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Price cap regulation with limited commitment*

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Abstract

We consider the price-cap regulation of a monopolistic network operator when the regulator has limited commitment. Operating the network requires fixed investments and the regulator has the opportunity to unilaterally revise the price cap at random times. When the regulator maximizes consumer surplus, he has an incentive to lower the price cap once the operator's fixed investments are sunk. This hold-up problem gives rise to two types of inefficiencies. In one type of equilibrium, the operator breaks even but strategically under-invests to induce the regulator to maintain the price cap. In another type of equilibrium the operator makes strictly positive profits and periods of high investment and high prices are followed by periods of low prices and capacity decline. Overall, the model suggests that the regulator's lack of commitment limits the deployment of network infrastructures.

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1 Introduction

The issue of cost recovery in telecommunications and other regulated industries has generated a rich debate over the last decades.[1] The concept of long-run incremental cost (LRIC) pricing emerged in the 90s, as a way to replicate the outcomes of a competitive market in which firms recover the rental cost of their assets by setting prices at levels that incorporate the cost of capital, operating expenses, and depreciation. Introduced in the United Kingdom in 1995 and subsequently adopted in the United States and Europe—with the European Union explicitly endorsing the LRIC in 2013—the framework was intended to provide incentives for cost minimization by decoupling allowed revenue from historical expenditures. Nonetheless, as early as the mid-1990s, economists such as Hausman et al. (1997) and Sidak and Spulber (1996) voiced concerns regarding the assumptions underpinning the LRIC. The method presupposes constant returns to scale in asset production, complete utilization of capacity, and a stable resale value for depreciated equipment, assumptions that are frequently violated in practice. Laffont and Tirole (2001) observed that the broad consensus in favor of the LRIC was supported by little economic argument. In particular, technological progress that lowers the competitive price of new assets over time should trigger adjustments to the conventional the LRIC formula.

In this perpetually changing environment, the issue of uncertainty, coupled with the irreversible nature of infrastructure investment, has become a central theme in the literature on price regulation. Telecommunications networks, characterized by significant sunk costs, face a substantial risk due to unpredictable demand and rapidly evolving technological standards. Incorporating a risk premium in regulated prices may not be enough to incentivize investment as the operator also needs to be compensated for the loss of flexibility inherent to any irreversible investment decision. This concept of option value is based seminal work by Dixit and Pindyck (1994) who propose a framework that captures the value of delaying investment until some of the uncertainty is resolved based on techniques developed for financial options.

In this article, we wish to explore further the risk created by regulatory opportunism for the regulated firm. While the issue of limited commitment by regulators has been

[1] See Auriol et al. (2021) for a recent and extensive discussion of practical issues in the regulation of public services, and Laffont and Tirole (2001) for the regulation of telecoms.

recognized, there is still little understanding of its implications for optimal price-cap regulation.[2] When the regulator is not able to commit on future regulation, the regulated firm faces a risk of "hold-up," whereby ex-post the regulator decreases the price cap below the LRIC. In this case a price cap at the LRIC cannot induce efficient investment even absent any other form of uncertainty on demand or technology.

To analyze this issue, we build a continuous time model involving a game between one regulator (she) choosing a price cap to maximize inter-temporal consumer surplus and one network operator (he) investing at any point in time to build or maintain capacity. The demand is known and there is no technological uncertainty. In this setting, the regulator can ensure efficient production if he can commit to keep a price cap at the LRIC. Indeed, if she can sell at that price, the operator breaks even on every capacity unit she builds, hence serves demand and makes zero profit. We then model limited commitment by introducing a Poisson process that randomly determines dates at which the regulator can unilateraly impose a new price cap on the firm.

In the first part of the paper, we allow for only one revision: the regulator sets a price cap at the beginning of the game, the operator builds and maintains capacity until the (random) time at which the regulator can change the price cap. At that stage, the regulator's only relevant choices are a price cap at the short run production cost which disincentivize any future investment, referred to as regulatory takin, or a price cap at the LRIC which provides incentives for efficient investment going forward. Turn now to the initial price cap. That price cap has to be at least the LRIC or else the operator does not invest. However, unlike in the second period (after the revision), the LRIC may not be sufficient to incentivize efficient investment: when the price cap is too low (though above the LRIC), the operator under-invests and rations demand. This is because a lower capacity increases the regulator's incentives to keep the price cap high (equal to LRIC) when the revision occurs. For higher levels of the price cap, the operator may invest to serve the demand despite the expectation of low price cap in the future. However the price cap necessary to compensate the operator for future regulatory taking increase when the regulator's commitment power weakens. The regulator may then prefer setting the initial

[2] See Weisman (2019) and Laffont and Tirole (2001), section 4.4.1.4. An illustration of the issue of limited commitment in regulation is provided by González Fanfalone and Crean (2022) OECD report (see box 4 on spectrum licenses in Mexico).

price cap at LRIC, at the cost of a strategic under-investment response by the operator. By contrast, when the level of commitment is high, the regulator offers a mark-up over the LRIC, inducing the operator to serve the demand until a regulatory taking occurs and investment stops.

Next, we study the robustness of these insights to a stationary version of the model where revision opportunities occur repeatedly. As in the one-revision case, there exists a Markov equilibrium where the price cap is at the LRIC and the operator strategically underinvests. Because the commitment power of the regulator is now constant over time, this constant-cap equilibrium also features a constant capacity. However, unlike in the one-revision case, an equilibrium at the LRIC does not exist when the regulator's commitment power weakens beyond a certain point. Indeed, if the regulator deviates to a low price cap, which triggers an investment freeze, he now has the possibility to adjust the price cap upwards in the next revision. When this course correction happens faster, capacity remains relatively high between the two revisions, while consumers benefit from the temporary price cut. Similarly, this deviation becomes profitable when depreciation is slow enough, which precludes a constant-cap constant-capacity Markov equilibrium. In that case, we show the existence of an equilibrium with regulatory cycles where periods of high caps and high investments alternate with periods of low caps and investment freezes. As in the one-revision case, this requires leaving a rent to the operator which then makes strictly positive profits in expectation.

Overall, the model shows that the lack of commitment on the regulator's part can generate a sub-optimal deployment of a network infrastructure through multiple channels. With regulatory cycles, the price is either too high to maximize gains from trade, or too low to incentivize any investment. On the other hand, when the regulator maintains the price cap at the competitive level –the LRIC– the operator strategically underinvests.

Literature

The literature on regulation under limited commitment remains scarce. Salant and Woroch (1992) model the relationship between a price cap regulator and a firm as a repeated game with no commitment in a discrete time model. They focus on equilibria supported by trigger strategies–i.e., any deviation is followed by a path with zero profit and no investment– and show that the most efficient stationary allocation can be approximated for low discount rate. By contrast we focus on Markov equilibria and limited commitment. Gilbert and Newbery (1994) also analyze a repeated game but for rate-of-return regulation. They argue that some "constitutional" commitment[3] may raise regulator credibility and allow to support efficient investment. We focus on price cap regulation that does not raise the issue of cost overrun, but we share the focus on limited commitment. Weisman (2019) focused on the regulation of entry and argued that some revenue sharing alleviates the investment issue caused by lack of commitment on the future level of concentration. We focus on price cap regulation instead and the trade-off between the price level and investment under limited commitment.

Pindyck (2007) also shows that in the presence of uncertainty, in their case about demand, inducing investment from an operator may require an upward adjustment to the traditional LRIC price to cover the potential cost of unused capacity, thereby shifting the effective retail price upward. Moreover, demand uncertainty creates an option value of waiting before making an irreversible investment. This intuition is taken further in Dobbs (2004) who considers continuous fluctuations in demand and the inexorable influence of technological progress. In this setting, the operator's investment policy is characterized by thresholds. When demand remains below a critical threshold, no inveastment occurs, but once demand reaches a certain level, investment is triggered to maintain prices at a level that just covers both the traditional cost and the option value premium. In line with Pindyck (2007), Dobbs (2004)'s analysis shows that if a price cap is set too stringently, the operator may choose to delay or even halt investment. Our model shares the feature that uncertainty, in our case about regulatory changes, can lead to underinvestment. However, uncertainty in our model is essentially strategic. Therefore the operator's reluctance to invest, and the corresponding upward price adjustment, are not generated by an option value to delay investment but rather by a strategic motive for the operator to protect the firm against regulatory taking.

Dobbs (2004)'s analysis has been extended to richer settings. Roques and Savva (2009) demonstrated that many of the intuitions derived from the monopolistic case remain valid when a small number of firms compete, although the strategic interactions among firms

[3] An obligation to pay a fair rate but only on used capacity.

introduce additional complexities. Evans and Guthrie (2005) incorporated quantity regulations alongside price caps, revealing that a combined regulatory approach can mitigate the operator's tendency to delay investment. Their work illustrates that imposing minimum service requirements can force the operator to invest more aggressively. In these more nuanced models, the optimal regulatory framework is one that carefully balances a dynamic price cap—with an upward adjustment to account for uncertainty—against minimum quantity requirements designed to prevent excessive rationing of demand. In our model as well, minimum service requirement would alleviate the rationing issue although we do not explore this possibility. A key difference between these models and ours is that we consider lumpy investment while they focus on a continuous stochastic investment process. As shown by Evans and Guthrie (2005), lumpy investment substantially complicates the dynamics.

As a final note, we abstract from asymmetric information issues that could distort firms' incentives to produce. In particular, Freixas et al. (1985), Baron and Besanko (1987) and Laffont and Tirole (1988) study the ratchet effect, according to which a low cost firm may refrain from reporting its cost if this information can be exploited in future regulation.

The rest of the paper is organized as follows. Section 2 presents the model, Section 3 analyses the case where the regulator can revise the cap only once, while Section 4 studies the stationary case where the probability of price cap revision is constant over time. Section 5 concludes, all proofs are in the Appendix.

2 Model

We model a game with two players, a regulator (he) and a monopolistic firm (she) which we interpret as a network operator. Time is continuous and at each time t, the operator can provide access to her network to consumers. As in Laffont and Tirole (2001), each unit of equipment delivers one unit of access to consumers, requires a one-time capital expenditure C and an operating flow cost c per unit of time (energy, maintenance). We let K_t denote the number of units of equipment in place at t, i.e., the operator's capacity. The operator can increase capacity K_t at any time (at unit cost C) and the equipment depreciates at a rate δ per unit of time. Given an access price a per unit of time, the operator faces a linear demand for access at each time t:

$$D(a) \equiv \overline{Q} - \alpha a. \tag{1}$$

The regulator starts by setting a price cap $\overline{a}$ at $t = 0$. However, we depart from Laffont and Tirole (2001) by allowing the regulator to unilaterally revise this price cap at a later time. The timing of this revision is determined by a Poisson process with intensity λ.[4] The inverse of λ is the average time before the regulator can revise the price cap so that a high λ means the regulator has low commitment power. We will start with the case where the regulator has a single opportunity to modify the price cap (at the first jump of the Poisson process) and then extend the analysis to the stationary case where the regulator can revise the cap multiple times (at every jump of the process).

All agents discount future payoffs at rate r, and we assume that the regulator's objective is to maximize the inter-temporal consumer surplus, where the per-period consumer surplus at price p if the demand is fully served is given by

$$S(p) \equiv \int_0^{D(p)} (P(q) - p)\, dq,$$

and

$$P(q) \equiv \frac{\overline{Q} - q}{\alpha}$$

is the inverse demand function. Because the access charge can be capped by the regulator and the operator can only serve demand if she has the corresponding capacity, demand can be rationed. The per-period consumer surplus at price p and for capacity $K \leq D(p)$ is then

$$\Sigma(p, K) = S(P(K)) + K(P(K) - p) \leq S(p), \tag{2}$$

with

$$\frac{\partial \Sigma(p, K)}{\partial K} = P(K) - p > 0 \;\text{ and }\; \frac{\partial \Sigma(p, K)}{\partial p} = -K < 0.$$

Note that Eq. 2 implies that when demand is rationed, the consumers that are served are

[4] We assume that the price cap is constant between two revision dates.

the ones with he highest willingness to pay.

For the linear demand, this corresponds to

$$\Sigma(p, K) = \frac{\overline{Q} - \alpha p}{\alpha} K - \frac{K^2}{2\alpha}.$$

Finally, as in Laffont and Tirole (2001), we can define the Long-Run Incremental Cost (LRIC) as the price such that the operator covers her investment and operating cost over the long term. That is, the LRIC is the access charge such that the operator's discounted profit from continuously delivering one unit of access is zero:

$$\int_0^{+\infty} e^{-rt}(a_{LRIC} - c - \delta C)dt - C = 0 \Leftrightarrow a_{LRIC} = c + (r + \delta)C. \quad (3)$$

In what follows, we assume the LRIC is lower than the monopoly price,

$$p^m \equiv \frac{1}{2}\left[\frac{\overline{Q}}{\alpha} + a_{LRIC}\right] > a_{LRIC} \iff \overline{Q} > \alpha a_{LRIC}. \quad (4)$$

Note that Eq. 4 also implies that demand is strictly positive when the price is equal to the LRIC. If the regulator had no commitment problem (if λ equalled zero), then setting the price at a_{LRIC} would induce the operator to serve the full demand $D(a_{LRIC})$ while leaving her with no profit and maximizing consumer surplus and overall surplus. This ensures that any inefficiency we identify in the analysis can be ultimately traced back to the regulator's commitment problem.

3 Single revision

We analyze here the case where the operator has a single opportunity to revise the price cap, at the first jump of the Poisson process. We proceed backward starting with the revision stage.

3.1 Revision stage

Consider first the operator's response to the revised price cap $\overline{a}$ at the revision stage. If $a_{LRIC} \leq \overline{a} \leq p^m$, then from Eq. 3 it is profitable for the firm to invest and serve all consumers that are willing to buy access at $\overline{a}$ given that the regulator has no other opportunity to revise the price cap in the future. If $c \leq \overline{a} < a_{LRIC}$, then building up capacity yields a strictly negative payoff for the firm, but given that the price cap is above the operating cost c, exploiting existing capacity is still profitable. Note that in this case, equipment depreciates with no offsetting investment, therefore capacity declines at rate δ over time. Finally, if $\overline{a} < c$, the operator stops offering access. This operator's optimal response implies there are only two relevant choices for the regulator. If the regulator wants the operator to maintain investment, he optimally sets the price cap $\overline{a}$ equal to a_{LRIC}. If instead the regulator wants to minimize the price for the consumer at the cost of disincentivizing investment, he sets $\overline{a} = c$. We show next that this trade-off depends on the operator's installed capacity K at the revision's stage.

Suppose first that the regulator chooses $\overline{a} = c$. Then for any capacity $K \leq D(c)$, consumer surplus going forward is

$$\int_0^{+\infty} e^{-rt} \Sigma(c, K e^{-\delta t})\, dt. \tag{5}$$

which is increasing in K. While consumers benefit from a low price $p = c$, they suffer from the lack of investment which creates increasing rationing over time as the equipment depreciates.

If the regulator chooses $\overline{a} = a_{LRIC}$, the operator invests if needed to obtain capacity $D(a_{LRIC})$ and maintains it to serve the full demand. The consumer surplus is therefore independent from the installed capacity K at the revision stage, equal to

$$\frac{S(a_{LRIC})}{r}. \tag{6}$$

To compare Eq. 5 and Eq. 6, note first that the operator's installed capacity at the revision stage K is at most $D(a_{LRIC})$. This is because the LRIC is the lowest price such that the operator has an incentive to build up capacity and therefore $D(a_{LRIC})$ is the largest demand she might serve. Second, as δ tends to 0, Eq. 5 tends to

$$\frac{\Sigma(c, D(a_{LRIC}))}{r},$$

when evaluated at $K = D(a_{LRIC})$, which is strictly larger than Eq. 6. It follows that if equipment depreciates slowly –δ is not too large– and the operator enters the revision stage with an installed capacity K close to $D(a_{LRIC})$, the regulator has an incentive to set a low price cap to c. Conversely, for δ large enough Eq. 5 tends to 0 and therefore the regulator chooses a price cap equal to the LRIC even if the operator has the maximal capacity $K = D(a_{LRIC})$. This captures that the commitment problem of the regulator is more severe when the operator's equipment is more durable: once the operator has incurred the investment cost KC and the equipment is in place for the long run, the regulator has an incentive to lower the price to increase consumer surplus. When δ increases, setting $a = c$ is increasingly costly for the operator because capacity depreciates more quickly and with it consumer surplus. This cost is particularly salient here because the regulator has no other opportunity to revise the price up, but the idea that the regulator's incentive to expropriate the operator ex post is higher when δ is smaller survives in the stationary model with multiple price cap revisions (Section 4). Using the linear demand specification in (1), we can derive a threshold on δ below which this hold-up problem bites.

Lemma 1. *There exists K^* such that if the operator's installed capacity K is lower than K^*, the regulator sets $a = a_{LRIC}$ and if $K > K^*$, the regulator sets $a = c$. There exists $\overline{\delta}$ such that $K^* < D(a_{LRIC})$ if and only if $\delta < \overline{\delta}$.*

The equilibrium level of K^* is given by the equation

$$\int_0^{+\infty} e^{-rt} \left[\Sigma(c, K^* e^{-\delta t}) - S\left(a_{LRIC}\right)\right] \, dt = 0. \tag{7}$$

which shows that K^* is increasing with the depreciation rate δ. As the depreciation increases, the short-run gain of low price c must increase to compensation the long-run loss.

3.2 Initial stage

Suppose $\delta \geq \overline{\delta}$. Recall that a necessary condition for the operator to be willing to install any capacity is $\overline{a} \geq a_{LRIC}$, which therefore has to hold at $t = 0$. Suppose the regulator sets the price cap at a_{LRIC} at $t = 0$. Then for any capacity (smaller than $D(a_{LRIC})$) the operator may install, the cap stays at a_{LRIC} at the revision stage. It follows that it is optimal for the operator to serve the full demand, and consumer and total inter-temporal surpluses are maximized. In other words, there is no commitment problem.

In the rest of this section, we focus on the case where δ is not too large.

Assumption (A1). $\delta < \overline{\delta}$.

3.2.1 The operator's investment strategy

Note first that the problem of the operator is stationary before the price revision, given a constant instantaneous probability this revision occurs. This implies that if the operator finds it optimal to invest up to a capacity K at $t = 0$, then it is also optimal to maintain this capacity until the price cap revision.

Second, as the regulator has to set $\overline{a} \geq a_{LRIC}$ at $t = 0$ to induce investment and from Lemma 1, the price cap stays at a_{LRIC} at the revision stage if $K \leq K^*$, it is optimal for the operator to install and maintain a capacity at least equal to K^* before the revision.

Suppose now the operator installs and maintains capacity $K > K^*$. This implies her profit after the revision is zero and therefore the profit per unit of equipment is

$$\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} (\overline{a} - c - \delta C)\, d\tau \right] dt - C = \frac{\overline{a} - (c + (r + \delta + \lambda)C)}{r + \lambda}. \qquad (8)$$

It follows that the regulator can induce investment beyond K^* only if he sets a price cap larger than an "augmented" LRIC, $c + (r + \delta + \lambda)C$ that increases with his lack of commitment power λ. On the other hand, if $\overline{a} \geq c + (r + \delta + \lambda)C$ and the operator finds it optimal to install $K > K^*$, then she should optimally satisfy the full demand since her per-unit profit Eq. 8 is then positive. Therefore if $\overline{a} \geq c + (r + \delta + \lambda)C$, the operator chooses between capacity K^* that generates an expected profit equal to

$$\begin{aligned}
&\int_0^{+\infty} \lambda e^{-\lambda t}\left[\int_0^t K^* e^{-r\tau}(\bar{a}-c-\delta C)\,d\tau\right] dt - CK^* \\
&+\int_0^{+\infty} \lambda e^{-\lambda t}\left[D(a_{LRIC})\int_t^{+\infty} e^{-r\tau}(a_{LRIC}-c-\delta C)\,d\tau - e^{-rt}C(D(a_{LRIC})-K^*)\right] dt \\
= \;& \frac{\bar{a}-c-(r+\delta)C}{r+\lambda}K^*, \qquad (9)
\end{aligned}$$

and investing capacity $D(\overline{a})$ which generates a profit

$$\begin{aligned}
&\int_0^{+\infty} \lambda e^{-\lambda t} D(\overline{a})\left[\int_0^t e^{-r\tau}(\overline{a}-c-\delta C)\,d\tau\right] dt - (D(\overline{a})C. \\
= \;& \frac{(\overline{a}-c-(r+\delta+\lambda)C)}{r+\lambda}D(\overline{a}). \qquad (10)
\end{aligned}$$

Let $\Delta(\overline{a})$ be the difference between the numerators of Eq. 10 and Eq. 9, i.e.,

$$\Delta(\overline{a}) = (\overline{a}-c-(r+\lambda+\delta)C)\,(D(\overline{a})-K^*) - \lambda CK^*. \qquad (11)$$

We get the following result,

Lemma 2. *If at $t=0$, $a_{LRIC} \leq \overline{a} \leq c+(r+\lambda+\delta)C$, then the operator installs capacity K^*. If $\overline{a} > c+(r+\lambda+\delta)C$, then the operator installs capacity K^* if $\Delta(\overline{a}) < 0$ and $D(\overline{a})$ if $\Delta(\overline{a}) \geq 0$. The operator maintains the capacity installed at $t=0$ until the price revision.*

3.2.2 The regulator's price cap decision

From Lemma 2, the regulator can induce investment beyond K^* only if $\Delta(\overline{a}) \geq 0$. Given that $\Delta(\overline{a})$ is decreasing in λ, this may not be achievable if λ is too high. More precisely, define

$$\lambda_1 := \min\left\{\lambda \mid \max_{a>c-(r+\lambda+\delta)C} \Delta(a) \geq 0\right\}.$$

Lemma 3. *If $\lambda > \lambda_1$, then $\Delta(\overline{a}) \leq 0$ for all feasible price cap, consequently*

- *at $t=0$, the regulator sets $a_{LRIC} = c+(r+\delta)C$ and the firm installs capacity K^* and maintains it until revision,*

- *at revision, the regulator keeps the cap at $a_{LRIC} = c+(r+\delta)C$ and the firm increases capacity to $D(a_{LRIC})$ and maintains it.*

Inducing the operator to invest beyond K^* requires setting a price cap higher than the augmented LRIC $c + (r + \delta + \lambda)C$ so that she can derives a higher profit for every additional unit of equipment before being expropriated at the revision stage. This rent per unit of equipment and per unit of time needs to increase with λ because when the commitment power of the regulator weakens (λ increases), the time period during which the operator can profit from it shortens. However, the demand per unit of time is fixed and demand declines as the price charged for access increases. This puts a cap on the surplus the regulator can transfer to the operator to incentivize investment, and this cap binds for $\lambda > \lambda_1$.

Suppose now $\lambda \leq \lambda_1$. In that case, the regulator can set $\overline{a}$ such that the operator builds up capacity $K > K^*$ but he may not find optimal to do so. The most efficient way for the regulator to incentivize $K > K^*$, is to set $\overline{a}$ equal to the smallest solution to

$$\Delta(\overline{a}) = 0 \tag{12}$$

We let $a^*(\lambda)$ be that smallest solution to Eq. 12. Notice that $\Delta(a)$ is concave (and quadratic), which implies that it is increasing at $a^*(\lambda)$. Also

$$\frac{\partial \Delta}{\partial \lambda}(a^*(\lambda)) = -\frac{D(a^*(\lambda)) - K^* - K^*C}{r+\lambda}$$

therefore $a(\lambda)$ is increasing. Moreover $a^*(\lambda) \to a_{LRIC}$ as $\lambda \to 0$. As discussed earlier, the rent per unit of equipment and unit of time to the operator needs to increase when the regulator's commitment power weakens.

Turn to consumer surplus. If the regulator chooses $a = a_{LRIC}$, then consumer surplus is

$$\begin{aligned}
& \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} \Sigma(a_{LRIC}, K^*) \, d\tau + e^{-rt} \frac{S(a_{LRIC})}{r} d\tau \right] dt \\
= \; & \frac{\Sigma(a_{LRIC}, K^*)}{r+\lambda} + \frac{\lambda S(a_{LRIC})}{r(r+\lambda)}
\end{aligned} \tag{13}$$

If instead the regulator chooses $\overline{a} = a^*(\lambda)$, then consumer surplus is

$$\frac{S(a^*(\lambda))}{r+\lambda} + \int_0^{+\infty} \lambda e^{-\lambda t} e^{-rt} \left[\int_0^{+\infty} e^{-r\tau} \Sigma(c, D(a^*(\lambda)) e^{-\delta\tau}) d\tau \right] dt \tag{14}$$

Note that when λ tends to 0 and therefore $a^*(\lambda)$ tends to a_{LRIC}, Eq. 14 is strictly higher than Eq. 13. It follows that for λ small enough, the regulator finds it optimal to set a price cap $\overline{a} = a^*(\lambda)$ above LRIC to induce the operator to invest in capacity beyond K^*. Intuitively, K^* is independent of λ as there is no further revision after the first one. As the regulator's commitment problem becomes less severe (λ becomes smaller), the average time before the revision is larger so that the firm is willing to take the risk of investing more and serves the full demand. Hence the regulator can induce sales close to the efficient level $D(a_{LRIC})$ with a small markup over the LRIC.

Combining this observation with Lemma 3 gives the following result.

Proposition 1. *Suppose $\delta < \overline{\delta}$. There exists $\lambda_0 \leq \lambda_1$ such that*

1. *if $\lambda > \lambda_1$,*
   - *at $t = 0$, the regulator sets $\overline{a} = a_{LRIC}$ and the firm installs capacity K^* and maintains it until revision,*
   - *at revision, the regulator keeps the cap at $\overline{a} = a_{LRIC}$ and the firm increases capacity to $D(a_{LRIC})$ and maintains it.*
2. *if $\lambda < \lambda_0$,*
   - *at $t = 0$, the regulator sets $\overline{a} = a^*(\lambda) > a_{LRIC}$, the firm invests $D(a^*(\lambda)) > K^*$ and maintains it until revision,*
   - *at revision, the regulator revises the cap down to $a = c$ and the firm stops investing.*

While Proposition 1 provides the regulator's optimal strategy when he either has strong commitment power or little commitment power, characterizing the equilibrium in the intermediate range $[\lambda_0, \lambda_1]$ is more challenging. This is because increasing the revision speed λ affects the regulator's payoff in two different ways. First, a higher λ increases the price cap $a^*(\lambda)$ necessary to induce the firm to fully serve demand at $t = 0$. This makes a price cap at the LRIC more attractive in comparison. On the other hand, a higher λ shortens the delay before the revision. For the regulator, this raises the payoff from his two possible strategies because consumer welfare after the revision is larger than before the revision since the regulator can fully commit at the revision stage. But that positive effect is larger when the regulator sets $\overline{a} = a^*(\lambda)$ at $t = 0$ than if he sets $\overline{a}$ equal to LRIC. To see why remember that in the latter case, the firm reaches the revision stage with a capacity K^* such that the regulator is indifferent between keeping the price cap at the LRIC (the equilibrium play) and setting $\overline{a} = c$ which discourages further investment. By contrast, if the regulator set $\overline{a} = a^*(\lambda)$ at $t = 0$, the firm reaches the revision stage with $K = D(a^*(\lambda)) > K^*$ which is such that the regulator strictly prefers setting $\overline{a} = c$. It follows that speeding up the revision favours the strategy where $\overline{a} = a^*(\lambda)$ at $t = 0$. Overall, increasing λ has opposing effects on the two relevant price cap strategies for the regulator.

Despite this ambiguity, whatever the optimal strategy the regulator ends up picking in that region, the lack of commitment generates an inefficiently low provision of service from the operator either because the regulator does not serve the full demand at the price cap, or because the price is first too high, which reduces demand, and then too low, which disincentivizes investment.

4 Multiple revisions

The model with one revision delivers two key connected intuitions. The first one is that the operator has a strategic motive to limit capacity to induce the regulator to maintain the price cap at the revision stage. This, in turn, implies that providing incentives to invest enough to serve demand requires a price cap above an augmented LRIC that incorporates a premium for the risk of a downward revision of the cap. We show in this section that these two effects are still at play in a stationary version of the model where the regulator has recurring opportunities to revise the price cap. In other words, these effects are not dependent on the regulator's ability to commit at the revision stage, which is effectively the case when only one revision is possible. On the other hand, we show that a persistent lack of commitment can have more detrimental effects on the level of investment that can be sustained in equilibrium.

To make the model stationary, we now assume that the regulator has an opportunity to revise the cap at each jump of the Poisson process. The model remains otherwise unchanged. We restrict attention to Markov perfect equilibria where at each revision, the strategy of the regulator is to set a price cap $a(K)$ that is a function of the operator's installed capacity K, and at each time t, the strategy of the operator is a capacity adjustment (an investment) $I(K, \overline{a})$, as a function of the currently installed capacity K and the price cap $\overline{a}$. We then define $W(K)$ as the equilibrium value for the regulator of reaching a revision stage when the operator's installed capacity is K and we let $V(\overline{a}, K)$ be the equilibrium value for the firm of holding capacity K when the price cap is $\overline{a}$.

Finally, we make the following assumptions

Assumption (A2). *When indifferent between two levels of investment, the operator chooses the largest.*

Assumption (A3). *When indifferent between two levels of price cap, the operator chooses the level inducing the largest investment.*

We can interpret these assumptions as focusing the analysis on equilibria where indifferences are resolved in favour of the action that maximizes surplus. From (A2), the operator, when indifferent, picks the action that maximizes consumers' instantaneous utility and from (A3), the regulator, when indifferent, picks the action that maximizes instantaneous overall surplus.

We analyze below two potential equilibria with particular interest: a stationary equilibrium and a cyclical equilibrium.

4.1 Equilibrium with constant cap and constant capacity

A natural candidate equilibrium is one in which on the equilibrium path, the price cap and the operator's capacity are constant. We let a^* and K^* denote these equilibrium values.

In this constant-cap constant-capacity equilibrium, the operator starts at $K = 0$ and immediately builds up to K^*, then maintain capacity at that level, which generates consumer surplus $\Sigma(a^*, K^*)$ per unit of time. so the regulator's value function on the equilibrium path is

$$W(0) = W(K^*) = \frac{\Sigma(a^*, K^*)}{r}.$$

Equilibrium conditions also impose that starting from zero capacity and given the price cap a^*, the operator builds capacity K^*, that is, $I(0, a^*) = K^*$. Therefore the operator's value function satisfies

$$V(0, a^*) = V(K^*, a^*) - CK^* = \frac{K^*(a^* - a_{LRIC})}{r}.$$

We show next that the operator behaves similarly off the equilibrium path. More precisely, given a price cap a^* she builds capacity up to K^* starting from any installed capacity $K \leq K^*$. Start with the following observation.

Lemma 4. *For any $\overline{a}$, $V(K, \overline{a}) - KC$ is non increasing in K.*

Indeed, suppose the operator starts with capacity $K < K'$. Then she has the option to invest $I = K' - K$ at cost CI, and therefore

$$V(K, \overline{a}) \geq V(K', \overline{a}) - C(K' - K),$$

which shows Lemma 4.

Suppose now that for some $K < K^*$ and at the price cap a^*, the operator strictly preferred installing a capacity different from K^*. This would imply

$$V(K, a^*) > V(K^*, a^*) - (K^* - K)C. \tag{15}$$

Then, since Lemma 4 implies $V(0, a^*) + KC \geq V(K, a^*)$, we would have

$$V(0, a^*) > V(K^*, a^*) - CK^*, \tag{16}$$

and therefore at the price cap a^* it would be suboptimal for an operator with no capacity to install K^*, a contradiction. This implies that at K, the operator is indifferent between adding capacity $I(K, a^*)$, her equilibrium strategy, and adding $K^* - K$,

$$V(K + I(K, a^*)) - I(K, a^*)C = V(K^*) - (K^* - K)C. \quad (17)$$

We then use (A2) to break this indifference and show

$$I(K, a^*) = K^* - K. \quad (18)$$

Indeed, $K + I(K, a^*) < K^*$ contradicts (A2) which states that the operator, when indifferent, chooses the higher capacity K^*. If $K + I(K, a^*) > K^*$, then Eq. 17 implies the operator is also indifferent between $K + I(K, a^*)$ and K^* when she starts with no capacity, therefore from (A2) chooses $I(0, a^*) = K + I(K, a^*) > K^*$, a contradiction.

Turn to the regulator. Eq. 18 implies he can induce capacity K^* with a price cap a^* starting from any $K < K^*$. We show in the Appendix that this is his equilibrium strategy. Intuitively, the regulator only cares about the installed capacity and the price cap after the revision. Therefore if he induced capacity $K + I(K, a(K))$ with a price cap $a(K) \neq a^*$ when the operator has capacity $K > 0$, he would set $a(0) = a(K)$ to induce the same capacity when the operator starts from $K = 0$, a contradiction. This analysis leads to our next intermediary result.

Lemma 5. *In any constant-cap constant-capacity equilibrium, for all $K \leq K^*$, $K + I(K, a^*) = K^*$ and $a(K) = a^*$.*

One main benefit of Lemma 5 is to pin down the value function off the equilibrium path. In particular, we can relate on- and off-path values for the operator if, following a deviation, she finds herself with a capacity $K < K^*$:

$$V(K, a^*) = V(K^*, a^*) - (K^* - K)C. \quad (19)$$

We show next we can leverage this relation to narrow down the set of possible equilibrium price caps a^*.

4.1.1 Necessary conditions and equilibrium price cap

One constraint on a^* is that if, at a revision stage, $K = K^*$ and the regulator sets $\overline{a} < a^*$, then the operator lets her capacity depreciate. If she does not, the regulator can improve consumer welfare by lowering the price while keeping the demand served by the operator at or above K^*. Intuitively, this implies a^* cannot be too high for otherwise, following a slight decrease in the price cap, the operator would be tempted to maintain capacity to take full advantage of a high price. We show next that this line of reasoning pushes a^* all the way down the a_{LRIC}.

Proposition 2. *In any constant-cap constant-capacity equilibrium,* $a^* = a_{LRIC}$.

Obviously, in a constant-cap equilibrium with strictly positive capacity, the price cap a^* cannot be strictly lower than a_{LRIC}. To understand why it cannot be strictly higher, suppose $a^* > a_{LRIC}$ and the regulator has an opportunity to revise the cap while the operator's capacity is at the equilibrium level K^*. Consider the deviation where the regulator slightly lowers the price cap to $\overline{a} \in (a_{LRIC}, a^*)$. As explained above, making that deviation unprofitable requires that the operator reacts by stopping to invest. For an intuition, consider the simpler case where the regulator can revise the price cap at fixed time intervals Δ_t. Then if the operator lets her capacity depreciate, her (equilibrium) payoff is

$$\int_0^{\Delta_t} e^{-rt} K^* e^{-\delta t}(\overline{a} - c) + e^{-r\Delta_t} V(K^* e^{-\delta \Delta_t}, a^*). \tag{20}$$

If, on the other hand, the operator maintains her capacity, her (deviation) payoff is

$$\int_0^{\Delta_t} e^{-rt} K^*(\overline{a} - c - \delta C) + e^{-r\Delta_t} V(K^*, a^*). \tag{21}$$

The key observation is that the operator's capacity remains below K^* in both cases. As a result, in a Markov equilibrium, the operator expects the regulator to revert to the price cap a^* in the next revision, as can be seen in the last term of Eq. 20 and of Eq. 21. Then using Eq. 19, the difference between the equilibrium payoff in Eq. 20 and the deviation payoff in Eq. 21 is

$$\left(\frac{1 - e^{(r+\delta)\Delta_t}}{r + \delta} - \frac{1 - e^{r\Delta_t}}{r}\right) K^*(\overline{a} - c - (r + \delta)C) < 0.$$

In other words, as long as the current and future price caps are strictly above LRIC, which the operator can ensure by keeping her capacity below K^*, her net profit from every unit of capacity is strictly positive. It follows that she is strictly better off maintaining her capacity at K^* than letting it depreciate. This, in turn, makes a slight decrease in the price cap a profitable deviation for the regulator. We show in the Appendix that a similar reasoning applies when the timing of the revision is stochastic.

4.1.2 Equilibrium existence

From Proposition 2, the only candidate equilibrium price cap is the LRIC. Given that the regulator plays $\overline{a} = c$ for $K > K^*$ and $\overline{a} = a_{LRIC}$ for $K \leq K^*$, the operator cannot expect a price cap higher than LRIC and her expected profit is therefore at most 0 for every unit of capacity she builds. Therefore the operator has no profitable deviation, and the final step for equilibrium existence is to examine deviations by the regulator.

The first potential deviation is to a price cap lower than a_{LRIC}. Since the firm stops investing if the price cap is strictly below a_{LRIC}, the only relevant deviation of this type is to a price cap equal to the operating cost c. This deviation is not profitable at K^* if

$$\begin{aligned} & \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} \Sigma(a_{LRIC}, K^*) \, d\tau + e^{-rt} W(K^*) \right] dt \\ \geq \ & \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} \Sigma(c, K^* e^{-\delta\tau}) \, d\tau + e^{-rt} W(K^*) \right] dt. \end{aligned}$$

Using the linear demand specification, this is equivalent to

$$\Leftrightarrow \frac{K^*}{\alpha(r+\lambda)} \left[\frac{\delta(\overline{Q} - \alpha c)}{r+\lambda+\delta} - \alpha(r+\delta)C - \frac{\delta K^*}{r+\lambda+2\delta} \right] \geq 0. \tag{22}$$

First, if Eq. 22 holds, then the same inequality is true for any $K < K^*$. It is therefore sufficient to rule out a deviation at K^*. Note then that if

$$\frac{\overline{Q} - \alpha c}{\alpha(r+\delta)C} \leq \frac{r+\lambda+\delta}{\delta}, \tag{23}$$

then Eq. 22 is violated for any strictly positive K^*. It follows that a necessary condition for a productive constant-cap constant-capacity equilibrium is

$$\frac{\overline{Q}-\alpha c}{\alpha(r+\delta)C} > 1+\frac{r}{\delta} \Leftrightarrow \overline{Q}-\alpha\left[c+(r+\delta)C\left(1+\frac{r}{\delta}\right)\right] > 0 \tag{24}$$

Our starting assumption is that demand is high enough to cover production costs, i.e., $\overline{Q}-\alpha(c+(r+\delta)C) = \overline{Q}-\alpha a_{LRIC} > 0$. Eq. 24 imposes a stronger condition, and that requirement is more difficult to satisfy when depreciation is slow, that is, when the ratio r/δ increases (keeping $r+\delta$ constant). As already discussed in the two-period case, slow depreciation exacerbates the regulator's commitment problem because capacity remains high even when the operator stops investing. When Eq. 24 is violated, this commitment problem is heightened to the point where no strictly positive K^* can be sustained in this type of equilibrium. If Eq. 24 holds a productive equilibrium may exist provided the regulator has enough commitment power, i.e., $\lambda \leq \overline{\lambda}$, where $\overline{\lambda}$ is such that Eq. 23 holds with equality.

On the other hand, if Eq. 22 holds at $K^* = D(a_{LRIC})$,

$$\begin{aligned} &\left[\frac{\delta(\overline{Q}-\alpha c)}{r+\lambda+\delta} - \alpha(r+\delta)C - \frac{\delta(\overline{Q}-\alpha c-\alpha(r+\delta)C)}{r+\lambda+2\delta}\right] \geq 0 \\ \Leftrightarrow \quad & \frac{\overline{Q}-\alpha c}{\alpha(r+\delta)C} \geq \left(\frac{r+\lambda+\delta}{\delta}\right)^2, \end{aligned} \tag{25}$$

then the operator can invest all the way up to $D(a_{LRIC})$ and serve all the demand, without inducing the regulator to lower the price cap to c. A necessary condition for Eq. 25 is

$$\frac{\overline{Q}-\alpha c}{\alpha(r+\delta)C} > \left(1+\frac{r}{\delta}\right)^2 \Leftrightarrow \overline{Q}-\alpha\left[c+(r+\delta)C\left(1+\frac{r}{\delta}\right)^2\right] > 0. \tag{26}$$

If Eq. 26 holds, we let $\underline{\lambda}$ denote the solution to Eq. 25 holding with equality. If Eq. 26 does not hold, we set $\underline{\lambda}=0$. In either case, note that $\underline{\lambda} < \overline{\lambda}$.

Finally if

$$\frac{r+\lambda+\delta}{\delta} < \frac{\overline{Q}-\alpha c}{\alpha(r+\delta)C} < \left(\frac{r+\lambda+\delta}{\delta}\right)^2, \tag{27}$$

then there exists a unique K^* in $(0, D(a_{LRIC}))$, such that Eq. 22 holds with equality,

$$K^* = (r + \lambda + 2\delta)\left(\frac{\bar{Q} - \alpha c}{r + \lambda + \delta} - \alpha\left(1 + \frac{r}{\delta}\right)C\right).$$

This capacity is then the only possible equilibrium capacity even though Eq. 22 holds strictly for lower capacities. This is because if Eq. 22 holds strictly and $K^* < D(a_{LRIC})$, then the operator deviates: she can slightly increase her capacity while keeping Eq. 22 satisfied, thereby making sure the price cap stays at least equal to a_{LRIC}.[5] This discussion leads to an intermediate result.

Lemma 6. *Suppose Eq. 24 holds. There exists* $\underline{\lambda} \geq 0$ *and* $\overline{\lambda} > \underline{\lambda}$ *such that if a constant-cap constant-capacity equilibrium exists,*

- *If* $\lambda \leq \underline{\lambda}$, *the operator's capacity is at the efficient level:* $K^* = D(a_{LRIC})$.
- *If* $\underline{\lambda} < \lambda \leq \overline{\lambda}$, *the operator's capacity is below the efficient level and strictly decreases in* λ *from* $K^* = D(a_{LRIC})$ *to* $K^* = 0$.

If $\lambda \geq \overline{\lambda}$, *there is no productive constant-cap constant-capacity equilibrium. Similarly, if Eq. 24 does not hold, there is no productive constant-cap constant-capacity equilibrium.*

Lemma 6 provides a first insight into the similarities and differences between the one-revision and multiple-revision cases. One intuition these two cases have in common is that the operator has a strategic incentive to keep capacity below demand to deter a decrease in the price cap. But the key difference is that in the one-revision model, the regulator has perfect commitment power at the revision stage and therefore K^* is independent from λ. This is no longer true when we allow for multiple revisions that happen at an average time interval $1/\lambda$. Then the expected time during which capacity depreciates following a one-time deviation to $\overline{a} = c$ shrinks with λ, making that deviation less harmful to consumers, therefore more attractive to the regulator. As a result, this stationary version of the model delivers the natural intuition that an equilibrium with a low price and a high capacity is harder to sustain when the regulator's commitment power weakens.

Suppose now Eq. 24 holds and $\lambda \in (\underline{\lambda}, \overline{\lambda})$. From Lemma 6, if a productive constant-cap constant-capacity equilibrium exists, then demand is rationed: $K^* < D(a_{LRIC})$. Because

[5] This deviation is not strictly profitable for the operator but is played under (A2).

of this inefficiency, the regulator could be tempted to increase the price cap above LRIC. This deviation is profitable for the regulator only if the operator does react to a price cap hike by increasing investment. Therefore a necessary condition is that if the regulator deviates to $\overline{a} > a_{LRIC}$, then $D(\overline{a}) > K^*$. We first show that if the regulator sets $\overline{a} > a_{LRIC}$ and the operator finds it optimal to increase capacity beyond K^*, then she serves the full demand $D(\overline{a})$. If she does so, her expected payoff is

$$\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(\overline{a} - c - \delta C) D(\overline{a}) d\tau + e^{-rt} V(D(\overline{a}), c) \right] dt - (D(\overline{a}) - K^*)C \tag{28}$$

to be compared with her payoff if she keeps her capacity at K^*,

$$\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(\overline{a} - c - \delta C) K^* d\tau + e^{-rt} V(K^*, a_{LRIC}) \right] dt. \tag{29}$$

Comparing Eq. 28 to Eq. 29 delivers the trade-off for the operator. Increasing capacity entails a capacity investment $(D(\overline{a}) - K^*)C$, but also implies that in the next revision, her capacity will be above K^* and she will therefore face a low price cap $\overline{a} = c$. On the other hand maintaining capacity at K^* guarantees $a^* = a_{LRIC}$ in the next revision.

Lemma 7. *There exists $\lambda^* \in [\underline{\lambda}, \overline{\lambda}]$ such that if $\lambda < \lambda^*$ the regulator has no profitable deviation to a price cap strictly above the LRIC. If Eq. 26 holds, then $0 < \underline{\lambda} < \lambda^* < \overline{\lambda}$.*

As in the one-revision case, pining down the region in which the regulator has an incentive to raise the price cap proves challenging. On the one hand, from Lemma 6, as λ increases K^* decreases to eventually reach zero, which makes a deviation to a higher price cap that induces higher investment more attractive to the regulator. But we also show that to induce investment beyond K^*, the regulator needs to set a price cap $\overline{a}$ higher than the augmented LRIC, $a_{LRIC} + \lambda C$. This price cap increases with λ which makes the deviation less attractive to the regulator. Therefore the regulator faces opposing incentives to deviate as his commitment power shrinks. We define λ^* as the lowest λ such that the regulator has an incentive to deviate. In the case where Eq. 26 holds and therefore $\underline{\lambda} > 0$, the second effect dominates for λ close enough to $\underline{\lambda}$ and therefore $\lambda^* > \underline{\lambda}$.

Combining Lemma 6 and Lemma 7 provides the final result in this section.

Proposition 3. *Assume that Eq. 26 holds (depreciation is fast enough), then there exist* $\underline{\lambda}$, λ^* *and* $\overline{\lambda}$ *with* $0 < \underline{\lambda} < \lambda^* < \overline{\lambda}$ *such that*

1. *if* $\lambda \leq \underline{\lambda}$, *the only constant-cap constant-capacity equilibrium features a price cap equal to* a_{LRIC} *and a capacity equal to demand,* $K^* = D(a_{LRIC})$. *This region is non-empty* ($\underline{\lambda} > 0$) *if and only if Eq. 26 holds.*

2. *if* $\underline{\lambda} < \lambda \leq \lambda^*$, *the only constant-cap constant-capacity equilibrium features a price cap equal to* a_{LRIC} *and a capacity strictly lower than demand,* $0 < K^* < D(a_{LRIC})$, *and decreasing in* λ.

3. *if* $\lambda > \overline{\lambda}$, *there is no productive constant-cap constant-capacity equilibrium.*

If Eq. 24 does not hold, there is no productive constant-cap constant-capacity equilibrium.

The proposition focuses on cases where the first-best is implemented for high commitment levels (i.e., Eq. 26 holds). When this is not the case but Eq. 24 holds, we can show there is a profitable deviation for λ close to 0–because the deviation requires only a slight deviation above a_{LRIC}–and if λ is close to $\overline{\lambda}$–because K^* is close to 0. Hence no constant-cap constant-capacity equilibrium exists in these two extreme cases. However we cannot exclude that such an equilibrium exists for intermediate values of λ.

We conclude with a few remarks on Proposition 3. As explained above, the condition in Eq. 26 is a requirement on r/δ being small enough, i.e., on depreciation being fast enough to exert a disciplining effect on the regulator's incentive to lower the price cap. Second, as in the one-revision model, there exists an intermediate range $(\lambda^*, \overline{\lambda})$ where we cannot fully characterize the equilibrium because the regulator faces conflicting incentives to increase the price cap above the LRIC, as discussed below Lemma 7. However, the form this equilibrium can take is not indeterminate. From Lemma 6, we know that in that region, if a productive equilibrium exists, then the price cap is the LRIC and the capacity is strictly lower than the equilibrium capacity at λ^*–therefore strictly lower than demand $D(a_{LRIC})$–and decreasing in λ. This is again consistent with the idea that lower commitment power makes it more difficult to sustain equilibria with low prices and high capacity. Finally, because the price cap is set at the LRIC, the operator makes zero profit in this equilibrium. Therefore the loss of consumer surplus is not caused by the regulator leaving a rent to the operator to provide incentives to invest, but rather by the strategic rationing of operator. We will see in the next section that this may not be true in every equilibrium.

4.2 Cyclical equilibrium

The analysis of the one-revision case shows a pattern where a high price cap (above LRIC) designed to incentivise high capacity can be followed by downward revision and an investment freeze. We study here whether a similar pattern exists in the stationary version of the model with multiple revisions. Specifically, we want to establish conditions under which periods of high prices and high capacity alternate with periods of low prices and capacity decay, which we refer to as a cyclical equilibrium.

As earlier, a natural intuition is that the regulator's incentive to lower the price cap is higher when the installed capacity is higher. In line with the notation used in previous equilibrium constructions, we let K^* denote the capacity above which the regulator gives up on incentivizing investment, and therefore sets a price cap $a^* = c$. When $K \leq K^*$, the regulator sets a price cap a^* higher than LRIC, which we denote a_h. Unlike in the constant-cap constant-capacity equilibrium where the operator always keeps her capacity at the threshold K^*, we look for an equilibrium where the operator, given a high price cap a_h, serves the demand. She does so even though $D(a_h) > K^*$ and she therefore exposes herself to a price cap cut in the next revision.

Overall, the equilibrium is defined by a threshold K^*, a (high) price cap a_h such that $a_h > a_{LRIC}$ and $D(a_h) > K^*$ and the following candidate equilibrium strategies.

1. The regulator sets $a^* = c$ if $K > K^*$ at the revision stage, and $a^* = a_h$ if $K \leq K^*$.

2. The operator

   (i) does not invest if $\overline{a} < a_{LRIC}$,

   (ii) does not invest if $a_{LRIC} \leq \overline{a} < a_h$ and $K > K^*$,

   (iii) sets and maintains capacity at K^* if $a_{LRIC} \leq \overline{a} < a_h$ and $K \leq K^*$,

   (iv) sets and maintains capacity at $D(\overline{a})$ if $a_h \leq \overline{a} \leq p^m$.

The operator's equilibrium strategy has indeed to satisfy a number of constraints. Capacity must drop below $D(a_h)$ when the price cap falls below a_h. Otherwise, the regulator would have no motive for increasing the price up to a_h. At the same time, given a price cap above a_{LRIC}, the operator should maintain her capacity at K^* since she knows that by staying at that level, she guarantees the price cap never falls below a_{LRIC} and she can therefore cover her costs. Finally, a_h has to be attractive enough that she wants to serve demand if the cap is at or above a_h, yet lower than the monopoly price p^m.

4.2.1 The regulator

In the conjectured equilibrium, the regulator sets $\overline{a} = a_h$ at a revision stage for any $K \leq K^*$, and the operator reacts by building up to $D(a_h)$. It follows that the regulator's expected payoff is constant in K as long as $K \leq K^*$, and we let $\underline{W}$ denote that value. For $K > K^*$, the regulator sets $\overline{a} = c$ and the operator lets her capacity depreciate. We let $\overline{W}(K)$ denote the regulator's value function at a revision stage when $K > K^*$. We can then write

$$\underline{W} = \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} S(a_h)\, d\tau + e^{-rt} \overline{W}(D(a_h)) \right] dt = \frac{S(a_h)}{\lambda + r} + \frac{\lambda \overline{W}(D(a_h))}{\lambda + r}. \quad (30)$$

That is, the regulator's value if $K \leq K^*$ is the expected consumer surplus at the price a_h until the next revision where capacity is at $D(a_h)$. Note also that at K^*, the regulator has to be indifferent between c and a_h: for the equilibrium conjecture to hold, he needs to weakly prefer a_h, and if he strictly prefers a_h, then there exists $K > K^*$ such that he also strictly prefers a_h. This implies

$$\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} \Sigma(c, K^* e^{-\delta \tau})\, d\tau \right] dt + \frac{\lambda \underline{W}}{r + \lambda} = \underline{W}, \quad (31)$$

Finally, turn to the regulator's value function when $K > K^*$. Define $t^*(K)$ as the time it takes for the operator's capacity to depreciate from K to K^*. So $t^*(K)$ is such that

$$K e^{-\delta t^*(K)} = K^* \Leftrightarrow e^{-rt^*(K)} = \left(\frac{K^*}{K} \right)^{\frac{r}{\delta}}. \quad (32)$$

Using this notation, we have that for $K > K^*$,

$$\overline{W}(K) = \int_0^{t^*(K)} e^{-rt}\Sigma(c, Ke^{-\delta t})\, dt + e^{-rt^*(K)}\underline{W}, \tag{33}$$

where the second term makes use of the indifference condition in Eq. 31 stating that once capacity reaches K^*, the expected value for the regulator of seeing that capacity further depreciate with a price cap $\overline{a} = c$ until the next revision is equal to $\underline{W}$. From Eq. 33, we can show (using Eq. 31) that $\overline{W}(.)$ is decreasing and tends to $\underline{W}$ as K tends to K^* from above.

Combining Eq. 30 and Eq. 33, we obtain a first relation between the two key equilibrium objects, a_h and K^*. Consider the following condition

$$c + \frac{\bar{Q} - \alpha c}{\alpha}\left(\frac{1}{1+\frac{r}{\delta}}\right)^2 < a_h < \frac{\overline{Q}}{\alpha}. \tag{34}$$

Lemma 8. *If Eq. 34 holds, then for any λ, Eq. 30, Eq. 31 and Eq. 33 define a unique $K_r^*(a_h)$, strictly lower than $D(a_h)$.*

Three remarks on the condition in Eq. 34. First, the right-hand-side inequality simply says that demand must be positive at the price a_h. Next, the reason why a_h cannot be too low is because the regulator would otherwise prefer keeping the price cap at a_h when the operator's capacity is at $D(a_h)$, rather than lowering it to $\overline{a} = c$. We will see later that incentivizing the operator to build up capacity to $D(a_h)$ also requires a_h to be high enough. Finally, Eq. 34 is more likely to be true when depreciation is slow (r/δ is large), unlike in the constant-cap, constant-capacity equilibrium that exists only if depreciation is sufficiently fast. Indeed, a slow depreciation rate reinforces the regulator's incentives to cut the price cap, as it makes an investment freeze less costly to consumers.

To close the regulator's side of the equilibrium derivation, we check for possible deviations. Obviously, given that the operator serves the demand at a_h, there is no incentive for the regulator to set $\overline{a} > a_h$. However, the regulator could have an incentive to set $\overline{a} = a_{LRIC}$ instead of the equilibrium play $\overline{a} = a_h$ when $K \leq K^*$. This would indeed induce a lower capacity $K^* < D(a_h)$ but also set a lower price for consumers. We show we can rule out this deviation for any λ if Eq. 24 does not hold. Recall that Eq. 24 is a condition on the depreciation rate being fast enough under which a productive constant-cap constant-capacity equilibrium does not exist for any λ (Proposition 3). That is, a cyclical equilibrium is more likely to exist when the commitment problem that slow depreciation creates for the regulator precludes equilibria with a stable price cap.

Lemma 9. *Suppose Eq. 24 does not hold (depreciation is slow enough), then given a_h and $K_r^*(a_h)$ defined in Lemma 8, the regulator has no profitable deviation.*

4.2.2 The operator

As in the regulator's case, the conjectured equilibrium strategy imposes some structure on the operator's value function. For example, for any $\overline{a} \in [a_{LRIC}, a_h)$, the operator keeps her capacity at K^* until the next revision, which implies

$$V(K^*, \overline{a}) = \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(\overline{a} - c - \delta C)K^* \, d\tau + e^{-rt}(V(D(a_h), a_h) - (D(a_h) - K^*)C) \right] dt$$

At the limit where $\overline{a}$ tends to a_h, we must have

$$\lim_{\overline{a} \to a_h} V(K^*, \overline{a}) = V(D(a_h), a_h) - (D(a_h) - K^*)C. \tag{35}$$

If the left-hand side in Eq. 35 was strictly larger, then at a_h the operator would prefer keeping her capacity at K^*. On the other hand, if the left-hand side was strictly lower, there would exist a price cap $\overline{a} < a_h$ such that at that price cap the operator would prefer building up to $D(\overline{a})$ rather than keeping her capacity at K^*. This would then be a profitable deviation for the regulator. Simplifying, Eq. 35 can be written as

$$V(D(a_h), a_h) = \frac{a_h - c - \delta C}{r} K^* + (D(a_h) - K^*)C \tag{36}$$

Moreover if the price cap is at a_h, the firm maintains capacity $D(a_h)$ until revision, which implies

$$V(D(a_h), a_h) = \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(a_h - c - \delta C)D(a_h) d\tau + e^{-rt} V(D(a_h), c) \right] dt \tag{37}$$

Finally, if the operator enters a revision stage with capacity $D(a_h)$, the regulator sets the price cap to c and the operator lets her capacity depreciate, which implies

$$V(D(a_h), c) = e^{-rt^*(D(a_h))} \int_0^{+\infty} \lambda e^{-\lambda t} e^{-rt}(V(D(a_h), a_h) - (D(a_h) - K^* e^{-\delta t})C)\, dt \quad (38)$$

Combining Eq. 36, Eq. 37 and Eq. 38 then provides a relation between K^* and a_h, which is the counterpart of Lemma 8. Consider the following condition

$$a_{LRIC} + \lambda C < a_h < \frac{\overline{Q}}{\alpha}. \quad (39)$$

Lemma 10. *If Eq. 39 holds, then Eq. 36, Eq. 37 and Eq. 38 define a unique $K_o^*(a_h)$.*

As earlier, the right-hand side of Eq. 39 simply ensures that demand is positive at a_h. On the left-hand side, the price cap a_h has to be strictly higher than the augmented LRIC, $a_{LRIC} + \lambda C$ that, intuitively, compensates the operator for the risk of a price cap cut. This is reminiscent of the equilibrium of the one-revision model in the case with high initial investment: the regulator needs to leave the operator with a rent that is sufficiently high before the cap decreases to induce investment. As in the one-revision model, the regulator's ability to do so is constrained by her commitment power: the augmented LRIC increases with λ and eventually reaches the monopoly price p^m that bounds the per-period rent the operator can obtain.

4.2.3 Equilibrium existence

The discussion above suggests that this equilibrium, like the constant-cap constant-capacity equilibrium, is more difficult to sustain when the regulator's commitment power is low (λ is high). But the key difference between these two types of equilibria is that the cyclical one is easier to sustain when depreciation is slow. This is captured in the condition in Lemma 9 that Eq. 24 does not hold. Taken together, these conditions on commitment power and depreciation speed lead to the main result of this section

Proposition 4. *Suppose Eq. 24 does not hold (depreciation is slow enough), then if λ is small enough, there exists a cyclical equilibrium. In that equilibrium, the price cap alternates between $a_h > a_{LRIC} + \lambda C$ and c.*

While the constant-cap constant-capacity equilibrium can be efficient for fast enough depreciation and high enough commitment power, the cyclical equilibrium is always inefficient when it exists. In phases where the capacity is high and the operator serves demand, the price cap is strictly higher than LRIC, and in phases where the price cap is low, capacity declines over time despite fundamental demand being high enough to cover costs. In addition to depressing overall surplus, the regulator's commitment problem also causes a redistribution of the remaining surplus from consumers to the operator, compared with the case where the price is at LRIC and the operator serves demand. To see this, recall from Eq. 39 that the price cap is strictly above the augmented LRIC, $a_{LRIC} + \lambda C$. Note then that the operator makes zero profit on every unit of investment if the price cap is at the augmented LRIC at the time of the investment and then permanently cut to $\overline{a} = c$ in the next revision. It follows the operator makes a strictly positive profit in expectation both because the price cap is stricly above the augmented LRIC and because over the lifetime of each capacity unit, periods of low and high price caps alternate. The reason why the equilibrium price cap is strictly above the augmented LRIC is because the operator's best alternative to high capacity $K = D(a_h)$ is not zero capacity but rather capacity K^* which protects her from regulatory capture. Therefore inducing investment beyond K^* requires not just a risk premium λC but also an extra rent.

5 Conclusion

We show how the lack of commitment of a regulator undermines the efficiency of a price-cap regulation. This commitment problem generically induces a network capacity below the optimum either because the price cap cannot be set at the competitive level, or because the operator strategically rations demand, out of concern for a future decrease of the price cap. This model could be used as a baseline to study the interplay between technological innovation and price cap regulation. While price cap revisions happen at exogenous random times in our model, they can be in practice motivated by the availability of a next-generation technology. In that case, the flexibility to adjust the price cap could generate efficiency gains alongside the commitment problems that this paper highlights.

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Appendix

Proof of Lemma 1

We have shown that the regulator payoff if $\overline{a} = c$, Eq. 5, is strictly increasing in K and it is zero for $K = 0$. Therefore $K^* < D(a_{LRIC})$ if and only if Eq. 5 is strictly larger than Eq. 6 when evaluated at $K = D(a_{LRIC})$. As we discussed above, this is true for δ small enough and false for δ high enough. Furthermore, using Eq. 1, Eq. 5 is equal

$$\int_0^{+\infty} e^{-rt}\left[\frac{1}{2\alpha}(Ke^{-\delta t})^2 + Ke^{-\delta t}\left(\frac{\overline{Q} - Ke^{-\delta t}}{\alpha} - c\right)\right] dt = \frac{K}{\alpha}\left[\frac{\overline{Q} - \alpha c}{r+\delta} - \frac{K}{2(r+2\delta)}\right] \tag{40}$$

Evaluated at $K = D(a_{LRIC})$, this is

$$\frac{\overline{Q} - \alpha(c + (r+\delta)C)}{\alpha}\left[\frac{\overline{Q} - \alpha c}{r+\delta} - \frac{\overline{Q} - \alpha(c + (r+\delta)C)}{2(r+2\delta)}\right] \tag{41}$$

The first term is decreasing in δ. Differentiating the second term with respect to δ yields

$$-\frac{\overline{Q} - \alpha c}{(r+\delta)^2} + \frac{\overline{Q} - \alpha c}{(r+2\delta)^2} - \alpha C \frac{r}{2(r+2\delta)^2} < 0.$$

Therefore Eq. 41 is decreasing which establishes the existence of the threshold $\overline{\delta}$.

Proof of Lemma 3

Let

$$\hat{\Delta}(\lambda) \equiv \max_{\overline{a} \geq c + (r+\delta+\lambda)C} (\overline{a} - c - (r+\delta+\lambda)C)(D(\overline{a}) - K^*) - \lambda K^* C$$

For λ large enough, $D(c + (r+\delta+\lambda)C) \leq K^*$, which implies $\hat{\Delta}(\lambda) < 0$. Suppose $D(c+(r+\delta+\lambda)C) > K^*$, then using the enveloppe, $\hat{\Delta}'(\lambda) = -C(D(a) - K_0) - K_0 C < 0$. Finally, $K^* < D(a_{LRIC})$ implies $\hat{\Delta}(0) > 0$. It follows there exists a unique $\lambda_1 > 0$ such that $\hat{\Delta}(\lambda) \leq 0$ if and only if $\lambda > \lambda_1$.

Proof of Lemma 5

We have shown in the text (see Eq. 16) that at $I(K, a^*) = K^* - K$.

Suppose there exists $K \leq K^*$ such that $a(K) \neq a^*$. For any $K' \geq K$

$$V(K + I(K, a(K)), a(K)) - I(K, a(K))C \geq V(K', a(K)) - K'C,$$

therefore at $\overline{a} = a(K)$, the operator weakly prefers $K + I(K, a(K))$ also when she has no capacity. The regulator weakly prefers capacity $K + I(K, a(K))$ with price cap $a(K)$ to K^* and a^*. Since K^* and a^* is the equilibrium when the operator starts from zero capacity, the regulator has to be indifferent between these two options whether the operator enters the revision stage with capacity K or 0.

Suppose $K + I(K, a(K)) > K^*$. From (A3), the regulator when indifferent should pick the cap that induces the higher capacity, here $a(K)$, which contradicts $a(0) = a^*$. Therefore $K + I(K, a(K)) \leq K^*$. Then $a(K) > a^*$ cannot be optimal for the regulator since we have just shown she can induce capacity K^* with a lower price cap a^*. Therefore $a(K) < a^*$. If $K + I(K, a(K)) = K^*$, then the regulator can induce at least K^* with a price cap lower than a^*, which contradicts $a(0) = a^*$. If $K + I(K, a(K)) < K^*$, from (A3), the regulator being indifferent should pick the price cap a^* when the operator has capacity K rather than $a(K)$.

Proof of Proposition 2

Suppose $a^* > a_{LRIC}$ and the firm reaches a revision stage with the equilibrium capacity K^*. Consider the deviation where the regulator then sets the price cap at $\overline{a} \in (a_{LRIC}, a^*)$. We show below that the operator's optimal response is to maintain her capacity at K^*. This implies that $\overline{a}$ is a strictly profitable deviation for the regulator.

Lemma 11. *Suppose that at a price cap $\overline{a} \in (a_{LRIC}, a^*)$, the operator finds it optimal to let her capacity depreciate from K^* to $K < K^*$. Then at K, the operator either maintain her capacity or let it further depreciate.*

Proof. By contradiction, suppose the operator lets her capacity depreciate to K and then finds it optimal to increase it to $K' \in (K, K^*]$. This implies $V(K', \overline{a}) \geq V(K, \overline{a}) + (K' -$

$K)C$, which, combined with Lemma 4, implies

$$V(K',\overline{a}) = V(K,\overline{a}) + (K'-K)C. \tag{42}$$

Let T be the time it takes for her capacity to depreciate from K' to K and consider the regulator's expected payoff when her capacity first reaches K',

$$\int_0^T \lambda e^{-\lambda t}\left[\int_0^t e^{-r\tau}K'e^{-\delta\tau}(\overline{a}-c)d\tau + e^{-rt}\left(V(K^*,a^*) - \left(K^* - Ke^{-\delta t}\right)C\right)\right]dt$$
$$+e^{-(r+\lambda)T}V(K,\bar{a}) \tag{43}$$

If instead she maintains her capacity at K', her expected payoff is

$$\int_0^T \lambda e^{-\lambda t}\left[\int_0^t e^{-r\tau}K'(\overline{a}-c-\delta C)d\tau + e^{-rt}\left(V(K^*,a^*) - (K^*-K')\,C\right)\right]dt$$
$$+e^{-(r+\lambda)T}V\left(K',\overline{a}\right) \tag{44}$$

Using Eq. 42 and $\overline{a} > a_{LRIC}$, Eq. 44 is strictly larger than Eq. 43 if

$$(r+\delta)\left(\frac{-e^{-\lambda T}\delta}{r\,(r+\delta)} + \lambda e^{-(r+\lambda)T}\left(\frac{1}{r\,(r+\lambda)} - \frac{e^{-\delta T}}{(r+\delta)\,(r+\lambda+\delta)}\right) + \frac{\delta}{(\lambda+r+\delta)\,(r+\lambda)}\right)$$
$$-\left[\frac{\lambda}{\lambda+r+\delta} + \frac{r+\delta}{\lambda+r+\delta}e^{-(r+\lambda+\delta)T} - e^{-(r+\lambda)T} - \frac{\lambda}{\lambda+r}\left(1-e^{-(r+\lambda)T}\right)\right]$$
$$-\delta\left(\frac{-e^{-\lambda T} + \frac{\lambda}{\lambda+r}e^{-(r+\lambda)T}}{r} + \frac{1}{\lambda+r}\right) \geq 0. \tag{45}$$

Note that Eq. 45 is equal to 0 at $T=0$ and when T tends to $+\infty$. Differentiating Eq. 45 with respect to T yields

$$e^{-\lambda T}e^{-rT}\left(-\lambda - r + (\lambda+r+\delta)\,e^{-\delta T}\right),$$

which is first strictly positive, then strictly negative. It follows that Eq. 45 is strictly positive. This implies the operator is strictly better off maintaining her capacity at K' than letting it depreciate to K and bringing it back to K' □

Lemma 11 implies we can restrict attention to strategies where the operator lets her capacity depreciate to some level $K < K^*$, and then stay at K until next revision (by Markov assumption). Her payoff is then

$$\int_0^{t^*(K)} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} K^* e^{-\delta \tau} (\overline{a} - c) d\tau + e^{-rt} V(K^* e^{-\delta t}, a^*) \right] dt$$
$$+ \int_{t^*(K)}^{+\infty} \lambda e^{-\lambda t} \left[\int_0^{t^*(K)} e^{-rt} K^* e^{-\delta t} (\overline{a} - c) d\tau + \int_{t(K)}^t e^{-rt} K (\overline{a} - c - \delta C) d\tau + e^{-rt} V(K, a^*) \right] dt,$$

where $t^*(K)$ is such that $K^* e^{-\delta t(K)} = K$. Using Eq. 19 this reduces to

$$K^* \left(\frac{\overline{a} - c - \delta C}{r + \lambda} \right) + \frac{\lambda}{r + \lambda} V(K^*, a^*) - \frac{\delta \left(K^* - K e^{-(\lambda + r) t^*(K)} \right)}{(\lambda + r)(r + \lambda + \delta)} (\overline{a} - a_{LRIC}) \tag{46}$$

If instead, the operator maintains capacity at K^*, her profit is

$$\begin{aligned} & \int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} (\overline{a} - c - \delta C) K^* \, d\tau + e^{-rt} V(a^*, K^*) \right] \\ = \quad & K^* \left(\frac{\overline{a} - c - \delta C}{\lambda + r} \right) + \frac{\lambda}{\lambda + r} V(K^*, a^*), \end{aligned}$$

which is larger than Eq. 46 as long as $\overline{a} > a_{LRIC}$. It follows that the regulator strictly improves her payoff by maintaining her capacity at K^*, rather than letting it depreciate. this, in turn, makes a deviation to $\overline{a} \in (a_{LRIC}, a^*)$ profitable for the regulator.

Proof of Lemma 7

Note first that a deviation to $\overline{\lambda} > a_{LRIC}$ is not relevant for if either $\lambda < \underline{\lambda}$, in which case the operator serves the demand at a_{LRIC} or if $\lambda > \underline{\lambda}$ in which case, from Lemma 6, a productive constant-cap constant-capacity equilibrium cannot exist.

Suppose now that $\lambda < \overline{\lambda}$ and suppose the regulator increases the price cap to $\overline{a} > a_{LRIC}$. For such a strategy to be valuable, it must be the case that $D(\overline{a}) > K^*$, otherwise the deviation would imply lower demand at higher price with a revision at a^* followed by K^*.

If the operator plays the strategy K^*, her payoff is

$$\begin{aligned}
&\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(\overline{a} - c - \delta C)K^* d\tau + e^{-rt}V(K^*, a^*) \right] dt \\
= \; & \frac{(\overline{a} - c - \delta C)K^*}{r+\lambda} + \frac{\lambda}{r+\lambda} V(K^*, a^*). \qquad (47)
\end{aligned}$$

If she increases capacity to $K > K^*$, her payoff is

$$\begin{aligned}
&\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau}(\overline{a} - c - \delta C)K d\tau + e^{-rt}V(K, c) \right] dt - (K - K^*)C \\
= \; & \frac{(\overline{a} - c - \delta C)K}{r+\lambda} + \frac{\lambda}{r+\lambda} V(K, c) - (K - K^*)C \qquad (48)
\end{aligned}$$

For $K > K^*$, define $t^*(K)$ such that $e^{-\delta t^*(K)}K = K^*$ (see Eq. 32 in the main text). As long as $K > K^*$ the price cap is at c and stays at c at revision. Therefore the operator expects zero profit before time $t^*(K)$ so that

$$\begin{aligned}
V(K, c) &= e^{-rt^*(K)} \int_0^{+\infty} \lambda e^{-\lambda t} e^{-rt} (V(K^*, a^*) - K^*C \left(1 - e^{-\delta t}\right)) \, dt \\
&= \left(\frac{K^*}{K}\right)^{\frac{r}{\delta}} \frac{\lambda}{\lambda + r + \delta} K^*C
\end{aligned}$$

Substituting in Eq. 48 and differentiating with respect to K,

$$\frac{\overline{a} - c - (r + \lambda + \delta)C}{r + \lambda} - \frac{\lambda^2}{(r+\lambda)(r+\lambda+\delta)} \frac{r}{\delta} \left(\frac{K^*}{K}\right)^{\frac{r}{\delta}+1} C \qquad (49)$$

This implies Eq. 48 is convex in K, and therefore if the operator finds it profitable to increase capacity following a deviation by the regulator to $\overline{a}$, she picks $K = D(\overline{a})$.

Then, comparing Eq. 47 and Eq. 48, a deviation $\overline{a} > a^*$ induces the operator to raise her capacity above to $D(\overline{a})$ if and only if

$$\overline{a} - c - (r + \lambda + \delta)C \geq \left(1 - \left(\frac{K^*}{D(\overline{a})}\right)^{\frac{r}{\delta}} \frac{\lambda}{\lambda + r + \delta}\right) \frac{\lambda K^* C}{D(\overline{a}) - K^*} \qquad (50)$$

Given $D(\overline{a}) > K^*$, this deviation exists only if $\overline{a} > a_{LRIC} + \lambda C$. We define λ^* as the lowest λ such that Eq. 50 holds and the regulator's deviation payoff is higher than her equilibrium payoff for some $\bar{a}$. Note that if $\underline{\lambda} > 0$ (Eq. 26 is true), then for λ higher but close enough to $\underline{\lambda}$, K^* is close $D(a_{LRIC})$ and therefore higher than $D(a_{LRIC} + \lambda C)$. In that case there is a non-empty region above $\underline{\lambda}$ where there is no deviation.

Proof of Lemma 8

Combining Eq. 30 and Eq. 33 we get

$$\frac{r+\lambda}{\lambda}\underline{W} - \frac{1}{\lambda}S(a_h) - \int_0^{t^*(D(a_h))} e^{-rt}\Sigma(c, D(a_h)e^{-\delta t})\, dt - e^{-rt^*(D(a_h))}\underline{W} = 0 \tag{51}$$

Differentiating the LHS of Eq. 51 with respect to K^* yields

$$\frac{r+\lambda}{\lambda}\frac{\partial \underline{W}}{\partial K^*} - \frac{\partial t^*(D(a^*))}{\partial K^*}e^{-rt^*(D(a^*))}\left[\Sigma(c, K^*) - r\underline{W}\right] > 0.$$

This is positive because Eq. 31 implies $\underline{W}$ is increasing in K^*, $\Sigma(c, K^*) - r\underline{W} > 0$, and finally, Eq. 32 implies $t^*(K)$ is decreasing in K^*.

For a given a^*, if $K^* = 0$, then $\underline{W} = 0$ and the LHS of Eq. 51 is negative. If $K^* = D(a^*)$, then the LHS of Eq. 51 is

$$\frac{r+\lambda}{\lambda}\underline{W} - \frac{1}{\lambda}S(a^*) - \underline{W} = \frac{1}{\lambda}(r\underline{W} - S(a^*)). \tag{52}$$

Therefore there exists a_h and $K^* < D(a_h)$ such that Eq. 51 holds if and only if Eq. 52 is strictly positive. Using Eq. 31, this is equivalent to

$$\int_0^{+\infty} \lambda e^{-\lambda t}\left[\int_0^t e^{-r\tau}\Sigma(c, D(a^*)e^{-\delta t})\, d\tau\right] dt > \frac{S(a^*)}{r+\lambda}. \tag{53}$$

Using the linear demand specification in Eq. 1, this is equivalent to

$$\int_0^{+\infty} \lambda e^{-\lambda t}\left[\int_0^t e^{-r\tau}\left[\frac{1}{2\alpha}D(a_h)^2 e^{-2\delta\tau} + \left(\frac{\bar{Q} - D(a_h)e^{-\delta\tau}}{\alpha} - c\right)D(a_h)e^{-\delta\tau}\right] d\tau\right] dt > \frac{D(a_h)^2}{2\alpha(r+\lambda)}.$$

which reduces to

$$a_h > c + \frac{\bar{Q} - \alpha c}{\alpha}\left(\frac{\delta}{r+\delta+\lambda}\right)^2 \tag{54}$$

Eq. 54 is true for any λ if Eq. 34 is true, in which case there exists a unique $K_r(a_h)$ such that Eq. 51 holds.

Proof of Lemma 9

For $K \leq K^*$ the regulator chooses between $\overline{a} = c$ with depreciation, $\overline{a} = a_{LRIC}$ with capacity maintained at K_h and the equilibrium $\overline{a} = a_h$ with capacity $D(a_h)$ and payoff $\underline{W}$. We know that at K^*, the regulator is indifferent between $\overline{a} = c$ and $\overline{a} = a_h$ (Eq. 31). Since the payoff from playing $\overline{a} = c$ is strictly decreasing in K, he strictly prefers $\overline{a} = a_h$ for $K < K^*$. The regulator prefers $\overline{a} = a_h$ to $\overline{a} = a_{LRIC}$ if an only if

$$\underline{W} \geq \frac{\Sigma\left(c + (r+\delta)C, K\right)}{r+\lambda} + \frac{\lambda \underline{W}}{r+\lambda}$$

Using Eq. 31, this holds for $K \leq K^*$ if and only if

$$\int_0^{+\infty} \lambda e^{-\lambda t} \left[\int_0^t e^{-r\tau} \Sigma(c, K^* e^{-\delta\tau})\, d\tau \right] dt \geq \frac{\Sigma\left(c + (r+\delta)C, K_0\right)}{r+\lambda}$$

Using the linear demand specification in Eq. 1, this is equivalent to

$$\frac{K^*}{r+2\delta+\lambda} \geq \frac{\overline{Q} - \alpha c}{r+\delta+\lambda} - \frac{\alpha(r+\delta)C}{\delta}. \tag{55}$$

The left-hand side of Eq. 55 is negative for any λ if and only if Eq. 24 is not true.

Next, suppose $K > K^*$. Setting $\overline{a} = a_h$ is dominated by $\overline{a} = c$ because $\overline{W}(K) > \underline{W}$. Setting $\overline{a} = c + (r+\delta)C$ is also dominated by $\overline{a} = c$. To see that notice that for both price caps the capacity depreciates to K^*. Then we have seen that at K^* the regulator prefers c to the LRIC. Moreover during the transition to K^*, the price is lower and therefore consumer surplus is higher when $\overline{a} = c$.

Proof of Lemma 10

Combining Eq. 36, Eq. 37 and Eq. 38, we get

$$\begin{aligned} & \frac{r+\lambda}{\lambda r}\left(a_h - a_{LRIC}\right)K^* - \frac{1}{\lambda}\left(a_h - a_{LRIC} - \lambda C\right)D\left(a_h\right) \\ = & \left(\frac{K^*}{D\left(a_h\right)}\right)^{\frac{r}{\delta}}\left[\frac{\lambda}{r\left(r+\lambda\right)}\left(a_h - a_{LRIC}\right)K^* + \frac{\lambda}{r+\lambda+\delta}CK^*\right] \end{aligned} \tag{56}$$

Rewrite Eq. 56 as

$$\begin{aligned} & \frac{r+\lambda}{\lambda r}\left(a_h - a_{LRIC}\right)\hat{X} - \hat{X}^{1+\frac{r}{\delta}}\left[\frac{\lambda}{r\left(r+\lambda\right)}\left(a_h - a_{LRIC}\right) + \frac{\lambda}{r+\lambda+\delta}C\right] \\ & -\frac{1}{\lambda}\left(a_h - a_{LRIC} - \lambda C\right) = 0 \end{aligned} \tag{57}$$

where $\hat{X} \equiv \frac{K^*}{D(a_h)}$ and is therefore in $(0,1)$. The LHS of Eq. 57 is concave in X. At $X = 0$, it takes value

$$-\frac{1}{\lambda}\left(a_h - a_{LRIC} - \lambda C\right)$$

and has a positive slope. At $X = 1$ it takes value

$$\frac{1}{r+\lambda}\left(a_h - a_{LRIC}\right) + C\frac{r+\delta}{r+\lambda+\delta},$$

which is strictly positive given $a_h \geq a_{LRIC}$. Therefore there exist a_h and $K^* < D(a_h)$ such that Eq. 56 holds if and only if $a_h > a_{LRIC} + \lambda C$, in which case there is a unique $K_o(a_h)$ such that Eq. 56 holds.

Proof of Proposition 4

Using Eq. 31 and the linear specification in Eq. 1, we can write Eq. 51 as

$$\begin{aligned} & \left(\frac{r+\lambda}{\lambda} - \bar{X}^{\frac{r}{\delta}}\right)\left(\frac{r+\lambda}{r}\left(\frac{\bar{Q}-\alpha c}{\alpha}\bar{X}\frac{1}{r+\lambda+\delta} - \frac{\bar{X}^2}{2\alpha}\frac{1}{r+\lambda+2\delta}D\left(a_h\right)\right)\right) \\ = & \frac{D\left(a_h\right)}{2\alpha\lambda} + \frac{1-\bar{X}^{1+\frac{r}{\delta}}}{r+\delta}\frac{\bar{Q}-\alpha c}{\alpha} - \frac{1-\bar{X}^{2+\frac{r}{\delta}}}{r+2\delta}\frac{D\left(a_h\right)}{2\alpha} \end{aligned} \tag{58}$$

where $\bar{X} \equiv \frac{D(a_h)}{K^*}$. From Lemma 8, Eq. 58 implicitly defines $\bar{X}(.)$ as a function of a_h on the interval $\left[c + \frac{\bar{Q}-\alpha c}{\alpha}\left(\frac{\delta}{r+\delta+\lambda}\right)^2, \frac{\overline{Q}}{\alpha}\right]$. We know from the Proof of Lemma 8 that $K^* = 0$ cannot be a solution to Eq. 51, which implies that for any $a_h \in \left[c + \frac{\bar{Q}-\alpha c}{\alpha}\left(\frac{\delta}{r+\delta+\lambda}\right)^2, \frac{\overline{Q}}{\alpha}\right]$, $\bar{X}(.) > 0$.

Next, Eq. 57 also implicitly defines $\bar{X}(.)$ as a function of a_h on the interval $\left[a_{LRIC} + \lambda C, \frac{\overline{Q}}{\alpha}\right]$. In addition, from the Proof of Lemma 10, $\hat{X}(a_{LRIC} + \lambda C) = 0$. Finally,

$$c + \frac{\bar{Q} - \alpha c}{\alpha}\left(\frac{\delta}{r+\delta+\lambda}\right)^2 < a_{LRIC} + \lambda C \Leftrightarrow \frac{\bar{Q} - \alpha c}{\alpha(r+\delta+\lambda)C} < \left(1 + \frac{r+\lambda}{\delta}\right)^2,$$

which is true if Eq. 24 does not hold.

This implies $\bar{X}(a_{LRIC} + \lambda C) > \hat{X}(a_{LRIC} + \lambda C)$. Therefore, a sufficient condition for the existence of $a_h \in \left(a_{LRIC} + \lambda C, \frac{\overline{Q}}{\alpha}\right)$ such that $\bar{X}(a_h) = \hat{X}(a_h)$ is

$$\bar{X}\left(\frac{\overline{Q}}{\alpha}\right) < \hat{X}\left(\frac{\overline{Q}}{\alpha}\right).$$

Eq. 58 implies

$$\frac{(r+\lambda)^2(r+\delta)}{r(r+\lambda+\delta)}\bar{X}\left(\frac{Q}{\alpha}\right) + \lambda\left(\bar{X}\left(\frac{Q}{\alpha}\right)\right)^{1+\frac{r}{\delta}}\left(1 - \frac{(r+\lambda)(r+\delta)}{r(r+\lambda+\delta)}\right) = \lambda, \tag{59}$$

and therefore $\bar{X}\left(\frac{Q}{\alpha}\right)$ tends to 0 as $\lambda \to 0$.

Similarly Eq. 57 implies

$$\frac{r+\lambda}{r}\left(\frac{Q}{\alpha} - a_{LRIC}\right)\hat{X}\left(\frac{Q}{\alpha}\right) - \left(\hat{X}\left(\frac{Q}{\alpha}\right)\right)^{1+\frac{r}{\delta}}\left[\frac{\lambda^2}{r\,(r+\lambda)}\left(\frac{Q}{\alpha} - a_{LRIC}\right) + \frac{\lambda^2}{r+\lambda+\delta}C\right]$$
$$= \left(\frac{Q}{\alpha} - a_{LRIC} - \lambda C\right) \tag{60}$$

and therefore $\hat{X}\left(\frac{Q}{\alpha}\right)$ tends to 1 as $\lambda \to 0$.

We conclude that if λ is small enough, there exists $a_h \in \left(a_{LRIC} + \lambda C, \frac{\overline{Q}}{\alpha}\right)$ such that $\bar{X}(a_h) = \hat{X}(a_h)$. At that solution, a_h and $K^* = D(a_h)\bar{X}(a_h)$ are solutions to Eq. 51 and Eq. 56 and form an equilibrium as long as Eq. 24 does not hold, which, from Lemma 9, guarantees the regulator has no profitable deviation.