WORKING PAPERS

N° 1564

JANUARY 2025

"Promotional Allowances: Loss Leading as an Incentive Device"

David Martimort and Jérôme Pouyet

Toulouse School of Economics

Promotional Allowances: Loss Leading as an Incentive Device*

David Martimort† Jerome Pouyet‡

January 21, 2025

Abstract

A retailer can boost demand for a manufacturer's product through unobservable activities. Promotional allowances, which are retrospective rebates tied to the success of the retailer's promotional efforts, can partially mitigate the resulting moral hazard problem. In equilibrium, the wholesale contract includes a retail price set below cost, complemented by a rebate for incremental units purchased when promotional efforts successfully increase sales. Loss leading thus emerges as an incentive mechanism, rather than a practice driven by anti-competitive or exploitative intent. A ban on below-cost pricing leads to higher retail prices and reduced promotional efforts.

Keywords: Vertical restraints; moral hazard; loss leading; promotional allowances; below-cost pricing.

JEL Code: L11, L42, L81.

*We thank people at Altermind, Catherine Bobtcheff, Zhijun Chen and Daniel Herrera for useful discussions and comments on earlier drafts. We are grateful to participants to IIOC (Boston, 2017), CRESSE (Heraklion, 2017), EARIE (Maastricht, 2017), EARIE (Amsterdam, 2024) and Paris IO Day (Paris, 2024) for useful comments. This paper has also been funded by the Agence Nationale de la Recherche under grant ANR-17-EURE-0010 (Investissements d'Avenir program). All errors remain ours.

†Toulouse School of Economics. E-mail: david.martimort@tse-fr.eu. Address: Toulouse School of Economics, 1 Esplanade de l'Université, 31080 Toulouse Cedex 06, France.

‡CY Cergy Paris Université, CNRS, THEMA and ESSEC Business School. E-mail: pouyet@essec.edu. Address: ESSEC Business School, 3 Avenue Bernard Hirsch, BP 50105, 95021 Cergy, France.

1. Introduction

Vertical relationships between producers and distributors are governed by various contractual arrangements, ranging from simple linear pricing to more sophisticated mechanisms. The literature on vertical restraints reflects this diversity, offering valuable insights into these practices.[1] However, one prevalent yet understudied practice is the use of promotional allowances.

Manufacturers primarily offer two types of allowances to retailers: slotting fees and promotional allowances. Slotting fees are fixed payments made in exchange for product placement or shelf space, particularly for new products (Lariviere and Padmanabhan, 1997; Marx and Shaffer, 2010). In contrast, promotional allowances are variable payments contingent on performance, rewarding retailers for activities such as advertising or offering discounts to increase sales.

The literature has primarily focused on the role of slotting fees along the value chain, with mixed findings regarding their competitive effects. Slotting fees have been shown to have either pro-competitive (Chu, 1992; Foros et al., 2009) or anti-competitive (Shaffer, 1991; Marx and Shaffer, 2007; Miklós-Thal et al., 2011) effects, depending on the context.[2] By comparison, the role and implications of promotional allowances have received less attention despite their practical relevance. For example, large retailers like Wal-Mart and Costco reportedly do not require slotting fees (Kuksov and Pazgal, 2007), yet promotional allowances account for the majority of vendor payments at retailers like Safeway.[3] Similarly, PepsiCo allocated approximately 30% of its 2009 net revenues to sales incentives and discounts (Allen et al., 2011).

This paper investigates the economics of promotional allowances by introducing two key features of manufacturer-retailer relationships. First, retailers promote manufacturers' products through various activities. While some of these activities can be specified contractually, others remain unverifiable, leading to a moral hazard problem, as highlighted by Rey and Tirole (1986). Retrospective rebates tied to the success of the retailer's promotional efforts can partially mitigate this moral hazard issue. Second, wholesale contracts face constraints in profit allocation, particularly when fixed fees or slotting allowances are absent or limited. As a result, wholesale contracts rely on a combination of wholesale prices and retrospective rebates to incentivize promotional efforts.

Our analysis provides several new insights. Some relate to the underlying rationale of using promotional allowances, while others address the consequences of regulatory interventions on such practices and, more broadly, on market conduct. First, below-cost pricing combined with rebates emerges as a tool to align retailers' incentives with manufacturers' goals without anti-competitive or exploitative intent. Second, the impact of banning below-cost pricing depends on the intensity of competition among manufacturers. Third, bans on below-cost pricing can unintentionally raise retail prices and reduce demand-enhancing promotional efforts. Fourth, both manufacturers and retailers

[1] See Rey and Vergé (2008) for a comprehensive survey.

[2] Piccolo and Miklós-Thal (2012) study how slotting fees impact collusion between downstream firms.

[3] According to Safeway's annual report to the SEC (2015, p. 28) (available at https://www.sec.gov/Archives/edgar/data/86144/000008614415000004/swy-1315x10k.htm#s8CD5A973515FABD6469F561F683E304A), vendor allowances totaled $2.5 billion in 2014, $2.4 billion in 2013, and $2.3 billion in 2012, with promotional allowances comprising the vast majority.

sometimes prefer relying on incentive allowances rather than on resale price maintenance agreements in order to coordinate the vertical value chain.

Below-Cost Pricing. A critical issue in this context is how manufacturers balance extracting retailers' profits with incentivizing their promotional efforts. Our analysis reveals that optimal wholesale contracts often induce below-cost pricing at the retail level. This is achieved through high wholesale prices that raise retailers' marginal costs, coupled with significant rebates contingent on promotional success.

The reasoning is intuitive. Below-cost pricing functions as a bonding mechanism: Retailers incur initial losses, motivating them to exert sufficient promotional effort to recoup these losses through rebates. Manufacturers, in turn, adopt a "stick and carrot" approach. The stick is the threat of negative profits if promotional efforts fail, while the carrot is the reward of rebates when efforts succeed. Such dynamics resemble marketing the product in two distinct markets. On the "base market," the retailer incurs losses, while, on the "extra market," rebates generate profits.

Sales-Below-Cost Laws. Sales-below-cost laws, often introduced to protect small producers, significantly alter such dynamics. These laws have a long history worldwide, with mixed assessments of their impact. Several European Union Member States and U.S. states prohibit below-cost pricing, though the scope and nature of these bans vary.[4]

When manufacturers lack bargaining power, wholesale prices are set at marginal cost, rendering bans on below-cost pricing irrelevant. However, when manufacturers possess significant market power, such bans impose constraints on retail pricing. Contrary to naive stances, these laws may reduce efficiency by raising retail prices and limiting demand-enhancing activities. Manufacturers can no longer rely on the stick of negative base profits to motivate retailers. Instead, they must increase rebates and grant retailers a moral hazard rent to incentivize promotional efforts. Echoing a fundamental principle of the Theory of Incentives,[5] manufacturers must reduce this rent, which, in turn, leads to a lower level of promotional effort from retailers. Additionally, bans on below-cost pricing limit manufacturers' ability to capture downstream profits. Under weak specifications about preferences and technologies, we show that such bans result in higher retail prices and reduced retailer effort.

Extensions and Applications. Vertical control through resale price maintenance (RPM) is another common practice in manufacturers-retailers relationships. Under RPM, manufacturers dictate both wholesale and retail prices but do not use incentive rebates tied to promotional success. We compare these mechanisms and delineate the conditions under which incentive rebates outperform RPM agreements. We also extend our model to two additional scenarios: first, the case of competition by an integrated retailer that does not rely on the manufacturers' product; second, the case of complementary products sold by the retailer. We show that our insights naturally extend to these scenarios.

Finally, our framework is applied to antitrust analysis. In 2014, several manufacturers of household and hygiene products were sanctioned by the French Competition Authority for collusion aimed at maintaining high retail prices and limiting payments to retailers.

[4]For example, Ireland's "Groceries Order" banned sales below net invoice prices, while France's *Loi Galland* prevents retailers from passing on anticipated rebates to consumers. Both regulations have been linked to higher food prices (Irish Competition Agency, 2005; Biscourp, Boutin, and Vergé, 2013).

[5]See Laffont and Martimort (2002, Chapter 4).

Using our analysis, we outline how to estimate the profit loss suffered by affected retailers under such conditions.

LITERATURE REVIEW. That retailers can increase the sales of manufacturers' products is a central tenet of the literature on vertical relationships. With simple contracts between manufacturers and retailers, and when retailers cannot fully appropriate the benefits of their efforts because of free-riding or spillovers, vertical restraints find a possible pro-competitive rationale. That argument, first made by Tesler (1960), has been extended to more general settings by Mathewson and Winter (1984), Rey and Tirole (1986), Krishnan and Winter (2007), Kastl, Piccolo and Martimort (2011), and Hunold and Muthers (2017). We do not consider externalities across retailers and focus, as in Winter (1993), on a vertical externality between the retailer and the manufacturer. Our main point of departure is that we assume that the retailer's effort has some observable, but random, impact on the demand for the manufacturers' product. Incentive payments, such as rebates conditional on performance, can thus be used.[6]

Several explanations have been pushed forward to explain below-cost pricing. Loss leading emerges as an advertising strategy in Ellison (2005): When add-on prices are unobserved firms may advertise a base good at a low price so as to sell add-ons at high unadvertised prices. Bliss (1988) views loss leading as a cross-subsidization strategy between products with different demand elasticities, an idea further developed in Beard and Stern (2008) and Ambrus and Weinstein (2008). Chen and Rey (2012, 2016) show that, with asymmetric competition between retailers, loss leading facilitates screening of consumers according to their shopping costs. Loss leading is a response to vertical opportunism in Allain and Chambolle (2011). Finally, Inderst and Obradovits (2021) examine how retailers' discounts on branded products, used as loss leaders, can shift consumer focus from quality to price, thereby eroding brand value. In stark contrast with all these papers, loss leading emerges in our simpler setting (one retailer, one product, perfectly informed consumers) as a disciplinary device to solve a simple (but overlooked) moral hazard problem on the retailer's side.

ORGANIZATION OF THE PAPER. Section 2 introduces the model and examines several useful benchmarks. Section 3 explores how the manufacturer uses rebates and wholesale payments to incentivize the retailer. Section 4 investigates the impact of banning below-cost pricing. Section 5 compares incentive allowances with resale price maintenance agreements. Section 6 extends the basic framework to more complex scenarios where the retailer faces competition or sells other complementary products in the final market. Section 7 applies the analysis to compute the damages suffered by retailers due to upstream cartels between manufacturers. Finally, Section 8 concludes. All proofs are provided in the Appendix and the Online Appendix.

2. MODEL AND BENCHMARKS

MODEL. We consider the bilateral relationship between an upstream manufacturer M, who produces at marginal cost c, and a downstream retailer R, whose cost is normalized to 0 without loss of generality. Given a retail price p, the demand for the good is denoted by $D(p)$, with $D'(p) < 0$ for all price p such that $D(p) > 0$.

[6] Lømo and Ulsaker (2016) view promotional allowances as fixed payments used discretionarily, in addition to two-part tariffs, by manufacturers in a relational contracting framework.

The retailer exerts promotional effort e, which can increase demand for the manufacturer's product. For example, the demand for standard products and services can be boosted by enhancing promotional efforts. For more complex products, the retailer might improve customer information and reduce search costs. The cost of exerting promotional effort is denoted by $\psi(e)$, where $\psi(\cdot)$ is assumed to be strictly increasing and convex ($\psi'(\cdot) > 0$, $\psi''(\cdot) > 0$) and $\psi(0) = 0$, ensuring interior solutions to all optimization problems below. Finally, we assume that effort e is non-verifiable, introducing moral hazard into the vertical relationship.

We normalize effort such that $e \in [0, 1]$. This normalization allows us to interpret e as the probability that consumer demand increases from $D(p)$ to $(1 + \theta)D(p)$, where $\theta \geq 0$ is a scaling parameter. With the complementary probability $1 - e$, the demand remains at its base level $D(p)$.[7] Since the increase in demand raises revenues, a positive shock in demand is observable by the manufacturer.

The manufacturer offers a wholesale contract, which consists of a per-unit wholesale price w paid by the retailer and a rebate z paid by the manufacturer on all incremental sales that result from the retailer's promotional effort. Specifically, if demand exceeds its base level, the retailer receives an additional payment of $\theta D(p)z$. The retailer can either accept or reject the contract. Upon acceptance, the retailer exerts promotional effort e and chooses a retail price p. When demand is high, the manufacturer pays the rebate z on all incremental units sold.[8]

We examine now several benchmarks that provide valuable insights into both the emergence and the role of promotional allowances.

INTEGRATION. Suppose that the manufacturer is vertically integrated with the retailer. We take the standard short-cut that integration gives access to information and facilitates control.[9] The integrated outcome maximizes the overall industry profit

$$\Pi_I(p, e) = (1 + \theta e)\pi(p, c) - \psi(e),$$

where $\pi(p, c) = (p - c)D(p)$ stands as the 'base profit' absent promotional effort.

Throughout, we shall assume that the following condition holds.

ASSUMPTION 1.
$$p + \frac{D(p)}{D'(p)} \textit{ is increasing.}$$

Assumption 1 holds for most of usual demand specifications (linear, exponential, constant elasticity, etc.). It ensures that $\pi(p, c)$ and $\Pi_I(p, e)$ are both quasi-concave in p.

The monopoly outcome (p^m, e^m) that maximizes the profit of the vertically integrated

---

[7] A more general structure would consider two different demand functions, $\underline{D}(p) < \overline{D}(p)$, depending on the success of the retailer's effort. The multiplicative structure implied by the scale parameter θ is chosen for tractability.

[8] An equivalent but more abstract formulation would be that the manufacturer offers two different wholesale prices, one when demand remains at its base level $D(p)$ and another one when demand jumps at $(1 + \theta)D(p)$. In practice, this solution may be hard to implement because wholesale prices are set before demand realizes.

[9] See for instance Arrow (1975) and Riordan (1990).

structure is thus readily obtained as follows:

$$p^m - c = -\frac{D(p^m)}{D'(p^m)}, \tag{2.1}$$

$$\psi'(e^m) = \theta\pi^m, \tag{2.2}$$

where $\pi^m = \pi(p^m, c)$ stands for the monopoly base profit. Since the promotional effort boosts demand multiplicatively, the monopoly price always maximizes profit whether the demand has been scaled up or not. The optimal effort thus simply trades off the marginal benefit coming from enjoying some extra monopoly profit $\theta\pi^m$ beyond the base level against the retailer's marginal disutility of effort.

Competitive Manufacturers. Assume that several manufacturers produce perfect substitutes and compete in wholesale contracts for the exclusivity of the retailer's services. Head-to-head competition between manufacturers is akin to shifting all the bargaining power towards the retailer. Competition thus drives the manufacturers' profit to zero, or

$$((w - c)(1 + \theta e) - \theta e z)\, D(p) = 0. \tag{2.3}$$

It is straightforward to see that the simple wholesale contract with a wholesale price that just covers the manufacturer's marginal cost and no rebate (i.e., $w^d = c$ and $z^d = 0$) ensures that manufacturers make zero profit.[10] Moreover, this solution clearly aligns the retailer's objectives with those of the vertically integrated structure so that the retail price and promotion effort coincide with integrated outcome, or $p^d = p^m$ and $e^d = e^m$. An immediate corollary is that, when manufacturers have no bargaining power, rebates are useless and a ban on below-cost pricing has no bite since $p^d > w^d$.[11]

No Rebates. Consider next a scenario where the manufacturer does not use rebates. The retailer would maximize

$$(p - w)D(p)(1 + \theta e) - \psi(e).$$

The optimal retail price p induced by a wholesale price w follows the familiar pass-through formula in the context of a double-marginalization scenario à la Spengler (1950), namely

$$w = p + \frac{D(p)}{D'(p)}.$$

Charging this wholesale price w leaves the retailer a positive profit worth[12]

$$(1 + \theta e)\varphi(p) - \psi(e),$$

where $\varphi(p) = -D^2(p)/D'(p)$ stems for the retail's base profit once the retailer has optimally chosen his retail price in response to the wholesale price. When Assumption 1

[10] See the Online Appendix. Superscript 'd' refers to the fact that with competitive manufacturers everything happens as if the downstream retailer had all the bargaining power.

[11] Retrospective payments given at the end of the accounting year are typically not accounted for to evaluate whether the retailer sells below his cost. The retailer is thus said to sell below cost when $p - w < 0$.

[12] Profits are expressed in terms of the retail price and effort, consistently with our approach that highlights retail decisions and not the wholesale contract that induces these decisions.

holds, a higher retail price decreases retail profit (i.e., $\varphi'(\cdot) < 0$).[13]

The optimal retail price that the manufacturer would like to induce by a convenient choice of w would thus maximize the sole manufacturer's profit, which can be written as follows

$$(\pi(p,c) - \varphi(p))(1 + \theta e).$$

We shall further assume that $\pi(p,c) - \varphi(p)$ is quasi-concave in p which requires a slightly stronger version of Assumption 1.

ASSUMPTION 2.

$$p + \frac{D(p)}{D'(p)} - (1-\lambda)\frac{\varphi'(p)}{D'(p)} \text{ is increasing for any } \lambda \in [0,1].$$

Again, this assumption, which we suppose to hold throughout, is satisfied for most of usual demand specifications (linear, exponential, constant elasticity, etc.).

The optimal retail price that the manufacturer wants to implement is thus defined as follows

$$\pi_p(\tilde{p}, c) = \varphi'(\tilde{p}).$$

From now on, we shall assume that the corresponding retail profit $\varphi(\tilde{p})$ does not suffice to induce the maximal effort level $e = 1$ that would best serve the manufacturer's interest.

ASSUMPTION 3.

$$(1+\theta)\varphi(\tilde{p}) < \psi(1).$$

When Assumption 3 holds, the manufacturer certainly wants to use a rebate to boost the retailer's incentives. Without a rebate, incentives provided by the sole share of the base profit that cannot be appropriated by the manufacturer do not suffice.

TWO-PART TARIFF. It is well known, at least since Dixit (1983) and Mathewson and Winter (1984), that the vertically integrated profit is also achieved when the manufacturer and the retailer are two independent units as long as the manufacturer uses a two-part tariff (w, F) to regulate the relationship with her retailer. To see why in the context of our model, suppose that the manufacturer charges a wholesale price equal to her marginal cost, $w = c$. The retailer is again the residual claimant for the choice of retail price and promotional effort. If, furthermore, the rebate is null ($z = 0$), the retailer de facto maximizes the industry overall profit (up to the constant fee), i.e.,

$$\Pi_I(p,e) - F.$$

The solution thus coincides with the integrated outcome (p^m, e^m). The manufacturer then optimally sets the fixed fee

$$F^m = (1 + \theta e^m)\pi^m - \psi(e^m)$$

to capture the whole downstream profit of the retailer. An immediate corollary, but an important one in view of the rest of our analysis, is that when fixed fees are available, rebates are useless.

[13]Indeed, $\varphi'(p) = -D(p)(2 - \frac{D''(p)D(p)}{(D'(p))^2}) = -D(p)\frac{d}{dp}(p + \frac{D(p)}{D'(p)}) < 0$, where the right-hand side inequality follows from Assumption 1.

Using the price and effort levels given by (2.1) and (2.2), the retailer's profit, gross of the fixed fee, may be rewritten as $\pi^m + R(e^m)$, where $R(e) = e\psi'(e) - \psi(e)$ is the retailer's moral hazard rent to use the jargon of the Theory of Incentives. This rent is the amount of profit that must be given up by the manufacturer to the retailer in order to induce a level of effort e.[14] Observe that, when failing to increase demand, the retailer only earns the base profit π^m and thus incurs a loss worth

$$\pi^m - F^m = -R(e^m) < 0. \tag{2.4}$$

The retailer's profit is thus fully extracted by the manufacturer because that loss is compensated by the positive payoff that the retailer receives following a boost in demand.

Rationale For Not Using Fixed Fees. Suppose now that the retailer incurs a positive fixed cost (which may stand for a specific investment in some contexts) whose value remains small enough so the base profit net of that cost remains positive. Optimal price and effort remain unchanged. Yet, the profit-extracting fee must now be adapted and reduced to compensate the retailer for that fixed cost. Had the fixed cost been private information for the retailer, new strategic opportunities would be opened. The retailer would like to manipulate the fixed cost so as to pay a lower fee and inflate net profits.[15] Within the framework of a full-fledged modeling of such asymmetric information, screening considerations suggest, at best, a limited role for such fixed fees. This in turn means that the lessons of our simple model, and in particular the prevalence of below-cost pricing, would carry over in such a full-fledged model.

Observe also that fixed fees are necessarily paid ex post, i.e., once downstream demand and effort disutility are known and profits are realized, especially if the retailer is cash-constrained in the first place. In practice, various shocks, whose values may be hard to contract upon ex ante, may affect demand and cost. The retailer might take advantage of those contractual loopholes, behave opportunistically and renege on his earlier commitment to pay back those fees. Avoiding the use of fixed fees is thus a response to such opportunism. Our goal in the present paper is certainly not to develop full-fledged models along those lines. Nevertheless, the arguments just sketched would offer some strong motivation for looking at models where fixed fees are not available and wholesale contracts are thereby incomplete.

3. Main Analysis

The Retailer's Problem. Suppose that the retailer operates under a wholesale contract (w, z). The retail price p and promotional effort e are optimally chosen so as to maximize his expected profit

$$(p - w)D(p)(1 + \theta e) + \theta e z D(p) - \psi(e). \tag{3.1}$$

With more compact notations, the retailer's profit in (3.1) can be rewritten as follows

$$\pi(p, w) + \theta e \pi(p, w - z) - \psi(e).$$

[14]See Laffont and Martimort (2002, Chapter 4). From the assumptions on $\psi(\cdot)$, we immediately deduce that $R(e) \geq 0$ with $R'(e) \geq 0$ for all effort level $e \in [0, 1]$.

[15]This point is clearly reminiscent of an argument often found in Regulatory Economics to justify that Ramsey-Boîteux pricing might have some appeal in some regulatory settings; see Laffont and Tirole (1993, Chapter 1).

Focusing on interior solutions, the first-order optimality conditions associated to the optimal retail price p and effort e are thus respectively given by[16]

$$\pi_p(p, w) + \theta e \pi_p(p, w - z) = 0, \tag{3.2}$$

$$\theta \pi(p, w - z) = \psi'(e). \tag{3.3}$$

We may develop the first-order condition (3.2) and rewrite it as follows

$$\frac{1}{1 + \theta e} \frac{p - w}{p} + \frac{\theta e}{1 + \theta e} \frac{p - w + z}{p} = -\frac{D(p)}{pD'(p)}. \tag{3.4}$$

Condition (3.4) shows that the inverse price-elasticity of demand is actually an average of the retail price-cost markups with and without rebate. As the impact of the promotional effort e on demand increases, the retail price becomes more responsive to the rebate z earned on incremental sales and, as such, decreases.

Consider now to the optimality condition (3.3) in terms of effort. Observe that decreasing the wholesale price w and increasing the rebate z increase the retailer's profit from incremental sales (since $\pi(p, w - z)$ is decreasing in its second argument), which boosts incentives to exert effort. Simultaneously, choosing a lower retail price maximizes this profit on incremental sales and also boosts effort.

THE MANUFACTURER'S PROBLEM. The optimality conditions (3.2) and (3.3) allow us to express the contracting variables (w, z) in terms of the pair (p, e) that the manufacturer induces from the retailer through the wholesale contract. This approach is reminiscent of the principal-agent literature where the focus is not necessarily on the contracting instruments used to implement a given effort profile from the agent but, instead, on this effort profile and on the cost for the principal of reaching it. So doing thus yields

$$w = p + \frac{D(p)}{D'(p)}(1 + \theta e) + \frac{e\psi'(e)}{D(p)}, \tag{3.5}$$

$$z = (1 + \theta e) \left(\frac{\psi'(e)}{\theta D(p)} + \frac{D(p)}{D'(p)} \right). \tag{3.6}$$

Equation (3.5) is particularly important in view of our forthcoming analysis of the role of a ban on below-cost pricing. Indeed, this condition delineates the key restriction on implementable pairs (p, e) that ensures a positive margin $p - w$.

Using Equations (3.5) and (3.6), we may express the manufacturer's and the retailer's profits, still in terms of the pair (p, e) to be implemented, respectively as follows

$$\Pi_M(p, e) = (\pi(p, c) - \varphi(p)) (1 + \theta e), \tag{3.7}$$

and

$$\Pi_R(p, e) = (1 + \theta e)\varphi(p) - \psi(e). \tag{3.8}$$

OPTIMAL WHOLESALE CONTRACT. This contract must maximize the manufacturer's

[16]Such a first-order approach is usual in Principal-Agent problems under asymmetric information. We discuss its validity in our setting in the Online Appendix.

profit $\Pi_M(p,e)$ subject to the retailer's participation condition

$$\Pi_R(p,e) \geq 0. \tag{3.9}$$

We shall assume that this problem is quasi-concave in (p,e) and denote by (w^u, z^u) the optimal wholesale contract for the manufacturer. Let also (p^u, e^u) be the corresponding retail price and effort level induced by such contract.

Proposition 1. *The optimal wholesale contract (w^u, z^u) satisfies the following properties.*

1. *The retailer makes zero profit*

$$\Pi_R(p^u, e^u) = (1+\theta e^u)\varphi(p^u) - \psi(e^u) = 0. \tag{3.10}$$

2. *There is below-cost-pricing when demand remains at its base level*

$$p^u - w^u < 0. \tag{3.11}$$

3. *Rebates are strictly positive*

$$z^u > 0. \tag{3.12}$$

The manufacturer would ideally like to implement the highest possible promotional effort and induce the heavily distorted retail price $\tilde{p}$ corresponding to the double marginalization scenario. Unfortunately, doing so would induce the retailer to make negative profits when Assumption 3 holds. The optimal contract moves along the retailer's beak-even condition towards a point that maximizes the manufacturer's profit. The wholesale price is used to extract profit whereas the rebate serves to induce the promotional effort. More precisely, using Equation (3.5) and the binding participation constraint (3.9), the retailer's base profit can be expressed as follows

$$(p^u - w^u)D(p^u) = -R(e^u) < 0. \tag{3.13}$$

This expression echoes the earlier formula (2.4) found in a scenario where fixed fees are available. In particular, Equation (3.13) implies below-cost pricing as stated in (3.11) and a negative base profit; the stick side of the mechanism. Instead, the rebate provides the moral hazard rent needed to induce effort; the carrot side. At the optimal wholesale contract, the retailer would not recover the loss on the base profit, $(p^u - w^u)D(p^u) < 0$ without exerting at least effort e^u. The loss on base profit thus acts as a bonding device triggering the demand-expanding effort.

We now turn to an important comparison.

Proposition 2. *In comparison with the vertically-integrated outcome, the retail price and the promotional effort both increase*

$$p^u > p^m \text{ and } e^u > e^m.$$

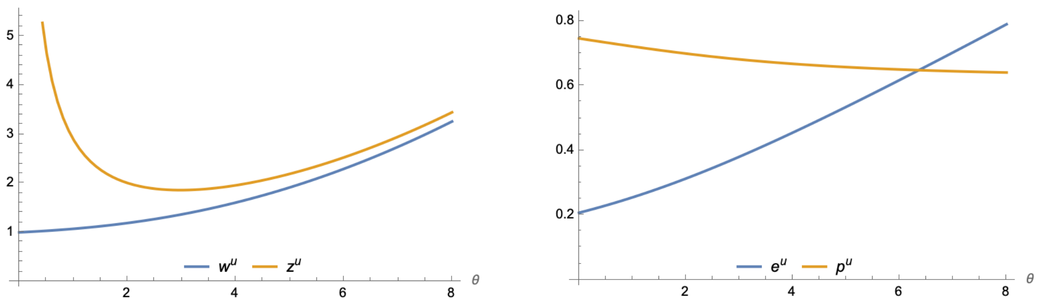
Even though fixed fees are no longer available, the manufacturer still needs to incentivize the retailer for the promotional effort on the one hand and extract the downstream profit on the other hand. Without a fixed fee, this extraction is incomplete but the residual $\varphi(p)$ left to the retailer can also play an incentive role. Increasing p beyond the monopoly outcome increases the retailer profit and thus relax the break even condition.

The upward distortion of effort is more subtle. To see why, it is useful to take as granted the result of Proposition 1 and denote by $E(p)$ the decreasing function implicitly defined through the binding break-even condition (3.9). The optimality condition on effort can then be rewritten in terms of p only as follows

$$\psi'(e^u) = \theta\pi(p^u, c) + (1 + \theta e^u)\frac{\pi_p(p^u, c)}{E'(p^u)}.$$

There are two forces that determine the effort distortion. On the one hand, the first term on the right-hand side captures how increasing the retail price from p^m to p^u decreases overall profit ($\pi(p^u, c) < \pi(p^m, c)$) and this tends to reduce the optimal effort. On the other hand, increasing the retail price along the break-even condition (3.9) also raises effort. This effect is captured in the second term above which is negative (since $\pi_p(p^u, c) < 0$ for $p^u > p^m$ and $E'(p^u) < 0$). Proposition 2 shows that this second effect always dominates.

COMPARATIVE STATICS. We now provide additional results on the optimal contract. This exercise is generally difficult because of the significant nonlinearities of the problem. Assuming a linear demand and a quadratic cost of effort helps us on this front. Indeed, we then determine numerically the optimal contract for $\theta \in [.1, 8]$ and vary θ by an increment of .1 for a total of 80 simulations.[17] Figure 1 represents the optimal contract for the manufacturer, and the corresponding price and effort level chosen by the retailer.



(a) Wholesale price and rebate. (b) Price and effort level.

Figure 1: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$, $c = 0$, and $\psi(e) = \frac{3}{2}e^2$), the optimal contract for the manufacturer (w^u, z^u) is represented in Panel (a) and the optimal price and effort level (p^u, e^u) are depicted in Panel (b).

As the benefit of effort increases (i.e., θ increases), the wholesale price w^u and the bonus z^u become more similar (Figure 1 (a)). This result is intuitive. Under those circumstances, there is less need to enlarge the gap between the wholesale price and the rebate to induce effort. More subtle is the fact that the rebate z^u is non-monotonic in the scale parameter. Indeed, there are potentially two effects from raising that rebate. First, it boosts efforts and this first effect is more significant when effort is more valuable. Second, raising the bonus also decreases the retail price, depresses demand and thus

[17]Simulations are performed using Mathematica and can be found on the second author's webpage.

reduces the residual profit left to the retailer. Figure 1 (a) shows that this second effect may dominate when effort has a relatively small marginal value.

An intuitive consequence of an increase in the benefits of effort, is that the equilibrium effort also increases (Figure 1 (b)). In turn, the fact that $E'(p) < 0$ implies that the retail price decreases under those circumstances.

4. Impact of Sales-Below-Cost Laws

We assume now that, by regulation, retail prices cannot be set below the retailer's cost. The following non-negativity constraint must thus always hold

$$p - w \geq 0. \tag{4.1}$$

Because the retail price margin satisfies the moral hazard incentive constraint (3.5), profits must at least cover the moral hazard rent needed to induce effort. The non-negativity constraint (4.1) becomes in fact a non-negative lower bound on the retailer's profit, or

$$\Pi_R(p, e) \geq R(e), \tag{4.2}$$

where the retailer's profit $\Pi_R(p, e)$ is still given by (3.8).

Since the retailer's profit may also be written as $(p-w)D(p)+R(e)$ (as we did to derive (3.13)), a first consequence of a ban of below-cost pricing appears immediately. The base profit $(p - w)D(p)$ that accrues to the retailer must be non-negative and, therefore, can no longer be used to extract the retailer's overall profit. Put differently, the retailer can no longer run a loss if the promotional effort fails to increase demand. Sticks are no longer available and only carrots can be used. Implementing a large rebate becomes the only channel to reward effort, and this is obviously costly for the manufacturer.[18] The optimum is characterized in the next proposition.

PROPOSITION 3. *Suppose there is a ban on below-cost pricing.*

1. *The retail price is equal to the wholesale price,*

$$p^b = w^b.$$

2. *The retailer makes a strictly positive profit equal to the moral hazard rent,*

$$\Pi_R(p^b, e^b) = R(e^b). \tag{4.3}$$

With a ban on below-cost pricing, the manufacturer's ability to extract the retailer's profit through the wholesale price is reduced because the base profit can no longer be negative. A moral hazard rent must be given up to the retailer.

Satisfying constraint (4.2) is thus clearly more demanding than satisfying the break-even condition (3.9) that prevails absent such a ban. A first intuition would then go as follows. Imposing a ban on below-cost pricing is akin to replacing the cost of effort $\psi(e)$ by a virtual disutility of effort $e\psi'(e) = \psi(e) + R(e)$ which is more costly. This calls for

[18] The contractual constraint on retail price imposed by regulation refers to an argument familiar from the moral hazard literature with limited liability; see Laffont and Martimort (2002, Chapter 4).

lowering the promotional effort. Moreover, lowering the retail price towards the monopoly level p^m also contributes to relaxing constraint (4.2). These are the direct effects of a ban on below-cost pricing.

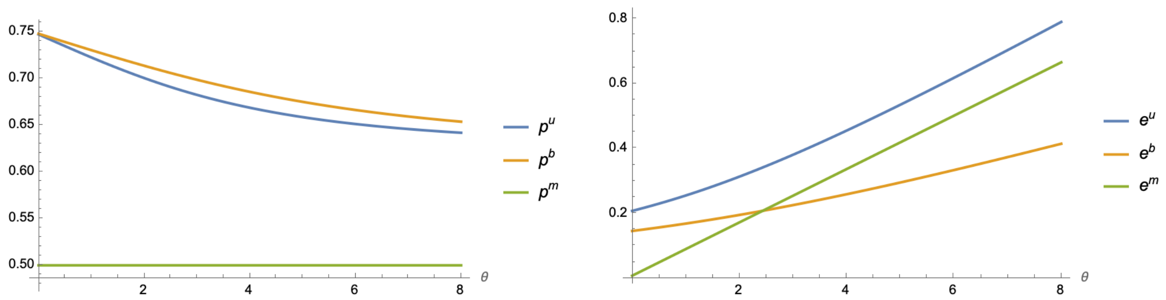
Indirect effects come from the fact that reducing effort affects marginal incentives to change the retail price, and vice versa. More formally, the value of the Lagrange multiplier associated to the retailer's participation constraint changes as one moves from one institutional environment to the other. A priori, replacing the break-even condition (3.9) by the more stringent condition (4.2) calls for increasing the value of the Lagrange multiplier. This intuition may be misleading, though, since the value of the multiplier depends on the equilibrium choices of price and effort. These indirect effects make the comparison between (p^u, e^u) and (p^b, e^b) difficult. Comparative statics can nevertheless be further explored in some specific environments.

Proposition 4. *Suppose that demand is given by $D(p) = (a - bp)^{\frac{1}{\delta}}$ (with $\delta \geq 1$)[19] and the disutility of effort is quadratic ($\psi(e) = \frac{\mu}{2}e^2$, $\mu > 0$). Imposing a ban on below-cost-pricing has the following consequences:*

1. *The effort decreases: $e^b < e^u$.*
2. *The retail price increases: $p^b > p^u$.*
3. *The value of the Lagrange multiplier of the retailer's participation constraint decreases: $\lambda^b < \lambda^u$.*

Given that a linear demand ($\delta = 1$) and a quadratic disutility of effort may be viewed as first-order approximations of more complex specifications of preferences and technologies, Proposition 4 strongly suggests that the retail price should increase following a ban of below-cost pricing even with more general specifications. Far from promoting competition in the hypothetical scenario where below-cost pricing would be used for predatory purposes, such a law may well harm consumers and reduce overall welfare.

Figure 2 provides a comparison of price and effort as function of the demand shock θ. It shows in particular that the effort level under a ban on below-cost pricing (e^b) may be below or above the efficient level (e^m). Keeping this in mind will be useful later on.



(a) Prices. (b) Effort levels.

Figure 2: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$, $c = 0$, and $\psi(e) = \frac{3}{2}e^2$), comparison of the optimal price (Panel (a)) and effort level (Panel (b)) with no ban on below-cost pricing (p^u and e^u), with a ban on below-cost pricing (p^b and e^b), and under integration (p^m and e^m).

[19]This class of demand, first used by Bulow and Pfleiderer (1983), has the property that the cost pass-through is constant and given by $1/(1 + \delta)$.

5. Resale Price Maintenance vs. Incentive Rebates

Vertical control in the form a resale price maintenance (RPM) agreement often arises in manufacturers-retailers relations.[20] From a competition policy perspective, RPM agreements may be viewed with a rule of reason approach, as in the U.S., or may be considered as a hardcore restraint, as in the E.U. We aim to address here the following question: how does a RPM agreement compare with respect to promotional allowances?

Under RPM, the manufacturer controls both the wholesale price w and the retail price p of the product. The retailer's only decision is to choose the level of promotion effort so as to maximize $(p-w)D(p)(1+\theta e)-\psi(e)$. The manufacturer's problem is to maximize $(w-c)D(p)(1+\theta e)$ subject to the retailer's participation constraint, and anticipating the choice of effort by the retailer.

Proposition 5. *Under a RPM agreement, the retail price is always set at the monopoly base level, or $p^{rpm}=p^m$. Moreover:*

- *If $\frac{\theta^2\pi^m}{\psi''(0)}<1$, the retailer exerts no effort and makes no profit.*
- *Otherwise, the retailer exerts a strictly positive effort $e^{rpm}<e^m$ and earns a strictly positive profit $\frac{\psi'(e^{rpm})}{\theta}+R(e^{rpm})$.*

Furthermore, assuming a linear demand and a quadratic cost of effort:

- *With no ban on below-cost pricing, the manufacturer prefers a RPM agreement over incentive rebates if and only if $\frac{\theta^2\pi^m}{\psi''(0)}$ is sufficiently small, whereas the retailer always prefers incentive rebates.*
- *With a ban on below-cost pricing, the manufacturer always prefers a RPM agreement, whereas the retailer always prefers incentive rebates.*

From the manufacturer's perspective, the benefit of a RPM is to limit distortions on the retail price, which always coincides with the monopoly price p^m. The downside is that there is only one pricing instrument, namely the wholesale price w, to pursue two conflicting objectives, namely extracting the retailer's profit and providing incentives to effort. As a result, the promotion effort is below the efficient level (and sometimes null). Intuitively, when the benefits associated to the promotion effort are small (for instance, when θ is small), the manufacturer should prefer a RPM agreement over a system of incentive rebates. By contrast, when these benefits are large, the manufacturer should prefer to give up the control of the retail price to the retailer and implement a system of incentive rebates. Observe that the retailer always prefers a system of incentive rebates over a RPM agreement. Provided that effort has a sufficiently strong impact on demand, both the manufacturer and the retailer view rebates as superior to a RPM agreement.

However, the presence of a ban on below-cost pricing impacts that conclusion. In that case, there is a strong disagreement between the manufacturer and the retailer on which sort vertical control (RPM or incentive rebates) ought to be implemented. Key to this disagreement is the fact that providing incentives is costly for the manufacturer when below-cost pricing is not allowed, which in turn provides a substantial moral hazard rent to the retailer.

[20] See Dertwinkel-Kalt and Wey (2024) for a general analysis of RPM agreements and Martimort and Piccolo (2007) for a comparison between RPM and quantity fixing agreements under asymmetric information.

6. EXTENSIONS

In this section, we extend our insights in two directions: first, when the retailer faces competition from an integrated retailer (Section 6.1); second, when the retailer also sells a product that is complementary to the manufacturer's good (Section 6.2).

6.1. Competing Against an Integrated Retailer

We now study how the competitive pressure exerted downstream by an integrated retailer impacts on the incentives of a manufacturer to use below-cost pricing with her own non-integrated retailer. This case is of a particular relevance in view of the substantial development of so-called "hard discounters" over the last past decades, one of their characteristics being that they mostly distribute their own brands or private labels.[21]

To do so, assume the non-integrated retailer (called retailer 1) faces a vertically-integrated competitor (called retailer 2), that is, retailer 2 produces and sells his own good. Retailers 1 and 2 produce differentiated products with demands $D_i(p_i, p_{-i})$, $i = 1, 2$. For simplicity, demand functions are assumed to be symmetric: $D_1(p_1, p_2) = D_2(p_2, p_1) \equiv D(p_1, p_2)$ for all prices (p_1, p_2). Still to avoid unnecessary notational burden, goods 1 and 2 are produced at the same marginal cost c.

Let also assume, for simplicity, that the contract (w_1, z_1) that governs the relationship between the manufacturer and the non-integrated retailer 1 is secret. That contract has thus no commitment value to impact downstream competition. We therefore look for a Nash equilibrium in which the integrated competitor takes as given the contract offered by this vertical structure and the retail price charged by the non-integrated retailer. Denoting by p_1 this price, the integrated retailer's best response $P_2(p_1)$ is then characterized as follows

$$P_2(p_1) - c = -\frac{D(P_2(p_1), p_1)}{\frac{\partial D}{\partial p_2}(P_2(p_1), p_1)}. \tag{6.1}$$

Under standard assumptions, which are detailed in the Appendix, the best response defined in (6.1) is upward-sloping with a slope smaller than 1.

COMPETITIVE MANUFACTURERS. We proceed as in our base scenario, the only change being that the vertical structure takes now the price p_2 charged by the integrated competitor as given. Mimicking our earlier findings, the best response contract offered by competitive manufacturers entails marginal cost pricing and no rebates, i.e. $w_1 = c$ and $z_1 = 0$. From this, it immediately follows that the best response in price $P_1(p_2)$ for non-integrated retailer 1 satisfies the following condition

$$P_1(p_2) - c = -\frac{D(P_1(p_2), p_2)}{\frac{\partial D}{\partial p_1}(P_1(p_2), p_2)}. \tag{6.2}$$

Equations (6.1) and (6.2) define the familiar upward-sloping best responses $P_2(p_1)$ and $P_1(p_2)$. Under familiar conditions, those best responses have slope less than 1. In other words, any increase in p_{-i} induces an increase in p_i along the best response that is of a

[21]See, for instance, Cleeren et al. (2010) for an account of the development of hard discount chains in Europe. In the U.S., the hard discount format has been relatively absent from the competitive landscape until recently, and is now quickly gaining market shares.

lower magnitude. At a Nash equilibrium, both Equations (6.1) and (6.2) hold and the symmetric Nash equilibrium $p_1^* = p_2^* = p^*$ solves

$$p^* - c = -\frac{D(p^*, p^*)}{\frac{\partial D}{\partial p_1}(p^*, p^*)}.$$

The optimal effort of the non-integrated retailer is then characterized as follows

$$\psi'(e^*) = \theta\pi^*,$$

where $\pi^* \equiv (p^* - c)D(p^*, p^*)$.

Monopoly Manufacturer. Suppose now that the manufacturer has all bargaining power in dealing with the retailer and behaves as an upstream monopoly, yet facing competition from the integrated retailer on the downstream market. The analysis again replicates some of our earlier findings but in an equilibrium context.

For future reference, we denote

$$\pi(p_1, p_2, c) = (p_1 - c)D(p_1, p_2)$$

and

$$\varphi(p_1, p_2) = -\frac{D^2(p_1, p_2)}{\frac{\partial D}{\partial p_1}(p_1, p_2)}.$$

Given the price p_2 charged by the integrated competitor, the manufacturer maximizes her profit

$$\Pi_M(p_1, p_2, e_1) = (1 + \theta e_1)\left(\pi(p_1, p_2, c) - \varphi(p_1, p_2)\right),$$

subject to the retailer's participation constraint

$$\Pi_{R_1}(p_1, p_2, e_1) = (1 + \theta e_1)\varphi(p_1, p_2) - \psi(e_1) \geq 0. \tag{6.3}$$

The following assumption generalizes Assumption 5 to this competitive environment. This condition again ensures that the retailer's participation constraint (6.3) is binding at equilibrium.

Assumption 4. *For all price p_2,*

$$(1 + \theta)\varphi(\tilde{P}_1(p_2), p_2)) < \psi(1),$$

where $\tilde{P}_1(p_2) = \arg\max_{p_1} \pi(p_1, p_2, c) - \varphi_1(p_1, p_2)$.

The best response of the non-integrated structure $P_1^u(p_2)$ is then defined as the solution to

$$\pi_{p_1}(p_1, p_2, c) - (1 - \lambda_1^u)\varphi_{p_1}(p_1, p_2) = 0, \tag{6.4}$$

where λ_1^u is the non-negative Lagrange multiplier associated to (6.3). That best response remains upward-sloping and has slope less than one despite the addition of the third term on the right-hand side of (6.4), provided that this term and its derivative are of a limited magnitude; an assumption that we will make from now on. Because $\varphi_{p_1}(p_1, p_2) \leq 0$ and $\lambda_1^u < 1$, the best response $P_1^u(p_2)$ defined by (6.4) lies above the best response $P_1(p_2)$

obtained when manufacturers are competitive. It then follows that, with a monopoly manufacturer, all retail prices raise at equilibrium in comparison with the scenario of competitive manufacturers. Yet, the price increase is stronger for the non-integrated retailer than for the vertically-integrated competitor.

Proposition 6. *With a monopoly manufacturer, all retail prices increase, but more so for the price charged by a non-integrated retailer:*

$$p_1^u > p_2^u > p^*.$$

Ban on Below-Cost-Pricing. The reasoning here should now be familiar from our previous analysis. The constraint on non-negative margin writes now as follows

$$p_1 \geq w_1.$$

This constraint still implies that the non-integrated retailer must receive a positive moral hazard rent $R(e_1)$. The latter's participation constraint writes thus as follows

$$\Pi_{R_1}(p_1, p_2, e_1) \geq R(e_1). \tag{6.5}$$

Observe that the moral hazard rent does not directly depend on the competitive pressure exerted by the integrated retailer. Denoting by λ_1^b the Lagrange multiplier of this constraint in the manufacturer's program, we can state the following proposition.

Proposition 7. *Suppose there is a ban on below-cost pricing.*

1. *Provided that $\lambda_1^b \leq \lambda_1^u$, all retail prices increase, but more so for the non-integrated retailer's price:*
$$p_1^b > p_1^u > p_1^* \text{ and } p_2^b > p_2^u > p_2^*,$$
with $p_1^b > p_2^b$.

2. *Assuming linear demands and a quadratic cost of effort, we further obtain that*
$$\lambda_1^b < \lambda_1^u \text{ and } e_1^b < e_1^u.$$

These results are much in line with our main analysis with no downstream competition. The ban on below-cost pricing leads the integrated competitor to price higher; downstream competition is thus softened, to the detriment of final customers.

6.2. Multi-Product Retailer With Demand Complementarities

We now evaluate the importance of below-cost-pricing in the more traditional multi-product context where below-cost pricing has already been proved useful. To illustrate, suppose that the retailer can also sell another good, say good 2. For notational simplicity, we assume that good 2 is also produced at marginal cost c. Good 1, whose demand is still given by $D(p_1)$ as before, is a loss leader in the following sense. Consumers may be eager to obtain the loss leader and, when doing so, they may also express a demand for good 2. Such complementarity is captured by assuming a simple multiplicative structure for the demand for good 2[22]

$$D_2(p_2, p_1) = D(p_1)\tilde{D}(p_2).$$

In this setting, the non-integrated structure stands ready to make a loss on the sales of good 1 if doing so sufficiently raises demand and profit on good 2.

The retailer's profit on good 2, conditional on selling good 1, writes then as follows

$$\tilde{\pi}(p_2, c) = (p_2 - c)\tilde{D}(p_2).$$

This profit is maximized at a monopoly price, independent of p_1, which is given by

$$\tilde{p}^m = \arg\max_{p_2} (p_2 - c)\tilde{D}(p_2).$$

Denoting by $\tilde{\pi}^m$ the corresponding monopoly profit, we may rewrite the retailer's overall downstream profit as follows

$$(p_1 - w_1 + \tilde{\pi}^m)D(p_1)(1 + \theta e) + \theta e z_1 D(p_1) - \psi(e). \tag{6.6}$$

COMPETITIVE MANUFACTURERS. As in our previous analysis, the wholesale contract $w_1 = c$ and $z_1 = 0$ maximizes the retailer's profit subject to the manufacturer's break-even constraint. The optimal retail price $p^m(\tilde{\pi}^m)$ and effort $e^m(\tilde{\pi}^m)$ are readily obtained from our previous analysis as follows

$$p^m(\tilde{\pi}^m) - c + \tilde{\pi}^m = -\frac{D(p^m(\tilde{\pi}^m))}{D'(p^m(\tilde{\pi}^m))}, \tag{6.7}$$

$$\psi'(e^m(\tilde{\pi}^m)) = \theta\pi^m(\tilde{\pi}^m), \tag{6.8}$$

where now $\pi^m(\tilde{\pi}^m) = \pi(p^m, c - \tilde{\pi}^m)$ stands for the base monopoly profit on good 1.

The comparison with the retail price and effort found in the single-good scenario (see (2.1) and (2.2)) is straightforward. As before, the retailer extracts all the profit from competitive manufacturers with a wholesale price equal to marginal cost. Then, conditionally on selling good 1, the retailer makes the monopoly profit on good 2. This profit acts as an implicit subsidy on the cost of producing good 1. This reduces the retail price, boosts the demand for and the profit earned on good 1, and raises in turn the promotional effort: $p^m(\tilde{\pi}^m) < p^m$, $\pi^m(\tilde{\pi}^m) > \pi^m$, and $e^m(\tilde{\pi}^m) > e^m$.

MONOPOLY MANUFACTURER. Consider first a scenario where there is no restriction on the wholesale price w_1 that can be charged by the manufacturer. The expression of the retailer's profits in (6.6) highlights that the retail price and effort level that are chosen by the retailer only depend on the *net* wholesale price

$$\tilde{w}_1 = w_1 - \tilde{\pi}^m,$$

which is the opportunity cost for the retailer of buying one extra unit of good 1. When not buying one extra unit of good 1, the retailer also foregoes the monopoly profit $\tilde{\pi}^m$

[22]For a model also featuring such a multiplicative demand structure, see Martimort, Pommey and Pouyet (2022) who develop a model of conditional sales. More complex demand structures could be entertained with no additional insight. The benefit of this multiplicative structure is that it allows us to import *mutatis mutandis* insights and techniques from our previous analysis.

made on good 2. This extra benefit can in fine be captured by the manufacturer whose perceived cost of producing good 1 is now

$$\tilde{c} = c - \tilde{\pi}^m.$$

From there, our previous analysis can be replicated mutatis mutandis. First, we need to modify Assumption 3 to account for this change of cost.

ASSUMPTION 5.

$$(1+\theta)\varphi(\tilde{p}(\tilde{\pi}^m)) < \psi(1),$$

where $\tilde{p}(\tilde{\pi}^m)$ *is now defined as* $\pi_p(\tilde{p}(\tilde{\pi}^m), \tilde{c}) = \varphi'(\tilde{p}(\tilde{\pi}^m))$.

Second, while the retailer's profit, when expressed in terms of the retail price and effort (p_1, e) implemented by the retailer, is kept unchanged as in (3.8), we observe that the manufacturer's profit accounts for the perceived cost of producing good 1 and can be written as follows

$$\Pi_M(p_1, e) = (\pi(p_1, \tilde{c}) - \varphi(p_1))(1 + \theta e). \tag{6.9}$$

The characterization of the optimal wholesale contract $(w^u(\tilde{\pi}^m), z^u(\tilde{\pi}^m))$ is thus readily obtained by applying Proposition 1 for the perceived cost $\tilde{c}$. In particular, a positive rebate for expanding demand on good 1 is again warranted. With respect to the single-good scenario, because the perceived cost of good 1 is now reduced, the manufacturer offers a lower wholesale price which induces in fine a lower retail price. A lower retail price increases the retailer's profit which boosts effort.

BAN ON BELOW-COST-PRICING. The analysis here also mimics our earlier findings. A ban on below-cost pricing imposes the condition

$$p_1 - w_1 \geq 0.$$

It in turn implies that the retailer's break-even condition writes now as

$$\Pi_R(p_1, e) \geq \tilde{\pi}^m D(p_1) + R(e). \tag{6.10}$$

The monopoly manufacturer now maximizes its profit $\Pi_M(p_1, e_1)$ as given in (6.9) subject to (6.10). Again, the retailer's participation constraint (6.10) is hardened and Assumption 5 suffices to ensure that this constraint is binding at the optimum. With respect to the single-good scenario, the main difference is that the rent that must be given up to the retailer depends directly both on the promotion effort (through the moral hazard rent $R(e)$ in (6.10)) and on the price of the loss leader (the term $\tilde{\pi}^m D(p_1)$ in (6.10)).

From the analysis of the single-good scenario performed in Section 4, we know that the comparison between price and effort levels with and without a ban on below-cost pricing is a priori ambiguous. That ambiguity remains with complementarity products. The new effect is that the price of the loss leader might be further increased in order to reduce the term $\tilde{\pi}^m D(p_1)$ in (6.10).

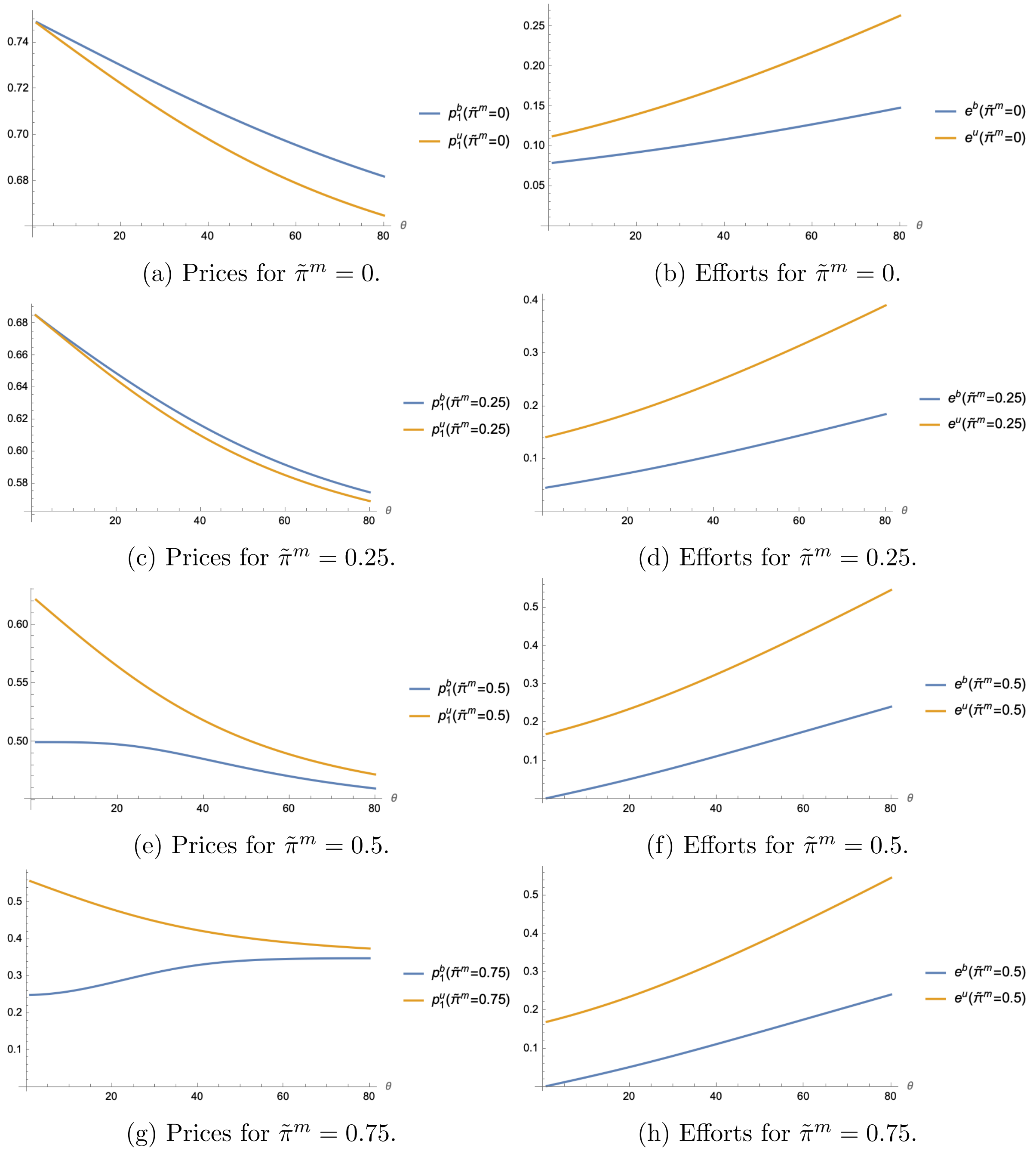
To illustrate, let us perform some numerical simulations, using $D(p) = 1 - p$, $c = 0$ and $\psi(e) = 5e^2$. There are two parameters of interests, θ and $\tilde{\pi}^m$. The profit on the complementary good 2, $\tilde{\pi}^m$, takes four values: 0, 0.25, 0.5 and 0.75. As before, θ

varies between 0.1 and 8 by increments of .1. We report in Figure 3 the corresponding simulations that help to understand the impact of a ban on below-cost pricing with complement goods.

When the profit on the complementary good, $\tilde{\pi}^m$, is zero, we recover the results of the single-good scenario. Regardless of whether below-cost pricing is permitted, the price decreases, and the efforts increases as the promotional effort becomes more effective (i.e., as θ increases). However, following a ban on below-cost pricing, the price increases, and the effort decreases.

As the profit from the complementary good, $\tilde{\pi}^m$, increases, the same comparative statics hold for the promotion effort. A closer examination of the graphs reveals that the difference between the effort under no ban and the effort under a ban on below-cost pricing also widens as $\tilde{\pi}^m$ increases. This is due to the participation constraint becoming more challenging to satisfy (see (6.10)), which indirectly necessitates a further downward distortion of the effort when a ban on below-cost pricing is enforced.

Turning on to the price of good 1, the comparative statics are qualitatively impacted by the presence of a complementary good. When $\tilde{\pi}^m$ is small, imposing a ban still results in a higher price for the loss leader. However, as $\tilde{\pi}^m$ increases, the price of the loss leader under a ban on below-cost pricing may fall below the price with no ban. This arises because constraint (6.10) is more difficult to satisfy. This is an interesting consequence of a ban on below-cost pricing: Such a ban may lead to a lower price for the loss leader when the product generates significant profits from complementary goods. In these cases, consumer surplus could increase following a ban on below-cost pricing, depending on how consumers value the price reduction on the loss leader relative to the reduction in promotional effort.



(a) Prices for $\tilde{\pi}^m = 0$. (b) Efforts for $\tilde{\pi}^m = 0$.

(c) Prices for $\tilde{\pi}^m = 0.25$. (d) Efforts for $\tilde{\pi}^m = 0.25$.

(e) Prices for $\tilde{\pi}^m = 0.5$. (f) Efforts for $\tilde{\pi}^m = 0.5$.

(g) Prices for $\tilde{\pi}^m = 0.75$. (h) Efforts for $\tilde{\pi}^m = 0.75$.

Figure 3: Price of the loss leader good and effort level with no ban (p_1^u and e^u) and with a ban on below-cost pricing (p_1^b and e^b) as function of θ and for different values of $\tilde{\pi}^m$.

7. Quantifying Damages Caused by Upstream Cartels

In 2014, the French Competition Authority condemned several manufacturers of cleansing and hygiene products for collusion over the years 2003 to 2006. These producers were sanctioned with a fine approaching €1B.[23] Some of these manufacturers had

[23]See "Décision n° 14-D-19 du 18 décembre 2014 relative à des pratiques mises en œuvre dans le secteur des produits d'entretien et des insecticides et dans le secteur des produits d'hygiène et de soins pour le corps," available at https://www.autoritedelaconcurrence.fr/fr/decision/relative-des-pratiques-mises-en-oeuvre-dans-le-secteur-des-produits-dentretien-et-des. Two sanctions have been pronounced: €345.2M for manufacturers of cleansing products, and €605.9M for manufactur-

already been sanctioned, in 2011, with a fine of €367.9M for colluding on the sales of consumer detergents, thereby confirming a decision taken by the European Commission.[24]

Collusion aimed not only to maintain artificially high retail prices for these products but also to restrict payments to retailers for their promotional activities. During that period, the *Loi Galland* was in effect, preventing retailers from selling below the net invoice price. This threshold could include any rebates agreed upon at the time of contracting between the retailer and the manufacturer, but it excluded any retrospective payments earned after the contracting date, such as promotional allowances. Furthermore, according to industry experts, fixed payments like slotting fees were negligible for these products. Although stylized, our model fits well with that case if one approximates collusive manufacturers as a monopoly. Our results thus provide some guidance on how to evaluate the damages suffered by retailers as a result of such collusion between manufacturers.

Absent collusion, the retailer appropriates the whole profit of the vertically integrated structure while, with collusion and a ban on below-cost pricing, he only appropriates the corresponding moral hazard rent. Formally, the retailer's loss from the collusion between manufacturers Δ is worth

$$\Delta \equiv \underbrace{(1+\theta e^m)\pi^m - \psi(e^m)}_{\substack{\text{Retailer's profit} \\ \text{with upstream competition}}} \quad - \underbrace{R(e^b),}_{\substack{\text{Moral hazard rent under manufacturers} \\ \text{collusion and no below-cost pricing}}}$$

which may be rewritten as follows (using Equation (2.2))

$$\Delta = \underbrace{\pi^m}_{\substack{\text{Monopoly} \\ \text{base profit}}} + \underbrace{R(e^m) - R(e^b).}_{\substack{\text{Difference in moral hazard rents} \\ \text{with and without collusion}}}$$

Damages are thus the sum of two terms. The first one is the monopoly level of the base profit that accrues to the retailer if, absent collusion, no effort was undertaken. The second term is the incremental value of the moral hazard rent as collusion would be deterred and effort would change from e^b to e^m, which may be positive or negative as illustrated in Figure 2.

Considering the specification $D(p) = (a - bp)^{\frac{1}{\delta}}$ and $\psi(e) = \frac{\mu}{2}e^2$, it is remarkable that damages can be expressed only in terms of values of the demand function before and after collusion has been deterred, namely $D(p^b)$ and $D(p^m)$. Indeed, the percentage of after-collusion profits that should be paid in terms of damages is worth

$$\frac{\Delta}{\pi^m} = 1 + \frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta} - (1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)} \tag{7.1}$$

where the Lagrange multiplier of (4.2) λ^b is given by $(1+\delta(1-\lambda^b))^{\frac{1}{\delta}} = \frac{D(p^m)}{D(p^b)}$. In other words, the evaluation of damages, a task often viewed as informationally demanding,

ers of hygiene products.

[24]See "Décision n° 11-D-17 du 8 décembre 2011 relative à des pratiques mises en œuvre dans le secteur des lessives," available at https://www.autoritedelaconcurrence.fr/fr/decision/relative-des-pratiques-mises-en-oeuvre-dans-le-secteur-des-lessives and "Commission Decision of 13.4.2011 relating to a proceeding under Article 101 of the Treaty on the Functioning of the European Union and Article 53 of the EEA Agreement (COMP/39579 – Consumer Detergents)," available at https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:C:2011:193:0014:0016:EN:PDF.

only requires the pre- and post-collusion demand levels if one is ready to adopt the above functional forms.

Adopting the same linear-quadratic specification as in Section 3, we can quantify the damages suffered by the retailer from upstream collusion between manufacturers using (7.1). These damages are obviously always strictly positive, but, perhaps less intuitively, non-monotonic with the demand shock θ. This comes from the fact that, first, damages depend on the difference $R(e^m) - R(e^b)$ as shown above and, second, that the difference $e^m - e^b$ is positive for low demand shocks but negative for large ones.

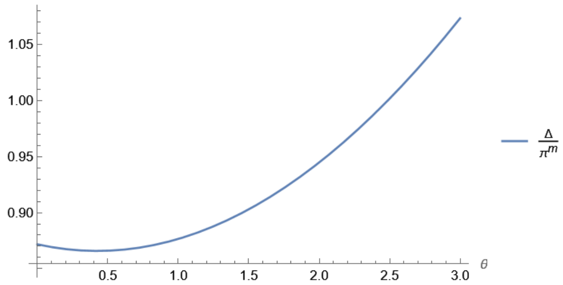


Figure 4: Damages $\frac{\Delta}{\pi^m}$ as function of the demand shock θ (with $D(p) = 1 - p$, $\psi(e) = \frac{3}{2}e^2$, $c = 0$).

8. Conclusion

Our analysis sheds light on the economics of promotional allowances, an oft-neglected component of the relationship between manufacturers and retailers. These allowances are used to provide incentives to retailers to boost the demand of manufacturers' products. Our analysis highlights how wholesale prices and promotional allowances are used in conjunction to foster that goal. We show in particular that manufacturers are willing to set high wholesale prices together with large rebates; this leads to losses for manufacturers whenever their promotion efforts are unsuccessful, and to large gains when they manage to boost demand.

Our analysis could be extended in various ways. First, we have taken for granted that fixed fees cannot be used in the vertical relationship. A possible avenue of research consists in studying situations where slotting fees can be used but do not allow to perfectly capture the retailers' profits, maybe because retailers have private information on demand parameters. In such environments, there exists an interesting interplay between incentivizing retailers to exert promotion efforts on the one hand and to reveal their private information on the other hand.

Second, we did not considered imperfect competition between manufacturers and the possibility for retailers to threaten a manufacturer not to sell her product. Such a threat might limit the manufacturer's ability to use stick-and-carrot strategies to incentivize the retailer and lead to further distortions.

These extensions are left for future research.

References

Allain, M-L. and C. Chambolle, 2011. "Anti-Competitive Effects of Resale-Below-Cost Laws," *International Journal of Industrial Organization*, 29: 373-385.

Allen, B., P. Farris, D. Mills and R. Sack, 2011. "Vendor Incentives: Out of the Shadows and into the Sunlight," *The CPA Journal*, June.

Ambrus, A. and J. Weinstein, 2008. "Price Dispersion and Loss Leaders," *Theoretical Economics*, 3: 525-537.

Arrow, K., 1975. "Vertical Integration and Communication." *The Bell Journal of Economics,* 173-183.

Beard, R. and M. Stern, 2008. "Continuous Cross Subsidies and Quantity Restrictions," *Journal of Industrial Economics*, 56: 840-861.

Biscourp, P., X. Boutin and T. Vergé, 2013. "The Effects of Retail Regulations on Prices: Evidence from the *Loi Galland*," *Economic Journal*, 123: 1279-1312.

Bliss, C., 1988. "A Theory of Retail Pricing," *Journal of Industrial Economics*, 36: 375-391.

Bulow, J. and P. Pfleiderer, 1983. "A Note on the Effect of Cost Changes on Prices," *Journal of Political Economy*, 91: 182-185.

Calvani, T, 2001. "Predatory Pricing and State Below-Cost Sales Statutes in the United States: An Analysis," Report for the Competition Bureau, Canada, available at http://www.competitionbureau.gc.ca/eic/site/cb-bc.nsf/eng/01292.html

Chen, Z., and P. Rey, 2012. "Loss Leading as an Exploitative Practice," *The American Economic Review*, 102: 3462-3482.

Chen, Z. and P. Rey, 2013. "Competitive Cross-Subsidization," TSE Working Papers 13-450, Toulouse School of Economics (TSE), revised Dec 2016.

Chu, W., 1992. "Demand Signaling and Screening in Channels of Distribution," *Marketing Science*, 11: 327-347

Cleeren, K., F. Verboven, M. Dekimpe and K. Gielens, 2010. "Intra- and Inter-Format Competition among Discounters and Supermarkets," *Marketing Science*, 29: 456-473.

Dertwinkel-Kalt, M. and C. Wey, 2024. "Resale Price Maintenance in a Successive Monopoly Model," *Journal of Industrial Economics*, 72: 729-761.

Dixit, A., 1983. "Vertical Integration in a Monopolistically Competitive Industry," *International Journal of Industrial Organization*, 1: 63-78.

Ellison, G., 2005. "A Model of Add-on Pricing," *The Quarterly Journal of Economics*, 120: 585-637.

Foros, Ø., H. Kind and J. Sand, 2009. "Slotting Allowances and Manufacturers' Retail Sales Effort," *Southern Economic Journal*, 76: 226-282.

Hunold, M. and J. Muthers, 2017. "Resale Price Maintenance and Manufacturer Competition for Retail Services," *The RAND Journal of Economics*, 48: 3-23.

Inderst, R. and M. Obradovits, 2024. "Price Promotions as a Threat to Brands," *Journal of Economics & Management Strategy*, 33: 53-77.

Irish Competition Agency, 2005. "Submission to the Minister for Enterprise, Trade and Employment on the Groceries Order," available at http://ccpc.ie/sites/default/files/s_05_006%20Groceries%20Order.pdf.

Kastl, J., D. Martimort and S. Piccolo, 2011. "When Should Manufacturers Want Fair Trade? New Insights from Asymmetric Information and Non-Market Externalities when Supply Chains Compete," *Journal of Economics and Management Strategy*, 20: 648-678.

Keirsbilck, B., 2012. "Does EU Economics Law Preclude National Prohibitions of Sales Below Cost?," *Erasmus Law Review*, 5: 253-266.

Krishnan, H. and R. Winter, 2007. "Vertical Control of Price and Inventory," *The American Economic Review*, 97: 1840-1857.

Kuksov, D. and A. Pazgal, 2007. "The Effects of Costs and Competition on Slotting Allowances," *Marketing Science*, 26: 259-267.

Laffont, J.-J. and D. Martimort, 2002. *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press.

Laffont, J.-J. and J. Tirole, J., 1993. *A Theory of Incentives in Procurement and Regulation.* MIT press.

Lariviere, M. and V. Padmanabhan, 1997. "Slotting Allowances and New Product Introductions," *Marketing Science*, 16(2): 112-128.

Lømo, T. L. and S. Ulsaker, 2016. "Promotional Allowances," University of Bergen, Department of Economics, Working Papers in Economics 08/16.

Martimort, D. and S. Piccolo, 2007. "Resale Price Maintenance Under Asymmetric Information," *International Journal of Industrial Organization*, 25(2): 315-339.

Martimort, D., G. Pommey and J. Pouyet, 2022. "Optimal Regulation Design of Airports: Investment Incentives and Impact of Commercial Services," *Transportation Research Part B: Methodological,* 164: 25-44.

Marx, L. and G. Shaffer, 2007. "Upfront Payments and Exclusion in Downstream Markets," *The RAND Journal of Economics*, 38: 823-843.

Marx, L. and G. Shaffer, 2010. "Slotting Allowances and Scarce Shelf Space," *Journal of Economics & Management Strategy*, 19: 575-603.

Mathewson, F. and R. Winter, 1984. "An Economic Theory of Vertical Restraints." *The RAND Journal of Economics*, 15: 27-38.

Miklós-Thal, J., P. Rey and T. Vergé, 2011. "Buyer Power and Intrabrand Coordination," *Journal of the European Economic Association*, 9(4): 721-741.

Piccolo, S. and J. Miklós-Thal, 2012. "Colluding Through Suppliers," *The RAND Journal of Economics*, 43: 492-513.

Rey, P. and T. Vergé, 2008. "Economics of Vertical Restraints," in *Handbook of Antitrust Economics*, Paolo Buccirossi (ed.), The MIT Press.

Rey, P. and J. Tirole, 1986. "The Logic of Vertical Restraints," *The American Economic Review*, 76: 921-939.

Riordan, M., 1990. "What Is Vertical Integration?," in *The Firm as a Nexus of Treaties,* O. Williamson, M. Aoki, B. Gustafsson (eds.), Sage Publishers.

Shaffer, G., 1991. "Slotting Allowances and Resale Price Maintenance: A Comparison of Facilitating Practices," *The RAND Journal of Economics*, 22: 120-135.

Shubik, M. and R. Levitan, 1980. *Market Structure and Behavior*, Harvard University Press.

Spengler J., 1950. "Vertical Integration and Antitrust Policy," *Journal of Political Economy*, 58: 347-352.

Telser, L., 1960. "Why Should Manufacturer Want Fair Trade," *Journal of Law and Economics*, 3: 86-105.

Winter, R., 1993. "Vertical Control and Price versus Non-price Competition," *The Quarterly Journal of Economics*, 108: 61-76.

APPENDIX

PROOF OF PROPOSITIONS 1 AND 2. The monopoly manufacturer's optimization problem can be written as follows:

$$\max_{(p,e)} \Pi_M(p,e) \equiv (\pi(p,c) - \varphi(p))\,(1+\theta e) \text{ subject to (3.9)}.$$

Let denote by λ^u the non-negative Lagrange multiplier of the retailer's break-even condition (3.9). The corresponding Lagrangian for this problem writes as follows:

$$\mathcal{L}(p,e,\lambda^u) = (\pi(p,c) - \varphi(p))\,(1+\theta e) + \lambda^u\,(\varphi(p)(1+\theta e) - \psi(e))\,.$$

The Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to p and e (for an interior solution) write respectively as

$$\pi_p(p^u,c) - (1-\lambda^u)\varphi'(p^u) = 0, \tag{A.1}$$

$$\theta\,(\pi(p^u,c) - (1-\lambda^u)\varphi(p^u)) = \lambda^u\psi'(e^u), \tag{A.2}$$

while the complementary slackness condition is

$$\lambda^u\,((1+\theta e^u)\varphi(p^u) - \psi(e^u)) = 0. \tag{A.3}$$

We now establish several properties of the optimum.

CLAIM 1. $\lambda^u > 0$ *and thus* (3.9) *is binding:*

$$(1+\theta e^u)\varphi(p^u) = \psi(e^u). \tag{A.4}$$

Proof. Suppose instead that $\lambda^u = 0$. Then (A.1) leads to $p^u = \tilde{p}$. Because $\pi(\tilde{p},c) > \varphi(\tilde{p})$, the optimal effort choice that the manufacturer would like to implement would be $e^u = 1$. The retailer's profit would then be equal to $(1+\theta)\varphi(\tilde{p}) - \psi(1)$, which is strictly negative when Assumption 3 holds; a contradiction. Therefore, at the optimum, it must be $\lambda^u > 0$. Condition (A.4) then follows from (A.3). □

CLAIM 2. $\lambda^u < 1$.

Proof. Suppose $\lambda^u = 1$. Then, from (A.1) and (A.2), we obtain $p^u = p^m$ and $e^u = e^m$. Observe now that, by definition of the monopoly price p^m, $\varphi(p^m) = \pi^m$, and e^m is such that $\pi^m = \psi'(e^m)/\theta$. Thus,

$$(1+\theta e^m)\varphi(p^m) - \psi(e^m) = (1+\theta e^m)\pi^m - \psi(e^m) = \pi^m + R(e^m) > 0. \tag{A.5}$$

(A.5) shows the retailer's participation constraint is strictly satisfied; a contradiction with Claim 1. Hence, by continuity, it must be that $\lambda^u \in (0,1)$. □

Claim 2 together with Assumption 2 implies that the Lagrangian is quasi-concave in p, and thus in (p,e) given the convexity of ψ and Claim 1.

CLAIM 3. $\tilde{p} > p^u > p^m$.

Proof. Together with Claim 2, (A.1) implies

$$\pi_p(p^u, c) = (1 - \lambda^u)\varphi'(p^u) < 0.$$

Because $\pi(p, c)$ is quasi-concave in p when Assumption 1 holds, we have

$$p^u > p^m.$$

Similarly, we have

$$\pi_p(p^u, c) - \varphi'(p^u) = -\lambda^u \varphi'(p^u) > 0.$$

Because $\pi(p, c) - \varphi(p)$ is also quasi-concave in p when Assumption 2 holds, we have $p^u < \tilde{p}$. □

Claim 4.

$$z^u > 0 \tag{A.6}$$

and

$$p^u - w^u = -R(e^u) < 0. \tag{A.7}$$

Proof. Let us rewrite the first-order condition (3.3) for the optimal solution (p^u, e^u) in a more explicit manner as

$$\theta(p^u - w^u + z^u)D(p^u) = \psi'(e^u). \tag{A.8}$$

From (A.8), we obtain:

$$p^u - w^u = \frac{\psi'(e^u)}{\theta D(p^u)} - z^u.$$

Inserting into (3.4) for the optimal solution (p^u, e^u) yields:

$$z^u \frac{D(p^u)}{1 + \theta e^u} = \frac{\psi'(e^u)}{\theta} - \varphi(p^u). \tag{A.9}$$

Using (A.1) and (A.2), we get:

$$\psi'(e^u) = \theta f(p^u) \tag{A.10}$$

where

$$f(p) = \pi(p, c) + \frac{\pi_p(p, c)}{\varphi'(p) - \pi_p(p, c)}(\pi(p, c) - \varphi(p)).$$

Using (A.10), (A.9) can be rewritten as follows:

$$z^u \frac{D(p^u)}{1 + \theta e^u} = \frac{\varphi'(p^u)(\pi(p^u, c) - \varphi(p^u))}{\varphi'(p^u) - \pi_p(p^u, c)}.$$

From Claim 1 and Equation (A.1), we get $\varphi'(p^u) - \pi_p(p^u, c) < 0$. Therefore, we have also $\varphi'(p^u) < \pi_p(p^u, c) < 0$ since $p^u > p^m$. Moreover, notice that $\pi(p, c) > \varphi(p) \Leftrightarrow p - c > -\frac{D(p)}{D'(p)} \Leftrightarrow p > p^m$. Since $p^u > p^m$, we have $\pi(p^u, c) > \varphi(p^u)$. Hence, the right-hand side of (A.10) is positive and (A.6) holds.

Inserting now (A.9) into (3.4), we obtain

$$(p^u - w^u)D(p^u) = (1 + \theta e^u)\varphi(p^u) - e^u \psi'(e^u).$$

Using the binding participation constraint (3.9) and $R(e) = e\psi'(e) - \psi(e)$, we obtain (A.7). □

CLAIM 5.

$$e^u > e^m. \tag{A.11}$$

Proof. Observe that

$$\psi'(e^m) = \theta f(p^m). \tag{A.12}$$

Taken together with (A.10), Condition (A.11) will be proved (thanks for the convexity of ψ) if $f(p^u) > f(p^m)$. We thus compute

$$f'(p) = (\pi(p,c) - \varphi(p))\frac{d}{dp}\left(\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)}\right).$$

We may write

$$\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)} = \frac{p + \frac{D(p)}{D'(p)} - c}{-\left(p + \frac{D(p)}{D'(p)} - \frac{\varphi'(p)}{D'(p)} - c\right)}.$$

Assumption 1 (resp. Assumption 2) ensures that the numerator (resp. denominator) is non-decreasing (resp. non-increasing). Hence, $f'(p) \geq 0$ whenever $\pi(p,c) - \varphi(p) \geq 0$ which holds for $p \geq p^m$ (with a strict inequality for $p > p^m$). Since $p^u > p^m$, we then obtain $f(p^u) > f(p^m)$ and thus $e^u > e^m$. □

This concludes the proof of Propositions 1 and 2. □

PROOF OF PROPOSITION 3. With a ban on below-cost pricing, the manufacturer's problem is to maximize $\Pi_M(p,e)$ subject to (4.2). We rewrite (4.2) as follows:

$$(1 + \theta e)\varphi(p) - e\psi'(e) \geq 0, \tag{A.13}$$

so that the manufacturer's problem may be expressed in a more compact form as follows:

$$\max_{(p,e)} (\pi(p,c) - \varphi(p))(1 + \theta e) \text{ subject to (A.13)}.$$

Denote by λ^b the Lagrange multiplier for constraint (A.13). The Lagrangian writes as follows:

$$\mathcal{L}(p,e,\lambda^b) = (\pi(p,c) - \varphi(p))(1 + \theta e) + \lambda^b\left((1 + \theta e)\varphi(p) - e\psi'(e)\right).$$

Assuming concavity of this Lagrangian in (p,e) and optimizing with respect to p and e respectively yields the following Karush-Khün-Tucker first-order necessary conditions:

$$\pi_p(p^b) - (1 - \lambda^b)\varphi'(p^b) = 0, \tag{A.14}$$

$$\theta\left(\pi(p^b) - (1 - \lambda^b)\varphi(p^b)\right) = \lambda^b\left(\psi'(e^b) + e^b\psi''(e^b)\right). \tag{A.15}$$

We now prove that (A.13) is binding. We proceed by contradiction. Suppose not, that is, $\lambda^b = 0$. Then, we would have $p^b = \tilde{p}$ and $e^b = 1$. These values do not satisfy the break-even condition (3.9) and, *a fortiori*, the more demanding constraint (4.2) when Assumption 5 holds. Therefore, (4.2), or equivalently (4.1), is binding at the optimum.

The last item in the proposition follows from observing that, when $e^b > 0$, we have $(1 + \theta e^b)\varphi(p^b) = e^b\psi'(e^b) > \psi(e^b)$. □

PROOF OF PROPOSITION 4. With $D(p) = (a - bp)^{\frac{1}{\delta}}$, (3.5) and (3.6) can respectively be expressed as

$$w = p + e^2\mu(a - bp)^{-\frac{1}{\delta}} - \frac{(a - bp)\delta(1 + \theta e)}{b}, \tag{A.16}$$

$$z = (1 + \theta e)\left(\frac{e\mu(a - bp)^{-\frac{1}{\delta}}}{\theta} - \frac{(a - bp)\delta}{b}\right). \tag{A.17}$$

From this, we can express the manufacturer's and the retailer's profit as function of (p, e) respectively as follows:

$$\Pi_M(p, e) = \frac{(1 + \theta e)(a - bp)^{\frac{1}{\delta}}(b(-c + \delta p + p) - a\delta)}{b}, \tag{A.18}$$

$$\Pi_R(p, e) = \frac{\delta(1 + \theta e)(a - bp)^{\frac{1}{\delta}+1}}{b} - \frac{\mu}{2}e^2. \tag{A.19}$$

Optimizing the Lagrangian with respect to p and e and solving yields the expressions of the price and the effort level as function of the multiplier λ^u, which we denote by $P^u(\lambda^u)$ and $E^u(\lambda^u)$:

$$P^u(\lambda^u) = \frac{a}{b} - \frac{a - bc}{b(1 + \delta)(1 + \delta(1 - \lambda^u))}, \tag{A.20}$$

$$E^u(\lambda^u) = \frac{\delta\theta}{b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^u}\left(\frac{1}{1 + \delta(1 - \lambda^u)}\right)^{\frac{1}{\delta}}. \tag{A.21}$$

Plugging these expressions into the binding retailer's participation constraint, we obtain that $\Pi_R(P^u(\lambda^u), E^u(\lambda^u)) = 0$ amounts to

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = \frac{(\lambda^u)^2(1 + \delta(1 - \lambda^u))^{\frac{1}{\delta}}}{1 + \delta - \lambda^u(2 + \delta)}. \tag{A.22}$$

The left-hand side in (A.22) is strictly positive. It can easily be shown that, for $\lambda^u \in [0, \frac{1+\delta}{2+\delta})$, the right-hand side is strictly increasing in λ^u and takes values in $[0, +\infty)$. Therefore, there exists a unique $\lambda^u \in [0, \frac{1+\delta}{2+\delta})$ which satisfies (A.22).

Consider now that there is a ban on below-cost-pricing. (w, z) as functions of (p, e) are still given by (A.16) and (A.17). Profits of the retailer and the manufacturer are still given by (A.18) and (A.19).

The Karush-Khün-Tucker first-order condition with respect to price is the same as in the case with no ban on below-cost-pricing. Therefore,

$$P^b(\lambda^b) = P^u(\lambda^b), \tag{A.23}$$

where P^u is given by (A.20). The Karush-Khün-Tucker first-order condition with respect to effort leads to $E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b)$ where E^u is given by (A.21). Replacing in the participation constraint (A.13), which must hold as an equality at the optimum, characterizes the Lagrange multiplier λ^b:

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = 2\frac{(\lambda^b)^2(1 + \delta(1 - \lambda^b))^{\frac{1}{\delta}}}{1 + \delta(1 - \lambda^b) - 2\lambda^b}. \tag{A.24}$$

The left-hand side in (A.24) is strictly positive. The right-hand side in (A.24) is, for $\lambda^b \in [0, \frac{1+\delta}{4+\delta})$, strictly increasing and takes values in $[0, +\infty)$. Hence, there exists a unique $\lambda^b \in (0, \frac{1+\delta}{4+\delta})$ such that (A.24) holds.

Comparing (A.22) and (A.24), it comes immediately that $\lambda^u > \lambda^b$. From (A.20), we obtain that $(P^u)'(\lambda) < 0$. Therefore, we have that: $p^b = P^b(\lambda^b) = P^u(\lambda^b) > P^u(\lambda^u) = p^u$.

It remains to compare effort levels. Remind that $e^u = E^u(\lambda^u)$ and $e^b = \frac{1}{2}E^u(\lambda^b)$, where $E^u(\cdot)$ is defined by (A.21). Since λ^u is implicitly defined by (A.22), we obtain the following simplification:

$$E^u(\lambda^u) = \frac{1}{\theta}\frac{2\lambda^u}{(1+\delta)-(2+\delta)\lambda^u}. \tag{A.25}$$

Similarly, since λ^b is implicitly defined by (A.24), we obtain:

$$E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b) = \frac{1}{\theta}\frac{2\lambda^b}{(1+\delta)-(2+\delta)\lambda^b}. \tag{A.26}$$

Since $\frac{2x}{(1+\delta)-(2+\delta)x}$ is strictly increasing in x and $\lambda^u > \lambda^b$, we have $e^u > e^b$. □

PROOF OF PROPOSITION 5. The retailer's profit is given by $(p-w)D(p)(1+\theta e)-\psi(e)$, which is strictly concave in e. Under a RPM agreement, given the wholesale contract (w, p), the retailer chooses an effort level such that

$$\theta(p-w)D(p) = \psi'(e). \tag{A.27}$$

The manufacturer's profit is given by $(w-c)D(p)(1+\theta e)$. Under a RPM agreement, the manufacturer's problem can be written as follows:

$$\max_{(p,w,e)} (w-c)D(p)(1+\theta e)$$

subject to the incentive contraint (A.27) and the retailer's participation constraint

$$(p-w)D(p)(1+\theta e)-\psi(e) \geq 0. \tag{A.28}$$

Denote by μ and λ the multipliers associated to (A.27) and (A.28) respectively. The Karush-Kühn-Tucker first-order necessary conditions for optimality write as follows

$$(w-c)(1+\theta e)D'(p) + (\lambda(1+\theta e)+\mu\theta)\big(D(p)+(p-w)D'(p)\big) = 0, \tag{A.29}$$

$$(1+\theta e)(1-\lambda)-\mu\theta = 0, \tag{A.30}$$

$$\theta(w-c)D(p)-\mu\psi''(e) = 0. \tag{A.31}$$

Using (A.29) and (A.30) leads to $D(p)+(p-c)D'(p) = 0$, or $p = p^m$. Two cases must then be considered depending on whether the participation constraint binds at the optimum.

Assume that the participation constraint is binding at the optimum, or $\lambda > 0$. Using (A.27), $(p-w)D(p)(1+\theta e) = \psi(e)$ rewrites as $\psi'(e)+\theta(e\psi'(e)-\psi(e)) = 0$, or $e = 0$. We also immediately deduce that $w = p^m$ and $\lambda = 1-\frac{\theta^2\pi^m}{\psi''(0)}$. Therefore, this case arises when $1 > \frac{\theta^2\pi^m}{\psi''(0)}$.

If the participation constraint is not binding at the optimum, then $\lambda = 0$. The promotion effort level is given by $\theta\pi^m = \psi'(e)+\frac{(1+\theta e)\psi''(e)}{\theta}$, which implies that $0 < e < e^m$. The optimal wholesale price is then given by $p^m - w = \frac{\psi'(e)}{\theta D(p^m)}$. The retailer makes a strictly positive profit given by $\frac{\psi'(e)}{\theta}+R(e)$.

COMPARISON OF PROFITS. We now compare profits assuming that $D(p) = a-bp$ and $\psi(e) = \frac{\mu}{2}e^2$. Let $\pi^m = \max_p (p-c)D(p) = \frac{1}{b}\big(\frac{a-bc}{2}\big)^2$. With a RPM agreement:

- If $\mu > \theta^2\pi^m$, then $\Pi_R^{rpm} = 0$ and $\Pi_M^{rpm} = \pi^m$.

- If $\mu \leq \theta^2\pi^m$, then $\Pi_R^{rpm} = \frac{1}{8\mu\theta^2}(\theta^2\pi^m - \mu)(\theta^2\pi^m + 3\mu)$ and $\Pi_M^{rpm} = \frac{(\theta^2\pi^m+\mu)^2}{4\mu\theta^2}$

No Ban on Below-Cost Pricing. Using the results derived in the Proof of Proposition 4, we obtain that with rebates and no ban on below-cost pricing, profits are given by

$$\Pi_R^u = 0,$$
$$\Pi_M^u = \frac{2(1-\lambda^u)\pi^m\left(\lambda^u\mu(2-\lambda^u)+\theta^2\pi^m\right)}{\lambda^u\mu(2-\lambda^u)^3},$$

where the multiplier $\lambda^u \in [0, \frac{2}{3})$ satisfies the following condition

$$\frac{\theta^2\pi^m}{\mu} = \frac{2(\lambda^u)^2(2-\lambda^u)}{2-3\lambda^u} \equiv h(\lambda^u).$$

$h(\lambda^u)$ is strictly increasing in λ^u over the relevant range. Therefore, $\lambda^u = h^{-1}(\frac{\theta^2\pi^m}{\mu})$ increases with $\frac{\theta^2\pi^m}{\mu}$. Moreover, $h^{-1}(1) \approx .454$.

Assume that $\frac{\theta^2\pi^m}{\mu} < 1$. Simple manipulations lead to

$$\Pi_M^u - \Pi_M^{rpm} \geq 0 \Leftrightarrow \frac{-2+3\lambda^u(2-\lambda^u)}{(2-\lambda^u)(2-3\lambda^u)}\pi^m \geq 0,$$

or $\lambda^u \geq 1 - 1/\sqrt{3} \approx .422$. Therefore, for $\frac{\theta^2\pi^m}{\mu} \leq h(.422) \approx .7698$, we have $\Pi_M^u \leq \Pi_M^{rpm}$ and for $1 > \frac{\theta^2\pi^m}{\mu} \geq .7698$, we have $\Pi_M^u \geq \Pi_M^{rpm}$.

Assume now that $\frac{\theta^2\pi^m}{\mu} \geq 1$ or $\lambda^u \geq .454$. Simple manipulations lead to

$$\Pi_M^u - \Pi_M^{rpm} \geq 0 \Leftrightarrow \frac{-4(\lambda^u)^6 + 16(\lambda^u)^5 - 28(\lambda^u)^4 + 16(\lambda^u)^3 - 9(\lambda^u)^2 + 12\lambda^u - 4}{8(2-\lambda^u)(\lambda^u)^2(2-3\lambda^u)}\pi^m \geq 0,$$

or $\lambda^u \geq .426$ (numerical approximation). Therefore, we always have $\Pi_M^u > \Pi_M^{rpm}$ in this case.

Ban on Below-Cost Pricing. Using again the results derived in the Proof of Proposition 4, we obtain that with rebates and a ban on below-cost pricing, profits are given by the following conditions

$$\Pi_R^b = \frac{\pi^m\left(8\mu(2-\lambda^b)(\lambda^b)^2 - \theta^2\pi^m(2-5\lambda^b)\right)}{8\mu(\lambda^b)^2(2-\lambda^b)^3},$$
$$\Pi_M^b = \frac{(1-\lambda^b)\pi^m\left(2\lambda^b\mu(2-\lambda^b)+\theta^2\pi^m\right)}{\lambda^b\mu(2-\lambda^b)^3},$$

where the multiplier $\lambda^b \in [0, \frac{2}{5})$ satisfies the following condition

$$\frac{\theta^2\pi^m}{\mu} = \frac{4(\lambda^b)^2(2-\lambda^b)}{2-3\lambda^b} = 2h(\lambda^b).$$

Assume that $\frac{\theta^2\pi^m}{\mu} < 1$. Simple manipulations, and using the fact that $\lambda^b \in [0, 2/5)$, lead to

$$\Pi_R^b - \Pi_R^{rpm} = \frac{\pi^m}{2(2-\lambda^b)(2-3\lambda^b)} \geq 0,$$
$$\Pi_M^b - \Pi_M^{rpm} = \left(\frac{2(1-\lambda^b)}{(2-\lambda^b)(2-3\lambda^b)} - 1\right)\pi^m \leq 0.$$

Assume that $\frac{\theta^2\pi^m}{\mu} \geq 1$. Simple manipulations, and using the fact that $\lambda^b \in [0, 2/5)$, lead to

$$\Pi_R^u - \Pi_R^{rpm} = \frac{-16(\lambda^b)^6 + 64(\lambda^b)^5 - 88(\lambda^b)^4 + 64(\lambda^b)^3 + 11(\lambda^b)^2 - 36\lambda^b + 12}{32(2-\lambda^b)(2-3\lambda^b)(\lambda^b)^2}\pi^m \geq 0.$$
$$\Pi_M^b - \Pi_M^{rpm} = -\frac{(1-2\lambda^b)^2(4(\lambda^b)^4 - 12(\lambda^b)^3 + 9(\lambda^b)^2 + 4\lambda^b + 4)}{16(2-\lambda^b)(2-3\lambda^b)(\lambda^b)^2}\pi^m \leq 0.$$

With a ban on below-cost pricing, the manufacturer always prefers a RPM agreement whereas the retailer favors incentive rebates. □

PROOF OF PROPOSITIONS 6 AND 7. Let denote by $\pi_1(p_1,p_2) = (p-c)D_1(p_1,p_2)$ and $\varphi(p_1,p_2) = \frac{(D(p_1,p_2))^2}{-\frac{\partial D}{\partial p_1}(p_1,p_2)}$. Let us assume that, for all relevant (p_1,p_2), $\frac{\partial^2 \pi}{\partial p_1^2}(p_1,p_2) < 0$ and $\frac{\partial^2 \pi}{\partial p_1 \partial p_2}(p_1,p_2) > 0$. This implies that the best response $P_2(p_1)$ defined in (6.2) is upward-sloping. Let us further assume that that best response has a slope smaller than 1. Observe that $\varphi_{p_1}(p_1,p_2) \leq 0$, which is a familiar condition from our analysis of the single-product scenario, follows from assuming concavity of the demand function in its own price, that is, $\frac{\partial^2 D}{\partial p_1^2}(p_1,p_2) \leq 0$. As is the main analysis, we shall assume that $\frac{\partial^2 \varphi}{\partial p_1^2}(p_1,p_2) \leq 0$, and we further impose that $\frac{\partial^2 \varphi}{\partial p_1 \partial p_2}(p_1,p_2) \geq 0$; these assumptions are satisfied with the linear demands system that we shall use later on.

No Ban on Below-Cost-Pricing. With no ban on below-cost-pricing, the manufacturer maximizes $\Pi_M(e_1,p_1,p_2) = (1+\theta e_1)(\pi_1(p_1,p_2) - \varphi(p_1,p_2))$ with respect to (e_1,p_1) and subject to (6.3). The Lagrangian associated to this problem is $(1+\theta e_1)[\pi_1(p_1,p_2) - \varphi(p_1,p_2)] + \lambda_1^u[(1+\theta e_1)\varphi(p_1,p_2) - \psi(e_1)]$. Optimizing with respect to p_1 leads to the following:

$$\pi_{p_1}(p_1,p_2) - (1-\lambda_1^u)\varphi_{p_1}(p_1,p_2) = 0.$$

The solution of that equation defines the best response in price $P_1^u(p_2)$. It is straightforward to show that, in equilibrium, $\lambda_1^u \in (0,1)$. Under our assumptions on $\varphi(\cdot)$, the best response $P_1^u(p_2)$ remains upward-sloping and lies above $P_1(p_2)$. This allows to prove Proposition 6.

Ban on Below-Cost-Pricing. Consider now the case of a ban on below-cost-pricing. The participation constraint of the retailer is now given by (6.5). The Lagrangian associated to this problem is $(1+\theta e_1)[\pi_1(p_1,p_2) - \varphi(p_1,p_2)] + \lambda_1^b[(1+\theta e_1)\varphi(p_1,p_2) - \psi(e_1) - R(e_1)]$. Optimizing with respect to p_1 leads to the following:

$$\pi_{p_1}(p_1,p_2) - (1-\lambda_1^b)\varphi_{p_1}(p_1,p_2) = 0.$$

The solution of that equation defines the best response in price $P_1^b(p_2)$. Assuming that $\lambda_1^b < \lambda_1^u$, the best response $P_1^b(p_2)$ lies above $P_1^u(p_2)$. This allows to prove the first item in Proposition 7.

Linear-Quadratic Specification. Let us now consider the following linear-quadratic specification of the model. Demand functions are given by $D_i(p_i,p_j) = \frac{1}{b^2-\gamma^2}[b(a-p_i) - \gamma(a-p_j)]$ for $i \neq j \in \{1,2\}$, with $b > \gamma \geq 0$. The cost of effort is given by $\psi(e_1) = \frac{\mu}{2}e_1^2$.

For a given p_2, the optimal price, effort level and Lagrange multiplier are given by

$$P_1^u(p_2, \lambda_1^u) = \frac{a(b-\gamma)(3-2\lambda_1^u) + bc + \gamma p_2(3-2\lambda_1^u)}{2b(2-\lambda_1^u)}, \tag{A.32}$$

$$E_1^u(\lambda) = \frac{2\lambda_1^u}{\theta(2-3\lambda_1^u)}, \tag{A.33}$$

$$\frac{(b(a-c)-\gamma(a-p_2))^2\theta^2}{8\mu b(b^2-\gamma^2)} = \frac{(2-\lambda_1^u)(\lambda_1^u)^2}{2-3\lambda_1^u}. \tag{A.34}$$

The best response of the integrated retailer is given by:

$$P_2(p_1) = \frac{1}{2b}(b(a+c)-\gamma(a-p_1)). \tag{A.35}$$

Combining (A.32) and (A.35) allows to obtain prices as function of the multiplier:

$$P_1(\lambda_1^u) = \frac{a(3-2\lambda_1^u)(b-\gamma)(2b+\gamma) + bc(\gamma(3-2\lambda_1^u)+2b)}{4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)},$$
$$P_2(\lambda_1^u) = \frac{a(b-\gamma)(4b+3\gamma-2\lambda_1^u(b+\gamma)) + bc(2b(2-\lambda_1^u)+\gamma)}{4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)}.$$

$P_2(\lambda_1^u)$ can be replaced in (A.34) to obtain the multiplier:

$$\frac{(\lambda_1^u)^2\left(4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)\right)^2}{(2-\lambda_1^u)(2-3\lambda_1^u)} = \frac{b\theta^2(a-c)^2(b-\gamma)(2b+\gamma)^2}{2\mu(b+\gamma)}. \tag{A.36}$$

The right-hand side in (A.36) is strictly positive. Notice that $\lambda_1^u \in (0,1)$ and $b > \gamma > 0$ implies that $4b^2(2-\lambda_1^u) - \gamma^2(3-2\lambda_1^u) > 0$. It is then straightforward to show that the left-hand side in (A.36) is strictly increasing in λ_1^u for $\lambda_1^u \in (0, \frac{2}{3})$ and takes values in $(0, +\infty)$. To summarize, there exists a unique $\lambda_1^u \in (0, \frac{2}{3})$ such that the participation constraint (6.3) is binding at equilibrium.

Consider now a ban on below-cost-pricing. We shall not detail the computations in that case as they are similar to the ones performed in the previous case. Simply notice that the binding participation writes at equilibrium as follows:

$$\frac{(\lambda_1^b)^2\left(4b^2(2-\lambda_1^b)-\gamma^2(3-2\lambda_1^b)\right)^2}{(2-\lambda_1^b)(2-3\lambda_1^b)} = \frac{b\theta^2(a-c)^2(b-\gamma)(2b+\gamma)^2}{4\mu(b+\gamma)}. \tag{A.37}$$

Comparing (A.36) and (A.37) immediately leads to $\lambda_1^b < \lambda_1^u$. □

ONLINE APPENDIX

COMPETITIVE MANUFACTURERS. The optimization problem with competitive manufacturers writes as follows:

$$\max_{(w,z,p,e)} \pi(p,w)(1+\theta e) + \theta ezD(p) - \psi(e) \text{ subject to (2.3), (3.5), (3.6).}$$

Inserting the values of (w, z) obtained from (2.3) into the maximand allows to restate this maximization problem as follows:

$$\max_{(p,e)} \pi(p,c)(1+\theta e) - \psi(e) \text{ subject to (3.5) and (3.6).}$$

Observe that (3.5) and (3.6) define the wholesale price and the rebate in terms of the final retail price and the effort but the formers do not enter the maximand. The maximand is thus maximized for the monopoly outcome (p^m, e^m). Inserting into (3.5) and (3.6) then gives the value of the wholesale price and the rebate that implement this outcome, namely $(w^d = c, z^d = 0)$. Finally, thanks to Assumption 1 and our assumption on the convexity of $\psi(e)$, the maximand is quasi-concave in (p, e) so that (p^m, e^m) is a global maximum.

Observe that there are actually other equilibrium wholesale contracts that lead to the same retail price, effort level and allocation of surplus. Any pair (w, z) such that (2.3) is binding leads the retailer to perform effort e^m and to charge a price p^m. Introducing a small degree of risk-aversion on the retailer's side selects $(w^d = c, z^d = 0)$ as the unique optimum.

QUANTIFYING ANTITRUST DAMAGES IN UPSTREAM COLLUSION CASES. Simple computations lead to the following expressions: $\pi^m = \frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$, $D(p^m) = (\frac{a-bc}{1+\delta})^{\frac{1}{\delta}}$, and $e^m = \frac{\theta}{\mu}\frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$. When manufacturers collude under a ban on below-cost pricing, using the proof of Proposition 4, we obtain:

$$p^b = \frac{a}{b} - \frac{a-bc}{b(1+\delta)(1+\delta(1-\lambda^b))},$$

$$D(p^b) = \left(\frac{a-bc}{(1+\delta)(1+\delta(1-\lambda^b))}\right)^{\frac{1}{\delta}},$$

$$e^b = \frac{\delta\theta}{2b\mu}\left(\frac{a-bc}{\delta+1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^b}\left(\frac{1}{1+\delta(1-\lambda^b)}\right)^{\frac{1}{\delta}},$$

where λ^b solves (A.24).

Then, the damage can be expressed as a function of λ^b only:

$$\frac{\Delta}{\pi^m} = 1 + \frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta} - (1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)},$$

where λ^b depends only on the ratio of the demand with and without collusion:

$$(1+\delta(1-\lambda^b))^{\frac{1}{\delta}} = \frac{D(p^m)}{D(p^b)}.$$

This concludes the proof. □

FIRST-ORDER APPROACH. We discuss the validity of the first-order approach used in the main analysis. When Assumption 1 holds, the retailer's profit function $\Pi_R(p,e) = \pi(p,w) + \theta e\pi(p, w-z) - \psi(e)$ is strictly concave in (p, e). The first-order conditions (3.2) and (3.3) characterize the retailer's choices of price and promotion effort, assuming that the promotion effort is strictly positive.

A possible deviation for the retailer is to choose $e = 0$ and to set the corresponding price, namely $\overline{p}(w) = \arg\max_p (p - w)D(p)$. Choosing $e = 0$ and $p = \overline{p}(w)$ provides the retailer with a profit worth $\overline{\Pi}_R(w) = (\overline{p}(w) - w)D(\overline{p}(w))$. Observe that $\overline{p}(w)$ is the solution of $h(\overline{p}(w)) = w$ with $h(\overline{p}(w)) = \overline{p}(w) + \frac{D(\overline{p}(w))}{D'(\overline{p}(w))}$. Under our assumptions on $D(\cdot)$, we have $h'(\cdot) > 0$ and thus $(h^{-1})' = \frac{1}{h'} > 0$. Hence, we have $\overline{p}(w) = h^{-1}(w)$ and $\overline{\Pi}_R(w) = \varphi(h^{-1}(w))$.

In the sequel, we show two results. First, for the class of demand functions $D(p) = (a-bp)^{\frac{1}{\delta}}$ with $\delta > 0$ and $a-bp \geq 0$, such a deviation towards no effort and the corresponding price is never profitable for the retailer if $\delta \geq 1$. Second, for an exponential demand function $D(p) = e^{a-bp}$ (which corresponds to the limit case of $D(p) = (a - bp)^{\frac{1}{\delta}}$ when δ goes to 0), the deviation towards no effort must be taken into account; but this does not change qualitatively our main results.

Consider that demand is given by $D(p) = (a - bp)^{\frac{1}{\delta}}$. It is immediate to show that $\overline{p}(w) = \frac{a\delta+bw}{b(1+\delta)}$, so that $D(\overline{p}(w)) = (\frac{a-bw}{1+\delta})^{\frac{1}{\delta}}$. Using (A.16)-(A.17) and (A.20)-(A.21), we can reconstruct the optimal wholesale price and rebate as functions of the multiplier λ^u, which we denote by $W^u(\lambda^u)$ and $Z^u(\lambda^u)$. Therefore, at the manufacturer's optimum, the retailer has no incentives to deviate towards no effort and the corresponding price if and only if $\overline{p}(W^u(\lambda^u)) - W^u(\lambda^u) \leq 0$ (or equivalently $D(\overline{p}(W^u(\lambda^u))) \leq 0$), which can be rewritten as follows:

$$\text{(B.1)} \qquad \frac{1}{1+\delta(1-\lambda^u)}\left(a - bc - \frac{\delta^2\theta^2(1-\lambda^u)((\delta+1)(1+\delta(1-\lambda^u)))^{-\frac{1}{\delta}}(a-bc)^{\frac{1}{\delta}+2}}{b(\delta+1)(\lambda^u)^2\mu}\right) \leq 0.$$

Since the multiplier λ^u is given by (A.22), (B.1) can be simplified to:

$$\text{(B.2)} \qquad \frac{1-\delta(1-\lambda^u)-2\lambda^u}{1+\delta(\delta+2)(1-\lambda^u)^2-2\lambda^u} \leq 0.$$

The left-hand side term in (B.2) is strictly decreasing in λ^u. Hence, a sufficient condition for (B.2) to always hold is that this inequality is satisfied for $\lambda^u = 0$, or $\frac{1-\delta}{(\delta+1)^2} \leq 0$, or $\delta \geq 1$.

Notice that (B.2) is never satisfied for $\delta = 0$. Our demand function $D(p) = (a - bp)^{\frac{1}{\delta}}$ is defined for $\delta > 0$ and gives at the limit case $\delta \to 0$ the log-linear/exponential demand. We now consider this case and assume that demand is given by $D(p) = e^{a-bp}$.

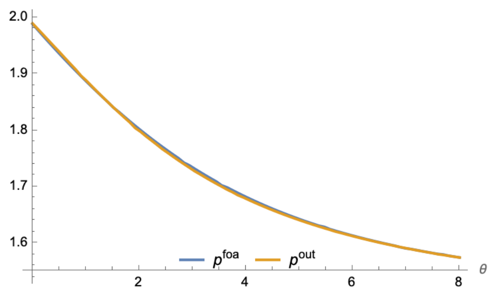
We can then adapt the methodology used in the proof of Propositions 1 and 2 to show that the manufacturer's problem writes as follows:

$$\max_{(p,e)} \left(\pi(p,c) - \varphi(p)\right)(1+\theta e) \text{ subject to } (1+\theta e)\varphi(p) - \psi(e) \geq \varphi(\overline{p}(W(p,e))),$$

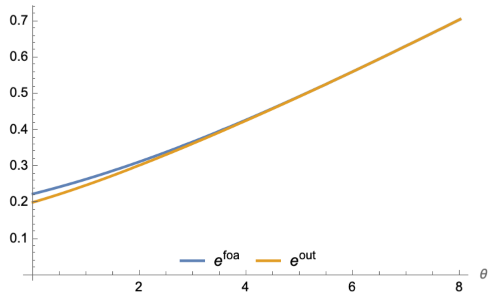
where w can be expressed as function of the pair (p, e) that the manufacturer wants to implement using (3.5), which leads to $\varphi(\overline{p}(W(p,e))) = \varphi\left(h^{-1}\left(h(p) + \frac{D(p)}{D'(p)}\theta e + \frac{e\psi'(e)}{D(p)}\right)\right)$.

Closed-form solutions of the optimum are not possible. We therefore rely on simulations using the following values of parameters: $a = b = 1$, $c = 0$, $\mu = 15$ and $\theta \in [.1, 8]$. We vary θ with an increment of .1 to obtain 80 simulations. For each simulation, we determine numerically the optimum using Mathematica.[25] While both the price and effort level are now chosen to limit the retailer's profit if it exerts no effort, those distortions do not appear to change qualitatively our results. Under a ban on below-cost pricing, simulations also show that the possibility to exert no effort and set the corresponding price is never relevant in equilibrium.

[25]The numerical simulations are available on the second author's webpage.



(a) Prices.



(b) Effort levels.

Figure B.1: With an exponential demand and a quadratic disutility of effort ($D(p) = e^{1-p}$ and $\psi(e) = \frac{15}{2}e^2$, $c = 0$), comparison of the optimal price (Panel (a)) and effort level (Panel (b)) under the first-order approach (p^{foa} and e^{foa}) and when the retailer can choose no effort (p^{out} and e^{out}).