function given by*

$$\tilde{\psi}(y,0,n) = \begin{cases} \left[\eta_n \tilde{y}^\epsilon - I\right](\frac{y}{\tilde{y}})^\gamma + \phi(y), & 0 < y \le \tilde{y}, \\ \eta_n y^\epsilon - I, & y \ge \tilde{y}, \end{cases}$$

where

$$\gamma := \frac{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2(\lambda + r)}}{\sigma^2} \ge \gamma_+, \tag{20}$$

and $\phi(\cdot)$ is positive and solves

$$\begin{cases} \phi(0) & = 0, \\ \frac{1}{2}\sigma^2 y^2 \phi''(y) + \mu y \phi'(y) - (r+\lambda)\phi(y) + \lambda \sum_{k=0}^{n-1} j(k)\bar{\psi}_k(y) & = 0, \quad \text{on } (0,\tilde{y}), \\ \phi(y) & = 0, \quad \text{for } y \ge \tilde{y}. \end{cases} \tag{21}$$

We now study how a greater sensitivity of the supplier base to the supply shock affects the buyer's investment decision. Again, $N_2 \preceq N_1$ signifies that more suppliers are likely to disappear after the shock if we consider the random variable N_2 in lieu of N_1. The following comparison result articulates that the buyer will invest later (and have a lower option value) if it expects more suppliers to disappear following the shock at time T:

Proposition 10 (Larger supplier-base impact) *Assume $v \ge 1$ and $N_2 \preceq N_1$. Furthermore, let $\tilde{\psi}_i$ and $\tilde{y}_i$ denote the value function and optimal investment threshold in Proposition 9 for the case with N_i, $i \in \{1, 2\}$. A larger likely negative impact on the supplier base ($N_2 \preceq N_1$) destroys value for the buyer ($\tilde{\psi}_2 \le \tilde{\psi}_1$) and delays its investment ($\tilde{y}_2 \ge \tilde{y}_1$).*

6 Buyer's investment under endogenous supply shock

The reliability of all n suppliers at all times may be challenged if fixed costs are significant. For instance, while owning harvesting machines is prohibitive for small or medium-sized coffee farms, renting them may be more affordable and overcome sunking large investment costs.[11] Other common fixed costs include shipping costs, which have been material from 2020 to 2022 and had a documented impact on coffee prices.[12] To sustain more upstream competition and mitigate double marginalization, the (monopsonic) buyer may use different schemes to keep (weaker) suppliers afloat. For instance, the buyer may subsidize a supplier that may disappear if its profit in Proposition 1 is not sufficient to cover its fixed cost.

6.1 Buyer's investment decision

Consider that one (unreliable) supplier rents the productive equipment (e.g., harvesters) at a cost $K > 0$ and intermittently supplies the buyer if its gross profit exceeds this rental cost. The other (reliable) supplier, e.g., because it made a sunk investment in this equipment, can supply the buyer at all times. In the case with economies of scale ($1-\delta < v < 1$), which is less relevant in the context of agriculture, the buyer is indifferent if one supplier temporarily drops because the upstream market is effectively monopolized (see Proposition 2). This, however, is not the case for linear production costs ($v = 1$) or if there are diseconomies of scale ($v \geq 1$), which we consider now.

The factors ν_n and a_n in Proposition 1 characterize the firms' *gross* profits, depending on the intensity of upstream competition (via the state variable n). From Proposition 6, we know that $a_2 \geq a_1$ for linear costs ($v = 1$) or in case of diseconomies of scale ($v > 1$). Unless the buyer intervenes, the unreliable supplier trades if and only if its gross profit $\nu_2 y^\epsilon$ exceeds the rental cost $K > 0$, i.e., if demand y exceeds $(K/\nu_2)^{\frac{1}{\epsilon}}$. The buyer thus earns a state-dependent profit,

$$\breve{\pi}_0(y) := \underbrace{a_1 y^\epsilon 1_{\{\nu_2 y^\epsilon - K < 0\}}}_{\text{sourcing from 1 supplier}} + \underbrace{a_2 y^\epsilon 1_{\{\nu_2 y^\epsilon - K \geq 0\}}}_{\text{sourcing from 2 suppliers}}, \tag{22}$$

[11] revistacultivar.com/news/characteristics-and-benefits-of-mechanized-coffee-harvesting
[12] openknowledge.fao.org/server/api/core/bitstreams/8135b05e-a013-4080-b8f6-a6ac5b02230a/content

and has a PV given by

$$\begin{aligned}\breve{u}_0(y) :=& \mathbb{E}\int_0^\infty e^{-rt}\breve{\pi}_0(Y_t^y)\mathrm{d}t \\ =& \frac{2}{(\gamma_+ - \gamma_-)\sigma^2}\left\{y^{\gamma_-}\int_0^y \frac{\breve{\pi}_0(z)}{z^{\gamma_-+1}}\mathrm{d}z + y^{\gamma_+}\int_y^\infty \frac{\breve{\pi}_0(z)}{z^{\gamma_++1}}\mathrm{d}z\right\} > 0,\end{aligned} \tag{23}$$

for γ_+ (resp., γ_-) again denoting the positive (resp., negative) root of $\mathcal{Q}(\cdot)$ in eq. (13).

6.2 Mitigating double marginalization by subsidizing the supply chain

The support a buyer can provide can take various other forms such as cash or investment subsidies (Babich 2010, Wadecki et al. 2013, Tang et al. 2014), buyer direct financing (Tang et al. 2017), and purchase-order financing (Tang et al. 2017, Jain et al. 2023). Cargill, a US-based multinational food corporation, recognizes that sustainability is essential to feed a rising global population: It financially supports its farmers for adopting regenerative agricultural practices and partners with technology firms to develop wind-propulsed bulk carriers meant to reduce shipping costs.[13] In the same vein, Nescafé supports coffee growers through the Nescafé Plan 2030 by providing agronomic training, distributing disease-resistant plantlets, incentivizing regenerative farming, and offering other financial tools to enhance productivity, build climate resilience, and stabilize farmers' incomes.[14]

If $v \geq 1$, the presence of an unreliable supplier is detrimental to the buyer in the sense that $a_1 y^\epsilon \leq \breve{\pi}_0(y) \leq a_2 y^\epsilon$ and $\alpha_1 y^\epsilon \leq \breve{u}_0(y) \leq \alpha_2 y^\epsilon$ for α_1 and α_2 given in Proposition 5. This result is similar in spirit to the result obtained when considering an exogenous shock affecting the structure of the supplier base. To mitigate the effect of an (endogenous) supply shock, the buyer can intervene to maintain more upstream competition. For instance, the buyer may choose to subsidize the unreliable supplier by paying a share $\eta \in [0, 1]$ of the rental cost K. Given this intervention, this supplier trades if and only if $\nu_2 y^\epsilon - (1-\eta)K \geq 0$. This share is considered herein a decision variable in each period.

[13] impact.economist.com/sustainability/decarbonising-agriculture-and-transportation/building-a-sustainable-resilient-global-food-system

[14] www.nestle.com/media/news/nescafe-plan-2030-progress-report-2024-regenerative-agriculture

This string of decisions by the buyer leads to a PV given by

$$\breve{u}(y) := \mathbb{E}\int_0^\infty e^{-rt}\breve{\pi}(Y_t^y)\mathrm{d}t$$
$$\text{where } \breve{\pi}(y) := \sup_{\eta\in[0,1]}\Big\{\underbrace{a_1 y^\epsilon 1_{\{\nu_2 y^\epsilon-(1-\eta)K<0\}}}_{\text{sourcing from 1 supplier}} + \underbrace{\big[a_2 y^\epsilon - \eta K\big]1_{\{\nu_2 y^\epsilon-(1-\eta)K\geq 0\}}}_{\text{sourcing from 2 suppliers}}\Big\}. \quad (24)$$

The following proposition provides key results:

Proposition 11 (Subsidizing unreliable suppliers?) *The buyer's profit in eq.* (24) *is given by* $\breve{\pi} = \breve{\pi}_0 + \zeta$*, where* $\breve{\pi}_0$ *is as in eq.* (22) *and* ζ *is given by*

$$\zeta(y) := \Big\{(\nu_2 + a_2 - a_1)y^\epsilon - K\Big\}1_{[(\frac{K}{\nu_2+a_2-a_1})^{\frac{1}{\epsilon}},(\frac{K}{\nu_2})^{\frac{1}{\epsilon}})}(y) \geq 0, \quad \forall y > 0.$$

The PV in eq. (24) *is* $\breve{u} = \breve{u}_0 + Z$*, for* $\breve{u}_0$ *in eq.* (23) *and*

$$Z(y) := \frac{2}{(\gamma_+ - \gamma_-)\sigma^2}\left\{y^{\gamma_-}\int_0^y \frac{\zeta(z)}{z^{\gamma_-+1}}\mathrm{d}z + y^{\gamma_+}\int_y^\infty \frac{\zeta(z)}{z^{\gamma_++1}}\mathrm{d}z\right\} > 0, \quad \forall y > 0.$$

Following Proposition 11, if both suppliers are reliable ($K \to 0$), then the buyer's profit $\breve{\pi}(y)$ simplifies to the profit $a_2 y^\epsilon$ in Proposition 1. For a larger fixed cost K, the buyer will not subsidize the unreliable supplier if the demand state y exceeds the level $(K/\nu_2)^{\frac{1}{\epsilon}}$, above which each party in the supply chain generates a *net* profit. For low demand (i.e. $0 < y \leq (\frac{K}{\nu_2+a_2-a_1})^{\frac{1}{\epsilon}}$), subsidizing ensures more upstream competition, but there is no net benefit from doing so and the buyer effectively prefers to source from one supplier only. Between these two threshold levels, the buyer will subsidize a share $\breve{\eta}(y) := 1 - \nu_2/Ky^\epsilon$ of the rental cost K. This share decreases in the demand state y, reflecting the buyer's desire to subsidize more if the reliable supplier is less profitable. With this contribution from the buyer, the reliable supplier barely breaks even but supplies. This intervention is sufficient to maintain more upstream competition. The buyer's subsidizing effort essentially boils down to consolidating (in the sense of its accounting treatment) the profit of the reliable supplier, as also reflected in the buyer's profit $(a_2 + \nu_2)y^\epsilon - K$. However, from an economic point of view, this is not tantamount to a vertical integration which would help the buyer avoid a double marginalization. Vertical integration is a commitment in all demand states. The profit decomposition $\breve{\pi} = \breve{\pi}_0 + \zeta$

stresses the benefit for the buyer to subsidize the unreliable supplier in times of need, as a way to maintain more upstream competition.

The NPV decomposition in Proposition 11 stresses the option-like feature of subsidizing the upstream market. Depending on the demand level, the buyer will optimally take over a share $\breve{\eta}(y) > 0$ of the unreliable supplier's fixed cost, a strategy that generates an additional benefit $Z \geq 0$. The presence of an unreliable supplier upstream is unfortunate for the buyer, as $\breve{u}(y) \leq \alpha_2 y^\epsilon$. However, subsidizing this firm is advised, as $\breve{u}(y) \geq \breve{u}_0(y) := \mathbb{E}\int_0^\infty e^{-rt}\breve{\pi}_0(Y_t^y)\mathrm{d}t \geq \alpha_1 y^\epsilon$, but should be conditional on the current market circumstances (as $\breve{\eta}(y) \not\equiv 1$ and $\breve{u}(y) \geq \alpha_2 y^\epsilon - K$).

In relation to the present values $\breve{u}_0(y)$ and $\breve{u}(y)$ in eqs. (23) and (24), respectively, we can consider (time-consistent) investment problems of the form

$$\breve{\psi}_0(y) := \sup_\tau \mathbb{E}e^{-r\tau}[\breve{u}_0(Y_\tau^y) - I] \quad \text{and} \quad \breve{\psi}(y) := \sup_\tau \mathbb{E}e^{-r\tau}[\breve{u}(Y_\tau^y) - I]. \tag{25}$$

These option values arguably differ from the benchmarks for the symmetric supplier base, given in eq. (14) and solved in Proposition 5. We establish:

Proposition 12 (Ranking for option values and optimal investment thresholds) *Assume $v \geq 1$. The real options problems in eq.* (25) *have (closed-form) threshold solutions $\breve{y}_0$ and $\breve{y}$ obtained by smooth fit. Compared to the benchmarks $\bar{\psi}_i$ and $\bar{y}_i$ that characterize the case with a symmetric, reliable supplier base, solved in Proposition 5, we have $\bar{\psi}_1 \leq \breve{\psi}_0 \leq \breve{\psi} \leq \bar{\psi}_2$ and $\bar{y}_1 \geq \breve{y}_0 \geq \breve{y} \geq \bar{y}_2$.*

Here again, the buyer will invest if the state of demand is sufficiently large, that is, above a demand value $\breve{y}_0$ or $\breve{y}$ that depends on whether the buyer can subsidize the unreliable supplier or not (the latter being indicated with the underscript 0). Following Proposition 12, the intermittent presence of an unreliable firm in the supplier base destroys option value for the buyer, as $\breve{\psi} \leq \bar{\psi}_2$, but this presence is still useful compared to the extreme case where the supplier base consists of a unique reliable firm, as $\breve{\psi}_0 \geq \bar{\psi}_1$. Subsidizing the unreliable supplier is also useful, as it generates more option value compared to the case where the buyer does not intervene ($\breve{\psi} \geq \breve{\psi}_0$). In this case, intermittent supply from an unreliable firm is not ideal for the buyer as it leads to a late investment (with $\breve{y} \geq \bar{y}_2$), but is still better than sourcing from a monopolistic supplier base (with $\bar{y}_1 \geq \breve{y}_0$). Subsidizing the unreliable supplier leads to earlier investment (as $\breve{y} \geq \breve{y}_0$).

7 Conclusion

We study the effect of upstream competition and supply shocks on a firm's investment decision. We analytically prove several key insights. If suppliers have linear costs or face diseconomies of scale, more upstream competition mitigates double marginalization, reduces equilibrium prices, improves a buyer's profitability, and hastens its investment. If there are economies of scale, the input market remains monopolized. If the upstream market is not already monopolized, supply shocks—whether due to exogenous events or to a lack of profitability for some suppliers who decide to cease operations—depress market conditions for the buyer ex post as the remaining suppliers wield more market power and double marginalization is exacerbated. A buyer anticipates such effects and postpones its investment: If more suppliers are likely to disappear following the shock, the buyer will delay its investment even further. The buyer may subsidize suppliers for them to remain afloat, while sustaining more competition upstream, which leads to earlier investment by the buyer.

Like any model, ours has limitations. First, it surmises complete information, with suppliers able to infer demand from the buyer and their rivals' best-reply functions (see, e.g., Simchi-Levi and Zhao 2003, Shen et al. 2019, on the role of information asymmetries on the terms of supply contracts). Second, we ignore the use of inventory which can serve operational (e.g., to circumvent backlogs due to, say, the Suez Canal Blockage, War in Ukraine, and Panama Canal drought) or strategic purposes (see, e.g., Guan et al. 2019). Third, we also ignored the possibility for suppliers to sell directly to end customers, which has become easier through online stores (see, e.g., Guan et al. 2019, Liu et al. 2021, on the notion of supplier encroachment). Fourth, a supply chain may involve more than two echelons (see, e.g., Ang et al. 2017, Birge et al. 2023). We leave these and other topics for future research.

References

Alizamir S, Iravani F, Mamani H (2019) An analysis of price vs. revenue protection: Government subsidies in the agriculture industry. *Management Science* 65(1).

Allon G, Van Mieghem JA (2010) Global dual sourcing: Tailored base-surge allocation to near-and offshore production. *Management Science* 56(1):110–124.

Ang E, Iancu DA, Swinney R (2017) Disruption risk and optimal sourcing in multitier supply networks. *Management Science* 63(8):2397–2419.

Babich V (2010) Independence of capacity ordering and financial subsidies to risky Suppliers. *Manufacturing and Service Operations Management* 12(4):583–607.

Babich V, Burnetas AN, Ritchken PH (2007) Competition and diversification effects in supply chains with supplier default risk. *Manufacturing and Service Operations Management* 9(2):123–146.

Billette De Villemeur E, Ruble R, Versaevel B (2014) Investment timing and vertical relationships. *International Journal of Industrial Organization* 33(1):110–123.

Bimpikis K, Candogan O, Ehsani S (2019) Supply disruptions and optimal network structures. *Management Science* 65(12):5504–5517.

Bimpikis K, Fearing D, Tahbaz-Salehi A (2018) Multisourcing and miscoordination in supply chain networks. *Operations Research* 66(4):1023–1039.

Birge JR, Capponi A, Chen PC (2023) Disruption and rerouting in supply chain networks. *Operations Research* 71(2):750–767.

Chevalier-Roignant B, Flath CM, Huchzermeier A, Trigeorgis L (2011) Strategic investment under uncertainty: A synthesis. *European Journal of Operational Research* 215(3):639–650.

Chevalier-Roignant B, Villeneuve S, Delpech F, Grapotte ML (2025) Coinvestment game under uncertainty. *Journal of Economic Dynamics and Control* 175(105098).

Chod J, Rudi N (2006) Strategic investments, trading, and pricing under forecast updating. *Management Science* 52(12):1913–1929.

Chod J, Trichakis N, Tsoukalas G (2019) Supplier diversification under buyer risk. *Management Science* 65(7).

Cohen MA, Cui S, Ernst R, Huchzermeier A, Kouvelis P, Lee HL, Matsuo H, Steuber M, Tsay AA (2018) Benchmarking global production sourcing decisions: Where and why firms offshore and reshore. *Manufacturing and Service Operations Management* 20(3):389–402.

Corbett CJ, Karmarkar US (2001) Competition and structure in serial supply chains with deterministic demand. *Management Science* 47(7):966–978.

Dada M, Petruzzi NC, Schwarz LB (2007) A newsvendor's procurement problem when suppliers are unreliable. *Manufacturing and Service Operations Management* 9(1):9–32.

De Wolf D, Smeers Y (1997) A stochastic version of a Stackelberg-Nash-Cournot equilibrium model. *Management Science* 43(2):190–197.

DeMiguel V, Xu H (2009) A stochastic multiple-leader Stackelberg model: Analysis, computation, and application. *Operations Research* 57(5):1220–1235.

Demirel S, Kapuscinski R, Yu M (2018) Strategic behavior of suppliers in the face of production disruptions. *Management Science* 64(2):533–551.

Dixit AK, Pindyck RS (1994) *Investment under Uncertainty* (Princeton University Press).

Fang F, Jiang B, Sun J (2023) Partial vertical ownership in the presence of downstream competition. *Production and Operations Management* 32(6):1692–1704.

Federgruen A, Yang N (2009) Optimal supply diversification under general supply risks. *Operations Research* 57(6):1451–1468.

Greenhut ML, Ohta H (1979) Vertical integration of successive oligopolists. *American Economic Review* 69(1):137–141.

Guan H, Gurnani H, Geng X, Luo Y (2019) Strategic inventory and supplier encroachment. *Manufacturing and Service Operations Management* 21(3):536–555.

Gurnani H, Gerchak Y (2007) Coordination in decentralized assembly systems with uncertain component yields. *European Journal of Operational Research* 176(3):1559–1576.

Gurnani H, Mehrotra A, Ray S (2013) *Supply chain disruptions: Theory and practice of managing risk*, volume 9780857297785 (Springer-Verlag London Ltd).

Huang X, Boyaci T, Gümüş M, Ray S, Zhang D (2016) United we stand or divided we stand? Strategic supplier alliances under order default risk. *Management Science* 62(5).

Huberts NF, Wen X, Dawid H, Huisman K, Kort P (2025) Double marginalization due to external financing: Capacity investment under uncertainty. *Management Science* 71(5):3749–3767.

Jain R, Reindorp M, Chockalingam A (2023) Buyer-backed purchase-order financing for SME supplier with uncertain yield. *European Journal of Operational Research* 307(2).

Jullien B, Reisinger M, Rey P (2023) Personalized pricing and distribution strategies. *Management Science* 69(3):1687–1702.

Kleindorfer PR, Saad GH (2005) Managing disruption risks in supply chains. *Production and Operations Management* 14(1):53–68.

Kouvelis P, Xu F (2021) A supply chain theory of factoring and reverse factoring. *Management Science* 67(10).

Li CL, Kouvelis P (1999) Flexible and risk-sharing supply contracts under price uncertainty. *Management Science* 45(10):1378–1398.

Liu B, Guan X, Wang Y (2021) Supplier encroachment with multiple retailers. *Production and Operations Management* 30(10):3523–3539.

Lücker F, Timonina-Farkas A, Seifert RW (2024) Balancing resilience and efficiency: A literature review on overcoming supply chain disruptions. *Production and Operations Management* 34(6):1495–1511.

Moon Y, Yao T, Park S (2011) Price negotiation under uncertainty. *International Journal of Production Economics* 134(2):413–423.

Osadchiy N, Gaur V, Seshadri S (2016) Systematic risk in supply chain networks. *Management Science* 62(6):1755–1777.

Salinger MA (1988) Vertical mergers and market foreclosure. *Quarterly Journal of Economics* 103(2):345–356.

Shen B, Choi TM, Minner S (2019) A review on supply chain contracting with information considerations: Information updating and information asymmetry. *International Journal of Production Research* 57(15-16):4898–4936.

Simchi-Levi D, Zhao Y (2003) The value of information sharing in a two-stage supply chain with production capacity constraints. *Naval Research Logistics* 50(8):888–916.

Sodhi MS, Son BG, Tang CS (2012) Researchers' perspectives on supply chain risk management. *Production and Operations Management* 21(1).

Spengler JJ (1950) Vertical integration and antitrust policy. *Journal of Political Economy* 58(4):347–352.

Swinney R, Netessine S (2009) Long-term contracts under the threat of supplier default. *Manufacturing and Service Operations Management* 11(1):109–127.

Tang CS, Alex Yang S, Wu J (2017) Sourcing from suppliers with financial constraints and performance risk. *Manufacturing and Service Operations Management* 20(1).

Tang SY, Gurnani H, Gupta D (2014) Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Production and Operations Management* 23(7):1198–1211.

Tirole J (1988) *The Theory of Industrial Organization* (The MIT Press).

Tomlin B (2006) On the value of mitigation and contingency strategies for managing supply chain disruption Risks. *Management Science* 52(5):639–657.

Trigeorgis L (1996) *Real options* (The MIT Press).

Trigeorgis L, Tsekrekos AE (2018) Real options in operations research: A review. *European Journal of Operational Research* 270(1):1–24.

Wadecki A, Babich V, Wu O (2013) Manufacturer competition and subsidies to suppliers. Gurnani H, Mehrotra A, Ray S, eds., *Supply chain disruptions: Theory and practice of managing risk*, chapter 6, 141–173 (Springer).

Yang SA, Birge JR, Parker RP (2015) The supply chain effects of bankruptcy. *Management Science* 61(10).

Mathematical proofs

A Proof of Proposition 1

We show the existence of a symmetric equilibrium below, when $v \geq 1$. For the inverse demand function in eq. (1) and an arbitrary input price w, we get the buyer's profit $\pi(q; y, w) = yq^{1-\delta} - wq$. It follows from the definition in eq. (2) that $(1-\delta)y\bar{q}(y,w)^{-\delta} = w$. The supplier i's optimization problem in eq. (3) now reads

$$\max_{q_i \geq 0} J(q_i, z), \quad \text{where} \quad J(q,z) := q\underbrace{(1-\delta)y(q+z)^{-\delta}}_{\text{market price}} - \frac{c}{v}q^v. \tag{26}$$

The map from a $z \geq 0$ to the solution(s) of that parametrized optimization problem is called "best-reply correspondence" (resp., "best-reply function" if the solution is unique for each z). We recall that $\delta \in [0,1]$. The partial derivatives of $q \mapsto J(q,z)$ in eq. (26) with respect to q are:

$$\begin{aligned} \frac{\partial J}{\partial q}(q,z) &= y(1-\delta)(q+z)^{-\delta-1}\big[(1-\delta)q + z\big] - cq^{v-1}, \\ \frac{\partial^2 J}{\partial q^2}(q,z) &= -y(1-\delta)\delta(q+z)^{-\delta-2}\big[(1-\delta)q + 2z\big] - c(v-1)q^{v-2}. \end{aligned} \tag{27}$$

Under the condition $v \geq 1$ and $0 \leq \delta \leq 1$, the function $q \mapsto J(q,z)$ is concave on $\mathbb{R}_+$ and attains its maximum value at $\bar{q}(z)$ solution of $\frac{\partial J}{\partial q}(\bar{q}(z), z) = 0$. We conjecture the existence of a symmetric equilibrium, that is, a quantity level q_n such that

$$z_n = (n-1)q_n \text{ and } \bar{q}(z_n) = q_n.$$

If this conjecture holds, then the individual supplier's output $\bar{q}_n$ solves $\frac{\partial J}{\partial q}(\bar{q}(z_n), z_n) = 0$ or equivalently,

$$y(1-\delta)[nq_n]^{-\delta-1}(n-\delta)q_n - cq_n^{v-1} = 0. \tag{28}$$

This gives eq.eq. (10) the result for $v \geq 1$.

From eq. (10), we get the aggregate quantity:

$$\bar{Q}_n(y) = n\bar{q}_n(y) = n^{\frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c} y \left[1 - \tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}} .$$

It follows that the equilibrium price in the input market in eq. (6) is given by

$$\begin{aligned}
\bar{w}_n(y) =& (1-\delta) y \bar{Q}_n(y)^{-\delta} \\
=& (1-\delta) y \left[n^{\frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c} y \left[1 - \tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}} \right]^{-\delta} \\
=& \left[(1-\delta) y\right]^{\frac{v-1}{\delta+v-1}} n^{-\delta \frac{v-1}{\delta+v-1}} \left(\tfrac{1}{c} \left[1 - \tfrac{\delta}{n}\right]\right)^{-\frac{\delta}{\delta+v-1}} , \\
=& (1-\delta) y^{\frac{v-1}{\delta+v-1}} n^{-\delta \frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c} \left[1 - \tfrac{\delta}{n}\right]\right)^{-\frac{\delta}{\delta+v-1}} ,
\end{aligned} \tag{29}$$

while the equilibrium price in the output market in eq. (7) is

$$\begin{aligned}
\bar{P}_n(y) &= y \bar{Q}_n(y)^{-\delta} \\
&= y^{\frac{v-1}{\delta+v-1}} n^{-\delta \frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c} \left[1 - \tfrac{\delta}{n}\right]\right)^{-\frac{\delta}{\delta+v-1}} .
\end{aligned}$$

Clearly, $\bar{w}_n = (1-\delta)\bar{P}_n < \bar{P}_n$. The buyer's equilibrium profit in eq. (8) now reads

$$\begin{aligned}
\bar{\pi}_n(y) &= \left[\bar{P}_n(y) - \bar{w}_n(y)\right] \bar{Q}_n(y) \\
&= (1 - 1 + \delta) \bar{P}_n(y) \bar{Q}_n(y) \\
&= \delta y \bar{Q}_n^{1-\delta} \\
&= \delta y \left[n^{\frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c} y \left[1 - \tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}} \right]^{1-\delta} \\
&= \delta \left(n^{v-1} \tfrac{1-\delta}{c} \left[1 - \tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}} .
\end{aligned} \tag{30}$$

The profit of one supplier in eq. (9) is here given by

$$
\begin{aligned}
\pi_n(y) =& \bar{w}_n \bar{q}_n(y) - C(\bar{q}_n) \\
=& (1-\delta) y^{\frac{v-1}{\delta+v-1}} n^{-\delta \frac{v-1}{\delta+v-1}} \left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{-\frac{\delta}{\delta+v-1}} \times n^{-\frac{\delta}{\delta+v-1}} \left(\tfrac{1-\delta}{c} y \left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}} \\
& - \tfrac{c}{v}\left[n^{-\frac{\delta}{\delta+v-1}} \left(\tfrac{1-\delta}{c} y \left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}}\right]^v \\
=& (1-\delta) y^{\frac{v}{\delta+v-1}} n^{-\frac{\delta}{\delta+v-1}v} \left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} - \tfrac{c}{v} n^{-\frac{\delta}{\delta+v-1}v} \left(\tfrac{1-\delta}{c} y \left[1-\tfrac{\delta}{n}\right]\right)^{\frac{v}{\delta+v-1}} \\
=& y^{\frac{v}{\delta+v-1}} n^{-\frac{\delta}{\delta+v-1}v} \left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} \left[1-\delta - \tfrac{c}{v}\left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{\delta+v-1}{\delta+v-1}}\right] \\
=& y^{\frac{v}{\delta+v-1}} n^{-\frac{\delta}{\delta+v-1}v} \left(\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} \tfrac{1-\delta}{v}\left[v-1+\tfrac{\delta}{n}\right]. \qquad (31)
\end{aligned}
$$

This completes the proof of Proposition 1.

Successive Cournot oligopolies with m buyers. Take $v \geq 1$. In the case with $m \in \mathbb{N}$ buyers, the demand function in the input market obtains from a symmetric Cournot-Nash equilibrium in the output market, i.e., from solving the fixed-point equation

$$
\mathrm{q} = \mathrm{R}\left((m-1)\mathrm{q}\right), \quad \text{where} \quad \mathrm{R}(z) := \arg\max_{q_i \geq 0} \left\{ q_i P(q_i + z, y) - w q_i \right\}, \qquad (32)
$$

where $m \in \mathbb{N}$ denotes the number of buyers and $z = \sum_{j=1}^{m} q_i \geq 0$.

We solve the fixed-point (32) for the specifications in eq. (1) and readily obtain that the suppliers face the inverse demand function

$$
q \mapsto y\left[1 - \frac{\delta}{m}\right] q^{-\delta}.
$$

Following the same methodology as earlier, we get that the output of one of the buyers is given by

$$
\bar{q}_{m,n} = \frac{n^{\frac{v-1}{\delta+v-1}}}{m} \left(\frac{1-\frac{\delta}{m}}{c} y \left[1 - \frac{\delta}{n}\right] \right)^{\frac{1}{\delta+v-1}}
$$

That buyer's profit is given by

$$
\begin{aligned}
\bar{\pi}_{m,n} &= \underbrace{\frac{n^{\frac{v-1}{\delta+v-1}}}{m}\left(\frac{1-\frac{\delta}{m}}{c}y\left[1-\frac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}}}_{\text{output}}\underbrace{y\left[1-1+\frac{\delta}{m}\right]n^{-\delta\frac{v-1}{\delta+v-1}}\left(\frac{1-\frac{\delta}{m}}{c}y\left[1-\frac{\delta}{n}\right]\right)^{-\frac{\delta}{\delta+v-1}}}_{\text{margin}} \\
&= \frac{\delta}{m^2}\left(n^{v-1}\frac{1-\frac{\delta}{m}}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}}.
\end{aligned}
$$

We note that

$$
\begin{aligned}
\bar{q}_{1,n} &= n^{\frac{v-1}{\delta+v-1}}\left(\frac{1-\delta}{c}y\left[1-\frac{\delta}{n}\right]\right)^{\frac{1}{\delta+v-1}} \\
\bar{\pi}_{1,n} &= \delta\left(n^{v-1}\tfrac{1-\delta}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}},
\end{aligned}
$$

which confirms results in Proposition 1.

We have

$$
\begin{aligned}
\frac{\partial \bar{\pi}_{m,n}}{\partial m} =& -2\frac{\delta}{m^3}\left(n^{v-1}\frac{1-\frac{\delta}{m}}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}} \\
& -\frac{\delta^2}{m^3}\frac{1-\delta}{\delta+v-1}\left(n^{v-1}\frac{1-\frac{\delta}{m}}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}-1} y^{\frac{v}{\delta+v-1}} \\
=& -\left(n^{v-1}\frac{1-\frac{\delta}{m}}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}-1} y^{\frac{v}{\delta+v-1}}\frac{\delta}{m^3}\left\{2n^{v-1}\frac{1-\frac{\delta}{m}}{c}\left[1-\frac{\delta}{n}\right]+\delta\frac{1-\delta}{\delta+v-1}\right\} < 0.
\end{aligned}
$$

Hence,

Proposition 13 (Supply-chain equilibrium with n suppliers and m buyers) *Given the specifications of eq.* (1) *with $v \geq 1$, where there are m buyers sourcing from n suppliers, the profit of each buyer, $\bar{\pi}_{m,n}$, satisfies*

$$
0 \leq \bar{\pi}_{m,n}(y) := \tfrac{\delta}{m^2}\left(n^{v-1}\tfrac{1-\delta/m}{c}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}} \leq \bar{\pi}_n(y), \quad \forall m \in \mathbb{N}.
$$

As expected, horizontal competition among buyers depresses the individual profit of each buyer compared to the monopsonic benchmark of Proposition 1. Our paper focuses on the strategic interactions along the supply chain, not at one specific echelon. From that perspective, we leverage the supply chain model with 1 buyer in Proposition 1, rather than a setup with m buyers.

B Proof of Proposition 2

For notational simplicity, the dependence on the variable y is suppressed hereafter. First, using the notation of Proposition 1, a supplier, who anticipates that its competitors will not produce, faces the following problem:

$$\max_q J(q, 0).$$

By computing the derivatives of this function up to the third order, we show that $J(\cdot, 0)$ is concave and attains its maximum at $\bar{q}_1$, which corresponds to the best reply of 0.

Take a $z > 0$. It thus remains to check that $q \mapsto J_1(q) := J(q, z) = (1-\delta)y(z+q)^{-\delta}q - \frac{c}{v}q^v$ in eq. (26) attains a maximum on $[0, \infty)$ at 0. Similarly, as in the proof of Proposition 1, we have

$$J_1'(q) = q^{v-1}\big[(1-\delta)y(z+q)^{-(1+\delta)}(z+(1-\delta)q)q^{1-v} - c\big].$$

Thus, the function J_1' has the same sign as the function

$$\theta(q) = (1-\delta)y(z+q)^{-(1+\delta)}\big\{z+(1-\delta)q\big\}q^{1-v} - c.$$

Because $1 - v > 0$ and $\alpha = v - (1-\delta) > 0$ under the economies of scale assumption, we have

$$\lim_{q \to 0+} \theta(q) = \lim_{q \to \infty} \theta(q) = -c.$$

A straightforward but tedious computation shows that the function θ' has the same sign as the polynomial function of degree 2 given by

$$pol(q) = (1-\delta)\alpha q^2 + (2\alpha + \delta(1-v))zq - (1-v)z^2.$$

Because $pol(0) < 0$, θ' changes sign only once on $(0, \infty)$ which implies that θ and thus J_1' are nonpositive. As a consequence, the function J_1 decreases attaining its maximum on $[0,\infty)$ at 0. A strategy profile where more than one supplier produces a nonzero quantity thus cannot be a Nash equilibrium.

The results for the equilibrium input and output prices, as well as for the buyer's profit, are immediate.

C Proof of Proposition 3

Case $1-\delta \le v < 1$. The result is immediate from Proposition 2.

Case $v \ge 1$. We note that $n \mapsto 1 - \delta/n$ is increasing. We get from Proposition 1 that

$$\frac{\bar{Q}_{n+1}}{\bar{Q}_n} = \underbrace{\left(1+\frac{1}{n}\right)}_{\ge 1}^{\frac{v-1}{\delta+v-1}} \underbrace{\left(\frac{1-\frac{\delta}{n+1}}{1-\frac{\delta}{n}}\right)}_{\ge 1}^{\frac{1}{\delta+v-1}} \ge 1 \text{ (both exponents are positive)},$$

$$\frac{\bar{w}_{n+1}}{\bar{w}_n} = \left(1+\frac{1}{n}\right)^{-\delta\frac{v-1}{\delta+v-1}} \left(\frac{1-\frac{\delta}{n+1}}{1-\frac{\delta}{n}}\right)^{-\frac{\delta}{\delta+v-1}} \le 1 \text{ (both exponents are negative)}$$

$$\frac{\bar{P}_{n+1}}{\bar{P}_n} = \left(1+\frac{1}{n}\right)^{-\delta\frac{v-1}{\delta+v-1}} \left(\frac{1-\frac{\delta}{n+1}}{1-\frac{\delta}{n}}\right)^{-\frac{\delta}{\delta+v-1}} \le 1 \text{ (both exponents are negative)}$$

$$\frac{a_{n+1}}{a_n} = \left(1+\frac{1}{n}\right)^{v-1} \times \left(\frac{1-\frac{\delta}{n+1}}{1-\frac{\delta}{n}}\right)^{\frac{1-\delta}{\delta+v-1}} \ge 1 \text{ (both exponents are positive)}.$$

This concludes the proposition.

D Proof of Proposition 4

We first consider the case of a vertically integrated firm where the buyer profit is given by

$$\Pi(y) := \max_{q\ge 0} \Big\{ qP(y,q) - C(q) \Big\} \in \mathbb{R}_+. \tag{33}$$

Given the specifications in eq. (1), we want to maximize $q \mapsto J(q) := yq^{1-\delta} - \frac{cq^v}{v}$ over $[0,\infty)$. For $v \ge 1$, we have $J''(q) = -(1-\delta)\delta y q^{-\delta-1} - c(v-1)q^{v-2} \le 0$ for all $q > 0$. So $q \mapsto J'(q) =$

$(1-\delta)yq^{-\delta} - cq^{v-1}$ decreases on $(0,\infty)$ from ∞ to $-c$ (if $v=1$) and $-\infty$ (if $v>1$). The maximizer obtains from the first-order condition and is explicitly given by $\left(\frac{1-\delta}{c}y\right)^{\frac{1}{\delta+v-1}}$. After simplifications, we obtain

$$J\left(\left[\tfrac{1-\delta}{c}y\right]^{\frac{1}{\delta+v-1}}\right) = \frac{\delta+v-1}{v}\left(\frac{1-\delta}{c}\right)^{\frac{1-\delta}{\delta+v-1}} y^{\frac{v}{\delta+v-1}},$$

which gives the following expression for $\Pi(y)$

$$\Pi(y) := \frac{1}{\epsilon}\left(\frac{1-\delta}{c}\right)^{\frac{1-\delta}{\delta+v-1}} y^{\epsilon}.$$

We introduce

$$\chi := \frac{\Pi(y)}{\bar{\pi}_n(y)}, \tag{34}$$

which captures whether the buyer is better off vertically integrating (if $\chi > 1$) or not (if $\chi \le 1$). Given the expressions for $\Pi(y)$ in Proposition 4 and $\bar{\pi}_n(y)$ in Proposition 1, we get

$$\begin{aligned}\chi(n,v) =& \frac{\frac{1}{\epsilon}\left(\frac{1-\delta}{c}\right)^{\frac{1-\delta}{\delta+v-1}}}{\delta\left(n^{v-1}\frac{1-\delta}{c}\left[1-\frac{\delta}{n}\right]\right)^{\frac{1-\delta}{\delta+v-1}}} \text{ from the expressions for } \Pi(y) \text{ and Proposition 1} \\ =& \tfrac{\delta+v-1}{\delta v}\left(n^{v-1}\left[1-\tfrac{\delta}{n}\right]\right)^{\frac{\delta-1}{\delta+v-1}} \text{ from the definition of } \epsilon.\end{aligned}$$

After simplifications,

$$\frac{\partial\chi}{\partial n}(n,v) = -\frac{1-\delta}{\delta v}n^{v-3}\left(n^{v-1}[1-\tfrac{\delta}{n}]\right)^{-\frac{v}{v+\delta-1}}\Big\{n(v-1)-\delta(v-2)\Big\}.$$

In the <u>case with $v=1$</u>, we have

$$\begin{aligned}\frac{\partial\chi}{\partial n}(n,1) =& -(1-\delta)n^{-2}\left(1-\tfrac{\delta}{n}\right)^{-\frac{1}{\delta}} < 0 \\ \lim_{n\to\infty}\chi(n,1) =& \lim_{n\to\infty}\left[1-\tfrac{\delta}{n}\right]^{1-\frac{1}{\delta}} = 1.\end{aligned}$$

Hence $\chi(1,1) > \chi(2,1) > \cdots > 1$.

In the case with $v > 1$,

$$\frac{\partial\chi}{\partial n}(n,v) = -\underbrace{\tfrac{v-1}{v}\tfrac{1-\delta}{\delta}n^{v-3}\left(n^{v-1}[1-\tfrac{\delta}{n}]\right)^{-\frac{v}{v+\delta-1}}}_{\geq 0}\left\{n-\delta\tfrac{v-2}{v-1}\right\},$$

$$\chi(1,v) = \frac{\delta+v-1}{\delta v}(1-\delta)^{\frac{\delta-1}{\delta+v-1}} > 1$$

$$\lim_{n\to\infty}\chi(n,v) = 0.$$

Define $n_\star := \inf\{n \in \mathbb{N} \big| n \geq \delta\frac{v-2}{v-1}\}$. It follows from the above that $n \mapsto \chi(n,v)$ is above 1 for any $n \in \{1,\ldots,n_\star\}$. Furthermore, there exists a finite $\tilde{n} \in \{n_\star,\ldots\}$ such that $1 > \chi(n,v) > 0$ for all $n \geq \tilde{n}$. This completes the proof.

E Proof of Proposition 5

Step 0 – Present value. The term $\bar{u}_n(y)$ in eq. (12) then becomes

$$\bar{u}_n(y) = a_n\mathbb{E}^y\int_0^\infty e^{-rt}Y_t^\epsilon \mathrm{d}t.$$

We now consider the stochastic process $(Y_t^\epsilon)_t$. By the Itô-Döblin formula,

$$\begin{aligned}\mathrm{d}Y_t^\epsilon &= \left[\tfrac{1}{2}\sigma^2Y_t^2\epsilon(\epsilon-1)Y_t^{\epsilon-2} + \mu Y_t\epsilon Y_t^{\epsilon-1}\right]\mathrm{d}t + \sigma Y_t\epsilon Y_t^{\epsilon-1}\mathrm{d}Z_t \\ &= \mathrm{m}(\epsilon)Y_t^\epsilon\mathrm{d}t + \sigma\epsilon Y_t^\epsilon\mathrm{d}Z_t,\end{aligned}$$

where

$$\mathrm{m}(\epsilon) := \tfrac{1}{2}\sigma^2\epsilon(\epsilon-1)+\mu\epsilon. \tag{35}$$

So $(Y_t^\epsilon)_t$ follows a GBM. It follows by standard properties of GBMs that

$$\mathbb{E}^yY_t^\epsilon = y^\epsilon e^{\mathrm{m}(\epsilon)t} \text{ and } \mathbb{E}^y\int_0^\infty e^{-rt}Y_t^\epsilon\mathrm{d}t = y^\epsilon\int_0^\infty e^{\mathcal{Q}(\epsilon)t}\mathrm{d}t, \tag{36}$$

for $\mathcal{Q}(\cdot)$ given in eq. (13), converges to

$$\mathbb{E}^y \int_0^\infty e^{-rt} Y_t^\epsilon \mathrm{d}t = -\frac{y^\epsilon}{\mathcal{Q}(\epsilon)} \text{ iff } \mathcal{Q}(\epsilon) < 0.$$

Step 1 – Dynamic programming equation. Equation (14) describes the classical problem of McDonald and Siegel (1986). We drop the index n in the notation α_n and introduce the differential operator

$$\mathcal{L} := \frac{1}{2}\sigma^2 y^2 \frac{\partial^2}{\partial y^2} + \mu y \frac{\partial}{\partial y} - r\mathbb{I}, \tag{37}$$

with $\mathbb{I}$ denoting the identity operator. The dynamic programming equation for the problem in eq. (14) is a variational inequality (VI), namely

$$\begin{cases} \max\left\{\alpha y^\epsilon - I - \bar{\psi}(y); \mathcal{L}\bar{\psi}(y)\right\} & = 0, \quad \text{a.e. } y > 0, \\ \lim_{y \downarrow 0} \bar{\psi}(y) & = 0, \\ \lim_{y \uparrow \infty} \dfrac{\bar{\psi}(y)}{\alpha y^\epsilon} & = 1. \end{cases} \tag{38}$$

We have

$$\mathcal{L}(\alpha \cdot^\epsilon - I)(y) \equiv -[r - \mathrm{m}(\epsilon)]\alpha y^\epsilon + rI \tag{39}$$

for $\mathrm{m}(\cdot)$ given in eq. (35). If $r > \mathrm{m}(\epsilon)$, $y \mapsto \mathcal{L}(\alpha \cdot^\epsilon - I)(y)$ is monotone decreasing on $(0, \infty)$ from $rI > 0$ to $-\infty$, so it has a unique root, denoted $y^\star$. We conjecture that $\{\bar{\psi} > \alpha \cdot^\epsilon - I\} = (0, \bar{y}) \subset (0, y^\star)$, where $\bar{y}$ obtains by smooth fit. If this conjecture holds, then $\bar{\psi}(\cdot)$ solves the free-boundary problem (FBP)

$$\begin{aligned} \bar{\psi}(0+) &= 0, \\ \mathcal{L}\bar{\psi}(y) &= 0, \quad \forall y \in (0, \bar{y}), \\ \bar{\psi}(\bar{y}) &= \alpha \bar{y}^\epsilon - I, \\ \bar{\psi}'(\bar{y}) &= \alpha \epsilon y^{\epsilon - 1}. \end{aligned}$$

The function $\mathcal{Q}(\cdot)$ in eq. (13) is convex, attains its minimum at the point $\gamma_\star := -\frac{\mu - \frac{1}{2}\sigma^2}{\sigma^2}$, and satisfies $\mathcal{Q}(\pm\infty) = \infty$. Further, because $\mathcal{Q}(1) = \mathrm{m}(\epsilon) - r < 0$, the minimum is necessarily a negative minimum and $\mathcal{Q}(\cdot)$ has a positive root γ_+, which is unique because $\mathcal{Q}(\cdot)$ is monotone increasing on $(\max\{1;\gamma_\star\},\infty)$. Standard computations lead us to conclude that the function $\bar{\psi}(\cdot)$ given in Proposition 5 solves the FBP.

It remains to verify that this $\bar{\psi}(\cdot)$ solves the variational ineq. (38). We look at the two intervals.

$\underline{(\bar{y},\infty)}$. For $\bar{\psi}$ to solve the VI in this interval, we must have $\mathcal{L}\bar{\psi} = \mathcal{L}(\alpha\cdot^{\epsilon} - I)(y) \leq 0$. From eq. (39) and the expression for $\bar{y}$,

$$\begin{aligned}\mathcal{L}(\alpha\cdot^{\epsilon} - I)(\bar{y}) = &= I[r - \mathrm{m}(\epsilon)]\Big[\frac{1}{1-\frac{\epsilon}{\gamma_+}} - \frac{1}{1-\frac{\mathrm{m}(\epsilon)}{r}}\Big] \\ &= \underbrace{-I[r-\mathrm{m}(\epsilon)]}_{<0}\int_{\frac{\mathrm{m}(\epsilon)}{r}}^{\frac{\epsilon}{\gamma_+}} \underbrace{\frac{1}{(1-\zeta)^2}}_{>0}\,\mathrm{d}\zeta.\end{aligned}$$

But, it follows from eq. (13) after simplifications that

$$\mathcal{Q}\left(\tfrac{r\epsilon}{\mathrm{m}(\epsilon)}\right) = \frac{1}{2}\frac{r-\mathrm{m}(\epsilon)}{[\mu+\frac{1}{2}\sigma^2(\epsilon-1)]^2}r\sigma^2 > 0 \text{ because } r > \mathrm{m}(\epsilon).$$

Because $\mathcal{Q}(\cdot)$ is monotone increasing on $(\max\{1;\gamma_\star\},\infty)$ and $\mathcal{Q}(\infty)=\infty$, the root γ satisfies $\gamma < \frac{r\epsilon}{\mathrm{m}(\epsilon)}$. It immediately follows $\frac{\epsilon}{\gamma_+} > \frac{\mathrm{m}(\epsilon)}{r}$. Hence, $\mathcal{L}(\alpha\cdot^{\epsilon} - I)(\bar{y}) < 0$. So the FBP's solution $\bar{\psi}(\cdot)$ verifies the VI in the interval $(\bar{y},\infty)$.

$\underline{(0,\bar{y})}$. We want to verify that $\bar{\psi}(y) \geq \alpha y^{\epsilon} - I$. We note that $\bar{\psi}(\cdot)$ also reads

$$\bar{\psi}(y) = \frac{\alpha\epsilon}{\gamma}y^{\gamma}\bar{y}^{\epsilon-\gamma} \text{ in the interval } (0,\bar{y}).$$

We define

$$\bar{\Psi}(y) := \bar{\psi}(y) - \alpha y^{\epsilon} + I, \tag{40}$$

In the interval $(0,\bar{y})$,

$$\bar{\Psi}'(y) = \alpha\epsilon\left[y^{\gamma-1}\bar{y}^{\epsilon-\gamma} - y^{\epsilon-1}\right] = \alpha\epsilon y^{\epsilon-1}\left[\left(\frac{y}{\bar{y}}\right)^{\gamma-\epsilon} - 1\right].$$

We note that

$$\mathcal{Q}(\epsilon) = \frac{1}{2}\sigma^2\epsilon(\epsilon-1) + \mu\epsilon - r = -[r - \mathrm{m}(\epsilon)],$$

for $\mathrm{m}(\cdot)$ given in eq. (35). Given the assumption $r > \mathrm{m}(\epsilon)$, it follows that $\mathcal{Q}(\epsilon) < 0$ and, so, given the behavior of $\mathcal{Q}(\cdot)$, we have $\gamma_+ > \epsilon$. It follows that

$$\bar{\Psi}'(\cdot) < 0 \text{ on } (0, \bar{y}). \tag{41}$$

Further, $\bar{\Psi}(\bar{y}) = 0$ by value matching. Hence, $\bar{\Psi}(\cdot)$ necessarily decreases on $(0, \bar{y})$ from a positive value and vanishes at the right boundary. It follows that $\bar{\psi}(y) \geq \alpha y^\epsilon - I$ and, so, that the FBP's solution $\bar{\psi}(\cdot)$ solves the VI in this interval as well.

Step 2 – Verification theorem. We conclude with the verification theorem. Let $\bar{\psi}$ be a supersolution of the variational ineq. (38). For an arbitrary stopping time τ, it follows from Dynkin's formula that

$$\begin{aligned}\bar{\psi}(y) &= \mathbb{E}^y\Big[e^{-r\tau}\underbrace{\bar{\psi}(Y_\tau)}_{\geq \alpha Y_\tau^\epsilon - I} - \int_0^\tau e^{-rt}\underbrace{\mathcal{L}\bar{\psi}(Y_t)}_{\leq 0}\,\mathrm{d}t\Big]\\ &\geq \mathbb{E}^y e^{-r\tau}\big\{\alpha Y_\tau^\epsilon - I\big\}.\end{aligned}$$

Then, a supersolution of the VI exceeds the value function. Let $\bar{\psi}(\cdot)$ denote the classical solution of the VI and take $\bar{\tau} := \inf\big\{t \geq 0 \big| \bar{\psi}(Y_t) \geq \alpha Y_t - I\big\}$. Proceeding similarly, we obtain that the solution of the VI is the smallest supersolution and coincides with the value function in eq. (14).

The result for $\bar{\phi}_n(\cdot)$ follows from standard results on first-stopping times and GBMs. This concludes the proof of Proposition 5.

F Proof of Proposition 6

Assume that the conditions in Proposition 5 are met. If $a_{n+1} \geq a_n$, it follows from eq. (15) that $\alpha_{n+1} Y_\tau^\epsilon - I \geq \alpha_n Y_\tau^\epsilon - I$ and, so, that

$$\bar{\psi}_{n+1}(y) \geq \mathbb{E}^y e^{-r\bar{\tau}_n} \Big[\alpha_{n+1} Y_{\bar{\tau}_n}^\epsilon - I\Big] \geq \mathbb{E}^y e^{-r\bar{\tau}_n} \Big[\alpha_n Y_{\bar{\tau}_n}^\epsilon - I\Big] = \bar{\psi}_n(y),$$

where the last equality comes from the optimality of the stopping time $\bar{\tau}_n$ for the value function $\bar{\psi}_n$. Furthermore, $\bar{y}_N \leq \bar{y}_n$ by monotonicity of the map $a \mapsto \left(\frac{\gamma_+}{\gamma_+ - \epsilon} \frac{I}{a}\right)^{\frac{1}{\epsilon}}$.

On the other hand, let us define $\bar{\Psi}_n(y) = \bar{\psi}_n(y) - (\bar{u}_n(y) - I)$. Clearly, $\bar{\Psi}_n(y) \geq \bar{\Psi}_{n+1}(y)$ for $y \geq \bar{y}_{n+1}$ because $\bar{y}_{n+1} < \bar{y}_n$. If we define $\Delta := \bar{\Psi}_n - \bar{\Psi}_{n+1}$, we thus have $\Delta(y_{n+1}) \geq 0$ and $\Delta(0) = 0$. Moreover, we have for $y \in [0, \bar{y}_{n+1}]$

$$\begin{aligned}
\mathcal{L}\Delta(y) &= \mathcal{L}(\bar{u}_{n+1} - \bar{u}_n) \\
&= -\big[r - \mathrm{m}(\epsilon)\big]\big[\alpha_{n+1} - \alpha_n\big] y^\epsilon \text{ from eq. (39)} \\
&\leq 0 \text{ from Proposition 3 and eq. (15).}
\end{aligned}$$

Applying Dynkin's formula, we obtain upon defining τ_0 (resp. $\tau_{\bar{y}_{n+1}}$) the hitting times of 0 (resp. $\bar{y}_{n+1}$),

$$\begin{aligned}
0 &\leq \mathbb{E}\left[e^{-r(\tau_0 \wedge \tau_{\bar{y}_{n+1}})} \Delta(Y_{\tau_0 \wedge \tau_{\bar{y}_{n+1}}})\right] \\
&= \Delta(y) + \mathbb{E}\left[\int_0^{\tau_0 \wedge \tau_{\bar{y}_{n+1}}} e^{-rs} \mathcal{L}\Delta(Y_s)\, ds\right] \\
&\leq \Delta(y).
\end{aligned}$$

This completes the proof.

G Proof of Proposition 7

Present value. We will show that the term $\tilde{u}_n(\cdot)$ in eq. (16) has an explicit expression. First, upon recalling the definition of $\mathrm{m}(\epsilon)$ in eq. (35) and using independence,

$$\begin{aligned}
\mathbb{E}\int_0^T e^{-rs} a_n Y_s^\epsilon \mathrm{d}s &= \int_0^\infty \Big(\int_0^t e^{-rs} a_n \mathbb{E}[Y_s^\epsilon]\mathrm{d}s\Big)\lambda e^{-\lambda t}\mathrm{d}t \\
&= a_n y^\epsilon \int_0^\infty e^{-(r-\mathrm{m}(\epsilon))s}\int_s^\infty \lambda e^{-\lambda t}\mathrm{d}t\mathrm{d}s \text{ by Fubini's theorem} \\
&= a_n y^\epsilon \int_0^\infty e^{-(r+\lambda-\mathrm{m}(\epsilon))s}\mathrm{d}s \\
&= \frac{a_n}{r+\lambda-\mathrm{m}(\epsilon)} y^\epsilon.
\end{aligned}$$

We note the distribution of N by $j(k) := \mathbb{P}(N = k) \in [0, 1]$ with $\sum_{k=0}^{n-1} j(k) = 1$. For the second term, we have by independence again,

$$\begin{aligned}
\mathbb{E}e^{-rT}\alpha_N Y_T^\epsilon &= \sum_{k=1}^{n}\int_0^\infty e^{-rt}\alpha_k j(k)\mathbb{E}Y_t^\epsilon \lambda e^{-\lambda t}\mathrm{d}t \\
&= \left[\frac{\lambda}{r+\lambda-\mathrm{m}(\epsilon)}\sum_{k=1}^{n}\alpha_k j(k)\right] y^\epsilon.
\end{aligned}$$

Combining the two terms yields

$$\begin{aligned}
\frac{\tilde{u}_n(y)}{y^\epsilon} &= \frac{a_n}{\lambda-\mathcal{Q}(\epsilon)} + \frac{\lambda}{\lambda-\mathcal{Q}(\epsilon)}\mathbb{E}\alpha_N \text{ from eqs. (13) and (35)} \\
&= \frac{a_n}{\lambda-\mathcal{Q}(\epsilon)} + \frac{\lambda}{\lambda-\mathcal{Q}(\epsilon)}\alpha_n + \frac{\lambda}{\lambda-\mathcal{Q}(\epsilon)}\big\{\mathbb{E}\alpha_N - \alpha_n\big\} \\
&= \frac{\alpha_n}{\lambda-\mathcal{Q}(\epsilon)}\Big\{-\mathcal{Q}(\epsilon)+\lambda\Big\} + \frac{\lambda}{\lambda+\mathcal{Q}(\epsilon)}\big\{\mathbb{E}\alpha_N - \alpha_n\big\} \text{ from eq. (15)} \\
&= \eta_n^{\lambda,N} \text{ given in eq. (17).}
\end{aligned}$$

Comparative statics with respect to λ. It is immediate that $\partial\eta_n/\partial\lambda \leq 0$. Hence, $\lambda_2 \geq \lambda_1$ implies $\eta_n^{\lambda_2,N} \leq \eta_n^{\lambda_1,N}$.

Comparative statics with respect to N. Consider two random variables N_1 and N_2 modeling the number of surviving suppliers after the exogenous disruption. In the case indexed by 2, we expect more suppliers to disappear after the disruption, in the sense that N_2 is stochastically dominated by N_1 (noted $N_2 \preceq N_1$ and understood in the first-order sense). We note that

$$\eta_n^{\lambda,N_2} - \eta_n^{\lambda,N_1} = \frac{\lambda}{\lambda - \mathcal{Q}(\epsilon)}\mathbb{E}[\alpha_{N_2} - \alpha_{N_1}].$$

Because it holds from Proposition 3 and eq. (15) that $\alpha_{n+1} \geq \alpha_n$ and because we assume $N_2 \preceq N_1$, it follows that $\eta_n^{\lambda,N_2} \leq \eta_n^{\lambda,N_1}$. This completes the proof.

H Proof of Proposition 8

Step 0 – Problem setting. At time 0, the buyer would source from a (known) set of n homogeneous suppliers. A disruption will affect the set of suppliers the buyer can source from. The profit at the time of investment τ depends on whether the disruption occurred before or after the investment. Let T denote the disruption date, which we assume exponentially distributed with parameter $\lambda > 0$ and independent of the Brownian filtration $\mathbb{F}$. We define the process $H = (H_t)_t$ by

$$H_t = 1_{\{T \leq t\}}, \quad t \geq 0, \tag{42}$$

a process which takes the value 1 is the disruption already occurred and 0 otherwise. At time $t \geq 0$, the number of remaining, potentially operating suppliers is given by

$$N_t = n(1 - H_t) + NH_t, \tag{43}$$

where N is a random variable with a distribution $j(\cdot)$ over $\{0, \ldots, n-1\}$, i.e., $j(k) = \mathbb{P}(N = k) \in (0,1)$ with $\sum_{k=0}^{n-1} j(k) = 1$. It follows that, at the time T of disruption, at least one potential supplier disappears.

The progressive enlargement of $\mathbb{F}$ with T is defined as $\mathbb{G} := (\mathcal{G}_t)_{t\geq 0}$ with

$$\mathcal{G}_t = \bigcap_{s>t} \left(\mathcal{F}_s \vee \sigma(T \wedge s)\right).$$

The filtration $\mathbb{G}$ is the smallest right-continuous filtration such $\mathbb{F} \subset \mathbb{G}$ and T is a $\mathbb{G}$-stopping time.

According to (Aksamit and Jeanblanc 2017, Remark 4.41), the σ-algebra $\mathcal{G}_t$ coincides with $\mathcal{F}_t \vee \sigma(T)$ on $[T, \infty)$. Because the random variable T is independent of the Brownian motion $B = (B_t)_{t\geq 0}$, B is a Brownian motion with respect to the enlarged filtration $\mathbb{G}$. Hereafter, we denote $\mathcal{T}_{\mathbb{G}}$ the set of $\mathbb{G}$-stopping times. The processes $(H_t)_t$ and $(N_t)_t$ are $\mathbb{G}$-adapted.

If the buyer decides to invest at the stopping time $\tau \in \mathcal{T}_{\mathbb{G}}$, he receives the amount $u(Y_\tau, H_\tau, N_\tau)$ where

$$u(y, h, k) := \tilde{u}_n(y) \times (1 - h) + \bar{u}_k(y) \times h, \tag{44a}$$

where $\bar{u}_k(\cdot)$ is given in eq. (12)—with a closed-form expression in Proposition 5—and $\tilde{u}_n(y)$, given the strong Markov property, reads

$$\tilde{u}_n(y) := \mathbb{E}\Big[\int_0^T e^{-rs}\overline{\pi}_n(Y_s)\,\mathrm{d}s + e^{-rT}\Big\{\sum_{k=0}^{n-1} j(k)\bar{u}_k(Y_T)\Big\}\Big], \tag{44b}$$

which is a rewriting the expression in eq. (16), where the expectation operator in the latter expression also account for distribution of the random variable K. By convention, we set $\bar{u}_0(\cdot) \equiv 0$.

Step 1 – Value process and principle of optimality. We introduce the value process $(\Psi_t)_t$ given by

$$\Psi_t = \operatorname{ess}\sup_{\tau \in \mathcal{T}_{\mathbb{G}}(\sqcup)} \mathbb{E}\left[e^{-r(\tau - t)}\{u(Y_\tau, H_\tau, N_\tau) - I\}|\mathcal{G}_t\right], \tag{45}$$

where $\mathcal{T}_{\mathbb{G}}(t)$ is the set of $\mathbb{G}$-stopping times with values in $[t, \infty)$. We denote $\mathcal{T}_{\mathbb{G}} = \mathcal{T}_{\mathbb{G}}(0)$. According to optimal stopping theory (El Karoui 1981, Peskir and Shiryaev 2006), the process $(e^{-rt}\Psi_t)_t$ is the smallest $\mathbb{G}$-supermartingale that dominates the payoff process $(e^{-rt}\{u_n(Y_t, H_t, N_t) - I\})_t$. Furthermore, Ψ_t can be written as $\Psi_t = \tilde{\psi}(Y_t, H_t, N_t)$, where

$$\begin{aligned} &\tilde{\psi}(y, 1, k) = \bar{\psi}_k(y) \text{ in eq. (14)}, \\ &\tilde{\psi}(y, 0, n) = \sup_{\tau \in \mathcal{T}_{\mathbb{G}}} \mathbb{E}\left[e^{-r\tau}\big(\tilde{u}_n(Y_\tau) - I\big)(1 - H_\tau) + H_\tau e^{-r\tau}\Big(\sum_{k=0}^{n-1} j(k)\big\{\bar{u}_k(Y_\tau) - I\big\}\Big)\right]. \end{aligned} \tag{46}$$

We are in a position to establish the following lemma:

Lemma 1 (Principle of optimality) *The value function in eq.* (46) *can be written as*

$$\tilde{\psi}(y,0,n) = \sup_{\tau \in \mathcal{T}_{\mathbb{F}}} \mathbb{E}\Big[e^{-(r+\lambda)\tau}\big(\tilde{u}_n(Y_\tau) - I\big) + H_\tau e^{-rT} \sum_{k=0}^{n-1} j(k)\bar{\psi}_k(Y_T)\Big]. \tag{47}$$

Furthermore,

$$\tilde{\psi}(y,0,n) \geq \Big\{\eta_n (y \vee y_\star)^\epsilon - I\Big\} \left(\tfrac{y}{y \vee y_\star}\right)^\gamma \text{ with } y_\star := \left(\tfrac{\gamma}{\gamma-1}\tfrac{I}{\eta_n}\right)^{\frac{1}{\epsilon}} \text{ and } \gamma \text{ given in eq. (20).} \tag{48}$$

Proof of Lemma 1 We write the objective functional of the value function $\tilde{\psi}(y,0,n)$ in eq. (46) valued at a $\mathbb{G}$-stopping time τ, in two parts,

$$\begin{aligned} J_1(\tau) :=& \mathbb{E} e^{-r\tau}\big(\tilde{u}_n(Y_\tau) - I\big)(1 - H_\tau) \\ J_2(\tau) :=& \mathbb{E} H_\tau e^{-r\tau}\Big(\sum_{k=0}^{n-1} j(k)\big\{\bar{u}_k(Y_\tau) - I\big\}\Big) \end{aligned}$$

and factorize the stopping time τ as

$$\tau = \sigma 1_{\{\sigma < T\}} + \sigma(T,N,\cdot)1_{\{\sigma \geq T\}},$$

where σ and the family $(\sigma(u,k,\cdot))_{u \geq 0, k \leq n-1}$ are $\mathbb{F}$-stopping times. Using independence between T and the Brownian motion, we first have

$$\begin{aligned} J_1(\tau) &= \mathbb{E}\left[e^{-r\sigma}\big(\tilde{u}_n(Y_\sigma) - I\big)1_{\{\sigma < T\}}\right] \\ &= \mathbb{E}\left[\int_\sigma^\infty \lambda e^{-\lambda t} \mathrm{d}t e^{-r\sigma}\big(\tilde{u}_n(Y_\sigma) - I\big)\right] \\ &= \mathbb{E}\left[e^{-(r+\lambda)\sigma}\big(\tilde{u}_n(Y_\sigma) - I\big)\right]. \end{aligned}$$

For the second part, it follows from the variational ineq. (38) characterizing $\bar{\psi}_k$ that

$$\begin{aligned} J_2(\tau) &\leq \mathbb{E}\left[e^{-r\tau}\sum_{k=0}^{n-1} j(k)\bar{\psi}_k(Y_\tau)H_\tau\right] \text{ as } \bar{\psi}_k \geq \bar{u}_k - I \\ &\leq \mathbb{E}\left[e^{-rT}\sum_{k=0}^{n-1} j(k)\bar{\psi}_k(Y_T)H_\tau\right] \text{ because } \mathcal{L}\bar{\psi}_k \leq 0 \text{ a.e.} \end{aligned}$$

We thus deduce from eq. (14) that

$$\tilde{\psi}(y,0,n) \leq \sup_{\tau\in\mathcal{T}_{\mathbb{G}}} \mathbb{E}\left[e^{-(r+\lambda)\tau}\big(\tilde{u}_n(Y_\tau) - I\big) + H_\tau e^{-rT}\sum_{k=1}^{n} j(k)\bar{\psi}_k(Y_T)\right]. \tag{49}$$

To show the reverse inequality, let us consider the $\mathbb{G}$-stopping time

$$\tau = \mathfrak{o}1_{\{\sigma<T\}} + \hat{\mathfrak{o}}(T,N,\cdot)1_{\{\sigma>T\}},$$

where

$$\hat{\mathfrak{o}}(u,k,\cdot) = \inf\left\{s\geq u \middle| Y_u \exp\big((\mu - \tfrac{1}{2}\sigma^2)(s-u) + \sigma W_{s-u}\big) \geq \bar{y}_k\right\}$$

is optimal for the optimal stopping value $\bar{\psi}_k$ (see Proposition 5). We thus have

$$\begin{aligned} J_2(\tau) =& \sum_{k=0}^{n-1} j(k)\mathbb{E}\Big[e^{-r\hat{\mathfrak{o}}(T,k,\cdot)}\big\{\bar{u}_k(Y_{\hat{\mathfrak{o}}(T,k,\cdot)}) - I\big\}H_\tau\Big] \\ =& \sum_{k=0}^{n-1} j(k)\mathbb{E}\Big[e^{-rT}\bar{\psi}_k(Y_T)H_\tau\Big] \text{ by Proposition 5.} \end{aligned}$$

This proves the expression in eq. (47).

Finally, similarly to the proof of Proposition 5, we can establish that $\tau_\star := \inf\{t\geq t \big| Y_t \geq y_\star\}$ is the optimal stopping time for the optimal stopping problem $\sup_\tau \mathbb{E}e^{-(r+\lambda)\tau}\{\eta_n y^\epsilon - I\}$. Setting $\tau_\star$ on the right-hand side of eq. (47) and noticing that $\bar{\psi}_k \geq 0$, we get ineq. (48).

Step 2 – Formulation as a classical optimal stopping problem. We introduce the functions

$$\begin{aligned} R_k(y) :=& \mathbb{E}\int_0^\infty e^{-(r+\lambda)t}\lambda\bar{\psi}_k(Y_t)\mathrm{d}t, \\ \bar{R}_k(y) :=& \mathbb{E}\int_0^\infty e^{-(r+\lambda)t}\lambda\big(\bar{u}_k - \bar{u}_n\big)(Y_t)\mathrm{d}t, \end{aligned} \tag{50}$$

which can be rewritten as in Proposition 8. It follows from eq. (44) and the strong Markov property that

$$\tilde{u}_n(y) = \bar{u}_n(y) + \sum_{k=0}^{n-1} j(k)\mathbb{E}\Big[e^{-rT}\big(\bar{u}_k - \bar{u}_n\big)(Y_T)\Big],$$
$$= \bar{u}_n(y) + \sum_{k=0}^{n-1} j(k)\bar{R}_k(y) \text{ from eq. (50).} \tag{51}$$

Furthermore, we have

$$\mathbb{E}\Big[e^{-rT}\bar{\psi}_k(Y_T)H_\tau\Big] = \mathbb{E}\int_0^\tau e^{-rt}\bar{\psi}_k(Y_t)\lambda e^{-\lambda t}\mathrm{d}t \text{ by independence of } T \text{ and } B \text{ and eq. (42),}$$
$$= R_k(y) - \mathbb{E}e^{-(r+\lambda)\tau}R_k(Y_\tau) \text{ by the strong Markov property.} \tag{52}$$

It then follows from Lemma 1 and eqs. (51) and (52) that

$$\tilde{\psi}(y,0,n) = \sum_{k=0}^{n-1} j(k)R_k(y) + G(y), \tag{53}$$

where

$$G(y) := \sup_{\tau\in\mathcal{T}} \mathbb{E}e^{-(r+\lambda)\tau}g(Y_\tau) \text{ for } g(y) := \bar{u}_n(y) - I + \sum_{k=0}^{n-1} j(k)(\bar{R}_k - R_k)(y). \tag{54}$$

Step 3 – Reformulation. We now want to express $\tilde{\psi}(y,0,n)$ in a way more amenable to an economic interpretation. We have

$$\mathbb{E}\int_0^\infty \lambda e^{-(r+\lambda)t}\bar{u}_k(Y_t)\mathrm{d}t = \alpha_k\lambda\int_0^\infty e^{-(r+\lambda)t}\mathbb{E}Y_t^\epsilon \mathrm{d}t \text{ from Proposition 5}$$
$$= \alpha_k\lambda y^\epsilon \int_0^\infty e^{-[r+\lambda-\mathrm{m}(\epsilon)]t}\mathrm{d}t \text{ from eq. (36)}$$
$$= \alpha_k y^\epsilon \frac{\lambda}{r+\lambda-\mathrm{m}(\epsilon)} \text{ by integration}$$
$$= \alpha_k \frac{\lambda}{\lambda-\mathcal{Q}(\epsilon)} y^\epsilon \text{ from eq. (13).}$$

This allows us to write $\bar{R}_k(y)$ in eq. (50) as

$$\bar{R}_k(y) = \big\{\alpha_k - \alpha_n\big\} \frac{\lambda}{\lambda - \mathcal{Q}(\epsilon)} y^{\epsilon}.$$

For the function $g(\cdot)$ in eq. (54), we get from Proposition 5 that

$$\begin{aligned} g(y) =& \alpha_n y^{\epsilon} - I + \frac{\lambda}{\lambda - \mathcal{Q}(\epsilon)} y^{\epsilon} \sum_{k=0}^{n-1} j(k) \big\{\alpha_k - \alpha_n\big\} - \sum_{k=0}^{n-1} j(k) R_k(y) \\ =& \eta_n y^{\epsilon} - I - \sum_{k=0}^{n-1} j(k) R_k(y) \text{ for } \eta_n \text{ defined by eq. (17).} \end{aligned}$$

This completes the proof.

I Proof of Proposition 9

We consider the optimal stopping problem in eq. (54). We introduce the differential operator $\mathcal{A} := \frac{1}{2}\sigma^2 y^2 \frac{\partial^2}{\partial y^2} + \mu y \frac{\partial}{\partial y}$ and study the differentiable function $y \mapsto (\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big] g)(y)$ for g given in eq. (54). By the Feynman-Kac theorem, the resolvents in eq. (50) solve

$$\begin{aligned} \big[\mathcal{A} - (r+\lambda)\mathbb{I}\big] R_k(y) &= -\lambda \bar{\psi}_k(y), & \forall y > 0, \\ \big[\mathcal{A} - (r+\lambda)\mathbb{I}\big] \bar{R}_k(y) &= -\lambda \big(\bar{u}_k - \bar{u}_n\big)(y), & \forall y > 0. \end{aligned} \tag{55}$$

Hence,

$$\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big] \big[\bar{R}_k - R_k\big](y) = \lambda \left(\bar{\psi}_k - \bar{u}_k\right)(y) + \lambda \bar{u}_n(y).$$

It follows that

$$\begin{aligned} \big[\mathcal{A} - (r+\lambda)\mathbb{I}\big] g(y) &= \big[\mathcal{A} - r\mathbb{I}\big] \bar{u}_n(y) + (r+\lambda) I + \lambda \sum_{k=0}^{n-1} j(k) \left(\bar{\psi}_k - \bar{u}_k\right)(y) \\ &= -a_n y^{\epsilon} + rI + \lambda \sum_{k=0}^{n-1} j(k) \bar{\Psi}_k(y), \end{aligned} \tag{56}$$

because of $\sum_{k=0}^{n-1} j(k) = 1$ and the definition of $\bar{\Psi}_k$ in eq. (40). Hence,

$$\Big(\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]g\Big)'(y) = -\epsilon a_n y^{\epsilon-1} + \lambda \sum_{k=0}^{n-1} j(k)\bar{\Psi}_k'(y).$$

But we know from eq. (41) that $\bar{\Psi}_k' < 0$ on $(0, \bar{y}_k)$ and $= 0$ on $(\bar{y}_k, \infty)$. So, the function $y \mapsto \big(\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]g\big)(y)$ decreases on $(0,\infty)$. It follows from Proposition 1 that $\bar{\pi}_n(\cdot)$ vanishes at 0 and diverges to ∞ at ∞. Furthermore, it follows from Proposition 5 that $\bar{\Psi}_k$ vanishes at 0 and ∞. Hence, $y \mapsto \big(\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]g\big)(y)$ is strictly positive at 0 and goes to $-\infty$ at ∞. As it is continuous and monotone, there exists a unique $y_\dagger$ such that $\big(\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]g\big)(y_\dagger) = 0$.

Because of the behavior of $y \mapsto (\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]g)(y)$, it follows from Villeneuve (2007) that the optimal stopping strategy is a threshold strategy. We now introduce the parameter γ in eq. (20). We obtain, as usual, that the function $y \mapsto H(y,\lambda) := g(y,\lambda)y^{-\gamma(\lambda)}$ attains a local maximum at the free boundary $\tilde{y}(\lambda) \in (y_\dagger, \infty)$. The value function in eq. (46) is thus of the form

$$\phi(y,0,n) = \sum_{k=0}^{n-1} j(k)R_k(y) + g\big(y \vee \tilde{y}\big)\left(\tfrac{y}{y\vee\tilde{y}}\right)^{\gamma}, \tag{57}$$

for $g(\cdot)$ defined in eq. (54) and $\tilde{y}$ the $\arg\max$ of $y \mapsto g(y)y^{-\gamma}$ in $(y_\dagger, \infty)$.

It also follows from eq. (57) that

$$\tilde{\psi}(y,0,n) = \Big\{\eta_n(y\vee\tilde{y})^{\epsilon} - I\Big\}\left(\tfrac{y}{y\vee\tilde{y}}\right)^{\gamma} + \sum_{k=0}^{n-1} j(k)\left\{R_k(y) - R_k(y\vee\tilde{y})\left(\tfrac{y}{y\vee\tilde{y}}\right)^{\gamma}\right\}.$$

We define $\tilde{\tau}(y) := \inf\big\{t \geq 0 \big| Y_t \geq \tilde{y}\big\}$. The second right-hand term has the probabilistic representation

$$\begin{aligned} R_k(y) - R_k(y\vee\tilde{y})\left(\tfrac{y}{y\vee\tilde{y}}\right)^{\gamma} &= R_k(y) - \mathbb{E}e^{-(r+\lambda)\tilde{\tau}}R_k(Y_{\tilde{\tau}}) \\ &= -\mathbb{E}\int_0^{\tilde{\tau}} e^{-(r+\lambda)t}\big[\mathcal{A} - (r+\lambda)\mathbb{I}\big]R_k(Y_t)\mathrm{d}t \text{ by Dynkin's formula} \\ &= +\lambda\mathbb{E}\int_0^{\tilde{\tau}} e^{-(r+\lambda)t}\bar{\psi}_k(Y_t)\mathrm{d}t \text{ from eq. (55)} \\ &=: \phi_k(y). \end{aligned}$$

The function $\phi(\cdot) := \sum_{k=0}^{n-1} j(k)\phi_k(\cdot)$ solves the second-order, linear ordinary differential eq. (21). Because $\bar{\psi}_k \geq 0$, it follows from the maximum principle that $\phi_k(\cdot) \geq 0$ and so $\phi \geq 0$. We can thus write the value function as

$$\tilde{\psi}(y,0,n) = \begin{cases} [\eta_n \tilde{y}^\epsilon - I](\frac{y}{\tilde{y}})^\gamma + \phi(y), & 0 < y \leq \tilde{y}, \\ \eta_n y^\epsilon - I, & y \geq \tilde{y}, \end{cases}$$

This completes the proof.

J Proof of Proposition 11

Step 0 - Case without intervention by the buyer. The derivatives of $\breve{u}_0(\cdot)$ in eq. (23) are

$$\begin{aligned} \breve{u}_0'(y) =& \frac{2}{(\gamma_+ - \gamma_-)\sigma^2} \left\{ \gamma_- y^{\gamma_- - 1} \int_0^y \frac{\breve{\pi}_0(z)}{z^{\gamma_- + 1}} \mathrm{d}z + \gamma_+ y^{\gamma_+ - 1} \int_y^\infty \frac{\breve{\pi}_0(z)}{z^{\gamma_+ + 1}} \mathrm{d}z \right\}, \\ \breve{u}_0''(y) =& \frac{2}{(\gamma_+ - \gamma_-)\sigma^2} \left\{ \gamma_-(\gamma_- - 1) y^{\gamma_- - 2} \int_0^y \frac{\breve{\pi}_0(z)}{z^{\gamma_- + 1}} \mathrm{d}z + \gamma_+(\gamma_+ - 1) y^{\gamma_+ - 2} \int_y^\infty \frac{\breve{\pi}_0(z)}{z^{\gamma_+ + 1}} \mathrm{d}z \right\} \\ & - \frac{2}{\sigma^2} \frac{\breve{\pi}_0(y)}{y^2}. \end{aligned}$$

The function $\breve{u}_0'(\cdot)$ is continuous. However, because $\breve{\pi}_0(\cdot)$ in eq. (22) has a positive jump at $(K/\nu_2)^{\frac{1}{\epsilon}}$, the function $\breve{u}_0''(\cdot)$ and consequently $\mathcal{L}\breve{u}_0$ have negative jumps at $(K/\nu_2)^{\frac{1}{\epsilon}}$. By substitution, we have $\mathcal{L}\breve{u}_0 + \breve{\pi}_0 = 0$ almost everywhere. The probabilistic representation in eq. (23) obtains by the Feymann-Kac theorem.

Step 1 - Profit. We can rewrite the profit expression in eq. (24) as

$$\begin{aligned}
\breve{\pi}(y) &= \sup_{\eta\in[0,1]} \Big\{a_1 y^\epsilon 1_{\{\eta<1-\frac{\nu_2 y^\epsilon}{K}\}} + \big[a_2 y^\epsilon - \eta K\big] 1_{\{\eta\geq 1-\frac{\nu_2 y^\epsilon}{K}\}}\Big\} \\
&= \max\Big\{a_1 y^\epsilon; \sup_{\eta\in[\frac{1-\nu_2 y^\epsilon}{K},\infty)\cap[0,1]} \big\{a_2 y^\epsilon - \eta K\big\}\Big\} \\
&= \max\Big\{a_1 y^\epsilon; a_2 y^\epsilon - \Big(1-\tfrac{\nu_2 y^\epsilon}{K}\Big)^+ K\Big\} \\
&= \max\Big\{a_1 y^\epsilon; a_2 y^\epsilon - (\nu_2 y^\epsilon - K)^-\Big\} \\
&= \begin{cases} \max\Big\{a_1 y^\epsilon; (a_2+\nu_2) y^\epsilon - K\Big\}, & 0<y<(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}, \\ \max\big\{a_1 y^\epsilon; a_2 y^\epsilon\big\}, & y \geq (\frac{K}{\nu_2})^{\frac{1}{\epsilon}}. \end{cases}
\end{aligned}$$

But we know from Proposition 1 that $a_2 \geq a_1$. Hence, $\max\big\{a_1 y^\epsilon; a_2 y^\epsilon\big\} = \max\big\{a_1; a_2\big\} y^\epsilon = a_2 y^\epsilon$. Consequently,

$$\breve{\pi}(y) = \begin{cases} \max\Big\{a_1 y^\epsilon; (a_2+\nu_2) y^\epsilon - K\Big\}, & 0<y<(\frac{K}{\nu_2})^{\frac{1}{\epsilon}} \\ a_2 y^\epsilon, & y \geq (\frac{K}{\nu_2})^{\frac{1}{\epsilon}}. \end{cases}$$

This function $\breve{\pi}(\cdot)$ is continuous everywhere including at $\Big(\frac{K}{\nu_2+a_2-a_1}\Big)^{\frac{1}{\epsilon}}$ and at $(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}$. Now, using eq. (22), we write

$$(\breve{\pi}-\breve{\pi}_0)(y) = \begin{cases} \Big[(\nu_2+a_2-a_1) y^\epsilon - K\Big]^+, & 0<y<(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}, \\ 0, & y \geq (\frac{K}{\nu_2})^{\frac{1}{\epsilon}}. \end{cases}$$

Hence, the expression for $\breve{\pi}(y)$ in Proposition 11. We note that $\breve{\pi}(\cdot)$ is continuous, while both $\breve{\pi}_0$ and ζ are discontinuous at $(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}$. Indeed, ζ vanishes at $(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}$ and is strictly positive at the left of $(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}$. This is a negative jump for Z, so by differentiating the closed-form expression for Z, we get that Z'' (and hence $\mathcal{L}Z$) has a negative jump at $(\frac{K}{\nu_2})^{\frac{1}{\epsilon}}$.

Step 2 - Profit estimates. There are several estimates. *First*, taking $\eta = 1$ arbitrarily in the optimization problem of eq. (24) is suboptimal, whence $\breve{\pi}(y) \geq a_2 y^\epsilon - K$. *Second*, taking $\eta = \breve{\eta}$, where $\breve{\eta}$ is the level at which the supremum is attained, and using the result in Proposition 1 that

$a_2 \geq a_1$, we get that

$$\begin{aligned}\breve{\pi}(y) &= a_1 y^{\epsilon} 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K<0\}} + \left[a_2 y^{\epsilon} - \breve{\eta}K\right] 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K\geq 0\}} \\ &\leq a_1 y^{\epsilon} 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K<0\}} + a_2 y^{\epsilon} 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K\geq 0\}} \\ &\leq a_2 y^{\epsilon} 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K<0\}} + a_2 y^{\epsilon} 1_{\{\nu_2 y^{\epsilon}-(1-\breve{\eta})K\geq 0\}} \\ &= a_2 y^{\epsilon}.\end{aligned}$$

Third, setting $\eta = 0$ arbitrarily in eq. (24) yields $\breve{\pi} \geq \breve{\pi}_0$ for $\breve{\pi}_0$ defined in eq. (22). Because $a_2 \geq a_1$, we clearly have $\breve{\pi}_0 \geq a_1 y^{\epsilon}$. In summary,

$$a_1 y^{\epsilon} \leq \breve{\pi}_0(y) \leq \breve{\pi}(y) \leq a_2 y^{\epsilon}, \quad \forall y > 0. \tag{58}$$

Step 3 - Net present value & estimates. Using classical arguments, $\breve{u} = \breve{u}_0 + Z$, where $\breve{u}_0$ and Z solve the ordinary second-order differential equation:

$$\mathcal{L}\breve{u}_0(y) + \breve{\pi}_0(y) = 0 \quad \text{and} \quad \mathcal{L}Z(y) + \zeta(y) = 0, \quad \forall y > 0, \tag{59}$$

respectively. From the general theory of linear ODEs, $\breve{u}_0$ and Z are not $C^2(\mathbb{R}_+)$ but only $C^1(\mathbb{R}_+)$ and have an explicit solutions, with Z given in Proposition 11.

From the results in Step 2, we get the estimates for $\breve{u}$ in Proposition 11.

K Proof of Proposition 12

Step 1 – Ranking of value functions. From (i) the profit ranking in eq. (58), (ii) the definition of $\bar{u}_n$ in eq. (15), and (iii) the probabilistic representations of $\breve{u}_0$ and $\breve{u}$ in eqs. (23) and (24), respectively, we get $\bar{u}_1 \leq \breve{u}_0 \leq \breve{u} \leq \bar{u}_2$. We then get from (iv) the definition of $\bar{\psi}_i$ in eq. (14) and (v) of $\breve{\psi}_0$ and $\breve{\psi}$ in eq. (25) and (vi) Theorem 1a) in Appendix L that $\bar{\psi}_1 \leq \breve{\psi}_0 \leq \breve{\psi} \leq \bar{\psi}_2$. We note that this ranking is consistent with Proposition 6, which proved that $\bar{\psi}_n \leq \bar{\psi}_{n+1}$.

Step 2 – Ranking of optimal stopping times. For the benchmark cases, we get from eqs. (15)

and (59) that

$$\begin{aligned}\mathcal{L}(\alpha_i y^\epsilon - I) =& rI + \mathcal{Q}(\epsilon)\alpha_i y^\epsilon = rI - a_i y^\epsilon,\\ \mathcal{L}(\breve{u}_0(y) - I) =& rI - \breve{\pi}_0(y), \\ \text{and } \mathcal{L}(\breve{u}(y) - I) =& rI - \breve{\pi}(y),\end{aligned} \tag{60}$$

respectively. To sum up, we have

$$\begin{aligned} & a_1 y^\epsilon \leq \breve{\pi}_0(y) \leq \breve{\pi}(y) \leq a_2 y^\epsilon \text{ from ineq. (58)} \\ \iff & rI - a_1 y^\epsilon \geq rI - \breve{\pi}_0(y) \geq rI - \breve{\pi}(y) \geq rI - a_2 y^\epsilon \\ \iff & \mathcal{L}(\alpha_1 y^\epsilon - I) \geq \mathcal{L}(\breve{u}_0(y) - I) \geq \mathcal{L}(\breve{u}(y) - I) \geq \mathcal{L}(\alpha_2 y^\epsilon - I) \text{ from ineq. (60)} \\ \implies & \bar{y}_1 \geq \breve{y}_0 \geq \breve{y} \geq \bar{y}_2 \text{ from Theorem 1c) in Appendix L,} \end{aligned}$$

a result consistent with Proposition 6 where we already proved that $\bar{y}_n \geq \bar{y}_{n+1}$.

We also get from eq. (59) and Theorem 1b) closed-form expressions for $\breve{\psi}_0$ and $\breve{\psi}$ in Proposition 12. This completes the proof.

L Abstract comparison theorem

Previous results relied on a comparison theorem, which we provide and prove below. For a pay-off function $u_i : \mathbb{R}_+ \to \mathbb{R}$ with $i \in \{1, 2\}$, consider the optimal stopping problem $\psi_i(y) := \sup_\tau \mathbb{E} e^{-r\tau} u_i(Y_\tau^y)$, where $(e^{-rt} Y_t)_t$ denotes a GBM with the infinitesimal operator $\mathcal{L}$. Let $\mathbb{S}_i := \{\psi_i = u_i\}$ denote the stopping set and τ_i be the first entrance time in $\mathbb{S}_i$ for the GBM $(Y_t)_t$. We make

Assumption 1 *The functions $u_i \in C^2(\mathbb{R}_+)$ satisfy:*

i. The function $\mathcal{L}u_i \in C^0(\mathbb{R}_+)$ is nonincreasing with $\lim_{y \downarrow 0} \mathcal{L}u_i(y) > 0$ and $\lim_{y \to \infty} \mathcal{L}u_i(y) < 0$.

ii. The relationship $\mathcal{L}u_2 \geq \mathcal{L}u_1$ is satisfied everywhere.

We have:

Theorem 1 (Abstract comparison theorem for stopping times) *The following relationships hold:*

a) If $u_2 \geq u_1$ everywhere, then $\psi_2 \geq \psi_1(y)$.

b) Under Assumption 1i, the stopping set $\mathbb{S}_i$ is of the form (y_i, ∞) with y_i satisfying smooth fit—that is, $\psi_i(y_i) = u_i(y_i)$ and $\psi_i'(y_i) = u_i'(y_i)$. Furthermore,

$$\psi_i(y) = \begin{cases} u_i(y_i)\left(\frac{y}{y_i}\right)^{\gamma_+}, & 0 < y < y_i, \\ u_i(y), & \textit{otherwise.} \end{cases}$$

c) Under Assumption 1ii, it holds that $\mathbb{S}_2 \subseteq \mathbb{S}_1$. Consequently, if Assumption 1i is also satisfied, then the optimal thresholds satisfy $y_2 \geq y_1$.

We now provide a proof. For Theorem 1a), we assume that $u_2 \geq u_1$ everywhere, then

$$\begin{aligned} \psi_2(y) :=& \mathbb{E}e^{-r\tau_2}u_2(Y_{\tau_2}) \text{ by definition} \\ \geq& \mathbb{E}e^{-r\tau_1}u_2(Y_{\tau_1}) \text{ by optimality of } \tau_2 \\ \geq& \mathbb{E}e^{-r\tau_1}u_1(Y_{\tau_1}) \text{ because } u_2 \geq u_1 \\ =:& \psi_1(y) \text{ by definition.} \end{aligned}$$

To obtain Theorem 1b), we make Assumption 1i. The optimality of a threshold policy is proven in Villeneuve (2007). The regularity of u_i is sufficient for the smooth-fit principle to hold.

For Theorem 1c), we note that

$$\begin{aligned} (\psi_2 - u_2)(y) &\geq \mathbb{E}e^{-r\tau_1}u_2(Y_{\tau_1}^y) - u_2(y) \text{ by optimality of } \tau_2 \\ &= \mathbb{E}\int_0^{\tau_1} e^{-rs}\mathcal{L}u_2(Y_s^y)\mathrm{d}s \text{ by Dynkin's formula} \\ &\geq \mathbb{E}\int_0^{\tau_1} e^{-rs}\mathcal{L}u_1(Y_s^y)\mathrm{d}s \text{ by Assumption 1ii} \\ &= \psi_1(y) - u_1(y) \text{ by Dynkin's formula and optimality of } \tau_1 \\ &\geq 0 \text{ because } \psi_i \text{ exceeds the obstacle } u_i. \end{aligned}$$

Consequently, if a y is in $\{y > 0 \big| (\psi_2 - u_2)(y) = 0\} =: \mathbb{S}_2$, it is also in $\{y > 0 \big| (\psi_1 - u_1)(y) = 0\} =: \mathbb{S}_1$ or $\mathbb{S}_2 \subseteq \mathbb{S}_1$. If, in addition, $\mathbb{S}_i$ is of the form (y_i, ∞), then $y_1 \leq y_2$.

References

Aksamit A, Jeanblanc M (2017) *Enlargement of Filtration with Finance in View* (Springer).

El Karoui N (1981) *Les aspects probabilistes du contrôle stochastique* (Berlin, Germany: Springer), 876 edition.

McDonald R, Siegel D (1986) The value of waiting to invest. *Quarterly Journal of Economics* 101(4):707–728.

Peskir G, Shiryaev AN (2006) *Optimal stopping and free-boundary problems* (Birkhäuser Verlag).

Villeneuve S (2007) On threshold strategies and the smooth-fit principle for optimal stopping problems. *Journal of Applied Probability* 44(1):181–198.