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“Tying with Network Effects”

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Tying with Network Effects*

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Abstract

We develop a leverage theory of tying in markets with network effects. When a monopolist in one market cannot perfectly extract surplus from consumers, tying can be a mechanism through which unexploited consumer surplus is used as a demand-side leverage to create a "quasi-installed base" advantage in another market characterized by network effects. Our mechanism does not require any precommitment to tying; rather, tying emerges as a best response that lowers the quality of tied-market rivals. While tying can lead to exclusion of tied-market rivals, it can also expand use of the tying product, leading to ambiguous welfare effects.

1 Introduction

The leverage theory of tying typically considers the following scenario: There is a monopolistic firm in one market (say A). This firm, however, faces competition in another

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market (say B). According to this theory, the monopolistic firm in market A can monopolize market B using the leverage provided by its monopoly power in market A through tying or bundling arrangements. The Chicago School, however, criticized this theory and proposed instead price discrimination as the main motivation for tying. The gist of the Chicago school criticism is based on the so-called "one monopoly theorem," which states that "[a] seller cannot get two monopoly profits from one monopoly." (Blair and Kaserman, 1985).

We demonstrate that in the presence of imperfect rent extraction in a monopolized market and network effects in a market where the monopolist faces competition, tying can be a mechanism through which unexploited consumer surplus in the monopolized market is used as a demand-side leverage to create a strategic "quasi-installed base" advantage in the competing market, raising the perceived quality of the monopolist's tied product and lowering the perceived quality of its tied market rivals.

In markets with network effects, consumer utility consists of stand-alone benefits and network benefits (Katz and Shapiro (1986)). Under independent pricing, all firms compete on a level playing field with respect to network effects. Even though markets with strong network effects are typically characterized by tipping equilibria in which all consumers choose the same product, yielding maximal network benefits, the network-benefits component can be competed away in equilibrium to consumers' benefit. With tying, however, the tying firm can use unexploited consumer surplus in the tying market in competition against a rival firm in a tied market. We show that this advantage allows the tying firm to lock in consumers who have a high value for the tying product, ensuring that it captures the network effect and enabling it to win in the tied market even against a more efficient rival.

More precisely, consider a situation in which there are two markets, A and B. Firm 1 is a monopolist of product A and sells its product $B1$ in market B against a rival, firm 2, that produces product $B2$. Consumers in the monopolized market A are heterogeneous and some consumers receive surplus in this market under independent pricing. In such a scenario, if firm 1 offers only a bundle that ties purchase of its product A to purchase of its product $B1$, consumers with high valuations for product A may prefer to purchase the bundle even if all other consumers purchase the rival firm's product $B2$. The existence of such consumers ensures a guaranteed market share in market B for firm 1, which is akin to firm 1 having an installed base. This advantage in terms of the quasi-installed base can in turn induce consumers with a lower valuation for product A to purchase the bundle instead of buying $B2$. We show that a process of iterated elimination of dominated strategies can lead to tipping toward the monopolist's bundle.

Notably, and in contrast to much of the literature on the strategic use of tying (which we discuss below), this mechanism does not rely on the ability to precommit to tying, such as through technological bundling. Rather, the incentive to leverage unexploited consumer surplus in the tying market to degrade the relative quality of rivals in the tied market makes tying a best response by the monopolist absent any precommitment.[1]

We first develop our theory in the context of independent products to illustrate how network effects in the tied good market may provide incentives to tie. Specifically, we provide a sufficient condition for the monopolist to tie in equilibrium and thereby monopolize the tied-good market. In situations in which the tying good market is covered (i.e., all consumers purchase the tying product under independent pricing), we show that pure bundling is an optimal strategy for the monopolist. When the tying good market is not covered under independent pricing, firm 1 may instead find it optimal to tie using a mixed bundling strategy in which consumers can choose between buying an $A/B1$ bundle and buying product $B1$ only. This mixed bundling enables firm 1 to screen consumers with respect to their willingness to pay for the monopolized product A while maximizing the network effects for its product $B1$. When the number of consumers buying the bundle is large enough, firm 1 is able to sell even its inferior $B1$ at a profit as a stand-alone product against product $B2$.

We then extend our analysis to the case of complementary products. With pure monopoly in the tying product market, we confirm the Chicago School critique that tying cannot be a leverage mechanism even with network effects. However, pure monopoly with absolutely no competitive products is rare. We show that in the presence of an inferior alternative to the tying good we can restore our mechanism with parallel results to the independent-products case; we formally demonstrate the equivalence of the complementary-products case to the independent-products case, with the inferior alternative in the tying market playing the same role as does the no-purchase option in the independent products case.

As we noted above, the literature on tying as an anticompetitive foreclosure mechanism has focused most on situations in which a monopolist firm commits to use of a tying strategy, as first developed in Whinston (1990).[2] If the market structure in the tied-good market is oligopolistic with scale economies, tying can be an effective and profitable

[1] Our mechanism can still be effective and attractive for the monopolist when precommitment is possible, but precommitment is not necessary (nor assumed in our analysis).

[2] Fumagalli et al. (2018) provide an excellent survey of tying as an exclusionary practice along with discussions of major antitrust cases.

strategy to alter market structure by making continued operation unprofitable for tied-good rivals. This effect occurs because a commitment to tying leads the monopolist to price aggressively in order to ensure sales of the valuable product A. However, in the main model in Whinston (1990), inducing the exit of the rival firm is essential for the profitability of tying arrangements. Thus, if the competitor has already paid the sunk cost of entry and there is no avoidable fixed cost, tying cannot be a profitable strategy. In contrast, our mechanism requires neither commitment power of the tying firm nor exit of the rival.[3] This is important because, while the commitment assumption makes sense when firms employ technological ties, in many tying cases the tie is a pricing choice that seems to involve little commitment.[4]

Our analysis can provide a theory of harm for tying cases in which network effects play a crucial role in determining the market winner. It also sheds light on recent cases in which tying is contractual rather than technical. For example, our model can help explain the European Commission's ongoing antitrust investigation into Microsoft's tying of its communication and collaboration product, Teams, to Office 365 and Microsoft 365.[5] In our model, Microsoft's productivity software (Office 365/Microsoft 365) serves as the tying product, while the market for communication and collaboration tools includes competitors such as Slack, Zoom, and alfaview GmbH.

Communication and collaboration software, by its nature, exhibits direct network effects. Moreover, the tying in this case is contractual rather than technical, as evidenced by Microsoft's subsequent unbundling of Teams from Office 365 in response to the Commission's concerns. This case illustrates how the effects and conditions analyzed in our theory can be mapped to real-world scenarios.

After presenting our model and explaining the mechanism through which tying affects competition, we will discuss (in Section 7) the Microsoft Teams case in greater detail, along with other high-profile tying cases to demonstrate how our framework provides insight into tying practices involving network effects.

The idea of using unexploited consumer surplus as a leverage mechanism appears in some other papers. Burstein (1960) and Greenlee et al. (2008) analyze a setting in which the monopolist in the tying product market sells to consumers with multiunit demands and is unable to fully extract consumer surplus with linear pricing. By tying,

[3] If network effects are absent, we replicate his result that bundling is not profitable if firm 2's exit is not induced.

[4] In some cases, reputational concerns might lead to an element of commitment.

[5] Office 365 is a cloud-based suite of productivity applications, including Outlook, Word, PowerPoint, and Excel. Microsoft 365 encompasses Office 365 along with additional services and features, such as identity and access management, threat protection, and endpoint/app management.

even to competitively-supplied tied goods, the monopolist can require buyers to purchase additional products at elevated prices. In essence, tying serves as a substitute for a fixed fee.[6] In contrast, in our model consumers have single-unit demands for the tying good and so tying cannot serve this function. Ide and Montero (2024) develop a related theory in the context of a vertical relationship, where upstream suppliers sell their products through competing distributors. In their model, the leverage mechanism hinges on consumers' one-stop shopping behavior—distributors cannot afford to exclude the "must-have" product if their rivals carry it. As a result, their mechanism operates through distribution channels in vertical relationships and is fundamentally different from ours, which does not rely on one-stop shopping.

Calzolari and Denicolò's (2015) theory of exclusive dealing is also based on uncaptured consumer surplus with multiunit buyers. They consider a single-market situation in which there is a dominant firm with a competitive advantage over a competitive fringe of rivals, but buyers are able to obtain information rents due to private information even if they deal exclusively with the dominant firm. Without exclusive dealing, the dominant firm needs to compete for each marginal unit of a buyer's demand; in contrast with exclusive dealing, the dominant firm competes for the entire volume demanded by a buyer. This change enables the dominant firm to exclude rivals by leveraging the information rents left on inframarginal units. Thus, the dominant firm is able to exclude rivals with a lower discount with the imposition of exclusive deals. Exclusive dealing serves as a more profitable pricing mechanism despite the fact that it has no effects on the prices or qualities offered by the dominant firm's rivals. In contrast, in our model with heterogeneous consumers with single-unit demands, our mechanism leverages the network effect provided by inframarginal tying good consumers who are "committed" to the bundle in order to lower the network benefits provided by the tied-good rivals and thereby monopolize the tied-good market.

Carlton and Waldman (2002) is the paper most closely related to ours. They also consider a model in which network effects exist in the tied-good market and they note that tying can be an effective strategy in such cases without commitment. They consider a dynamic two-period model in which the tying and tied goods are perfect complements. In period 1 there is a monopoly in the tying "primary" market, while in period 2 the tied ("complementary good") market rival can potentially enter the primary market. In contrast to our focus on tying as a mechanism to profitably monopolize the tied-good market, the purpose of tying by the primary good monopolist in their model is to preserve

[6] However, tying is less efficient than a fixed fee since it causes distortions in the tied markets.

its market power by preventing entry into the primary market, even though it may entail short-run losses. As well, in our model these effects operate within a single period.[7]

Our analysis is also related to work by Caillaud and Jullien (2003) on competition between two-sided platforms. Caillaud and Jullien show how a firm can offer customers on one side a sufficient discount to lock in their business, thereby leading customers on the other side to join as well (a "divide-and-conquer" strategy). In our analysis, by tying a dominant firm can lock in consumers with a high willingness-to-pay for its monopolized product to the use of its competitive product, with the resulting network effects leading to monopolization of the latter market. In contrast to the offering of a discount in Caillaud and Jullien (2003), the leveraging of unexploited surplus in our analysis is free for the dominant firm.

The rest of the paper is organized in the following way. In Section 2, we illustrate the main intuitions behind our tying mechanism through a simple example with discrete consumer types. In Section 3, we describe our baseline model for independent products with a more general demand structure. In Section 4, we analyze this model and provide conditions under which tying (offering only a bundle and possibly product $B1$ for sale, and not offering A by itself) emerges in equilibrium and leads to monopolization of market B. In Section 5, we consider complementary products, and show that we can derive parallel results to the independent products case if we assume an inferior alternative to the monopolized tying good. In Section 6, we explore three extensions to test the robustness of our baseline model: (i) consumer multihoming, (ii) asymmetric network effects across products, and (iii) product differentiation with partial market foreclosure. We also examine the role of consumer beliefs and coordination in shaping market outcomes. We discuss the application of our results to recent antitrust cases in Section 7 and offer concluding remarks in Section 8.

2 An Illustrative Example

To explain the main mechanism and intuition behind our model, we provide an illustrative example. There are two markets A and B. Market A is served by a monopolist, firm 1. In market B, firm 1 and firm 2 compete. These two product markets are independent. Firms' production costs are normalized to zero in all markets. There are two consumers.

[7] On pp. 206-7, Carlton and Waldman suggest that tying could be profitable in the presence of network effects even "if the alternative producer cannot enter the primary good market," although without providing any analysis. Our paper can be viewed as following up on this point of theirs.

2.1 Market A

The two consumers are heterogeneous in terms of their valuations for product A. One is a high (H) type consumer and the other is a low (L) type consumer. Each consumer's willingness to pay for product A is given by u_k, where $k = H, L$, with $u_H = u_L + s > u_L > 0$ so $s > 0$. We assume that $u_L > s$. This implies that the optimal monopoly price in market A is $p_A^* = u_L$ and consumer H receives a surplus of s. An important feature of market A is that there is variation in consumers' willingnesses to pay for A so that firm 1 is unable to extract the whole surplus in that market despite its monopoly power.

2.2 Market B

Market B is characterized by network effects and firm 1's product $B1$ is inferior to firm 2's product $B2$. Products $B1$ and $B2$ are not compatible with each other. In this market, we assume that the two consumers have the same preferences. More specifically, firm i's product Bi provides a stand-alone value of v_i to consumers, where $v_2 > v_1 > 0$. We define the quality difference $\Delta \equiv v_2 - v_1$. If the two consumers purchase the same product Bi, there are additional network benefits of $n > 0$, yielding a total value of $v_i + n$. In other words, given that a consumer buys product Bi, her gross surplus is v_i if she is the only consumer buying the product and is the larger amount $v_i + n$ if the other consumer buys the same product. We assume that these potential network effects are larger than the quality difference $n > \Delta$. Thus, locking in the network benefits can allow firm 1 to overcome its disadvantage in market B.

2.3 Independent Pricing Equilibrium

Consider first the market equilibrium when the two products are sold independently by firm 1. In this case, the two markets can be analyzed separately. As noted above, firm 1 will charge $p_A^* = u_L$ in market A. In market B, given any prices (p_{B1}, p_{B2}), there is no continuation equilibrium in which the two consumers choose different products; in equilibrium, either both consumers purchase $B1$ or both purchase $B2$. We assume, as is common in the literature, that consumers coordinate on their Pareto-optimal continuation equilibrium. In addition, we restrict attention to equilibria in undominated price offers. Then, we have a unique equilibrium in which firm 2 charges $p_{B2} = \Delta$, firm 1 charges $p_{B1} = 0$, and both consumers buy product $B2$.[8] Hence, consumers capture the full

[8] As usual, we break consumer indifference in favor of the more efficient firm.

network benefits for themselves, and firm 2 captures only its quality advantage as profit. In this case, firm 1 earns nothing in market B and an overall profit of

$$\Pi_1^* = 2u_L.$$

2.4 Equilibrium with Tying

Now suppose that firm 1 engages in tying: it requires that any consumer who purchases its monopolized product A also buy product $B1$ (and only $B1$), and sells the bundle at price P.[9]

We show that if there is sufficient unexploited surplus in market A – specifically if $s > 2n$ – then firm 1 has a profitable deviation to offering only a bundle starting from the independent pricing equilibrium described above.[10] By doing so it leverages the unexploited consumer surplus enjoyed by consumer H in market A to monopolize market B. The argument shows that when firm 1 ties in this fashion there is a unique continuation equilibrium in which consumers' choices to buy the bundle are pinned down by iterated dominance.

The leverage mechanism with two discrete-type consumers operates in two steps. First, tying allows firm 1 to leverage the surplus from the monopoly product A that is enjoyed by consumer H to gain purchases of $B1$. Once consumer H is secured to buy the bundle, firm 1 achieves a strategic advantage in selling to consumer L as if it had consumer H as an installed-base of its product. These network benefits allow firm 1 to induce consumer L to buy the bundle as well.

To see this point, suppose that firm 1 deviates from the independent pricing equilibrium and offers only a bundle at price $P = u_L + \varepsilon$ where $\varepsilon > 0$. Observe, first, that if both consumers buy the bundle, then firm 1's profit will strictly exceed its profit in the independent pricing equilibrium in which its profit is $2u_L$. Moreover, as we now show, both consumers will indeed buy the bundle for small enough ε.

First, with $P = u_L + \varepsilon$, for small enough ε it is a dominant strategy for consumer H to

[9] We assume here that consumers will not buy both the bundle and product $B2$ and then use product $B2$ instead of $B1$. Firm 1 can prevent this behavior with a contract that prevents use of $B2$. Alternatively, such a contractual requirement will not be necessary when there are large enough production costs of product $B1$ and $B2$, which would make purchase of the bundle for the purpose of using only product A undesirable. In Section 6.1, we also extend our analysis to the case in which some share of consumers can multihome, using both products $B1$ and $B2$. We show that as long as some consumers single home, the mechanism we identify continues to operate.

[10] The assumption $s > 2n$ is a sufficient condition for tying to be profitable, but is stronger than necessary. We make this assumption for simplicity to reduce the number of cases to consider.

purchase the bundle: she prefers the bundle even under the most unfavorable condition that $B2$ is offered for free and consumer L purchases $B2$, since

$$(u_H + v_1) - P > v_2 + n \Leftrightarrow s > \Delta + n + \varepsilon, \tag{1}$$

which is satisfied for small enough ε given that $s > 2n > \Delta + n$. This works because of the unextracted surplus consumer H enjoys from buying A at price $p_A = u_L$.

Given that consumer H purchases the bundle, for small enough ε it is also optimal for consumer L to purchase the bundle even if $B2$ is offered for free since

$$(u_L + v_1) + n - P > v_2 \Leftrightarrow n > \Delta + \varepsilon \tag{2}$$

given that $n > \Delta$. Thus, by deviating in this fashion firm 1 is certain to monopolize market B and increases its profit.

Observe that firm 1's use of tying to monopolize market B does not rely on any commitment by firm 1; rather, tying is simply a best response to firm 2's price p_{B2} that successfully leverages consumer H's desire for A. Note also that variation in consumers' willingnesses to pay for A is essential: if both consumers had the same value for A, so that $u_H = u_L \equiv u$, firm 1's deviation to offering the bundle at slightly above the market A monopoly price (so that $P = u + \varepsilon$), would lead to no sales of the bundle (consumer purchases of $B2$ would be the same as if firm 1 had offered $B1$ independently at price $p_{B1} = P - u = \varepsilon$).[11]

In the next section, we examine in a more general model the equilibrium that results when firm 1 can tie in this fashion. When, as here, firm 1 would sell product A to all consumers under independent pricing (the case of "full coverage"), the equilibrium involves firm 1 offering only a bundle and monopolizing both markets. In the example above, that equilibrium outcome involves firm 2 offering to sell product $B2$ at cost ($p^*_{B2} = 0$) and firm 1 selling its bundle at the price $P = u_L + (n - \Delta)$.[12]

[11]Firm 1's use of tying here has the flavor of a "divide and conquer" strategy, reminiscent of that in Caillaud and Jullien (2003), as firm 1's bundle offer makes purchase from firm 1 a dominant strategy for consumer H, which then leads consumer L to follow along in buying from firm 1. However, tying is more effective here than would be discriminatory offers by firm 1 for just $B1$ (if possible). While, given $p_{B2} = \Delta$, firm 1 could make purchase of $B1$ a dominant strategy for one consumer by offering that consumer a price slightly less than $p_{B1} = -n$, it could then charge the other consumer at most $p_{B1} = n$, and so it cannot make a profit in this example with such offers. In essence, such a divide-and-conquer attempt just in market B involves use of a costly discount to the first consumer, while the tying strategy costlessly takes advantage of consumer H's unexploited surplus.

[12]By tying firm 1 is therefore able to achieve the same profit as when under independent pricing consumers fail to coordinate on their Pareto-preferred continuation equilibrium and all buy $B1$ instead

3 The Independent-Products Model

In this section, we lay out a more general model of tying in markets with network effects. As in the illustrative example in the previous section, we study here the case in which products A and B are independent and can be used separately as the baseline model. (We discuss the case of complements in Section 5.)

Market A is monopolized by firm 1. In market B, there are direct network effects, firm 1 and firm 2 compete, and consumers have homogeneous valuations: their willingness to pay for each firm's product is given by $v_1 + \beta N_1 > 0$ and $v_2 + \beta N_2 > 0$, respectively, where $v_1 > 0, v_2 > 0$, $\beta > 0$, and N_i represents the number of consumers using firm i's product Bi.[13] We normalize the total number of consumers to 1. All marginal costs are normalized to zero.

At the heart of our leverage mechanism is "unexploited consumer surplus" in the tying market that can be used in competition with a competitor in another market. If in market A there are high-valuation consumers who receive sufficiently large consumer surpluses, they may be willing to purchase the bundle (rather than product $B2$ only) even if all other consumers purchase $B2$. The existence of such high-valuation consumers in market A provides a demand-side leverage for firm 1 in market B akin to having an installed base. If network effects are sufficiently strong, this strategic advantage more than makes up for any quality disadvantage of firm 1 and enables firm 1 to monopolize market B, extracting the resulting network effects as profit.

More specifically, we assume that consumers' valuations for product A, denoted u, are distributed on $[\alpha, \overline{u}]$, where α represents the lower bound for the consumers' valuations.[14] It will be convenient to define a consumer's "type" as $x = u - \alpha$, which we assume is distributed on $[0, \overline{x}]$, where $\overline{x} \equiv \overline{u} - \alpha$, according to a c.d.f. $G(\cdot)$ with a strictly positive density $g(\cdot)$.[15] Hence, a consumer of type x's valuation for A is $\alpha + x$.

Let p_A be the price of product A. With a change of variables of $\widehat{p}_A \equiv p_A - \alpha$, we have a demand function $D(\widehat{p}_A) = 1 - G(\widehat{p}_A)$ in market A. We assume that $G(\cdot)$ satisfies the monotone hazard rate condition, that is, $\frac{g(\cdot)}{1-G(\cdot)}$ is strictly increasing. In market A, with

at price $p^*_{B1} = n - \Delta$.

[13]We can also incorporate network effects in market A; however, they play no role in our results. For details, see the Online Appendix.

[14]For example, consider a product that has a basic functionality plus some additional features. We can imagine a situation in which the basic functionality provides the same utility of α to all consumers, but additional features may generate different levels of extra utility to consumers, which is distributed on $[0, \overline{u} - \alpha]$.

[15]We admit the possibility that $\overline{x} = \infty$.

independent pricing firm 1 chooses $\widehat{p}_A$ to maximize

$$\max_{\widehat{p}_A \geq 0} \ \Pi_A(\widehat{p}_A) \equiv (\widehat{p}_A + \alpha) \left[1 - G(\widehat{p}_A)\right]. \tag{3}$$

The monotone hazard rate assumption implies that the solution to (3) is unique and satisfies the first-order condition if it is interior.

Remark 1 *Our model can also be applied to two-sided markets where in market A firm 1 is a two-sided platform that receives advertising revenue whenever it is chosen by a consumer. If we assume that there is an associated advertising revenue of $\alpha > 0$ for each consumer in market A and consumers' valuations for product A are distributed on $[0, \overline{x}]$, then our one-sided market model is isomorphic to a two-sided model with additional advertising revenue per consumer. Under this interpretation, the price $\widehat{p}_A (= p_A - \alpha)$ is the price paid by consumers while α is a negative marginal cost of the firm, as reflected in problem (3).*

In market B, we make the following assumption:

Assumption 1: $\Delta \equiv v_2 - v_1 > 0$ and $\Delta < \beta < \frac{1}{2g(x)}$ for all $x \in [0, \overline{x}]$.

As in Section 2, the condition that $\Delta > 0$ means that firm 2's product $B2$ has higher quality than firm 1's product $B1$. The assumption that $\beta > \Delta$ means that network effects are sufficiently important relative to the quality differential Δ: if all consumers buy product $B1$ from firm 1, then its (network-augmented) quality $v_1 + \beta$ becomes greater than that of product $B2$, v_2. Last, the assumption that $\beta < 1/\left[2g(x)\right]$ for all x is a stability condition for interior equilibrium in the tying regime and guarantees a unique cut-off type when firm 1 offers a pure bundle (see the proof of Lemma 1).[16] This stability condition also ensures sensible comparative statics when firm 1 offers a bundle: increases in the bundle price P reduce demand for the bundle.

We will consider two simultaneous-pricing games and compare them. In one, firm 1 can offer only single-product prices p_A and p_{B1} (the "independent-pricing" game), while in the other it is free to also offer a bundle at price P. In the latter case, no pre-commitment is involved; rather, if firm 1 offers a bundle, it does so only because it is a best response to firm 2's price p_{B2}.

[16] If $G(\cdot)$ is uniform, $\beta < \frac{1}{2g(x)} = \frac{\overline{x}}{2}$ is the necessary and sufficient condition for an interior equilibrium to be stable. For general distributions, $\beta < \frac{1}{2g(x)}$ for all $x \in [0, \overline{x}]$ is a sufficient, but not necessary, condition for the stability of an interior equilibrium because the violation of the condition implies only local instability.

4 Analysis of the Independent-Products Model

In this section, we examine the impact of allowing tying in the independent-products model. In the rest of the paper, we restrict attention to Pareto-undominated Nash equilibrium (NE) consumer responses.[17] In addition, when multiple undominated NE consumer responses exist to the firms' price offers, we assume that the worst such consumer response for firm 1 arises. Because the presence of multiple undominated responses matters only when firm 1 ties, in this way we "stack the deck" against firm 1's ability to succeed when it utilizes a tying strategy. We continue to restrict attention to equilibria in which the firms make undominated price offers (e.g., firm 2 will never set $p_{B2} < 0$).[18]

4.1 Independent-Pricing Game

In the absence of tying, as in Section 2 the two markets can be analyzed separately as we assume independent products.

In market B, all consumers have the same preference. Because we restrict attention to Pareto-undominated NE consumer responses to the firms' price offers, in the unique equilibrium all consumers purchase product $B2$, firm 1 sets a zero price ($p^*_{B1} = 0$), and firm 2 charges $p^*_{B2} = \Delta$.[19] Thus, when consumers coordinate their purchase responses with neither firm having an advantage in network effects, all consumers end up purchasing product $B2$ at a price at which the consumers capture the network benefits.

In market A, independent pricing may or may not result in all consumers purchasing product A. We denote the solution to the monopoly pricing problem (3) by $\widehat{p}^*_A$ (and write the actual monopoly price as $p^*_A \equiv \widehat{p}^*_A + \alpha$). Under the monotone hazard assumption on $G(\cdot)$, when

$$\alpha \geq \frac{1 - G(0)}{g(0)} = \frac{1}{g(0)} \tag{4}$$

firm 1 sets $\widehat{p}^*_A = 0$ (or, equivalently, $p^*_A = \alpha$) so that all consumers buy A (there is "full

[17] Specifically, we say that a NE response dominates another NE response if all consumers are weakly better off in the former NE and some are strictly better off.

[18] We can derive identical results by instead employing coalition-proof Nash equilibrium (CPNE) of the consumer response (Bernheim, Peleg, and Whinston, 1987) as a refinement. In addition, we can derive similar results even when further stacking the deck against firm 1's tying strategy by assuming that consumers choose $B2$ even if it constitutes a Pareto-dominated Nash equilibrium under tying. Under this assumption, our results rely solely on iterated dominance, and the profitability of tying does not depend on any coordination assumptions. See the Appendix for more details on this alternative approach.

[19] As is standard in the literature, we make the tie-breaking assumption in favor of the firm that can offer the highest consumer surplus to avoid the open set problem.

coverage"), while $\widehat{p}_A^* > 0$ (or, equivalently, $p_A^* > \alpha$) otherwise. In the latter case, firm 1 sets a price of $p_A^* = \widehat{p}_A^* + \alpha$, where $\widehat{p}_A^*$ satisfies the following condition:

$$\widehat{p}_A^* = \frac{1 - G(\widehat{p}_A^*)}{g(\widehat{p}_A^*)} - \alpha \ (> 0). \tag{5}$$

In both cases, the mass of consumers buying product A without tying is given by $1 - G(\widehat{p}_A^*)$.

Hence, without tying, firm 1 receives a profit of

$$\Pi_1^* = \begin{cases} \alpha & \text{if } \widehat{p}_A^* = 0 \\ (\alpha + \widehat{p}_A^*)(1 - G(\widehat{p}_A^*)) = \frac{\left[1 - G(\widehat{p}_A^*)\right]^2}{g(\widehat{p}_A^*)} & \text{if } \widehat{p}_A^* > 0 \end{cases}$$

while firm 2's profit is the same as in Section 2:

$$\Pi_2^* = \Delta.$$

4.2 Tying Equilibrium

We now derive the equilibrium when firm 1 is allowed to tie, requiring purchase (and use) of product $B1$ in order to acquire product A.[20] We will show that, under certain conditions, in this equilibrium firm 1 offers for sale either only a bundle (at price P) or possibly the bundle and product $B1$ (at price p_{B1}), and by doing so monopolizes market B. Moreover, in cases of "full coverage," in which in the independent-pricing game all consumers would buy product A at the monopoly price $p_A^* = \alpha$, firm 1 offers only the bundle. Specifically, we make the following additional assumption, which we will show is a sufficient condition for firm 1 to monopolize market B when tying is permitted:[21]

Assumption 2: $(1 - G(\widehat{p}_A^*)) > (\frac{\beta+\Delta}{2\beta})$

[20] As we noted earlier, the requirement that the consumer cannot purchase both the bundle and $B2$ and then use $B2$ instead of $B1$ is not necessary if the cost of producing $B1$ and $B2$ (which we have set here to zero) is sufficiently high; if it is, no consumer who has purchased an $A/B1$-bundle will also find it worthwhile to pay a price above cost for product $B2$. When it is not, some consumers may wish to "multihome" by buying both the bundle and $B2$. In Section 6.1 we discuss the case in which some consumers can costlessly multihome.

[21] For ease of exposition, we state Assumption 2 for the endogenous independent-pricing monopoly price for A, $\widehat{p}_A^*$. Assumption 2 holds, in terms of the primitive model parameters, whenever $(1 - G(\frac{1}{g(0)} - \alpha)) > (\frac{\beta+\Delta}{2\beta})$ since the monotone hazard rate property and first-order condition (5) imply that

$$G(\frac{1}{g(0)} - \alpha) \geq G(\frac{1 - G(\widehat{p}_A^*)}{g(\widehat{p}_A^*)} - \alpha) = G(\widehat{p}_A^*).$$

Assumption 2 implies that if all consumers of product A under independent pricing purchase the bundle, then product $B1$ offers a higher utility than product $B2$ even if all remaining consumers purchase $B2$: that is,

$$\beta(1 - G(\widehat{p}_A^*)) > \Delta + \beta G(\widehat{p}_A^*). \tag{6}$$

In Lemma 2 below, we show that condition (6) implies that firm 1 is able to charge p_A^* plus this resulting utility differential for a bundle and sell it to the same consumers that were buying only product A (and enjoying positive surplus) under independent pricing, increasing its profit over independent pricing.[22]

Observe that Assumption 2, which we show is sufficient for firm 1 to use tying to monopolize market B, requires that under independent pricing firm 1 sells A to more than half of the consumers: $(1 - G(\widehat{p}_A^*)) > (\frac{\beta+\Delta}{2\beta}) > \frac{1}{2}$. It is necessarily satisfied whenever the full coverage condition (4) holds, since then $(1 - G(\widehat{p}_A^*)) = 1$. More generally, Assumption 2 is more likely to be satisfied if α is large (so A is very valuable to consumers) and $\frac{\Delta}{\beta}$ is small (so network effects are large relative to vertical product differentiation).

It is useful to first consider how consumers would react to firm 1 offering only a pure bundle. The following lemma shows that the consumer response is pinned down by iterated dominance.[23]

Lemma 1 *When firm 1 offers only a bundle for sale, given prices of P for the bundle and $p_{B2} < v_2$ for product $B2$, and defining $\widehat{P} \equiv P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*
(i) If $(\widehat{P} - p_{B2}) \in (\beta - \Delta, \overline{x} - \beta - \Delta)$, consumers whose valuation for A is higher than $\widetilde{X} \in (0, \overline{x})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{X}$ purchase $B2$, where $\widetilde{X}$ satisfies

$$\Psi(\widetilde{X}) \equiv \widetilde{X} + \beta[1 - 2G(\widetilde{X})] - \Delta - (\widehat{P} - p_{B2}) = 0. \tag{7}$$

(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, all consumers purchase the bundle (i.e., $\widetilde{X} = 0$).
(iii) If $(\widehat{P} - p_{B2}) \geq \overline{x} - \beta - \Delta$, all consumers purchase $B2$ only (i.e., $\widetilde{X} = \overline{x}$) .

[22] Assumption 2 is analogous to the $s > 2n$ condition in the example of Section 2 and lies at the core of the "unexploited surplus" idea that drives our leverage mechanism in the presence of network effects.

[23] When there is a unique consumer response that survives iterated elimination of dominant strategies, it is also the unique Nash equilibrium of consumer responses.

Remark 2 *Note that* $\beta - \Delta < \overline{x} - \beta - \Delta$ *since* $1 = \int_0^{\overline{x}} g(x)dx \leq [\max_x g(x)]\overline{x}$ *implies that* $\overline{x} \geq 1/[\max_x g(x)] > 2\beta$*, where the last inequality follows from Assumption 1.*

Proof. See the Appendix. ■

To understand Lemma 1, consider case (i) in which the cutoff type $\widetilde{X}$ who is indifferent between the bundle and $B2$ is interior (i.e., takes a value between 0 and $\overline{x}$). Let $\psi(x, X)$ denote the payoff gain from purchasing the bundle over purchasing $B2$ for a type x consumer (i.e., whose willingness to pay for A is $\alpha + x$) if all other players whose types are higher than X choose the bundle while all the remaining consumers choose $B2$:

$$\psi(x, X) \equiv x + \beta[1 - 2G(X)] - \Delta - (\widehat{P} - p_{B2}). \tag{8}$$

Because $(\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$ in case (i), type $\overline{x}$ (and therefore a set of types just below $\overline{X}$) strictly prefers the bundle even if all other consumers buy $B2$:

$$\psi(\overline{x}, \overline{x}) = \overline{x} - \beta - \Delta - (\widehat{P} - p_{B2}) > 0.$$

Given that these very high types see choosing the bundle as their dominant strategy, increasing the network benefits of product $B1$ and lowering those of product $B2$, a set of somewhat lower types will prefer it even if all remaining types choose $B2$. This starts a decreasing sequence of types who, by iterated dominance, prefer the bundle.

Likewise, because $(\widehat{P} - p_{B2}) > \beta - \Delta$ in case (i), type $x = 0$ (and therefore a set of types just above 0) strictly prefers $B2$ even if all other consumers buy the bundle:

$$\psi(0, 0) = \beta - \Delta - (\widehat{P} - p_{B2}) < 0.$$

Given that these very low types see choosing $B2$ as their dominant strategy, a set of somewhat higher types will prefer it even if all remaining types choose the bundle. This starts an increasing sequence of types who, by iterated dominance, prefer $B2$.

We show in the proof that iterated dominance then implies a unique interior cutoff $\widetilde{X}$ such that $\Psi(\widetilde{X}) \equiv \psi(\widetilde{X}, \widetilde{X}) = 0$, which is condition (7).

A consequence of Lemma 1 is that even if $p_{B2} = 0$, by choosing $P = \alpha + \beta - \Delta$, firm 1 can induce all consumers to buy the bundle and realize a profit of $P = \alpha + \beta - \Delta$, which is strictly larger than the profit under independent pricing when full coverage is optimal. Intuitively, by offering only a bundle firm 1 is able to leverage the presence of consumers with a high value for product A to gain a network benefit advantage over firm 2 in market

B. Consumers with a high value for product A want to buy the bundle regardless of what other consumers are doing, which in turn creates network benefits for $B1$ that make consumers with somewhat lower values buy the bundle, and so on.

More generally, Lemma 1 implies that if tying is allowed and Assumptions 1 and 2 hold, then firm 1 can offer the bundle at a price that induces the same consumers who buy A under independent pricing to buy the bundle and that exceeds $\widehat{p}_A^*$. To see this, define the bundle price that implements cut-off type $\widetilde{X} \in (0, \overline{x})$ between the bundle and $B2$ given price p_{B2} as $\alpha + \widehat{P}(\widetilde{X}|p_{B2})$ where

$$\widehat{P}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \widetilde{X} + \beta[1 - 2G(\widetilde{X})] - \Delta. \tag{9}$$

We then have:

Lemma 2 *Under Assumptions 1 and 2, given any $p_{B2} \geq 0$, firm 1 can sell the bundle to consumer types $x \geq \widehat{p}_A^*$ at a price $\widehat{P}(\widehat{p}_A^*|p_{B2})$ that exceeds $\widehat{p}_A^*$.*

Proof. From (9) we have

$$\begin{aligned} \widehat{P}(\widehat{p}_A^*|p_{B2}) &= p_{B2} + \widehat{p}_A^* + (\beta - \Delta) - 2\beta G(\widehat{p}_A^*) \\ &\geq \widehat{p}_A^* + [\beta(1 - G(\widehat{p}_A^*)) - (\Delta + \beta G(\widehat{p}_A^*))] \\ &> \widehat{p}_A^*, \end{aligned}$$

where the strict inequality follows from Assumption 2 (which implies (6)). ■

Lemma 2 implies that an independent-pricing equilibrium cannot exist since, facing $p_{B2} = \Delta$, firm 1 could deviate to offering only a bundle and could sell it to the same consumers as those who buy A under independent pricing but at a higher price. In addition, since such a strategy is always available to firm 1, in any equilibrium it must earn strictly more profit than in the independent-pricing equilibrium. Hence, we have:[24]

[24]Assumption 2 (and its equivalent, condition (6)), is sufficient for tying to be a profitable leverage mechanism, but it is not necessary. In proving Lemma 2 we showed that, under Assumption 2, firm 1 has a profitable deviation away from independent pricing that sells the bundle to the same set of consumers who would buy product A at the independent-pricing equilibrium price p_A^*, taking advantage of these consumers' unexploited consumer surplus in market A. However, when Assumption 2 does not hold, firm 1 may nonetheless have a profitable deviation that sells the bundle to a different set of consumers. To see this in a simple setting, consider again the example of Section 2. If the willingness to pay difference s was instead slightly larger than u_L, then the monopoly price for A would be instead u_H with only consumer H buying A and enjoying no surplus. In that case, the equivalent of condition (6) fails to hold as the network benefit generated by consumer H alone (zero) does not overcome firm 1's quality disadvantage in market B (Δ). As a result, firm 1 cannot profitably deviate in a manner that sells the bundle only

Corollary 1 *If tying is allowed and Assumptions 1 and 2 hold, no independent-pricing equilibrium exists and, in any equilibrium, firm 1 earns strictly greater profit than in the independent-pricing equilibrium.*

We next state our main result, which establishes that under Assumptions 1 and 2 no sales of *B2* occur in equilibrium – i.e., that firm 1 monopolizes market B in any equilibrium. The equilibrium outcome involves sales of the bundle to consumers above a cut-off type $\widetilde{x}^*$ (which equals $\widehat{P}^* - p^*_{B1}$ in the equilibrium) and sales of $B1$ to consumers below this type. In some cases, the cut-off type is $\widetilde{x}^* = 0$, so firm 1 sells only a bundle.[25]

Proposition 1 *If tying is allowed and Assumptions 1 and 2 hold, the unique equilibrium involves firm 1 tying (selling only the bundle and possibly product* $B1$*) and fully monopolizing market* B*. The cut-off type* $\widetilde{x}^*$ *between sales of the bundle and sales of* $B1$ *solves*

$$\underset{\widetilde{x}}{Max}\ \Pi_A(\widetilde{x}) + \beta[1 - G(\widetilde{X}(\widetilde{x}))] - \Delta, \tag{10}$$

where $\Pi_A(\cdot)$ *is the market A profit function defined in* (3) *and* $\widetilde{X}(\cdot)$ *is a strictly increasing function defined by the relation*

$$\widetilde{x} \equiv \widetilde{X} - \beta G(\widetilde{X}). \tag{11}$$

The set of types buying the bundle contains the set of types that buy product A under independent pricing (i.e., $\widetilde{x}^* \leq \widehat{p}^*_A$*). This relation is strict, with the exception that when there is full coverage under independent pricing the two sets coincide (*$\widetilde{x}^* = \widehat{p}^*_A = 0$*). Firm 2 makes no sales and sets price* $p^*_{B2} = 0$*, while firm 1's prices are:*

(i) When firm 1 sells only the bundle (i.e., $\widetilde{x}^* = 0$*):* $P^* = \alpha + (\beta - \Delta)$

(ii) When firm 1 sells both the bundle and $B1$ *(i.e.,* $\widetilde{x}^* > 0$*):*[26]

$$\begin{aligned} P^* &= \alpha + \widetilde{X}(\widetilde{x}^*) + \beta[1 - 2G(\widetilde{X}(\widetilde{x}^*))] - \Delta \\ p^*_{B1} &= \beta[1 - G(\widetilde{X}(\widetilde{x}^*))] - \Delta. \end{aligned}$$

to consumer H as this would require selling it at price $P = u_H - \Delta$. Nonetheless, firm 1 *does* have a profitable deviation in which (as in Section 2) it sells the bundle to both consumers at price $P = u_L + \varepsilon$. In this case, in which consumer H does not have unexploited surplus, making purchase of the bundle a dominant strategy for consumer H is costly (it requires sale of the bundle at price $P = u_L + \varepsilon < u_H$), but this is more than made up by the profits on consumer L. We thank Vincenzo Denicolò for this observation.

[25]In the proof, we show that firm 1 does not make separate sales of product A in any equilibrium.

[26]Note that these prices converge to those in Case (i) as $\widetilde{x}^* \to 0$

Firm 1's equilibrium profit exceeds its profit under independent pricing.

Proof. See the Appendix. ■

To see the idea behind the proof of Proposition 1, recall that $\widetilde{X}$ is the consumer type that is indifferent between the bundle and $B2$ when firm 2 charges p_{B2} and firm 1 charges $\alpha + \widehat{P}(\widetilde{X}|p_{B2})$ for the bundle. Now consider whether, when charging $\alpha + \widehat{P}(\widetilde{X}|p_{B2})$ for the bundle, firm 1 can profitably get consumers below $\widetilde{X}$ to buy $B1$ instead of $B2$. Given bundle price $\alpha + \widehat{P}(\widetilde{X}|p_{B2})$, the highest price p_{B1} that induces all consumers below $\widetilde{X}$ to purchase $B1$ rather than $B2$ (given our assumption about consumers' responses) is given by

$$p_{B1}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \beta(1 - G(\widetilde{X})) - \Delta, \tag{12}$$

since, at this price, consumers below $\widetilde{X}$ are indifferent between the two outcomes in which they all buy $B1$ and they all buy $B2$. Note that the term $\beta(1-G(\widetilde{X}))$ is the network benefit advantage firm 1 has over firm 2 because of consumers who are buying the bundle, while Δ is the quality advantage that firm 2 has over firm 1. To induce purchase of $B1$ rather than $B2$, firm 1's price for $B1$ can exceed p_{B2} by at most the difference between these two amounts; specifically, at any higher price for $B1$ there is an undominated NE consumer response in which all consumers below $\widetilde{X}$ buy $B2$ and this NE response is worse for firm 1 than any other undominated NE (here we use our "stack-the-deck" assumption). In the proof of the proposition, we show that in any equilibrium firm 1 will charge $p_{B1}(\widetilde{X}|p_{B2})$ for $B1$, undercutting firm 2. As a result, firm 2 makes no sales and sets $p^*_{B2} = 0$.

Given the two prices $\widehat{P}(\widetilde{X}|p_{B2})$ and $p_{B1}(\widetilde{X}|p_{B2})$, we denote by $\widetilde{x}(\widetilde{X})$ the consumer type that is indifferent between the bundle and $B1$ when all consumers buy either the bundle or $B1$. Using (9), this is the type for which

$$\widetilde{x}(\widetilde{X}) = \widehat{P}(\widetilde{X}|p_{B2}) - p_{B1}(\widetilde{X}|p_{B2}) = \widetilde{X} - \beta G(\widetilde{X}). \tag{13}$$

It is given by (11) and is a strictly increasing function by Assumption 1.

It is convenient to instead work with the inverse of $\widetilde{x}(\widetilde{X})$, which is the function $\widetilde{X}(\widetilde{x})$ described in the statement of Proposition 1. Figure 1 depicts the resulting purchase regions for a case in which firm 1 optimally prices so that sales of both the bundle and $B1$ occur. In the proof of Proposition 1, we show that firm 1's optimal choice of prices for the bundle and $B1$ implements the $\widetilde{x}^*$ that solves problem (10).

Since the term $\beta[1 - G(\widetilde{X}(\widetilde{x}))]$ in problem (10) is a weakly decreasing differentiable function of $\widetilde{x}$, it follows that at any solution to (10) we must have $\Pi'_A(\widetilde{x}^*) \geq 0$, which implies that $\widetilde{x}^* \leq \widehat{p}_A^*$ – i.e., that the set of types buying the bundle contains the set of types who buy A under independent pricing. (We show as well that the relationship is strict, except in the case of full coverage.) Intuitively, firm 1 wants to sell more of the bundle than it sells of A under independent pricing because by doing so it gains a network benefit advantage that allows it to charge more for product $B1$.

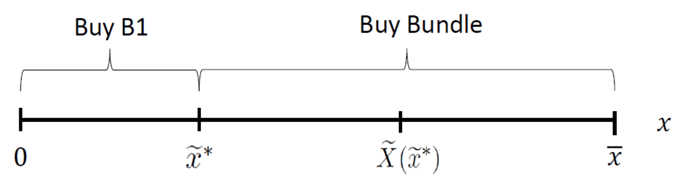

Figure 1: Consumers' equilibrium purchase decisions as a function of their type x for product A: types above $\widetilde{X}(\widetilde{x}^*)$ prefer the bundle over $B2$ by iterated dominance, types above $\widetilde{x}^*$ buy the bundle, and types below $\widetilde{x}^*$ buy $B1$.

When firm 1 makes sales of both the bundle and $B1$, its mixed bundling strategy enables it to screen consumers with more price instruments while still maintaining the ability to leverage the presence of high-value consumers for its monopoly product A to the competitive market B: as in the case of pure bundling, it is as if firm 1 has an installed base advantage when competing in market B, which ensures firm 1's market dominance and enables it to expropriate the resulting network benefits of consumers in market B.

Notice that in this equilibrium, firm 2 lowers its price by Δ compared to the independent pricing equilibrium. Despite this fact, under our assumptions (that state that network effects are large enough) firm 1's profit is greater in this equilibrium than under independent pricing as we noted in Corollary 1: the benefits from leveraging high-value consumers in market A to gain a network benefit advantage in market B are large enough to offset the effect of firm 2 pricing more aggressively.

4.3 Welfare Effects of Tying

We now investigate welfare implications of tying in our model. In the case in which market A is covered under independent pricing (i.e., $\widehat{p}_A^* = 0$), bundling is profitable, but always welfare-reducing, lowering aggregate surplus by the amount Δ, as it results in substitution of the inferior product $B1$ for the superior product $B2$. Regarding consumer surplus, as tying reduces aggregate surplus by Δ while changing total industry profit by $(\alpha + (\beta - \Delta)) - (\alpha + \Delta) = \beta - 2\Delta$, in the case of full coverage under independent pricing it reduces consumer surplus by $(\beta - \Delta)$: firm 1 rather than consumers now captures the network benefits (β), but partially offsetting this loss for consumers is the fact that firm 2 lowers its price by Δ.

Consider, instead, the case in which market A is not covered under independent pricing (i.e., $\widehat{p}_A^* > 0$). There is then an opposing aggregate welfare effect of tying: it expands the use of product A since $\widetilde{x}^* < \widehat{p}_A^*$. Its welfare impacts thus can be ambiguous. More precisely, the welfare reduction in market B, which equals Δ, must be compared to the increase in welfare in market A. Letting W denote aggregate welfare under independent pricing and $\widetilde{W}$ denote welfare when tying is allowed:[27]

$$\widetilde{W} - W = \underbrace{\int_{\widetilde{x}^*}^{\widehat{p}_A^*} (x + \alpha) g(x) dx}_{\text{Market Expansion Effect in } A} - \underbrace{\Delta}_{\text{Efficiency Loss in } B}$$

To explore further how the aggregate surplus effect of tying depends on key parameters of the model, consider first case (ii) of Proposition 1 in which market A is not fully covered even with tying (i.e., $0 < \widetilde{x}^* < \widehat{p}_A^*$). In this case, because the objective function in (10) has increasing differences in $(-\widetilde{x}, \beta)$, $\widetilde{x}^*$ is decreasing in β, whereas $\widehat{p}_A^*$ is independent of β. This implies that in case (ii) the market expansion effect is positively related to β. An increase in α, on the other hand, can affect the aggregate surplus impact of tying in case (ii) because it changes the curvature of the function $\Pi_A(\cdot)$, and thus alters the degree to which $\widetilde{x}^*$ is less than $\widehat{p}_A^*$.

In contrast, in case (i) of Proposition 1 — in which market A is fully covered under tying (i.e., $\widetilde{x}^* = 0$) — $\widetilde{x}^*$ is invariant in both α and β whereas $\widehat{p}_A^*$ is decreasing in α and unaffected by β. As a result, the expansion in purchases of product A, $\widehat{p}_A^* - \widetilde{x}^*$, decreases in α and is unaffected by β.

We now investigate the effects of tying on consumer surplus when market A is not

[27] Similarly, we denote consumer surplus in these two situations by CS and $\widetilde{CS}$.

covered in the independent-pricing equilibrium. Consumer surplus under independent pricing can be written as the sum of consumer surplus in market A and market B.

$$CS = \underbrace{\int_{\widetilde{p}_A^*}^{\overline{x}} [1 - G(x)]dx}_{CS_A} + \underbrace{v_1 + \beta}_{CS_B = (v_2+\beta) - \Delta}$$

In the case of tying, it is useful to imagine that all consumers buy product $B1$ at price $p_{B1}^* = \beta[1 - G(\widetilde{X}(\widetilde{x}^*))] - \Delta$, while the consumers who buy the bundle (which are all consumers when $\widetilde{x}^* = 0$) do so at price

$$P^* - p_{B1}^* = [\alpha + \widetilde{X}(\widetilde{x}^*) + \beta[1 - 2G(\widetilde{X}(\widetilde{x}^*))] - \Delta] - p_{B1}^* = \alpha + \widetilde{x}^*.$$

Thus,

$$\widetilde{CS} = \underbrace{\left[\int_{\widetilde{x}^*}^{\overline{x}} [1 - G(x)]dx\right]}_{\widetilde{CS}_A} + \underbrace{[v_1 + \beta - p_{B1}^*]}_{\widetilde{CS}_B}$$

and so we have

$$\widetilde{CS} - CS = \underbrace{\int_{\widetilde{x}^*}^{\widetilde{p}_A^*} [1 - G(x)]dx}_{\widetilde{CS}_A - CS_A > 0} + \underbrace{\left[\Delta - \beta[1 - G(\widetilde{X}(\widetilde{x}^*))]\right]}_{\widetilde{CS}_B - CS_B < 0}$$

Once again, the effects of tying on total consumer surplus depend on the relative magnitudes of two opposite effects in markets A and B.

As an illustration, Figure 2 provides welfare and consumer surplus comparisons in the case in which $G(\cdot)$ is a uniform distribution with $\overline{x} = 1$ and $\Delta = 0.05$. Assumption 1 is satisfied for β belonging to $(0.05, 0.5)$ and Assumption 2 is satisfied in all colored areas. Under independent pricing, full coverage in market A arises if and only if $\alpha \geq 1$ whereas full coverage under tying occurs for the area on the right side of the downward-sloping dashed curve. In the red (green) area, tying reduces (raises) both welfare and consumer surplus. In the orange (yellow) area, tying reduces (raises) consumer surplus but raises (reduces) welfare.

As noted above, tying reduces aggregate welfare if there is full coverage under independent pricing. Even if there is partial coverage under independent pricing, tying reduces aggregate welfare for α larger than a certain threshold. Otherwise, for a given α, tying reduces aggregate welfare if β is smaller than a threshold as the market expansion effect increases with β.

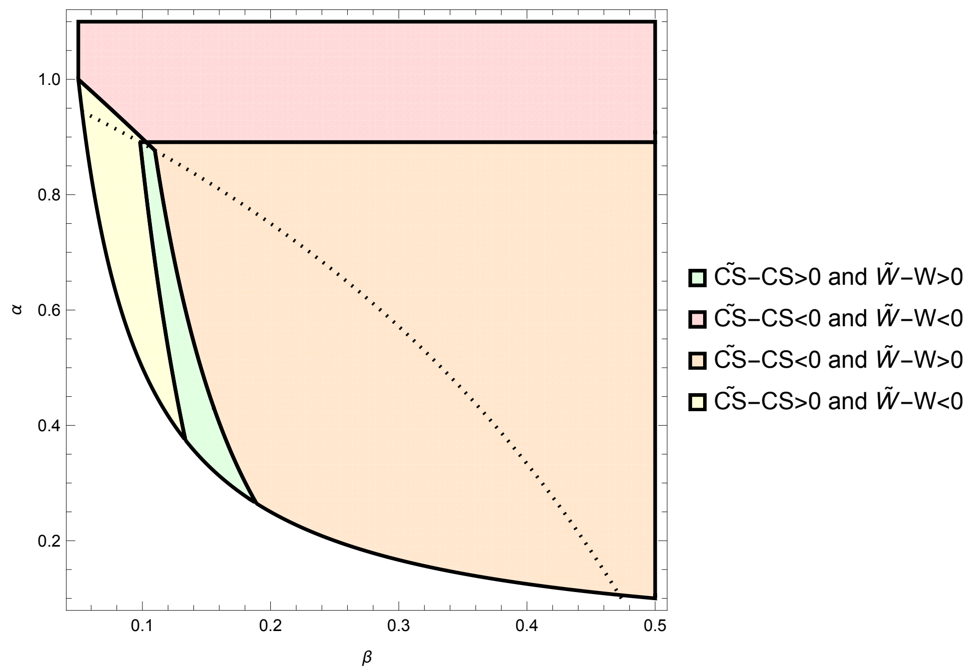

Figure 2: The effect of tying on welfare and consumer surplus

Regarding the comparison of consumer surplus, tying reduces consumer surplus if there is full coverage under independent pricing, as we noted above. Otherwise, for a given α, tying reduces consumer surplus if β is larger than a threshold as β is captured by firm 1 under tying whereas it is captured by consumers under independent pricing. This is why tying reduces consumer surplus in almost all of the area in which full coverage occurs under tying.

Overall, for α relatively high (including the area of full coverage under independent pricing), tying reduces both welfare and consumer surplus. Otherwise, in the orange area, where β is relatively large, tying reduces consumer surplus while increasing welfare.

5 Complementary Products

In this section, we consider complementary products. In line with the Chicago school logic, we first show that tying is not a profitable strategy as a leverage mechanism to suppress competition in complementary product markets. However, as in Whinston (1990), we show that if there is an inferior competitively-supplied alternative to the tying product then results that parallel those for the independent products case re-emerge. One major difference from Whinston (1990) is, once again, we do not rely on the commitment assumption and subsequent exit of the rival firm in the tied product market.

5.1 The Basic Model: The Chicago School Argument

We consider a setting that parallels the baseline model, where firm 1 is a monopolist in market A, except that products A and B are now complementary. For the purpose of exposition, consider product A as the primary product whereas B is an add-on product, that is, for the use of product B, product A is necessary; without A, product B is of no use.[28] For instance, product A can be considered as an operating system whereas B is application software.

When products are sold independently, consumers can use one of the two system products, $(A, B1)$ and $(A, B2)$, depending on which firm's product B is used, or product A only. Let us denote consumers' valuations for product A by u, and their valuations for the combined products $A/B1$ and $A/B2$ are respectively given by $u + (v_1 + \beta N_1)$ and $u + (v_2 + \beta N_2)$, where $u = (\alpha + x)$ with x distributed on $[0, \overline{x}]$ according to a cumulative distribution function $G(\cdot)$ with a strictly positive density $g(\cdot)$. We assume that $G(\cdot)$ satisfies the monotone hazard rate condition and that $\Delta \equiv v_2 - v_1 > 0$ as in the independent products case. To simplify the analysis, we also assume that condition (4) holds, which guarantees full market coverage in market A under independent pricing.

We first show that for the complementary products case, firm 1 has no incentive to tie as it can benefit from the presence of product $B2$. We maintain the same parametric assumptions (i.e., Assumption 1) made in the independent products model.[29]

Observe that if firm 1 ties then firm 2 is unable to make any sales. In that case, firm 1 would optimally sell a bundle at price $P = \alpha + v_1 + \beta$.[30] Firm 1's profit would then be $\alpha + v_1 + \beta$. However, firm 1 can earn more by adopting independent pricing that induces consumers to use product $B2$ instead of product $B1$, but extracts the resulting increase in aggregate surplus through a higher price of product A. In fact, there is a continuum of Nash equilibria due to firm 1's ability to "price squeeze" and extract a portion of the surplus Δ (Choi and Stefanadis, 2001) which are parameterized by $\lambda \in [0, 1]$, the degree of price squeeze exercised by firm 1:

$$p_A = \alpha + v_1 + \beta + \lambda\Delta,\ p_{B1} = -\lambda\Delta,\ p_{B2} = (1 - \lambda)\Delta$$

Firm 1's profit is then given by $\Pi_1 = \alpha + v_1 + \beta + \lambda\Delta$ so that all equilibria under independent pricing yield a higher profit than that under tying unless $\lambda = 0$ (in which

[28]Similar points can be made if products A and B are instead perfect complements that must be used in fixed proportions.

[29]Recall that condition (4) implies Assumption 2.

[30]For this result, we actually need a less stringent assumption than (4), namely $\alpha + v_1 + \beta > 1/g(0)$.

case the profits are the same), establishing the Chicago school argument.[31]

5.2 An Inferior Alternative Product in the Tying Market

We now suppose that there is an inferior alternative product in market A that is competitively supplied at the marginal cost of zero.[32] We call firm 1's product in the tying market $A1$ while the alternative product is called $A2$. To maintain mathematical isomorphism between the complementary and independent product cases, we normalize consumers' valuations for the combined products that include this alternative $A2/B1$ and $A2/B2$ to $(v_1 + \beta N_1)$ and $(v_2 + \beta N_2)$, respectively.[33] Thus, $\alpha + x$ represents the added value that product $A1$ brings to a system over use of product $A2$ for a consumer of type x.[34]

5.2.1 Independent-Pricing Game

In the presence of product $A2$, a consumer of type x chooses $A1$ over $A2$ if and only if

$$\alpha + x - p_{A1} \geq 0.$$

Thus, firm 1's sales of product A are positive only if $p_{A1} < \overline{u}$ and, as in the independent products case, equal $1 - G(\widehat{p}_{A1})$ where $\widehat{p}_{A1} \equiv p_{A1} - \alpha$. We then get the following result:

Proposition 2 *Suppose that products A and B are complementary and that Assumption 1 and condition (4) hold. When tying is prohibited and there is an inferior competitively-supplied alternative in market A, the equilibrium is identical to the one for the independent products case. Specifically:*

(i) In market A, firm 1 charges $p^*_{A1} = \alpha$ *and sells* $A1$ *to all consumers*

[31] If product $B2$ instead were supplied competitively, then the outcome would be equivalent to $\lambda = 1$ as firm 1 would be able to extract all the surplus gain from use of $B2$. We note that our leverage mechanism applies as well to the case in which $B2$ is competitively supplied, both here and in the independent products case.

[32] The assumption of a competitively-supplied alternative is for simplicity. It can be supplied by a firm with market power.

[33] We can allow a more general utility specification by assuming that consumers' valuations for the combined products $A2/Bj$ are given by $u' + (v_j + \beta N_j)$ for $j = 1, 2$ with $u' < u$. For instance, we can assume that $u' = \alpha' + (1 - \theta)x$ with $\alpha' < \alpha$ and $1 > \theta \geq 0$ without qualitatively changing any results, where $(\alpha - \alpha')$ and θ represent the degree of quality inferiority for the alternative product $A2$.

[34] In the Online Appendix, we provide an extension in which the added value that product $A1$ brings to a system over use of product $A2$ has two components: it increases the stand-alone value of A by $\alpha + x$ to a consumer of type x and also increases the value added by product Bi.

(ii) In market B, firm 1 charges $p_{B1}^* = 0$ and firm 2 charges $p_{B2}^* = \Delta$ and all consumers buy product $B2$.

Firm 1 earns $\Pi_1^* = \alpha$ and firm 2 earns $\Pi_2^* = \Delta$.

5.2.2 Tying

In the presence of tying, let P be the price of the $A1/B1$ bundle of firm 1 and let p_{B2} be the price of firm 2's product $B2$. As above, product $A2$ is provided competitively at the price of zero. As in the analysis of the independent-products model, we restrict attention to Pareto-undominated NE consumer responses and, when multiple undominated NE responses exist, to the worst such consumer response for firm 1. In the presence of product $A2$ in the tying market, the case of complementary products is isomorphic to the case of independent products and there is a unique equilibrium, which involves all consumers purchasing the $A1/B1$ bundle, as described in the following proposition that parallels case (i) of Proposition 1 for the independent products case:

Proposition 3 *Suppose that products A and B are complementary and that Assumption 1 and condition (4) hold. When tying is allowed and there is an inferior competitively-supplied alternative in market A, there is a unique equilibrium in which all consumers purchase the $A1/B1$ bundle and the equilibrium prices are given by*

$$P^* = \alpha + \widehat{P}^* = \alpha + (\beta - \Delta) \text{ and } p_{B2}^* = 0.$$

Moreover, firm 1's profit, equal to $\alpha + (\beta - \Delta)$, exceeds that under independent pricing. Both consumer surplus and social welfare decrease:

$$\begin{aligned} \widetilde{CS} &= CS^* - (\beta - \Delta) < CS^* \\ \widetilde{AS} &= AS^* - \Delta < AS^*. \end{aligned}$$

6 Extensions and Discussions

In this section, we extend our baseline model of independent products to examine the effects of consumer multihoming, asymmetric network effects across products, and product differentiation, ensuring the robustness of our main qualitative results. We highlight the main results and their intuition; further details and proofs are in the Online Appendix. We also discuss the role of consumer beliefs in our model.

6.1 Consumers' Multihoming

So far, we have analyzed tying assuming consumers do not multihome in market B. Here we extend the model of Section 3 to allow for the possibility of multihoming by consumers. More specifically, we endow a positive fraction $\mu \in (0, 1)$ of consumers with the ability to multihome at no cost, while the remaining consumers single-home, as in our baseline model. We assume that the ability to multihome is independent of a consumer's valuation for product A.

As in Doganoglu and Wright (2006) and Jullien, Pavan and Rysman (2021), a multihoming consumer's gross market B surplus from consumption of both $B1$ and $B2$ equals the higher of the standalone benefits from the two products, $\max\{v_1, v_2\}$, plus network benefits she can derive from the total number of consumers she can interact with through use of $B1$ and $B2$ (equal to the network size of $B1$ plus the network size of $B2$ minus the number of multihomers who consume both B1 and B2), which can be written as $\max\{v_1, v_2\} + \beta(N_1 + N_2 - N_m)$, where N_m denotes the number of multihoming consumers.

We focus on the case of full market coverage in market A; α is sufficiently large so that firm 1 serves all consumers with the price of $p_A = \alpha$ in market A under independent pricing, i.e., condition (4) holds. In market B, we maintain the current Assumption 1.

6.1.1 Independent Pricing with Multihoming Consumers

Under the full market coverage condition (4) in market A, firm 1 serves all consumers in the market; hence, the profit of firm 1 in market A is α.

To analyze the market equilibrium in market B, we first need to derive NE consumer responses given price offers (p_{B1}, p_{B2}) when some consumers can multihome. In the Online Appendix, we characterize Pareto undominated NE consumer responses and show that in any equilibrium we must have $(p_{B1}, p_{B2}) = (0, \Delta)$ and all consumers buying only $B2$. Thus, under independent pricing, firm 1's profit is α and firm 2's profit is Δ, as in the case of the baseline model without multihoming.

6.1.2 Tying with Multihoming Consumers

We focus on the case of pure bundling to examine how our results generalize with the presence of consumer multihoming and establish conditions under which firm 1 optimally bundles and all consumers buy the bundle. We begin by examining consumer responses to firms' price offers $(\widehat{P}, p_{B2})$. Observe that when all consumers buy the bundle any p_{B2} strictly greater than Δ results in zero profit for firm 2, as no multihoming consumer will buy $B2$. Therefore, in what follows, we consider only price offers by firm 2 with $p_{B2} \in [0, \Delta]$ and consumer responses in which all multihoming consumers purchase $B2$.[35]

The analysis of consumer responses is somewhat intricate, as it requires accounting for the incentives of both single-homing and multihoming consumers. Nonetheless, we are able to extend Lemma 1 to the case where a positive fraction of consumers can multihome under the following assumption (see the Online Appendix for details and the role of Assumption M):

Assumption M: $\overline{x} > (2 - \mu)\beta + \Delta$

More specifically, we establish that there is a unique NE consumer response in cutoff strategies, which prescribes the following pattern of consumer responses when $p_{B2} \in [0, \Delta]$: At very high values of $\widehat{P}$ no consumers buy the bundle. As $\widehat{P}$ falls, multihomers of type $\overline{x}$ are the first to find purchase of the bundle (in addition to $B2$) to be a dominant strategy. They do so if $\widehat{P} < \overline{x}$ since multihomers' benefit from buying the bundle (given that they will buy $B2$) is just their value from A. As $\widehat{P}$ falls further and additional multihomers buy the bundle (by iterated dominance), single-homers of type $\overline{x}$ come to find purchase of the bundle to be optimal (regardless of what any other single-homers and the remaining multihomers do). This happens at a bundle price $\widehat{P}$ (which we denote by $\overline{P}_{int}$), that is above $\overline{x} - \beta - (\Delta - p_{B2})$, the bundle price $\widehat{P}$ at which single-homers of type $\overline{x}$ find it dominant to buy the bundle if *no* other consumers are doing so. As $\widehat{P}$ declines further, consumer responses are interior until $\widehat{P}$ falls to a level (which we denote by $\underline{P}^{int}$) at which multihomers of type $x = 0$ find it optimal to buy the bundle given the other single- and multihoming consumers who are definitely buying the bundle. Because some single-homing consumers still are buying $B2$, $\underline{P}^{int}$ is strictly below $(1 - \mu)\beta$, the bundle price at which a multihomer of type $x = 0$ would find it optimal to buy the bundle if *all* other consumers were buying the bundle. Finally, when $\widehat{P}$ falls to $(1 - \mu)\beta - (\Delta - p_{B2})$ all single-homers also find buying the bundle to be optimal given that all other consumers definitely are doing so.[36]

We examine conditions under which there is an equilibrium in which firm 1 sells the

[35]When $p_{B2} = \Delta$ multihomers are indifferent about buying $B2$ in addition to the bundle, but in equilibrium they must for otherwise firm 2 would deviate to a slightly lower price.

[36]Note that when $p_{B2} = \Delta$, single-homers and multihomers all come to buy the bundle at the same price $\widehat{P}$ and we have $\underline{P}^{int} = (1 - \mu)\beta$.

bundle to all consumers, setting a bundle price of $P^* = \alpha + (1-\mu)\beta$, and firm 2 sells $B2$ to all multihomers by setting $p^*_{B2} = \Delta$. In this outcome, the network size of the bundle is 1, while that of $B2$ is μ. With multihoming, tying results in *partial* market foreclosure, as firm 2 continues to sell to multihoming consumers.

We consider, in turn, firm 1's and firm 2's incentives to deviate from these price offers, and establish the following result:

Proposition 4 *Suppose that fraction $\mu \in (0,1)$ of consumers can multihome without any cost and that Assumptions 1 and M as well as the full coverage condition are satisfied. If $\mu \gtrsim 0.52$,*[37] *equilibrium prices with tying are*

$$P^* = \alpha + (1-\mu)\beta \text{ and } p^*_{B2} = \Delta,$$

all consumers buy the bundle, and all multihoming consumers buy both the bundle and B2. Tying raises firm 1's profit relative to independent pricing but reduces firm 2's profit as well as consumer and total welfare.

Proof. See the Online Appendix.

Note that in the equilibrium characterized in Proposition 4, firm 2's price is identical to its price under independent pricing. This observation shows that the independent pricing equilibrium cannot be sustained, as firm 1 would have a profitable deviation by offering the bundle, yielding a higher payoff than under independent pricing.

Proposition 4 demonstrates that the core mechanism identified in our baseline model—whereby firm 1 profitably uses tying to leverage its market power in market A into market B—continues to operate even in the presence of multihoming consumers. However, the gain to firm 1 is more limited in this setting. With multihoming, tying generates a quasi-installed base advantage only by capturing single-homing consumers through bundle sales, yielding a value advantage of $(1-\mu)\beta$. When *all* consumers can multihome at no cost, this advantage disappears.

At the same time, multihoming introduces a countervailing effect that benefits the tying firm: it softens competition, as firms are no longer locked in an all-or-nothing contest

[37] This condition is sufficient for our results to hold under any distribution of $G(.)$, and in most cases is considerably stronger than necessary (see the proof of the proposition in the Online Appendix). For instance, in the case in which $G(\cdot)$ is a uniform distribution over $[0,\overline{x}]$, this equilibrium exists if and only if

$$\frac{\mu}{1-\mu} \geq \frac{\Delta}{\overline{x}} \frac{1-\beta\frac{\mu}{\overline{x}+\mu}}{1-\frac{\beta}{\overline{x}}\left[2(1-\mu)+\frac{\mu(\overline{x}-(1-\mu))}{\overline{x}+\mu}\right]}, \quad (14)$$

which is satisfied for $\overline{x}$ large.

for consumers in market B. Intuitively, when a subset of consumers multihome, tying segments the market. If there is a sufficient share of multihoming consumers, firm 2 focuses exclusively on serving multihoming consumers rather than attempting to attract single-homing consumers as well. The condition on μ in Proposition 4 serves as a sufficient condition to ensure that firm 2 has no incentive to attract single-homing consumers through a price cut. This gives rise to two somewhat surprising observations. First, the presence of multihoming consumers can make tying more profitable for firm 1 than if all consumers single home. This occurs if

$$(1-\mu)\beta > \beta - \Delta \Leftrightarrow \Delta > \mu\beta. \tag{15}$$

Second, under the same condition, the presence of multihoming consumers reduces consumer surplus: Since the net surplus that multihomers obtain from buying $B2$ is zero. Then, what matters for the consumer surplus comparison is the price of the bundle, which is higher under multihoming when (15) holds.[38] So we have:

Corollary 2 *Under the conditions of Proposition 4, the presence of multihoming consumers increases the tying firm 1's profit and reduces consumer surplus if and only if* $\Delta > \mu\beta$.

The reason for this result is that with a sufficient share of multihoming consumers, firm 2 does not lower its price in response to firm 1's tie, as noted above.

6.2 Asymmetric Network Effects

Here we extend the analysis of the baseline model with independent products by considering the case in which $B1$ and $B2$ can also differ in their network effects β_1 and β_2, where $\beta_i > 0$ represents the network benefit parameter for B_i, $i = 1, 2$. The assumption that $B2$ is the superior product in market B can be stated as $v_1 + \beta_1 < v_2 + \beta_2$, or $\Delta_v + \Delta_\beta > 0$, where $\Delta_v \equiv (v_2 - v_1)$ and $\Delta_\beta \equiv (\beta_2 - \beta_1)$.

We modify Assumption 1 as follows:

Assumption 1A: $\Delta_v + \Delta_\beta > 0$, $\beta_1 > \Delta_v$, and $\beta_1 + \beta_2 < \frac{1}{g(x)}$ for all $x \in [0, \overline{x}]$.

The second part of Assumption 1A (i.e., $\beta_1 > \Delta_v$) says that network effects for $B1$ can more than offset its stand-alone value disadvantage versus $B2$. The last part of

[38] The fact that multihomers receive no surplus is reminiscent of Armstrong and Wright (2007) in the context of two-sided markets, where the multihoming side captures none of the gains from trade.

Assumption 1A ensures that the demand for the bundle decreases with an increase in the bundle price. As in the analysis of multihoming consumers, in market A we assume the full market coverage condition (4).

6.2.1 Independent pricing

Consider the competition in market B. By a similar argument as in the main text, the equilibrium in market B has $p_{B1} = 0$ and $p_{B2} = (v_2 + \beta_2) - (v_1 + \beta_1) = \Delta_v + \Delta_\beta$, with all consumers buying $B2$.

Thus, under independent pricing, the total profit of firm 1 is α and the total profit of firm 2 is $(v_2 + \beta_2) - (v_1 + \beta_1) = \Delta_v + \Delta_\beta > 0$.

6.2.2 Tying

We extend Lemma 1 as follows:

Lemma 3 *With asymmetric network effects, when firm 1 offers only a bundle for sale, given prices of P for the bundle and $p_{B2} < v_2$ for product $B2$, and defining $\widehat{P} = P - \alpha$, the unique outcome in consumers' choices that survives iterated deletion of dominated strategies is as follows:*[39]

(i) If $\widehat{P} - p_{B2} \in (\beta_1 - \Delta_v, \overline{x} - \beta_2 - \Delta_v)$, consumers whose valuation for A is higher than $\widetilde{X} \in (0, \overline{x})$ purchase the bundle while consumers whose valuation is lower than $\widetilde{X}$ purchase $B2$, where $\widetilde{X}$ satisfies

$$\widetilde{X} + \beta_1(1 - G(\widetilde{X})) - \beta_2 G(\widetilde{X}) - \Delta_v = (\widehat{P} - p_{B2}); \tag{16}$$

(ii) If $\widehat{P} - p_{B2} \leq \beta_1 - \Delta_v$, all consumers purchase the bundle (i.e., $\widetilde{X} = 0$);

(iii) If $\widehat{P} - p_{B2} \geq \overline{x} - \beta_2 - \Delta_v$, all consumers purchase $B2$ only (i.e., $\widetilde{X} = \overline{x}$).

Proof. See the Online Appendix.

Lemma 3(ii) implies that no independent pricing equilibrium exists: Given firm 2's independent pricing equilibrium price of $p_{B2} = \Delta_v + \Delta_\beta$, firm 1 can increase its profit by offering only the bundle at a price of $P = \alpha + \widehat{P} = \alpha + (\beta_1 - \Delta_v) + (\Delta_v + \Delta_\beta) = \alpha + \beta_2$ and monopolizing market B.

The following proposition summarizes the market equilibrium under tying in the presence of asymmetric network effects across $B1$ and $B2$.

[39] A similar argument to that in Remark 3 in the main text establishes that $\beta_1 - \Delta_v < \overline{x} - \beta_2 - \Delta_v$.

Proposition 5 *With asymmetric network effects, under Assumption 1A and the full coverage assumption, tying results in an equilibrium in which all consumers buy the bundle with prices equal to*

$$P^* = \alpha + \beta_1 - \Delta_v \text{ and } p^*_{B2} = 0.$$

Firm 1's profit is larger than under independent pricing. Tying reduces firm 2's profit as well as consumer and aggregate welfare.

Proof. See the Online Appendix.

As in our baseline model, firm 1's profit under tying exceeds α, its profit under independent pricing, while firm 2 earns zero profit. Aggregate welfare falls as consumers purchase the inferior product $B1$ in market B. Regarding consumer surplus, recall that under independent pricing, consumers are indifferent between coordinating on $B1$ at $p_{B1} = 0$ and coordinating on $B2$ at $p_{B2} = \Delta_v + \Delta_\beta$. To compute consumer surplus, we can decompose $P^* = \alpha + \beta_1 - \Delta_v$ into a price of α for A and a price of $\beta_1 - \Delta_v$ for $B1$. So when keeping the price of A the same as with independent pricing, it is as if tying raised the price of $B1$ from 0 to $\beta_1 - \Delta_v > 0$.

Remark 3 *Observe that when $v_1 = v_2 = 0$, so that firm 2's advantage in market B stems solely from network effects (i.e., $\beta_2 > \beta_1$), firm 1's bundle price with tying, $P^* = \alpha + \beta_1$, implicitly charges a price of β_1 for $B1$, which is firm 1's monopoly price in market B; i.e., the price it would charge if firm 2 did not exist. This is because when firm 1 bundles it completely eviscerates firm 2's advantage, as iterated dominance leads firm 2's network size to be zero.*

6.3 Tying with Product Differentiation and Partial Market Foreclosure in the Tied Market

In our baseline model, we assumed vertical differentiation but with homogeneous consumer preferences in the tied market. This led to complete market foreclosure with tying in the tied market. However, if we allow horizontal product differentiation in the tied market, we can have an outcome in which tying can be profitable without complete market foreclosure. In the Online Appendix, we demonstrate the profitability of tying with partial market foreclosure in a Hotelling model with network effects. The intuition is that with product differentiation, full market foreclosure—i.e., attracting even consumers with a strong preference for firm 2's product $B2$ and a weak preference for product A—can become too costly.

6.4 Changes in Consumer Beliefs with Tying

In our baseline model, we demonstrated that our results hold even when firm 1 faces unfavorable consumer beliefs in market B under tying—effectively "stacking the deck" against its ability to leverage market power from market A. Specifically, we assumed that consumers coordinate on an equilibrium in which firm 2 wins in the presence of multiple undominated NE consumer responses. However, in practice, tying firms are often large and well-established. If tying can shift consumer beliefs—i.e., influence how consumers coordinate—in favor of firm 1, this introduces an additional channel through which tying can be profitable in the presence of network effects. While this mechanism is more mechanical than the one developed in our model, it reinforces the idea that tying can be a powerful strategic tool in such environments. In fact, tying can be profitable even when the monopolist already fully extracts surplus in the tying market.

To illustrate, consider a scenario in which consumers in market A are homogeneous, each with a valuation of u for product A, and firm 1 is able to extract all consumer surplus, whereas market B is as in our baseline model. In the absence of tying, firm 2 is able to win market B by setting a price of Δ, leaving firm 1 no profit from product B as in our original setup. Now suppose firm 1 sells only the bundle, and by bundling, it shifts consumer beliefs in its favor. Given firm 1 is the monopolist and all these consumers already buy A, they may reasonably expect everyone else to purchase $B1$ too if it is tied together, and this expectation can be self-fulfilling in equilibrium. Under such favorable consumer beliefs, firm 1 can set the bundle price at $u + \beta - \Delta > u$; even if firm 2 lowers its price to zero, consumers will still choose the bundle from firm 1. Thus, tying enables firm 1 to capture additional surplus from market B solely through belief coordination.

As a related point, as noted in footnote 12, if firm 1 enjoys favorable consumer beliefs—such that consumers fail to coordinate on their Pareto-preferred equilibrium and instead purchase $B1$ under independent pricing—then firm 1 can already achieve the same profit as in the tying equilibrium. This indicates that consumer beliefs play a certain role in our mechanism: the absence of favorable beliefs toward firm 1 in market B under independent pricing is a necessary condition for tying to serve as a profitable strategy in our framework. In contrast, if firm 2 enjoys favorable consumer beliefs and wins market B under independent pricing despite $v_1 > v_2$ (i.e., $\Delta < 0$), tying can be a mechanism to restore market efficiency.

7 Applications

Our analysis offers a theory of harm for tying in markets with network effects. In this section, we examine how our framework can be applied to prominent antitrust cases.

7.1 The European Commission's Case Against Microsoft Teams

As we discussed briefly in the Introduction, our model helps explain the European Commission's investigation into Microsoft's tying of its communication and collaboration product, Teams, to Office 365 and Microsoft 365. The Commission initiated its investigation on July 27, 2023, following a complaint from Slack Technologies, Inc. The Commission preliminarily found that Microsoft holds a dominant position in the productivity software market with Office 365 and Microsoft 365. The Commission's concern is that "Microsoft may have granted Teams a **distribution advantage** by not giving customers the choice whether or not to acquire access to Teams" (emphasis in the original) when they purchase its productivity suites.[40] In response to the Commission's concerns, Microsoft began unbundling Teams from its Microsoft 365 and Office 365 suites in the European Economic Area (EEA) and Switzerland on October 1, 2023.[41] After six months, in April 2024, Microsoft extended this unbundling globally, separating Teams from the rest of the Office suite.

In our model, Microsoft's productivity software (Office 365/Microsoft 365) acts as the tying product. Within the market for communication and collaboration tools, Microsoft competes with other companies such as Slack, Zoom, and alfaview GmbH.[42] This case illustrates how the effects and conditions analyzed in our theory can be mapped to a real-world scenario. First, communication and collaboration software, by its nature, exhibits direct network effects. Second, the tying in this case is contractual rather than technical, as demonstrated by Microsoft's subsequent unbundling of Teams from Office 365 in response to the Commission's concerns. Third, all products in the bundle are likely

[40] See the European Commission Press Release "Commission sends Statement of Objections to Microsoft over possibly abusive tying practices regarding Teams," released on June 25, 2024. Available at https://ec.europa.eu/commission/presscorner/detail/en/ip_24_3446

[41] See *Microsoft's EU Policy Blog* by Nanna-Louise Linde, Vice President, Microsoft European Government Affairs, available at https://blogs.microsoft.com/eupolicy/2023/08/31/european-competition-teams-office-microsoft-365/

[42] After the initial complaint by Slack, the Commission received a second complaint regarding Teams by alfaview GmbH (see the EC Press Release on June 25, 2024). In addition, Zoom CEO Eric Yuan has asked the US Federal Trade Commission (FTC) to investigate Microsoft's bundling of Microsoft Teams with its Office 365 and Microsoft 365 products (see Bloomberg News, September 5, 2023).

relatively independent, with only limited complementarities between them.[43]

Our model provides a microfoundation for the channel through which the "distribution advantage" highlighted by the EC emerges. To accommodate the diverse range of productivity software users, Microsoft offers a variety of subscription options for its productivity suite, with different tiers. However, due to the limited number of menu options, price discrimination cannot fully capture all consumer surplus, leading to "unexploited" surplus for consumers in the form of informational rent. Users who derive significant value from Office 365 will continue to purchase it after it is bundled with Teams as a dominant strategy. These users form a "quasi-installed base" for Teams and encourage other users to opt for the bundle, creating a cascading effect through the process of iterated dominance with network effects.[44]

Carlton and Waldman (2002) develop a theory of tying based on network effects, where the primary objective of tying is to preserve monopoly power in the tying market, rather than to extend that power into the tied product market. In addition, they consider the case of complementary products, where the monopolist's tying product is essential, meaning that all uses of the tied product require the tying product. This framework is particularly relevant to the Microsoft case involving the tying of its Windows operating system with the Internet Explorer browser in 2001 (United States v. Microsoft Corp., 253 F.3d 34 (D.C. Cir. 2001)). In that instance, the tying was aimed at maintaining Microsoft's dominance in the operating system market by preventing Netscape from competing effectively in the browser market. This was crucial because Netscape had the potential to evolve and enter the operating system market itself, challenging Microsoft's position.

In contrast, the current tying case involving Office 365/Microsoft 365 and Teams concerns products that appear to be fairly independent, with only limited complementarities. In particular, Office 365/Microsoft 365 is not essential to use Teams. Moreover, the goal seems to align more closely with the leverage theory, which involves extending Microsoft's monopoly power from the productivity software market into the communication and collaboration software market, rather than preserving its dominance in the productivity software market. As a result, the Carlton and Waldman (2002) model, which focuses on blocking entry to protect monopoly power in the tying market, is less applicable to the Teams tying case.

[43] Using them together may offer some benefits such as file sharing within a team, workflow integration, and easier task transitions (akin to the convenience of one-stop shopping).

[44] Large companies with dedicated IT departments are likely to find it easy to multihome. Thus, the ability to use tying to leverage a quasi-installed base advantage likely depends on the number of individual and small businesses who are unlikely to multihome.

Carlton and Waldman (2011) offer an alternative perspective on the leverage theory of tying, challenging the Chicago School's "one monopoly profit theorem" for complementary products in dynamic contexts involving product upgrades. They argue that when upgrade profits are significant, but the monopolist lacks the ability to commit to future actions, tying and directly selling the upgrades may be essential for capturing these profits. However, this logic applies only to the sales model and does not extend to leasing. In fact, they show that leasing and marketing products individually yield at least the same payoff as tying, and potentially a higher one. Consequently, their theory does not apply to the Teams tying case, as Office 365/Microsoft 365 operates as a subscription service, which is effectively equivalent to leasing.[45]

7.2 Google Android Case

Our model can also shed light on the recent antitrust investigation concerning Google's practices in its MADA (Mobile Application Distribution Agreement) contracts. In particular, the European Commission concluded that Google engaged in illegal tying by requiring Android OEM "manufacturers to pre-install the Google search app ..., as a condition for licensing Google's app store (the Play Store)."[46] Google's Play Store can be considered as the tying product as a "must-have" app, with other third-party app stores being inferior alternatives. In the search market, Google faces competition from other search engines and there is evidence of network effects, in particular, stemming from the fact that search results quality increases with the scale of queries received by a search engine.[47] In addition, the tie was not technological, but rather purely contractual, and raises the question of why Google would not have simply paid the OEMs for Android pre-installation.[48]

[45] Carlton and Waldman (2011) develop a second model that incorporates switching costs along with upgrades, demonstrating that tying can be profitable when leasing is involved. However, the logic in their second model hinges on a "price squeeze" that arises from the essentiality of the tying good in the usage of the tied product. This is not applicable to the case of tying involving Teams, where the tying good (Office 365) is not essential for the tied product's (Teams) usage.

[46] See the European Commission Press Release "Antitrust: Commission fines Google €4.34 billion for illegal practices regarding Android mobile devices to strengthen dominance of Google's search engine," released on July 18, 2018. Available at http://europa.eu/rapid/press-release_IP-18-4581_en.htm.

[47] See, for example, He et al. (2017), Schaefer and Sapi (2023), and Klein et al. (2023) for empirical evidence of network effects in Internet search. The District Court's opinion in *US v. Google* also cites this fact, stating that "[a]t every stage of the search process, user data is a critical input that directly improves quality." (US v. Google, Case No. 20-cv-3010, United States District Court for the District of Columbia) In the US case, the DOJ did not include the tie of the Play Store in its monopolization claims, instead simply focusing on the exclusivity that providing the Play Store for free secured.

[48] While the literature on bundling provides an answer by showing that with heterogeneous valuations tying can indeed be a best response as a price discrimination mechanism (as the Chicago School claimed),

Our model suggests that by leveraging what would otherwise be unexploited surplus from OEMs' use of the app store, this tying may be a way for Google to lock in part of the search market, thereby reducing the quality of rivals.[49] What sets our theory apart is that tying can be a strategic best response precisely because it undermines the perceived quality of competitors in the tied market by limiting the network benefits they can offer.

De Cornière and Taylor (2021) propose an alternative theory of upstream tying in the Android case, based on capacity constraints at the downstream device level (or retail shelf space) and demand complementarity. Specifically, they argue that each phone can have only one default search engine and that consumer demand for a phone without Google Play would be low. Their analysis shows that tying can reduce upstream rivals' willingness to offer slotting fees to device OEMs, thereby softening competition from the rival suppliers of the tied good.[50] While our theory and theirs emphasize different aspects of tying, they can be seen as complementary, as the presence of network effects in the tied good would further reinforce their argument.

7.3 Tying of Windows Media Player

The model also has relevance for the *Microsoft* case in Europe (IP/04/382) in 2004. The European Commission held Microsoft guilty of an abuse of dominant position by "tying its Windows Media Player (WMP), a product where it faced competition, with its ubiquitous Windows operating system."[51] Microsoft had a near monopoly position in the PC operating system market with over 90 percent market share. We can consider Linux

in these cases tying might be viewed as "innocent" and any effects on rivals inadvertent. See Section IIC of Whinston (1990) and Peitz (2008) for examples. Peitz's(2008) model differs in several ways from ours. First, in his article the market A monopolist commits to its pricing structure (pure bundling versus independent pricing) prior to a price choice stage (but only after the potential rival's entry decision, in contrast to the main commitment model in Whinston (1990)). Second, in Peitz (2008) bundling reduces welfare only when it affects the potential entrant's entry decision and affects the market structure whereas our model does not require any market structure change. Third, bundling is profitable irrespective of entry in his model, i.e., bundling is always used. In our model, in contrast, bundling is profitable only when there is competition in the tied-good market. Otherwise, independent pricing is more profitable. Thus, in our model bundling is useful only as an exclusionary strategy.

[49] One difference from our model, however, is that the "buyers" are distributors (phone OEMs/carriers) not consumers. Thus, while the search "purchase" can be interpreted as determining which search engine gets to be the pre-installed default on the distributor's device, distributors are not consumers with single-unit demands and more complicated pricing than linear pricing may be possible.

[50] See Ide and Montero (2024) for a related argument.

[51] Microsoft's tie of its media player to Windows may have involved a technological tie, but as we noted above the effects we highlight would apply in that case as well. The case also involved Microsoft's conduct of "deliberately restricting interoperability between Windows PCs and non-Microsoft work group servers." https://ec.europa.eu/commission/presscorner/detail/en/IP_04_382

as an inferior alternative to Microsoft's Windows OS in the tying market. The media player market can be considered as the tied market in which Microsoft faced competition (from firms such as RealPlayer) and network effects are critical. More precisely, the media player market can be considered a two-sided market with indirect network effects. If more content is provided in the format of a particular company, then more consumers will use the company's Media Player to access such content. Moreover, if more consumers select a particular company's Media Player, then content providers have a greater incentive to make their content available in the format of the company.[52,53] Our model assumes direct network effects in the tied market, but can be considered as capturing such feedback effects of two-sided markets in a reduced form.[54]

8 Conclusion

In this paper, we have developed a leverage theory of tying in markets with network effects. We first analyze incentives to tie for independent products. When a monopolist in one market cannot fully extract the whole surplus from consumers, tying can be a mechanism through which unexploited consumer surpluses in one market are used as a demand-side leverage to create a strategic "quasi-installed base" advantage in another market characterized by network effects. Our mechanism does not require any pre-commitment to tying, and hence can apply to cases in which a firm employs a purely contractual tie. Tying can lead to the exclusion of more efficient rival firms in the tied market, but can also in some cases expand purchase of the tying good if the tying market is not fully covered with independent pricing. We also extend our analysis to the complementary products case. By allowing the existence of inferior alternatives as in Whinston (1990), we show that the setup of complementary products is mathematically identical to that of independent products. We also discuss welfare implications of tying.

[52]As described in the *Official Journal of the European Union* (6.2.2007): "The decision then explains why tying in this particular case is liable to foreclose competition....WMP's ubiquitous presence induces content providers and software developers to rely primarily on Windows Media technology. Consumers will in turn prefer to use WMP, since a wider array of complimentary software and content will be available for that product. Microsoft's tying reinforces and distorts these 'network effects' to its advantage, thereby seriously undermining the competitive process in the media player market."

[53]The Korean Fair Trade Commission also fined Microsoft 33 billion won (US$32 million) for abusing its market dominant position by bundling Windows OS with its instant messaging (IM) program as well as WMP. For the messenger case, the tied market market is characterized by direct network effects, as in our model.

[54]See Choi and Jeon (2021) for an analysis of tying that explicitly accounts for indirect network effects in two-sided markets.

We conclude with comments about three possible extensions of our analysis. First, in our model firm 2 faces no fixed costs of either entry or remaining in the market. If it did, the fact that firm 1 would monopolize market B through tying if firm 2 is in the market would mean that firm 2 would choose not to be active (either not enter, or exit if it already had). Firm 1 would then monopolize both markets but, in the absence of firm 2, would use independent pricing. If so, the fact that firm 1 *could* tie would unambiguously reduce both aggregate and consumer surplus regardless of whether market A is covered under independent pricing.

Second, we developed our model in the context of one-sided markets. However, as we noted, our model is mathematically equivalent to one with a two-sided tying market with advertising revenues (with a reinterpretation of α as per-consumer advertising revenue). This may have important implications for recent antitrust debates on two-sided digital platforms. We showed that welfare impacts of tying depend on the relative magnitudes of positive market expansion effects and negative market foreclosure effects of more efficient firms. When advertising revenue is important (i.e., α is high) and services would under independent pricing be provided for free (hence, the market would be covered), as is common for many digital platforms, our model indicates that there are greater incentives to engage in tying to leverage unexploited consumer surplus. In that case, however, there are no socially beneficial market expansion effects. Therefore, the effects of tying are more likely to be welfare-reducing in such a case. In addition, the negative effects on consumer surplus will be more pronounced as network effects in the tied market become more important. This implies that careful scrutiny may be warranted when ad-financed digital platforms engage in tying with other products or services characterized by network effects.

Finally, and more conceptually, in our model tying immediately leads firm 1 to monopolize market B through iterated dominance. We conjecture that tying could be a profitable best response without commitment in dynamic models in which consumer choices are sticky. Specifically, by leveraging unexploited consumer surplus a dominant firm could shift some consumers to its bundled offering immediately, with the increased network benefits for the firm and reduced network benefits for rivals leading additional consumers to shift in the following periods, mimicking over time the iterated shift in consumers' expectations of network benefits that happens in the static model we have studied here.

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Appendix

Proofs

Proof of Lemma 1: Because $p_{B2} < v_2$, in any equilibrium all consumers will make some purchase. Recall that the payoff gain from purchasing the bundle over purchasing $B2$ for a type x consumer (i.e., whose willingness to pay for A is $\alpha + x$) if all other players whose types are higher than X choose the bundle while all the remaining consumers choose $B2$ is

$$\psi(x, X) \equiv x + \beta[1 - 2G(X)] - \Delta - (\widehat{P} - p_{B2}).$$

Note that $\psi(x, X)$ is continuous in x and X, increasing in x, and decreasing in X. Recall as well the function $\Psi(X)$:

$$\Psi(X) \equiv \psi(X, X) = X + \beta[1 - 2G(X)] - \Delta - (\widehat{P} - p_{B2}). \tag{17}$$

Note that $\Psi(X)$ is strictly increasing in X because by Assumption 1

$$\Psi'(X) = 1 - 2\beta g(X) > 0,$$

and that when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, we have $\Psi(0) < 0 < \Psi(\overline{x})$. Therefore, in any equilibrium response by consumers the "cut-off" type $\widetilde{X}$ who is indifferent between the bundle and product $B2$ (if interior) must be the unique solution to $\Psi(\widetilde{X}) = 0$. As in the analysis of global games, we use an induction argument to show that the choices of types above and below $\widetilde{X}$ are in fact pinned down by iterated dominance.[55] Observe that:

(i) If $(\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, then even when all other consumers are expected to choose $B2$ (i.e., the cut-off type is $\overline{X}^0 = \overline{x}$), it is optimal to choose the bundle for any consumer whose type is higher than $\overline{X}^1 = \beta + \Delta + (\widehat{P} - p_{B2}) < \overline{x} = \overline{X}^0$. Given that at least a measure of $1 - G(\overline{X}^1)$ consumers choose the bundle, we can derive another cut-off value $\overline{X}^2 < \overline{X}^1$. Note that $\overline{X}^n$ is a decreasing sequence in $[0, \overline{x}]$. Similarly, if $\beta - \Delta < (\widehat{P} - p_{B2})$, then even when all other consumers are expected to choose the bundle (i.e., the cut-off type $\underline{X}^0 = 0$), it is optimal to choose $B2$ for any consumer whose type is lower than $\underline{X}^1 = -\beta + \Delta + (\widehat{P} - p_{B2}) > 0 = \underline{X}^0$. Given that at least a measure of $G(\underline{X}^1)$ consumers choose $B2$, we can derive another cut-off value $\underline{X}^2 > \underline{X}^1$. Note that $\underline{X}^n$ is an increasing sequence in $[0, \overline{x}]$. Thus, when $\beta - \Delta < (\widehat{P} - p_{B2}) < \overline{x} - \beta - \Delta$, the

[55] For an excellent survey of global games, see Morris and Shin (2010).

continuity of $\psi(x, X)$ and the way the two sequences $\overline{X}^n$ and $\underline{X}^n$ are constructed imply that $\psi(\underline{X}, \underline{X}) = \psi(\overline{X}, \overline{X}) = 0$, where $\overline{X} = \lim_{n\to\infty} \overline{X}^n$ and $\underline{X} = \lim_{n\to\infty} \underline{X}^n$. Given that there is a unique $\widetilde{X}$ such that $\Psi(\widetilde{X}) = 0$, it must be that $\underline{X} = \overline{X} = \widetilde{X}$.

(ii) If $(\widehat{P} - p_{B2}) \leq \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{X} = 0$ because $\Psi(0) \geq 0$.

(iii) Similarly, if $(\widehat{P} - p_{B2}) \geq \overline{x} - \beta - \Delta$, the process of iterated deletion of dominated strategies leads to $\widetilde{X} = \overline{x}$ because $\Psi(\overline{x}) \leq 0$. ■

Proof of Proposition 1: We begin with some preliminary observations before establishing the claims in Proposition 1.

A consequence of Corollary 1 is that in any equilibrium (i) firm 1 must offer a bundle discount (i.e., $P < p_A + p_{B1}$) and (ii) make no separate sales of product A. A bundle discount is necessary because in its absence consumer purchases and firm profits would be the same as with independent-product pricing, but Corollary 1 rules out such an equilibrium outcome.

To see why firm 1 must not be making separate sales of product A, we first describe consumer purchase decisions when faced with a bundle discount offer from firm 1. Figure 3 depicts the purchase decisions of three sets of consumers in this case, who are distinguished by their value u for product A:[56]

Consumer Set I ($u < P - p_{B1} < p_A$): These consumers buy (at most) either $B1$ or $B2$. For these consumers, buying only $B1$ is better than buying the bundle since $u < P - p_{B1}$, and buying either only A or both A and $B2$ is unattractive since $u < p_A$. Note that these consumers' preferences for $B1$ and $B2$, including whether to make a purchase at all, are independent of the level of u.

Consumer Set II ($u \in (P - p_{B1}, p_A)$): These consumers buy (at most) either $B2$ or the bundle. For these consumers, buying the bundle is better than buying only $B1$ since $u > P - p_{B1}$, and buying only A or both A and $B2$ is unattractive since $u < p_A$. These consumers' preferences for the bundle versus both buying only $B2$ or buying nothing depend on the value of u, with a consumer in this set more likely to buy the bundle for higher values of u.

Consumer Set III ($P - p_{B1} < p_A < u$): These consumers either buy the bundle or buy A and (possibly) $B2$. For these consumers, buying the bundle is better

[56] We do not specify the purchase decisions of consumer types $u = P - p_{B1}$ and $u = p_A$ as these are measure zero sets and so their purchase decisions do not affect either firm's profit.

than buying only $B1$ since $u > P - p_{B1}$, while a consumer not buying the bundle will certainly purchase A since $u > p_A$. These consumers' preferences for the bundle versus purchase of A and (possibly) $B2$ are independent of the level of u.

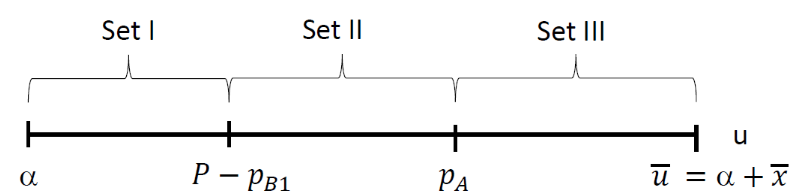

Figure 3: Consumer purchase decisions as a function of their value $u = \alpha + x$ for product A

Using this fact, we have:

Lemma 4 *If tying is allowed and Assumptions 1 and 2 hold, in any equilibrium firm 1 makes no separate sales of product A.*

Proof. Any separate sales of product A to a positive measure of consumers must be to consumers in set III who then all weakly prefer the better of buying only A or buying both A and $B2$ over buying the bundle.

Suppose, first, that buying A only is strictly the better of these two options for these consumers – i.e., that purchase of $B2$ yields strictly negative surplus in the equilibrium. Then the consumer of type $u = p_A$ earns no surplus and must enjoy as well non-positive surplus from the bundle. All types in set II below this type therefore enjoy strictly negative surplus from the bundle, including type $u = P - p_{B1}$. But this then implies that purchase of only product $B1$ gives strictly negative surplus as well. Hence, sales of A and possibly the bundle to consumers in set III are the only sales that firm 1 is making, while firm 2 (which offers strictly negative surplus) is making no sales. We next argue that firm 1 cannot be making sales of each of A and the bundle to positive measures of these consumers: If it were, then all consumers in set III are indifferent between buying A only and buying the bundle. As a result, there would then be a Pareto superior NE consumer response in which all consumers in set III buy the bundle (increasing the network size of $B1$).[57] Thus, firm 1's sales of A to consumers in set III must be the *only* sales firm 1 is making and so its profit is no greater than in the independent-pricing equilibrium, a contradiction to Corollary 1.

Suppose, instead, that buying both A and $B2$ is weakly best for a positive measure of consumers in set III who buy A separately. If so, consumer type $u = p_A$ weakly prefers buying $B2$ only to buying the bundle. All other types in set II then strictly prefer buying $B2$ to buying the bundle. In turn, consumer type $u = P - p_{B1}$ strictly prefers buying $B2$ over buying $B1$, as does every consumer in set I. Thus, again, firm 1's only sales are of product A and possibly the bundle to consumers in set III, while firm 2 may be making sales to consumers in sets I and II in addition to its sales to consumers in set III. In this case, firm 1 cannot be making any sales of the bundle, for if it were there would be a Pareto dominating NE consumer response in which all of the consumers purchasing the bundle instead purchase A and $B2$ (increasing the network size of $B2$, the only product that would be purchased). Thus, we again conclude that firm 1's sales of A are the only sales it is making, yielding a contradiction to Corollary 1. ■

Lemma 4 implies that all consumers buy one of the bundle, $B1$, or $B2$. We can also note the following:

Lemma 5 *If tying is allowed and Assumptions 1 and 2 hold, in any equilibrium all consumers not purchasing the bundle must be making the same choice between products $B1$ and $B2$.*

Proof. If there is a positive measure of consumers purchasing $B1$ and a positive measure purchasing $B2$ all of these consumers must be indifferent between these two options. There would then be a Pareto dominating NE consumer response is which they all purchase $B1$ (thereby increasing the network size of $B1$). ■

We now establish the claims in Proposition 1. We again denote by $\widetilde{X}$ the consumer type that is indifferent between the bundle and $B2$ when consumer types above $\widetilde{X}$ buy the bundle and those below $\widetilde{X}$ buy $B2$, given by $\Psi(\widetilde{X}) = 0$ (see (17)). We denote by $\widetilde{x}$

[57] In this NE consumer response some consumers in set II would also buy the bundle and some consumers in set I might buy $B1$ only.

the consumer type that is indifferent between the bundle and $B1$ when consumer types above $\widetilde{x}$ buy the bundle and those below $\widetilde{x}$ buy $B1$. That is, $\widetilde{x}$ satisfies

$$\widetilde{x} + v_1 - \widehat{P} = v_1 - p_{B1}$$

or equivalently

$$\widetilde{x} = \widehat{P} - p_{B1}$$

We next argue that no sales of $B2$ occur in equilibrium. To see this, suppose instead that all of the consumers who do not buy the bundle buy $B2$ and that $\widetilde{X}^*$ is the cut-off consumer type in the equilibrium. In that case, we can show that firm 1 would have a profitable deviation to undercut firm 2's sales of $B2$, switching firm 2's customers to product $B1$.

To establish this fact, note first that if given p_{B2} we have $\widehat{P} \leq \widehat{P}(0|p_{B2})$, it follows immediately that firm 2 makes no sales. So suppose instead that $\widehat{P} > \widehat{P}(0|p_{B2}) > 0$ and consider the consumer responses for various prices of p_{B1} when firm 1 offers both the bundle and $B1$ for sale with a bundle price of $\widehat{P}$ and price for $B1$ of $p_{B1} \geq 0$:

- $\mathbf{p_{B1} < p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta}$: Since all consumer types above $\widetilde{X}^*$ prefer the bundle over $B2$ by iterated dominance[58] even if all other consumer types below $\widetilde{X}^*$ buy $B2$ the payoff to a consumer whose type is below $\widetilde{X}^*$ from buying $B1$ ($v_1 + \beta(1 - G(\widetilde{X}^*)) - p_{B1}$) exceeds the payoff from buying $B2$ ($v_2 + \beta G(\widetilde{X}^*) - p_{B2}$).[59] Thus, the unique NE consumer response has all such consumers buy $B1$.

- $\mathbf{p_{B1} \in [p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta, p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta)}$: In this case there is a NE consumer response in which all consumer types below $\widetilde{X}^*$ buy either $B1$ or the bundle (consumers below a cut-off type $\widetilde{x}$ purchase $B1$) and another NE in which they all buy $B2$. (All consumer types below $\widetilde{X}^*$ buying $B2$ is a NE consumer response because $v_1 + \beta(1 - G(\widetilde{X}^*)) - p_{B1} < v_2 + \beta G(\widetilde{X}^*) - p_{B2}$.) The former NE response gives a larger payoff to all consumers than the latter NE response: in the former NE response, consumer-types below $\widetilde{X}^*$ have a payoff of at least $(v_1 + \beta - p_{B1})$, exceeding the largest possible payoff from buying $B2$ in any NE, $(v_2 + \beta G(\widetilde{X}^*) - p_{B2})$, since no types above $\widetilde{X}^*$ buy $B2$, while all consumer types above $\widetilde{X}^*$ have a larger

[58] The argument of Lemma 1 can be extended to the case in which firm 1 also offers $B1$ for sale: iterated dominance implies that all types above $\widetilde{X}$ buy either the bundle or $B1$.

[59] Note that the fact that firm 2 is making sales of $B2$ implies that $p_{B2} \leq v_2 + \beta G(\widetilde{X}^*)$ so consumer surplus is non-negative.

payoff in the former NE because of greater network benefits. So the former NE consumer response Pareto dominates the one in which types below $\widetilde{X}^*$ buy $B2$.

- $\mathbf{p_{B1} = p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta}$: In this case there is a NE consumer response in which all consumer types below $\widetilde{X}^*$ buy either $B1$ or the bundle, where the type $\widetilde{x}$ that is indifferent between buying the bundle and buying $B1$ satisfies relation (11) in the text.

  There is also another NE consumer response in which all consumer types below $\widetilde{X}^*$ buy $B2$. While these two NE give consumer types below $\widetilde{x}$ the same payoff, consumers with types above $\widetilde{x}$ strictly prefer the former NE response, so the NE in which all consumers buy either the bundle or $B1$ Pareto dominates the one in which types below $\widetilde{X}^*$ buy $B2$.[60]

- $\mathbf{p_{B1} > p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta}$: In this case there is a NE consumer response in which all consumers with types below $\widetilde{X}^*$ buy $B2$ and, by iterated dominance and the definition of $\widetilde{X}^*$, it is the unique consumer response in which no sales of $B1$ occur. If there is another NE consumer response in which some consumers buy $B1$ (these must be consumer types for which $x \leq \widetilde{x}(\widetilde{X}^*) \leq \widetilde{X}^*$ since otherwise these consumers prefer the bundle over purchase of $B1$) the payoff of these consumers would be no greater than $(v_1 + \beta - p_{B1})$, which is strictly less than their payoff in the NE response where all consumer-types below $\widetilde{X}^*$ buy $B2$, $(v_2 + \beta G(\widetilde{X}^*) - p_{B2})$. Hence, the NE response in which all consumer-types below $\widetilde{X}^*$ buy $B2$ is not Pareto dominated. Moreover, because firm 1 still sells the bundle to all consumer types above $\widetilde{X}^*$ and (we show below) $p_{B1} > 0$, firm 1's profits are lower in the NE response in which all consumer-types below $\widetilde{X}^*$ buy $B2$. By our ("stack-the-deck") assumption that consumers coordinate on the worst undominated NE response for firm 1, all consumer types below $\widetilde{X}^*$ would buy $B2$.

A first implication of these consumer responses is that if consumers are buying $B2$ in an equilibrium then $B2$ offers a strictly positive surplus:

$$p_{B2} < v_2 + \beta G(\widetilde{X}^*).$$

[60]If we instead used a criterion of *strict* Pareto dominance so that the NE consumer response in which all consumer types below $\widetilde{X}^*$ buy $B2$ was not dominated, then our selection of the worst equilibrium for firm 1 among undominated consumer responses would create an openness problem when we consider firm 1's optimal prices for $B1$.

To see this, note first that the fact that consumers are buying $B2$ implies that $p_{B2} \leq v_2 + \beta G(\widetilde{X}^*)$. Moreover, if instead $p_{B2} = v_2 + \beta G(\widetilde{X}^*)$ then firm 1 would have a profitable deviation that undercuts firm 2 by setting a price for $B1$ satisfying

$$p_{B1} = p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta = v_2 + \beta - \Delta = v_1 + \beta > 0.$$

With this deviation, consumers buy either $B1$ or the bundle, with the indifferent consumer being type $\widetilde{x} < \widetilde{X}^*$ satisfying (11).[61] So the deviation expands sales of the bundle (which has $\widehat{P} > 0$) and also makes sales of $B1$ at a strictly positive price.

Next, to show that firm 1 has a profitable deviation that undercuts firm 2 we show that if all lower-type consumers buy $B2$ then in any equilibrium the cut-off type $\widetilde{X}^*$ who is indifferent between the bundle and product $B2$ would have $\widetilde{X}^* \leq \widehat{p}_A^*$. If $\widetilde{X}^* = 0$, this is necessarily the case, so suppose that $\widetilde{X}^* > \widehat{p}_A^* \geq 0$, contrary to our claim. Then for any cut-off type $\widetilde{X} \in [\widetilde{X}^* - \varepsilon, \widetilde{X}^*]$ for some $\varepsilon > 0$, by offering only the bundle for sale at price $\widehat{P}(\widetilde{X}|p_{B2})$ firm 1 can ensure (by iterated dominance[62]) sales of the bundle to all types above $\widetilde{X}$. Thus, it must be that

$$\widetilde{X}^* = \underset{\widetilde{X} \in [\widetilde{X}^* - \varepsilon, \widetilde{X}^*]}{\arg\max} \; \Pi_1(\widetilde{X}|p_{B2}) \equiv [\widehat{P}(\widetilde{X}|p_{B2}) + \alpha] \cdot (1 - G(\widetilde{X})).$$

Substituting we have:

$$\begin{aligned}
\Pi_1(\widetilde{X}|p_{B2}) &= [p_{B2} + \widetilde{X} + \beta(1 - 2G(\widetilde{X})) - \Delta + \alpha](1 - G(\widetilde{X})) \\
&= (\widetilde{X} + \alpha)(1 - G(\widetilde{X})) + [p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta](1 - G(\widetilde{X})) \\
&= \Pi_A(\widetilde{X}) + [p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta](1 - G(\widetilde{X}))
\end{aligned}$$

Observe that by Corollary 1 we have $\Pi_1(\widetilde{X}^*|p_{B2}) > \Pi_A(\widetilde{X}^*)$, which implies that

$$[p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta] > 0 \tag{18}$$

Then:

$$[d\Pi_1(\widetilde{X}|p_{B2})/d\widetilde{X}]_{\widetilde{X}=\widetilde{X}^*} = \Pi_A'(\widetilde{X}^*) - g(\widetilde{X}^*)[p_{B2} + \beta(1 - 2G(\widetilde{X}^*)) - \Delta] - 2\beta g(\widetilde{X}^*)(1 - G(\widetilde{X}^*)) < 0$$

[61] This follows from the discussion in the main text around (13).

[62] This follows in a similar fashion to the proof of Lemma 1 in which $p_{B2} < v_2 + \beta G(\widetilde{X}^*)$ replaces the assumption that $p_{B2} < v_2$ to ensure that all consumer types enjoy a strictly positive surplus.

where the inequality follows because $\Pi'_A(\widetilde{X}^*) < 0$ when $\widetilde{X}^* > \widehat{p}^*_A \geq 0$ and from (18). Thus, we have a contradiction and we can conclude that $\widetilde{X}^* \leq \widehat{p}^*_A$.

Given the consumer responses we described above, observe that firm 1 can undercut firm 2 and switch the consumers buying $B2$ to all buying $B1$ by offering price $p_{B1} = p_{B2} + \beta(1 - G(\widetilde{X}^*)) - \Delta$. This deviation would be profitable (and would be firm 1's best deviation) because, as above, sales of the bundle expand and sales of $B1$ occur at a positive price since

$$\begin{aligned} p_{B1} &= p_{B2} + \beta[1 - G(\widetilde{X}^*)] - \Delta \\ &\geq \beta - \Delta - 2\beta G(\widehat{p}^*_A) \\ &> 0, \end{aligned}$$

where the first inequality follows because $p_{B2} \geq 0$, $\widetilde{X}^* \leq \widehat{p}^*_A$, and $\beta G(\widehat{p}^*_A) \geq 0$, while the second inequality follows from (6). Thus, we conclude that firm 2 cannot be making any sales of $B2$.

Since firm 1 sets p_{B1} at the highest level such that consumers do not switch to buying $B2$, firm 2 must be setting $p^*_{B2} = 0$ or otherwise it would have a profitable deviation that lowers p_{B2} slightly.

Finally, we identify the equilibrium purchases and prices when firm 1 makes sales of the bundle and also possibly of product $B1$, while firm 2 makes no sales and sets $p^*_{B2} = 0$.

As seen above, when firm 1 charges $\widehat{P}(\widetilde{X}|0) = \widetilde{X} + \beta(1 - 2G(\widetilde{X})) - \Delta$ for the bundle, the most firm 1 can charge for $B1$ and secure low-type consumers' business for $B1$ instead of $B2$ is $p_{B1} = \beta(1 - G(\widetilde{X})) - \Delta$. Among all prices for $B1$ less than or equal to this amount, this price for $B1$ is best for firm 1 since any lower price reduces revenues from sales of $B1$ and shifts sales from the higher-priced bundle to the lower-priced $B1$.

If firm 1 charges this amount, types above the cutoff $\widetilde{x}$ satisfying relation (11) buy the bundle, while lower types buy $B1$. We first observe from (11) that $\widetilde{X} = 0$ implies $\widetilde{x} = 0$ and vice versa and that there is a monotonic relationship between $\widetilde{X}$ and $\widetilde{x}$ as $1 - \beta g(\widetilde{X}) > 0$ from Assumption 1. As defined in the proposition, we denote by $\widetilde{X}(\widetilde{x})$ the level of $\widetilde{X}$ satisfying (11) given $\widetilde{x}$ and we denote by $\widetilde{x}(\widetilde{X})$ its inverse.

We now focus on firm 1's optimal choice of $\widetilde{x}$ which solves

$$\underset{\widetilde{x}}{Max}\ [\alpha + \widetilde{X}(\widetilde{x}) + \beta(1 - 2G(\widetilde{X}(\widetilde{x}))) - \Delta](1 - G(\widetilde{x})) + [\beta(1 - G(\widetilde{X}(\widetilde{x}))) - \Delta]G(\widetilde{x})$$

or equivalently

$$\underset{\widetilde{x}}{Max}\ [\alpha+\widetilde{X}(\widetilde{x})+\beta(1-G(\widetilde{X}(\widetilde{x})))-\beta G(\widetilde{X}(\widetilde{x}))-\Delta)](1-G(\widetilde{x}))+[\beta(1-G(\widetilde{X}(\widetilde{x})))-\Delta]G(\widetilde{x})$$

Using the fact that $\widetilde{x}=\widehat{P}-p_{B1}=\widetilde{X}(\widetilde{x})-\beta G(\widetilde{X}(\widetilde{x}))$, this becomes

$$\underset{\widetilde{x}}{Max}\ (\alpha+\widetilde{x})(1-G(\widetilde{x}))+\beta(1-G(\widetilde{X}(\widetilde{x})))-\Delta. \tag{19}$$

whose solution $\widetilde{x}^*$ is the solution to (10). Observe that the fact that firm 1's equilibrium profit must exceed its independent-pricing profit (Corollary 1) implies that we must have $\beta(1-G(\widetilde{X}(\widetilde{x}^*))>0$. Hence, we have $\widetilde{X}(\widetilde{x}^*)<\overline{x}$, which (along with the fact that $g(x)>0$ for all x) implies that $\beta(1-G(\widetilde{X}(\widetilde{x})))$ has a strictly negative derivative at $\widetilde{x}^*$. The remaining conclusions of the Proposition follow from the discussion in the main text.

An Alternative Approach without Any Coordination Assumptions under Tying

To illustrate the robustness of the leverage mechanism with network effects in our model, we show that the same qualitative results can be derived with the use of only iterated dominance under tying. Specifically, we now assume that, when multiple NE consumer responses exist, the consumers' response is the worst such NE for firm 1. Relative to our selection assumption in the main text, this assumption further stacks the deck against firm 1's use of tying: as in the main text, it results in the unique Pareto-undominated NE consumer response under independent pricing, but now the consumer response under tying can be even worse for firm 1 than in the main text (a dominated NE consumer response can result if it yields the lowest profit for firm 1). We demonstrate that profitable market foreclosure with tying is guaranteed even under this more pessimistic scenario against the use of tying by firm 1.

As in the main text, given $p_{B2}\geq 0$, define the bundle price that implements a cut-off type $\widetilde{X}\in[0,\overline{x}]$ between the bundle and *B2* as

$$\widehat{P}(\widetilde{X}|p_{B2})\equiv p_{B2}+\widetilde{X}+\beta(1-2G(\widetilde{x}))-\Delta.$$

Then, all types above $\widetilde{X}$ prefer the bundle over *B2* by iterated dominance. Given that bundle price, the highest price p_{B1} that makes it a *dominant strategy* for a consumer type below $\widetilde{X}$ to purchase *B1* over *B2* is

$$p^D_{B1}(\widetilde{X}|p_{B2}) \equiv p_{B2} + \beta(1 - 2G(\widetilde{X})) - \Delta.$$

At this price, any consumer type below $\widetilde{X}$ prefers *B1* over *B2* even if all other consumers below $\widetilde{X}$ purchase *B2*. Thus, this pricing strategy guarantees the foreclosure of *B2*. Furthermore, from the proof of Lemma 2, we know $\widehat{P}(\widehat{p}^*_A|p_{B2}) > \widehat{p}^*_A$ and $p^D_{B1}(\widehat{p}^*_A|p_{B2}) > 0$: note that $\widehat{P}(\widehat{p}^*_A|p_{B2}) > \widehat{p}^*_A$ is equivalent to $p^D_{B1}(\widehat{p}^*_A|p_{B2}) > 0$. Therefore, firm 2 makes no sales and sets $p_{B2} = 0$ in equilibrium and firm 1 can realize a higher profit than under independent pricing by choosing $\widehat{P} = \widehat{P}(\widehat{p}^*_A|0)$ and $p_{B1} = p^D_{B1}(\widehat{p}^*_A|0)$.[63]

As in the analysis of the main text, given the two prices $\widehat{P}(\widetilde{X}|p_{B2})$ and $p^D_{B1}(\widetilde{X}|p_{B2})$, let $\widetilde{x}(\widetilde{X})$ denote the consumer type that is indifferent between the bundle and $B1$ when all consumers buy the bundle or $B1$. Then, we have $\widetilde{x}(\widetilde{X}) = \widetilde{X}$. When firm 1 implements a cutoff $\widetilde{x}$ by prices $\widehat{P}(\widetilde{x}|0)$ and $p^D_{B1}(\widetilde{x}|0)$, its profit is given by

$$\begin{aligned}\Pi_1(\widetilde{x}) &= \underbrace{[\alpha + \widetilde{x} + \beta(1 - 2G(\widetilde{x})) - \Delta]\,(1 - G(\widetilde{x}))}_{\text{Profit from Bundle Sale}} + \underbrace{[\beta(1 - 2G(\widetilde{x})) - \Delta]\,G(\widetilde{x})}_{\text{Profit from Sale of B1}} \\ &= (\alpha + \widetilde{x})(1 - G(\widetilde{x})) + \beta(1 - 2G(\widetilde{x})) - \Delta \\ &= \Pi_A(\widetilde{x}) + \beta(1 - 2G(\widetilde{x})) - \Delta\end{aligned}$$

As $\beta(1 - 2G(\widetilde{x}))$ strictly decreases with $\widetilde{x}$, by proceeding in a similar manner as in the proof of Proposition 1, we can establish the following Proposition.

Proposition 6 *Suppose that firm 2 enjoys favorable beliefs in that whenever there exist multiple NE consumer responses, the worst such response for firm 1 results. If tying is allowed and Assumptions 1 and 2 hold, the unique equilibrium involves firm 1 tying (offering for sale only the bundle and possibly product B1) and fully monopolizing market B. The cut-off type* $\widetilde{x}^*$ *between sales of the bundle and sales of B1 solves*

$$\underset{\widetilde{x}}{Max}\ \Pi_A(\widetilde{x}) + \beta(1 - 2G(\widetilde{x})), \tag{20}$$

where $\Pi_A(.)$ *is the market A profit function defined in (3).*

When there is full coverage under independent pricing, all consumers buy the bundle ($\widetilde{x}^* = 0$) *and when there is not full coverage under independent pricing, the set of types*

[63] Note that since $\widehat{P}(\widehat{p}^*_A|0) - p^D_{B1}(\widehat{p}^*_A|0) = \widehat{p}^*_A$, firm 1 sells the bundle to all consumer types above $\widehat{p}^*_A$, and $B1$ to those below $\widehat{p}^*_A$.

buying the bundle strictly contains the set of types that buy product A under independent pricing (i.e., $\widetilde{x}^* < \widehat{p}_A^*$*). The equilibrium prices under tying are:*

(i) When firm 1 sells only the bundle (i.e., $\widetilde{x}^* = 0$*), the bundle price is* $P^* = \alpha + (\beta - \Delta)$ *while firm 2 sets* $p_{B2}^* = 0$*.*

(ii) When firm 1 sells both the bundle and $B1$ *(i.e.,* $\widetilde{x}^* > 0$*), prices are*

$$
\begin{aligned}
P^* &= \alpha + \widetilde{x}^* + \beta(1 - 2G(\widetilde{x}^*)) - \Delta \\
p_{B1}^* &= \beta(1 - 2G(\widetilde{x}^*)) - \Delta \\
p_{B2}^* &= 0.
\end{aligned}
$$

Firm 1's equilibrium profit exceeds its profit under independent pricing.