WORKING PAPERS

N° 1638

April 2025

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Lock-in in Renewable Energy Generation under Constraining Capacities and Heterogenous Conversion Performances [1]

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April 14, 2025

[1]This work started originally under the impulsion and strong feed-in of Michel Moreaux. Michel passed in November 2021, I miss him.

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Abstract

The theme of the 'energy transition' away from fossil fuels toward clean renewable energy has attracted a lot of attention in the context of climate change mitigation. However the emergence of a new energy system raises its own problems. An aggressive carbon pricing or a renewables subsidisation policy can result in fast investment in poor performing energy conversion capacities. Once installed the industry will remain locked-in in these inferior technical options especially if capital investments are submitted to adjustment costs. With the help of a stylized fully dynamic model, we show the following. Without an access cost to primary energy (e.g. solar radiation) the industry can run more performing equipments even if they are both more costly to operate and more costly to build provided a sufficiently strong energy demand. With this preliminary result in hands we assume next convex access costs to primary energy, due for example to limited space access constraints. The high performing energy conversion technique has now a productivity advantage. However for a small energy demand it can remain optimal for the industry to first deploy high performance equipments together with low performing ones before dismantling their stock of high performing equipments. Despite the increase of the marginal access cost to primary energy coming alongside the deployment of production capacities, thus inducing a fall of the cost gap between the two technologies, the capital price of the high performing equipments can fall down to zero before the capital price of low performing ones because of the building costs gap, implying that the industry should scrap in the end its high performing equipments while still investing in low cost (and low performing) ones. This 'transition inside the transition' problem provides also interesting insights concerning the regulation of the energy transition towards renewable energy. It suggests that avoiding lock-in in renewable energy provision is more a matter of speed of increase of the carbon price than just the fixation of its level at any moment.

Keywords: Renewable energy; Energy transition; Lock-in; Capacity constraints; Adjustment costs.

JEL classifications: D25, O33, Q42, Q54, Q55.

1 Introduction

Lock-in in poorly efficient, or in carbon emitting, energy conversion technologies is a prominent issue in the context of climate change regulation promoting the transition toward a carbon-free energy system. Many studies have tried to measure the importance of this effect in the energy sector and the manufactured goods industries. All conclude to significant investment inertia impeding the efforts of the governments to deploy clean alternatives to harmful fossil fuels combustion. Lock-in is the consequence of inertia of the capital structure of industries relying on long operational life duration equipments. The relevant literature has stressed the need for a much more active carbon pricing policy to overcome capital structure inertia.

However the need for a fast transition to clean energy can have its own drawbacks. Incentivising the energy industry to adopt present state-of-the-art technologies in renewable energy conversion means that further technological advances converting more efficiently energy are at risk of never being adopted or with a significant delay. The subsidisation of renewable energy conversion and/or carbon pricing can thus be the cause of lock-in to inferior energy conversion techniques. Say differently, in addition to the issue of the energy transition away from fossil fuels toward carbon-free renewable energy, a topic relatively well explored today, we should pay more attention to the issue of the 'transition inside the transition', that is the consequences of the emergence of a new industrial structure of energy provision, with its own problems and dynamics. The purpose of this paper is to explore this issue by means of a stylised model.

The investment decision problem of the industry is at the core of the lock-in phenomenon. The investor must make a guess on the future profitability of its investments inducing a dynamically recursive ingredient in the capital accumulation policy of the industry. The capital price at the equilibrium on the equipments markets is thus made endogenous to the investors guess because of external and internal adjustment costs. But on the other hand the relative competitiveness of different capital vintages, which determines the technology choice, is itself endogenous to the capital price formation process. This suggests the adoption of a putty-clay formulation of the capital structure, an assumption widely used in the relevant literature. To avoid the intricacies of this choice of capital structure, we adopt several simplifying assumptions.

We dispense from the study of the innovation process in new technologies. There exists a large literature in this field, although adopting a putty-putty view of the capital structure and thus overlooking the issue of capital inertia in technology adoption. We assume that at the beginning of the planning horizon the industry can invest in two possible types of technology embodying equipments: a low performing energy conversion one and a high performing one, the needed R&D effort to invent the two techniques having already taken place. The low performing equipments are both cheaper to operate and cheaper to build. Thus the competitive advantage of a high performing technology can show only when there exist access constraints to renewable crude energy, e.g. solar radiation.

In the same spirit of simplification, we avoid discussing a complete vintage capital model through the following assumptions. We assume that keeping the equipments in operational status requires paying a maintenance cost. Once properly maintained, the equipments can run forever but if maintenance is not applied at any moment, the equipment will be definitively out-of-order. Thus the capital scraping problem becomes an optimal maintenance policy problem, the depreciation rate of the capital stock being subject to choice through the maintenance decision. Thanks to this simplification we are able to fully characterise not only the building process of capacities but also their scraping over time.

Renewable energy production is submitted to access cost constraints. For solar or wind farms these costs identify to the opportunity cost of land for other uses, agricultural or residential. The negative externalities resulting from the presence of windmills on estate prices fall also in this category. For wind farms in sea areas, the access cost corresponds to the extra costs of energy transportation and the possible negative externalities for ship transportation or fishing activities. We assume that the marginal access cost to crude renewable energy is an increasing function of the consumed amount of crude energy. With the passage of time the deployment of equipments will raise the access cost, incentivizing the industry to adopt the high performing technology. It suggests that the justification for adoption would be the increasing pressure of access cost constraints on crude energy conversion, making lock-in in poor performing techniques a transitory phenomenon that could happen only at the beginning of the development of the industry. We show that the issue is a bit more subtle than that because of the interplay of the energy demand with the building, operating and capital maintaining cost structure in a dynamic context.

Firstly it should be remarked that even without any access costs constraints, it can be perfectly rational for the industry to invest in the high performing technology. With a sufficiently large demand, it is well known that the two types of equipments can coexist despite their cost gap because of capacity constraints. The gap between the capital prices is bounded by the cost gap in rental value. Thus investment in the more expensive technology can be justified with a sufficiently large demand for investments. This is a consequence of adjustments costs, splitting the investment decision between low performing and high performing ones may be cost minimising for the industry on the equipment markets, even if it would prefer to run only low performing equipments. We clarify this point in the paper and give the precise conditions under which complete or incomplete specialisation of equipments could occur.

Concerning the dynamics of capital accumulation we show that either the industry accumulates only low cost equipments or either both types of equipments initially. We show that in this case investments in high performing equipments must end strictly before investments in low performing ones. Then the high performing equipments are maintained while the stock of low performing equipments continues to grow. We are far from the narrative of a progressive adoption of the best technique, the high performing equipments having to be accumulated right from the beginning and their share in the total productive capital will shrink during the maintenance phase of the equipments. Lastly, with a sufficiently large demand the high performing equipments will be maintained in the long run, but with a small demand, they will be ultimately dismantled, the industry relying entirely in the long run on the low performing (and low cost) technology. Without access costs to primary energy, the high performing technique has no economic advantage. However it can expand and even survive in the long run depending on the joint dynamics of the opportunity costs of the capacity constraints and the building costs.

When the access to the primary resource is costly, the long run cost contour of the industry becomes not convex because of our different operating cost assumption on the two techniques. We hence first proceed to the convexification of this contour to build the aggregate cost function at the industry-wide scale. The resulting marginal cost function is made of three segments: (1) An increasing one for small output levels where low performing equipments should predominate; (2) A flat segment for an intermediate range of outputs, where the relative proportion of high performing equipments increases with the output level; (3) Another increasing segment for large output levels where high performing equipments predominate. Crossing the resulting energy supply curve of the industry with the energy demand we distinguish three possible situations in the long run. Either the energy demand is small and encounters the energy supply in the first increasing segment of the marginal cost curve, where the low-performing technology dominates. Either supply meets the demand in the intermediate zone with a mix of equipments of the two kinds. Either the energy demand is sufficiently strong for supply to meet the demand at a point of the third segment of the cost curve, where high performing equipments dominate. We call 'small', 'medium' and 'big' demand cases these three possibilities.

Having identified the demand typology determining the long run marginal energy provision cost of the industry we turn to the determination of the long run capital composition. The difficulty now is that this composition results from possibly very different capital accumulation histories, involving in particular the potential scraping of capacities of one or the other type, either completely of partially. To overcome this difficulty we identify in the capital endowments space critical frontiers, we call the *scraping borders*, corresponding to the upper envelope of capital stocks that can be maintained in the long run. Because the long run cost of running capacities depends on the demand type, these borders are demand specific and we construct them for the three types of energy demands: low, medium or high.

Next we construct two other frontiers corresponding to the capital stocks pairs above which there should be no more investments in either low or high performing equipments respectively. These borders can overlap or intersect themselves and they can also overlap or intersect with the scraping borders, resulting in a rich set of optimal long run capital compositions. Taking benefit of the Markovian structure of our dynamic optimisation problem, we are next able to describe the complete set of optimal histories of capital accumulation converging to any long run capital composition. These histories are some sequence of at most three time episodes: (1) Simultaneous accumulation of capacities in the two types of capital; (2) Accumulation of capital of one type, the other one being maintained at some constant level; (3) Scraping of one type of capital alongside accumulation of the other type of capital. Observe that scraping one type of equipments while maintaining at a constant level the other technology-embedding capital stock is never optimal.

We describe the whole set of optimal sequences starting from any initial distribution of capital stocks and show that they differ according to the type of energy demand. More precisely we show that with a small energy demand, where we expect low cost but also low performing equipments to dominate, only high performing units can be scraped in some scenarios, and the scraping process can be complete or incomplete. With a medium energy demand, scraping of the two types of equipment can happen but can never be complete, the capital composition of the industry converging toward some mix of the two techniques. Last with a big energy demand, where we expect the high performing equipments to dominate, only the scraping of low performing equipments can occur, either complete or incomplete.

The main takeaway of the study are the following. The outcome of the 'energy transition inside the transition' has no reason to be a smooth process even if the access to space constraints put an ever increasing pressure favouring the adoption of more efficient energy conversion techniques. Depending on the relative cost advantages and the energy demand, the history of the transition can be made of moves in different technological directions dragging the energy system back and forth toward better conversion techniques.

In many scenarios, the gradual accumulation of production capacities, whatever their composition, induces a progressive fall of the energy price resulting from an ever larger supply of renewable energy. However when scraping of some type of capacities occur, it can happen at a higher rate than new capacities are built, inducing a fall of the energy supply and thus a price hike. That the price of a commodity whose production is submitted to time changing capacity constraints can move up and down is familiar in the vintage capital literature as a consequence of the complex structural adjustment process of the demography of capital units. In the present model, this feature of the energy price dynamics is more surprising.

Another robust conclusion of the analysis is the path dependency of the optimal capital accumulation trajectories in the two technologies. Even if unique for any given vector of initial capital stocks, these trajectories evolve and converge to very different long run situations according to the initial vector. Path dependency is a common feature of endogenous directed technical change models in growth macroeconomics. The often made assumption of a linear knowledge accumulation function gives a Ricardian flavour to the resulting innovation dynamics, helping the conservation over time of the initial comparative advantage of one research direction onto the other. However the relevant literature has shown that the path dependency phenomenon is a more general character of directed technical change. Our analysis confirms the importance of this feature when capital inertia and lock-in possibilities are properly accounted for.

The analysis also provides some insights on the energy transition issue in the context of climate change. We can reinterpret the energy demand for renewable energy as the residual energy demand net of the energy supply from fossil energy sources. The objective of a CO_2 emissions reduction policy is to favour a move away from fossil fuels crude energy conversion to clean renewable energy through regulatory measures like carbon pricing. The result in the present context will be a progressive shift of the residual demand curve from a small demand situation up to a big demand situation. The scraping borders and limit investment borders in the two techniques being demand dependent, a complex pattern of reallocation of equipments during the transition should emerge. The analysis suggests however that if the upward move of the energy demand stabilises ultimately in a big demand scenario, high performing equipments should dominate in the long run capital composition.

The next section 2 describes our workhouse model. In section 3 we explore the benchmark situation where renewable energy is abundant so that the competitive advantage of using a highly efficient energy conversion technique does not exist. Section 4 studies the costly primary energy case. The last section 5 concludes.

Related literature

Assessing the determinants of energy use is a long-standing topic in production economics (Atkeson and Kehoe, 1999, Pindyck and Rothenberg, 1983).

Work in this area uses a putty-clay capital structure to stress the importance of adjustment costs between different capital goods. The studies conclude generally to a strong inertia of the energy demand determinants resulting from the long lasting effects of capital vintage initial choice. In a recent study, Hawkins-Pierot and Wagner (2024) add to these effects the role of initial productivity differentials in explaining fuel demand inertia.

The issue of lock-in has also attracted a lot of attention in the innovation literature. Hassler *et al.*, (2021), stress the role of relative scarcity of natural resources and thus their respective price dynamics on directed technical change, Acemoglu *et al.*, (2023), do a similar job for the energy transition in the context of climate change regulation. While Hassler *et al.* focus more on the use of energy in the industry, Acemoglu *et al.* stress the negative effect of shale gas extraction on innovation in the renewable energy sector. Most of this literature uses disembodied capital assumptions, see Lennox and Witajewski-Baltvilks, (2017), for an extension to embodied formulation.

The problem of 'carbon lock-in' in the climate economics literature is a long-standing concern dating back to Unruh, (2000), The survey by Seto *et al.*, (2016), provides a good account of this literature. The problem is also explored at a more general level in Fowlie *et al.*, (2016). The literature has mainly focused on the lock-in in fossil fuel energy conversion techniques impeding the transition to carbon-free alternatives. The issue of lock-in inside the renewable industry itself has attracted far less attention up to now.

There exists a long-standing line of research which models the energy transition to renewables as an investment problem under capacity constraints and adjustment costs (Wirl, 1991, Amigues *et al.*, 2015, Coulomb *et al.*, 2020, Pommeret and Schubert, 2022). To our knowledge, our contribution is the first one to apply this framework to heterogenous types of renewable energy production installations, differing in both their exploitation costs and building costs.

2 The model

Useful energy production

The energy industry can deliver useful energy (UE) to the final users from a renewable energy source, say solar, by means of two different techniques. One technique, we index by h, has a high energy conversion performance. The other one, indexed by l, is a poor energy converter. We denote by r_i, $i = h, l$, the transformation rate of solar energy radiation measured in energy units into UE with the technology i, equivalently the number of units of solar energy needed to produce one unit of UE. Since a one-to-one conversion of crude solar energy into UE is impossible, $r_i > 1$. On the other hand the the technology h being more energy efficient than the technology l, $r_h < r_l$. Denote by $\bar{r}_i$ the energy conversion rate of the technology i, $\bar{r}_i = 1/r_i$, $i = h, l$.

Let q_i, $i = h, l$, denote the UE production rate from the technique i. Producing UE from any technology requires specific equipments or productive capital. Denote by K_i the installed capital in the UE production sector i, $i = h, l$. UE generation with the technology i requires also variable inputs v_i, $i = h, l$, in addition to the converted renewable energy flow y_i. The UE production function with a technology i is a Leontiev one:

$$q_i = \min\{K_i, \bar{s}_i v_i, \bar{r}_i y_i\} \qquad i = h, l \ .$$

We denote by s_i, $i = h, l$, the amount of the variable input needed to produce one unit of UE with the technology i and $\bar{s}_i = 1/s_i$. The most efficient technique is more costly to operate: $s_l < s_h$. The variable input price, p_v, is assumed to be constant and we denote by a_i the unitary cost of the variable input: $a_i \equiv p_v s_i$, $i = h, l$.[1] Since $s_l < s_h$, $a_l < a_h$.

Accessing the solar energy source has also a specific cost, like land hiring expenditures to install windmills, solar panels or allocate arable land to biofuels production. Let $y = y_h + y_l$, be the aggregate solar energy flow used to produce UE from the two available technologies and denote by $C_y(y)$ the access cost to solar radiation, with $C_y(0) = 0$. $C_y(y)$ is a twice continuously

[1] Technical efficiency requires that $q_i = \bar{s}_i v_i$, equivalently $v_i = q_i s_i$, $i = h, l$. Thus the total variable cost is $p_v v_i = p_v s_i q_i$ and the unitary cost per UE unit is $a_i = p_v s_i$, $i = h, l$.

differentiable function, strictly increasing, $c_y(y) \equiv dC_y(y)/dy > 0$, convex, $c_y'(y) \equiv d^2C_y(y)/dy^2 > 0$, and $\underline{c}_y \equiv c_y(0^+) > 0$.

UE final demand

$q = q_h + q_l$ denotes the aggregate UE production. UE consumption by the final users generates a gross surplus and let $u(q)$ denote the gross surplus function. $u(q)$ is a twice continuously differentiable function, strictly increasing, $u'(q) \equiv du(q)/dq > 0$, concave, $u''(q) \equiv d^2u(q)/dq^2 < 0$, and satisfies $u'(0^+) < +\infty$. We denote by $p(q) = u'(q)$ the marginal surplus, or inverse demand function, and by $q^d(p)$ the direct demand function.

Capacities dynamics and capital costs

Under our assumptions, UE production is submitted to the capacity constraint $q_i \leq K_i$, $i = h, l$, so we refer indifferently to K_i as the installed capital in the UE production sector using the technology i or the UE production capacity with the technology i. Capacities may be enlarged by investment in specific equipments to the technology i. Let $k_i(t)$ denote the investment rate in new capacity of type i at time t and $\delta_i(t)$ the scrapping proportional rate of the installed capacity of type i at time t. The capacities dynamics are given by:

$$\dot{K}_i(t) \quad = \quad k_i(t) - \delta_i(t)K_i(t) \qquad\qquad i = h, l \ ,$$

provided that $\delta_i(t)$ be continuous at time t.

The total investment cost in type i capacity, $N_i(k_i)$, is a twice continuously differentiable function, strictly increasing, $n_i(k_i) \equiv dN_i(k_i)/dk_i > 0$, convex, $n_i'(k_i) \equiv d^2N_i(k_i)/dk_i^2 > 0$ and $\underline{n}_i \equiv n_i(0^+) > 0$. In addition we assume that $\underline{n}_l < \underline{n}_h$ and that $d\left[n_h(k) - n_l(k)\right]/dk > 0$, so that the marginal investment cost in type h capacity is higher than the marginal investment cost in type l capacity for any positive investment level k.

Keeping capacities in service requires maintenance. Let m_i denote the instantaneous unitary maintenance cost of a type i piece of equipment. We assume that any piece of equipment not benefiting from maintenance at any given time is definitively out of service. Scrapping out-of-service equipments is costless.

3 The cost free primary energy case with two technologies

Since solar energy is abundant, the comparative advantage of the high conversion performance technology does not show. Being also more costly to operate, it seems unlikely that it can be used to produce useful energy. However, demand rationing resulting from the use of only low performing equipments can open a room for the costly high performing technique.

Let $K^0 = K_h^0 + K_l^0$ be the sum of the initial capacities inherited from the past at time $t = 0$ and denote by c_i the full marginal operating cost, in short the O&M unit cost of the technology i: $c_i \equiv a_i + m_i$, $i = h, l$. We assume that $c_l < c_h$ and that $u'(0^+) > c_h$.

3.1 Inherited capacities and optimal initial scrapping

It is never optimal to operate a capacity K_i whose O&M unit cost is higher than its marginal surplus. Thus define $K^{(h)}$ as the solution of $u'(K) = c_h$ and $K^{(l)}$ as the solution of $u'(K) = c_l$, $K^{(h)} < K^{(l)}$ because $c_l < c_h$, and consider the cases illustrated in Figure 1.

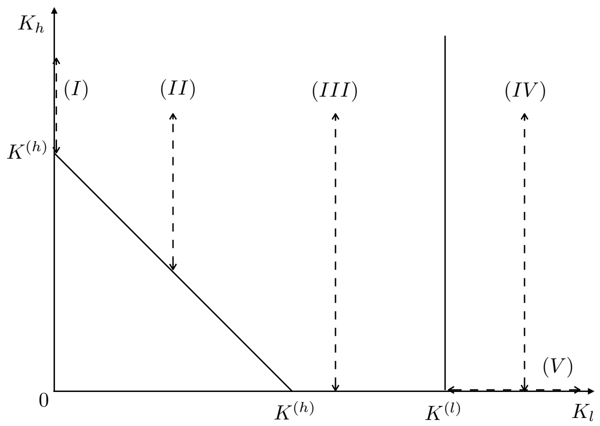

Figure 1: **Initial Scrapping**

For initial endowments $K_h^0 > K^{(h)}$ and $K_l^0 = 0$, like (I) in the Figure, the best is to scrap initially this part $K_h^0 - K^{(h)}$ of the initial endowment in capacity h. For initial endowments $0 < K_l^0 < K^{(h)}$ and $K_h^0 > K^{(h)} - K_l^0$, like (II) in the Figure, the best is to scrap initially this part $K_h^0 - (K^{(h)} - K_l^0)$ of the initial endowment in capacity h. For initial endowments $K^{(h)} \leq K_l^0 \leq K^{(h)}$ and $K_h^0 > 0$, like (III) in the Figure, the best is to scrap initially the whole inherited capacity h, K_h. For initial endowments $K_l^0 > K^{(l)}$ and $K_h^0 \geq 0$, like (IV) and (V) in the Figure, the best is to scrap initially the whole capacity h, K_h^0, and this part $K_l^0 - K^{(l)}$ of the inherited capacity l.

The previous discussion has shown that it may be worth keeping inherited high cost capacities, and thus exploiting them at least temporarily, provided that the available stock of low cost equipments be sufficiently low, even if the O&M costs are linear and the industry does not face capacity constraints. This is an immediate consequence of optimal demand rationing by the industry. We are going to show that the result extends to industry development under endogenously dynamic capacity constraints, although in addition to be more costly to operate, high performing equipments are also more costly to build.

3.2 The social planner problem $(S.P.1)$

From now, we assume for the sake of brevity that the initial endowments are scrapping free, that is either $K^0 \leq K^{(h)}$ or either $K^{(h)} < K_l^0 \leq K^{(l)}$ and $K_h^0 = 0$. The investment, scrapping and exploitation policies solve the following problem.

$$\begin{aligned}
\max_{\{(k_i,q_i,\delta_i),i=h,l\}} \quad & \int_0^\infty \left\{ u\left(\sum_{i=h,l} q_i(t)\right) - \sum_{i=h,l} a_i q_i(t) - \sum_{i=h,l} m_i K_i(t) \right. \\
& \left. - \sum_{i=h,l} N_i(k_i(t)) \right\} e^{-\rho t} dt \\
s.t. \quad & \dot{K}_i(t) = k_i(t) - \delta_i(t) K_i(t) \ , \ K_i(0) = K_i^0 \geq 0 \ , \ K_i(t) \geq 0 \ , \\
& K_i(t) - q_i(t) \geq 0 \ , \ q_i(t) \geq 0 \ , \ \delta_i(t) \geq 0 \ , \ k_i(t) \geq 0 \ , \ i = h, l \ .
\end{aligned}$$

Denote by $\lambda_i(t)$, $i = h, l$, the co-state variables associated to the motion of the capital stocks, by $\nu_i(t)$, $i = h, l$, the Lagrange multipliers associated to the positivity constraints on the capital stocks, $K_i(t) \geq 0$, by $\eta_i(t)$, $i = h, l$, the Lagrange multipliers associated to the capacity constraints $K_i(t) - q_i(t) \geq 0$, by $\gamma_{iq}(t)$, $i = h, l$, the Lagrange multipliers associated to the positivity constraints on q_i, by γ_{ik}, $i = h, l$, the Lagrange multipliers associated to the positivity constraints on investments, $k_i(t) \geq 0$, and by $\gamma_{i\delta}$, $i = h, l$, the Lagrange multipliers associated to the non negativity of the scrapping rates, $\delta_i(t) \geq 0$. Then the Hamiltonian and the Lagrangian of the $(S.P.1)$ problem read (dropping time dependency):

$$\begin{aligned} \mathcal{H} &= u\left(\sum_{i=h,l} q_i\right) - \sum_{i=h,l} a_i q_i - \sum_{i=h,l} m_i K_i - \sum_{i=h,l} N_i(k_i) + \sum_{i=h,l} \lambda_i \left[k_i - \delta_i K_i\right] \\ \mathcal{L} &= \mathcal{H} + \sum_i \nu_i K_i + \sum_i \eta_i \left[K_i - q_i\right] + \sum_i \gamma_{iq} q_i + \sum_i \gamma_{ik} k_i + \sum_i \gamma_{i\delta} \delta_i \ . \end{aligned}$$

The following f.o.c's result:

$$\frac{\partial \mathcal{L}}{\partial q_i} = 0 \iff u'\left(\sum_{j=h,l} q_j\right) = a_i + \eta_i - \gamma_{iq} \quad , \ i = h, l \tag{3.1}$$

$$\frac{\partial \mathcal{L}}{\partial k_i} = 0 \iff n_i(k_i) = \lambda_i + \gamma_{ik} \quad , \ i = h, l \tag{3.2}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_i} = 0 \iff \lambda_i K_i = \gamma_{i\delta} \quad , \ i = h, l \ , \tag{3.3}$$

to which must be added the usual complementary slackness conditions. When time differentiable, the dynamics of the co-state variables obey:

$$\dot{\lambda}_i = \rho \lambda_i - \frac{\partial \mathcal{L}}{\partial K_i} \iff \dot{\lambda}_i = (\rho + \delta_i)\lambda_i + m_i + \eta_i - \nu_i \ , \ i = h, l \ , \tag{3.4}$$

together with:

$$\nu_i \geq 0 \ , \ K_i \geq 0 \text{ and } \nu_i K_i = 0 \ . \tag{3.5}$$

Last, the following transversality condition must hold:

$$\lim_{t \uparrow \infty} e^{-\rho t} \sum_{i=h,l} \lambda_i(t) K_i(t) = 0 \ . \tag{3.6}$$

It will prove useful in the sequel to derive alternative expressions of the dynamics of the co-state variables. First, if $q_i > 0$, then $\gamma_{iq} = 0$ and from

(3.1):

$$\eta_i = u'(q_h + q_l) - a_i \quad , \; i = h, l \; .$$

Substitute for η_i in (3.4) to get, with $\nu_i = 0$ since $K_i \geq q_i > 0$:

$$\dot{\lambda}_i = (\rho + \delta_i)\lambda_i - [u'(q_h + q_l) - (a_i + m_i)] \quad , \; i = h, l \; .$$

If furthermore $\delta_i = 0$, then:

$$\dot{\lambda}_i = \rho\lambda_i - [u'(q_h + q_l) - (a_i + m_i)] \quad , \; i = h, l \; . \tag{3.7}$$

Remark that $\eta_i = u'(q_h + q_l) - a_i = p - a_i$ is the gross operating margin (g.o.m) on equipments of type i, from which it must be deduced the maintenance cost of equipments to get the net operating margin (n.o.m):

$$\eta_i - m_i = p - (a_i + m_i) = p - c_i \quad , \; i = h, l \; .$$

Thus an equivalent expression of (3.7) writes:

$$\dot{\lambda}_i = \rho\lambda_i - [p - c_i] \quad , \; i = h, l \; . \tag{3.8}$$

Before entering the computation of the solution to the $(S.P.1)$ problem, it is worth noting that $\underline{n}_l < \underline{n}_h$ implies that in the long run there cannot be investment in capacity h without investments in capacity l taking place. Thus either there is only investments in capacity l at the long run limit, or either investments in both types of capacities. This suggests the following study plan. First we determine for any given K_h^0 what should be the limit capacity, K_l, in the long run. Having in hand a relationship between the capacity limit for l type equipments and any given K_h, we next solve an auxiliary optimisation problem of investment in capacity l for this K_h, now assumed to be nil. The solution to this problem allows determining the critical capacity in h equipments such that no investments in this type of capacities could take place, for a given initial capacity, K_l^0. This gives another limit relationship between capacities now applying to type h equipments. Lastly, we use the two limit investment conditions for types l and h to picture the possible optimal dynamics in the capital endowments space.

3.3 Total capacity limits for investments in technology l

Let $\bar{K}^{(l)}$ be the solution of:

$$\underline{n}_l \quad = \quad \frac{1}{\rho}\left[u'(K) - c_l\right] \ .$$

If the industry has accumulated a capacity $\bar{K}^{(l)}$, investing in l type equipments would yield a rental equivalent net operational margin unable to cover the minimum investment cost. Thus $\bar{K}^{(l)}$ stands as the upper bound on the industry capital stock allowing for further investment in capacity l. Since $\underline{n}_l > 0$, then $\bar{K}^{(l)} < K^{(l)}$ because $K^{(l)}$ solves $u'(K) - c_l = 0$.

We want to determine the long run capital composition when the industry has at its disposal at least the inherited total capacity $\bar{K}^{(l)}$. Two cases must be distinguished according to $\bar{K}^{(l)} < K^{(h)}$ or not. Let us denote by L_l the line in the capacities plane, (K_l, K_h) of equation $K_h = \bar{K}^{(t)} - K_l$, defining the upper border for an active investment policy in l type equipments.

3.3.1 The case $\bar{K}^{(l)} < K^{(h)}$

This case in illustrated in Figure 2. For initial endowments (K_h^0, K_l^0) such that either, $\bar{K}^{(l)} \leq K_h^0 \leq K^{(h)}$, or either, $K^{(h)} < K_l^0 \leq K^{(l)}$ and $K_h^0 = 0$, then the best is to stay indefinitely at (K_h^0, K_l^0).

3.3.2 The case $K^{(h)} < \bar{K}^{(l)}$

This case is illustrated in Figure 3. For initial endowments (K_h^0, K_l^0) such that:

- $K^0 = K^{(h)}$ and $K_h^0 > 0$, like (I) in Figure 3, then the best is first to substitute capacities l for capacities h one for one by investing in l and scrapping h up to reduce the capacity h down to 0, and next to carry on the investment in l to attain the capacity $K_l = \bar{K}^{(l)}$.

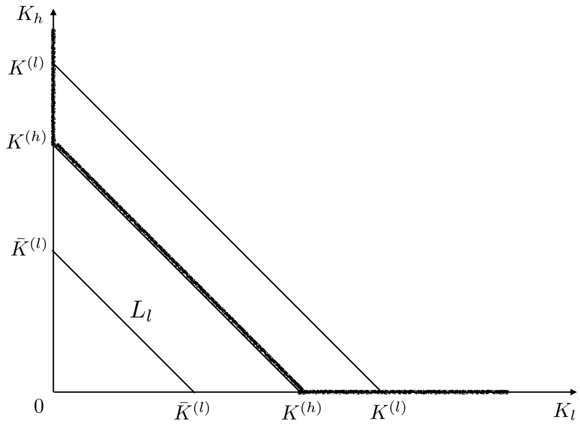

Figure 2: **The Case $\bar{K}^{(l)} < K^{(h)}$.**

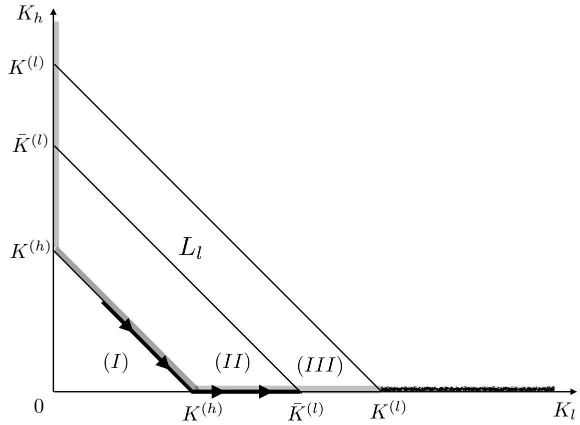

Figure 3: **The Case $K^{(h)} < \bar{K}^{(l)}$.**

- $K^{(h)} < K_l^0 < \bar{K}^{(l)}$ and $K_h^0 = 0$, like (II) in the Figure, the best is to invest only in technology l to increase the capacity of this technology up to $\bar{K}^{(l)}$.
- $\bar{K}^{(l)} \leq K_l^0 \leq K^{(l)}$ and $K_h^0 = 0$, like (III) in the Figure, the best is to stay indefinitely at (K_h^0, K_l^0).

3.4 Determination of the optimal path

The objective is now to determine the optimal path in the (K_h, K_l) plane, that is build the phase diagram in this space. The previous analysis has shown that we have to make a distinction between the cases $\bar{K}^{(t)} < K^{(h)}$ and $\bar{K}^{(l)} > K^{(h)}$. In the first case we know that once the capital composition has reached through investments some point along the border L_l in the capacities space, then the industry should stop investing and maintain forever the attained capacities. In the second case we know that the high performing equipments will be in excess before the border L_l is attained inducing a progressive scraping of type h equipments.

3.4.1 The case $\bar{K}^{(l)} < K^{(h)}$

What we have to determine is: What to do starting from (K_h^0, K_l^0) under the line $L_l : K_h + K_l = \bar{K}^{(l)}$ (see Figure 2), because for (K_h^0, K_l^0) such that $K^0 \geq \bar{K}^{(l)}$, the best is to stay at this initial endowment as shown in 3.3.1. Thus the problem is to determine how to attain the L_l border in the capacities space and stay at the point which is attained.

Note that since (1): (K_h^0, K_l^0) is located below the line L_l; (2): this line is itself located below the line $K_h^0 + K_l^0 = K^{(h)}$, and (3): the asymptotic state is located along the line L_l, then no accumulated capacity K_h is ever dismantled. It follows from (3.8) that:

$$\lambda_l^*(t) - \lambda_h^*(t) \quad = \quad \int_t^\infty [p(\tau) - c_l] e^{-\rho\tau} d\tau - \int_t^\infty [p(\tau) - c_h] e^{-\rho\tau} d\tau = \frac{1}{\rho} [c_h - c_l] \ ,$$

where $\lambda_i^*(t)$, $i = h, l$, is the solution of the problem $(S.P.1)$.

Consider the pivotal case $(K_h^0, K_l^0) = (0, 0)$ and the following auxiliary problem $(A.P.1)$ in which $K_h(t)$ is constrained to stay at the level 0:

$$\begin{aligned} \max_{k_l, q_l} \quad & \int_0^\infty \{u(q_l(t)) - a_l q_l(t) - m_l K_l(t) - N_l(k_l(t))\} e^{-\rho t} dt \\ & \dot{K}_l(t) = k_l(t) \ , \ K_l(0) = K_l^0 = 0 \\ & K_l(t) - q_l(t) \geq 0 \ , \ q_l(t) \geq 0 \text{ and } k_l(t) \geq 0 \ . \end{aligned}$$

The scrapping rate $\delta_l(t)$ can be discarded since any accumulated capacity l is never scrapped. Let us denote by $\lambda_l^*(t;(0,0))$ the value of λ_l at time t, solution of $(A.P.1)$.[2] In the space (K_l, λ_l), the optimal path $\{(K_l^*(t), \lambda_l^*(t)), t \geq 0\}$ is illustrated in Figure 4. Although $K_h(t) = 0$, $t \geq 0$, we can deduce from

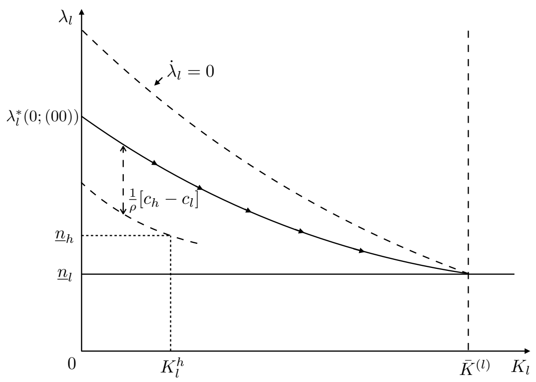

Figure 4: **Optimal path for the** $(A.P.1)$ **problem.**

$\lambda_l^*(t;(0,0))$ what is the marginal value of the nil capacity in technology h. For the same reason than for the solution of $(S.P.1)$, we have:

$$\lambda_h^*(t;(0,0)) \quad = \quad \max\left\{\lambda_l^*(t;(0,0)) - \frac{1}{\rho}\left[c_h - c_l\right], 0\right\} \; .$$

Hence either:

A.) $\lambda_l^*(0;(0,0)) - \frac{1}{\rho}\left[c_h - c_l\right] \leq \underline{n}_h$ in which case it is not optimal to invest in capacity h starting from $(K_h^0, K_l^0) = (0,0)$ so that *a fortiori* it is never optimal to invest in technology h for whatever (K_h^0, K_l^0). The phase diagram in the (K_h, K_l) space is illustrated in the Figure 5 below.

B.) Or $\lambda_l^*(0;(0,0)) - \frac{1}{\rho}\left[c_h - c_l\right] \geq \underline{n}_h$ and since $\underline{n}_h > \underline{n}_l$ there exists some critical level of K_l denoted by K_l^h, $0 < K_l^h < \bar{K}^{(l)}$, such that, starting from $K_h^0 = 0$ and $K_l^0 \geq K_l^h$, it is never optimal to invest in capacity h; while

[2] Note that:

$$\lambda_l^*(0;(0,0)) = \int_0^\infty \left[u'(K_l^*(t)) - c_l\right] e^{-\rho t} dt \; .$$

The assumption $u'(0) < \infty$ allows the integral to be well defined.

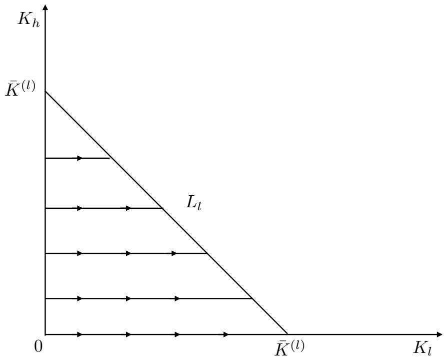

Figure 5: **Phase diagram. Case** $\bar{K}^{(l)} < K^{(h)}$ **and** $\lambda_l^*(0;(0,0)) - \frac{1}{\rho}[c_h - c_l] \leq \underline{n}_h$**.**

starting from $K_h^0 = 0$ and $K_l^0 < K_l^h$ it would be worth investing in the technology h (see Figure 6). In this case there exists a frontier $\hat{K}_h(K_l)$, with $\hat{K}_h(K_l^h) = 0$, such that for $K_h^0 < \hat{K}_h(K_l)$ and $K_h^0 < K_l^h$ it is optimal to invest in technology h as illustrated in Figure 6. It can be shown that:

$$\frac{d\hat{K}_h(K_l)}{dK_l} < 0 \quad \text{and } \hat{K}_h(0) < \bar{K}^{(l)} .$$

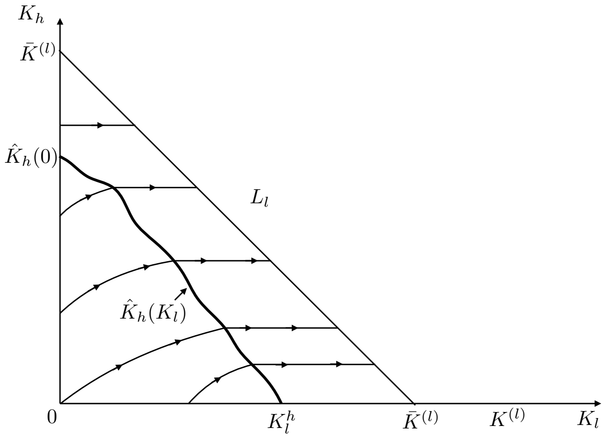

Figure 6: **Phase diagram. Case** $\bar{K}^{(l)} < K^{(h)}$ **and** $\lambda_l^*(0;(0,0)) - \frac{1}{\rho}[c_h - c_l] \geq \underline{n}_h$**.**

Remark: Let $\lambda_l^*(t)$ be the path of $\lambda_l(t)$ solution of $(S.P.1)$ with initial conditions located below the curve $\hat{K}_h(K_l)$ so that there exists an initial phase $[0, \bar{t}_h)$, during which both $k_h(t) > 0$ and $k_l(t) > 0$. At such a time t the determination of the investment rates is illustrated in the Figure 7.

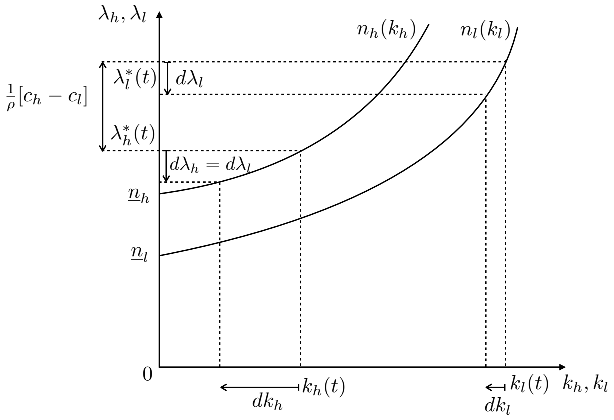

Figure 7: **Determination of k_l and k_h.**

When both investment rates are positive then $k_l^*(t) > k_h^*(t)$. An immediate implication is that in the (K_h, K_l) space when both K_h and K_l are increasing, the slope of the trajectory curve is lower than 1. Furthermore the slopes of the curves are decreasing under the additional assumption that $n_i(k_i)$, $i = h, l$, are convex functions. This case is illustrated in Figure 7.

3.4.2 The case $K^{(h)} < \bar{K}^{(l)}$

We know that once the line $K_h + K_l = K^{(h)}$ is attained, then the optimal policy is to scrap entirely the h type equipments, that is follow the line downward to $(K^{(h)}, 0)$. Thus the problem is to determine, starting from an initial endowments pair (K_h^0, K_l^0) such that $K_h^0 + K_l^0 < K^{(h)}$, whether it is optimal to invest or not in the technology h. The test is the same as in the preceding case: solve the auxiliary program $(A.P.1)$. However now there is no simple relationship between λ_h and λ_l like in the preceding case since all the pieces of equipments of the technology h will be scrapped. The marginal value at $t = 0$ of the nil capacity, that we denote by $\lambda_h^*(0; (0, 0))$, is now:

$$\lambda_h^*(0; (0, 0)) \;\; = \;\; \int_0^{\underline{t}_h} \left[u'(K^*(t)) - c_h \right] e^{-\rho t} dt \ ,$$

where $K_l^*(t)$ is the solution of $(A.P.1)$ and $\underline{t}_h$ is the time at which $K_l^*(t) = K^{(h)}$, that is the time at which $u'(K^*(\underline{t}_h)) = c_h$, assuming that $u'(0) > c_h$.[3] Then:

A. Either $\lambda_h^*(0; (0, 0)) \leq \underline{n}_h$ and it is never optimal to invest in the technology h starting from $(0, K_l^0)$, hence *a fortiori* from any (K_h^0, K_l^0). The corresponding phase diagram in the (K_h^0, K_l^0) is drawn in Figure 8.

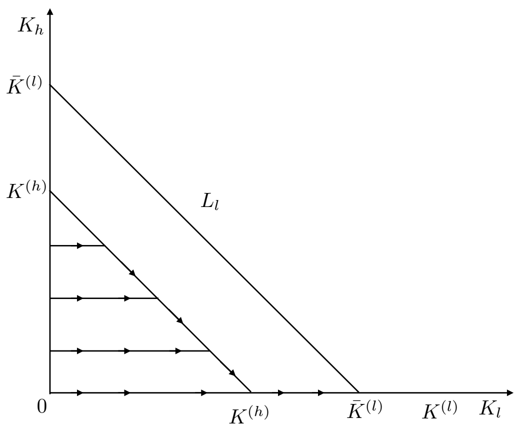

Figure 8: **Phase diagram, $K^{(h)} < \bar{K}^{(l)}$ and $\lambda_h^*(0) \leq \underline{n}_h$.**

B. Or $\lambda_h^*(0; (0, 0)) > \underline{n}_h$ and the solution of $(A.P.1)$ is not the solution of $(S.P.1)$. There exists some time t_l^h, $0 < t_l^h < \underline{t}_h$, such that:

$$\underline{n}_h \;\; = \;\; \int_{t_l^h}^{\underline{t}_h} \left[u'(K_l^*(t) - c_h \right] e^{-\rho(t - t_l^h)} dt \ ,$$

where $K_l^*(t)$ is the solution of $(A.P.1)$. Let us denote by K_l^h, the capacity in technology l at the time $t_l^h = K_l^*(t_l^h) = K_l^h$. Like in the preceding case 3.4.1, there exists a frontier $\hat{K}_h(K_l)$ of no investment in technology h such that $\hat{K}_h(K_l^h) = 0$ as shown in Figure 9.

[3] Assuming that $u'(0) \leq c_h$, then clearly it is never optimal to invest in the technology h and any inherited capital K_h should be immediately scrapped.

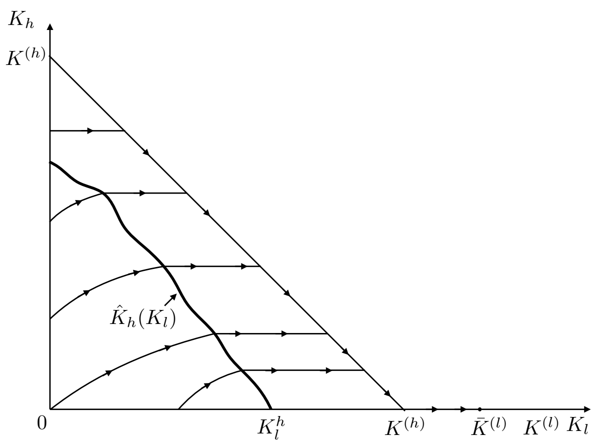

Figure 9: **Phase diagram,** $K^{(h)} < \bar{K}^{(l)}$ **and** $\lambda_h^*(0) > \underline{n}_h$**.**

When it is optimal to invest in both technologies at time t, necessarily $\lambda_h(t) < \lambda_l(t)$ and the investment scheme is similar to the one illustrated in Figure 7 excepted that $\lambda_l(t) - \lambda_h(t)$ is smaller than $[c_h - c_l]/\rho$ and decreases during the simultaneous investment phase when *in fine* the capacities in technology h must be dismantled.

3.5 Concluding comments

We have shown that when the solar resource is abundant, and thus more performing energy conversion techniques have no economic value, it may be optimal in some circumstances to exploit and invest in the high performing renewable energy conversion technology although it is both more costly to run and build. That exploiting more costly equipments may be optimal is a consequence of a low availability of cheap equipments combined with a sufficiently large useful energy demand, a standard substitutability argument under capacity constraints.

That it can be optimal to invest in high building costs equipments may appear more surprising at first sight. Note that absent any difference in operating costs, $c_h = c_l$, then $\lambda_l(t) = \lambda_h(t)$ and even with differences in the investment costs functions, the minimization of the cost of new capacities implies to invest in both technologies when $\lambda_l(t) = \lambda_h(t) > \underline{n}_h$. The Figure 10 illustrates this point.

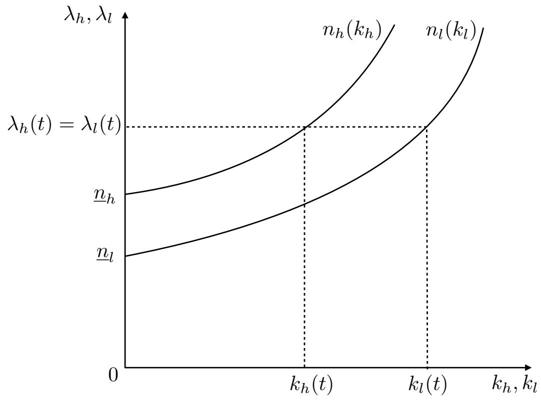

Figure 10: **Investment rates when $c_h = c_l$.**

Introducing a not too strong difference between the respective operating costs of the two technologies, and hence a positive differential between the rents of the capacities l and h does not change this investment logic. Although $\lambda_l(t)$ and $\lambda_h(t)$ are endogenously determined, it is possible to show that:

- The higher is the UE demand (that is u'), the higher is $\lambda_l(t)$ for given other parameters.
- The higher is the discount rate, ρ, the lower is the difference $\lambda_l(t) - \lambda_h(t)$, for given other parameters. A more impatient society will be less sensitive to the cost gap between the two technologies and thus the rent disadvantage of capacities h with respect to capacities l will be lower.

4 The costly primary resource case

We now assume that the primary resource is costly. In a first step, we examine the excess capacities issue and determine the economically relevant capacities absent any building cost consideration. To assess the economic relevance of capacities we build the aggregate cost function of the industry combining optimally the two technologies. Because of our linearity assumptions on the O&M cost structure, the cost contour is not convex at the industry-wide scale. Thus we convexify the cost structure in order to build the aggregate marginal cost function, and thus the energy supply curve. The energy supply curve is made of three segments, an increasing one for low production levels, a flat one for an intermediate range of outputs and last another increasing one for large output levels.

Depending on the segment of the supply curve at which supply meets demand, we identify three demand configurations: what we call the 'small' demand case, when intersection occurs in the first supply segment, the 'medium' demand case when intersection occurs in the second flat segment, and the 'big' demand case when it occurs in the last segment. We show that low cost equipments dominate if the demand is 'small', that there exists a definite mix of high and low performing establishes with a 'medium' demand while the high cost equipments dominate with a 'big' demand.

We next construct the corresponding scrapping frontiers of equipments for these three demand types in the capacities plane. With these scrapping maps in hand, we turn to the characterization of the optimal investment plans in the two energy conversion techniques. To achieve this aim, we complement the analysis in the primal plane (K_h, K_l) with a parallel analysis in the dual plane (λ_h, λ_l). Then we describe the optimal investment scenarios when the industry energy supply faces the three possible types of energy demand.

4.1 Inherited capacities and optimal initial scrapping

Like in the preceding section the objective is to determine the capacities which would be useless even if their building costs would not have to be borne. We first determine the efficient combination of technologies as a function of the total U.E. quantity to produce before taking the demand into account.

4.1.1 Efficient combination of the two technologies

Let $\text{t.o.c.}(q_h, q_l)$ denote the total operating cost of the pair (q_h, q_l), $\text{t.o.c.}(q)$ denote (by a slight abuse of notations) the total operating cost of q, that is the value of $\text{t.o.c.}(q_h, q_l)$ when the dispatching $q_h + q_l = q$ is optimized, and $\text{t.o.c.}_i(q)$ denote the total operating cost of q by using only the technology i, $i = h, l$:

$$\text{t.o.c.}(q_h, q_l) \equiv \sum_{i=h,l} c_i q_i + C_y \left(\sum_{i=h,l} r_i q_i \right) \tag{4.1}$$

$$\text{t.o.c.}_h(q) \equiv \text{t.o.c.}(q, 0) = c_h q + C_y (r_h q) \tag{4.2}$$

$$\text{t.o.c.}_l(q) \equiv \text{t.o.c.}(0, q) = c_l q + C_y (r_l q) \ . \tag{4.3}$$

We define and denote the different marginal operating costs as follows:

$$\text{m.o.c}_h(q_h, q_l) \equiv \frac{\partial}{\partial q_h} \text{t.o.c.}(q_h, q_l) = c_h + r_h c_y \left(\sum_{i=h,l} r_i q_i \right) \tag{4.4}$$

$$\text{m.o.c}_l(q_h, q_l) \equiv \frac{\partial}{\partial q_l} \text{t.o.c.}(q_h, q_l) = c_l + r_l c_y \left(\sum_{i=h,l} r_i q_i \right) \tag{4.5}$$

$$\text{m.o.c}_h(q) \equiv \frac{d}{dq} \text{t.o.c.}_h(q) = c_h + r_h c_y (r_h q) \tag{4.6}$$

$$\text{m.o.c}_l(q) \equiv \frac{d}{dq} \text{t.o.c.}_l(q) = c_l + r_l c_y (r_l q) \ . \tag{4.7}$$

For the second order derivatives we use the following notations:

$$\text{m.o.c}_{hh}(q_h, q_l) \equiv \frac{\partial}{\partial q_h}\text{m.o.c.}_h(q_h, q_l) = \frac{\partial^2}{\partial q_h^2}\text{t.o.c.}(q_h, q_l) = r_h^2 c_y' \left(\sum_{i=h,l} r_i q_i \right) \tag{4.8}$$

$$\text{m.o.c}_{hl}(q_h, q_l) \equiv \frac{\partial}{\partial q_l}\text{m.o.c.}_h(q_h, q_l) = \frac{\partial^2}{\partial q_h \partial q_l}\text{t.o.c.}(q_h, q_l) = r_h r_l c_y' \left(\sum_{i=h,l} r_i q_i \right)$$

$$= \text{m.o.c}_{lh}(q_h, q_l) \tag{4.9}$$

$$\text{m.o.c}_{ll}(q_h, q_l) \equiv \frac{\partial}{\partial q_h}\text{m.o.c.}_l(q_h, q_l) = \frac{\partial^2}{\partial q_l^2}\text{t.o.c.}(q_h, q_l) = r_l^2 c_y' \left(\sum_{i=h,l} r_i q_i \right) \tag{4.10}$$

and:

$$\text{m.o.c.}_h'(q) \equiv \frac{d}{dq}\text{m.o.c.}_h(q) = \frac{d^2}{dq^2}\text{t.o.c.}_h(q) = r_h^2 c_y'(r_h q) \tag{4.11}$$

$$\text{m.o.c.}_l'(q) \equiv \frac{d}{dq}\text{m.o.c.}_l(q) = \frac{d^2}{dq^2}\text{t.o.c.}_l(q) = r_l^2 c_y'(r_l q) \ . \tag{4.12}$$

To determine t.o.c.(q), m.o.c.$(q) \equiv (d/dq)$t.o.c.(q) and m.o.c'(q) $\equiv (d/dq)$m.o.c.$(q) = (d^2/dq^2)$t.o.c.(q) we must solve the following cost minimization problem:

$$\begin{aligned} \max_{q_i, i=h,l} \quad & -\left\{ \sum_i c_i q_i + C_y \left(\sum_i r_i q_i \right) \right\} \\ s.t. \quad & \sum_i q_i - q \geq 0 \text{ and } q_i \geq 0 \ , \ i = h, l \ , \end{aligned}$$

whose Lagrangian is:

$$\mathcal{L} \ = \ -\left\{ \sum_i c_i q_i + C_y \left(\sum_i r_i q_i \right) \right\} + \gamma_q \left[\sum_i q_i - q \right] + \sum_i \gamma_i q_i \ .$$

The f.o.c's are:

$$\frac{\partial \mathcal{L}}{\partial q_i} = 0 \iff c_i + r_i c_y \left(\sum_j r_j q_j \right) = \gamma_q + \gamma_i \ , \ i, j = h, l \ , \tag{4.13}$$

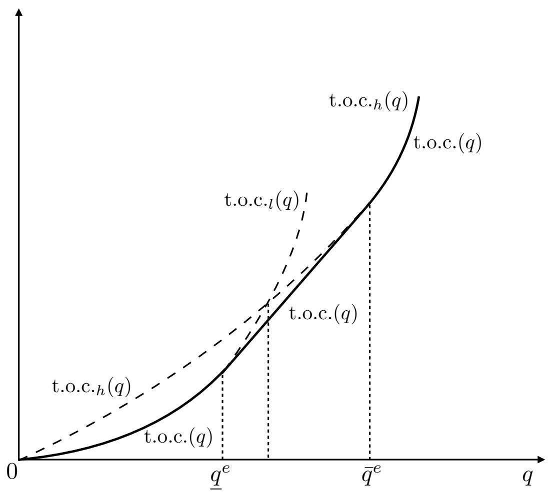

Figure 11: **UE industry total operating cost function t.o.c.**(q)**.**

together with the usual complementary slackness conditions.

The total operating cost function is illustrated on the Figure 11 and we now present how to build the cost function. Under the assumption $c_l + r_l \underline{c}_y < c_h + r_h \underline{c}_y$, for small production levels the cost of the technology l is lower than the cost of any combination of the two technologies or the cost of the technology h, that is:

$$\text{t.o.c.}(q) = \text{t.o.c.}_l(q) < \text{t.o.c.}(q_h, q - q_h)\ ,\ \ 0 < q_h \leq q\ . \tag{4.14}$$

In terms of the dual variables the marginal cost m.o.c.(q) is given by γ_q, hence by (4.13) and (4.14):

$$\text{m.o.c.}(q) = \gamma_q = c_l + r_l c_y(r_l q) = \text{m.o.c.}_l(q) = \text{m.o.c.}_l(0, q) \tag{4.15}$$

$$\text{m.o.c.}_h(0^+, q) = c_h + r_h c_y(r_l q) = \gamma_q + \gamma_l > c_l + r_l c_y(r_l q) = \text{m.o.c.}_l(0, q) \tag{4.16}$$

Differentiating (4.15) and denoting by m.o.c'(q) the derivative of m.o.c.(q) yields:

$$\text{m.o.c.}'(q) \equiv \frac{d}{dq}\text{m.o.c.}(q) = \frac{d\gamma_q}{dq} = r_l^2 c_y'(r_l q) > 0\ , \tag{4.17}$$

and differentiating (4.16) we get:

$$\text{m.o.c.}_{hl}(0^+,q) = r_h r_l c_y'(r_l q) < r_l^2 c_y'(r_l q) = \text{m.o.c}_{ll}(0,q) = \frac{d\gamma_q}{dq} \quad (4.18)$$

Thus as q and γ_q increase, the m.o.c.$_h(0^+,q) = c_h + r_h c_y(r_l q)$ also increases but, by (4.18), less than γ_q. Let $\underline{q}^e$ (e for 'equal' marginal costs) be the U.E. production rate at which $\gamma_q =$ m.o.c.$_h(0^+,q)$, hence m.o.c.$_h(0^+,q)$ =m.o.c.$_l(0,q)$, that is $c_h + r_h c_y(r_h.0 + r_l \underline{q}^e) = c_l + r_l c_y(r_h.0 + r_l c_l)$. The set of (q_h, q_l) dispatches along the two technologies for which the marginal costs are equal is defined by:

$$\text{m.o.c.}_h(q_h,q_l) = c_h + r_h c_y(r_h q_h + r_l q_l) = c_l + r_l c_y(r_h q_h + r_l q_l) = \text{m.o.c.}_l(q_h,q_l)$$

that is:

$$\frac{c_h - c_l}{r_l - r_h} \;=\; c_y(r_h q_h + r_l q_l)\ . \quad (4.19)$$

Let us denote by y^e the solution of $(c_h - c_l)/(r_l - r_h) = c_y(y)$. The locus of production mix (q_h, q_l) satisfying (4.19) is the line $q_l^e(q_h)$:

$$q_l^e(q_h) = \frac{1}{r_l} y^e - \frac{r_h}{r_l} q_h\ ,\ 0 \le q_h \le \frac{1}{r_h} y^e \quad (4.20)$$

Let us denote by $\bar{q}^e$ the production rate y^e/r_h. Thus the set of U.E. production rates for which both technologies must be operated is the open interval $(\underline{q}^e, \bar{q}^e)$. For U. E. production rates within the interval, the marginal operating cost is constant:

$$\text{m.o.c.}(q) = \gamma_q = c_h + r_h c_y(y^e) = c_l + r_l c_y(y^e)\ ,\ \underline{q}^e < q < \bar{q}^e \quad (4.21)$$

$$\frac{d\gamma_q}{dq} = 0\ ,\ \underline{q}^e < q < \bar{q}^e\ . \quad (4.22)$$

Clearly, for production rates $q > \bar{q}^e$ only the technology h must be operated:

$$\text{m.o.c.}(q) = c_h + r_h c_y(r_h q) = \gamma_q\ ,\ q > \bar{q}^e \quad (4.23)$$

$$\frac{d\gamma_q}{dq} = r_h^2 c_y(y) \quad q > \bar{q}^e \quad (4.24)$$

The function m.o.c.(q) is illustrated in Figure 12. The optimal mix of productions as a function of the U.E. production rate is illustrated in Figure 13. To sum up:

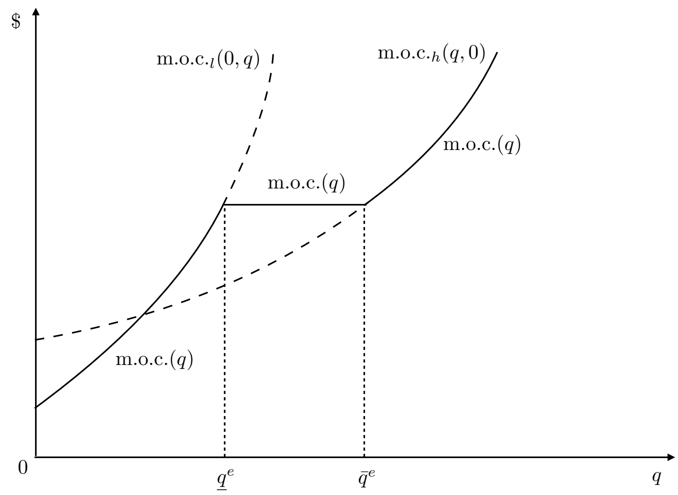

Figure 12: **Marginal cost function of the U.E. industry.**

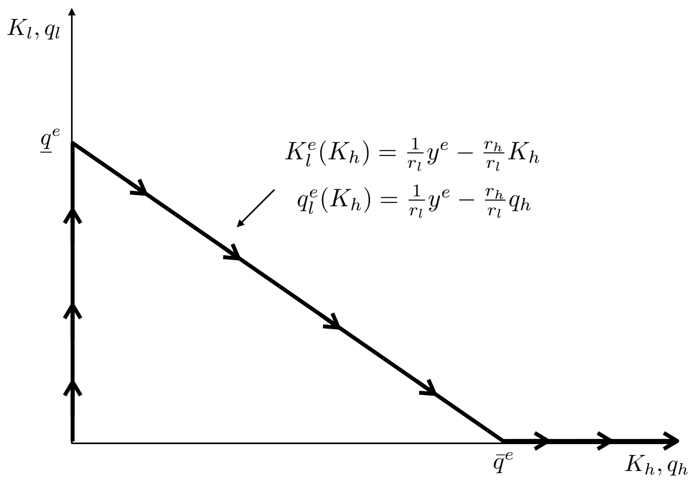

Figure 13: **Optimal dispatch as function of q.**

Proposition P. 1 • *As a function of q, the U.E. quantity to produce,* *the optimal dispatch is:*

$$\left.\begin{array}{lll} q_h = 0 & , \; q_l = q & , \; 0 < q \le \underline{q}^e = y^e/r_l \\[2ex] q_h = \dfrac{r_l[q - \underline{q}^e]}{r_l - r_h} & , \; q_l = \dfrac{r_l\underline{q}^e - r_h q}{r_l - r_h} & , \; \underline{q}^e < q < \bar{q}^e = y^e/r_h \\[2ex] q_h = q & , \; q_l = 0 & , \; \bar{q}^e < q \end{array}\right\} \quad (4.25)$$

- *As a function of q, the marginal operating cost is:*

$$\left.\begin{array}{llr} m.o.c.(q) & = c_l + r_l c_y(r_l q) & \\ & = m.o.c._l(0, q) & \\ and & m.o.c.'(q) = r_l^2 c_y'(r_l q_l) & 0 < q < \underline{q}^e \\ m.o.c.(q) & = c_l + r_l c_y(y^e) & \\ & = m.o.c._l\left(\dfrac{r_l[q - \underline{q}^e]}{r_l - r_h}, \dfrac{r_l\underline{q}^e - r_h q}{r_l - r_h}\right) & \\ & = m.o.c._h\left(\dfrac{r_l[q - \underline{q}^e]}{r_l - r_h}, \dfrac{r_l\underline{q}^e - r_h q}{r_l - r_h}\right) & \\ & = c_h + r_h c_y(y^e) & \\ and & m.o.c.'(q) = 0 & \underline{q}^e < q < \bar{q}^e \\ m.o.c.(q) & = c_h + r_h c_y(r_h q) & \\ & = m.o.c._h(q, 0) & \\ and & m.o.c.'(q) = r_h^2 c_y'(r_h q_h) & \bar{q}^e < q \end{array}\right\} \quad (4.26)$$

- *As a function of q, the total operating cost is:*

$$\left.\begin{aligned}
t.o.c.(q) &= t.o.c._l(q) = c_l q + C_y(r_l q) && 0 < q < \underline{q}^e \\
t.o.c.(q) &= t.o.c._l(\underline{q}^e) + [c_l + r_l c_y(y^e)](q - \underline{q}^e) && \\
&= t.o.c._l(\underline{q}^e) + [c_h + r_h c_y(y^e)](q - \underline{q}^e) && \underline{q}^e < q < \bar{q}^e \\
t.o.c.(q) &= t.o.c._h(q) = c_h q + C_y(r_h q) && \bar{q}^e < q
\end{aligned}\right\} \quad (4.27)$$

To determine what has to be initially scrapped we must take into account the U.E. demand. We distinguish according to the demand be 'small', 'medium' or 'big'. We denote respectively by q^s, q^m and q^b the corresponding optimal U.E. production levels absent any building cost.

- By 'small' demand we mean that $u(q)$ is such that:

$$u'(q^s) = c_l + r_l c_y(r_l q^s) \quad \text{and} \quad q^s < \underline{q}^e \ .$$

- by 'medium' demand we mean that $u(q)$ is such that:

$$u'(q^m) = c_l + r_l c_y(y^e) = c_h + r_h c_y(y^e) \quad \text{and} \quad \underline{q}^e < q^m < \bar{q}^e \ .$$

- by 'big' demand we mean that $u(q)$ is such that:

$$u'(q^b) = c_h + r_h c_y(r_h q^b) \quad \text{and} \quad \bar{q}^e < q^b \ .$$

The 3 possibilities are illustrated in the Figure 14.

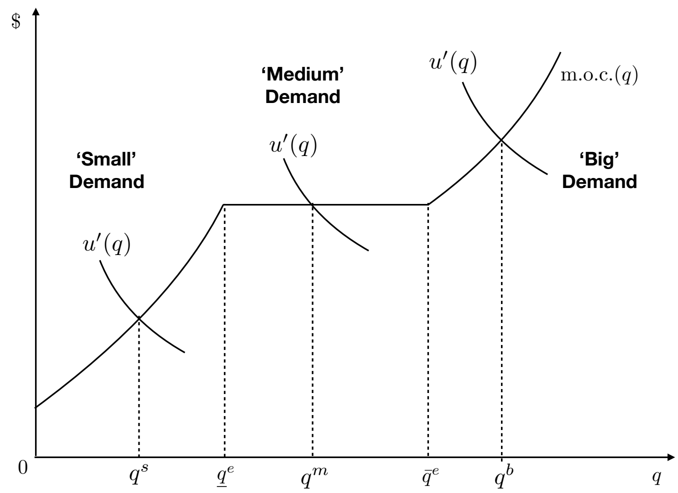

Figure 14: **Types of demand.**

4.1.2 The 'small' demand case

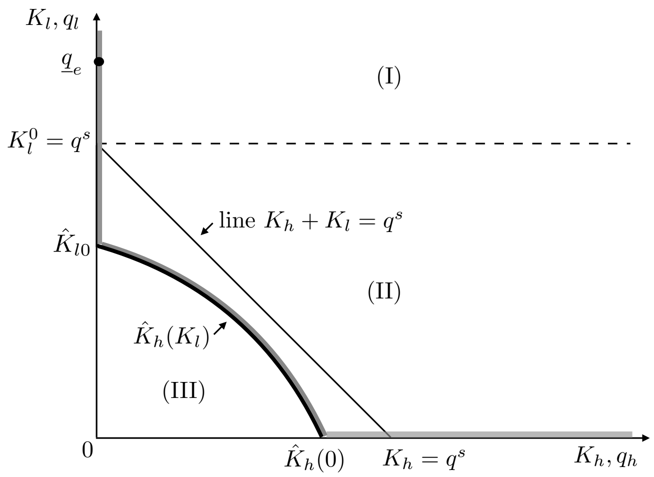

Figure 15: **Initial scrapping: the 'small' demand case.**

The scrapping frontier in this case is illustrated in the Figure 15. Clearly for any (K_h^0, K_l^0) such that $K_l^0 > q^s$ and $K_h^0 > 0$, like in the zone (I), then:

- This part $K_l^0 - q^s > 0$ of the initial endowment in capacity l;
- The whole initial endowment K_h^0 in capacity h,

have to be scrapped. The problem arises with initial endowments (K_h^0, K_l^0) : $K_l^0 < q^s$ and $K_h^0 > 0$, like in the zones (II) or (III) in the Figure 15. To determine what to do with K_h^0 if $K_l^0 < q^s$, let us consider the following auxiliary problem $(A.P.2)$ since clearly, no part of K_l^0 has to be scrapped.

$$\begin{array}{ll} (A.P.2) & \\ \max\limits_{K_h} & u\left(K_h + K_l^0\right) - c_h K_h - C_y\left(r_h K_h + r_l K_l^0\right) \\ s.t. & K_h \geq 0 \end{array}$$

The f.o.c's are:

$$u'\left(K_h + K_l^0\right) = c_h + r_h c_y\left(r_h K_h + r_l K_l^0\right) - \gamma_h \tag{4.28}$$

$$\gamma_h \geq 0 \ , \ K_h \geq 0 \ \text{ and } \gamma_h K_h = 0 \ . \tag{4.29}$$

We denote by $\hat{K}_h(K_l^0)$ the solution of $(A.P.2)$ as a function of K_l^0, a parameter in this problem. Assuming that $\hat{K}_y(K_l^0) > 0$ and differentiating (4.28) with $\gamma_h = 0$, we obtain:

$$\frac{d\hat{K}_h}{dK_l^0} = -\frac{u'' - r_h r_l c_y'}{u'' - r_h^2 c_y'} < 0 \implies \left|\frac{d\hat{K}_h}{dK_l^0}\right| > 1 \ . \tag{4.30}$$

Note that we have $\hat{K}_h(0) < q^s$, as illustrated in the Figure 16, because for $K_l^0 = 0$, (4.28) reads:

$$u'(K_h) \quad = \quad \text{m.o.c.}_h(K_h, 0) = c_h + r_h c_y(r_h K_h) \ .$$

Both $|d\hat{K}_h/dK_l^0| > 1$ and $\hat{K}_h(0) < q_s$ imply that $\hat{K}_h(K_l^0)$ is nil for some $K_l^0 < q^s$ as illustrated in Figure 15. Let us denote by $\hat{K}_{l0}$ this level of K_l^0:

$$\hat{K}_{l0} \quad = \quad \min\left\{K_l : \hat{K}_h(K_l) = 0\right\} \ . \tag{4.31}$$

To save trivialities, we assume that in the case of a 'small' demand, the initial endowment (K_h^0, K_l^0) satisfies:

- Either $K_h^0 \leq \hat{K}_h(K_l^0)$ and $K_l^0 \leq \hat{K}_{l0}$, the zone (III) in Figure 15;
- Or $K_h^0 = \hat{K}_h(K_l^0) = 0$ and $\hat{K}_{l0} < K_l^0 \leq K^s \equiv q^s$.

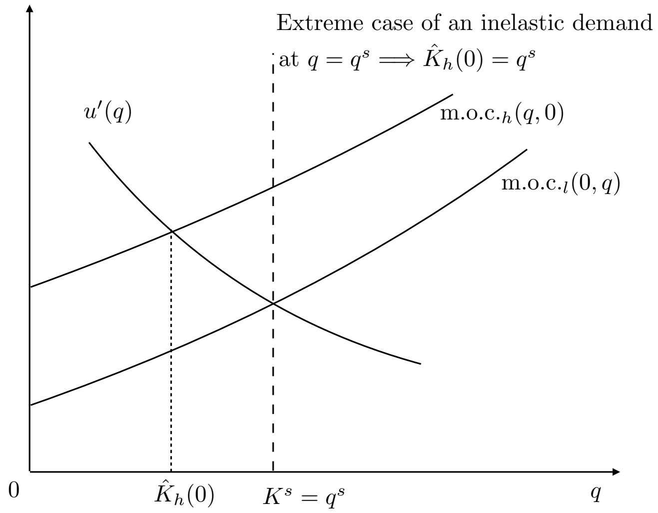

Figure 16: **Determination of $\hat{K}_h(0)$.**

4.1.3 The 'medium' demand case

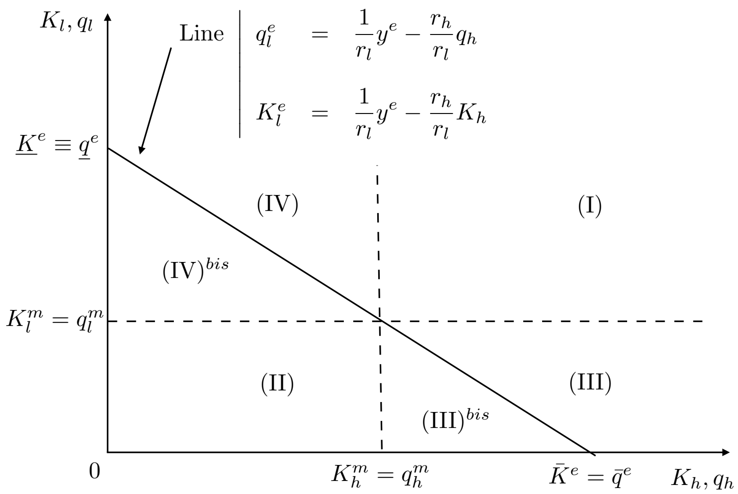

Figure 17: **The 'medium' demand case.**

This case is illustrated in Figure 17. In Figure 17 the optimal dispatch of

q^m, (q_h^m, q_l^m), $q_h^m + q_l^m = q^m$, is given by (4.25) with $q = q^m$:

$$q_h^m = \frac{r_l\left(q^m - \underline{q}^e\right)}{r_l - r_h} \quad \text{and} \quad q_l^m = \frac{r_l \underline{q}^e - r_h q^m}{r_l - r_h} \ .$$

Clearly, for initial endowments (K_l^0, K_h^0) : $K_h^0 > K_h^m$ and $K_l^0 > K_l^m$, like (I) in Figure 17, the best is to scrap initially this part $K_h^0 - K_h^m$ of the capacity h and this part $K_l^0 - K_l^m$ of the capacity l. Also for initial endowments (K_h^0, K_l^0) : $K_h^0 < K_h^m$ and $K_l^0 < K_l^m$, like (II) in Figure 17, the best is to maintain both K_h^0 and K_l^0. The problem arises with endowments like (III) or $(III)^{bis}$, where $K_h^0 > K_h^m$ but $K_l^0 < K_l^m$, or like (IV) or $(IV)^{bis}$, where $K_h^0 < K_h^m$ but $K_l^0 > K_l^m$.

4.1.3.1. *Case $K_h^0 > K_h^m$ and $K_l^0 < K_l^m$*

Since $K_l^0 < K_l^m$, K_l^0 must be maintained, the problem is what to do with K_h^0? Consider again the auxiliary problem $(A.P.2)$ and its solution $\hat{K}_h(K_l^0)$, $0 \leq K_l^0 \leq K_l^m$. Once more we have:

$$\hat{K}_h(K_l^0) > 0 \implies \frac{d\hat{K}_h}{dK_l^0} = -\frac{u'' - r_h r_l c_y'}{u'' - r_h^2 c_y'} < 0 \text{ and } \left|\frac{d\hat{K}_h}{dK_l^0}\right| > 1 \ .$$

Note that for $K_l^0 = 0$, (4.28) reads: $u'(K_h) = \text{m.o.c.}_h(K_h, 0) = c_h + r_h c_y(r_h K_h)$. Hence, as illustrated in Figure 18:

$$\hat{K}_h(0) < \bar{K}^e \equiv \bar{q}^e \quad \text{and} \quad \hat{K}_h(0) > q^m = K_h^m + K_l^m \ ,$$

Next clearly: $\hat{K}_h(K_l^m) = K_h^m$. The reason is that, by definition, (K_h^m, K_l^m) satisfies:

$$u'(K_h^m + K_l^m) \ = \ u'(q^m) = c_h + r_h c_y \left(r_h K_h^m + r_l K_l^m\right) \ ,$$

that is the f.o.c. (4.28) for $K_l^0 = K_l^m$. Last easy calculations show that:

$$1 < \left|\frac{d\hat{K}_h}{dK_l^0}\right| \ < \ \frac{r_l}{r_h} \ .$$

The frontier is illustrated in Figure 19.

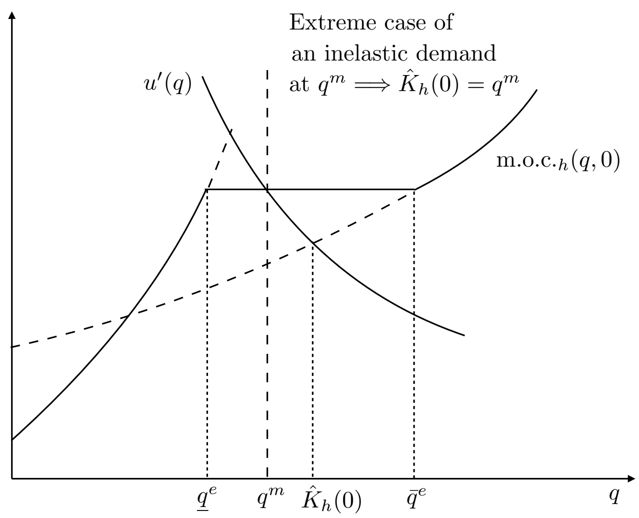

Figure 18: **Determination of $\hat{K}_h(0)$. The 'medium' demand case.**

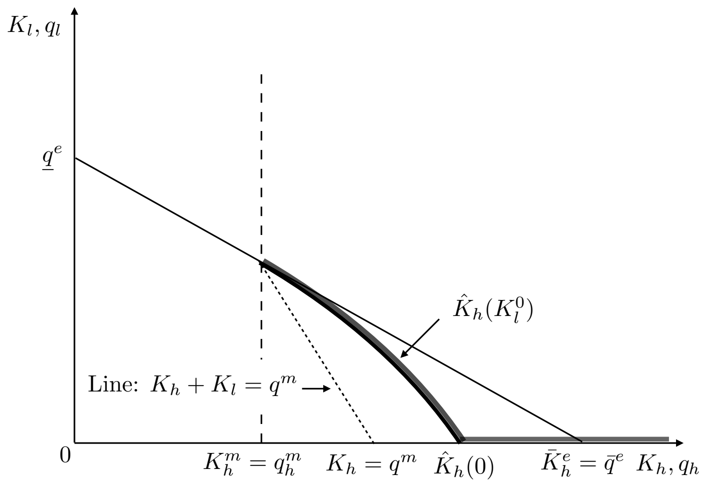

Figure 19: **The $\hat{K}_h(K_l^0)$ frontier.**

4.1.3.2. *Case* $K_h^0 < K_h^m$ *and* $K_l^0 > K_l^m$.

Clearly since $K_h^0 < K_h^m$, K_h^0 must be maintained. The problem is what to do with K_l^0? The argument is similar to the argument of the preceding paragraph: Solve the following auxiliary problem $(A.P.3)$ with $K_h^0 < K_h^m$.

$$
\begin{array}{ll}
(A.P.3) & \\
\max\limits_{K_l} & u\left(K_h^0 + K_l\right) - c_l K_l - C_y\left(r_h K_h^0 + r_l K_l\right) \\
s.t. & K_l \geq 0 \ .
\end{array}
$$

The f.o.c's are:

$$
u'(K_h^0 + K_l) = c_l + r_l c_y\left(r_h K_h^0 + r_l K_l\right) \tag{4.32}
$$

$$
\gamma_l \geq 0 \ , \ K_l \geq 0 \ \text{ and } \gamma_l K_l = 0 \ . \tag{4.33}
$$

We denote $\hat{K}_l(K_h^0)$ the solution of $(A.P.3)$ as a function of K_h^0, a parameter in this problem. Assuming that $K_h > 0$ and differentiating (4.32) with $\gamma_l = 0$, we obtain:

$$
\frac{d\hat{K}_l}{dK_h^0} = -\frac{u'' - r_h r_l c_y'}{u'' - r_l^2 c_y'} < 0 \implies \left|\frac{d\hat{K}_l}{dK_h^0}\right| < 1 \ .
$$

Note that for $K_h^0 = 0$, (4.32) reads: $u'(K_l) = \text{m.o.c.}_l(0, K_l) = c_l + r_l c_y(r_l K_l)$, hence: $\hat{K}_l(0) > \underline{K}^e \equiv \underline{q}^e$ and $\hat{K}_l(0) < q^m = K_h^m + K_l$, as illustrated in the Figure 20.

Next clearly: $\hat{K}_l(K_h^m) = K_l^m$. The reason is similar to the reason for which $\hat{K}_h(K_l^m) = K_h^m$. Thus the curves $\hat{K}_l(.)$ and $\hat{K}_h(.)$ cross themselves at the point (K_h^m, K_l^m) in the plane (K_h, K_l). Last easy calculations show that:

$$
1 > \left|\frac{d\hat{K}_l}{dK_h^0}\right| > \frac{r_h}{r_l} \ .
$$

Putting together $\hat{K}_l(K_h^0)$ and $\hat{K}_h(K_l^0)$ we obtain the frontier illustrated in the Figure 21. To save trivialities we assume that the initial endowments (K_h^0, K_l^0) satisfy:

- Either $K_h^0 \leq K_h^m$ and $K_l^0 \leq \hat{K}_l(K_h^0)$;
- Or either $K_l^0 \leq K_l^m$ and $K_h^0 \leq \hat{K}_h(K_l^0)$.

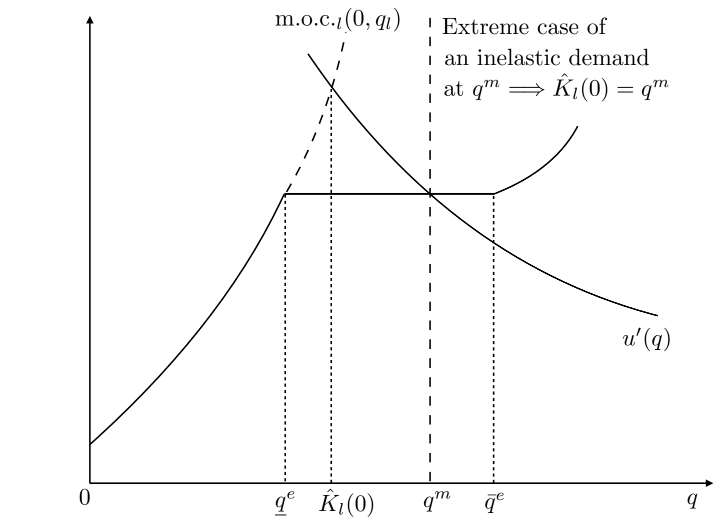

Figure 20: **Determination of $\hat{K}_l(0)$.**

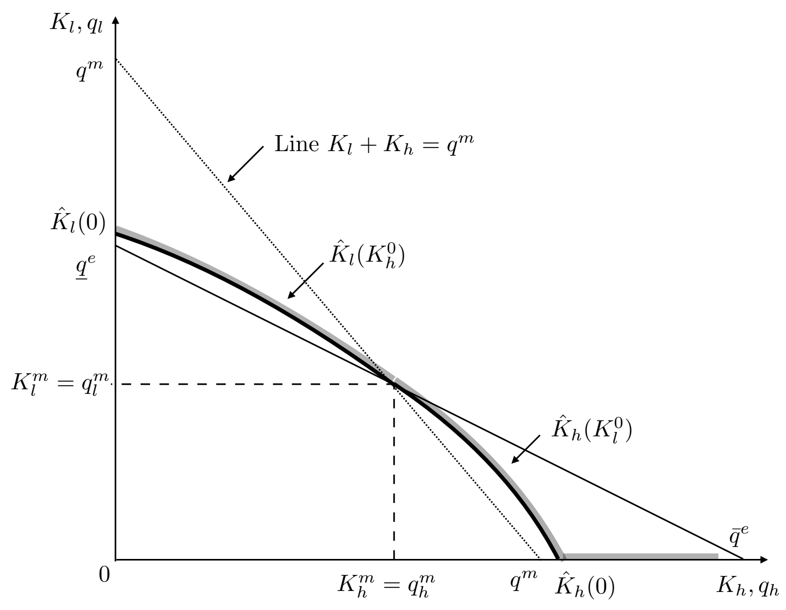

Figure 21: **Initial scrapping frontier: The 'medium' demand case.**

4.1.4 The 'big' demand case

This case is illustrated in the Figure 22. The UE production level $q^b > \bar{q}^e$ is dispatched as follows: $q_l^b = 0$ and $q_h^b \equiv K_h^b = q^b$. For initial endowments

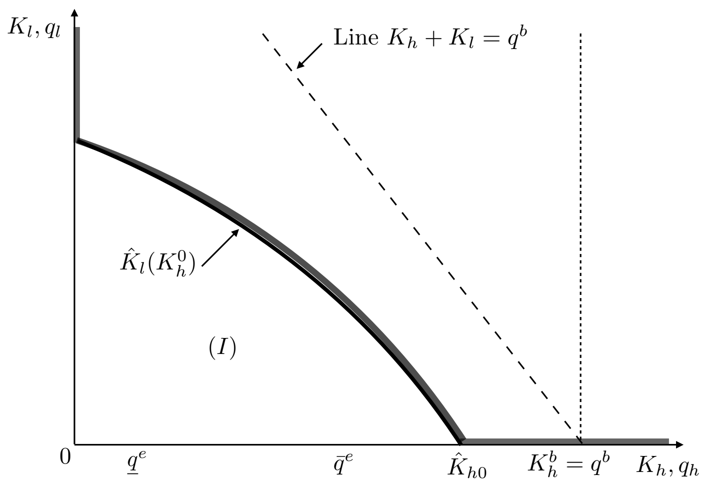

Figure 22: **Initial scrapping frontier: The 'big' demand case.**

(K_h^0, K_l^0) : $K_h^0 > K_h^b$ and $K_l^0 \geq 0$, the best is to scrap initially this part $K_l^0 - K_h^b$ of the capacity h and the whole endowment in capacity l, K_l^0. The problem arises with endowments (K_h^0, K_l^0) : $K_h^0 < K_h^b$ and $K_l^0 < K_l^b \equiv q_b - K_h^b$, like the endowments in the zone (I) of the Figure 22.

Clearly K_h^0 must be maintained. It remains to determine what to do with K_l^0. We have to resort to the problem $(A.P.3)$. Denoting $\hat{K}_l(K_h^0)$ its solution as a function of K_h^0 we have, as shown previously:

$$\frac{d\hat{K}_l}{dK_h^0} = -\frac{u'' - r_h r_l c_y'}{u'' - r_l^2 c_y'} < 0 \implies \left|\frac{d\hat{K}_l}{dK_h^0}\right| < 1 \ .$$

As illustrated in the Figure 23:

$$\hat{K}_l(0) \quad < \quad q^b = K_h^b$$

Note that there exists $\hat{K}_{h0} < K_h^b$ such that $\hat{K}_l(K_h^0) = 0$ for $K_h^0 \geq \hat{K}_{h0}$, more precisely:

$$\hat{K}_{h0} \quad = \quad \min\left\{K_h^0 : \hat{K}_l(K_h^0) = 0\right\} \ . \tag{4.34}$$

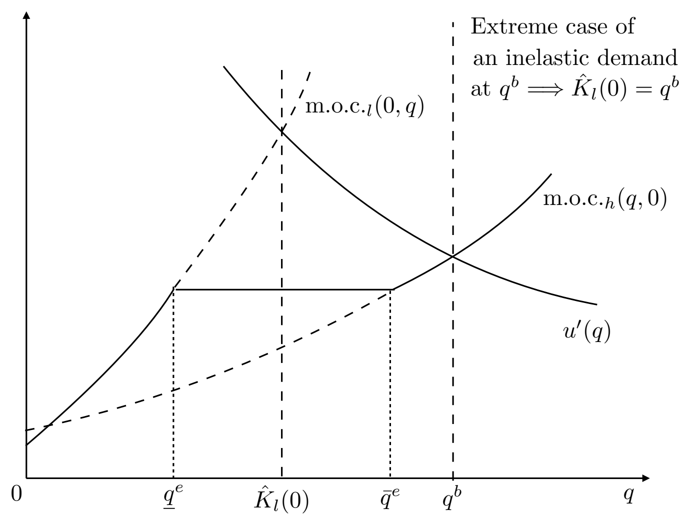

Figure 23: **Determination of $\hat{K}_l(0)$.**

The reason is that for $K_h^0 > 0$, m.o.c.$_l(K_h^0, q_l) >$ m.o.c.$_h(K_h^0 + q_l, 0)$, $q_l > 0$. More precisely for $q_l > 0$, there exists $\Delta K_h > 0$ and Δm.o.c. > 0, such that for $K_h^0 \in \left(K_h^b - \Delta K_h, K_h^b\right)$, m.o.c.$_l(K_h^0, q_l) >$ m.o.c.$_h(K_h^0 + q_l, 0) + \Delta$m.o.c.. This implies that m.o.c.$_l(K_h^0, q_l) > u'\left(K_h^0 + q_l\right)$, hence $\hat{K}_l(K_h^0) = 0$ as illustrated in the Figure 24.

This is itself a consequence of the fact that, for $K_h^0 = \bar{q}^e$, then (see Figure 25):

$$\begin{aligned} \text{m.o.c.}_l(K_h^0, 0^+) &= \text{m.o.c.}_l(\bar{q}^e, 0) = c_l + r_l c_y(r_h \bar{q}^e) = c_l + r_l c_y(y^e) \\ &= c_h + r_h c_y(y^e) = c_h + r_h c_y(r_h \bar{q}^e) = c_h + r_h c_y(r_h K_h^0) \\ &= \text{m.o.c.}_h(K_h^0, 0) \ . \end{aligned}$$

Since $r_l > r_h$ and $c_y(y)$ is increasing, then for $K_h^0 = \bar{q}^e + \Delta K$, $\Delta K > 0$, $y = r_h K_h^0 > r_h \bar{q}^e = y^e$, hence (see Figure 24):

$$\begin{aligned} \text{m.o.c.}_l(K_h^0, 0^+) &= c_l + r_l c_y\left(r_h\left[\bar{q}^e + \Delta K\right]\right) \\ &> c_h + r_h c_y\left(r_h\left[\bar{q}^e + \Delta K\right]\right) = \text{m.o.c.}_h\left(K_h^0, 0\right) \ ; \end{aligned}$$

so that, for the same K_h^0 and $q_l > 0$, then:

$$r_h\left[\bar{q}^e + \Delta K\right] + r_l q_l \ > \ r_h\left[\bar{q}^e + \Delta K + q_l\right] \ ,$$

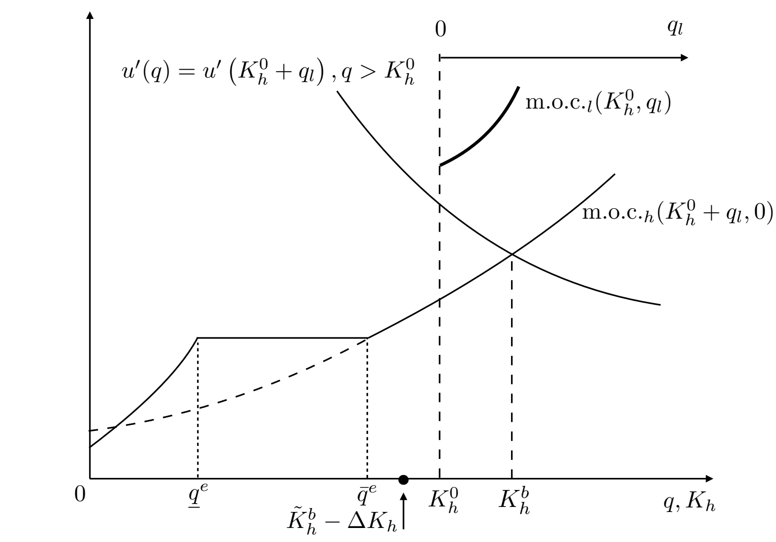

Figure 24: **Determination of $\hat{K}_{h0}$.**

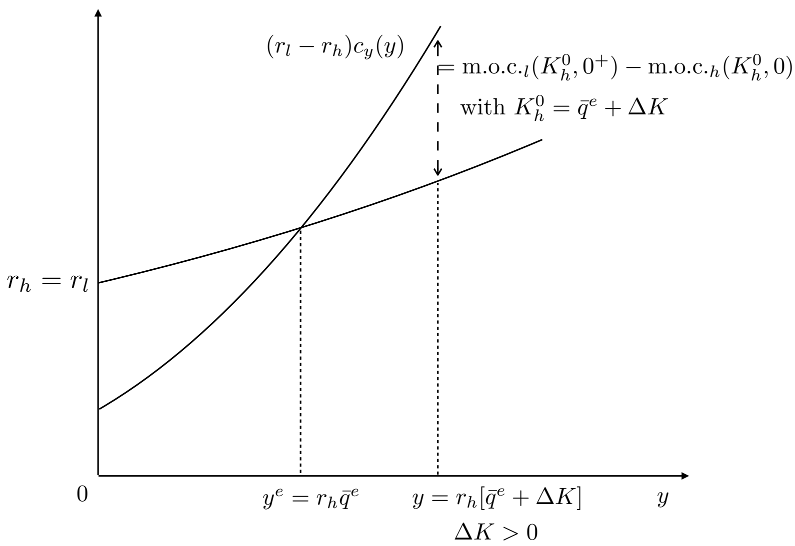

Figure 25: $m.o.c._l(K_k^0, q_l) - m.o.c._h(K_h^0 + q_l, 0) > 0$**,** $q_l \geq 0$**.**

hence:

$$
\begin{aligned}
\text{m.o.c.}_l(K_h^0, q_l) &= c_l + r_l c_y\left(r_h\left[\bar{q}^e + \Delta K\right] + r_l q_l\right) \\
&> c_h + r_h c_y\left(r_h\left[\bar{q}^e + \Delta K + q_l\right]\right) \\
&= \text{m.o.c.}_h\left(K_h^0 + q_l, 0\right)\ ,\ q_l > 0\ .
\end{aligned}
$$

To save trivialities we assume, in the 'big' demand case, that the initial endowments, (K_h^0, K_l^0) satisfy:

- Either $K_h^0 \leq K_{h,\min}^0$ and $K_l^0 \leq \hat{K}_l(K_h^0)$;
- Or either $K_{h,\min}^0 < K_h^0 \leq K_h^b = q^b$ and $K_l^0 = 0$.

4.2 The social planner problem $(S.P.2)$

An optimal plan is a path $\{(q_i(t), k_i(t), \delta_i(t)), i = h, l\}_0^\infty$ solving the following problem $(S.P.2)$:

$(S.P.2)$

$$\begin{aligned}
\max_{\{q_i(t),k_i(t),\delta_i(t),i=h,l\}} \quad & \int_0^\infty \left\{ u\left(\sum_{i=h,l} q_i(t)\right) - \sum_{i=h,l} [a_i q_i(t) + m_i K_i(t) + N_i(k_i(t))] \right. \\
& \left. -C_y\left(\sum_{i=h,l} r_i q_i(t)\right)\right\} e^{-\rho t} dt \\
s.t. \quad & \dot{K}_i(t) = k_i(t) - \delta_i(t) K_i(t) \ , \ K_i(0) = K_i^0 \geq 0 \ , \ K_i(t) \geq 0 \\
& K_i(t) - q_i(t) \geq 0 \ , q_i(t) \geq 0 \ , \ k_i(t) \geq 0 \ \text{ and } \ \delta_i(t) \geq 0 \\
& i = h, l \ .
\end{aligned}$$

Keeping the same notations as in the sub-section 3.2 for the dual variables, the current value Hamiltonian and Lagrangian of the problem $(S.P.2)$ read:

$$\begin{aligned}
\mathcal{H} &= u\left(\sum_{i=h,l} q_i\right) - \sum_{i=h,l} [a_i q_i + m_i K_i + N_i(k_i)] - C_y\left(\sum_{i=h,l} r_i q_i\right) \\
& \quad + \sum_{i=h,l} \lambda_i [k_i - \delta_i K_i] \\
\mathcal{L} &= \mathcal{H} + \sum_{i=h,l} [\nu_i K_i + \eta_i [K_i - q_i] + \gamma_{iq} q_i + \gamma_{ik} k_i + \gamma_{i\delta} \delta_i] \ .
\end{aligned}$$

The following f.o.c's result:

$$\frac{\partial \mathcal{L}}{\partial q_i} = 0 \iff u'\left(\sum_{j=h,l} q_j\right) = a_i + r_i c_y \left(\sum_{j=h,l} r_j q_j\right) + \eta_i - \gamma_{iq} \ , \ i = h,l \tag{4.35}$$

$$\frac{\partial \mathcal{L}}{\partial k_i} = 0 \iff n_i(k_i) = \lambda_i + \gamma_{ik} \quad , \ i = h,l \tag{4.36}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_i} = 0 \iff \lambda_i K_i = \gamma_{i\delta} \quad , \ i = h,l \ , \tag{4.37}$$

together with the usual complementary slackness conditions. When time differentiable, the dynamics of the co-state variables must satisfy:

$$\dot{\lambda}_i = \rho\lambda_i - \frac{\partial \mathcal{L}}{\partial K_i} \iff \dot{\lambda}_i = (\rho + \delta_i)\lambda_i + m_i - \eta_i - \nu_i \ , \ i = h,l \tag{4.38}$$

together with:

$$\nu_i \geq 0 \ , \ K_i \geq 0 \text{ and } \nu_i K_i = 0 \ . \tag{4.39}$$

Last, the transversality condition at infinity is:

$$\lim_{t\uparrow\infty} e^{-\rho t} \sum_{i=h,l} \lambda_i(t) K_i(t) \ = \ 0 \ . \tag{4.40}$$

Like in the case of a costless primary resource it will prove useful to derive alternative expressions of the dynamics of the co-state variables.

For $q_i > 0$ then $\gamma_{iq} = 0$ and from (4.35):

$$\begin{aligned} \eta_i &= u'(q_h + q_l) - [a_i + r_i c_y (r_h q_h + r_l q_l)] \\ &= p - [a_i + r_i c_y (r_h q_h + r_l q_l)] \ , i = h,l \ . \end{aligned} \tag{4.41}$$

Thus η_i is the gross operating margin of the technology i, $\eta_i(q_h, q_l) = \text{g.o.m.}_i(q_h, q_l)$, where the primary resource is paid its marginal cost $c_y(r_h q_h + r_l q_l)$.

The net operational margin, $\text{n.o.m.}_i(q_h, q_l)$, which takes into account the marginal maintenance cost of capital is:

$$\begin{aligned} \text{n.o.m.}_i(q_h, q_l) &\equiv \eta_i(q_h, q_l) - m_i \\ &= u'(q_h + q_l) - [a_i + m_i + r_i c_y(r_h q_h + r_l q_l)] \\ &= u'(q_h + q_l) - [c_i + r_i c_y(r_h q_h + r_l q_l)] \\ &= p - [c_i + r_i c_y(r_h q_h + r_l q_l)] = p - \text{m.o.c.}_i(q_h, q_l) \end{aligned} \tag{4.42}$$

Substituting for η_i in (4.38) with $\nu_i = 0$ since $K_i \geq q_i > 0$, we obtain:

$$\begin{aligned}\dot{\lambda}_i &= (\rho + \delta_i)\lambda_i - \{u'(q_h + q_l) - [c_i + r_i c_y(r_h q_h + r_l q_l)]\} \\ &= (\rho + \delta_i)\lambda_i - \{p - \text{m.o.c.}_i(q_h, q_l)\} \quad i = h, l \ . \end{aligned} \tag{4.43}$$

If furthermore $\delta_i = 0$ then:

$$\begin{aligned}\dot{\lambda}_i &= \rho\lambda_i - \{u'(q_h + q_l) - [c_i + r_i c_y(r_h q_h + r_l q_l)]\} \\ &= \rho\lambda_i - \{p - \text{m.o.c.}_i(q_h, q_l)\} \quad i = h, l \ . \end{aligned} \tag{4.44}$$

4.3 General properties of optimal investment plans

Under the assumptions made in 4.1.2-4.1.4 on the initial endowments with respect to the demand type, an optimal investment plan is a sequence of the following elementary tails.

- *Simultaneous investment phase.* $k_i(t) > 0$, $i = h, l$, during such a time phase and $\lambda_i(t) > \underline{n}_i$. The industry accumulates both types of equipments and the useful energy price decreases.

- *One sector investment phases without scrapping.* Either the industry accumulates only low performing equipments, maintaining the stock of high performing ones, $k_l(t) > 0$, $\dot{K}_h(t) = 0$, $\lambda_l(t) > \underline{n}_l$ and $0 < \lambda_h(t) < \underline{n}_h$ during such a time phase, either it accumulates only high performing equipments, maintaining its stock of low performing ones, $k_h(t) > 0$, $\dot{K}_l(t) = 0$, $\lambda_h(t) > \underline{n}_h$ and $0 < \lambda_l(t) < \underline{n}_l$ during such a time phase. The useful energy price decreases throughout the both types of time phases.

- *One sector investment phase with scrapping.* Either the industry accumulates low performing equipments while scrapping high performing ones. Such a policy corresponds to a move along the scrapping border in the (K_h, K_l) plane. During such a time phase, $\lambda_l(t) > \underline{n}_l$ and $\lambda_h(t) = 0$. Denoting by $\hat{K}_h(K_l)$ the implicit equation of the scrapping border, $\dot{K}_h(t) = (d\hat{K}_h/dK_l)k_l(t)$ defines the scrapping rate of high performing equipments as:

$$\delta_h(t) = -\frac{d\hat{K}_h}{dK_l}\frac{k_l(t)}{K_h(t)} \ .$$

Either the industry accumulates high performing equipments while scrapping low performing ones. During such a time phase, $\lambda_h(t) > \underline{n}_h$ and $\lambda_l(t) = 0$. Denoting by $\hat{K}_l(K_h)$ the equation defining the scrapping border in the (K_l, K_h) plane, $\dot{K}_l(t) = (d\hat{K}_l/dK_h)k_h(t)$ and the scrapping rate of low performing equipments is given by:

$$\delta_l(t) \quad = \quad -\frac{d\hat{K}_l}{dK_h}\frac{k_h(t)}{K_l(t)} \; .$$

We now show that the net operational margin on type i equipments, n.o.m$_i(t)$, $i = h, l$, a net margin that we shall denote by $\beta_i(t)$, is a time decreasing function throughout any type of phase.

During a simultaneous investment phase or during a one sector active investment phase without scrapping, the useful energy price being time decreasing while the marginal energy cost being time increasing, it is immediate that the net operational margin should be time decreasing. Thus only remain to be considered the two possible types of scrapping phases. During a scrapping phase of high performing equipments, $\beta_h(t) = 0$ and time differentiating $\beta_l(t)$ yields:

$$\dot{\beta}_l(t) \quad = \quad \left\{ \left[u'' - r_l^2 c_y'\right] + \left[u'' - r_l r_h c_y'\right] \frac{d\hat{K}_h}{dK_l} \right\} k_l(t)$$

Our previous analysis has shown that such a scrapping phase is only possible in the 'small' demand case or in the 'medium' demand case when $K_h > K_h^m$ and $K_l < K_l^m$ at the beginning of the phase. In both cases $d\hat{K}_h/dK_l = -(u'' - r_h r_l c_y')/(u'' - r_h^2 c_y')$, so that:

$$\dot{\beta}_l(t) \quad = \quad \frac{k_l(t)}{u'' - r_h^2 c_y'} \left[(u'' - r_l^2 c_y')(u'' - r_h^2 c_y') - \left(u'' - r_l r_h c_y'\right)^2 \right] \; .$$

The sign of $\dot{\beta}_l(t)$ is the opposite of the sign of the expression into brackets, that is of:

$$\begin{aligned}
& \left(r_h^2 c_y' + |u''|\right)\left(r_l^2 c_y' + |u''|\right) - \left(r_l r_h c_y' + |u''|\right)^2 \\
= \; & \left(r_l r_h c_y'\right)^2 + r_h^2 c_y' |u''| + r_l^2 c_y' |u''| + |u''|^2 - \left(r_l r_h c_y'\right)^2 \\
& -2 r_l r_h c_y' |u''| - |u''|^2 \\
= \; & \left(r_h^2 + r_l^2 - 2 r_l r_h\right) c_y' |u''| = \left(r_l - r_h\right)^2 c_y' |u''| > 0 \; .
\end{aligned}$$

Hence $\beta_h(t) = 0$ and $\dot{\beta}_l(t) < 0$ during a scrapping phase of high performing equipments.

During a scrapping phase of low performing equipments, $\beta_l(t) = 0$ and:

$$\dot{\beta}_h(t) = \left\{ \left[u'' - r_l r_h c'_y\right] \frac{d\hat{K}_l}{dK_h} + \left[u'' - r_h^2 c'_y\right] \right\} k_h(t) \ .$$

Such a time phase is only possible in the 'medium' demand case if $K_h < K_h^m$ and $K_l > K_l^m$ at the beginning of the phase or in the 'big' demand case. In all cases $d\hat{K}_l/dK_h = -(u'' - r_h r_l c'_y)/(u'' - r_l^2 c'_y)$, and:

$$\dot{\beta}_h(t) = \frac{k_h(t)}{u'' - r_l^2 c'_y} \left[-(u'' - r_l r_h c'_y)^2 + (u'' - r_h^2 c'_y)(u'' - r_l^2 c'_y)\right] < 0 \ ,$$

from our previous computations.

Note that during a scrapping phase of high performing equipments replaced by low performing ones, $|d\hat{K}_h/dK_l| > 1$ implies that $\dot{K}(t) = k_l(t)(1 - |d\hat{K}_h/dK_l|) < 0$, thus $\dot{q}(t) < 0$ and $\dot{p}(t) > 0$, the useful energy price should rise throughout a scrapping phase of high performing equipments.

The net operational margin $\beta_i(t)$ being time decreasing, if at some time $\bar{t}$, $\rho\lambda_i(\bar{t}) > \beta_i(\bar{t})$, so that $\dot{\lambda}_i(\bar{t}) > 0$, then $\dot{\lambda}_i(t) > 0$, $\forall t \geq \bar{t}$. This would imply that $k_i(t)$ should increase, thus $K_i(t)$ should converge toward the scrapping border, implying in turn that $\lambda_i(t)$ should converge to zero, a contradiction. Thus we can conclude that $\dot{\lambda}_i(t) < 0$ if $\lambda_i(t) > 0$.

Proposition P. 2 *Throughout any investment phase, with or without scrapping of equipments, the net operational margin on both types of equipments decreases over time while positive, $\dot{\beta}_i(t) < 0$ if $\beta_i(t) > 0$, $i = h, l$. The same applies to the marginal value of an investment if positive, $\dot{\lambda}_i(t) < 0$ if $\lambda_i(t) > 0$, $i = h, l$.*

4.4 The dual plane (λ_h, λ_l)

We can apply the results of the Proposition 2 to describe the optimal sequence of phases in the dual plane (λ_h, λ_l) (see Figure 26).

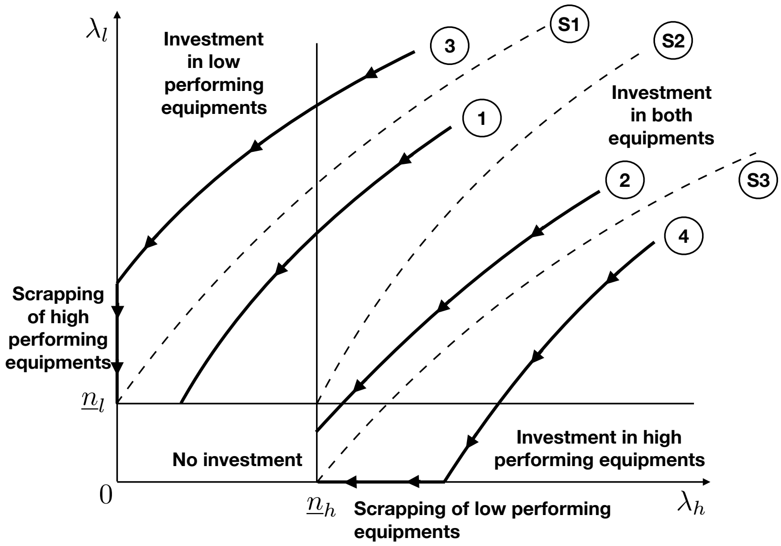

Figure 26: **The dual plane** (λ_h, λ_l)**.**

Above the separating curve $(S1)$, the dual variables paths correspond to a sequence of at most three time phases: a first phase of simultaneous investment until the vertical border $\underline{n}_h$ is attained, a phase of investment in only low equipments until $\lambda_h = 0$, last a scrapping phase of high performing equipments corresponding to a move along the vertical axis. In-between the separating curves $(S1)$ and $(S2)$, the dual variables paths correspond to investment plans composed of at most two phases, a phase of simultaneous investment followed by a phase of investment in low performing equipments. In-between the separating curves $(S2)$ and $(S3)$, the dual paths correspond to a sequence of a simultaneous investment phase followed by an investment phase in high performing equipments only. Below the separating curve $(S3)$, the dual variables trajectories correspond to investment plans composed of at most three phases: a phase of simultaneous investment, a phase of investment in high performing equipments and a scrapping phase of low performing equipments.

Because of the time continuity of the dual variables, the industry cannot switch directly from a simultaneous investment phase to a scrapping phase, investment phases in only one type of equipments occurring either initially or immediately after the end of a simultaneous investment phase. Note that scrapping phases can result either in an incomplete scrapping or a complete scrapping of the equipments stocks, in this last case the industry can continue to invest in the type of equipments that have not been scrapped before.

4.5 The 'small' demand case

We first examine the dynamics of the dual variables in the plane (λ_h, λ_l), describing the different scenarios of evolution of these variables. We next study their counterparts in the primal space (K_h, K_l).

4.5.1 Optimal dynamics in the dual plane (λ_h, λ_l)

Denote by $\varphi_i \equiv c_i + r_i c_y$, the full marginal operating cost, f.m.o.c.$_i$, of the technology i, $i = h, l$. In the 'small' demand case, $\varphi_l < \varphi_h$ for any aggregate supply level below q^s. We first show that this implies that the dual variables associated to an optimal investment plan must satisfy $\lambda_l(t) > \lambda_h(t)$. If there is no scrapping of high performing equipments, we deduce from:

$$\lambda_i(t) \;=\; \int_t^{\infty} \beta_i(\tau) e^{-\rho(\tau - t)} d\tau = \int_t^{\infty} \left[p(\tau) - \varphi_i(\tau) \right] e^{-\rho(\tau-t)} d\tau \quad i = h, l \ ,$$

that:

$$\lambda_l(t) - \lambda_h(t) \;=\; \int_t^{\infty} \left[\varphi_h(\tau) - \varphi_l(\tau) \right] e^{-\rho(\tau-t)} d\tau \ > 0 \ .$$

Let t_h^{δ} the time at which the industry begins to scrap high performing equipments in an investment scenario including a scrapping phase. At time t_h^{δ}, $\lambda_h(t_h^{\delta}) = 0$ and $\lambda_l(t_h^{\delta}) > \underline{n}_l > 0$, the industry accumulating low performing equipments during the scrapping phase. After t_h^{δ}, $\lambda_l(t) > \lambda_h(t) = 0$ holds trivially. Before t_h^{δ}:

$$\lambda_l(t) - \lambda_h(t) \;=\; \int_t^{t_h^{\delta}} \left[\varphi_h(\tau) - \varphi_l(\tau) \right] e^{-\rho(\tau-t)} d\tau + \lambda_l(t_h^{\delta}) e^{-\rho(t_h^{\delta} - t)} \ > 0 \ .$$

In the small demand case, the full marginal operating cost of low performing equipments is always lower than the full marginal operating cost of high performing ones along an optimal investment path. This implies that the marginal benefit of an investment in low performing pieces of equipments is higher than the marginal benefit of an investment high performing pieces of equipment at any time.

Denote by $\lambda_l^*(\lambda_h)$ the implicit relationship between λ_l and λ_h along an optimal investment plan in the dual plane (λ_h, λ_l). We now show that the slope of this implicit curve, $d\lambda_l^*/d\lambda_h$ must be larger than one. It is immediate that $d\lambda_l^*/d\lambda_h = \dot{\lambda}_l/\dot{\lambda}_h$. Furthermore, time differentiating:

$$\dot{\varphi}_h(t) - \dot{\varphi}_l(t) \quad = \quad -(r_l - r_h)c_y'\dot{y}(t) < 0 \ .$$

The marginal operating cost gap between low and high performing equipments declines over time because of the progressive expansion of the useful energy supply level resulting from the accumulation of capacities. This implies that $\varphi_h(t) - \varphi_l(t) > \varphi_h(\tau) - \varphi_l(\tau)$, $\tau > t$. In an investment scenario without scrapping:

$$\begin{aligned} \lambda_l(t) - \lambda_h(t) &= \int_t^\infty \left[\varphi_h(\tau) - \varphi_l(\tau)\right] e^{-\rho(\tau-t)} d\tau \\ &< \frac{1}{\rho}\left[\varphi_h(t) - \varphi_l(t)\right] \ , \end{aligned}$$

implies that (remembering that $\dot{\lambda}_h(t) < 0$, as shown before):

$$\begin{aligned} & \rho\lambda_l(t) - [p(t) - \varphi_l(t)] < \rho\lambda_h(t) - [p(t) - \varphi_h(t)] \\ \Longrightarrow \quad & \dot{\lambda}_l(t) < \dot{\lambda}_h(t) \ \Longrightarrow \ \frac{\dot{\lambda}_l(t)}{\dot{\lambda}_h(t)} > 1 \\ \Longrightarrow \quad & \frac{d\lambda_l^*(\lambda_h)}{d\lambda_h} > 1 \ . \end{aligned}$$

In an investment scenario with scrapping, for $t < t_h^\delta$:

$$\begin{aligned} & \lambda_l(t) - \lambda_h(t) = \int_t^{t_h^\delta} \left[\varphi_h(\tau) - \varphi_l(\tau)\right] e^{-\rho(\tau-t)} d\tau + \lambda_l(t_h^\delta) e^{-\rho(t_h^\delta - t)} \\ & < [\varphi_h(t) - \varphi_l(t)] \frac{1 - e^{-\rho(t_h^\delta - t)}}{\rho} + \lambda_l(t_h^\delta) e^{-\rho(t_h^\delta - t)} \\ \Longrightarrow \quad & \rho\lambda_l(t) + \varphi_l(t) < \rho\lambda_h(t) + \varphi_h(t) + [\rho\lambda_l(t_h^\delta) - (\varphi_h(t) - \varphi_l(t))] e^{-\rho(t_h^\delta - t)} \ . \end{aligned}$$

The marginal cost gap being time decreasing, $\varphi_h(t_h^\delta) - \varphi_l(t_h^\delta) < \varphi_h(t) - \varphi_l(t)$ implies that $-(\varphi_h(t) - \varphi_l(t)) < -(\varphi_h(t_h^\delta) - \varphi_l(t_h^\delta))$. We thus deduce from the previous inequality that:

$$\begin{aligned}\rho\lambda_l(t) - [p(t) - \varphi_l(t)] &< \rho\lambda_h(t) - [p(t) - \varphi_h(t)] \\ &\quad +[\rho\lambda_l(t_h^\delta) - (\varphi_h(t_h^\delta) - \varphi_l(t_h^\delta))]e^{-\rho(t_h^\delta - t)} \ .\end{aligned}$$

Adding and subtracting $p(t_h^\delta)$ while taking into account that $p(t_h^\delta) = \varphi_h(t_h^\delta)$ at the beginning of the scrapping phase of high performing equipments yields:

$$\begin{aligned}\dot\lambda_l(t) &< \dot\lambda_h(t) + [\rho\lambda_l(t_h^\delta) - [p(t_h^\delta) - \varphi_l(t_h^\delta)] + p(t_h^\delta) - \varphi_h(t_h^\delta)]e^{-\rho(t_h^\delta - t)} \\ &= \dot\lambda_h(t) + \dot\lambda_l(t_h^\delta)e^{-\rho(t_h^\delta - t)} < \dot\lambda_h(t) \ ,\end{aligned}$$

since $\dot\lambda_l(t_h^\delta) < 0$ as shown previously. We conclude that $d\lambda_l^*/d\lambda_h > 1$ before t_h^δ in an investment scenario with scrapping of high performing equipments.

The trajectory labeled (1) on the Figure 26 illustrates the optimal path of the dual variables in an investment scenario without scrapping composed of a first phase of simultaneous investment (to the right of the $\underline{n}_h$ vertical) followed by an investment phase in low performing equipments only. The trajectory labeled (3) on the same Figure shows the optimal dual path corresponding to an investment scenario with scrapping.

4.5.2 Optimal investment dynamics in the phase plane (K_h, K_l)

We now turn to the study of the investment dynamics in the plane (K_h, K_l). Positive investment rates in type l equipments, require capacities located below the border L_l defined as:

$$L_l \ : \ \{(K_h, K_l)|\ u'(K_l + K_h) - r_l c_y(r_h K_h + K_l) = c_l + \rho\underline{n}_l\} \ .$$

Denote by $\bar K_l^l(K_h)$ the implicit relation defining the L_l border and $\bar K_{l0}^l \equiv \bar K_l^l(0)$. We have first to locate $\bar K_{l0}^l$ with respect to $\hat K_{l0}$, the intercept of the scrapping frontier with the vertical axis. Since $r_h < r_l$, $u'(K) - r_l c_y(r_l K) < u'(K) - r_h c_y(r_l K)$ for any positive K, so that three possibilities arise, pictured in Figure 27.

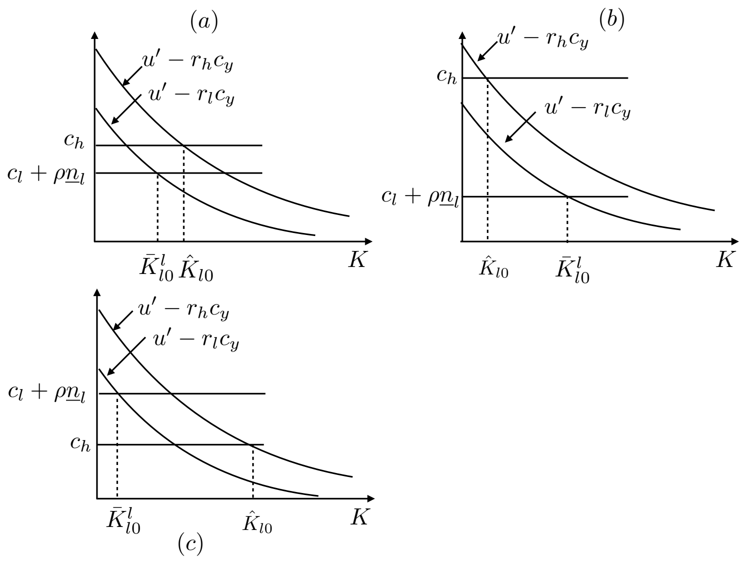

Figure 27: **Comparing $\bar{K}^l_{l0}$ and $\hat{K}_{l0}$.**

In the cases (a) and (c), $\bar{K}^l_{l0} < \hat{K}_{l0}$ while $\bar{K}^l_{l0} > \hat{K}_{l0}$ in the case (b). We thus have to consider the two possibilities. Note also that $\bar{K}^l_{l0}$ solving $u'(K) - r_l c_y(r_l K) = c_l + \rho \underline{n}_l$ and q^s solving $u'(q) - r_l c_y(r_l q) = c_l$, we have $\bar{K}^l_{l0} < q^s$.

Differentiating gets:

$$\frac{d\bar{K}^l_l(K_h)}{dK_h} = -\frac{r_h r_l c'_y + |u''|}{r_l^2 c'_y + |u''|} < 0 ,$$

and:

$$r_h < r_l \implies \left|\frac{d\bar{K}^l_l(K_h)}{dK_h}\right| < 1 .$$

The slopes of the L_l border and of the scrapping frontier being lower than one in absolute value, we have to compare the slopes. In the small demand case:

$$\begin{aligned}\left|\frac{d\hat{K}_l}{dK_h}\right| - \left|\frac{d\bar{K}_l}{dK_h}\right| &= \frac{u'' - r_h^2 c'_y}{u'' - r_h r_l c'_y} - \frac{u'' - r_h r_l c'_y}{u'' - r_l^2 c'_y} \\ &= \frac{(u'' - r_h^2 c'_y)(u'' - r_l^2 c'_y) - \left(u'' - r_h r_l c'_y\right)^2}{(u'' - r_h r_l c'_y)(u'' - r_l^2 c'_y)} > 0 .\end{aligned}$$

Note that if $\hat{K}_{l0} < \bar{K}^l_{l0}$ then the slope of the L_l border being lower than the slope of the scrapping frontier, the L_l border is located above the scrapping frontier. Denote by $\hat{K}_{h0} \equiv \hat{K}_h(0)$ and let $\bar{K}^l_{h0}$ be the solution of $u'(K) - r_l c_y(r_h K) = c_l + \rho \underline{n}_l$. We have to compare $\hat{K}_{h0}$ and $\bar{K}^l_{h0}$ in the case $\bar{K}^l_{l0} < \hat{K}_{l0}$. The same reasoning as for $\bar{K}^l_{l0}$ shows that the two possibilities $\hat{K}_{h0} < \bar{K}^l_{h0}$ or $\hat{K}_{h0} > \bar{K}^l_{h0}$ may arise. We conclude that the three cases (A), (B) and (C), have to be considered, illustrated in the Figure 28.

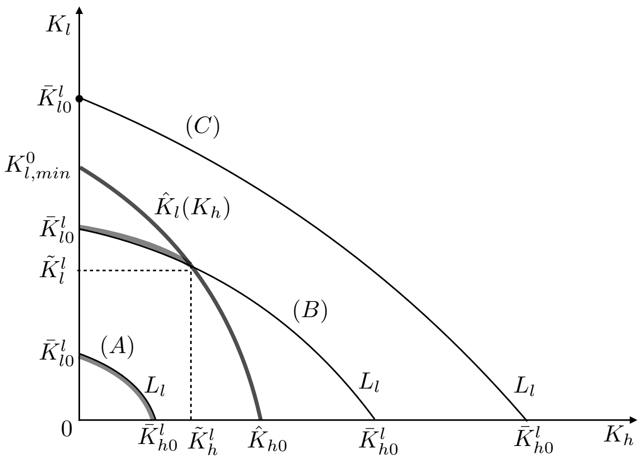

Figure 28: **Possible positions of the L_l border in the 'small' demand case.**

In the case (A) the investment process ends at any point of the L_l border depending on the initial conditions. In the case (B), the investment process ends along that part of the L_l border located at the left of the intersection point between the L_l border and the scrapping frontier, a part pictured as a grey curve on the Figure. In the case (C), the investment process ends on the vertical axis at the point $(0, \bar{K}_{l0})$.

There exists another border, we denote by $\bar{L}_h$, such that there is no more investment in type h equipments. This border is located below the L_l border. To check this point let t_h be the time at which stops the investment process in high performing equipments. Then, in a no scrapping of high performing equipments scenario, the accumulated stock of high performing equipments at time t_h, $K_h(t_h)$, will be maintained forever and the value of the last built high performing piece of equipment having to be equal to $\underline{n}_h$:

$$\underline{n}_h = \int_{t_h}^{\infty} \left[u'(K_h(t_h) + K_l(t)) - c_h - r_h c_y(r_h K_h(t_h) + r_l K_l(t))\right] e^{-\rho(t-t_h)} dt \ . \tag{4.45}$$

Since $K_l(t)$ increases over time after t_h, $K_l(t_h) < K_l(t)$, $t > t_h$ and:

$$\begin{aligned} & u'(K_h(t_h) + K_l(t)) - c_h - r_h c_y(r_h K_h(t_h) + r_l K_l(t)) \\ < \ & u'(K_h(t_h) + K_l(t_h)) - c_h - r_h c_y(r_h K_h(t_h) + r_l K_l(t_h)) \\ < \ & u'(K_h(t_h) + K_l(t_h)) - c_l - r_l c_y(r_h K_h(t_h) + r_l K_l(t_h)) \ . \end{aligned}$$

The last inequality results from the fact that $c_l + r_l c_y < c_h + r_h c_y$ in the small demand case. We thus conclude that:

$$\begin{aligned} \rho \underline{n}_l < \rho \underline{n}_h &= \rho \int_{t_h}^{\infty} \left[u'(K_h(t_h) + K_l(t)) - c_h - \right. \\ & \qquad \left. r_h c_y(r_h K_h(t_h) + r_l K_l(t))\right] e^{-\rho(t-t_h)} dt \\ &< u'(K_l(t_h) + K_h(t_h)) - c_l - r_l c_y(r_h K_h(t_h) + r_l K_l(t_h)) \ , \end{aligned}$$

which can be possible only if the pair $(K_h(t_h), K_l(t_h))$ is located below L_l, the investment frontier for the low performing type of equipments.

Furthermore:

$$0 < \rho \underline{n}_h < u'(K_h(t_h) + K_l(t_h)) - c_h - r_h c_y(r_h K_h(t_h) + r_l K_l(t_h)) \ ,$$

shows that the pair $(K_h(t_h), K_l(t_h))$ must also be located below the scrapping frontier.

In an investment scenario with scrapping of high performing equipments, $\lambda_h(t) = 0$, $t \geq t_h^\delta$. The formula (4.45) writes:

$$\underline{n}_h = \int_{t_h}^{t_h^\delta} \left[u'(K_h(t_h) + K_l(t)) - c_h - r_h c_y(r_h K_h(t_h) + r_l K_l(t))\right] e^{-\rho(t-t_h)} dt \ .$$

Since the stock of high performing equipments is maintained throughout the time interval $[t_h, t_h^\delta)$ at the level $K_h(t_h)$ and $K_l(t)$ grows, the same reasoning as for the no scrapping scenario applies, leading to the same conclusion.

We thus face the three following possibilities:

1. (A): the L_l border is located below the scrapping frontier and above the $\bar{L}_h$ border;

2. (B): the L_l border cuts from below the scrapping frontier while the $\bar{L}_h$ border remains located below the scrapping frontier;

3. (C): the L_l border is located above the scrapping frontier while the $\bar{L}_h$ border is located below the scrapping frontier.

The optimal investment plans in these different situations are easily determined. They are pictured in the Figure 29.

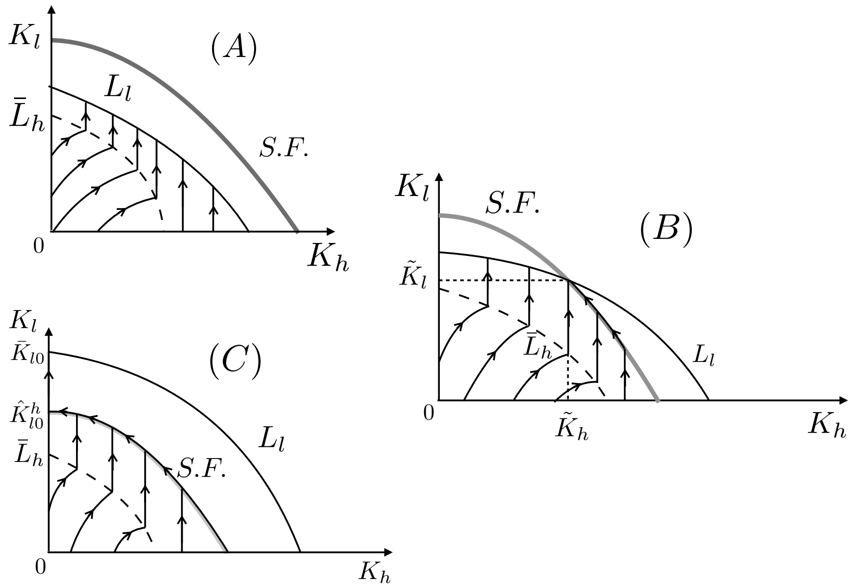

Figure 29: **Optimal investment paths in the 'small' demand case.**

In the case (A), the L_l border being located below the scrapping border, no high performing equipments have to be scrapped. As in the abundant solar case, either investment occurs only in low performing equipments and there is no investment in high performing ones, either there is a first time period, $[0, t_h)$, of simultaneous investment in both types of equipments until the border $\bar{L}_h$ is attained, followed by an infinite duration time period of investment in only low performing equipments, $[t_h, \infty)$.

In the case (B), two main types of investment scenarios can arise, depending on the initial conditions. The first one is similar to the case (A), that is composed of a first investment phase in the two types of equipment, $[0, t_h)$, followed by a second phase of investment in low performing equipments only, $[t_h, \infty)$. The capital trajectory ends at some point of that part of the L_l border located between the point $(0, \bar{K}_{l0})$ and the intersection point between the L_l border and the scrapping frontier, a point we denote by $(\tilde{K}_h, \tilde{K}_l)$.

The second one is a bit more intricate. Depending on the initial capital pair (K_h^0, K_l^0), it starts with an active investment phase in the two types of equipments, $[0, t_h)$, followed by a phase of investment in only low performing equipments, $[t_h, t_h^\delta)$, or it starts with a phase of investment in low performing equipments only, $[0, t_h^\delta)$, the stock of high performing ones being maintained at its initial level, K_h^0, until t_h^δ. In both cases, the capital accumulation plan ends with a scrapping phase of high performing equipments, $[t_h^\delta, \infty)$, the capital stocks converging asymptotically toward the intersection point between the L_l border and the scrapping frontier, $(\tilde{K}_h, \tilde{K}_l)$.

The case (C) differs from the case (B) by the fact that the whole stock of high performing equipments is scrapped. The capital accumulation process starts with either a simultaneous investment phase, $[0, t_h)$, followed by an investment phase in low performing equipments only, $[t_h, t_h^\delta)$, or starts with a phase of investment in low performing equipments, $[0, t_h^\delta)$. Next, the industry scraps progressively the whole stock of high performing equipments during a time period, $[t_h^\delta, \bar{t}_h)$. The capital stocks trajectory moves along the scrapping frontier until the point $(0, \hat{K}_{l0}^h)$ is attained at time $\bar{t}_h$. During a last time phase, $[\bar{t}_h, \infty)$, the industry accumulates only low performing equipments, the stock of these equipments converging asymptotically towards $\bar{K}_{l0}$.

Note that the dual variable $\lambda_l(t)$ is continuous but not time differentiable at the end of the scrapping process of high performing equipments. Denote by $\nu_h(t)$ the Lagrange multiplier associated to the positivity constraint $\hat{K}_h(K_l(t)) \geq 0$. Through the complementary slackness condition, $\nu_h = 0$ if $\hat{K}_h > 0$. Since at time $\bar{t}_h$, the end of the scrapping phase of high performing equipments, $\nu_h(\bar{t}_h) \geq 0$ and $\hat{K}_h(\bar{t}_h) = 0$:

$$\begin{aligned}
\lim_{t \uparrow \bar{t}_h} \dot{\lambda}_l(t) &= \rho\lambda_l(\bar{t}_h) - [p(\bar{t}_h) - c_l - r_l c_y(y(\bar{t}_h))] - \nu_h(\bar{t}_h) \\
&\leq \rho\lambda_l(\bar{t}_h) - [p(\bar{t}_h) - c_l - r_l c_y(y(\bar{t}_h))] \\
&= \lim_{t \downarrow \bar{t}_h} \dot{\lambda}_l(t) \ .
\end{aligned}$$

The function $\lambda_l(t)$ is not time differentiable at $\bar{t}_h$, the end time of the scrapping phase. In Appendix A.1 we develop an algorithmic argument able to compute the characteristics of the different scenarios.

4.6 The 'medium' demand case

Denote by $(C.E)$ the line $K_l = y_e/r_l - (r_h/r_l)K_h$, corresponding to the equalization of the f.m.o.c's of the two technologies. Below the $(C.E)$ line, f.m.o.c.$_l$ < f.m.o.c$_h$, the full marginal operating cost of the low performing technique is lower than the full marginal operating cost of the high performing one, equivalently $\varphi_l < \varphi_h$. The contrary happens above the $(C.E)$ line, the high performing technique being also cheaper to operate, that is $\varphi_l > \varphi_h$.

The investment frontier of high performing equipments, we denote by L_h is defined by:

$$L_h \ : \ u'(K_h + K_l) - c_h - r_h c_h(r_h K_h + r_l K_l) = \rho \underline{n}_h \ .$$

It is easily checked that the slope of the L_h border is strictly higher than the slope of the L_l border in absolute value. Let $\bar{K}_l^h(K_h)$ denote the implicit equation of the L_h border in the plane (K_h, K_l) and denote by $\bar{K}_{h0}^h$ the solution of $\bar{K}_l^h(K_h) = 0$. We first show that $\bar{K}_{h0}^h < \bar{K}_{h0}^l$. Since $\varphi_h > \varphi_l$ for any K_h along the horizontal axis:

$$\begin{aligned}
& u'(\bar{K}_{h0}^h) - \rho\underline{n}_h = c_h + r_h c_y(r_h \bar{K}_{h0}^h) > c_l + r_l c_y(r_h \bar{K}_{h0}^h) \\
= \ & \left[u'(\bar{K}_{h0}^l) - \rho\underline{n}_l - r_l c_y(r_h \bar{K}_{l0}^l)\right] + r_l c_y(r_h \bar{K}_{h0}^h) \\
\Longrightarrow \ & u'(\bar{K}_{h0}^h) > u'(\bar{K}_{h0}^l) + r_l \left[c_y(r_h \bar{K}_{h0}^h) - c_y(r_h \bar{K}_{h0}^l)\right] + \rho(\underline{n}_h - \underline{n}_l) \\
> \ & u'(\bar{K}_{h0}^l) + r_l \left[c_y(r_h \bar{K}_{h0}^h) - c_y(r_h \bar{K}_{h0}^l)\right] \\
\Longrightarrow \ & u'(\bar{K}_{h0}^h) - u'(\bar{K}_{h0}^l) > r_l \left[c_y(r_h \bar{K}_{h0}^h) - c_y(r_h \bar{K}_{h0}^l)\right] \ .
\end{aligned}$$

If $\bar{K}^h_{h0} > \bar{K}^l_{h0}$, $u'(\bar{K}^h_{h0}) - u'(\bar{K}^l_{h0}) < 0$ and $c_y(r_h\bar{K}^h_{h0}) - c_y(r_h\bar{K}^l_{h0}) > 0$ yield a contradiction.

Secondly if an intersection point between the L_l and the L_h border exists in the positive capital stocks domain, then $\varphi_h(K_h, K_l) + \rho\underline{n}_h = \varphi_l(K_h, K_l) + \rho\underline{n}_l$ implies that $\varphi_h < \varphi_l$ since $\underline{n}_l < \underline{n}_h$. Thus if the curves $\bar{K}^l_l(K_h)$ and $\bar{K}^h_l(K_h)$ cross themselves in the positive domain, the intersection point, we denote by $(\bar{K}^0_h, \bar{K}^0_l)$, must be located above the $(C.E)$ line.

Next, $u'(\bar{K}^l_{l0}) - c_l - r_l c_y(r_l\bar{K}^l_{l0}) = \rho\underline{n}_l > 0$ and $u'(\hat{K}_{l0}) - c_l - r_l c_y(r_l\hat{K}_{l0}) = 0$ shows that $\bar{K}^l_{l0} < \hat{K}_{l0}$. The curve $\bar{K}_l(K_h)$ being decreasing, $\bar{K}^0_l < \bar{K}^l_{l0}$ implies that the intersection point $(\bar{K}^0_h, \bar{K}^0_l)$, if it exists, must be located below the scrapping frontier.

Furthermore, if $\bar{K}^l_{l0} < \underline{q}_e$, the L_l and L_h borders cannot cross, since this would imply that the intersection point between the two borders would have to be located below the $(C.E.)$ line, a contradiction. Since $\bar{K}^h_{h0} < \bar{K}^l_{h0}$, we also conclude that the L_h border is located below the L_l border and $\bar{K}^h_{l0} < \bar{K}^l_{l0} < \underline{q}_e$.

If $\underline{q}_e < \bar{K}^l_{l0} < \hat{K}_{l0}$, $\bar{K}^h_{l0}$ can be lower than $\bar{K}^l_{l0}$ in which case the L_h border is located below the L_l border or $\bar{K}^h_{l0}$ is located above $\bar{K}^l_{l0}$ in which case the L_h and L_l cross themselves in the positive domain. It may even be the case that $\hat{K}_{l0} < \bar{K}^h_{l0}$ and the L_h border cuts from above the scrapping border since the slope of the L_h border is higher than the slope of the scrapping frontier in absolute value. Last it can also be possible that $\hat{K}_{h0} < K^l_{h0}$, in which case the L_l border cuts from below the scrapping frontier.

If the L_l and the L_h borders do not cross in the positive domain, we conclude that the L_l border must be located above the L_h border. Thus only the L_l border is relevant to build the optimal investment plan. In this case, the optimal scenarios are similar to those of the small demand case.

If the L_l border is located below the scrapping frontier, no equipments of any type are dismantled. The economy accumulates both types of equipments during a first time period and next only low performing equipments throughout an infinite time interval, the stock of high performing equipments being maintained during this time phase. If the L_l border cuts from below the scrapping frontier, a three phases optimal scenario is also possible, depending on the initial conditions. In this case, the optimal investment plan ends with an infinite duration scrapping phase of high performing equipments until the intersection point, $(\tilde{K}_h, \tilde{K}_l)$, between the L_l border and the scrapping frontier is attained. Note that in the 'medium' demand case, it cannot be optimal to scrap entirely the stock of high performing equipments since $\bar{K}^l_{l0} < \hat{K}_{l0}$ prevents the L_l border to be located entirely above the scrapping frontier.

The novel feature of the 'medium' demand case with respect to the 'small' demand one is thus the possibility of an intersection between the L_l and the L_h border. For the ease of exposition, we contrast the case $\bar{K}^h_{l0} < \hat{K}_{h0}$ and $\bar{K}^l_{h0} < \hat{K}_{h0}$, where the L_l and L_h borders are both located below the scrapping frontier, a situation where no equipments of any type must be scrapped, from the case $\bar{K}^h_{l0} > \hat{K}_{h0}$ and $\bar{K}^l_{h0} > \hat{K}_{h0}$, where depending on the initial conditions, the optimal investment plan can end with a scrapping phase of either low performing or high performing equipments.

4.6.1 The L_l and L_h borders are located below the scrapping frontier

The following Figure 30 illustrates the different borders configurations in this situation.

There exists an accumulation trajectory in the capacities plane (K_h, K_l) converging in infinite time toward the intersection point $(\bar{K}^0_h, \bar{K}^0_l)$ of the L_l and L_h borders. This trajectory is the solution of an active simultaneous investment program in the two types of equipment followed throughout an infinite time interval, that is $k_i(t) > 0$, $i = h, l$, $t \geq 0$. The separating curve $(S2)$ is the dual image of this trajectory in the phase plane (λ_h, λ_l). In the long run, $\lambda_h(t)$ converges towards $\underline{n}_h$ and $\lambda_l(t)$ converges towards $\underline{n}_l$. We denote by $(S.I)$ the corresponding curve in the phase plane (K_h, K_l), a separating curve distinguishing initial conditions such that the industry will first accumulate both types of equipments and next will accumulate only low performing equipments or only high performing equipments, the other type stock of equipments being maintained.

The investment scenarios in the present case are pictured in the Figure

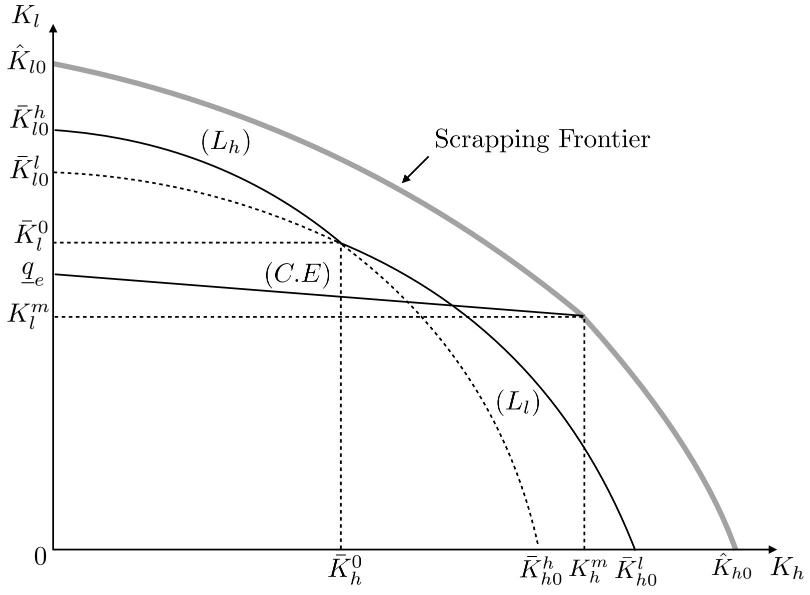

Figure 30: **Investment borders located below the scrapping frontier in the 'medium' demand case.**

31.

Below the $(S.I)$ separating curve, the optimal plan is at most a two phases path. During a first phase, $[0, t_h)$, the industry accumulates both types of equipments. The capacities expand until the border $\bar{L}_h$ is attained. At time t_h, $\lambda_h(t_h) = \underline{n}_h$ and $\lambda_l(t_h) > \underline{n}_l$. Let $(\bar{K}_h, K^h_l)$ denote the capacities pair attained at time t_h. If this pair is located below the $(C.E)$ line, the full marginal operating cost of low performing equipments is lower than the f.m.o.c of high performing equipments during the whole phase of simultaneous investment. If this pair is located above the $(C.E)$ line, then low performing equipments are cheaper to operate than high performing ones during a first time period, $[0, t^e_h)$, while high performing equipments are cheaper to operate during a second time period, $[t^e_h, t_h)$. This does not prevent the industry to stop accumulating high performing equipments at time t_h. The reason is that such equipments remain more costly to build.

During a second time phase of infinite duration, $[t_h, \infty)$, the industry accumulates only low performing equipments, the stock of high performing

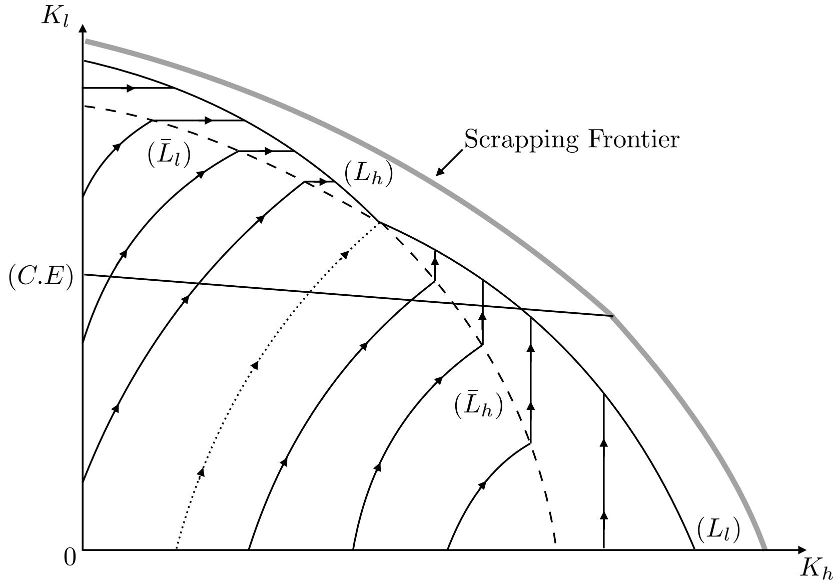

Figure 31: **Optimal investment plans without scrapping in the 'medium' demand case.**

ones being maintained at the level $\bar{K}_h$. The investment process ends on the L_l border at a capacity pair $(\bar{K}_h, \tilde{K}_l)$.

The geometry of the $\bar{L}_h$ shown on the Figure 31 can be explained as follows. Remember that the operational margins decline over time for both types of equipments, thus $\beta_h(t_h) \geq \beta_h(t)$, $t \geq t_h$. Thus:

$$\begin{aligned} \lambda_h(t_h) = \underline{n}_h &= \int_{t_h}^{\infty} \beta_h(t) e^{-\rho(t-t_h)} dt \\ &\leq \frac{\beta_h(t_h)}{\rho} \,. \end{aligned}$$

Thus $\beta_h(t_h) \geq \rho \underline{n}_h$ which implies that the $\bar{L}_h$ border is located below the L_h border, whose equation is $\beta_h = \rho \underline{n}_h$. The L_h border is itself located below the L_l border for capacities pairs located at the right and below the pair $(\bar{K}_h^0, \bar{K}_l^0)$. The $\bar{L}_h$ border initiates from the pair $(\bar{K}_h^0, \bar{K}_l^0)$ since $t_h \to \infty$ when moving along the $\bar{L}_h$ border when K_h is decreased towards $\bar{K}_h^0$. This implies that $\lambda_h(t_h)$ converges towards $\underline{n}_h$ and that $\beta_h(t_h)$ converges towards $\rho \underline{n}_h$. Thus the $\bar{L}_h$ border intersects the L_h border and thus also the L_l border at $(\bar{K}_h^0, \bar{K}_l^0)$.

Above the $(S.I)$ separating curve, the optimal investment plan is also at most a two phases path. During a first phase, $[0, t_l)$, the energy industry accumulates both types of equipments. The capital stocks increase until the border $\bar{L}_l$ is attained. The geometry of the $\bar{L}_l$ border is similar to those of the $\bar{L}_h$ border. β_l being time decreasing after t_l, $\beta_l(t_l) \geq \beta_l(t)$, $t \geq t_l$, implies that $\beta_l(t_l) \geq \rho \underline{n}_l$ and thus that the $\bar{L}_l$ border is located below the L_l border, whose equation reads $\beta_l = \rho \underline{n}_l$. To the left of the intersection point $(\bar{K}_h^0, \bar{K}_l^0)$, the L_l border is located below the L_h border. The same argument as for the $\bar{L}_h$ border shows that the $\bar{L}_l$ border intersects the L_l border, and thus the L_h border, at the point $(\bar{K}_h^0, \bar{K}_l^0)$.

Now at time t_l, $\lambda_h(t_l) > \underline{n}_h$ and $\lambda_l(t_l) = \underline{n}_l$. Let $(K_h^l, \bar{K}_l)$ denote the capital stocks pair accumulated at time t_l. By construction such pairs are located above the $(C.E)$ line. Thus depending on the initial conditions, the simultaneous investment phase is either a phase when high performing equipments are cheaper to operate than low performing equipments, either a sequence of a first sub-phase, $[0, t_l^e)$, when the f.m.o.c of low performing equipments is lower than the f.m.o.c of high performing equipments, followed by a second sub-phase, $[t_l^e, t_l)$, during which high performing equipments are cheaper to operate than low performing ones.

Throughout a second period of infinite duration, $[t_l, \infty)$, the industry maintains at the level $\bar{K}_l$ its stock of low performing equipments and accumulates high performing installations until the L_h border is attained asymptotically, the capital accumulation process ending at a pair $(\tilde{K}_h, \bar{K}_l)$.

The trajectory labelled #1 on the Figure 32 illustrates in the dual plane the first case of a capital accumulation trajectory located below the separating curve $(S.I)$. The trajectory cuts the vertical $\underline{n}_h$ when stops the accumulation of high performing equipments. Then the trajectory converges asymptotically toward the pair $(\tilde{\lambda}_h, \underline{n}_h)$ where $\tilde{\lambda}_h > 0$. The trajectory labelled #2 on the Figure illustrates the second case of a capital accumulation trajectory located above the separating curve $(S.I)$. The trajectory cuts the $\underline{n}_l$ horizontal when stops the accumulation of low performing equipments. Then the trajectory converges towards $(\underline{n}_h, \tilde{\lambda}_l)$ where $\tilde{\lambda}_l > 0$.

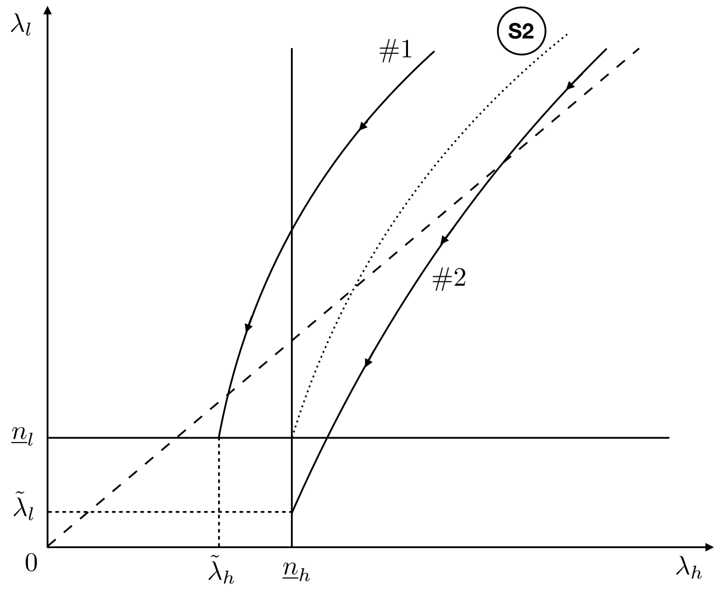

Figure 32: **Dual variables trajectories of optimal investment plans without scrapping in the 'medium' demand case.**

The cost gap $\varphi_h(t) - \varphi_l(t)$ declines along the optimal plan like in the 'small' demand case. It becomes negative when the capital trajectory moves above the $(C.E)$ line. Thus the same type of argument as developed for the 'small' demand situation shows that the slope of the optimal dual variables trajectory $\lambda_l^*(\lambda_h)$ is larger than one, as illustrated on the Figure 32.

4.6.2 The L_l and L_h borders cut the scrapping frontier

Of course, it may be possible that only one border cuts the scrapping frontier. For the sake of brevity we focus on a case where both borders intersect the scrapping frontier. The main difference with the preceding scenarios is that the partial scrapping of capacities can be optimal. For trajectories initiated below the separating curve $(S.I)$, the optimal investment plan is composed of at most three phases. The first phase, $[0, t_h)$, is a simultaneous investment phase ending on the border $\bar{L}_h$. Then begins a second phase of accumulation of low performing equipments, $[t_h, t_h^\delta)$, the stock of high performing equipments being maintained at a constant level, $\bar{K}_h$. This phase ends when the scrapping border of high performing equipments is attained. The last investment phase, $[t_h^\delta, \infty)$, combines the scrapping of high performing equipments with the accumulation of low performing units until the pair $(\tilde{K}_h^l, \tilde{K}_l^l)$ is attained, the intersection point between the scrapping frontier and the L_l border.

Trajectories located above the separating curve $(S.I)$ exhibit symmetric characteristics. Depending on the initial conditions, the investment plan begins with a simultaneous investment phase, $[0, t_l)$. This phase is followed by a phase of accumulation of high performing equipments, the stock of low performing ones being maintained, until the scrapping frontier is attained at a time t_l^δ. The investment plan ends with and infinite duration phase, $[t_l^\delta, \infty)$, during which the industry scraps low performing equipments while accumulating high performing ones until the pair $(\tilde{K}_h^h, \tilde{K}_l^h)$ is attained, the intersection point between the L_h border and the scrapping frontier. The following Figure 33 illustrates the capital accumulation dynamics in the phase plane (K_h, K_l).

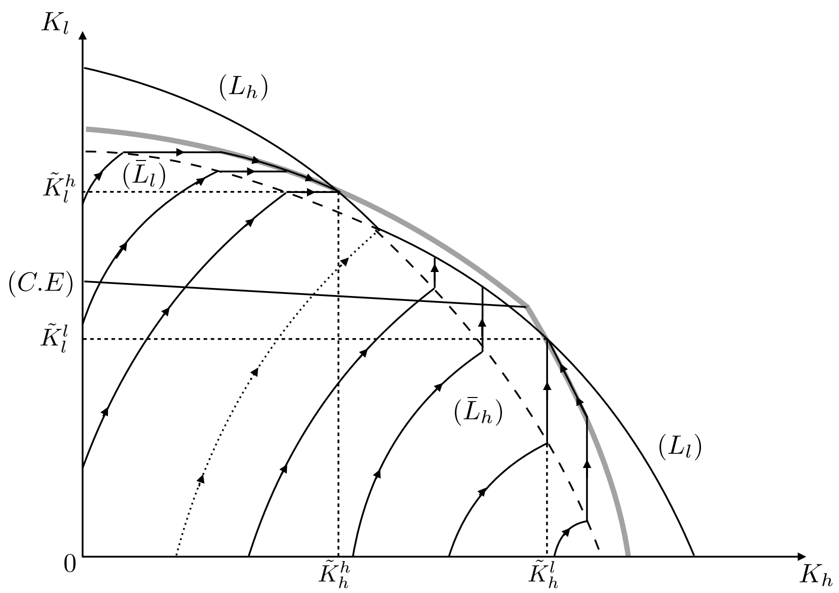

Figure 33: **Optimal investment plans with scrapping in the 'medium' demand case.**

The trajectory labelled #1 on the Figure 34 pictures in the dual plane (λ_h, λ_l), the case of an optimal investment plan with scrapping of high performing equipments. The right part of the trajectory shows the declining move of the marginal benefit from an investment in the two techniques until the vertical axis is attained at a pair $(0, \lambda_l^\delta)$ at the beginning of the scrapping phase of high performing equipments. Then the dual variables trajectories declines along the vertical axis until the point $(0, \underline{n}_l)$ is attained asymptotically.

The trajectory labelled #2 on the Figure illustrates the case of an optimal investment plan with scrapping of low performing equipments. The right part of the trajectory shows the declining move of $\{\lambda_h(t), \lambda_l(t)\}$ during the time interval $[0, t_l^\delta)$ until the pair $(\lambda_h^\delta, 0)$ is attained at the beginning of the scrapping phase of low performing equipments. Then the trajectory moves along the horizontal axis, converging asymptotically toward the point $(\underline{n}_h, 0)$.

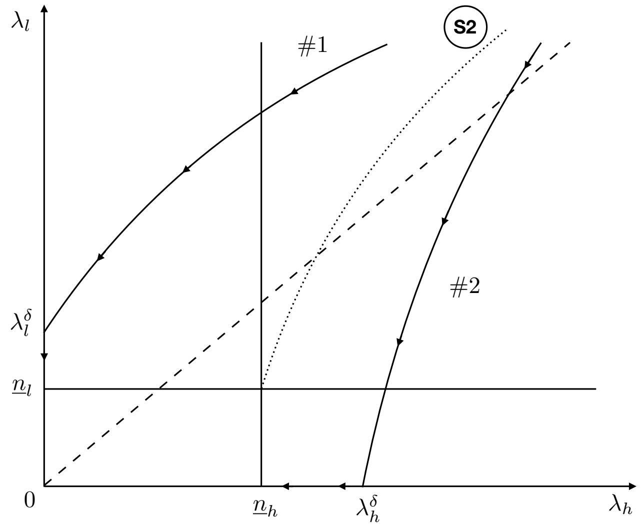

Figure 34: **Dual variables trajectories of optimal investment plans with scrapping in the 'medium' demand case.**

Because the slope of the scrapping frontier is lower than one in absolute value, the industry builds more high performing installations than it dismantles low performing ones during a scrapping phase of low performing equipments while the contrary happens during a scrapping phase of high performing equipments, the industry dismantling more type h equipments than it builds type l units. Therefore, the UE production rate increases with the expansion of aggregate capacities in the case of a scrapping of low performing equipments while it decreases when the industry scraps high performing production units. Such opposite moves of the production rate induce opposite moves of the UE price, the energy price being decreasing during a scrapping phase of low performing equipments and increasing during a scrapping phase of high performing installations.

These contrasting moves can be explained when considering the dynamics of $y(t)$, the primary energy consumption rate, during each kind of scrapping phase. When the industry scraps low performing equipments while building high performing ones:

$$\left|\frac{d\hat{K}_l}{dK_h}\right| > \frac{r_h}{r_l} \implies \dot{y} = k_h \left[r_h - r_l \left|\frac{d\hat{K}_l}{dK_h}\right| \right] < 0 \ .$$

The replacement of low performing units by high performing units in a more than one-to-one way allows saving the primary resource, and the consumption of primary energy declines during a low performing equipments scrapping phase. The f.m.o.c's of the two types of equipments, $c_i + r_i c_y(y)$, $i = h, l$, thus decline with time, inducing a parallel decline of the energy price.

The contrary happens when the industry replaces high performing equipments by low performing units in a less than one-to-one way, since in this case:

$$\left|\frac{d\hat{K}_h}{dK_l}\right| < \frac{r_l}{r_h} \implies \dot{y} = k_l \left[-r_h \left|\frac{d\hat{K}_h}{dK_l}\right| + r_l \right] > 0 \ .$$

The primary energy consumption rate, $y(t)$, increases during a scrapping phase of high performing equipments replaced by a lower amount of low performing units. Thus the f.m.o.c's of the two types of installations increase over time, inducing a parallel rise of the useful energy price, $p(t)$.

Note that the dual variables paths, $\lambda_i(t)$, $i = h, l$, having to be time continuous at time t_i, the time at which stops the investment process in type i equipments, $q(t)$, is time differentiable at time t_i, implying that the useful energy price path, $p(t)$, is also time differentiable when stops the accumulation of equipments of type i.

At the beginning of a scrapping phase of low performing equipments, the time continuity of $\lambda_h(t)$ at the time t_l^δ implies that $k_h(t)$ is time continuous at t_l^δ, and:

$$\dot{K}(t_l^{\delta-}) = k_h(t_l^\delta) + 0 \quad > \quad k_h(t_l^\delta)\left[1 - \left|\frac{d\hat{K}_l}{dK_h}\right|\right] = \dot{K}(t_l^{\delta+}) \ .$$

Hence, it can be concluded from $\dot{p}(t) = u''(K(t))\dot{K}(t)$, that $|\dot{p}(t_l^{\delta-})| > |\dot{p}(t_l^{\delta+})|$.

At the beginning of a scrapping phase of high performing equipments, $\dot{p}(t_h^{\delta-}) < 0$ and $\dot{p}(t_h^{\delta+}) > 0$. Denote by $\tilde{p} = u'(\tilde{K}_h + \tilde{K}_l)$, the long run useful energy price level. The following Figure 35 illustrates the useful energy price dynamics in investment scenarios with scrapping of either low performing equipments (on the top panel) or either high performing ones (on the bottom panel).

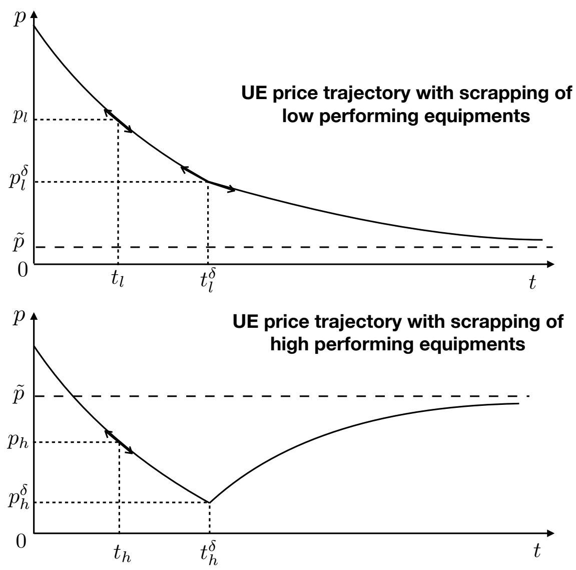

Figure 35: **Useful energy price paths with scrapping in the 'medium' demand case.**

4.7 The 'big' demand case

This case is similar with the 'small' demand case, the dynamics of high performing equipments sharing analog characteristics to those of low performing equipments with a 'small' energy demand. First, since the equation of the scrapping border is given by $\beta_l = 0$ while the equation of the L_l border is given by $\beta_l = \rho \underline{n}_l > 0$, we can conclude that the L_l border is entirely located below the scrapping frontier. Secondly, denoting by $\bar{L}_l$ the border defining the upper limit on accumulated low performing equipments for a given level of high performing ones, $\bar{K}_l(K_h)$ its implicit equation, and by t_l the time at which this border is attained, $\lambda_l(t_l) = \underline{n}_l$ and $\beta_l(t) < \beta_l(t_l)$, $t > t_l$, implies that:

$$\begin{aligned}
\rho \underline{n}_l &= \rho \int_{t_l}^{\infty} \beta_l(t) e^{-\rho(t-t_l)} dt \\
&> \beta_l(t_l) = u'\left(K_h(t_l) + \bar{K}_l(K_h(t_h))\right) - c_l - r_l c_y \left(r_h K_k(t_l) + r_l \bar{K}_l(K_h(t_l))\right) \; ,
\end{aligned}$$

showing that the $\bar{L}_l$ border is located entirely below the L_l border and thus under the scrapping frontier.

Turning to the L_h border, the investment border in high performing equipments, we have to compare the intercept $\bar{K}_{h0}^h$ with $\hat{K}_{h0}$, the intercept of the scrapping frontier with the horizontal axis, and also to compare the intercept $\bar{K}_{l0}^h$ with $\hat{K}_{l0}$, the intercept of the scrapping frontier with the vertical axis. The next Figure 36 shows that the possibilities $\bar{K}_{h0}^h \lesseqgtr \hat{K}_{h0}$ together with $\bar{K}_{l0}^h \lesseqgtr \hat{K}_{l0}$ have to be considered.

Furthermore, we have already shown the the slope of the L_h border is higher in absolute value than the slope of the scrapping frontier. Thus if the L_h border cuts the scrapping frontier, it must do so from above. This implies that the case $\bar{K}_{l0}^h < \hat{K}_{l0}$ and $\bar{K}_{h0}^h > \hat{K}_{h0}$ must be excluded. We thus conclude that three cases have to be distinguished.

- (A): The L_h border is located below the scrapping frontier, that is $\bar{K}_{h0}^h < \hat{K}_{h0}$ and $\bar{K}_{l0}^h < \hat{K}_{l0}$;
- (B): The L_h border cuts from above the scrapping frontier at a point $(\tilde{K}_h, \tilde{K}_l)$, that is $\bar{K}_{h0}^h < \hat{K}_{h0}$ and $\bar{K}_{l0}^h > \hat{K}_{l0}$;

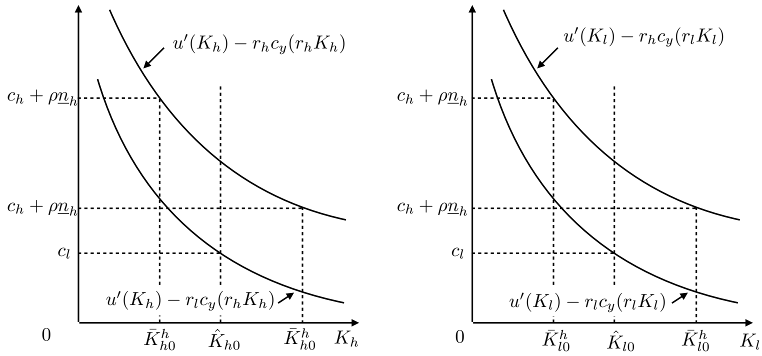

Figure 36: **Comparing the intercepts of the L_h border and the scrapping frontier in the 'big' demand case.**

- (C): The L_h border is located above the scrapping frontier, that is $\bar{K}^h_{h0} > \hat{K}_{h0}$ and $\bar{K}^h_{l0} > \bar{K}_{l0}$.

The different investment scenarios in these three cases are pictured on the Figure 37.

The optimal plans in the case (A) are at most two phases paths. During a first time phase, $[0, t_l)$, the industry invests in both types of equipments. At time t_l, the industry stops accumulating low performing equipments and $\lambda_l(t_l) = \underline{n}_l$. Next, the industry invests only in high performing equipments during an infinite duration time interval, $[t_l, \infty)$. The low performing capacities are maintained at a constant level, $\bar{K}_l$, during this time phase. In the long run, the production capacities trajectory converges towards the L_l border in the phase plane. Let $\tilde{K}_h$, the solution of $\bar{K}^h_l(K_h) = \bar{K}_l$, denote the long run value of K_h. The value of an investment in high performing equipments converges towards $\underline{n}_h$ while $\lambda_l(t)$ converges toward $\tilde{\lambda}_l$, given by:

$$\tilde{\lambda}_l \;=\; \frac{\tilde{\beta}_l}{\rho} = \frac{1}{\rho}\left[u'(\tilde{K}_h + \bar{K}_l) - c_l - r_l c_y\left(r_h \tilde{K}_l + r_l \bar{K}_l\right)\right] \, .$$

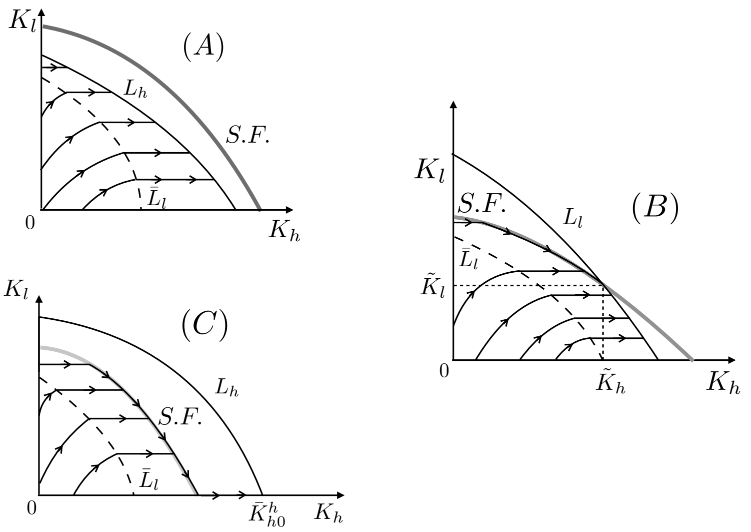

Figure 37: **Optimal investment plans in the 'big' demand case.**

The optimal investment plans in the case (B) are at most three phases paths. Like in the preceding case (A), the investment process starts with a first phase, $[0, t_l)$, of accumulation of both types of equipments. Then during a second time phase, $[t_l, t_l^\delta)$, the industry maintains the previously accumulated low performing production capacity and continues to invest in high performing equipments. At the end of this time phase, $\lambda_l(t_l^\delta) = 0$. Next the industry begins to scrap low performing equipments while investing in high performing ones in a more than one-to-one way during a last time interval $[t_l^\delta, \infty)$. The capacities trajectory follows the scrapping frontier in the phase plane. Since the industry builds more high performing units than it dismantles low performing ones during the time phase, the total production capacity increases and the price of useful energy declines. In the long run, the production capacities levels tend to $(\tilde{K}_h, \tilde{K}_l)$, the intersection point between the L_h border and the scrapping frontier. The value of an investment in high performing capacities converges down to $\underline{n}_h$.

The case (C) differs from the case (B) by the fact that the industry scraps entirely its low performing production capacity. The optimal investment plans in this case are at most four phases paths. During a first time phase, $[0, t_l)$, the industry accumulates both types of equipments. Next it maintains the previously accumulated low performing production capacity while accumulating more high performing units during the second time period, $[t_l, t_l^\delta)$. Next, the capital stocks trajectory initiates a move along the scrapping frontier until $(\hat{K}_{h0}, 0)$. The industry scraps the whole stock of low performing installations during a third time phase, $[t_l^\delta, \bar{t}_l)$, and $K_l(\bar{t}_l) = 0$. Last, during a fourth time phase, $[\bar{t}_l, \infty)$, the industry continues to accumulate high performing equipments. The stock of such equipments converges toward $\bar{K}_{h0}^h$ in the long run and $\lambda_h(t)$ converges down to $\underline{n}_h$.

5 Concluding remarks

Even without productivity advantage, a more expensive technology can expand and survive because of demand rationing under capacity constraints. For this to be the case the demand must be sufficiently strong to absorb the cost disadvantage. We show that this well known property extends to dynamic capacities accumulation contexts even if the more expensive to operate technology is also the more costly to develop, a consequence of investment costs convexity. With this observation in hand we have described the optimal investment policy when in addition to capacity constraints the industry faces an access constraint to its primary resource. The more efficient technology has now a productivity advantage.

However we show that the demand must still be sufficiently strong for this advantage to manifest. With a small demand it may be optimal that after a first expansion phase, the high performing equipments be dismantled, the industry ending using only low cost and low performing equipments. We show also that the constant rise of capacities that triggers down the energy price is not the cause of abandonment of the high performing technology. Actually during a scraping phase of high performing equipments, the replacement rate by low performing equipments must be less than one, leading to a contraction of the energy supply and hence a rise of the energy price.

Observe also that the result is obtained while assuming increasing marginal costs of access to primary energy. Hence it is at first sight surprising that after accumulating high performing equipments, the industry should dismantle them, since the increasing pressure of access costs favours more and more the efficient technology against the less efficient one through the constant fall of the cost advantage of the least efficient technology. The minimisation of the investment costs with convex adjustment costs requires to split investments between the two types of equipment sustaining the deployment of the most efficient equipments. However it can be shown that the capital prices of both kinds of equipments should fall over time, exposing more and more the high performing ones to their building and operating costs disadvantage, inducing first an investment stop in these equipments while the industry keeps investing in low cost equipments, and finally the scraping of the high performing equipment stock when their capital value has fallen down to zero.

We show also that the conventional wisdom demand explanation keeps its validity in the present dynamic context. With a sufficiently large energy demand, the high performing technology will dominate and eventually eliminate the low performing alternative thanks to a combination of higher energy prices and higher access costs due to a large consumption of primary energy.

The study provides interesting insights on the problem of the energy transition. Mitigating the global warming issue means transitioning from a fossil fuels based energy system toward a clean renewables one. In term of the present model, the energy demand can be interpreted as the residual demand faced by the renewable energy conversion sector, net of the fossil fuels based energy supply. The energy transition from fossils to renewables thus corresponds to a progressive rise of the residual energy demand schedule, moving typically from what we have a called a 'small' demand situation toward a 'big' demand situation. We have shown that in the 'small' demand situation the industry can either maintain in the long run some proportion of high performing equipments or either scraps entirely this type of equipments to rely only on low cost installations. This is an illustration of the lock-in problem in poor performing energy conversion techniques. However if the energy demand moves to the medium situation during the energy transition, we have also shown that despite some scraping, the industry always maintain the two types of equipments in the long run. If the demand moves to the big demand situation, high performing equipments will dominate and even eliminate in the long run the low cost equipments.

The lock-in phenomenon should thus characterize the beginning of the transition and tend to disappear with the passage of time. The conclusion differs from the directed technical change literature findings, where because of the increasing returns on knowledge accumulation, the economy can be trapped in the wrong research direction from the climate change mitigation point of view, thus requiring strong policy interventions to alleviate the problem. In the present model, there is no increasing returns effects and the increasing pressure of space access constraints favours the high performing equipments at the expense of the low performing ones. What is only needed for a large adoption of the best technologies is an increasing pattern of the carbon price. Thus the regulation issue lies more in the capacity of the governments to maintain a constant increasing pace of carbon pricing than the present day level of the price.

In principle, carbon pricing should be made endogenous to the energy transition itself. Thus the progressive upward shift of the demand should be made endogenous to the capital accumulation policy in the renewable energy industry. It could hence be the case that optimal carbon pricing alleviates or even make disappear the lock-in issue. We leave this problem for future research.

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Appendix

A.1 Computation of the optimal scenarios

We focus on scenarios with investment in high performing equipments. They may be of three types:

1. A two phases scenario composed of a first phase, $[0, t_h)$, of accumulation of the two types of equipments, followed by an accumulation phase of only low performing equipments, $[t_h, \infty)$;
2. A three phases scenario composed of a first phase, $[0, t_h)$, of simultaneous investment in the two types of equipments, followed by a phase, $[t_h, t_h^\delta)$, of accumulation of low performing equipments, the stock of high performing production capacities being maintained throughout the phase, and ended by an investment phase in low performing equipments, $[t_h^\delta, \infty)$, with a partial scrapping of the stock of high performing equipments;
3. A four phases scenario composed of a first phase of simultaneous investment, $[0, t_h)$, followed by an accumulation phase of low performing equipments, $[t_h, t_h^\delta)$, the stock of high performing equipments being maintained, next an investment phase in low performing equipments, $[t_h^\delta, \bar{t}_h)$, during which the whole stock of high performing equipments is scrapped, last an investment phase in only low performing equipments, $[\bar{t}_h, \infty)$.

A.1.1 The two phases scenario

We now present an algorithmic argument to compute the optimal investment plan characteristics. Denote $\bar{K}_h \equiv K_h(t_h)$, the maximum size of the stock of high performing equipments accumulated at the time t_h. For a given pair

$(\bar{K}_h, t_h)$, the optimal investment plan after t_h is the solution of the following $(A.P.)_l$ problem:

$$\begin{aligned}
\max_{\{k_l(t)\}} \quad & \int_{t_h}^{\infty} \left[u(\bar{K}_h + K_l(t)) - c_l K_l(t) - c_h \bar{K}_h - C_y(r_h \bar{K}_h + r_l K_l(t))\right. \\
& \left. -N_l(k_l(t))\right] e^{-\rho(t-t_h)} dt \\
s.t. \quad & \dot{K}_l(t) = k_l(t) \\
& k_l(t) \geq 0 \ .
\end{aligned}$$

Let us first describe the asymptotic evolutions of the capital stocks and the dual variables. The low performing capital stock converges in the long run towards $\tilde{K}_l$, the K_l level solution of:

$$u'\left(\bar{K}_h + K_l\right) - c_l - r_l c_y\left(r_h \bar{K}_h + r_l K_l\right) \quad = \quad \rho \underline{n}_l \ .$$

This long run capital stock depends on $\bar{K}_h$, denote by $\tilde{K}_l(\bar{K}_h)$ the corresponding implicit relationship by a slight abuse of notation. Taking into account our previous computations:

$$\frac{d\tilde{K}_l(\bar{K}_h)}{d\bar{K}_h} < 0 \quad \text{and} \quad \left|\frac{d\tilde{K}_l}{d\bar{K}_h}\right| < 1 \ .$$

We have already shown in the paragraph 4.5.1 that $\dot{\lambda}_l(t) < \dot{\lambda}_h(t)$. Since $\dot{\lambda}_l(t) < 0$ and $\dot{\lambda}_h(t) < 0$, this is equivalent to $|\dot{\lambda}_l(t)| > |\dot{\lambda}_h(t)| \geq 0$. Taking the limits, $\lim_{t\uparrow\infty} |\dot{\lambda}_l(t)| = 0$ implies that $\lim_{t\uparrow\infty} |\dot{\lambda}_h(t)| = 0$. Let $\tilde{\beta}_h$ denote the asymptotic level of the operational margin on high performing equipments, then:

$$\lim_{t\uparrow\infty} |\dot{\lambda}_h(t)| = 0 \quad \Longrightarrow \quad \lim_{t\uparrow\infty} \lambda_h(t) = \tilde{\beta}_h / \rho \ .$$

Since $\tilde{\beta}_h$ depends on $\tilde{K}_l$ and on $\bar{K}_h$, denote by a slight abuse of notations, $\tilde{\beta}_h(\bar{K}_h)$ the corresponding implicit relationship. Let $\tilde{\lambda}_h \equiv \lim_{t\uparrow\infty} \lambda_h(t) =$

$\tilde{\beta}_h/\rho$. Differentiating and taking into account our previous derivations yield:

$$\begin{aligned}
\frac{d\tilde{\lambda}_h}{d\bar{K}_h} &= \frac{1}{\rho}\frac{d\tilde{\beta}_h}{d\bar{K}_h} \\
&= \frac{1}{\rho}\left[(u'' - r_h^2 c_y') + (u'' - r_h r_l c_y')\frac{d\tilde{K}_l}{d\bar{K}_h}\right] \\
&= \frac{1}{\rho(u'' - r_l^2 c_y')}\left[(u'' - r_h^2 c_y')(u'' - r_l^2 c_y') - (u'' - r_h r_l c_y')^2\right] \\
&= -(r_l - r_h)^2 \frac{u'' c_y'}{\rho(u'' - r_l^2 c_y')} < 0 \ .
\end{aligned}$$

The capacities levels and their marginal values converge asymptotically towards the vector $(\bar{K}_h, \tilde{K}_l, \tilde{\lambda}_h, \underline{n}_l)$. The long run level of accumulated low performing capacities is a decreasing function of $\bar{K}_h$, $d\tilde{K}/d\bar{K}_h < 0$, and the long run marginal value of high performing equipments is also a decreasing function of $\bar{K}_h$, $d\tilde{\lambda}_h/d\bar{K}_h < 0$.

The transitory dynamics of $K_l(t)$ and $\lambda_l(t)$ during the time phase $[t_h, \infty)$ are a general solution of the autonomous differential system:

$$\begin{aligned}
\dot{K}_l(t) &= k_l(\lambda_l(t)) \\
\dot{\lambda}_l(t) &= \rho\lambda_l(t) - \left[u'\left(\bar{K}_h + K_l(t)\right) - c_l - r_l c_y\left(r_h \bar{K}_h + r_l K_l(t)\right)\right] \ ,
\end{aligned}$$

where $k_l(\lambda_l(t))$ is implicitly defined by the optimal investment condition: $\lambda_l(t) = n_l(k_l(t))$.

Let $\lambda_l^*(K_l; \bar{K}_h)$ denote the implicit relationship between λ_l and K_l along the trajectory solution of the above system. It is immediate that $d\lambda_l^*/dK_l = \dot{\lambda}_l/\dot{K}_l < 0$. Furthermore for a given t and a given $K_l(t)$:

$$\frac{\partial \beta_l(t)}{\partial \bar{K}_h} = u'' - r_l r_h c_y' < 0 \ ,$$

implies that:

$$\frac{\partial \lambda_l^*(K_l(t); \bar{K}_h)}{\partial \bar{K}_h} = \int_t^\infty \frac{\partial \beta_l(\tau)}{\partial \bar{K}_h} e^{-\rho(\tau - t)} d\tau < 0 \ .$$

The effect of a larger stock of high performing equipments at time t_h is to shift down the whole λ_l trajectory in the phase plane (K_l, λ_l). The trajectory of $\lambda_h(t) = \lambda_h^*(\lambda_l; \bar{K}_h)$ is also shifted down since $\partial\beta_h/\partial K_h = u'' - r_h^2 c_y' < 0$.

On the other hand, it can be shown that the slope of the optimal curve $\lambda_l^*(\lambda_h)$ describing the optimal trajectory in the dual plane is increased by a higher $\bar{K}$. Consider a given pair (λ_h, λ_l) in the dual plane. For any fixed pair of dual variables:

$$\begin{aligned}
\int_t^\infty \frac{d\beta_l}{d\bar{K}_h} e^{-\rho(\tau-t)} d\tau &= 0 \\
\int_t^\infty \frac{d\beta_h}{d\bar{K}_h} e^{-\rho(\tau-t)} d\tau &= 0 \ .
\end{aligned}$$

Since this should apply to any time t, it results that:

$$\frac{d\beta_l}{d\bar{K}_h} - \frac{d\beta_h}{d\bar{K}_h} = 0 \ .$$

Denote:

$$B_{ll} \equiv \frac{\partial \beta_l}{\partial K_l} = u'' - r_l^2 c_y' < 0 \qquad B_{lh} \equiv \frac{\partial \beta_l}{\partial K_h} = u'' - r_l r_h c_y' < 0$$

$$B_{hh} \equiv \frac{\partial \beta_h}{\partial K_h} = u'' - r_h^2 c_y' < 0 \ .$$

Then the above equality is equivalent to:

$$[B_{ll} - B_{lh}] \frac{dK_l}{d\bar{K}_h} + [B_{lh} - B_{hh}] = 0 \ ,$$

which yields:

$$\frac{dK_l}{d\bar{K}_h} = \frac{B_{hh} - B_{lh}}{B_{ll} - B_{lh}} = \frac{|B_{lh}| - |B_{hh}|}{|B_{ll}| - |B_{lh}|} < 0 \ . \tag{A.1.1}$$

Next:

$$\begin{aligned}
\frac{d}{d\bar{K}_h}\left(\frac{d\lambda_l^*(\lambda_h)}{d\lambda_h}\right)_{(\lambda_l,\lambda_h)\text{ given}} &= \frac{d}{d\bar{K}_h}\left(\frac{\dot{\lambda}_l}{\dot{\lambda}_h}\right) = \frac{d}{d\bar{K}_h}\left(\frac{\beta_l - \rho\lambda_l}{\beta_h - \rho\lambda_h}\right) \\
&= \frac{1}{|\dot{\lambda}_h|^2}\left[\frac{d\beta_l}{d\bar{K}_h}|\dot{\lambda}_h| - \frac{d\beta_h}{d\bar{K}_h}|\dot{\lambda}_l|\right] \\
&= \frac{1}{|\dot{\lambda}_h|}\left[\left(B_{ll}\frac{dK_l}{d\bar{K}_h} + B_{lh}\right)\right. \\
&\quad \left. - \left(B_{lh}\frac{dK_l}{d\bar{K}_h} + B_{hh}\right)\frac{|\dot{\lambda}_l|}{|\dot{\lambda}_h|}\right] \ .
\end{aligned}$$

Denoting by $\pi_\lambda \equiv d\lambda_l^*/d\lambda_h$, the slope of the dual variables trajectories and taking (A.1.1) into account, the sign of $d\pi_\lambda/d\bar{K}_h$ is the sign of the following expression:

$$\begin{aligned}
& \left(B_{ll} - B_{lh}\pi_\lambda\right)\frac{dK_l}{d\bar{K}_h} + \left(B_{lh} - B_{hh}\pi_\lambda\right) \\
= \quad & \frac{\left(B_{ll} - B_{lh}\pi_\lambda\right)\left(B_{hh} - B_{lh}\right) + \left(B_{lh} - B_{hh}\pi_\lambda\right)\left(B_{ll} - B_{lh}\right)}{B_{ll} - B_{lh}} \\
= \quad & \frac{B_{ll}B_{hh} - B_{ll}B_{lh} - B_{lh}B_{hh}\pi_\lambda + \left(B_{lh}\right)^2\pi_\lambda}{B_{ll} - B_{lh}} \\
& + \frac{B_{ll}B_{lh} - \left(B_{lh}\right)^2 - B_{ll}B_{hh}\pi_\lambda + B_{lh}B_{hh}\pi_\lambda}{B_{ll} - B_{lh}} \\
= \quad & \frac{B_{ll}B_{hh} + \left(B_{lh}\right)^2\pi_\lambda - \left(B_{lh}\right)^2 - B_{ll}B_{hh}\pi_\lambda}{B_{ll} - B_{lh}} \\
= \quad & \frac{\left(B_{ll}B_{hh} - \left(B_{lh}\right)^2\right)\left(1 - \pi_\lambda\right)}{B_{ll} - B_{lh}} > 0 \ .
\end{aligned}$$

The above positive sign results from first $B_{ll}B_{hh} - (B_{lh})^2 > 0$ as shown before, second from $\pi_\lambda > 1$, and last from $B_{ll} < B_{lh}$.

Since first the slope of the trajectory $\lambda_l^*(\lambda_h)$ is enlarged by a higher level of $\bar{K}_h$ and second $\tilde{\lambda}_h$ is shifted down by a larger $\bar{K}_h$, we conclude that an increase of $\bar{K}_h$ shifts upward the whole trajectory $\lambda_l^*(\lambda_h)$ inside the interval $[0, \underline{n}_h)$. It results that $\lambda_l^h \equiv \lambda_l(\bar{t}_h)$ is also shifted up.

We now show that the low performing equipment stock at time t_h, we denote by K_l^h, is a decreasing function of $\bar{K}_h$. First the null isocline $\dot{\lambda}_l = 0$, defined by $\lambda_l = \beta_l/\rho$, being shifted down by an increase of $\bar{K}_h$ in the phase plane (K_l, λ_l), a standard property of autonomous differential systems is that the whole set of trajectories solving the differential system in this plane is also shifted down. Thus the optimal trajectory $\lambda_l^*(K_l)$ is shifted down. Since we have already shown that $\lambda_l^h = \lambda_l^*(K_l^h)$ is shifted up by an increase of $\bar{K}_h$, we conclude that K_l^h must be decreased by an increase of $\bar{K}_h$. Thus K_l^h is a decreasing function of $\bar{K}_h$.

We conclude that the $\bar{L}_h$ border is a decreasing curve in the (K_h, K_l) plane. The vector of the dynamic variables at time t_h can be written equivalently as $(\bar{K}_h, K_l^h(\bar{K}_h), \underline{n}_h, \lambda_l^h(\bar{K}_h))$.

Next consider the first phase $[0, t_h)$. The vector $(\bar{K}_h, K_l^h, \underline{n}_h, \lambda_l^h)$ gives a particular solution of the differential system governing the motion of $(K_h(t), K_l(t), \lambda_h(t), \lambda_l(t))$ during this phase. Let $K_{i0} \equiv K_i(0)$, $i = h, l$, be the corresponding initial stock levels of equipments of both types. These stocks are functions of $(\bar{K}_h, t_h)$. It is immediate that:

$$\frac{\partial K_{h0}}{\partial t_h} < 0 \quad \text{and} \quad \frac{\partial K_{l0}}{\partial t_h} < 0 \ .$$

The differential system before t_h being time autonomous and K_l^h being a decreasing function of $\bar{K}_h$, it is immediate that for a fixed time duration t_h:

$$\frac{\partial K_{h0}}{\partial \bar{K}_h} > 0 \quad \text{and} \quad \frac{\partial K_{l0}}{\partial \bar{K}_h} < 0 \ .$$

Let $K_{i0}(\bar{K}_h, t_h)$, $i = h, l$, denote the functions so implicitly defined. The optimal vector $(\bar{K}_h, t_h)$ solves the system of initial conditions:

$$\begin{aligned} K_{h0}(\bar{K}_h, t_h) &= K_h^0 \\ K_{l0}(\bar{K}_h, t_h) &= K_l^0 \ . \end{aligned}$$

Let Δ be the determinant of the Jacobian matrix of this system. Our previous computations yield:

$$\Delta \ = \ \underbrace{\frac{\partial K_{h0}}{\partial \bar{K}_h}}_{(>0)} \underbrace{\frac{\partial K_{l0}}{\partial t_h}}_{(<0)} - \underbrace{\frac{\partial K_{h0}}{\partial t_h}}_{(<0)} \underbrace{\frac{\partial K_{l0}}{\partial \bar{K}_h}}_{(<0)} < 0.$$

The system of initial conditions defines a unique vector $(\bar{K}_h, t_h)$ ending the computation procedure of the optimal investment plan in a two phases scenario.

A.1.2 The three phases scenario

Let t_h^δ be the time at which the scrapping period of high performing equipments begins and (K_h^δ, K_l^δ) be the initial equipments vector at the beginning of the scrapping phase. During the scrapping period, the move of the capital stocks along the scrapping border $\hat{K}_l(K_h)$ implies that :

$$\begin{aligned} \dot{K}_l(t) &= k_l(t) = \frac{d\hat{K}_l}{dK_h}\dot{K}_h(t) = -\frac{d\hat{K}_l}{dK_h}\delta_h(t)K_h(t) \\ &= \frac{u'' - r_h^2 c_y'}{u'' - r_h r_l c_y'}\delta_h(t)K_h(t) \quad t \geq t_h^\delta \ . \end{aligned}$$

Thus, once the optimal capital accumulation path $\{(K_h^*(t), K_l^*(t)), t \geq t_h^\delta\}$ has been determined, the optimal scrapping rate path of high performing equipments, $\delta_h^*(t)$, $t \geq t_h^\delta$, is defined by:

$$\delta_h^*(t) \quad = \quad \frac{u''(K_h^*(t)+K_l^*(t)) - r_h r_l c_y'(r_h K_h^*(t) + r_l K_l^*(t))}{u''(K_h^*(t)+K_l^*(t)) - r_h^2 c_y'(r_h K_h^*(t) + r_l K_l^*(t))} \frac{k_l^*(t)}{K_h^*(t)} \; . \tag{A.1.2}$$

Denote by $\hat{K}_h(K_l)$ the inverse of the function $\hat{K}_l(K_h)$. Consider the following $A.P_h^\delta$ program, or *optimal scrapping program* of high performing equipments:

$$\begin{aligned}
\max_{\{k_l(t)\}} \quad & \int_{t_h^\delta}^{\infty} \Big[u(\hat{K}_h(K_l(t)) + K_l(t)) - c_l K_l(t) - c_h \hat{K}_h(K_l(t)) \\
& -C_y\left(r_h \hat{K}_h(K_l(t)) + r_l K_l(t)\right) - N_l(k_l(t))\Big] e^{-\rho(t-t_\delta)} dt \\
s.t. \quad & \dot{K}_l(t) = k_l(t) \quad , \; K_l(t_\delta) = K_l^\delta \\
& k_l(t) \geq 0 \; , \; \hat{K}_h(K_l(t)) \geq 0 \; .
\end{aligned}$$

The constraint $k_l(t) \geq 0$ can be discarded. In the three phases scenario under examination, the constraint $\hat{K}_h(K_l(t)) \geq 0$ can also be discarded since some strictly positive stock of high performing equipments will be kept operating forever.

Denote by λ_l^δ the co-state variable associated to the low performing stock of equipments. The f.o.c's of the $A.P_h^\delta$ program read:

$$n_l(k_l(t)) \quad = \quad \lambda_l^\delta(t) \tag{A.1.3}$$

$$\begin{aligned}
\dot{\lambda}_l^\delta(t) \quad = \quad & \rho\lambda_l^\delta(t) - \Bigg[u'(K(t))\frac{d\hat{K}_h}{dK_l} + u'(K(t)) - c_h\frac{d\hat{K}_h}{dK_l} - c_l \\
& -r_h c_y \frac{d\hat{K}_h}{dK_l} - r_l c_y\Bigg]
\end{aligned} \tag{A.1.4}$$

Collecting terms, the equation of motion of $\lambda_l^\delta(t)$ can be rewritten as:

$$\dot{\lambda}_l^\delta(t) \quad = \quad \rho\lambda_l^\delta(t) - \left\{[u'(K(t)) - c_l - r_l c_y] + [u'(K) - c_h - r_h c_y]\frac{d\hat{K}_h}{dK_l}\right\} \; .$$

Since $u' - c_h - r_h c_y = 0$ along the scrapping frontier, this last equation simplifies in:

$$\begin{aligned} \dot{\lambda}_l^\delta(t) &= \rho\lambda_l^\delta(t) - \Big[u'(K_l(t) + \hat{K}_h(K_l(t))) - c_l \\ &\quad -r_l c_y \left(r_h \hat{K}_h(K_l(t)) + r_l K_l(t)\right)\Big] \ . \end{aligned} \tag{A.1.5}$$

We deduce from (A.1.5) that the null isocline $\dot{\lambda}_l^\delta = 0$ defines a relationship between λ_l^δ and K_l, that we denote by $\lambda_l^{\delta 0}(K_l)$. Differentiating and taking (4.30) into account:

$$\begin{aligned} \frac{d\lambda_l^{\delta 0}(K_l)}{dK_l} &= \frac{1}{\rho}\left\{ \left[u'' - r_l^2 c_y'\right] + \left[u'' - r_h r_l c_y'\right] \frac{d\hat{K}_h}{dK_l} \right\} \\ &= \frac{1}{\rho}\left\{ \left[u'' - r_l^2 c_y'\right] + \left[u'' - r_h r_l c_y'\right] \frac{u'' - r_l r_h c_y'}{u'' - r_h^2 c_y'} \right\} \ . \end{aligned}$$

Since $u'' - r_h^2 c_y' < 0$, the sign of $d\lambda_l^{\delta 0}/dK_l$ is the opposite of the sign of the following expression:

$$\begin{aligned} & \left[u'' - r_l^2 c_y'\right]\left[u'' - r_h^2 c_y'\right] - \left[u'' - r_h r_l c_y'\right]^2 \\ = \ & (u'')^2 + \left(r_h r_l c_y'\right)^2 - r_h^2 u'' c_y' - r_l^2 u'' c_y' - (u'')^2 - \left(r_h r_l c_y'\right)^2 + 2 r_h r_l u'' c_y' \\ = \ & -\left(r_h^2 + r_l^2 - 2 r_h r_l\right) u'' c_y' \\ = \ & -\left(r_l - r_h\right)^2 u'' c_y' > 0 \ . \end{aligned}$$

Hence $d\lambda_l^{\delta 0}/dK_l < 0$. Furthermore $\lim_{t\uparrow\infty} \lambda_l^{\delta 0}(K_l(t)) = \lim_{t\uparrow\infty} \lambda_l^\delta(t) = \underline{n}_l$ and $\lim_{t\uparrow\infty} K_l(t) = \tilde{K}_l$. Since $\dot{\lambda}_l(t) < 0$, the optimal trajectory is located below the null isocline in the plane (K_l, λ_l) and the variables dynamics converge asymptotically toward the vector $(\tilde{K}_h, \tilde{K}_l, 0, \underline{n}_l)$. Denote by $\lambda_l^\delta = \lambda_l^*(K_l)$ the implicit relationship so defined between λ_l^δ and K_l in the phase plane (λ_l, K_l). Since $\dot{\lambda}_l^\delta < 0$ and $\dot{K}_l > 0$, $d\lambda_l^*/dK_l < 0$.

Note that the dynamics of $\lambda_l(t)$ being defined by the same equation before and after t_h^δ, the function $\lambda_l(t)$ is time differentiable at the time t_h^δ.

During the time phase $[t_h, t_h^\delta)$, the stock of high performing equipments is maintained at a constant level we denote by $\bar{K}_h$, so that $K_h(t_h^\delta) = K_h^\delta = \bar{K}_h$. At time t_h^δ, the vector of characteristic variables is given by $(\bar{K}_h, K_l^\delta, 0, \lambda_l^\delta)$, where $\lambda_l^\delta \equiv \lambda_l^\delta(t_h^\delta)$ by a slight abuse of notations. This vector is a function of $\bar{K}_h$ only. On the one hand, $K_l^\delta = \hat{K}_l(\bar{K}_h)$ defines implicitly $K_l^\delta(\bar{K}_h)$ and $dK_l^\delta/d\bar{K}_h < 0$. On the other hand, λ_l^δ being a function of K_l^δ, itself a function of $\bar{K}_h$, λ_l^δ is a function of $\bar{K}_h$ and:

$$\frac{d\lambda_l^\delta}{d\bar{K}_h} = \underbrace{\frac{d\lambda_l^*}{dK_l}}_{(<0)} \underbrace{\frac{dK_l^\delta}{d\bar{K}_h}}_{(<0)} > 0 \ .$$

We can conclude as such so far concerning the optimal scrapping phase. To a given level of $\bar{K}_h$, the size of the stock of high performing equipments at time t_h, also the maintained level of this stock between t_h and t_h^δ, is associated a unique solution of the $A.P^\delta$ program from any time t_h^δ to be determined later. The industry replaces progressively high performing equipments by low performing ones after t_h^δ. The investment rate in low performing equipments progressively slows down and tends asymptotically to zero. The stocks of equipments converge toward the pair $(\tilde{K}_h, \tilde{K}_l)$ defined as the intersection between the low performing equipments investment frontier L_l and the scrapping frontier.

Note that because $|d\hat{K}_h/dK_l| > 1$, the economy scraps a higher proportion of high performing equipments than it builds low performing ones when the capital pair moves along the scrapping frontier. This implies that $\dot{K}(t) < 0$ during the scrapping phase and thus the energy supply, $q(t)$, should decrease implying in turn a constant rise of the energy price, $p(t)$, during the last scrapping phase.

The vector of characteristic variables at the beginning of the scrapping phase, $(\bar{K}_h, K_l^\delta, 0, \lambda_l^\delta)$ is a function of $\bar{K}_h$, the maintained stock of high performing equipments before the scrapping phase. We have shown that K_l^δ is a decreasing function of $\bar{K}_h$ and that λ_l^δ is an increasing function of $\bar{K}_h$.

Now turn to the study of the phase $[t_h, t_h^\delta)$ of accumulation of low performing equipments, the stock of high performing ones being maintained at the level $\bar{K}_h$. First it can be shown that, as in the two phases scenario, the slope of the $\lambda_l^*(\lambda_h)$ trajectory in the dual plane (λ_h, λ_l) is shifted up by an upward shift of $\bar{K}_h$. $d\lambda_l^\delta/d\bar{K}_h > 0$ implies that:

$$\frac{d\beta_l}{d\bar{K}_h} - \frac{d\beta_h}{d\bar{K}_h} < 0 \ .$$

It is easily verified that the same argument as for the two phases scenario leads to the same conclusion.

Since on the one hand, λ_l^δ is shifted up by a larger $\bar{K}_h$ while on the second hand, the slope of the dual trajectory $\lambda_l^*(\lambda_h)$ is increased, the whole dual trajectory is shifted up. We conclude that $\lambda_l^h \equiv \lambda_l(t_h)$ defined by $\lambda_l^*(\underline{n}_h) = \lambda_l^h$ is also increased by a larger $\bar{K}_h$. The trajectory $\lambda_l^*(K_l)$ being shifted down by a larger $\bar{K}_h$, this property of autonomous differential systems not depending on the length, finite or infinite, of the time phase, we also conclude that K_l^h must be decreased by an increase of $\bar{K}_h$. Thus the vector of characteristic variables at at the beginning of the time phase $[t_h, t_h^\delta)$, $(\bar{K}_h, K_l^h, \underline{n}_h, \lambda_l^h)$ is a function of $\bar{K}_h$ only.

The time needed for the system to move from its position at time t_h, $(\bar{K}_h, K_l^h, \underline{n}_h, \lambda_l^h)$, to its position at time t_h^δ, $(\bar{K}_h, K_l^\delta, 0, \lambda_l^\delta)$, is a function of $\bar{K}_h$ only. Thus once $\bar{K}_h$ and t_h are determined, t_h^δ is also determined. Last the same argument as developed for the two phases scenario determines the optimal pair $(\bar{K}_h, t_h)$ as a function of the initial capital endowments pair (K_h^0, K_l^0).

A.1.3 The four phases scenario

Let $\bar{t}_h$ denote the time at which the whole stock of high performing equipments has been scrapped, also the beginning time of the last phase, $[\bar{t}_h, \infty)$ of accumulation of low performing equipments. Let $\bar{\lambda}_l^h$ be the value of λ_l at the same time. By construction, $\bar{K}_l(\bar{t}_h) = \hat{K}_l(0)$. Since $\lambda_l(K_l)$ is a decreasing function of K_l during the last phase, $[\bar{t}_h, \infty)$, $\bar{\lambda}_l^h = \lambda_l^*(\hat{K}_l(0))$ determines $\bar{\lambda}_l^h$. This determines in turn the vector of characteristic variables at time $\bar{t}_h$: $(0, \hat{K}_l(0), 0, \bar{\lambda}_l^h)$.

We can apply the same kind of algorithmic argument developed for the three phases scenario. For a given pair $(\bar{K}_h, t_h^\delta)$, $K_h^\delta = \hat{K}_l(\bar{K}_h)$ is determined. To the particular solution $(\hat{K}_l(0), \bar{\lambda}_l^h)$ is associated a general solution of the differential system governing the motion of $K_l(t)$ and $\lambda_l(t)$ during the scrapping phase. This determines λ_l^δ as the solution of $\lambda_l^\delta = \lambda_l^*(\hat{K}_l(\bar{K}_h))$. Then $\bar{t}_h$ is determined by a given t_h^δ. The vector of characteristic variables at time t_h^δ

is thus determined as a function of $\bar{K}_h$. Then the same algorithmic argument as for the three phases scenario, for the phases $[t_h, t_h^\delta)$ and $[0, t_h)$ allows to determine the optimal investment plan.