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"Digital Payments Interoperability with Naïve Consumers"

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Digital Payments Interoperability with Naïve Consumers*

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Abstract

We consider a model in which consumers live in isolated villages and need to send money to each other. Each village has (at most) one digital payment provider, which acts as a bridge to other villages. With fully rational consumers interoperability is beneficial: it raises financial inclusion, which in turn increases consumer surplus. With behavioural consumers who have imperfect information or incorrect beliefs about off-net fees, interoperability can reduce consumer welfare. Policies that cap transaction fees have an ambiguous effect on consumers, depending on how the cap is implemented, whether consumers are rational, and on how asymmetric providers are in terms of coverage.

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1 Introduction

Access to digital financial services is generally seen as a fundamental tool for financial inclusion (Demirguc-Kunt et al. (2018)). For many consumers, the first step is to move away from cash-based transactions and adopt digital payments, which are increasingly available even absent formal banking services, through payment platforms or mobile operators (Hoernig & Bourreau (2016)).

As markets for digital payments develop, a key question is how to expand access and usage, while at the same time promoting competition among service providers. Allowing consumers of a given network to transact with consumers in another network—i.e., inducing interoperability across payment platforms—is commonly seen as an important way to promote these goals (Arabehety et al. (2016), Beck & De La Torre (2007), Scott-Morton et al. (2023)).[1]

A second key issue relates to consumer protection. As mentioned, digital payments are often the entry port into financial services and some consumers may lack financial literacy or experience to fully assess their costs and benefits (see, e.g., Garz et al. (2021)). A series of empirical observations motivates this view. First, fee structures for digital payments can be complex. They include many dimensions (for sending and receiving money, for on-net and off-net transactions), they can take different forms (e.g. a percentage of the transaction amount, a fixed amount within a given interval), and each provider can freely choose how to frame its pricing structure.[2] Second, consumers are often unaware of the fees they face and report incurring unexpected fees.[3] This opens the possibility that consumers' unawareness is exploited by service providers through complex and unfavourable pricing schemes (Annan (2022)).

In this paper, we investigate firms' pricing strategies and their impact on transaction volumes and consumers' welfare under different market scenarios. We analyze the effect of introducing interoperability across platforms, showing how it may vary depending

[1]We focus on platform-level interoperability in this paper; other forms of interoperability are discussed in Bianchi et al. (2023).

[2]For example, in Kenya, Safaricom and Airtel use slab pricing but with different transaction bands; in Ghana, MTN uses slab pricing for low and high transaction amounts and a fixed percentage fee for intermediate amounts while G-Money uses slab pricing for on-net transactions and a mix of slab pricing and percentage fees for off-net transactions; in Uganda, instead, MTN and Airtel use slab pricing and essentially the same transaction bands. See, e.g., Bianchi et al. (2023).

[3]For example, according to Annan (2022), only 48% of mobile money customers in Ghana know the official fees they should pay to transfer money; in Uganda, according to IPA (2021), 83% of customers don't know the fees charged by their provider or estimate them with errors exceeding 10% and in Kenya 72% of customers report learning the actual fees only after carrying out a transaction.

on consumers' sophistication. We also highlight how standard regulatory interventions, such as fee caps, need not improve consumers' welfare.

We develop a model in which two digital payment providers serve consumers located in different villages. Each provider sets fees for sending and receiving money. Consumers are heterogeneous in their valuation of digital transactions, as driven for example by taste, access to alternative ways to transfer money, or distance from the service provider.[4] When deciding to make a transaction, consumers take into account the fees incurred both by the sender and by the receiver and complete the transaction only if their net valuation is positive. Since transactions may occur among family members or friends, it is natural to assume that senders (at least, partly) take into account also the fees incurred by the receiver; empirical evidence suggests this is indeed the case (Economides & Jeziorski (2017)).

Absent interoperability, each consumer can only send and receive money from those in the same network. In our baseline analysis, we assume that each village only accommodates one network, which makes each provider a local monopolist. Standard results in terms of monopoly pricing and consumer surplus follow immediately.

We first analyse the effects of interoperability in a market with fully rational consumers. Interoperability expands the set of feasible transactions by allowing consumers to also transfer money off-net. We show that interoperability does not induce any change in on-net fees or the surplus that consumers derive from such transactions. At the same time, providers charge larger fees for off-net transactions. The reason is that, relative to on-net transactions, the demand for off-net transactions of a given provider is less sensitive to its own fees, as it is partly determined by the other provider's (receiving) fee. In other words, high (receiving) fees for off-net transactions negatively affects the demand of the other provider, but providers do not internalize this effect. The fact that off-net fees tend to exceed on-net fees is well documented. Brunnermeier et al. (2023) for example, show that across African countries on-net fees are on average 4% of the transaction value, while off-net fees are 11%.

Despite high off-net fees, interoperability generally increases consumer surplus in a market with fully rational agents, and more so when the market is *less* concentrated.[5] The benefits from off-net transactions are larger when there is no dominant player in the market, and they are maximized when providers have equal market shares as in this case the proportion of off-net transactions is maximal.

[4]Even for digital payments, consumers may have to travel to meet a local agent providing cash-in and cash-out services, which may entail substantial transportation costs (Grzybowski et al. (2023)).

[5]In our model, both firms operate and jointly cover the market. The effects of interoperability may be different in a market in which new firms may enter and increase market coverage.

We contrast these effects to those arising when consumers are not fully rational in the sense of not being able to perfectly observe and understand firms' pricing schedules. In particular, consumers may be poorly informed about the exact pricing of off-net transactions, with which they may be less familiar.

We start with a rather minimal departure from full information, assuming that consumers do not know the fee set by the other network provider for receiving money off-net, and they consider that this fee is the same as the one set by their own network provider. At the same time, consumers are assumed to perfectly observe the off-net fees incurred when sending money.

We show that in this setting we have two asymmetric pure strategy equilibria, in which fees for receiving money off-net are equal to zero for one provider (say, provider A) and they equal the maximal consumers' valuation for the other provider (say, provider B). This generates many off-net transactions from A to B, since A's consumers mistakenly underestimate the fees incurred to receive money on network B and, symmetrically, few transactions from B to A, since B's consumers mistakenly overestimate the fees to receive money on network A. As a result, it can be shown that introducing interoperability makes consumers worse off. The reason is that consumers tend to avoid off-net transactions that would generate a large surplus (those from B to A) while at the same time performing lots of transactions that generate a negative surplus (those from A to B).

We then investigate the robustness of these insights in a setting where consumers simply use the on-net fees charged by their own network to assess off-net fees. If consumers in network i incur charges $n_{s,i}$ to send and $n_{r,i}$ to receive money on-net, they believe that it will also cost them $n_{s,i}$ to send money off-net and will cost consumers in the other network $n_{r,i}$ to receive off-net.

In line with the literature on shrouded attributes (Gabaix & Laibson (2006)), we show that firms set low on-net fees to attract consumers and large off-net fees to extract consumers' surplus. The level of on-net fees typically depends on a firm's market share: the lower the market share, the more important are off-net transactions, and the lower are the on-net fees set by the firm. Relative to the case with no interoperability, on-net transactions are cheaper, which tends to increase consumers' surplus. However, off-net transactions are typically priced above consumers' valuations, which tends to decrease their surplus. We show in a simple example that the latter effect may dominate: consumers would be better off in a market *without* interoperability.

Interoperability is often advocated as a way to promote competition and decrease fees; Brunnermeier et al. (2023) document that indeed this tends to be the case. Interestingly, our model is consistent with this view only when consumers have biased beliefs both on senders and on receivers fees, in which case a reduction in on-net fees is not welfare improving but rather a way to attract consumers and exploit them with larger off-net fees (which as documented in Brunnermeier et al. (2023) tend to significantly exceed on-net fees).

Motivated by the previous results that providers tend to overcharge consumers for off-net transactions, we consider the effects of various regulatory interventions. We start by considering absolute caps on off-net fees. We show that, even if in some instances providers may respond by increasing on-net fees, the net effect on consumers' welfare is generally positive. We then consider a regulation which mandates that fees for off-net transactions should be the same as for on-net transactions. We show that, as expected, this tends to lower off-net fees. At the same time, however, providers respond by increasing on-net fees. The resulting effects on consumers' surplus depend crucially on the type of biases consumer exhibit. When consumers are fully rational, the regulation generally increases their surplus. We show in a simple example that the gain in surplus is however minimal relative to the gain experienced when introducing interoperability. When consumers have biased beliefs only about receiving fees (the first behavioral setting considered above), the regulation has an ambiguous effect on their welfare. In our example, when firms have sufficiently different market shares, and so on-net transactions are more frequent than off-net transactions, the effect on increased on-net fees dominates, thereby making consumers worse off relative to the case of unregulated interoperability. Conversely, when consumers have biased beliefs both about sending and receiving fees (the second behavioral setting considered above), the regulation always improves their surplus in our example, and its effect can be substantial, largely compensating the loss in surplus induced by interoperability.

Finally, we consider a regulation which imposes that either the senders or the receivers should incur no fees to transact. This also corresponds to the common case in digital payments where only one of the parties pay the fee. We show that this regulation has no impact on consumers' welfare when consumers have biased beliefs, and so introducing interoperability may be harmful to consumers even when this regulation is in place.

Our paper relates to the literature on optimal pricing of communications (e.g., Laffont et al. (1998)) and specifically to models in which receivers can be charged (e.g., Hermalin & Katz (2004)) and in which operators can discriminate between on-net and off-net fees (e.g., Jeon et al. (2004)).[6] Our model differs as it incorporates

[6] Similar issues have also been analyzed in the literature on ATM charges (e.g., Donze & Dubec (2006), Massoud & Bernhardt (2002)).

some specificities of digital payments (e.g., that senders may internalize the utility of receivers) and especially as it focuses on the effects of interoperability in markets with behavioral consumers, which to our knowledge have not been considered in this literature.

We also connect with the behavioural IO literature on shrouded pricing (e.g. Ellison (2005) and Gabaix & Laibson (2006)) in which some product dimensions—in our case, off-net fees—are less salient to consumers due for example to limited attention or to limited experience with those fees. In line with this literature, we find that less salient off-net fees are highly priced and low on-net fees may be used to attract consumers. The mechanisms in our case are however quite different. In the shrouded pricing literature, interoperability can be viewed as allowing consumers to buy add-on and base goods from different providers, which may increase competition and decrease prices in the add-on markets. In our case, interoperability instead increases the number of dimensions that consumers should pay attention to, and it opens the possibility of consumers' mistakes and so may have detrimental effects on their welfare.

In this sense, our insights are also related to the literature on consumers' confusion (e.g., Chioveanu & Zhou (2013), Piccione & Spiegler (2012)) and in particular to a recent literature in finance which models the pricing and complexity of (innovative) financial products when consumers may not fully understand their value (Carlin (2009), Carlin & Manso (2011), Thakor & Merton (2023)).[7] While firms in our model cannot choose the way in which fees are framed, they can exploit consumers' confusion by overcharging off-net transactions. Our distinctive focus is on how the effects of interoperability and of regulatory interventions vary as consumers' confusion takes different forms.[8]

2 Model

There are two digital payment networks A and B, and a unit mass of isolated "villages". Network A is present in $\alpha \in (0,1)$ of the villages, and network B is present in the remaining $1-\alpha$ villages. In each village, consumers wish to transfer money to consumers in one other randomly chosen village. Doing the transfer via digital money rather than cash generates a total benefit v to the sender and receiver. For example, v may capture gains from using digital money such as added security, and also non-monetary costs such as time taken to travel to cash-in and cash-out agents. We assume that v is

[7] See Beshears et al. (2018) and Garz et al. (2021) for recent reviews.

[8] We refer to Bianchi et al. (2023) for further discussion of how the insights developed in the behavioral IO literature can be applied to the market for digital payments.

distributed on $[\underline{v}, \bar{v}]$ according to a CDF $G(v)$, with associated log-concave density $g(v)$. To simplify the exposition we assume that $\underline{v} \leq 0$.[9]

The marginal cost to the networks of transferring money between villages is zero. We denote by $n_{s,i}$ and $n_{r,i}$ the fees charged by network $i = A, B$ to respectively send and receive money "on-net", i.e., transfer money between two villages in which it is present. We similarly let $f_{s,i}$ and $f_{r,i}$ denote the fees charged by network $i = A, B$ to respectively send and receive money "off-net", i.e., send money to or receive money from a village where the other network is present. All fees are assumed to be non-negative and are set simultaneously by the two networks. The sender chooses between cash and digital money to maximize the joint benefit of the transaction net of any fees that need to be paid.

Using this framework, we consider a shift from no interoperability, such that only on-net transactions are feasible, to interoperability, such that off-net transactions are also feasible. We will do this both for the case where consumers are rational, and where they exhibit behavioral biases concerning off-net transaction fees (which we define more precisely later on).

Remark: An alternative interpretation for villages is that α consumers are loyal to A and $1-\alpha$ are loyal to B—and consumers want to transact on their preferred network.

3 Rational Consumers

We begin by considering the benchmark case in which senders are rational and fully informed about all fees.

No interoperability First, suppose that only on-net transactions are possible. Network A is able to carry out a mass α^2 of transactions, and each of these transactions occurs if and only if $v - n_{s,A} - n_{r,A} \geq 0$, i.e., when the net benefit offered by A relative to cash is positive. Similarly network B is able to carry out a mass $(1-\alpha)^2$ of transactions, each of which occurs if and only if $v - n_{s,B} - n_{r,B} \geq 0$. Hence, the two networks' optimization problems are completely separable and are given by respectively

$$\max_{n_{s,A}, n_{r,A}} (n_{s,A} + n_{r,A})[1 - G(n_{s,A} + n_{r,A})] \quad \text{and} \quad \max_{n_{s,B}, n_{r,B}} (n_{s,B} + n_{r,B})[1 - G(n_{s,B} + n_{r,B})].$$

The following result is then immediate. (All omitted proofs are in the appendix.)

[9] This assumption rules out corner solutions where all transactions occur via digital money.

Proposition 1. When there is no interoperability, both networks set their total on-net fees equal to t^*, where t^* is the unique solution to

$$t^* = \frac{1 - G(t^*)}{g(t^*)}, \tag{1}$$

i.e., $n^*_{s,A} + n^*_{r,A} = n^*_{s,B} + n^*_{r,B} = t^*$.

Notice that only a network's *total* on-net fee can be determined. This is because only the total fee (rather than its split between sender and receiver fees) matters for the network's per-transaction revenue and for the sender's decision. We can then write total consumer surplus from mobile money (relative to cash) as

$$CS^{NI} = [\alpha^2 + (1-\alpha)^2] \int_{t^*}^{\bar{v}} (v - t^*) dG(v),$$

where the superscript NI denotes no interoperability.

Example. Throughout the paper, we will illustrate some of our results using a running uniform distribution example in which $G(v) = v$. In this example, it is easy to calculate that $t^* = 1/2$, such that each network realizes half of its feasible transactions, and that relative to cash, digital payments generate additional consumer surplus of $[\alpha^2 + (1-\alpha)^2]/8$.

Interoperability Now suppose that both on-net and off-net transactions are possible. In this case network A's optimization problem can be written as

$$\begin{aligned} \max_{n_{s,A}, n_{r,A}, f_{s,A}, f_{r,A}} \quad & \alpha^2 (n_{s,A} + n_{r,A})[1 - G(n_{s,A} + n_{r,A})] \\ & + \alpha(1-\alpha) f_{s,A}[1 - G(f_{s,A} + f_{r,B})] + \alpha(1-\alpha) f_{r,A}[1 - G(f_{s,B} + f_{r,A})]. \end{aligned}$$

The first line is profit from on-net transactions, and is unaffected by interoperability. The second line is profit from off-net transactions. Specifically, a mass $\alpha(1-\alpha)$ of transactions from network A to network B are now possible, and each of these transactions occurs if and only if $v - f_{s,A} - f_{r,B} \geq 0$, i.e., if the benefit net of A's sender fee and B's receiver fee is positive. Similarly, a mass $\alpha(1-\alpha)$ of transactions from network B to network A are also now possible, and each occurs if and only if $v - f_{s,B} - f_{r,A} \geq 0$. Proceeding similarly, we can write network B's optimization problem as

$$\begin{aligned} \max_{n_{s,B}, n_{r,B}, f_{s,B}, f_{r,B}} \quad & (1-\alpha)^2 (n_{s,B} + n_{r,B})[1 - G(n_{s,B} + n_{r,B})] \\ & + \alpha(1-\alpha) f_{s,B}[1 - G(f_{s,B} + f_{r,A})] + \alpha(1-\alpha) f_{r,B}[1 - G(f_{s,A} + f_{r,B})]. \end{aligned}$$

Jointly solving these optimization problems, we obtain the following result.

Proposition 2. When there is interoperability:
i) Both firms set their total on-net fees equal to t^*, as defined in equation (1).
ii) Both firms set their off-net sender and receiver fees to $t^{**}/2$, where $t^{**} \in (t^*, 2t^*)$ is the unique solution to

$$\frac{t^{**}}{2} = \frac{1 - G(t^{**})}{g(t^{**})}. \tag{2}$$

According to Proposition 2, all fees are independent of α, i.e., independent of the degree of network asymmetry. The reason is that each network has four fees that it can vary, but only three different transaction types—on-net transactions, transactions from A to B, and transactions from B to A. This relative abundance of fee instruments means that each network can price the different transactions independently; hence the fees charged for a given transaction type are independent of the number of those transactions, and thus also independent of α. Since on- and off-net transactions are entirely separable, the former are priced in the same way as without interoperability, and so incur a total fee of t^* as defined in equation (1). Meanwhile off-net transactions incur a total fee of t^{**}, as defined in equation (2), because both the sender and receiver each have to pay $t^{**}/2$. Proposition also 2 shows that $t^{**} > t^*$. Intuitively, each network ignores the negative impact of an increase in its off-net fees on demand (and hence profits) of the rival network; this leads networks to charge more overall than they do for on-net transactions.

Note that since interoperability has no effect on the pricing of on-net transactions, but allows new off-net transactions to occur, it unambiguously raises transaction volumes as well as consumer surplus (relative to using cash), which is given by the following expression:

$$CS^I = [\alpha^2 + (1-\alpha)^2] \int_{t^*}^{\bar{v}} (v - t^*) dG(v) + \underbrace{2\alpha(1-\alpha) \int_{t^{**}}^{\bar{v}} (v - t^{**}) dG(v)}_{\text{Additional consumer surplus}},$$

where the superscript I denotes interoperability.
Example. In our running uniform distribution example, on-net transactions incur a total fee of $t^* = 1/2$, while off-net transactions incur a total fee of $t^{**} = 2/3$. Interoperability increases consumer surplus (relative to using cash) by $\alpha(1-\alpha)/9$.

Although interoperability expands the total number of transactions, off-net transactions are relatively low due to their high fees. We later study the effects of policy interventions that could be used to put downward pressure on off-net transaction fees.

4 Behavioral Consumers

We now suppose consumers are behavioral in the sense that they exhibit biases when computing off-net transaction fees. We consider two cases, depending on whether this bias applies only to receiver fees, or to both sender and receiver fees. Throughout this section we assume that consumers are perfectly informed about on-net transaction sender and receiver fees, e.g., because they are more accustomed to making and receiving such transactions. We will also make the following assumption:

Assumption 1. Networks may not set any fee above $\bar{v}$.

A standard assumption in behavioral models is that fees are capped (e.g., by policy, or because if fees are too high consumers may complain ex post); in our model, a natural cap would be $\bar{v}$, which we henceforth impose.[10]

4.1 Bias only on Receiver Fees

We start with a rather minimal departure from the rational benchmark and assume that consumers *only* exhibit a behavioral bias on the fee to receive money off-net. In particular, we assume that senders of both on- and off-net transactions perfectly observe and understand the fee they pay. But we also assume that consumers in network i's village believe that if they send money to network $j \neq i$'s village, consumers there will incur $f_{r,i}$ to receive the money, i.e., network i's off-net receiver fee is "projected" onto the other network.

We start by considering network A's optimization problem. Since consumers face no bias with respect to on-net transactions, network A's profit from these transactions is the same as in the case without interoperability. On the other hand, transactions from A to B now occur with probability $1 - G(f_{s,A} + f_{r,A})$ because senders from network A correctly understand it will cost them $f_{s,A}$ to send the money, but (in general, incorrectly) believe that consumers in the other village will incur $f_{r,A}$ to receive the money. Meanwhile, using a similar reasoning, transactions from B to A now occur with probability $1 - G(f_{s,B} + f_{r,B})$. Hence we can write network A's optimization problem

[10] An alternative interpretation for our model is the following. Suppose that $\bar{v}$ is the maximum value of transacting via digital money, and that $\bar{v} - v$ reflects the time and effort taken to travel to the mobile money agent. In that case, it is natural that at least receiver fees are capped at $\bar{v}$, because once a consumer has traveled to an agent, she is willing to pay up to $\bar{v}$ rather than abandon the transaction and use cash.

as

$$\max_{n_{s,A},n_{r,A},f_{s,A},f_{r,A}} \alpha^2(n_{s,A}+n_{r,A})[1-G(n_{s,A}+n_{r,A})]$$
$$+\alpha(1-\alpha)f_{s,A}[1-G(f_{s,A}+f_{r,A})]+\alpha(1-\alpha)f_{r,A}[1-G(f_{s,B}+f_{r,B})],$$

Using a similar logic, we can also write network B's optimization problem as

$$\max_{n_{s,B},n_{r,B},f_{s,B},f_{r,B}} (1-\alpha)^2(n_{s,B}+n_{r,B})[1-G(n_{s,B}+n_{r,B})]$$
$$+\alpha(1-\alpha)f_{s,B}[1-G(f_{s,B}+f_{r,B})]+\alpha(1-\alpha)f_{r,B}[1-G(f_{s,A}+f_{r,A})].$$

We first notice that each network's revenues from on- and off-net fees are completely separable, and hence on-net transactions are priced (in total) at t^*, as they would be absent interoperability. Focusing on network A and its choice of off-net fees, notice that for fixed $F_A \equiv f_{s,A}+f_{r,A}$ and fixed $F_B \equiv f_{s,B}+f_{r,B}$, network A's choice over $f_{s,A}$ and $f_{r,A}$ is generically "bang-bang". Specifically, if $F_A < F_B$, such that network A sends more off-net transactions than it receives, it is optimal to set $f_{s,A}=F_A$ and $f_{r,A}=0$; if, on the other hand, $F_A > F_B$, such that network A receives more off-net transactions than it sends, it is optimal to $f_{s,A}=0$ and $f_{r,A}=F_A$. Using that insight, we can then rewrite network A's profit from off-net transactions as

$$\pi_A(F_A,F_B)=\begin{cases}\alpha(1-\alpha)F_A[1-G(F_A)] & \text{if } F_A<F_B\\ \alpha(1-\alpha)F_A[1-G(F_B)] & \text{if } F_A\geq F_B\end{cases}.$$

This profit function has a particularly simple form. In particular, suppose we fix F_B. As we vary F_A over the interval $[0,F_B]$ network A's profit is proportional to $F_A[1-G(F_A)]$ and hence is quasiconcave; therefore if $F_B \leq t^*$ profit here is strictly increasing, and otherwise it is hump-shaped and decreasing in F_A as $F_A \to F_B$. Note that network A's profit kinks up at $F_A=F_B$, and is then strictly increasing in F_A up until the point where $F_A=\bar{v}$. Using this logic, it is then straightforward to prove the following result:

Lemma 1. Fixing F_{-i} for $i=A,B$, network i's best response is to set $F_i=t^*$ if $G(F_{-i}) > 1-t^*[1-G(t^*)]/\bar{v}$, and to set $F_i=\bar{v}$ if this inequality is reversed (and otherwise network i is indifferent between charging t^* and $\bar{v}$).

Notice that even if $\alpha \neq 1/2$, such that the networks have asymmetric coverage levels, their best responses are symmetric. Intuitively, if say network B's total off-net fee is relatively high, such that network A's customers receive relatively little money off-net, network A concentrates on sender fees, and given consumers' behavioral bias, acts like a monopolist absent interoperability, thereby charging $F_A=t^*$. However, if instead network B's off-net fee is relatively low, such that network A's customers receive relatively a lot of money off-net, network A exploits this by setting as high an off-net receiver fee as it can (given our assumption earlier that individual fees are capped at $\bar{v}$). Using the above lemma, we can then characterize the equilibrium fees. (We focus exclusively on equilibria in pure strategies.)

Proposition 3. Suppose consumers have biased beliefs about off-net receiver fees. Then:
i) On-net fees are the same as without interoperability: $n_{s,A} + n_{r,A} = n_{s,B} + n_{r,B} = t^*$.
ii) There are two pure strategy equilibria for off-net fees. In each equilibrium, one network $i = A, B$ sets $f_{s,i} = t^*$ and $f_{r,i} = 0$, while the other network sets $f_{s,-i} = 0$ and $f_{r,-i} = \bar{v}$.

Notice that there is no symmetric pure strategy equilibrium. The reason is simply that, as seen in Lemma 1, networks' optimal fees are binary, either low at t^* or high at $\bar{v}$, and when one network has a low fee the other prefers a high fee (and vice versa). This substitutability in fees is what drives the existence of asymmetric equilibrium. For example, suppose that network A decides to set $F_A = t^*$, which from earlier arguments implies that $f_{s,A} = t^*$ and $f_{r,A} = 0$. This fee profile leads to a relatively large volume of off-net transactions from A to B. To exploit this, network B therefore optimally sets $f_{s,B} = 0$ and $f_{r,B} = F_B = \bar{v}$. Given this fee profile, there are zero off-net transactions from B to A, so network A focuses on off-net sender transactions, and so has an incentive to set a low fee in order to stimulate such transactions (and given the structure of consumers' beliefs about off-net receiver fees, it turns out it is indeed optimal to set the sender fee equal to t^*).

Turning to consumer surplus, notice that interoperability is unambiguously bad for consumers. The reason is that on-net transactions are priced the same as without interoperability, and hence generate the same consumer surplus. However, now there is a mass $\alpha(1-\alpha)[1-G(t^*)]$ of off-net transactions from the network which sets $N_i = t^*$ to the other network that sets $N_{-i} = \bar{v}$, and these transactions incur a total fee of $\bar{v} + t^*$ which is more than the value (relative to cash) of any transaction. We also notice that there are no transactions from the network charging $N_{-i} = \bar{v}$ to the one charging $N_i = t^*$, since consumers in network $-i$ observe a fee $f_{r,-i} = \bar{v}$ and wrongly believe that it also therefore costs $\bar{v}$ to receive money off-net in village i, and hence send no money off-net. Thus consumer surplus equals

$$\frac{\alpha^2 + (1-\alpha)^2}{8} + \alpha(1-\alpha)\int_{t^*}^{\bar{v}} (v - t^* - \bar{v})dG(v).$$

Example Return to the running example with $G(v) = v$. On-net fees equal $n_{s,A}+n_{r,A} = n_{s,B}+n_{r,B} = 1/2$, while for off-net fees one network charges $1/2$ to send and 0 to receive money, while the other network charges 0 to send and $\bar{v}$ to receive money. Consumer surplus equals

$$\frac{\alpha^2 + (1-\alpha)^2 - 3\alpha(1-\alpha)}{8}.$$

4.2 Biases on both Sender and Receiver Fees

We now consider the case where senders exhibit biases when computing how much off-net transactions will cost to send and receive. In particular, we assume that consumers use the on-net fees charged by the network provider in their village to form beliefs about off-net fees: if their village's network charges $n_{s,i}$ to send and $n_{r,i}$ to receive money on-net, they believe that it will also cost them $n_{s,i}$ to send money off-net and will cost consumers in the other village $n_{r,i}$ to receive off-net.

Start by considering network A's optimization problem. As we have seen previously, since consumers face no bias with respect to on-net transactions, these transactions generate the same profit as in the case without interoperability. On the other hand, transactions from A to B now occur with probability $1-G(n_{s,A}+n_{r,A})$ because senders from network A (in general, incorrectly) believe that they will pay $n_{s,A}$ to send money and that the consumer in the other village will pay $n_{r,A}$ to receive it. Meanwhile transactions from B to A now occur with probability $1 - G(n_{s,B} + n_{r,B})$, because senders in the B village similarly use network B's on-net fees to form their belief about off-net fees. Hence we can write network A's optimization problem as

$$\begin{aligned}\max_{n_{s,A},n_{r,A},f_{s,A},f_{r,A}} \quad & \alpha^2(n_{s,A}+n_{r,A})[1-G(n_{s,A}+n_{r,A})] \\ & +\alpha(1-\alpha)f_{s,A}[1-G(n_{s,A}+n_{r,A})] + \alpha(1-\alpha)f_{r,A}[1-G(n_{s,B}+n_{r,B})],\end{aligned}$$

Using a similar logic, we can also write network B's optimization problem as

$$\begin{aligned}\max_{n_{s,B},n_{r,B},f_{s,B},f_{r,B}} \quad & (1-\alpha)^2(n_{s,B}+n_{r,B})[1-G(n_{s,B}+n_{r,B})] \\ & +\alpha(1-\alpha)f_{s,B}[1-G(n_{s,B}+n_{r,B})] + \alpha(1-\alpha)f_{r,B}[1-G(n_{s,A}+n_{r,A})].\end{aligned}$$

Given consumers' behavioral bias, off-net transaction fees have no impact on off-net transaction volumes. Hence the two networks set their off-net fees as high as possible, i.e., they set them equal to $\bar{v}$ given Assumption 1. On the other hand, on-net fees now play a dual role: aside from affecting profit from on-net transactions, they also influence the volume and hence profitability of off-net transactions. Notice that, as we have seen before, only total on-net transactions fees $n_{s,A} + n_{r,A}$ and $n_{s,B} + n_{r,B}$ rather than their composition can be pinned down. It is straightforward to prove the following:

Proposition 4. Suppose consumers have biased beliefs about off-net sender and receiver fees. Then:
i) All off-net fees are set as high as possible: $f_{s,A} = f_{r,A} = f_{s,B} = f_{r,B} = \bar{v}$.
ii) If $\alpha/(1-\alpha) \leq \bar{v}g(0)$ then network A sets $n_{s,A} + n_{r,A} = 0$, and otherwise it sets the unique $n_{s,A} + n_{r,A} \in (0, t^*)$ that solves

$$\alpha \left[\frac{1 - G(n_{s,A} + n_{r,A})}{g(n_{s,A} + n_{r,A})} \right] - \alpha(n_{s,A} + n_{r,A}) - (1-\alpha)\bar{v} = 0.$$

iii) If $\alpha/(1-\alpha) \geq 1/[g(0)\bar{v}]$ then network B sets $n_{s,B} + n_{r,B} = 0$, and otherwise it sets the unique $n_{s,B} + n_{r,B} \in (0, t^*)$ that solves

$$(1-\alpha) \left[\frac{1 - G(n_{s,B} + n_{r,B})}{g(n_{s,B} + n_{r,B})} \right] - (1-\alpha)(n_{s,B} + n_{r,B}) - \alpha\bar{v} = 0.$$

Recall that, absent interoperability, networks charge a total fee of t^* for on-net transactions. When consumers have biased beliefs, interoperability causes the networks to price below t^* for on-net transactions. The reason is that by doing this, they increase demand for off-net transactions, which they then exploit by pricing such transactions at $\bar{v}$. Indeed, when off-net transactions are sufficiently important in relative terms for a network—which means $\alpha/(1-\alpha)$ is sufficiently low for network A, or sufficiently high for network B—that network will set its on-net fees to zero to encourage as many off-net transaction as possible. Otherwise on-net transactions will be priced between 0 and t^*, and become more expensive as on-net transactions become more important relative to off-net transactions.

Relative to the case with no interoperability, interoperability now introduces a trade-off. On the one hand, on-net transactions are cheaper, which is of course good for consumers. On the other hand, however, consumers overpay for off-net transactions, because the total fee incurred by the sender and receiver is $2\bar{v}$, whereas the highest willingness-to-pay to transact off-net is $\bar{v}$. Total consumer surplus is given by:

$$CS^I = \alpha^2 \int_{n_{s,A}+n_{r,A}}^{\bar{v}} (v - n_{s,A} - n_{r,A}) dG(v) + (1-\alpha)^2 \int_{n_{s,B}+n_{r,B}}^{\bar{v}} (v - n_{s,B} - n_{r,B}) dG(v)$$
$$+\alpha(1-\alpha) \int_{n_{s,A}+n_{r,A}}^{\bar{v}} (v - 2\bar{v}) dG(v) + \alpha(1-\alpha) \int_{n_{s,B}+n_{r,B}}^{\bar{v}} (v - 2\bar{v}) dG(v).$$

In order to further highlight fees and consumer surplus in this setting, we return to our running uniform distribution example.

Example. Suppose that $G(v) = v$. Clearly, $f_{s,A} = f_{r,A} = f_{s,B} = f_{r,B} = 1$. On-net fees can be calculated as follows:

$$n_{s,A} + n_{r,A} = \begin{cases} 0 & \text{if } \alpha \leq 1/2 \\ \frac{2\alpha-1}{2\alpha} & \text{if } \alpha > 1/2 \end{cases} \quad \text{and} \quad n_{s,B} + n_{r,B} = \begin{cases} \frac{1-2\alpha}{2(1-\alpha)} & \text{if } \alpha \leq 1/2 \\ 0 & \text{if } \alpha > 1/2 \end{cases}.$$

Notice that network A's on-net fee increases in α, because for higher α it faces relatively fewer off-net transactions, and so as explained above, has less incentive to bait consumers with a low on-net fee; network B's fees have the opposite comparative static. Moreover, as, say, $\alpha \to 1$, network A prices as if there were no interoperability, and sets $n_{s,A} + n_{r,A} = t^* = 1/2$, while network B sets $n_{s,B} + n_{r,B} = 0$. One can also compute that consumer surplus equals

$$CS^I = \begin{cases} \frac{1-2\alpha}{8(1-\alpha)} - 2\alpha(1-\alpha) & \text{if } \alpha \leq 1/2 \\ \frac{2\alpha-1}{8\alpha} - 2\alpha(1-\alpha) & \text{if } \alpha > 1/2 \end{cases}.$$

It is then easy to check that in this example, interoperability *reduces* consumer surplus. In other words, the high fees for off-net transfers dominate the reduction in on-net fees that networks use to bait consumers into thinking that off-net transactions are cheaper than they actually are.

5 Capping Fees

The previous analysis has shown that overpricing is especially severe for off-net fees when consumers have biased beliefs about them. A natural regulation in this setting is to impose fee caps, and different countries have adopted caps of different forms (see, e.g., CGAP (2021)). In this section, we explore the effect of several types of regulation, mandating that certain fees cannot exceed some cap, showing how the ultimate impact of these regulations on consumers depends crucially on whether consumers are rational or have biased beliefs, and on which type of biased beliefs. We also highlight how these measures may have unintended effects.

5.1 Absolute caps on off-net fees

We have so far assumed (Assumption 1) that networks cannot set any fee above $\bar{v}$ and, as seen, this constraint is often binding when consumers have wrong beliefs about off-net fees. A first natural policy is to reduce the maximal fee below $\bar{v}$. If consumers have biased beliefs only about off-net receiver fees, there is no interaction between the equilibrium off-net and on-net fees (see Proposition 3); hence, a marginal decrease in the maximal fee would decrease off-net fees, leaving on-net fees unaffected, and thus increase consumers' welfare. If consumers have biased beliefs both about off-net sender and receiver fees, equilibrium on-net fees depend negatively on off-net fees (see Proposition 4); hence, a marginal decrease in the maximal fee would decrease off-net fees and at the same time increase on-net fees. Although this "rebalancing" of fees makes it hard to assess the impact of a cap for a general distribution, in our running uniform example it is easy to see that consumers benefit whenever the cap is sufficiently aggressive.

5.2 Off-net fees bounded by on-net fees

Suppose that under interoperability, each network $i = A, B$ is required by regulation to i) charge the same fee to send money on- and off-net (i.e., $n_{s,i} = f_{s,i}$) and ii) also charge the same fee to receive money on- and off-net (i.e., $n_{r,i} = f_{r,i}$).[11]

5.2.1 Rational Consumers

Suppose that consumers are fully rational. Given this "equal fee regulation", we can rewrite the networks' optimization problems from earlier purely as a function of on-net fees, i.e, as

$$\begin{aligned}\max_{n_{s,A},n_{r,A}} \quad & \alpha^2(n_{s,A}+n_{r,A})[1-G(n_{s,A}+n_{r,A})] \\ & +\alpha(1-\alpha)n_{s,A}[1-G(n_{s,A}+n_{r,B})]+\alpha(1-\alpha)n_{r,A}[1-G(n_{s,B}+n_{r,A})]\end{aligned}$$

for network A, and as

$$\begin{aligned}\max_{n_{s,B},n_{r,B}} \quad & (1-\alpha)^2(n_{s,B}+n_{r,B})[1-G(n_{s,B}+n_{r,B})] \\ & +\alpha(1-\alpha)n_{s,B}[1-G(n_{s,B}+n_{r,A})]+\alpha(1-\alpha)n_{r,B}[1-G(n_{s,A}+n_{r,B})].\end{aligned}$$

[11]We note that it is important that this regulation applies to both sender and receiver fees. To illustrate, consider the case of rational consumers, and consider a regulation which forces networks to charge the same sender fee on- and off-net, but places no restriction on receiver fees. In this case, the two networks will end up playing the same equilibrium as in Proposition 2. In particular, they will charge $t^{**}/2$ to send money on- and off-net, charge $t^* - t^{**}/2 > 0$ to receive money on-net, and charge $t^{**}/2$ to receive money off-net. This is possible because even with the regulation, the number of fee instruments is equal to the number of transaction types.

for network B. For this (and some later) analysis we need to impose the following stronger assumption on the the distribution G, to ensure that the networks' optimization problems are well behaved:

Assumption 2. For all $n_{-i} \in [0, \bar{v})$, $n_i \left[1 - G\left(n_i + n_{-i}\right)\right]$ is concave in $n_i \in [0, \bar{v} - n_{-i}]$.

Notice that this assumption is satisfied provided the density g is either increasing, or not decreasing too fast; it is therefore satisfied by our running example where $G(v) = v$. Using this assumption, we are then able to prove the following result:

Lemma 2. Suppose Assumption 2 holds. With equal fee regulation, there is a unique equilibrium in which network $i = A, B$ sets the same fee for sending and receiving money (i.e., $n_{s,i} = n_{r,i} = n_i$). These fees solve the following equations:

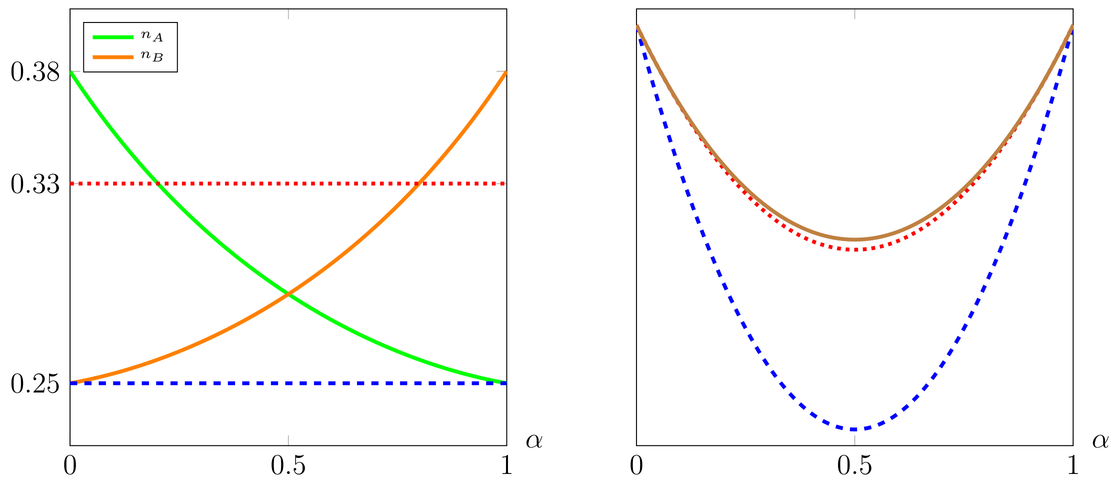
$$\alpha[1 - G(2n_A) - 2n_A g(2n_A)] + (1-\alpha)[1 - G(n_A + n_B) - n_A g(n_A + n_B)] = 0 \quad (3)$$

$$(1-\alpha)[1 - G(2n_B) - 2n_B g(2n_B)] + \alpha[1 - G(n_A + n_B) - n_B g(n_A + n_B)] = 0. \quad (4)$$

Although networks are not obliged to price sender and receiver fees in the same way, it turns out that under our additional regularity condition it is optimal for them to do this. Interestingly, this implies that the total fee required to make an off-net transaction is the same regardless of whether the sender is in an A or a B village. Note that equilibrium fees now depend on α, because networks have three distinct types of transaction but only two fee instruments, so the relative importance of different types of transaction (which depends on α) affects fees. The following result explores how the two networks' fees compare, and how this depends on α:

Proposition 5. The equilibrium fees characterized in Lemma 2 have the following properties:
i) Fees are higher than their "average" level absent interoperability, i.e., $n_A, n_B > t^*/2$.
ii) The regulation reduces total fees paid for off-net transactions, i.e., $n_A + n_B < t^{**}$.
iii) Network A's fee is larger, i.e., $n_A > n_B$, if and only if $\alpha \in (0, 1/2)$.

The equal fee regulation brings on- and off-net fees closer together. The regulation is successful at reducing the total fees paid for off-net transactions, since $n_A + n_B < t^{**}$, and thus increases the volume of off-net transactions. However, this comes at the cost of raising the total fees paid for on-net transactions, since $2n_A, 2n_B > t^*$. The legislation therefore generates a trade-off, by raising consumer surplus from off-net transactions



(a) Equilibrium fees

(b) Consumer surplus

Figure 1: Market outcomes with rational consumers
(The dashed curves are outcomes without interoperability, the dotted curves are outcomes with "unregulated" interoperability, and the thick curves are outcomes with equal fee regulation)

but reducing it from on-net transactions. Total consumer surplus can be written as

$$CS^R = \underbrace{\alpha^2 \int_{2n_A}^{\bar{v}} (v - 2n_A)dG(v) + (1-\alpha)^2 \int_{2n_B}^{\bar{v}} (v - 2n_B)dG(v)}_{\text{Lower on-net surplus with regulation}}$$
$$\underbrace{+2\alpha(1-\alpha) \int_{n_A+n_B}^{\bar{v}} (v - n_A - n_B)dG(v)}_{\text{Higher off-net surplus with regulation}},$$

where the superscript R denotes interoperability with equal fee regulation. The proposition also shows that relative fees across the two networks vary with respect to α in a natural way. Specifically, when $\alpha = 1/2$, such that the networks have symmetric coverage, all fees are identical, and hence on- and off-net transactions cost the same for consumers. However when, for example, $\alpha \in (0, 1/2)$, such that network A has lower coverage, it charges higher fees compared to network B. The intuition is that in this case network A has relatively more off-net transactions, and so does not internalize the effect of higher n_A on reduced transactions (and hence profits) for network B. Network B, on the other hand, has relatively more on-net transactions, and so has more incentive to moderate its fees. The fact that in this case $n_A > n_B$ implies that transactions on network A are the most expensive, transactions on network B are least expensive, and the cost of off-net transactions (in either direction) is now intermediate. More structure is required in order to assess the impact of the regulation on consumer surplus. We therefore return to our running uniform distribution example.

Example. In the case where $G(v) = v$, the equilibrium fees are

$$n_A = \frac{3-\alpha}{8+3\alpha-3\alpha^2} \quad \text{and} \quad n_B = \frac{2+\alpha}{8+3\alpha-3\alpha^2}.$$

Figure 1(a) plots these fees, along with the average fees $t^*/2 = 1/4$ for on-net transactions and $t^{**}/2 = 1/3$ for off-net transactions that we computed earlier. Notice that n_A is decreasing in α while n_B is increasing in α. When $\alpha = 1/2$ both networks set their sender and receiver fees equal to 2/7. When, for example, $\alpha \to 0$ then network B charges $n_B = 1/4$ while network A charges $n_A = 3/8$; fees are symmetric and so the opposite of these as $\alpha \to 1$. (Intuitively, as $\alpha \to 0$ almost all transactions take place on B's network, and so network B sets each fee to be exactly one half of the $t^* = 1/2$ that it would charge absent interoperability. Meanwhile as $\alpha \to 0$, network A performs almost no transactions, but in relative terms almost all of them are off-net; since network B charges $n_B = 1/4 < t^{**}/2$, network A's off-net demand is larger than it was in Proposition 2, so it responds by increasing its fees above the $t^{**}/2 = 1/3$ that it would charge for off-net transactions absent the legislation.) Continuing with this uniform distribution example, one can also compute that consumer surplus is

$$\frac{(2-\alpha)(1+\alpha)(8-7\alpha+7\alpha^2)}{2(8+3\alpha-3\alpha^2)^2}.$$

Figure 1(b) plots this consumer surplus, along with consumer surplus without interoperability and with "unregulated" interoperability. Consumers are always worst off when there is no interoperability. Although it is hard to see from the figure, conditional on having interoperability, the equal fee regulation benefits consumers unless α is less than around 0.06 or above around 0.94. In other words, for most values of α, the reduction in off-net fees induced by the regulation outweighs the increase in on-net fees. We notice however that the gain in consumer surplus tends to be small relative to the case of unregulated interoperability.

5.2.2 Behavioral Consumers

Consider now the effects of the same "equal fee regulation" in a market in which consumers hold biased beliefs both on sender and on receiver fees, as in Section 4.2.[12] Given consumers' endogenous beliefs, we can again rewrite the networks' optimization

[12]This is made for simplicity of exposition; as we show later, assuming that consumers are only biased about receiver fees does not affect our analysis.

problems as a function of on-net fees only. Network A's problem then becomes

$$\max_{n_{s,A},n_{r,A}} \alpha^2(n_{s,A}+n_{r,A})[1-G(n_{s,A}+n_{r,A})]$$
$$+\alpha(1-\alpha)n_{s,A}[1-G(n_{s,A}+n_{r,A})]+\alpha(1-\alpha)n_{r,A}[1-G(n_{s,B}+n_{r,B})],$$

while network B's problem becomes

$$\max_{n_{s,B},n_{r,B}} (1-\alpha)^2(n_{s,B}+n_{r,B})[1-G(n_{s,B}+n_{r,B})]$$
$$+\alpha(1-\alpha)n_{s,B}[1-G(n_{s,B}+n_{r,B})]+\alpha(1-\alpha)n_{r,B}[1-G(n_{s,A}+n_{r,A})].$$

In order to solve for equilibrium fees, it is useful to let $N_A \equiv n_{s,A}+n_{r,A}$ and $N_B \equiv n_{s,B}+n_{r,B}$ denote the two networks' total on-net fees. As a first step, we can then fix these total fees, and solve for how they should be optimally distributed across the sender and receiver sides. To ease the exposition, in the text we focus on network A, whose optimization problem can be re-expressed using this new notation as follows:

$$\max_{N_A,n_{s,A}} \alpha^2 N_A[1-G(N_A)]+\alpha(1-\alpha)\left\{n_{s,A}[1-G(N_A)]+(N_A-n_{s,A})[1-G(N_B)]\right\}.$$

Observe that, fixing N_A, generically network A's optimal solution is "bang-bang":

Lemma 3. Fixing $N_A \in (0,\bar{v})$, network A optimally sets $n_{s,A}=0$ if $N_A > N_B$, and optimally sets $n_{s,A}=N_A$ if $N_A < N_B$. (In the special case where $N_A = N_B$ network A is indifferent over all $n_{s,A}$.)

The lemma is very intuitive. Consider, for example, the case where $N_A > N_B$, or equivalently $1-G(N_A) < 1-G(N_B)$, such that there are more off-net transactions from B to A than in the other direction. The lemma says that network A should optimally set $n_{s,A}=0$ and $n_{r,A}=N_A$, i.e., it rips off consumers who receive off-net on its network because they are more numerous than the consumers who send off-net from its network. Using Lemma 3 we can then write network A's profit solely as a function of total on-net fees:

$$\pi_A(N_A,N_B)=\begin{cases}\alpha N_A[1-G(N_A)] & \text{if } N_A<N_B \\ \alpha^2 N_A[1-G(N_A)]+\alpha(1-\alpha)N_A[1-G(N_B)] & \text{if } N_A\ge N_B\end{cases}.$$

Using this profit function, we can now solve for network A's best response N_A given any conjectured N_B that it expects network B to charge. Specifically, suppose we fix some $N_B \in (0,\bar{v})$. As we vary N_A over the interval $[0,N_B]$, network A's profit $\alpha N_A[1-G(N_A)]$ is quasiconcave; hence if $N_B \le t^*$ then network A's profit is monotonically increasing in N_A over this interval, and otherwise it first increases and then decreases in N_A. At the point where $N_A = N_B$ network A's profit can be seen to kink upwards. Then, if we impose our earlier Assumption 2, network A's profit is concave as we vary N_A over the interval $[N_B, \bar{v}]$. Using these observations, we can then prove the following:

Lemma 4. Suppose Assumption 2 holds. There exists a critical threshold $\overline{N_B} \in (t^*, \bar{v})$ such that if $N_B > \overline{N_B}$ then network A optimally sets $N_A = t^*$, and if $N_B < \overline{N_B}$ then network A optimally sets the unique $N_A > \max\{N_B, t^*\}$ that solves

$$\alpha[1 - G(N_A) - N_A g(N_A)] + (1 - \alpha)[1 - G(N_B)] = 0.$$

(If $N_B = \overline{N_B}$ then network A is indifferent between t^* or the $N_A > \max\{N_B, t^*\}$ that solves the last equation.)

The lemma shows that when N_B is relatively large—such that customers on network A will receive relatively few off-net transactions—network A prefers to frontload all fees on the sender side, and set its sender fee to the same level as absent interoperability. On the other hand, when N_B is relatively small—such that customers on network A will receive relatively many off-net transactions—network A prefers to load all fees on the receiver side, and set its receiver fee above t^* (and also above N_B) to take advantage of these off-net transactions. Notice that network A's best response is never equal to N_B; intuitively, this is caused by the kink in network A's profit function around the point $N_A = N_B$ that we described earlier.

We can also fix N_A and perform the same exercise for network B. We again find that for a given N_A network B's allocation of its total fee N_B between the sender and receiver sides is "bang-bang". Similarly, we can also derive a cutoff value on N_A that determines whether network B sets N_B to t^* or to a value above N_A. And, importantly, we also find that network B never finds it optimal to set $N_B = N_A$ due to a kink in its profit function.

Using the above results, we can now solve for equilibrium fees. We focus throughout on pure strategy equilibrium. As a first step, notice that there is no symmetric pure strategy Nash equilibrium—because this would require the two networks to charge the same total fee, but we have argued above that neither network best responds by matching the other network's total fee. Instead the game has at least one (but no more than two) asymmetric equilibrium, as the following proposition shows:

Proposition 6. Suppose consumers have biased beliefs about off-net sender and receiver fees, and that Assumption 2 holds. Under equal fee regulation, there are two

candidate asymmetric pure strategy Nash equilibria, at least one of which exists for any $\alpha \in (0,1)$:
i) One candidate equilibrium has network A charge $n_{s,A} = N_A = t^*$ and $n_{r,A} = 0$, and network B charge $n_{s,B} = 0$ and $n_{r,B} = N_B \in (t^*, \bar{v})$ which is the unique solution to

$$(1-\alpha)[1 - G(N_B) - N_B g(N_B)] + \alpha[1 - G(t^*)] = 0.$$

A sufficient condition for this equilibrium to exist is $\alpha \geq 1/2$.
ii) Another candidate equilibrium has network B charge $n_{s,B} = N_B = t^*$ and $n_{r,B} = 0$, and network A charge $n_{s,A} = 0$ and $n_{r,A} = N_A \in (t^*, \bar{v})$ which is the unique solution to

$$\alpha[1 - G(N_A) - N_A g(N_A)] + (1-\alpha)[1 - G(t^*)] = 0.$$

A sufficient condition for this equilibrium to exist is $\alpha \leq 1/2$.

Unfortunately it is not possible, without imposing more structure on G, to determine necessary and sufficient conditions for the two equilibria to exist.[13] (We provide such conditions shortly, in our running uniform distribution example.) However the sufficient conditions provided in the proposition are quite intuitive. For instance, consider part i) of the proposition, which characterizes a putative equilibrium in which network A sets a low fee while network B sets a high fee, and shows that such an equilibrium is guaranteed to exist if $\alpha \geq 1/2$. Intuitively, when $\alpha \geq 1/2$, (in relative terms) network A is less dependent and network B is more dependent on off-net transactions. It is therefore natural that network A should set $N_A = n_{s,A} = t^*$ and focus more on on-net transactions. Given consumers' behavioral bias, this in turn stimulates a relatively high volume of off-net transactions from A to B. It is then natural that network B should set $N_B = n_{r,B} > t^*$ and focus on exploiting consumers who receive off-net on its network.

Now consider the impact of the equal fee regulation on consumer surplus, which is equal to

$$CS^R = \alpha^2 \int_{n_{s,A}+n_{r,A}}^{\bar{v}} (v - n_{s,A} - n_{r,A}) dG(v) + (1-\alpha)^2 \int_{n_{s,B}+n_{r,B}}^{\bar{v}} (v - n_{s,B} - n_{r,B}) dG(v)$$
$$+\alpha(1-\alpha) \int_{n_{s,A}+n_{r,A}}^{\bar{v}} (v - n_{s,A} - n_{r,B}) dG(v) + \alpha(1-\alpha) \int_{n_{s,B}+n_{r,B}}^{\bar{v}} (v - n_{s,B} - n_{r,A}) dG(v).$$

The regulation again creates a trade-off. On the one hand, as with rational consumers,

[13]Nevertheless it is easy to see that for α in a neighborhood of 0 or 1 only one equilibrium exists. For example, note that as $\alpha \to 0$, the putative equilibrium N_B in Proposition 6i) tends to t^*, but then it follows from Lemma 4 that network A would prefer to deviate and set N_A strictly above t^*.

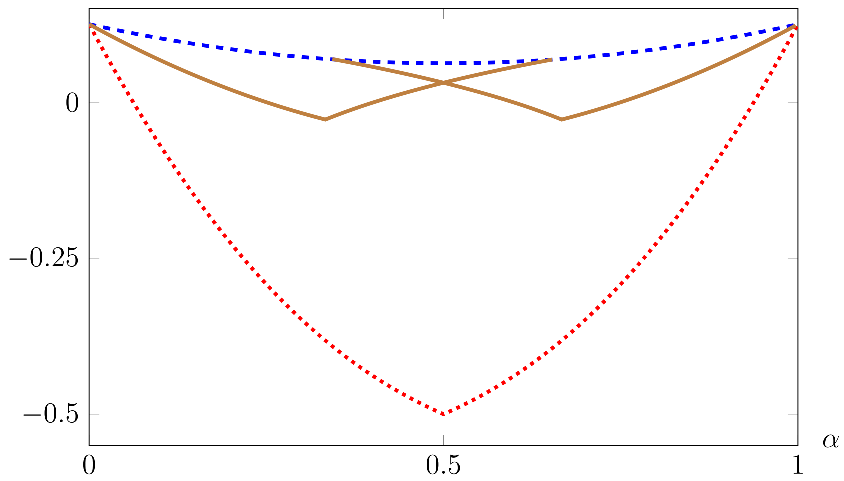


Figure 2: Consumer surplus under a behavioral bias on off-net fees
(The dashed curve is surplus without interoperability, the dotted curve is surplus with "unregulated" interoperability, and the thick curve is surplus with equal fee regulation)

the regulation (weakly) raises the fees paid by on-net consumers, and strictly increases them on one of the two networks. On the other hand, off-net transactions are now cheaper: instead of incurring a total fee of $\bar{v}$, transfers in one direction incur zero sender and receiver fees, while transfers in the other direction incur a sender fee of t^* and a receiver fee between t^* and $\bar{v}$. As usual, it is difficult to make further progress for a general G, and so we now switch to the example.

Example. Return to our running example with $G(v) = v$. Consider the first possible equilibrium described in Proposition 6. One can check that this equilibrium exists if and only if $\alpha \geq 6 - 4\sqrt{2}$, and that the total fees charged by the two networks are

$$N_A^{R1} = \frac{1}{2} \quad \text{and} \quad N_B^{R1} = \begin{cases} \frac{2-\alpha}{4(1-\alpha)} & \text{if } 6 - 4\sqrt{2} < \alpha < 2/3 \\ 1 & \text{if } \alpha \geq 2/3 \end{cases},$$

leading to consumer surplus

$$CS^{R1} = \begin{cases} \frac{4-8\alpha+\alpha^2+2\alpha^3}{32(1-\alpha)} & \text{if } 6 - 4\sqrt{2} < \alpha < 2/3 \\ \frac{\alpha(4\alpha-3)}{8} & \text{if } \alpha \geq 2/3 \end{cases},$$

where $R1$ denotes that this is the first possible equilibrium with regulation. Next, consider the second possible equilibrium described in Proposition 6. One can check that this equilibrium exists if and only if $\alpha \leq -5 + 4\sqrt{2}$, and that the total fees charged by the two networks are

$$N_A^{R2} = \begin{cases} 1 & \text{if } \alpha < 1/3 \\ \frac{1+\alpha}{4\alpha} & \text{if } 1/3 \leq \alpha \leq -5 + 4\sqrt{2} \end{cases} \quad \text{and} \quad N_B^{R2} = \frac{1}{2},$$

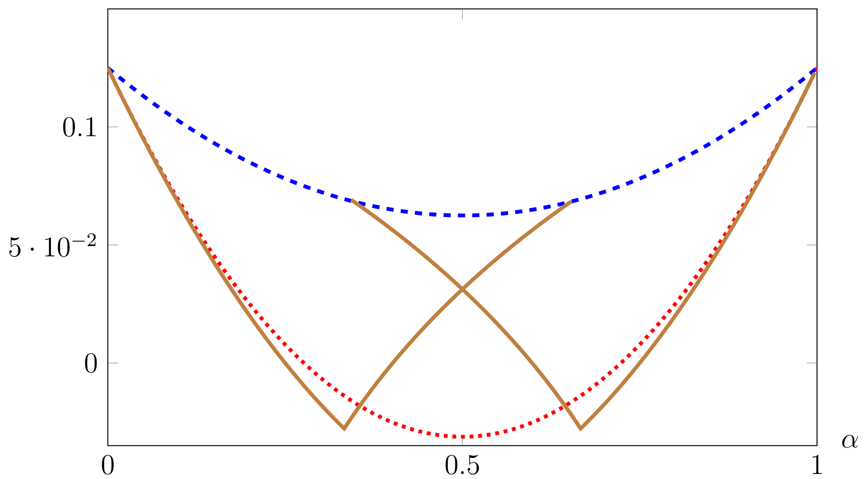


Figure 3: Consumer surplus under a behavioral bias on off-net receiver fees
(The dashed curve is surplus without interoperability, the dotted curve is surplus with "unregulated" interoperability, and the thick curve is surplus with equal fee regulation)

leading to consumer surplus

$$CS^{R2} = \begin{cases} -\frac{(1-\alpha)(4\alpha-1)}{8} & \text{if } \alpha < 1/3 \\ \frac{-1+(7-2\alpha)\alpha^2}{32\alpha} & \text{if } 1/3 \leq \alpha \leq -5+4\sqrt{2} \end{cases},$$

where $R2$ denotes that this is the second possible equilibrium with regulation. Figure 2 plots consumer surplus in these two equilibria, as well as for the case without interoperability and with "unregulated" interoperability. Conditional on having interoperability, the regulation always improves consumer surplus. However, except for a very thin range of parameters, consumers are actually better off without interoperability.

Remark: Bias only on Receiver fees

Suppose that consumers have biased beliefs only on receiver fees. Notice that the networks have exactly the same optimization problems as when they faced regulation and the behavioral bias was on both sender and receiver off-net fees. Hence equilibrium fees are exactly the same as in Proposition 6, and consumer surplus is the same as the expression given in that section. We can therefore turn straight to our running example.
Example Again consider the case $G(v) = v$. Figure 3 plots consumer surplus. For almost all values of α consumers are better off without interoperability, due to the high off-net fees that networks charge. Moreover, conditional on interoperability, the equal fee regulation introduces a trade-off, since it raises on-net transaction fees but reduces off-net transaction fees. For intermediate values of α the regulation benefits consumers, but otherwise it harms them.

5.3 Caps on Senders and Receivers Fees

As an alternative policy, suppose that regulation only imposes a cap either on sender or receiver fees, and let us set this cap to zero. This also allows us to study settings in which only one party pays the fee, not necessarily because of regulation.

Assume as before that the sender cares about the total fee incurred for the transaction, irrespective of who formally pays it. It is immediate to observe that, absent interoperability, each network charges $n^* = t^*$ as in the previous analysis, regardless of who pays the fee. A transaction occurs if and only if $v \geq t$, so a network's (on-net) transaction profit is proportional to $t[1 - G(t)]$ which is maximized at t^* by definition.

We now consider the effect of the policy in a market with interoperability, distinguishing again the case with rational or behavioral consumers.

5.3.1 Rational consumers

Under interoperability, networks charge one fee for on-net transactions and another fee for off-net transactions. The two transaction types are separable. On-net transactions are again priced at t^*, irrespective of whether it is the sender or receiver that pays.

Differently from the previous analysis, notice that now off-net transactions are also priced at t^*, again regardless of whether the sender or receiver pays. The reason is that whichever network collects the fee for a given off-net transaction charges t and gets profits proportional to $t[1 - G(t)]$, which is again maximized at t^*. Intuitively, there is no double marginalisation problem now since only one side is choosing the total fee *and* is collecting all of it. Hence the cap unambiguously benefits consumers, and further increases the benefits of interoperability.

5.3.2 Behavioural consumers when senders pay

Suppose that only senders pay the fee. Suppose that when sending money off-net, senders believe that the fee will be the same as the on-net one. Clearly it is then optimal to set $f = \bar{v}$, and then network A will set n_A in order to

$$\max \ \alpha^2 n_A \left[1 - G\left(n_A\right)\right] + \alpha\left(1 - \alpha\right) \bar{v} \left[1 - G\left(n_A\right)\right],$$

while network B will set n_B in order to

$$\max \ \left(1 - \alpha\right)^2 n_B \left[1 - G(n_B)\right] + \alpha\left(1 - \alpha\right) \left[1 - G\left(n_B\right)\right].$$

Notice that these are the same payoff functions as in Section 4.2; hence, equilibrium fees here will the same as in Proposition 4. That is, the regulation has no impact on fees in this setting.

5.3.3 Behavioural consumers when receivers pay

Suppose instead that only receivers pay the fee. Suppose that senders believe that whatever they pay to receive off-net, is what consumers on the other network will also pay to receive off-net. Network A say does not care about the volume of $A \to B$ transactions, and hence sets f_A to maximize its revenue from $B \to A$ transactions, which are independent of f_A. Meanwhile, on-net transactions can be treated separately and are again therefore priced at t^*. It then follows that equilibrium fees are $n^* = t^*$ and $f^* = \bar{v}$.[14] Interoperability is unambiguously bad: it has no effect on on-net fees, but causes consumers to over-pay for off-net transactions.

6 Conclusion

We have developed a simple model of digital payments across networks. We have shown that introducing interoperability may reduce consumers' welfare in a setting in which consumers are poorly informed about the fees for off-net transactions. While off-net transactions tend to be overpriced, regulations that impose caps on fees need not make consumers better off.

Our model is deliberately stylized and it can be enriched along many important dimensions. One could introduce the possibility that new firms enter the market and/or that existing firms may invest to expand their coverage. Moreover, rather than assuming that each provider acts as a local monopolist, one may consider the possibility that (some) villages are served by different providers, thereby allowing providers to compete with each other to attract consumers. We expect that interoperabilty may have interesting, and possibly different, effects on consumers' welfare in these settings.

[14] The fees would be exactly the same if senders believed that whatever they pay to receive on their own network, is what receivers on the other network will pay to receive their money.

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Appendix

Proof of Proposition 1. Letting t denote its total on-net fee, a network wishes to $\max_t t[1-G(t)]$. Note that log-concavity of g implies log-concavity of $1-G$, which in turn implies that the network's profit is log-concave and hence also quasiconcave. Taking the first order condition and rearranging gives equation (1). Note that log-concavity of $1-G$ implies that the right-hand side of (1) is decreasing in t^* and hence it has a unique solution. □

Proof of Proposition 2. First, consider on-net transaction fees. It is immediate that network A's profit is maximized by setting $n_{s,A}+n_{r,A}=t^*$, and network B's profit is maximized by setting $n_{s,B}+n_{r,B}=t^*$. Second, consider off-net fees associated with transfers from network A to network B. Log-concavity of $1-G$ ensures that both networks' profits from these transactions are quasiconcave in $f_{s,A}$ and $f_{r,B}$ respectively. Taking first order conditions, we obtain

$$f_{s,A}=\frac{1-G(f_{s,A}+f_{r,B})}{g(f_{s,A}+f_{r,B})} \quad \text{and} \quad f_{r,B}=\frac{1-G(f_{s,A}+f_{r,B})}{g(f_{s,A}+f_{r,B})}.$$

Hence $f_{s,A}=f_{r,B}$. Setting the two fees equal to $t^{**}/2$, and substituting this in to one of these equations and simplifying, we find that

$$\frac{t^{**}}{2}=\frac{1-G(t^{**})}{g(t^{**})}. \tag{5}$$

This equation has a unique solution because log-concavity of $1-G$ implies that the right-hand side is decreasing in t^{**}. Third, the same steps can be used to establish that $f_{s,B}=f_{r,A}=t^{**}$. Finally, we prove that $t^{**}\in(t^*,2t^*)$. On the way to a contradiction, suppose that $t^{**}\le t^*$: the left-hand side of (5) would be strictly less than t^*, while using log-concavity of $1-G$ the right-hand side of (5) would be weakly greater than $[1-G(t^*)]/g(t^*)\equiv t^*$, which is impossible. Similarly, on the way to a contradiction, suppose that $t^{**}\ge 2t^*$: the left-hand side of (5) would weakly exceed t^*, while using log-concavity of $1-G$ the right-hand side would be strictly smaller than $[1-G(t^*)]/g(t^*)\equiv t^*$, which is again impossible. Hence $t^{**}\in(t^*,2t^*)$ as claimed. □

Proof of Lemma 1. We prove this for network A. (The steps for network B are exactly the same and hence omitted.) First, suppose $F_B\le t^*$. Clearly network A's profit is globally increasing in F_A and hence maximized at $F_A=\bar{v}$. Second, then, suppose $F_B>t^*$. Following arguments in the text, network A's profit is maximized at either $F_A=t^*$, whereupon it earns $\alpha(1-\alpha)t^*[1-G(t^*)]$, or at $F_A=\bar{v}$, whereupon it earns $\alpha(1-\alpha)\bar{v}[1-G(F_B)]$. The former exceeds the latter if and only if the inequality in the lemma is satisfied. Note that this inequality is not satisfied if $F_B=t^*$, but is satisfied if $F_B=\bar{v}$. □

Proof of Proposition 3. Part i) was already explained in the text, so here we focus on off-net fees in part ii). Using Lemma 1, we must have $F_A \in \{t^*, \bar{v}\}$ and $F_B \in \{t^*, \bar{v}\}$. Moreover, again using Lemma 1, if network A sets $F_A = t^*$ then network B best responds with $F_B = \bar{v}$, and if $F_B = \bar{v}$ then network A best responds with $F_A = t^*$—and hence this constitutes an equilibrium. Similarly, if network A sets $F_A = \bar{v}$ then network B best responds with $F_B = t^*$, and if $F_B = t^*$ then network A best responds with $F_A = \bar{v}$—hence this also constitutes an equilibrium. Moreover, it is clear there are no other pure strategy equilibria. □

Proof of Proposition 4. Part i) of the proposition (concerning off-net fees) follows from arguments in the text. Now consider on-net fees. Note that only the total fees $n_{s,A}+n_{r,A}$ and $n_{s,B} + n_{r,B}$ can be determined. Consider network A. The derivative of its profit with respect to $n_{s,A} + n_{r,A}$ is proportional to

$$\alpha \left[\frac{1 - G(n_{s,A} + n_{r,A})}{g(n_{s,A} + n_{r,A})} \right] - \alpha(n_{s,A} + n_{r,A}) - (1 - \alpha)\bar{v}.$$

Note that this is strictly decreasing in $n_{s,A}+n_{r,A}$ given that $1-G$ is log-concave, and is strictly negative as $n_{s,A}+n_{r,A} \to \bar{v}$. Therefore if it is weakly negative as $n_{s,A}+n_{r,A} \to 0$, the optimum has $n_{s,A} + n_{r,A} = 0$, and otherwise the optimum is the unique $n_{s,A} + n_{r,A}$ which sets the above equation to zero. This explains part ii) of the proposition. Part iii) is proved in exactly the same way. □

Proof of Lemma 2. We proceed using the following steps.

Step 1. Differentiating network A's profit with respect to $n_{s,A}$ and $n_{r,A}$ gives respectively the following two expressions:

$$\begin{aligned} &\alpha^2[1 - G(n_{s,A} + n_{r,A}) - (n_{s,A} + n_{r,A})g(n_{s,A} + n_{r,A})] \quad (6)\\ &+\alpha(1 - \alpha)[1 - G(n_{s,A} + n_{r,B}) - n_{s,A}g(n_{s,A} + n_{r,B})], \quad \text{and} \\ &\alpha^2[1 - G(n_{s,A} + n_{r,A}) - (n_{s,A} + n_{r,A})g(n_{s,A} + n_{r,A})] \quad (7)\\ &+\alpha(1 - \alpha)[1 - G(n_{s,B} + n_{r,A}) - n_{r,A}g(n_{s,B} + n_{r,A})]. \end{aligned}$$

Similarly, differentiating B's profit respect to $n_{s,B}$ and $n_{r,B}$ gives us

$$\begin{aligned} &(1 - \alpha)^2[1 - G(n_{s,B} + n_{r,B}) - (n_{s,B} + n_{r,B})g(n_{s,B} + n_{r,B})] \quad (8)\\ &+\alpha(1 - \alpha)[1 - G(n_{s,B} + n_{r,A}) - n_{s,B}g(n_{s,B} + n_{r,A})], \quad \text{and} \\ &(1 - \alpha)^2[1 - G(n_{s,B} + n_{r,B}) - (n_{s,B} + n_{r,B})g(n_{s,B} + n_{r,B})] \quad (9)\\ &+\alpha(1 - \alpha)[1 - G(n_{s,A} + n_{r,B}) - n_{r,B}g(n_{s,A} + n_{r,B})]. \end{aligned}$$

Step 2. Next, we prove that $n_{s,A}, n_{r,A}, n_{s,B}, n_{r,B} < \bar{v}$. We do this for $n_{s,A}$ and $n_{r,A}$ (the proof for $n_{s,B}$ and $n_{r,B}$ is identical and therefore omitted). It cannot be optimal for network A to choose $n_{s,A}, n_{r,A} \geq \bar{v}$; it would make zero transactions and so earn zero profit, whereas if it deviated to $n_{s,A} + n_{r,A} \in (0, \bar{v})$ it would earn strictly positive profit from on-net transactions, which is a contradiction. Similarly it cannot be optimal for network A to choose $0 \leq n_{s,A} < \bar{v} \leq n_{r,A}$; it would earn $\alpha(1-\alpha)n_{s,A}[1-G(n_{s,A}+n_{r,B})]$, whereas if it deviated to $n_{r,A} \in (0, \bar{v} - n_{s,A})$ it would earn at least an additional $\alpha^2(n_{s,A} + n_{r,A})[1 - G(n_{s,A} + n_{r,A})] > 0$. Using the same logic it also cannot be optimal for network A to choose $0 \leq n_{r,A} < \bar{v} \leq n_{s,A}$.

Step 3. Next, we prove that $n_{s,A}, n_{r,A}, n_{s,B}, n_{r,B} > 0$. We again do this for $n_{s,A}$ and $n_{r,A}$. It cannot be optimal for network A to choose $n_{s,A}, n_{r,A} = 0$; it would earn zero profit, whereas if it deviated to $n_{s,A} + n_{r,A} \in (0, \bar{v})$ it would earn strictly positive profit, a contradiction. Similarly it cannot be optimal for network A to choose $n_{s,A} = 0 < n_{r,A}$. On the way to a contradiction, suppose this is optimal. Then the derivative of network A's profit with respect to $n_{s,A}$ should be weakly negative around $n_{s,A} = 0$, i.e., using equation (6), it should be that

$$\alpha^2[1 - G(n_{r,A}) - n_{r,A}g(n_{r,A})] + \alpha(1 - \alpha)[1 - G(n_{r,B})] \leq 0.$$

However, since $n_{r,B} < \bar{v}$ (from the previous step) this inequality can only hold if $1 - G(n_{r,A}) - n_{r,A}g(n_{r,A}) < 0$. Hence the first term in (7) must also be strictly negative. Moreover, note that $1 - G(n_{r,A}) - n_{r,A}g(n_{r,A}) < 0$ if and only if $n_{r,A} > t^*$; this, combined with log-concavity of G implies that the second term in (7) is either zero (if $n_{s,B} + n_{r,A} > \bar{v}$) or strictly negative (otherwise). Hence we conclude that $n_{r,A} \in (0, \bar{v})$ (by assumption) and (7) is strictly negative, such that network A's profit is locally strictly decreasing in $n_{r,A}$. But this is impossible, and hence a contradiction. Using a similar argument it also cannot be optimal for network A to choose $n_{r,A} = 0 < n_{s,A}$.

Step 4. The previous two steps imply that we have an interior solution, and hence equations (6)-(9) should bind with equality. Setting (6) and (7) to zero and combining them, we obtain

$$1 - G(n_{s,A} + n_{r,B}) - n_{s,A}g(n_{s,A} + n_{r,B}) = 1 - G(n_{s,B} + n_{r,A}) - n_{r,A}g(n_{s,B} + n_{r,A}). \quad (10)$$

Similarly, setting (8) and (9) to zero and combining them, we obtain

$$1 - G(n_{s,A} + n_{r,B}) - n_{r,B}g(n_{s,A} + n_{r,B}) = 1 - G(n_{s,B} + n_{r,A}) - n_{s,B}g(n_{s,B} + n_{r,A}). \quad (11)$$

Adding these last two equations together, we have that

$$\begin{aligned} & 2[1 - G(n_{s,A} + n_{r,B})] - (n_{s,A} + n_{r,B})g(n_{s,A} + n_{r,B}) \\ = \; & 2[1 - G(n_{s,B} + n_{r,A})] - (n_{s,B} + n_{r,A})g(n_{s,B} + n_{r,A}). \end{aligned} \quad (12)$$

We will use this equation to argue that $n_{s,A} + n_{r,B} = n_{s,B} + n_{r,A} < \bar{v}$. As a first step, notie that we must have $n_{s,A} + n_{r,B}, n_{s,B} + n_{r,A} < \bar{v}$. On the way to a contradiction, suppose that $n_{s,A} + n_{r,B}, n_{s,B} + n_{r,A} \geq \bar{v}$, which means that each network makes zero off-net transactions. Consider network A. It must be that $n_{s,A} + n_{r,A} \in (0, \bar{v})$, otherwise network A would be earning zero profit, which by earlier arguments is impossible. However we also know from Step 2 that $n_{r,B} < \bar{v}$, and so network A could keep its total on-net fee $n_{s,A} + n_{r,A}$ unchanged but rebalance the fees in such a way that $n_{s,A} + n_{r,B} < \bar{v}$. This would leave its profit from on-net transactions unchanged, but allow it to earn strictly positive profit from off-net transactions from its own network to network B, a contradiction.

Next, note that Assumption 2 implies that the left-hand side of (12) is strictly decreasing in $n_{s,A} + n_{r,B} < \bar{v}$, and that the right-hand side of (12) is strictly decreasing in $n_{s,B} + n_{r,A} < \bar{v}$. Hence it must be that $n_{s,A} + n_{r,B} = n_{s,B} + n_{r,A} < \bar{v}$.

Step 5. The previous step has established that $n_{s,A} + n_{r,B} = n_{s,B} + n_{r,A} < \bar{v}$. Substituting this into (10) and (11), we obtain that $n_{s,A} = n_{r,A}$ and $n_{s,B} = n_{r,B}$. Writing $n_{s,A} = n_{r,A} = n_A$ and $n_{s,B} = n_{r,B} = n_B$ and substituting this back into, say, equations (6) and (8) and setting them equal to zero gives us (3) and (4).

Step 6. Finally, we need to show that profits are maximized at the solution we have just derived. Consider network A. Letting π_A denote its profit, the elements in its Hessian matrix are:

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial (n_{s,A})^2} &= -\alpha^2[2g(n_{s,A} + n_{r,A}) + (n_{s,A} + n_{r,A})g'(n_{s,A} + n_{r,A})] \\
&\quad -\alpha(1-\alpha)[2g(n_{s,A} + n_{r,B}) + n_{s,A}g'(n_{s,A} + n_{r,B})], \\
\frac{\partial^2 \pi}{\partial (n_{r,A})^2} &= -\alpha^2[2g(n_{s,A} + n_{r,A}) + (n_{s,A} + n_{r,A})g'(n_{s,A} + n_{r,A})] \\
&\quad -\alpha(1-\alpha)[2g(n_{s,B} + n_{r,A}) + n_{r,A}g'(n_{s,B} + n_{r,A})], \\
\frac{\partial^2 \pi}{\partial n_{s,A} \partial n_{r,A}} &= -\alpha^2[2g(n_{s,A} + n_{r,A}) + (n_{s,A} + n_{r,A})g'(n_{s,A} + n_{r,A})].
\end{aligned}$$

Given Assumption 2 it is easy to check that the Hessian is negative definite at the equilibrium fees (because, as established above, $n_{s,A} + n_{r,B} = n_{s,B} + n_{r,A} < \bar{v}$ at the equilibrium), and otherwise the Hessian is either negative definite or negative semi-definite. □

Proof of Proposition 5. The proof proceeds using three steps.

Step 1. We prove that in equilibrium $n_A + n_B \in (t^*, t^{**})$. On the way to a contradiction, suppose that $n_A + n_B \leq t^*$. At least one of n_A and n_B must then be weakly less than $t^*/2$. Suppose without loss of generality that $n_A \leq t^*/2$, in which case the first term in (3) is weakly positive. In the preceding proof we argued that $n_A + n_B < \bar{v}$, and hence $g(n_A + n_B) > 0$. Therefore the second term in (3) is strictly larger than $1-\alpha$ multiplied by $1 - G(n_A + n_B) - (n_A + n_B)g(n_A + n_B)$, which is itself weakly positive given the supposition that $n_A + n_B \leq t^*$. But then the left-hand side of (3) is strictly positive, which is impossible. On the way to another contradiction, suppose that $n_A + n_B \geq t^{**}$. At least one of n_A and n_B must then be weakly greater than $t^{**}/2$. Suppose without loss of generality that $n_A \geq t^{**}/2$. Then, since $t^{**} > t^*$, we have $n_A > t^*/2$ and thus the first term in (3) is strictly negative. Again, we also know that $g(n_A + n_B) > 0$. Therefore the second term in (3) is weakly smaller than $1 - G(n_A + n_B) - t^{**}g(n_A + n_B)/2$, which itself is weakly negative given the supposition that $n_A + n_B \geq t^{**}$. But then the left-hand side of (3) is strictly negative, which is impossible.

Step 2. We prove how the ranking of n_A and n_B depends on α. To do this, multiply (3) by α, multiply (4) by $1-\alpha$, then subtract one from the other to get

$$\begin{aligned}
&\alpha^2[1 - G(2n_A) - 2n_A g(2n_A)] - (1-\alpha)^2[1 - G(2n_B) - 2n_B g(2n_B)] \\
=\ &\alpha(1-\alpha)(n_A - n_B)g(n_A + n_B). \qquad (13)
\end{aligned}$$

Suppose $\alpha \in (0, 1/2)$. We prove that $n_A > n_B$. On the way to a contradiction, suppose that $n_A \leq n_B$. The right-hand side of (13) is then weakly negative. In addition, the supposition that $n_A \leq n_B$ implies that $1 - G(2n_A) - 2n_A g(2n_A) \geq 1 - G(2n_B) - 2n_B g(2n_B)$; moreover, at least the right-hand side of this inequality must be negative, given that $n_A \leq n_B$ and $n_A + n_B > t^*$ jointly imply that $n_B > t^*$. But then the left-hand side of (13) is strictly positive, which is a contradiction.

Using the same steps, one can prove that if $\alpha \in (1/2, 1)$ then $n_A < n_B$, and that if $\alpha = 1/2$ then $n_A = n_B$.

Step 3. Finally, we prove that $n_A, n_B > t^*/2$. Consider first the case $\alpha \in (0, 1/2)$, which from the previous step implies that $n_A > n_B$. On the way to a contradiction, suppose that $n_B \leq t^*/2$. The first term in (4) must be weakly positive, and hence the second term in (4) must be weakly negative. The latter in turn implies that the second term in (3) is strictly negative, given that we have shown earlier that $n_A + n_B < \bar{v}$ and hence $g(n_A + n_B) > 0$. But this implies that the first term in (3) must be strictly positive, which can only be true if $n_A \leq t^*/2$. However then we have $n_A + n_B \leq t^*$, which contradicts what we showed in Step 1 of this proof. Hence $n_A, n_B > t^*/2$. The case $\alpha \in (1/2, 1)$ can be proved in the same way. Consider finally the case $\alpha = 1/2$, which from the previous step implies $n_A = n_B$. In Step 1 we concluded that $n_A + n_B > t^*$, so it follows immediately that $n_A = n_B > t^*/2$. □

Proof of Lemma 3. The derivative of network A's profit function (in the equation directly before the lemma) with respect to $n_{s,A}$ is $G(N_B) - G(N_A)$, which is independent of $n_{s,A}$. The claimed "bang-bang" result then follows immediately. □

Proof of Lemma 4. First, note that if $N_B \leq t^*$ then it is strictly dominated for network A to set $N_A < N_B$ given that its profit $\alpha N_A[1 - G(N_A)]$ is strictly increasing over this range. Hence network A will optimally set

$$N_A = \arg \max_{N_A \geq N_B} \alpha^2 N_A[1 - G(N_A)] + \alpha(1 - \alpha)N_A[1 - G(N_B)]. \tag{14}$$

Taking a first-order condition, and canceling an α term, we obtain

$$\alpha[1 - G(N_A) - N_A g(N_A)] + (1 - \alpha)[1 - G(N_B)] = 0. \tag{15}$$

(Second-order conditions are satisfied, and this equation also has a unique solution $N_A > t^*(> N_B)$, given Assumption 2.) Second, consider $N_B \in (t^*, \bar{v}]$. Conditional on setting $N_A < N_B$, network A should optimally set $N_A = t^*$ and earn profit $\alpha t^*[1 - G(t^*)]$ which is independent of N_B. Conditional on setting $N_A \geq N_B$, network A's optimized profit is strictly decreasing in N_B by a simple revealed preference argument. (In particular, consider N'_B and $N''_B > N'_B$, and let N'_A and N''_A be the associated optimal total fees set by network A. Consider a decrease in N_B from N''_B to N'_B: fixing $N_A = N''_A$, network A's profit strictly increases, and then since it can reoptimize, network A's profit further weakly increases.) Moreover, as $N_B \to \bar{v}$, network A's profit from setting $N_A \geq N_B$ is zero, which is strictly less than $\alpha t^*[1 - G(t^*)]$. On the other hand, at $N_B = t^*$, it is clear that $\lim_{N_A \downarrow N_B} \partial \pi_A (N_A, N_B) / \partial N_A > 0$ and so network A is strictly better off setting $N_A > N_B$. These arguments together are sufficient to establish the existence of the cutoff $\overline{N_B}$, and to conclude that $\overline{N_B} \in (t^*, \bar{v})$.

Finally, consider properties of N_A when $N_B \in (t^*, \overline{N_B})$. Network A's optimal fee satisfies (14) and thus also (15). Clearly, given that for $N_B \in (t^*, \overline{N_B})$ we have established that $N_A \geq N_B$, it follows that $N_A > t^*$. The lemma also claims that $N_A > N_B$. On the way to a contradiction, suppose not, in which case $N_A = N_B$. But then network A's profit is

$$\alpha N_A[1 - G(N_A)] < \alpha t^*[1 - G(t^*)],$$

where the inequality uses the fact that $N_A = N_B > t^*$, as well as the fact that $t[1-G(t)]$ is by definition maximized at t^*. But this implies that network A would prefer to set $N_A = t^* < N_B$, which is a contradiction. $\square$

Proof of Proposition 6. We first sketch the proof of the equivalent of Lemma 4 for network B. Network B's problem can be written as

$$\max_{N_B, n_{s,B}} (1-\alpha)^2 N_B[1-G(N_B)] + \alpha(1-\alpha)\left\{n_{s,B}[1 - G(N_B)] + (N_B - n_{s,B})[1 - G(N_A)]\right\},$$

whereupon it is clear that for a fixed N_B the optimal $n_{s,B}$ is generically "bang-bang", and so network B's profit can be written as

$$\pi_B(N_A, N_B) = \begin{cases} (1-\alpha)N_B[1 - G(N_B)] & \text{if } N_B < N_A \\ (1-\alpha)^2 N_B[1 - G(N_B)] + \alpha(1-\alpha)N_B[1 - G(N_A)] & \text{if } N_B \geq N_A \end{cases}.$$

Following the same steps as in the proof of Lemma 4, we can then show that there exists a cutoff $\overline{N_A} \in (t^*, \bar{v})$ such that if $N_A > \overline{N_A}$ then network B optimally sets $N_B = t^*$, while if $N_A < \overline{N_A}$ then network B optimally sets

$$N_B = \arg\max_{N_B} (1-\alpha)^2 N_B[1 - G(N_B)] + \alpha(1-\alpha)N_B[1 - G(N_A)], \qquad (16)$$

or equivalently, the unique solution to the following equation

$$(1-\alpha)[1 - G(N_B) - N_B g(N_B)] + \alpha[1 - G(N_A)] = 0. \qquad (17)$$

We now derive the pure strategy equilibria of the game. We have already argued in the text that there is no such equilibrium with $N_A = N_B$.

Consider equilibria with $N_A < N_B$. We know from Lemma 4 that we must have $N_A = t^*$ and $N_B > \overline{N_B}$. Following arguments in the first part of this proof, it is immediate that network B is behaving optimally given that $N_A = t^*$, and that network B optimally chooses the unique N_B that solves (17), i.e.,

$$(1-\alpha)[1 - G(N_B) - N_B g(N_B)] + \alpha[1 - G(t^*)] = 0. \qquad (18)$$

It remains to check, however, that this N_B satisfies $N_B > \overline{N_B}$, such that network A is optimizing as well. A sufficient condition for this is

$$\left.\frac{\partial \pi_A(N_A, N_B)}{\partial N_A}\right|_{N_A = N_B} \leq 0 \iff \alpha[1 - G(N_B) - N_B g(N_B)] + (1-\alpha)[1 - G(N_B)] \leq 0.$$

Using (18), this simplifies down to

$$(1-\alpha)^2[1-G(N_B)] \leq \alpha^2[1-G(t^*)].$$

Given that $t^* < N_B$ and hence $1-G(t^*) > 1-G(N_B)$, a sufficient condition for this to hold is that $\alpha \geq 1/2$.

Now consider equilibria with $N_A > N_B$. We know from earlier in this proof that we must have $N_B = t^*$ and $N_A > \overline{N_A}$. Using Lemma 4 it is immediate that network A is pricing optimally given that $N_B = t^*$, and that N_A solves (15), i.e.,

$$\alpha[1-G(N_A)-N_A g(N_A)] + (1-\alpha)[1-G(t^*)] = 0. \tag{19}$$

It remains to check, then, that this N_A satisfies $N_A > \overline{N_A}$. Again, a sufficient condition for this is that

$$\left.\frac{\partial \pi_B(N_A, N_B)}{\partial N_B}\right|_{N_B=N_A} \leq 0 \iff (1-\alpha)[1-G(N_A)-N_A g(N_A)] + \alpha[1-G(N_A)] \leq 0.$$

Using (19) this simplifies to

$$\alpha^2[1-G(N_A)] \leq (1-\alpha)^2[1-G(t^*)],$$

for which a sufficient condition is $\alpha \leq 1/2$.

We have therefore established that there are at most two asymmetric pure strategy Nash equilibria. We have derived a sufficient condition $\alpha \geq 1/2$ for one of them, and a sufficient condition $\alpha \leq 1/2$ for the other of them, to exist. We are therefore guaranteed that there exists at least one such equilibrium. (And for α in a neighborhood of 1/2 both equilibria must exist.) □