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Dispersed Information, Nominal Rigidities and Monetary Business Cycles: A Hayekian Perspective *

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Abstract

We study the propagation of nominal shocks in a dispersed information economy where firms learn from and respond to information generated by their activities in product and factor markets. We prove the existence of a "Hayekian benchmark", defined by conditions under which imperfect information has *no* effect on equilibrium outcomes. This occurs under fairly general conditions when prices are flexible, i.e. without nominal frictions, informational frictions are irrelevant. With sticky prices, however, this irrelevance obtains only if there are no strategic complementarities in pricing and aggregate and idiosyncratic shocks are equally persistent. With complementarities and/or differences in persistence, the interaction of nominal and informational frictions slows down price adjustment, amplifying real effects from nominal shocks (relative to a full information model with only nominal frictions). In a calibrated model, the amplification is most pronounced over the medium to long term. In the short run, market generated information leads to substantial aggregate price adjustment, even though firms may be completely unaware of changes in aggregate conditions.

JEL Classifications: D80, E31, E32, E40

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1 Introduction

Models of the monetary transmission mechanism attribute an important role to adjustment frictions for prices and wages in order to account for delays and inefficiencies in how markets respond to macroeconomic shocks. These frictions come either in the form of limits to the available information, or in the form of nominal rigidities, i.e. sticky prices or menu costs of price adjustment. Rich literatures have developed around each of these themes.[1]

In this paper, we offer a novel perspective on the propagation of nominal shocks, which focuses on the interaction between these two sources of frictions, but also takes seriously the informational role of markets and prices in coordinating economic outcomes. We consider a standard model of production and price adjustment by monopolistically competitive firms who use their own activities in product and factor markets to learn about firm-specific and aggregate shocks to demand and input costs. Since these market signals reflect the combined effects of these shocks, they naturally induce incomplete and dispersed information about aggregate conditions. Nevertheless, we argue that conditioning on market information leads to equilibrium outcomes that closely resemble, and that in special cases are exactly equivalent to the equilibrium outcomes that would arise if all firms could perfectly disentangle firm-specific and aggregate shocks. In other words, aggregate shocks are absorbed into prices and production decisions to a remarkably large degree long before the firms even become aware that these shocks have occurred.

We first consider the case where firm decisions are completely flexible and respond to market information without delay. We show that under these conditions, the equilibrium with incomplete information and conditioning on market information perfectly replicates the full information equilibrium with flexible prices. In this equilibrium, firms are able to perfectly infer their own marginal costs and revenues –and hence determine their optimal pricing or production decisions– from the information provided by markets, and therefore act *as if* they had perfect information.

[1] Both strands are too large to fully survey here, so we restrict ourselves to a few representative examples. Informational frictions as a source of monetary non-neutrality dates back at least to the seminal work of Phelps (1970) and Lucas (1972). Recent contributions include Woodford (2003), Mankiw and Reis (2002), Angeletos and La'O (2009; 2020), Angeletos et al. (2016), Angeletos and Huo (2021), La'O and Tahbaz-Salehi (2022), Mackowiak and Wiederholt (2009; 2010), Coibion and Gorodnichenko (2012) and Alvarez et al. (2011). Nominal rigidities have also been studied extensively. Notable contributions on their quantitative significance include Ball and Romer (1990), Rotemberg and Woodford (1997), Christiano et al. (2005), Golosov and Lucas (2007), Nakamura and Steinsson (2008), and Eichenbaum et al. (2011).

We term this equivalence result the 'Hayekian benchmark', because it is reminiscent of the idea first developed by Hayek in his influential (1945) essay: *"(in) a system where knowledge of the relevant facts is dispersed, prices can act to coordinate... The most significant fact about the system is the economy of knowledge with which it operates, how little the individual participants need to know in order to be able to take the right action."* Hayek (1945) thus emphasizes the parsimony of knowledge with which the price system guides individual participants to take decisions that are not only in their own best interest, but ultimately lead to a socially efficient allocation of resources, despite the lack of central organization and communication of all information to market participants.[2,3]

We then introduce nominal rigidities into the firms' pricing problem through different forms of sticky prices, menu costs or other lags in price adjustment that render the firm's optimal pricing decisions forward-looking. Our second result provides conditions for a dynamic Hayekian benchmark: with nominal rigidities and conditioning on market information, incomplete information remains irrelevant –i.e. equilibrium price adjustment with incomplete information is the same as under full information– if and only if two additional conditions are satisfied: (i) there are no pricing complementarities, i.e. each firm's optimal pricing decision is independent of prices set by the other firms, and (ii) firm-specific and aggregate shocks have the same degree of persistence. With nominal rigidities, firms' optimal pricing decisions depend on current and expected future marginal costs. The firms' current market conditions allow them to perfectly infer their current marginal costs. The dynamic Hayekian benchmark result obtains if and only if the current marginal cost offers the best possible forecast of future marginal costs, or equivalently, marginal costs follow a first-order Markov process. This Markov property however arises only if there are no pricing complementarities and firm-specific and aggregate shocks have equal persistence. In line with Hayek (1945), firms then do not need to know whether their marginal costs are driven by firm-specific shocks, aggregate shocks,

[2]Hayek (1945) referred to a 'price system' without being explicit about the underlying market structure. His insight is implicitly invoked in models where the information structure is left unspecified, but to our knowledge it has not been articulated in a formal model with well-defined market and information structures. We formalize Hayek's argument for monopolistically competitive economies by applying the notion of "supply function equilibrium" in a macroeconomic context, following Flynn et al. (2024).

[3]This is rather different from Grossman and Stiglitz (1980) who emphasize the role of prices in aggregating and publicizing information. Our arguments do not rely on market prices being fully revealing or even public signals. In fact, firms' signals could be very poor indicators of the true nature of shocks hitting the economy, but, as long as they provide an accurate indication of the firm's individually optimal decisions, the Hayekian benchmark obtains.

or the other firms' pricing decisions, in order to form expectations about future marginal costs and set prices optimally.

These Hayekian benchmark results provide a point of departure for analyzing the channels through which incomplete information affects firm decisions and aggregate dynamics by identifying conditions under which it has no bearing on equilibrium outcomes.[4] In particular, incomplete information only matters to the extent that firms remain uncertain about their own optimal pricing or production plans. With flexible prices or real time adjustment, market conditions completely remove this uncertainty, restoring the full information equilibrium. With nominal rigidities, conditions for the Hayekian benchmark are far more stringent, but they also depend on parameters that can in principle be estimated: the extent of price rigidities, the persistence of firm-specific and aggregate demand and cost shocks, and the strength of strategic complementarities in pricing decisions. The Hayekian benchmark therefore also provides guidance on the parameters that govern how much incomplete information dampens or reinforces nominal rigidities.

Our third set of results characterizes departures from the Hayekian benchmark in two canonical models of nominal rigidities, Calvo pricing and menu costs. For tractability and expositional clarity, we focus on the limiting case when aggregate shocks are small in comparison to idiosyncratic shocks. In this limit, agents cease to update their beliefs about aggregate conditions since they attribute all fluctuations in their market signals to idiosyncratic shocks.[5] This limit thus provides a lower bound on the adjustment of prices to aggregate nominal shocks under more realistic informational assumptions.

We show that aggregate price adjustment is slowed down and real effects of monetary shocks are amplified relative to the full information case whenever idiosyncratic shocks are less persistent and/or pricing decisions are strategic complements. The short-run and long-run adjustment dynamics both depend on the idiosyncratic and aggregate shocks' respective degrees of persistence, along with the degree of strategic complementarity.

[4] In this respect our result is similar to other well-known irrelevance results whose main role is to provide a conceptual point of departure for understanding phenomena whose existence is ruled out by the irrelevance benchmark.

[5] To be clear, the observation that agents cease to learn about aggregate shocks at the small shocks limit follows from Bayes' Rule, and is not an artefact of additional assumption about beliefs. An alternative way to interpret this limit is to consider a stationary environment with idiosyncratic shocks only, and introduce an "MIT shock", i.e. an unexpected aggregate change, but without ever informing agents that this shock has occurred.

The role of differential persistence is straight-forward: When nominal shocks are small, firms attribute most (and, in the small shocks limit, all) of the variation in their sales and wage signals to idiosyncratic factors. In other words, firms systematically confound aggregate for idiosyncratic shocks and adjust their prices in response to the former as if the latter had occurred. When idiosyncratic factors are less persistent than the aggregate shock, forward-looking firms make smaller price adjustments, relative to a situation where the nature of the shock is known. But since the aggregate shock moves all firms' marginal costs and optimal prices in the same direction, the collective price adjustment can still absorb a significant fraction of the aggregate nominal shock, even if firms are not aware of its occurrence. If firm-specific and aggregate shocks are both permanent, then a version of the Hayekian benchmark holds in the long run, i.e. eventually aggregate nominal shocks are fully absorbed into prices.[6]

The role of strategic complementarities is more subtle. On the one hand, strategic complementarities introduce a well-known forward-looking element into pricing decisions, as firms seek to forecast the future path of price adjustment. When firms learn that aggregate conditions have changed, they still refrain from fully adjusting to these news because they expect that other firms will not fully adjust either - and hence dampen their own responses.[7] This channel is already present with full information, and strongest in that case because firms learn immediately about the change in aggregate conditions. On the other hand, with incomplete information and learning from market conditions, complementarities also slow down the rate at which aggregate prices and shocks enter the firm level market conditions and their marginal costs, thus amplifying the sluggishness in firm-level and aggregate price adjustment. This informational role for complementarities is novel and quite different from the well-known strategic channel emphasized both in the full information sticky price models as well as in the dispersed information models cited in footnote 1. In our model, and especially when firm-specific shocks are an order of magnitude larger than aggregate ones, the strategic channel is practically absent because firms attach almost zero probability to a change in

[6]These arguments extend the "setting the right prices for the wrong reasons" channel of Hellwig and Venkateswaran (2009) to an economy with nominal rigidities.

[7]With full information, the complementarity propagates lack of adjustment from firms that have their prices fixed to firms who have the ability to adjust, while in the dispersed information models, the complementarity amplifies an initial lack of adjustment to available information because firms are not exactly certain what other firms have observed.

aggregate conditions. Instead, the informational channel is solely responsible for slowing down the pass-through from aggregate shocks to firm-level marginal costs and price adjustments.

Finally, we calibrate models with nominal rigidities (one with Calvo pricing and the other with menu costs) to match moments of price adjustment at the micro level and ask how much, quantitatively, the incomplete information channel contributes to monetary non-neutrality. In both specifications, the Hayekian mechanism remains quite powerful with a significant proportion of aggregate shocks absorbed into prices even though firms never learn the true nature of the aggregate shock. At short horizons, i.e. over the first 2-3 quarters upon impact, the response of prices to aggregate nominal shocks is similar to a full information information economy that is subject to the same nominal rigidity, suggesting that the contribution of incomplete information to short run monetary non-neutrality is quite modest. Incomplete information has larger effects on aggregate price adjustment at medium to long horizons, but this is under the rather stark assumption that firms do not learn about aggregate monetary conditions through other channels.[8]

Our work bears a direct relation to a large and growing work using models with heterogeneous information to study business cycles.[9] Much of this literature tends to model information as abstract, noisy signals of the exogenous shocks.[10] Therefore, market prices and allocations do not have any part to play in the aggregation and transmission of information, ruling out the Hayekian mechanism by assumption. Our results, especially the static benchmark, highlight the central role of this assumption in generating meaningful effects from informational frictions in those papers. The presence of market-generated information makes it much harder to make information frictions relevant - in the price-setting context, we need both additional (i.e. nominal) frictions as well as assumptions about the structure of preferences/production and shocks hitting the economy.[11]

[8] As robustness checks, we also consider a variation of our model with fat-tailed idiosyncratic shocks as in Midrigan (2011). This dampens the so-called selection effect, reducing the impact of short run price adjustment and hence reinforcing the informational role of pricing complementarities. We also consider a variation where aggregate shocks are serially correlated and show how in this case the informational channel of complementarities becomes more powerful in delaying short run price adjustment.

[9] Besides the works cited in footnote 1, Hellwig (2005), Angeletos and La'O (2010), Amador and Weill (2010), Lorenzoni (2009), Hellwig and Veldkamp (2009), Nimark (2008), Paciello and Wiederholt (2014), Graham and Wright (2010), Melosi (2014), Angeletos and La'o (2013), Atolia and Chahrour (2020), Angeletos and Lian (2018) and Chahrour and Ulbricht (2023). have also contributed to this literature.

[10] Important exceptions are Amador and Weill (2010), Hellwig and Venkateswaran (2009), Graham and Wright (2010), Atolia and Chahrour (2020), Benhabib et al. (2015) and Chahrour and Gaballo (2021). Mackowiak and Wiederholt (2009) also consider endogenous signals in an extension to their baseline model.

[11] Our results also have implications for another important branch of the dispersed information literature - one that

Our analysis builds on the canonical New Keynesian framework widely used to study the dynamics of price adjustment. Our work complements this literature by analyzing the interaction of informational frictions with various assumptions about nominal rigidity, including both time-dependent, as in Taylor (1980) or Calvo (1983), and state-dependent models, as in Caplin and Leahy (1991) and Golosov and Lucas (2007). Gorodnichenko (2010) analyzes a similar model with incomplete information and nominal frictions, but focuses on externalities affecting nominal adjustment through information acquisition. In his paper, the prospect of learning from market prices reduces firms' incentives to acquire costly information. At our Hayekian benchmark, this trade-off can be particularly extreme - markets provide firms with all the information they need, so additional information is worthless and will not be acquired at a positive cost in equilibrium. Alvarez et al. (2011) also study the interaction of nominal and informational frictions, but they introduce additional, exogenous observation costs, limiting the Hayekian channel at the heart of this paper.

Finally, our calibration draws on recent work documenting price adjustment at the micro level using large scale data sets of individual price quotes. The moments we target - the cross-sectional dispersion and time series properties of prices as well as the frequency and magnitude of price changes - are taken from the work of Bils and Klenow (2004), Nakamura and Steinsson (2008), Klenow and Kryvstov (2008), Burstein and Hellwig (2007) and Midrigan (2011).

The rest of the paper proceeds as follows. Section 2 introduces the model and establishes the Hayekian benchmark results, both with and without nominal adjustment frictions. Section 3 analyzes departures from the Hayekian benchmark in the case with Calvo pricing. Section 4 provide quantitative results for a calibrated version of our economy with Calvo pricing or menu costs, along with robustness checks and extensions. Section 5 concludes.

2 Model

In this section, we lay out a dynamic stochastic general equilibrium model, where firms' pricing and production decisions are constrained by both information and nominal frictions. In order to keep

studies the welfare effects of additional information. In Morris and Shin (2002), Hellwig (2005) and Angeletos and Pavan (2007), additional information can reduce social welfare, due to misalignment of social and private incentives for coordination. Our analysis suggests that the applicability of these insights to market economies crucially depends on departures from the conditions characterizing the Hayekian benchmark.

the focus squarely on the impact of these two frictions on firms' pricing and production decisions, we deliberately keep the household side of the economy as close to the New Keynesian benchmark as possible. In particular, we assume that there is a representative household with perfect information about aggregate and market-specific shocks as well as access to complete contingent claims markets.[12]

Production: The economy has a single final good, which is produced by a *fully informed*, perfectly competitive firm using a continuum of intermediate goods:

$$Y_t = \left(\int B_{it}^{\frac{1}{\theta}} \, Y_{it}^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}} \quad ,$$

where B_{it} is an idiosyncratic demand shock for intermediate good i and $\theta > 1$ denotes the elasticity of substitution. Optimization by the final goods producer implies the usual demand function

$$Y_{it} = B_{it} Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} \quad , \tag{1}$$

where the aggregate price level P_t is given by

$$P_t = \left(\int B_{it} P_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad . \tag{2}$$

Each intermediate good i is produced by a monopolistic firm using labor of type i as the sole input, according to a decreasing returns to scale production function:

$$Y_{it} = (\delta N_{it})^{\frac{1}{\delta}} \qquad\qquad \delta > 1. \tag{3}$$

Each type of labor is traded in a competitive factor market, with a market-clearing wage W_{it}. Intermediate firms' expected nominal profits, discounted at state prices are then given by

$$\mathbb{E}_{it} \left[\lambda_t \left(P_{it} Y_{it} - W_{it} N_{it} \right) \right] \quad , \tag{4}$$

where λ_t is the stochastic discount factor of the household, $\mathbb{E}_{it}$ is the expectation conditional on i's information at time t (to be specified just below), and Y_{it}, P_{it}, and N_{it} must jointly satisfy equations (1) and (3).

Information and Equilibrium: The information set of firm i at the time it makes its period t decision is denoted $\mathcal{I}_{it}$. For most of our analysis, we restrict attention to two polar cases. Under

[12] We defer a complete description of preferences till later in this section.

dispersed information, firms only observe (histories of) signals generated by their market activities - in particular, their price P_{it}, their sales Y_{it} and wages W_{it}. Under *full information*, all firms observe (histories of) all shocks, both aggregate and idiosyncratic. Obviously, these are extreme assumptions but they will permit a stark demonstration of the effect of informational assumptions on equilibrium outcomes.[13]

Beliefs about fundamentals (i.e. the underlying shocks) and aggregate outcomes will typically be very different in the two economies, except in the special case where the market signals allow the firm to infer the underlying shocks exactly. However, we are not interested in beliefs *per se* but in outcomes, namely prices and quantities. We will say that dispersed information is *relevant* if prices and quantities are different under dispersed information compared to an otherwise identical economy with full information. If this is not the case, i.e. prices and quantities in the two economies coincide, then we will say that dispersed information is *irrelevant.*

Models with learning from market conditions encounter a circularity problem: the firm conditions its production or pricing decisions on information about market conditions (i.e. equilibrium price or demand) that are themselves a function of the firms' pricing or production decisions. To over-come the circularity in a model-consistent way, we appeal to the concept of *Supply Function Equilibrium*. In a supply function equilibrium, firms commit to a supply function $Y_{it} = y(\cdot)$, i.e. a quantity schedule as a function of its output price P_{it} and wage (or input price) W_{it}. When this function is invertible, it can be equivalently interpreted as a "price function equilibrium", in which firms announce the prices at which they are willing to supply any given quantity of their intermediate good demanded by final goods producers.[14] The supply function choice nests pure quantity choice (Y_{it} is independent of price or demand conditions, or fully inelastic in the short-run) and pure price-setting (Y_{it} is infinitely elastic at some pre-determined price P_{it}) as special cases but generalizes the range of possible prices to any intermediate degree of price responses to wage or demand conditions. The firms' endogenous response to wages and demand conditions then emerges

[13] In addition, following the heterogeneous information literature, we assume throughout that the economic structure, i.e. all structural parameters and stochastic properties of shocks, are common knowledge among firms.

[14] Flynn et al. (2024) allow firms to choose any implicit function $f(\cdot)$ of prices and quantities s.t. $f(P_{it}, Q_{it}) = 0$. This formulation nests both pure price-setting and pure quantity choice (inelastic supply) as extreme cases. We abstract from this extra layer of generality and restrict attention to invertible supply functions, as the extra generality does not add any additional insights in our context.

as the outcome of their profit maximizing production plan.

The concept of supply function equilibrium is borrowed from the Industrial Organization literature (e.g. Vives, 2011) where it captures the notion that firms adjust their profit-maximizing production or pricing strategies in real time to their market conditions. Flynn et al. (2024) introduce this concept into macroeconomic analysis of aggregate demand and supply, and offer a compelling justification for its use in general equilibrium models.[15] We depart from Flynn et al. (2024) by assuming that supply functions can be conditioned not only on output prices but also on wages (input costs). As we will see below, the firms' ability to flexibly respond to information concerning both their demand and marginal cost conditions is critical in allowing firms to implement the full information outcome in equilibrium despite having only very limited information about the aggregate shocks to the economy.

2.1 Flexible Prices: A benchmark result

We start with an important benchmark, in which firms make production and pricing decisions in every period, i.e. they choose their supply function to maximize static expected profits. We interpret this benchmark as an environment without nominal rigidities or sticky prices (flexible price benchmark), since the only friction possibly affecting the firms' adjustment of prices and output to idiosyncratic and aggregate shocks stems from lack of information.

Under full information, the firms then simply maximize their nominal profits in each state of the world, i.e. they set Y_{it} to maximize $P_{it}Y_{it} - W_{it}N_{it}$ subject to (1) and (3). As is well known, the firm's optimal decisions set P_{it} and Y_{it} to equalize its marginal cost and revenue:

$$\underbrace{\frac{\theta - 1}{\theta} P_{it}}_{\text{Marginal Revenue}} = \underbrace{W_{it} \, Y_{it}^{\delta-1}}_{\text{Marginal Cost}} \tag{5}$$

$$= W_{it} \left(B_{it} C_t P_t^{\theta}\right)^{\delta-1} P_{it}^{-\theta(\delta-1)} \tag{6}$$

We compare this full information outcome with the supply function equilibrium in which firms' choices are conditioned only on $\mathcal{I}_{it} = \{P_{it}, Y_{it}, W_{it}\}$. In this setting, a supply function equilibrium

[15] See Section 2.1 of Flynn (2024) for a discussion of the assumption of supply function choice, and its appeal in specific market settings, which applies equally well to our model. An earlier version of our paper that predates Flynn et al. (2024) by several years established our main results in a pricing context without explicit reference to supply function choice. We decided to adopt their formulation because it offers an internally consistent foundation for the whole modeling approach, along with a sharper characterization of the role of different informational assumptions.

consists of supply functions $Y_{it} = y(\cdot)$ that are measurable w.r.t. the information set $\mathcal{I}_{it}$ and maximize (4) subject to (1) and (3), wages W_{it}, prices P_{it}, final good prices P_t and production choices by the final goods producer $\{Y_{it}, Y_t\}$ that are consistent with the intermediate goods demand (1) and price index (2), and consumption and labor supply choices of the household and a pricing kernel $\{\lambda_t\}$, such that household choices are optimal and markets clear.

Our first result states that, in the absence of nominal frictions, informational frictions do not have *any* effect on equilibrium allocations, as long as firm decisions condition on market information.

Proposition 1 *Suppose firms decide on production and prices every period. Then, in a supply function equilibrium, dispersed information is not relevant, i.e. equilibrium prices and quantities under dispersed information are identical to those under full information.*

To explain the intuition behind this striking result, we proceed in two steps. We first identify conditions on the information structure $\{\mathcal{I}_{it}\}$ under which the full information equilibrium remains sustainable with incomplete information. Specifically, we show that this is the case whenever the information set $\mathcal{I}_{it}$ allows every firm to infer its own full information best response. This is necessary and sufficient for the full information equilibrium strategies to be feasible, and by definition of an equilibrium, the full information strategies then remain mutual best responses.

Second, we show that observing their own wage rate and sales or output price is indeed sufficient to implement full information best responses, even if the actual information in this setting is much coarser. Under dispersed information, the firm is assumed to have access to two contemporaneous signals - its own sales Y_{it} and wage rate W_{it}. From equation (1), the demand signal is informationally equivalent to $B_{it} C_t P_t^{\theta}$. Along with the directly observed wage signal, this gives the firm all the information it needs to accurately forecast its own marginal cost and revenue in equation (5), and therefore infer its best response. By the first step, the full information equilibrium is obtained. Importantly, this conclusion is independent of the nature of the stochastic process for each of the shocks or preferences of the household, which we have so far left unspecified. Note also that this is an equilibrium in dominant strategies and hence extremely robust to perturbations of behavior by other firms.

This result formalizes the insight from Hayek's quotation in the introduction: the information gained through market activities allows firms to perfectly align marginal costs and revenues, thereby coordinating allocations and prices on the full information outcome, even if this information remains very noisy about aggregate conditions. It is a stark demonstration of the *economy of knowledge* of the market system that Hayek emphasized - once individual actors incorporate the information from their market interactions, the full information allocation obtains, and any further information becomes redundant from the perspective of individual profit maximization and equilibrium allocations. We will from now on refer to the conditions under which the dispersed information is irrelevant as the "Hayekian benchmark".

One implication of the Hayekian benchmark for nominal price adjustment is that money remains neutral (i.e. nominal shocks are instantly and fully absorbed into nominal prices) even if firms are unable to observe these shocks in real time. This result stands in stark contrast to the findings of a rather large body of work starting with the seminal contribution of Lucas (1972) that emphasizes the importance of incomplete information for incomplete nominal adjustment. Other examples with learning from market conditions include Amador and Weill (2009) or Flynn et al. (2024). The key distinction between our model and theirs is that they typically incorporate a single signal from market conditions - say demand for their product at a given price (or the price a firm can charge for a given quantity supplied), and this signal must be used to infer both aggregate demand and aggregate supply conditions.[16] The equilibrium elasticity of aggregate supply then factors in whether a given change in market conditions is more likely to be driven by demand or supply conditions, i.e. the relative magnitude of these different types of shocks. In contrast, our analysis shows that when firms operate simultaneously in input and output markets, market conditions in both can be used to separately identify demand and supply or marginal cost conditions, and thereby fully adjust to both types of shocks within the same period.

Another strand of the literature emphasizes the role of complementarity in prices and heterogeneity in information for the propagation of nominal shocks. For example, in Mankiw and Reis (2002), Woodford (2001), Mackowiak and Wiederholt (2008) or Angeletos and Lao (2012), or other

[16]In Flynn et al. (2024), this restriction on market signals is incorporated by the assumption that their supply function cannot be conditioned on other external sources of information, say about input prices - in our case the wage the firm has to pay its workers. In Lucas (1972) or Amador and Weill (2010), there are no explicit factor markets for firms to learn about changes in aggregate supply conditions – each firm is owned and operated by a single worker.

similar models, dispersed information generates substantial delay in price adjustment and consequently important propagation from nominal shocks to real aggregate demand and output. In contrast, such propagation does not occur in our model, even if pricing decisions remain complementary across intermediate firms and information about aggregate conditions is dispersed. The reason is that in all these models dispersed information about aggregate conditions is closely tied to uncertainty about firms' demand and marginal costs. In our setting instead, uncertainty about aggregate conditions is disconnected from uncertainty about firms' marginal costs and revenues, because the latter are perfectly revealed through market signals. In many of the other models in the literature, such uncertainty remains either through abstract specifications of the information structure – signals are modeled as arbitrary combinations of fundamental shocks and observational noise, sometimes endogenized as a choice of the firm, such as in the rational inattention framework. This rules out the Hayekian mechanism by assumption.

Proposition 1 has a number of additional implications. First, at the Hayekian benchmark, any additional information about the aggregate economy, including direct information about the shocks themselves, is irrelevant for the firm's decision. This holds irrespective of the quality or the public-versus-private content of that information. It then follows that the results in Morris and Shin (2002) or Angeletos and Pavan (2007) about the welfare implications of more information do not apply when firms respond to market-generated information in real time: Additional information, whether public or private, is simply irrelevant for equilibrium allocations and therefore, for welfare. Information provision or transparency has non-trivial welfare effects only if there is a meaningful departure from the Hayekian benchmark, through which firms actually have a motive to take additional information into consideration when making pricing and production decisions.

Second, some authors, e.g. Coibion and Gorodnichenko (2012) or Melosi (2012), use data on individual forecasts and forecast dispersion to estimate the impact of information frictions for aggregate dynamics. Our result offers a note of caution on inferences drawn from observed heterogeneity in beliefs about the relevance of informational frictions in an economy. At the Hayekian benchmark, our economy features a great deal of cross-sectional dispersion in beliefs about aggregate conditions, but the extent of this dispersion in beliefs has absolutely no effects on allocations or aggregate dynamics. What's more, any changes in the cross-sectional dispersion of beliefs have no bearing on aggregate outcomes, and are hence uninformative of information frictions when the conditions of the Hayekian benchmark are satisfied.

In summary, to argue that information frictions and belief dispersion affect business cycle propagation, one needs to argue for departures from the Hayekian benchmark that interfere with the firm's ability to perfectly infer its marginal costs from the available market information and set ex post optimal prices. Proposition 1 suggests two possible directions for such departures: firms must either be unable to directly infer their demand and cost conditions from market-generated signals, i.e. in real time their information structure is different from or coarser than $\mathcal{I}_{it} = \{P_{it}, Y_{it}, W_{it}\}$, or unwilling to do so because of internal adjustment frictions (i.e. sticky prices or menu costs). The former route is implicitly taken by Flynn et al. (2024) to generate non-trivial departures of aggregate supply from the flexible price benchmark. We will explore the second alternative –introducing sticky prices or menu costs, which turns the firms' decision problem into a dynamic one with need to determine not just current but also future fundamentals and optimal prices and possible delay in the transmission of market signals– in the next sub-section. Before going there, we explore extensions to discuss the robustness and generality of the Hayekian benchmark for general (non-CES) input demand systems.

Beyond CES demand: So far, we have assumed that input demand admits a CES structure (1), production has a constant elasticity of marginal cost to output (3), and firms take wages as given (infinitely elastic labor supply). A direct implication of these assumptions, which are common in the literature, is that optimal mark-ups are constant, and therefore the firm can track its optimal price perfectly by just tracking its nominal marginal costs. But the firm is able to do so perfectly by tracking its nominal wage W_{it} and its demand signal $B_{it}C_tP_t^{\theta}$. Readers may be concerned that the properties of the Hayekian benchmark are too tightly linked to the rather specific CES structure of the current work.

To address this concern, consider the case where demand is given by a general function $Y_{it} = \mathcal{Y}(P_{it}, B_{it}, P_t, Y_t)$. In this case, the Hayekian benchmark obtains if the set of variables in the firm's supply function includes not only prices P_{it} and wages W_{it} as before, but also the (local) curvature of the demand function, specifically, $\theta_{it} \equiv \frac{P_{it}}{Y_{it}} \frac{\partial \mathcal{Y}}{\partial P_{it}}$. Then, the firm's optimal schedule takes the form

$$P_{it} = \frac{\theta_{it}}{\theta_{it} - 1} \underbrace{W_{it} Y_{it}^{\delta - 1}}_{\text{Marginal Cost}} , \tag{7}$$

which is exactly its the optimal strategy under full information. Thus, under these conditions, the imperfect nature of the firm's information once again turns out to be irrelevant; this is in particular true in the perfect competition limit in which θ_{it} becomes arbitrarily large.

The condition that market signals allow a firm to infer both the level of its demand schedule and its elasticity or local curvature from the available market information may seem very restrictive. It is satisfied whenever each pair $\{P_{it}, Y_{it}\}$ is consistent with a unique firm-level demand schedule against which the firm optimizes - or equivalently, if demand shifters do not allow for different demand curves intersecting at any pair $\{P_{it}, Y_{it}\}$. This is true for example if $\mathcal{Y}$ is weakly separable between P_{it} and (B_{it}, P_t, Y_t), i.e. $Y_{it} = \mathcal{Y}(P_{it}, g(B_{it}, P_t, Y_t))$ for some function $g(B_{it}, P_t, Y_t)$ of firm-specific and aggregate demand conditions. Departures from the full information equilibrium arise only from uncertainty about desired mark-ups, not because of uncertainty about marginal costs.

Beyond Competitive Labor Markets: Similarly, our analysis so far has assumed a competitive market for labor, where the firm takes W_{it} as given. To characterize the Hayekian benchmark when firms have market power in the labor market, consider the problem of a firm facing an upward sloping labor supply function, $W_{it} = \mathcal{W}(N_{it}, Z_{it}, Y_t, P_t, ...)$. This flexible specification allows for the wage to depend on both idiosyncratic shocks (Z_{it}) and aggregate conditions ($Y_t, P_t, ...$). A natural notion of supply function competition in this case applies to both product and labor markets, i.e. the firm sets both production and labor input as functions of prices and wages. One can show that, so long as the labor supply elasticity is known (paralleling the assumption we made on the output demand side) is known, the firm is able to perfectly track its optimal price, i.e. the Hayekian benchmark obtains in this case as well.

Technology Shocks: Lastly, we assumed that the firm faced no uncertainty about its technology, given by equation (3), and thus assumed that variation in marginal costs resulted from variation in input prices (or wages). We can extend our analysis to include technology shocks by assuming that firms produce according to $Y_{it} = A_{it} (\delta N_{it})^{1/\delta}$, where A_{it} may depend on idiosyncratic and aggregate shocks to labor productivity. The Hayekian benchmark result generalizes to economies with stochastic productivity as long as A_{it} is included in firm i's information set $\mathcal{I}_{it}$, i.e. the firm faces no technological uncertainty, and firms pricing decisions respond to technology shocks in real time. These observations contrast with Flynn et al. (2024) who allows for idiosyncratic and aggregate technology shocks, while limiting firms' ability to respond to these shocks in real time.

2.2 Sticky Prices

In this subsection, we introduce sticky prices or nominal adjustment frictions into our model and ask to what extent the Hayekian Benchmark remains relevant when prices are sticky and firm decisions become forward-looking. We first introduce additional structure on household preferences and stochastic processes in order to be able to close the model with sticky prices in general equilibrium.

Preferences: The representative household maximizes its lifetime utility over consumption C_t and real balances M_t/P_t, as well as disutility of effort over a measure 1 continuum of labor types N_{it},

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\psi}}{1-\psi} + \ln \frac{M_t}{P_t} - \int_0^1 Z_{it} N_{it} di \right),$$

where $\beta \in (0,1)$, $\psi > 0$, N_{it} and Z_{it} denote, respectively, labor supply and an idiosyncratic preference shock for labor of type i. $\mathbb{E}_t(\cdot)$ denotes the representative household's expectations as of date t. Assuming that the household has access to a complete contingent claims market, the household's life-time budget constraint is

$$M_0 \geq \mathbb{E}_0 \sum_t \lambda_t \{C_t P_t + i_t M_t - \int_0^1 W_{it} N_{it} di - \Pi_t - T_t\} ,$$

where λ_t denotes the economy's stochastic discount factor used to price nominal balances, Π_t and T_t denote aggregate corporate profits and taxes or transfers (in nominal terms), W_{it} denotes the nominal wage for labor of type i, and the term $i_t M_t$ denotes the household's opportunity costs of holding monetary balances at date t. The first order conditions of this problem are

$$\lambda_t = \beta^t \frac{C_t^{-\psi}}{P_t} = \beta^t \frac{1}{i_t M_t} = \beta^t \frac{Z_{it}}{W_{it}} = (1+i_t)\mathbb{E}_t \lambda_{t+1} . \tag{8}$$

Along with the budget constraint and a law of motion for i_t that is determined by monetary policy, these equations characterize the solution to the household's problem. Throughout this paper, we focus on the special case where $\ln M_t$ follows a random walk with drift μ:

$$\ln M_{t+1} = \ln M_t + \mu + u_t ,$$

where u_t is an iid random variable, distributed $N(0, \sigma_u^2)$. This implies that interest rates are constant at a level $\hat{\imath}$, defined by $(1+\hat{\imath})^{-1} = \beta \mathbb{E}_t \left(M_t / M_{t+1} \right) = \beta exp \left(-\mu + \sigma_u^2/2 \right)$, which we assume to be strictly less than 1. Along with the FOC, we can then write state prices, consumption, and wages as follows:

$$\lambda_t = \beta^t \frac{1}{M_t} \hat{\imath} \ , \tag{9}$$

$$C_t = \left(\frac{M_t}{P_t} \right)^{\frac{1}{\psi}} \ , \tag{10}$$

$$W_{it} = M_t Z_{it}. \tag{11}$$

Besides this characterization of the household's equilibrium behavior through static equations for aggregate consumption and type-specific wages, the constant interest rate also eliminates the effects of nominal interest rates as a public signal of aggregate economic activity.[17]

Idiosyncratic shocks: Next, we describe the stochastic processes for the preference shock Z_{it} and the demand shock B_{it}. We assume that they follow AR(1) processes (in logs) with normally distributed innovations:[18]

$$b_{it} = \rho_b \cdot b_{it-1} + u_{it}^b$$

$$z_{it} = \rho_z \cdot z_{it-1} + u_{it}^z$$

where u_{it}^b, u_{it}^z are mean-zero, normally distributed random variables with variances σ_b^2 and σ_z^2.

Nominal frictions: Next, we introduce nominal frictions/adjustment lags into the firm's price-setting problem. For concreteness, we consider four canonical models of nominal frictions. In each case information sets are the same as in the version without nominal frictions, i.e. we allow decisions to be conditioned on contemporaneous market signals:

$$\mathcal{I}_{it}^{\text{Full}} = \{M_{t-s}, P_{t-s}, B_{it-s}, Z_{it-s}\}_{s=0}^{\infty} \quad \text{and} \quad \mathcal{I}_{it}^{\text{Disp}} = \{Y_{it-s}, W_{it-s}\}_{s=0}^{\infty}.$$

- **Case I: Prices set every period, but N periods in advance:** Here, the firm uses the information available in period t to set its price for period $t+N$ (equivalently, firms set the

[17] This specification closely tracks Hellwig and Venkateswaran (2009). See Hellwig (2005) and Nakamura and Steinsson (2008) for other micro-founded models with similar equilibrium relationships.

[18] The natural logs of capital-lettered variables, are denoted by the corresponding small letters, e.g. for any variable X, we write $x = \ln X$.

price for period t based on information from period $t-N$, i.e. prices are adjusted every period, but information is observed with an N period lag). The firm's problem is

$$\max_{P_{it+N}} \quad \mathbb{E}_{it}\left[\lambda_{t+N}\left(P_{it+N}Y_{it+N}-W_{it+N}N_{it+N}\right)\right]. \tag{12}$$

- **Case II: Prices set once every N periods:** In every reset period t, the firm solves

$$\max_{P_{it}} \quad \mathbb{E}_{it} \quad \sum_{s=0}^{N-1}\left[\lambda_{t+s}\left(P_{it}Y_{it+s}-W_{it+s}N_{it+s}\right)\right]. \tag{13}$$

- **Case III: Prices set as in Calvo (1983):** Every period, with probability ξ, the firm can change its price. This probability is independent over time and across firms. Thus, the probability that the firm's price remains unchanged for exactly T periods is given by $(1-\xi)^{T-1}\xi$. In every reset period, the firm solves

$$\max_{P_{it}} \quad \mathbb{E}_{it} \quad \sum_{T=1}^{\infty}\ (1-\xi)^{T-1}\xi\ \sum_{s=0}^{T-1}\ \left[\lambda_{t+s}\left(P_{it}Y_{it+s}-W_{it+s}N_{it+s}\right)\right]\ . \tag{14}$$

- **Case IV: Prices set subject to fixed menu costs:** In every period, firms decide whether to pay a fixed cost $\mathcal{C}>0$, in terms of labor units, to adjust their price. Starting in period t with a price P_{it-1} and an information set $\mathcal{I}_{it}$, the firm sets a contingent price sequence $\{P_{it+s}\}$ that maximize its value $V(P_{it-1},\mathcal{I}_{it})$, which is characterized by:

$$V(P_{it-1},\mathcal{I}_{it}) = \max_{\{P_{it+s}\}} \quad \mathbb{E}_{it}\left\{\sum_{s=0}^{\infty}\lambda_{t+s}\left[P_{it+s}Y_{it+s}-W_{it+s}N_{it+s}-\mathcal{C}W_{it+s}\cdot\mathbb{I}_{P_{it+s}\neq P_{it+s-1}}\right]\right\}, \tag{15}$$

where

$$\mathbb{I}_{P_{it}\neq P_{it-1}} = \begin{cases} 1 \text{ if } P_{it+s}\neq P_{it+s-1} \\ 0 \text{ if } P_{it+s}=P_{it+s-1}\end{cases}$$

denotes an indicator variable that equals 1 if and only if the firm adjusts its price.

In contrast to the flexible price benchmark, we assume that firms set prices (not quantities), but we stay close to the "supply function equilibrium" concept in allowing firms to condition their pricing decision on the concurrent market conditions. Note that all four cases reduce to our static benchmark as nominal rigidities disappear ($N=0$ in Case I, $N=1$ in Case II, $\xi=1$ in Case III and $\mathcal{C}=0$ in Case IV), in which case we recover the Hayekian Benchmark. Hence dispersed information becomes relevant only in combination with nominal rigidities.

2.3 Both information and nominal frictions: A dynamic irrelevance result

In this subsection, we discuss a dynamic version of our information irrelevance result for the four cases laid out above. Our notion of irrelevance is the same as before. In all four cases, the equilibrium generates propagation of nominal shocks to the real economy even under full information (i.e. assuming the realizations of the underlying shocks are common knowledge), because prices are sticky. We then compare the dispersed information economy with sticky prices to an identical economy subject to the same nominal friction but under full information. We say that dispersed information is irrelevant for aggregate propagation if conditioning pricing decisions on current and past market conditions is sufficient for replicating the equilibrium of the full information economy with sticky prices, i.e. moving from full to dispersed information does not alter macroeconomic adjustment dynamics. Conversely we are interested in conditions under which dispersed information amplifies monetary non-neutrality relative to the full information case.

In the absence of nominal frictions, dispersed information turned out to be irrelevant because firms' signals allowed them to perfectly infer their (marginal) revenues and costs. Two complications arise in extending that logic to a dynamic environment. First, firms now have to forecast future revenues and costs using current and past market conditions. Second, profits are weighted by an aggregate stochastic discount factor, λ_t. For the informational friction to be irrelevant, both forecasts of future profits and their relative weight in the firm's objective (determined by expected $\frac{\lambda_{t+s}}{\lambda_t}$) must be the same under dispersed and full information.

We start by deriving conditions under which the irrelevance result of Proposition 1 survives the introduction of nominal frictions. These conditions will be more stringent, but they deliver a general insight - nominal frictions make dispersed information relevant only if they introduce motives to disentangle different types of aggregate and idiosyncratic shocks. This occurs when (a) aggregate and idiosyncratic shocks differ in their persistence or (b) general equilibrium linkages generate a direct link between aggregate prices and firm-level costs and revenues (or, in the language of the pricing literature, there are strategic interactions in pricing decisions). Conversely, dispersed information remains irrelevant if current marginal cost and demand conditions provide the best available predictor of not just current but also future marginal costs and optimal prices.

The following proposition, the dynamic analogue of the benchmark result in Proposition 1, presents conditions under which this holds.

Proposition 2 *Consider economies subject to the frictions listed in cases I through IV above. Then, dispersed information is irrelevant if*

1. *there are no pricing complementarities, i.e.* $\theta = \frac{1}{\psi}$ *, and*
2. *idiosyncratic shocks are permanent, i.e.* $\rho_b = \rho_z = 1$.

To see the intuition behind these conditions, we use (1) and (11) to rewrite revenues and costs:

$$\text{Total Revenue}_t = \left(B_{it} C_t P_t^{\theta}\right) P_{it}^{1-\theta} = \left(B_{it} M_t^{\frac{1}{\psi}} P_t^{\theta - \frac{1}{\psi}}\right) P_{it}^{1-\theta}$$

$$\text{Total Cost}_t = \frac{1}{\delta} W_{it} \left(B_{it} C_t P_t^{\theta}\right)^{\delta} P_{it}^{-\theta\delta} = \frac{1}{\delta} \left(M_t Z_{it}\right) \left(B_{it} M_t^{\frac{1}{\psi}} P_t^{\theta - \frac{1}{\psi}}\right)^{\delta} P_{it}^{-\theta\delta} \quad .$$

The aggregate price level P_t affects revenues and costs through demand and wages as reflected in the two terms $B_{it} M_t^{\frac{1}{\psi}} P_t^{\theta - \frac{1}{\psi}}$ and $M_t Z_{it}$. The first term shows that aggregate prices enter demand for a firm's product through a relative price effect as well as an aggregate demand effect. When $\theta = \frac{1}{\psi}$, these two effects exactly cancel each other, so aggregate prices have no net influence on a firm's current or future demand. Thus, under this parametric restriction, profits are functions solely of the firm's price P_{it} and particular combinations of exogenous shocks - specifically, $B_{it} M_t^{\frac{1}{\psi}}$ and $M_t Z_{it}$.

The second condition – on persistence of shocks – ensures that the market signals are sufficient to forecast these combinations. When shocks are equally persistent (recall that the monetary shock is permanent), the most recent realizations of $B_{it} M_t$ and $M_t Z_{it}$ contain all the relevant information for characterizing the conditional distributions of $B_{it+s} M_{t+s}^{\frac{1}{\psi}}$ and $M_{t+s} Z_{it+s}$. Therefore, the additional information available to firms in the full information economy (i.e. the realizations of the underlying shocks M_t, B_{it} and Z_{it}) is irrelevant for the firm's price-setting decision.

Finally, from (9), the growth rate of the discount factor bears a one-to-one relationship to the growth rate of money supply. Since M_t is a random walk, this growth rate is iid. This implies that the joint distribution of $\left\{\frac{\lambda_{t+s}}{\lambda_t}\right\}$ and the relevant processes for costs and revenues $\left\{B_{it+s} M_{t+s}^{\frac{1}{\psi}},\ M_{t+s} Z_{it+s}\right\}$ are identical under both informational assumptions.

Comparing Proposition 2 with Proposition 1, we observe that the conditions for the Hayekian benchmark are far more restrictive when firm decisions are forward-looking due to nominal rigidities. In such environments, although firms face no uncertainty about their current marginal costs, their optimal pricing decisions also require forecasting future marginal costs, which depend on both the stochastic properties of idiosyncratic and aggregate shocks, and the expected future pricing decisions by other firms. The additional conditions in Proposition 2 are necessary as well as sufficient to guarantee that current marginal costs offer the best available forecast of future optimal prices (i.e. they are 1st-order Markov, or in this specific case, a random walk), thus reducing the dynamic forecasting problem to a static one. Any departures from these conditions then open the door for dispersed information to affect - and potentially amplify - the real effects of aggregate nominal shocks. The strength of such amplification in turn depends on the strength of nominal rigidities, and the magnitude of departures from the Hayekian benchmark conditions, both of which can potentially be calibrated using micro data on firms' price adjustment.

To summarize, in this section, we have established two theoretical benchmarks in assessing the role of dispersed information in a market economy. First, in the absence of information lags or nominal rigidities, markets play a very effective role in coordinating equilibrium outcomes on an allocation that mirrors the full information outcome. Second, lags or frictions induce departures from this benchmark whenever there are strategic interactions in decisions or differences in the dynamic properties of underlying shock processes that give firms a strong rationale to separate firm-specific from aggregate market conditions.

3 Departures from the Hayekian Benchmark

In this section, we explore how incomplete information amplifies the effects of nominal rigidities in propagating nominal shocks, when the conditions for the dynamic Hayekian benchmark are not satisfied, i.e. when there are pricing complementarities and/or firm-specific and aggregate shocks don't have the same degree of persistence. We focus on the model with Calvo pricing (Case III), for which we are able to obtain insightful analytical results that provide intuition for numerical simulations of both the Calvo model and the menu cost model (Case IV) with incomplete information in the next section.

We show two results: First, incomplete information and nominal adjustment frictions amplify each other, i.e. the real effects of nominal shocks are larger when both frictions act in combination, whenever idiosyncratic shocks are mean-reverting and/or pricing decisions are strategic complements. Second, even if agents face extreme uncertainty about aggregate conditions (i.e. aggregate shocks are an order of magnitude smaller than firm-specific ones), the aggregate real effects of nominal spending shocks can be substantially muted by the firms' response to their own market signals, generating substantial aggregate price adjustment in both the short and the long-run. In other words, when firms respond to their own market conditions, aggregate prices may be much more responsive to aggregate nominal shocks than firms' expectations about these shocks.

For analytical tractability, we work with a log-quadratic approximation of the firm's profit function. Using $\hat{x}_t$ to denote the deviation of the variable x_t from its steady state value, we can write profits as (see Appendix for details):

$$(1-\theta+\theta\delta)\sum_{t=0}^{\infty}\beta^t\left[\mathcal{P}^*_{it+s}\hat{p}_{it+s}-\frac{1}{2}\hat{p}^2_{it+s}\right]+\text{Terms independent of }\{\hat{p}_{it+s}\}\quad, \tag{16}$$

where $\mathcal{P}^*_{it+s}$ denotes the (log of the) static optimum, i.e. the optimal price under perfect information and fully flexible prices. It is a linear combination of aggregate variables (money supply, $\hat{m}_{t+s}$ and the aggregate price level, $\hat{p}_{t+s}$) as well as the two idiosyncratic shocks ($\hat{b}_{it+s}$ and $\hat{z}_{it+s}$),

$$\mathcal{P}^*_{it+s}\equiv(1-r)\,\hat{m}_{t+s}+r\hat{p}_{t+s}+\left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\hat{b}_{it+s}+\left(\frac{1}{1-\theta+\theta\delta}\right)\hat{z}_{it+s}\quad. \tag{17}$$

The parameter $r\equiv\frac{\delta-1}{1-\theta+\theta\delta}\left(\theta-\frac{1}{\psi}\right)$ summarizes the effect of the aggregate price on an individual firm's target and thus indexes the degree of strategic complementarity in pricing decisions.

In every reset period, the firm solves:

$$\max_{\hat{p}_{it}}\quad\sum_{T=1}^{\infty}(1-\xi)^{T-1}\xi\sum_{s=0}^{T-1}\beta^s\,\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\hat{p}_{it}-\frac{1}{2}\hat{p}^2_{it}\right].$$

The optimal reset price is

$$\hat{p}^*_{it}=(1-\beta+\beta\xi)\sum_{s=0}^{\infty}(1-\xi)^s\beta^s\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right]=(1-\beta+\beta\xi)\,\mathbb{E}_{it}\left[\mathcal{P}^*_{it}\right]+\beta\,(1-\xi)\,\mathbb{E}_{it}\left[\hat{p}^*_{it+1}\right]\quad. \tag{18}$$

Information: As before, under full information, the firm observes the realization of all aggregate and idiosyncratic shocks, whereas under dispersed information the firm's information set consists only of a history of its revenues and wages. Again, from (1) and (11), we see that this is informationally equivalent to observing a history of the following two signals:

$$
\begin{aligned}
s_{it}^{1} &= \frac{1}{\psi}\hat{m}_t + \left(\theta - \frac{1}{\psi}\right)\hat{p}_t + \hat{b}_{it} \\
s_{it}^{2} &= \hat{m}_t + \hat{z}_{it} \ .
\end{aligned} \tag{19}
$$

The contemporaneous observation of these two signals allows the firm to infer its static optimum perfectly, since

$$
\mathcal{P}_{it}^{*} = \left(\frac{\delta - 1}{1 - \theta + \theta\delta}\right) s_{it}^{1} + \left(\frac{1}{1 - \theta + \theta\delta}\right) s_{it}^{2} \ , \tag{20}
$$

and therefore, $\mathbb{E}_{it}\left[\mathcal{P}_{it}^{*}\right] = \mathcal{P}_{it}^{*}$.

We are interested in comparing the response of this economy to innovations in money supply under dispersed information to its full information counter-part. Equation (18) shows that this leads to a complex filtering problem, where at each date t, the firms who adjust their price must form forecasts over the entire future sequence of reset prices, based on current and past signals of their demand and wages.

Averaging equation (18) over i and letting $\hat{p}_t^{*} = \int \hat{p}_{it}^{*} di$ denote the average optimal re-set price, $\mathcal{P}_t^{*} = \int \mathcal{P}_{it}^{*} di = (1 - r)\,\hat{m}_t + r\hat{p}_t$ the average static optimum, and $\overline{\mathbb{E}}_t\,(x) = \int \mathbb{E}_{it}\,(x)\,di$ the average expectation of any random variable x, across all firms i, we obtain the following representation of the average optimal reset price:

$$
\hat{p}_t^{*} = (1 - \beta + \beta\xi)\,\mathcal{P}_t^{*} + \beta\,(1 - \xi)\,\overline{\mathbb{E}}_t\left(\hat{p}_{t+1}^{*}\right) + \beta\,(1 - \xi)\,\overline{\mathbb{E}}_t\left(\hat{p}_{it+1}^{*} - \hat{p}_{t+1}^{*}\right). \tag{21}
$$

Using standard manipulations, this expression can also be restated as a modified version of the New Keynesian Philips Curve:

$$
\pi_t = \frac{\xi}{1 - \xi}\left[\mathcal{P}_t^{*} - \hat{p}_t - \beta\,(1 - \xi)\left(\mathcal{P}_t^{*} - \overline{\mathbb{E}}_t\,(\hat{p}_t)\right)\right] + \beta\overline{\mathbb{E}}_t\,(\pi_{t+1}) + \beta\xi\overline{\mathbb{E}}_t\left(\hat{p}_{it+1}^{*} - \hat{p}_{t+1}^{*}\right), \tag{22}
$$

where $\pi_t = \hat{p}_t - \hat{p}_{t-1}$ denotes inflation in period t. This representation is valid for both the full information and dispersed information cases. The first two terms in the average reset price (21) and the New Keynesian Philips Curve (22) are the standard ones: they capture the response of inflation to real marginal costs and expected future price increases. The third term in both equations, which scales with $\overline{\mathbb{E}}_t\left(\hat{p}^*_{it+1} - \hat{p}^*_{t+1}\right)$, is novel and and captures the idea that firms also update their expectations about firm-specific shocks in response to aggregate changes.

Under full information, agents completely separate forecasting idiosyncratic from aggregate conditions, and $\hat{p}_t$ is common knowledge. Hence $\overline{\mathbb{E}}_t\left(\hat{p}^*_{it+1} - \hat{p}^*_{t+1}\right)$ averages out to 0 in the cross-section. In addition, $\overline{\mathbb{E}}_t\left(\hat{p}_t\right) = \hat{p}_t$, and so the first term reduces to the standard representation of real marginal costs, and equation (22) reduces to the textbook New Keynesian Philips Curve.

With dispersed information, the response of inflation to real marginal costs is adjusted to account for the fact that the current price level $\hat{p}_t$ is no longer common knowledge, and therefore $\overline{\mathbb{E}}_t\left(\hat{p}_t\right) \neq \hat{p}_t$. More importantly, firms now form forecasts about both future inflation and their market conditions, i.e. the term $\hat{p}^*_{it+1} - \hat{p}^*_{t+1}$ that is orthogonal to aggregate conditions. Since their forecasts are based on market signals that confound idiosyncratic and aggregate shocks, this average forecast of own market conditions is generally different from zero, and contributes to the response of average prices to market conditions. The impact of dispersed information on price adjustment then comes down to a "race" between (i) the fact that firms update more slowly about future inflation (the standard channel, in the second term), and (ii) the fact that firms' forecasts of their own market conditions are systematically affected by aggregate shocks (the third term). This last term is the dynamic analogue of the "setting the right prices for the wrong reasons" channel that we explored in Hellwig and Venkateswaran (2009) in a static economy.

Now, as shown by Proposition 2, if $r = 0$ (or $\theta = 1/\psi$) and $\rho_b = \rho_z = 1$, the dispersed and perfect information equilibria coincide: in that case, it is straight-forward to check that $\mathcal{P}^*_{it}$ follows a Martingale, and therefore, $\hat{p}^*_{it} = \mathcal{P}^*_{it}$ under both perfect and dispersed information, since future movements in the optimal reset price are completely unpredictable. However, the full information case attributes inflation dynamics entirely to forward-looking expectations about future inflation (the second term), while the dispersed information case attributes some of the same price adjustment dynamics to the third term – yet when summing up the contributions of the different terms, the total effect on inflation dynamics is the same. We now discuss how the strength of the $\overline{\mathbb{E}}_t\left(\hat{p}^*_{it+1} - \hat{p}^*_{t+1}\right)$ term varies as we depart from the Hayekian benchmark.

Small shocks limit: To simplify our analysis, we consider the limiting case of the firms' decision problem when nominal shocks become arbitrarily small, i.e. as $\sigma_u^2 \to 0$. Whereas for any positive finite level of σ_u^2, the firms gradually update their expectations about the aggregate state according to Bayes' Rule using the information contained in their market signals, this learning becomes slower and slower as σ_u^2 becomes smaller, and completely stops in the limit as $\sigma_u^2 \to 0$. In this limit, it is rational for firms, over any finite horizon, to attribute all fluctuations in their market conditions to firm-specific shocks, and respond accordingly. One implication of this limit is that $\overline{\mathbb{E}}_t\left(\pi_{t+1}\right)$ and $\overline{\mathbb{E}}_t\left(\hat{p}^*_{t+1}\right)$ both converge to zero in equations (21) and (22), and hence short-run inflation dynamics are entirely shaped by firms' expectations about their own market conditions.

We view this limit as an approximation to the firms' full Bayesian updating problem – an empirically plausible one, since estimates for aggregate nominal disturbances are at least an order of magnitude smaller than idiosyncratic demand and cost shocks. At the same time, this limit is theoretically convenient, since it illustrates the power of market-generated information even if firms (asymptotically) do not update their beliefs about aggregate conditions at all. In Section 4.3, we discuss implications of relaxing this assumption and explore how both the short-run and the long-run adjustment properties change when firms have additional sources of information through which they can learn about aggregate conditions. In both cases, the small-shocks limit remains a good approximation for the full Bayesian updating problem, especially in the short-run.

Consider a small permanent nominal shock in period t, i.e. $u_{t+s} = \Delta > 0$ for $s = 0, 1, 2, ...$ This change in money supply enters the two signals of the firm according to (19). However, since the likelihood of such shocks is arbitrarily small, each firm attributes all changes in its signals to the idiosyncratic components $\hat{b}_{it}$ and $\hat{z}_{it}$. For expositional simplicity, we will also assume that both the idiosyncratic shocks are equally persistent, i.e. $\rho_b = \rho_z = \rho$.

We guess (and verify) that the expected aggregate price level in periods following the shock, under both dispersed and full information, can be represented in the form

$$\hat{p}_{t+s} = \tau_1 \; \Delta + \tau_2 \; \hat{p}_{t+s-1} \quad , \tag{23}$$

where the coefficients τ_1 and τ_2 will vary with the information structure. We can thus compare the price adjustment under different informational assumptions by comparing these coefficients. After

s periods of the shock, the aggregate price level then adjusts to

$$\hat{p}_{t+s} = \frac{\tau_1}{1-\tau_2}\left(1-\tau_2^s\right)\ \Delta\ . \tag{24}$$

With this representation, the coefficient τ_1 summarizes how much prices adjust on impact, τ_2 summarizes how fast they converge to their steady-state level, and $\frac{\tau_1}{1-\tau_2}$ summarizes how much prices adjust to aggregate conditions in the long-run. If $\tau_1 = 1-\tau_2$, the nominal shock will eventually be fully absorbed into prices.

Under dispersed information, the firm's time-t expectation of its future target is

$$\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right] = \mathbb{E}_{it}\left[\left(\frac{\delta-1}{1-\theta+\theta\delta}\right)s^1_{it+s} + \left(\frac{1}{1-\theta+\theta\delta}\right)s^2_{it+s}\right].$$

Now, using the fact that firms attribute all shocks to idiosyncratic factors, so that $\mathbb{E}_{it}m_{t+s} = \mathbb{E}_{it}p_{t+s} = 0$ and $\mathbb{E}_{it}\hat{b}_{it+s} = \rho^s s^1_{it}$ and $\mathbb{E}_{it}\hat{z}_{it+s} = \rho^s s^2_{it}$, we obtain $\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right] = \rho^s\mathcal{P}^*_{it}$. Substituting into (18) and simplifying, we then obtain the following expression for the optimal reset price under dispersed information:

$$\hat{p}^*_{it} = \left[\frac{1-\beta(1-\xi)}{1-\beta\rho(1-\xi)}\right]\mathcal{P}^*_{it}. \tag{25}$$

Then, a few lines of algebra yield $\left(\tau_1^{Disp}, \tau_2^{Disp}\right)$, collected in following result.

Proposition 3 *As aggregate shocks become arbitrarily small, i.e.* $\sigma_u^2 \to 0$, *the expected aggregate price level in period* $t+s$ *in the dispersed information economy evolves according to* (23) *with*

$$\tau_1^{Disp} = \xi\frac{(1-r)}{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta} - \xi r} \qquad \tau_2^{Disp} = (1-\xi)\frac{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta}}{\frac{1-(1-\xi)\beta\rho}{1-(1-\xi)\beta} - \xi r}\ .$$

Moreover, for all horizons, $\hat{p}_{t+s}$ *is strictly decreasing in* r *and increasing in* ρ.

It is straightforward to verify that both τ_1^{Disp} and τ_2^{Disp} are increasing in ρ. Then, equation (24) implies that the aggregate price adjustment under dispersed information is also increasing in ρ. This is intuitive - when aggregate shocks are small, firms attribute them almost entirely to idiosyncratic factors and therefore, expect them to decay at the rate ρ. When $\rho < 1$, this misattribution leads firms to adjust their prices by less than they would under full information. Therefore, aggregate price adjustment is slower and smaller, the larger is the gap between the persistence of idiosyncratic and aggregate shocks.

Likewise, it is easy to check that τ_1^{Disp} is decreasing in r while τ_2^{Disp} is increasing in r: stronger complementarity reduces the response of prices on impact but makes the overall response more persistent. To derive the comparative statics of $\hat{p}_{t+s}$ with respect to r, it is therefore useful to rewrite the RHS of (24) as the product of two terms - the long-run adjustment $\tau_1/(1-\tau_2)\,\Delta$ and the rate of convergence $(1-\tau_2^s)$. For the long-run adjustment, we observe that

$$\frac{\tau_1^{Disp}}{1-\tau_2^{Disp}} = \frac{1}{1+\frac{\beta(1-\xi)}{1-\beta+\beta\xi}\left(\frac{1-\rho}{1-r}\right)} \;\leq\; 1 \,. \tag{26}$$

In other words, prices never fully adjust to the nominal shock under dispersed information, unless idiosyncratic shocks are permanent ($\rho = 1$), or prices are completely flexible ($\xi = 1$). The strength of long-run non-neutrality depends on the difference in persistence (lower ρ) and strategic complementarities (higher r). In fact, the $\frac{1-\rho}{1-r}$ term in the denominator shows how complementarities amplify the effects of differences in persistence in the long run. In addition, higher persistence τ_2 also implies that stronger complementarities (higher r) delay the speed at which prices converge to their long-term level, resulting in lower price adjustment at all horizons.

Equation (26) provides a measure of the extent to which the Hayekian mechanism remains in operation despite the presence of adjustment frictions and complementarities: it shows that in the long run aggregate prices can adjust substantially, and in some cases completely, even without agents being aware of the changes in the underlying aggregate conditions ! On the other hand, it also shows how sticky prices, complementarities and differential degrees of persistence can interact to prevent full adjustment in the long run. Note, however, that this long-run non-neutrality result relies on the rather stark assumption that over any finite horizons, firms never learn about the true nature of the aggregate shock - for any positive variance of supply shocks, Bayesian updating still generates convergence to full price adjustment, but in the limit, learning is so slow that the the speed of convergence - beyond the adjustment to market-generated information, is infinitely slow. As we will see in section 4.3, it is sensitive to the introduction of additional information about aggregate conditions.[19] The residual adjustment and speed of convergence to neutrality then will be governed by the availability and informativeness of these additional signals.

[19] It is important to point out that there are two limits in equation (26) and the order is crucial for long-run non-neutrality. More precisely, the term on the left hand side of (26) is $\lim_{s\to\infty}\lim_{\sigma_u^2\to 0}\ \frac{d\hat{p}_{t+s}}{du_t}$. If we reverse the order of the limits, we recover neutrality.

The mechanism through which complementarities influence price adjustment under dispersed information is novel and distinct from the one in play both in full information sticky price models and in the dispersed information models of Woodford (2003) or Mankiw and Reis (2002). In those settings, complementarities dampen price adjustment because they tie a firm's willingness to adjust its price to the prices set by the other firms. When an aggregate nominal shock occurs, firms receive news that aggregate conditions have changed, but they refrain from fully adjusting to these news because they expect that other firms will not fully adjust either and they do not want to be too far out of line with their peers –and hence dampen their responses.[20] This mechanism is captured by how the complementarity affects the endogenous inflation expectations in the second term in equations (21) and (22).

Under dispersed information, this strategic interaction channel is muted because firms are slow to learn about aggregate conditions, hence they do not adjust their expectations about how other firms will react to an aggregate nominal shock – in fact, at the small shocks limit, it doesn't occur to them that an aggregate shock may have shifted the long-run average level of prices, i.e. the second term in equation (21) and (22) goes to zero. Instead, firms form expectations about their own market conditions –the last term in equations (21) and (22)– and the complementarity governs how fast and how much the aggregate shock shifts firms' market signals and therefore their views about their own market conditions. More specifically, averaging equation (17) across i, we obtain $\int \mathcal{P}_{it}^{*} \; di = (1-r)\,\hat{m}_t + r\hat{p}_t$. Recall that $\mathcal{P}_{it}^{*}$ perfectly summarizes the information in firms' own market signals, so the complementarity parameter r governs the sensitivity of the average static optimum w.r.t. the aggregate price level; in other terms it decomposes the response of optimal prices to aggregate nominal shocks into a direct effect $1-r$ of the nominal shock on individual firm's optimal prices, and an indirect feedback effect r from the aggregate price level, which together determine how much and how fast firms' expectations about their own market conditions respond to the aggregate nominal shock. Since $\hat{p}_t$ reflects the aggregate shock only slowly (while the effect on $\hat{m}_t$ is immediate), stronger complementarities slow the rate at which the shock enters firm-

[20]With full information, the complementarity thus propagates lack of adjustment from firms that have their prices fixed to firms who have the ability to adjust, while in the dispersed information models, the complementarity amplifies an initial lack of adjustment to available information because firms are not exactly certain what other firms have observed, but at its core the two mechanisms are very similar.

level information and through that, the rate at which it is incorporated into prices. With dispersed information, strategic complementarities thus slow down price adjustment through an informational feedback channel that is quite distinct from the strategic effect of complementarities emphasized by the existing literature on nominal price adjustment.

We conclude with two polar cases to isolate the effects of persistence and complementarities. First, without complementarities ($r = 0$), the coefficients in proposition 3 simplify to

$$\tau_1^{Disp} = \xi \frac{1 - \beta (1 - \xi)}{1 - \beta\rho (1 - \xi)} \qquad\qquad \tau_2^{Disp} = 1 - \xi \ .$$

With full information, $\tau_1^{Full} = \xi$ and $\tau_2^{Full} = 1 - \xi$, and therefore $\tau_1^{Disp} < \tau_1^{Full}$ if and only if $\rho < 1$. In other words, without complementarities, informational frictions do not have any effect on the speed of adjustment, but lower the long-term adjustment level when firm-specific shocks are less persistent than aggregate shocks. The following result is immediate.

Claim 1 *Suppose $r = 0$, but $\rho < 1$. Then, prices in the dispersed information economy respond less to nominal shocks than at the full information benchmark.*

Next, to see the role of complementarities, we consider the case where all shocks are permanent ($\rho = 1$). From Proposition 3 we obtain:

$$\tau_1^{Disp} = \frac{\xi (1 - r)}{1 - \xi r} \qquad\qquad \tau_2^{Disp} = \frac{1 - \xi}{1 - \xi r} \ .$$

Directly, we see that $\tau_1^{Disp} / \left(1 - \tau_2^{Disp}\right) = \tau_1^{Full} / \left(1 - \tau_2^{Full}\right) = 1$. In the Appendix, we show that $\tau_2^{Disp} > \tau_2^{Full}$. In other words, with complementarities alone, informational frictions do not affect the long-run level of adjustment, but slow down convergence.

Claim 2 *Suppose $\rho = 1$, but $r > 0$. Then, prices in the dispersed information economy respond more slowly to nominal shocks than at the full information benchmark.*

Since overall price adjustment is increasing in ρ under dispersed information, this result also shows that regardless of parameter values, overall price adjustment is smaller and slower in the dispersed information economy relative to its full information counterpart. Information frictions thus amplify non-neutrality.

4 Quantitative results

In this section, we explore the quantitative effects of incomplete information for aggregate price adjustment under reasonable parameter values, focusing on the Calvo and menu cost models (Cases III and IV). We follow a well-worn path from the pricing literature to parameterize the model. The time period is set to a week and accordingly, $\beta = 0.999$ targets an annual interest rate of 5 percent. We set the elasticity of substitution $\theta = 5$, the curvature of the utility function $\psi = 2$ and the (inverse of the) returns to scale in production $\delta = 2$. Together, they imply complementarity $r = 0.75$. This is towards the higher end of the range of micro estimates – see, for example, Burstein and Hellwig (2007) – but slightly lower than the estimates based on aggregate data – see Rotemberg and Woodford (1997).

We then set the parameters relating to firm-level price adjustment, i.e. the variances and autocorrelations of firm-specific demand and wage shocks $\left(\rho_b, \sigma_b^2\right)$ and $\left(\rho_z, \sigma_z^2\right)$, and the adjustment frequency ξ in the Calvo model or the menu cost $\mathcal{C}$ in the menu cost model to minimize the squared deviations of the following simulated moments from their empirical counterparts (which are listed in parentheses) are minimized – frequency of price changes (25%) average absolute size of price changes (12%), standard deviation (in logs) of prices (0.08) and the autocorrelation of log prices (0.67). The first two targets are in line with estimates reported by a number of studies - Bils and Klenow (2004), Nakamura and Steinsson (2008) and Klenow and Krvystov (2008). The second is derived from statistics reported by Burstein and Hellwig (2007) and the autocorrelation target from Midrigan (2011).[21] To simplify the calibration, we further set $\rho_b = \rho_z = \rho$ and $\sigma_b^2 = \sigma_z^2 = \sigma^2$.[22]

4.1 Calvo model with information frictions

In the Calvo model, ξ is chosen so that 25 percent of prices are changed every month. Our estimate for ρ is 0.95.

The results are presented in Table 1 and Figure 1. The table reports the response of aggregate prices (normalized by the size of the shock, so 1 corresponds to full adjustment) under full and

[21] Using additional moments (e.g. percentiles of the price change distribution) does not change our parameter estimates or our results materially.

[22] Allowing cost and demand shocks to have different magnitudes and targeting as an additional moment the covariance between price changes and relative quantities as in Burstein and Hellwig (2007) would not significantly alter our results.

	Horizon (weeks)			
	1	12	24	52
Baseline				
– Full Information	0.03	0.35	0.58	0.84
– Dispersed Information	0.01	0.10	0.17	0.24
– Relative (Dispersed/Full)	0.31	0.30	0.29	0.28
Equal Persistence				
– Full Information	0.03	0.35	0.58	0.84
– Dispersed Information	0.02	0.20	0.36	0.62
– Relative (Dispersed/Full)	0.52	0.57	0.62	0.73
No complementarities				
– Full Information	0.07	0.58	0.82	0.98
– Dispersed Information	0.04	0.35	0.49	0.59
– Relative (Dispersed/Full)	0.60	0.60	0.60	0.60

Table 1: Response of p_t to a shock, $\Delta = 1$.

dispersed information while the figure plots the corresponding responses of (the log of) real consumption spending, (i.e. $c_t = \hat{m}_t - \hat{p}_t$) also normalized by the size of the shock. The first panel of the table is the baseline version and shows that under full information, aggregate prices reflect only 35% of the shock after 12 weeks, reflecting the bite of the nominal friction. More interestingly, even without any direct knowledge about the aggregate shock, the aggregate price level in the dispersed information economy shows considerable responsiveness (about one-third of the full information response). The left panel of Figure 1 shows the impulse response function of real consumption spending for this parameterization. It reveals that at short horizons, the deviation from the Hayekian benchmark is small, but at longer horizons, the gap is more meaningful.

The middle and right panels of the figure isolate the effects of differences in persistence and strategic complementarities, respectively. The middle panel maintains $r = 0.75$, but assumes that the idiosyncratic shocks are permanent while the right panel keeps $\rho = 0.95$ but sets $r = 0$. These two polar cases have similar implications in the short run – the response of prices as well as economic activity do not look too different in the dispersed information economy compared to the corresponding full information economy. Over longer horizons, however, the two panels look very different. When all shocks are equally persistent, the amplified effect on real consumption spend-

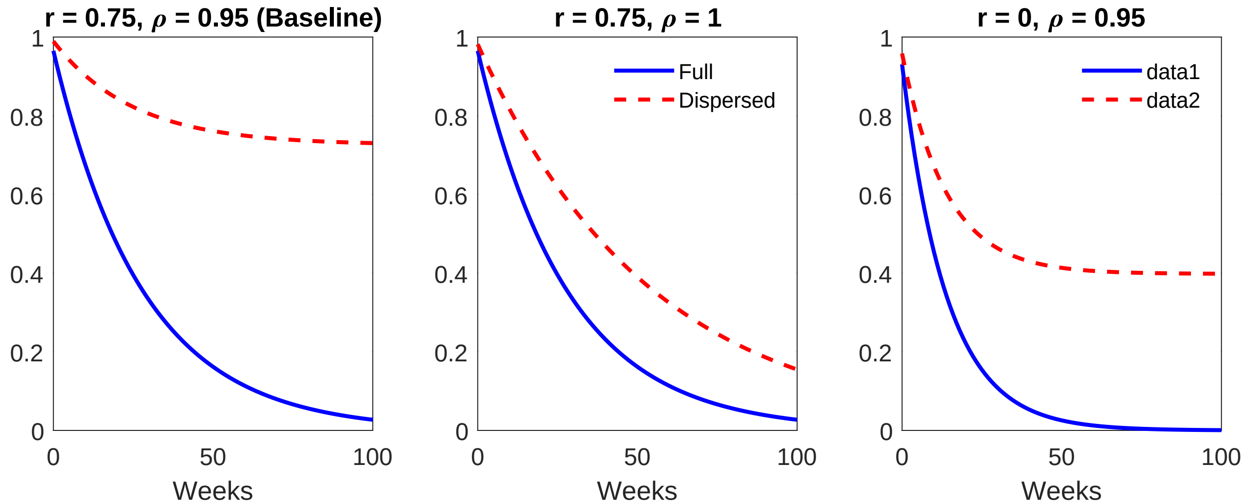

Figure 1: Impulse responses of real consumption spending to a nominal shock under Calvo pricing.

ing from dispersed information remains modest and decays relatively quickly. The fact that this occurs despite the relatively high degree of complementarity indicates that equilibrium interactions (and the coordination motives they give rise to) do not, by themselves, generate a substantial role for informational frictions. When idiosyncratic shocks are more transitory than aggregate shocks, however, dispersed information leads to a much more prolonged adjustment than under full information.[23] Finally, comparing the left and right panels, we note that strategic complementarities do exert a more significant influence when $\rho < 1$. In other words, the interaction of these two elements – complementarities and differences in persistence – can induce non-trivial deviations from the Hayekian benchmark at longer horizons.

4.2 Menu costs with information frictions

We next evaluate the role of incomplete information in a calibrated model with menu costs and dispersed information. In contrast to the Calvo model studied in the previous section, we work with full-blown non-linear expressions for revenues and costs, rather than a quadratic approximation. This precludes analytical solutions, but allows a more robust quantitative exercise.

[23] Recall that, in this limiting case, prices, even in the long run, do not fully incorporate the shock when $\rho < 1$.

	Description	Value
Parameters		
	Time period	1 week
β	Discount factor	0.996
ρ	Autocorrelation of idiosyncratic shocks	0.92
σ	Standard deviation of idiosyncratic shocks	0.155
$\mathcal{C}$	Menu cost	0.235

Table 2: Calibration summary (Menu costs with information frictions)

We maintain our focus on the limiting case of small aggregate shocks. Specifically, we compute the stationary distribution of prices without aggregate shocks and then subject the economy to a small permanent shock to money supply. The resulting changes in firms' signals are attributed entirely to idiosyncratic factors.

Table 2 collects the calibrated parameter values. Note that the estimates for the persistence of idiosyncratic shocks, ρ, is slightly higher than in the Calvo case studied in the previous section. It is also higher than the corresponding number in the baseline calibration of Golosov and Lucas (2007). This difference arises because we target two new moments, namely autocorrelation and dispersion of prices. Since differences in persistence between aggregate and idiosyncratic shocks are an important source of departures from the Hayekian benchmark, our baseline calibration makes it harder for informational frictions to be relevant.

As before, we compare impulse response functions of real consumption spending under full and dispersed information.[24] These are plotted in Figure 2. The real effects from nominal shocks are generally smaller than under Calvo pricing. This is due to the well-known 'selection effect' - see Golosov and Lucas (2007). A positive aggregate shock alters the mix of firms adjusting prices towards those increasing prices, which increases the responsiveness of the aggregate price level to the shock. But, in terms of the relevance of informational frictions, the overall pattern is similar to Figure 1. The left panel, which presents our baseline calibration, shows that dispersed information amplifies the effect of the nominal shock in the short run, but the increase is somewhat modest.

[24] To generate these functions, we simulated 1000 runs of an economy with 10000 firms for 1200 periods, with the realization of the money growth rate for the 1000^{th} period fixed at 0.0072. The graphs show the average response of output (normalized by the aggregate shock). We verified numerically that our results are robust to varying these simulation parameters.

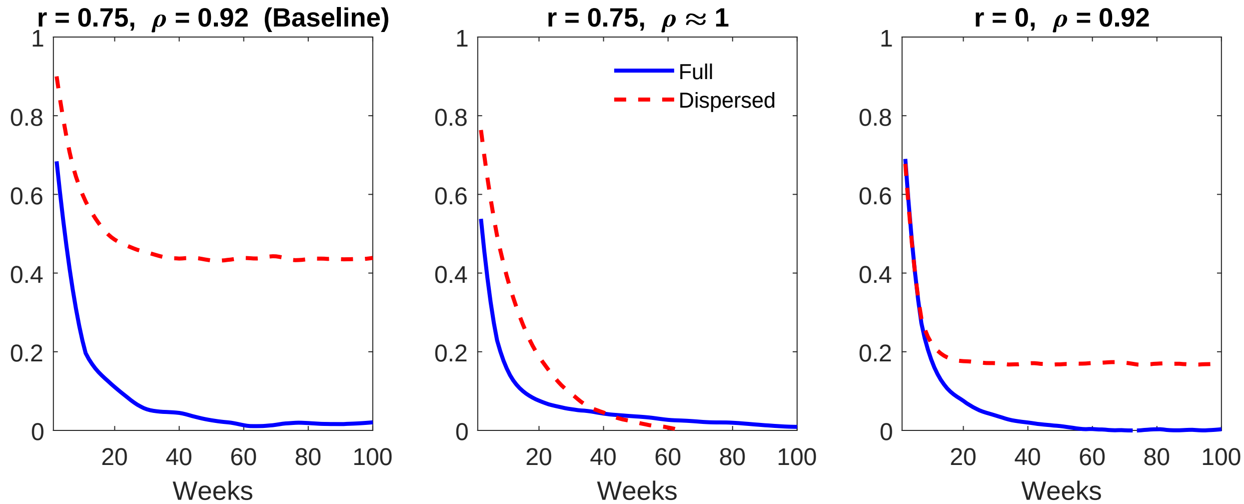

Figure 2: Impulse responses of real consumption spending to a nominal shock with menu costs.

Again, we see that complementarities by themselves do not induce significant departures from the Hayekian benchmark (the middle panel), but when combined with differences in persistence, they have significant medium term effects (comparing the left panel to the panel). Comparing Figure 2 to 1, we also observe that the additional selection effect results in substantially stronger long-run adjustment than with Calvo pricing. The reason is that the stronger short-run response influences firms' market signals and through that channel amplifies the long-run response of prices to the aggregate shock, relative to the case without selection effects. Hence in our baseline calibration, more than half of the nominal shock is absorbed into prices even though firms treat the information in their market signals as being purely driven by firm-specific conditions.

4.3 Discussion

We examine the robustness of the numerical findings in sections 4.1 and 4.2 to variations in information structure, shock processes and parameter values. In the interest of brevity, we will present results of these exercises under either Calvo pricing or menu costs, using the corresponding results from sections 4.1 or 4.2 as our baseline.

Information structure I: Our analysis so far has assumed that learning, at all horizons, occurs

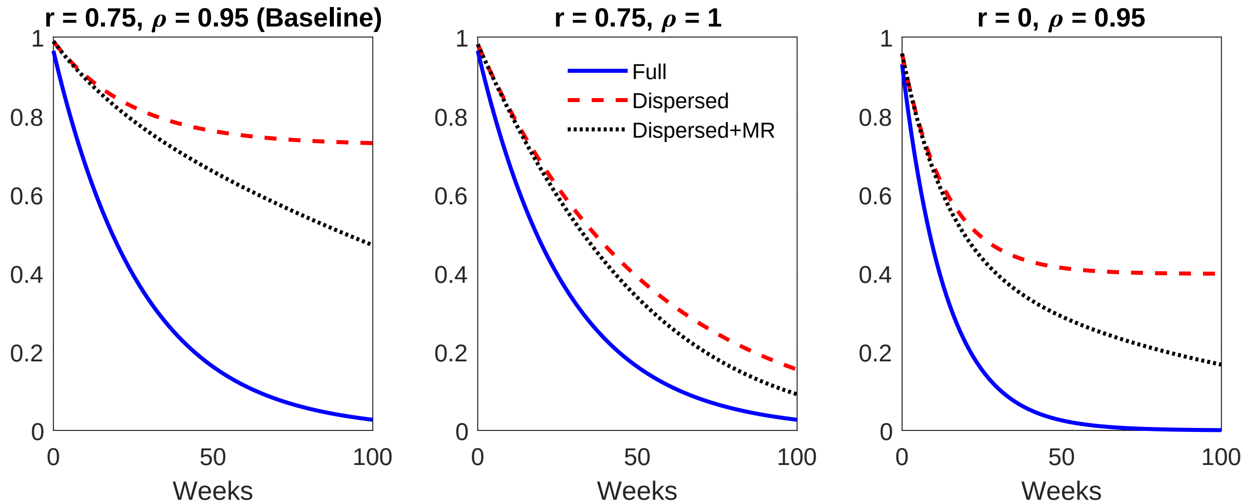

Figure 3: Impulse responses of real consumption spending to a nominal shock under Calvo pricing with additional information.

exclusively from market-generated signals. This extreme assumption maximizes the potential for dispersed information to be relevant and isolated the bite of the Hayekian mechanism. However, this clearly becomes less and less reasonable at longer horizons. We therefore relax it by introducing direct learning about aggregate shocks. Specifically, we assume that, every period, a fraction ϕ of firms receive a perfectly informative signal of the aggregate shock. This is in the spirit of the sticky information models of Mankiw and Reis (2002). We set ϕ at a relatively small value (0.01) so as to maintain the focus on learning from market signals over short horizons. The impulse response functions with this additional source of learning are marked 'Dispersed + MR' in Figure 3, which also presents the corresponding lines from Figure 1 (marked 'Dispersed'). We see that additional information leaves the short run implications essentially unchanged, but attenuates (and ultimately, eliminates) the gap between full and dispersed information over the medium-to-long term. This only really matters when $\rho < 1$. When shocks are equally persistent, as in the center panel, the Hayekian benchmark applies in the long run and the effect of this additional information is negligible.

Information structure II: Our numerical analysis also focused on the limiting case of arbitrarily small aggregate shocks. Relaxing this assumption raises formidable technical challenges. The first is the well-known 'curse of dimensionality' which arises in models where the cross-sectional distribution is a relevant state variable. Here, this problem is compounded by the 'infinite regress' problem highlighted by Townsend (1983): When actions are strategically linked, firms' optimal decisions depend on the entire structure higher-order expectations (i.e. their beliefs about others' beliefs, their beliefs about others' beliefs about their beliefs...). Thus, the entire structure of higher-order beliefs becomes an state variable. Combined with the non-linearities in the policy functions in the menu cost model, this high dimensionality of the state vector makes aggregation a very challenging task. Finally, the presence of a dynamic filtering problem with endogenous signals makes it difficult to directly apply Kalman filter techniques. In an earlier version of this paper, we overcame these challenges in a simplified version of the menu cost model presented above, by combining the approximation methods in Krusell and Smith (1998) with standard filtering techniques.[25] The results, especially in the short run, were very close to the limiting case analyzed in this paper. Over a longer horizon, however, the true nature of the shock is ultimately revealed. As with the direct learning above, this has the effect of shrinking (and ultimately, eliminating) the longer term gap between dispersed and full information cases, but for reasonable calibrations learning about aggregate conditions remains very slow.

Alternative shock processes (aggregate) : Next, we introduce serial correlation in innovations to money supply. This specification is commonly used in the literature, partly because it causes the real effect of a nominal shock to peak a few quarters after impact (under the random walk assumption, real effects are highest in the period of impact).[26] Specifically, we assume that

$$
\begin{aligned}
m_t &= m_{t-1} + u_t \\
u_t &= \rho_u u_{t-1} + v_t \qquad\qquad v_t \sim N\left(0, \sigma_v^2\right).
\end{aligned}
$$

Again, we focus on the limit as $\sigma_v^2 \to 0$. Figure 4 plots the response of output and inflation to a shock v_t, normalized by the eventual increase in money supply, assuming Calvo pricing.[27] In

[25] See Hellwig and Venkateswaran (2012).

[26] See discussion in Woodford (2003).

[27] If $v_t = \Delta$, the eventual increase in $m_t = \frac{\Delta}{1-\rho_u}$. We use a value of $\rho_u = 0.95$, which corresponds to a quarterly autocorrelation in money growth of about 0.5, consistent with the estimates in Christiano et al. (2005).

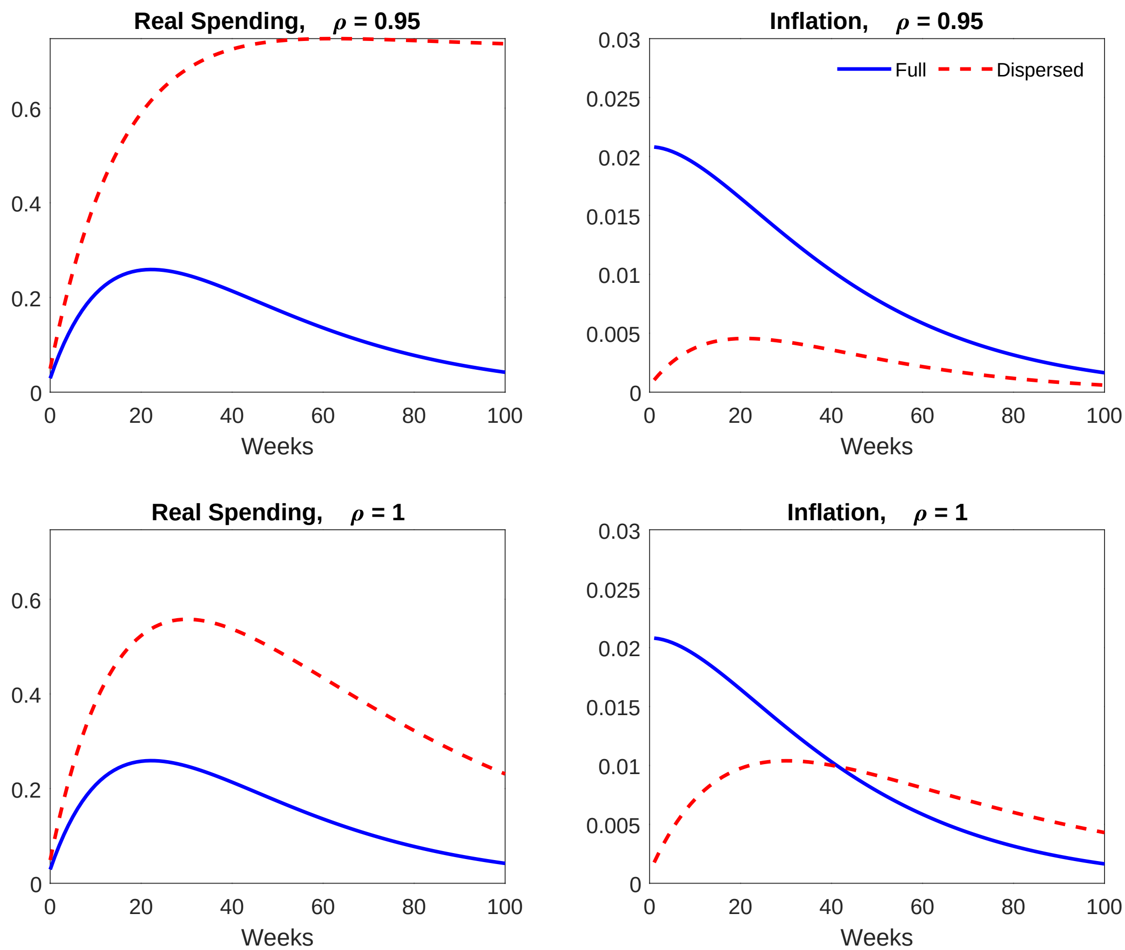

Figure 4: Impulse responses to a positive nominal shock with Calvo pricing and persistent shocks to the money growth rate.

the top two panels, we maintain our baseline calibration for the persistence of the idiosyncratic shock, $\rho = 0.95$, and plot the impulse response of output and real consumption spending on the left, and of inflation to on the right. In comparison to the random walk case in Figure 1, the overall pattern is similar - dispersed information leads to higher real effects at all horizons but the most striking amplification occurs at longer horizons. In the bottom panels, we set $\rho = 1$, making firm-specific shocks fully persistent. Now, prices ultimately fully reflect the shock, though the speed of adjustment is slower than in the full information economy. More interestingly, however, there is still significant short run amplification from the information friction. To see the source of this amplification, note that, under full information, the effects of the current shock on future money growth is correctly anticipated by firms when they reset prices. Under dispersed information, however, firms interpret changes in their signals as arising from (iid) innovations to their idiosyncratic factors. This channel is missing when $\rho_u = 0$, i.e. when all shocks were random walks. This explains why the bottom left panel in Figure 4 is markedly different from the corresponding one in Figure 1.

The sluggish response of prices to aggregate shocks contrasts with the (relatively) rapid and permanent adjustment of prices to idiosyncratic shocks. The model with serial correlation in money supply and fully persistent idiosyncratic shocks thus matches the empirical evidence in Boivin et al. (2009) and Maćkowiak et al. (2009) who find that prices reflect sector-specific shocks much more rapidly than aggregate shocks.

Alternative shock processes (idiosyncratic): We now consider the implications of an alternative calibration strategy for idiosyncratic shocks in the menu cost model. Midrigan (2011) shows in a full information menu cost model that accounting for the large observed heterogeneity in price changes weakens the selection effect at work in Golosov and Lucas (2007) and therefore, leads to slower price adjustment and larger real effects. We make two modifications to our menu cost model and follow the calibration strategy from Midrigan (2011) to explore how the selection effect interacts with dispersed information. First, in every period, each firm faces zero costs of changing prices with probability ϕ. Second, the two idiosyncratic shocks now evolve according to the following process

$$\begin{aligned} x_{it} &= x_{it-1} && \text{with probability } 1-\varpi \\ &= \rho x_{it-1} + u_{it}^{x} && \text{with probability } \quad \varpi \qquad\qquad x = \{b, z\}\,. \end{aligned}$$

We parameterize this version of the model as follows. We hold the preference and production parameters at their baseline values. To pin down the idiosyncratic shock and menu cost parameters $(\rho, \sigma^2, \phi, \varpi, \mathcal{C})$, we target, as before, (a) an average absolute monthly price change of 12% (b) a frequency of monthly price changes of 25% (c) a standard deviation of prices of 8%, but also the 10th, 25th, 50th, 75th and 90th percentile of price changes[28]. The bottom left panel of Figure 5 plots the impulse response of real consumption spending to a positive nominal shock in this version. The top left panel is taken from Figure 2 and contains the corresponding results from the menu cost model in section 4.2. We see that, relative to that baseline version, this alternative calibration strategy noticeably increases the relevance of dispersed information, particularly at longer horizons. The intuition rests on the weakening of the selection effect, which occurs exactly for the reasons pointed out by Midrigan (2011). A calibration strategy which matches the heterogeneity in price changes has a much smaller measure of 'marginal' firms close to their adjustment thresholds. As a result, the effect of the change in the mix of adjusting firms induced by a nominal shock is also smaller. While this feature tends to raise real effects in both the full and dispersed information economies, it is amplified through an informational channel in the latter. Slower adjustment of aggregate prices implies that firm-level signals reflect the aggregate shock slowly (through the presence of strategic complementarities, see discussion in Section 4.1). This exacerbates the sluggishness in response of prices and leads to larger, more persistent real effects. To bring this out more clearly, we repeat the exercise without strategic complementarities, i.e. with $r = 0$ and present results in the right two panels. Now, the relevance of dispersed information under the Midrigan calibration is not that different from what we saw in section 4.2.

Parameter choices: Finally, in Figure 6, we examine the effects of varying the degree of price stickiness, ξ. The left panel reproduces the baseline version, where the target for monthly price

[28]The targets are taken from Midrigan (2011) are 0.03, 0.05, 0.09, 0.13 and 0.21. The resulting parameter estimates are $(\rho, \sigma, \phi, \varpi, \mathcal{C}) = (0.8, 0.315, 0.35, 0.05, 1)$.

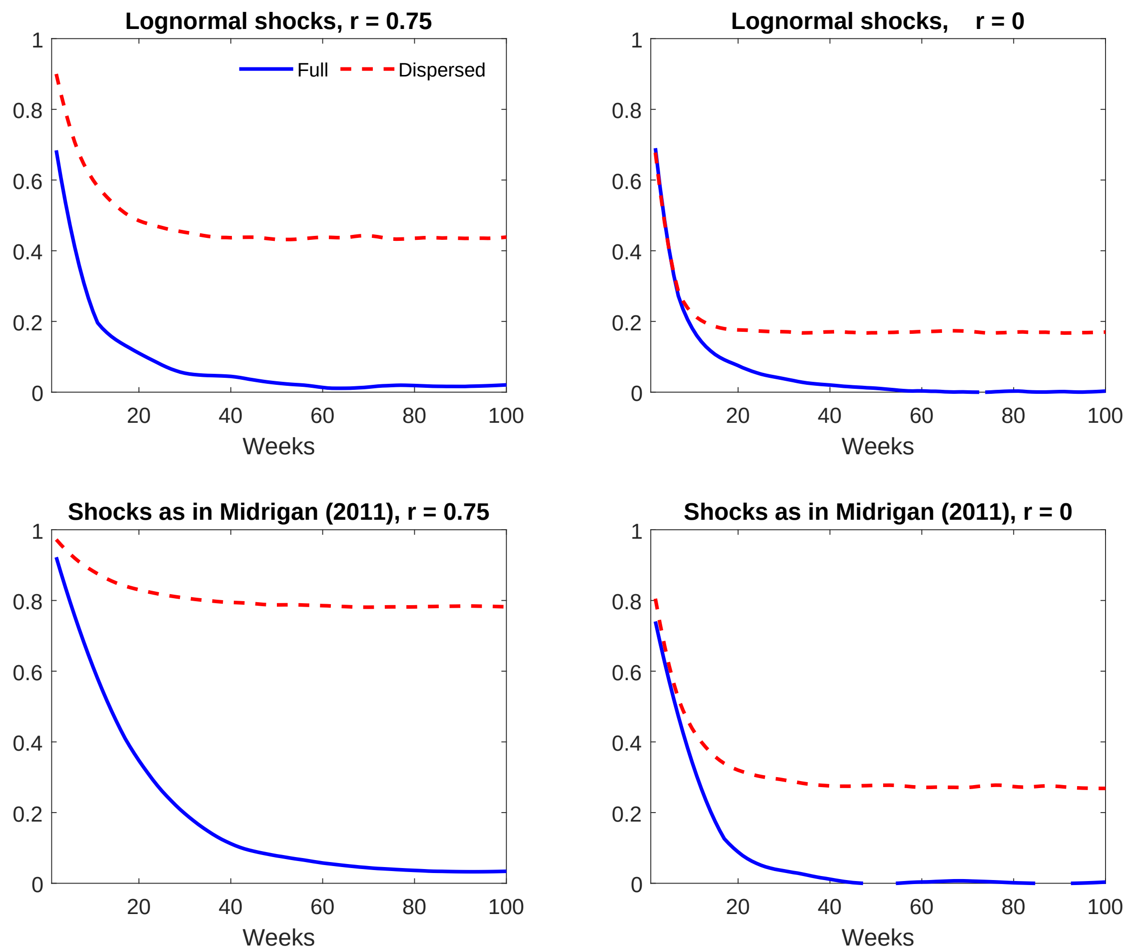

Figure 5: Impulse responses of real consumption spending to a nominal shock in a menu cost model. The top two panels are reproduced from Figure 2.

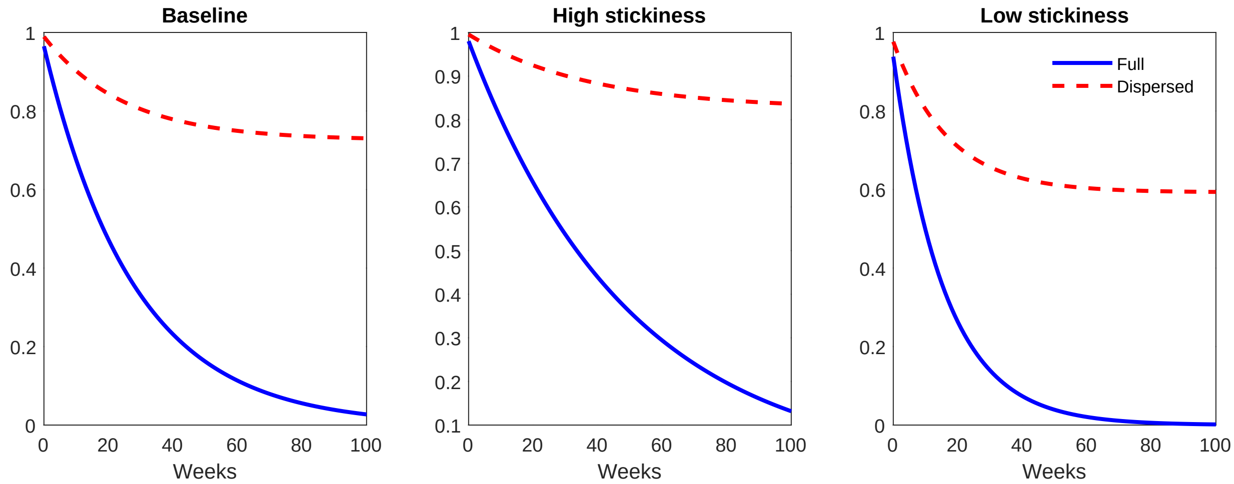

Figure 6: Impact of a positive monetary shock on real consumption spending under Calvo pricing. The baseline panel is reproduced from Figure 1.

changes was 25 percent. In the other two panels, that target is changed to 15 and 40 percent. The pattern is all three figures is similar, showing that our main conclusions about the relevance of dispersed information are robust to reasonable changes in the degree of stickiness.

5 Conclusion

In this paper we offer a novel perspective on the respective roles of incomplete information and sticky prices for monetary non-neutrality. At the heart of our analysis is the assumption that firms update and respond to the information that emerges from their market activities. We establish the existence of a "Hayekian benchmark", at which the incorporation of information solely from market activities results in the same outcome as if everyone had full information. This occurs when prices are completely flexible or prices are sticky but there are no pricing complementarities and aggregate and idiosyncratic shocks have the same degree of persistence. Our framework, particularly our modeling of firm-level information as emerging from market activities, further has the advantage that it tightly connects the micro- and macroeconomic properties of price adjustment. By calibrating the key parameters to match micro moments of price adjustment, we were thus able to explore quantitatively the potential for complementarities and differential rates of persistence to generate monetary non-neutrality.

The original signal extraction model by Lucas (1972) already contains two key elements that we identified as important for departures from the Hayekian benchmark: forward-looking decisions and differential degrees of persistence.[29] Lucas (1972) considers an OLG model in which agents produce while young and save their earnings to consume when they are old, and when deciding how much to produce the young are unable to distinguish *temporary* fluctuations in the relative price of their output from *permanent* changes in aggregate money supply, with the nominal price of their output providing a noisy signal that doesn't separate between the two. Wallace (1992) simplifies the exposition of Lucas' signal extraction model by replacing the relative price fluctuations with an aggregate real shock to the size of each generation. One can show that if real and monetary shocks are both permanent (i.e. population growth, rather than population levels is iid over time, just like money supply), they have the same impact on the agents' expected future purchasing power and hence their current incentives to produce, and therefore the equilibrium of the incomplete information economy studied by Wallace is the same as with perfect information. Hence when agents need to forecast the future real value of their labor income from the nominal price level, as in the models studied by Lucas (1972) and Wallace (1992), differential persistence of real and monetary shocks is critical for generating meaningful non-neutrality from the signal extraction problem, consistent with the conditions of our dynamic Hayekian benchmark result.

The departures from the Hayekian benchmark can also be interpreted through the lens of the beauty contest analogy in Keynes (1936), where he writes : *"Thus, certain classes of investment are governed by the average expectation of those who deal on the stock market, as revealed in the price, rather than by the genuine expectations of the professional entrepreneur."* Keynes draws the distinction between the valuation signals generated in markets and the underlying "fundamental" values (those of the professional entrepreneur) that emerges in a context of uncertainty about economic conditions. Although Keynes focused on investment, not pricing decisions, the beauty contest analogy implicitly refers to the same key elements that govern departures from the Hayekian

[29] We thank our discussant, Yu-Ting Chiang, for drawing our attention to this connection.

benchmark: forward-looking investment decisions which require agents to forecast future market conditions, complementarities that introduce a need to forecast other investors' expectations, and the focus on stock markets in which there tends to be a significant and persistent *common value* component. Our framework thus also allows us to draw a formal connection between the rather contrasting views on the informational role of markets and prices expressed in the writings of Hayek and Keynes, and to identify the implicit assumptions on which these views are based.

There are several avenues for future research. To keep the focus squarely on the firms' price-setting problem, we abstracted, as does most of the New Keynesian literature, from frictions on households. A natural next step is to examine the strength of the Hayekian mechanism when both households and firms are imperfectly informed. An alternative direction consists in adding other shocks, especially ones that interfere with the Hayekian mechanism.[30] Finally, our analysis reveals the crucial role played by forward-looking decisions in generating a role for informational frictions in a market economy. These insights about the interaction of dynamic decisions and imperfect information are also applicable to other forward-looking decisions, among other things, investment, savings, or hiring.[31]

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[30] An obvious example would be mark-up shocks. When the optimal markup is stochastic, firms' sales and wage signals are no longer sufficient for their optimal price, even in the absence of nominal frictions.

[31] For other work exploring implications of market-based information, see Atolia and Chahrour (2020) for a multi-sector investment model and Venkateswaran (2014) for a labor market application.

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Appendix A Proofs of results

A.1 Proposition 1:

The result follows immediately from the fact that the full information optimal price (5) is a supply function that is measurable with respect to the firm's information set under dispersed information.

A.2 Proposition 2:

Total revenue and cost in period t are

$$\begin{aligned}\text{Total Revenue}_{it} &= \ B_{it}M_t^{\frac{1}{\psi}}P_t^{\theta-\frac{1}{\psi}}\ P_{it}^{1-\theta} \\ \text{Total Cost}_{it} &= \frac{1}{\delta}\ M_t Z_{it}\ (B_{it}M_t^{\frac{1}{\psi}}P_t^{\theta-\frac{1}{\psi}})^{\delta}\ P_{it}^{-\theta\delta}\end{aligned}$$

When $\theta = \frac{1}{\psi}$, these variables no longer depend on P_t. We will exploit this property in our discussion of each of the cases below. Since the full information and dispersed information economies only differ in the intermediate producers' information in the pricing problem, it suffices to show that dispersed information does not affect their optimal prices to show that it is irrelevant for equilibrium allocations.

Case I: When $\theta = \frac{1}{\psi}$, the first order condition of the firm is

$$\underbrace{\frac{\theta-1}{\theta}\ P_{it+N}^{-\theta}\ \mathbb{E}_{it}[\lambda_{t+N}B_{it+N}M_{t+N}^{\frac{1}{\psi}}]}_{\text{Exp. Marginal Revenue}} = \underbrace{P_{it+N}^{-\theta\delta-1}\mathbb{E}_{it}[\lambda_{t+N}M_{t+N}Z_{it+N}(B_{it+N}M_{t+N}^{\frac{1}{\psi}})^{\delta}\ ]}_{\text{Exp. Marginal Cost}}$$

Since $\lambda_{t+N} = \hat{\imath}\beta^{t+N}/M_{t+N}$, the optimal price given by

$$P_{it+N}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{\mathbb{E}_{it}[(B_{it+N}M_{t+N}^{\frac{1}{\psi}})^{\delta}\ Z_{it+N}]}{\mathbb{E}_{it}[B_{it+N}M_{t+N}^{\frac{1}{\psi}-1}]} = \frac{\theta}{\theta-1} \cdot \frac{\mathbb{E}_{it}\left(e^{\delta b_{it+N}+\delta/\psi m_{t+N}+z_{it+N}}\right)}{\mathbb{E}_{it}\left(e^{b_{it+N}+(1/\psi-1)m_{t+N}}\right)}.$$

If shocks are permanent, i.e. $\rho_b = \rho_z = 1$, this becomes

$$P_{it+N}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{(B_{it}M_t^{\frac{1}{\psi}})^{\delta} \cdot \mathbb{E}_{it}\left(Z_{it}e^{\tilde{V}_{it}}\right)}{(B_{it}M_t^{\frac{1}{\psi}}) \cdot \mathbb{E}_{it}\left(M_t^{-1} \cdot e^{\tilde{U}_{it}}\right)}$$

where $\tilde{U}_{it}$ and $\tilde{V}_{it}$ are functions of $\{u_{t+s}, u_{it+s}, v_{it+s}\}_{s=1}^{N}$, which are independent of both $\mathcal{I}_{it}^{Full}$ and $\mathcal{I}_{it}^{Disp}$. Hence we have

$$P_{it+N}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{(B_{it}M_t^{\frac{1}{\psi}})^{\delta}\mathbb{E}_{it}\left(Z_{it}\right)}{(B_{it}M_t^{\frac{1}{\psi}})\mathbb{E}_{it}\left(M_t^{-1}\right)} \cdot \frac{\mathbb{E}_{it}\left(e^{\tilde{V}_{it}}\right)}{\mathbb{E}_{it}\left(e^{\tilde{U}_{it}}\right)}$$

Because of independence, the ratio $\mathbb{E}_{it}\left(e^{\tilde{V}_{it}}\right)/\mathbb{E}_{it}\left(e^{\tilde{U}_{it}}\right)$ is the same under full and dispersed information. Moreover, since $Z_{it} = W_{it}/M_t$, it also follows that $\mathbb{E}_{it}\left(Z_{it}\right) = W_{it}/\mathbb{E}_{it}\left(M_t^{-1}\right)$ under both full information and dispersed information. Hence under both informational assumptions the optimal price can be written as

$$P_{it+N}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{(B_{it}M_t^{\frac{1}{\psi}})^{\delta}W_{it}}{(B_{it}M_t^{\frac{1}{\psi}})} \cdot \frac{\mathbb{E}_{it}\left(e^{\tilde{V}_{it}}\right)}{\mathbb{E}_{it}\left(e^{\tilde{U}_{it}}\right)},$$

with all terms being the same under full information and dispersed information. Therefore, the optimal price is the same in both economies.

Case II: When a price is set for N periods at a time, the optimal price is characterized by

$$P_{it}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{\sum_{s=0}^{N}\mathbb{E}_{it}[\lambda_{t+s}(B_{it+s}M_{t+s}^{\frac{1}{\psi}})^{\delta}\ \ M_{t+s}Z_{it+s}]}{\sum_{s=0}^{N}\mathbb{E}_{it}[\lambda_{t+s}B_{it+s}M_{t+s}^{\frac{1}{\psi}}]} = \frac{\theta}{\theta-1} \cdot \frac{\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}[(B_{it+s}M_{t+s}^{\frac{1}{\psi}})^{\delta}\ Z_{it+s}]}{\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}[B_{it+s}M_{t+s}^{\frac{1}{\psi}-1}]}.$$

When all shocks are permanent, we rewrite the numerator and denominator as

$$\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}[(B_{it+s}M_{t+s}^{\frac{1}{\psi}})^{\delta}\ Z_{it+s}] = e^{\delta(b_{it}+\frac{1}{\psi}m_t)}\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}\left[Z_{it}e^{\hat{V}_{it+s}}\right] = e^{\delta(b_{it}+\frac{1}{\psi}m_t)}\mathbb{E}_{it}\left[Z_{it}\right]\sum_{s=0}^{N}\beta^s\mathbb{E}_{it}e^{\hat{V}_{it+s}}$$

$$\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}[B_{it+s}M_{t+s}^{\frac{1}{\psi}-1}] = e^{b_{it}+\frac{1}{\psi}m_t}\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}\left[M_t^{-1}e^{\hat{U}_{it+s}}\right] = e^{b_{it}+\frac{1}{\psi}m_t}\mathbb{E}_{it}\left[M_t^{-1}\right]\sum_{s=0}^{N}\beta^s\ \mathbb{E}_{it}e^{\hat{U}_{it+s}}$$

where the first equality uses the fact that $b_{it} + \frac{1}{\psi}m_t$ is fully observable under dispersed information, and the second equality uses the fact that Z_{it} and M_t^{-1} are independent of the random variables

$\hat{V}_{it+s}$ and $\hat{U}_{it+s}$ that only depend on future realizations of shocks. As in Case I, $W_{it} = Z_{it}M_t$ implies that $W_{it}\mathbb{E}_{it}\left[M_t^{-1}\right] = \mathbb{E}_{it}\left[Z_{it}\right]$ under both full and dispersed information. Substituting both expressions into the FOC for the optimal price, we obtain

$$P_{it}^{1-\theta+\theta\delta} = \frac{\theta}{\theta-1} \cdot \frac{(B_{it}M_t^{\frac{1}{\psi}})^{\delta} W_{it}}{(B_{it}M_t^{\frac{1}{\psi}})} \cdot \frac{\sum_{s=0}^{N} \beta^s \mathbb{E}_{it} e^{\hat{V}_{it+s}}}{\sum_{s=0}^{N} \beta^s \, \mathbb{E}_{it} e^{\hat{U}_{it+s}}}$$

The first term is constant, representing the mark-up, the second term is fully observed under both full and dispersed information, and the third ratio only depends on future realizations of shocks and is hence again the same under full and dispersed information. Thus, the firm's optimal pricing decision is not affected by dispersed information.

Case III: With Calvo pricing, the optimal reset price is given by

$$\begin{aligned} P_{it}^{1-\theta+\theta\delta} &= \frac{\theta}{\theta-1} \cdot \frac{\sum_{T=1}^{\infty} \, (1-\xi)^{T-1}\xi \, \left(\sum_{s=0}^{T-1} \, \mathbb{E}_{it}[\lambda_{t+s} \, (B_{it+s}M_{t+s}^{\frac{1}{\psi}})^{\delta} \;\; M_{t+s}Z_{it+s}]\right)}{\sum_{T=1}^{\infty} \, (1-\xi)^{T-1}\xi \, \left(\sum_{s=0}^{T-1} \mathbb{E}_{it}[\lambda_{t+s} \, B_{it+s}M_{t+s}^{\frac{1}{\psi}}]\right)} \\ &= \frac{\theta}{\theta-1} \cdot \frac{\sum_{T=1}^{\infty} \, (1-\xi)^{T-1}\xi \, \left(\sum_{s=0}^{T-1} \, \beta^s \, \mathbb{E}_{it}[(B_{it+s}M_{t+s}^{\frac{1}{\psi}})^{\delta} \;\; Z_{it+s}]\right)}{\sum_{T=1}^{\infty} \, (1-\xi)^{T-1}\xi \, \left(\sum_{s=0}^{T-1} \beta^s \, \mathbb{E}_{it}[B_{it+s}M_{t+s}^{\frac{1}{\psi}-1}]\right)} \end{aligned}$$

where ξ is the (exogenous) probability of resetting prices in any given period. It is easy to see that the logic of the proof for Case II goes through exactly.

Case IV: The firm's nominal profits $P_{it}Y_{it} - W_{it}N_{it}$ can be written as a function of its price P_{it} and the concurrent market conditions $\mathcal{I}_{it}^{Market} = \left(B_{it}M_t^{1/\psi}P_t^{\theta-1/\psi}, W_{it}\right)$, i.e. we write $\Pi\left(P_{it}, \mathcal{I}_{it}^{Market}\right)$. Under full information ($\mathcal{I}_{it}^{Full} \equiv \{B_{it-s}, M_{t-s}, Z_{it-s}\}_{s=0}^{\infty}$), the firm's optimal pricing strategy solves the following Bellman equation, for $v\left(P_{it-1}, \mathcal{I}_{it}^{Full}\right) \equiv V\left(P_{it-1}, \mathcal{I}_{it}^{Full}\right)/\lambda_t$:

$$v\left(P_{it-1}, \mathcal{I}_{it}^{Full}\right) = \max_{P_{it}} \left\{ \Pi\left(P_{it}, \mathcal{I}_{it}^{Market}\right) - \mathcal{C}W_{it} \cdot \mathbb{I}_{P_{it}\neq P_{it-1}} + \beta\mathbb{E}\left(\frac{M_t}{M_{t+1}} v\left(P_{it}, \mathcal{I}_{it+1}^{Full}\right) \middle| \mathcal{I}_{it}^{Full} \right) \right\}.$$

Let $\mathcal{V}^{Full}$ denote the set of functions $w\left(P_{it}, \mathcal{I}_{it}^{Full}\right)$ that are measurable with respect to $\mathcal{I}_{it}^{Full}$, and let $\mathcal{T}^{Full} : \mathcal{V}^{Full} \to \mathcal{V}^{Full}$ denote the full information Bellman operator on $\mathcal{V}^{Full}$:

$$\mathcal{T}^{Full} w\left(P_{it}, \mathcal{I}_{it}^{Full}\right) = \max_{P_{it}} \left\{ \Pi\left(P_{it}, \mathcal{I}_{it}^{Market}\right) - \mathcal{C}W_{it} \cdot \mathbb{I}_{P_{it}\neq P_{it-1}} + \beta\mathbb{E}\left(\frac{M_t}{M_{t+1}} w\left(P_{it}, \mathcal{I}_{it+1}^{Full}\right) \middle| \mathcal{I}_{it}^{Full} \right) \right\}.$$

Let $\mathcal{V}^{Disp} \subseteq \mathcal{V}^{Full}$ denote the set of functions $w\left(P_{it}, \mathcal{I}_{it}^{Disp}\right)$ that are measurable with respect to $\mathcal{I}_{it}^{Disp} \equiv \{B_{it-s}M_{t-s}^{1/\psi}P_{t-s}^{\theta-1/\psi}\,,\; W_{it-s}\}_{s=0}^{\infty}$, and $\mathcal{V}^{Market} \subseteq \mathcal{V}^{Disp}$ the set of functions $w\left(P_{it}, \mathcal{I}_{it}^{Market}\right)$

that are measurable with respect to the contemporaneous market signals $\mathcal{I}_{it}^{Market}$. We show that when $\theta = 1/\psi$ (no complementarities) and $\rho_b = \rho_z = 1$ (equal persistence), the full information Bellman operator $\mathcal{T}^{Full}$ maps $\mathcal{V}^{Market}$ into itself, and therefore the unique fixed point of $\mathcal{T}^{Full}$ must reside in $\mathcal{V}^{Market}$. It follows that the optimal pricing strategy under full information is measurable with respect to $\mathcal{I}_{it}^{Market}$, and hence remains implementable under dispersed information; hence dispersed information remains irrelevant for optimal price adjustment.

Formally, when $\theta = 1/\psi$ and $\rho_b = \rho_z = 1$ (equal persistence), we have, for any $w \in \mathcal{V}^{Market}$,

$$\mathbb{E}\left(\frac{M_t}{M_{t+1}} w\left(P_{it}, \mathcal{I}_{it+1}^{Market}\right) \middle| \mathcal{I}_{it}^{Full}\right) = \mathbb{E}\left(e^{-u_{t+1}} w\left(P_{it}, B_{it+1} M_{t+1}^{1/\psi}, W_{it+1}\right) \middle| \mathcal{I}_{it}^{Full}\right)$$
$$= \mathbb{E}\left(\widetilde{w}\left(P_{it}, \mathcal{I}_{it}^{Market}, u_{t+1}, u_{it+1}^b, u_{it+1}^z\right) \middle| \mathcal{I}_{it}^{Full}\right),$$

where $\widetilde{w}\left(P_{it}, \mathcal{I}_{it}^{Market}, u_{t+1}, u_{it+1}^b, u_{it+1}^z\right) \equiv e^{-u_{t+1}} w\left(P_{it}, B_{it} M_t^{1/\psi} e^{u_{t+1}/\psi + u_{it+1}^b}, W_{it} e^{u_{t+1} + u_{it+1}^z}\right)$. Now, since $\left(u_{t+1}, u_{it+1}^b, u_{it+1}^z\right)$ is independent of $\mathcal{I}_{it}^{Full}$, it follows that

$$\mathbb{E}\left(\widetilde{w}\left(P_{it}, \mathcal{I}_{it}^{Market}, u_{t+1}, u_{it+1}^b, u_{it+1}^z\right) \middle| \mathcal{I}_{it}^{Full}\right) = \mathbb{E}\left(\widetilde{w}\left(P_{it}, \mathcal{I}_{it}^{Market}, u_{t+1}, u_{it+1}^b, u_{it+1}^z\right) \middle| \mathcal{I}_{it}^{Market}\right).$$

Substituting this expression into the Bellman operator yields

$$\mathcal{T}^{Full} w\left(P_{it}, \mathcal{I}_{it}^{Full}\right)$$
$$= \max_{P_{it}} \left\{\Pi\left(P_{it}, \mathcal{I}_{it}^{Market}\right) - \mathcal{C} W_{it} \cdot \mathbb{I}_{P_{it} \neq P_{it-1}} + \beta \mathbb{E}\left(\widetilde{w}\left(P_{it}, \mathcal{I}_{it}^{Market}, u_{t+1}, u_{it+1}^b, u_{it+1}^z\right) \middle| \mathcal{I}_{it}^{Market}\right)\right\}.$$

Since $W_{it} \in \mathcal{I}_{it}^{Market}$, it follows that the optimal price P_{it}, as well as $\mathcal{T}^{Full} w\left(P_{it}, \mathcal{I}_{it}^{Full}\right)$ are both measurable w.r.t. $\mathcal{I}_{it}^{Market}$, which implies that $\mathcal{T}^{Full}$ maps $\mathcal{V}^{Market}$ into itself. Therefore, $\mathcal{T}^{Full}$ admits a unique fixed point $v^* \in \mathcal{V}^{Market} \subseteq \mathcal{V}^{Full}$, and the optimal price policy $P^*\left(P_{it-1}, \mathcal{I}_{it}^{Full}\right)$ under full information is also measurable w.r.t. $\mathcal{I}_{it}^{Market}$. Hence dispersed information is again irrelevant, in the case of menu costs, no complementarities and equal persistence.

A.3 Firm's objective under Calvo pricing

We begin by deriving a log-quadratic approximation of the firm's discounted profits:

$$(1 - \theta + \theta\delta) \sum_{s=0}^{\infty} \beta^{t+s} \left\{\left[\left(\frac{\delta - 1}{1 - \theta + \theta\delta}\right)\left(\hat{y}_{t+s} + \hat{b}_{it+s} + \theta \hat{p}_{t+s}\right) + \left(\frac{1}{1 - \theta + \theta\delta}\right) \hat{w}_{it+s}\right] \hat{p}_{it+s} - \frac{1}{2} \hat{p}_{it+s}^2\right\}$$

where $\hat{x}$ denotes the log-deviation of the variable x from its steady state value. Using the equilibrium conditions from the household's problem, we substitute $\hat{y}_{t+s} = (\hat{m}_{t+s} - \hat{p}_{t+s})/\psi$ and $\hat{w}_{it+s} = \hat{m}_{t+s} + \hat{z}_{it+s}$ into the expression for profits to obtain the expression in the text

$$(1-\theta+\theta\delta)\sum_{s=0}^{\infty}\beta^{t+s}\left\{\mathcal{P}^*_{it+s}\hat{p}_{it+s} - \frac{1}{2}\hat{p}^2_{it+s}\right\}$$

where

$$\begin{aligned}\mathcal{P}^*_{it+s} &\equiv (1-r)\,\hat{m}_{t+s} + r\hat{p}_{t+s} + \left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\hat{b}_{it+s} + \left(\frac{1}{1-\theta+\theta\delta}\right)\hat{z}_{it+s} \\ r &= \frac{\delta-1}{1-\theta+\theta\delta}\left(\theta-\frac{1}{\psi}\right)\end{aligned}$$

In every reset period, the firm solves

$$\max_{\hat{p}_{it}} \quad \sum_{T=1}^{\infty}(1-\xi)^{T-1}\xi \sum_{s=0}^{T-1}\beta^s \left(\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right]\hat{p}_{it} - \frac{1}{2}\hat{p}^2_{it}\right)$$

A.4 Proof of proposition 3

From (14), the average rest price in period t is

$$\int \hat{p}^*_{it}\ di = (1-\beta+\beta\xi)\sum_{s=0}^{\infty}(1-\xi)^s\beta^s\int\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right]di.$$

As aggregate shocks become arbitrarily small, firms attribute all variations in their signals to the idiosyncratic shocks, i.e. $\mathbb{E}_{it}\left(\hat{b}_{it}|s^1_{it},s^2_{it}\right) = s^1_{it}$ and $\mathbb{E}_{it}\left(\hat{z}_{it}|s^1_{it},s^2_{it}\right) = s^2_{it}$. Then, the expectation of the future target $\mathcal{P}^*_{it+s}$ is

$$\begin{aligned}\mathbb{E}_{it}\left[\mathcal{P}^*_{it+s}\right] &= \left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\rho^s\mathbb{E}_{it}\hat{b}_{it} + \left(\frac{1}{1-\theta+\theta\delta}\right)\rho^s\mathbb{E}_{it}\hat{z}_{it} \\ &= \left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\rho^s s^1_{it} + \left(\frac{1}{1-\theta+\theta\delta}\right)\rho^s s^2_{it}\end{aligned}$$

Integrating across resetting firms, we have $\int\mathbb{E}_{it}\left(\hat{b}_{it+s}\right)di = \rho^s s^1_{it} = \rho^s\left[\frac{1}{\psi}\hat{m}_t + \left(\theta-\frac{1}{\psi}\right)\hat{p}_t\right]$ and $\int\mathbb{E}_{it}\left(\hat{z}_{it+s}\right)di = \rho^s\Delta_{t+s}$. Substituting these integrals into the condition for the average reset price, we obtain

$$\begin{aligned}\int\hat{p}^*_{it}\ di &= (1-\beta+\beta\xi)\sum_{s=0}^{\infty}(1-\xi)^s\beta^s\rho^s\left\{\left(\frac{\delta-1}{1-\theta+\theta\delta}\right)\left[\frac{1}{\psi}\hat{m}_t + \left(\theta-\frac{1}{\psi}\right)\hat{p}_t\right] + \frac{\hat{m}_t}{1-\theta+\theta\delta}\right\} \\ &= \frac{1-\beta+\beta\xi}{1-(1-\xi)\beta\rho}\left[(1-r)\,\hat{m}_t + r\hat{p}_t\right]\end{aligned}$$

Substituting into the law of motion for the aggregate price,

$$\hat{p}_t = \xi \int \hat{p}^*_{it} \quad di \quad + \quad (1-\xi)\,\hat{p}_{t-1} = \xi \frac{1-\beta+\beta\xi}{1-(1-\xi)\beta\rho} \left[(1-r)\,\Delta + r\hat{p}_t\right] + (1-\xi)\,\hat{p}_{t-1}$$

Collecting terms, we arrive at the coefficients in Proposition 3.

Proof of Claim 1: This follows directly from the expressions for τ_1^{Full} and τ_1^{Disp}.

Proof of Claim 2: By a standard guess-and-verify approach (see, for example, Galí (2009)), the response coefficients in the full information economy satisfy

$$\tau_1^{Full} = \frac{\sqrt{\left[1-\beta+\chi\,(1-r)\right]^2 + 4\beta\chi\,(1-r)} - (1-\beta+\chi\,(1-r))}{2\beta}$$

where $\chi = \frac{\xi}{1-\xi}\,(1-\beta+\beta\xi)$, and $\tau_2^{Full} = 1-\tau_1^{Full}$. Now, it's easy to check that

$$\tau_1^{Full} > \frac{\sqrt{4\beta\chi\,(1-r)}}{2\beta} = \sqrt{\frac{\xi}{1-\xi}\,(1-\beta+\beta\xi)\,\frac{(1-r)}{\beta}} > \sqrt{\frac{\xi^2\,(1-r)}{1-\xi}}$$

Then,

$$\begin{aligned}\left(\tau_1^{Full}\right)^2 - \left(\tau_1^{Disp}\right)^2 \quad &> \quad \xi^2\left(\frac{1-r}{1-\xi}\right) - \xi^2\left(\frac{1-r}{1-\xi r}\right)^2 = \frac{\xi^2\,(1-r)\left(r^2\xi^2 - 3r\xi + r + \xi\right)}{(1-\xi)\,(1-\xi r)^2} \\ &= \quad \frac{\xi^2\,(1-r)}{(1-\xi)\,(1-\xi r)^2}\left[\xi^2\,(1-r)^2 + r\,(1-\xi)^2 + (1-r)\,\xi\,(1-\xi)\right] > 0\end{aligned}$$

This implies $\tau_2^{Disp} = 1-\tau_1^{Disp} > 1-\tau_1^{Full} = \tau_2^{Full}$.