WORKING PAPERS

N° 1569

September 2024

"Platform Transaction Fees and Freemium Pricing"

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September 2024

Abstract

We study transaction fees applied by marketplace platforms where sellers (e.g., app developers) adopt freemium pricing. An ad valorem transaction fee reduces quality distortions introduced by the price-discriminating seller, thereby increasing consumer surplus. Moreover, a small fee increases welfare, implying that the agency model may be socially preferable to integration between platform and seller. However, the platform may set the equilibrium fee above the socially optimal level. Providing devices needed to access the marketplace (e.g., phones) induces the platform to raise the fee, whereas providing a product that competes with the seller induces a lower fee.

JEL Classification: D4, D21, L11, H22

Keywords: Transaction fees, price discrimination, marketplace platforms, apps

*We thank Anna Rita Bennato, Gary Biglaiser, Marc Bourreau, Subho Chowdhury, Paul Dobson, Luke Garrod, Enrique Ide, Doh-Shin Jeon, Bruno Jullien, Hans Jarle Kind, Wynne Lam, Yassine Lefouili, David Martimort, Lijun Pan, Jens Prufer, Matthew Rablen, Florian Schuett, Shiva Shekar, Piercarlo Zanchettin, Chris Wilson and participants to presentations at EARIE 2024 (Amsterdam) Csef-Igier Symposium on Economics and Institutions 2024 (Capri), UB Microeconomics Workshop 2024 (Barcelona), Workshop on the Economics of Platforms 2024 (Rome), SEO 2024 (Amsterdam), SEM 2024 (Tilburg), Economics of Digitization Seminar 2024 (Télécom Paristech) and the Digital Economics Conference 2024 (Toulouse School of Economics) for useful comments and suggestions. Funding from Agence Nationale de la Recherche (ANR) under grant ANR-17-EURE-0010 (Investissements d'Avenir program) is gratefully acknowledged. All errors are ours.

1 Introduction

Digital marketplace platforms, such as the App Store, Google Play, Microsoft Store and Play Station Store, commonly apply fees to monetary transactions between sellers and consumers.[1] These fees are quite controversial. While platforms maintain that they are key to the viability of their services, many sellers claim that fees unfairly squeeze their profits and force them to raise prices, thereby harming consumers. These complaints have prompted several investigations by regulators, who have questioned the legitimacy of the rules applied by platforms.[2] Moreover, because the platforms often sell their own products on their marketplace, regulators have raised concerns that transaction fees may be anti-competitive, by putting third-party sellers at a disadvantage.[3]

Transaction fees are a key component of the "agency model" of vertical relations, where sellers set final prices to consumers and platforms retain a share of their revenue via such fees (Johnson, 2017; Foros et al., 2017). We contribute to the study of this model by considering the sophisticated pricing policies adopted by sellers in digital marketplaces. Notably, sellers of digital goods, e.g., mobile apps, often adopt second-degree price discrimination, more specifically in the form of *freemium* pricing: the basic version is free of charge (though users may pay a nonmonetary price, e.g., by having to see ads), whereas a monetary price applies to the premium version (possibly in the form of repeated purchases over time).[4] In this context, even if a fee on monetary transactions applies formally to all versions of the product, it effectively targets the premium one only. Moreover, marketplace platforms are typically part of ecosystems, so platforms have multiple sources of revenue besides the transaction fees. Given these observations, the incentives to set these fees and how sellers respond to them are far from obvious.

Conventional wisdom suggests that charging a fee on a product should result in a higher equilibrium price and in lower supply, imposing a burden on consumers and producers, and

[1]Apple and Google currently charge sellers 30 percent of the price consumers pay to download apps, initial subscriptions or in-app purchases, and 15 percent for repeated subscriptions.

[2]See the recent lawsuit brought by Epic Games against Apple and Google. Moreover, Apple and Google have been designated as gatekeepers by the European Commission under the Digital Market Act for their app stores (see https://ec.europa.eu/commission/presscorner/detail/en/ip_23_4328). An investigation has been launched on Apple for violating the Digital Market Act in its Apple Store (see https://ec.europa.eu/commission/presscorner/detail/en/IP_24_3433.)

[3]This is one of the main concerns reported by the Dutch Competition Authority (ACM, 2019, chpt. 3 and 4), in the context of the mobile app market. Similar concerns were raised in a recent antitrust lawsuit against Google brought by multiple US States (see https://www.courtlistener.com/docket/60042641/522/state-of-utah-v-google-llc/).

[4]This strategy is extremely common for mobile apps in categories including music and video streaming, gaming, and data storage. As reported by ACM (2019), freemium accounts for more than 90% of revenue from the games category of apps in the App Store. Statista (see http://tinyurl.com/5aj4cdtj) indicates that freemium is also widely used in other app categories, such as news and social networking.

compounding the classic distortions arising from supplier market power. Our first result is that this presumption does not hold for an ad valorem transaction fee that targets only one version of the product sold by a price-discriminating firm, as in the case of freemium pricing. Specifically, a fee on the top version can alleviate the distortion imposed by the seller, which originates from the classic trade-off between rent extraction and efficiency (Laffont and Martimort, 2002).

To understand the above claim, consider a monopolist offering two versions of its product, intended for two types of consumers that differ in their marginal utility from the product's quality (or size). The seller must ensure that consumers self-select on the intended version. The incentive compatibility constraint limits the amount of revenue that can be extracted from the high types, who receive an information rent. Facing this constraint, the seller sets the quality intended for the high types at the efficient level, and distorts the quality of the low version, so that the low types' marginal utility exceeds the marginal cost, to reduce the information rent (Maskin and Riley, 1984). By taking away a percentage of the revenue extracted from the high types, an ad valorem fee on the top version makes generating revenue from the high types relatively less attractive to the seller. Hence, the incentive to distort the quality of the basic version diminishes. The drawback is that the fee distorts the quality of the top version. However, starting from the laissez-faire equilibrium, this second distortion is small in magnitude, so consumer surplus and social welfare increase. This mechanism is robust to including competition among sellers and more than two consumer types. However, although these findings indicate that a positive level of the transaction fee is socially desirable, they do not guarantee that the level chosen by the platform (that maximizes its profit) will be socially optimal. The equilibrium level of the fee may indeed exceed the optimum one.

Our findings contribute to the study of the agency model and, more generally, of vertical relations in digital markets. In our basic setting, the platform is purely an intermediary connecting consumers to the seller and collecting a percentage of its revenue. If the platform and the seller were integrated, this firm would simply maximize the seller's profit (i.e., implement the equilibrium with no fee). Given that consumer surplus and welfare can be higher when the platform applies an ad valorem fee than with no fee at all, the agency model with separated firms can be socially preferable to an integrated one. This contrasts with the usual prescription that vertical integration increases welfare due to the otherwise imperfect coordination between upstream and downstream firms (Tirole, 1988).

Next, we consider that marketplace platforms are part of ecosystems, so transaction fees interact with other sources of revenue for the platform. We first let the platform sell a device essential to accessing the marketplace (e.g., a smartphone). By effectively charging consumers for access to the market via the price of the device, the platform internalizes the effect of the fee on their surplus. As established above, this effect is positive given freemium pricing, which induces the platform to set a higher fee. This is in contrast to standard settings, dating at least as far back as Oi (1971), where access and transaction charges are substitutes. In our context, access and transaction charges are instead complements. We find a similar result when the platform captures part of the advertising revenue the seller makes from the free version.[5] The fee induces the seller to raise the quality of the free version and, thus, the non-monetary price (volume of ads) that consumers are willing to tolerate.

We then address the role of transaction fees on "hybrid" platforms that sell their own products on the marketplace. For example, Apple and Google offer music and video streaming apps that compete with third-party ones. As mentioned, a major concern is that platforms may use fees to force third-party sellers to raise prices and weaken competition to their own product. We find that, given freemium pricing, the platform prefers a lower transaction fee than if it had no competing product to sell, thereby making the seller better off. The intuition is that, as argued above, a marginal increase in the fee induces the third-party seller to adjust its offer such that consumers get more surplus. Hence, the fee increases the competitive pressure on the platform's product from the seller, as long as the latter adopts the freemium model.

In the last part of the analysis, we consider the implications of letting the seller distribute its product outside the platform. Although doing so conceivably increases transaction costs for consumers (for instance, because they have to register to a new payment system rather than use the platform's), the advantage for the seller is to avoid the transaction fee. We show that, while the seller sets a lower price for its product outside the platform, the effects of the transaction fee are similar to those described above. In particular, the quality of the low version increases, while the price of the top version decreases, when distributed either inside or outside the platform. We also endogenize the number of sellers and consumers, showing that transaction fees may induce a virtuous circle, thereby increasing participation from both sides to the platform. This is because, although the fee reduces the per-consumer profit of each supplier, consumers get more surplus from interacting with suppliers.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 introduces our basic model. In Section 4 we consider transaction fees as part of several sources of revenue for the platforms, focusing on the interaction between transaction fees and the sale of devices (Section 4.1), ads (Section 4.2) and the platforms' own products (Section 4.3). Section 5 studies further extensions, including one where the seller can bypass the platform to reach consumers. Section 6 summarizes the main policy implications of the analysis and concludes.

[5] Some platforms (e.g., Google) also operate as intermediaries in the online advertising market. See the recent report by CMA (2020).

2 Literature

There is an extensive literature studying price discrimination (Tirole, 1988; Laffont and Martimort, 2002; Stole, 2007), that has recently focused on two-sided markets. Lin (2020) and Jeon et al. (2022) study second-degree discrimination, focusing on how a platform's incentives and ability to screen participants on one side depend on the externalities generated on the other side. de Cornière et al. (2024) study third-degree price discrimination by a platform hosting different types of sellers. Wang and Wright (2017) show that ad valorem fees allow efficient price discrimination across goods with different costs and values, unlike unit fees. We consider the effects of transaction fees when the sellers on the platform, rather than the platform itself, engage in price discrimination.

Our study also contributes to the literature studying marketplace platforms that bring together buyers and sellers (Baye and Morgan, 2001; Karle et al., 2020). A branch of this literature focuses on the "agency model" in vertical relations, which is typical of platforms (Johnson, 2017; Foros et al., 2017). The literature has found that this model may perform better than other vertical arrangements (e.g., wholesale) in terms of welfare and consumer surplus. However, due to the usual lack of coordination between upstream and downstream firms, welfare and consumer surplus would be higher with an integrated firm. Our findings confirm the welfare-superiority of ad valorem fees compared to unit fees in vertical relations, but we show that when sellers adopt freemium pricing, consumer surplus and welfare can be *higher* when the platform applies the ad valorem fee than without the fee. This result implies that a vertically integrated firm may reduce welfare.

Another branch of this literature focuses on the relation between transaction fees and other sources of revenue for the platform, such as the sale of devices and/or ads (see, e.g., Etro, 2021; Gaudin and White, 2021). Unlike previous papers, we find that, when one accounts for freemium pricing by sellers, a transaction fee can be complementary to the price of an essential device (equivalent to an access fee to the marketplace). Anderson and Bedre-Defolie (2024) study the market for apps endogenizing participation of both consumers and app sellers. The authors focus on the effects due to cross-market externalities and not on freemium pricing.

More recently, the literature has considered "hybrid" marketplace platforms (Hagiu et al., 2020, 2022). Anderson and Bedre-Defolie (2021) consider monopolistic competition among sellers and a platform that provides a range of competing products. They find that, compared to a pure marketplace, a hybrid platform may set higher transaction fees to steer consumers towards its products. Focusing on the case of freemium sellers, we show that the platform does not necessarily gain from using the fee as an anti-competitive instrument, because the competitive pressure from third-party sellers may increase with the fee. Tremblay (2022) shows that, when entering a market, a platform tends to reduce transaction fees applied to other sellers in that market. Competition from the platform reduces the sellers' output and, hence, their willingness to pay. This result is fairly consistent with our findings, though our setting, and the mechanism behind our results, are different.

Finally, our paper makes also a contribution to the literature on the incidence of indirect taxes, which is a classical topic in economics (Fullerton and Metcalf, 2002). Many previous studies have looked at tax incidence in imperfectly competitive markets, focusing primarily on single-product firms that adopt linear pricing (Anderson et al., 2001; Weyl and Fabinger, 2013; Miklós-Thal and Shaffer, 2021). A fundamental result in this literature is that taxes depress supply and increase prices.[6] Some recent studies have shown that this result may not hold in multiproduct settings (see Armstrong and Vickers, 2023; D'Annunzio and Russo, 2024). Only a handful of studies have investigated taxation in markets with second-degree price discrimination (Laffont, 1987; Jensen and Schjelderup, 2011; D'Annunzio et al., 2020). In this paper, we consider firms implementing freemium pricing. Our main results stem from considering a tax that (de facto) targets only some versions.[7]

3 Baseline model

To demonstrate the central mechanism to our results, we propose a simple model of second-degree price discrimination with freemium pricing. Consider a monopolist (the "seller") providing a single good (e.g., an app). There are two types of consumers, differing in their preference for this good, captured by the parameter θ_i with $i = H, L$ (where H stands for "high" and L for "low"). A consumer of type i gets the following net utility from consuming the good

$$U_i(p,q,x) \equiv u(q,\theta_i) - p - \alpha x, \; i = H, L, \tag{1}$$

where q is the quality of the good, p is the monetary price, x is the nonmonetary price (e.g., ads shown on the app, or the volume of personal data collected) and $\alpha > 0$ is the disutility from every non-monetary unit. We assume that $\theta_H > \theta_L$, $\frac{\partial u}{\partial q} > 0$, $\frac{\partial^2 u}{\partial q^2} < 0$, $\frac{\partial u}{\partial \theta} > 0$ and $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. The parameter θ_i is private information. The total number of consumers is normalized to one and $\nu \in (0,1)$ is the share of type-H consumers. Although we refer to q as the quality of the good, this variable can also be interpreted as quantity (Mussa and Rosen, 1978; Maskin and Riley, 1984).[8]

[6] There are very few exceptions to this result. These include Cremer and Thisse (1994), who show that taxation can increase welfare in a vertically differentiated oligopoly, and Carbonnier (2014), who studies price-dependent tax schedules

[7] McCalman (2010) considers a similar type of differentiation when analyzing trade tariffs applied on a foreign monopolist seller. Tariffs have conventional effects on equilibrium quantities and prices in his model, although they can increase domestic welfare (that is, when the surplus of the seller is ignored).

[8] For instance, in the case of an app, q may indicate the number of functions, amount of data storage or actions

We assume the seller offers to consumers two bundles, (q_i, p_i, x_i), to choose from, each intended for one consumer type. These bundles can be interpreted as two versions of the product differing in their features and/or functionalities, sold at different prices. We assume that the seller adopts freemium pricing, providing a basic version for free (which consumers "pay" for with their attention and/or personal data), and a premium one, for which consumers pay a monetary price.[9] That is, we assume $x_H = 0$ and $p_L = 0$. The seller derives some exogenous revenue, r, for every unit of the nonmonetary price on the L-version (i.e., the advertising rate or the price of data).

A platform connects the seller to consumers and applies an ad valorem fee, t, to the monetary revenue collected by the seller. The fee does not apply to revenue from ads or the exploitation of consumer data. This is consistent with the transaction fees currently applied by digital marketplaces (e.g. the App Store and Google Play Store). Given this fee, the seller's problem is

$$\max_{q_H, p_H, p_L, q_L, x_L} \quad \pi = v\left((1-t)\, p_H - c q_H\right) + (1-v)\left(r x_L - c q_L\right), \tag{2}$$

$$s.t. \quad u_H - p_H \geq u_{HL} - \alpha x_L, \tag{3}$$

$$u_L - \alpha x_L \geq u_{LH} - p_H, \tag{4}$$

$$u_H - p_H \geq 0, \tag{5}$$

$$u_L - \alpha x_L \geq 0, \tag{6}$$

where $u_i \equiv u(q_i, \theta_i)$, $i = H, L$, $u_{HL} \equiv u(q_L, \theta_H)$ and $u_{LH} \equiv u(q_H, \theta_L)$. The parameter $c > 0$ is the unit cost of providing q (e.g., additional app functionalities).[10] In the above problem, (3) and (4) are the incentive compatibility constraints, while (5) and (6) are the participation constraints for H and L-type consumers, respectively (we normalize the utility from no consumption to zero).

For simplicity, assume that the platform sustains no cost and that the fee is its only source of revenue (we relax this assumption in Section 4). Hence, the platform's profit is

$$\pi_P = v t p_H. \tag{7}$$

The timing is as follows: the platform sets t. Next, the seller designs the two pairs (p_i, q_i) with $i = L, H$. Finally, consumers choose which version to buy, if any. Throughout the analysis we denote

available for a given price.

[9] We consider a static model, implying that we do not distinguish between payments made at the time the consumer acquires the good (e.g., downloading the app) or while using the good (e.g., in app purchases). Moreover, we disregard the intensive margin, i.e. how much the consumers use the app. Given these aspects, the monetary price p_i encompasses payments for subscription and in-app purchases.

[10] We assume this cost is proportional to the number of consumers for ease of exposition. Our results would not change if the cost was independent of the number of consumers, i.e., if total cost were $cq_H + cq_L$.

the equilibrium variables (given the fee) with the superscript e.

We briefly discuss our setup before proceeding with the analysis. We assume the seller adopts freemium pricing because our objective is to study the effects of the transaction fee given this widely adopted pricing strategy. In Appendix A, we provide a foundation to this assumption by making the seller's choice of monetary and nonmonetary price endogenous. We show that this choice depends on the relation between the revenue the seller can generate from the nonmonetary price and the disutility from such price, for each consumer type.[11] We consider a monopolist seller and two consumer types to simplify the exposition. In Appendix C, we provide extensions allowing for more than two types and for competition among sellers, showing that the results we present below are robust to such modifications.[12] The assumption of a monopolist platform captures the fact that each marketplace is a monopolist in a given ecosystem. A further simplifying assumption is that the number of consumers and sellers on the platform is given. We relax this assumption in Section 5.1. Finally, in Appendix D we consider the case where the seller applies a pure monetary price to all versions of the product, i.e., $x_H = 0$ and $x_L = 0$. In this scenario, the fee t applies equally to all versions. As a result, the effect of the fee on the quality of both versions, consumer surplus and welfare are quite standard.

3.1 Equilibrium and effects of the transaction fee

Following standard steps (Laffont and Martimort, 2002), one can show that only (3) and (6) are binding at the allocation that solves the seller's problem. Therefore, we ignore (4) and (5), and set

$$p_H = u_H - u_{HL} + u_L, \quad x_L = \frac{u_L}{\alpha}. \tag{8}$$

Hence, we can rewrite the seller's problem in (2) as

$$\max_{q_H, q_L} \quad \pi = v\left((1-t)\left(u_H - u_{HL} + u_L\right) - cq_H\right) + (1-v)\left(\tfrac{r}{\alpha}u_L - cq_L\right). \tag{9}$$

In the above expression, $u_{HL} - u_L$ represents the high types' information rent, which the seller must grant to prevent them from choosing the version intended for the low types. There is, instead, no

[11] See Sato (2019) and Zennyo (2020) for previous studies on freemium as a form of price discrimination. Unlike these papers, we do not study the profitability of freemium per se, but we are interested in the implications of platform fees when the app sellers adopt this pricing strategy. Accordingly, the objective of Appendix A is to derive sufficient conditions such that freemium pricing arises in equilibrium.

[12] We consider a setting with discrete types because it fits the market we consider (where a small number of versions is available) better than one with a continuum of types.

rent left to the low types. We thus get the following expressions for consumer surplus

$$CS_H = u_{HL} - u_L, \qquad CS_L = 0. \tag{10}$$

The equilibrium qualities, q_i^e, solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} = v\left(\frac{\partial u_H}{\partial q_H}(1-t) - c\right) = 0, \tag{11}$$

$$\frac{\partial \pi}{\partial q_L} = v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t) + (1-v)\left(\frac{r}{\alpha}\frac{\partial u_L}{\partial q_L} - c\right) = 0. \tag{12}$$

Setting the fee aside ($t = 0$), these equations indicate that the seller offers an efficient version to the high types, in the sense that the marginal utility these consumers get from quality equals the marginal cost. Given $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$, the seller distorts the quality of version intended for the low types downwards. This is to reduce the information rent left to the high types.

Let us now study the effects of the fee. Obviously, the seller's profit decreases with it. However, the effects on equilibrium variables, consumer surplus and welfare are less straightforward. We show in Appendix B.1 that

$$\frac{\partial q_H^e}{\partial t} < 0, \quad \frac{\partial q_L^e}{\partial t} > 0. \tag{13}$$

Interestingly, the quality of the L-version *increases* with the fee. To see why, recall that the seller distorts q_L downward to reduce the information rent of the high types. The fee takes part of the revenue earned from selling to these consumers away, without affecting the revenue from the L-version directly, because the price for such version is nonmonetary. As a consequence, the fee reduces the incentive to distort q_L (see (12)). By the same token, the fee induces a downward distortion in q_H. However, starting from the laissez-faire equilibrium, the magnitude of this distortion is of second order (see Section 3.2).

The effects of the fee on prices mirror those on quality. Starting from (8) and given (13), we have

$$\frac{\partial p_H^e}{\partial t} < 0, \quad \frac{\partial x_L^e}{\partial t} > 0. \tag{14}$$

Intuitively, the fee on the H-version reduces its price. This is due not only to the fact that q_H^e decreases, but also to the increase in q_L^e, which raises the information rent. On the other hand, the fee increases the price of the L-version by an amount equal to the willingness to pay of the low types for the increase in quality. Thus, the high types benefit from the fee, while the net surplus of the low types remains equal to zero (see (10)). Overall, therefore, consumer surplus *increases* with t. We have

$$\frac{\partial CS_H^e}{\partial t} = \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L^e}{\partial t} > 0, \quad \frac{\partial CS_L^e}{\partial t} = 0. \tag{15}$$

Proposition 1. *When the seller adopts freemium pricing, the quality and price of the H-version decrease with the transaction fee charged by the platform, whereas the quality of the L-version increases. Moreover, while the profit of the seller decreases, consumer surplus increases.*

3.2 Effect of transaction fee on social welfare

Consider now the effects of the fee on social welfare. The welfare function, obtained as the sum of consumer surplus and firms' profits, boils down to total surplus in this market

$$W = v\left(u_H - cq_H\right) + (1-v)\left(\frac{r}{\alpha}u_L - cq_L\right). \tag{16}$$

Differentiating (16) and given the first-order conditions of the seller's problem, we obtain[13]

$$\frac{\partial W}{\partial t} = v\frac{\partial q_H^e}{\partial t}\frac{\partial u_H}{\partial q_H}t + (1-v)\frac{\partial q_L^e}{\partial t}\left(\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)(1-t)\right). \tag{17}$$

It is useful to start by evaluating the above derivative at the laissez-faire equilibrium, where $t = 0$. This provides an indication of the effect of introducing a fee, as opposed to no fee at all. Given (13), we obtain

$$\left.\frac{\partial W}{\partial t}\right|_{t=0} = v\frac{\partial q_L^e}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0. \tag{18}$$

This expression shows that welfare increases when introducing the fee. Hence, having no transaction fee cannot be socially optimal. As shown above, this fee reduces the distortion that the seller imposes on the L-version, while the distortion on the H-version is of second-order.

3.3 Equilibrium and optimal transaction fee

The above finding does not imply that *any* fee level would be socially desirable. In general, we cannot expect the platform to set a fee equal to (or lower than) the socially optimal level. To establish this, consider first that the equilibrium fee, t^e, maximizes the platform's revenue

$$\pi_P = vtp_H^e = vt\left(u_H^e - u_{HL}^e + u_L^e\right). \tag{19}$$

[13] In these expressions, the derivatives of utility with respect to q_i are evaluated at the equilibrium values, q_i^e, given t.

Assuming an interior solution, t^e satisfies the following first-order condition

$$\frac{\partial \pi_P}{\partial t} = v\left(u_H^e - u_{HL}^e + u_L^e\right) - vt\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right) = 0. \tag{20}$$

The platform sets t^e taking into account the revenues from the fee (first term in (20)) and how increasing the fee affects the incentives of the supplier to decrease p_H (second term in (20)). The welfare-maximizing fee, t^W, is such that $\frac{\partial W}{\partial t} = 0$ in (17). Assuming concavity of the welfare function, we compare t^e and t^W by evaluating $\frac{\partial W}{\partial t}$ in $t = t^e$ (using (20)). We get

$$\left.\frac{\partial W}{\partial t}\right|_{t=t^e} = -v\left(u_H^e - u_{HL}^e + u_L^e\right) + v\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L}{\partial t}. \tag{21}$$

The first term on the right hand side is negative, while the second one is positive. Indeed, seller's revenues from selling to the high types intuitively decrease with the fee. On the other hand, consumer surplus in (10) increases. Hence, the fee has opposite effects on the two groups of users on the platform: it makes the seller worse off and the consumers better off, implying that the comparison depends on which effect dominates.[14]

Proposition 2. *When the seller adopts freemium pricing, introducing a transaction fee (starting from the no-fee equilibrium) increases welfare. However, the equilibrium level of the fee can be too high compared to the optimal one.*

3.4 Implications for the analysis of vertical structures

3.4.1 Vertical integration and separation

In the analysis so far, we have considered a market structure where the seller and the platform are independent, and they adopt the "agency" model of vertical relations (Johnson, 2017; Foros et al., 2017), where the seller retains control over final prices and the platform takes a share of the revenue. A vertically integrated firm would instead maximize the sum of seller and platform profit, given by (9) and (19) respectively. This sum boils down to the profit the seller earns gross of the fee. The firm would therefore implement the same prices and quality levels as when $t = 0$.

[14] A caveat regarding the analysis is that, given it induces a reduction in q_H^e and an increase in q_L^e, a sufficiently high t might result in these two variables being equal. At that point, the seller could not implement a price schedule such that $p_H > \alpha x_L$, because the constraint (3) would be violated. In response, the seller might stop serving the low-type consumers and offer a single version that targets the high-types (setting $p_H = u_H$). Let this threshold fee level be $\bar{t}$. Characterizing $\bar{t}$ is quite cumbersome, but qualitatively there is little loss in ignoring it. We assume that the platform sets a transaction fee such that the seller serves both types and the above effects apply. We come back to this point in Section 5.

Propositions 1 and 2 show that, given freemium pricing, welfare and consumer surplus increase with the fee applied by the platform if the latter is within the range $\left[0, t^{W}\right]$, although the seller's profit decreases. Hence, the fact that the platform does not internalize the effect of its fee on the supplier's profit can improve efficiency and consumer surplus. We conclude that the agency model in vertical relations may be efficient, implying that separation can dominate integration in our setting (provided that $t^{e} < t^{W}$). This finding contrasts with the common prescription that, by removing the frictions caused by the lack of coordination between suppliers and retailers, vertical integration increases market performance (Tirole, 1988). This result may have interesting implications for the analysis of mergers in markets where sellers adopt freemium pricing.

Proposition 3. *Given freemium pricing, vertical integration between the seller and the platform can reduce welfare and consumer surplus.*

3.4.2 Ad valorem vs. unit transaction fees

We have considered an ad valorem transaction fee, consistently with the policy applied by platforms like Apple and Google. Previous literature has compared ad valorem to unit fees in vertical relations, often finding ad valorem ones to be more efficient (see, e.g., Wang and Wright, 2017; Gaudin and White, 2021).[15] To address this question, in Appendix B.2 we study the effects of a unit transaction fee. If the fee is proportional to q_i, the seller's profit is $\pi = v\left(p_H - (c+\tau)\,q_H\right) + (1-v)\left(x_L - (c+\tau)\,q_L\right)$. Clearly, unit fees are similar to an increase in the cost of production. Accordingly, we find that q_i in each version decreases with the unit fee, so that the effect on consumer and total surplus is negative. [16]

4 Transaction fees in an ecosystem

We now study the incentives of the platform to set transaction fees when it manages an ecosystem. More specifically, transaction fees are typically one of several sources of revenue to the platform. For example, Apple and Google sell the devices required to access the marketplace (e.g., smartphones and tablets), in addition to imposing transaction fees on the marketplace for apps. Platforms like Google are also involved in the sale of ads displayed on the "ad-funded" versions of the apps, acting

[15] This finding is consistent with the literature on commodity taxation, which shows that ad valorem taxes are less distortionary than unit taxes in imperfectly competitive markets (Delipalla and Keen, 1992; Anderson et al., 2001; Auerbach and Hines, 2002).

[16] Alternatively, we could model the unit fee as proportional to each bundle sold to consumers, implying that profits are $\pi = v\left(p_H - cq_H - \tau\right) + (1-v)\left(x_L - cq_L - \tau\right)$. Given our assumptions, these fees would not have any effect on the equilibrium variables.

as intermediaries between advertisers and app sellers. Also, both Google and Apple provide apps competing with other apps sold in their marketplace by independent sellers. We propose several extensions to the model above to study these issues.

4.1 Device sales

Suppose that consumers need a device, sold by the platform at a price p_D, to access the marketplace. We normalize the consumers' outside option and the marginal cost of the device to zero. Consumers also get an intrinsic utility d from owning the device. We assume consumers do not observe their own preference parameter for the seller's product, θ, prior to acquiring the device and observing the products available on the marketplace. Moreover, consumers have rational expectations regarding the seller's product and can thus compute their expected surplus. The timing is as follows: at stage 1, the platform sets t and p_D. At stage 2, consumers decide whether to buy the device, observe their type, and the seller sets the price and quality of its products. At stage 3, consumers who bought the device observe the seller's products and decide which version to buy, if any.

We solve the model in Appendix B.4 and here provide an overview of the results. At stage 3, the solution to the seller's problem is the same as in Section 3. To see why, recall that consumers do not observe its products until they have bought the device, so the seller takes the size of the market as given when designing and pricing such products at stage 2. Given (10), the expected consumer surplus from the seller's product at stage 3 is $E(CS) = v(u^e_{HL} - u^e_L)$, where $u^e_i \equiv (q^e_i, \theta_i)$ and $u_{HL} \equiv u(q^e_H, \theta_L)$. At stage 1, the platform recovers this expected surplus through the price of the device (recall that consumers do not know their preference for apps at this stage), which is effectively an access charge, setting

$$p_D = d + v(u^e_{HL} - u^e_L).$$

This is the highest price such that consumers buy the device. When choosing t, therefore, the platform maximizes the following

$$\pi_P = d + v(u^e_{HL} - u^e_L) + tvp^e_H. \tag{22}$$

By Proposition 1, consumer surplus from accessing the marketplace (see (10)) increases in t. Hence, the price of the device increases in t as well (second term in (22)). Therefore, the equilibrium fee, that is set by the platform to maximize (22), is *larger* than when the platform does not sell any device. From the platform's perspective, transaction and access charges are *complements* rather than substitutes. The intuition is that the platform internalizes the effect of the transaction fee on consumer surplus. This result contrasts with previous literature which suggests that access and transaction charges should be substitute instruments from the platform's perspective (see, e.g., Etro, 2021; Gaudin and White, 2021).

It is interesting to compare the equilibrium and optimal fee in this extended setting. Given our assumptions, welfare writes as in (16), with an additional fixed term d. Hence, the optimal fee is the same as in the baseline model, which satisfies $\frac{\partial W}{\partial t} = 0$ in (17). The equilibrium fee chosen by the platform, t^{ed}, solves the following equation

$$\frac{\partial \pi_P}{\partial t} = v\left(u_H^e - u_{HL}^e + u_L^e\right) - vt\left(\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) - \frac{\partial q_H}{\partial t}\frac{\partial u_H}{\partial q_H}\right) + v\frac{\partial q_L}{\partial t}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) = 0. \tag{23}$$

Computing $\frac{\partial W}{\partial t}$ in t^{ed} characterized in equation (23), we find that

$$\left.\frac{\partial W}{\partial t}\right|_{t=t^{ed}} = -v\left(u_H^e - u_{HL}^e + u_L^e\right) < 0.$$

We conclude that the equilibrium fee is higher than the optimal one when the platform can recover consumer surplus by charging for the essential device. The reason is that, although the platform internalizes the effect on the fee on consumers, it does not internalize the loss for the seller.[17]

Proposition 4. *When sellers adopt the freemium model, if the platform can charge consumers for access to the marketplace (e.g., by selling an essential device), it sets a higher transaction fee than in the absence of this source of revenue. Moreover, the equilibrium fee exceeds the socially optimal one.*

4.2 Ad sales

App developers generally rely on an intermediary to sell their ad space. In the baseline model of Section 3, we implicitly assume this intermediary is unrelated to the marketplace platform. Suppose now the platform and the intermediary are part of the same firm. For instance, Google owns the Play Store app marketplace and is also the largest online advertising intermediary. Therefore, the platform may also earn revenues from the ads embedded in the apps sold in its marketplace.

[17] As a final remark, note that violating the constraint $t < \bar{t}$ would not necessarily be beneficial to the platform when it sells an essential device. The reason is that through p_D the platform internalizes the loss consumers would suffer if the seller decided to serve only the H-types. To see this, recall that $p_H^e = u_H^e - u_{HL}^e + \frac{\alpha_H}{\alpha_L} u_L^e$ if the seller serves both types, so (22) can be written as $\pi_P = tvu_H^e + d + v(1-t)\left(u_{HL}^e - \frac{\alpha_H}{\alpha_L} u_L^e\right)$. By contrast, if the seller serves only the H types, it sets $p_H^e = u_H^e$ (but the quality level q_H^e does not change). Hence, the platform cannot set p_D above d, and its total profit is $\pi_P = tvu_H^e + d$.

To capture this aspect, suppose the platform obtains an exogenous revenue r_P for every ad that the seller displays on its app. The seller's revenue per ad, r, is to be interpreted as net of the advertising intermediation fees and is the same as in Section 3.[18]

Proceeding backwards, the solution to the seller's problem is as in our baseline model. Given $x_L^e = \frac{u_L^e}{\alpha}$, the profit of the platform is

$$\pi_P = tvp_H^e + r_P(1-v)\left(\frac{u_L^e}{\alpha}\right). \tag{24}$$

The platform accounts for the effect of the transaction fee on the advertising revenue. As established in Proposition 1, the quality of the ad-funded, L-version of the seller's product, q_L^e, increases with t. Consequently, u_L^e increases, and so does the L-consumers' willingness to pay for the product. The ad revenue therefore *increases* with t. Hence, the platform prefers a higher level of t than when it does not also act as an advertising intermediary (see Appendix B.4).

In this setting, the comparison with the optimal fee is ambiguous as in the baseline model. This is because the term $r_P(1-v)\left(\frac{u_L^e}{\alpha}\right)$ enters symmetrically both the profit function of the platform and total welfare, implying that both equilibrium and optimal fees are equally shifted upward.

Proposition 5. *When sellers adopt the freemium model, if the platform captures some of the revenue from advertising on the apps, it sets a higher transaction fee than in the absence of this source of revenue.*

4.3 Transaction fees on hybrid platforms

It is common for platforms to sell their own products on the marketplace they host. The literature has referred to these platforms as "hybrids" between marketplace and seller (see, e.g., Anderson and Bedre-Defolie, 2021; Hagiu et al., 2020, 2022). For example, Apple and Google provide apps that compete with (possibly well established) third-party ones in, e.g., video and music streaming, office utilities and cloud storage. Third-party apps are subject to the transaction fee, which potentially puts them at a disadvantage with respect to the platform's own products. Furthermore, platforms often make their own products prominent on their marketplaces and/or devices.[19] In this section, we

[18] To focus on the effects of interest, we assume the supplier's net revenue per ad is the same in the two scenarios. As pointed out in recent market studies, the way large ad intermediaries set their fees is fairly complex and obscure (see, e.g., CMA, 2020). This issue is beyond the scope of our investigation, so we assume the revenues r_P and r are exogenous.

[19] For instance, Apple and Google pre-install some of their own apps on smartphones and tablets running the respective operating systems.

investigate how these aspects affect the incentives of a hybrid platform when setting the transaction fee.

Consider the model of Section 3, but assume the platform also provides a product that competes with the third-party one. We assume that a share $s \in [0,1]$ of "loyal" consumers only buys the seller's product, if any. This is consistent with the third-party seller having an established user base. The other consumers obtain the same utility (see (1)) from either product. The distribution of θ is independent of whether consumers are loyal or not. We assume that all consumers observe the platform's product at no cost, because it is prominent. By contrast, non-captive consumers must incur a small search cost to observe the seller's product (but they have rational expectations). The platform has the same production cost as the seller and obtains the same advertising/data rate on this product, r_i^P. As in previous sections, we concentrate on the "freemium" scenario where platform and seller charge no monetary price for the basic version of their product, restricting attention to $t \in (0, \bar{t})$.

The timing is as follows. At stage 1, the platform sets t and the characteristics (p_i^P, x_i^P and q_i^P) of its product. At stage 2, the seller sets the features (p_i, x_i and q_i) of its own product. At stage 3, consumers land on the marketplace and observe the platform's product. Unless they are loyal, they decide whether and which version to buy or search the third-party one. Finally, at stage 4, non-loyal consumers who searched and loyal ones observe the third-party product and decide which version of this product to buy, if any.

We describe the main findings here and relegate the analysis to Appendix B.5. In equilibrium, only loyal consumers buy the third-party product. The values of p_i^e, x_i^e and q_i^e chosen by the seller are the same as in Section 3.[20] These consumers obtain the surplus given in (10). The non-loyal consumers buy the platform's product and its profit function can be written as

$$\pi_P = (1-s)\left(v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(u_L^P \frac{r}{\alpha} - cq_L^P\right)\right) + t\left(svp_H^e\right). \tag{25}$$

The last term is the revenue from the transaction fee, whereas the other term captures the profit the platform gets from selling its own product. The latter is constrained by the expected surplus, CS_H^e,that the H-types would get from the seller's product. The reason is that, to attract the non-captive consumers, the platform must ensure they get the same surplus they would get from the third-party product, conditional on their type. Note that CS_H^e increases with t, as shown in Proposition 1. That is, the fee makes the seller's product *more* attractive to consumers and thus

[20] The seller treats the share of consumers that search as given and is effectively a monopolist for all consumers that are loyal and for those who search its product, because consumers observe the features of its product only after searching. Therefore, the seller's problem is identical to that in the baseline model.

more competitive with the platform's own product. Therefore, provided the seller adopts freemium pricing, the equilibrium fee the platform sets is *smaller* than the rate it would choose if it did not sell its own product. Indeed, a lower fee relaxes price competition between the applications on the marketplace.[21]

We remark that the comparison with the optimal fee is the same as in equation (21) and it is still ambiguous.

Proposition 6. *Given the seller adopts the freemium model, the fee chosen by a hybrid platform is lower than the fee that a pure marketplace platform (i.e., without a product that competes with the seller's) would choose.*

5 Extensions

In this section, we provide three extensions to the baseline model presented in Section 3. First, we endogenize the number of sellers ad consumers. Second, we allow the seller to bypass the platform's marketplace.

5.1 Endogenous number of consumers and sellers

We relax the assumption that the number of consumers and sellers on the platform is exogenous in Appendix B.3. For simplicity, we assume each seller is a monopolist in its own app category. Given the number of sellers, the fee increases the surplus that each consumer gets from interacting with a seller (see Proposition 1), which gives consumers a greater incentive to visit the platform. Hence, although the fee reduces the profit per consumer, its overall effect on sellers can be positive if this effect is compensated by the expansion in the number of consumers. On the other hand, even if consumer surplus per seller increases with the fee, the net effect on consumers can be negative if the fee induces too many sellers to abandon the platform. Therefore, the transaction fee may result in either a smaller or larger total number of consumers and sellers joining the platform. For some values of the parameters, the fee creates a virtuous circle such that both consumers and app sellers on the marketplace increase.

[21] As discussed above, a fee equal to or above the threshold $\bar{t}$ would induce the seller to abandon freemium pricing and just serve the H-types, extracting their entire surplus, so that $CS_H^e = CS_L^e = 0$. While setting the fee at this level would increase the platform's profit from its own product, it would not necessarily increase the platform's total profit, because the revenue generated by the fee may decrease. In any case, it cannot be optimal for the platform to set t so high that the third-party seller is foreclosed, because $t = \bar{t}$ is sufficient to ensure that consumers get zero surplus from the seller's product. Hence, raising the fee further would not make the platform's product more profitable. A complete analysis of the conditions such that the platform sets $t > \bar{t}$ would have to account for all the above factors, and we forgo it because it is not central to our results.

5.2 Allowing the seller to bypass the platform

Traditionally, marketplaces for apps did not allow sellers to distribute their apps outside the platforms and collect payments without using the platform's proprietary system. Some regulators, such as the European Commission, have recently considered forcing the platforms to allow app developers to implement their own in-app payment systems and to distribute their apps via alternative channels.[22] Taking stock of these recent developments, in this section we briefly consider whether allowing the app seller to transact with consumers outside the platform may change the effects of the transaction fee considered above.

Suppose the platform allows the seller to distribute its app and collect payments from consumers independently, thereby avoiding the transaction fee. Suppose there is a share of consumers, $(1-b) \in [0,1]$, who acquires the seller's product outside the platform, though they sustain a finite cost $\gamma \geq 0$ when doing so. This cost captures the fact that using an alternative system may require, for instance, to set up a new password and re-enter payment data. The other consumers are unwilling to use the alternative distribution channel, so they only transact on the platform. These may be consumers who are less tech savvy or more time constrained, so they would incur a too high cost in using an alternative system. For simplicity, we assume that b and θ are independently distributed. We assume that the seller can charge different prices to consumers inside and outside the platform, but the (quality of) the products sold must be the same.

Suppose the seller wants to induce consumers to use its alternative channel to acquire the H-version, despite the transaction cost (this must be true as long as t is high enough). Instead, the seller has nothing to gain from distributing the low version of its product outside the platform, because that version is not subject to the transaction fee. As we show in Appendix B.6, the seller sets $x_L = \frac{u_L r}{\alpha}$, and charges $p_H = u_H - u_{HL} + u_L$ for the top version of the app when purchased on the platform, as in the baseline setting. The seller also discounts the price charged outside the platform, p_H^o, in order to compensate consumers for the transaction cost, i.e., $p_H^o = p_H - \gamma$. The seller's problem therefore can be written as

$$\begin{aligned} \max_{q_H, q_L} \quad \pi = & \, v\left(b\left(1-t\right)\left(u_H - u_{HL} + u_L\right) + \left(1-b\right)\left(u_H - u_{HL} + u_L - \gamma\right) - cq_H\right) + \\ & + \left(1-v\right)\left(\tfrac{r}{\alpha}u_L - cq_L\right) = \\ & v\left(\left(1-bt\right)\left(u_H - u_{HL} + u_L\right) - \left(1-b\right)\gamma - cq_H\right) + \left(1-v\right)\left(\tfrac{r}{\alpha}u_L - cq_L\right). \end{aligned} \tag{26}$$

[22] At the time of this writing, in response to the recent European regulations (European Union, 2022a,b), Apple opened up to the possibility for app developers to bypass the App store. However, apps distributed to EU customers via the App Store that use independent in-app payments systems are liable to pay a reduced transaction fee (10-20 percent versus 30 percent) and a per-installation fee of 0.5 Euros. The new rules imposed by Apple are under investigation by the European Commission (see https://ec.europa.eu/commission/presscorner/detail/en/IP_24_3433).

This expression suggest that the effects of t on the profits of the seller, and on the choice of price and quality, are qualitatively similar to the baseline model.

Proposition 7. *If the platform allows the seller to distribute its app using an alternative channel, the effect of the transaction fee on quality, consumer surplus and welfare is similar to the baseline model.*

6 Policy implications and concluding remarks

We have studied markets hosted by platforms where sellers adopt freemium pricing. Our leading example is the market for mobile apps. We have established that, given freemium pricing, an ad valorem fee applied on transactions between consumers and sellers can reduce the monetary price of the product, increase the quality of the basic version thereby increasing consumer surplus and, potentially, welfare. However, the platform may set the fee above the socially optimal level, in which case the losses for the seller exceed the gains for consumers. These results suggest that, while it may be necessary to regulate the level of transaction fees, it is not obvious that these fees harm consumers and market performance.

The results above indicate that, a vertical structure where the platform and the seller with market power use the agency model may be more efficient than one where they are integrated, provided that the transaction fee is low enough. This is an interesting insight for regulators when considering the implications of vertical mergers between app providers and platforms (e.g., the recent Microsoft-Activision merger).

Our analysis also offers some interesting insights on the relation between transaction fees and other sources of revenue for marketplace platform. This is an important point because the main marketplaces are run by firms, as Google and Apple, that are part of ecosystems. We have shown that the price of devices that consumers need to access the marketplace (e.g., smartphones) is a complement to the transaction fee: a higher fee allows the platform to raise the price for the device, because consumers get more surplus from interacting with sellers as a result. Similarly, the fee also increases when the marketplace earns a share of the revenues from ads shown in the free version of the app. Thus, device-funded and ad-funded platforms have stronger incentives to raise transaction fees than in the absence of these sources of revenue. Hence, the presence of these sources of revenues makes the app provider worse off. This contrasts with the conclusions of previous literature (e.g., Etro, 2021) that did not consider freemium pricing.

Finally, we have shown that, on a hybrid platform that provide products competing with third-party sellers (e.g., Apple Music or Youtube Premium), a higher transaction fee may strengthen price competition between the platform's and third-party products. Thus, contrary to common wisdom, competing with third-party sellers gives the platform an incentive to decrease the transaction fee. This is an interesting insight considering the current debate on the potentially anticompetitive role of transaction fees.

Overall, our analysis demonstrates that considering the sophisticated pricing models of sellers in digital markets, such as freemium, is essential to evaluate the effects of the pricing strategies adopted by platforms and, hence, design regulatory responses. Moreover, the paper suggests that the presence of other sources of revenues may interact in an unforeseen way with revenues from transaction fees in the marketplace for apps, implying that policy makers should attentively consider these interactions in ecosystems when considering regulatory intervention on transaction fees in a marketplace.

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Appendix

A Endogenous freemium pricing

We now provide an extended version of the model of Section 3, where we allow the seller to choose between monetary and nonmonetary prices for each version of its good. The objective is to establish that the effects of a transaction fee that we have shown in Propositions 3 carry through to this more general setting.

Assume a consumer of type i sustain a disutility $\alpha_i > 0$ for every non-monetary unit paid. Thus, the utility of a type-i consumer is

$$U_i(p,q,x) = u(q,\theta_i) - p - \alpha_i x,\ i = H,L. \tag{27}$$

For simplicity, we assume perfect correlation (positive or negative) between the parameters θ and α. The seller earns a revenue r_i for every unit of non-monetary price on version i. Given these assumptions, and using the same notation for utility as in the baseline model, the seller's problem is

$$\max_{q_H,p_H,x_H,q_L,p_L,x_L} \quad \pi = v((1-t)p_H + r_H x_H - cq_H) + (1-v)((1-t)p_L + r_L x_L - cq_L), \tag{28}$$

$$s.t. \quad u_H - p_H - \alpha_H x_H \geq u_{HL} - p_L - \alpha_H x_L, \tag{29}$$

$$u_L - p_L - \alpha_L x_L \geq u_{LH} - p_H - \alpha_L x_H, \tag{30}$$

$$u_H - p_H - \alpha_H x_H \geq 0, \tag{31}$$

$$u_L - p_L - \alpha_L x_L \geq 0. \tag{32}$$

We assume that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$ holds, i.e., the difference between the disutility from ads of the high and low types is small relative to the difference in their marginal utility from product quality. This assumption guarantees that the incentive compatibility constraint of the high type is satisfied. In Appendix A.1 we show that under this assumption, the usual constraints (29) and (32) bind at equilibrium, so we have

$$p_H + \alpha_H x_H = u_H - u_{HL} + \alpha_H x_L + u_L - \alpha_L x_L, \quad p_L + \alpha_L x_L = u_L. \tag{33}$$

Consequently, we can rewrite the seller's problem as

$$\begin{aligned} \max_{q_H,x_H,q_L,x_L} \quad \pi = v((1-t)(u_H - \alpha_H x_H - u_{HL} + \alpha_H x_L + u_L - \alpha_L x_L) + r_H x_H - cq_H) + \\ + (1-v)((1-t)(u_L - \alpha_L x_L) + r_L x_L - cq_L). \end{aligned} \tag{34}$$

Given the linearity of the objective in x_H and x_L, the solution is such that

$$\begin{cases} x_L = \frac{u_L}{\alpha_L},\ x_H = \frac{1}{\alpha_H}\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right) & \text{if} \quad \alpha_L - \frac{v}{1-v}\left(\alpha_H - \alpha_L\right) \leq \frac{r_L}{(1-t)}, \text{ and } \alpha_H \leq \frac{r_H}{(1-t)} \\ x_L = 0,\ x_H = \frac{1}{\alpha_H}\left(u_H - u_{HL} + u_L\right) & \text{if} \quad \alpha_L - \frac{v}{1-v}\left(\alpha_H - \alpha_L\right) > \frac{r_L}{(1-t)}, \text{ and } \alpha_H \leq \frac{r_H}{(1-t)} \\ x_L = \frac{u_L}{\alpha_L},\ x_H = 0 & \text{if} \quad \alpha_L - \frac{v}{1-v}\left(\alpha_H - \alpha_L\right) \leq \frac{r_L}{(1-t)}, \text{ and } \alpha_H > \frac{r_H}{(1-t)}, \\ x_L = 0,\ x_H = 0 & \text{if} \quad \alpha_L - \frac{v}{1-v}\left(\alpha_H - \alpha_L\right) > \frac{r_L}{(1-t)}, \text{ and } \alpha_H > \frac{r_H}{(1-t)}. \end{cases}$$

In words, the seller offers version i for free if and only if the revenue r_i is large enough compared to the disutility α_i. Although the model contemplates many possible cases, to concentrate on the most empirically relevant one we assume henceforth that $\frac{r_H}{(1-t)} < \alpha_H$ and $\alpha_L - \frac{v}{1-v}\left(\alpha_H - \alpha_L\right) \leq \frac{r_L}{(1-t)}$. Given these conditions, we have *freemium* pricing: the high quality version is offered for a monetary price and free of ads (i.e., $p_H > 0$ and $x_H = 0$), while the low quality version is offered for free with ads (i.e., $p_L = 0$ and $x_L > 0$). At equilibrium, we have:

$$\begin{aligned} p_L = 0, &\quad p_H = u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L, \\ x_L = \frac{u_L}{\alpha_L}, &\quad x_H = 0. \end{aligned}$$

We can therefore write the expressions for consumer surplus in this setting as

$$CS = vCS_H + (1-v)CS_L = v\left(u_{HL} - \frac{\alpha_H}{\alpha_L}u_L\right), \tag{35}$$

where $CS_H = u_{HL} - \frac{\alpha_H}{\alpha_L}u_L$ and $CS_L = 0$. Note that the condition $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$ guarantees that CS_H is strictly positive.

Given these assumptions, we can rewrite the seller's problem as

$$\max_{q_H, q_L} \quad \pi = v\left((1-t)\left(u_H - u_{HL} + \frac{\alpha_H}{\alpha_L}u_L\right) - cq_H\right) + (1-v)\left(\frac{u_L r_L}{\alpha_L} - cq_L\right). \tag{36}$$

The above expression is fundamentally identical to (9). Hence, the analysis follows along the same lines as in Section 3.

A.1 Binding constraints in problem (28)

As a first step, we show that (29) and (32) imply that (31) holds. Constraint (32) can be rewritten as $x_L \leq u(q_L, \theta_L) - \alpha_L x_L$. Setting x_L at the upper bound of this constraint gets the right hand side of (29) as close as possible to zero. Hence, if $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$, constraint (31) must be implied by (29). Given the linearity of the problem in x_L, we can anticipate that either $x_L = 0$ or $x_L = u(q_L, \theta_L)/\alpha_L$ holds at the solution. In the former case, $u(q_L, \theta_H) - u(q_L, \theta_L) + \alpha_L x_L - \alpha_H x_L \geq 0$

is satisfied because $u(q_L,\theta_H) > u(q_L,\theta_L)$ by assumption. In the latter case, the constraint boils down to $u(q_L,\theta_H) - \alpha_H u(q_L,\theta_L)/\alpha_L \geq 0$, which is satisfied given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L}$. Summing up, we can ignore constraint (31) and anticipate that (29) must be binding at the solution of (28).

In the second step, we show that (29) being binding implies that (30) is slack and can be ignored. Given the linearity of the problem, we can anticipate that if (29) binds, either $x_H = 0$ or $x_H = \frac{u(q_H,\theta_H)-u(q_L,\theta_H)+x_L+\alpha_H x_L}{\alpha_H}$ hold. Suppose first that $x_H = 0$, so that $p_H = u(q_H,\theta_H) - u(q_L,\theta_H) + u(q_L,\theta_L)$. Plugging these expressions in the right hand side of (30) we get after some rearrangements: $u(q_H,\theta_L) - u(q_L,\theta_L) - (u(q_H,\theta_H) - u(q_L,\theta_H))$. This expression is strictly negative by assumption, which implies that (30) is slack. Suppose now that $x_H = \frac{u(q_H,\theta_H)-u(q_L,\theta_H)+x_L+\alpha_H x_L}{\alpha_H}$ and $p_H = 0$. Plugging these expressions in (30) we get

$$u(q_L,\theta_L) - x_L - \alpha_L x_L \geq u(q_H,\theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H,\theta_H) - u(q_L,\theta_H) + x_L + \alpha_H x_L\right).$$

Suppose the solution is such that $x_L = 0$ and $x_L = u(q_L,\theta_L)$. The above constraint can then be written after some rearrangements as

$$0 \geq u(q_H,\theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H,\theta_H) - u(q_L,\theta_H) + u(q_L,\theta_L)\right).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \iff \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H,\theta_H)}{u(q_L,\theta_H)} \geq \frac{u(q_H,\theta_L)}{u(q_L,\theta_L)},$$

which holds strictly by our assumptions on utility. Finally, suppose that the solution is such that $x_L = u(q_L,\theta_L)/\alpha_L$ and $x_L = 0$. The constraint (30) can then be written as

$$0 \geq u(q_H,\theta_L) - u(q_L,\theta_L) - \frac{\alpha_L}{\alpha_H}\left(u(q_H,\theta_H) - u(q_L,\theta_H)\right).$$

The last term in brackets on the right hand side is positive. Hence, given the assumption that $\frac{u_{HL}}{u_L} > \frac{\alpha_H}{\alpha_L} \iff \frac{\alpha_L}{\alpha_H} > \frac{u_L}{u_{HL}}$, the constraint must hold if it holds when $\frac{\alpha_L}{\alpha_H} = \frac{u_L}{u_{HL}}$. Plugging this expression in the constraint, we have after some rearrangements that

$$\frac{u(q_H,\theta_H)}{u(q_L,\theta_H)} \geq \frac{u(q_H,\theta_L)}{u(q_L,\theta_L)},$$

which holds strictly by our assumptions on utility.

B Proofs of results not given in the text

B.1 Establishing the signs of the derivatives in (13) and (14)

By totally differentiating the first-order conditions of the monopolist's problem in(11) and (12), we find that

$$\frac{\partial q_H}{\partial t} = -\frac{\frac{\partial^2\pi}{\partial q_L^2}\frac{\partial^2\pi}{\partial q_H \partial t} - \frac{\partial^2\pi}{\partial q_L \partial t}\frac{\partial^2\pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_L}{\partial t} = -\frac{\frac{\partial^2\pi}{\partial q_H^2}\frac{\partial^2\pi}{\partial q_L \partial t} - \frac{\partial^2\pi}{\partial q_H \partial t}\frac{\partial^2\pi}{\partial q_H \partial q_L}}{H}.$$

where $H \equiv \frac{\partial^2\pi}{\partial q_L^2}\frac{\partial^2\pi}{\partial q_H^2} - \left(\frac{\partial^2\pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2\pi}{\partial q_H^2} < 0$, $\frac{\partial^2\pi}{\partial q_L^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2\pi}{\partial q_H \partial q_L} = 0$,and $\frac{\partial^2\pi}{\partial q_L \partial t} = \frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right) > 0$, $\frac{\partial^2\pi}{\partial q_H \partial t} = \frac{\partial u_H}{\partial q_H} > 0$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t}\right) = \operatorname{sgn}\left(\frac{\partial^2\pi}{\partial q_L^2}\frac{\partial u_H}{\partial q_H}\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2\pi}{\partial q_H^2}\frac{v}{1-v}\left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\right) > 0.$$

Let us now compute the derivatives of the equilibrium prices $p_H = u_H - u_{HL} + u_L$ and $x_L = \alpha u_L$ with respect to t. Given $\frac{\partial u}{\partial q} > 0$ and $\frac{\partial^2 u}{\partial q \partial\theta} > 0$, we have

$$\frac{\partial p_H}{\partial t} = \frac{\partial u_H}{\partial q_H}\frac{\partial q_H}{\partial t} < 0, \qquad \frac{\partial x_L}{\partial t} = \alpha\frac{\partial u_L}{\partial q_L}\frac{\partial q_L}{\partial t} > 0.$$

B.2 Unit transaction fee in the freemium model

Suppose the seller is subject to unit fees, denoted by τ_i, $i = H, L$. The profit function is

$$\pi = v\left(p_H - (c+\tau_H)\,q_H\right) + (1-v)\left(rx_L - (c+\tau_L)\,q_L\right). \tag{37}$$

The seller maximizes this function subject to (29)-(32). We find that (29) and (32) are binding, hence prices are (33). Replacing these prices in (37), we solve it with respect to (q_L, q_H, x_L, p_H).

The equilibrium qualities solve the following system of equations

$$\frac{\partial\pi}{\partial q_H} := v\left(\frac{\partial u_H}{\partial q_H} - c - \tau_H\right) = 0, \tag{38}$$

$$\frac{\partial \pi}{\partial q_L} := v\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{HL}}{\partial q_L}\right) + (1-v)\left(\frac{r}{\alpha}\frac{\partial u_L}{\partial q_L} - c - \tau_L\right) = 0. \tag{39}$$

The effect of either fee is thus similar to that of an increase in the cost of the respective version. As we show below, we get

$$\frac{\partial q_L^e}{\partial \tau_L} < 0, \qquad \frac{\partial q_H^e}{\partial \tau_L} = 0, \qquad \frac{\partial q_H^e}{\partial \tau_H} < 0, \qquad \frac{\partial q_L^e}{\partial \tau_H} = 0. \tag{40}$$

The effect of these fees on consumer surplus, and welfare, can only be negative. Intuitively, similar results apply with a uniform unit fee, i.e., $\tau_L = \tau_H = \tau$.

Proof. By totally differentiating the first-order conditions of the monopolist's problem in(38) and (39), we find that

$$\frac{\partial q_i}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_j}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau_i} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau_i} - \frac{\partial^2 \pi}{\partial q_i \partial \tau_i}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0$, $\frac{\partial^2 \pi}{\partial q_j^2} < 0$, $\frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_L} = 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_H} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau_H} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau_L} = -(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau_H}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0 \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial \tau_L}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

$$\frac{\partial q_L}{\partial \tau_H} = 0, \frac{\partial q_H}{\partial \tau_L} = 0$$

Consider now a uniform unit fee $\tau_L = \tau_H = \tau$. By totally differentiating the first-order conditions of the monopolist's problem in(38) and (39), we find that

$$\frac{\partial q_i}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \quad \frac{\partial q_j}{\partial \tau} = -\frac{\frac{\partial^2 \pi}{\partial q_i^2}\frac{\partial^2 \pi}{\partial q_j \partial \tau} - \frac{\partial^2 \pi}{\partial q_i \partial \tau}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \; i,j = H,L, \; j \neq i.$$

where $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial \tau} = -v$ and $\frac{\partial^2 \pi}{\partial q_L \partial \tau} = -(1-v)$. Hence, we have

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial \tau}\right) = \operatorname{sgn}\left(\frac{\partial^2 \pi}{\partial q_L^2} v\right) < 0,$$

$$\text{sgn}\left(\frac{\partial q_L}{\partial \tau}\right) = \text{sgn}\left(\frac{\partial^2 \pi}{\partial q_H^2}(1-v)\right) < 0.$$

B.3 Endogenous number of consumers and sellers

Let the number of consumers and sellers that join the platform be n_c and n_s, respectively. Each seller provides a different product category, so each is a monopolist in its own category. Each consumer interacts with all sellers available on the platform. Sellers are symmetric in terms of their production cost. Therefore, each seller's profit function (conditional on joining the platform) is given by

$$\pi = n_c v((1-t)p_H - cq_H) + n_c(1-v)(rx_L - cq_L).$$

The timing is as follows: at stage 1, the platform sets t. At stage 2, sellers and consumers decide whether to join the platform and each seller sets p_i, q_i and x_i, for $i = H, L$. At stage 3, consumers observe the features of the products available from each seller and decide which to buy, if any. We assume each seller takes n_c as given when deciding whether to join the platform and choosing the values of p_i, q_i and x_i, $i = H, L$. This is because consumers do not observe the features of the products available on the platform prior to joining (but have rational expectations). We denote the values that these variables take in equilibrium with superscript e. Consumers derive the same utility from consuming each good of each seller on the platform and observe their type only after joining. Consumer take n_s as given when deciding whether to join the platform.

To capture the fact that sellers and consumers differ in the opportunity cost of joining the platform, we assume that $n_s = \phi_s(\pi)$, where $\phi_s(.)$ is an increasing and continuously differentiable function. Denoting the surplus that each consumer expects to get when joining the platform by $E(CS)$, we assume that $n_c = \phi_c(n_s E(CS))$, where $\phi_c(.)$ is an increasing and continuously differentiable function.

We solve the model backwards. At stage 3, consumers obtain the same surplus from each seller as in the baseline model, i.e., $CS_H^e = u_{HL}^e - u_L^e$ and $CS_L^e = 0$. To see why, consider that at stage 2, each seller faces the same problem as in (36), except that the total number of consumers is n_c. Since this number is taken as given, the solution is identical to (36) and we get the same values of q_i^e, p_i^e and x_i^e. Each consumer expects to obtain the surplus $n_s E(CS) = n_s(vCS_H^e + (1-v)CS_L^e) = n_s vCS_H^e$, whereas each seller gets the profit in (36), that we denote by π^e after replacing for the equilibrium values q_H^e and q_L^e (given t), and multiplied by n_c. Hence, we have $n_s = \phi_s(n_c \pi^e)$ and $n_c = \phi_c(n_s vCS_H^e)$. Starting from these expressions, we can write the following derivatives

$$\frac{\partial n_s}{\partial t} = \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0, \qquad \frac{\partial n_c}{\partial t} = \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0,$$

$$\frac{dn_s}{dt} = \frac{\partial n_s}{\partial t} + \phi_s' \frac{dn_c}{dt} \pi^e, \qquad \frac{dn_c}{dt} = \frac{\partial n_c}{\partial t} + \phi_c' v CS_H^e \frac{dn_s}{dt}.$$

Combining the above derivatives and rearranging, we obtain

$$\frac{dn_s}{dt} = \frac{\phi_s' \left(n_c \frac{\partial \pi^e}{\partial t} + \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} \pi^e \right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}, \qquad \frac{dn_c}{dt} = \frac{\phi_c' \left(n_s v \frac{\partial CS_H^e}{\partial t} + \phi_s' n_c \frac{\partial \pi^e}{\partial t} v CS_H^e \right)}{1 - \phi_c' \phi_s' v CS_H^e \pi^e}.$$

These derivatives show that, if the number of consumers is much less responsive than the number of sellers (i.e., $\phi_s' \to 0$), then $\frac{dn_s}{dt} \to 0$, whereas $\frac{dn_c}{dt} \to \phi_c' n_s v \frac{\partial CS_H^e}{\partial t} > 0$. By contrast, if the number of sellers is much less responsive than the number of consumers, we have $\frac{dn_c}{dt} \to 0$, whereas $\frac{dn_s}{dt} \to \phi_s' n_c \frac{\partial \pi^e}{\partial t} < 0$. Furthermore, both total derivatives can be positive, provided the magnitude of $\frac{\partial \pi^e}{\partial t}$ is small enough compared to that of $v \frac{\partial CS_H^e}{\partial t}$.

B.4 Analysis with device-selling platform and ad-selling platform

B.4.1 Device sales (Section 4.1)

We solve the model by backward induction. Consider Stage 3. Let $n \in [0,1]$ be the number of consumers who bought the device at Stage 2. At Stage 3, these consumers observe θ and select the version of the product based on the same utility function as in (1). Consider now Stage 2. Each consumer gets an expected payoff equal to $d + E(CS) - p_D$ when buying the device, where $E(CS)$ is the ex-ante expected surplus a consumer gets from the seller's products. It follows that all consumers buy the device, i.e., $n = 1$, if and only if $d + E(CS) \geq p_D$, whereas $n = 0$ otherwise. The latter scenario cannot be optimal to the platform, so we restrict attention to $p_D \leq d + E(CS)$ and $n = 1$.

Given that the share of high types, v, is the same as in the baseline model, the profit of the seller is isomorphic to (2), except that it is multiplied by n. Note that the seller takes n as given, because consumers make their decision whether to buy the device prior to observing the values of the variables p_i, q_i and x_i, for $i = H, L$. It follows that the solution to the seller's problem, given p_D and t, is the same as in the model of Section 3. Hence, at stage 3, H-type consumers get the same surplus as in (10) and their expected surplus at stage 2 is $E(CS) = v(u_{HL}^e - u_L^e)$, where $u_i^e \equiv (q_i^e, \theta_i)$ and $u_{HL} \equiv u(q_H^e, \theta_L)$ and the superscript e denotes the values chosen by the seller in equilibrium (given t).

Finally, consider Stage 1. The solution to the platform's problem must be such that $p_D = d + E(CS)$. When choosing t, therefore, the platform maximizes (22). Compare this expression to (19), and notice that $u^e_{HL} - u^e_L$ increases with t. Hence the derivative of (22) with respect to t is everywhere greater than the derivative of (19) (recall that we focus on the $0 \leq t < \bar{t}$ interval). Thus, one can apply the results of Milgrom and Shannon (1994) to conclude that the equilibrium level of the fee must be higher when the platform sells the device than when it does not.

B.4.2 Ad sales (Section 4.2)

Compare the platform's profit in expression (24) to (19) and notice that $r_P(1-v)\left(\frac{u^e_L}{\alpha}\right)$ increases with t, while the term tvp^e_H is identical in the two expressions. Hence the derivative of (22) with respect to t is everywhere greater than the derivative of (19) (recall that we focus on $0 \leq t < \bar{t}$ interval). Thus, one can apply the results by Milgrom and Shannon (1994) to conclude that the equilibrium level of the fee must be higher when the platform sells the ads than when it does not.

B.5 Analysis with hybrid platform

The game is described in Section 4.3. At stage 3, consumers can buy the platform's product or search the seller's. Consumers of type i who search expect to get the surplus $CS^e_i - \sigma$, where σ is the search cost and CS^e_i is the surplus conditional on the equilibrium values of p_i, x_i and q_i (that the seller chooses at stage 2, given t), that we shall denote with the superscript e. Recall that consumers have a rational expectation about this surplus, but they need to search to observe the characteristics of the seller's product. The search cost is small, i.e., $\sigma \to 0$, and thus omitted in the expressions that follow. In equilibrium, no loyal consumers buy the platform's product, while the non-loyal search it if and only if $CS^e_i \geq CS^P_i$. Therefore, all consumers of type i are available to the seller if $CS^e_i \geq CS^P_i$, while only a share s is available otherwise.

Consider now stage 2. The seller chooses p_i, x_i and q_i, given t, p^P_i, x^P_i, q^P_i, and the share of consumers that is available. Non-loyal consumers do not observe the equilibrium values of p_i, x_i and q_i prior to searching, but only have a rational expectation about such values. Hence, the seller treats the shares of consumers that are available as given when choosing these variables (since the loyal consumers only buy the seller's product by definition). Let these shares be S_H and S_L among, respectively, high- and low-type consumers. We have $S_H = vs$ if $CS^e_H < CS^P_H$, and $S_H = v$ otherwise. Similarly, $S_L = s(1-v)$ if $CS^e_L < CS^P_L$, and $S_L = 1 - v$ otherwise. The constraints faced by the seller are the same as in the baseline model (i.e., (3)-(6)). Given the same the value of r, the seller adopts the freemium pricing scheme, with prices set as in expression (8). Specifically, $p_H = u_H - u_{HL} + u_L$ and $x_L = u_L r/\alpha$ must hold, where $u_i \equiv (q_i, \theta_i)$ and $u_{HL} \equiv u(q_H, \theta_L)$. Hence, the seller's problem reduces to

$$\max_{q_H, q_L} \quad \pi = S_H\left((1-t)\left(u_H - u_{HL} + u_L\right) - cq_H\right) + S_L\left(\tfrac{u_L r}{\alpha} - cq_L\right). \tag{41}$$

Note that if $S_H = vs$ and $S_L = (1-v)s$, or if $S_H = v$ and $S_L = (1-v)$, the objective is isomorphic (up to a multiplicative constant) to (2), so the two problems must have the same solution. In words, the seller faces the same problem as in the baseline model when either all consumers or only the captive ones search.

Whenever $t < \bar{t}$, the seller serves both consumer types, high-type consumers get a surplus $CS_H^e = u_{HL}^e - u_L^e$ in equilibrium, where $u_i^e \equiv (q_i^e, \theta_i)$ and so on, whereas low type consumers get $CS_L^e = 0$. If $t \geq \bar{t}$, the seller only sells a single version of its product, targeting the high-types and sets $p_H = u_H$, so that $CS_H^e = CS_L^e = 0$. Consumers would of course obtain the same levels of expected surplus if t was so large that the seller simply dropped out of the market. Observe that the solution to the seller's problem only depends on the platform's decisions at stage 1 through t and the surpluses CS_i^P (which affect the shares S_H and S_L).

Focus now on stage 1. We assume that the platform wants to serve all consumer types with its product. The platform's problem is therefore

$$\max_{t, q_H^P, p_H^P, x_H^P, q_L^P, x_L, x_L^P} \quad \pi_P = (1-S_H)\left(p_H^P - cq_H^P\right) + (1-v)(1-S_L)\left(rx^P - cq_L^P\right) + tS_H p_H.$$

with $S_H = vs$ and $S_L = (1-v)s$ (i.e., only the loyal consumers buy from the seller). The platform must satisfy the following constraints

$$u\left(q_H^P, \theta_H\right) - p_H^P \geq u\left(q_L^P, \theta_H\right) - \alpha x_L^P, \tag{42}$$

$$u\left(q_L^P, \theta_L\right) - \alpha x_L^P \geq u\left(q_H^P, \theta_L\right) - p_H^P, \tag{43}$$

$$u\left(q_H^P, \theta_H\right) - p_H^P \geq \max\left(0, CS_H^e\right), \tag{44}$$

$$u\left(q_L, \theta_L\right) - \alpha_L x_L \geq \max\left(0, CS_L^e\right). \tag{45}$$

The first two constraints are incentive compatibility constraints. The last two constraints are participation constraints: each (non-loyal) consumer type must receive at least the surplus it can expect to get by searching the third-party seller's product. Following standard procedures, and noting that $\max(0, CS_H^e) = CS_H^e$, while $\max(0, CS_L^e) = 0$, we have

$$\begin{aligned} p_H^P &= \min\left(u_H^P - u_{HL}^P + u_L^P, CS_H^e\right), \\ \alpha x_L^P &= u_L^P, \end{aligned}$$

where $u_i^P \equiv \left(q_i^P, \theta_i\right)$ and $u_{HL}^P \equiv u\left(q_H^P, \theta_L\right)$. Note that in this setting the incentive compatibility constraint for the H-type is not necessarily binding in equilibrium, because the third-party seller's product tightens the participation constraints. Assuming that $x_H^P = 0$ and $x_L^P = u_L^P/\alpha$ as in the baseline model, we have

$$\begin{aligned} p_H^P &= \min\left(u_H^P - u_{HL}^P + u_L^P, u_H^P - CS_H^e\right), \\ \alpha x_L^P &= u_L^P. \end{aligned} \tag{46}$$

Suppose that the H-type's incentive compatibility constraint binds, that is, that $u_H^P - u_{HL}^P + u_L^P \leq u_H^P - CS_H^e$, so that $p_H^P = u_H^P - u_{HL}^P + u_L^P$. The platform's problem would then reduce to

$$\max_{t, q_H^P, q_L^P} \quad (1-s)\left(v\left(u_H^P - u_{HL}^P + u_L^P - cq_H^P\right) + (1-v)\left(\frac{u_L^P r}{\alpha} - cq_L^P\right)\right) + stvp_H^e.$$

Since p_H^e does not depend on q_i^P, the pair $\left(q_{H0}^P, q_{L0}^P\right)$ that solves this problem must be the same as the pair solving (41) when $t = 0$ (and $S_H = vs$ and $S_L = (1-v)s$ hold). Hence, conditional on $t = 0$, the surplus of the high types, $u_{HL0}^P - u_{L0}^P$ equals CS_H^e. However, by Proposition 1, CS_H^e increases with t, for any $0 \leq t < \bar{t}$. Hence, for any $0 < t < \bar{t}$, we have $u_{HL0}^P - u_{L0}^P < CS_H^e$, so $u_H^P - u_{HL}^P + u_L^P > u_H^P - CS_H^e$ must hold. That is, the H-type's participation constraint binds. Finally, when $t \geq \bar{t}$, $CS_H^e = 0$, so $u_H^P - u_{HL}^P + u_L^P \leq u_H^P - CS_H^e$ must hold. This is because the seller only serves the H-types in this case, and extract all their surplus.

Summing up, we can write the platform's problem as

$$\max_{q_H^P, q_L^P, t} \quad \pi_P = \begin{cases} (1-s)\left(v\left(u_H^P - CS_H^e - cq_H^P\right) + (1-v)\left(\frac{u_L^P r}{\alpha} - cq_L^P\right)\right) + stvp_H & \text{if } 0 \leq t < \bar{t}, \\ (1-s)\left(v\left(u_H^P - u_{HL}^P + u_L^P - cq_H^P\right) + (1-v)\left(\frac{u_L^P r}{\alpha} - cq_L^P\right)\right) + stvp_H & \text{if } t \geq \bar{t}. \end{cases} \tag{47}$$

Let us first focus on the case where $0 \leq t < \bar{t}$. Comparing the platform's profit in expression (47) to (19), and noting that CS_H^e increases with t, while $\left(q_H^P, q_L^P\right)$ do not depend on it. Note also that the term tvp_H^e is identical in the two expressions, for any t. Hence the derivative of (47) with respect to t is everywhere smaller than the derivative of (19). Thus, one can apply the results by Milgrom and Shannon (1994) to conclude that the equilibrium level of the fee must be smaller when the platform sells its own product than when it does not (again, conditional on the solution being such that $0 \leq t < \bar{t}$).

However, we cannot exclude the possibility that the platform prefers a fee such that $t \geq \bar{t}$ when selling its own product, and a fee such that $0 \leq t < \bar{t}$ when it is a pure marketplace. This is because there is a discrete increase in the revenue from selling the product when t reaches the level $\bar{t}$, compared to when $0 < t < \bar{t}$, as established above. Nevertheless, by setting $t \geq \bar{t}$ the platform already ensures that no consumer gets a positive surplus when buying from the seller, because the latter only serves the high types and captures all their surplus (setting $p_H = u_H$), so that $CS_H^e = 0$. Therefore setting t to a level such that the seller makes zero net profit, and exits the market, cannot be optimal: the platform would then earn the same profit from the sale of its product as in the second row of (47), but forgo the fee revenue $tvsp_H^e$.

B.6 Allowing for alternative payment systems

Given our assumptions, we set $p_L = 0$, $x_H = 0$ and can write the seller's problem as follows

$$
\begin{aligned}
\max_{q_H,p_H,x_H,q_L,x_L,x_L,p_H^0,x_L} \quad & \pi = b\left(v\left((1-t)\,p_H - cq_H\right) + (1-v)\left(rx_L - cq_L\right)\right) + \\
& + (1-b)\left(v\left(p_H^o - cq_H\right) + (1-v)\left(rx_L - cq_L\right)\right), \\
s.t. \quad & u\left(q_H,\theta_H\right) - p_H \geq u\left(q_L,\theta_H\right) - \alpha x_L, && (48)\\
& u\left(q_H,\theta_H\right) - p_H^o - \gamma \geq u\left(q_H,\theta_H\right) - p_H, && (49)\\
& u\left(q_L,\theta_L\right) - \alpha x_L \geq u\left(q_H,\theta_L\right) - p_H, && (50)\\
& u\left(q_L,\theta_L\right) - \alpha x_L \geq u\left(q_H,\theta_L\right) - p_H^0 - \gamma, && (51)\\
& u\left(q_H,\theta_H\right) - p_H \geq 0, && (52)\\
& u\left(q_H,\theta_H\right) - p_H^o - \gamma, \geq 0, && (53)\\
& u\left(q_L,\theta_L\right) - \alpha x_L \geq 0. && (54)
\end{aligned}
$$

In the above problem, in addition to the usual incentive compatibility and participation constraints for the consumers who would transact with the seller only inside the platform, we include incentive constraints for those consumers that would be willing to transact with the seller outside of it, sustaining the cost γ. Specifically, constraint 49 requires that high type consumers who are willing to transact outside the platform prefer to do so rather than go through the platform. Constraint (51) requires that low type consumers prefer to choose the version of the app intended for them than the high version (accessed outside the platform). Finally, constraint (53) requires that high type consumers that transact outside the platform get non-negative utility. Clearly, (49) is satisfied optimally by setting $p_H^o = p_H - \gamma$, so constraints (51) and (53) are redundant. As a result, we can use the same arguments as in the baseline model to show that the solution to the seller's problem is such that $p_H = u_H - u_{HL} + u_L$ and $x_L = u_L/\alpha$. Hence, we can write the objective as (26). The remainder of the analysis follows from the main text.

C Robustness checks

C.1 More than two types

We assume there are three types of consumers, characterized by the preference parameter $\theta \in \{\theta_H, \theta_M, \theta_L\}$, with $\theta_H > \theta_M > \theta_L$. Let v_H, v_M and v_L be the shares of consumers of type H, M and L, respectively, with $v_H + v_M + v_L = 1$. Furthermore, to avoid "bunching" of types we assume that $\frac{v_L}{v_M} < \frac{v_L + v_M}{v_H}$, i.e., that the distribution of types satisfies the monotone hazard rate property (Laffont and Martimort, 2002, p.90). The model is otherwise identical to our baseline setup.

The seller offers to consumers three bundles, (q_i, p_i, x_i), each intended for one type. As in the baseline model, we assume only the version of the good intended for the H-type has a monetary price, whereas the others are not. That is $p_L = p_M = 0$ and $x_H = 0$. These bundles must satisfy six incentive constraints (two for each type)

$$u(q_i, \theta_i) - p_i - \alpha x_i \geq u(q_j, \theta_i) - p_j - \alpha x_j, \quad i, j = L, M, H \quad i \neq j,$$

and three participation constraints (one per each type)

$$u(q_i, \theta_i) - p_i - \alpha x_i \geq 0, \quad i = L, M, H.$$

Following standard steps (Laffont and Martimort, 2002), one can show that, in equilibrium, there are two binding incentives constraints (the ones such that a higher type want to mimic a lower type) and one binding participation constraint (the one of low types). From these binding constraints, we can derive the equilibrium prices conditional on quality levels. Hence, we can restrict attention to the following problem for the seller

$$\max_{(q_i,p_i,x_i)} \quad \pi = \sum_{i=L,M,H} v_i\left((1-t)\,p_i - rx_i - cq_i\right), \tag{55}$$

$$s.t. \quad p_H = u_H + u_M + u_L - u_{ML} - u_{HM}, \tag{56}$$

$$\alpha x_M = u_M + u_L - u_{ML}, \tag{57}$$

$$\alpha x_L = u_L, \tag{58}$$

$$p_L = p_M = x_H = 0. \tag{59}$$

where $u_i \equiv u(q_i,\theta_i)$ for each $i = L,M,H$, and $u_{ij} \equiv u\left(q_j,\theta_i\right)$ for each $i,j = L,M,H$ with $i \neq j$. Plugging the above constraints in the objective function, the problem reduces to

$$\max_{(q_i,p_i,x_i)} \pi = v_H\left((1-t)\left(u_H + u_M + u_L - u_{ML} - u_{HM}\right) - cq_H\right) +,$$

$$+ v_M\left(\frac{r}{\alpha}\left(u_M + u_L - u_{ML}\right) - cq_M\right) + v_L\left(\frac{r}{\alpha}u_L - cq_L\right).$$

Hence, we derive the following first-order conditions

$$\frac{\partial \pi}{\partial q_H} := v_H\left(\frac{\partial u_H}{\partial q_H}(1-t) - c\right) = 0, \tag{60}$$

$$\frac{\partial \pi}{\partial q_M} := v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)(1-t) + v_M\left(\frac{r}{\alpha}\frac{\partial u_M}{\partial q_M} - c\right) = 0, \tag{61}$$

$$\frac{\partial \pi}{\partial q_L} := v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)(1-t) + v_M\frac{r}{\alpha}\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right) + v_L\left(\frac{r}{\alpha}\frac{\partial u_L}{\partial q_L} - c\right) = 0. \tag{62}$$

These equations indicate that, if t=0, the quality levels q_L and q_M are distorted downwards. Totally differentiating the above equations and taking into account that cross-profits derivatives are zero ($\frac{\partial^2\pi}{\partial q_i \partial q_j} = 0$ for $i,j = L,M,H$ with $i \neq j$), we find that

$$\frac{\partial q_H}{\partial t} = -\frac{\begin{vmatrix} \frac{\partial^2\pi}{\partial q_L^2} & \frac{\partial^2\pi}{\partial q_L \partial q_M} & \frac{\partial^2\pi}{\partial q_L \partial t} \\ \frac{\partial^2\pi}{\partial q_M \partial q_L} & \frac{\partial^2\pi}{\partial q_M^2} & \frac{\partial^2\pi}{\partial q_M \partial t} \\ \frac{\partial^2\pi}{\partial q_H \partial q_L} & \frac{\partial^2\pi}{\partial q_H \partial q_M} & \frac{\partial^2\pi}{\partial q_H \partial t} \end{vmatrix}}{D} = -\frac{\frac{\partial^2\pi}{\partial q_L^2}\frac{\partial^2\pi}{\partial q_M^2}\frac{\partial^2\pi}{\partial q_H \partial t}}{D} \leq 0,$$

where D is the determinant of the Hessian matrix, which is negative by second order conditions, $\frac{\partial^2\pi}{\partial q_i^2} < 0$ for $i = L,M,H$ also by second order conditions, and $\frac{\partial^2\pi}{\partial q_H \partial t} = -v_H\frac{\partial u_H}{\partial q_H} < 0$. Following similar steps, we find that the derivatives of q_M and q_L with respect to t are, respectively, such that

$$\operatorname{sgn}\left(\frac{\partial q_M}{\partial t}\right) = \operatorname{sgn}\left(-v_H\left(\frac{\partial u_M}{\partial q_M} - \frac{\partial u_{HM}}{\partial q_M}\right)\right) \geq 0, \qquad \operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-v_H\left(\frac{\partial u_L}{\partial q_L} - \frac{\partial u_{ML}}{\partial q_L}\right)\right) \geq 0 \ .$$

These signs follow from the assumption that $\frac{\partial^2 u}{\partial q \partial \theta} > 0$. This establishes that the effect of the ad valorem fee applied to the H-bundle is such that the quantity of the other two bundles increases, reducing the distortion applied by the seller. This is consistent with the results of our baseline model. Moreover, the effects on quality level imply that the fee reduces the price of the H-version , but raises the price of the L-version, while the effect on the price of the M-version is ambiguous. Given these effects, it is straightforward to show that the surplus of H- and M-type consumers increases, while that of L-consumers does not change. Moreover, welfare increases with the fee starting from $t = 0$.

C.2 Duopoly

Consider two symmetric sellers, indexed by $s \in \{1,2\}$ and four consumer types, indexed by $i \in \{H_1, L_1, H_2, L_2\}$, differing in (i) their intensity of preferences for the good and (ii) their preference for the two sellers. The utility when buying from seller s is $u_s(q, \theta_i) - p - \alpha x$, where p is the monetary price, x is the nonmonetary price, θ_i is the preference parameter and x the disutility from the nonmonetary price. Let v_i be the share of consumers of type i, with $\sum_{i=H_1,L_1,H_2,L_2} v_i = 1$, and assume that each consumer buys from at most one seller. We assume the utility function satisfies the following conditions:

$$u_1(q, \theta_{H_1}) > u_1(q, \theta_{L_1}) > u_1(q, \theta_{L_2}) > u_1(q, \theta_{H_2}) = 0, \quad \forall q > 0,$$

$$u_2(q, \theta_{H_2}) > u_2(q, \theta_{L_2}) > u_1(q, \theta_{L_1}) > u_1(q, \theta_{H_1}) = 0, \quad \forall q > 0,$$

$$\frac{\partial u_1}{\partial q}(q, \theta_{H_1}) > \frac{\partial u_1}{\partial q}(q, \theta_{L_1}) > \frac{\partial u_1}{\partial q}(q, \theta_{L_2}) > \frac{\partial u_1}{\partial q}(q, \theta_{H_2}) = 0, \quad \forall q > 0,$$

$$\frac{\partial u_2}{\partial q}(q, \theta_{H_2}) > \frac{\partial u_2}{\partial q}(q, \theta_{L_2}) > \frac{\partial u_2}{\partial q}(q, \theta_{L_1}) > \frac{\partial u_2}{\partial q}(q, \theta_{H_1}) = 0, \quad \forall q > 0.$$

These conditions imply a perfect correlation between the preference for one seller and the intensity of preference for the good it supplies (Spulber, 1989; Stole, 2007). For simplicity, we assume only the "low" types are willing to buy from either seller, whereas the "high" types do not get any utility from buying from their least preferred seller.

Let (q_i, p_i, x_i) denote the bundle that a seller proposes to consumers of type i. Given the condition that consumers self-select on the intended bundle, there is no loss in proceeding under the assumption that seller 1 only offers bundles intended for the couple of consumer types that prefer its product, i.e., H_1 and L_1, whereas seller 2 only serves H_2 and L_2. Moreover, as in Section 3, we focus on the situation where both sellers adopt freemium pricing, i.e. $p_{Ls} = 0$ and $x_{Hs} = 0$, for any s. We are now going to state the constraints that the sellers face regarding each type of consumer. Considering a seller s, we have the following incentives and participation constraints that apply to the H_s-bundle:

$$u_s\left(q_{H_s}, \theta_{H_s}\right) - p_{H_s} \geq u_s\left(q_{L_s}, \theta_{H_s}\right) - \alpha x_{L_s},\ s = 1,2, \tag{63}$$

$$u_s\left(q_{H_s}, \theta_{H_s}\right) - p_{H_s} \geq u_{s'}\left(q_{L_{s'}}, \theta_{H_s}\right) - \alpha x_{L_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{64}$$

$$u_s\left(q_{H_s}, \theta_{H_s}\right) - p_{H_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{H_s}\right) - p_{H_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{65}$$

$$u_s\left(q_{H_s}, \theta_{H_s}\right) - p_{H_s} \geq 0,\ s = 1,2. \tag{66}$$

Constraint (63) must hold in order for H_s types not to choose the bundle offered to L_s consumers by the same seller . The next two constraints, (64) and (65), must hold to avoid that H_s types buy any of the bundles offered by the other seller, s'. Finally, (66) must hold for H_s types to prefer the bundle intended for them to not participating in the market at all.

Symmetrically, the constraints that apply to the L_s-bundle are as follows

$$u_s\left(q_{L_s}, \theta_{L_s}\right) - \alpha x_{L_s} \geq u_s\left(q_{H_s}, \theta_{L_s}\right) - p_{H_s},\ s = 1,2, \tag{67}$$

$$u_s\left(q_{L_s}, \theta_{L_s}\right) - \alpha x_{L_s} \geq u_{s'}\left(q_{L_{s'}}, \theta_{L_s}\right) - \alpha x_{L_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{68}$$

$$u_s\left(q_{L_s}, \theta_{L_s}\right) - \alpha x_{L_s} \geq u_{s'}\left(q_{H_{s'}}, \theta_{L_s}\right) - p_{H_{s'}},\ s, s' = 1,2,\ s' \neq s, \tag{69}$$

$$u_s\left(q_{L_s}, \theta_{L_s}\right) - \alpha x_{L_s} \geq 0,\ s = 1,2. \tag{70}$$

Constraint (67) must hold in order for L_s types not to choose the bundle offered to H_s consumers by seller s. The next two constraints, (64) and (65), must hold to avoid that L_s types buy from the other seller. Finally, (66) must hold for L_s types to prefer the bundle intended for them to not participating in the market at all.

The problem of seller s is

$$\max_{q_{H_s}, p_{H_s}, q_{L_s}, p_{L_s}} \quad \pi = v_{H_s}\left[(1-t)\, p_{H_s} - c q_{H_s}\right] + v_{L_s}\left[(1-t_L)\, r x_{L_s} - c q_{L_s}\right],\ s = 1,2, \tag{71}$$

subject to constraints (63)-(70). In the above expression, r is the revenue from the nonmonetary price, that we assume to be symmetric for the two sellers.

We are now going to solve seller s's problem characterized above, focusing on symmetric equilibria. Our first step is to establish which constraints are going to be binding in equilibrium to determine equilibrium prices. Given $u_{s'}\left(q_{L_{s'}},\theta_{H_s}\right)=u_{s'}\left(q_{H_{s'}},\theta_{H_s}\right)=0$, constraints (64) and (65) cannot be binding, because of the participation constraints in (66). Furthermore, given (70), and that $u_s\left(q_{L_s},\theta_{H_s}\right)>u_s\left(q_{L_s},\theta_{L_s}\right)$, constraint (66) cannot be binding either. Hence, the equilibrium must be such that (63) is binding. We have

$$p_{H_s}=\alpha x_{L_s}+u_s\left(q_{H_s},\theta_{H_s}\right)-u_s\left(q_{L_s},\theta_{H_s}\right),\ s=1,2. \tag{72}$$

Given (72), we can write the constraints (67), after some rearrangements, as

$$u_s\left(q_{H_s},\theta_{H_s}\right)-u_s\left(q_{L_s},\theta_{H_s}\right)\geq u_s\left(q_{H_s},\theta_{L_s}\right)-u_s\left(q_{L_s},\theta_{L_s}\right),\ s=1,2,$$

which must hold strictly by the assumption that $\frac{\partial u_s}{\partial q}\left(q,\theta_{H_s}\right)>\frac{\partial u_s}{\partial q}\left(q,\theta_{L_s}\right)$. Hence, these constraints cannot be binding. Consider now the constraints (69). These can be rewritten, using (72) and after a few rearrangements as

$$u_{s'}\left(q_{H_{s'}},\theta_{H_{s'}}\right)-u_{s'}\left(q_{L_{s'}},\theta_{H_{s'}}\right)-\alpha x_{L_s}\geq u_{s'}\left(q_{H_{s'}},\theta_{L_s}\right)-u_s\left(q_{L_s},\theta_{L_s}\right)-\alpha x_{L_{s'}},\ s=1,2.$$

In a symmetric equilibrium (where $x_{L_s}=x_{L_{s'}}$ and $q_{L_s}=q_{L_{s'}}$), this inequality must hold strictly by the assumption that $\frac{\partial u_{s'}}{\partial q}\left(q,\theta_{H_{s'}}\right)>\frac{\partial u_s}{\partial q}\left(q,\theta_{L_s}\right)>\frac{\partial u_{s'}}{\partial q}\left(q,\theta_{L_s}\right)$. Therefore, the only constraints that can be binding are (68) and (70). We have

$$\alpha x_{L_s}=u_s\left(q_{L_s},\theta_{L_s}\right)-max\left(0,u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right)-\alpha x_{L_{s'}}\right)\ s,s'=1,2,s'\neq s. \tag{73}$$

Given (72) and (73), we can therefore write the the problem of seller s as

$$\begin{aligned}\max_{q_{H_s},q_{L_s}}\quad \pi_s=\ & v_{H_s}\left[\left(1-t\right)\left(u_s\left(q_{L_s},\theta_{L_s}\right)-max\left(0,u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right)-\alpha x_{L_{s'}}\right)+u_s\left(q_{H_s},\theta_{H_s}\right)-u_s\left(q_{L_s},\theta_{H_s}\right)\right)-cq_{H_s}\right] \\ & +v_{L_s}\left[\left(\frac{r}{\alpha}u_s\left(q_{L_s},\theta_{L_s}\right)-max\left(0,u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right)-\alpha x_{L_{s'}}\right)\right)-cq_{L_s}\right],\ \ s,s'=1,2,s'\neq s.\end{aligned} \tag{74}$$

Observe that $u_{s'}\left(q_{L_{s'}},\theta_{L_s}\right)-\alpha x_{L_{s'}}$ does not depend on q_{H_s} nor on q_{L_s}. The first-order conditions of this problem are

$$\frac{\partial\pi}{\partial q_{H_s}}:=\frac{\partial u_s\left(q_{H_s},\theta_{H_s}\right)}{\partial q_{H_s}}\left(1-t\right)-c=0\ s=1,2, \tag{75}$$

$$\frac{\partial \pi}{\partial q_{L_s}} := v_{H_s}\left(-\frac{\partial u_s\left(q_{L_s}, \theta_{H_s}\right)}{\partial q_{L_s}} + \frac{\partial u_s\left(q_{L_s}, \theta_{L_s}\right)}{\partial q_{L_s}}\right)(1-t) + v_{L_s}\left(\frac{r}{\alpha}\frac{\partial u_s\left(q_{L_s}, \theta_{L_s}\right)}{\partial q_{L_s}} - c\right) = 0 \; s = 1,2. \tag{76}$$

The key observation is that these equations have the same form as (11) and (12), which implies that the effects of fees must be also be the same, and so are the implications for optimal policy.

D Monetary prices

To provide a benchmark for the main results, we here consider a version of the model without freemium selling. That is, we assume the seller applies a pure monetary price to all versions, i.e., $x_H = 0$ and $x_L = 0$. Following standard steps (Laffont and Martimort, 2002), one can show that only (3) and (6) are binding at the allocation that solves the seller's problem. Therefore, we ignore (4) and (5), and set

$$p_H = u_H - u_{HL} + u_L, \quad p_L = u_L. \tag{77}$$

Hence, we can rewrite the seller's problem in (2) as

$$\max_{q_H, q_L} \quad \pi = v\left(\left(1-t\right)\left(u_H - u_{HL} + u_L\right) - c q_H\right) + \left(1-v\right)\left(\left(1-t\right) u_L - c q_L\right). \tag{78}$$

Remark that the transaction fee equally hits the two versions of the product because they are exchanged for a monetary price. In the above expression, $u_{HL} - u_L$ represents the high types' information rent, which the seller must grant to prevent them from choosing the version intended for the low types. There is, instead, no rent left to the low types. We thus get the following expressions for consumer surplus

$$CS_H = u_{HL} - u_L, \qquad CS_L = 0.$$

The equilibrium qualities, q_i^e, solve the following system of equations

$$\frac{\partial \pi}{\partial q_H} = v\left(\frac{\partial u_H}{\partial q_H}\left(1-t\right) - c\right) = 0, \tag{79}$$

$$\frac{\partial \pi}{\partial q_L} = v\left(-\frac{\partial u_{HL}}{\partial q_L} + \frac{\partial u_L}{\partial q_L}\right)(1-t) + (1-v)\left((1-t)\frac{\partial u_L}{\partial q_L} - c\right) = 0. \tag{80}$$

With no transaction fee ($t = 0$), these equations indicate that the seller offers an efficient version to the high types, in the sense that the marginal utility these consumers get from quality equals the marginal cost. Given $\frac{\partial u_{HL}}{\partial q_L} > \frac{\partial u_L}{\partial q_L}$, the seller distorts the quality of version intended for the low types downwards. This is to reduce the information rent left to the high types. The distortion in quality of the low version is thus due to the standard rent extraction vs. efficiency trade-off (Maskin and Riley, 1984; Laffont and Martimort, 2002).

The effects of the fee in this scenario are not surprising. The fee reduces the profit of the seller, which follows from the envelope theorem. By totally differentiating the above first-order conditions of the monopolist's problem with respect to a uniform fee t, we find that

$$\frac{\partial q_i}{\partial t} = -\frac{\frac{\partial^2 \pi}{\partial q_j^2}\frac{\partial^2 \pi}{\partial q_i \partial t} - \frac{\partial^2 \pi}{\partial q_i \partial t}\frac{\partial^2 \pi}{\partial q_H \partial q_L}}{H}, \tag{81}$$

where $H \equiv \frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H^2} - \left(\frac{\partial^2 \pi}{\partial q_H \partial q_L}\right)^2 > 0, \quad \frac{\partial^2 \pi}{\partial q_j^2} < 0, \quad \frac{\partial^2 \pi}{\partial q_i^2} < 0$ by second order conditions. Moreover, $\frac{\partial^2 \pi}{\partial q_H \partial q_L} = 0$, $\frac{\partial^2 \pi}{\partial q_H \partial t} = -v\frac{\partial u_H}{\partial q_H} < 0$ and $\frac{\partial^2 \pi}{\partial q_L \partial t} = -\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right) < 0$. Hence,

$$\operatorname{sgn}\left(\frac{\partial q_H}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_L^2}\frac{\partial^2 \pi}{\partial q_H \partial t}\right) = \operatorname{sgn}\left(-v\frac{\partial u_H}{\partial q_H}\right) < 0,$$

$$\operatorname{sgn}\left(\frac{\partial q_L}{\partial t}\right) = \operatorname{sgn}\left(-\frac{\partial^2 \pi}{\partial q_H^2}\frac{\partial^2 \pi}{\partial q_L \partial t}\right) = \operatorname{sgn}\left(-\left(\frac{\partial u_L}{\partial q_L} - v\frac{\partial u_{HL}}{\partial q_L}\right)\right) < 0.$$

There is a reduction in quality for all versions because the fee has fundamentally the same effect as an increase in the cost c. As for the effect on prices, given (77) and (81), we have

$$\frac{\partial p_H^e}{\partial t} \gtrless 0, \quad \frac{\partial p_L^e}{\partial t} < 0. \tag{82}$$

The price of the L-version decreases due to the reduction in quality of that version. The effect on the price of the H-version can be positive or negative, because the utility from the high version decreases as q_H goes down, but the information rent decreases as q_L goes down.

The latter observation implies that the surplus of H-type consumers decreases, while the surplus of L-type consumers does not change because the participation constraint is binding:

$$\frac{\partial CS_H^e}{\partial t} = \left(\frac{\partial u_{HL}}{\partial q_L} - \frac{\partial u_L}{\partial q_L}\right)\frac{\partial q_L^e}{\partial t} < 0, \quad \frac{\partial CS_L^e}{\partial t} = 0. \tag{83}$$

Furthermore, the introduction of a small ad valorem fee has negative effects on welfare

$$\left.\frac{\partial W}{\partial t}\right|_{t=0} = \frac{\partial q_H}{\partial t}v\left(\frac{\partial u_H}{\partial q_H} - c\right) + \frac{\partial q_L}{\partial t}(1-v)\left(\frac{\partial u_L}{\partial q_L} - c\right) < 0.$$