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"The Power of Persistence: How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes"

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The Power of Persistence

How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes

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Abstract: This paper explores how the persistence of demand shocks interacts with monetary policy in New Keynesian frameworks. We identify two key propagation channels: a *permanent income channel*, which amplifies the effects of persistent shocks, and a *real interest rate channel*, which goes in the opposite direction. The balance between these forces depends critically on the aggressiveness of the central bank's response to inflation, giving rise to distinct monetary policy regimes. Under accommodative policies, persistence magnifies the response of output, while aggressive policies dampen these effects. In the intermediate regime, a hump-shaped relationship emerges between persistence and the response of output. Our analysis extends to medium-scale DSGE models, featuring capital accumulation, household heterogeneity, behavioral frictions, working capital, nominal wage and price rigidities, revealing that these dynamics are remarkably robust.

Keywords: Persistence, Demand Shocks, Monetary Policy, New Keynesian Model.

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1. Introduction

Understanding the macroeconomic effects of aggregate shocks and their transmission mechanisms lies at the core of dynamic macroeconomic analysis. While much of the literature has focused on identifying the channels through which these shocks operate, their persistence emerges as a critical factor in shaping their overall impact. This is especially true in forward-looking rational expectations models, where, provided agents value the future positively, the persistence of shocks significantly amplifies their effects.

For instance, in Hall's (1978) partial equilibrium permanent income model with a constant real interest rate, agents' willingness to smooth their consumption profile prompt only modest adjustments in consumption following temporary fluctuations in labor income. In contrast, highly persistent or permanent changes lead to far greater consumption responses. This amplification mechanism is foundational in modern macroeconomic theory (see, *e.g.* Lucas, 1976, Sargent, 1978, for similar effects in other settings) and manifests itself because, in a forward-looking rational expectations model, agents front-load the expected present value of the future effects of the shocks. This mechanism operates purely through a permanent income channel, which is the sole transmission mechanism in the partial equilibrium framework of Hall's (1978) model. In a general equilibrium version of the model —one of the backbone of modern macroeconomics, the real interest rate responds to the shock and ought to counteract the permanent income channel. However, in standard one sector Real Business Cycle (RBC) models (*e.g.* Kydland and Prescott, 1982, King et al., 1988, King and Rebelo, 1999), real interest rate adjustments are typically insufficient (see Beaudry and Guay, 1996, Boldrin et al., 2001) to offset the effects of the permanent income channel on consumption decisions, and the result remains robust: greater persistence leads to larger responses. This conclusion extends to a broader class of models that fail to generate sufficient volatility in real interest rates.

In the New Keynesian (NK) model (*e.g.* Woodford, 2003, Galí, 2015), the interplay between nominal rigidities and an interest rate rule amplifies the volatility of both the nominal and real interest rates, which ought to curb the relative importance of the permanent income channel and therefore alter the positive relationship between persistence and volatility —in particular in face of demand and monetary policy shocks which play a key role in driving fluctuations in these models. Despite the extensive literature on the NK framework —encompassing the role of nominal rigidities, fiscal and monetary policy, and exogenous shocks— there remains a gap in understanding how the persistence of shocks shapes short-term economic fluctuations. This oversight is striking given the critical role of persistence in amplifying or dampening the effects of shocks, whether they originate from demand, supply, or monetary policy. This paper is an attempt to fill this gap. More specifically, we ask: *How does the persistence of demand shocks —more particularly those affecting the dynamic IS curve— (DIS shocks) interact with monetary policy to influence the short run aggregate fluctuations?* A key insight from our analysis is that the relationship between the persistence of demand shocks and their short-run impact on output (hereafter referred to as the output multiplier) is far from straightforward. It hinges fundamentally —and in a non trivial way— on the aggressiveness of monetary policy. Our study provides a useful decomposition to understand the key transmission mechanisms that mediate the effects of persistence under different monetary regimes. This framework helps to clarify why persistent shocks do not always lead to amplified economic effects. It also underscores the importance of tailoring monetary policy to the nature and persistence of shocks, also offering valuable insights for policymakers.

We start by exploring the link between short-term output fluctuations and shock persistence within the baseline Representative Agent New Keynesian (RANK) model (see, *e.g.* Galí, 2015). A key advantage of this version of the NK model, in its log-linear formulation, is its purely forward-looking structure, which allows for a straightforward closed-form solution. This feature makes it possible to analytically examine how the output multiplier responds to the persistence of a demand shock shifting the dynamic IS equation—a DIS demand shock. Crucially, we show that this relationship is tied to the stance of monetary policy.

When a demand shock hits the economy, its propagation and impact on output are shaped by two competing channels. The first, which we call the *permanent income channel*, reflects how the net present value of future income flows affects households consumption and savings decisions. Closely related to the permanent income effect described by Hall (1978), this channel amplifies the response of consumption and output as the persistence of the demand shock increases, holding all else equal. The second channel, which we call the *real interest rate channel*, works in the opposite direction. As inflation rises in response to the shock, the central bank raises nominal interest rates to maintain price stability, which drives up real interest rates discouraging current consumption. This channel acts through intertemporal substitution motives and gives more traction to less persistent shocks. The dynamic interplay between these two forces—the permanent income channel, which amplifies the effects of persistence, and the real interest rate channel, which dampens them—ultimately shapes the impact of the persistence of demand shocks on short-run economic outcomes.

Crucially, we show that, depending on how strongly the central bank reacts to inflation, one of these forces dominates, giving rise to distinct policy regimes. More specifically, we identify three regimes of monetary policy, each defined by how aggressively the central bank responds to inflation. In an *accommodative regime*, in which the central bank reacts weakly to inflation, the permanent income channel dominates. Persistent shocks are particularly powerful in this environment because the central bank is not too aggressive towards inflation, not raising the nominal interest rate, and hence the real interest rate too forcefully. By doing so, it restrains intertemporal substitution motives, which leave the permanent income channel dominate. Greater persistence leads to greater output gains. On the other side of the spectrum, in an *aggressive regime*, the central bank responds very strongly to inflation, prioritizing price stability above all else. The real interest rate channel dominates completely, overshadowing the permanent income channel. Persistent shocks, which might otherwise have stimulated the economy, are counteracted by the higher real interest rates. The output multiplier decreases with the degree of persistence of demand shocks. Importantly, when monetary policy lies in an intermediate regime, the relationship between persistence and output becomes more complex. At first, the permanent income channel remains strong, and the effects of persistence on output continue to grow. However, as persistence increases further, the real interest rate channel starts to counteract the permanent income channel. We then characterize a degree of persistence, which we call the ρ-*max*, above which the output multiplier starts to decline. In other words, in this intermediate regime, the relationship between the persistence of demand shocks and the output multiplier exhibits a hump-shaped pattern. This regime captures the delicate balance between the two channels and highlights the non-linear nature of persistence effects.

We then show that these findings extend to other settings. They straightforwardly extend to monetary policy shocks, for the same reasons, as they play a similar role to DIS demand shock in the baseline NK model. To some extent, they also carry to supply shocks, but, while demand shocks generate a hump-shaped relationship between persistence and the output multiplier, the hump-shaped pattern shows up in the inflation multiplier. Likewise, they also extend to other monetary policy rules, reacting to both current inflation and the output gap, or their forecasts. Importantly, we show that the presence of the hump in the intermediate regime does not hinge on specific microfoundations. In particular, we set up a general, though still analytically tractable, model emcompassing models featuring, among others, heterogeneous agents, behavioral frictions and cognitive biases, life cycle dynamics, financial frictions, preference for wealth. We then show that our main result remains valid in all these versions and is actually a generic feature of the NK model.

We finally assess the quantitative relevance of our analytical results by evaluating their robustness within a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model featuring capital accumulation.[1] The model relaxes some of the strict assumptions of the purely forward-looking baseline framework by incorporating additional propagation mechanisms widely used in the literature. More precisely, the model builds on Christiano et al. (2005), Gabaix (2020) and Bilbiie et al. (2022), thereby creating a rich framework that includes heterogeneous households, working capital, introducing a monetary cost channel in the New Keynesian Phillips Curve (NKPC), both price and wage nominal rigidities. Importantly, it incorporates cognitive discounting, where households and firms form expectations imperfectly, introducing discounting into the dynamic IS equation and the NKPC. Our main findings highlight the robustness of the hump-shaped relationship between the persistence of the demand shock and the output multiplier observed in simpler models. The introduction of backward-looking components—such as capital accumulation, price and wage indexation, and working capital—does not overturn the analytical predictions but instead refines them. Specifically, the findings show that under accommodative monetary policy, the permanent income channel dominates, amplifying the effects of persistence. Conversely, under aggressive policy stances, the real interest rate channel becomes the primary force, suppressing the output response as persistence increases. The intermediate regime, where both channels interact more equally, gives rise to a hump-shaped relationship between persistence and output, reinforcing the non-linear dynamics suggested by the theory.

[1] Rupert and Šustek (2019) insist on the importance of capital accumulation for some key properties of the NK model, as, compared to Galí's (2015) textbook three-equation NK model, it introduces a disconnect between consumption and output.

Our results highlight the critical role of structural features, such as nominal rigidities and behavioral frictions, in shaping these dynamics. It emphasizes the importance of understanding these mechanisms for crafting effective monetary policy in economies facing persistent demand shocks.

Related Literature: This paper contributes to the extensive literature that studies the transmission of demand shocks in New Keynesian models. We relate foremost to four strands of the literature.

First, our theoretical analysis is rooted in the canonical RANK framework of Woodford (2003) and Galí (2015). However, our insights apply to a larger class of tractable New Keynesian models that extend the RANK model by incorporating household heterogeneity through cyclical income inequality (Bilbiie, 2008, Broer et al., 2020, Bilbiie, 2020, Cantore and Freund, 2021, Debortoli and Galí, 2024) or cyclical income risk (Werning, 2015, Acharya and Dogra, 2020, Ravn and Sterk, 2020, Bilbiie, Forthcoming), a departure from the full information rational expectation assumption (Angeletos and Lian, 2018, Farhi and Werning, 2019, Gabaix, 2020, Pfäuti and Seyrich, 2023, Meichtry, 2023, Gallegos, 2024), wealth in the utility and preferences over liquidity (Campbell et al., 2017, Michaillat and Saez, 2021), life cycle dynamics through a Blanchard-Yaari perpetual youth structure (Del Negro et al., 2023), household debt and default premia (Beaudry and Portier, 2018), or a marginal cost channel of monetary policy (Ravenna and Walsh, 2006, Surico, 2008, Beaudry et al., 2024). To the extent that tractable heterogeneous agent NK models (THANK) models are capable of approximating the dynamics of more quantitative incomplete market models, our results also extend to these environments (see, among many others, McKay et al. (2016), Kaplan et al. (2018), Auclert (2019), Bayer et al. (2019), Hagedorn et al. (2019a,b), Auclert and Rognlie (2018)).

Second, our decomposition of the propagation of demand shocks into a permanent income channel and a real interest rate channel connects to a strand of the literature questioning whether the transmission mechanisms in NK models genuinely operate through the real interest rate channel. Some argue that observed consistency between output and real interest rate responses could be a result of specific parameterizations rather than structural properties (Barsky et al., 2007, Rupert and Šustek, 2019, Brault and Khan, 2022). Relative to this literature, we show that the real interest rate channel is indeed present in a wide class of NK models, but its sign and strength are heavily influenced by the interaction between monetary policy and the persistence of shocks.

Third, our quantitative analysis is based on a medium-scale DSGE model that builds upon Christiano et al. (2005), Galí et al. (2007) and Bilbiie et al. (2022). This model incorporates essential features identified in the literature as critical for shaping monetary policy transmission, such as those highlighted in Christiano et al. (2005) and Smets and Wouters (2003, 2007). By using both sticky prices and sticky wages, our framework also relates to Colciago (2011) and Ascari et al. (2017), which extend Erceg et al. (2000) to model sticky wages in the context of heterogeneous households and limited asset market participation.

Finally, we contribute to the growing body of research examining how systematic monetary policy affects the propagation of macroeconomic shocks (Barnichon and

Mesters, 2023, McKay and Wolf, 2023, Hack et al., 2024). Unlike these empirical studies, though, we adopt both a theoretical and quantitative perspective, focusing on the role of time-invariant monetary policy in shaping macroeconomic dynamics.

Paper Outline: The paper proceeds as follows. Section 2 provides analytical insights from the Representative Agent New Keynesian model and lays out our main result in this simple framework. Section 3 offers a discussion of the results, showing how they extend to other shocks and qualifying further the role of monetary policy. This section also discusses how our results extend to a more general New Keynesian framework. Section 4 offers a quantitative analysis and further decomposes the role of various standard mechanisms for the results. A last section concludes. For expositional purposes, all proofs and additional insights are reported in the online appendix to this paper.

2. Demand Shock Persistence in the RANK Model

In this section, we examine the macroeconomic effects of an expansionary demand shock using the textbook three-equation New Keynesian model. We show that the persistence of demand shocks influences the response of output —the output multiplier— through two key propagation channels. The first is a permanent income channel, which amplifies the multiplier and strengthens as persistence increases. The second is a real interest rate channel, which dampens the multiplier and increases in magnitude as persistence rises. We identify three monetary policy regimes that navigate this trade-off and derive, in closed form, a persistence threshold that maximizes the short-run output stimulus as a weighted combination of both channels.

2.1 The Economic Environment

Consider the standard discrete time sticky price RANK economy (see e.g. Woodford, 2003, Galí, 2015), whose equilibrium dynamics are summarized by the following four (log-)linear equations

$$y_t = \mathbb{E}_t\left[y_{t+1}\right] - \left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \bar{r}\right) + \xi_t\ , \tag{1}$$

$$\pi_t = \beta \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa_y y_t\ , \tag{2}$$

$$i_t = \bar{r} + \phi_\pi \pi_t\ , \tag{3}$$

$$\xi_t = \rho \xi_{t-1} + \varepsilon_{\xi,t}\ , \tag{4}$$

where y, π and i denote respectively output, the inflation rate and the nominal interest rate. ξ is a demand shock. $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on the information set available at period t.

Equation (1) specifies the *dynamic IS equation* (DIS) and captures intertemporal consumption-saving decisions, that fundamentally depend on the real interest rate gap $r_t - \bar{r} \equiv i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}$, where $\bar{r} > 0$ is the natural rate of interest.[2] Equation (2) defines

[2] Implicit in this specification is that household's preferences are represented by a time-separable logarithmic utility function and output is used for consumption purposes only. In the absence of technology

the *New Keynesian Phillips Curve* (NKPC), which describes the price-setting behavior of intermediary firms. The sensitivity of current inflation to expected future inflation is governed by the discount factor $\beta \in (0, 1)$, while a marginal cost channel links current inflation to output via $\kappa_y \geq 0$. The exact micro-foundations of the NKPC are immaterial for our main results, although, in some instances, we will find it convenient to refer explicitly to specific micro-foundations (*e.g.* Rotemberg, 1982, Calvo, 1983) to put perspective on some results.[3] Equation (3) describes the behavior of a Central Bank, which follows a simple interest rate rule à la Taylor (1993) and adjusts the nominal interest rate to stabilize current inflation. Importantly, the equilibrium path of the economy is locally determinate if the *Taylor principle* applies, *i.e.*, $\phi_\pi > 1$. Finally, Equation (4) specifies the stochastic process for the *exogenous DIS-demand shock* ξ_t, modeled as an autoregressive process of order one, AR(1), with a positive auto-correlation coefficient $\rho \in [0, 1)$.[4]

The following proposition reports the solution of the model at the rational expectations equilibrium.

PROPOSITION 1. *Under the Taylor principle ($\phi_\pi > 1$), the locally determinate solution of the model writes*

$$y_t = \mathcal{M}_y(\rho)\xi_t\,, \quad \pi_t = \mathcal{M}_\pi(\rho)\xi_t\,, \quad i_t = \bar{r} + \mathcal{M}_i(\rho)\xi_t\,, \quad \textit{and} \quad r_t = \bar{r} + \mathcal{M}_r(\rho)\xi_t\,,$$

where $\mathcal{M}_y(\rho)$, $\mathcal{M}_\pi(\rho)$, $\mathcal{M}_i(\rho)$ and $\mathcal{M}_r(\rho)$ are given by

$$\mathcal{M}_y(\rho) = \frac{1-\beta\rho}{(1-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,, \qquad \mathcal{M}_\pi(\rho) = \frac{\kappa_y}{(1-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,,$$

$$\mathcal{M}_i(\rho) = \frac{\phi_\pi\kappa_y}{(1-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,, \qquad \mathcal{M}_r(\rho) = \frac{(\phi_\pi-\rho)\kappa_y}{(1-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,.$$

The impact multipliers $\mathcal{M}_x(\rho)$, for $x \in \{y, \pi, i, r\}$, are such that the proposed decision rules solve system (1)-(3). In the sequel, particular attention will be devoted to the behavior of the output multiplier, $\mathcal{M}_y(\rho)$ —the impact effect of a DIS-demand shock on output— as the degree of persistence of the shock, ρ, varies.

2.2 Main Results

It is widely understood that, in rational expectations forward-looking models, more persistent shocks have larger effects on macroeconomic outcomes, particularly output.

shocks, the natural rate, r_t^n, is equal to the discount rate $\bar{r} \equiv -\ln\beta > 0$, where $\beta \in (0, 1)$ is the household's discount factor.

[3]In the case of state contingent pricing à la Rotemberg (1982), and assuming quadratic price adjustment costs, $\kappa_y = \frac{\varepsilon-1}{\psi}\frac{1+\varphi}{1-\alpha}$ where $\varphi > 0$ is the inverse Frisch labor supply elasticity, $\alpha \in [0, 1)$ is the degree of decreasing returns to scale, $\varepsilon > 1$ is the demand elasticity and $\psi > 0$ is the price adjustment cost parameter controlling for the degree of price stickiness. In the case of time dependent pricing à la Calvo (1983), $\kappa_y = \Theta\frac{(1-\theta)(1-\beta\theta)}{\theta}$ where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}\left(1+\frac{\varphi+\alpha}{1-\alpha}\right)$ and $\theta \in (0, 1)$ is the probability of not resetting the price in a given period. The key difference for our results is the role played by β in determining the slope of the NKPC.

[4]A rationale for this shock can be found in Fisher (2015). An alternative is to follow Smets and Wouters (2007) who interpret $e^{-\xi_t}R_t$ as the effective nominal return on bonds, resulting in a nominal bond premium of $(e^{-\xi_t}-1)R_t$ or a real bond premium of $(e^{-\xi_t}-1)R_t/\pi_{t+1}$, where $\ln R_t = i_t$. In that case, ξ_t is referred to as a bond premium shock.

This occurs because agents front-load their reactions based on the expected discounted future effects of the shock, with greater persistence increasing its present value and triggering a stronger response. However, the following proposition shows that, even in this baseline model, this relationship may break down depending on the extent to which monetary authorities respond to inflation.

PROPOSITION 2. *Consider a forward-looking and upward-sloped NKPC, i.e., $\beta > 0$ and $\kappa_y > 0$. There exist two cut-off values, $\underline{\phi}_\pi$ and $\overline{\phi}_\pi$, of the central bank's degree of reaction to the inflation gap*

$$\underline{\phi}_\pi \equiv \beta^{-1}\left(1+(1-\beta)^2\kappa_y^{-1}\right) \quad \textit{and} \quad \overline{\phi}_\pi \equiv \beta^{-1}\left(1+\kappa_y^{-1}\right), \quad \textit{where} \quad \overline{\phi}_\pi > \underline{\phi}_\pi > 1\,,$$

such that the impact multiplier of output to a DIS-demand shock satisfies:

(a) *If $\phi_\pi \leq \underline{\phi}_\pi$, $\mathcal{M}_y$ increases monotonously in ρ.*

(b) *If $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, $\mathcal{M}_y$ is hump-shaped in ρ, i.e.,*

$$\exists\, \rho^* \equiv \beta^{-1}\left(1-\sqrt{(\phi_\pi\beta-1)\,\kappa_y}\right) \textit{ such that } \mathcal{M}_y'(\rho) \gtreqless 0 \textit{ if } \rho \lesseqgtr \rho^*.$$

(c) *If $\phi_\pi \geq \underline{\phi}_\pi$, $\mathcal{M}_y$ decreases monotonously in ρ.*

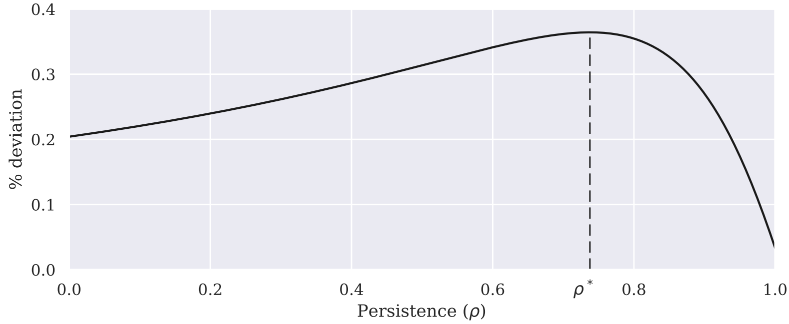
Proposition 2 identifies three regimes arising from the interaction between monetary policy —the Taylor rule— and a persistent discretionary DIS-demand shock. Part (a) of the proposition shows that under a sufficiently accommodative monetary policy — *e.g.*, $\phi_\pi \leq \underline{\phi}_\pi$ — increasing the persistence of the DIS-demand shock unambiguously amplifies the impact response of output. In contrast, Part (c) demonstrates that when the central bank is sufficiently aggressive towards inflation — *e.g.*, $\phi_\pi \geq \overline{\phi}_\pi$ — greater persistence unambiguously dampens the response of output. In the intermediate case, where $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, an increase in persistence initially amplifies output before reducing it beyond a threshold ρ^*. Hence, contrary to the common wisdom, beyond a given persistence level, ρ^*, increasing persistence reduces the impact effect of a demand shock. Hereafter, to ease exposition, we will dub this threshold level of persistence the ρ-max.

Figures 1 and 2 provide a numerical illustration of Proposition 2. For illustrative purposes, we set $\beta = 0.99$, $\kappa_y = 0.15$ and use, following Taylor (1993), a degree of aggressiveness with respect to the inflation gap of $\phi_\pi = 1.5$. Given these numbers, the two policy thresholds, $\underline{\phi}_\pi$ and $\overline{\phi}_\pi$, take the values $\underline{\phi}_\pi = 1.011$ and $\overline{\phi}_\pi = 7.744$, implying that the Taylor rule coefficient lies in the middle regime.

Figure 1 reports the relationship between the output impact multiplier and the persistence of the DIS-demand shock. In line with the common wisdom, the more persistent the shock, the larger the response of output on impact for levels of persistence below $\rho^* = 0.737$. Above this value, the impact multiplier decreases with the level of persistence. Interestingly, the impact response of output for a white noise shock is larger, $\mathcal{M}_y(0) = 0.2$, than for a random walk shock, $\mathcal{M}_y(1) = 0.03$.[5]

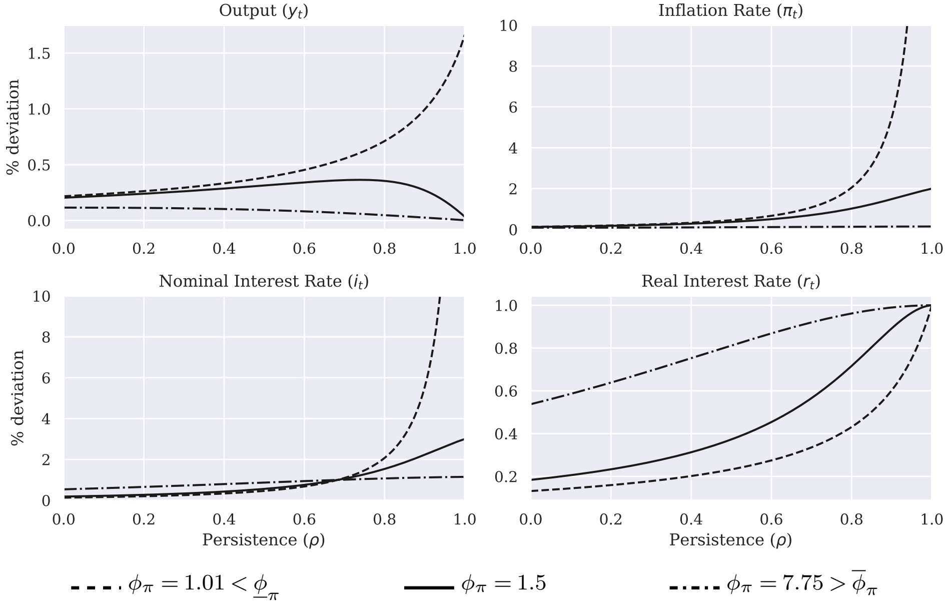
[5]See Section 3.1 for a more detailed analysis of this result.

FIGURE 1. Impact Output Multiplier and Persistence



Note: The impact multiplier is expressed in percentage deviation from steady state. This figure is obtained assuming $\beta = 0.99$, $\kappa_y = 0.15$, $\phi_\pi = 1.5$ and a shock of 25 basis points.

FIGURE 2. Impact Multipliers and Persistence



Note: The impact multipliers are denoted in terms of percentage deviation from steady state. Impact multipliers for inflation, the nominal interest rate, and the real interest rate are expressed in annual terms. This figure is obtained assuming $\beta = 0.99$, $\kappa_y = 0.15$ and a shock of 25 basis points.

Figure 2 reports the impact effect of a positive 25 basis points DIS-demand shock on output, the inflation rate, the nominal and real interest rate as the persistence of the shock is varied. To illustrate the three regimes from Proposition 2, beyond the benchmark case $\phi_\pi = 1.5$, we also consider the cases $\phi_\pi = 1.01$ and $\phi_\pi = 7.75$. The latter two values lie, respectively, below and above the thresholds $\underline{\phi}_\pi$ and $\overline{\phi}_\pi$, implying that, as stated in Proposition 2, the output multiplier is strictly increasing for $\phi_\pi = 1.01$ and strictly decreasing for $\phi_\pi = 7.75$. The hump-shaped pattern we find on output is not present for the inflation rate, and the interest rates. In fact, the annualized responses of inflation and the nominal interest strongly increases in ρ when output monotonously increases.

What are the driving forces at play in this result? The hump-shaped behavior of the short-run output multiplier is the outcome of two competing forces: *(i)* a *permanent income (PI) channel* increasing the multiplier as the persistence of the shock rises and *(ii)* a *real interest rate (RIR) channel* that plays in the opposite direction. To see this more clearly, it is useful to consider the dynamic IS equation (1) at the general equilibrium solution to get:

$$\mathcal{M}_y(\rho) = \rho\mathcal{M}_y(\rho) - \mathcal{M}_r(\rho) + 1 \, .$$

Rearranging term, we obtain:

$$\mathcal{M}_y(\rho) = \frac{1}{1-\rho} - \frac{\mathcal{M}_r(\rho)}{1-\rho} = \underbrace{\overline{\mathcal{M}}_y(\rho)}_{\text{PI Channel}} - \underbrace{\overline{\mathcal{M}}_r(\rho)}_{\text{RIR Channel}} \, . \tag{5}$$

The PI channel, $\overline{\mathcal{M}}_y(\rho) \equiv 1/(1-\rho)$, captures the expected discounted impact of the demand shock on agent's income. This is similar to the permanent income effect of a shock in the standard Hall's (1978) consumption-savings model, assuming a constant real interest rate. Therefore, it encapsulates the partial equilibrium effects of the shock. As such it is totally independent from monetary policy aspects and nominal rigidities, and, as in the standard permanent income model, is strictly increasing in the degree of persistence, i.e., $\overline{\mathcal{M}}_y'(\rho) > 0$.

The RIR channel, $\overline{\mathcal{M}}_r(\rho) \equiv \mathcal{M}_r(\rho)/(1-\rho)$, captures the expected discounted sum of the reaction of the real interest rate to the shock. This term is fundamentally determined by the interplay between persistence, monetary policy and the degree of nominal rigidities. As such, it encapsulates all the general equilibrium effects of the shock. To the extent that $\mathcal{M}_r(\rho)$ increases with the degree of persistence, ρ, the RIR channel plays in the opposite direction to the PI channel. Proposition 3 confirms that, as soon as the Taylor principle holds in this economy, this condition is met in the baseline RANK economy.

PROPOSITION 3. *The real interest rate increases monotonously in the persistence of a DIS-demand shock, i.e., $\mathcal{M}_r'(\rho) > 0$, if and only if the Taylor principle holds.*

In light of Proposition 3, the rationale behind the three regimes can be explained as follows. A higher persistence, on the one hand, increases the real interest rate multiplier, thereby reducing output, *ceteris paribus*. On the other hand, higher persistence raises the expected present value of future income, *ceteris paribus*, prompting an even larger increase in current consumption, and therefore output, due to the forward-looking behavior of agents. When monetary policy is not too aggressive ($\phi_\pi \leq \underline{\phi}_\pi$), the weak response of the nominal interest to the increase in the inflation rate mitigates the RIR channel, allowing the PI channel to dominate. As a result, the output multiplier increases with persistence. In contrast, when monetary policy aggressively targets inflation ($\phi_\pi \geq \overline{\phi}_\pi$), the stronger response of the nominal interest rate amplifies the RIR channel, causing it to outweigh the PI channel. Consequently, the output multiplier decreases as persistence rises. In the intermediate regime, both forces interact, giving rise to the hump-shaped pattern. At low persistence levels, the PI channel dominates, whereas the RIR channel becomes the dominant force at high persistence levels.

To better understand the role of the NKPC structure in shaping the real interest rate response and monetary policy regimes, it is useful to consider two extreme cases. First, if prices are fully rigid,[6] the NKPC is flat ($\kappa_y = 0$), resulting in an impact output multiplier of $\mathcal{M}_y = 1/(1-\rho)$ and a real interest rate multiplier of $\mathcal{M}_r = 0$. In this case, only the PI channel is active, and the output impact multiplier increases monotonically with shock persistence:

$$\mathcal{M}_y'(\rho) = \overline{\mathcal{M}}_y'(\rho) = \frac{1}{(1-\rho)^2} > 0\,.$$

Second, consider the extreme case of a static NKPC ($\pi_t = \kappa_y y_t$), the output impact multiplier reduces to $\mathcal{M}_y = [1-\rho+(\phi_\pi-\rho)\kappa_y]^{-1}$ and the real interest rate multiplier to $\mathcal{M}_r = (\phi_\pi-\rho)\kappa_y\,[1-\rho+(\phi_\pi-\rho)\kappa_y]^{-1}$. In this case, it is easy to check that persistence increases both channels, but the PI channel remains dominant, ensuring monotonicity as seen from the decomposition:

$$\mathcal{M}_y'(\rho) = \underbrace{\frac{1}{(1-\rho)^2}}_{\overline{\mathcal{M}}_y'(\rho)>0} - \underbrace{\frac{(1-\rho+\kappa_y(\phi_\pi-\rho))^2-(1+\kappa_y)(1-\rho)^2}{(1-\rho)^2(1-\rho+\kappa_y(\phi_\pi-\rho))^2}}_{\overline{\mathcal{M}}_r'(\rho)>0} = \frac{1+\kappa_y}{(1-\rho+\kappa_y(\phi_\pi-\rho))^2} > 0\,.$$

In the baseline model, the interplay between the PI and RIR explains the thresholds in persistence. Parameters such as κ_y and ϕ_π reinforce the RIR channel, lowering regime thresholds and ρ-max. A higher discount factor, β, increases the sensitivity of current to future inflation, thus strengthening the real interest rate channel but, in the Calvo (1983) case, lowers the slope of the NKPC, κ_y, thus weakening the RIR channel. This creates a trade-off that impacts regime thresholds and persistence cutoffs depending on the initial NKPC slope.

How prevalent is the hump-shaped regime? This can be simply measured by the size of the intermediate regime, $\Delta_\pi \equiv \overline{\phi}_\pi - \underline{\phi}_\pi = \kappa_y^{-1}(2-\beta)$, which depends fundamentally on the discount factor, β, and the slope of the NKPC, κ_y, that is inversely related to the degree of price stickiness.

COROLLARY 1. *The size of the hump-shaped region, Δ_π, satisfies the following comparative statics:*

1. $\dfrac{\partial \Delta_\pi}{\partial \kappa_y} = -\dfrac{2-\beta}{\kappa_y^2} < 0\,,$

[6] This obtains when $\theta = 1$ in Calvo (1983), or $\psi \to +\infty$ in Rotemberg (1982).

2. $\dfrac{\partial \Delta_\pi}{\partial \beta} = -\dfrac{\kappa_y + (2-\beta)\dfrac{\partial \kappa_y}{\partial \beta}}{\kappa_y^2}$.

The first part of Corollary 1 indicates that a flatter NKPC – identically greater price stickiness – makes the intermediate monetary policy regime unambiguously more prevalent. The second part of the corollary calls for more discussion. Let us first consider state dependent pricing à la Rotemberg (1982), in this case the slope of the NKPC is independent from β and $\partial\kappa_y/\partial\beta = 0$. Accordingly, the size of the intermediate region decreases with the discount factor. The Calvo (1983) case is slightly more intricate as the slope of the NKPC depends, negatively, on the discount factor. Using the specification laid out in footnote 3, we have

$$\frac{\partial \Delta_\pi}{\partial \beta} = -\frac{1-2\theta}{\kappa_y(1-\beta\theta)} \lesseqgtr 0 \text{ if } \theta \lesseqgtr \frac{1}{2}\,.$$

Hence the intermediate region widens with the discount factor when prices are sufficiently sticky.

3. Discussion and Extensions

This section provides a broader perspective on our main findings. First, we emphasize their connection to the demand-side nature of the shocks. Specifically, we examine the dynamics that arise in response to monetary policy shocks and supply shocks. Additionally, this section qualifies the extent to which these results depend on the type of monetary policy rule adopted by the central bank. Finally, it shows how these insights extend to a broader class of New Keynesian models.

3.1 Shocks

So far, our main focus has been placed on DIS–demand shocks. This section investigates the extent with which our results extend to other usual shocks in the literature.

3.1.1 *Monetary Policy Shocks:* All preceding results naturally extend to the case of a standard monetary policy shock —a shock to the Taylor rule. We opt to focus on the DIS-demand shock for the sake of clarity of the exposition only. The output and inflation multipliers associated with monetary policy shocks are identical to those obtained with the DIS-demand shock. They however differ for the interest rates. In New Keynesian models, the nominal interest rate increases on impact if the persistence of the expansionary monetary policy shock is sufficiently high. However, our findings regarding monetary regimes and the ρ-max are independent of this observation, as they are driven by the response of the real interest rate, not the response of the nominal interest rate. This distinction is critical because the ρ-max can differ from the threshold at which the nominal interest rate response changes sign. By contrast, the DIS-demand shock avoids this issue entirely: the nominal interest rate increases monotonically with persistence and does not reverse its sign. Consequently, the DIS-demand shock provides a cleaner framework to analyze the opposing roles of the PI and RIR channels, even though decomposition (5) applies in essentially the same way.

Monetary policy shocks, interpreted as discretionary deviation from the rule, are useful to consider though as they help to put perspective on some policy implications of our results. The results of Proposition 2 indeed indicate that a central bank with the dual mandate of stabilizing both inflation and output, will face no trade-off when it relies on discretionary interventions that are weakly persistent ($\mathcal{M}'_y > 0$ and $\mathcal{M}'_\pi > 0$ for $\rho < \rho^*$). In contrast, when it relies on highly persistent discretionary interventions ($\rho > \rho^*$), a trade-off emerges between stabilizing output and inflation ($\mathcal{M}'_y < 0$ and $\mathcal{M}'_\pi > 0$). The next proposition strengthens this result.

PROPOSITION 4. *If $\phi_\pi > \beta^{-1}\left(1 + (1-\beta)\kappa_y^{-1}\right) > \underline{\phi}_\pi$, the impact output multiplier to a monetary policy shock satisfies $\lim\limits_{\rho\to 0} \mathcal{M}_y(\rho) > \lim\limits_{\rho\to 1} \mathcal{M}_y(\rho)$.*

This proposition shows that when the central bank adopts a sufficiently aggressive policy against inflation, output reacts more strongly to a purely transient shock than to a permanent one. In contrast, inflation's response increases consistently with the persistence of the shock. In this context, a central bank aiming to implement a negative discretionary intervention to stabilize inflation while limiting the impact on output would benefit more from choosing a permanent intervention rather than a purely transient one.

3.1.2 *Cost-Push Shocks:* We now consider a version of the model (1)–(4) in which we replace the DIS-demand shock with a cost-push shock

$$y_t = \mathbb{E}_t\left[y_{t+1}\right] - \left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - \bar{r}\right) \,, \tag{6}$$

$$\pi_t = \beta \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa_y y_t + \nu_t \,, \tag{7}$$

$$i_t = \bar{r} + \phi_\pi \pi_t \,, \tag{8}$$

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t} \,, \tag{9}$$

where ν_t is a cost-push shock with corresponding AR(1) persistence $\rho_\nu \in [0,1)$. In that case, the following proposition obtains.

PROPOSITION 5. *Under the Taylor principle, the impact multipliers of output, $\mathcal{M}_y(\rho_\nu)$, and inflation, $\mathcal{M}_\pi(\rho_\nu)$ to a cost-push shock are given by*

$$\mathcal{M}_y(\rho_\nu) = \frac{-(\phi_\pi - \rho_\nu)}{(1-\rho_\nu)(1-\beta\rho_\nu) + (\phi_\pi - \rho_\nu)\kappa_y} \,, \quad \textit{and} \quad \mathcal{M}_\pi(\rho_\nu) = \frac{1-\rho_\nu}{(1-\rho_\nu)(1-\beta\rho_\nu) + (\phi_\pi - \rho_\nu)\kappa_y} \,.$$

We have:

(a) *The output impact multiplier, $\mathcal{M}_y(\rho_\nu)$, decreases monotonically with ρ_ν.*

(b) *The inflation impact multiplier, $\mathcal{M}_\pi(\rho_\nu)$, satisfies:*

(b.1) *If* $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$, $\mathcal{M}_\pi(\rho_\nu)$ *is humped-shaped in* ρ_ν, *i.e.*

$$\exists\, \rho_\nu^* \equiv 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} \text{ such that } \mathcal{M}_\pi'(\rho_\nu) \gtreqless 0 \text{ for } \rho_\nu \lesseqgtr \rho_\nu^* .$$

(b.2) *If* $\phi_\pi \geq 1 + \frac{\beta}{\kappa_y}$, $\mathcal{M}_\pi(\rho_\nu)$ *decreases monotonically in* ρ_ν.

The first statement of Proposition 5 establishes that, unlike the DIS-demand shock, a cost-push shock does not lead to the emergence of three monetary policy regimes or a hump-shaped relationship between shock persistence and the output impact multiplier. Instead, the output impact multiplier decreases monotonically with the persistence of the shock. The focus shifts to the inflation impact multiplier, where non-monotonic behavior arises. A decomposition similar to (5) applied on the NKPC helps shedding light on the forces at work. Evaluating the NKPC at the general equilibrium yields:

$$\mathcal{M}_\pi(\rho_\nu) = \beta\rho_\nu\mathcal{M}_\pi(\rho_\nu) + \kappa_y\mathcal{M}_y(\rho_\nu) + 1 \,.$$

Rearranging the term, we get

$$\mathcal{M}_\pi(\rho_\nu) = \underbrace{\frac{1}{1-\beta\rho_\nu}}_{\text{PE}} + \underbrace{\frac{\kappa_y\mathcal{M}_y(\rho_\nu)}{1-\beta\rho_\nu}}_{\text{GE}} \,.$$

As with a DIS-demand shock, monetary policy balances two opposing channels. The first pertains to a partial equilibrium (PE) effect, which corresponds to the present value of future expected effects of the cost-push shock assuming a constant marginal cost. Hence, the more persistent the shock, the larger the effect of the shock on inflation. The second component, GE, corresponds to the present value of future expected effects of the cost-push shock encapsulating all the general equilibrium effects affecting the marginal cost. A positive cost-push shock puts upward pressure on prices, which depresses demand —as witnessed by the presence of the output multiplier in the GE term— production and hence the labor demand. This puts downward pressure on wages and so on the marginal cost. Accordingly, this GE effect goes in the opposite direction to the PE effect, creating the hump-shaped relationship between persistence and the response of inflation. When monetary policy becomes extremely aggressive, the central bank is hiking the interest to such an extent that the recession it triggers makes the GE effect fully dominate the determination of the inflation multiplier.

3.1.3 *Technology Shocks:* We now consider a version of the model (1)–(4) in which we replace the DIS-demand shock with a technology shock

$$\widehat{y}_t = \mathbb{E}_t\left[\widehat{y}_{t+1}\right] - \left(i_t - \mathbb{E}_t\left[\pi_{t+1}\right] - r_t^n\right) \,, \tag{10}$$

$$\pi_t = \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \kappa_y\widehat{y}_t \,, \tag{11}$$

$$i_t = \bar{r} + \phi_\pi\pi_t \,, \tag{12}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \,, \tag{13}$$

where $\widehat{y}_t \equiv y_t - y_t^n$ denotes the output gap between actual output y_t and the level of output, y_t^n, attained in the flexible price allocation. Moreover, $r_t^n \equiv \bar{r} + \mathbb{E}_t[\Delta a_{t+1}]$ denotes the natural interest rate, where a_t is a technology shock with corresponding AR(1) persistence $\rho_a \in [0,1)$. In that case, the following proposition obtains.

PROPOSITION 6. *Under the Taylor principle, the impact multipliers of output, $\mathcal{M}_y(\rho_a)$, and inflation, $\mathcal{M}_\pi(\rho_a)$ to a technology shock are given by*

$$\mathcal{M}_y(\rho_a) = \frac{\kappa_y(\phi_\pi - \rho_a)}{(1-\beta\rho_a)(1-\rho_a)+\kappa_y(\phi_\pi-\rho_a)} + \vartheta_y \;, \quad \textit{and} \quad \mathcal{M}_\pi(\rho_a) = \frac{-\kappa_y(1-\rho_a)}{(1-\beta\rho_a)(1-\rho_a)+\kappa_y(\phi_\pi-\rho_a)} \;,$$

where ϑ_y is a constant collecting terms independent of ρ_a. We have:

(a) *The output impact multiplier, $\mathcal{M}_y(\rho_a)$, increases monotonically with ρ_a.*

(b) *The inflation impact multiplier, $\mathcal{M}_\pi(\rho_a)$, satisfies:*

(b.1) *If $1 < \phi_\pi < 1 + \frac{\beta}{\kappa_y}$, $\mathcal{M}_\pi(\rho_a)$ is U-shaped in ρ_a, i.e.*

$$\exists\, \rho_a^* \equiv 1 - \sqrt{\frac{\kappa_y(\phi_\pi - 1)}{\beta}} \; \textit{such that} \; \mathcal{M}'_\pi(\rho_a) \lesseqgtr 0 \; \textit{for} \; \rho_a \lesseqgtr \rho_a^*.$$

(b.2) *If $\phi_\pi > 1 + \frac{\beta}{\kappa_y}$, $\mathcal{M}_\pi(\rho_a)$ increases monotonically in ρ_a.*

The first statement of Proposition 6 establishes that, unlike the DIS-demand shock, a technology shock does not lead to the emergence of three monetary policy regimes or a hump-shaped relationship between shock persistence and the output impact multiplier. Instead, the output impact multiplier increases monotonically with the persistence of the shock. The focus shifts again to the inflation impact multiplier, where non-monotonic behavior arises. As with a DIS-demand shock, monetary policy balances two opposing channels. A positive technology shock reduces the output gap, lowering inflation. Since the output gap exhibits similar comparative statics to output in response to shock persistence, the *output gap channel* increases inflation with persistence. Conversely, with forward-looking inflation ($\beta > 0$), a more persistent technology shock amplifies the drop in inflation via an *inflation expectation channel*. If monetary authorities adopt a moderately aggressive stance toward inflation (Part *b.1* of the proposition), a U-shaped relationship in the inflation impact multiplier emerges. However, under sufficiently aggressive policy (Part *b.2* of the proposition), the inflation expectation channel weakens, and inflation increases monotonically with shock persistence, driven by stronger feedback effects on the output gap.

3.2 The Role of Monetary Policy

The previous section highlighted the interaction between a simple monetary policy rule involving solely the objective of inflation stabilization and the persistence of DIS-demand shocks in shaping their impact effect on output dynamics. This section considers rules that extend the benchmark monetary policy rule along three dimensions. First, we jointly allow for inflation and output feedback. Second, we study forward-looking rules. Third, we consider a real interest rate rule. Proposition 7 summarizes our findings in the case of $\phi_\pi > 1$.[7]

PROPOSITION 7. *Under the Taylor principle, the following results apply:*

(a) *Assume the monetary authorities react to both the current inflation rate and the current output gap, i.e.* $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t$. *Then, Proposition 2 continues to hold identically.*

(b) *Assume the monetary authorities react to the one-step ahead expected inflation and output gap, i.e.* $i_t = \bar{r} + \phi_\pi \mathbb{E}_t[\pi_{t+1}] + \phi_y \mathbb{E}_t[y_{t+1}]$, *with* $\phi_\pi > 1$.[8]

- *If* $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$, *there exist two regime thresholds given by* $\underline{\phi}_\pi \equiv 1 + (1-\phi_y)(1-\beta)^2\kappa_y^{-1}$ *and* $\overline{\phi}_\pi \equiv 1 + (1-\phi_y)\kappa_y^{-1}$ *such that the relationship between the output impact multiplier and the degree of persistence* ρ *(i) is increasing if* $\phi_\pi \leq \underline{\phi}_\pi$, *(ii) is decreasing if* $\phi_\pi \geq \overline{\phi}_\pi$, *and (iii) displays a hump-shaped pattern if* $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$. *In this latter case, the* ρ*-max is given by* $\rho^* = \beta^{-1}\left(1 - \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right)$.
- *If* $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$, *only the upper regime survives and the output multiplier decreases monotonically with* ρ.

(c) *Assume the monetary authorities follow a real interest rate rule, i.e.* $i_t = r_t + \mathbb{E}_t[\pi_{t+1}]$, *where* $r_t = \bar{r}$. *Then, only the lower monetary policy regime survives and the output impact multiplier increases monotonically with* ρ.

Part (a) of Proposition 7 shows that when monetary authorities stabilize current inflation, output stabilization concerns do not alter the monetary policy thresholds $(\underline{\phi}_\pi, \overline{\phi}_\pi)$ or the ρ–max (ρ^*), as established in Proposition 2. This obtains from the fact that $\phi_y > 0$ affects symmetrically the PI and the RIR channels in the DIS equation. However, increasing ϕ_y reduces the impact multipliers of output and inflation.

In contrast, Part (b) reveals that when monetary policy reacts to one-step-ahead expectations of inflation and the output gap, with $\phi_\pi > 1$, output stabilization plays a significant role in determining how persistence shapes the short-run output response to a DIS-demand shock. Three monetary policy regimes still obtain as long as the central bank's output gap response remains moderate, i.e., $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$. Within the hump-shaped regime, the ρ-max decreases with ϕ_y as stronger output stabilization weakens the PI channel. When ϕ_y becomes sufficiently large, i.e. $1 \leq \phi_y <$

[7]The assumption $\phi_\pi > 1$ is made only for expositional purposes, to guarantee the local determinacy of the equilibrium and to ease comparison with the results of Proposition 2. However, in the case of a forward-looking rule, determinacy can also occur with $\phi_\pi \leq 1$ and Section A.1 in the online appendix offers a full statement of the proposition, allowing for $\phi_\pi \leq 1$.

[8]See Proposition 4 in Bullard and Mitra (2002) for a discussion of local determinacy of the equilibrium path for this particular rule.

$\min\{1+\beta^{-1}, 2-\frac{\phi_\pi-1}{1+\beta}\kappa_y\}$, the hump-shaped regime disappears and output declines monotonically with ρ. In this case, the RIR channel dominates strictly the PI channel.

Finally, Part (c) emphasizes the critical role of the RIR channel in moderating the trade-off between the PI and RIR channels. When the central bank controls the real interest rate directly – holding it constant as in Hall's (1978) model – this trade-off vanishes. Consequently, the impact of a DIS-demand shock on output increases monotonically with ρ, as only the PI channel remains active.[9]

Recently, *robust real rate rules* (see Holden, 2024) of the form $i_t = r_t + \phi_\pi \pi_t$, with $\phi_\pi > 1$, have gained attention. These rules are particularly compelling because they remain robust to key features of modern monetary theory, such as household heterogeneity or departure from strict rationality, while ensuring equilibrium stability. This robustness arises because inflation dynamics obtain from the monetary policy rule and the Fisher equation rather than the dynamic IS equation. In other words, this class of rules effectively eliminates both the PI and RIR channels. As a result, they represent an extreme case compared to the dynamics discussed in Proposition 7, bypassing the tension between these two channels.[10]

This observation highlights that our conclusions regarding the hump-shaped behavior of persistent DIS shocks depend fundamentally on how simple policy rules balance the PI and RIR channels. Specifically, under robust real rate rules, DIS demand shocks are entirely absorbed by adjustments in the nominal interest rate, leaving output and inflation completely unaffected. Conversely, loose monetary policy shocks result in expansions of both output and inflation. Interestingly, the output response to such shocks is non-monotonic in persistence, depending on whether ϕ_π is greater or less than the inverse of the sensitivity of current to expected inflation within the NKPC.

3.3 A General New Keynesian Model

In this section, we extend the analytical results of the previous section to a more general tractable model and show that the insights related to the three output regimes of monetary policy carry over to a larger class of New Keynesian models.

3.3.1 *A General Tractable Framework* In Section 2, we outlined the trade-offs inherent to the parsimonious benchmark RANK economy. Here, we extend the analysis to a more general, although tractable, formulation of the NK model:[11]

$$y_t = \zeta_f \mathbb{E}_t[y_{t+1}] - \zeta_r(i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}) + \xi_t\,, \tag{14}$$

$$\pi_t = \beta_f \mathbb{E}_t[\pi_{t+1}] + \kappa_y y_t, \tag{15}$$

[9] This result also applies for rules of the form $i_t = \bar{r} + \mathbb{E}_t[\pi_{t+1}] + \phi_y y_t$, where the sign of output feedback parameter (ϕ_y) captures the cyclicality of the real interest rate (see Angeletos et al., 2024a,b).

[10] In a related vein, Rupert and Šustek (2019) question whether monetary policy shocks propagate via the RIR channel in a broad class of New Keynesian models with physical capital. Compared to their analysis, our findings emphasize that the presence of a RIR channel critically hinges on the feedback rule adopted by the central bank.

[11] The online appendix considers an even more general version of the model in which we introduce a cost channel of monetary policy in the NKPC.

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t \ , \tag{16}$$

$$\xi_t = \rho \xi_{t-1} + \varepsilon_{\xi,t} \ . \tag{17}$$

This representation differs from the benchmark logarithmic RANK model in several ways. First, we introduce two new parameters within the DIS equation (14), ζ_f and ζ_r. The parameter $\zeta_f > 0$ denotes the elasticity of current aggregate demand to expected income. We refer to a *compounded* DIS equation in the case of $\zeta_f > 1$, whereas we refer to a *discounted* DIS equation in the case of $\zeta_f < 1$. The parameter $\zeta_r > 0$ denotes the elasticity of aggregate demand to the real interest rate, which depends in practice, among others, on the elasticity of intertemporal substitution (EIS). Second, we alter the NKPC equation (15) by allowing the elasticity of current inflation to future inflation, i.e., $\beta_f \in [0,1)$, to differ from the discount factor β. Importantly, this representation nests the benchmark RANK model with logarithmic preferences from Section 2 if $\zeta_f = \zeta_r = 1$, $\beta_f = \beta$ and $\phi_y = 0$.[12]. Importantly, in Section 3.3.2 we will discuss how many extensions of the baseline RANK economy admit a representation of the form (14)-(17) in terms of deep structural parameters for $(\zeta_f, \zeta_r, \beta_f, \kappa_y)$. In this sense, we consider the former parameters as *sufficient statistics, i.e.*, their exact structural composition is irrelevant for the derivation of the following analytical results.

Throughout this section, we will assume $\beta_f \in (0,1)$ and $\kappa_y > 0$, which amounts to impose a forward-looking behavior of the NKPC and ensures it is upward slopping. The following proposition follows.

PROPOSITION 8. *Under local determinacy, i.e.*

$$\phi_\pi > \widetilde{\phi} \equiv \max\left(1 + \frac{(1-\beta_f)(\zeta_f - 1 - \zeta_r \phi_y)}{\zeta_r \kappa_y}, \frac{\zeta_f \beta_f - 1 - \zeta_r \phi_y}{\zeta_r \kappa_y}\right)$$

the solution of the model takes the form

$$y_t = \mathcal{M}_y(\rho)\xi_t, \quad \pi_t = \mathcal{M}_\pi(\rho)\xi_t, \quad i_t = \bar{r} + \mathcal{M}_i(\rho)\xi_t, \quad \textit{and} \quad r_t = \bar{r} + \mathcal{M}_r(\rho)\xi_t \ ,$$

where $\mathcal{M}_y$, $\mathcal{M}_\pi$, $\mathcal{M}_i$ and $\mathcal{M}_r$ are given by:

$$\mathcal{M}_y(\rho) = \frac{1-\beta_f \rho}{\Gamma_\xi} > 0 \ , \quad \textit{and} \quad \mathcal{M}_\pi(\rho) = \frac{\kappa_y}{\Gamma_\xi} > 0 \ ,$$

$$\mathcal{M}_i(\rho) = \frac{\phi_\pi \kappa_y + \phi_y(1-\beta_f \rho)}{\Gamma_\xi} > 0 \ , \quad \textit{and} \quad \mathcal{M}_r(\rho) = \frac{(\phi_\pi - \rho)\kappa_y + \phi_y(1-\beta_f \rho)}{\Gamma_\xi} \gtreqless 0 \ ,$$

with $\Gamma_\xi \equiv (1-\beta_f \rho)(1-\rho\zeta_f + \zeta_r \phi_y) + (\phi_\pi - \rho)\zeta_r \kappa_y$. Finally, the sign of $\mathcal{M}_r(\rho)$ is determined by $\phi_\pi \gtreqless \rho - \phi_y(1-\beta_f \rho)/\kappa_y$.

[12]As general as it is, this version does not introduce additional sources of endogenous persistence like capital accumulation, price indexation or interest rate smoothing. Doing so would expand the state space and would considerably complicates the analytics without adding substantial analytical insights. We will consider such extensions in our quantitative exercise of Section 4.

In the lines of Bilbiie (Forthcoming) for a tractable HANK model and Gabaix (2020) for a behavioral version of the RANK model, Proposition 8 first extends the Taylor principle to environments featuring a compounded or discounted DIS equation and/or a behavioral NKPC.[13] Specifically, in the presence of a discounted DIS equation ($\zeta_f < 1$), determinacy does not necessarily require the Taylor principle to hold—*i.e.* local determinacy can obtain under $\widetilde{\phi} < 1$.[14] Conversely, in the case of compounding ($\zeta_f > 1$), the Taylor principle is reinforced, as $\widetilde{\phi} \gg 1$. The proposition then establishes that, provided the equilibrium is locally determinate, a positive DIS shock generates a demand-driven boom ($\mathcal{M}_y > 0$ and $\mathcal{M}_\pi > 0$), that commands an increase in the nominal interest rate. The real interest rate necessarily rises with the shock when the DIS equation is either standard ($\zeta_f = 1$) or exhibits compounding ($\zeta_f > 1$) since in those cases we have $\phi_\pi > \widetilde{\phi} > 1 > \rho - \phi_y(1 - \beta_f \rho)/\kappa_y$. However, when the DIS equation features discounting ($\zeta_f < 1$), $\widetilde{\phi}$ is not necessarily greater than 1, and a positive response of the real interest rate requires monetary policy to be sufficiently aggressive with respect to inflation. We then get the following proposition.

PROPOSITION 9. *Proposition 2 still holds in the general model, replacing β by β_f and κ_y by $\kappa_y^g \equiv (\zeta_r/\zeta_f) \cdot \kappa_y$.*

Proposition 9 generalizes Proposition 2 with respect to the monetary policy regime thresholds $\underline{\phi}_\pi$ and $\overline{\phi}_\pi$, as well as the output-maximizing persistence ρ^*. In the sequel, we will denote these quantities $\underline{\phi}_\pi^g$ and $\overline{\phi}_\pi^g$ and ρ^{g*} to indicate they pertain to the general formulation of the model. Notably, the proposition establishes that their formal expressions share an identical analytical structure when β is replaced by β_f and κ_y is scaled by ζ_r/ζ_f. If ζ_r increases relative to ζ_f, the RIR channel gains in importance relative to the PI channel. As such, regime thresholds and output-maximizing persistence fall. In other words, the results obtained in Proposition 2 are not the outcome of particular assumptions placed on the primitives of the model, or on the micro-foundations underlying it, but are fundamentally related to the generic structure of the NK model. Note that, similarly, Proposition 7 generalizes in the same manner as κ_y is replaced by κ_y^g.

Note that, just like Proposition 2 holds in this economy, so does decomposition (5), which is, in fact, a generic property of the (log-linear) Euler equation:

$$\mathcal{M}_y(\rho) = \frac{1}{1-\zeta_f\rho} - \frac{\zeta_r \mathcal{M}_r(\rho)}{1-\zeta_f\rho} = \underbrace{\overline{\mathcal{M}}_y(\rho)}_{\text{PI Channel}} - \underbrace{\overline{\mathcal{M}}_r(\rho)}_{\text{RIR Channel}} \quad . \tag{18}$$

The decomposition needs to be adjusted for discounting, ζ_f, and the interest rate elasticity, ζ_r, but its interpretation remains identical to that obtained in the baseline model.

[13] The condition actually reduces to the RANK Taylor principle, *i.e.*, $(1-\beta)\phi_y + (\phi_\pi - 1)\kappa_y > 0$ if $\zeta_f = \zeta_r = 1$ and $\beta_f = \beta$.

[14] In that case, the condition for the real interest rate multiplier to increase monotonically with persistence is more stringent.

3.3.2 *Towards a Structural Interpretation* The results presented in Proposition 9 are derived from a general formulation of the New Keynesian framework deliberately silent about the structural micro-foundations of the key *sufficient statistics* $(\zeta_f, \zeta_r, \beta_f, \kappa_y)$. While this abstraction highlights the broad applicability of the reduced-form representation, it forbids a structural interpretation of our results. Therefore, the primary objective of this section is twofold. First, we establish that a wide range of NK models can be represented in the form of equations (14)-(17). Second, we illustrate how specific mechanisms influence the thresholds for systematic monetary policy regimes and the ρ-max. In particular, the recent macroeconomic literature increasingly recognizes the importance of heterogeneity across households, behavioral frictions, and life-cycle dynamics in shaping economic outcomes and policy effectiveness. We explore how these factors interact with monetary policy and aggregate dynamics, emphasizing their role in modifying equilibrium thresholds and persistence measures within NK frameworks.[15]

Household Preferences and Consumption Smoothing: Household preferences play a central role in determining the responsiveness of consumption to interest rates and inflation. A natural starting point is the standard constant relative risk aversion (CRRA) framework, where the elasticity of inter-temporal substitution (EIS) affects both the elasticity of the real interest rate in the DIS equation and the slope of the NKPC. A higher EIS strengthens the RIR channel through greater consumption smoothing but simultaneously flattens the NKPC. This interaction highlights a fundamental trade-off: monetary policy must balance its influence on demand with its impact on inflation expectations. In practice, the dominance of the RIR channel suggests that economies with higher EIS require less aggressive policy responses to stabilize inflation and output in the short-run. In other words, a higher EIS decreases $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Recent extensions of CRRA preferences introduce wealth dependence, reflecting the idea that relative wealth influences utility. For instance, Michaillat and Saez (2021) addresses anomalies observed at the zero lower bound by incorporating wealth in the utility function into a RANK framework. In this setup, a discounted DIS equation emerges when the marginal utility of wealth is positive, effectively dampening the PI channel. Hence, just like a higher EIS, wealth in the utility function decreases $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Household Heterogeneity and Income Inequality: Empirical evidence (see, *e.g.*, Campbell and Mankiw, 1989, 1991) indicates that consumption-saving behavior deviates from the predictions of the standard Euler equation. Models incorporating household heterogeneity – such as the two-agent New Keynesian (TANK) framework of Bilbiie (2008, 2020) – provide insights into these deviations, particularly through cyclical variations in income inequality. In these economies, cyclical income inequality between savers and hand-to-mouth households significantly alters the transmission of monetary policy. Counter-cyclical income inequality, by increasing the real interest rate elasticity, ζ_r, amplifies the impact of monetary policy while pro-cyclical inequality dampens it. Notably, as cyclical inequality approaches some critical thresholds, the ρ-max approaches zero, underscoring the destabilizing potential of income disparities.

[15] Formal statements and proofs of the following intuitions are reported to the technical appendix for exposition purposes.

In addition to cyclical inequality, cyclical income risk alters household consumption-saving behavior as well. For instance, in the pseudo-representative agent New Keynesian (PRANK) framework of Acharya and Dogra (2020), pro-cyclical income risk generates discounted dynamic IS (DIS) behavior, whereas counter-cyclical income risk produces compounded DIS dynamics. Models that integrate both cyclical inequality and income risk, such as the tractable heterogeneous agent New Keynesian (THANK) model (see, e.g., Bilbiie, Forthcoming), show that counter-cyclical inequality amplifies the RIR channel relative to PI channels. This effect is particularly pronounced when a specific condition on the share of hand-to-mouth households (λ) and the persistence of saver status (s) is met ($\lambda + s > 1$). Under these conditions, counter-cyclical (and, conversely, pro-cyclical) income inequality reduces (or increases) $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Importantly, contemporaneous income risk has a limited impact, as it affects uniformly the PI and RIR channels. In contrast, expectations of future income risk play a much more significant role, altering the relative strength of these channels and thereby shaping dynamic responses. Specifically, future counter-cyclical (or pro-cyclical) income risk raises (or lowers) $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Behavioral Frictions and Cognitive Biases: Models that depart from the Full Information Rational Expectation (FIRE) assumption by introducing limited foresight through cognitive discounting (see, e.g., Gabaix, 2020) reduce the sensitivity of the allocation to both the PI and inflation expectation channels. This framework modifies policy dynamics in two ways. First, by introducing discounting into the dynamic IS equation ($\zeta_f < 1$), it weakens the PI channel. Second, by reducing inflation persistence ($\beta_f < \beta$), it dampens the RIR channel, diminishing the role of inflation expectations in the NKPC. Interestingly, the latter effect typically dominates, calling for a more aggressive monetary policy to stabilize inflation. As a result, cognitive discounting increases $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Further research, such as Angeletos and Lian (2018), extends these findings by exploring the role of incomplete information and higher-order beliefs. These frameworks yield dynamics akin to those generated by discounted Euler equations, strengthening the link between cognitive frictions and monetary non-neutrality. Hybrid models that combine household heterogeneity with deviations from FIRE (see, e.g., Pfäuti and Seyrich, 2023, Meichtry, 2023, Gallegos, 2024) reveal how market incompleteness and cognitive frictions, sticky or dispersed information jointly influence monetary policy transmission. Specifically, these elements amplify the RIR channel while dampening the PI channel. Consequently, they lead to a reduction in $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*}.

Life-Cycle Dynamics and Stochastic Mortality: Recent studies (see, e.g., Del Negro et al., 2023) incorporate perpetual youth structures (Blanchard, 1985, Yaari, 1965) into medium-scale DSGE models, primarily to address the forward guidance puzzle. In these frameworks, stochastic death probabilities act on three key margins. First, they lead to discounted DIS equations ($\zeta_f < 1$), reflecting households' shorter effective planning horizons. Second, they reduce the sensitivity of current inflation to inflation expectations ($\beta_f < \beta$). Third, they steepen the slope of the NKPC. These findings align with insights from Eggertsson et al. (2019), which highlight how life-cycle effects reduce the persistence of expectations, thereby weakening the efficacy of forward guidance policies. Demographic considerations in these models generally increase $\underline{\phi}_{\pi}^{g}$, while their effects on $\overline{\phi}_{\pi}^{g}$ and ρ^{g*} depend heavily on the degree of price stickiness.

Debt, Default, and Borrowing Constraints: Borrowing constraints and default risk offer another mechanism through which household heterogeneity affects macroeconomic dynamics. Models such as Beaudry and Portier (2018) introduce information asymmetries between borrowers and lenders, resulting in borrowing costs that depend on debt levels. These costs impact both the PI channel and the RIR channel. Specifically, borrowing constraints cause $\zeta_f < 1$ and bound ζ_r from above —strictly below the EIS. While these dynamics alter household consumption behavior, they leave monetary policy thresholds and the ρ-max essentially unchanged compared to standard CRRA models, as the ratio ζ_f/ζ_r is independent of the sensitivity of borrowing rates to debt.

Overall, these findings highlight the pivotal role of household preferences, heterogeneity, and frictions in shaping macroeconomic responses. Whether through cyclical income inequality and risk, behavioral biases, life-cycle dynamics, or borrowing constraints, these factors reshape the transmission channels of monetary policy and influence policy regimes and the ρ-max.

4. Quantitative Insights from a Medium-Scale DSGE Model

The previous sections provided analytical insights into the interplay between the persistence of DIS-demand shocks and monetary policy within a purely forward-looking framework. While this approach offers sharp intuition, it abstracts from several empirically relevant features of modern economies. In this section, we complement the analytical results by assessing their robustness in a medium-scale DSGE model that relaxes the purely forward-looking structure and incorporates additional propagation mechanisms commonly used in the DSGE literature.[16] Specifically, we build upon Christiano et al. (2005), Gabaix (2020), and Bilbiie et al. (2022) to extend the standard RANK framework and provide a flexible and empirically relevant environment for analyzing the dynamic effects of persistence and monetary policy. By bridging analytical and quantitative approaches, this extension not only validates the theoretical predictions but also uncovers new mechanisms shaping short-run trade-offs.

4.1 The Model Economy

The economy is populated by infinitely-lived heterogeneous households, final and intermediate good firms, a financial intermediary, a government, and a central bank. Throughout, we depart from the classical full information rational expectation (FIRE) assumption and introduce cognitive discounting in the DIS and NKPC equations of the model (see the generalized analytical model of Section 3.3.1).

[16] For expositional purposes, this section totally focuses on DIS-demand shocks. We also experimented with monetary policy shocks, cost-push shocks and technology shocks and recovered results inline with those obtained in the previous section.

4.1.1 *Households* The economy features a unit mass of infinitely-lived households, indexed by $i \in [0,1]$, with preferences over consumption C_t^i and labor L_t^i represented by the following utility function:

$$\mathcal{U}(C_t^i, L_t^i) = \frac{{C_t^i}^{1-\sigma}}{1-\sigma} - \upsilon \frac{{L_t^i}^{1+\varphi}}{1+\varphi} \,,$$

where σ^{-1} is the inter-temporal elasticity of substitution (EIS), $\upsilon > 0$ is a labor disutility parameter, and φ^{-1} is the Frisch elasticity. Households discount the future at the constant rate $\beta \in (0,1)$. Finally, households supply a variety of differentiated labor input $L^i(l)$. Importantly, wage-setting decisions are made by labor type specific unions indexed by $l \in (0,1)$, detailed below. While household share the same preferences, they are heterogeneous with respect to their access to financial markets. Households are subject to idiosyncratic shocks that make them switch between two states — saver, S, and hand-to-mouth, H, households. Savers participate in asset markets on which they hold a portfolio consisting of three types of assets: *(i)* liquid bonds, *(ii)* shares of illiquid real stocks of intermediary good firms, and *(iii)* illiquid physical capital. Hand-to-mouth households cannot participate in asset markets and insure against income risk by holding liquid bonds. Transitions between states are modeled as a Markov process with probabilities $\mathbb{P}(H_{t+1} \mid S_t) = 1 - s$ and $\mathbb{P}(S_{t+1} \mid H_t) = 1 - h$, where $s, h \in (0,1)$ govern persistence. The stationary equilibrium ensures the unconditional mass of hand-to-mouth households is $\lambda = \frac{1-s}{2-h-s}$.[17]

At the beginning of period t, all households of a given group (S or H) pool resources, hence fully insuring each other and observe the aggregate shock prior to making their consumption and saving decisions. At the end of the period, they observe their period $t+1$ state on the asset market, and pool their liquid assets only with their peers. For instance, let B_{t+1}^S (resp. B_{t+1}^H) denote the aggregate real bond holdings by all savers (resp. hand-to-mouth households) at the beginning of period $t+1$. B_{t+1}^S comprises all the individual liquid assets holdings, b_{t+1}^S of the $(1-\lambda) \times s$ period t savers that remain savers in $t+1$, to which add all the individual liquid assets holdings, b_{t+1}^H of the $\lambda \times (1-h)$ period t hand-to-mouth households that become savers in $t+1$. Accordingly, we have

$$B_{t+1}^S = (1-\lambda) s b_{t+1}^S + \lambda(1-h) b_{t+1}^H \,. \tag{19}$$

Similarly, B_{t+1}^H comprises all the individual liquid assets holdings, b_{t+1}^h of the $\lambda \times h$ period t hand-to-mouth households that remain hand-to-mouth in $t+1$, to which add all the individual liquid assets holdings, b_{t+1}^s of the $(1-\lambda) \times (1-s)$ period t savers that become hand-to-mouth households in $t+1$,

$$B_{t+1}^H = \lambda h b_{t+1}^H + (1-\lambda)(1-s) b_{t+1}^S \,. \tag{20}$$

Physical capital is illiquid, cannot be transferred across states and accumulates as

$$K_{t+1}^S = (1-\delta) K_t^S + \Phi\left(\frac{I_t^S}{K_t^S}\right) K_t^S \,, \tag{21}$$

[17] In this setup, transition probabilities also represent the proportion of households switching states.

where $\delta \in (0,1]$ denotes the constant capital depreciation rate and $\Phi(\cdot)$ describes how investment in final goods is turned into capital goods. This technology is increasing with investment intensity ($\Phi'(\cdot) > 0$) and exhibits decreasing returns to scale ($\Phi''(\cdot) \leq 0$), a manifestation of capital adjustment costs. We further impose $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$ such that these costs are not operative in the deterministic steady state. Note that, since only savers have access to capital markets, we have $K_t = (1-\lambda)K_t^S$ and $I_t = (1-\lambda)I_t^S$.

Saver: Saver households enter period t with the bond holdings, $B_t^S/(1-\lambda)$, carried from period $t-1$ and which yield a real return $\xi_{t-1}R_{t-1}/\pi_t$, where R is the gross nominal interest rate, π is the inflation rate between $t-1$ and t, and ξ is our DIS demand shock. In the sequel, we will refer to it as the DIS demand shock, rather than to a bond premium shock, to insure consistency with Section 2. The shock is assumed to follow an exogenous AR(1) process defined in Equation (4). On top of these liquid assets, savers also carry physical capital K_t^S that yield an after tax return $(1-\tau^K)R_t^K$, where $\tau^K \in [0,1)$ denotes the constant capital income tax and R_t^K is the real return on capital. Finally, their capital income also comprises $\omega_t^S/(1-\lambda)$ stocks each yielding an after tax dividend $(1-\tau^D)D_t$, with $\tau^D \in [0,1)$, and that resell at price q_t. The savers also receive a transfer F_t^S from a financial intermediary and pay a lump sum tax T_t^S to the government. Finally, they supply a variety of differentiated labor inputs $L_t^S(l)$ at a nominal wage $W_t(l)$, which generates a labor income $y_t^S = \int_0^1 W_t(l)L_t^S(l)\mathrm{d}l/P_t$. These revenues finance consumption and investment expenditures, C_t^S and I_t^S, and savers' financial investment in the form of liquid bonds, b_{t+1}^S and stocks, ω_{t+1}^S, so as to maximize their intertemporal utility. Accordingly, the optimal plan of a saver solves the Bellman equation

$$V^S(B_t^S, \omega_t^S, K_t^S) = \max_{\{C_t^S,\, I_t^S,\, b_{t+1}^S,\, \omega_{t+1}^S\}} \mathcal{U}(C_t^S, L_t^S) + \beta \widetilde{\mathbb{E}}_t \left[V^S(B_{t+1}^S, \omega_{t+1}^S, K_{t+1}^S) + \frac{\lambda}{1-\lambda} V^H(B_{t+1}^H) \right]$$

subject to

$$C_t^S + b_{t+1}^S + q_t \frac{\omega_{t+1}^S}{1-\lambda} + I_t^S = y_t^S + \frac{\xi_{t-1}R_{t-1}}{\pi_t} \frac{B_t^S}{1-\lambda} + \left(q_t + (1-\tau^D)D_t \right) \frac{\omega_t^S}{1-\lambda}$$
$$+ (1-\tau^K)R_t^K K_t^S - T_t^S + F_t^S, \tag{22}$$

$$b_{t+1}^S \geq 0 \quad \text{and} \quad (19) - (21)\,,$$

where the value function accounts for the possibility that, in period $t+1$, the saver may become a hand-to-mouth household.[18] $\widetilde{\mathbb{E}}_t$ denotes the expectation operator that may deviate from the full information rational expectation operator as discussed below.

Hand-to-mouth: The problem of hand-to-mouth households is similar to that of savers, to the notable exception that they do not have access to capital markets, implying that they cannot accumulate neither capital nor stocks and receive transfers from the

[18] The next period hand-to-mouth value function is scaled by $\frac{\lambda}{1-\lambda}$ as the state variable is written in terms of total island bond holdings. As such, the relative transition probability $\frac{\lambda}{1-\lambda}$ maps into the corresponding individual transitions probabilities $(s, 1-s)$.

government ($T_t^H \leq 0$). Just like savers, hand-to-mouth households decide consumption expenditures, C_t^H, their liquid bond holdings, b_{t+1}^H, so as to maximize their intertemporal utility. Accordingly, the optimal plan of a saver solves the Bellman equation

$$V^H(B_t^H) = \max_{\{C_t^H, b_{t+1}^H\}} \mathcal{U}(C_t^H, L_t^H) + \beta\widetilde{\mathbb{E}}_t\left[V^H(B_{t+1}^H) + \frac{1-\lambda}{\lambda}V^S(B_{t+1}^S, \omega_{t+1}^S, K_{t+1}^S)\right]$$

subject to

$$C_t^H + b_{t+1}^H = y_t^H + \frac{\xi_{t-1}R_{t-1}}{\pi_t}\frac{B_t^H}{\lambda} - T_t^H, \tag{23}$$

$$b_{t+1}^H \geq 0 \quad \text{and} \quad (19) - (20)\,,$$

where $y_t^H \equiv \int_0^1 W_t(l)L_t^H(l)\mathrm{d}l/P_t$ denotes real labor income of hand-to-mouth agents.

Finally, we select an equilibrium on the financial market by placing the following assumptions on asset holdings.

ASSUMPTION 1.

$(A1.a)$ *There is perfect insurance across households in the same state, but not across states.*

$(A1.b)$ *Stocks and physical capital are illiquid and cannot be transferred across states.*

$(A1.c)$ *Liquid bond holdings are weakly positive and the positivity constraint of hand-to-mouth households binds in each period.*

$(A1.d)$ *No bonds are traded in equilibrium.*

$(A1.a)$–$(A1.d)$ jointly apply and ensure a tractable representation of the bond market equilibrium. Under $(A1.a)$–$(A1.b)$, all savers (resp. hand-to-mouths) make equivalent consumption and saving decisions. $(A1.c)$ guarantees that H households are strictly hand-to-mouth, while $(A1.d)$ imposes a zero liquidity limit (see e.g. Krusell et al., 2011).

4.1.2 *Expectation Formation:* We follow Gabaix (2014, 2020) and introduce cognitive discounting on the effects of aggregate shocks.[19] Consider a random variable X_t and let $\overline{X}_t$ be the default value that agents assign to this variable in a given period t.[20] Let $\widehat{X}_{t+1} \equiv X_{t+1} - \overline{X}_t$ be the deviation of the future realization X_{t+1} from the default value as of period t. The behavioral expectation operator, $\widetilde{\mathbb{E}}_t[\cdot]$, is then defined as

$$\widetilde{\mathbb{E}}_t\left[X_{t+1}\right] = \widetilde{\mathbb{E}}_t\left[\overline{X}_t + \widehat{X}_{t+1}\right] = \overline{X}_t + \mu\mathbb{E}_t\left[\widehat{X}_{t+1}\right]\,, \tag{24}$$

[19] Notice that saver households have positive wealth holdings due to the two asset structure, even under the zero liquidity limit. To ensure comparability of the cognitive discounting across a larger class of models, we differ from Gabaix (2020) by assuming that the behavioral expectation operator (24) is defined with respect to the state comprising only the aggregate shock, not individual wealth. In mathematical terms, we assume $\widetilde{\mathbb{E}}_t\left[X_{t+1}^S(\mathcal{S}_{t+1}, K_{t+1}^S)\right] = \widetilde{\mathbb{E}}_t\left[X_{t+1}^S(\mathcal{S}_{t+1}, 0)\right]$ for some exogenous aggregate state vector $\mathcal{S}_t$. This assumption is innocuous in the sense that it maps into the same counterparts of the *sufficient statistics* (ζ_f, β_f) and only changes their values without affecting their expression.

[20] For example, agents may have in mind the steady value of X_t as a default value, in which case $\overline{X}_t = \overline{X}$.

where $\mathbb{E}_t$ is the standard conditional expectation operator used under FIRE and $\mu \in [0,1]$ is the cognitive discounting parameter. If $\mu = 1$ then $\widetilde{\mathbb{E}}_t\left[X_{t+1}\right] = \mathbb{E}_t\left[X_{t+1}\right]$ and expectations are rational. In contrast, if $\mu = 0$ then expectations are fully anchored to the default value $\overline{X}_t$. Subsequently, we follow Gabaix (2020) and assume that households correctly perceive the contemporaneous real interest rate on bond holdings and physical capital. We denote the cognitive discounting parameter of the self-insurance equation by μ_b and the one of the physical capital investment equation accordingly by μ_k.[21]

4.1.3 *Firms* The economy comprises a continuum $j \in [0,1]$ of intermediary and a continuum of completely identical final good firms.

Final Good Sector: The final good sector is populated by a continuum of identical firms that act in a competitive environment. They purchase intermediate goods, $X_t(j)$, at price $P_t(j)$ on a competitive market and produce a homogeneous good, Y_t, that is sold for consumption and investment purposes at a price P_t and produced with the constant return technology

$$Y_t = \left(\int_0^1 X_t(j)^{\frac{\epsilon_p}{\epsilon_p - 1}} \mathrm{d}j\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \quad \text{with} \quad \epsilon_p > 1\,.$$

Profit maximization leads to the constant elasticity demand for intermediate good j

$$X_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t\,,$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} \mathrm{d}j\right)^{\frac{1}{1-\epsilon_p}}$ denotes the aggregate price level.

Intermediate Good Sector: Each intermediate good j is produced by means of capital, $K_t(j)$, and labor $L_t(j)$ according to a constant returns to scale technology described by the production function

$$X_t(j) = A K_t(j)^{\alpha} L_t(j)^{1-\alpha}\,, \quad \text{with} \quad A > 0 \text{ and } \alpha \in (0,1)\,.$$

The labor input used by firm j is a composite of differentiated labor types and defined by $L_t(j) = \left(\int_0^1 L_t(j,l)^{\frac{\epsilon_w}{\epsilon_w - 1}} \mathrm{d}l\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$.

Intermediary good firms rent capital and labor in perfectly competitive factor markets. However, the firm is subject to working capital and workers must be paid ahead of production. Consequently, every intermediate firm j borrows its nominal wage bill $W_t L_t(j) = \int_0^1 W_t(l) L_t(j,l)\mathrm{d}l$ from a financial intermediary and repays the loan at the end of period at a interest rate R_t. For simplicity, we assume that the financial intermediary has access to an inexhaustible credit line with the central bank at no interest

[21] Heterogeneity in cognitive discount factors is meant to capture a combination of heterogeneous attention to specific variables and a uniform cognitive discounting of the state, leading to a *term structure of attention*, see Gabaix (2020), Section V.A.

payments, and that it redistributes its aggregate profits at the end of period to savers —*i.e.*, $F_t^S = (R_t - 1)W_t L_t$.

Intermediary goods firms behave under monopolistic competition, i.e., firm j has local monopoly power and sets its price as in Calvo (1983). For instance, only a fraction $1-\theta_p$ can adjust their price in each period. The remaining firms cannot re-optimize their price and instead partially adjust their prices to past inflation according to $P_{t+\tau|t}(j) = P_{t+\tau-1|t}(j)\Xi_{t+\tau-1}^{\chi_p}$, where $\Xi_t = \pi_{t-1}\Xi_{t-1}$ $\forall t \geq 1$ with $\Xi_0 = 1$, and $\chi_p \in [0,1]$ reflects the degree of indexation to past inflation. The firms that are allowed to re-adjust their price do so by maximizing their discounted expected profits

$$\max_{P_t^*} \widetilde{\mathbb{E}}_t^p \left[\sum_{\tau=0}^{\infty} \theta_p^\tau \Psi_{t,t+\tau} Y_{t+\tau|\tau}(j) \left(\frac{P_t^* \Xi_{t+\tau-1}^{\chi_p}}{P_{t+\tau}} - MC_{t+\tau} \right) \right] \; s.t. \; Y_{t+\tau|\tau}(j) = \left(\frac{P_t^* \Xi_{t+\tau-1}^{\chi_p}}{P_{t+\tau}} \right)^{-\epsilon_p} Y_{t+\tau} \, ,$$

where $\Psi_{t,t+\tau} \equiv \beta^\tau \left(C_{t+\tau}^S / C_t^S \right)^{-\sigma}$ is the savers' stochastic discount factor, $MC_{t+\tau}$ is the marginal cost of each intermediate firm and $\widetilde{\mathbb{E}}_t^p[\cdot]$ denotes the behavioral expectation operator of intermediate firms with cognitive discounting parameter μ_p.[22] Finally, aggregating across firms and using the Poisson process associated with the Calvo fairy, the aggregate price index satisfies

$$P_t^{1-\epsilon_p} = \theta_p \left(P_{t-1} \pi_{t-1}^{\chi_p} \right)^{1-\epsilon_p} + (1-\theta_p)(P_t^*)^{1-\epsilon_p} \, .$$

4.1.4 *Nominal Wage Rigidities:* Just like prices, nominal wages are subject to nominal rigidities. Following Colciago (2011), we assume that each variety (or skill) of labor, $l \in [0,1]$, supplied by households is represented by a labor union. Households are uniformly distributed across these unions.[23] Each skill-specific union aggregates labor across households and sets nominal wages subject to Calvo (1983)–type nominal rigidities. In any given period, only a fraction $1-\theta_w$ of labor skill-specific unions can re-optimize their wages. The remaining fraction adjusts wages based on past price inflation, such that $W_{t+\tau|t}(l) = W_{t+\tau-1|t}(l)\Xi_{t+\tau-1}^{\chi_w}$, where $\chi_w \in [0,1]$ reflects the degree of wage indexation to inflation, and Ξ_t was defined earlier. When allowed to re-optimize, the union sets wages to maximize the utilitarian expected welfare of its members – both savers and hand-to-mouth workers – subject to the aggregate demand for labor of type l.[24] The union's optimization problem is:

$$\max_{W_t^*(l)} \widetilde{\mathbb{E}}_t^w \left[\sum_{\tau=0}^{\infty} (\beta\theta_w)^\tau \left(\lambda \mathcal{U}(C_{t+\tau}^H, L_{t+\tau}^H) + (1-\lambda)\mathcal{U}(C_{t+\tau}^S, L_{t+\tau}^S) \right) \right]$$

[22]Cost minimization of intermediary firms implies that $\frac{K_t(j)}{L_t(j)} = \frac{\alpha}{1-\alpha} \frac{W_t R_t}{P_t R_t^K}$. As all firms are symmetric, real marginal costs are $MC_t(j) = MC_t = \Psi(R_t^k)^\alpha \left((W_t/P_t)R_t\right)^{1-\alpha}$, where $\Psi \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}A^{-1}$.

[23]This assumption implies that the individual quantity of hours worked is equally distributed across households, *i.e.*, $L_t^i = L_t^H = L_t^S = L_t$. Note that the aggregate hours worked of a given individual are given by $L_t^i \equiv \int_0^1 L_t^i(l)\mathrm{d}l = \int_0^1 (W_t(l)/W_t)^{-\epsilon_w} L_t^d dl$. The common labor market income of each household is therefore given by $\int_0^1 W_t(l)L_t^i(l)dl = L_t^d \int_0^1 W_t(l)^{1-\epsilon_w}/W_t^{-\epsilon_w} dl$.

[24]The aggregate demand for labor of type l is given by $L_t(l) = \int_0^1 L_t(j,l)\mathrm{d}j$. Using the fact that $L_t(j,l) = (W_t(l)/W_t)^{-\epsilon_w} L_t(j)$, we have $L_t(l) = (W_t(l)/W_t)^{-\epsilon_w} \int_0^1 L_t(j)\mathrm{d}j = (W_t(l)/W_t)^{-\epsilon_w} L_t$.

$$s.t.\ L_{t+\tau}(l) = \left(\frac{W_t^*(l)\Xi_{t+\tau-1}}{W_{t+\tau}}\right)^{-\epsilon_w} L_{t+\tau}\ , \quad (22)\ \&\ (23)\ .$$

where $\widetilde{\mathbb{E}}_t^w[\cdot]$ denotes the behavioral expectation operator of unions with a cognitive discounting factor μ_w, and W_t is the aggregate wage given by $W_t \equiv \left(\int_0^1 W_t(l)^{1-\epsilon_w}\mathrm{d}l\right)^{\frac{1}{1-\epsilon_w}}$. In a symmetric equilibrium where all unions set the same wage, $W_t(l) = W_t$, and applying the Calvo fairy, nominal wage dynamics simplify to:

$$W_t^{1-\epsilon_w} = \theta_w \left(W_{t-1}\pi_{t-1}^{\chi_w}\right)^{1-\epsilon_w} + (1-\theta_w)\left(W_t^*\right)^{1-\epsilon_w}\ .$$

4.1.5 *Policy Authorities* The fiscal authority redistributes tax revenues across households, while maintaining a balanced budget without resorting to debt issuance:

$$\lambda T_t^H + (1-\lambda)T_t^S = \tau^D D_t + \tau^K R_t^K K_t\ .$$

In addition, the monetary authority follows a simple Taylor rule of the form:

$$R_t = R\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\ ,$$

where $\phi_\pi > 1$ and $\phi_y > 0$.

4.1.6 *Aggregation and Market Clearing* Finally, goods and input markets clear in all periods such that

$$L_t = L_t^H = L_t^S = \int_0^1 L_t(l)dl\ , \quad K_t = \int_0^1 K_t(j)dj\ , \quad Y_t(j) = X_t(j)\,\forall\, j\ , \quad Y_t = C_t + I_t\ .$$

4.2 Parametrization

We parametrize the model to match some key features of the US economy. Table 1 summarizes the baseline parameters used in the calibration.

Parameters pertaining to household preferences are set to reflect standard values used in the literature. The discount factor is set to $\beta = 0.99$, implying an annual real interest rate of 4%. Agents are assumed to have logarithmic preferences over consumption ($\sigma = 1.00$), as in Christiano et al. (2005).[25] The inverse Frisch labor supply elasticity is fixed at $\varphi = 1.00$, a value consistent with the meta-analysis of Chetty et al. (2013). To match a steady-state share of work time at one-third of available time, we set the labor disutility parameter to $\upsilon = 5.00$.

Following Bilbiie et al. (2022), we assume that saver households transition to the hand-to-mouth state with a 2% probability each quarter ($s = 0.98$). The share of hand-to-mouth households is calibrated at $\lambda = 0.30$, closely aligning with the time-series average reported by Kaplan et al. (2014) based on SCF data from 1989-2010. This implies a value for the probability of remaining a hand-to-mouth household of $h = 0.95$.

[25] Recent studies, such as Jakobsen et al. (2020) and Holm et al. (2024), suggest a higher EIS in the range of $2-3$, i.e., $\sigma \in [0.33, 0.50]$, while Heathcote et al. (2010) specifies $\sigma = 1.50$ (EIS of 0.67). A value of 1.00 is therefore in the middle of the range of commonly used values.

TABLE 1. Parameters in the Benchmark US Economy

PARAMETER	SYMBOL	VALUE	SOURCE/TARGET
Household Preferences			
Discount factor	β	0.99	annual nominal interest rate of 4%
Elasticity of inter-temporal substitution	σ	1.00	logarithmic utility
Labor dis-utility	υ	5.00	calibrated to match 1/3 time at work
Frisch elasticity	φ	1.00	Meta-analysis Chetty et al. (2013)
Household Heterogeneity			
Idiosyncratic risk	s	0.98	Bilbiie et al. (2022)
Hand-to-mouth share	λ	0.30	Kaplan et al. (2014)
Expectation Formation			
Cognitive discounting HH's bond DIS	μ_b	0.85	Gabaix (2020)
Cognitive discounting HH's capital DIS	μ_k	1.00	benchmark value
Cognitive discounting firm's price setting	μ_p	0.85	Gabaix (2020)
Cognitive discounting unions's wage setting	μ_w	0.85	Gabaix (2020)
Intermediate Good Production			
TFP	A	1.00	benchmark value
Capital share	α	0.33	US capital income share
Investment			
Depreciation	δ	0.025	Christiano and Eichenbaum (1992)
Jermann (1998) auxiliary parameter 1	a_J	0.500	benchmark value to match $\eta = 2.00$
Jermann (1998) auxiliary parameter 2	b_J	0.158	benchmark value to match $\Phi'(\delta) = 1$
Jermann (1998) auxiliary parameter 3	c_J	-0.025	benchmark value to match $\Phi(\delta) = \delta$
Price and Wage Setting			
Price markup	ϵ_p	6.00	price markup of 20%
Wage markup	ϵ_w	6.00	wage markup of 20%
Calvo price reset probability	θ_p	0.65	three quarter average price duration
Calvo wage reset probability	θ_w	0.75	annual average wage duration
Price indexation	χ_p	0.30	Smets and Wouters (2007)
Wage indexation	χ_w	0.30	Smets and Wouters (2007)
Government Policy			
Capital income tax	τ^K	0.20	US marginal long-term capital gains tax
Dividend tax	τ^D	0.20	US federal corporate income tax
Systematic Monetary Policy			
Taylor rule inflation	ϕ_π	1.50	benchmark value
Taylor rule output	ϕ_y	0.125	benchmark value

The physical capital accumulated by savers depreciates at a rate of $\delta = 0.025$ per quarter, consistent with a 10% annual depreciation estimated by Christiano and Eichenbaum (1992). Following Jermann (1998), capital adjustment costs are modeled as

$$\Phi\left(I_t/K_t\right) = b_J(1-a_J)^{-1}(I_t/K_t)^{1-a_J} + c_J\ .$$

The elasticity of Tobins Q with respect to investment, denoted by η, determines the adjustment cost parameters. We set $\eta = 2.00$, close to the unit elasticity assumed by Galí et al. (2007). The remaining parameters are calibrated to insure that capital adjustment costs are inoperative in the deterministic steady state ($\Phi'(\delta) = 1$ and $\Phi(\delta) = \delta$), implying $a_J = \eta^{-1}$, $b_J = \delta^{a_J}$, and $c_J = \delta - b_J(1-a_J)^{-1}\delta^{1-a_J}$.

Consistently with the values used in Gabaix (2020) and Pfäuti and Seyrich (2023), saver households and firms are assigned a uniform cognitive discounting factor of $\mu_b = \mu_p = \mu_w = 0.85$, governing consumption-saving decisions, as well as price and wage setting. However, we assume no cognitive discounting in the accumulation of physical capital by savers, setting $\mu_k = 1.00$. This assumption implies that capital decisions are consistent with rational expectations.

On the firm side, and without loss of generality, we normalize steady-state total factor productivity (TFP) to $A = 1$. The capital elasticity of intermediate production function is set to $\alpha = 0.33$.

Price and wage markups are set at 20%, implying $\epsilon_p = \epsilon_w = 6$. These values align with Christiano et al. (2005) and Galí (2015), although the wage markup is slightly lower than the $\epsilon_w = 21$ used by Christiano et al. (2005).

Nominal rigidities in price and wage adjustments are captured through Calvo probabilities. Following Galí (2015), we set $\theta_p = 0.65$ and $\theta_w = 0.75$, reflecting greater wage stickiness relative to prices. These values are slightly higher than those in Christiano et al. (2005), who use $\theta_p = 0.60$ and $\theta_w = 0.65$, but preserve the same ranking. Partial indexation to past inflation is calibrated conservatively, with $\chi_p = \chi_w = 0.30$ —the level of price indexation estimated for the US economy by Smets and Wouters (2007).

We calibrate fiscal parameters to align with U.S. tax policies. The capital income tax rate is set at $\tau^K = 0.20$, approximating the marginal long-term capital gains tax for high-income brackets. Dividend taxes are also set at $\tau^D = 0.20$, reflecting the U.S. federal corporate income tax rate. We choose a time-invariant lump-sum transfer from saver to hand-to-mouth households, i.e., $T_t^S = T^S$, such that the steady state consumption of both household types is equalized given the capital and dividend taxes in place, i.e., $C^S = C^H$. The monetary policy uses in the Taylor rule a feedback to inflation of $\phi_\pi = 1.50$ and a feedback to output of $\phi_y = 0.125$.

Finally, the persistence of the DIS shock is set to $\rho = 0.60$ to reflect a moderately persistent shock.

Given this calibration, the equilibrium is locally determinate.[26]

4.3 Multipliers and Persistence

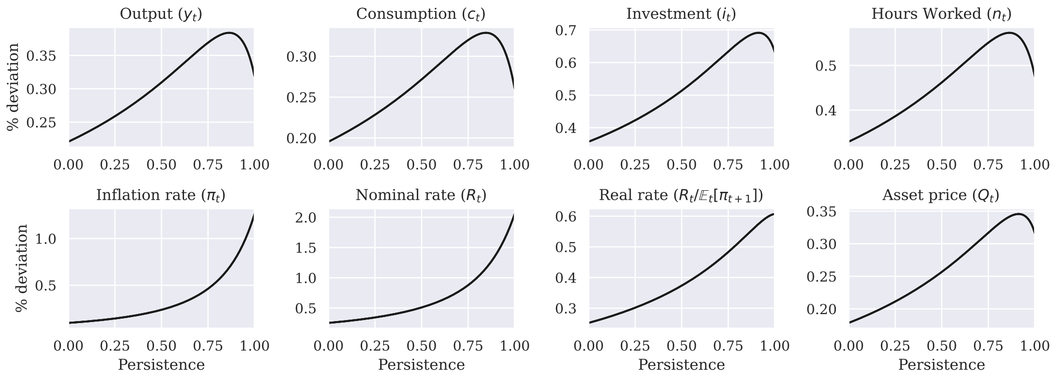
Figure 3 depicts the relationship between the impact multipliers and the persistence of the DIS demand shock.[27] As is clear from the figure, the multiplier of output, consumption, investment, hours worked, and asset prices (Tobin's Q) display a hump-shaped pattern with respect to shock persistence, confirming that the analytical findings from Section 2 extend to more complex environments. In particular, the introduction of a backward looking component (capital accumulation, price and nominal wage indexation) or a cost channel in the NKPC does not affect our main result *qualitatively*.

Figure 4 illustrates the existence of the various monetary policy regimes, as determined by the thresholds $\underline{\phi}_\pi^q$ and $\overline{\phi}_\pi^q$, where the superscript q indicates that these thresholds pertain to the quantitative model. Panel (a) of the figure reports the ρ-max as a function of the degree of aggressiveness towards inflation. It clearly shows that for weakly

[26] Section B.1 of the online appendix reports a detailed determinacy analysis of the equilibrium path.

[27] The interested reader is referred to the online appendix for impulse response functions.

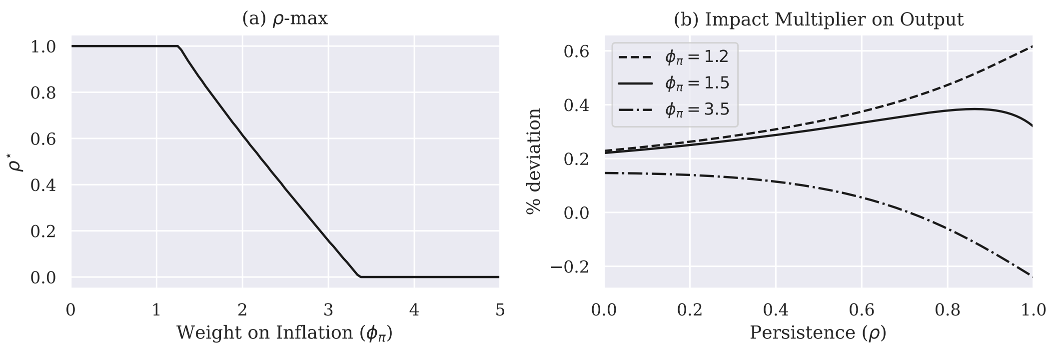
FIGURE 3. Impact Multipliers and the Persistence of DIS Demand Shocks



Legend: The impact IRFs are denoted in terms of percentage deviation from steady state. Impact IRFs of the nominal interest rate, inflation, and the real interest rate are expressed in annual terms.

aggressive policies ($\phi_\pi < \underline{\phi}_\pi^q = 1.25$), the degree of persistence maximizing the output impact multiplier is 1, as, in that regime, the PI channel dominates and the multiplier increases with persistence (see Panel (b) of the figure, $\phi_\pi = 1.2$). Conversely, when the central bank becomes very aggressive towards inflation ($\phi_\pi > \overline{\phi}_\pi^q = 3.4$), the RIR channel dominates, the degree of persistence maximizing the output impact multiplier is 0 and the output impact multiplier is decreasing in ρ (see Panel (b) of the figure, $\phi_\pi = 3.5$). In between these values, the multiplier exhibits a hump-shared pattern as the relative weight of the two channels varies with the degree of persistence of the shock.

FIGURE 4. Output Impact Multipliers and the Persistence of DIS Demand Shocks



Note: The impact IRFs are denoted in terms of percentage deviation from steady state.

Figure 4 only gives a partial picture of the various monetary policy regimes that can exist in this economy.

A positive DIS shock increases both output and inflation, prompting the central bank to respond by raising the nominal interest rate. This, in turn, increases the marginal costs of monetary policy within the NKPC, exerting additional upward pressure on inflation. When monetary policy reacts aggressively enough to inflation ($\phi_\pi > 7.95$), the resulting increase in the real interest rate becomes so pronounced that the initially positive DIS shock exerts a contractionary effect on output. In this scenario, where the RIR channel strictly dominates, the reversal in the output response introduces a fourth regime in which the relationship between persistence and the multiplier is U-shaped, with ρ-max reaching either 0 or 1 depending on the skewness of the U-shape. When monetary policy is even more aggressive ($\phi_\pi > 15.60$), closely approximating strict inflation targeting, a fifth regime emerges. In this regime, inflation and output become almost unresponsive to shocks, resulting in a ρ-max of 0.[28] These two additional regimes are not reported in Figure 4 for two main reasons. First, such extreme levels of monetary policy responsiveness to inflation are not observed empirically. Second, at such high values of ϕ_π, the economy closely approximates strict inflation targeting, diminishing the significance of demand shocks to a negligible level.[29]

4.4 Quantitative Drivers of Monetary Policy-Persistence Regimes

This section tries to shed light on the mechanisms underlying the emergence of distinct monetary policy-persistence regimes. To this end, we systematically re-compute regime thresholds, as well as the ρ-max and impact multipliers, while selectively deactivating specific model features. Our findings underscore the remarkable robustness of multiple regimes across model variants. The results, are presented in Table 2 and are organized around three types of experiments. First, we shut down the working capital, that in turn switches off the marginal monetary cost channel in the NKPC. Then, we vary structural parameters that directly determine the parameters in the DIS and NKPC equations — the equivalent to β_f, ζ_f and ζ_r in our analytical model of Section 3.3.1. Finally, we investigate the role played by the type of nominal rigidities for our results.

The working capital assumption implies that firms' marginal costs are affected directly by interest rates, introducing a monetary policy cost channel. In equilibrium, this channel, in combination with the Taylor rule, increases the sensitivity of the NKPC to output, effectively steepening it. When the working capital assumption is removed, this monetary policy cost channel is eliminated, leading to a flattening of the NKPC, which is akin to an increase in price stickiness. As a result, as discussed in Section 2, the PI channel becomes relatively stronger than the RIR channel. This shift has two immediate implications. First, it raises the intermediate regime in which the output multiplier exhibits a hump-shaped relationship with persistence, as the threshold $\underline{\phi}_\pi^q$ increases from 1.25 in the benchmark model to 1.95 in the version without working capital. Second, it expands the range of this regime, with $\overline{\phi}_\pi^q$ increasing significantly from 3.4 to nearly 15. Given the central bank's inflation response coefficient of 1.5 in our parametrization,

[28] It is important to note that these extreme regimes do not occur in response to a monetary policy shock. In this case, the response of the nominal interest rate—and by extension, the real interest rate—is subdued, ensuring that output always remains expansionary regardless of the aggressiveness of monetary policy.

[29] For a complete picture, readers can refer to Figure B.3 in the online appendix.

TABLE 2. Monetary Policy Regimes, Impact Multipliers and $\rho-$max

MODEL VARIANT	PARAMETER	REGIME		IMPACT IRF	
		$\underline{\phi}_\pi^q$	$\overline{\phi}_\pi^q$	ρ^*	$\mathcal{M}_y^*$
Benchmark Model		1.25	3.40	0.86	0.38
Cost Channel of Monetary Policy					
No Working Capital		1.95	14.90	0.99	0.65
Behavioral Parameters					
Rational Expectations	$\mu_{b,p,w} = 1.00$	–	3.25	0.74	0.39
Higher EIS	$\sigma = 0.33$	1.30	2.75	0.78	0.51
Less HtM Households	$\lambda = 0.01$	1.15	3.15	0.78	0.37
No Idiosyncratic Risk	$s = 1.00$	1.25	3.40	0.87	0.39
No Capital Redistribution	$\tau^K = 0.00$	1.25	3.40	0.86	0.37
No Dividend Redistribution	$\tau^D = 0.00$	1.30	3.40	0.88	0.43
Nominal Rigidities					
No Price/Wage Indexation	$\chi_{p,w} = 0.00$	1.20	3.60	0.85	0.38
No Wage Indexation	$\chi_w = 0.00$	1.25	3.55	0.89	0.41
No Price Indexation	$\chi_p = 0.00$	1.20	3.40	0.81	0.36
Flexible Wages (only)	$\theta_w = 0.00$	1.15	1.80	0.31	0.13
Flexible Prices (only)	$\theta_p = 0.00$	0.70	0.95	0.00	-0.27

the economy without working capital falls within the lower regime, where the PI channel strictly dominates the RIR channel, causing ρ-max to reach 1. Additionally, the flattening of the NKPC implies an increase in the output impact multiplier when working capital is removed, nearly doubling from 0.38 to 0.65.

Behavioral expectations significantly dampen the inflation expectation channel in the NKPC —a drop in β_f in the analytical version of Section 3.3.1— and reduce the PI channel in the DIS equation —a drop in ζ_f in the analytical version of Section 3.3.1. Therefore, shifting to rational expectations has two direct consequences. First, it substantially strengthens the PI channel in the DIS equation. Second, it amplifies the role of inflation expectations in the NKPC, thereby strengthening the RIR channel —the same response of monetary policy exerts a larger effect on consumption and output. In our parameterization, the latter effect dominates, hence requiring a less aggressive monetary policy stance to stabilize inflation. Thus, the lower threshold $\underline{\phi}_\pi^q$ decreases so much that the lower monetary policy regime effectively disappears. The relatively stronger RIR channel also causes ρ-max to decline from 0.86 in the benchmark model to 0.74.

In the lines of Jakobsen et al. (2020) and Holm et al. (2024), we increase the intertemporal elasticity of substitution. This variation in the IES strengthens the RIR channel in the DIS equation —an increase in ζ_r in the analytical version of Section 3.3.1—and reduces the sensitivity of the NKPC to output —κ_y in the analytical model, thereby strengthening the role of inflation expectations. As discussed above, both effects call for a less aggressive monetary policy. Consequently, *ceteris paribus*, the monetary policy regime thresholds $\underline{\phi}_\pi^q$ and $\overline{\phi}_\pi^q$ decrease. However, this decline is partially offset by the monetary policy cost channel in the NKPC, which, as formally analyzed, increases the sensitivity of the NKPC to output in equilibrium and plays in the opposite direction. Under our parametrization, this results in a slight increase in the lower threshold $\underline{\phi}_\pi^q$, while the upper threshold, $\overline{\phi}_\pi^q$, decreases. This adjustment narrows the range of values over which the output multiplier exhibits a hump-shaped relationship with respect to persistence. The dominance of the RIR channel leads to a lower ρ-max, 0.78, together with a larger impact multiplier, 0.5.

The direct effect of the presence of hand-to-mouth households is to diminish the influence of both the PI and RIR channels. However, among others Werning (2015) and Bilbiie (Forthcoming) showed that, when income risk and inequality are countercyclical, indirect general equilibrium effects lead to a departure from the "as if" representative agent benchmark and reverse this statement: both channels are reinforced. By nearly eliminating these households and the associated indirect effect of hand-to-mouth behavior, permanent income effects regain prominence and both channels recede. For our calibration, the PI channel weakens relatively more than the RIR channel. As previously discussed, a less aggressive monetary policy is required to achieve the same level of stabilization, resulting in a reduction in both monetary policy regime thresholds $\underline{\phi}_\pi^q$ and $\overline{\phi}_\pi^q$ —specifically, to 1.15 and 3.15, respectively, for our parametrization. While this adjustment is accompanied by a decrease in ρ-max, the concurrent weakening of the PI and RIR channels leaves the impact multiplier unaffected.

Cyclical inequality, idiosyncratic income risk, and redistribution policies have only minimal effects on regime thresholds, the ρ-max, and the output impact multiplier. From the perspective of the analytical model in Section 3.3.1, each of these mechanisms alters the ratio between the discounting component of the DIS equation (ζ_f in the analytical model) and the DIS elasticity with respect to the real interest rate (ζ_r in the analytical model). However, each parameter contributes, quantitatively, very little to these elasticities and, more critically, to their ratio. As demonstrated in Proposition 9, this ratio plays a pivotal role in determining the regime thresholds, the ρ-max, and the output impact multiplier. The result then follows.

Last, we examine the role of nominal rigidities for our results. We start by investigating the role of price and wage indexation, which act as an endogenous source of persistence in inflation and wage dynamics. Results reported in the lower panel of Table 2 clearly show that indexation, be it in wages and/or prices, does not play an important quantitative role for the monetary regime threshold, the ρ-max and the multiplier. The main reason for this result lies in the fact that, while indexation may create some propagation of nominal rigidities, it does not fundamentally affect the transmission of monetary policy on impact.

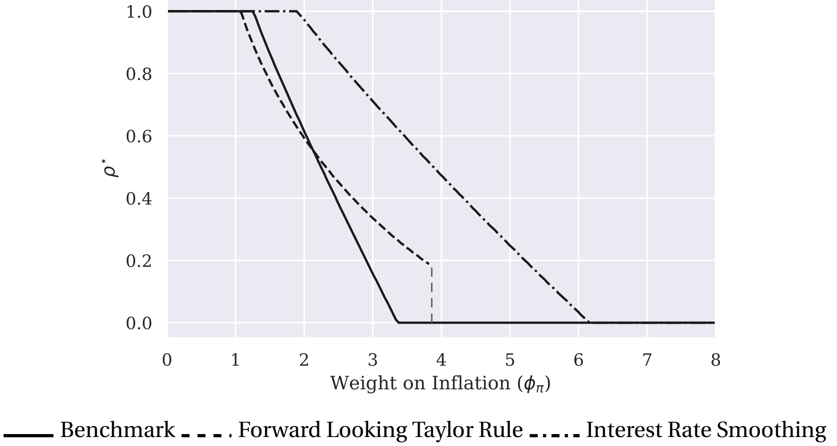
In contrast, the combined presence of nominal price and wage rigidity plays a significant role. When nominal wages are flexible and prices are sticky, real wages—and consequently, firms' marginal costs—become more sensitive to changes in demand. In this context, monetary policy gains greater effectiveness. As a result, the upper regime threshold decreases sharply, dropping from $\overline{\phi}_{\pi}^{q} = 3.40$ to $\overline{\phi}_{\pi}^{q} = 1.80$. This reduction is accompanied by a lower ρ-max ($\rho^* = 0.31$).

In the case of flexible prices and sticky wages, the inflation rate becomes significantly more responsive compared to the previous scenario. Since the central bank targets price inflation —and not wage inflation— the RIR channel is much stronger than in an economy characterized by flexible wages and sticky prices. As a result, both the upper and lower monetary policy thresholds drop sharply. Specifically, the lower threshold decreases from $\underline{\phi}_{\pi}^{q} = 1.30$ to 0.70, while the upper threshold drops from $\overline{\phi}_{\pi}^{q} = 3.40$ to 0.95. This sharp reduction is accompanied by a collapse of ρ-max to $\rho^* = 0.00$, as the RIR channel overwhelmingly dominates the PI channel in the DIS equation. This dominance leads to a pronounced recessionary response in output.

4.5 The Role of Monetary Policy

The preceding discussions have highlighted the importance of monetary policy in shaping how persistence affects the output impact multiplier. In this section, we examine how adopting a forward-looking monetary policy rule or introducing interest rate smoothing can modify these results. Figure 5 illustrates the emergence of the regimes and the determination of the ρ-max as a function of the aggressiveness of various monetary policies towards inflation.

FIGURE 5. ρ-max and the Monetary Policy



We begin by examining the case in which the central bank adopts a forward-looking Taylor rule, expressed as:

$$R_t = R\left(\frac{\mathbb{E}_t[\pi_{t+1}]}{\pi}\right)^{\phi_\pi}\left(\frac{\mathbb{E}_t[Y_{t+1}]}{Y}\right)^{\phi_y},$$

where the central banker is assumed to have rational expectations regarding future output and inflation. In the absence of the monetary policy cost channel, Proposition 7 established that expected output stabilization counteracts the PI channel, causing both monetary policy regime thresholds to recede. However, in this version of the model, the inclusion of working capital introduces a monetary policy cost channel in the NKPC. This channel amplifies the inflation response, necessitating a more aggressive Taylor rule to achieve the same degree of inflation stabilization. Consequently, under our calibration, the upper threshold rises from $\phi_\pi = 3.40$ to 3.87.[30] This effect is less pronounced for the lower threshold, which declines from 1.25 to 1.05.

Next, we analyze the case where the central bank aims to smooth nominal interest rate fluctuations by adopting a rule of the form:

$$R_t = R_{t-1}^{\rho_R}\left[R\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_R},$$

where $\rho_R \in [0,1]$ represents the degree of interest rate inertia or smoothing. At one extreme, this rule simplifies to the baseline contemporaneous Taylor rule when $\rho_R = 0$, while $\rho_R \to 1$ implies complete interest rate smoothing. For this analysis, we select an intermediate value, $\rho_R = 0.50$, while maintaining $\phi_y = 0.125$. Under these assumptions, for a DIS demand shock, the regime thresholds increase to $\underline{\phi}_\pi^g = 1.89$ and $\overline{\phi}_\pi^g = 6.15$. Interest rate smoothing prevents the RIR channel from being fully operative at the time the shock hits the economy. As a result, the RIR channel affects the economy only under a more aggressive Taylor rule with respect to inflation. Consequently, both monetary regime thresholds shift upward. Under our benchmark calibration with $\phi_\pi = 1.5$, the economy operates in the upper regime, where the PI channel dominates entirely. In this case, ρ-max reaches its upper limit, *i.e.*, $\rho^* = 1$. Note however that estimated values of ϕ_π greater than 2 often obtain in various models (see *e.g.* Clarida et al., 2000), which would reignite the hump-shaped mechanism.

5. Concluding Remarks

This paper attempts to shed light on how the persistence of DIS demand shocks interacts with monetary policy to shape short-term macroeconomic outcomes within the New Keynesian framework. We start by studying a simplified RANK model to derive closed-form solutions. We identify two key propagation mechanisms: a permanent income channel, which amplifies the effects of persistent shocks, and a real interest rate

[30] The discontinuity in ρ-max arises because, in the presence of the monetary policy cost channel, the output multiplier becomes negative when $\phi_\pi > 3.87$, reaching its maximum at $\rho^* = 0$.

channel, which works in the opposite direction and encapsulates all general equilibrium effects pertaining to monetary policy and nominal rigidities. The interaction between these two forces depends on the central bank's monetary policy stance. Under an accommodative policy, persistent demand shocks amplify the output response due to the dominance of the permanent income channel. In contrast, aggressive policies suppress output responses as the real interest rate channel becomes predominant. In an intermediate policy regime, towards which most empirically estimated degrees of monetary policy aggressiveness to inflation point to, the relationship between persistence and output exhibits a hump-shaped pattern.

We extend the analysis to medium-scale DSGE models incorporating household heterogeneity, behavioral frictions, nominal price and wage rigidities, working capital and capital accumulation. Our results confirmed the robustness of the hump-shaped persistence-output relationship across richer and more compelling settings. The nuanced interplay between propagation channels underscores the importance of tailoring policy responses to the nature and persistence of demand shocks.

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Online Appendix (Not For Publication)

The Power of Persistence

How Demand Shocks and Monetary Policy Shape Macroeconomic Outcomes

The Online Appendix consist of two main parts: first, the Analytical Appendix A that contains proofs and derivations of the analytical insights; second, the Quantitative Appendix B that provides additional results to the quantitative model.

Specifically, Appendix A.1 contains the RANK model, Appendix A.2 the general New Keynesian model, Appendix A.3 the extension with a cost channel of monetary policy, Appendix A.4 provides formal results towards a structural interpretation of the general NK model, Appendix A.5 contains further analytical characterizations, and Appendix A.6 some numerical results derived from the analytical model. Regarding the Quantitative Appendix, Appendix B.1 provides determinacy regions of various model variants, Appendix B.2 provides all impulse response function to our baseline calibration, and Appendix B.3 contains a complete overview over the various monetary regimes.

Appendix A: Analytical Appendix

A.1 Representative Agent New Keynesian Model

Subsequently, we provide derivations with a contemporaneous Taylor rule that reacts to inflation and output, i.e., $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t$. The expressions in the main body of the text are obtained by setting $\phi_y = 0$.

A.1.1 *Proof Proposition 1*

Proof. Substituting the Taylor rule into the DIS equation (1) results in

$$(1+\phi_y)y_t = \mathbb{E}_t\left[y_{t+1}\right] + \mathbb{E}_t\left[\pi_{t+1}\right] - \phi_\pi \pi_t + \xi_t \,.$$

We guess and verify that the solution takes the form $y_t = \mathcal{M}_y \xi_t$ and $\pi_t = \mathcal{M}_\pi \xi_t$, such that the previous equation together with the NKPC can be rewritten as

$$\begin{aligned}(1+\phi_y-\rho)\,\mathcal{M}_y &= -\left(\phi_\pi-\rho\right)\mathcal{M}_\pi + 1\,,\\ (1-\beta\rho)\,\mathcal{M}_\pi &= \kappa_y \mathcal{M}_y\,.\end{aligned}$$

Substituting the latter into the former equation gives us

$$(1+\phi_y-\rho)\,\mathcal{M}_y = -\left(\phi_\pi-\rho\right)\frac{\kappa_y}{1-\beta\rho}\mathcal{M}_y + 1\,,$$

which finally yields

$$\mathcal{M}_y = \frac{1-\beta\rho}{(1+\phi_y-\rho)\,(1-\beta\rho) + (\phi_\pi-\rho)\,\kappa_y} > 0\,.$$

The impact multipliers for inflation, the nominal interest rate, and the real interest rate then follow by

$$\mathcal{M}_\pi = \frac{\kappa_y}{(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,,$$

$$\mathcal{M}_i = \frac{\phi_\pi\kappa_y+\phi_y(1-\beta\rho)}{(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,,$$

$$\mathcal{M}_r = \frac{(\phi_\pi-\rho)\kappa_y+\phi_y(1-\beta\rho)}{(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y} > 0\,,$$

where the sign of $\mathcal{M}_r$ follows from the Taylor principle $(\phi_\pi-1)\kappa_y+\phi_y(1-\beta)>0$, i.e.,

$$\text{sgn}(\mathcal{M}_r) = (\phi_\pi-\rho)\kappa_y+\phi_y(1-\beta\rho) = (\phi_\pi-1)\kappa_y+\phi_y(1-\beta)+(\kappa_y+\beta\phi_y)(1-\rho) > 0\,.$$

□

A.1.2 *Proof Proposition 2*

PROOF. Taking the derivative of $\mathcal{M}_y$ with respect to ρ yields

$$\frac{\partial\mathcal{M}_y}{\partial\rho} = \frac{-\beta[(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y]+(1-\beta\rho)[(1-\beta\rho)+\beta(1+\phi_y-\rho)+\kappa_y]}{[(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y]^2}\,.$$

Defining $x\equiv(1+\phi_y-\rho)(1-\beta\rho)+(\phi_\pi-\rho)\kappa_y$, the previous expression simplifies to

$$x^2\frac{\partial\mathcal{M}_y}{\partial\rho} = (1-\beta\rho)^2+(1-\beta\phi_\pi)\kappa_y\,.$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ_ξ, i.e.,

$$\text{sgn}\left(\frac{\partial\mathcal{M}_y}{\partial\rho}\right) = a\rho^2+b\rho+c\,,$$

where the auxiliary parameters are given by

$$a\equiv\beta^2\,,\quad b\equiv-2\beta\,,\quad c\equiv1+(1-\beta\phi_\pi)\kappa_y\,.$$

The corresponding roots are

$$\rho^{+,-} = \frac{-b\pm\sqrt{\Delta}}{2a}\,,$$

where the discriminant $\Delta\equiv b^2-4ac$ is given by

$$\Delta = 4\beta^2-4\beta^2[1+(1-\beta\phi_\pi)\kappa_y] = 4\beta^2(\beta\phi_\pi-1)\kappa_y\,.$$

Consequently, the following case distinction applies. First, $\phi_\pi<\beta^{-1}$ implies $\Delta<0$ and the second order polynomial has two distinct complex roots. Second, $\phi_\pi=\beta^{-1}$ implies

$\Delta = 0$ and the second order polynomial has a unique real root that is given by $-\frac{b}{2a} = \beta^{-1} > 1$. Third, $\phi_\pi > \beta^{-1}$ implies that the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \frac{2\beta \pm \sqrt{4\beta^2(\beta\phi_\pi - 1)\kappa_y}}{2\beta^2} = \beta^{-1}\left(1 \pm \sqrt{(\beta\phi_\pi - 1)\kappa_y}\right) .$$

Evidently, $\rho^+ > 1$ as $\beta \in (0,1)$ and $\phi_\pi > \beta^{-1}$ such that the only feasible candidate on $\rho \in (0,1)$ is

$$\rho^* \equiv \rho^- = \beta^{-1}\left(1 - \sqrt{(\beta\phi_\pi - 1)\kappa_y}\right) .$$

Notice that $\rho^* \in (0,1)$ if $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, where we have

$$\underline{\phi}_\pi \equiv \beta^{-1}\left(1 + (1-\beta)^2\kappa_y^{-1}\right) , \quad \text{and} \quad \overline{\phi}_\pi \equiv \beta^{-1}\left(1 + \kappa_y^{-1}\right) .$$

As $f(\rho) \equiv a\rho^2 + b\rho + c$ is a strictly convex function in ρ, statement (b) of Proposition 2 follows.

To show statement (a) of Proposition 2, notice that sgn $\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$ is strictly decreasing in ϕ_π. Hence, substituting in the lower bound $\underline{\phi}_\pi$, we obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi} = \beta^2\rho^2 - 2\beta\rho + 1 + (1 - \beta\underline{\phi}_\pi)\kappa_y = (1-\beta\rho)^2 - (1-\beta)^2 > 0 ,$$

where the last inequality applies as $\rho \in [0,1)$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ for all $\phi_\pi \leq \underline{\phi}_\pi$.

Finally, to show statement (c) we analogously substitute in for the upper bound $\overline{\phi}_\pi$ to obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \overline{\phi}_\pi} = \beta^2\rho^2 - 2\beta\rho + 1 + (1 - \beta\overline{\phi}_\pi)\kappa_y = \beta\rho\left[\beta\rho - 2\right] < 0$$

where the last inequality applies as $\rho \in [0,1)$. Notice that the previous inequality is strict for $\rho > 0$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$ for all $\phi_\pi \geq \underline{\phi}_\pi$. □

A.1.3 *Proof Proposition 3*

PROOF. Recall that the impact multiplier of the real interest rate is given by

$$\mathcal{M}_r = \frac{(\phi_\pi - \rho)\,\kappa_y + \phi_y\,(1-\beta\rho)}{(1+\phi_y-\rho)\,(1-\beta\rho) + (\phi_\pi - \rho)\,\kappa_y} .$$

As before, defining $x \equiv (1+\phi_y-\rho)\,(1-\beta\rho) + (\phi_\pi-\rho)\,\kappa_y$, we obtain

$$x^2\frac{\partial \mathcal{M}_r}{\partial \rho} = -\left(\kappa_y + \beta\phi_y\right)\left[(1+\phi_y-\rho)\,(1-\beta\rho) + (\phi_\pi-\rho)\,\kappa_y\right]$$
$$+\left((1-\beta\rho) + \beta(1+\phi_y-\rho) + \kappa_y\right)\left[(\phi_\pi-\rho)\,\kappa_y + \phi_y\,(1-\beta\rho)\right] .$$

The previous expression can be further simplified to

$$x^2 \frac{\partial \mathcal{M}_r}{\partial \rho} = \kappa_y \left(\phi_\pi - \rho\right) \left[1+\beta-2\beta\rho\right] - \kappa_y (1+\phi_y-\rho)\left(1-\beta\rho\right) + \phi_y \left(1-\beta\rho\right)\left[\left(1-\beta\rho\right)+\kappa_y\right] .$$

Collecting terms in ϕ_y yields

$$\begin{aligned}
x^2 \frac{\partial \mathcal{M}_r}{\partial \rho} =& \phi_y \left(1-\beta\rho\right)^2 + \kappa_y \Big(\left[1+\beta-2\beta\rho\right]\left(\phi_\pi-\rho\right) - \left(1-\rho\right)\left(1-\beta\rho\right) \Big) \\
=& \phi_y \left(1-\beta\rho\right)^2 + \kappa_y \Big(\left[1+\beta-2\beta\rho\right]\phi_\pi - \left[1+\beta-2\beta\rho\right]\rho - \left[1-\beta\rho-\rho+\beta\rho^2\right] \Big) \\
=& \phi_y \left(1-\beta\rho\right)^2 + \kappa_y \Big(\left[1+\beta-2\beta\rho\right]\phi_\pi - 1 + \beta\rho^2 \Big) .
\end{aligned}$$

As a result, we obtain

$$\frac{\partial \mathcal{M}_r}{\partial \rho} = \frac{\phi_y \left(1-\beta\rho\right)^2 + \kappa_y \Big(\left[1+\beta-2\beta\rho\right]\phi_\pi - 1 + \beta\rho^2 \Big)}{x^2} .$$

Let us denote the numerator of the previous expression by $f(\rho)$. Note that $f(\rho)$ is a second order polynomial in ρ, i.e., $f(\rho) = a\rho^2 + b\rho + c$, where

$$\begin{aligned}
a &= \beta\left(\beta\phi_y + \kappa_y\right) , \\
b &= -2\beta\left(\phi_y + \phi_\pi\kappa_y\right) , \\
c &= \left(\beta\phi_\pi - 1\right)\kappa_y + \phi_y + \phi_\pi\kappa_y .
\end{aligned}$$

Notice that we obtain

$$\begin{aligned}
&\lim_{\rho\to 0} f(\rho) = \phi_y + \left(\phi_\pi - 1\right)\kappa_y + \beta\phi_\pi\kappa_y = \phi_y(1-\beta) + \left(\phi_\pi - 1\right)\kappa_y + \beta\left(\phi_y + \phi_\pi\kappa_y\right) > 0 , \\
&\lim_{\rho\to 1} f(\rho) = (1-\beta)\left[\phi_y(1-\beta) + \left(\phi_\pi - 1\right)\kappa_y\right] > 0 ,
\end{aligned}$$

where the signs are ensured by the Taylor principle and the restriction that ϕ_π and ϕ_y are non-negative parameters. Consider now the following case distinction and let Δ denote the discriminant of $f(\rho)$. First, if $\Delta < 0$, then $f(\rho)$ does not change its sign on $\rho \in [0,1)$, i.e., $\frac{\partial \mathcal{M}_r}{\partial \rho} > 0$ applies throughout. Second, if $\Delta = 0$, the unique root of $f(\rho)$ is given by $\frac{\phi_y + \phi_\pi\kappa_y}{\beta\phi_y + \kappa_y} > 1$, where the strict inequality follows in turn from the Taylor principle. As such, $\frac{\partial \mathcal{M}_r}{\partial \rho} > 0$ applies throughout. Third, if $\Delta > 0$, both distinct real roots of $f(\rho)$ are characterized by

$$\begin{aligned}
\rho^{+,-} &= \frac{2\beta\left(\phi_y + \phi_\pi\kappa_y\right) \pm \sqrt{4\beta^2\left(\phi_y + \phi_\pi\kappa_y\right)^2 - 4\beta\left(\beta\phi_y + \kappa_y\right)\left[\left(\beta\phi_\pi - 1\right)\kappa_y + \left(\phi_y + \phi_\pi\kappa_y\right)\right]}}{2\beta\left(\beta\phi_y + \kappa_y\right)} \\
&= \frac{\phi_y + \phi_\pi\kappa_y}{\beta\phi_y + \kappa_y} \pm \frac{1}{\beta\phi_y + \kappa_y}\sqrt{\left(\phi_y + \phi_\pi\kappa_y\right)^2 - \beta^{-1}\left(\beta\phi_y + \kappa_y\right)\left[\phi_y + \phi_\pi\kappa_y - \left(1-\beta\phi_\pi\right)\kappa_y\right]} .
\end{aligned}$$

Notice that $\rho^+ > 1$ by the Taylor principle and $\Delta > 0$. Moreover, we have $\rho^- > 1$ as well if

$$
\begin{aligned}
&[(1-\beta)\phi_y + (\phi_\pi - 1)\kappa_y]^2 > (\phi_y + \phi_\pi\kappa_y)^2 - \beta^{-1}(\beta\phi_y + \kappa_y)[\phi_y + \phi_\pi\kappa_y - (1-\beta\phi_\pi)\kappa_y] \\
\Leftrightarrow &[\phi_y + \phi_\pi\kappa_y - (\beta\phi_y + \kappa_y)]^2 > (\phi_y + \phi_\pi\kappa_y)^2 - \beta^{-1}(\beta\phi_y + \kappa_y)[\phi_y + \phi_\pi\kappa_y - (1-\beta\phi_\pi)\kappa_y] \\
\Leftrightarrow &-2(\phi_y + \phi_\pi\kappa_y)(\beta\phi_y + \kappa_y) + (\beta\phi_y + \kappa_y)^2 > -\beta^{-1}(\beta\phi_y + \kappa_y)[\phi_y + \phi_\pi\kappa_y - (1-\beta\phi_\pi)\kappa_y] \\
\Leftrightarrow &(\beta\phi_y + \kappa_y)\left[\beta\phi_y + \kappa_y - 2(\phi_y + \phi_\pi\kappa_y) + \beta^{-1}[\phi_y + \phi_\pi\kappa_y - (1-\beta\phi_\pi)\kappa_y]\right] > 0 \\
\Leftrightarrow &(\beta\phi_y + \kappa_y)(\beta^{-1} - 1)[\phi_y(1-\beta) + (\phi_\pi - 1)\kappa_y] > 0\,,
\end{aligned}
$$

which is satisfied throughout as $\beta \in [0,1)$ and the Taylor principle holds. As a result, we have that that both roots are strictly larger than unity. Also, recognize that $f(\rho)$ is strictly decreasing in ρ on $\rho \in [0,1)$ as $2\beta[\phi_y(\beta\rho - 1) + \kappa_y(\rho - \phi_\pi)] < 0$. As a result, $\frac{\partial \mathcal{M}_r}{\partial \rho} > 0$ applies throughout. Combined with the previous result, the real interest rate thus strictly increases in the persistence of a DIS-demand shock. □

A.1.4 *Proof Corollary 1*

PROOF. To begin with, it follows from the regime thresholds of Proposition 2 that

$$
\Delta_\pi \equiv \overline{\phi}_\pi - \underline{\phi}_\pi = \beta^{-1}\left(1 + \kappa_y^{-1}\right) - \beta^{-1}\left(1 + (1-\beta)^2\kappa_y^{-1}\right) = \frac{2-\beta}{\kappa_y}\,.
$$

As a result, we obtain

$$
\frac{\partial \Delta_\pi}{\partial \kappa_y} = -\frac{2-\beta}{\kappa_y^2}\,, \quad \text{and} \quad \frac{\partial \Delta_\pi}{\partial \beta} = -\frac{\kappa_y + (2-\beta)\dfrac{\partial \kappa_y}{\partial \beta}}{\kappa_y^2}\,.
$$

In the case of Rotemberg pricing, we have $\frac{\partial \kappa_y}{\partial \beta} = 0$. Contrary, in the case of Calvo pricing, we obtain $\frac{\partial \kappa_y}{\partial \beta} = -\kappa_y \frac{\theta}{1-\beta\theta}$ such that

$$
\frac{\partial \Delta_\pi}{\partial \beta} = -\frac{\kappa_y + (2-\beta)\frac{\partial \kappa_y}{\partial \beta}}{\kappa_y^2} = -\frac{1-(2-\beta)\frac{\theta}{1-\beta\theta}}{\kappa_y} = -\frac{(1-\beta\theta)-(2-\beta)\theta}{\kappa_y(1-\beta\theta)} = -\frac{1-2\theta}{\kappa_y(1-\beta\theta)}\,.
$$

As a result,

$$
\text{sign}\left(\frac{\partial \Delta_\pi}{\partial \beta}\right) \gtreqless 0 \quad \Leftrightarrow \quad \theta \gtreqless \frac{1}{2}\,.
$$

□

A.1.5 *Proof Proposition 4*

PROOF. Proposition 4 follows by comparing

$$
\lim_{\rho \to 0} \mathcal{M}_y = \frac{1}{1 + \phi_y + \phi_\pi\kappa_y} \quad \text{and} \quad \lim_{\rho \to 1} \mathcal{M}_y = \frac{1-\beta}{\phi_y(1-\beta) + (\phi_\pi - 1)\kappa_y}\,,
$$

where $\lim_{\rho \to 0} \mathcal{M}_y > \lim_{\rho \to 1} \mathcal{M}_y$ applies if $\phi_\pi > \beta^{-1}\left(1 + (1-\beta)\kappa_y^{-1}\right)$, where the right hand side is strictly larger than $\underline{\phi}_\pi$ and strictly smaller than $\overline{\phi}_\pi$. This concludes the proof. □

A.1.6 *Proof Proposition 5*

PROOF. The proof follows the same steps as the ones of Propositions 1 and 2. Specifically, using the MOUC we obtain the impact multipliers for output ($\mathcal{M}_y(\rho_\nu)$) and inflation ($\mathcal{M}_\pi(\rho_\nu)$) as indicated in the main text. Statement (a) follows then from

$$\frac{\partial \mathcal{M}_y}{\partial \rho_\nu} = -\frac{(1-\beta\rho_\nu)(\phi_\pi - 1) + (\phi_\pi - \rho_\nu)\beta(1-\rho_\nu)}{\left[(1-\rho_\nu)(1-\beta\rho_\nu) + (\phi_\pi - \rho_\nu)\kappa_y\right]^2} < 0\,,$$

where the sign is determined by the Taylor principle. To show statement (b), note that

$$\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu} = \frac{\beta(1-\rho_\nu)^2 + \kappa_y\left[1-\rho_\nu - (\phi_\pi - \rho_\nu)\right]}{\left[(1-\rho_\nu)(1-\beta\rho_\nu) + (\phi_\pi - \rho_\nu)\kappa_y\right]^2}\,.$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ_ν, i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu}\right) = a\rho_\nu^2 + b\rho_\nu + c\,,$$

where the auxiliary parameters are given by

$$a \equiv \beta\,, \quad b \equiv -2\beta\,, \quad c \equiv \beta + (1-\phi_\pi)\kappa_y\,.$$

The corresponding roots are

$$\rho_\nu^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a}\,,$$

where the discriminant $\Delta \equiv b^2 - 4ac$ is given by

$$\Delta = 4\beta^2 - 4\beta\left[\beta + (1-\phi_\pi)\kappa_y\right] = 4\beta(\phi_\pi - 1)\kappa_y\,.$$

Consequently, the following case distinction applies. First, $\phi_\pi < 1$ implies $\Delta < 0$ and the second order polynomial has two distinct complex roots. Second, $\phi_\pi = 1$ implies $\Delta = 0$ and the second order polynomial has a unique real root that is given by $-\frac{b}{2a} = 1$. Third, $\phi_\pi > 1$ implies that the second order polynomial has two real-valued roots:

$$\rho_\nu^{+,-} = \frac{2\beta \pm \sqrt{4\beta(\phi_\pi - 1)\kappa_y}}{2\beta} = 1 \pm \sqrt{\frac{(\phi_\pi - 1)\kappa_y}{\beta}}\,.$$

Evidently, $\rho_\nu^+ > 1$ as $\phi_\pi > 1$ such that the only feasible candidate on $\rho_\nu \in (0,1)$ is

$$\rho_\nu^* \equiv \rho_\nu^- = 1 - \sqrt{\frac{(\phi_\pi - 1)\kappa_y}{\beta}}\,.$$

Note that $\rho_\nu^* \in (0,1)$ if $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, where we have

$$\underline{\phi}_\pi \equiv 1\,, \quad \text{and} \quad \overline{\phi}_\pi \equiv 1 + \frac{\beta}{\kappa_y}\,.$$

As $f(\rho_\nu) \equiv a\rho_\nu^2 + b\rho_\nu + c$ is strictly convex in ρ_ν, statement $(b.1)$ of Proposition 5 follows.

Finally, to show statement $(b.2)$ note that sgn $\left(\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu}\right)$ strictly decreases in ϕ_π. We then substitute in for the upper bound $\overline{\phi}_\pi$ to obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu}\right)|_{\phi_\pi = \overline{\phi}_\pi} = \beta\rho_\nu^2 - 2\beta\rho_\nu + \beta + (1-\overline{\phi}_\pi)\kappa_y = \beta\rho_\nu\left[\rho_\nu - 2\right] < 0$$

where the last inequality applies as $\rho_\nu \in [0,1)$. Notice that the previous inequality is strict for $\rho_\nu > 0$. As a result, we have $\frac{\partial \mathcal{M}_\pi}{\partial \rho_\nu} < 0$ for all $\phi_\pi \geq \overline{\phi}_\pi$.

□

A.1.7 *Proof of Proposition 6*

PROOF. Subsequently, we provide derivations with a contemporaneous Taylor rule that reacts to inflation and output, i.e., $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y \hat{y}_t$. The expressions in the main body of the text are obtained by setting $\phi_y = 0$. Under logarithmic preferences, the impact multiplier for output and inflation are derived in Galí (2015) and given by

$$\mathcal{M}_y = 1 - \frac{(1-\rho_a)(1-\beta\rho_a)}{\left[1-\rho_a+\phi_y\right](1-\beta\rho_a) + \kappa_y(\phi_\pi - \rho_a)} = \frac{(1-\beta\rho_a)\phi_y + \kappa_y(\phi_\pi-\rho_a)}{\left[1-\rho_a+\phi_y\right](1-\beta\rho_a) + \kappa_y(\phi_\pi - \rho_a)}\,,$$

$$\mathcal{M}_\pi = -\kappa_y \frac{1-\rho_a}{\left[1-\rho_a+\phi_y\right](1-\beta\rho_a) + \kappa_y(\phi_\pi - \rho_a)}\,.$$

To begin with, we show Statement (a). Defining the auxiliary variable $x \equiv \left[1-\rho_a+\phi_y\right](1-\beta\rho_a) + \kappa_y(\phi_\pi - \rho_a)$ and taking the comparative static with respect to the persistence ρ_a yields

$$\begin{aligned} x^2\frac{\partial \mathcal{M}_y}{\partial \rho_a} =& -(\beta\phi_y + \kappa_y)\left[(1-\rho_a+\phi_y)(1-\beta\rho_a) + \kappa_y(\phi_\pi-\rho_a)\right] \\ &+ \Big(\beta(1-\rho_a+\phi_y) + (1-\beta\rho_a) + \kappa_y\Big)\left[(1-\beta\rho_a)\phi_y + \kappa_y(\phi_\pi - \rho_a)\right]\,. \end{aligned}$$

Collecting terms simplifies the previous expression to

$$\begin{aligned} x^2\frac{\partial \mathcal{M}_y}{\partial \rho_a} =& (1-\beta\rho_a)\Big\{(1-\beta\rho_a)\phi_y + \kappa_y(\phi_\pi-\rho_a) - (\beta\phi_y+\kappa_y)(1-\rho_a+\phi_y)\Big\} \\ &+ (\beta\phi_y+\kappa_y)\Big\{(1-\beta\rho_a)\phi_y + \kappa_y(\phi_\pi-\rho_a) - \kappa_y(\phi_\pi-\rho_a)\Big\} \\ &+ \beta(1-\rho_a)\Big\{(1-\beta\rho_a)\phi_y + \kappa_y(\phi_\pi-\rho_a)\Big\}\,, \end{aligned}$$

which can be further simplified to

$$x^2\frac{\partial \mathcal{M}_y}{\partial \rho_a} = (1-\beta\rho_a)\phi_y\Big\{1-\beta\rho_a+\beta\phi_y+\kappa_y+\beta(1-\rho_a)-\beta(1-\rho_a+\phi_y)-\kappa_y\Big\}$$
$$+\kappa_y\Big\{\beta(1-\rho_a)(\phi_\pi-\rho_a)+(1-\beta\rho_a)(\phi_\pi-\rho_a)-(1-\rho_a)(1-\beta\rho_a)\Big\}\,.$$

The previous equation can be rewritten to

$$x^2\frac{\partial \mathcal{M}_y}{\partial \rho_a} = (1-\beta\rho_a)^2\phi_y+\kappa_y(\phi_\pi-1)\Big\{1-\beta\rho_a+\beta(1-\rho_a)\Big\}+\kappa_y\beta(1-\rho_a)^2\,.$$

Recall that the equilibrium is locally determinate under $(1-\beta)\phi_y+\kappa_y(\phi_\pi-1)>0$ such that we have

$$\begin{aligned}
\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho_a}\right) &= (1-\beta\rho_a)^2\phi_y+\kappa_y(\phi_\pi-1)\Big\{1-\beta\rho_a+\beta(1-\rho_a)\Big\}+\kappa_y\beta(1-\rho_a)^2\\
&> (1-\beta\rho_a)^2\phi_y-(1-\beta)\phi_y\Big\{1-\beta\rho_a+\beta(1-\rho_a)\Big\}+\kappa_y\beta(1-\rho_a)^2\\
&= \phi_y\Big\{(1-\beta\rho_a)^2-\Big(1-\beta\rho_a-\beta(1-\rho_a)\Big)\Big(1-\beta\rho_a+\beta(1-\rho_a)\Big)\Big\}+\kappa_y\beta(1-\rho_a)^2\\
&= (\phi_y\beta+\kappa_y)\beta(1-\rho_a)^2\\
&> 0\,,
\end{aligned}$$

which shows that the impact output multiplier monotonously increases in ρ_a.

To prove Statement (b), the derivative of $\mathcal{M}_\pi$ with respect to ρ_a yields

$$\begin{aligned}
x^2\frac{\partial \mathcal{M}_\pi}{\partial \rho_a} &= \kappa_y\Big\{\Big([1-\rho_a+\phi_y]\,(1-\beta\rho_a)+\kappa_y(\phi_\pi-\rho_a)\Big)-(1-\rho_a)\Big(\beta(1-\rho_a+\phi_y)+(1-\beta\rho_a)+\kappa_y\Big)\Big\}\\
&= \kappa_y\Big\{(1-\beta\rho_a)\phi_y+\kappa_y(\phi_\pi-\rho_a)-(1-\rho_a)\Big(\beta(1-\rho_a+\phi_y)+\kappa_y\Big)\Big\}\\
&= \kappa_y\Big\{-\beta(1-\rho_a)^2+(1-\beta)\phi_y+\kappa_y(\phi_\pi-1)\Big\}\,.
\end{aligned}$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ_a, i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_\pi}{\partial \rho_a}\right) = a\rho_a^2+b\rho_a+c\,,$$

where the auxiliary parameters are given by

$$a\equiv-\beta\,,\quad b\equiv 2\beta\,,\quad c\equiv-\beta+(1-\beta)\phi_y+\kappa_y(\phi_\pi-1)\,.$$

The corresponding roots are

$$\rho_a^{+,-} = \frac{-b\pm\sqrt{\Delta}}{2a}\,,$$

where the discriminant $\Delta \equiv b^2 - 4ac$ is given by

$$\Delta = 4\beta^2 - 4(-\beta)\left[-\beta + (1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)\right] = 4\beta\left[(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)\right] .$$

Consequently, $\Delta > 0$ holds by the determinacy condition and the second order polynomial has two real-valued roots that are given by

$$\rho_a^{+,-} = \frac{-2\beta \pm \sqrt{4\beta\left[(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)\right]}}{-2\beta} = 1 \pm \sqrt{\frac{(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)}{\beta}} .$$

Evidently, $\rho_a^+ > 1$ holds by the determinacy condition, i.e., the only feasible candidate on $\rho_a \in (0,1)$ is

$$\rho_a^* \equiv \rho_a^- = 1 - \sqrt{\frac{(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)}{\beta}} .$$

Note that $\rho_a^* \in (0,1)$ if $0 < (1-\beta)\phi_y + \kappa_y(\phi_\pi - 1) < \beta$, which is equivalent to $\phi_\pi < \overline{\phi}_\pi$, where

$$\overline{\phi}_\pi \equiv 1 + \frac{\beta - (1-\beta)\phi_y}{\kappa_y} .$$

As $f(\rho_a) \equiv a\rho_a^2 + b\rho_a + c$ is a strictly concave function in ρ_a, statement $(b.1)$ of Proposition 6 follows. To show statement $(b.2)$, notice that $(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1) \geq \beta$ implies

$$\begin{aligned}
\operatorname{sgn}\left(\frac{\partial \mathcal{M}_\pi}{\partial \rho_a}\right) = f(\rho_a) &= -\beta(1-\rho_a)^2 + (1-\beta)\phi_y + \kappa_y(\phi_\pi - 1) \\
&\geq \beta\left[1 - (1-\rho_a)^2\right] \\
&\geq 0 ,
\end{aligned}$$

where the last inequality applies as $\rho_a \in [0,1)$ and is strict for $\rho_a > 0$. As a result, we have $\frac{\partial \mathcal{M}_\pi}{\partial \rho_a} > 0$ for all parameter combinations characterized by $(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1) \geq \beta$. □

A.1.8 *Proof Proposition 7* To begin with, we offer a full statement of the Proposition 7. Specifically, under a forward-looking Taylor rule, three sub-cases arise depending on the aggressiveness of inflation and output feedback, i.e., the following proposition complements statement (b) in the main text by two sub-cases, $(b.2)$ $\phi_\pi = 1$ and $(b.3)$ $0 \leq \phi_\pi < 1$. Overall, a forward-looking Taylor behaves similar to a contemporaneous rule if inflation feedback is sufficiently strong and output feedback sufficiently moderate.

PROPOSITION 10. *The following results apply.*

(a) *Assume the monetary authorities react to both the current inflation rate and the current output gap, i.e.* $i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t$*, Proposition 2 continues to hold identically.*

(b) *Assume the monetary authorities react to the one-step ahead expected inflation and output gap, i.e.* $i_t = \bar{r} + \phi_\pi \mathbb{E}_t[\pi_{t+1}] + \phi_y \mathbb{E}_t[y_{t+1}]$. *The subsequent case distinction applies.*

(b.1) *Consider the case* $\phi_\pi > 1$*:*

- *If* $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$, *there exist two regime thresholds given by* $\underline{\phi}_\pi \equiv 1 + (1-\phi_y)(1-\beta)^2\kappa_y^{-1}$ *and* $\overline{\phi}_\pi \equiv 1 + (1-\phi_y)\kappa_y^{-1}$ *such that the relationship between the output impact multiplier and the degree of persistence (i) is increasing if* $\phi_\pi \leq \underline{\phi}_\pi$, *(ii) is decreasing if* $\phi_\pi \geq \overline{\phi}_\pi$, *and (iii) displays a hump-shaped pattern if* $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$. *In this latter case, the degree of persistence that maximizes the output multiplier is given by* $\rho^* = \beta^{-1}\left(1 - \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right)$.
- *If* $1 \leq \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$, *only the upper regime survives and the output multiplier decreases monotonically with* ρ.

(b.2) *Consider the case* $\phi_\pi = 1$*:*

- *If* $0 < \phi_y < 1$, *the output multiplier increases monotonously in* ρ.
- *If* $\phi_y = 1$, *the output multiplier is constant as* ρ *is varied.*
- *If* $1 < \phi_y < 2$, *the output multiplier decreases monotonously in* ρ.

(b.3) *Consider the case* $0 \leq \phi_\pi < 1$*:*

- *If* $\frac{1-\phi_\pi}{1-\beta}\kappa_y < \phi_y \leq \min\{1, 1+\beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$ *applies, the output multiplier increases monotonously in* ρ.
- *If* $\max\{1, \frac{1-\phi_\pi}{1-\beta}\kappa_y\} < \phi_y < \min\{1 + \beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$, *there exist two regime thresholds given by* $\underline{\phi}_\pi \equiv 1 + (1-\phi_y)\kappa_y^{-1}$ *and* $\overline{\phi}_\pi \equiv 1 + (1-\phi_y)(1-\beta)^2\kappa_y^{-1}$ *such that the relationship between the output impact multiplier and the degree of persistence (i) is increasing if* $\phi_\pi \leq \underline{\phi}_\pi$, *(ii) is decreasing if* $\phi_\pi \geq \overline{\phi}_\pi$, *and (iii) displays a u-shaped pattern if* $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$. *In this latter case, the degree of persistence that minimizes the output multiplier is given by* $\rho^* = \beta^{-1}\left(1 - \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right)$.

(c) *Assume the monetary authorities follow a real interest rate rule, i.e.* $i_t = r_t + \mathbb{E}_t[\pi_{t+1}]$, *where* $r_t = \bar{r}$. *Then, only the lower monetary policy regime survives and the output impact multiplier increases monotonically with* ρ.

To begin with, consider the case $\phi_\pi = 1$. There are three sub-cases. First, if $0 < \phi_y < 1$, the output multiplier monotonously increases in persistence. This is the case as the income expectation channel is positive and dominates the real interest rate channel. Second, if $\phi_y = 1$ applies, the partial equilibrium income expectation channel and the output feedback in the monetary policy rule offset each other, implying that output is constant in persistence. Third, if $1 < \phi_y < 2$, the output multiplier monotonously decreases in persistence. This is the case as the income expectation channel is negative and enforces the real interest rate channel. Notice that fixing the output feedback $\phi_y = 1$ is highly pedagogical as it allows us to isolate and link output as well as the real interest rate response to the magnitude of inflation feedback ϕ_π in a more transparent manner. Specifically, if $\phi_\pi < 1$ holds, output increases monotonously in ρ as either the real interest rate falls in ρ or the general equilibrium income expectation effect dominates the rise in real interest rates. Moreover, if additionally $\phi_\pi = 1$ holds, the overall income expectation and the real interest channel eliminate each other such that output is constant. Finally, if $\phi_\pi > 1$ applies, output falls in ρ as either the real interest rate increases in ρ or the general equilibrium income expectation effect overcompensates for the fall in the real interest rate.

Second, consider the case $(b.3)$, in which inflation feedback is moderate, i.e., $0 \leq \phi_\pi < 1$. If additionally output feedback is moderate, i.e., $\frac{1-\phi_\pi}{1-\beta}\kappa_y < \phi_y < \min\{1, 1+\beta^{-1}, 2 - \frac{\phi_\pi - 1}{1-\beta}\kappa_y\}$, output increases monotonously in ρ. This is the case, as the income expectation channel and the real interest rate channel move in the same direction. In contrast, if output feedback is sufficiently strong, i.e., $\max\{1, \frac{1-\phi_\pi}{1-\beta}\kappa_y\} < \phi_y < \min\{1+\beta^{-1}, 2 - \frac{\phi_\pi - 1}{1-\beta}\kappa_y\}$, the income expectation channel flips sign and we obtain three regimes: output increases monotonously in ρ for $\phi_\pi < \underline{\phi}_\pi$, is u-shaped in ρ for $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, and decreases monotonously in ρ for $\phi_\pi > \overline{\phi}_\pi$. The u-shaped regime arises from two opposing channels: the income expectation channel now decreases output for higher persistence values, while the real interest rate channel increases output for higher persistence values. As such, the real interest rate channel dominates the income expectation channel for $\phi_\pi < \underline{\phi}_\pi$, whereas the income expectation channel dominates the real interest channel for $\phi_\pi > \overline{\phi}_\pi$. We illustrate local determinacy properties and the monetary policy regimes for statement (b) of Proposition 7 in Figure A.1.

PROOF. Subsequently, we proceed by case distinction.

Statement (a): The proof of statement (a) is contained in the proof of Proposition 2.

Statement (b): To prove statement (b) we proceed in three steps. First, we derive the output impact multiplier. Second, we provide determinacy conditions. Third, we conduct a case distinction.

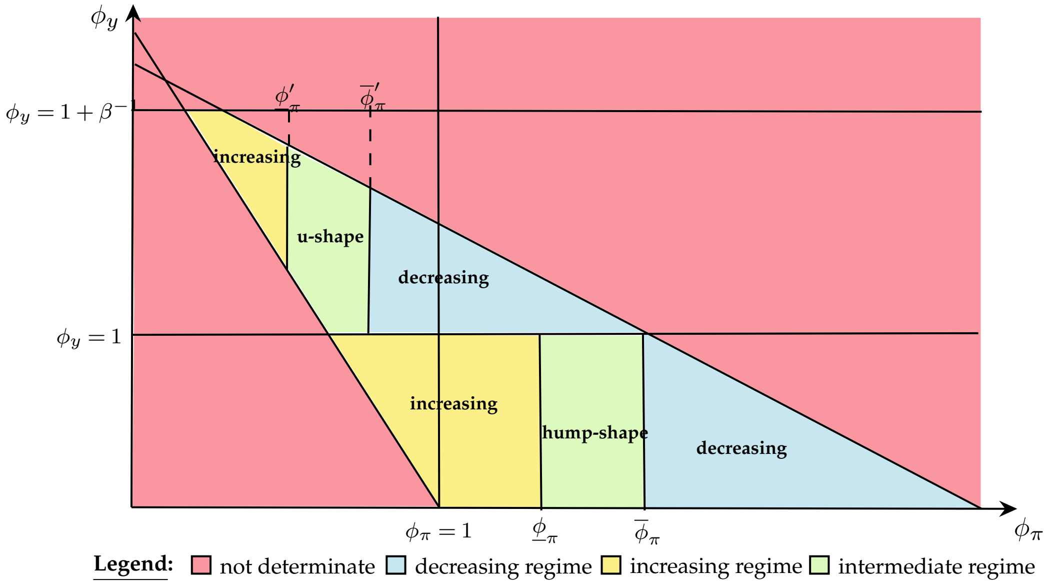
Impact Multiplier. Substituting the *forward-looking* Taylor rule into the DIS equation (1) results in

$$y_t = (1-\phi_y)\mathbb{E}_t\left[y_{t+1}\right] - (\phi_\pi - 1)\,\mathbb{E}_t\left[\pi_{t+1}\right] + \xi_t \,.$$

As before, we guess and verify that the solution takes the form $y_t = \mathcal{M}_y\xi_t$ and $\pi_t = \mathcal{M}_\pi\xi_t$, such that the previous equation together with the NKPC can be rewritten as

$$\left[1-(1-\phi_y)\rho\right]\mathcal{M}_y = -\left(\phi_\pi - 1\right)\rho\mathcal{M}_\pi + 1\,,$$
$$\left(1-\beta\rho\right)\mathcal{M}_\pi = \kappa_y\mathcal{M}_y\,.$$

FIGURE A.1. Determinacy and Monetary Policy Regimes under a Forward-Looking Taylor Rule



Substituting the latter into the former equation gives us

$$[1 - (1 - \phi_y)\rho]\,\mathcal{M}_y = -(\phi_\pi - 1)\,\frac{\kappa_y \rho}{1 - \beta\rho}\mathcal{M}_y + 1\,,$$

which finally yields

$$\mathcal{M}_y = \frac{1 - \beta\rho}{[1 - (1 - \phi_y)\rho]\,(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho} > 0\,.$$

The impact output multiplier is strictly positive as the denominator $\mathcal{M}_y^d$ can be rewritten as follows

$$\begin{aligned}\mathcal{M}_y^d &= (1 - \rho)(1 - \beta\rho) + \rho\,[\phi_y(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)] \\ &= (1 - \rho)(1 - \beta\rho) + \rho\,[\phi_y(1 - \beta) + \kappa_y(\phi_\pi - 1) + \phi_y\beta(1 - \rho)] > 0\,,\end{aligned}$$

where the strictly inequality is due to the determinacy condition $\phi_y(1-\beta)+\kappa_y(\phi_\pi-1) > 0$. The impact multipliers for inflation, the nominal interest rate, and the real interest rate then follow by

$$\mathcal{M}_\pi = \frac{\kappa_y}{[1 - (1 - \phi_y)\rho]\,(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho} > 0\,,$$

$$\mathcal{M}_i = \frac{\rho\,[\phi_\pi\kappa_y + \phi_y\,(1 - \beta\rho)]}{[1 - (1 - \phi_y)\rho]\,(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho} \geq 0\,,$$

$$\mathcal{M}_r = \frac{\rho\,[(\phi_\pi - 1)\kappa_y + \phi_y\,(1 - \beta\rho)]}{[1 - (1 - \phi_y)\rho]\,(1 - \beta\rho) + \kappa_y(\phi_\pi - 1)\rho} \geq 0\,,$$

where the sign of $\mathcal{M}_r$, in turn, follows from a similar reasoning as above, i.e., $(1-\beta)\phi_y + (\phi_\pi - 1)\kappa_y > 0$, with strict inequality for $\rho > 0$.

Determinacy. Following Proposition 4 on page 1121 in Bullard and Mitra (2002), we summarize local equilibrium determinacy conditions for a model with a forward-looking monetary policy rule in Lemma 1.

LEMMA 1 (BULLARD AND MITRA (2002)). *Under interest rate rules with forward expectations, the necessary and sufficient conditions for a rational expectations equilibrium to be unique are*

$$\phi_y < 1 + \beta^{-1}\,, \tag{D1}$$

$$\kappa_y(\phi_\pi - 1) + (1+\beta)\phi_y < 2(1+\beta)\,, \tag{D2}$$

$$\kappa_y(\phi_\pi - 1) + (1-\beta)\phi_y > 0\,. \tag{D3}$$

Comparative Statics. Defining $x \equiv [1-(1-\phi_y)\rho]\,(1-\beta\rho) + \kappa_y(\phi_\pi - 1)\rho$ and taking the derivative of $\mathcal{M}_y$ with respect to ρ yields

$$\begin{aligned} x^2\frac{\partial \mathcal{M}_y}{\partial \rho} =& -\beta\Big([1-(1-\phi_y)\rho]\,(1-\beta\rho) + \kappa_y(\phi_\pi - 1)\rho\Big) \\ & - (1-\beta\rho)\Big(-(1-\phi_y)(1-\beta\rho) - \beta\,[1-(1-\phi_y)\rho] + \kappa_y(\phi_\pi - 1)\Big)\,, \end{aligned}$$

which can be simplified to

$$x^2\frac{\partial \mathcal{M}_y}{\partial \rho} = (1-\phi_y)(1-\beta\rho)^2 - \kappa_y(\phi_\pi - 1)\,.$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ, i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2 + b\rho + c\,,$$

where the auxiliary parameters are given by

$$a \equiv (1-\phi_y)\beta^2\,, \quad b \equiv -2(1-\phi_y)\beta\,, \quad \text{and} \quad c \equiv 1 - \phi_y - \kappa_y(\phi_\pi - 1)\,.$$

The corresponding roots are

$$\rho^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a}\,,$$

where the discriminant $\Delta \equiv b^2 - 4ac$ is given by

$$\Delta = 4(1-\phi_y)^2\beta^2 - 4(1-\phi_y)\beta^2\,[1-\phi_y - \kappa_y(\phi_\pi - 1)] = 4\beta^2\kappa_y(\phi_\pi - 1)(1-\phi_y)\,.$$

From here we proceed by case distinction:

1. First, consider the case $\phi_\pi > 1$. In this case (D1)-(D3) imply that the model is locally determinate iff $0 \leq \phi_y < \min\{1+\beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$. To begin with, consider the sub-case in which $0 \leq \phi_y < \min\{1, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$ such that $\Delta > 0$, $a > 0$, and $b < 0$. As such, the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \frac{2(1-\phi_y)\beta \pm \sqrt{4\beta^2\kappa_y(\phi_\pi - 1)(1-\phi_y)}}{2(1-\phi_y)\beta^2} = \beta^{-1}\left(1 \pm \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right) .$$

Evidently, $\rho^+ > 1$ as $\beta \in (0,1)$ and $\phi_\pi > 1$ such that the only feasible candidate on $\rho \in (0,1)$ is

$$\rho^* \equiv \rho_\xi^- = \beta^{-1}\left(1 - \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right) .$$

Notice that $\rho^* \in (0,1)$ if $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, where we have

$$\underline{\phi}_\pi \equiv 1 + (1-\phi_y)(1-\beta)^2\kappa_y^{-1} , \quad \text{and} \quad \overline{\phi}_\pi \equiv 1 + (1-\phi_y)\kappa_y^{-1} .$$

As $f(\rho) \equiv a\rho^2 + b\rho + c$ is a strictly *convex* function in ρ, the first part of statement $(b.1)$ follows. As before, note that sgn $\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$ is strictly decreasing in ϕ_π. Hence, substituting in the lower bound $\underline{\phi}_\pi$, we obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi} = (1-\phi_y)\left[(1-\beta\rho)^2 - (1-\beta)^2\right] > 0 ,$$

where the last inequality applies as $\rho \in [0,1)$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ for all $\phi_\pi \leq \underline{\phi}_\pi$. Similarly, we substitute in for the upper bound $\overline{\phi}_\pi$ to obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \overline{\phi}_\pi} = (1-\phi_y)\left[\beta^2\rho^2 - 2\beta\rho\right] = (1-\phi_y)\beta\rho\left[\beta\rho - 2\right] < 0$$

where the last inequality applies as $\rho \in [0,1)$. Notice that the previous inequality is strict for $\rho > 0$. As a result, $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0 \; \forall \phi_\pi \geq \underline{\phi}_\pi$. This completes the derivation of the first part of statement $(b.1)$.

   To show the second part of statement $(b.1)$, consider the sub-case in which $1 \leq \phi_y < \min\{1+\beta^{-1}, 2 - \frac{\phi_\pi - 1}{1+\beta}\kappa_y\}$ such that $\Delta \leq 0$, $a \leq 0$, $b \geq 0$, and $c < 0$. As such, the second order polynomial is strictly negative on $\rho \in [0,1)$.

2. Second, consider the case $\phi_\pi = 1$. In this case (D1)-(D3) imply that the model is locally determinate iff $0 < \phi_y < 2$. Three sub-cases arise: first, if $0 < \phi_y < 1$, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$; second, if $\phi_y = 1$, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} = 0$; and third, if $1 < \phi_y < 2$, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$.

3. Finally, consider the case $0 \leq \phi_\pi < 1$. In this case (D1)-(D3) imply that the model is locally determinate iff $\frac{1-\phi_\pi}{1-\beta}\kappa_y^{-1} < \phi_y < \min\{1+\beta^{-1}, 2 + \frac{1-\phi_\pi}{1+\beta}\kappa_y\}$. There arise, in turn, two sub-cases. First, if additionally $\frac{1-\phi_\pi}{1-\beta}\kappa_y^{-1} < \phi_y \leq \min\{1, 1+\beta^{-1}, 2 +$

$\frac{1-\phi_\pi}{1+\beta}\kappa_y\}$ applies, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$. Second, if additionally $\max\{1, \frac{1-\phi_\pi}{1-\beta}\kappa_y^{-1}\} < \phi_y \leq \min\{1+\beta^{-1}, 2+\frac{1-\phi_\pi}{1+\beta}\kappa_y\}$ applies, we have $\Delta > 0$, $a < 0$, $b > 0$, and $f(\rho) \equiv \text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2 + b\rho + c$ is strictly *concave* and has two real-valued roots that are given by

$$\rho^{+,-} = \beta^{-1}\left(1 \pm \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right) .$$

Evidently, $\rho^+ > 1$ as $\beta \in (0,1)$, $0 \leq \phi_\pi < 1$, and $\phi_y > 1$ such that the only feasible candidate on $\rho_\xi \in (0,1)$ is

$$\rho^* \equiv \rho_\xi^- = \beta^{-1}\left(1 - \sqrt{\frac{\phi_\pi - 1}{1-\phi_y}\kappa_y}\right) .$$

Note that $\rho^* \in (0,1)$ if $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$, where we have

$$\underline{\phi}_\pi \equiv 1 + (1-\phi_y)\kappa_y^{-1} , \quad \text{and} \quad \overline{\phi}_\pi \equiv 1 + (1-\phi_y)(1-\beta)^2\kappa_y^{-1} .$$

As $f(\rho)$ is a strictly *concave* function in ρ, the second part of statement $(b.3)$ follows. As before, note that $\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$ is strictly decreasing in ϕ_π. Substituting in the lower bound $\underline{\phi}_\pi$, we thus get

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi} = (1-\phi_y)\beta\rho\left[\beta\rho - 2\right] > 0$$

where the last inequality applies as $\rho \in [0,1)$. Note that the previous inequality is strict for $\rho > 0$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ for all $\phi_\pi \leq \underline{\phi}_\pi$. Similarly, we substitute in for the upper bound $\overline{\phi}_\pi$ and finally obtain

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \overline{\phi}_\pi} = (1-\phi_y)\left[(1-\beta\rho)^2 - (1-\beta)^2\right] < 0 ,$$

where the last inequality applies as $\rho \in [0,1)$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$ for all $\phi_\pi \geq \underline{\phi}_\pi$. This completes the derivation of statement $(b.3)$.

Statement (c): Under a real interest rate rule, the impact output multiplier is given by $\mathcal{M}_y = (1-\rho)^{-1}$, which is monotonously increasing in ρ. □

A.2 General New Keynesian Model

A.2.1 *Proof Proposition 8*

PROOF. We proceed in three steps.

Determinacy. For a derivation of the determinacy condition, see, for instance, Proposition 3 in Gabaix (2020), respectively Proposition 1 in Bilbiie (Forthcoming).

Multipliers. Substituting the Taylor rule (16) into the DIS equation (14) results in

$$(1+\phi_y\zeta_r)y_t = \zeta_f \mathbb{E}_t\left[y_{t+1}\right] + \zeta_r \mathbb{E}_t\left[\pi_{t+1}\right] - \zeta_r\phi_\pi\pi_t + \xi_t \,.$$

As before, we guess and verify that the solution takes the form $y_t = \mathcal{M}_y\xi_t$ and $\pi_t = \mathcal{M}_\pi\xi_t$, such that the previous equation together with the NKPC can be rewritten as

$$\begin{aligned}\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{M}_y &= -\zeta_r\left(\phi_\pi - \rho\right)\mathcal{M}_\pi + 1\,,\\ \left(1-\beta_f\rho\right)\mathcal{M}_\pi &= \kappa_y\mathcal{M}_y\,.\end{aligned}$$

Substituting the latter into the former equation gives us

$$\left(1+\phi_y\zeta_r - \rho\zeta_f\right)\mathcal{M}_y = -\zeta_r\left(\phi_\pi - \rho\right)\frac{\kappa_y}{1-\beta_f\rho}\mathcal{M}_y + 1\,,$$

which finally yields

$$\mathcal{M}_y = \frac{1-\beta_f\rho}{\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\rho\zeta_f\right) + \left(\phi_\pi-\rho\right)\zeta_r\kappa_y}\,.$$

From the previous equation the impact multipliers for inflation, the nominal as well as the real interest rate follow recursively, as provided in the main text.

Sign. To determine the sign of the multipliers, we derive a reduced form system of output and inflation dynamics of the following form

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \mathbf{A}_T\begin{pmatrix}\mathbb{E}_t\left[y_{t+1}\right] \\ \mathbb{E}_t\left[\pi_{t+1}\right]\end{pmatrix} + \mathbf{B}_T\xi_t\,,$$

where

$$\begin{aligned}\mathbf{A}_T &\equiv \frac{1}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}\begin{pmatrix}\zeta_f & \zeta_r(1-\phi_\pi\beta_f) \\ \zeta_f\kappa_y & \beta_f\left(1+\phi_y\zeta_r\right)+\zeta_r\kappa_y\end{pmatrix}\,,\\ \mathbf{B}_T &\equiv \frac{1}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}\begin{pmatrix}1 \\ \kappa_y\end{pmatrix}\,.\end{aligned}$$

Using the fact that $\mathbb{E}_t\left[y_{t+1}\right] = \rho y_t$ and $\mathbb{E}_t\left[\pi_{t+1}\right] = \rho\pi_t$ hold on the equilibrium path, we can write

$$\begin{aligned}\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} &= \left(\mathbf{I}_2 - \rho\mathbf{A}_T\right)^{-1}\mathbf{B}_T\xi_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}}\begin{pmatrix}1-\rho A_{22} & \rho A_{12} \\ \rho A_{21} & 1-\rho A_{11}\end{pmatrix}\mathbf{B}_T\xi_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12}A_{21}}\begin{pmatrix}(1-\rho A_{22})B_{11} + \rho A_{12}B_{21} \\ \rho A_{21}B_{11} + (1-\rho A_{11})B_{21}\end{pmatrix}\xi_t\,.\end{aligned}$$

Notice that we have

$$
\begin{aligned}
(1-\rho A_{22})B_{11}+\rho A_{12}B_{21} =& \left(1-\rho\frac{\beta_f\left(1+\phi_y\zeta_r\right)+\zeta_r\kappa_y}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}\right)\frac{1}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y} \\
&+\rho\frac{\zeta_r(1-\phi_\pi\beta_f)}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}\frac{\kappa_y}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y} \\
=&\frac{1-\rho\beta_f}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}>0\,,
\end{aligned}
$$

where the strict inequality follows under Assumption 2 combined with $1>\rho\beta_f$ and $(\phi_\pi,\phi_y)\in\mathbb{R}^2_+$. As a result, this implies that $\text{sgn}(\mathcal{M}_y)=\text{sgn}(\det(\mathbf{I}_2-\rho_\xi\mathbf{A}_T))$. One can show that

$$
\begin{aligned}
A_{11}A_{22}-A_{12}A_{21} &= \frac{\zeta_f\beta_f}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}>0\,, \\
A_{11}+A_{22} &= \frac{\zeta_f+\beta_f\left(1+\phi_y\zeta_r\right)+\zeta_r\kappa_y}{1+\phi_y\zeta_r+\phi_\pi\zeta_r\kappa_y}>0\,.
\end{aligned}
$$

Moreover, we can rewrite

$$
\begin{aligned}
\det(\mathbf{I}_2-\rho\mathbf{A}_T) &= (1-\rho A_{11})(1-\rho A_{22})-\rho^2 A_{12}A_{21} \\
&= 1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right)\,.
\end{aligned}
$$

As $A_{11}+A_{22}>0$ applies, we can write

$$
\begin{aligned}
\det(\mathbf{I}_2-\rho\mathbf{A}_T) &= 1-\rho\left(A_{11}+A_{22}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right) \\
&> 1-\rho\left(1+A_{11}A_{22}-A_{12}A_{21}\right)+\rho^2\left(A_{11}A_{22}-A_{12}A_{21}\right) \\
&= (1-\rho)\left[1-\rho\left(A_{11}A_{22}-A_{12}A_{21}\right)\right] \\
&> 0\,,
\end{aligned}
$$

where the first strict inequality is due to the determinacy condition $A_{11}+A_{22}<1+A_{11}A_{22}-A_{12}A_{21}$ and the second strictly inequality is due to the determinacy condition $A_{11}A_{22}-A_{12}A_{21}<1$. As a result, this implies

$$
\mathcal{M}_y=\frac{1-\rho\beta_f}{\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\rho\zeta_f\right)+\left(\phi_\pi-\rho\right)\zeta_r\kappa_y}>0\,.
$$

From here, the sign of the inflation and nominal interest rate multipliers follow. Moreover, the sign of the real interest rate multiplier depends on the sign of its numerator, which is positive if $\phi_\pi\geq\rho-(1-\beta_f\rho)\phi_y/\kappa_y$. □

A.2.2 *Proof Proposition 9*

PROOF. The proof follows the same steps as the one for Proposition 2. Taking the derivative of $\mathcal{M}_y$ with respect to ρ yields

$$\frac{\partial \mathcal{M}_y}{\partial \rho} = \frac{-\beta_f\left[\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\zeta_f\rho\right)+\left(\phi_\pi-\rho\right)\zeta_r\kappa_y\right]}{\left[\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\zeta_f\rho\right)+\left(\phi_\pi-\rho\right)\zeta_r\kappa_y\right]^2} + \frac{\left(1-\beta_f\rho\right)\left[\zeta_f\left(1-\beta_f\rho\right)+\left(1+\phi_y\zeta_r-\zeta_f\rho\right)\beta_f+\zeta_r\kappa_y\right]}{\left[\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\zeta_f\rho\right)+\left(\phi_\pi-\rho\right)\zeta_r\kappa_y\right]^2}\,.$$

Defining $x \equiv \left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\zeta_f\rho\right)+\left(\phi_\pi-\rho\right)\zeta_r\kappa_y$, we can write

$$\begin{aligned} x^2\frac{\partial \mathcal{M}_y}{\partial \rho} =& -\beta_f\left(1+\phi_y\zeta_r-\zeta_f\rho\right)\left(1-\beta_f\rho\right)-\beta_f\left(\phi_\pi-\rho\right)\zeta_r\kappa_y \\ &+(1-\beta_f\rho)\zeta_f\left(1-\beta_f\rho\right)+\left(1-\beta_f\rho\right)\left(1+\phi_y\zeta_r-\zeta_f\rho\right)\beta_f \\ &+(1-\beta_f\rho)\zeta_r\kappa_y\,, \end{aligned}$$

which can be rewritten to

$$\begin{aligned} x^2\frac{\partial \mathcal{M}_y}{\partial \rho} &= \left[1-\beta_f\phi_\pi\right]\zeta_r\kappa_y+\zeta_f(1-\beta_f\rho)\left(1-\beta_f\rho\right)-\left(1+\phi_y\zeta_r-\zeta_f\rho\right)\left[\beta_f\left(1-\beta_f\rho\right)-(1-\beta_f\rho)\beta_f\right] \\ &= \left[1-\beta_f\phi_\pi\right]\zeta_r\kappa_y+\zeta_f\left[1-2\beta_f\rho+\beta_f^2\rho^2\right]\,. \end{aligned}$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ, i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2+b\rho+c\,,$$

where the auxiliary parameters are given by

$$a \equiv \zeta_f\beta_f^2\,, \qquad b \equiv -2\zeta_f\beta_f\,, \qquad c \equiv \zeta_f+\left[1-\beta_f\phi_\pi\right]\zeta_r\kappa_y\,.$$

The corresponding roots are

$$\rho^{+,-} = \frac{-b\pm\sqrt{\Delta}}{2a}\,,$$

where the discriminant $\Delta \equiv b^2-4ac$ is given by

$$\Delta = 4\zeta_f^2\beta_f^2-4\zeta_f\beta_f^2\left[\zeta_f+\left(1-\beta_f\phi_\pi\right)\zeta_r\kappa_y\right] = 4\zeta_f\zeta_r\beta_f^2(\beta_f\phi_\pi-1)\kappa_y\,.$$

Consequently, the following case distinction applies. First, $\phi_\pi < \beta_f^{-1}$ implies $\Delta < 0$ and the second order polynomial has two distinct complex roots. Second, $\phi_\pi = \beta_f^{-1}$ implies $\Delta = 0$ and the second order polynomial has a unique real root that is given by $-\frac{b}{2a} = \beta_f^{-1} > 1$. Third, $\phi_\pi > \beta_f^{-1}$ implies that the second order polynomial has two real-valued roots that are given by

$$\rho^{+,-} = \frac{2\zeta_f\beta_f\pm\sqrt{4\zeta_f\zeta_r\beta_f^2(\beta_f\phi_\pi-1)\kappa_y}}{2\zeta_f\beta_f^2} = \beta_f^{-1}\left(1\pm\sqrt{(\beta_f\phi_\pi-1)\frac{\zeta_r}{\zeta_f}\kappa_y}\right)\,.$$

Evidently, $\rho^+ > 1$ as $\beta_f \in (0,1)$ and $\phi_\pi > \beta_f^{-1}$ such that the only feasible candidate on $\rho \in (0,1)$ is

$$\rho^{g*} \equiv \rho^- = \beta_f^{-1}\left(1 \pm \sqrt{(\beta_f\phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y}\right) .$$

Note that $\rho^{g*} \in (0,1)$ if $\underline{\phi}_\pi^g < \phi_\pi < \overline{\phi}_\pi^g$, where we have

$$\underline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1 + (1-\beta_f)^2\frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right) , \quad \text{and} \quad \overline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1 + \frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right) .$$

As $f(\rho) \equiv a\rho^2 + b\rho + c$ is strictly convex in ρ, statement (b) of Proposition 9 follows.

To show statement (a) of Proposition 9, note that sgn $\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$ is strictly decreasing in ϕ_π. Hence, substituting in the lower bound $\underline{\phi}_\pi^g$, we obtain

$$\begin{aligned}
\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi^g} =& \zeta_f\beta_f^2\rho^2 - 2\zeta_f\beta_f\rho + \zeta_f + \left(1 - \beta_f\underline{\phi}_\pi^g\right)\zeta_r\kappa_y \\
=& \zeta_f\beta_f^2\rho^2 - 2\zeta_f\beta_f\rho + \zeta_f - (1-\beta_f)^2\zeta_f \\
=& \zeta_f\left[(1-\beta_f\rho)^2 - (1-\beta_f)^2\right] \\
>& 0 ,
\end{aligned}$$

where the last inequality applies as $\rho \in [0,1)$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ for all $\phi_\pi \leq \underline{\phi}_\pi^g$.

Finally, to show statement (c) we analogously substitute in for the upper bound $\overline{\phi}_\pi^g$ to obtain

$$\begin{aligned}
\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \overline{\phi}_\pi^g} =& \zeta_f\beta_f^2\rho^2 - 2\zeta_f\beta_f\rho + \zeta_f + \left(1 - \beta_f\overline{\phi}_\pi^g\right)\zeta_r\kappa_y \\
=& \zeta_f\beta_f^2\rho^2 - 2\zeta_f\beta_f\rho \\
=& \zeta_f\beta_f\rho\left[\beta_f\rho - 2\right] \\
\leq& 0 ,
\end{aligned}$$

where the last inequality applies as $\rho \in [0,1)$. Notice that the previous inequality is strict for $\rho > 0$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$ for all $\phi_\pi \geq \overline{\phi}_\pi^g$. □

A.3 Extension: New Keynesian Model with Cost Channel of Monetary Policy

A.3.1 *Environment* Subsequently, we further extend the general New Keynesian model of Section 3.3.1 by a cost channel of monetary policy. Specifically, equilibrium dynamics are now described by the following four log-linear equations

$$y_t = \zeta_f\mathbb{E}_t[y_{t+1}] - \zeta_r(i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}) + \zeta_r\xi_t , \tag{25}$$

$$\pi_t = \beta_f\mathbb{E}_t[\pi_{t+1}] + \kappa_y y_t + \kappa_r(i_t - \mathbb{E}_t[\pi_{t+1}] - \bar{r}) - \kappa_r\xi_t , \tag{26}$$

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t \, , \tag{27}$$

$$\xi_t = \rho \xi_{t-1} + \varepsilon_{\xi,t} \, . \tag{28}$$

This representation introduces a cost channel of monetary policy into the NKPC equation (26) that contemporaneously links inflation to the real interest rate, with corresponding elasticity $\kappa_r > 0$. Moreover, relative to the main text, we scale up the demand shock in the DIS and NKPC equations by ζ_r, respectively by κ_r, to ensure that output and inflation dynamics coincide under a bond premium shock with the ones under a standard monetary policy shock. This particular assumption is innocuous for our subsequent results.

Parameter Restrictions Throughout this subsection, we impose the following parameter restrictions.

ASSUMPTION 2 (PATMAN CONDITION). *Assume $\beta_f > \kappa_r$ and $\kappa_y > \zeta_r^{-1}\kappa_r > \zeta_r^{-1}\kappa_r(1-\zeta_f)$.*

Assumption 2 restricts the strength of the cost channel of monetary policy. The constraint $\beta_f > \kappa_r$ ensures that the NKPC is forward looking in a positive sense, i.e., current inflation reacts increases with next period inflation expectations. In addition, the constraint $\kappa_y > \zeta_r^{-1}\kappa_r$ is referred to as reverse temporary equilibrium *Patman* condition, whereas $\kappa_y > \zeta_r^{-1}\kappa_r(1-\zeta_f)$ is referred to as general equilibrium *Patman* condition if the persistence of the real interest rate tightening approaches unity. If the former constraint applies, the latter does so as well, and a persistent positive surprise in the *real* interest rate decreases inflation, i.e., the economy behaves in an upward sloping NKPC environment. We refer the interested reader to Beaudry et al. (2024) who provide an in-depth discussion of Assumption 2.

A.3.2 *Main Results* We now state our main results. To begin with, we provide determinacy conditions. Next, we derive impact multipliers. Finally, we provide a characterization of the three monetary policy regimes. Note that all the results in the main text are obtained after eliminating the cost channel of monetary policy, i.e., by specifying $\kappa_r = 0$

Local Determinacy We summarize the necessary and sufficient conditions that ensure locally stable model dynamics in Proposition 11.

PROPOSITION 11. *Let us define auxiliary parameters $\underline{\underline{\phi}}_\pi^d$, $\underline{\phi}_\pi^d$, and $\overline{\phi}_\pi^d$, where*

$$\underline{\underline{\phi}}_\pi^d \equiv \max\left\{1 + \frac{(1-\beta_f)(\zeta_f - 1 - \phi_y\zeta_r)}{\zeta_r\kappa_y + (\zeta_f - 1)\kappa_r}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} ,$$

$$\underline{\phi}_\pi^d \equiv \max\left\{\frac{\zeta_f + \beta_f(1+\phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} ,$$

$$\overline{\phi}_\pi^d \equiv -1 + \frac{(1+\beta_f)(1+\zeta_f+\phi_y\zeta_r)}{(1+\zeta_f)\kappa_r - \zeta_r\kappa_y} \, .$$

Under Assumption 2, local determinacy properties are summarized by the following case distinction:

(a) *If* $\zeta_r\kappa_y \geq (1+\zeta_f)\kappa_r$, *the model has a determinate equilibrium iff* $\phi_\pi > \underline{\underline{\phi}}^d_\pi$.

(b) *If* $\zeta_r\kappa_y < (1+\zeta_f)\kappa_r$, *the model has a determinate equilibrium iff* $\underline{\phi}^d_\pi < \phi_\pi < \overline{\phi}^d_\pi$.

Proposition 11 extends the Taylor principle to environments that combine a compounded or discounted DIS equation with a cost channel of monetary policy. Thus, it shares common elements with similar conditions derived in pure tractable heterogeneous agent New Keynesian models (Acharya and Dogra, 2020, Bilbiie, Forthcoming), in models that feature bounded rationality (Gabaix, 2020), or models with a cost channel of monetary policy (Surico, 2008, Beaudry et al., 2024). Specifically, Proposition 11 generalizes the results of Surico (2008) to environments that feature a compounded or discounted DIS equation. Moreover, relative to Beaudry et al. (2024) who provide determinacy conditions with a discounted DIS equation and a cost channel of monetary policy in a continuous-time environment, Proposition 11 highlights the emergence of two particular cases characterized by the sign of $\zeta_r/(1+\zeta_f) - \kappa_r/\kappa_y$. The latter expression reflects the relative strength of a standard demand channel of monetary policy relative to the cost channel.

According to Proposition 11, determinacy depends on the relative strength of the demand and the cost channel. Specifically, if the demand channel is sufficiently strong, i.e., $\zeta_r\kappa_y \geq (1+\zeta_f)\kappa_r$, determinacy requires a sufficiently strong reaction of systematic monetary policy to contemporaneous inflation, that is $\phi_\pi > \underline{\underline{\phi}}^d_\pi$. Regarding the lower bound, the first condition dominates the second one for reasonable calibrations, such that we discuss it subsequently in more detail. For simplicity, we first consider the case $\phi_y = 0$, i.e., central banks react endogenously to inflation only. In this case, a compounded DIS equation ($\zeta_f > 1$), respectively a discounted DIS equation ($\zeta_f < 1$), requires a stronger (respectively weaker) reaction to fight inflation as compared to the standard Taylor principle. This effect, however, decreases in the strength of the marginal cost channel κ_r. The underlying rationale is as follows. A compounded DIS equation amplifies ceteris paribus output and inflation, and requires a stronger central bank reaction. On the other hand, inflation is stabilized through the marginal cost channel, which requires a weaker reaction. In contrast, if the cost channel is sufficiently strong, i.e., $\zeta_r\kappa_y < (1+\zeta_f)\kappa_r$, determinacy requires a sufficiently strong - but not too aggressive - reaction of systematic monetary policy to contemporaneous inflation, that is $\underline{\phi}^d_\pi < \phi_\pi < \overline{\phi}^d_\pi$. Intuitively, a strong cost channel raises inflation in response to a rise in the real interest rate. If the Taylor rule-feedback with respect to inflation is too aggressive, inflation and output dynamics will therefore further diverge and not stabilize. Overall, lower and upper thresholds are relaxed in ϕ_y, while the comparative statics regarding ζ_r, κ_y, and κ_r are ambiguous an depend, among others, on whether the DIS equation features compounding or discounting.

Impact Multipliers The following Proposition follows.

PROPOSITION 12. *If determinacy conditions from Proposition 11 are met, the model solution takes the form*

$$y_t = \mathcal{M}_y(\rho)\xi_t, \quad \pi_t = \mathcal{M}_\pi(\rho)\xi_t, \quad i_t = \bar{r} + \mathcal{M}_i(\rho)\xi_t, \quad \textit{and} \quad r_t = \bar{r} + \mathcal{M}_r(\rho)\xi_t\ ,$$

where $\mathcal{M}_y$, $\mathcal{M}_\pi$, $\mathcal{M}_i$ and $\mathcal{M}_r$ are recursively given by

$$\mathcal{M}_y = \frac{1-\rho\beta_f}{(1+\phi_y\zeta_r-\rho\zeta_f)\left[1-\rho\beta_f-\kappa_r(\phi_\pi-\rho)\right]\zeta_r^{-1}+(\kappa_y+\phi_y\kappa_r)(\phi_\pi-\rho)}\,, \tag{M1'}$$

$$\mathcal{M}_\pi = \frac{\kappa_y+\phi_y\kappa_r}{1-\rho\beta_f-\kappa_r(\phi_\pi-\rho)}\mathcal{M}_y - \frac{\kappa_r}{1-\rho\beta_f-\kappa_r(\phi_\pi-\rho)}\,, \tag{M2'}$$

$$\mathcal{M}_i = \phi_\pi\mathcal{M}_\pi + \phi_y\mathcal{M}_y\,, \tag{M3'}$$

$$\mathcal{M}_r = (\phi_\pi-\rho_\xi)\mathcal{M}_\pi + \phi_y\mathcal{M}_y\,. \tag{M4'}$$

Under Assumption 2, impact multipliers for output, inflation, and the nominal interest rate are strictly positive. Moreover, the impact multiplier of the real interest rate is strictly positive if $\phi_\pi > \phi_\pi^{r+}$ and negative if $\phi_\pi \le \phi_\pi^{r+}$, where

$$\phi_\pi^{r+} \equiv \rho - \frac{\phi_y\zeta_r(1-\beta_f)}{\zeta_r\kappa_y-\kappa_r(1-\rho\zeta_f)}\,.$$

As stated above, Assumption 2 restricts the strength of the cost channel of monetary policy. As such, deflationary forces due to falling real interest rates in response to an expansionary DIS-demand shock are limited and inflation rises at impact. Moreover, the sign of the impact real interest rate multiplier depends on the strength of inflation feedback within the Taylor rule. The real interest rate rises only if monetary policy reacts sufficiently aggressive to inflation, and falls otherwise. The former condition is always met within the second determinacy regime characterized by $\zeta_r\kappa_y < (1+\zeta_f)\kappa_r$. In contrast, in the first determinacy regime characterized by $\zeta_r\kappa_y \ge (1+\zeta_f)\kappa_r$ the real interest rate always rises under a compounded DIS equation, and may fall under a discounted DIS equation. This is the case as the latter allows for a sufficiently low inflation sensitivity to ensure local determinacy.

Monetary Policy Regimes We provide a characterization of monetary policy regimes in Proposition 13.

PROPOSITION 13. *Consider a forward-looking and upward-sloping NKPC, i.e., $\beta_f > 0$ and $\kappa_y > 0$. Moreover, let Assumption 2 hold, i.e., $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$. There exist two cut-off values, $\underline{\phi}_\pi^g$ and $\overline{\phi}_\pi^g$, of the Central Bank's degree of reaction to the inflation gap*

$$\underline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1+(1-\beta_f)^2\frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y+(\zeta_f-\beta_f)\kappa_r}\right) \quad \textit{and} \quad \overline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1+\frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y+(\zeta_f-\beta_f)\kappa_r}\right)\,,$$

with $\overline{\phi}_\pi^g > \underline{\phi}_\pi^g > 1$ such that the impact multiplier of output to a DIS-demand shock satisfies:

(a) *If $\phi_\pi \le \underline{\phi}_\pi^g$, $\mathcal{M}_y$ increases monotonously in ρ.*

(b) *If $\underline{\phi}_\pi^g < \phi_\pi < \overline{\phi}_\pi^g$, $\mathcal{M}_y$ is hump-shaped in ρ, i.e.,*

$$\exists\, \rho^{g*} \equiv \beta_f^{-1}\left(1-\sqrt{(\phi_\pi\beta_f-1)\frac{\beta_f\zeta_r\kappa_y+(\zeta_f-\beta_f)\kappa_r}{(\beta_f-\kappa_r)\zeta_f}}\right) \textit{ such that } \mathcal{M}_y'(\rho) \gtreqless 0 \textit{ if } \rho \lesseqgtr \rho^{g*}.$$

(c) *If* $\phi_\pi \geq \underline{\phi}^g_\pi$, $\mathcal{M}_y$ *decreases monotonously in* ρ.

Proposition 13 generalizes Proposition 2 regarding monetary policy regime thresholds $\underline{\phi}^g_\pi$ and $\overline{\phi}^g_\pi$ as well as the output-maximizing persistence ρ^{g*}. In particular, their formal expressions follow an identical analytical structure and are isomorphic when replacing κ_y in Proposition 2 with $\kappa^g_y = \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}$. Notice that the latter term is strictly positive under Assumption 2, i.e., $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$.

Corollary 2 provides comparative statics of regime thresholds and the output maximizing persistence.

COROLLARY 2. *Consider the hump-shaped regime of Proposition 13 characterized by* $\underline{\phi}^g_\pi < \phi_\pi < \overline{\phi}^g_\pi$. *The following comparative statics apply under Assumption 2.*

(a) *If* $\kappa_r = 0$ *applies, both regime thresholds* $(\underline{\phi}^g_\pi, \overline{\phi}^g_\pi)$ *as well as the output maximizing persistence* (ρ^{g*}) *strictly decrease in* β_f, κ_y, ζ_r, *and strictly increase in* ζ_f.

(b) *If* $\kappa_r > 0$ *applies, both regime thresholds* $(\underline{\phi}^g_\pi, \overline{\phi}^g_\pi)$ *as well as the output maximizing persistence* (ρ^{g*}) *strictly decrease in* κ_y, ζ_r, *and strictly increase in* ζ_f. *Moreover, they strictly decrease (respectively, strictly increase) in* κ_r *if* $\zeta_f + \zeta_r\kappa_y > \beta_f$ *(respectively,* $\zeta_f + \zeta_r\kappa_y < \beta_f$*).*

In the case of a positive marginal cost channel, i.e., $\kappa_r > 0$, comparative statics with respect to ζ_r and ζ_f are unaffected under Assumption 2, i.e., if $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$ applies, and reversed otherwise. Moreover, regime thresholds and output-maximizing persistence both fall in κ_r if $\zeta_f + \zeta_r\kappa_y > \beta_f$, which always holds under a compounded DIS equation. Intuitively, the marginal cost channel flattens the NKPC and hence reinforces the future income channel relative to the real interest rate channel.

For completeness, we compare the limits of the impact output multiplier in Proposition 14 and provide an extended condition for its asymmetric shape around the persistence threshold ρ^{g*}.

PROPOSITION 14. *If* $\phi_\pi > 1 + (1 - \beta_f)\frac{\kappa_y\zeta_r + \zeta_f - \kappa_r}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r} > \underline{\phi}^g_\pi$ *applies, the impact output multiplier to a DIS shock is asymmetric in its persistence, i.e.,* $\lim_{\rho\to 0} \mathcal{M}_y > \lim_{\rho \to 1} \mathcal{M}_y$.

A.3.3 *Proofs*

Proof Proposition 11

PROOF. We closely follow the steps in Bullard and Mitra (2002) to analyze determinacy properties of the model. For convenience, we drop the DIS-demand shock ξ_t when deriving the reduced dynamic system. To begin with, substituting the Taylor rule (27) into the DIS equation (25) and the NKPC (26) yields

$$y_t = \zeta_f \mathbb{E}_t[y_{t+1}] - \zeta_r\left(\phi_\pi \pi_t + \phi_y y_t - \mathbb{E}_t[\pi_{t+1}]\right), \tag{29}$$

$$\pi_t = \beta_f \mathbb{E}_t[\pi_{t+1}] + \kappa_y y_t + \kappa_r\left(\phi_\pi \pi_t + \phi_y y_t - \mathbb{E}_t[\pi_{t+1}]\right), \tag{30}$$

where the latter equation can be rearranged to

$$\pi_t = \frac{\beta_f - \kappa_r}{1-\phi_\pi \kappa_r}\mathbb{E}_t\left[\pi_{t+1}\right] + \frac{\kappa_y + \phi_y \kappa_r}{1-\phi_\pi \kappa_r} y_t \,.$$

Substituting, in turn, the latter equation into (29) results in

$$(1+\phi_y \zeta_r)\, y_t = \zeta_f \mathbb{E}_t\left[y_{t+1}\right] + \zeta_r \mathbb{E}_t\left[\pi_{t+1}\right] - \phi_\pi \zeta_r \left[\frac{\beta_f - \kappa_r}{1-\phi_\pi \kappa_r}\mathbb{E}_t\left[\pi_{t+1}\right] + \frac{\kappa_y + \phi_y \kappa_r}{1-\phi_\pi \kappa_r} y_t\right] ,$$

$$\Leftrightarrow \left[(1+\phi_y \zeta_r)\left(1-\phi_\pi \kappa_r\right) + \phi_\pi \zeta_r \left(\kappa_y + \phi_y \kappa_r\right)\right] y_t = \zeta_f \left(1-\phi_\pi \kappa_r\right)\mathbb{E}_t\left[y_{t+1}\right] + \zeta_r (1-\phi_\pi \beta_f)\mathbb{E}_t\left[\pi_{t+1}\right] ,$$

which finally yields

$$y_t = \frac{\zeta_f \left(1-\phi_\pi \kappa_r\right)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[y_{t+1}\right] + \frac{\zeta_r (1-\phi_\pi \beta_f)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[\pi_{t+1}\right] . \tag{E.1}$$

We now substitute the latter equation into (30) to obtain

$$\begin{aligned}
(1-\phi_\pi \kappa_r)\pi_t =& \frac{\kappa_y + \phi_y \kappa_r}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\left(\zeta_f \left(1-\phi_\pi \kappa_r\right)\mathbb{E}_t\left[y_{t+1}\right] + \zeta_r (1-\phi_\pi \beta_f)\mathbb{E}_t\left[\pi_{t+1}\right]\right) \\
&+ (\beta_f - \kappa_r)\mathbb{E}_t\left[\pi_{t+1}\right] \\
=& \frac{\left(\beta_f - \kappa_r\right)\left(1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)\right) + \left(\kappa_y + \phi_y \kappa_r\right)\zeta_r (1-\phi_\pi \beta_f)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[\pi_{t+1}\right] \\
&+ \frac{\left(\kappa_y + \phi_y \kappa_r\right)\zeta_f (1-\phi_\pi \kappa_r)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[y_{t+1}\right] \\
=& \frac{\left[\beta_f \left(1+\phi_y \zeta_r\right) + \zeta_r \kappa_y - \kappa_r\right]\left(1-\phi_\pi \kappa_r\right)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[\pi_{t+1}\right] + \frac{\left(\kappa_y + \phi_y \kappa_r\right)\zeta_f (1-\phi_\pi \kappa_r)}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[y_{t+1}\right] ,
\end{aligned}$$

which finally provides us with

$$\pi_t = \frac{\beta_f \left(1+\phi_y \zeta_r\right) + \zeta_r \kappa_y - \kappa_r}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[\pi_{t+1}\right] + \frac{\left(\kappa_y + \phi_y \kappa_r\right)\zeta_f}{1+\phi_y \zeta_r + \phi_\pi \left(\zeta_r \kappa_y - \kappa_r\right)}\mathbb{E}_t\left[y_{t+1}\right] . \tag{E.2}$$

We can thus write the system of equations (E.1)-(E.2) as

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \mathbb{E}_t\left[y_{t+1}\right] \\ \mathbb{E}_t\left[\pi_{t+1}\right] \end{pmatrix} ,$$

where

$$\mathbf{A}_T \equiv \frac{1}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \begin{pmatrix} \zeta_f (1-\phi_\pi \kappa_r) & \zeta_r (1-\phi_\pi \beta_f) \\ \zeta_f (\kappa_y + \phi_y \kappa_r) & \beta_f \left(1+\phi_y \zeta_r\right) + \zeta_r \kappa_y - \kappa_r \end{pmatrix} .$$

The solution is locally determinate if both eigenvalues of $\mathbf{A}_T$ are inside the unit circle. The characteristic polynomial of $\mathbf{A}_T$ is defined by $p(\lambda) \equiv \det(\lambda \mathbf{I} - \mathbf{A}_T) = \lambda^2 + a_1 \lambda + a_0$, where

$$a_0 \equiv \frac{\zeta_f (1-\phi_\pi \kappa_r)\left[\beta_f \left(1+\phi_y \zeta_r\right) + \zeta_r \kappa_y - \kappa_r\right] - \zeta_r \zeta_f (1-\phi_\pi \beta_f)(\kappa_y + \phi_y \kappa_r)}{\left[1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)\right]^2} = \frac{\zeta_f (\beta_f - \kappa_r)}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} ,$$

$$a_1 \equiv -\frac{\zeta_f (1-\phi_\pi \kappa_r) + \beta_f \left(1+\phi_y \zeta_r\right) + \zeta_r \kappa_y - \kappa_r}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} .$$

Both eigenvalues are inside the unit circle if and only if $|a_0| < 1$ and $|a_1| < 1 + a_0$ (see page 28 in LaSalle (1986)). Under Assumption 2 we have the parameter constraint $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$ such that the first condition is satisfied if

$$\phi_\pi > \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r} .$$

From here, we proceed by case distinction:
Case 1. First, consider the case in which the numerator of a_1 is weakly positive, i.e., $\zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r \geq 0$, which is equivalent to

$$\phi_\pi \leq \frac{\zeta_f + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r} .$$

In this case, the second determinacy condition, i.e., $|a_1| < 1 + a_0$, is satisfied if

$$\begin{aligned}
&1 + \frac{\zeta_f(\beta_f - \kappa_r)}{1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r)} > \frac{\zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r)} \\
&\Leftrightarrow 1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r) + \zeta_f(\beta_f - \kappa_r) > \zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r \\
&\Leftrightarrow (\phi_\pi - 1)\left[\zeta_r\kappa_y + \kappa_r(\zeta_f - 1)\right] > \left(1 - \beta_f\right)\left(\zeta_f - 1 - \phi_y\zeta_r\right) \\
&\Leftrightarrow \phi_\pi > 1 + \frac{\left(1 - \beta_f\right)\left(\zeta_f - 1 - \phi_y\zeta_r\right)}{\zeta_r\kappa_y + \kappa_r(\zeta_f - 1)} .
\end{aligned}$$

As a result, the equilibrium is locally determinate if $\underline{\underline{\phi}}_\pi^d < \phi_\pi < \overline{\overline{\phi}}_\pi^d$, where

$$\begin{aligned}
\underline{\underline{\phi}}_\pi^d &\equiv \max\left\{1 + \frac{\left(1 - \beta_f\right)\left(\zeta_f - 1 - \phi_y\zeta_r\right)}{\zeta_r\kappa_y + \kappa_r(\zeta_f - 1)}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} \\
\overline{\overline{\phi}}_\pi^d &\equiv \frac{\zeta_f + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r} .
\end{aligned}$$

Case 2. Second, consider the case in which the numerator of a_1 is strictly negative, i.e., $\zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r < 0$, which is equivalent to

$$\phi_\pi > \overline{\overline{\phi}}_\pi^d \equiv \frac{\zeta_f + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r} .$$

In this case, the second determinacy condition, i.e., $|a_1| < 1 + a_0$, is satisfied if

$$\begin{aligned}
&1 + \frac{\zeta_f(\beta_f - \kappa_r)}{1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r)} > -\frac{\zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r}{1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r)} \\
&\Leftrightarrow 1 + \phi_y\zeta_r + \phi_\pi(\zeta_r\kappa_y - \kappa_r) + \zeta_f(\beta_f - \kappa_r) > -\left[\zeta_f(1 - \phi_\pi\kappa_r) + \beta_f(1 + \phi_y\zeta_r) + \zeta_r\kappa_y - \kappa_r\right] \\
&\Leftrightarrow (\phi_\pi + 1)\left[\zeta_r\kappa_y - \kappa_r(1 + \zeta_f)\right] > -(1 + \beta_f)\left[\zeta_f + 1 + \phi_y\zeta_r\right] .
\end{aligned}$$

Depending on the sign of $\zeta_r\kappa_y - \kappa_r(1 + \zeta_f)$, two sub-cases emerge:

(a) If $\min\{\beta_f, \zeta_r\kappa_y, \zeta_r\kappa_y(1+\zeta_f)^{-1}\} > \kappa_r$, the previous condition can be rewritten as

$$\phi_\pi > -1 - \frac{(1+\beta_f)\left(\zeta_f + 1 + \phi_y\zeta_r\right)}{\zeta_r\kappa_y - \kappa_r(1+\zeta_f)} ,$$

and the model has a determinate solution if

$$\begin{aligned}\phi_\pi &> \max\left\{-1 - \frac{(1+\beta_f)\left(\zeta_f + 1 + \phi_y\zeta_r\right)}{\zeta_r\kappa_y - \kappa_r(1+\zeta_f)}, \frac{\zeta_f + \beta_f\left(1+\phi_y\zeta_r\right) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} \\ &= \max\left\{\frac{\zeta_f + \beta_f\left(1+\phi_y\zeta_r\right) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} .\end{aligned}$$

(b) If $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r > \zeta_r\kappa_y(1+\zeta_f)^{-1}$, the previous condition can be rewritten as

$$\phi_\pi < -1 + \frac{(1+\beta_f)\left(\zeta_f + 1 + \phi_y\zeta_r\right)}{\kappa_r(1+\zeta_f) - \zeta_r\kappa_y} ,$$

and the model has a determinate equilibrium if

$$\max\left\{\frac{\zeta_f + \beta_f\left(1+\phi_y\zeta_r\right) + \zeta_r\kappa_y - \kappa_r}{\zeta_f\kappa_r}, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} < \phi_\pi < -1 + \frac{(1+\beta_f)\left(\zeta_f + 1 + \phi_y\zeta_r\right)}{\kappa_r(1+\zeta_f) - \zeta_r\kappa_y} .$$

As a result, combining both cases, we obtain the following determinacy regions:

1. If $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$ and $\zeta_r\kappa_y \geq \kappa_r(1+\zeta_f)$, the model is locally determinate if

$$\underline{\underline{\phi}}_\pi^d < \phi_\pi \leq \overline{\overline{\phi}}_\pi^d \quad \text{and} \quad \phi_\pi > \max\left\{\overline{\overline{\phi}}_\pi^d, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} .$$

   It is straightforward to show that $\overline{\overline{\phi}}_\pi^d > \underline{\underline{\phi}}_\pi^d$ under Assumption 2 and $\zeta_r\kappa_y \geq \kappa_r(1+\zeta_f)$ such that both conditions can be combined to $\phi_\pi > \underline{\underline{\phi}}_\pi^d$.

2. If $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$ and $\zeta_r\kappa_y < \kappa_r(1+\zeta_f)$, the model is locally determinate if

$$\max\left\{\overline{\overline{\phi}}_\pi^d, \frac{\zeta_f(\beta_f - \kappa_r) - 1 - \phi_y\zeta_r}{\zeta_r\kappa_y - \kappa_r}\right\} \equiv \underline{\phi}_\pi^d < \phi_\pi < \overline{\phi}_\pi^d \equiv -1 + \frac{(1+\beta_f)\left(\zeta_f + 1 + \phi_y\zeta_r\right)}{\kappa_r(1+\zeta_f) - \zeta_r\kappa_y} .$$

This concludes the proof of Proposition 11. □

Proof Proposition 12

PROOF. To begin with, we derive the impact multipliers. Substituting the Taylor rule (27) into the DIS equation (25) results in

$$\begin{aligned} y_t &= \zeta_f\mathbb{E}_t\left[y_{t+1}\right] - \zeta_r\left(\phi_\pi\pi_t + \phi_y y_t - \mathbb{E}_t\left[\pi_{t+1}\right]\right) + \zeta_r\xi_t , \\ \Leftrightarrow (1+\phi_y\zeta_r)y_t &= \zeta_f\mathbb{E}_t\left[y_{t+1}\right] + \zeta_r\mathbb{E}_t\left[\pi_{t+1}\right] - \zeta_r\phi_\pi\pi_t + \zeta_r\xi_t . \end{aligned}$$

Similarly, substituting the Taylor rule (27) into the NKPC equation (26) yields

$$\pi_t = \beta_f \mathbb{E}_t[\pi_{t+1}] + \kappa_y y_t + \kappa_r(\phi_\pi \pi_t + \phi_y y_t - \mathbb{E}_t[\pi_{t+1}]) - \kappa_r \xi_t \,,$$
$$(1-\phi_\pi \kappa_r)\pi_t = (\beta_f - \kappa_r)\mathbb{E}_t[\pi_{t+1}] + (\kappa_y + \phi_y \kappa_r) y_t - \kappa_r \xi_t \,.$$

As before, we guess and verify that the solution takes the form $y_t = \mathcal{M}_y \xi_t$ and $\pi_t = \mathcal{M}_\pi \xi_t$, such that the previous two equations can be rewritten as

$$(1+\phi_y \zeta_r - \rho \zeta_f)\mathcal{M}_y = -\zeta_r(\phi_\pi - \rho)\mathcal{M}_\pi + \zeta_r \,,$$
$$(1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r))\mathcal{M}_\pi = (\kappa_y + \phi_y \kappa_r)\mathcal{M}_y - \kappa_r \,.$$

The latter equation can be rewritten as

$$\mathcal{M}_\pi = \frac{\kappa_y + \phi_y \kappa_r}{1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r)}\mathcal{M}_y - \frac{\kappa_r}{1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r)} \,.$$

Substituting this equation in turn into the one for the impact output multiplier results in

$$(1+\phi_y \zeta_r - \rho \zeta_f)\mathcal{M}_y = -\zeta_r(\phi_\pi - \rho)\left[\frac{\kappa_y + \phi_y \kappa_r}{1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r)}\mathcal{M}_y - \frac{\kappa_r}{1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r)}\right] + \zeta_r \,,$$

which can be restated as

$$\left[(1+\phi_y \zeta_r - \rho \zeta_f)(1-\phi_\pi \kappa_r - \rho(\beta_f - \kappa_r)) + \zeta_r(\phi_\pi - \rho)(\kappa_y + \phi_y \kappa_r)\right]\mathcal{M}_y = (1-\rho\beta_f)\zeta_r \,,$$

which ultimately yields

$$\mathcal{M}_y = \frac{1-\rho\beta_f}{(1+\phi_y \zeta_r - \rho\zeta_f)\left[1-\rho\beta_f - \kappa_r(\phi_\pi - \rho)\right]\zeta_r^{-1} + (\phi_\pi - \rho)(\kappa_y + \phi_y \kappa_r)} \,.$$

From the previous equation the impact multipliers for inflation, the nominal as well as the real interest rate follow recursively, as provided in the main text.

We then study the sign of the impact multipliers. First, we derive, as in the Proof of Proposition 11, a reduced form system of output and inflation dynamics, i.e.,

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \mathbf{A}_T \begin{pmatrix} \mathbb{E}_t[y_{t+1}] \\ \mathbb{E}_t[\pi_{t+1}] \end{pmatrix} + \mathbf{B}_T \xi_t \,,$$

where

$$\mathbf{A}_T \equiv \frac{1}{1+\phi_y \zeta_r + \phi_\pi(\zeta_r \kappa_y - \kappa_r)} \begin{pmatrix} \zeta_f(1-\phi_\pi \kappa_r) & \zeta_r(1-\phi_\pi \beta_f) \\ \zeta_f(\kappa_y + \phi_y \kappa_r) & \beta_f(1+\phi_y \zeta_r) + \zeta_r \kappa_y - \kappa_r \end{pmatrix} ,$$

$$\mathbf{B}_T \equiv \frac{1}{1+\phi_y \zeta_r + \phi_\pi(\zeta_r \kappa_y - \kappa_r)} \begin{pmatrix} \zeta_r \\ \zeta_r \kappa_y - \kappa_r \end{pmatrix} .$$

As $\mathbb{E}_t[y_{t+1}] = \rho y_t$ and $\mathbb{E}_t[\pi_{t+1}] = \rho \pi_t$ holds in a rational expectations equilibrium, we can write

$$\begin{aligned}\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} &= (\mathbf{I}_2 - \rho \mathbf{A}_T)^{-1} \mathbf{B}_T \xi_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12} A_{21}} \begin{pmatrix} 1-\rho A_{22} & \rho A_{12} \\ \rho A_{21} & 1-\rho A_{11} \end{pmatrix} \mathbf{B}_T \xi_t \\ &= \frac{1}{(1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12} A_{21}} \begin{pmatrix} (1-\rho A_{22}) B_{11} + \rho A_{12} B_{21} \\ \rho A_{21} B_{11} + (1-\rho A_{11}) B_{21} \end{pmatrix} \xi_t \,.\end{aligned}$$

Notice that we have

$$\begin{aligned}(1-\rho A_{22}) B_{11} + \rho A_{12} B_{21} = &\left(1 - \rho \frac{\beta_f (1+\phi_y \zeta_r) + \zeta_r \kappa_y - \kappa_r}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)}\right) \frac{\zeta_r}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \\ &+ \rho \frac{\zeta_r (1-\phi_\pi \beta_f)}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \frac{\zeta_r \kappa_y - \kappa_r}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \\ = &\frac{\zeta_r (1-\rho \beta_f)}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} > 0 \,,\end{aligned}$$

where the strict inequality follows under Assumption 2 combined with $1 > \rho \beta_f$ and $(\phi_\pi, \phi_y) \in \mathbb{R}_+^2$. As a result, this implies that $\text{sgn}(\mathcal{M}_y) = \text{sgn}(\det(\mathbf{I}_2 - \rho \mathbf{A}_T))$. Recall from the proof of Proposition 11 that

$$\begin{aligned}A_{11} A_{22} - A_{12} A_{21} &= \frac{\zeta_f (\beta_f - \kappa_r)}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \,, \\ A_{11} + A_{22} &= \frac{\zeta_f (1-\phi_\pi \kappa_r) + \beta_f (1+\phi_y \zeta_r) + \zeta_r \kappa_y - \kappa_r}{1+\phi_y \zeta_r + \phi_\pi (\zeta_r \kappa_y - \kappa_r)} \,.\end{aligned}$$

Moreover, we can rewrite

$$\begin{aligned}\det(\mathbf{I}_2 - \rho \mathbf{A}_T) &= (1-\rho A_{11})(1-\rho A_{22}) - \rho^2 A_{12} A_{21} \\ &= 1 - \rho (A_{11} + A_{22}) + \rho^2 (A_{11} A_{22} - A_{12} A_{21}) \,.\end{aligned}$$

There arise now two cases. First, consider the case where $A_{11} + A_{22} \leq 0$, i.e., $\phi_\pi \geq \overset{=d}{\phi}_\pi$. In this case, we have $\det(\mathbf{I}_2 - \rho \mathbf{A}_T) > 0$ and consequently $\mathcal{M}_y > 0$. Second, consider the case where $A_{11} + A_{22} > 0$, i.e., $\phi_\pi < \overset{=d}{\phi}_\pi$. In this case, we can write

$$\begin{aligned}\det(\mathbf{I}_2 - \rho \mathbf{A}_T) &= 1 - \rho (A_{11} + A_{22}) + \rho^2 (A_{11} A_{22} - A_{12} A_{21}) \\ &> 1 - \rho (1 + A_{11} A_{22} - A_{12} A_{21}) + \rho^2 (A_{11} A_{22} - A_{12} A_{21}) \\ &= (1-\rho) [1 - \rho (A_{11} A_{22} - A_{12} A_{21})] \\ &> 0 \,,\end{aligned}$$

where the first strict inequality is due to the determinacy condition $A_{11} + A_{22} < 1 + A_{11}A_{22} - A_{12}A_{21}$ and the second strictly inequality is due to the determinacy condition $A_{11}A_{22} - A_{12}A_{21} < 1$. As a result, this implies

$$\mathcal{M}_y = \frac{1 - \rho\beta_f}{\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\left[1 - \rho\beta_f - \kappa_r\left(\phi_\pi - \rho\right)\right]\zeta_r^{-1} + \left(\phi_\pi - \rho\right)\left(\kappa_y + \phi_y\kappa_r\right)} > 0\,.$$

Let us denote the denominator of $\mathcal{M}_y$ by $\mathcal{M}_y^d$. To determine the sign of the impact inflation multiplier, we can rearrange terms to obtain

$$\begin{aligned}
\mathcal{M}_\pi &= \frac{\kappa_y + \phi_y\kappa_r}{1 - \phi_\pi\kappa_r - \rho\left(\beta_f - \kappa_r\right)}\mathcal{M}_y - \frac{\kappa_r}{1 - \phi_\pi\kappa_r - \rho\left(\beta_f - \kappa_r\right)} \\
&= \frac{1}{\left[1 - \rho\beta_f - \kappa_r(\phi_\pi - \rho)\right]\mathcal{M}_y^d}\left[(\kappa_y + \phi_y\kappa_r)(1 - \rho\beta_f) - \kappa_r\mathcal{M}_y^d\right]\,.
\end{aligned}$$

As a result, the sign of $\mathcal{M}_\pi$ is determined by the sign of $\Delta_\pi \equiv (\kappa_y + \phi_y\kappa_r)(1 - \rho\beta_f) - \kappa_r\mathcal{M}_y^d$, i.e.,

$$\begin{aligned}
\Delta_\pi &= (\kappa_y + \phi_y\kappa_r)(1 - \rho\beta_f) - \kappa_r\left\{\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\left[1 - \rho\beta_f - \kappa_r\left(\phi_\pi - \rho\right)\right]\zeta_r^{-1} + \left(\phi_\pi - \rho\right)\left(\kappa_y + \phi_y\kappa_r\right)\right\} \\
&= (\kappa_y + \phi_y\kappa_r)\left[1 - \rho\beta_f - \kappa_r(\phi_\pi - \rho)\right] - \left(\kappa_r\zeta_r^{-1}\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\right)\left[1 - \rho\beta_f - \kappa_r(\phi_\pi - \rho)\right] \\
&= \left(\kappa_y + \phi_y\kappa_r - \kappa_r\zeta_r^{-1}\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\right)\left[1 - \rho\beta_f - \kappa_r(\phi_\pi - \rho)\right] \\
&= \left(\kappa_y - \kappa_r\zeta_r^{-1}\left(1 - \rho\zeta_f\right)\right)\left[1 - \rho\beta_f - \kappa_r(\phi_\pi - \rho)\right]\,.
\end{aligned}$$

Consequently, we have

$$\mathcal{M}_\pi = \frac{\kappa_y - \kappa_r\zeta_r^{-1}\left(1 - \rho\zeta_f\right)}{\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\left[1 - \rho\beta_f - \kappa_r\left(\phi_\pi - \rho\right)\right]\zeta_r^{-1} + \left(\phi_\pi - \rho\right)\left(\kappa_y + \phi_y\kappa_r\right)}\,,$$

which is strictly positive if $\zeta_r\kappa_y > \kappa_r\left(1 - \rho\zeta_f\right)$. The latter condition is referred to as *general equilibrium Patman condition* (see Beaudry et al. (2024)) and is always satisfied under Assumption 2. Finally, the sign of the impact real interest multiplier can be determined by

$$\begin{aligned}
\mathcal{M}_r &= (\phi_\pi - \rho)\mathcal{M}_\pi + \phi_y\mathcal{M}_y \\
&= \frac{(\phi_\pi - \rho)\left[\kappa_y - \kappa_r\zeta_r^{-1}\left(1 - \rho\zeta_f\right)\right] + \phi_y(1 - \rho\beta_f)}{\left(1 + \phi_y\zeta_r - \rho\zeta_f\right)\left[1 - \rho\beta_f - \kappa_r\left(\phi_\pi - \rho\right)\right]\zeta_r^{-1} + \left(\phi_\pi - \rho\right)\left(\kappa_y + \phi_y\kappa_r\right)}\,,
\end{aligned}$$

which is strictly positive if

$$\phi_\pi > \phi_\pi^{r+} \equiv \rho - \frac{\phi_y\zeta_r(1 - \rho\beta_f)}{\zeta_r\kappa_y + \kappa_r\left(\rho\zeta_f - 1\right)}\,.$$

Notice that ϕ_π^{r+} is strictly increasing in ρ with corresponding limit

$$\lim_{\rho\to 1}\phi_\pi^{r+} \equiv \overline{\phi}_\pi^{r+} = 1 - \frac{\phi_y\zeta_r(1 - \beta_f)}{\zeta_r\kappa_y + \kappa_r\left(\zeta_f - 1\right)}\,.$$

Recall from Proposition 11 that the model's determinacy regions are as followed: if $\zeta_r\kappa_y \geq \kappa_r(1+\zeta_f)$ applies, the model is determinate for $\phi_\pi > \underline{\underline{\phi}}^d_\pi$; in contrast, if $\zeta_r\kappa_y < \kappa_r(1+\zeta_f)$ applies, the model is determinate for $\max\left\{\overline{\overline{\phi}}^d_\pi, \frac{\zeta_f(\beta_f-\kappa_r)-1-\phi_y\zeta_r}{\zeta_r\kappa_y-\kappa_r}\right\} < \phi_\pi < -1+\frac{(1+\beta_f)(\zeta_f+1+\phi_y\zeta_r)}{\kappa_r(1+\zeta_f)-\zeta_r\kappa_y}$. Note that Assumption 2 together with $\beta_f \in [0,1)$ implies that $\overline{\overline{\phi}}^d_\pi > 1 > \overline{\phi}^{r+}_\pi$. As such, $\text{sgn}(\mathcal{M}_r) > 0$ in the case of $\zeta_r\kappa_y < \kappa_r(1+\zeta_f)$. In contrast, it is straightforward to show that $\overline{\phi}^{r+}_\pi > \underline{\underline{\phi}}^d_\pi$ if $\zeta_f < 1$ and $\overline{\phi}^{r+}_\pi \leq \underline{\underline{\phi}}^d_\pi$ if $\zeta_f > 1$, i.e., $\text{sgn}(\mathcal{M}_r)$ is always positive under a compounded DIS equation, and maybe be negative under a discounted DIS equation depending on the persistence. □

Proof Proposition 13

PROOF. Taking the derivative of $\mathcal{M}_y$ with respect to ρ yields

$$\frac{\partial \mathcal{M}_y}{\partial \rho} = \frac{-\beta_f\left[\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left[1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right]\zeta_r^{-1}+\left(\phi_\pi-\rho\right)\left(\kappa_y+\phi_y\kappa_r\right)\right]}{\left[\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left[1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right]\zeta_r^{-1}+\left(\phi_\pi-\rho\right)\left(\kappa_y+\phi_y\kappa_r\right)\right]^2}$$
$$+\frac{\left(1-\rho\beta_f\right)\left[\zeta_f\zeta_r^{-1}\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)+\zeta_r^{-1}\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left(\beta_f-\kappa_r\right)+\kappa_y+\phi_y\kappa_r\right]}{\left[\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left[1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right]\zeta_r^{-1}+\left(\phi_\pi-\rho\right)\left(\kappa_y+\phi_y\kappa_r\right)\right]^2}.$$

Defining $x \equiv \left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left[1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right]\zeta_r^{-1}+\left(\phi_\pi-\rho\right)\left(\kappa_y+\phi_y\kappa_r\right)$, we can write

$$\begin{aligned}
x^2\frac{\partial \mathcal{M}_y}{\partial \rho} = &-\beta_f\zeta_r^{-1}\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)-\beta_f\left(\phi_\pi-\rho\right)\left(\kappa_y+\phi_y\kappa_r\right)\\
&+(1-\rho\beta_f)\zeta_f\zeta_r^{-1}\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)+(1-\rho\beta_f)\zeta_r^{-1}\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\left(\beta_f-\kappa_r\right)\\
&+\left(1-\rho\beta_f\right)\left(\kappa_y+\phi_y\kappa_r\right),
\end{aligned}$$

which can be rewritten to

$$\begin{aligned}
x^2\frac{\partial \mathcal{M}_y}{\partial \rho} &= \left[1-\beta_f\phi_\pi\right]\left(\kappa_y+\phi_y\kappa_r\right)+\zeta_f\zeta_r^{-1}(1-\rho\beta_f)\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)\\
&\quad-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\zeta_r^{-1}\left[\beta_f\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)-(1-\rho\beta_f)(\beta_f-\kappa_r)\right]\\
&= \left[1-\beta_f\phi_\pi\right]\left(\kappa_y+\phi_y\kappa_r\right)+\zeta_f\zeta_r^{-1}(1-\rho\beta_f)\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)\\
&\quad-\left(1+\phi_y\zeta_r-\rho\zeta_f\right)\zeta_r^{-1}\kappa_r\left[1-\beta_f\phi_\pi\right]\\
&= \left[1-\beta_f\phi_\pi\right]\left(\kappa_y-\kappa_r\zeta_r^{-1}\right)+\zeta_f\zeta_r^{-1}\left[\left(1-\beta_f\phi_\pi\right)\rho\kappa_r+\left(1-\rho\beta_f\right)\left(1-\rho\beta_f-\kappa_r\left(\phi_\pi-\rho\right)\right)\right]\\
&= \left[1-\beta_f\phi_\pi\right]\left(\kappa_y-\kappa_r\zeta_r^{-1}\right)+\zeta_f\zeta_r^{-1}\left[1-\phi_\pi\kappa_r-2(\beta_f-\kappa_r)\rho+\beta_f(\beta_f-\kappa_r)\rho^2\right].
\end{aligned}$$

As a result, the sign of the previous expression is determined by a second order polynomial in ρ, i.e.,

$$\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right) = a\rho^2+b\rho+c\,,$$

where the auxiliary parameters are given by

$$
\begin{aligned}
a &\equiv \zeta_f \zeta_r^{-1} \beta_f (\beta_f - \kappa_r) , \\
b &\equiv -2\zeta_f \zeta_r^{-1} (\beta_f - \kappa_r) , \\
c &\equiv \zeta_f \zeta_r^{-1} (1 - \phi_\pi \kappa_r) + \left[1 - \beta_f \phi_\pi\right] \zeta_r^{-1} (\zeta_r \kappa_y - \kappa_r) .
\end{aligned}
$$

The corresponding roots are

$$
\rho^{+,-} = \frac{-b \pm \sqrt{\Delta}}{2a} ,
$$

where the discriminant $\Delta \equiv b^2 - 4ac$ is given by

$$
\begin{aligned}
\Delta &= 4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r)^2 - 4\left(\frac{\zeta_f}{\zeta_r}\right) \beta_f(\beta_f - \kappa_r) \left[\left(\frac{\zeta_f}{\zeta_r}\right)(1 - \phi_\pi \kappa_r) + \left(1 - \beta_f \phi_\pi\right) \zeta_r^{-1} (\zeta_r \kappa_y - \kappa_r)\right] \\
&= -4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r) \kappa_r + 4\left(\frac{\zeta_f}{\zeta_r}\right) \beta_f(\beta_f - \kappa_r) \left[\left(\frac{\zeta_f}{\zeta_r}\right) \phi_\pi \kappa_r + \left(\beta_f \phi_\pi - 1\right) \zeta_r^{-1} (\zeta_r \kappa_y - \kappa_r)\right] \\
&= 4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r)(\beta_f \phi_\pi - 1)\kappa_r + 4\left(\frac{\zeta_f}{\zeta_r}\right) \beta_f(\beta_f - \kappa_r) \left(\beta_f \phi_\pi - 1\right) \zeta_r^{-1} (\zeta_r \kappa_y - \kappa_r) \\
&= 4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r)^2 (\beta_f \phi_\pi - 1) \left[\frac{\kappa_r}{\beta_f - \kappa_r} + \frac{\beta_f}{\beta_f - \kappa_r} \frac{1}{\zeta_f} (\zeta_r \kappa_y - \kappa_r)\right] \\
&= 4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r)^2 (\beta_f \phi_\pi - 1) \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f} .
\end{aligned}
$$

As $\min\{\beta_f, \zeta_r \kappa_y\} > \kappa_r$ under Assumption 2, the last ratio is strictly positive. Consequently, the following case distinction applies. First, $\phi_\pi < \beta_f^{-1}$ implies $\Delta < 0$ and the second order polynomial has two distinct complex roots. Second, $\phi_\pi = \beta_f^{-1}$ implies $\Delta = 0$ and the second order polynomial has a unique real root that is given by $-\frac{b}{2a} = \beta_f^{-1} > 1$. Third, $\phi_\pi > \beta_f^{-1}$ implies that the second order polynomial has two real-valued roots that are given by

$$
\begin{aligned}
\rho^{+,-} &= \frac{2\zeta_f \zeta_r^{-1}(\beta_f - \kappa_r) \pm \sqrt{4\left(\frac{\zeta_f}{\zeta_r}\right)^2 (\beta_f - \kappa_r)^2 (\beta_f \phi_\pi - 1) \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}}}{2\zeta_f \zeta_r^{-1} \beta_f (\beta_f - \kappa_r)} \\
&= \beta_f^{-1} \left(1 \pm \sqrt{(\beta_f \phi_\pi - 1) \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}}\right) .
\end{aligned}
$$

Evidently, $\rho^+ > 1$ as $\beta_f \in (0,1)$ and $\phi_\pi > \beta_f^{-1}$ such that the only feasible candidate on $\rho \in (0,1)$ is

$$
\rho^{g*} \equiv \rho^- = \beta_f^{-1} \left(1 - \sqrt{(\beta_f \phi_\pi - 1) \frac{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}}\right) .
$$

Note that $\rho^{g*} \in (0,1)$ if $\underline{\phi}_\pi^g < \phi_\pi < \overline{\phi}_\pi^g$, where we have

$$\underline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1 + (1-\beta_f)^2 \frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right)\,, \quad \text{and} \quad \overline{\phi}_\pi^g \equiv \beta_f^{-1}\left(1 + \frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right)\,.$$

As $f(\rho) \equiv a\rho^2 + b\rho + c$ is strictly convex in ρ, statement (b) of Proposition 13 follows.

To show statement (a) of Proposition 13, notice that sgn $\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)$ is strictly decreasing in ϕ_π. Hence, substituting in the lower bound $\underline{\phi}_\pi^g$, we obtain

$$\begin{aligned}
\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \underline{\phi}_\pi^g} =& \frac{\zeta_f}{\zeta_r}\beta_f(\beta_f-\kappa_r)\rho^2 - 2\frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\rho + \frac{\zeta_f}{\zeta_r}\left(1-\underline{\phi}_\pi^g\kappa_r\right) + \left(1-\beta_f\underline{\phi}_\pi^g\right)\left(\kappa_y - \frac{\kappa_r}{\zeta_r}\right) \\
=& \frac{\zeta_f}{\zeta_r}\beta_f(\beta_f-\kappa_r)\rho^2 - 2\frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\rho \\
&+ \frac{\zeta_f}{\zeta_r}\left(1 - \frac{\kappa_r}{\beta_f}\left[1 + (1-\beta_f)^2\frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right]\right) \\
&- (1-\beta_f)^2\frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\left(\kappa_y - \frac{\kappa_r}{\zeta_r}\right) \\
=& \frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\left[\beta_f\rho^2 - 2\rho + \beta_f^{-1} - (1-\beta_f)^2\frac{\zeta_r\kappa_y - \kappa_r + \zeta_f\dfrac{\kappa_r}{\beta_f}}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right] \\
=& \frac{\zeta_f}{\zeta_r}\frac{\beta_f-\kappa_r}{\beta_f}\left[\left(\rho\beta_f\right)^2 - 2\rho\beta_f + 1 - (1-\beta_f)^2\frac{\beta_f\zeta_r\kappa_y + \left(\zeta_f-\beta_f\right)\kappa_r}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right] \\
=& \frac{\zeta_f}{\zeta_r}\frac{\beta_f-\kappa_r}{\beta_f}\left[(1-\rho\beta_f)^2 - (1-\beta_f)^2\right] > 0\,,
\end{aligned}$$

where the last inequality applies as $\min\{\beta_f, \zeta_r\kappa_y\} > \kappa_r$ under Assumption 2 and $\rho \in [0,1)$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} > 0$ for all $\phi_\pi \leq \underline{\phi}_\pi^g$.

Finally, to show statement (c) we analogously substitute in for the upper bound $\overline{\phi}_\pi^g$ to obtain

$$\begin{aligned}
\text{sgn}\left(\frac{\partial \mathcal{M}_y}{\partial \rho}\right)|_{\phi_\pi = \overline{\phi}_\pi^g} =& \frac{\zeta_f}{\zeta_r}\beta_f(\beta_f-\kappa_r)\rho^2 - 2\frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\rho + \frac{\zeta_f}{\zeta_r}\left(1-\overline{\phi}_\pi^g\kappa_r\right) + \left(1-\beta_f\overline{\phi}_\pi^g\right)\left(\kappa_y - \frac{\kappa_r}{\zeta_r}\right) \\
=& \frac{\zeta_f}{\zeta_r}\beta_f(\beta_f-\kappa_r)\rho^2 - 2\frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\rho \\
&+ \frac{\zeta_f}{\zeta_r}\left(1 - \frac{\kappa_r}{\beta_f}\left[1 + \frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right]\right) \\
&- \frac{(\beta_f-\kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\left(\kappa_y - \frac{\kappa_r}{\zeta_r}\right) \\
=& \frac{\zeta_f}{\zeta_r}(\beta_f-\kappa_r)\left[\beta_f\rho^2 - 2\rho + \beta_f^{-1} - \frac{\zeta_r\kappa_y - \kappa_r + \zeta_f\dfrac{\kappa_r}{\beta_f}}{\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r}\right] \\
=& \frac{\zeta_f}{\zeta_r}\left(\beta_f-\kappa_r\right)\rho\left[\rho\beta_f - 2\right] \leq 0
\end{aligned}$$

where the last inequality applies as $\min\{\beta_f, \zeta_r \kappa_y\} > \kappa_r$ under Assumption 2 and $\rho \in [0,1)$. Note that the previous inequality is strict for $\rho > 0$. As a result, we have $\frac{\partial \mathcal{M}_y}{\partial \rho} < 0$ for all $\phi_\pi \geq \overline{\phi}_\pi^g$. □

Proof Corollary 2

PROOF. The proof proceeds by case distinction. We first show Statement (a) and afterwards Statement (b):

Statement (a). In the case of $\kappa_r = 0$, the regime thresholds and $\rho-$max reduce to

$$\underline{\phi}_\pi^g = \beta_f^{-1}\left(1 + (1-\beta_f)^2 \frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right),$$

$$\overline{\phi}_\pi^g = \beta_f^{-1}\left(1 + \frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right),$$

$$\rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y}\right).$$

As a result, we obtain for the lower regime threshold:

$$\frac{\partial \underline{\phi}_\pi^g}{\partial \beta_f} = -\beta_f^{-2}\left(1 + (1-\beta_f)^2 \frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right) - 2\beta_f^{-1}(1-\beta_f)\frac{\zeta_f}{\zeta_r}\kappa_y^{-1} < 0\,, \quad \frac{\partial \underline{\phi}_\pi^g}{\partial \zeta_r} = -\beta_f^{-1}(1-\beta_f)^2 \frac{\zeta_f}{\zeta_r^2}\kappa_y^{-1} < 0\,,$$

$$\frac{\partial \underline{\phi}_\pi^g}{\partial \kappa_y} = -\beta_f^{-1}(1-\beta_f)^2 \frac{\zeta_f}{\zeta_r}\kappa_y^{-2} < 0\,, \quad \frac{\partial \underline{\phi}_\pi^g}{\partial \zeta_f} = \beta_f^{-1}(1-\beta_f)^2 \frac{1}{\zeta_r}\kappa_y^{-1} > 0\,.$$

Similarly, we obtain for the upper regime threshold:

$$\frac{\partial \overline{\phi}_\pi^g}{\partial \beta_f} = -\beta_f^{-2}\left(1 + \frac{\zeta_f}{\zeta_r}\kappa_y^{-1}\right) < 0\,, \quad \frac{\partial \overline{\phi}_\pi^g}{\partial \zeta_r} = -\beta_f^{-1}\frac{\zeta_f}{\zeta_r^2}\kappa_y^{-1} < 0\,,$$

$$\frac{\partial \overline{\phi}_\pi^g}{\partial \kappa_y} = -\beta_f^{-1}\frac{\zeta_f}{\zeta_r}\kappa_y^{-2} < 0\,, \quad \frac{\partial \overline{\phi}_\pi^g}{\partial \zeta_f} = \beta_f^{-1}\frac{1}{\zeta_r}\kappa_y^{-1} > 0\,,$$

and for the output-maximizing persistence:

$$\frac{\partial \rho^{g*}}{\partial \beta_f} = -\beta_f^{-2}\left(1 - \sqrt{(\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y}\right) - \frac{1}{2}\beta_f^{-1}\left((\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y\right)^{-\frac{1}{2}}\phi_\pi \frac{\zeta_r}{\zeta_f}\kappa_y < 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \zeta_r} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y\right)^{-\frac{1}{2}}(\beta_f \phi_\pi - 1)\frac{1}{\zeta_f}\kappa_y < 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \kappa_y} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y\right)^{-\frac{1}{2}}(\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f} < 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \zeta_f} = \frac{1}{2}\beta_f^{-1}\left((\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f}\kappa_y\right)^{-\frac{1}{2}}(\beta_f \phi_\pi - 1)\frac{\zeta_r}{\zeta_f^2}\kappa_y > 0\,.$$

Statement (b). In the case of $\kappa_r > 0$, the regime thresholds and $\rho-$max are

$$\underline{\phi}^g_\pi = \beta_f^{-1}\left(1 + (1-\beta_f)^2 \frac{(\beta_f - \kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}\right) ,$$

$$\overline{\phi}^g_\pi = \beta_f^{-1}\left(1 + \frac{(\beta_f - \kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}\right) ,$$

$$\rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}}\right) .$$

As a result, we obtain for the lower regime threshold:

$$\frac{\partial\underline{\phi}^g_\pi}{\partial\zeta_r} = -\beta_f^{-1}(1-\beta_f)^2 \frac{(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2}\beta_f\kappa_y < 0 ,$$

$$\frac{\partial\underline{\phi}^g_\pi}{\partial\kappa_y} = -\beta_f^{-1}(1-\beta_f)^2 \frac{(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2}\beta_f\zeta_r < 0 ,$$

$$\begin{aligned}\frac{\partial\underline{\phi}^g_\pi}{\partial\zeta_f} &= \beta_f^{-1}(1-\beta_f)^2 \frac{(\beta_f - \kappa_r)\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] - \kappa_r(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\ &= \beta_f^{-1}(1-\beta_f)^2\beta_f(\beta_f - \kappa_r)\frac{\zeta_r\kappa_y - \kappa_r}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial\underline{\phi}^g_\pi}{\partial\kappa_r} &= \beta_f^{-1}(1-\beta_f)^2 \frac{-\zeta_f\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] - \zeta_f(\zeta_f - \beta_f)(\beta_f - \kappa_r)}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\ &= \beta_f^{-1}(1-\beta_f)^2\beta_f\zeta_f\frac{-\zeta_r\kappa_y - \zeta_f + \beta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} .\end{aligned}$$

Similarly, we obtain for the upper regime threshold:

$$\frac{\partial\overline{\phi}^g_\pi}{\partial\zeta_r} = -\beta_f^{-1}\frac{(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2}\beta_f\kappa_y < 0 ,$$

$$\frac{\partial\overline{\phi}^g_\pi}{\partial\kappa_y} = -\beta_f^{-1}\frac{(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2}\beta_f\zeta_r < 0 ,$$

$$\begin{aligned}\frac{\partial\overline{\phi}^g_\pi}{\partial\zeta_f} &= \beta_f^{-1}\frac{(\beta_f - \kappa_r)\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] - \kappa_r(\beta_f - \kappa_r)\zeta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\ &= (\beta_f - \kappa_r)\frac{\zeta_r\kappa_y - \kappa_r}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial\overline{\phi}^g_\pi}{\partial\kappa_r} &= \beta_f^{-1}\frac{-\zeta_f\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] - \zeta_f(\zeta_f - \beta_f)(\beta_f - \kappa_r)}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\ &= \zeta_f\frac{-\zeta_r\kappa_y - \zeta_f + \beta_f}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} .\end{aligned}$$

Finally, we obtain for the output maximizing persistence:

$$\frac{\partial \rho^{g*}}{\partial \zeta_r} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{\beta_f\kappa_y}{(\beta_f - \kappa_r)\zeta_f} < 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \kappa_y} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{\beta_f\zeta_r}{(\beta_f - \kappa_r)\zeta_f} < 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \zeta_f} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{\kappa_r(\beta_f - \kappa_r)\zeta_f - (\beta_f - \kappa_r)\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]}{\left[(\beta_f - \kappa_r)\zeta_f\right]^2}$$

$$= \frac{1}{2}(\beta_f - \kappa_r)\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{\zeta_r\kappa_y - \kappa_r}{\left[(\beta_f - \kappa_r)\zeta_f\right]^2} > 0\,,$$

$$\frac{\partial \rho^{g*}}{\partial \kappa_r} = -\frac{1}{2}\beta_f^{-1}\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{(\zeta_f - \beta_f)(\beta_f - \kappa_r)\zeta_f + \zeta_f\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right]}{\left[(\beta_f - \kappa_r)\zeta_f\right]^2}$$

$$= -\frac{1}{2}\left((\beta_f\phi_\pi - 1)\frac{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}{(\beta_f - \kappa_r)\zeta_f}\right)^{-\frac{1}{2}} \frac{\zeta_f\left[\zeta_f + \zeta_r\kappa_y - \beta_f\right]}{\left[(\beta_f - \kappa_r)\zeta_f\right]^2}\,,$$

which completes the proof of Corollary 2. □

Proof Proposition 14

PROOF. In the limit, we obtain the following two expressions

$$\lim_{\rho\to 0}\mathcal{M}_y = \frac{1}{(1+\phi_y\zeta_r)(1-\phi_\pi\kappa_r)\zeta_r^{-1} + \phi_\pi(\kappa_y + \phi_y\kappa_r)}\,,$$

$$\lim_{\rho\to 1}\mathcal{M}_y = \frac{1-\beta_f}{(1+\phi_y\zeta_r - \zeta_f)\left[1-\beta_f - \kappa_r(\phi_\pi - 1)\right]\zeta_r^{-1} + (\phi_\pi - 1)(\kappa_y + \phi_y\kappa_r)}\,.$$

The proof of Proposition 12 states that $\lim_{\rho\to 0}\mathcal{M}_y > 0$ and $\lim_{\rho\to 1}\mathcal{M}_y > 0$. As a result, a purely transitory DIS-demand shock is more expansionary than a completely permanent one, i.e., $\lim_{\rho\to 0}\mathcal{M}_y > \lim_{\rho\to 1}\mathcal{M}_y$, if

$$\begin{aligned}&(1+\phi_y\zeta_r - \zeta_f)\left[1-\beta_f - \kappa_r(\phi_\pi - 1)\right]\zeta_r^{-1} + (\phi_\pi - 1)(\kappa_y + \phi_y\kappa_r) > \\ &(1-\beta_f)\left[(1+\phi_y\zeta_r)(1-\phi_\pi\kappa_r)\zeta_r^{-1} + \phi_\pi(\kappa_y + \phi_y\kappa_r)\right]\,,\end{aligned}$$

which can be rearranged to

$$\begin{aligned}&(1-\zeta_f)\left[1-\beta_f - \kappa_r(\phi_\pi - 1)\right]\zeta_r^{-1} + (\phi_\pi - 1)\kappa_y > (1-\beta_f)\left[(1-\phi_\pi\kappa_r)\zeta_r^{-1} + \phi_\pi\kappa_y\right] \\ \Leftrightarrow\ &(1-\zeta_f)\left[1-\beta_f - \kappa_r(\phi_\pi - 1)\right] + (\phi_\pi - 1)\zeta_r\kappa_y > (1-\beta_f)\left[1 + \phi_\pi(\zeta_r\kappa_y - \kappa_r)\right] \\ \Leftrightarrow\ &\phi_\pi\left[-(1-\zeta_f)\kappa_r + \zeta_r\kappa_y - (1-\beta_f)(\zeta_r\kappa_y - \kappa_r)\right] > (1-\beta_f) + \zeta_r\kappa_y - (1-\zeta_f)(1-\beta_f+\kappa_r) \\ \Leftrightarrow\ &\phi_\pi\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] > \zeta_r\kappa_y + \zeta_f(1-\beta_f) - (1-\zeta_f)\kappa_r \\ \Leftrightarrow\ &\phi_\pi\left[\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r\right] > \beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r + (1-\beta_f)\left[\zeta_r\kappa_y + \zeta_f - \kappa_r\right] \\ \Leftrightarrow\ &\phi_\pi > 1 + (1-\beta_f)\frac{\zeta_r\kappa_y + \zeta_f - \kappa_r}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}\,,\end{aligned}$$

which can be shown to be larger than $\underline{\phi}^g_\pi$ and smaller than $\overline{\phi}^g_\pi$ if $\beta_f > 0$. □

A.4 Towards a Structural Interpretation

In this section, we provide formal statements and proofs for the sensitivity of monetary regime thresholds and ρ-max in case specific micro-foundations act on multiple sufficient statistics at the same time. We summarize our findings in Proposition 15 and refer to the main text for a discussion of the driving forces at play in generating these result.

PROPOSITION 15. *The following statements apply.*

(a) *Household Preferences: a higher elasticity of inter-temporal substitution or wealth in the utility decrease $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*}.*

(b) *Household Heterogeneity: counter-cyclical (respectively, pro-cyclical) income risk increases (respectively, decreases) $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*}. Moreover, in a THANK economy, counter-cyclical (respectively, pro-cyclical) income inequality decreases (respectively, increases) $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*} if $\lambda^{HtM} + s > 1$, where λ^{HtM} is the share of hand-to-mouth households and s the probability to stay a saver household.*

(c) *Behavioral Frictions: a departure from the full information rational expectation assumption via cognitive discounting increases $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*}.*

(d) *Life-Cycle: life-cycle dynamics with stochastic death increase $\underline{\phi}^g_\pi$ unambiguously, while it increases $\overline{\phi}^g_\pi$ and ρ^{g*} if $1 \geq \frac{\theta\beta_f}{1-\theta\beta_f}\frac{\zeta_f}{\kappa_y}$, i.e., prices are sufficiently flexible.*

(e) *Household Debt and Default: information asymmetries leading to interest rate spreads between borrowers and lenders leave $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*} unaffected if $\kappa_r = 0$ and increases all of them if $\kappa_r > 0$.*

(f) *Long-Run Interest Rate: a higher steady state real interest rate decreases (respectively, increases) $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*} if $\zeta_f + \zeta_r\kappa_y - \beta_f \geq (\beta_f - \kappa_r)\zeta_r$ (respectively, $\zeta_f + \zeta_r\kappa_y - \beta_f < (\beta_f - \kappa_r)\zeta_r$).*

Proof Proposition 15

PROOF. Subsequently, we prove all statements in the corresponding order.

Statement (a). Following Chapter 3 in Galí (2015) the RANK model with an arbitrary elasticity of inter-temporal substitution is characterized by

$$\zeta_f = 1\,, \quad \zeta_r = \sigma^{-1}\,, \quad \beta_f = \beta\,, \quad \kappa_y = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\,, \quad \text{and} \quad \kappa_r = 0\,.$$

In this case, we can define $\Xi \equiv \frac{1}{\zeta_r\kappa_y}$ such that we have

$$\underline{\phi}^g_\pi = \beta^{-1}\left(1 + (1-\beta)^2\Xi\right)\,, \quad \overline{\phi}^g_\pi = \beta^{-1}\left(1+\Xi\right)\,, \quad \text{and} \quad \rho^{g*} = \beta^{-1}\left(1 - \sqrt{(\phi_\pi\beta - 1)\Xi^{-1}}\right)\,.$$

The first part of Statement (a) follows immediately from $\frac{\partial \Xi}{\partial \sigma} = \frac{1}{(\zeta_r \kappa_y)^2} \frac{\lambda}{\sigma^2} \frac{\varphi+\alpha}{1-\alpha} > 0$. Following Online Appendix C in Michaillat and Saez (2021), the WUNK model with logarithmic utility over consumption is, in turn, characterized by

$$\zeta_f = \frac{\beta}{\beta + u'(0)y^n}\,, \quad \zeta_r = 1\,, \quad \beta_f = \beta\,, \quad \kappa_y = \lambda\left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)\,, \quad \text{and} \quad \kappa_r = 0\,,$$

where we work with Calvo price setting rather than Rotemberg adjustment costs. In this case, we can define $\Xi \equiv \frac{\zeta_f}{\kappa_y}$ such that we have

$$\underline{\phi}^g_\pi = \beta^{-1}\left(1 + (1-\beta)^2 \Xi\right)\,, \quad \overline{\phi}^g_\pi = \beta^{-1}\left(1+\Xi\right)\,, \quad \text{and} \quad \rho^{g*} = \beta^{-1}\left(1 - \sqrt{(\phi_\pi \beta - 1)\Xi^{-1}}\right)\,.$$

The second part of Statement (a) follows from $\frac{\partial \Xi}{\partial u'(0)} = -\frac{1}{\kappa_y} \frac{\beta}{(\beta+u'(0)y^n)^2} y^n < 0$.

Statement (b). Following Section 2 in Bilbiie (Forthcoming) the tractable HANK economy based on a spender-saver dichotomy with cyclical income inequality is characterized by

$$\zeta_f = 1 + (\chi - 1)\frac{1-s}{1-\lambda^{HtM}\chi}\,, \quad \zeta_r = \frac{1}{\sigma}\frac{1-\lambda^{HtM}}{1-\lambda^{HtM}\chi}\,, \quad \beta_f = \beta\,, \quad \kappa_y = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\,, \quad \text{and} \quad \kappa_r = 0\,,$$

where the measure of cyclical inequality, χ, is given by $\chi = 1 + \varphi(1 - \frac{\tau^D}{\lambda^{HtM}})$. In this case, we can define $\Xi \equiv \frac{\zeta_f}{\zeta_r \kappa_y}$ such that we have

$$\underline{\phi}^g_\pi = \beta^{-1}\left(1 + (1-\beta)^2 \Xi\right)\,, \quad \overline{\phi}^g_\pi = \beta^{-1}\left(1+\Xi\right)\,, \quad \text{and} \quad \rho^{g*} = \beta^{-1}\left(1 - \sqrt{(\phi_\pi \beta - 1)\Xi^{-1}}\right)\,.$$

The following comparative statics apply

$$\frac{\partial \Xi}{\partial \chi} = \frac{\sigma}{\kappa_y}\frac{1-s-\lambda^{HtM}}{1-\lambda^{HtM}}\,,$$
$$\frac{\partial \Xi}{\partial s} = \frac{\sigma}{\kappa_y}\frac{1-\chi}{1-\lambda^{HtM}}\,,$$
$$\frac{\partial \Xi}{\partial \lambda^{HtM}} = \frac{\sigma}{\kappa_y}\frac{(1-\chi)s + (1-\lambda^{HtM})(1-\lambda^{HtM}-s)\frac{\partial \chi}{\partial \lambda^{HtM}}}{(1-\lambda^{HtM})^2}\,,$$

As a result, $\underline{\phi}^g_\pi, \overline{\phi}^g_\pi$, and ρ^{g*} decrease in χ if $s + \lambda^{HtM} > 1$, and decrease in s if $\chi > 1$. Moreover, they decrease in λ^{HtM} if $(1-\chi)s + (1-\lambda^{HtM})(1-\lambda^{HtM}-s)\frac{\partial \chi}{\partial \lambda^{HtM}} < 0$, which is satisfied if cyclical inequality if sufficiently countercyclical, i.e., χ large enough. Regarding cyclical income risk, we follow Bilbiie (Forthcoming) in making the following case distinction. First, we consider the case where the probability to stay in the saver state depends on the *current* aggregate state of the economy, i.e., $s(Y_t)$. In this case, one arrives at the following representation

$$\tilde{\zeta}_f = \frac{1}{1-\eta}\left[1 + (\chi-1)\frac{1-\tilde{s}}{1-\lambda^{HtM}\chi}\right]\,, \quad \tilde{\zeta}_r = \frac{1}{1-\eta}\frac{1}{\sigma}\frac{1-\lambda^{HtM}}{1-\lambda^{HtM}\chi}\,, \quad \tilde{s} = \frac{s}{s+(1-s)\Gamma^\sigma}\,, \quad \eta = \frac{s_y Y}{1-s}(1-\Gamma^{-\sigma})(1-\tilde{s})\frac{1}{\sigma}\frac{1-\lambda^{HtM}}{1-\lambda^{HtM}\chi}\,.$$

where $\Gamma = \frac{c^S}{c^H}$ is a measure of steady state consumption inequality. Notice that β_f, κ_y, and κ_r take the same values as before. It is straightforward to see that pro- or counter-cyclical risk, i.e., $\eta \neq 0$ leaves $\frac{\tilde{\zeta}_f}{\tilde{\zeta}_r}$ unchanged, i.e., $\frac{\tilde{\zeta}_f}{\tilde{\zeta}_r} = \frac{\zeta_f}{\zeta_r}$. As a result, the comparative statics of $\underline{\phi}^g_\pi, \overline{\phi}^g_\pi$, and ρ^{g*} do not depend on the cyclicality of income risk and are unaffected otherwise up to replacing s by $\tilde{s}$. Second, we consider the case where the probability to stay in the saver state depends on the *future* aggregate state of the economy, i.e., $s(Y_{t+1})$. In this case, one arrives at the following representation

$$\tilde{\zeta}_f = 1 + (\chi - 1)\frac{1-\tilde{s}}{1-\lambda^{HtM}\chi} + \eta\,, \quad \tilde{\zeta}_r = \frac{1}{\sigma}\frac{1-\lambda^{HtM}}{1-\lambda^{HtM}\chi}\,, \quad \tilde{s} = \frac{s}{s+(1-s)\Gamma^\sigma}\,, \quad \eta = \frac{s_y Y}{1-s}(1-\Gamma^{-\sigma})(1-\tilde{s})\frac{1}{\sigma}\frac{1-\lambda^{HtM}}{1-\lambda^{HtM}\chi}\,.$$

Finally, defining $\Xi \equiv \frac{\tilde{\zeta}_f}{\tilde{\zeta}_r \kappa_y}$ we arrive at the following comparative statics

$$\begin{aligned}
\frac{\partial \Xi}{\partial \chi} &= \frac{\sigma}{\kappa_y}\frac{1-\tilde{s}-\lambda^{HtM}}{1-\lambda^{HtM}}\,,\\
\frac{\partial \Xi}{\partial s} &= \frac{1}{\kappa_y}\frac{1}{(s+(1-s)\Gamma^\sigma)^2}\left[s_y Y(\Gamma^\sigma - 1)^2 + \sigma\frac{1-\chi}{1-\lambda^{HtM}}\Gamma^\sigma\right]\,,\\
\frac{\partial \Xi}{\partial s_y} &= \frac{1}{\kappa_y}\frac{Y}{1-s}(1-\Gamma^{-\sigma})(1-\tilde{s})\,,\\
\frac{\partial \Xi}{\partial \lambda^{HtM}} &= \frac{\sigma}{\kappa_y}\frac{(1-\chi)\tilde{s} + (1-\lambda^{HtM})(1-\lambda^{HtM}-\tilde{s})\frac{\partial \chi}{\partial \lambda^{HtM}}}{(1-\lambda^{HtM})^2}\,,
\end{aligned}$$

from where the results in the main text follow. Notice that the PRANK economy of Acharya and Dogra (2020) leads to $\zeta_f = 1 - \frac{\sigma}{2}(\frac{R-1}{R})^2\frac{\partial \sigma^2(y*)}{\partial y}$ such that the comparative statics are qualitatively similar to the ones with respect to s_y when s depends on the future aggregate state, i.e., procyclical risk reduces $\tilde{\zeta}_f$ and Ξ.

Statement (c). Following Proposition 2 in Gabaix (2020) the behavioral RANK model is characterized by

$$\zeta_f = \overline{m} \qquad \text{and} \qquad \beta_f = \overline{m}\left(\theta + \frac{1-\beta\theta}{1-\beta\theta\overline{m}}(1-\theta)\right)\,.$$

Notice that we have

$$\frac{\partial \beta_f}{\partial \overline{m}} > 0\,, \quad \text{and} \quad \frac{\partial \beta_f}{\partial \overline{m}}\frac{\overline{m}}{\beta_f} = 1 + \frac{\beta\theta\overline{m}}{1-\beta\theta\overline{m}}\frac{(1-\beta\theta)(1-\theta)}{\theta(1-\beta\theta\overline{m}) + (1-\beta\theta)(1-\theta)} \geq 1\,.$$

In this case, we can define $\Xi \equiv \frac{\zeta_f}{\zeta_r \kappa_y}$, with $\frac{\partial \Xi}{\partial \overline{m}} = \frac{1}{\zeta_r \kappa_y}$ such that we have

$$\underline{\phi}^g_\pi = \beta_f^{-1}\left(1 + (1-\beta_f)^2\Xi\right)\,, \quad \overline{\phi}^g_\pi = \beta_f^{-1}\left(1+\Xi\right)\,, \quad \text{and} \quad \rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\phi_\pi\beta_f - 1)\Xi^{-1}}\right)\,.$$

From there, we obtain

$$\frac{\partial \underline{\phi}^g_\pi}{\partial \overline{m}} = \left[-\beta_f^{-2}\left(1+(1-\beta_f)^2\Xi\right) - 2\beta_f^{-1}(1-\beta_f)\Xi\right]\frac{\partial \beta_f}{\partial \overline{m}} + \beta_f^{-1}(1-\beta_f)^2\frac{\Xi}{\overline{m}}$$

$$= \left[-\beta_f^{-2} - 2\beta_f^{-1}(1-\beta_f)\Xi\right]\frac{\partial\beta_f}{\partial\overline{m}} + \beta_f^{-1}(1-\beta_f)^2\frac{\Xi}{\overline{m}}\left[1 - \frac{\partial\beta_f}{\partial\overline{m}}\frac{\overline{m}}{\beta_f}\right]$$

$$< 0\,.$$

Moreover, we have

$$\frac{\partial\overline{\phi}_\pi^g}{\partial\overline{m}} = -\beta_f^{-2}\left(1+\Xi\right)\frac{\partial\beta_f}{\partial\overline{m}} + \beta_f^{-1}\frac{\Xi}{\overline{m}} = -\beta_f^{-2}\frac{\partial\beta_f}{\partial\overline{m}} + \beta_f^{-1}\frac{\Xi}{\overline{m}}\left[1 - \frac{\partial\beta_f}{\partial\overline{m}}\frac{\overline{m}}{\beta_f}\right] < 0\,.$$

Finally, we show that

$$\frac{\partial\rho^{g*}}{\partial\overline{m}} = \left[-\beta_f^{-2}\left(1 - \left[(\phi_\pi\beta_f - 1)\Xi^{-1}\right]^{\frac{1}{2}}\right) - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f - 1)\Xi^{-1}\right]^{-\frac{1}{2}}\phi_\pi\Xi^{-1}\right]\frac{\partial\beta_f}{\partial\overline{m}}$$

$$+ \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f - 1)\Xi^{-1}\right]^{-\frac{1}{2}}(\phi_\pi\beta_f - 1)\frac{\Xi^{-1}}{\overline{m}}$$

$$= -\beta_f^{-2}\left(1 - \left[(\phi_\pi\beta_f - 1)\Xi^{-1}\right]^{\frac{1}{2}}\right)\frac{\partial\beta_f}{\partial\overline{m}} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f - 1)\Xi^{-1}\right]^{-\frac{1}{2}}\Xi^{-1}\left[\phi_\pi\frac{\partial\beta_f}{\partial\overline{m}} - \frac{\phi_\pi\beta_f - 1}{\overline{m}}\right]$$

$$< 0\,,$$

where the last inequality follows by the observation that

$$\phi_\pi\frac{\partial\beta_f}{\partial\overline{m}} - \frac{\phi_\pi\beta_f - 1}{\overline{m}} = \frac{1}{\overline{m}} + \frac{\phi_\pi\beta_f}{\overline{m}}\left[\frac{\partial\beta_f}{\partial\overline{m}}\frac{\overline{m}}{\beta_f} - 1\right] > 0\,.$$

This completes the proof of Statement (c).

Statement (d). Following Del Negro et al. (2023) the NK model with a perpetual youth structure and no habit persistence admits the following representation

$$\zeta_f = \left(1 + \tfrac{p}{1-p}\tfrac{1-\beta(1-p)}{1+\varphi}\tfrac{s}{c}\right)^{-1}, \quad \zeta_r = 1\,, \quad \beta_f = \beta\zeta_f\,, \quad \kappa_y = \lambda_{\beta_f}\left(1 + \tfrac{\varphi+\alpha}{1-\alpha}\right), \quad \text{and} \quad \kappa_r = 0\,,$$

where $p \in [0,1)$ denotes the probability to die and s/c is the wealth to consumption steady state ratio. In this case, we can define $\Xi \equiv \frac{\zeta_f}{\kappa_y}$ such that we have

$$\underline{\phi}_\pi^g = \beta_f^{-1}\left(1 + (1-\beta_f)^2\Xi\right), \quad \overline{\phi}_\pi^g = \beta_f^{-1}\left(1+\Xi\right), \quad \text{and} \quad \rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\phi_\pi\beta_f - 1)\Xi^{-1}}\right).$$

Notice that the following auxiliary results apply:

$$\frac{\partial\zeta_f}{\partial p} = -\zeta_f^2\frac{s}{c}\frac{1}{1+\varphi}\frac{1-\beta(1-p)^2}{(1-p)^2}\,,$$

$$\frac{\partial\beta_f}{\partial p} = \beta\frac{\partial\zeta_f}{\partial p}\,,$$

$$\frac{\partial\kappa_y}{\partial p} = -\kappa_y\frac{\beta\theta}{1-\beta\theta\zeta_f}\frac{\partial\zeta_f}{\partial p}\,,$$

$$\frac{\partial\Xi}{\partial p} = \frac{1}{\kappa_y(1-\beta\theta\zeta_f)}\frac{\partial\zeta_f}{\partial p}\,.$$

Using the previous auxiliary results, we obtain

$$\begin{aligned}
\frac{\partial \underline{\phi}_\pi^g}{\partial p} &= \left[-\beta_f^{-2}\left(1+(1-\beta_f)^2\Xi\right) - 2\beta_f^{-1}(1-\beta_f)\Xi\right]\beta\frac{\partial \zeta_f}{\partial p} + \beta_f^{-1}(1-\beta_f)^2\frac{1}{\kappa_y(1-\beta\theta\zeta_f)}\frac{\partial \zeta_f}{\partial p} \\
&= \left[-\beta_f^{-2} - 2\beta_f^{-1}(1-\beta_f)\Xi\right]\beta\frac{\partial \zeta_f}{\partial p} + \beta_f^{-1}(1-\beta_f)^2\frac{\theta\beta_f}{1-\theta\beta_f}\frac{1}{\kappa_y}\frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-2}\beta\frac{\partial \zeta_f}{\partial p} + \left[\frac{1-\beta_f}{\beta_f}\frac{\theta\beta_f}{1-\theta\beta_f} - 2\right]\frac{1-\beta_f}{\kappa_y}\frac{\partial \zeta_f}{\partial p} \\
&> 0\,,
\end{aligned}$$

where the terminal strict inequality holds as $\theta(1+\beta_f)\leq 2$ due to $\theta\leq 1$ and $\beta_f<\beta<1$. In a similar vein, we obtain for the upper regime threshold

$$\frac{\partial \overline{\phi}_\pi^g}{\partial p} = -\beta_f^{-2}\left(1+\Xi\right)\beta\frac{\partial \zeta_f}{\partial p} + \beta_f^{-1}\frac{1}{\kappa_y(1-\beta\theta\zeta_f)}\frac{\partial \zeta_f}{\partial p}\,.$$

The previous expression can be rewritten by substituting in for Ξ as

$$\begin{aligned}
\frac{\partial \overline{\phi}_\pi^g}{\partial p} &= -\beta_f^{-2}\beta\frac{\partial \zeta_f}{\partial p} + \frac{\beta_f^{-1}}{\kappa_y}\left[\frac{1}{1-\beta\theta\zeta_f} - \frac{\beta\zeta_f}{\beta_f}\right]\frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-2}\beta\frac{\partial \zeta_f}{\partial p} + \frac{\beta_f^{-1}}{\kappa_y}\frac{\theta\beta_f}{1-\beta\theta\zeta_f}\frac{\partial \zeta_f}{\partial p}\,.
\end{aligned}$$

Finally, the former expression can be simplified to

$$\frac{\partial \overline{\phi}_\pi^g}{\partial p} = -\beta_f^{-1}\frac{\partial \zeta_f}{\partial p}\left[\frac{\beta}{\beta_f} - \frac{1}{\kappa_y}\frac{\theta\beta_f}{1-\beta\theta\zeta_f}\right] = -\frac{1}{\beta_f\zeta_f}\frac{\partial \zeta_f}{\partial p}\left[1 - \frac{\zeta_f}{\kappa_y}\frac{\theta\beta_f}{1-\theta\beta_f}\right]\,.$$

As a result, we obtain that $\frac{\partial \overline{\phi}_\pi^g}{\partial p} \gtreqless 0$ if $1 \gtreqless \frac{\zeta_f}{\kappa_y}\frac{\theta\beta_f}{1-\theta\beta_f}$. To conclude the proof of statement (d), we finally study the comparative statics of the output-maximizing persistence

$$\begin{aligned}
\frac{\partial \rho^{g*}}{\partial p} &= -\beta_f^{-1}\rho^{g*}\beta\frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f-1)\frac{\kappa_y}{\zeta_f}\right]^{-\frac{1}{2}}\left\{\phi_\pi\beta\frac{\kappa_y}{\zeta_f}\frac{\partial \zeta_f}{\partial p} + (\phi_\pi\beta_f-1)\frac{\frac{\partial \kappa_y}{\partial p}\zeta_f - \frac{\partial \zeta_f}{\partial p}\kappa_y}{\zeta_f^2}\right\} \\
&= -\beta_f^{-1}\rho^{g*}\beta\frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f-1)\frac{\kappa_y}{\zeta_f}\right]^{-\frac{1}{2}}\left\{\phi_\pi\beta\frac{\kappa_y}{\zeta_f}\frac{\partial \zeta_f}{\partial p} - (\phi_\pi\beta_f-1)\frac{\kappa_y}{1-\theta\beta_f}\frac{\partial \zeta_f}{\partial p}\frac{1}{\zeta_f^2}\right\} \\
&= -\beta_f^{-1}\rho^{g*}\beta\frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f-1)\frac{\kappa_y}{\zeta_f}\right]^{-\frac{1}{2}}\left\{\phi_\pi\beta - (\phi_\pi\beta_f-1)\frac{1}{1-\theta\beta_f}\frac{1}{\zeta_f}\right\}\frac{\kappa_y}{\zeta_f}\frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-1}\rho^{g*}\beta\frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f-1)\frac{\kappa_y}{\zeta_f}\right]^{-\frac{1}{2}}\left\{\phi_\pi\beta_f - (\phi_\pi\beta_f-1)\frac{1}{1-\theta\beta_f}\right\}\frac{\kappa_y}{\zeta_f^2}\frac{\partial \zeta_f}{\partial p} \\
&= -\beta_f^{-1}\rho^{g*}\beta\frac{\partial \zeta_f}{\partial p} - \frac{\beta_f^{-1}}{2}\left[(\phi_\pi\beta_f-1)\frac{\kappa_y}{\zeta_f}\right]^{-\frac{1}{2}}\left\{1 - (\phi_\pi\beta_f-1)\frac{\theta\beta_f}{1-\theta\beta_f}\right\}\frac{\kappa_y}{\zeta_f^2}\frac{\partial \zeta_f}{\partial p}\,.
\end{aligned}$$

Notice that the sign of the previous expression is positive if $1 \geq (\phi_\pi \beta_f - 1)\frac{\theta\beta_f}{1-\theta\beta_f}$. Substituting in for the upper bound $\overline{\phi}^g_\pi = \beta_f^{-1}\left(1 + \frac{\zeta_f}{\kappa_y}\right)$ yields the sufficient condition $1 \geq \frac{\theta\beta_f}{1-\theta\beta_f}\frac{\zeta_f}{\kappa_y}$.

Statement (e). Following Beaudry and Portier (2018) the NK model with an upward sloping interest rate schedule admits the following representation

$$\zeta_f = \frac{\sigma}{\sigma + \epsilon_p}\,, \quad \zeta_r = \frac{1}{\sigma+\epsilon_p}\,, \quad \beta_f = \beta\,, \quad \kappa_y = \lambda\left(1 + \frac{\varphi+\alpha}{1-\alpha}\right)\,, \quad \text{and} \quad \kappa_r \geq 0\,,$$

where $\epsilon_p \in [0,\infty)$ captures the increase in borrowing costs depending on the level of debt. Notice that κ_r does not depend on ϵ_p. In this case, we can define $\Xi \equiv \frac{(\beta_f - \kappa_r)\zeta_f}{\beta_f\zeta_r\kappa_y + (\zeta_f - \beta_f)\kappa_r}$ such that we have

$$\underline{\phi}^g_\pi = \beta_f^{-1}\left(1 + (1-\beta_f)^2\Xi\right)\,, \quad \overline{\phi}^g_\pi = \beta_f^{-1}\left(1+\Xi\right)\,, \quad \text{and} \quad \rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\phi_\pi\beta_f - 1)\Xi^{-1}}\right)\,.$$

Notice that we have

$$\frac{\partial\zeta_f}{\partial\epsilon_p} = -\frac{1}{\sigma+\epsilon_p}\zeta_f\,, \qquad \text{and} \qquad \frac{\partial\zeta_r}{\partial\epsilon_p} = -\frac{1}{\sigma+\epsilon_p}\zeta_r\,.$$

As such, we obtain

$$\begin{aligned}\frac{\partial\Xi}{\partial\epsilon_p} &= \frac{(\beta_f - \kappa_r)\frac{\partial\zeta_f}{\partial\epsilon_p}\left[\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r\right] - (\beta_f-\kappa_r)\zeta_f\left[\beta_f\kappa_y\frac{\partial\zeta_r}{\partial\epsilon_p} + \frac{\partial\zeta_f}{\partial\epsilon_p}\kappa_r\right]}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r\right]^2} \\ &= (\beta_f-\kappa_r)\beta_f\frac{\frac{\partial\zeta_f}{\partial\epsilon_p}\left[\zeta_r\kappa_y - \kappa_r\right] - \frac{\partial\zeta_r}{\partial\epsilon_p}\zeta_f\kappa_y}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r\right]^2}\,.\end{aligned}$$

Substituting in the expression for $\frac{\partial\zeta_f}{\partial\epsilon_p}$ and $\frac{\partial\zeta_r}{\partial\epsilon_p}$, we finally obtain

$$\frac{\partial\Xi}{\partial\epsilon_p} = \frac{(\beta_f-\kappa_r)\beta_f}{\sigma+\epsilon_p}\frac{\zeta_f\kappa_r}{\left[\beta_f\zeta_r\kappa_y + (\zeta_f-\beta_f)\kappa_r\right]^2} \geq 0\,,$$

from which the results stated in the main text directly follow.

Statement (f). Following Beaudry et al. (2024) the NK model with a cost channel of monetary policy admits the following representation

$$\zeta_f = \frac{\sigma}{\sigma+\epsilon_p}\,, \quad \zeta_r = \frac{1}{\sigma+\epsilon_p}\,, \quad \beta_f = \beta\,, \quad \kappa_y = \kappa\left(\frac{\frac{1}{a}\frac{W}{P}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}}\right)\,, \quad \text{and} \quad \kappa_r = \kappa\left(\frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}}\right)\,,$$

where $\epsilon_p \in [0,\infty)$ captures the increase in borrowing costs depending on the level of debt, W/P is the real wage, a and b capture the degree of substitutability between input goods within the Leontief technology, $1+i$ is the gross nominal rate, and $1+\pi$ gross price inflation. Notice that $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta} > 0$ does not depend on the nominal interest rate. In this case, we can define $\Xi \equiv \frac{(\beta_f - \kappa_r)\zeta_f}{\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r}$ such that we have

$$\underline{\phi}_\pi^g = \beta_f^{-1}\left(1 + (1-\beta_f)^2 \Xi\right), \quad \overline{\phi}_\pi^g = \beta_f^{-1}\left(1 + \Xi\right), \quad \text{and} \quad \rho^{g*} = \beta_f^{-1}\left(1 - \sqrt{(\phi_\pi \beta_f - 1)\Xi^{-1}}\right).$$

Subsequently, we define $r \equiv \frac{1+i}{1+\pi}$. Moreover, notice that the following relations hold

$$\frac{\partial \kappa_y}{\partial r} = -\kappa \frac{\frac{\beta}{ab}\frac{W}{P}}{\left[\frac{1}{a}\frac{W}{P} + \frac{\beta}{b} r\right]^2}, \qquad \text{and} \qquad \frac{\partial \kappa_r}{\partial r} = \kappa \frac{\frac{\beta}{ab}\frac{W}{P}}{\left[\frac{1}{a}\frac{W}{P} + \frac{\beta}{b} r\right]^2}.$$

As a result, we obtain

$$\begin{aligned}
\frac{\partial \Xi}{\partial r} &= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r}\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right] - (\beta_f - \kappa_r)\zeta_f \left[\beta_f \zeta_r \frac{\partial \kappa_y}{\partial r} + (\zeta_f - \beta_f)\frac{\partial \kappa_r}{\partial r}\right]}{\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\
&= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r}\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r + (\beta_f - \kappa_r)(\zeta_f - \beta_f)\right] - (\beta_f - \kappa_r)\zeta_f \beta_f \zeta_r \frac{\partial \kappa_y}{\partial r}}{\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\
&= \frac{-\zeta_f \frac{\partial \kappa_r}{\partial r}\left[\beta_f \zeta_r \kappa_y + \beta_f(\zeta_f - \beta_f)\right] - (\beta_f - \kappa_r)\zeta_f \beta_f \zeta_r \frac{\partial \kappa_y}{\partial r}}{\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\
&= -\zeta_f \beta_f \frac{\frac{\partial \kappa_r}{\partial r}\left[\zeta_r \kappa_y + (\zeta_f - \beta_f)\right] + (\beta_f - \kappa_r)\zeta_r \frac{\partial \kappa_y}{\partial r}}{\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \\
&= -\frac{\zeta_f \beta_f}{\left[\beta_f \zeta_r \kappa_y + (\zeta_f - \beta_f)\kappa_r\right]^2} \frac{\kappa \frac{\beta}{ab}\frac{W}{P}}{\left[\frac{1}{a}\frac{W}{P} + \frac{\beta}{b} r\right]^2} \left\{\zeta_r \kappa_y + \zeta_f - \beta_f - (\beta_f - \kappa_r)\zeta_r\right\}.
\end{aligned}$$

As a result, we obtain that $\frac{\partial \Xi}{\partial r} \lesseqqgtr 0$ if $\zeta_r \kappa_y + \zeta_f - \beta_f \gtreqqless (\beta_f - \kappa_r)\zeta_r$. This completes the proof of the statement in the main text. □

A.5 Additional Results

A.5.1 *Comparative Statics in RANK* We summarize comparative static results of the regime thresholds of systematic monetary policy as well as the impact output maximizing persistence in Corollary 3. Relative to Corollary 1 from the main text, which focused on the size of the intermediate regime (Δ_π), we consider subsequently levels.

COROLLARY 3. *Consider the hump-shaped regime of Proposition 2 characterized by $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$. The following comparative statics apply.*

(a) *The lower and upper thresholds* $(\underline{\phi}_\pi, \overline{\phi}_\pi)$ *strictly decreases in* κ_y. *Moreover,* $\underline{\phi}_\pi$ *strictly decreases in* β*, whereas* $\overline{\phi}_\pi$ *strictly decreases (respectively, increases) in* β *if* $\kappa_y > \frac{2\beta\theta-1}{1-\beta\theta}$ *(respectively,* $\kappa_y \leq \frac{2\beta\theta-1}{1-\beta\theta}$*).*

(b) *The persistence threshold* ρ^* *strictly decreases in* κ_y *and* ϕ_π. *Moreover,* ρ^* *strictly decreases in* β *if* $\kappa_y > \frac{2\beta\theta-1}{1-\beta\theta}$*, whereas* $\exists\, \phi^*_\pi \in \left(\underline{\phi}_\pi, \overline{\phi}_\pi\right)$ *such that* $\frac{\partial\rho^*}{\partial\beta} < 0\ \forall\phi_\pi \in \left(\underline{\phi}_\pi, \phi^*_\pi\right)$ *and* $\frac{\partial\rho^*}{\partial\beta} \geq 0\ \forall\phi_\pi \in \left[\phi^*_\pi, \overline{\phi}_\pi\right)$ *if* $\kappa_y \leq \frac{2\beta\theta-1}{1-\beta\theta}$.

Proof Corollary 3

PROOF. To show Statement (a) note that the comparative statics of $\underline{\phi}_\pi$ and $\overline{\phi}_\pi$ with respect to κ_y follow in a straightforward manner. To compute the comparative statics with respect to β, we make use of the following relation: $\frac{\partial\kappa_y}{\partial\beta} = -\frac{\theta}{1-\beta\theta}\kappa_y$. We then obtain

$$\begin{aligned}
\frac{\partial\underline{\phi}_\pi}{\partial\beta} &= -\beta^{-2}\left(1+(1-\beta)^2\kappa_y^{-1}\right) - 2\beta^{-1}(1-\beta)\kappa_y^{-1} - \beta^{-1}(1-\beta)^2\kappa_y^{-2}\frac{\partial\kappa_y}{\partial\beta} \\
&= -\beta^{-2}\left(1+(1-\beta)^2\kappa_y^{-1}\right) - 2\beta^{-2}(1-\beta)^2\frac{\beta}{1-\beta}\kappa_y^{-1} + \beta^{-2}(1-\beta)^2\kappa_y^{-1}\frac{\beta\theta}{1-\beta\theta} \\
&= -\beta^{-2}\left[1+(1-\beta)^2\kappa_y^{-1}\left(1+2\frac{\beta}{1-\beta}-\frac{\beta\theta}{1-\beta\theta}\right)\right] \\
&< 0\,,
\end{aligned}$$

where the terminal inequality is due to

$$1+2\frac{\beta}{1-\beta}-\frac{\beta\theta}{1-\beta\theta} = \frac{1+\beta}{1-\beta}-\frac{\beta\theta}{1-\beta\theta} = \frac{(1+\beta)(1-\beta\theta)-\beta\theta(1-\beta)}{(1-\beta)(1-\beta\theta)} = \frac{1-\beta\theta+\beta(1-\theta)}{(1-\beta)(1-\beta\theta)} > 0\,.$$

Following similar steps, we obtain

$$\begin{aligned}
\frac{\partial\overline{\phi}_\pi}{\partial\beta} &= -\beta^{-2}\left(1+\kappa_y^{-1}\right) - \beta^{-1}\kappa_y^{-2}\frac{\partial\kappa_y}{\partial\beta} = -\beta^{-2}\left(1+\kappa_y^{-1}\right) + \beta^{-2}\kappa_y^{-1}\frac{\beta\theta}{1-\beta\theta} \\
&= -\beta^{2}\left[1+\kappa_y^{-1}\left(1-\frac{\beta\theta}{1-\beta\theta}\right)\right],
\end{aligned}$$

which is strictly negative (respectively, positive) if $\kappa_y > \frac{2\beta\theta-1}{1-\beta\theta}$ (respectively, $\kappa_y \leq \frac{2\beta\theta-1}{1-\beta\theta}$).

To show Statement (b) notice that the comparative statics of ρ^* with respect to κ_y and ϕ_π follow in a straightforward manner. Moreover, regarding the comparative statics with respect to β we have

$$\frac{\partial\rho^*}{\partial\beta} = -\beta^{-2}\left[1-\left((\phi_\pi\beta-1)\kappa_y\right)^{\frac{1}{2}}\right] - \frac{\beta^{-1}}{2}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\left[\phi_\pi\kappa_y-(\beta\phi_\pi-1)\frac{\theta}{1-\beta\theta}\kappa_y\right].$$

To begin with, taking the limit yields

$$
\begin{aligned}
\lim_{\phi_\pi \downarrow \underline{\phi}_\pi} \operatorname{sgn}\left(\frac{\partial \rho^*}{\partial \beta}\right) &= -\beta^{-2}\left[1-\left((\underline{\phi}_\pi\beta-1)\kappa_y\right)^{\frac{1}{2}}\right] - \frac{\beta^{-1}}{2}\left((\underline{\phi}_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\left[\underline{\phi}_\pi\kappa_y - (\beta\underline{\phi}_\pi-1)\frac{\theta}{1-\beta\theta}\kappa_y\right] \\
&= -\beta^{-1} - \frac{1}{2}\frac{\beta^{-1}}{1-\beta}\left[\beta^{-1}\kappa_y + \beta^{-1}(1-\beta)^2 - \frac{\theta}{1-\beta\theta}(1-\beta)^2\right] \\
&= -\beta^{-1}\left[\frac{1}{2}\frac{\beta^{-1}}{1-\beta}\kappa_y + 1 + \frac{1}{2}\frac{1-\beta}{\beta}\left(1-\frac{\beta\theta}{1-\beta\theta}\right)\right] \\
&= -\beta^{-1}\left[\frac{1}{2}\frac{\beta^{-1}}{1-\beta}\kappa_y + 1 + \frac{1}{2}\frac{1-\beta}{\beta}\frac{1-2\beta\theta}{1-\beta\theta}\right] \\
&< 0\,,
\end{aligned}
$$

where the strict inequality is due to

$$
1+\frac{1}{2}\frac{1-\beta}{\beta}\frac{1-2\beta\theta}{1-\beta\theta} = \frac{2\beta(1-\beta\theta)+(1-\beta)(1-2\beta\theta)}{2\beta(1-\beta\theta)} = \frac{1-\beta\theta+\beta(1-\theta)}{2\beta(1-\beta\theta)} > 0\,.
$$

Similarly, we obtain

$$
\lim_{\phi_\pi \uparrow \overline{\phi}_\pi} \operatorname{sgn}\left(\frac{\partial \rho^*}{\partial \beta}\right) = -\frac{\beta^{-1}}{2}\left[\beta^{-1}\kappa_y + \beta^{-1} - \frac{\theta}{1-\beta\theta}\right] = -\frac{\beta^{-2}}{2}\left[1+\kappa_y - \frac{\beta\theta}{1-\beta\theta}\right]\,,
$$

which is strictly negative (respectively, positive) if $\kappa_y > \frac{2\beta\theta-1}{1-\beta\theta}$ (respectively, $\kappa_y \leq \frac{2\beta\theta-1}{1-\beta\theta}$). Moreover, notice that we have

$$
\begin{aligned}
\frac{\partial\left(\frac{\partial \rho^*}{\partial \beta}\right)}{\partial \phi_\pi} =& \frac{\beta^{-2}}{2}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\beta\kappa_y + \frac{1}{4}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{3}{2}}\kappa_y\left[\phi_\pi\kappa_y - (\beta\phi_\pi-1)\frac{\theta}{1-\beta\theta}\kappa_y\right] \\
&- \frac{\beta^{-1}}{2}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\kappa_y\left[1-\frac{\beta\theta}{1-\beta\theta}\right] \\
=& \frac{1}{2}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\kappa_y\frac{\theta}{1-\beta\theta} + \frac{1}{4}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{3}{2}}\kappa_y\left[\phi_\pi\kappa_y - (\beta\phi_\pi-1)\frac{\theta}{1-\beta\theta}\kappa_y\right] \\
=& \frac{1}{2}\left((\phi_\pi\beta-1)\kappa_y\right)^{-\frac{1}{2}}\kappa_y\left\{\frac{1}{2}\frac{\phi_\pi}{\phi_\pi\beta-1} + \frac{\theta}{1-\beta\theta} - \frac{1}{2}\frac{\theta}{1-\beta\theta}\kappa_y\right\}\,.
\end{aligned}
$$

To determine the sign of the previous expression, notice that $\frac{\phi_\pi}{\phi_\pi\beta-1}$ is strictly decreasing in ϕ_π. Therefore, the derivative can switch signs at most once on $\phi_\pi \in \left(\underline{\phi}_\pi, \overline{\phi}_\pi\right)$. As a result, we arrive at the following case distinction:

1. If $\kappa_y > \frac{2\beta\theta-1}{1-\beta\theta}$, i.e., $\lim_{\phi_\pi \uparrow \overline{\phi}_\pi} \operatorname{sgn}\left(\frac{\partial \rho^*}{\partial \beta}\right) < 0$, then $\frac{\partial \rho^*}{\partial \beta} < 0\; \forall \phi_\pi \in \left(\underline{\phi}_\pi, \overline{\phi}_\pi\right)$.
2. If $\kappa_y \leq \frac{2\beta\theta-1}{1-\beta\theta}$, i.e., $\lim_{\phi_\pi \uparrow \overline{\phi}_\pi} \operatorname{sgn}\left(\frac{\partial \rho^*}{\partial \beta}\right) \geq 0$, then $\exists\; \phi_\pi^* \in \left(\underline{\phi}_\pi, \overline{\phi}_\pi\right)$ such that $\frac{\partial \rho^*}{\partial \beta} < 0\; \forall \phi_\pi \in \left(\underline{\phi}_\pi, \phi_\pi^*\right)$ and $\frac{\partial \rho^*}{\partial \beta} \geq 0\; \forall \phi_\pi \in \left(\phi_\pi^*, \overline{\phi}_\pi,\right)$.

□

A.5.2 *Cumulative Output Gains* In this section, we examine how the degree of persistence shapes cumulative output gains in the context of a RANK economy. Proposition 16 summarizes our findings.

PROPOSITION 16. *The cumulative output multiplier monotonously increases in the persistence of a DIS-demand shock, i.e.,* $\frac{\partial C_y(1)}{\partial \rho} > 0$.

Propositions 2 and 16 together imply that the persistence of the DIS-demand shock possibly engenders a short-versus long-run stabilization trade-off. First, a more persistent DIS-demand shock drives short-run and cumulative output in the same direction if $\phi_\pi < \underline{\phi}_\pi$. Second, a more persistent DIS-demand shock drives short-run and cumulative output in the same direction for $\rho < \rho^*$ and in opposite directions for $\rho > \rho^*$ if $\underline{\phi}_\pi < \phi_\pi < \overline{\phi}_\pi$. Finally, a more persistent DIS-demand shock unambiguously reduces short-run output gains at the expense of higher cumulative output gains if $\phi_\pi > \overline{\phi}_\pi$.

Proof Proposition 16

PROOF. The cumulative output multiplier is given by

$$C_y(1) = \frac{\mathcal{M}_y}{1-\rho} = \frac{1-\beta\rho}{(1-\rho)(1+\phi_y-\rho)(1-\beta\rho)+\kappa_y(\phi_\pi-\rho)(1-\rho)} \,.$$

Defining $\overline{x} \equiv (1-\rho)\,(1+\phi_y-\rho)\,(1-\rho\beta)+\kappa_y\,(\phi_\pi-\rho)\,(1-\rho)$ we obtain its comparative static

$$\begin{aligned}
\overline{x}^2\frac{\partial C_y(1)}{\partial \rho} =& -\beta\left[(1-\rho)\,(1+\phi_y-\rho)\,(1-\rho\beta)+\kappa_y\,(\phi_\pi-\rho)\,(1-\rho)\right] \\
& -(1-\beta\rho)\Big\{-(1-\rho)\,(1-\rho\beta)-(1+\phi_y-\rho)\left[1-\beta\rho+\beta(1-\rho)\right]\Big\} \\
& -(1-\beta\rho)\Big\{-\kappa_y\left[1-\rho+\phi_\pi-\rho\right]\Big\}\,,
\end{aligned}$$

which can be further simplified to

$$\begin{aligned}
\overline{x}^2\frac{\partial C_y(1)}{\partial \rho} =& -\beta\kappa_y(\phi_\pi-\rho)(1-\rho)+2(1-\rho)(1-\beta\rho)^2+\phi_y(1-\beta)^2 \\
& +\kappa_y(1-\beta\rho)\left[1-\rho+\phi_\pi-\rho\right]\,.
\end{aligned}$$

The previous expression can be rewritten as

$$\begin{aligned}
\overline{x}^2\frac{\partial C_y(1)}{\partial \rho} &= \kappa_y\left[(\phi_\pi-\rho)(1-\beta)+(1-\rho)(1-\beta\rho)\right]+2(1-\rho)(1-\beta\rho)^2+\phi_y(1-\beta\rho)^2 \\
&= \kappa_y\left[\phi_\pi(1-\beta)+1-2\rho+\beta\rho^2\right]+\left(2-2\rho+\phi_y\right)\left[1-2\beta\rho+(\beta\rho)^2\right] \\
&= -2\beta^2\rho^3+\left[4+\kappa_y+\beta(2+\phi_y)\right]\beta\rho^2-2\left[1+\kappa_y+\beta(2+\phi_y)\right]\rho+\kappa_y\left[\phi_\pi(1-\beta)+1\right]+2+\phi_y\,.
\end{aligned}$$

As a result, we have

$$\frac{\partial C_y(1)}{\partial \rho} = \frac{-2\beta^2\rho^3+\left[4+\kappa_y+\beta(2+\phi_y)\right]\beta\rho^2-2\left[1+\kappa_y+\beta(2+\phi_y)\right]\rho+\kappa_y\left[\phi_\pi(1-\beta)+1\right]+2+\phi_y}{\left[(1-\rho)\,(1+\phi_y-\rho)\,(1-\rho\beta)+\kappa_y\,(\phi_\pi-\rho)\,(1-\rho)\right]^2}\,.$$

The sign of the previous expression is determined by the numerator, which constitutes a third order polynomial in ρ. Thus, it can have at most three roots and change its sign thrice on $\rho \in [0,1)$. To begin with, let us define the numerator by $C_y^n(1) \equiv \psi_a \rho^3 + \psi_b \rho^2 + \psi_c \rho + \psi_d$, where

$$
\begin{aligned}
\psi_a &= -2\beta^2 \, , \\
\psi_b &= \beta \left[4 + \kappa_y + \beta(2+\phi_y)\right] \, , \\
\psi_c &= -2\left[1 + \kappa_y + \beta(2+\phi_y)\right] \, , \\
\psi_d &= \kappa_y \left[\phi_\pi(1-\beta) + 1\right] + 2 + \phi_y \, .
\end{aligned}
$$

It follows that $\lim\limits_{\rho \to -\infty} C_y^n(1) > 0$ and $\lim\limits_{\rho \to \infty} C_y^n(1) < 0$. In particular, we have

$$
\begin{aligned}
\lim_{\rho \to 0} C_y^n(1) &= \kappa_y \left[\phi_\pi(1-\beta) + 1\right] + 2 + \phi_y > 0 \, , \\
\lim_{\rho \to 1} C_y^n(1) &= (1-\beta)\left[(1-\beta)\phi_y + \kappa_y(\phi_\pi - 1)\right] > 0 \, ,
\end{aligned}
$$

where the sign of the last term is determined by the Taylor principle. To complete the proof, we proceed by case distinction. First, consider the case where $C_y^n(1)$ has a unique triple root and, thus, flips sign only once. This implies that $C_y^n(1) > 0$ on $\rho \in [0,1)$. Second, consider the case where $C_y^n(1)$ has two distinct roots, one of them double. Again, this implies that it flips sign only once and, thus, $C_y^n(1) > 0$ on $\rho \in [0,1)$. Last, consider the case where $C_y^n(1)$ has three distinct roots. We show that both local extrema are located on $(1,\infty)$ such that $C_y^n(1)$ is strictly decreasing but positive on $\rho_\xi \in [0,1)$. They are characterized by

$$
3\psi_a \rho^2 + 2\psi_b \rho + \psi_c = 0 \, ,
$$

with corresponding roots

$$
\hat{\rho}^{+,-} = \frac{-2\psi_b \pm \sqrt{4\psi_b^2 - 12\psi_a\psi_c}}{6\psi_a} = \frac{-\psi_b \pm \sqrt{\psi_b^2 - 3\psi_a\psi_c}}{3\psi_a} \, .
$$

To begin with, we show that both roots are real, i.e.,

$$
\begin{aligned}
\psi_b^2 - 3\psi_a\psi_c &= \beta^2\left[4 + \kappa_y + \beta(2+\phi_y)\right]^2 - 12\beta^2\left[1 + \kappa_y + \beta(2+\phi_y)\right] \\
&= \beta^2 \left\{ \left[4 + \kappa_y + \beta(2+\phi_y)\right]^2 - 12\left[1 + \kappa_y + \beta(2+\phi_y)\right] \right\} \\
&= \beta^2 \left\{ 16 + 8\left[\kappa_y + \beta(2+\phi_y)\right] + \left[\kappa_y + \beta(2+\phi_y)\right]^2 - 12\left[1 + \kappa_y + \beta(2+\phi_y)\right] \right\} \\
&= \beta^2 \left\{ 4 - 4\left[\kappa_y + \beta(2+\phi_y)\right] + \left[\kappa_y + \beta(2+\phi_y)\right]^2 \right\} \\
&= \beta^2 \left\{ 2 - \left[\kappa_y + \beta(2+\phi_y)\right] \right\}^2 > 0 \, .
\end{aligned}
$$

As $\psi_a < 0$, the smaller root is given by

$$\hat{\rho}^- = \frac{-\psi_b + \sqrt{\psi_b^2 - 3\psi_a\psi_c}}{3\psi_a} .$$

We then show that the latter is strictly larger than unity, i.e., $\hat{\rho}^- > 1$, which is equivalent to

$$3\psi_a + \psi_b > \sqrt{\psi_b^2 - 3\psi_a\psi_c} .$$

Notice that $3\psi_a + \psi_b = \beta\left[\kappa_y + \beta\phi_y + 4(1-\beta)\right] > 0$ such that the previous inequality can be rewritten as

$$(3\psi_a + \psi_b)^2 > \psi_b^2 - 3\psi_a\psi_c \quad \Leftrightarrow \quad 3\psi_a^2 + 2\psi_a\psi_b + \psi_a\psi_c > 0 \quad \Leftrightarrow \quad \psi_a\left(3\psi_a + 2\psi_b + \psi_c\right) > 0 .$$

As $\psi_a < 0$, the terminal inequality holds true because

$$\begin{aligned} 3\psi_a + 2\psi_b + \psi_c &= -6\beta^2 + 2\beta\left[4 + \kappa_y + \beta(2+\phi_y)\right] - 2\left[1 + \kappa_y + \beta(2+\phi_y)\right] \\ &= 2\beta\left[4 + \kappa_y + \beta(\phi_y - 1)\right] - 2\left[1 + \kappa_y + \beta(2+\phi_y)\right] \\ &= 8\beta + 2\beta\kappa_y + 2\beta^2(\phi_y - 1) - 2 - 2\kappa_y - 4\beta - 2\beta\phi_y \\ &= 4\beta - 2\kappa_y(1-\beta) - 2\beta\phi_y(1-\beta) - 2 - 2\beta^2 \\ &= -2(1-\beta)\left[\kappa_y + \beta\phi_y + 1 - \beta\right] < 0 , \end{aligned}$$

which concludes the proof. □

A.6 Numerical Example

We conclude this section with a brief numerical example based on the general analytical model. Specifically, we determine the respective monetary policy regime thresholds as well as the output-maximizing persistence ρ-max of Proposition 13. Three key take-away results stand out: first, the hump-shaped impact output response occurs for the bulk of reasonable calibrations; second, ρ-max lies in a plausible range for $0.65 - 0.75$ for most models; third, only models that act on the forward-looking NKPC channel (i.e., β_f) are close to have a monotonic impact output multiplier response in the persistence of the DIS demand shock. We gather our results in Table A.1.

Specifically, we choose the following parameter values. We specify throughout all model variants $\kappa_y = 0.15$. Moreover, the elasticity of current aggregate demand with respect to future income takes two values, i.e., $\zeta_f \in \{0.85, 1.00\}$, where the former corresponds to a discounted DIS equation (as obtained under a WUNK or a behavioral NK model) and the latter to a RANK economy. Additionally, we specify the real interest rate elasticity as $\zeta_r \in \{1.00, 1.50\}$, where the former realization reflects, for instance, logarithmic preferences and the latter additional amplification from a T(H)ANK model. We set the sensitivity of current inflation with respect to future inflation to $\beta_f \in \{0.99, 0.80\}$. Notice that the first value is the standard discount factor, whereas the latter is the effective

TABLE A.1. Determinacy, monetary policy regimes, and ρ-max.

MODEL	PARAMETERS					DETERMINACY	REGIME	PERSISTENCE
	ζ_f	ζ_r	β_f	κ_r	κ_y	$\phi_\pi \in \left(\underline{\phi}^d, \overline{\phi}^d\right)$	$\phi_\pi \in \left(\underline{\phi}_\pi^g, \overline{\phi}_\pi^g\right)$	ρ^{g*}
A. RANK Models								
RANK	1.00	1.00	0.99	0.00	0.15	$(1.00, \infty)$	$(1.01, 7.74)$	0.74
WUNK	0.85	1.00	0.99	0.00	0.15	$(0.99, \infty)$	$(1.01, 6.73)$	0.71
Behavioral RANK 1	1.00	1.00	0.80	0.00	0.15	$(1.00, \infty)$	$(1.58, 9.58)$	0.99
Behavioral RANK 2	0.85	1.00	0.80	0.00	0.15	$(0.80, \infty)$	$(1.53, 8.33)$	0.99
B. HANK Models								
TANK	1.00	1.50	0.99	0.00	0.15	$(1.00, \infty)$	$(1.01, 5.50)$	0.68
WU-TANK	0.85	1.50	0.99	0.00	0.15	$(0.99, \infty)$	$(1.01, 4.83)$	0.65
Behavioral TANK 1	1.00	1.50	0.80	0.00	0.15	$(1.00, \infty)$	$(1.47, 6.81)$	0.98
Behavioral TANK 2	0.85	1.50	0.80	0.00	0.15	$(0.87, \infty)$	$(1.44, 5.97)$	0.96
C. RANK Cost Models								
RANK	1.00	1.00	0.99	0.05	0.15	$(1.00, \infty)$	$(1.01, 7.38)$	0.73
WUNK	0.85	1.00	0.99	0.05	0.15	$(0.99, \infty)$	$(1.01, 6.71)$	0.71
Behavioral RANK 1	1.00	1.00	0.80	0.05	0.15	$(1.00, \infty)$	$(1.54, 8.46)$	0.99
Behavioral RANK 2	0.85	1.00	0.80	0.05	0.15	$(0.79, \infty)$	$(1.51, 7.76)$	0.99
D. HANK Cost Models								
TANK	1.00	1.50	0.99	0.05	0.15	$(1.00, \infty)$	$(1.01, 5.26)$	0.67
WU-TANK	0.85	1.50	0.99	0.05	0.15	$(0.99, \infty)$	$(1.01, 4.75)$	0.65
Behavioral TANK 1	1.00	1.50	0.80	0.05	0.15	$(1.00, \infty)$	$(1.45, 6.18)$	0.97
Behavioral TANK 2	0.85	1.50	0.80	0.05	0.15	$(0.86, \infty)$	$(1.42, 5.61)$	0.95

discount factor under cognitive discounting. In the presence of a marginal cost channel of monetary policy we set, consistent with Assumption 2, $\kappa_r = 0.05$. As a result, the condition $\zeta_r \kappa_y \geq (1+\zeta_f)\kappa_r$ is satisfied throughout such that determinacy is guaranteed if the inflation feedback in the Taylor rule is sufficiently aggressive, i.e., $\phi_\pi > \underline{\phi}_\pi^d$. Finally, we specify monetary policy in terms of a contemporaneous interest rate rule with $\phi_\pi = 1.50$ and $\phi_y = 0.00$.

To begin with, consider the RANK model as our benchmark economy. Under this calibration, the model locates in the hump-shaped regime of Proposition 13 as $\underline{\phi}_\pi^g < \phi_\pi < \overline{\phi}_\pi^g$, where $\underline{\phi}_\pi = 1.01$ and $\overline{\phi}_\pi = 7.74$. The output-maximizing persistence is given by $\rho^{g*} = 0.74 > 0.50$, which corresponds to the baseline value used by Galí (2015) for monetary shocks. As such, the persistence can be further increased to maximize the impact output stimulus. When considering deviations from the RANK benchmark, it can be seen that a discounted DIS equation, i.e., $\zeta_f = 0.85$, decreases $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*} only sightly. In contrast, $\beta_f = 0.80 < 0.99$ increases $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*} substantially such that the impact output multiplier becomes monotonously increasing in the persistence of the DIS demand shock. These insights carry over to a class of heterogeneous agent models that admit a higher real interest rate sensitivity, i.e., $\zeta_r = 1.50$. In this model class $\underline{\phi}_\pi^g$, $\overline{\phi}_\pi^g$, and ρ^{g*} decrease slightly relative to their RANK counterparts. Note that similar results would be obtained in the class of models that jointly increase ζ_r, ζ_f, e.g., HANK models with idiosyncratic counter-cyclical income risk, whenever the rise in ζ_r outweighs the one in ζ_f. Finally, adding a marginal cost channel of monetary policy only slightly reduces $\underline{\phi}^g_\pi$, $\overline{\phi}^g_\pi$, and ρ^{g*}.

APPENDIX B: Quantitative Appendix

B.1 Local Determinacy

Our model features a locally determinate equilibrium under the benchmark calibration. Recall that the systematic monetary policy parameters (ϕ_π, ϕ_y) have a dual role in our analysis. On the one hand, they ensure a determinate equilibrium. On the other hand, they are crucial in determining the monetary policy regimes. Out of this reason, we show the determinacy region for the benchmark economy and various model variants in Figure B.1. Most of the quantitative findings are guided by our analytical determinacy discussion from Proposition 11.[31]

To begin with, let us consider the determinacy properties of the benchmark economy in Sub-Figure B.1a. Due to behavioral expectation formation in form of cognitive discounting, the benchmark model features a discounted bond DIS equation such that its dynamics are locally determinate even under an interest rate peg, i.e., $\phi_\pi = \phi_y = 0$ (see Gabaix (2020)). Importantly, the model is indeterminate if ϕ_π is sufficiently high and, simultaneously, ϕ_y too low in the presence of a cost channel of monetary policy (see Beaudry et al. (2024)). This interaction is well-illustrated in Sub-Figure B.1b, i.e., the model is locally determinate in the absence of a cost channel of monetary policy ($\psi = 0.00$) on the entire parameter space.

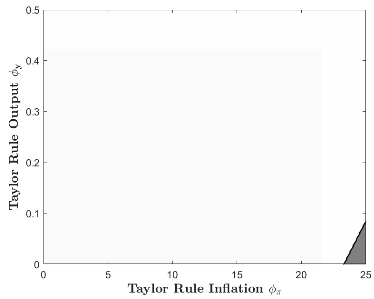
Moreover, in Sub-Figure B.1c we show that the departure from rational expectations crucially alters the (in)determinacy regions. Under rational expectations (i.e., $\mu_b = \mu_p = \mu_w = 1.0$) idiosyncratic income risk leads to a compounded bond DIS equation and hence requires a more aggressive Taylor feedback to inflation for determinacy (Acharya and Dogra, 2020, Bilbiie et al., 2022, Bilbiie, Forthcoming), i.e., ϕ_π needs to be strictly positive. What is more, under rational expectations the indeterminacy from the cost channel of monetary policy arises under much lower values of the inflation feedback, i.e., $\phi_\pi \approx 4$, provided that the output feedback ϕ_y takes a value near zero.

In Sub-Figure B.1c we plot the determinacy region under a larger elasticity of intertemporal substitution ($\sigma = 0.33$). Two observations stand out: on the one hand, the indeterminacy region due to the cost channel of monetary policy disappears (or occurs at a much higher value for ϕ_π); on the other hand, the behavioral expectation formation is not sufficient anymore to ensure determinacy under an interest rate peg.

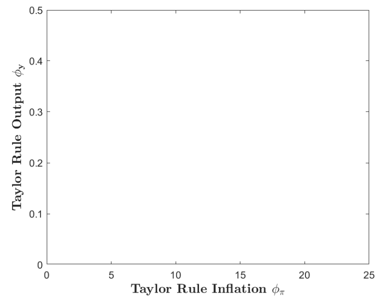
Finally, we study the impact of two modelling choices related to price and wage setting mechanisms. First, we simultaneously shut off partial price and wage indexation ($\chi_p = \chi_w = 0$). As such, the determinacy region becomes much smaller. This is the case

[31] As the regimes in the analytical section were only determined by the Taylor feedback with respect to inflation, we choose a much larger grid for ϕ_π as for ϕ_y in Figure B.1.

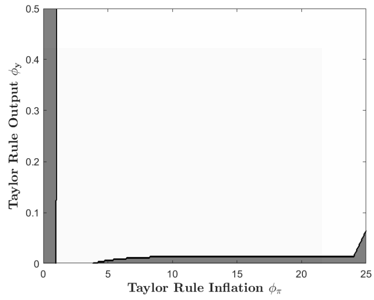
FIGURE B.1. Regions of local determinacy and indeterminacy for several model variants.



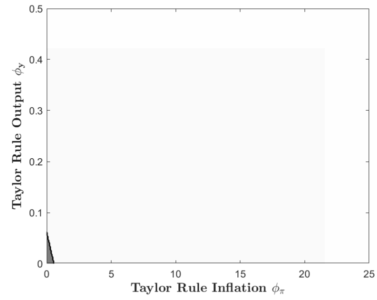
(a) baseline calibration: Table 1



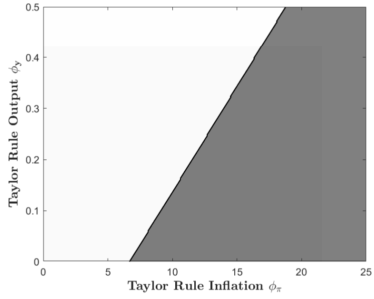
(b) no cost channel: $\psi = 0.00$



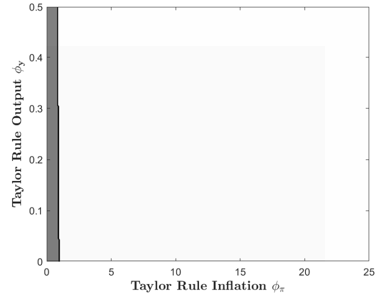
(c) rational expectations: $\mu_b = \mu_p = \mu_w = 1.00$



(d) higher EIS: $\sigma = 0.33$



(e) no price & wage indexation: $\chi_p = \chi_w = 0.00$



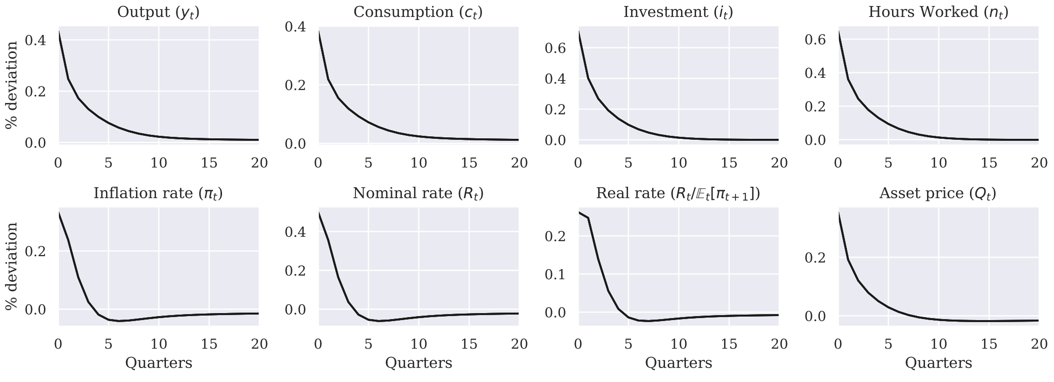
(f) flexible wages: $\theta_w = \chi_w = 0.00$

Note: The grey shaded area denotes the model implied indeterminacy region, whereas the white region denotes the determinacy region. Each model variant is obtained by changing only the parameter value denoted inside the sub-caption, while keeping the baseline values of Table 1 otherwise unchanged.

as the contemporaneous and forward-looking drivers of inflation gain in weight in the NKPC, which increases the nominal interest rate response and, hence, the active forces of the cost channel. Second, we study a model variant with flexible wages ($\theta_w = \chi_w = 0$). In this case, behavioral expectations are no longer sufficient to restore determinacy under an interest rate peg. The reasoning is as follows: the real wage response and the aggregate demand complementarity between saver and hand-to-mouth households are much more pronounced under flexible wages such that the monetary authority must fight inflation in a sufficiently aggressive manner (see Colciago (2011)). As a byproduct, indeterminacy from the cost channel disappears as well (or occurs at a much higher value for ϕ_π).

B.2 Impulse Response Functions

FIGURE B.2. Impulse Response Functions of DIS Demand Shocks



Legend: The IRFs are denoted in terms of percentage deviation from steady state. IRFs of the nominal interest rate, inflation, and the real interest rate are expressed in annual terms.

B.3 Monetary Regimes

FIGURE B.3. Monetary Policy Regimes Bond Premium and Monetary Policy Shocks