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Interoperability and Quality Provision in Digital Payments*

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Abstract

We develop a model in which digital payment providers compete by setting fees and investing in the quality of their service. Consumers' valuation of the service depends on the fraction of other consumers who joins the same network. Providers' fees and quality investment, together with consumers' transport cost, endogenously determine the degree of market coverage and consumer surplus. We show that an unregulated monopolist charges high fees and limits its investment, resulting in lower market coverage and lower consumer surplus. Inducing competition increases consumer surplus by reducing service fees and increasing quality investments, and these effects are compounded by network externalities. Introducing interoperability allows a larger volume of transactions, but at the same time it weakens the competitive pressure on providers by limiting the strength of network effects, resulting in ambiguous welfare effects. Finally, we show how lack of interoperability is more likely to be associated to a monopolistic market structure.

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1 Introduction

In many network industries, operators need to develop physical or digital network infrastructures, and that requires significant investments both for expanding the network coverage and for assuring the quality of the service provided. These industries are also often characterized by (positive) network externalities, whereby the benefits that consumers derive from joining one network may increase with the number of consumers using the same network.

In this paper, we consider a market with network externalities and study how competition affect prices, infrastructure investments, and ultimately welfare. We are particularly interested in analyzing the effects of introducing interoperability among service providers, whereby consumers of a given network are allowed to transact with consumers in another network. While this is often seen as a tool to promote competition (Arabehety et al. (2016), Beck & De La Torre (2007), Scott-Morton et al. (2023)), its broader effects on providers' incentives to invest in network coverage and quality are much less understood (Brunnermeier et al. (2023)). We are also interested in analyzing how interoperability may affect the market structure, possibly promoting entry of new operators. The main application of our analysis is the market for digital payments; our insights however can be applied more generally to other network industries such as internet and telecommunication.

Specifically, we consider a market in which two providers offer payment services. Consumers have a downward sloping demand for these services and the surplus they derive from joining a given network is proportional to the mass of consumers who subscribe to the same network. Providers may be differentiated both horizontally and vertically. We model horizontal differentiation by assuming that consumers need to incur a transportation cost to access a given network, which can be interpreted as usual both as a taste parameter (some consumers are particularly attached to a given operator) or in terms of physical distance from a given network (consumers need to travel to be covered by the network or, in the case of mobile money, to reach a mobile money agent). Moreover, service providers can invest in improving the quality of their service, a measure of vertical differentiation. Formally, investments in quality can be seen as reducing transportation costs to the network uniformly across consumers, making it more attractive for all consumers to join a given network. We also assume it is prohibitively costly to join both networks.

Providers set prices that may include both fixed and marginal fees. The former can be interpreted not only as a (possibly negative) fee to access the network but also as the net benefits of ancillary services (e.g. telephony in the case of mobile money) that may be bundled with the payment service or, even more generally, as a (reduced form) representation of the surplus each consumer gets from joining the network on a long-term basis.

Given firm' prices and investments in network quality, consumers decide which network to join (if any) considering their transportation costs and the benefits they expect from joining a given network, which in itself depends on how many other consumers do the same. Each network's market coverage, which we define as the fraction of consumers who join a given network, is then determined endogenously as the result of the joint decisions of firms and consumers.

Our main interest is in comparing firms' pricing and investment strategies and the associated market coverage and consumer surplus across various market structures. We start with a first-best scenario in which a budget-unconstrained regulator maximizes social welfare. We contrast this benchmark with the outcomes attained by an unregulated monopoly and by duopolistic competition. We then analyze the effects of network introducing interoperability both in terms of pricing and investment but also in terms of promoting entry of new providers.

Our main results can be summarized as follows. In the first-best scenario, consumers should face a negative access fee (i.e., receive a subsidy) to participate in the market. The reason is that, due to the positive network externality, expanding participation increases the surplus enjoyed by each consumer. At the same time, marginal payments should be set at marginal costs, implying that the investment costs should be covered with public funding. The shadow cost of public funding determines how close the regulator can get to the first-best outcome. When the cost goes to zero, the first-best outcome is reached. As the cost grows very large, one comes close to a setting with an unregulated monopoly, a scenario which we consider next.

Consider then a market dominated by an unregulated monopolist. We show that the monopolist sets a larger access fee and undertakes a lower quality investment relative to the social optimum. As result, market coverage and consumers' surplus are lower than in the first-best. This effect is compounded by network externalities: when market coverage is low, each consumer finds it less attractive to join the network. Since consumers' participation is not fully elastic, however, limiting coverage is a way to increase the monopolist's profits.

Third, we consider a competitive market in which two symmetric service providers operate and the network is not interoperable. We show that competition induces a more efficient outcome. Each firm competes for attracting consumers by reducing the access fee. Competition is fierce. Attracting the marginal consumer allows a given provider not only to increase its profit (a direct business stealing effect) but also to increase the surplus of all consumers on its network, and at the same time to decrease the surplus of all consumers on its competitor's network. These two effects are driven by network externalities, which reinforce each firm's incentives to attract consumers and so increase competition over access fees. A similar mechanism also induces firms to increase their quality investment. By offering a higher quality service, the firm can attract consumers on its network, which again induces both a direct and a network effect on its profits.

We then introduce interoperability among service providers, which allows consumers to transact within and across networks. We allow for varying degrees of interoperability, from a scenario in which off-net transactions are possible but still subject to frictions (e.g. due to higher costs for off-net than for on-net transactions) to an ideal case in which consumers transact in the same way with all other consumers irrespective of their network. In this setting, interoperability has two effects. First, it improves consumers' surplus by expanding the set of transactions each consumer can have access to. At the same time, however, interoperability softens competition among providers. The reason is that since now consumers have access to both networks, each firm has less incentives to increase its own network. Using the above terminology, each firm still faces a direct business stealing effect, but now network externalities are much weaker, and this induces firms to compete less aggressively both on fees and on investment levels.

As a result, we show that interoperability can increase consumer surplus only if it is associated with a significant increase in the volume of transactions. Keeping the volume of transactions fixed, only the weakening of network effects operates, resulting in a lower consumer surplus relative to the case in which the two networks are not interoperable. At the same time, as we show with a simple example, the effect of increased volumes of transactions may be strong enough to dominate and lead to an increased consumer surplus.

Another important dimension of interoperability is that it can affect the market structure. To investigate this possibility, we consider two ex-ante symmetric firms deciding whether or not to enter the market, which requires paying a fixed entry cost. We show that markets with lower degrees of interoperability are more likely to feature a monopolistic service provider and, conversely, increasing interoperability increases the incentives for firms to enter the market. In this sense, interoperability has clearly a pro-competitive effect. Notice however that the reason why interoperability facilitates entry is that it allows firms to *soften* price competition once in the market.

This paper builds on a substantial literature in IO and competition policy that investigates the effects of various regulatory interventions on firms' incentives to invest in network infrastructures (see e.g. Vogelsang (2003); Cambini & Jiang (2009); Briglauer et al. (2014) for comprehensive reviews). We also relate to the literature on optimal pricing of communications (e.g. Laffont et al. (1997), Laffont & Tirole (2001), Hermalin & Katz (2004)) and of payments (e.g. Donze & Dubec (2009), Massoud & Bernhardt (2002)). Bianchi et al. (2023) provide a discussion of how the insights developed in this literature can be applied to the markets for digital payments.

Our paper is also related to the literature on competition between platforms (see Armstrong (2006) and Jullien & Sand-Zantman (2021) for an overview). This literature is mostly interested in the effects of competition on pricing in two-sided markets while the sole network externality we consider is on the consumers' side. On the other hand, we broaden the scope of the analysis by also investigating the consequences of competition on investments and market coverage. Related to interoperability, the received theory is indeed that interoperability intensifies competition (Crémer et al. (2000)) but this result is due to the fact that investment and market coverage are fixed. When investment is endogenized, the size of the overall network becomes a public good, which is underprovided, and this underprovision may actually soften competition.

Most closely related to our study, Björkegren (2022) studies the development of mobile phone market in Rwanda to quantify the effect of competition on investment and welfare. He shows that competition tends to lower prices and to increase the investment to expand market coverage, and that the latter effect may be weakened by a regulation mandating interoperability. Brunnermeier et al. (2023) analyse the effects of introducing interoperability across several African countries. They show that interoperability, interpreted as increased competition among providers, has reduced fees but at the same it has also reduced coverage. Since operating a mobile infrastructure can have important variable costs, providers prefer to cut coverage when profit margins are decreased due increased competition. Our framework is different in two respects. First, we distinguish the effects of inducing competition from those of introducing interoperability, showing that the latter may result in softer competition. Second, we consider investment in networks that increase the quality of the network for all consumers, rather than investments affecting only the marginal consumer. As we show, firms' incentives may differ substantially for these types of investments.

2 The Model

Consider two firms, respectively denoted by F and F^*, which provide their services to a population of consumers whose mass is normalized to one. These firms invest in network coverage so as to facilitate access to their services. A transaction is charged at a unit price p (resp. p^*) by F (resp. F^*). For simplicity, both firms are symmetric and face the same marginal cost of service c. Possibly, F (resp. F^*) may also charge its consumers a fixed fee T (resp. T^*). We have in mind a rather general interpretation of such fees, which should be viewed not only as a subscription fee but also as the net benefits of ancillary services that firms are offering to their customers, a relevant interpretation when those firms are say telecom or internet providers that also offer other services. Those fees can also be viewed as a reduced form for how the surplus is shared between the service provider and its customers in an ongoing relationship. We make no assumption on the sign of this fee, allowing also for subsidies to poach customers.

CONSUMERS. Given a unitary price p, a given consumer has a demand $D(p)$ for transactions with any other connected consumer he can join on his affiliated network N (where $D' < 0$). There is a volume Q of such connected consumers to this network.[1] The consumer's overall demand on network N is thus $QD(p)$. We denote the demand-elasticity by $\varepsilon_D(p) = -\frac{pD'(p)}{D(p)}$ and assume that $-\frac{D(p)}{D'(p)}$ is non-increasing to ensure quasi-concavity of some of the maximization problems below.

We shall thus define overall consumer surplus as

$$QS(p) = Q \int_p^{\infty} D(v)dv. \tag{2.1}$$

where $S(p)$ denotes the *per transaction* surplus. The fact that the per capita surplus (2.1) depends on the coverage Q captures the existing network externality that is an important feature of the market for mobile money. As more consumers are affiliated

[1]Later, we will consider the scenario where the consumer may also transact with other consumers who are actually connected to the alternative network N^*.

to network N, this per capita surplus certainly increases.

A key aspect of this market is that, in most rural areas, consumers face significant transportation costs to access neighborhoods that might be covered by the network. Accordingly, we denote by ε (resp. ε^*) the transportation cost to network N (resp. N^*) that is incurred by a given consumer. Consumers are heterogeneous in terms of those costs. Our model has of course a broader applicability. In more developed economies, those transportation costs can instead be viewed as intrinsic shocks on preferences or as psychological biases that could have been induced by advertising campaigns. These biases thus capture a dimension of horizontal differentiation across firms.

The transportation costs ε and ε^* are independently and identically drawn from a distribution H which has an atomless and positive density $h = H'$ and support $[0, \bar{\varepsilon}]$.[2] We also assume that H satisfies the familiar *Monotone Hazard Rate Property*, i.e., $\frac{d}{dy}\left(\frac{H(y)}{h(y)}\right) \geq 0$.[3]

By investing i in market coverage, F can reduce the transportation costs that consumers incur when willing to reach its own network. An alternative interpretation is that i could also be a vertical improvement in the quality of the service, maybe under the form of better after-sales services, customized offers, additional benefits on mobile telecommunications that the service provider could also offer. Formally, the net surplus per consumer from visiting and transacting through network N thus writes as

$$QS(p) - T + i - \varepsilon.$$

In the scenario where F is the sole service provider, the next best option for consumers is not to transact at all, an option that yields zero utils to consumers. From an *ex ante* viewpoint, i.e., before transportation costs realize, the overall consumer surplus

[2] To facilitate some of the derivations below, we might sometimes suppose that the distribution H has unbounded support (i.e., $\bar{\varepsilon} = +\infty$). The analysis could be generalized to the case where transportation costs towards different networks are correlated with some additional technicalities.

[3] Bagnoli & Bergstrom (2005).

writes thus as

$$\mathbb{E}_{\varepsilon}\big[\max\{QS(p) - T + i - \varepsilon, 0\}\big] = \int_0^{R(Q,p,i-T)} H(\varepsilon)d\varepsilon. \tag{2.2}$$

The quantity $R(Q, p, i - T) = QS(p) + i - T$ denotes the transportation cost of the marginal consumer who is just indifferent between joining or not network N. It is worth stressing that this threshold $R(Q, p, i - T)$ depends on the difference between F 's investment to expand market coverage and the lump-sum payment T it charges to consumers which, of course, reduces participation. Later, we will further investigate how those tools can be jointly used by the service providers under various competitive scenarios.

The randomness in the participation decision of consumers allows us to define F 's market share, often referred to as *market coverage* in the sequel, as

$$Q = H\left(R(Q, p, i - T)\right). \tag{2.3}$$

The so obtained market share Q thus comes as a fixed-point. Given a pricing policy (p, T) and an investment i, condition (2.3) determines a market share. A sufficient condition to ensure existence, uniqueness of such a solution and simple comparative statics, is that the following Assumption holds:

Assumption 1.

$$1 - S(c)\left(\max_y h(y)\right) > 0.$$

Throughout, similar assumptions will be made in somehow more complex environments.

FIRMS. F offers its clientele a price package (p, T) and, simultaneously, undertakes an observable investment i that expands its market coverage. Investing i costs $\varphi(i)$. We again posit that, when in duopoly, firms F and F^* are symmetric in this respect. The function φ is increasing and convex ($\varphi' > 0$, $\varphi'' > 0$) and it satisfies the usual Inada condition $\varphi(0) = \varphi'(0) = 0$ to ensure interior solutions.

Market structures. In the sequel, we shall consider the following market structures.

- *Fully regulated industry.* In this hypothetical benchmark, a regulator runs the service on her own so as to maximize social welfare with no restriction in the choice of the policy instruments (p, T, i) she may use.

- *Unregulated monopoly.* Firm F is the sole service provider, and it is left unregulated.

- *Duopolistic competition.* F and F^* compete in price package and investment. Consumers decide to visit either network or none. Either firm can be a monopoly on a subset of captive consumers while they may compete more fiercely on those consumers who are less captive and may subscribe to either network.

Finally, the nature of competition depends on whether or not the two networks are inter-operable, that is, whether firms allow reciprocal access to their own network, possibly because they are forced to do so by regulation. interoperability a priori increases the volume of transactions that each individual may undertake, but it also affects prices and the incentives to invest in network quality.

3 Optimal Regulation

In this section, we consider a benchmark scenario with a single regulated firm F and in which the market is only partially covered. This scenario serves as an important benchmark for analyzing how various forms of competition might modify prices, market coverage and welfare.

Given that there is a mass Q of consumers connected to network N and that each of those consumers has an overall demand for the firm's services $QD(p)$, we may write

F 's profit when a monopolist as

$$Q^2\Pi(p) + QT - \varphi(i), \tag{3.1}$$

where $\Pi(p) = (p - c)D(p)$ stands for the per transaction profit.

Anticipating some of our findings below, we should first notice that the optimal regulation of the monopolist may require marginal cost pricing and so no profit per transaction. Unfortunately, marginal cost pricing is not compatible with covering the cost of investment in market coverage. Accordingly, a public subsidy z is necessarily needed to ensure that the service provider breaks even. Of course, and it is an issue of tantamount importance, this subsidy is socially costly when public funds are themselves costly.[4] Denoting by $\mu \geq 0$ the (marginal) cost of public funds and adding up consumers surplus, firm's profit and the budgetary cost of the subsidy yields the following expression of social welfare

$$Q^2\Pi(p) + QT - \varphi(i) + z - (1 + \mu)z + \int_0^{R(Q,p,i-T)} H(\varepsilon)d\varepsilon. \tag{3.2}$$

The optimal regulation of the monopolist is then summarized in the next proposition.

Proposition 1. *Suppose that Assumption 1 holds. The optimal regulation entails the following properties.*

- *Marginal cost pricing:*

$$\hat{p} = c. \tag{3.3}$$

- *The marginal cost of investment is equal to the monopolist's market share:*

$$\varphi'(\hat{i}) = \hat{Q}. \tag{3.4}$$

[4]This issue is of tantamount importance in developing countries. See Auriol & Warlters (2012) for some estimates of the social cost of public funds in African countries.

- *The optimal access fee is proportional to the volume of transactions:*

$$\hat{T} = \hat{Q}\left(\frac{\mu}{(1+\mu)h(\hat{R})} - S(c)\right). \tag{3.5}$$

- *The optimal subsidy together with the revenues raised from the access fee covers the cost of investment:*

$$\hat{z} + \hat{T}\hat{Q} = \varphi(\hat{i}) \tag{3.6}$$

where market coverage solves

$$\hat{Q} = H(\hat{R}) \tag{3.7}$$

with

$$\hat{R} = \hat{Q}S(c) + \hat{i} - \hat{T}. \tag{3.8}$$

To understand the above results, let us first consider the polar case where public funds are costless, i.e., $\mu = 0$. Then, the optimal regulation certainly achieves allocative efficiency. First, efficiency requires to operate under marginal cost pricing. Second, increasing investment by a positive marginal amount di or decreasing the fee by $dT = -di$ are two strategies that should have the same impact in raising consumers' participation by an amount $h(\hat{i} + \hat{Q}S(c) - \hat{T})di$. At the optimum, these two enhancing-participation strategies should thus have the same marginal cost for society. Raising investment by di has a marginal cost $\varphi'(\hat{i})di$ for the regulated monopolist while reducing the fee for each participating consumer foregoes revenues $\hat{Q}di$. As a result, the marginal cost of investment is, at the optimum, equal to market coverage at the optimum as requested by (3.4).

Lastly, consider expanding market coverage $\hat{Q}$ by an amount dQ. Given that the price per transaction is set at marginal cost, the firm makes no profit per transaction. On the other hand, increasing market coverage also increases the positive network externality enjoyed by each consumer. The impact on expected consumer surplus is positive and given by $H\left(\hat{Q}S(c)+\hat{i}-\hat{T}\right)S(c)dQ$.[5] At the same time, expanding market coverage also increases revenues from charging an access fee by $\hat{T}dQ$. At the optimum, it must thus be that those two marginal effects just cancel out. Using the definition of market coverage obtained in (3.7), we thus obtain

$$\hat{T} = -\hat{Q}S(c) < 0.$$

In other words, when public funds are costless, consumers participation should always be subsidized by the firm. Although a priori surprising, this effect is best understood by noticing that expanding consumers participation increases the overall surplus enjoyed by each participating consumer thanks to the positive network externality. At the optimum, the subsidy $-\hat{T} > 0$ should be equal to the overall surplus that a given consumer enjoys from transacting at marginal cost with a population of mass $\hat{Q}$ of other consumers who are themselves also connected to network N.

Because the regulated firm has to pay for the cost of investment together with the cost of inducing consumers participation but charges a per transaction price equal to marginal cost, its profit would be negative without any public subsidy. The firm can thus only break even if it receives a lump-sum subsidy $\hat{z}$ that covers those costs. This subsidy is defined as

$$\hat{z} = \varphi(\hat{i}) - \hat{T}\hat{Q}. \tag{3.9}$$

When public funds are costly, using such subsidy now comes with a budgetary cost. This burden can be diminished by reducing consumers participation. When $-\hat{T}$ decreases, market coverage $\hat{Q}$ also diminishes which in turn makes it less attractive to invest (i.e., $\hat{i}$ decreases). All those effects compound to reduce the required subsidy $\hat{z}$ and the budgetary cost of subsidizing the firm.

[5]The expectation is taken before transportation costs realize.

4 Unregulated Monopoly

Consider now the monopolistic scenario where F is the sole firm acting on the market and is left unregulated. When there is a mass Q of consumers who join network N, the monopolist gets an overall profit worth

$$Q^2\Pi(p) + QT - \varphi(i).$$

The next proposition summarizes the choice of policy instruments that such an unregulated monopoly would make.

Proposition 2. *Suppose that Assumption 1 holds. The unregulated monopolist chooses a unit price p^m, an investment level i^m, and a fee T^m that satisfy the following properties.*

- *Marginal cost pricing prevails:*

$$p^m = c, \tag{4.1}$$

- *The marginal cost of investment is equal to market coverage:*

$$\varphi'(i^m) = Q^m, \tag{4.2}$$

- *The access fee is proportional to the volume of transactions:*

$$T^m = Q^m \left(\frac{1}{h(R^m)} - S(c) \right) \tag{4.3}$$

where market coverage Q^m solves

$$Q^m = H\left(R^m\right) \tag{4.4}$$

with

$$R^m = Q^m S(c) + i^m - T^m. \tag{4.5}$$

The unregulated monopoly solution bears lots of similarities with the optimal regulation. In fact, in the extreme scenario of very large costs of public funds (i.e., μ converges towards $+\infty$), the weight of the firm's profit in the social welfare objective would become predominant and the planner, mostly concerned by raising enough revenue to cover the cost of investment, moves the optimal regulation closer to the monopoly solution. Here also, the unit price charged for each transaction remains equal to marginal cost when access fees are feasible. It comes at no surprise, the monopolist actually only relies on the fee T to extract consumer surplus; thereby inducing less distortions than what it would be by raising the price per transaction above marginal cost.

Interestingly, the monopolist adopts the same rule for the optimal investment than what the regulator would recommend. The investment decision is always optimal given market coverage. The marginal cost of investment must equal market coverage. The point is that the monopolist nevertheless covers less of the market than what an optimal regulation would recommend and thus invests less than what would be optimal.

Notice that the monopolist always charges a positive access fee $T^m > 0$. This immediately follows from Assumption 1. More intuitively, the unregulated monopolist can always guarantee itself zero profit by not investing, not charging any fee and fixing a unit price equal to marginal cost. Henceforth, a positive profit can only be reached if the monopolist charges a positive fee that must at least cover the cost of the investment which, from (4.2) is positive, whenever the market has some positive coverage, i.e.,

$$T^m Q^m \geq \varphi(i^m) > 0.$$

To further understand the optimality condition (4.3), consider increasing the access fee by an infinitesimal amount dT. This variation induces a small change in market coverage which is worth dQ. Differentiating the definition of market coverage (2.3), we actually deduce that an infinitesimal increase in the fee has a negative impact on market coverage which is given by

$$(1 - h(R^m)S(c))\, dQ = -h(R^m)dT < 0. \tag{4.6}$$

On the other hand, the variations of dQ and dT above should leave the monopolist's profit unchanged when it chooses its strategy optimally, i.e.,

$$T^m dQ + Q^m dT = 0. \tag{4.7}$$

Gathering (4.6) and (4.7) finally yields (4.3). This latter condition shows that the choice T^m actually results from two compounding effects. First, increasing the access fee increases profit when consumers participation (induced by their transportation costs) is not fully elastic. This *Participation Effect* is captured by the first term on the right-hand side of (4.3). Second, reducing the fee also increases the mass of consumers connected to network N. It improves network externality and makes it more attractive for any given consumer to join the network; a *Network Effect* which is captured by the second term on the right-hand side of (4.3).

The next proposition unveils how the unregulated monopolist distorts investment and market coverage away from the optimum.

Proposition 3. *Suppose that*

$$2QS(c) - H^{-1}(Q) + \varphi'^{-1}(Q) \tag{4.8}$$

is non-increasing in Q. The unregulated monopolist chooses

- *less market coverage than socially optimal*

$$Q^m < \hat{Q}, \tag{4.9}$$

- *a lower investment level*

$$i^m < \hat{i}, \tag{4.10}$$

- *a greater access fee*

$$T^m > \hat{T}. \tag{4.11}$$

To maximize profit, the monopolist relies on a two-part tariff. Each transaction is priced at marginal cost and an access fee is used to capture consumer surplus. The access fee chosen by an unregulated monopoly results from a familiar trade-off. Raising the fee by an infinitesimal amount increases profit on all infra-marginal consumers whose transportation costs are low enough to make them still willing to join network N. On the other hand, the marginal consumer now prefers to opt out. Because the monopolist is not concerned by consumer surplus, the access fee is now too high and participation is reduced. This in turn makes the monopolist less willing to invest.

RUNNING EXAMPLE. For the sake of providing useful comparative statics, we consider here a simple and rather tractable example with zero marginal cost $c = 0$, an exponential demand $D(p) = De^{-\frac{p}{a}}$ (which also yields the following expressions of surplus $S(p) = Dae^{-\frac{p}{a}}$ and profit $\Pi(p) = Dpe^{-\frac{p}{a}}$), a quadratic cost of investment $\varphi(i) = \varphi\frac{i^2}{2}$, and an exponential distribution of transportation costs with density $h(\varepsilon) = \frac{1_{\varepsilon \geq 0}}{\lambda}e^{-\frac{\varepsilon}{\lambda}}$. With this latter specification, notice that λ stands for the average transportation costs which is a measure of mobility frictions (or "competitiveness") on the market.

With those specifications at hand, we show[6] that

$$Q^m = 1 - e^{-\frac{R^m}{\lambda}} \tag{4.12}$$

where

$$\frac{\frac{R^m}{\lambda}}{1 - e^{-\frac{R^m}{\lambda}}} = \frac{2Da + \frac{1}{\varphi}}{\lambda} - e^{\frac{R^m}{\lambda}}. \tag{4.13}$$

From this, simple comparative statics follow.

[6]See the proof of Corollary 1 in the Appendix.

Corollary 1. *Under monopoly, market coverage Q^m increases with*

- *greater demand (i.e., D or a larger),*
- *lower cost of investment (i.e., φ lower),*
- *lower average transportation costs (i.e., λ higher).*

Proceeding as above, it is straightforward to check that

$$\frac{\frac{\hat{R}}{\lambda}}{1-e^{-\frac{\hat{R}}{\lambda}}} = \frac{2Da+\frac{1}{\varphi}}{\lambda} - \frac{\mu}{1+\mu}e^{\frac{\hat{R}}{\lambda}}. \tag{4.14}$$

Because the marginal cost of public funds μ is finite, the right-hand side of (4.14) is thus lower than the right hand side of (4.13). From there, it immediately follows that

$$R^m < \hat{R} \tag{4.15}$$

and thus (4.9), (4.10) and (4.11) also hold when Condition (4.8) is replaced, for the sake of this numerical application, by the parametric condition

$$3\lambda < 2Da + \frac{1}{\varphi}. \tag{4.16}$$

5 Duopolistic Competition without Interoperability

We now investigate to what extent competition between duopolists operating different networks might modify price and investment levels. We shall consider different scenarios depending on whether consumers affiliated to a given network can also transact with those affiliated to the competing network. We start with the simple case where networks are not inter-operable. Consumers can only transact with their peers who are connected to the same network.

We also consider scenarios where firms have not sufficiently invested to cover the whole market. The reason is twofold. On the one hand, this scenario makes comparison with the monopoly scenario somehow easier. On the other hand, partial coverage is what is found in practice and is of much relevance from an empirical perspective. Finally, we also suppose hereafter that consumers do not multi-home; the cost of joining both network being prohibitive.

With partial coverage, a given consumer now visits network N (resp. N^*) not only if he derives greater surplus net of transportation costs by doing so but also if this net surplus remains non-negative. This consumer is thus attracted by network N whenever

$$R(Q,p,i-T)-\varepsilon \geq \max\left\{R(Q^*,p^*,i^*-T^*)-\varepsilon^*,0\right\}.$$

From there, F's market share can be decomposed into two subsets. Consumers who are too far away from network N^* and close enough to N (i.e., $\varepsilon \leq R(Q,p,i-T)$ and $\varepsilon^* \geq R(Q^*,p^*,i^*-T^*)$) form a captive demand for F. The overall mass of such a captive demand is thus

$$\int_0^{R(Q,p,i-T)} h(\varepsilon)\left(1-H(R(Q^*,p^*,i^*-T^*)\right)d\varepsilon.$$

Consumers who are not so far away from network N^* but still closer to N (i.e., $\varepsilon \leq R(Q,p,i-T)$ and $R(Q^*,p^*,i^*-T^*)-R(Q,p,i-T)+\varepsilon \leq \varepsilon^* \leq R(Q^*,p^*,i^*-T^*)$) form a less captive demand who is now sensitive to F^*'s offer. The overall mass of this less captive demand is

$$\int_0^{R(Q,p,i-T)} h(\varepsilon)\left(H\left(R(Q^*,p^*,i^*-T^*)\right)-H\left(R(Q^*,p^*,i^*-T^*)-R(Q,p,i-T)+\varepsilon\right)\right)d\varepsilon.$$

Gathering those two expressions finally yields the following expression of F's overall demand:

$$Q=\int_0^{R(Q,p,i-T)} h(\varepsilon)\left(1-H\left(R(Q^*,p^*,i^*-T^*)-R(Q,p,i-T)+\varepsilon\right)\right)d\varepsilon. \qquad (5.1)$$

It is worth pointing out that, at any symmetric equilibrium (indexed by the superscript N), we have

$$Q^N = \int_0^{R(Q^N,p^N,i^N-T^N)} h(\varepsilon)\,(1-H(\varepsilon))\,d\varepsilon = \frac{1}{2}G(R(Q^N,p^N,i^N-T^N)) \tag{5.2}$$

where $G(\eta) = H(\eta)(2-H(\eta))$ is the cumulative distribution of the *min* statistics $\eta = \min\{\varepsilon,\varepsilon^*\}$. For future reference, let aslo denote $g(\eta) = 2h(\eta)\,(1-H(\eta))$ the corresponding density function.

Monopolistic Benchmark. The intuition for (5.2) can be best understood when thinking about a monopolist, acting as a merger between F and F^* and choosing the same policy for the two firms, namely $(p,T,i) = (p^*,T^*,i^*) = (p^c,T^c,i^c)$. In that hypothetical scenario where the net surplus is the same on both networks, a customer always chooses the closest firm and thus incurs a transportation cost $\eta = \min\{\varepsilon,\varepsilon^*\}$ towards that nearby network. By symmetry, the covered market is split equally across firms so that, each firm's market coverage becomes

$$Q^m = \frac{1}{2}G(R(Q^m,p^m,i^m-T^m)). \tag{5.3}$$

It is straightforward to replicate the results of Proposition 2 in this context. Again, this monopolist would choose to charge a price equal to marginal cost and investment level as in (4.2). The access fee is slightly modified to account for the fact that each consumer enjoys a network externality from only half of the overall market coverage. This leads to replace (4.3) with:

$$T^m = Q^m\left(\frac{2}{g(R^m)} - S(c)\right) \tag{5.4}$$

where market coverage Q^m and the net surplus R^m still solve (5.2) and (4.5). The formula (5.4) is an important benchmark to understand how competition between networks affects access fees and market coverage.

Symmetric Equilibrium. To ensure existence and uniqueness of a solution Q to (5.1), we also posit that the following Assumption, of similar nature as Assumption 1, holds:

Assumption 2.

$$1 - S(c)\left(\max_y h(y) + \max_z \int_0^{+\infty} h(\varepsilon)h(z+\varepsilon)d\varepsilon\right) > 0.$$

The next proposition characterizes the duopolistic outcome.

Proposition 4. *Suppose that the market is only partially covered and there is no interoperability. Suppose also that Assumption 2 holds. There exists a unique symmetric Nash equilibrium characterized by the following properties.*

- *Both firms charge a price per transaction equal to marginal cost,*

$$p^N = c. \tag{5.5}$$

- *Both firms choose the same level of investment:*

$$\varphi'(i^N) = Q^N. \tag{5.6}$$

- *Both firms charge the same access fee:*

$$T^N = Q^N\left(\frac{2}{g(R^N) + 2\int_0^{R^N} h^2(\varepsilon)d\varepsilon} - S(c)\right) \tag{5.7}$$

  where each firm's market coverage is given by

$$Q^N = \frac{1}{2}G(R^N) < \frac{1}{2} \tag{5.8}$$

  with

$$R^N = Q^N S(c) + i^N - T^N. \tag{5.9}$$

The duopolistic scenario shares some common features with the unregulated monopoly solution. There is no distortion on the unit price of a transaction which is set at marginal cost. Reducing prices is an inefficient way of transferring surplus to attract consumers because it would come with a deadweight loss. Instead, duopolists prefer to attract consumers by reducing access fees. Competition then boils down to attracting the marginal consumer, who is just indifferent between joining either network, with a lower fee. As a result, those fees tend to be lower than in a monopolist context.

To better understand the choice of an access fee in a duopolistic context, it is again useful to consider how a marginal change dT in F's access fee impacts market coverage and profit. Importantly, decreasing T^N by an amount dT certainly attracts consumers towards network N; an illustration of the *Participation Effect* under duopoly. However a marginal reduction of the fee also decreases the mass of consumers who might visit the competing network N^*, which dampens the network externality there. This indirect *Network Effect* reduces even further the surplus that consumers get on network N^* and thus makes network N even more attractive.

Formally, differentiating (5.1) with respect to T, starting from a symmetric equilibrium (i.e., $p = p^* = c$, $i = i^* = i^N$ and $T = T^* = T^N$), we observe that reducing $T = T^N$ by dT has an impact on F's market share Q^N which is now increased by dQ. By differentiating (5.2) around this symmetric equilibrium, we have

$$\left(1 - S(c)\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right)\right) dQ = -\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right) dT \tag{5.10}$$

where the bracketed term on the left-hand side is positive when Assumption 2 holds. At the same time, such marginal perturbations dT and dQ should leave F's profit unchanged at the equilibrium, which requires

$$T^N dQ + Q^N dT = 0. \tag{5.11}$$

Gathering (5.10) and (5.11) finally yields (5.7).

It is interesting to compare the equilibrium fees with the one that would be chosen by a monopoly jointly owning the two networks, i.e., (5.4). For a given market coverage Q^N, the fee chosen under duopoly is systematically lower. This comes from a *business stealing effect*. To understand this effect, observe that the monopoly would decrease both fees T and T^* by the same amount when maximizing joint profits. It would thus not change the relative size of each firm's market share over the part of their demand which is non-captive. The only impact of such joint decrease would be to increase market coverage on the captive demands of both firms. Instead, with a unilateral deviation consisting in reducing his own fee T, firm F not only increase its captive demand but it also attracts more consumers away from F^* and thereby raises its own profit at the expense of its competitor. Formally, we have

$$T^N < Q^N \left(\frac{2}{g(R^N)} - S(c) \right). \tag{5.12}$$

Running Example. It is straightforward to compute $G(\eta) = 1 - e^{-\frac{2\eta}{\lambda}}$ and thus $g(\eta) = \frac{2}{\lambda} e^{-\frac{2\eta}{\lambda}}$. Also, we may compute $\int_0^\eta h^2(\varepsilon)d\varepsilon = \frac{1-e^{-\frac{2\eta}{\lambda}}}{2\lambda}$. With those specifications, we show in the Appendix that there exists a duopolistic equilibrium whenever

$$3\lambda < 2Da + \frac{1}{\varphi}, \tag{5.13}$$

an assumption that we maintain throughout.

Simple and intuitive comparative statics again follow; replicating our findings of Corollary 1 for the monopoly scenario.

Corollary 2. *Suppose that (5.13) holds. Under duopolistic competition with no interoperability, market coverage Q^N increases with*

- *a greater demand (i.e., D or a larger),*
- *a lower cost of investment (i.e., φ lower),*

- *a lower average transportation cost (i.e., λ lower).*

Next Corollary finally compares this duopolistic solution with the monopoly outcome in the framework of our Running Example.

Corollary 3. *In comparison with the monopoly scenario, duopolistic competition with no interoperability entails greater overall surplus, more participation and more investment:*

$$R^N > R^m, \quad Q^N > Q^m, \quad i^N > i^m. \tag{5.14}$$

6 Duopolistic Competition with interoperability

Consider now the scenario where customers may transact not only with peers connected to their own network N but also with customers connected to the competing network N^*. Formally, we will thus write consumer's gross surplus when connected to N as

$$(Q + \gamma Q^*)\, S(p),$$

where γ measures the degree of network interoperability. It can be viewed as a reduced form for how difficult or costlier it is to transact off-network; γ=0 representing the case with no interoperability and γ=1 the case with perfect interoperability.

We proceed as above and consider a scenario where, even with interoperability, the market remains partially covered. A consumer is thus attracted by network N whenever

$$R(Q + \gamma Q^*, p, i - T) - \varepsilon \geq \max \left\{ R(Q^* + \gamma Q, p^*, i^* - T^*) - \varepsilon^*, 0 \right\}.$$

Mimicking our previous approach, we find the expression of F's market coverage as:

$$Q = \int_0^{R(Q+\gamma Q^*,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^* + \gamma Q, p^*, i^* - T^*) - R(Q + \gamma Q^*, p, i - T) + \varepsilon\right)\right) d\varepsilon. \tag{6.1}$$

At any symmetric equilibrium, the equilibrium market coverage of each duopolist is now given by

$$Q^N = \int_0^{R((1+\gamma)Q^N,p^N,i^N-T^N)} h(\varepsilon)\,(1-H(\varepsilon))\,d\varepsilon = \frac{1}{2}G(R((1+\gamma)Q^N,p^N,i^N-T^N)) \tag{6.2}$$

To obtain simple comparative statics in what follows, we shall posit

Assumption 3.

$$1 - S(c)\left(\frac{1}{2}\max_y g(y) + \int_0^{+\infty} h^2(\varepsilon)d\varepsilon\right) > 0.$$

This condition also amounts to say that there is a unique solution Q to (6.2), in the neighborhood where firms share the market equally, and that this solution is non-decreasing with $i - T$.

The next proposition characterizes a symmetric equilibrium in this context.

Proposition 5. *Consider a market that is only partially covered and in which interoperability is feasible. Suppose also that Assumption 2 holds. There exists a unique symmetric Nash equilibrium characterized by the following properties.*

- *Both firms charge a price equal to marginal cost and choose an investment level as in (5.6).*
- *Both firms charge the same access fee:*

$$T^N = Q^N\left(\frac{2}{g(R^N)+2\int_0^{R^N}h^2(\varepsilon)d\varepsilon} - \left(1 - \frac{2\gamma\int_0^{R^N}h^2(\varepsilon)d\varepsilon}{g(R^N)+2\int_0^{R^N}h^2(\varepsilon)d\varepsilon}\right)S(c)\right). \tag{6.3}$$

  where Q^N and R^N are respectively given by (5.8) and

$$R^N = (1+\gamma)Q^N S(c) + i^N - T^N. \tag{6.4}$$

To understand the determinants of the choice of the fee at equilibrium, remember that, when networks are not inter-operable, the *Business-Stealing Effect* dampens the choice of the fee in comparison with the unregulated monopolistic scenario. With interoperability, each firm now somehow benefits from the other expanding its own network. Indeed, by expanding its own network, F^* increases the overall surplus that any consumer gets when connecting to F's own network. It allows F to extract even more from this consumer with a fee; the only source of revenues for F when price per transaction is equal to marginal cost. Of course, F^* does not internalize this profit gain that accrues to F. Hence, F^* may not be reducing enough its own fee to attract consumers in comparison with the monopolistic scenario where this externality would be internalized. At equilibrium, fees end up being too high in comparison with a scenario without interoperability since

$$T^N > Q^N \left(\frac{2}{g(R^N) + 2\int_0^{R^N} h^2(\varepsilon) d\varepsilon} - S(c) \right).$$

A first consequence of interoperability is thus to soften competition in access fees. This first effect indirectly reduces overall market coverage and investment. Yet, with interoperability, consumers always enjoy a greater volume of transactions since they have access to a larger network as it can be seen on the right-hand side of (6.4). This second force directly raises expected consumer surplus and it also increases market coverage and investment.

Running Example. This second effect always dominates in the framework of our example as shown in next Corollary.

Corollary 4. *Under duopolistic competition and interoperability,market coverage Q^N increases with*

- *a greater demand (i.e., D or a larger),*
- *a lower cost of investment (i.e., φ lower),*

- *a lower average transportation cost (i.e., λ lower).*

- *a greater degree of interoperability (i.e., γ larger).*

7 Endogenous Market Structures

Casual evidence suggests that market structures significantly vary across countries. In some countries, a situation close to the monopoly scenario prevails. Elsewhere, a more competitive playing field may prevail. To account for this variability, we slightly expand our model and suppose that firms face the same entry cost K. Let suppose that K is distributed according to a cumulative distribution function K over the positive real line. Although firms are ex ante symmetric, the decision whether to entry or not may create an ex post asymmetry. For some equilibria of the entry game and for some values of K, a single firm may be ex post the sole provider of the service.

To see how it can be so, we now define an entry game and shall study its equilibria. The timing of this game is straightforward. First, firms simultaneously decide whether to entry or not. Second, given those entry decisions, firms compete in price/quality packages.

Let denote the equilibrium profits when a monopoly or a duopoly structure emerges for the second stage of this game respectively as Π^m and $\Pi^N(\gamma)$ where we now make the dependence on the interoperability parameter explicit. Because the price per transaction is equal to marginal cost under all scenarios, those profits just follow from charging access fees to the connected consumers. This observation leads to the following expressions of those profits:

$$\Pi^m = Q^m T^m \text{ and } \Pi^N(\gamma) = Q^N(\gamma) T^N(\gamma).$$

From now on, we assume that $\Pi^m > \Pi^N(\gamma)$, a condition that necessarily holds when γ is small enough. Deriving the unique equilibrium of the entry game is straightforward.

- For $K \in \left[0, \Pi^N(\gamma)\right]$, both firms enter with probability one. The duopoly outcome follows, with profits taking different expressions depending on the degree of interoperability.

- For $K \in \left[\Pi^N(\gamma), \Pi^m\right]$, the unique equilibrium entails a symmetric mixed strategy. Each firm is indifferent between entering or not and enters with probability ε such that his opponent is also indifferent:

$$\varepsilon \Pi^N(\gamma) + (1 - \varepsilon)\Pi^m = K. \tag{7.1}$$

- For $K \geq \Pi^m$, no firms enter and the service is not provided. Of course, this case is not so interesting in view of explaining actual market structures.

This analysis uncovers an important insight on the role of interoperability in the framework of our Running Example.

Corollary 5. *As the interoperability parameter γ slightly increases above 0, the likelihood that a duopolistic market structure emerges increases.*

8 Conclusion

We have developed a simple model in which firms compete in prices and quality provision and consumers enjoy network externalities. We have contrasted the resulting consumers' welfare in a monopolistic and in a duopolistic scenario, and highlighted the effects of introducing interoperability on prices, quality investments and incentives' for providers to enter the market.

This model has considered the simplest setting to address these questions. There are many ways in which the model can be extended, a natural one would be for example to consider asymmetric service providers, describing a market with one large incumbent and many typically smaller potential entrants. Despite its simplicity, our model delivers a rich set of testable predictions on the effects of competition and interoperability on prices, quality and market coverage. Confronting these predictions with the data is in our view a very important avenue for future research.

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A1 Proofs of Main Results

Lemma A.1. *Suppose that Assumption 1 holds. Then, there exists a unique solution to* $\tilde{Q}(p, i - T)$ *to (2.3) which is non-decreasing in* $i - T$ *and non-increasing in* p.

Proof of Lemma A.1. The function $Q - H(R(Q,p,i-T)$ is increasing in Q when

$$1 - h(R(Q,p,i-T))S(p) > 0.$$

A sufficient condition for this to hold is Assumption 1. Observe that $H(R(0,p,i-T)) > 0$ and $H(R(1,p,i-T)) < 1$. Thus, there exists a unique solution $\tilde{Q}(p, i-T)$ to (2.3). Comparative statics immediately follow. □

Proof of Proposition 1. Because Assumption 1 holds, there exists a unique solution Q to (2.3). Because $\mu > 0$, the social welfare function (3.2) is maximized when the subsidy z is just low enough to allow the firm to break even; that is,

$$Q^2\Pi(p) + QT - \varphi(i) + z = 0.$$

Accordingly, we may thus rewrite social welfare as

$$(1+\mu)\left(Q^2\Pi(p) + QT - \varphi(i)\right) + \int_0^{R(Q,p,i-T)} H(\varepsilon)d\varepsilon. \tag{A.1}$$

We now maximize (A.1) subject to the definition of market coverage (2.3) as a constraint. Denoting by λ the Lagrange multiplier for (2.3), the corresponding Lagrangean writes as

$$(1+\mu)\left(Q^2\Pi(p) + QT - \varphi(i)\right) + \int_0^{R(Q,p,i-T)} H(\varepsilon)d\varepsilon + \lambda\left(H(QS(p)+i-T) - Q\right).$$

Assuming concavity of this expression, the necessary and sufficient conditions for optimality with respect to (p, i, T, Q) can respectively be written, as

$$(1+\mu)\left((p-c)D'(p) + D(p)\right)Q^2 - QD(p)H(R(Q,p,i-T)) = \lambda Qh(R(Q,p,i-T))D(p), \tag{A.2}$$

$$(1+\mu)\varphi'(i) = H(R(Q,p,i-T)) + \lambda h(R(Q,p,i-T)), \tag{A.3}$$

$$(1+\mu)Q = H(R(Q,p,i-T)) + \lambda h(R(Q,p,i-T)), \tag{A.4}$$

$$(1+\mu)\left(2\Pi(p)Q+T\right)+S(p)H(R(Q,p,i-T)) = \lambda\left(1-h(R(Q,p,i-T))S(p)\right). \quad \text{(A.5)}$$

Inserting (A.4) into (A.2) and simplifying using (2.3) yields (3.3). Inserting (A.3) into (A.4) yields (3.4). From (2.3) and (A.4), we also deduce

$$\lambda = \frac{\mu Q}{h(R(Q,p,i-T))} \geq 0. \quad \text{(A.6)}$$

Inserting (3.3), (A.4) and (A.6) into (A.5) and again using (2.3) yields (3.5). □

Proof of Proposition 2. The proof proceeds as the Proof of Proposition 1. Because Assumption 1 holds, there exists a unique solution Q to (2.3). Denoting by λ the multiplier for (2.3), the corresponding Lagrangean writes as:

$$\mathcal{L}(Q,p,i,T) = \left(Q^2\Pi(p)+QT-\varphi(i)\right)+\lambda\left(H(R(Q,p,i-T))-Q\right).$$

Assuming concavity of this expression, the necessary and sufficient conditions for optimality with respect to (p,i,T,Q) can respectively be written, as

$$\left((p-c)D'(p)+D(p)\right)Q^2 = \lambda Q h(R(Q,p,i-T))D(p), \quad \text{(A.7)}$$

$$\varphi'(i) = \lambda h(R(Q,p,i-T)), \quad \text{(A.8)}$$

$$Q = \lambda h(R(Q,p,i-T)), \quad \text{(A.9)}$$

$$\left(2\Pi(p)Q+T\right) = \lambda\left(1-h(R(Q,p,i-T))S(p)\right). \quad \text{(A.10)}$$

Inserting (A.8) into (A.9) yields (4.1). Inserting (A.9) into (A.8) yields (4.2). Finally, inserting (4.1) into (A.10) and using (A.9) yields (4.3). □

Proof of Proposition 3. Taking into account the investment rules (3.4) and (4.2) and the definition of market coverage

$$Q = H\left(QS(c)+i-T\right), \quad \text{(A.11)}$$

that applies under both scenarios, we may rewrite the objectives of an unregulated monopoly and of a planner (concerned with social welfare) in terms of market coverage Q only respectively as

$$\Pi(Q) = Q(QS(c) + \varphi'^{-1}(Q) - H^{-1}(Q)) - \varphi(\varphi'^{-1}(Q)) \tag{A.12}$$

and

$$\mathcal{W}(Q) = \Pi(Q) + \frac{1}{1+\mu}\int_0^{H^{-1}(Q)} H(\varepsilon)d\varepsilon. \tag{A.13}$$

Notice that Q^m and $\hat{Q}$ are given by the follow first-order necessary conditions:

$$2Q^m S(c) - H^{-1}(Q^m) + \varphi'^{-1}(Q^m) = \frac{Q^m}{h(H^{-1}(Q^m))}, \tag{A.14}$$

$$2\hat{Q}S(c) - H^{-1}(\hat{Q}) + \varphi'^{-1}(\hat{Q}) = \frac{\mu}{1+\mu}\frac{\hat{Q}}{h(H^{-1}(\hat{Q}))} < \frac{\hat{Q}}{h(H^{-1}(\hat{Q}))}. \tag{A.15}$$

Observe that the right-hand sides of (A.14) and (A.15) are both non-decreasing in Q when H satisfies $MHRP$ since

$$\frac{d}{dQ}\left(\frac{Q}{h(H^{-1}(Q))}\right) = \frac{1}{h(H^{-1}(Q))}\frac{d}{dy}\left(\frac{H(y)}{h(y)}\right)_{y=H^{-1}(Q)} > 0$$

That (4.8) holds also implies that $\Pi(Q)$ and $\mathcal{W}(Q)$ are both quasi-concave in Q. The necessary conditions (A.14) and (A.15) are thus also sufficient.

Comparing the right-hand sides of (A.14) and (A.15) and using concavity of $\Pi(Q)$, we immediately obtain (4.9). From this, (4.10) also follows.

That (4.8) holds also implies that

$$\zeta(Q) = QS(c) - H^{-1}(Q) + \varphi'^{-1}(Q)$$

is non-increasing in Q. Observe that $T^m = \zeta(Q^m)$ and $\hat{T} = \zeta(\hat{Q})$. Then, (4.9) also implies (4.11). $\square$

Proof of Corollary 1. The optimality conditions (4.2), (4.3) respectively become:

$$\varphi i^m = Q^m, \tag{A.16}$$

$$T^m = Q^m \left(\lambda e^{\frac{R^m}{\lambda}} - Da\right) \tag{A.17}$$

while (4.4) and (4.5) can be rewritten as (4.12) and (4.13). It can be readily checked that the left-hand (resp. right-hand) side of (4.13) is an increasing (resp. decreasing) function of $\frac{R^m}{\lambda}$ only. Whenever (5.13) holds, Condition (4.13) thus defines a unique positive solution $\frac{R^m}{\lambda}$. Comparative statics are then immediate. □

Lemma A.2. *Suppose that Assumption 2 holds. Then, there exists a unique solution to $\tilde{Q}(Q^*, p, i - T)$ to (5.1) which is non-decreasing in $i - T$ and non-increasing in p.*

Proof of Lemma A.2. The function

$$Q - \int_0^{R(Q,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^*, p^*, i^* - T^*) - R(Q, p, i - T) + \varepsilon\right)\right) d\varepsilon$$

is increasing in Q when

$$1 - S(p)\Big(h(R(Q, p, i - T))\left(1 - H\left(R(Q^*, p^*, i^*)\right)\right)$$

$$+ \int_0^{R(Q,p,i-T)} h(\varepsilon) h\left(R(Q^*, p^*, i^* - T^*) - R(Q, p, i - T) + \varepsilon\right) d\varepsilon\Big) > 0.$$

A sufficient condition for this to hold is thus given by Assumption 2. Observe that

$$\int_0^{R(0,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^*, p^*, i^* - T^*) - R(0, p, i - T) + \varepsilon\right)\right) d\varepsilon > 0$$

and

$$\int_0^{R(1,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^*, p^*, i^* - T^*) - R(1, p, i - T) + \varepsilon\right)\right) d\varepsilon < 1.$$

Thus, there exists a unique solution $\tilde{Q}(Q^*, p, i - T)$ to (5.1). Comparative statics immediately follow.

Observe also that Assumption 2 also implies

$$1 - S(c)\left(\max_y \frac{1}{2}g(y) + \int_0^{+\infty} h^2(\varepsilon)d\varepsilon\right) > 0. \tag{A.18}$$

□

Proof of Proposition 4. In a duopoly and when access fees are feasible, we may write F 's profit as

$$Q^2\Pi(p) + QT - \varphi(i) \tag{A.19}$$

where Q is given by (5.1). Because Assumption 2 holds, there exists a unique solution Q to (5.1).

We maximize the expression of F 's profit given in (A.19) subject to the definition of F's market share (5.1). Denoting again by λ the Lagrange multiplier for (5.1), we form the corresponding Lagrangean as

$$\mathcal{L}(Q, p, i - T) = Q^2\Pi(p) + QT - \varphi(i)$$

$$+\lambda\left(\int_0^{R(Q,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^*, p^*, i^* - T^*) - R(Q, p, i - T) + \varepsilon\right)\right)d\varepsilon - Q\right).$$

Assuming concavity of the Lagrangean, we obtain the following necessary and sufficient conditions with respect to (p, i, Q, T) at a symmetric equilibrium (p^N, i^N, Q^N, T^N) as:

$$\left((p^N - c)D'(p^N) + D(p^N)\right)(Q^N)^2 = \lambda\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right)Q^N D(p^N), \tag{A.20}$$

$$\varphi'(i^N) = \lambda\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right), \tag{A.21}$$

$$2\Pi(p^N)Q^N + T^N = \lambda\left(1 - \left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right)S(p^N)\right), \tag{A.22}$$

$$Q^N = \lambda \left(\frac{1}{2} g(R^N) + \int_0^{R^N} h^2(\varepsilon) d\varepsilon \right), \tag{A.23}$$

where Q^N and R^N are respectively given by (5.2) and (5.9). Inserting (A.23) into (A.31), (A.21) and (A.22) respectively yields (5.5), (5.6) and (5.7) respectively. □

Proof of Corollary 2. With our specifications, the optimality conditions (5.6), (5.8) and (5.9) respectively now become:

$$\varphi i^N = Q^N, \tag{A.24}$$

$$Q^N = \frac{1}{2}\left(1 - e^{-\frac{2R^N}{\lambda}}\right), \tag{A.25}$$

and

$$\frac{\frac{2R^N}{\lambda}}{1 - e^{-\frac{2R^N}{\lambda}}} = \frac{2Da + \frac{1}{\varphi}}{\lambda} - \frac{2}{1 + e^{-\frac{2R^N}{\lambda}}}. \tag{A.26}$$

It can be readily checked that the left-hand (resp. right-hand) side of (A.26) is an increasing (resp. decreasing) function of $\frac{R^N}{\lambda}$ only. Whenever Condition (5.13) holds, Condition (A.26) thus defines a unique positive solution $\frac{R^N}{\lambda}$.

From there, the comparative statics as stated in Corollary 2 follow. Increasing $\frac{2Da+\frac{1}{\varphi}}{\lambda}$ shifts up the solution $\frac{R^N}{\lambda}$ to (A.26); which increases market coverage by (A.25). □

Proof of Corollary 3. The monopoly outcome is defined in (4.1)-(4.2)-(5.3) and (5.4). Those optimality conditions now become:

$$\varphi i^m = Q^m, \tag{A.27}$$

$$Q^m = \frac{1}{2}\left(1 - e^{-\frac{2R^m}{\lambda}}\right), \tag{A.28}$$

and

$$\frac{2\frac{R^m}{\lambda}}{1 - e^{-\frac{2R^m}{\lambda}}} = \frac{2Da + \frac{1}{\varphi}}{\lambda} - e^{\frac{2R^m}{\lambda}}. \tag{A.29}$$

Notice now that, for $R^m \geq 0$, we have $e^{\frac{2R^m}{\lambda}} \geq \frac{2}{1+e^{-\frac{2R^m}{\lambda}}}$. Hence, the right-hand side of (A.26) is greater than the right-hand side of (A.29). The statement in Corollary 3 immediately follows. □

Lemma A.3. *Suppose that Assumption 2 holds. Then, there exists a unique solution to* $\tilde{Q}(Q^*, p, i-T)$ *to (6.1) which is non-decreasing in* $i-T$ *and non-increasing in* p.

Proof of Lemma A.3. The function

$$Q - \int_0^{R(Q+\gamma Q^*,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^* + \gamma Q, p^*, i^* - T^*) - R(Q + \gamma Q^*, p, i-T) + \varepsilon\right)\right) d\varepsilon$$

is increasing in Q when

$$1 - S(p)\Big(h(R(Q + \gamma Q^*, p, i-T))\left(1 - H\left(R(Q^* + \gamma Q, p^*, i^*)\right)\right)$$

$$+ \int_0^{R(Q+\gamma Q^*,p,i-T)} h(\varepsilon) h\left(R(Q^* + \gamma Q, p^*, i^* - T^*) - R(Q + \gamma Q^*, p, i-T) + \varepsilon\right) d\varepsilon\Big) > 0.$$

A sufficient condition for this to hold is thus given by Assumption 2. Observe that

$$\int_0^{R(\gamma Q^*,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^*, p^*, i^* - T^*) - R(\gamma Q^*, p, i-T) + \varepsilon\right)\right) d\varepsilon > 0$$

and

$$\int_0^{R(1+\gamma Q^*,p,i-T)} h(\varepsilon)\left(1 - H\left(R(Q^* + \gamma, p^*, i^* - T^*) - R(1 + \gamma Q^*, p, i-T) + \varepsilon\right)\right) d\varepsilon < 1.$$

Thus, there exists a unique solution $\tilde{Q}(Q^*, p, i-T)$ to (6.1). Comparative statics immediately follow. □

Proof of Proposition 5. Each consumer who joins network N expresses a demand for F 's services which is worth

$$(Q + \gamma Q^*) D(p).$$

There is a mass Q of such consumers so that, we may write F 's profit as

$$Q\left(Q+\gamma Q^{*}\right)\Pi(p)+QT-\varphi(i). \tag{A.30}$$

From Lemma A.3, there exists a unique solution Q to (6.1).

Denoting again by λ the Lagrange multiplier for (B.21), we form the corresponding Lagrangean as

$$\mathcal{L}(Q,p,i,T)=Q\left(Q+\gamma Q^{*}\right)\Pi(p)+QT-\varphi(i)$$
$$+\lambda\left(\int_{0}^{R(Q+\gamma Q^{*},p,i-T)}h(\varepsilon)\left(1-H\left(R(Q^{*}+\gamma Q,p^{*},i^{*}-T^{*})-R(Q+\gamma Q^{*},p,i-T)+\varepsilon\right)\right)d\varepsilon-Q\right).$$

Assuming concavity of the Lagrangean, we obtain the following necessary and sufficient conditions with respect to (p,i,Q,T) at a symmetric Nash equilibrium:

$$\left((p^{N}-c)D'(p^{N})+D(p^{N})\right)(1+\gamma)(Q^{N})^{2}=\lambda\left(\frac{1}{2}g(R^{N})+\int_{0}^{R^{N}}h^{2}(\varepsilon)d\varepsilon\right)(1+\gamma)Q^{N}D(p^{N}), \tag{A.31}$$

$$\varphi'(i^{N})=\lambda\left(\frac{1}{2}g(R^{N})+\int_{0}^{R^{N}}h^{2}(\varepsilon)d\varepsilon\right), \tag{A.32}$$

$$(2+\gamma)\Pi(p^{N})Q^{N}+T^{N}=\lambda\left(1-\left(\frac{1}{2}g(R^{N})+(1-\gamma)\int_{0}^{R^{N}}h^{2}(\varepsilon)d\varepsilon\right)S(p^{N})\right). \tag{A.33}$$

Again (5.5) holds. Inserting into (A.31) yields

$$Q^{N}=\lambda\left(\frac{1}{2}g(R^{N})+\int_{0}^{R^{N}}h^{2}(\varepsilon)d\varepsilon\right). \tag{A.34}$$

Inserting (A.34) into (A.32) and (A.33) respectively yields (5.6) and (6.3). □

Proof of Corollary 4. The optimality conditions (A.24) and (A.25) still hold, together with

$$\frac{\frac{2R^{N}}{\lambda}}{1-e^{-\frac{2R^{N}}{\lambda}}}=\frac{2Da+\frac{1}{\varphi}}{\lambda}-\frac{2}{1+e^{-\frac{2R^{N}}{\lambda}}}+\gamma\frac{\frac{2Da}{\lambda}}{1+e^{\frac{2R^{N}}{\lambda}}}. \tag{A.35}$$

It can be readily checked that the right-hand side of (A.35) remains a decreasing function of $\frac{R^N}{\lambda}$ only. Whenever (5.13) holds, Condition (A.35) thus defines a unique positive solution $\frac{R^N}{\lambda}$.

Our earlier findings in Corollary 2 still hold. More interesting is the role of interoperability. As γ increases, the third corrective term the right-hand side of (A.35), which is positive, also increases. We can thus conclude that $\frac{R^N}{\lambda}$ and Q^N are non-decreasing function of γ. $\square$

Proof of Corollary 5. . To prove the result, it suffices to show that $\Pi^N(\gamma)$ is non-decreasing in γ in the neighborhood of $\gamma = 0$. To this end, we remember the following expression for $T^N(\gamma)$ and $R^N(\gamma)$:

$$T^N(\gamma) = Q^N(\gamma)\left(\frac{2\lambda}{1+e^{-\frac{2R^N(\gamma)}{\lambda}}} - Da\left(1-\gamma\frac{1-e^{-\frac{2R^N(\gamma)}{\lambda}}}{1+e^{-\frac{2R^N(\gamma)}{\lambda}}}\right)\right) \tag{A.36}$$

and

$$R^N(\gamma) = Q^N(\gamma)\left(\frac{2Da+\frac{1}{\varphi}}{\lambda} - \frac{2}{1+e^{-\frac{2R^N(\gamma)}{\lambda}}} + \gamma\frac{\frac{2Da}{\lambda}}{1+e^{\frac{2R^N}{\lambda}}}\right) \tag{A.37}$$

where

$$Q^N(\gamma) = \frac{1}{2}\left(1-e^{-\frac{2R^N(\gamma)}{\lambda}}\right). \tag{A.38}$$

We can thus compute

$$\dot{\Pi}^N(0) = 2\frac{\dot{Q}^N(0)}{Q^N(0)}\Pi^N(0) + (Q^N(0))^2\left(\frac{4\lambda e^{-\frac{2R^N(0)}{\lambda}}}{(1+e^{-\frac{2R^N(0)}{\lambda}})^2} + Da\frac{1-e^{-\frac{2R^N(0)}{\lambda}}}{1+e^{-\frac{2R^N(0}{\lambda}}}\right) \tag{A.39}$$

which is positive since $\dot{Q}^N(0) > 0$. $\square$

B1 Appendix B: No Access Fee

Suppose for the sake of this Appendix that access fees are not feasible, i.e., $T \equiv 0$. The sole source of revenues for firms now comes from charging per transaction prices above marginal cost.

We investigate how this pricing restriction impacts on market coverages and service quality under various scenarios.

B1.1 Unregulated Monopoly

Proposition B.1. *Suppose that access fees are not available. Suppose also that Assumption 1 holds. The unregulated monopolist F chooses a price per transaction p^m and an investment i^m such that*

$$\frac{p^m - c}{p^m} = \frac{1}{\varepsilon_D(p^m)} \left(1 - \frac{\varphi'(i^m)}{Q^m}\right) \tag{B.1}$$

and

$$\Pi(p^m) = \frac{\varphi'(i^m)}{2Q^m} \left(\frac{1}{h(R^m)} - S(p^m)\right) \tag{B.2}$$

where the market coverage Q^m is now

$$Q^m = H\left(R^m\right) \tag{B.3}$$

and where

$$R^m = Q^m S(p^m) + i^m. \tag{B.4}$$

Proof of Proposition B.1. When access fees are not available, the monopolist maximizes

$$Q^2\Pi(p) - \varphi(i)$$

subject to the definition of market coverage

$$Q = H(i + QS(p)). \tag{B.5}$$

Because Assumption 1 holds, there exists a unique solution Q to (B.5).

We form the corresponding Lagrangean, still denoting by λ the multiplier for (B.5), as

$$\mathcal{L}(Q,p,i) = Q^2\Pi(p) - \varphi(i) + \lambda(H(i+QS(p)) - Q).$$

Assuming concavity of this Lagrangean, the necessary and sufficient optimality conditions with respect to p, i and Q respectively are written as:

$$((p^m - c)D'(p^m) + D(p^m))\, Q^m + \lambda h\,(R^m))\, D(p^m) = 0, \tag{B.6}$$

$$\varphi'(i^m) = \lambda h(R^m), \tag{B.7}$$

$$2\Pi(p^m)Q^m = \lambda\,(1 - h(R^m)S(p^m)) \tag{B.8}$$

where R^m solves (B.4). Inserting (B.7) into (B.6) and (B.8) respectively yields (B.1) and (B.2). □

When access fees are not feasible, the monopolist can only cover its investment by charging a price per transaction above marginal cost as specified in (B.1). Of course, the wedge between price and marginal cost is greater with a less elastic demand.

Because the right-hand side of (B.1) is necessarily positive to ensure that the unregulated monopolist covers its costs with no access fee, it also follows that

$$\varphi'(i^m) < Q^m.$$

In other words, the monopolist now underinvests with respect to the socially optimal investment rule. Reducing investment makes it easier to cover cost with revenues from the service. Hence, the price-cost margin is inversely related to the investment level.

To understand the optimality condition (B.2) in more details, let us now consider a marginal change di in the level of investment starting from i^m. In the absence of access fee, the market coverage under monopoly is given by (B.3). Differentiating this expression while keeping the monopoly price p^m as fixed shows how market coverage responds to a marginal increase in investment, namely

$$(1 - h(R^m)S(p^m))\, dQ = h(R^m)di. \tag{B.9}$$

Because Assumption 1 holds, the left-hand side of (B.9) is necessarily positive. Henceforth, market coverage necessarily increases with the firm's investment.

At the same time, the marginal changes di and dQ should leave the monopoly's profit unchanged at the optimum. This condition thus writes as

$$2\Pi(p^m)Q^m dQ = \varphi'(i^m)di. \tag{B.10}$$

Gathering (B.9) and (B.10) yields (B.2).

Running Example (Continued). Our specifications allows us to derive simple and intuitive comparative statics.

Corollary B.1. *Under monopoly and with no fixed fee, both the price ratio $\frac{p^m}{a}$ and market coverage Q^m increase with*

- *a greater demand (i.e., D or a larger),*
- *a lower cost of investment (i.e., φ lower),*
- *a lower average transportation cost (i.e., λ higher).*

Proof of Corollary B.1. The optimality conditions (B.1), (B.2) and (B.3) can be used to get

$$\frac{p^m}{a} = 1 - \varphi\frac{i^m}{Q^m} \tag{B.11}$$

together with a pair of equations that specify $\frac{p^m}{a}$ and $\frac{R^m}{\lambda}$, namely

$$\frac{1 + \frac{p^m}{a}}{1 - \frac{p^m}{a}} e^{-\frac{p^m}{a}} = \frac{\lambda}{Da} e^{\frac{R^m}{\lambda}} \tag{B.12}$$

and

$$\frac{\frac{R^m}{\lambda}}{1-e^{-\frac{R^m}{\lambda}}} = 2Dae^{-\frac{p^m}{a}} + \frac{1}{\varphi}\left(1 - \frac{p^m}{a}\right). \tag{B.13}$$

All changes suggested in the statement of Corollary B.1 shift upwards the right-hand side of (B.13) and downwards the right-hand side of (B.12); which leads to both an increase in $\frac{R^m}{\lambda}$ and in the ratio $\frac{p^m}{a}$. □

■

B1.2 Duopolistic Competition without Interoperability

Monopolistic Benchmark. We again start with a quick reminder of the monopoly benchmark in the present framework, again thinking about a monopolist as a merger between F and F^* and choosing the same policy for the two firms, namely $(p, T, i) = (p^*, T^*, i^*) = (p^c, T^c, i^c)$. Remind that, in that scenario, a customer always chooses the closest firm and thus incurs a transportation cost $\eta = \min\{\varepsilon, \varepsilon^*\}$ towards that nearby network. By symmetry, the covered market is split equally across firms so that, each firm's market coverage is again given by (5.3). The next proposition is replicating *mutatis mutandis* the results of Proposition B.1 in this context,

Proposition B.2. *Suppose that access fees are not available. Suppose also that Assumption 1 holds. The unregulated monopolist F chooses a price per transaction p^m and an investment i^m such that*

$$\frac{p^m - c}{p^m} = \frac{1}{\varepsilon_D(p^m)}\left(1 - \frac{\varphi'(i^m)}{Q^m}\right) \tag{B.14}$$

and

$$\Pi(p^m) = \frac{\varphi'(i^m)}{2Q^m}\left(\frac{2}{g(R^m)} - S(p^m)\right) \tag{B.15}$$

where the market coverage Q^m is still given by (5.3) and where

$$R^m = Q^m S(p^m) + i^m. \tag{B.16}$$

Proof of Proposition B.2. It is identical to that of Proposition B.1 and thus omitted. □

Symmetric Equilibrium. The logic in this scenario is well known from our previous analysis of the unregulated scenario. The unit price is upward distorted to generate revenues but competition now limits this upwards force.

Proposition B.3. *Suppose that the market is partially covered, there is no interoperability and access fees are not feasible. There exists a unique symmetric Nash equilibrium where firms choose a price per transaction p^N and an investment i^N such that*

$$\frac{p^N - c}{p^N} = \frac{1}{\varepsilon_D(p^N)} \left(1 - \frac{\varphi'(i^N)}{Q^N}\right) \tag{B.17}$$

and

$$\Pi(p^N) = \frac{\varphi'(i^N)}{2Q^N} \left(\frac{2}{g(R^N) + 2\int_0^{R^N} h^2(\varepsilon)d\varepsilon} - S(p^N) \right) \tag{B.18}$$

where Q^N still satisfies (5.8) and where

$$R^N = Q^N S(p^N) + i^N. \tag{B.19}$$

Proof of Proposition B.3. We maximize the expression of F 's profit

$$Q^2 \Pi(p) - \varphi(i) \tag{B.20}$$

subject to the definition of F 's market share (taking into account that $T = T^* = 0$) as

$$Q = \int_0^{R(Q,p,i)} h(\varepsilon) \left(1 - H(R(Q^*, p^*, i^*) - R(Q, p, i) + \varepsilon)\right) d\varepsilon. \tag{B.21}$$

Because Assumption 2 holds, there exists a unique solution Q to (B.21).

Denoting again by λ the Lagrange multiplier for (B.21), we form the corresponding Lagrangean as

$$\mathcal{L}(Q,p,i) = Q^2\Pi(p) - \varphi(i)$$

$$+\lambda\left(\int_0^{R(Q,p,i)} h(\varepsilon)\left(1 - H\left(R(Q^*,p^*,i^*) - R(Q,p,i) + \varepsilon\right)\right)d\varepsilon - Q\right).$$

Assuming concavity of the Lagrangean, we obtain the following necessary and sufficient conditions with respect to (p,i,Q,T) at a symmetric Nash equilibrium:

$$\left((p^N - c)D'(p^N) + D(p^N)\right)(Q^N)^2 = \lambda\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right)Q^N D(p^N), \quad \text{(B.22)}$$

$$\varphi'(i^N) = \lambda\left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right), \quad \text{(B.23)}$$

$$2\Pi(p^N)Q^N = \lambda\left(1 - \left(\frac{1}{2}g(R^N) + \int_0^{R^N} h^2(\varepsilon)d\varepsilon\right)S(p^N)\right) \quad \text{(B.24)}$$

where Q^N and R^N are respectively defined by (5.8) and (B.19). Inserting (B.23) into (B.22) and (B.24) respectively yields (B.17) and (B.18). □

The equilibrium conditions above bear some similarity with the case of an unregulated monopoly. As the latter, a duopolist keeps some captive demand when the market is not fully covered. As a result, it can further raise its unit price above marginal cost.

Running Example (Continued). Here also, simple and intuitive comparative statics follow.

Corollary B.2. *Consider a scenario with duopolistic competition, no fixed fees and no interoperability. Both the equilibrium price ratio $\frac{p^N}{a}$ and market coverage Q^N increase with*

- *a greater demand (i.e., D or a larger),*
- *a lower cost of investment (i.e., φ lower),*

- *a lower average transportation cost (i.e., λ higher).*

Proof of Corollary B.2. With our specifications, the optimality conditions (B.18), (5.8) and (B.19) can be used to get

$$\frac{p^N}{a} = 1 - \varphi \frac{i^N}{Q^N} \tag{B.25}$$

together with a pair of equations that specify $\frac{p^N}{a}$ and $\frac{R^N}{\lambda}$, namely

$$\frac{1 + \frac{p^N}{a}}{1 - \frac{p^N}{a}} e^{-\frac{p^N}{a}} = \frac{\frac{2\lambda}{Da}}{1 + e^{-\frac{2R^N}{\lambda}}} \tag{B.26}$$

and

$$\frac{\frac{2R^N}{\lambda}}{1 - e^{-\frac{2R^N}{\lambda}}} = \frac{Da}{\lambda} e^{-\frac{p^N}{a}} + \frac{1}{\lambda\varphi}\left(1 - \frac{p^N}{a}\right). \tag{B.27}$$

It can be readily checked that (B.26) implicitly defines an upward sloping relationship $\frac{R^m}{\lambda} = \Phi_1\left(\frac{p^m}{a}\right)$ while (B.27) implicitly defines a downward sloping relationship $\frac{R^m}{\lambda} = \Phi_2\left(\frac{p^m}{a}\right)$. Provided that

$$\Phi_2(0) > \Phi_1(0) = \lambda ln\left(\frac{Da}{\lambda}\right) \tag{B.28}$$

holds, there is a unique pair $\left(\frac{R^m}{\lambda}, \frac{p^m}{a}\right)$ that solves (B.12)-(B.13). □

The comparison with the monopoly scenario is now straightforward.

Corollary B.3. *In comparison with the monopoly scenario, duopolistic competition with no fixed fees and no interoperability entails greater overall surplus, more participation and more investment:*

$$R^N > R^m, \quad Q^N > Q^m, \quad i^N > i^m. \tag{B.29}$$

Proof of Corollary B.3. With a monopoly, the optimality conditions are now given by

$$\frac{p^m}{a} = 1 - \varphi \frac{i^m}{Q^m} \tag{B.30}$$

together with a pair of equations that specify $\frac{p^m}{a}$ and $\frac{R^m}{\lambda}$, namely

$$\frac{1+\frac{p^m}{a}}{1-\frac{p^m}{a}}e^{-\frac{p^m}{a}} = \frac{\lambda}{Da}e^{\frac{2R^m}{\lambda}} \tag{B.31}$$

and

$$\frac{\frac{2R^m}{\lambda}}{1-e^{-\frac{2R^m}{\lambda}}} = \frac{Da}{\lambda}e^{-\frac{p^m}{a}} + \frac{1}{\lambda\varphi}\left(1-\frac{p^m}{a}\right). \tag{B.32}$$

The claim in the Corollary is a direct consequence of comparing the right-hand sides of (B.26) and (B.31). □

■

B1.3 Duopolistic Competition without Interoperability

The analysis hereafter somehow mixes our previous findings. interoperability again softens competition while, in the absence of access fees, the unit prices for transaction are now distorted.

Proposition B.4. *Suppose that the market is only partially covered, there is interoperability, but access fees are not feasible. Suppose also that Assumption 2 holds. There exists a unique symmetric Nash equilibrium where firms choose a price per transaction p^N and an investment i^N such that*

$$\frac{p^N-c}{p^N} = \frac{1}{\varepsilon_D(p^N)}\left(1-\frac{\varphi'(i^N)}{Q^N}\right), \tag{B.33}$$

$$\Pi(p^N) = \frac{\varphi'(i^N)}{(2+\gamma)Q^N}\left(\frac{2}{g(R^N)+2\int_0^{R^N}h^2(\varepsilon)d\varepsilon} - \left(1-\frac{2\gamma\int_0^{R^N}h^2(\varepsilon)d\varepsilon}{g(R^N)+2\int_0^{R^N}h^2(\varepsilon)d\varepsilon}\right)S(p^N)\right) \tag{B.34}$$

Q^N and R^N are respectively given by (5.8) and

$$R^N = (1+\gamma)Q^N S(p^N) + i^N. \tag{B.35}$$

Proof of Proposition B.4. We may now write F 's profit as

$$Q\left(Q+\gamma Q^{*}\right)\Pi(p)-\varphi(i). \tag{B.36}$$

With no access fee, we find the expression of F's market coverage as:

$$Q=\int_{0}^{R(Q+\gamma Q^{*},p,i)} h(\varepsilon)\left(1-H\left(R(Q^{*}+\gamma Q,p^{*},i^{*})-R(Q+\gamma Q^{*},p,i)+\varepsilon\right)\right)d\varepsilon. \tag{B.37}$$

From Lemma A.3, there again exists a unique solution Q to (B.37).

Denoting again by λ the Lagrange multiplier for (B.21), we form the corresponding Lagrangean as

$$\mathcal{L}(Q,p,i)=Q\left(Q+\gamma Q^{*}\right)\Pi(p)-\varphi(i)$$
$$+\lambda\left(\int_{0}^{R(Q+\gamma Q^{*},p,i)} h(\varepsilon)\left(1-H\left(R(Q^{*}+\gamma Q,p^{*},i^{*})-R(Q+\gamma Q^{*},p,i)+\varepsilon\right)\right)d\varepsilon-Q\right).$$

Assuming concavity of the Lagrangean, we obtain the following necessary and sufficient conditions with respect to (p,i,Q) at a symmetric Nash equilibrium:

$$\left((p^{N}-c)D'(p^{N})+D(p^{N})\right)(1+\gamma)(Q^{N})^{2}=\lambda\left(\frac{1}{2}g(R^{N})+\int_{0}^{R^{N}} h^{2}(\varepsilon)d\varepsilon\right)(1+\gamma)Q^{N}D(p^{N}), \tag{B.38}$$

$$\varphi'(i^{N})=\lambda\left(\frac{1}{2}g(R^{N})+\int_{0}^{R^{N}} h^{2}(\varepsilon)d\varepsilon\right), \tag{B.39}$$

$$(2+\gamma)\Pi(p^{N})Q^{N}=\lambda\left(1-\left(\frac{1}{2}g(R^{N})+(1-\gamma)\int_{0}^{R^{N}} h^{2}(\varepsilon)d\varepsilon\right)S(p^{N})\right) \tag{B.40}$$

where

$$R^{N}=(1+\gamma)Q^{N}S(p^{N})+i^{N}. \tag{B.41}$$

Inserting (B.39) into (B.38) and (B.40) respectively yields (B.33) and (B.34). □

Running Example (Continued). We are particularly interested in the impact of γ.

Corollary B.4. *Consider a scenario with duopolistic competition, no fixed fee and* *interoperability and suppose that market coverage in the absence of interoperability is small. Increasing interoperability (i.e., making γ larger) increases market coverage Q^N but has an ambiguous impact on the equilibrium price ratio $\frac{p^N}{a}$.*

We argue in the proof below that a price decrease with interoperability is more likely to arise when the price before interoperability is already large and there is little market coverage.

Proof of Corollary B.4. The optimality condition (B.25) still holds. Conditions (5.8) and (B.34) can again be used to get a pair of equations satisfied by $\frac{p^N}{a}$ and $\frac{R^N}{\lambda}$, namely

$$\frac{1+(1+\gamma)\frac{p^N}{a}}{1-\frac{p^N}{a}}e^{-\frac{p^N}{a}} = \frac{\frac{2\lambda}{Da}}{1+e^{-\frac{2R^N}{\lambda}}} + \frac{\gamma}{Da}e^{-\frac{p^N}{a}}\left(\frac{1-e^{-\frac{2R^N}{\lambda}}}{1+e^{-\frac{2R^N}{\lambda}}}\right) \tag{B.42}$$

and

$$\frac{\frac{2R^N}{\lambda}}{1-e^{-\frac{2R^N}{\lambda}}} = (1+\gamma)\frac{Da}{\lambda}e^{-\frac{p^N}{a}} + \frac{1}{\lambda\varphi}\left(1-\frac{p^N}{a}\right). \tag{B.43}$$

It can again be readily checked that the left-hand side of (B.42) is increasing in $\frac{p^N}{a}$ over the range $[0,1)$ while the right hand-side remains increasing in $\frac{R^N}{\lambda}$ and decreasing in $\frac{p^N}{a}$. It uniquely defines an upward sloping relationship $\frac{R^N}{\lambda} = \Phi_1\left(\frac{p^N}{a},\gamma\right)$. In turn, the left-hand side of (B.43) is increasing in $\frac{R^N}{\lambda}$ while the right hand-side is decreasing $\frac{p^N}{a}$. It thus uniquely defines a downward sloping relationship $\frac{R^m}{\lambda} = \Phi_2\left(\frac{p^m}{a},\gamma\right)$. Provided that

$$\Phi_2(0,\gamma) > \Phi_1(0,\gamma), \tag{B.44}$$

a condition that will be supposed to hold, there is a unique pair $(\frac{R^N}{\lambda}, \frac{p^m}{a})$ that thus solves (B.42)-(B.43).

We are particularly interested in the impact of γ. The first impact of increasing γ is to shift upwards the locus $\frac{R^N}{\lambda} = \Phi_1\left(\frac{p^N}{a},\gamma\right)$. Similarly, increasing γ increases the right-hand side of (B.43) and thus also shifts upwards the locus $\frac{R^N}{\lambda} = \Phi_2\left(\frac{p^N}{a},\gamma\right)$. The equilibrium values of $\frac{R^N}{\lambda}$ necessarily increases, and thus necessarily market participation Q^N, while $\frac{p^N}{a}$ may either increase or decrease. It is more likely to decrease when the upward shift of the locus $\frac{R^N}{\lambda} = \Phi_1\left(\frac{p^N}{a}, \gamma\right)$ is less significant, which is more likely to arise when the price before interoperability is already large and there is little market coverage. $\square$

$\blacksquare$