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“Buying Components”

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Buying Components*

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Abstract

We study how the organizational structure of producers affects competition between systems. We model systems as differentiated bundles of complementary components, where components within each system are produced either by a single firm (integration) or by two distinct firms (disintegration). When information about buyers' preferences is symmetric, disintegration typically increases prices and reduces total welfare as the less efficient system gains market share relative to integration. In addition, when buyers' preferences are private information, disintegration magnifies the quality distortions suppliers introduce to screen buyers and further reduces the market share of the more efficient system. Overall, the analysis suggests that technological standards that facilitate the combination of components from different suppliers can have adverse effects.

KEYWORDS. Composite goods, suppliers organization, competition, double marginalization.

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1 Introduction

We examine the implications of producers' integration and disintegration for the competition between systems composed of multiple complementary components. We model asymmetric, differentiated system producers and allow for buyers' private information. Our analysis shows that disintegration introduces two types of distortions, at the extensive margin (who purchases each system) and at the intensive margin (the quality of each competing system).

Markets for composite goods that combine multiple complementary inputs are ubiquitous in modern industries (e.g., airplane or car manufacturing, computers and server infrastructure). Our focus is on technological and regulatory changes that facilitate the combination of components from different producers into a single system. One interesting application is the telecom industry where the market for mobile network infrastructures has been historically dominated by a small number of integrated solutions. However, the virtualization of infrastructures lowers barriers to compatibility as intelligence tasks migrate from components to centralized cloud services. The introduction of OpenRAN that defines standardized interfaces should further facilitate the combination of equipment and software from different vendors into the same network. While OpenRAN is expected to have a positive impact on procurement cost and quality, there could be a concern that misalignment of sellers of components of the same system–which may result in excessive prices and insufficient quality– undermines these benefits. This is the core question we address in this paper.

In what follows we consider buyers who choose between two systems. Each system involves two components that are pure complements: the overall quality of a system is equal to the minimal quality of the components. Systems differ in their overall cost of production and are horizontally differentiated along the Hotelling line.

In the benchmark, each system is produced by an integrated firm that produces both components and offers a contract specifying a price for each quality level of the system. The buyer then chooses where to buy and which quality. In the separation (or disintegration) case, each component of each system is produced by a different supplier. Each supplier then proposes a tariff for its component. Then the buyer chooses a system and buys the two components from the two suppliers of this system. We focus on the issue of disintegration by assuming that the market structure is technologically neutral: the cost of providing a component of a given quality is independent of the integration or disintegration of suppliers. We discuss in the conclusion the implications for a more general setup with free entry that could lower the cost of some components under disintegration.

In the first part of the paper we assume that the buyer's preference for quality is public information (while his Hotelling location is private). In this case the contract proposed by each integrated supplier induces an efficient quality choice (conditional on the system), which can be implemented with a standard two-part tariff. When suppliers are disintegrated, the truthful equilibrium refinement selected an equilibrium where quality is also efficient. In other words, separation of component producers does not generate a distortion at the intensive margin. However, disintegration introduces a distortion at the extensive margin. Already in the integrated case, the market power generated by product differentiation leads to excessive sales of the costlier system. This distortion is exacerbated by the separation of suppliers. This additional distortion follows from the same logic as the classic Cournot effect for monopolists selling complementary goods. In that setting, each producer fails to internalize the positive effect of its demand for the producer of the complementary good and prices are therefore higher than what maximizes the producers' joint profit for each system. However, the Cournot effect can benefit rather than harm producers in our competitive setting. Indeed, the price increase induced by disintegration now soften competition between the two systems and can therefore raise total industry profits. At the same time, it causes the most efficient system to further lose market share. As a result, separation is detrimental to welfare absent other sources of efficiency gains (e.g., entry).

We then turn to the case where the buyer has private information about his preference for quality. Specifically, the marginal value the buyer attaches to quality is captured by a parameter β that is privately learnt by the buyer after contracting. The model is therefore a game of competition with ex-ante contracting. In this setting, the integrated structure features a quality distortion if the buyer is risk-averse. We capture risk aversion using Yaari (1987) dual theory of choice under risk which preserves the tractability of the model. Precisely, each buyer assigns a mass to the worst possible state of the world, in our case the ex-post realization that he has a low valuation for the quality of the product he contracted upon ex ante. We show that quality is distorted downward and that the magnitude of the distortion increases with risk aversion. This occurs because of a tension for the seller between insuring the buyer against the lowest realization of his payoff and limiting the informational rents to all the other buyer's types. Note that the distortion at the extensive margin–excessive sales of the costlier system–also operates, as in the symmetric information case, in fact is now exacerbated because the welfare differential between the cheaper and the costlier systems is reduced, due to reduced quality. The key result in this version with asymmetric information is that separating the two supplier pairs at each end of the Hotelling line now exacerbates quality distortions (in addition to distortions at the extensive margin). The reason is that each supplier does not internalize the loss of revenue of the other supplier when reducing its own quality.[1]

Literature Review. How to organize the supply of complementary goods in a procure-

[1] This effect did exist when the buyer's preferences are known because, under the refinement of truthful equilibrium, the marginal tariff equals the marginal cost so that suppliers are indifferent to quality choices of the buyer.

ment context is a key question that has only received scarce attention in the literature. Baron and Besanko (1992) and Gilbert and Riordan (1995) have considered the scenario where a buyer purchases perfect complements from two suppliers who are privately informed on their own costs, which are independent random variables. Two organizational forms are explored. For unbundled supply (separation), the buyer, who is endowed with all bargaining power, design contracts with two independent suppliers. As a result of the standard rent-efficiency trade-off familiar from the screening literature,[2] marginal costs have to be replaced by Myersonian virtual costs at the optimum.[3] Because of perfect complementarity between components, the buyer's marginal benefit should thus be equal to the sum of virtual costs of the suppliers; a phenomenon of compounded distortions akin to the familiar double marginalization of IO textbooks. For bundled supply (integration), a single supplier knows both cost parameters and optimal contracting only entails one distortion. Baron and Besanko (1992) and Gilbert and Riordan (1995) demonstrate that, under a broad range of circumstances the *virtual cost of the sum is lower than the sum of the virtual costs*, and thus integration is preferred by the buyer. Baron and Besanko (1992), Melumad et al. (1995) and Laffont and Martimort (1998) have considered a third scenario where one supplier subcontracts with the other and those authors have shown that such delegation entails no further agency cost in comparison with the unbundling scenario when this subcontract is observable and inputs are complements. Severinov (2008) highlights the limit of this result in the case of substitutable inputs. Finally, Baron and Besanko (1999) and Dequiedt and Martimort (2005) have analyzed the suppliers' incentives to consolidate by sharing and/or gathering cost information before production takes place. When consolidation is costly, an extra cost of integration might tilt the optimal organizational form towards unbundled supply.

We depart from this literature on at least three grounds. First, suppliers are no longer privately informed on costs while, instead, the buyer might own private information on his own preferences. Contracting frictions might thus a priori come from two sources. First, those frictions might follow from the fact that suppliers non-cooperatively design supply contracts when they remain non-integrated. Second, frictions might follow from the suppliers' concerns for screening the buyer's willingness to pay for quality as in the standard screening literature (Musa and Rosen, 1978; Maskin and Riley, 1984; Varian, 1989; Wilson, 1993). As a second departure from the existing literature, we thus adopt the same allocation of bargaining power as in this screening literature. Suppliers are ready to sell components with a wide range of possible levels of quality before the buyer decides whose suppliers to visit and which quality target to select. Actually, giving all bargaining power to suppliers seems the most relevant assumption if one wants to depict more competitive scenarios where those suppliers are willing to attract buyers. Modeling

[2]Laffont and Martimort (2002, Chapter 2).
[3]Myerson (1981).

competition as a bidding game among suppliers (those being integrated or not) is thus the third significant departure from the existing literature.

In our context, buyers are not only vertically differentiated in terms of their willingness to pay for quality but they are also horizontally differentiated. Supplier pairs selling complementary inputs by means of nonlinear price schedules are located at both sides of an Hotelling line as in Stole (1995). In this respect, our model belongs to the broader literature on price discrimination in competitive environments which is exhaustively surveyed in Stole (2007). Competition occurs along the line between supplier pairs willing to attract the buyer on either side of the market but it also occurs at each sides between suppliers under non-integration. To model those bidding games both between and within supplier pairs, we leverage the literature on competition in bidding schedules that was initiated by Bernheim and Whinston (1986). Building on the techniques developed in Martimort and Stole (2012) and Martimort, Semenov and Stole (2017), we characterize important and meaningful contract equilibria. Those equilibria satisfy the well-known refinement of *truthfulness* under complete information on the buyer's vertical preference parameter and the (admittedly less well known) requirement of *maximality* as defined in Martimort, Semenov and Stole (2017) under asymmetric information.

Under complete information on vertical preferences, suppliers at each end of the market provide efficient quality even when those suppliers remain independent and do not coordinate pricing. Competition between suppliers selling complementary goods at each end of the market thus does not suffer from the multiple monopolies fallacy. At first glance, this result may be surprising. Yet, a closer look shows that it is completely in lines with the complete information menu auction literature initiated by Bernheim and Whinston (1986). At a truthful equilibrium, each upstream supplier offers a two-part tariff where each additional unit of quality is sold at marginal cost. This schedule reflects *truthfully* the seller's preferences across feasible trades. Given those tariffs, the buyer then perfectly coordinates upstream sellers when making her purchases and efficiency along the whole supply chain immediately follows. Competition on the horizontal dimension between suppliers pairs then just erodes fixed fees.

Under asymmetric information on vertical preferences, quality at both sides of the market is reduced in response to standard screening considerations. Now, the maximal equilibrium features the familiar flavor of double marginalization. Quality distortions are compounded along the supply chain when suppliers of complement goods do not cooperate; a result that echoes findings in Martimort and Stole (2009)'s model of competition with nonlinear pricing but which is here also embedded into a broader competition environment that also entails horizontal differentiation. Competition at each sides of the market between complement suppliers then weakens horizontal competition between pairs.

Biglaiser and Ma (2003) is probably the paper which is closest to ours. Those authors also analyze a model where suppliers (integrated or not) compete for the buyer's purchases. Our analysis is in some sense complementary to theirs. In Biglaiser and Ma (2003), goods are substitutes, the buyer's marginal rate of substitution between those goods is her private information which is binary, and transportation costs depend on whether suppliers are integrated or not. Instead, we analyze the case of perfect complements, together with a modeling of the buyer's vertical preferences which is a piece of private information on vertical continuously distributed. Finally, we do not make any restriction on the set of feasible contracts.

Organization of the paper. Section 2 presents the model. Section 3 discusses the simple scenario where the buyer's vertical preference parameter is known by the sellers on both sides of the market, whether they are integrated or not. Section 4 addresses the case where this vertical preference parameter is the buyer's private information. Section 5 briefly concludes and discusses possible extensions. Proofs are relegated to an Appendix.

2 The Model

Technology and Preferences. A buyer (hereafter referred to as the agent, sometimes denoted by *he* or B) seeks to procure one unit of a composite good composed of two perfectly complementary intermediate components (indexed by $i = 1, 2$). To achieve this, B approaches pairs of suppliers who are ready to provide these essential components.

The supplier pairs are located at the two endpoints of a standard $[0, 1]$ Hotelling line. Supplier S_i (resp. S_i^*) produces component i at location 0 (resp. 1). Let q_i (resp. q_i^*) denote the quality of component i at location 0 (resp. 1). Due to perfect complementarity, the quality q (resp. q^*) of the composite good is given by $q = \min\{q_1, q_2\}$ (resp. $q^* = \min\{q_1^*, q_2^*\}$). For technical convenience, we assume that all quality levels belong to the interval $\mathcal{Q} = [0, \overline{q}]$, where the upper bound $\overline{q}$ is chosen to be sufficiently large to guarantee interior solutions under all scenarios considered below.

Let denote by θ_i (resp. θ_i^*) S_i (resp. S_i^*)'s marginal cost. For future reference, we denote by $\theta = \theta_1 + \theta_2$ (resp. $\theta^* = \theta_1^* + \theta_2^*$) the overall marginal cost at location 0 (resp. 1). In the sequel, we will be particularly interested by settings where suppliers are asymmetric. Without loss of generality, we will denote as *strong* (resp. *weak*) suppliers, those suppliers located at 0 (resp. 1) and assume that $\theta \leq \theta^*$. For future reference, we define a measure of the competitive advantage of the strong side of the market as $\delta_\theta = \theta^* - \theta \geq 0$. We also define the average cost on this market as $\vartheta = \frac{\theta+\theta^*}{2}$.

B's preferences entails both horizontal and vertical differentiation. First, B's preferences are characterized by his location $x \in [0, 1]$, a parameter of horizontal differentiation.

For simplicity, we assume linear transportation costs. Buying from 0 (resp. 1) when located at x thus costs rx (resp. $r(1-x)$) to the buyer. This parameter x is not observed by suppliers and drawn from a common knowledge distribution F with density $f = F'$. We assume that the density function f is symmetric around $\frac{1}{2}$ (which in particular implies $F\left(\frac{1}{2}\right) = \frac{1}{2}$). Following the screening literature,[4] we also assume that the (generalized) *Monotone Hazard Rate Property* holds, that is,

$$\frac{F-\kappa}{f} \text{ is non-decreasing for } \kappa \in \{0,1\}. \tag{2.1}$$

Second, B's preferences are also characterized by a parameter β that represents his vertical preferences for quality. For tractability reasons, we assume that B has quadratic preferences on quality. His net surplus from consumption at 0 (net of transportation costs) can thus be written as

$$\beta q - \frac{q^2}{2} - rx - \sum_{i=1}^{2} t_i \tag{2.2}$$

where t_i is B's payment to S_i a component of quality $q_i = q$. Similarly, B's net surplus from consumption at 1 can be expressed as

$$\beta q^* - \frac{(q^*)^2}{2} - r(1-x) - \sum_{i=1}^{2} t_i^* \tag{2.3}$$

where t_i^* is S_i^*'s price for quality $q_i^* = q^*$.

The parameter β is drawn from a common knowledge distribution G with positive density $g = G'$ on a support $\mathcal{B} = \left[\underline{\beta}, \overline{\beta}\right]$. We will first consider that β is known to suppliers, then we will examine the case where it is private information known only to the buyer. Let $\beta^e = \mathbb{E}_G(\beta)$ denote the mean of this distribution. We again impose another familiar *Monotone Hazard Rate Property*:

$$\frac{1-G}{g} \text{ non-increasing.} \tag{2.4}$$

To ensure that a positive quality is always offered in equilibrium under all circumstances below, we finally require that $\underline{\beta}$ be not too small, that is

$$\underline{\beta} > \theta^* + \frac{1}{g(\underline{\beta})}. \tag{2.5}$$

Contracts and Organizations of the Supply Chains. Supplier S_i (resp. S_i^*) stands ready to provide a quality level q (resp. q^*) at prices $T_i(q)$ (resp. $T_i^*(q^*)$). Those nonlinear prices are thus commitments from the suppliers; leaving to B the choice of

[4] Bagnoli and Bergstrom (2005).

which quality level he wants to procure given those schedules. *A priori*, no restrictions on these price schedules are placed beyond the following weak requirements. First, those schedules are lower semi-continuous to ensure existence of a maximizer to the buyer's optimization problem. Second, those schedules are defined on the full domain $\mathcal{Q}$, that is, there is a price to supply any feasible quality level.

Under *Integration*, S_1 and S_2 (resp. S_1^* and S_2^*) act as a merger, say S (resp. S^*). They jointly decide on an aggregate non-linear price $\overline{T}(q) = \sum_{i=1}^{2} T_i(q)$ (resp. $\overline{T}^*(q^*) = \sum_{i=1}^{2} T_i^*(q^*)$) to maximize the overall profit of the supply chain.

Under *Non-Integration*, S_1 and S_2 (resp. S_1^* and S_2^*) non-cooperatively decide of their respective nonlinear price $T_1(q)$ and $T_2(q)$ (resp. $T_1^*(q^*)$ and $T_2^*(q^*)$) with the sole objective of maximizing their own profits.

Whether integrated or not, suppliers on each side of the Hotelling line still collectively compete to satisfy the buyer's needs. Therefore, our model involves both competition between brands located in different venues and possibly cooperation or not depending on the market structure under scrutiny at a given location.

Benchmark. The efficient level of quality and the overall surplus (gross of transportation costs) at 0 are respectively defined as

$$q^{fb}(\beta) = \arg\max_{q\in\mathcal{Q}}(\beta-\theta)q-\frac{q^2}{2} = \beta-\theta \text{ and } W^{fb}(\beta) = \max_{q\in\mathcal{Q}}(\beta-\theta)q-\frac{q^2}{2} = \frac{(\beta-\theta)^2}{2}. \quad (2.6)$$

Similar definitions (indexed with a star) for $q^{*fb}(\beta)$ and $W^{*fb}(\beta)$ apply at location 1. We assume that β is large enough to ensure full coverage of the market in all circumstances below.

For future reference, we also define the surplus difference between venues as $\Delta W^{fb}(\beta) = W^{fb}(\beta) - W^{*fb}(\beta)$. This surplus difference reflects the comparative advantage of suppliers at 0. It is always non-negative thanks to the fact that strong suppliers generate more surplus:

$$\Delta W^{fb}(\beta) = \delta_\theta\left(\beta - \vartheta\right) > 0. \quad (2.7)$$

3 Competition at the Extensive Margin

Suppose first that the buyer's vertical preference parameter β is common knowledge and verifiable. This informational environment might actually reflect a scenario where the buyer and his suppliers have entertained long-lasting relationships so that suppliers have been able to perfectly learn the buyer's vertical preference parameter over time. Suppliers can thus use price schedules of the form $T_i(q,\beta)$ (resp. $T_i^*(q^*,\beta)$) that are contingent on this parameter.

3.1 Integrated Suppliers

When suppliers S and S^* are integrated at both sides of the Hotelling line, the sole source of competition comes from their horizontal differentiation.

3.1.1 Preliminaries

For future reference, we denote B's net surplus and quality choice at venue 0 as $(U(\beta), q(\beta))$. Formally, we have

$$U(\beta) = \max_{q \in \mathcal{Q}} \beta q - \frac{q^2}{2} - \overline{T}(q, \beta) \text{ and } q(\beta) = \arg\max_{q \in \mathcal{Q}} \beta q - \frac{q^2}{2} - \overline{T}(q, \beta) \tag{3.1}$$

where $\overline{T} = \sum_{i=1}^{2} T_i$ denotes the aggregate price schedule paid when the composite good is purchased at location 0. When suppliers are integrated, this aggregate price schedule is jointly chosen to maximize profits along the vertical chain. Of course, similar starred notations and definitions apply for the allocation $(U^*(\beta), q^*(\beta))$ that prevails at venue 1.

Given the surplus at each location, those x such that

$$x \leq X(\beta, \overline{T}, \overline{T}^*) = \frac{1}{2} + \frac{1}{2r}\left(U(\beta) - U^*(\beta)\right), \tag{3.2}$$

buy from S. Accordingly we identify S's market share with the marginal buyer $X(\beta, \overline{T}, \overline{T}^*)$[5]. Then S^*'s market share is $1 - F(X(\beta, \overline{T}, \overline{T}^*))$.

Equipped with this characterization of market shares, we may write S's profit as

$$\Pi(\beta, \overline{T}, \overline{T}^*) = (\overline{T}(q(\beta), \beta) - \theta q(\beta))F(X(\beta, \overline{T}, \overline{T}^*)). \tag{3.3}$$

S's profits can also be expressed in terms of the induced allocations $(U(\beta), q(\beta))$ and $(U^*(\beta), q^*(\beta))$ reached at both sides of the market as

$$\Pi(\beta, \overline{T}, \overline{T}^*) = \left((\beta - \theta)q(\beta) - \frac{(q(\beta))^2}{2} - U(\beta)\right) F\left(\frac{1}{2} + \frac{1}{2r}\left(U(\beta) - U^*(\beta)\right)\right).$$

By modifying the price charged for its own composite good, S can always undo the impact of S^*'s own price on demand and thereby determine the identity of the marginal customer $X(\beta, \overline{T}, \overline{T}^*)$, indifferent between both venues. In other words, $X = X(\beta, \overline{T}, \overline{T}^*)$ becomes the relevant strategic variable in the horizontal competition between integrated suppliers and this variable is jointly controlled. In the parlance of Martimort and Stole (2012), the game between S and S^* is an *aggregative game*, with X being the relevant aggregate and each supplier's objective depending on its own payment schedule and the

[5] *Stricto sensu*, S's market share should be defined as the mass $F(X(\beta, \overline{T}, \overline{T}^*))$ but abusing terminology raises no semantic issues thereafter.

aggregate only. This fact is illustrated by the expression of S's profits in (3.3) above.

Similarly, we may define S^*'s profit as

$$\Pi^*(\beta,\overline{T},\overline{T}^*) = (\overline{T}^*(q^*(\beta),\beta) - \theta^* q^*(\beta))(1 - F(X(\beta,\overline{T},\overline{T}^*)))$$

or

$$\Pi^*(\beta,\overline{T},\overline{T}^*) = \left((\beta-\theta^*)q^*(\beta) - \frac{(q^*(\beta))^2}{2} - U^*(\beta)\right)\left(1 - F\left(\frac{1}{2} + \frac{1}{2r}\left(U(\beta) - U^*(\beta)\right)\right)\right).$$

3.1.2 Equilibrium

Under integration, an equilibrium of the game is thus entirely characterized by a pair of aggregate price schedules $(\overline{T}^m, \overline{T}^{*m})$ together with a market share $X^m(\beta)$ that those tariffs jointly induce. At equilibrium, neither integrated supplier wants to deviate by offering another aggregate price schedule and, so doing, induce another market segmentation.

For future reference and slightly abusing notations, it is thus useful to express S's profit in terms of its own market share $F(X)$, the quality it supplies and the surplus profile offered by its rival S^* as

$$\Pi(\beta,\overline{T},\overline{T}^*) = \left((\beta-\theta)q(\beta) - \frac{(q(\beta))^2}{2} - U^{*m}(\beta) - r(2X-1)\right)F(X). \tag{3.4}$$

Expressing also S^*'s profits in terms of its own market share yields

$$\Pi^*(\beta,\overline{T},\overline{T}^*) = \left((\beta-\theta^*)q^*(\beta) - \frac{(q^*(\beta))^2}{2} - U^m(\beta) + r(2X-1)\right)(1 - F(X)). \tag{3.5}$$

Quality Levels and Market Shares. Following Martimort and Stole (2012), the *Principle of Aggregate Concurrence* applies in our context.[6] At equilibrium, both integrated suppliers should agree on which market segmentation $X^m(\beta)$ should prevail. Of course, on their respective market shares, integrated suppliers remain free to choose how much quality to provide.

Proposition 1. *Suppose that suppliers are integrated on both sides of the market and there is complete information on the vertical differentiation parameter β. There exists a unique equilibrium and it entails the following features.*

1. *Quality is efficient on both sides of the market:*

$$q^m(\beta) = q^{fb}(\beta) \text{ and } q^{*m}(\beta) = q^{*fb}(\beta). \tag{3.6}$$

[6] See also Bernheim and Whinston (1986a) for an earlier example of this *Principle*.

2. The market segmentation $X^m(\beta)$ *satisfies*

$$\frac{\Delta W^{fb}(\beta)}{2r} + \frac{1}{2} = X^m(\beta) + \frac{2F(X^m(\beta)) - 1}{f(X^m(\beta))}. \tag{3.7}$$

$X^m(\beta)$ *increases with* δ_θ *and belongs to* $\left[\frac{1}{2}, 1\right]$ *whenever*

$$0 \leq \frac{\delta_\theta}{2r}(\beta - \vartheta) \leq \frac{1}{2} + \frac{1}{f(1)}. \tag{3.8}$$

On their respective market shares, suppliers offer the efficient quality level so as to make their own venue as attractive as possible for the buyer. More quality is found at location 0 because suppliers have a comparative advantage there. Everything happens as if integrated suppliers were competing for the buyer's needs by bidding the net surplus that they can provide and the best way of doing so is not to distort the quality level that they can respectively supply. Surplus to the buyer is shifted by means of lower prices with no consequences on quality level on each market segment.[7] The sole channel for competition here is at the extensive margin. The size of the market that each of those suppliers secures at equilibrium and the overall surplus reached on each market segments are determined separately.

The choice of the market share that the integrated supplier S wants to cater follows from a simple trade-off. Starting from the equilibrium market share $X^m(\beta)$, consider the benefits for S of raising the price charged for the efficient quality level $q^{fb}(\beta)$ by a marginal amount dT. This perturbation increases S's expected profit by a term of first-order magnitude, namely

$$F(X^m(\beta))dT.$$

On the other hand, more consumers now prefer to visit S^*. The mass of those lost consumers is $f(X^m(\beta))\frac{dT}{2r}$ and the overall profit loss for S is thus

$$\left(W^{fb}(\beta) - U^{*s}(\beta) - r(2X^m(\beta) - 1)\right) f(X^m(\beta))\frac{dT}{2r}.$$

At equilibrium, losses and gains should compensate each other so that

$$W^{fb}(\beta) - U^{*m}(\beta) - r(2X^m(\beta) - 1) = 2r\frac{F(X^m(\beta))}{f(X^m(\beta))}. \tag{3.9}$$

A similar condition applies for S^* and thus

$$W^{*fb}(\beta) - U^m(\beta) + r(2X^m(\beta) - 1) = 2r\frac{1 - F(X^m(\beta))}{f(X^m(\beta))}. \tag{3.10}$$

[7] See Armstrong and Vickers (2001) for a similar insight in a related model.

Using (3.9), (3.10) and the definition of market share $X^m(\beta)$ coming from (3.2.1) finally yields (3.7). In particular, we observe that, when suppliers on both sides of the market are of equal strength, each caters one half of the overall demand. Instead, as suppliers at 0 enjoy a more significant competitive advantage, they also serve a greater market share.

TWO-PART TARIFFS AND PROFITS. Under complete information on β, there is no loss of generality in assuming that S (resp S^*) charges a two-part tariff of the form

$$T^m(q,\beta) = \theta q + C^m(\beta) \text{ (resp. } T^{*m}(q^*,\beta) = \theta^* q^* + C^{*m}(\beta)). \tag{3.11}$$

To see why, remember that, had it acted as a monopolist with no threat of competition, S could as well recommend to the buyer to procure the efficient quality level $q^{fb}(\beta)$ and extract the whole buyer's gross surplus up to the point where the buyer's move to location 0 is no longer attractive. This solution could as well be replicated with a two-part tariff of the form (3.11). When offered such scheme, B chooses the level of quality he wants to procure and pays each extra unit of quality at its marginal cost. Because B is de facto made residual claimant for the overall surplus of the relationship with S, he chooses the first-best level of quality. Then, a fee $W^{fb}(\beta)$ is used by the monopolist to extract all the corresponding surplus from B.

When integrated suppliers compete over the Hotelling line, the same logic applies. Each supplier has in its best-response correspondence a two-part tariff of the form (3.11). With those schemes, each supplier is indifferent over the quality level that might be chosen by B. Henceforth, the fee $C^m(\beta)$ (resp. $C^{*m}(\beta)$) corresponds to the equilibrium profit level that S (resp. S^*) can secure.

Proposition 2. *Suppose that suppliers are integrated on both sides of the market and there is complete information on β. S and S^*'s equilibrium fees satisfy*

$$C^m(\beta) = 2r\frac{F(X^m(\beta))}{f(X^m(\beta))} \geq 2r\frac{1-F(X^m(\beta))}{f(X^m(\beta))} = C^{*m}(\beta). \tag{3.12}$$

Equilibrium expected profits for S and S^* respectively also satisfy

$$\Pi^m(\beta) = 2r\frac{(F(X^m(\beta)))^2}{f(X^m(\beta))} \geq 2r\frac{(1-F(X^m(\beta)))^2}{f(X^m(\beta))} = \Pi^{*m}(\beta). \tag{3.13}$$

When integrated suppliers at both sides of the market are no longer symmetric, the strong supplier located at 0 gets a greater market share and can charge a higher fee for its services. Since both this fee charged and the market share are greater on the strong side of the market, a strong integrated supplier makes more profit than its weaker rival.

3.2 Non-Integrated Suppliers

Consider now the scenario where suppliers on both sides remain non-integrated. Those suppliers now choose non-cooperatively their respective price schedules. There is no competition both across and within locations. Across location, suppliers offer composite goods which are substitutes. At each location, suppliers compete by offering components which are complementary.

3.2.1 Preliminaries

The game played by non-integrated suppliers at a given venue is an *intrinsic common agency game*, to use the expression coined by Bernheim and Whinston (1986a). Because they supply essential components, both suppliers are needed for S to build a composite good. Hence, both price schedules T_1 and T_2 should be accepted at once by B if he considers purchasing at location 0. The sole outside option for B is thus to move to location 1 and buy both components from non-integrated suppliers located there. In sharp contrast with Bernheim and Whinston (1986a) who considered deterministic models of common agency under complete information, this outside option is now stochastic thanks to the fact that B's decision to buy from this outside option depends on his own location on the line which is itself a random and non-contractible variable.

Formally, we may now write S_i's profit as

$$\Pi_i(\beta, T_i, T_{-i}, \overline{T}^*) = (T_i(q(\beta), \beta) - \theta_i q(\beta))F(X(\beta, T_i + T_{-i}, \overline{T}^*)).$$

Profit can also be written in terms of the allocations obtained at each venue as

$$\Pi_i(\beta, T_i, T_{-i}, \overline{T}^*) = \left((\beta - \theta_i)q(\beta)) - \frac{(q(\beta))^2}{2} - T_{-i}(q(\beta), \beta) - U(\beta)\right) F\left(\frac{1}{2} + \frac{1}{2r}\left(U(\beta) - U^*(\beta)\right)\right).$$

The bracketed terms stands for S_i's share of the bilateral payoff that this supplier can reach in its relationship with B when he visits location 0. The second term is the probability that B visits that location.

3.2.2 Truthful Equilibrium

An equilibrium of the game under non-integration on both sides of the market can now be defined as an array of price schedules $(T_1^s, T_2^s, T_1^{*s}, T_2^{*s})$ that induce a pair of allocations $(U^s(\beta), q^s(\beta))$ and $(U^{*s}(\beta), q^{*s}(\beta))$ at both locations such that each supplier does not find it valuable to deviate with an alternative price schedule that would induce another allocation on his own side of the market and another market segmentation.

Again, we follow our previous approach that stresses the role of the market share as an aggregate strategic variable. Slightly abusing notations, we now express S_i's profits in terms of the market share that the choice of its own price schedule induces as

$$\Pi_i(\beta, T_i, T^s_{-i}, \overline{T}^{*s}) = \left((\beta-\theta_i)q(\beta)) - \frac{(q(\beta))^2}{2} - T^s_{-i}(q(\beta),\beta) - U^{*s}(\beta) - r(2X-1)\right) F(X). \tag{3.14}$$

Given the conjectured schedule T^s_{-i} that is offered by the complementary supplier S_{-i} at equilibrium, a very similar logic to that found to compute the best response of an integrated supplier should still apply. First, S_i would like to induce the buyer to choose a quality level $q^s(\beta)$ that maximizes the bilateral payoff of the coalition it forms with the buyer, namely

$$q^s(\beta) \in \arg\max_{q\in\mathcal{Q}} (\beta-\theta_i)q - \frac{q^2}{2} - T^s_{-i}(q,\beta). \tag{3.15}$$

Second, S_i would like to induce a market share $X^s(\beta)$ that maximizes (3.14), i.e.,

$$(\beta-\theta_i)q^s(\beta) - \frac{(q^s(\beta))^2}{2} - T^s_{-i}(q^s(\beta),\beta) - U^{*s}(\beta) - r(2X^s(\beta)-1) = 2r\frac{F(X^s(\beta))}{f(X^s(\beta))}. \tag{3.16}$$

Conditions (3.15) and (3.16) characterize S_i's best responses both at the intensive margin (the choice of quality) and at the extensive margin (the induced choice of market share).

As far as the intensive margin is concerned, it is straightforward to check that S_i can always implement the quality level $q^s(\beta)$ by using again a two-part tariff, or *truthful schedule* of the form

$$T^s_i(q,\beta) = \theta_i q + C^s_i(\beta) \tag{3.17}$$

and letting the buyer choose optimally the quality level he requests. In other words, such a two-part tariff where each extra unit of quality is paid at marginal cost can always be found within S_i's best-response correspondance. Following Bernheim and Whinston (1986b),[8] focusing on two-part/truthful schedules of the form (3.17) is thus akin to an equilibrium refinement. We will follow most of the literature when adopting this mild refinement.[9]

[8] The notion of truthfulness was developed by these authors in contexts where common agency is delegated, i.e., the buyer can always refuse any single contract. This option requires that a price schedule cannot be negative on its equilibrium range but, when positive, a truthful price schedule perfectly reflects the supplier's marginal cost. In our context, the buyer needs to buy complementary components from each supplier when he visits a given location and contracts have to be jointly accepted. Common agency is intrinsic. Still, at the margin, a trufthul price schedule again reflects the supplier's marginal cost but now on its full range.

[9] Yet, we notice that a plethora of other equilibria exists. Consider first forcing schedules such that $T_i(q) = T_i < +\infty$ if $q = \hat{q} \in \mathcal{Q}$ and $T_i(q) = +\infty$ for $q \neq \hat{q}$. By offering such a schedule, S_i can essentially force the agent to buy only quality $\hat{q}$ at a finite price. It is immediate to check that any quality level $\hat{q}$ can be supported at some equilibrium with such forcing contracts. Yet, only $\hat{q} = q^{fb}(\beta)$ turns out to be chosen if the equilibrium set is refined with a Pareto-dominance criterion or with a coalition-proofness criterion (Berheim et al., 1987). This dominant equilibrium is also selected with the intuitively simpler truthfulness requirement.

Turning to the extensive margin, we notice the similarity of (3.16) with (3.9). Again, S_i would like to raise its own price up to the point where the expected profit gain on nearby captive customers compensates the corresponding loss of demand.

Quality and Market Shares. We summarize our findings in the next proposition.

Proposition 3. *Suppose that suppliers are non-integrated on both sides of the market and there is complete information on the vertical differentiation parameter β. There exists a unique equilibrium where suppliers charge truthful tariffs. This equilibrium entails the following features.*

1. *Quality is efficient at both locations:*

$$q^s(\beta) = q^{fb}(\beta) \text{ and } q^{*s}(\beta) = q^{*fb}(\beta). \tag{3.18}$$

2. *The market segmentation $X^s(\beta)$ satisfies*

$$\frac{\Delta W^{fb}(\beta)}{2r} + \frac{1}{2} = X^s(\beta) + 2\frac{2F(X^s(\beta)) - 1}{f(X^s(\beta))}. \tag{3.19}$$

$X^s(\beta)$ increases with δ_θ and belongs to $\left[\frac{1}{2}, X^m(\beta)\right]$.

The benefit of relying on the truthfulness refinement as a selection criterion is that it implies that, as under integration, on both sides of the market non-cooperating suppliers agree on providing the efficient quality. This similarity facilitates the comparison between the integration and the non-integration scenarios. Intuitively, when non-integrated suppliers offer two-part tariffs of the form (3.17), the buyer ends up being residual claimant for the choice of the quality level. He thus fully internalizes the impact of this choice on the costs of the different components. Efficiency immediately follows. As we will see, this reasoning relies however on the fact that the preference of the buyer for quality (β) is known.

Importantly, when suppliers remains non-integrated, the market is more balanced; $X^s(\beta)$ comes closer to $\frac{1}{2}$ than $X^m(\beta)$. Strong suppliers restrict their market share when acting non-cooperatively while weak suppliers expand their own. To understand this phenomenon, we have to come back on the well-known double-marginalization problem à la Cournot (1838).[10] In this textbook model, a monopolist selling one component does not take into account the loss of profit incurred by the firm selling the complementary component when it raises its own price. Monopolists selling complementary goods end up each charging a unit price for their component higher than the joint profit maximizing level so that they excessively contract the overall demand for their products. The first

[10]See Linnemeur (2022) for the correct intellectual origins of the double-marginalization effect.

difference is that, in our setting suppliers are using tariffs with marginal cost-pricing. Hence, conditionally on buying, there is no inefficiency in the level of quality offered at equilibrium whether suppliers are integrated or not. Yet, distortions come at the extensive margin. Under integration, suppliers would increase their joint fee up to the point where the gain of increased joint profit conditionally on selling is offset by the corresponding loss of profit coming with lower demand. Under non-integration, a given supplier does not take into account the impact of raising its own fee on the loss of profit of the complementary supplier. Each supplier thus charges too high a fee and demand is excessively restricted. Given that suppliers do not coordinate on the fees they charge, we may want to refer to this phenomenon as a double-surplus extraction instead of the more common wording of double-marginalization.

Had the market not been fully covered, adopting these non-cooperative strategies at both sides of the Hotelling line would lead to restricted services. A greater share of the market around that would remain uncovered. When the market is instead fully covered, B reacts to the higher fees non-cooperatively charged by suppliers on the strong side of the market by switching to the weak side. Although suppliers also do not cooperate on the weak side of the market, the so expanded demand for their products more than compensate for the double-surplus extraction at that mere location.

Profits and Welfare. Next proposition further investigates the consequences of non-integration on profits.

Proposition 4. *Suppose that suppliers are non-integrated on both sides of the market and there is complete information on the vertical differentiation parameter β. Suppliers at a given location charge the same fee*

$$C_i^s = C^s(\beta) \text{ (resp. } C_i^{*s} = C^{*s}(\beta)\text{)}, \quad i = 1, 2$$

where

$$C^s(\beta) = 2r\frac{F(X^s(\beta))}{f(X^s(\beta))} \geq 2r\frac{1 - F(X^s(\beta))}{f(X^s(\beta))} = C^{*s}(\beta), \quad i = 1, 2 \tag{3.20}$$

and make the same equilibrium profits

$$\Pi_i^s(\beta) = \Pi^s(\beta) \text{ (resp. } \Pi_i^{*s}(\beta) = \Pi^{*s}(\beta)\text{)}, \quad i = 1, 2$$

where

$$\Pi^s(\beta) = 2r\frac{(F(X^s(\beta)))^2}{f(X^s(\beta))} \geq 2r\frac{(1 - F(X^s(\beta)))^2}{f(X^s(\beta))} = \Pi^{*s}(\beta). \tag{3.21}$$

According to the proposition, the buyer of type β procuring from suppliers S_1 and S_2 ends up paying

$$\overline{T}^s\left(q^{fb}(\beta)\right) = 2C^s(\beta) + \theta q^{fb}(\beta).$$

With a truthful schedule, S_i's profit amounts to the fee C_i^s that it charges to each possible buyer it caters times the demand that it serves. Raising this fee by a marginal amount dC thus raises profit per capita by dC and thus increases overall profit by

$$F(X^s(\beta))dC.$$

At the same time, the loss from that demand being shifted towards the other location becomes

$$C^s f(X^s(\beta))\frac{dC}{2r}.$$

At equilibrium, those gains and losses compensate each other and both suppliers on the strong side of the market, since they agree on which market share to induce, charge the same fee. The left-hand (resp. right-hand) side equality of (3.20) immediately follows.

It turns out that, when suppliers are non-integrated, the market is more balanced than with integration. Non-integrated suppliers on the strong side of the market lose market share while the reverse occurs for suppliers on the weak side of the market. Intuitively, because strong suppliers have a higher market share, they are more inclined to raise prices when de-integration causes suppliers on the weak side to increase their own prices. Our next result shows that this combination of higher prices and shift in market shares results in higher overall profit for suppliers.

Proposition 5. *Assume that $\delta_\theta = \theta^* - \theta$ is not too large. Then total profit is higher when suppliers are non-integrated:* $2\Pi^s(\beta) + 2\Pi^{*s}(\beta) > \Pi^m(\beta) + \Pi^{*m}(\beta)$.

To understand the intuition underlying Proposition 5 consider the case where suppliers have equal strengths on both sides of the market. Under integration, suppliers would share equally the market and thus charge $C^m(\beta) = \frac{r}{f(\frac{1}{2})}$ on both sides. Since demand is equally shared accross locations, the overall profit that accrues to the industry is thus worth $\Pi^m(\beta) = 2 \times \frac{r}{2f(\frac{1}{2})}$. Under non-integration, the market is still split evenly and each supplier on both side charges the very same fee $C^s(\beta) = C^m(\beta)$, which implies that the profit of the industry doubles: $\Pi^s(\beta) = 2\Pi^m(\beta)$. That is, non-integration soften competition, which shifts surplus from the buyer to the suppliers. It follows that this result extends to cases where the asymmetry δ_θ between suppliers is not too large. In the case where F is uniform, Proposition 5 holds for any δ_θ.

To address more broadly the welfare consequences of organizational choices, we now define the total welfare when the market is fully covered but split between suppliers at some $X \in [0, 1]$ as

$$\mathcal{W}(X) = \int_0^X \left(W^{fb}(\beta) - rx\right) f(x)dx + \int_X^1 \left(W^{*fb}(\beta) - r(1-x)\right) f(x)dx. \tag{3.22}$$

In particular, $\mathcal{W}(X^m(\beta))$ stands for welfare under integration while $\mathcal{W}(X^s(\beta))$ is reached under non-integration. These expressions take into account the fact, that in both scenarios, quality is efficiently provided on both sides of the market.

Next Proposition addresses more generally the comparison between organizational forms from an overall welfare viewpoint.

Proposition 6. *Total welfare are higher under integration:*

$$\mathcal{W}(X^m(\beta)) \geq \mathcal{W}(X^s(\beta)). \tag{3.23}$$

The logic behind this result is straightforward. Efficiency would require that the market be split at $X^{fb}(\beta)$ that maximizes (3.22) and we find

$$\frac{\Delta W^{fb}(\beta)}{2r} + \frac{1}{2} = X^{fb}(\beta). \tag{3.24}$$

When suppliers have unequal strengths on each side of the market, $X^{fb}(\beta)$ is greater than one half. It is also greater than $X^m(\beta)$ and *a fortiori* $X^s(\beta)$. Hence, the strong side of the market restricts too much its market share under integration but it does so even more under separation; which increases inefficiency.

Finally notice that, as prices increase and efficiency decreases under separation, the buyer surplus decreases with separation (at least for $\theta^* - \theta$ not too large).

Running Example. Suppose that F is uniform on Θ. It is straightforward to compute $X^m(\beta)$ and $X^s(\beta)$ when interior as

$$X^m(\beta) = \frac{1}{2} + \frac{\Delta W^{fb}(\beta)}{6r} \text{ and } X^s(\beta) = \frac{1}{2} + \frac{\Delta W^{fb}(\beta)}{10r}.$$

Similarly, the equilibrium fees under integration and non-integration respectively satisfy

$$C^m(\beta) = r + \frac{\Delta W^{fb}(\beta)}{3} \geq r - \frac{\Delta W^{fb}(\beta)}{3} = C^{*m}(\beta)$$

and

$$C^s(\beta) = r + \frac{\Delta W^{fb}(\beta)}{5} \geq r - \frac{\Delta W^{fb}(\beta)}{5} = C^{*s}(\beta).$$

Equilibrium profits under integration are finally given by

$$\Pi^m(\beta) = 2r\left(\frac{1}{2} + \frac{\Delta W^{fb}(\beta)}{6r}\right)^2 \geq 2r\left(\frac{1}{2} - \frac{\Delta W^{fb}(\beta)}{6r}\right)^2 = \Pi^{*m}(\beta)$$

Under non-integration, each supplier on the strong side of the market gets $\Pi^s(\beta)$ while it gets $\Pi^{*s}(\beta)$ on the weak side of the market with those profit levels being given by

$$\Pi^s(\beta) = 2r\left(\frac{1}{2} + \frac{\Delta W^{fb}(\beta)}{10r}\right)^2 \geq 2r\left(\frac{1}{2} - \frac{\Delta W^{fb}(\beta)}{10r}\right)^2 = \Pi^{*s}(\beta).$$

■

4 Competition at Both the Intensive and Extensive Margins

We now consider a scenario where B has private information on his vertical preference parameter β. This implies that prices can no longer be contingent on the vertical differentiation parameter β as this parameter is non-observable. In line with seminal work by Musa and Rosen (1978), Maskin and Riley (1984) and Wilson (1993) among others, non-linear pricing finds a theoretical foundation as a mechanism through which sellers screen buyers.

4.1 Incentive Compatibility

Consider the case where B visits the strong side of the market. We may rewrite his payoff from choosing within the offered aggregate tariff $\overline{T}$ that suppliers offer on that location as

$$U(\beta) = \max_{q \in \mathcal{Q}} \beta q - \frac{q^2}{2} - \overline{T}(q).^{11} \tag{4.1}$$

Accordingly, we define B's quality choice at venue 0 as

$$q(\beta) \in \arg\max_{q \in \mathcal{Q}} \beta q - \frac{q^2}{2} - \overline{T}(q). \tag{4.2}$$

We use similar starred notation for the allocation $(U^*(\beta), q^*(\beta))$ reached on the weak side of the market (location 1).

Standard results[12] show that B's net payoff $U(\beta)$ should be absolutely continuous, satisfy the following integral representation

$$U(\beta) = U(\underline{\beta}) + \int_{\underline{\beta}}^{\beta} q(\tilde{\beta}) d\tilde{\beta}, \tag{4.3}$$

[11] Because $\overline{T}$ is lower semi-continuous and $\mathcal{Q}$ is compact, the buyer's maximization problem has always a solution.

[12] See Rochet (1987) and Milgrom and Segal (2002) among others Lemma A.1 in the Appendix for details.

and be convex, a condition that can be written as

$$q \text{ non-decreasing.} \tag{4.4}$$

The integral representation (4.3) is a fundamental result in the mechanism design literature. It relates any non-decreasing quality profile q to the buyer's payoff $U(\beta)$ that such profile induces. This result is the basis for the trade-off between efficiency and rent extraction that pervades the nonlinear pricing literature. We will see that in our context, this trade-off will bite.

To understand the envelope condition (4.3), it is useful to consider the benefits that a buyer with preference parameter β gets by adopting the lower quality $q(\beta - d\beta)$ that is chosen by a marginally lower type $\hat{\beta} = \beta - d\beta$. By doing so, the buyer with type β would pay less and overall obtain a payoff that is above the payoff of a buyer of type $\hat{\beta}$ by approximately $q(\beta - d\beta)d\beta \approx q(\beta)d\beta$. A buyer with type β should thus receive an extra marginal rent worth $q(\beta)d\beta$ beyond what is already given to a type $\beta - d\beta$. The integral representation (4.3) shows how those rents offered to all infra-marginal types end up being compounded.

4.2 Market Shares

B decides to procure at either location 0 or 1 *ex ante*, i.e., before knowing the exact realization of his vertical preference parameter β. Following Yaari (1987) and Gershkov et al. (2023), we shall assume that the buyer's preferences for risky lotteries satisfy dual risk aversion, i.e., the buyer overweights low realizations of β in his assessment of the benefits of procuring at either location. Formally, assuming (weak) dual risk aversion amounts to considering that B located at x evaluates his expected utility from consuming at 0 (resp. 1) with the following criterion:

$$\mathbb{E}_H(U(\beta)) - rx \text{ (resp. } \mathbb{E}_H(U^*(\beta)) - r(1-x))$$

where the distribution G first-order stochastically dominates the distribution H used to compute this expectation.

To maintain tractability without losing any economic insight, we suppose that H is a simple transformation of G that we write as

$$H(\beta) = \begin{cases} \varepsilon & \text{if } \beta = \underline{\beta}, \\ \varepsilon + (1-\varepsilon)G(\beta) & \text{if } \beta \in (\underline{\beta}, \overline{\beta}] \end{cases} \tag{4.5}$$

where $\varepsilon \in (0, 1)$. In other words, B assigns a Dirac mass ε to the worst possible realization of his preference parameter. With the complementary probability $1 - \varepsilon$, other types are accounted for based on the prior beliefs G. In the polar case $\varepsilon = 1$, the buyer bases his purchases decision on the worst possible realization of his type $\underline{\beta}$. The other polar case $\varepsilon = 0$ amounts to assuming that the buyer is risk neutral.

Equipped with the specification (4.5) and the integral representation (4.3), a simple integration by parts yields

$$\mathbb{E}_H(U(\beta)) = U(\underline{\beta}) + (1-\varepsilon)\int_{\underline{\beta}}^{\overline{\beta}} q(\beta)(1-G(\beta))d\beta.$$

B will thus decide to move to location 0 for his purchases whenever

$$\mathbb{E}_H(U(\beta)) - rx \geq \mathbb{E}_H(U^*(\beta)) - r(1-x).$$

This condition can be expressed in terms of B's location x as

$$x \leq X(\overline{T}, \overline{T}^*) = \frac{1}{2} + \frac{1}{2r}\left(U(\underline{\beta}) - U^*(\underline{\beta}) + (1-\varepsilon)\mathbb{E}_G\left((q(\beta) - q^*(\beta))\frac{1-G(\beta)}{g(\beta)}\right)\right). \tag{4.6}$$

4.3 Equilibrium Under Integration

Under integration, S's expected profits can now be expressed as

$$\Pi(\overline{T}, \overline{T}^*) = \mathbb{E}_G(\overline{T}(q(\beta)) - \theta q(\beta))F(X(\overline{T}, \overline{T}^*)).$$

Following previous steps, this expression can be written in terms of the allocations $(U(\beta), q(\beta))$ and $(U^*(\beta), q^*(\beta))$ as[13]

$$\Pi(\overline{T}, \overline{T}^*) = \mathbb{E}_G\left(\left(\beta - \theta - \frac{1-G(\beta)}{g(\beta)}\right)q(\beta) - \frac{(q(\beta))^2}{2} - U(\underline{\beta})\right) \times F\left(X(\overline{T}, \overline{T}^*)\right).$$

We may finally express S's profit in terms of its own market share $F(X)$ (using 4.6) as

$$\begin{aligned}\Pi(\overline{T}, \overline{T}^*) =& \mathbb{E}_G\Big(\left(\beta - \theta - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^*(\beta) - \frac{(q(\beta))^2}{2} \\ &- U^*(\underline{\beta}) - r(2X-1)\Big)F(X).\end{aligned} \tag{4.7}$$

[13]Using $\mathbb{E}_G\left(\overline{T}(q(\beta))\right) = \mathbb{E}_G\left(\beta q(\beta) - \frac{q(\beta)^2}{2} - U(\beta)\right)$ and $\mathbb{E}_G\left(U(\beta)\right) = \mathbb{E}_G\left(U(\underline{\beta}) + \frac{1-G(\beta)}{g(\beta)}q(\beta)\right)$.

A similar expression is obtained for S^*'s profit under integration:

$$\Pi^*(\overline{T},\overline{T}^*) = \mathbb{E}_G\Big(\left(\beta - \theta^* - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right) q^*(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)} q(\beta) - \frac{(q^*(\beta))^2}{2} - U(\underline{\beta}) + r(2X-1)\Big)(1-F(X)). \tag{4.8}$$

Quality Distortions. To screen the buyer's preferences, integrated suppliers on both sides of the market distort the quality they offer. As in Mussa and Rosen (1978)'s monopolistic scenario, those distortions help extracting surplus from buyers with the highest willingness to consume quality. Reducing the quality offered to buyers with lower realizations of the vertical preference parameter β makes it less attractive for buyers with higher such vertical types to adopt the behavior of the latters and save on the price they pay for the composite good. Horizontal competition along the Hotelling line does not affect this basic insight. Competition determines the market shares that strong and weak suppliers respectively cover but, on a given market share, distortions are very much like in a monopolistic setting.

Yet, there is a first significant modeling difference between our setting and the monopolistic screening environment à la Mussa and Rosen (1978). In that paper, like in most of the nonlinear pricing literature, the buyer knows his type before contracting with the monopolist. Contracting takes place *ex post*. In contrast, in our model, the buyer ignores his type before choosing which suppliers to visit and thus contracting takes place *ex ante*. Had the buyer been risk neutral, the suppliers located at 0 could just offer a two-part tariff of the form

$$T(q) = \theta q + C \tag{4.9}$$

as under complete information on β. With such a scheme, the buyer would always choose the efficient level of quality when contracting at that venue. The fixed part of the tariff could be chosen by suppliers so as to optimally trade off the gains in profits obtained on all demand served when raising this fee against the corresponding loss of demand that follows such raise. The difficulty with this scheme is that the buyer may find paying this fee too costly in case his preference parameter turns out to be close to $\underline{\beta}$. This type would prefer leaving S when aware of his type. Our preference specification where the buyer overweights the probability that type $\underline{\beta}$ realizes (under $H(.)$) makes that risk costly ex ante for the buyer. Because the seller does not perceive this realization as very likely (under $G(.)$) it is efficient to increase the utility of the buyer in that adverse state to boost his demand. The cost of doing so is that this utility rise at the bottom transmits to all higher types β through incentive-compatibility constraints, as is apparent in (4.3). To limit that transmission which is governed by the quality level $q(\beta)$, S benefits from distorting quality downwards, all the more so for lower types. As a result, with a (dual) risk-averse buyer, there should now be a positive wedge between supplier S's marginal cost θ and the marginal price $T'(q)$ this supplier charges for each extra unit.

The second significant difference between this scenario and the monopolistic screening environment studied in Mussa and Rosen (1978) comes from the fact that, in our context, suppliers compete over the real line. Raising his own tariff certainly induces a loss of demand for S's goods but that demand can still be served by the rival supplier S^* at the other end of the line. Intuition built in earlier work by Stole (1995), Armstrong and Vickers (2001) and Rochet and Stole (2002) might suggest that such competition would reduce marginal prices towards marginal costs, eroding discriminatory power on both sides of the market. In contrast with ours, those papers consider models where competing suppliers wants to attract an already informed buyer. The above intuition turns out to be incorrect in our context where competition takes place *ex ante*, i.e., before the buyer learns his own type. Competing suppliers find it optimal to reduce the overall level of their own tariffs to attract buyers but still, they charge the same marginal price as if they were monopolists. In short, competition affects the extensive margin (the market shares) but not the intensive margin (the quality level).

These findings are presented in the next proposition.

Proposition 7. *Suppose that suppliers are integrated on both sides of the market and the buyer has private information on his demand parameter β. There exists a unique equilibrium between suppliers that entails the following features.*

1. *Quality is downward distorted below the first-best level on both sides of the market:*

$$q^m(\beta) = q^{fb}(\beta) - \varepsilon\frac{1-G(\beta)}{g(\beta)} \text{ and } q^{*m}(\beta) = q^{*fb}(\beta) - \varepsilon\frac{1-G(\beta)}{g(\beta)}. \tag{4.10}$$

2. *The market segmentation X^m satisfies*

$$\frac{\Delta W^{as}}{2r} + \frac{1}{2} = X^m + \frac{2F(X^m)-1}{f(X^m)} \tag{4.11}$$

where

$$\Delta W^{as} = \delta_\theta\left((1-\varepsilon)\beta^e + \varepsilon\underline{\beta} - \vartheta\right). \tag{4.12}$$

X^m decreases with ε and belongs to $\left[\frac{1}{2}, 1\right]$ whenever

$$0 \leq \frac{\delta_\theta}{2r}\left((1-\varepsilon)\beta^e + \varepsilon\underline{\beta} - \vartheta\right) \leq \frac{1}{2} + \frac{1}{f(1)}. \tag{4.13}$$

Some comments are in order. First, both suppliers distort the quality of the good they respectively offer below its first-best level. By reducing the whole spectrum of quality offered (and adjusting $U(\underline{\beta})$ accordingly), each supplier reduces the risk borne by B and makes it more attractive for B to visit its own location. From the point of view of sellers, everything happens as if the demand parameter β was now replaced by a lower virtual demand parameter equal to

$$\tilde{\beta}^m = \beta - \varepsilon \frac{1 - G(\beta)}{g(\beta)} \leq \beta. \tag{4.14}$$

In the extreme case where ε tends to 1, the buyer's decision is solely based on what each supplier offers to the worst realization of the buyer's type $\underline{\beta}$. Therefore suppliers compete to attract this low type, understanding that buyers with $\beta \geq \underline{\beta}$ then become *de facto* captive. Suppliers' incentives to extract rents from these higher types are then maximal, which generates the highest quality distortions.

Instead, when ε decreases towards 0, everything happens as if B was almost risk neutral. S offers a tariff which is almost of the form (4.9) and thus B, once informed on his vertical type, chooses an efficient quality level. The same applies on S^*'s side.

Because suppliers at both sides of the market charge the same marginal price and distort quality similarly, the difference in quality levels between both venues remain unchanged and only reflects the suppliers' comparative advantage, as under complete information on β. Yet, quality levels on both sides are lower because of screening distortions.

Market Shares. Under complete information on β, market shares depended on the difference in surplus across the two locations (see Equation (3.7)). A similar intuition applies here (Equation (4.11)) but the difference is now in the expected *virtual* surpluses that account for the downward adjustment in Equation (4.14).

Tariffs and Profits. As discussed above, under asymmetric information, each supplier charges a marginal price above its marginal cost. The resulting tariffs are characterized in the next Proposition.

Proposition 8. *Suppose that suppliers are integrated on both sides of the market and there is asymmetric information on the vertical parameter β. S and S^*'s equilibrium tariffs satisfy*

$$T^m(q) = \theta q + \varepsilon \int_{q^m(\underline{\beta})}^{q} \frac{1 - G(\gamma^m(\tilde{q}))}{g(\gamma^m(\tilde{q}))} d\tilde{q} + C^m \tag{4.15}$$

and

$$T^{*m}(q) = \theta^* q + \varepsilon \int_{q^m(\underline{\beta})}^{q} \frac{1 - G(\gamma^{*m}(\tilde{q}))}{g(\gamma^{*m}(\tilde{q}))} d\tilde{q} + C^{*m} \tag{4.16}$$

where $\gamma^m(q)$ and $\gamma^{*m}(q)$ are respectively the inverse functions for $q^m(\beta)$ and $q^{*m}(\beta)$ defined in (4.3) and where fees are given by

$$C^m = 2r \frac{F(X^m)}{f(X^m)} - \varepsilon \int_{\underline{\beta}}^{\overline{\beta}} \frac{(1 - G(\beta))^2}{g(\beta)} \left(1 - \varepsilon \frac{d}{d\beta} \left(\frac{1 - G(\beta)}{g(\beta)}\right)\right) d\beta \tag{4.17}$$

and

$$C^{*m} = 2r\frac{1-F(X^m)}{f(X^m)} - \varepsilon\int_{\underline{\beta}}^{\overline{\beta}} \frac{(1-G(\beta))^2}{g(\beta)}\left(1-\varepsilon\frac{d}{d\beta}\left(\frac{1-G(\beta)}{g(\beta)}\right)\right)d\beta. \tag{4.18}$$

The comparison of (4.15) and (4.16) with their complete information counterparts (3.11) shows two effects of asymmetric information. First, marginal prices are above marginal costs at both sides of the market as suppliers exert discriminatory power on their captive demand. Second, fixed fees, although similar to their complete information counterparts (3.12), are now lowered. However, the variable part of tarif is higher under asymmetric information.

Next proposition turns to equilibrium profits. Remarkably, those profits take the same expressions as under complete information and they only depend on the respective market shares of the suppliers.

Proposition 9. *Suppose that suppliers are integrated on both sides of the market and there is asymmetric information on β. Equilibrium profits for S and S^* are respectively such that*

$$\Pi^m = 2r\frac{(F(X^m))^2}{f(X^m)} \geq 2r\frac{(1-F(X^m))^2}{f(X^m)} = \Pi^{*m}. \tag{4.19}$$

Running Example. It is straightforward to compute X^m when interior as

$$X^m = \frac{1}{2} + \frac{\Delta W^{as}}{6r}.$$

Asymmetric information reduces the attractiveness of the strong side of the market. More precisely, we have

$$\Delta W^{as} \leq \mathbb{E}_G\left(\Delta W^{fb}(\beta)\right) = \delta_\theta(\beta^e - \vartheta)$$

and thus

$$X^m \leq \mathbb{E}_G\left(X^m(\beta)\right).$$

In other words, asymmetric information weakens the strong suppliers' comparative advantage and make suppliers on both sides more alike. In fact, asymmetric information dampens quality as shown above and, at lower quality levels, virtual surpluses on both sides of the market are more alike. To illustrate, suppose that G is uniform on $[1, 2]$. We compute

$$q^m(\beta) = (1+2\varepsilon)\beta - 2\varepsilon - \theta \text{ and } q^{*m}(\beta) = (1+2\varepsilon)\beta - 2\varepsilon - \theta^*.$$

Both quality levels remain non-negative when $1 \geq \theta^* \geq \theta$. From there, it also follows that

$$\gamma^m(q) = \frac{q+\theta+2\varepsilon}{1+2\varepsilon} \text{ and } \gamma^{*m}(q) = \frac{q+\theta^*+2\varepsilon}{1+2\varepsilon}.$$

Inserting into the expressions of marginal prices yields

$$\frac{dT^m(q)}{dq} - \theta = \frac{\varepsilon}{1+2\varepsilon}(2+2\varepsilon-\theta-q) \geq \frac{\varepsilon}{1+2\varepsilon}(2+2\varepsilon-\theta^*-q) = \frac{dT^{*m}(q)}{dq} - \theta^*.$$

This strong side of the market charges a lower price-cost margin than the weak side of the market. Hence, a higher quality is supplied at that venue. Finally, equilibrium fees satisfy

$$C^m = r + \delta_\theta\left(\frac{1+\varepsilon}{2} - \vartheta\right) - \frac{\varepsilon(1+\varepsilon)}{3}, \quad C^{*m} = r - \delta_\theta\left(\frac{1+\varepsilon}{2} - \vartheta\right) - \frac{\varepsilon(1+\varepsilon)}{3}.$$

We notice that, for $\varepsilon = 0$, $C^m = \mathbb{E}_G\left(C^m(\beta)\right)$, $C^{*m} = \mathbb{E}_G\left(C^{*m}(\beta)\right)$ and $X^m = \mathbb{E}_G\left(X^m(\beta)\right)$. In other words, when the buyer is risk neutral, suppliers still compete in two part tariffs with no (marginal) price-cost margin but, in expectations, this competition replicates what is achieved when β is known. ■

4.4 Maximal Equilibria Under Non-Integration

Under non-integration and asymmetric information, S_i's expected profits can now be expressed as

$$\Pi_i(T_i, T^s_{-i}, \overline{T}^{*s}) = \mathbb{E}_G(T_i(q(\beta)) - \theta_i q(\beta))F(X(T_i + T^s_{-i}, \overline{T}^{*s})).$$

This profit can be rewritten in terms of the allocations $(U(\beta), q(\beta))$ and $(U^*(\beta), q^*(\beta))$ at both sides of the market as

$$\Pi_i(T_i, T^s_{-i}, \overline{T}^{*s}) = \mathbb{E}_G\left(\left(\beta - \theta_i - \frac{1-G(\beta)}{g(\beta)}\right)q(\beta) - \frac{(q(\beta))^2}{2} - T^s_{-i}(q(\beta))) - U(\underline{\beta})\right)$$
$$\times F\left(\frac{1}{2} + \frac{1}{2r}\left(U(\underline{\beta}) - U^*(\underline{\beta}) + (1-\varepsilon)\mathbb{E}_G\left((q(\beta) - q^*(\beta))\frac{1-G(\beta)}{g(\beta)}\right)\right)\right).$$

We may finally express S_i's profit in terms of its own market share $F(X)$ that its choice of a tariff T_i induces, taking as given the other suppliers' tariffs on both sides of the markets, as

$$\Pi_i(T_i, T^s_{-i}, \overline{T}^{*s}) = \mathbb{E}_G\Big(\left(\beta - \theta_i - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^*(\beta) - \frac{(q(\beta))^2}{2}$$
$$- T^s_{-i}(q(\beta))) - U^*(\underline{\beta}) - r(2X-1)\Big)F(X). \quad (4.20)$$

A similar expression yields S_i^*'s profit under non-integration taking as given the other suppliers' tariffs on both sides of the markets:

$$\Pi_i^*(T_i, T_{-i}^{*s}, \overline{T}^s) = \mathbb{E}_G\Big(\left(\beta - \theta_i - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right) q^*(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)} q(\beta) - \frac{(q^*(\beta))^2}{2} \\ - T_{-i}^{*s}(q(\beta)) - U(\underline{\beta}) + r(2X-1)\Big)(1-F(X)). \tag{4.21}$$

Here also, suppliers on a given side of the market should agree on which quality level to induce at equilibrium. To illustrate, pointwise optimization of the maximand in (4.20) leads S_i to choose $q^s(\beta)$ that maximizes the bilateral virtual surplus of the coalition it forms with the buyer, namely

$$q^s(\beta) \in \arg\max_{q\in\mathcal{Q}} \left(\beta - \theta_i - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right) q - \frac{q^2}{2} - T_{-i}^s(q). \tag{4.22}$$

Second, S_i would like to induce a market segmentation X^s such that

$$X^s \in \arg\max_{X\in[0,1]} \mathbb{E}_G\Big(\left(\beta - \theta_i - \varepsilon\frac{1-G(\beta)}{g(\beta)}\right) q^s(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)} q^{*s}(\beta) - \frac{(q^s(\beta))^2}{2} \\ - T_{-i}^s(q^s(\beta))) - U^{*s}(\underline{\beta}) - r(2X-1)\Big) F(X). \tag{4.23}$$

Conditions (4.22) and (4.23) again characterize S_i's best responses both at the intensive and at the extensive margin.

Following Martimort, Semenov and Stole (2018), we observe that, if $q^s(\beta)$ solves the maximization problem (4.22) for i=1, 2, it also solves a maximization problem obtained by summing the maximands for each supplier, namely

$$q^s(\beta) \in \arg\max_{q\in\mathcal{Q}} \left(\beta - \theta - 2\varepsilon\frac{1-G(\beta)}{g(\beta)}\right) q - \frac{q^2}{2} + \left(\beta q - \frac{q^2}{2} - \overline{T}^s(q)\right) \tag{4.24}$$

where $\overline{T}^s(q) = \sum_{i=1}^2 T_i^s(q)$.

The power of this aggregation procedure across best responses is to allow us to identify the equilibrium choice of quality with the solution of a single maximization problem. Of course, the so obtained maximization problem still encompasses a fixed-point because the second-bracketed term on the right-hand side of (4.24) depends on the aggregated equilibrium tariff $\overline{T}^s$. Everything happens as if a *surrogate supplier* was offering the quality $q^s(\beta)$ so as to maximize this new maximand.

The similarity with the objective of an integrated supplier is twofold. First, because the new maximization problem for this surrogate is obtained by summation, its maximand coumpounds the distortion that each supplier would like to induce on its own. Each supplier wants to reduce the risk borne by the buyer and thus distort quality downwards accordingly but when he does so it does not take into account the impact of this distortion on the complementary supplier on its own location.

To see that, consider equation (4.22). When choosing the quality, the supplier S_1 accounts for the surplus of the consumer and its own profit, but perceives the payment to the other supplier S_2 as a cost. Downward distortion of quality implies that the slope of the tariff $T_2^s(q)$ proposed by S_2 is larger than its marginal cost θ_2. Hence the marginal cost perceived by S_1 is $\theta_1 + \frac{T_2^s(q)}{dq} > \theta$. In addition to–and because of–the distortion induced by the rent-efficiency trade-off, there is a tariff externality that exacerbates inefficiencies.

As a result, there is excessive quality distortion that is induced under non-integration. This double distortion due to asymmetric information is best seen by the new expression of the buyer's virtual preference parameter under non-integration which becomes

$$\tilde{\beta}^s = \beta - 2\varepsilon \frac{1 - G(\beta)}{g(\beta)} \leq \tilde{\beta}^m \leq \beta. \tag{4.25}$$

Second, the surrogate supplier's objective accounts for the buyer's surplus (the second bracketed term on the right-hand side of (4.24)).

Martimort, Semenov and Stole (2018) show that there exists a multiplicity of equilibria of this game which are fully characterized by the spectrum of quality $\overline{\mathcal{Q}}$ that is available to choose from. To illustrate, had S_1 requested infinite payments for quality levels outside some range $\overline{\mathcal{Q}}$, S_2 would be forced to induce quality choices within that range as well because both components they sell are needed when perfect complements.

Formally, Martimort, Semenov and Stole (2018) show that a version of the Enveloppe Theorem applies (even for non-differentiable tariffs). Because $q^s(\beta)$ is also the buyer's choice given the equilibrium tariffs offered, it maximizes the second bracketed term in (4.24). It follows that condition (4.24) can actually be reduced to

$$q^s(\beta) \in \arg\max_{q \in \overline{\mathcal{Q}}} \left(\beta - \theta - 2\varepsilon \frac{1 - G(\beta)}{g(\beta)} \right) q - \frac{q^2}{2}. \tag{4.26}$$

where $\overline{\mathcal{Q}} = q^s(\mathcal{B})$ is a range of quality levels available at equilibrium. In other words, (4.26) defines a set of incentive constraints for the surrogate principal. Moreover, this equilibrium characterization is not only necessary but is is also sufficient for any $\overline{\mathcal{Q}}$ possibly constraining the quality spectrum. Condition (4.24) determines a quality profile, which together with the integral representation of the buyer's payoff at location 0 and a boundary condition that determines market shares fully characterizes the aggregate tariff and thus completes the characterization of the equilibrium. This methodology will be used below.

Among all equilibria, the so called *maximal equilibrium* is obtained when the above maximization remains unconstrained. The maximal equilibrium entails thus a quality profile such that

$$q^s(\beta) \in \arg\max_{q \in \mathcal{Q}} \left(\beta - \theta - 2\varepsilon \frac{1-G(\beta)}{g(\beta)}\right) q - \frac{q^2}{2}. \tag{4.27}$$

The similarity with the objective of an integrated supplier S is straightforward. The only difference comes from the fact that now the buyer's virtual preference parameter entails twice the informational distortion as specified in (4.25).

We may proceed similarly when aggregating the choice of market shares induced by non-integrated suppliers located 0. Summing (4.23) that characterizes its choices for S_i and a similar condition for S_{-i} yields

$$X^s \in \arg\max_{X \in [0,1]} \mathbb{E}_G \Big(\left(\beta - \theta - \varepsilon \frac{1-G(\beta)}{g(\beta)}\right) q^s(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)} q^{*s}(\beta) - \frac{(q^s(\beta))^2}{2} \\ - U^{*s}(\underline{\beta}) + 2r\,(X^s - X) - r(2X-1)\Big) F(X). \tag{4.28}$$

Proposition 10. *Suppose that suppliers are non-integrated on both sides of the market and the buyer has private information on his vertical demand parameter β. There exists a unique maximal equilibrium between suppliers. This equilibrium entails the following features.*

1. *Quality is downward distorted below the integrated solution on both sides of the market:*

$$q^s(\beta) = q^{fb}(\beta) - 2\varepsilon \frac{1-G(\beta)}{g(\beta)} \text{ and } q^{*s}(\beta) = q^{*fb}(\beta) - 2\varepsilon \frac{1-G(\beta)}{g(\beta)}. \tag{4.29}$$

2. *The market segmentation X^s satisfies*

$$\frac{\Delta W^{as}}{2r} + \frac{1}{2} = X^s + 2\frac{2F(X^s)-1}{f(X^s)}. \tag{4.30}$$

X^s decreases with ε and belongs to $\left[\frac{1}{2}, X^m\right]$.

Quality Levels. Under non-integration, each supplier at a given location brings its own informational distortion in the provision of quality. As a result, the equilibrium quality level is twice as distorted as under integration. Competition at the intensive margin between complementary suppliers leads to excessively low quality levels.

Market Shares. Although the separation of suppliers magnifies the quality distortion, the welfare differential ΔW^{as} in (4.30) remains the same as in the integrated case. However, and by a reasoning similar to that arising under complete information, strong suppliers, when non-integrated, restrict their market shares by charging too high a fee for their product while their weak (non-integrated) opponents *a contrario* gain market shares; a second force that relies on the extensive margin of competition.

TARIFFS AND PROFITS. Next Propositions characterize tariffs and profits at the maximal equilibrium.

Proposition 11. *Suppose that suppliers are non-integrated on both sides of the market and there is asymmetric information on the vertical preference parameter β. S_i and S_i^*'s maximal equilibrium tariffs satisfy*

$$T_i^s(q) = \theta_i q + \varepsilon \int_{q^s(\underline{\beta})}^{q} \frac{1 - G(\gamma^s(\tilde{q}))}{g(\gamma^s(\tilde{q}))} d\tilde{q} + C^s \tag{4.31}$$

and

$$T_i^{*s}(q) = \theta_i^* q + \varepsilon \int_{q^s(\underline{\beta})}^{q} \frac{1 - G(\gamma^{*s}(\tilde{q}))}{g(\gamma^{*s}(\tilde{q}))} d\tilde{q} + C^{*s} \tag{4.32}$$

where $\gamma^s(q)$ and $\gamma^{*s}(q)$ are respectively the inverse functions for $q^s(\beta)$ and $q^{*s}(\beta)$ defined in (4.29) and where the fees are given by

$$C^s = 2r\frac{F(X^s)}{f(X^s)} - \varepsilon \int_{\underline{\beta}}^{\overline{\beta}} \frac{(1 - G(\beta))^2}{g(\beta)} \left(1 - \varepsilon \frac{d}{d\beta}\left(\frac{1 - G(\beta)}{g(\beta)}\right)\right) d\beta \tag{4.33}$$

and

$$C^{*s} = 2r\frac{1 - F(X^s)}{f(X^s)} - \varepsilon \int_{\underline{\beta}}^{\overline{\beta}} \frac{(1 - G(\beta))^2}{g(\beta)} \left(1 - \varepsilon \frac{d}{d\beta}\left(\frac{1 - G(\beta)}{g(\beta)}\right)\right) d\beta. \tag{4.34}$$

Next proposition turns to the expression of equilibrium profits. Remarkably, those profits take the same expressions as under complete information and they only depend on the respective market shares of the suppliers.

Proposition 12. *Suppose that suppliers are non-integrated on both sides of the market and there is asymmetric information on β. Equilibrium profits for S_i and S_i^* are respectively such that*

$$\Pi^s = 2r\frac{(F(X^s))^2}{f(X^s)} \geq 2r\frac{(1 - F(X^s))^2}{f(X^s)} = \Pi^{*s}. \tag{4.35}$$

RUNNING EXAMPLE. If F is uniform, It is straightforward to compute X^s, when it is interior, as

$$X^s = \frac{1}{2} + \frac{\Delta W^{as}}{10r}.$$

Asymmetric information reduces the attractiveness of the strong side of the market. More precisely, we have

$$X^s \leq \mathbb{E}_G\left(X^s(\beta)\right).$$

In other words, asymmetric information weakens the strong suppliers' comparative advantage and make suppliers on both sides more alike. ■

5 Concluding Remarks

This article highlights some potential adverse effects associated with the fragmentation of the supply chain of competing system goods. Separating the supply of complementary system components across several producers tends to exacerbate distortions driven by market power and informational frictions. First, mis-coordination between suppliers within the same system raises prices. While this Cournot effect hurts suppliers when there is a single system it can soften competition when there are competing systems. This effect lowers welfare as the most efficient system loses market share. An additional effect materializes under asymmetric information and (dual) risk aversion. In that case, disintegration exacerbates the downward quality distortions introduced by sellers to lower the cost of screening buyers. Indeed, suppliers do no internalize that quality distortions lower the demand for the complementary product within the system.

Overall, this analysis calls for caution when assessing the likely implications of interface standardization between components, which lies at the heart of Open RAN. Introducing this standard follows a logic where facilitating the combination of components from different producers in a single system lowers barriers to entry, leads to lower costs for consumers and stimulates innovation. Our analysis suggests that excessive fragmentation may erode this benefits and that the overall balance will depend critically on the structure of the market. Beyond the conclusions of this paper, the model we propose can serve as a building block to envision several scenarios.

A first extension of our framework would be to allow for entry. For example, suppose there are multiple potential entrants at each end of the Hotelling line, each facing a fixed cost of entry. If firms can only offer fully integrated systems Bertrand competition on each side of the market ensures that only the firm with the lowest cost supplies the system. Consequently, there is at most one entrant for each system—the one with the lowest total production cost (in the parlance of our model, the firm with the lowest θ for system S and the lowest θ^* for system S^*).

With Open RAN, however, components are standardized, allowing buyers to source different components from different suppliers. The same entry logic now applies at the component level. Ex post Bertrand competition again eliminates redundant entry, so for each component only the lowest-cost producer remains active. Provided that the minimum cost for the two components is not achieved by the same firm, there will be two independent suppliers contributing to each system. In this scenario, Open RAN reduces overall production costs, which must be weighed against the coordination inefficiencies highlighted in this paper. In our analysis, because of complementarity of components, only total system costs θ and θ^* matters (and not the repartition of the cost between suppliers). Thus, under this scenario, Open Ran will benefit the buyer if and only if the reduction in total cost of each system is strong enough.

To further highlight how the effect of disintegration depends on the market structure, consider a different scenario where there exists a competitive fringe that could supply of one component for each system at a lower cost, say $\hat{\theta}_1 < \theta_1$ and $\hat{\theta}_1^* < \theta_1^*$. Absent a standard such as Open Ran, this fringe may not operate as they cannot make their component compatible with the systems in place on the market. Suppose now that standardization makes this fringe component compatible. Then incumbent system suppliers should let the buyer procure component 1 in the competitive market at prices $\hat{\theta}_1$ and $\hat{\theta}_1^*$ and sell only component 2.[14] Under this scenario, competition for component one eliminates the coordination problem. Moreover each supplier of the second component, S_2 and S_2^*, captures the full profit from its system, i.e., it is in their best interest to disintegrate. The equilibrium would then be the same as without Open Ran but with lower costs, which would benefits both suppliers and the buyer.

Finally we may also envision a scenario where Open Ran does not result in more entry but allows the buyer to mix-and-match components from the two incumbents. Implications of this scenario for markets shares and quality levels should be the object of future investigations.

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Appendix: Proofs of the Main Results

Proof of Proposition 1. We prove each item in turn.

1. Maximizing the expression of $\Pi(\beta, \overline{T}, \overline{T}^*)$ given in (3.4) with respect to $q(\beta)$ and that of $\Pi^*(\beta, \overline{T}, \overline{T}^*)$ given in (3.5) with respect to $q^*(\beta)$ gives (3.6).

2. Maximizing the expression of $\Pi(\beta, \overline{T}, \overline{T}^*)$ given in (3.14) with respect to X yields the following necessary first-order condition for an interior solution:

$$\left((\beta-\theta)q(\beta)-\frac{(q(\beta))^2}{2}-r(2X^m(\beta)-1)-U^{*m}(\beta)\right)f(X^m(\beta))=2rF(X^m(\beta)). \quad \text{(A.1)}$$

This condition is also sufficient since the maximand in (3.14) is quasi-concave when the *Monotone Hazard Rate Property* (2.1) holds.

Similarly, maximizing the expression of $\Pi^*(\beta, \overline{T}, \overline{T}^*)$ given in (3.5) with respect to X yields the following necessary (and also sufficient by a similar argument) first-order condition for an interior solution:

$$\left((\beta - \theta^*)q^*(\beta) - \frac{(q^*(\beta))^2}{2} + r(2X^m(\beta) - 1) - U^m(\beta)\right) f(X^m(\beta)) = 2r(1 - F(X^m(\beta))). \tag{A.2}$$

Using (3.6), we rewrite (A.1) and (A.2) respectively as

$$\left(W^{fb}(\beta) - r(2X^m(\beta) - 1) - U^{*m}(\beta)\right) f(X^m(\beta)) = 2rF(X^m(\beta)) \tag{A.3}$$

and

$$\left(W^{*fb}(\beta) + r(2X^m(\beta) - 1) - U^m(\beta)\right) f(X^m(\beta)) = 2r(1 - F(X^m(\beta))). \tag{A.4}$$

Substracting (A.4) from (A.3) yields (3.7).

Because $\frac{2F(X)-1}{f(X)} = \frac{F(X)}{f(X)} + \frac{F(X)-1}{f(X)}$ is non-decreasing when the *Monotone Hazard Rate Property* (2.1) holds, $X^m(\beta)$ as defined in (3.7) increases with $\Delta W^{fb}(\beta)$. Moreover, $X^m(\beta) = \frac{1}{2}$ when $\Delta W^{fb}(\beta) = 0$ and $X^m(\beta)$ remains less than one when (3.8) holds.

□

Proof of Proposition 2. Note that the bracketed term in (A.3) can be written as

$$W^{fb}(\beta) - r(2X^m(\beta) - 1) - U^{*m}(\beta) = W^{fb}(\beta) - U^m(\beta) = T(q^m(\beta)) - \theta q^m(\beta)$$

or

$$C^m(\beta) = 2r\frac{F(X^m(\beta))}{f(X^m(\beta))} \tag{A.5}$$

Proceeding similarly for (3.5) yields the expression of $C^{*m}(\beta)$.

Because $X^m(\beta) \geq \frac{1}{2}$ and f is symmetric, we have $\frac{F(X^m(\beta))}{f(X^m(\beta))} \geq \frac{1-F(X^m(\beta))}{f(X^m(\beta))}$ and thus the inequality in (3.12) holds.

Observe that $\Pi^m(\beta) = C^m(\beta)F(X^m(\beta))$ and $\Pi^{*m} = C^{*m}(\beta)(1-F(X^m(\beta)))$. The inequality in (3.13) follows from observing that, first, $\frac{(F(X))^2}{f(X)}$ (resp. $\frac{(1-F(X))^2}{f(X)}$ is non-decreasing (resp. non-increasing) when the *Monotone Hazard Rate Property* (2.1) holds and second, $X^m(\beta) \geq \frac{1}{2}$.
□

Proof of Proposition 3. We prove each item in turn.

1. In any truthful equilibrium, S_{-i} offers a tariff of the form $T^s_{-i}(q) = \theta_{-i}q + C^s_{-i}$. Inserting this expression into S_i's best-response condition (3.15) yields

$$q^s(\beta) \in \arg\max_{q\in\mathcal{Q}} (\beta-\theta)q - \frac{q^2}{2} - C^s_{-i} = q^{fb}(\beta).$$

   A similar condition applies for S_{-i}'s best response. Hence, (3.18) holds.

2. Inserting (3.18) into S_i's best-response condition (3.16) yields

$$W^{fb}(\beta) - C^s_{-i} - r(2X^s(\beta)-1) - U^{*s}(\beta) = 2r\frac{F(X^s(\beta))}{f(X^s(\beta))}. \tag{A.6}$$

   This condition is also sufficient since the maximand is again quasi-concave when the *Monotone Hazard Rate Property* (2.1) holds.

   Similarly, S_{-i}'s best-response condition yields

$$W^{fb}(\beta) - C^s_{i} - r(2X^s(\beta)-1) - U^{*s}(\beta) = 2r\frac{F(X^s(\beta))}{f(X^s(\beta))}. \tag{A.7}$$

   Observe also that

$$U^s(\beta) = W^{fb}(\beta) - C^s_i - C^s_{-i}. \tag{A.8}$$

   Summing (A.6) and (A.7) and using (A.8) finally yields

$$W^{fb}(\beta) + U^s(\beta) - 2r(2X^s(\beta)-1) - 2U^{*s}(\beta) = 4r\frac{F(X^s(\beta))}{f(X^s(\beta))}. \tag{A.9}$$

   Turning to suppliers located at 1 and proceeding similarly, we find

$$W^{*fb}(\beta) + U^{*s}(\beta) + 2r(2X^s(\beta)-1) - 2U^{s}(\beta) = 4r\frac{1-F(X^s(\beta))}{f(X^s(\beta))}. \tag{A.10}$$

   Observe that

$$X^s(\beta) = \frac{1}{2} + \frac{1}{2r}(U^s(\beta) - U^{*s}(\beta)). \tag{A.11}$$

   Substracting (A.10) from (A.9) and using (A.11) finally yields (3.19).

   Because $\frac{2F(X)-1}{f(X)} = \frac{F(X)}{f(X)} + \frac{F(X)-1}{f(X)}$ is non-decreasing when the *Monotone Hazard Rate Property* (2.1) holds, $X^s(\beta)$ as defined in (4.3) increases with δ_θ. Moreover, $X^s(\beta) = \frac{1}{2}$ when $\delta_\theta = 0$ and

$$\frac{\Delta W^{fb}(\beta)}{2r} + \frac{1}{2} \geq X^s(\beta) + \frac{2F(X^s(\beta))-1}{f(X^s(\beta))}. \tag{A.12}$$

   Comparing with (3.7), we immediately deduce that $X^m(\beta) \geq X^s(\beta)$.

□

Proof of Proposition 4. The only important point is to check that suppliers on the same end of the market charge the same fee and make the same profit. To this end, observe that (A.6) and (A.7) immediately imply $C_1^s(\beta) = C_2^s(\beta)$. A similar proof would go for suppliers located at 1. Finally, the proof of the ranking between fees and profits is similar to the Proof of Proposition 2 and is thus left to the reader. □

Proof of Proposition 5. Consider first the case of integration. The overall profit of the industry in this scenario is

$$\Pi^m(\beta) + \Pi^{*m}(\beta) = 2r\left(\frac{(F(X^m(\beta)))^2}{f(X^m(\beta))} + \frac{(1-F(X^m(\beta)))^2}{f(X^m(\beta))}\right). \tag{A.13}$$

Consider now the case of non-integration. Because all suppliers on a given side of the market get the same profit, the overall profit of the industry writes then as

$$2\Pi^s(\beta) + 2\Pi^{*s}(\beta) = 4r\left(\frac{(F(X^s(\beta)))^2}{f(X^s(\beta))} + \frac{(1-F(X^s(\beta)))^2}{f(X^s(\beta))}\right). \tag{A.14}$$

When θ and θ^* are close than both $X^m(\beta)$ and $X^s(\beta)$ are close to $1/2$, so that the total profit under separation is almost the double of the total profit under integration.

Moreover, in the case of a uniform distribution we have

$$3X^m(\beta) - 1 = 5X^s(\beta) - 2 \tag{A.15}$$

so that the difference in profit is

$$2\Pi^s(\beta) + 2\Pi^{*s} - \Pi^m(\beta) - \Pi^{*m}(\beta) = 2r\left(\frac{-14X^m(\beta)^2 + 14X^m(\beta) + 9}{25}\right). \tag{A.16}$$

This is concave in $X^m(\beta)$ with values 0.5 and $\frac{9}{25}$ at $X^m(\beta) = 0.5$ and $X^m(\beta) = 1$ respectively. Hence it is positive for all $X^m(\beta)$. We thus have for F uniform:

$$2\Pi^s(\beta) + 2\Pi^{*s} > \Pi^m(\beta) + \Pi^{*m}(\beta) \tag{A.17}$$

which ends the proof. □

Proof of Proposition 6. Observe that

$$\mathcal{W}'(X) = (\Delta W^{fb}(\beta) - r(2X-1))f(X) \text{ and } \frac{d}{dX}\left(\frac{\mathcal{W}'(X)}{f(X)}\right) = -2r.$$

In other words, $\mathcal{W}(X)$ is quasi-concave and maximized at X^{fb} such that

$$X^{fb} = \frac{\Delta W^{fb}(\beta)}{2r} + \frac{1}{2}.$$

From Proposition 3

$$\frac{1}{2} \leq X^s(\beta) \leq X^m(\beta) \leq X^{fb}.$$

Using that $\mathcal{W}$ is quasi-concave then yields (3.23). □

Lemma A.1. *The buyer's payoff U is convex, absolutely continuous and satisfies the integral representation (4.3). Reciprocally, any allocation $(U(\beta), q(\beta))$ such that $U(\beta)$ is absolutely continuous and convex and satisfies (4.3), is such that (4.1) holds.*

Proof of Lemma A.1. NECESSITY. First, observe that U defined in (4.1) is convex as a maximum of linear functions of θ. Second, it immediately follows from Theorem 2 and Corollary 1 in Milgrom and Segal (2002), that U is absolutely (in fact Lipschitz) continuous and almost everywhere differentiable with

$$\dot{U}(\beta) = q(\beta) \tag{A.18}$$

holding at any point of differentiability. From there, the integral representation (4.3) follows.

SUFFICIENCY. Reciprocally, any allocation $(U(\beta), q(\beta))$ such that $U(\beta)$ is absolutely continuous and convex, with (4.3), $q(\beta) \in \partial U(\beta)$[15] is such that (4.1) holds. To prove this, consider any pair $(\beta, \hat{\beta}) \in \mathcal{B}^2$. We may then rewrite the integral representation (4.3) as

$$U(\beta) = U(\hat{\beta}) + \int_{\hat{\beta}}^{\beta} q(\tilde{\beta})d\tilde{\beta}.$$

Because U is convex, it admits a sub-differential ∂U and, since $q(\hat{\beta}) \in \partial U(\hat{\beta})$, we have

$$U(\beta) \geq U(\hat{\beta}) + q(\hat{\beta})(\beta - \hat{\beta}).$$

From there and the definition of U as in (4.3), (4.1) follows. □

Proof of Proposition 7. We now prove each item of the Proposition in turn.

1. Maximizing the expression of $\Pi(\overline{T}, \overline{T}^*)$ given in (4.7) with respect to $q(\beta)$ pointwise and that of $\Pi^*(\overline{T}, \overline{T}^*)$ given in (4.8) with respect to $q^*(\beta)$ also pointwise gives (4.3).

   Thanks to the *Monotone Hazard Rate Property* (2.4), q^m and q^{*m} so defined both satisfy the monotonicity condition (4.4).

[15] $\partial U(\beta)$ denotes the subdifferential of the convex function U at β, namely $\partial U(\beta) = \left\{q \text{ such that } U(\hat{\beta}) - U(\beta) \geq q(\hat{\beta} - \beta) \quad \forall \hat{\beta} \in \mathcal{B}\right\}$.

2. Maximizing the expression of $\Pi(\overline{T}, \overline{T}^*)$ given in (4.7) with respect to X yields the following necessary first-order condition for an interior solution:

$$\begin{aligned}&\mathbb{E}_G\left(\left(\beta-\theta-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^m(\beta)-(1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^{*m}(\beta)-\frac{(q^m(\beta))^2}{2}-U^*(\underline{\beta})\right)\\ =\;& r(2X^m-1)+2r\frac{F(X^m)}{f(X^m)}. \qquad \text{(A.19)}\end{aligned}$$

This condition is also sufficient since the maximand in (4.7) is quasi-concave when the *Monotone Hazard Rate Property* (2.1) holds.

Similarly, maximizing the expression of $\Pi^*(\overline{T}, \overline{T}^*)$ given in (4.8) with respect to X yields the following necessary (and also sufficient by a similar argument) first-order condition for an interior solution:

$$\begin{aligned}&\mathbb{E}_G\left(\left(\beta-\theta-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^{*m}(\beta)-(1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^{m}(\beta)-\frac{(q^{*m}(\beta))^2}{2}-U(\underline{\beta})\right)\\ =\;& -r(2X^m-1)+2r\frac{1-F(X^m)}{f(X^m)}. \qquad \text{(A.20)}\end{aligned}$$

Substracting (A.20) from (A.19) and simplifying using (4.6) yields

$$\frac{\Delta W^{as}}{2r}+\frac{1}{2}=X^m+\frac{2F(X^m)-1}{f(X^m)}. \qquad \text{(A.21)}$$

where

$$\begin{aligned}\Delta W^{as} \;=\;& \mathbb{E}_G\left(\left(\beta-\theta-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^m(\beta)-\frac{(q^m(\beta))^2}{2}\right)\\ &-\mathbb{E}_G\left(\left(\beta-\theta^*-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^{*m}(\beta)-\frac{(q^{*m}(\beta))^2}{2}\right).\end{aligned}$$

Observe that

$$\Delta W^{as}=\mathbb{E}_G\left(\frac{(q^m(\beta))^2}{2}-\frac{(q^{*m}(\beta))^2}{2}\right)=\delta_\theta\mathbb{E}_G\left(\frac{q^{fb}(\beta)+q^{*fb}(\beta)}{2}-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right),$$

which eventually yields (4.12).

Because $\frac{2F(X)-1}{f(X)}=\frac{F(X)}{f(X)}+\frac{F(X)-1}{f(X)}$ is non-decreasing when the *Monotone Hazard Rate Property* (2.1) holds, X^m as defined in (4.11) decreases with ε. Moreover, $X^m(\beta)=\frac{1}{2}$ when $\delta_\theta=0$ and $X^m(\beta)$ remains less than one when (4.13) holds.

□

Proof of Proposition 8. By definition, we have

$$U^m(\beta)=\beta q^m(\beta)-\frac{(q^m(\beta))^2}{2}-T^m(q^m(\beta))=\max_{q\in\mathcal{Q}}\beta q-\frac{q^2}{2}-T^m(q).$$

By a simple duality argument, we deduce that $T^m(q)$ that implements the allocation $(U^m(\theta), q^m(\theta))$ can be defined as

$$T^m(q) = \max_{\beta \in \mathcal{B}} \beta q - \frac{q^2}{2} - U^m(\beta) \text{ and } \gamma^m(q) = \arg\max_{\beta \in \mathcal{B}} \beta q - \frac{q^2}{2} - U^m(\beta). \tag{A.22}$$

Hence, $T^m(q) + \frac{q^2}{2}$ is the max of linear functions of q and is thus convex in q. From there, it follows that $T^m(q)$ is differentiable in q at any point where the correspondence γ^m is single-valued, but this last point follows from the fact that q^m satisfying (4.10), is strictly increasing since (2.1) holds. Hence, we may differentiate (A.22) and compute

$$T^{m'}(q^m(\beta)) = \beta - q^m(\beta).$$

Using (4.10), we finally obtain for $q = q^m(\beta)$ (or, equivalently $\gamma^m(q) = \beta$):

$$T^{m'}(q) = \gamma^m(q) - q = \theta + \varepsilon \frac{1 - G(\gamma^m(q))}{g(\gamma^m(q))}. \tag{A.23}$$

Integrating yields (4.15). A similar proof yields (4.16).

Using (4.6), we can rewrite (A.19) as

$$\mathbb{E}_G \left(\left(\beta - \theta - \frac{1 - G(\beta)}{g(\beta)} \right) q^m(\beta) - \frac{(q^m(\beta))^2}{2} - U^m(\underline{\beta}) \right) = 2r \frac{F(X^m)}{f(X^m)}.$$

Further simplifying using the integral representation (4.3) and an integration by parts yields

$$\mathbb{E}_G \left((\beta - \theta) q^m(\beta) - \frac{(q^m(\beta))^2}{2} - U^m(\beta) \right) = \mathbb{E}_G \left(T^m(q^m(\beta)) - \theta q^m(\beta) \right) = 2r \frac{F(X^m)}{f(X^m)}. \tag{A.24}$$

Using (4.15), we simplify further (A.24) as

$$C^m + \varepsilon \mathbb{E}_G \left(\int_{q^m(\underline{\beta})}^{q^m(\beta)} \frac{1 - G(\gamma^m(\tilde{q}))}{g(\gamma^m(\tilde{q}))} d\tilde{q} \right) = 2r \frac{F(X^m)}{f(X^m)}. \tag{A.25}$$

We compute

$$\mathbb{E}_G \left(\int_{q^m(\underline{\beta})}^{q^m(\beta)} \frac{1 - G(\gamma^m(\tilde{q}))}{g(\gamma^m(\tilde{q}))} d\tilde{q} \right) = \mathbb{E}_G \left(\int_{q^m(\underline{\beta})}^{\beta} \frac{1 - G(\tilde{\beta})}{g(\tilde{\beta})} \dot{q}^m(\tilde{\beta}) d\tilde{\beta} \right) = \int_{\underline{\beta}}^{\overline{\beta}} \frac{(1 - G(\beta))^2}{g(\beta)} \dot{q}^m(\beta) d\beta \tag{A.26}$$

where the first equality follows from a change of variables and the second from integrating by parts. Inserting (A.26) into (A.25) yields

$$C^m = 2r \frac{F(X^m)}{f(X^m)} - \varepsilon \int_{\underline{\beta}}^{\overline{\beta}} \frac{(1 - G(\beta))^2}{g(\beta)} \dot{q}^m(\beta) d\beta \tag{A.27}$$

Similarly, we would find

$$C^{*m} = 2r\frac{1-F(X^m)}{f(X^m)} - \varepsilon\int_{\underline{\beta}}^{\overline{\beta}} \frac{(1-G(\beta))^2}{g(\beta)}\dot{q}^{*m}(\beta)d\beta. \tag{A.28}$$

Using (4.10) and differentiating with respect to β yields

$$\dot{q}^m(\beta) = \dot{q}^{*m}(\beta) = 1 - \varepsilon\frac{d}{d\beta}\left(\frac{1-G(\beta)}{g(\beta)}\right) \tag{A.29}$$

Inserting (A.29) into (A.27) and (A.28) yields (4.17) and (4.18). □

Proof of Proposition 9. Finally, using (A.24) and observing that

$$\Pi^m = \mathbb{E}_G\left(T^m(q^m(\beta)) - \theta q^m(\beta)\right)F(X^m)$$

yields the left-hand side equality in (4.19). The right-hand side equality is obtained by a similar proof. The comparison is the same as in the Proof of Proposition 2. □

Proof of Proposition 10. We now prove each item of the Proposition in turn.

1. Maximizing the objective (4.26) also pointwise gives q^s (4.29). A similar condition applies for suppliers located at 1 which yields q^{*s} .

   Thanks to the *Monotone Hazard Rate Property* (2.4), q^s and q^{*s} so defined both satisfy the monotonicity condition (4.4).

2. Maximizing (4.28) with respect to X yields the following necessary first-order condition for an interior solution that is X^s itself yields:

$$\mathbb{E}_G\left(\left(\beta-\theta-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^s(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^{*s}(\beta) - \frac{(q^s(\beta))^2}{2} - U^{*s}(\underline{\beta})\right)$$
$$= r(2X^s-1) + 4r\frac{F(X^s)}{f(X^s)}. \tag{A.30}$$

   This condition is also sufficient since the maximand in (4.28) is quasi-concave in X when the *Monotone Hazard Rate Property* (2.1) holds.

   Maximizing a similar expression for S_i^* with respect to X yields the following necessary (and also sufficient by a similar argument) first-order condition for an interior solution:

$$\mathbb{E}_G\left(\left(\beta-\theta-\varepsilon\frac{1-G(\beta)}{g(\beta)}\right)q^{*s}(\beta) - (1-\varepsilon)\frac{1-G(\beta)}{g(\beta)}q^{s}(\beta) - \frac{(q^{*s}(\beta))^2}{2} - U^{s}(\underline{\beta})\right)$$
$$= -r(2X^s-1) + 4r\frac{1-F(X^s)}{f(X^s)}. \tag{A.31}$$

Substracting (A.31) from (A.30) and simplifying using (4.6) yields

$$\frac{\Delta W^{ass}}{2r} + \frac{1}{2} = X^s + 2\frac{2F(X^s) - 1}{f(X^m)}. \tag{A.32}$$

where

$$\Delta W^{ass} = \mathbb{E}_G \left(\left(\beta - \theta - \varepsilon \frac{1 - G(\beta)}{g(\beta)} \right) q^s(\beta) - \frac{(q^s(\beta))^2}{2} \right)$$
$$- \mathbb{E}_G \left(\left(\beta - \theta^* - \varepsilon \frac{1 - G(\beta)}{g(\beta)} \right) q^{*s}(\beta) - \frac{(q^{*s}(\beta))^2}{2} \right).$$

Observe that

$$\Delta W^{ass} = \delta_\theta \mathbb{E}_G \left(\frac{q^{fb}(\beta) + q^{*fb}(\beta)}{2} - \varepsilon \frac{1 - G(\beta)}{g(\beta)} \right) = \Delta W^{as}$$

Because $\frac{2F(X)-1}{f(X)} = \frac{F(X)}{f(X)} + \frac{F(X)-1}{f(X)}$ is non-decreasing when the *Monotone Hazard Rate Property* (2.1) holds, X^s as defined in (4.30) decreases with ε. Moreover, $X^s = \frac{1}{2}$ when $\delta_\theta = 0$ and X^s remains less than X^m.

□

Proofs of Propositions 11 and 12. The proofs are similar to those of Propositions 8 and 9 and thus omitted. □