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“Why Is Exclusivity in Broadcasting Rights  
Prevalent and Why Does Simple Regulation Fail?”

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# Why Is Exclusivity in Broadcasting Rights Prevalent and Why Does Simple Regulation Fail?\*

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## Abstract

Pay-TV firms compete both downstream to attract viewers and upstream to acquire broadcasting rights. Because profits inherited from downstream competition satisfy a convexity property, allocating rights to the dominant firm maximizes the industry profit. Such an exclusive allocation of rights emerges as a robust equilibrium outcome but may fail to maximize welfare. We analyze whether a ban on resale and a ban on package bidding may improve welfare. These corrective policies have no impact on the final allocation but lead to profit redistribution along the value chain.

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KEYWORDS. Broadcasting rights; upstream and downstream competition; exclusivity.

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## 1. INTRODUCTION

MOTIVATION. The pay-television (pay-TV henceforth) market is characterized by fierce competition all along the value chain. In the upstream market for contents, TV firms compete through bidding procedures to obtain must-have contents (sports events or premium movies) from content owners (sport leagues, major film studios, etc.). In the downstream market for viewers, TV firms compete in price and quality to attract viewers.

Upstream and downstream competition are deeply intertwined. Downstream, viewers choose which channels to purchase by comparing the price and quality of their respective offers, with the understanding that quality depends on the distribution of broadcasting rights inherited from the upstream stage. Upstream, TV firms express a willingness to pay for contents that depends on how those contents might help them gain market shares downstream.

A remarkable feature of those markets is that most often, a single TV firm ends up obtaining all the broadcasting rights for a class of premium contents. That such exclusivity emerges as a robust outcome raises a number of important questions. First, on the positive side, we may wonder which fundamental economic forces drive such concentration. Why would a content owner choose to pick a single winner and not spread out such rights across rivals to *in fine* touch a larger viewership? Is exclusivity a property that prevails across all equilibrium outcomes and for various modes of downstream competition? Is it robust to the auction format? Second, and taking a more normative stance, we may also ask whether such exclusivity harms welfare and, under those circumstances, whether simple regulatory constraints might have a role in improving this outcome.

To answer these questions, we present a simple model of two-sided competition that blends key features of the upstream and downstream markets. Building such an integrated model is a necessary step to assess the origins and consequences of exclusive agreements in those markets. On the one hand, a partial focus on the upstream bidding market would not suffice to understand bidding strategies since TV firms evaluate their willingness to pay for broadcasting rights with an eye on how the downstream allocation of viewers across channels impacts downstream profits. Acquiring exclusive rights certainly gives a winning firm a competitive wedge downstream by attracting more viewers. It also hurts this firm's competitors by reducing their own market shares, which is a standard feature of auctions with externalities. On the other hand, focusing on the downstream market alone would abusively reduce the analysis to a simple two-stage game with firms choosing the price and quality of their own programming without taking into account that the quality of contents<sup>1</sup> actually depends on how the upstream market is cleared, which itself depends on downstream profits and thus on the quality of programs.

CONVEXITY OF PROFIT FUNCTIONS. This circularity is at the core of our analysis. Its key implication is that there are strong reasons why exclusive agreements emerge and dominate other non-exclusive modes of distribution from the industry's viewpoint. The key property beyond this result is the *convexity* of the profit functions in the downstream market. A dominant firm downstream gains more from obtaining exclusive rights upstream than what it would lose if those rights were instead given to a weaker competitor.

Although the convexity property is shown to hold under a broader set of assumptions on actual modes of downstream competition, it is useful to illustrate this property in more

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<sup>1</sup>See Shaked and Sutton (1982, 1983) for some contributions on this front.

details, by adopting a modeling of downstream differentiation across TV channels, which is now well established in the literature. To this end, we consider two pay-TV firms located at the extreme points of a Hotelling segment.<sup>2</sup> Viewers are uniformly distributed along this line and face constant marginal transportation costs, which is a standard metaphor for the preferences bias that consumers may *a priori* have for each TV channel. When offering a better quality of content, a firm becomes more attractive and increases its market share. This firm may then charge a higher price and increase its profit. At the same time, its lower-quality competitor views its market share as shrinking, and thus reduces its own price and earns lower profits. Starting from a hypothetical situation where both suppliers would offer the same quality of content, for instance, because broadcasting rights are distributed evenly, an (exogenous) increase in the quality provided by one firm redistributes viewers, but the price increase for that firm just equals the price decrease for its rival. As a result of these joint changes in prices and market shares, the increase in profit of the high-quality firm more than offsets the loss in profit for the low-quality supplier. Henceforth, profit functions in the downstream market are necessarily convex.

**CONSTRAINED EFFICIENCY AND MONOTONIC BIDDING EQUILIBRIA.** This convexity property ensures that granting exclusivity for broadcasting rights is *constrained-efficient*; *i.e.*, it maximizes the industry's overall profit. Moreover, if firms are *a priori* different, because, for instance, one of them is better able to market premium programs or has already benefited from a captive viewership, then all rights should be given to this firm, which might reinforce its dominance in a dynamic context.

To avoid making any restrictions on the auction procedure, we view competition in this market as a menu auction in the spirit of Wilson (1979) and Bernheim and Whinston (1986a). Downstream firms submit nonnegative bidding schedules that stipulate bids for all possible distributions of rights that could be chosen by their upstream owner. To illustrate, this owner could choose to evenly allocate rights or to opt for exclusivity depending on the bids she may collect. Thanks to the convexity of profit functions, granting exclusivity on broadcasting rights is *constrained-efficient*; *i.e.*, it maximizes the industry's overall profit. That the equilibrium allocation is *constrained-efficient* is a robust finding. This finding holds throughout the whole set of equilibria of the bidding procedure that can be reached by means of monotonic bidding schedules.<sup>3</sup>

Of course, those equilibria differ in terms of the distribution of profits they induce across the industry. Although the weaker firm always obtains the same payoff, profits may be redistributed across equilibria from the dominant firm to the upstream seller of rights. Among all possible equilibria, the highest profit to the dominant firm is achieved by means of *truthful bidding schedules* (Bernheim and Whinston, 1986a). Those bidding schedules are undominated strategies that perfectly reflect the preferences of firms across the various possible allocations of rights. In such a *truthful equilibrium*, the dominant firm bids a Vickrey-Clarke-Groves payment that compensates the seller for the foregone opportunity of not having sold to the weaker firm. In other equilibria (based on weakly dominated strategies for the weaker firm), the dominant firm may end up paying even more to obtain exclusivity. This property of the truthful equilibrium thus echoes the

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<sup>2</sup>Armstrong (1999), Gabszewicz et al. (2001, 2002, 2004), Harbord and Ottaviani (2001), Dukes and Gal-Or (2003), Gal-Or and Dukes (2003), Anderson and Coate (2005), Peitz and Valletti (2008) and Stenneck (2014).

<sup>3</sup>Monotonicity means that a firm should bid more for an allocation that increases its profit, which is quite a weak restriction.

celebrated Chicago School argument, which posits that, if any exclusive agreement is ever signed, the seller has certainly been compensated for the foregone opportunity of not having chosen a more even distribution of rights across downstream competitors.

IS REGULATION WARRANTED? In this respect, an important issue is to determine whether those gains that pertain to the industry can be redistributed to viewers. The presumption often made in related antitrust cases is that they are. Coming back to our Hotelling illustration, the fact that viewers are charged by TV firms a price that is independent of their location means that those firms are unable to grasp all consumer surplus and, as a result, that the industry's most preferred allocation of rights may not maximize welfare. In this scenario, exclusive agreements might also be considered, perhaps more pessimistically, as reducing downstream competition for viewers and thus hindering consumer surplus. That the industry equilibrium may fail to be welfare-maximizing thus *a priori* calls for some sort of public intervention.

As a result of this tension between the possible efficiency gains of exclusivity for the industry and its negative consequences on viewers, competition authorities throughout the world have taken different postures. In North America, case law specific to sports media views exclusive agreements as *a priori* legal. In the recent *Spinelli v NFL* case, the Court held that “because the benefits of exclusive licensing agreements are well-recognized,” the arrangements were presumptively legal.<sup>4</sup> In 2015, the Canadian Competition Bureau approved a 12-year exclusive distribution agreement between the NHL and Rogers Broadcasting, arguing that the deal would not foreclose competition and that pro-competitive gains in the form of quality investments would benefit viewers.<sup>5</sup> The attitude of the EU competition authority toward such exclusivity agreements differs, as exemplified by the landmark UEFA Champions League decision.<sup>6</sup> There, the joint-selling arrangement initially notified by the UEFA implied that all broadcasting rights for this elite soccer clubs competition were sold to a single broadcaster in each Member State and on an exclusive basis for periods up to four years. The European Commission negotiated several important changes to the initial agreement, including that open, transparent and nondiscriminatory tenders must be used to sell those rights, that several packages have to be offered and no single bidder can acquire all packages exclusively, and that exclusivity is limited in time (up to three years as a general rule).<sup>7</sup>

To the extent that corrective policies are certainly called for, policy recommendations would be at risk of being based on a particular selection of the best-response correspondence and thus nonrobust without having at hand a full characterization of equilibria payoffs. An important take-away of our analysis is that such characterization actually shows that the light-handed tools available to competition authorities to prevent dominance have no impact on the allocation of rights and very minor consequences on the distribution of profits across the industry. Thus, exclusivity seems hard to avoid.

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<sup>4</sup>Spinelli v NFL, 96 F Supp 3d 81 (SDNY 2015), available at <https://law.justia.com/cases/federal/appellate-courts/ca2/17-0673/17-0673-2018-09-11.html>.

<sup>5</sup>See Bachelor et al. (2020).

<sup>6</sup>See 2003/778/EC: Commission Decision of 23 July 2003 relating to a proceeding pursuant to Article 81 of the EC Treaty and Article 53 of the EEA Agreement (COMP/C.2-37.398 – Joint selling of the commercial rights of the UEFA Champions League), available at <https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32003D0778>.

<sup>7</sup>See Toft (2003) for a detailed account of the negotiations between UEFA and the European Commission.

**LIGHT-HANDED REGULATORY CONSTRAINTS.** To show this result, we analyze the impact of several regulatory constraints that might be imposed on the bidding game in view of improving welfare. Typically, those constraints may be a ban on the resale of the rights or a ban on package bidding when multiple rights might be allocated. To motivate a regulatory view on resale and package bidding, the case of the French soccer league is in order. In 2018, broadcasting rights for the next four seasons were on sale. These rights were divided into seven packages, and participants could not make any global offer (*i.e.*, a joint bid for several lots). Mediapro ended up getting the lion's share with five lots, which made an overall very attractive package. beIN and the telecom operator Free each obtained a package. Last but not least, the traditional distributor of those programs over the last couple of decades, Canal+, returned empty-handed. Mediapro's price was quickly judged by many observers as being excessive, and industry experts also reported that its initial plan was to resell some of the rights acquired earlier on to its main competitor, namely, Canal+.<sup>8</sup> However, after the tender, Canal+ and beIN promptly sealed an exclusive distribution and sublicensing deal for the broadcasting of the lot obtained by beIN. Mediapro subsequently failed to fulfill its payment obligations to the French soccer league, thereby putting their agreement and the financial health of the league itself at risk. The rights initially obtained by Mediapro were then reallocated in a separate tender, with Amazon obtaining all the rights against a consortium of bidders formed by Canal+ and beIN at a much discounted price relative to Mediapro's initial offer.<sup>9</sup>

*Resale of Rights.* Pay-TV markets often feature resales of rights, sometimes even between firms that were earlier competitors at the bidding stage. Resale has often been viewed by practitioners as a means for a dominant firm to increase its monopoly power by buying back rights sold earlier on (maybe wrongly so) to a competitor.

Resale comes with pros and cons. First, any constrained-inefficient allocation of rights can be renegotiated with resale so that, absent informational problems in bargaining and any further transaction costs, the final allocation is necessarily constrained-efficient. Henceforth, resale may be unattractive from a social welfare viewpoint and should be considered with caution by competition authorities. Second, when it is allowed, the possibility of resale is anticipated by bidders earlier on. They actually bid for getting an *interim* allocation of rights that might be renegotiated. Such renegotiation transforms the bidding game into a constant-sum game. Whoever finally gets the good ends up paying whoever gets it at the *interim* stage. Bidding for an *interim* allocation becomes a fierce head-to-head auction that erodes much of the dominant firm's profit. These profits are now pocketed by the initial owner of rights. Henceforth, any ban on resale has a redistributive impact along the supply chain, although it does not change the final allocation of rights. With or without resale, the final allocation is necessarily constrained-efficient.

*Package Bidding and Multiple Rights.* We next consider a setting where multiple rights can be sold simultaneously. As alluded to in the above example of the French soccer

<sup>8</sup>See, for instance, <https://www.lequipe.fr/Football/Article/Droits-tv-la-strategie-ratee-demediapro-pour-rivaliser-avec-canal-et-bein-sports/1186223>.

<sup>9</sup>For a detailed account of the tender and the legal disputes following the subsequent withdrawal of Mediapro, see Décision no. 21-D-12 du 11 juin 2021 relative à des pratiques mises en oeuvre par la Ligue de Football Professionnel dans le secteur de la vente de droits de diffusion télévisuelle de compétitions sportives, available at [https://www.autoritedelaconcurrence.fr/sites/default/files/integral\\_texts/2021-06/21d12\\_0.pdf](https://www.autoritedelaconcurrence.fr/sites/default/files/integral_texts/2021-06/21d12_0.pdf).

league, this scenario fits well with most procedures taking place for sports events. In those circumstances, competition authorities have sometimes suggested that the dominance of key players could be undermined by splitting upstream auctions into smaller tender procedures in which broadcasting rights of lesser size would be for sale.<sup>10</sup> Again, the costs and benefits of such procedures and their welfare consequences should be assessed with a careful look at outcomes that follow from splitting and a comparison with the scenario of a package auction where all rights are on sale at once.

Our analysis starts with the case of package bidding, where firms are allowed to freely bid for any combination of the rights for sale. Again, convexity of the downstream profit functions prevails when multiple rights are at stake and constrained efficiency calls for giving all rights to the dominant firm. This outcome is achieved in all monotonic equilibria, although different distributions of profits can be sustained. Among those possible distributions, the one that most favors the dominant firm is achieved with truthful bidding schedules. The incremental value for the dominant firm of obtaining a second set of rights, when already holding one set of rights, is large. This dominant firm is thus eager to make an aggressive bid for the whole package of exclusivity rights.

We then consider the outcome when auction markets for each set of rights are split. To this end, we first borrow from the vertical restraints literature and define a *pairwise-proof allocation* as the result of a *market-by-market bidding equilibrium*.<sup>11</sup> Bidding schedules are found for a given set of rights with the expectation that a constrained-efficient allocation of those rights that would favor the dominant firm arises for other rights. Focusing on (undominated) truthful outcomes on each market, in order to favor the dominant firm, we characterize such allocation. Of course, it is again constrained-efficient. From the logic of Vickrey-Clarke-Groves payments, but now applied market by market, the dominant firm needs only to compensate the owner for the foregone opportunity of not selling rights on a single market. These opportunities correspond to the weaker firm's willingness to pay for exclusivity in only one market, expecting that the dominant firm already gets exclusivity in the other. Due to the convexity of the weaker firm's profit function, this willingness to pay is quite low. In a pairwise-proof allocation, the dominant firm ends up paying little for exclusivity on all rights.

One could *a priori* conclude that competition authorities would, perhaps contrary to their intention, stack the deck in favor of the dominant firm by banning package bidding. In this respect, banning package bidding and banning resale would have the same effect. The above intuition is actually wrong. Indeed, we first demonstrate that the pairwise-proof allocation that is implemented with truthful bidding schedules market-by-market is not immune to multilateral deviations in both markets at the same time.<sup>12</sup> Intuitively, and it again follows from the convexity of profit functions, the weaker firm could outbid the dominant firm, which pays too little, and obtain exclusivity in both markets by making a more aggressive bid for the package. Policy recommendations relying on pairwise-proofness would thus be misleading.

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<sup>10</sup>In the above-mentioned 2018 tender organized by the French soccer league, seven lots were allocated sequentially.

<sup>11</sup>Crémer and Riordan (1987), Horn and Wolinsky (1988), O'Brien and Shaffer (1992), Hart and Tirole (1990) and McAfee and Schwartz (1994). See also Collard-Wexler et al. (2019) for an up-to-date overview of the literature.

<sup>12</sup>In the literature on vertical restraints, this possibility is well known, at least since McAfee and Schwartz (1995), Segal and Whinston (2003), and Rey and Vergé (2004).

Rather than imposing *pairwise-proofness* in tandem with truthfulness on each market, we thus only require that bidding schedules should be additively separable across auctions when those auctions are split and characterize equilibria on all markets at once. Robustness to multilateral deviations is then ensured for free with this requirement. The additivity constraint *a priori* seems to limit the expression of the willingness to pay for bundles, which are so attractive in this environment (thanks again to the convexity of profit functions). As such, one may expect that this would make it difficult for the dominant firm to gain exclusivity in all markets. This intuition is again wrong. We demonstrate that *all* distributions of profits available in any monotonic equilibrium with package auctions remain feasible when the additivity restriction is imposed. This conclusion holds because, with the discrete allocation of rights (each set of rights being allocated to either downstream firm), there is much leeway in finding bids that force the dominant firm to pay what it is worth obtaining exclusivity on all rights while also preventing the initial owner from choosing other allocations.

ORGANIZATION OF THE PAPER. Section 2 gives a brief account of the relevant literature. Section 3 presents the different players (sellers of rights in the upstream market, pay-TV firms, viewers) and describes the upstream and downstream markets in which they interact. Section 4 discusses the structure of downstream profits and their important convexity property. In Section 5, we characterize monotonic equilibria and the corresponding distributions of profits and surplus. Section 6 considers the possibility of resale. Section 7 extends our framework to allow for multiple broadcasting rights and the possible complementarities between those rights from the point of view of downstream firms. Section 8 briefly recaps our findings.

## 2. LITERATURE REVIEW

This paper touches on several trends in the literature. On the more applied side, models of pay-TV downstream competition have most often relied on the Hotelling-based approach established by Gabszewicz et al. (2001, 2002, 2004), Dukes and Gal-Or (2003), Gal-Or and Dukes (2003), Anderson and Coate (2005) and Peitz and Valletti (2008). These contributions examine a number of issues, including content differentiation, advertising intensities and comparisons of welfare under pay-TV and free-to-air (advertising-financed) television. However, none of these articles considers content exclusivity.

More specifically, the literature on exclusivity in the pay-TV market is thin, to say the least. Earlier works were mainly informed by British experiences on the subject. For instance, Armstrong (1999) discusses the possibility that two pay-TV networks could be implicitly colluding by exchanging content. This author analyzes incentives to sign exclusive agreements in this context. Harbord and Ottaviani (2001) analyze contractual agreements, particularly in the context of the resale of broadcasting rights. The fine structure of these contracts (fixed payments or more complex schedules depending on market shares) plays a crucial role in assessing their anti-competitive properties. Stenneck (2014) shows how exclusive distribution can stimulate specific investments by distributors and benefit all viewers, even those who *a priori* do not have access to the premium offer so distributed. Sonnac (2012) explains the strategic nature of exclusivity clauses in the pay-TV market. Weeds (2016) examines incentives for exclusivity in a dynamic setting and argues that switching costs confer benefits to a vertically integrated operator, which may thus forego the static benefits of selling contents to a rival competitor. In



our paper, exclusivity arises because it maximizes industry profit, and (unrestricted) upstream auction procedures have the property of reaching this outcome.

Taking a broader theoretical perspective, this paper contributes on several fronts. The idea that exclusivity is an equilibrium outcome when it is constrained-efficient is already present under various forms in the vertical contracting literature. Aghion and Bolton (1987), Choné and Linnemer (2015) and Calzolari and Denicolò (2015) develop this point but also stress some of its limits under various technological and informational scenarios. Closer to our perspective is certainly the framework proposed by Bernheim and Whinston (1998). In a model where two firms (principals) bid to attract an agent's services, those authors also show that in the absence of frictions, exclusive dealing, when it arises, maximizes the profit of the grand coalition. Bedre-Defolie and Biglaiser (2022) study, as we do, competition for the exclusive use of a superior input, but in a context where the input lowers the cost of improving the quality of a firm's product. Qualities are exogenous in our setting and we are interested in a different set of issues related specifically to broadcasting rights: the possibility of resale of the right between downstream competitors and competition for multiple rights. Lastly, but less related to our analysis, Esö, Nocke and White (2010) also study a general model of competition for a scarce resource and show that scarcity may induce asymmetric industry structure downstream.

To assess the possible consequences of constraints on the auction formats, we need a full characterization of equilibria in such bidding games. This step, although much reminiscent, goes beyond the analysis of menu auction games performed in Bernheim and Whinston (1986a), who instead uniquely focus on truthful equilibria. To do so, we rely on some general techniques developed by Martimort and Stole (2012) in their study of aggregate games, *i.e.*, a more general class of games that includes menu auctions and other contracting scenarios as special cases.<sup>13</sup> In our set up, those menu auctions also entail downstream externalities, and our paper contributes to the relevant literature as well (Jéhiel and Moldovanu, 2000; Varma, 2002; Brocas, 2013; Assef and Chade, 2008; Molnar and Virag, 2008; Martimort and Pouyet, 2020; among many others).

Last, our paper, in regard to developing a comparative analysis of package bidding and split auctions for multiple rights, also speaks to the literature on vertical restraints (Crémer and Riordan, 1987; Horn and Wolinsky, 1988; O'Brien and Shaffer, 1992; Hart and Tirole, 1990; McAfee and Schwartz, 1994; Collard-Wexler et al., 2019) and borrows from there the notion of pairwise-proofness. In our context, we show the limit of this approach that fails to account for simultaneous deviations in all markets at once when bidding schedules are truthful on each market taken separately. The possibility that multilateral deviations may kill a putative pairwise-proof allocation has been well known since at least the works of McAfee and Schwartz (1995), Segal and Whinston (2003), and Rey and Vergé (2004). To restore the existence of equilibria under all circumstances, we retain an additivity constraint on bidding schedules that capture the idea that bidding takes place market by market but no longer impose the truthfulness criterion on each of those markets.

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<sup>13</sup>Bernheim and Whinston (1986b) already pointed out that common agency games have this aggregate property.

### 3. MARKETS, PLAYERS, PRELIMINARY RESULTS

STRUCTURE OF THE INDUSTRY. Two pay-TV firms,  $F_0$  and  $F_1$ , compete in the downstream market for viewers. They offer program packages that include some basic service (free-to-air TV or classic movies, for instance) plus possibly some premium programs (sports events or blockbusters). Competition in the downstream market thus entails both a quality (the content of those packages) and a price component. In the upstream market,  $F_0$  and  $F_1$  bid to acquire premium broadcasting rights from a supplier  $A$ . Those rights may entail either an exclusive or a joint distribution of the premium contents by service providers.

ALLOCATIONS OF RIGHTS AND CONSEQUENCES ON DOWNSTREAM PROFITS. Let the set of feasible allocations of rights be denoted by  $\mathcal{A} = \{0, 1, c, \emptyset\}$ , where 0 (resp. 1) stands for exclusivity to  $F_0$  (resp.  $F_1$ ),  $c$  stands for joint distribution, and  $\emptyset$  stands for no allocation of rights at all, which is an issue that is of course weakly dominated but included in our description for the sake of completeness.

An allocation of rights  $a$  can be parametrized by two quality parameters  $\alpha_0$  and  $\alpha_1$  that respectively pertain to firm  $F_0$  or  $F_1$  respectively. We express their respective profits as  $\tilde{\Pi}_0(\alpha_0, \alpha_1)$  and  $\tilde{\Pi}_1(\alpha_0, \alpha_1)$ . The parameter  $\alpha_i$  describes  $F_i$ 's quality on the downstream market for viewers which is inherited from upstream competition for the acquisition of broadcasting rights. If rights are jointly allocated to both firms, firms have the same quality parameter  $\alpha$  and earn the same profit, which we denote by  $\Pi(c) \equiv \tilde{\Pi}_0(\alpha, \alpha) = \tilde{\Pi}_1(\alpha, \alpha)$ . If rights are allocated exclusively to  $F_0$  (resp.  $F_1$ ), then profits are given by  $\Pi_0(0) \equiv \tilde{\Pi}_0(\alpha + \Delta_0, 0)$  and  $\Pi_1(0) \equiv \tilde{\Pi}_1(\alpha + \Delta_0, 0)$  (resp.  $\Pi_0(1) \equiv \tilde{\Pi}_0(0, \alpha + \Delta_1)$  and  $\Pi_1(1) \equiv \tilde{\Pi}_1(0, \alpha + \Delta_1)$ ). The parameter  $\Delta_i > 0$  captures how firm  $F_i$  benefits from obtaining exclusivity on the broadcasting rights compared to the scenario of a joint distribution of those rights. To summarize, the following string of inequalities hold for  $F_0$ 's and  $F_1$ 's profit functions respectively,  $\Pi_0(0) > \Pi(c) > \Pi_0(1)$  and  $\Pi_1(1) > \Pi(c) > \Pi_1(0)$ .

DOMINANCE. Dominance on the downstream market by firm  $F_0$  is modeled by assuming that  $\Delta_0 > \Delta_1$ . In words,  $F_0$  benefits more than  $F_1$  from obtaining exclusivity over the broadcasting rights.

THE CONVEXITY PROPERTY AND EFFICIENCY NOTIONS. The next assumption turns out to be useful for our analysis.

ASSUMPTION 1. *Firms' profits are such that*

$$\Pi_i(i) + \Pi_{-i}(i) > 2\Pi(c), \quad \forall i \in \{0, 1\}.$$

Assumption 1 essentially states that the industry profit is maximized when broadcasting rights are allocated exclusively to either firm rather than when they are jointly allocated without any exclusivity clause to both firms. In that sense, the industry profit is convex in the allocation of broadcasting rights. The general intuition underlying Assumption 1 is as follows. Consider as a starting point a situation of joint distribution of broadcasting rights where firms offer the same quality level to their customers and thus equally share the market. A situation of exclusive distribution creates an asymmetry between firms, since one of them now offers viewers a higher-quality product. The high-quality firm increases both its price and its market share, whereas the reverse happens for the low-quality firm. Assumption 1 means that moving from joint to exclusive distribution of broadcasting rights increases more the high-quality firm's profit than it decreases

the low-quality firm's, leading to an overall higher industry profit. We postpone to the next section a more in-depth discussion of the plausible market structures that support Assumption 1.

In the sequel, we are interested in the efficiency properties of the equilibrium allocations of rights. A *constrained efficiency* criterion only considers the industry's viewpoint, thus maximizing the total profit of the grand coalition  $A - F_0 - F_1$ . An *efficiency* criterion instead assesses the consequences of such an allocation on overall welfare, including consumer surplus; which is an objective that is in line with that of competition authorities.

LEMMA 1. *The allocation  $a^* = (0)$  is the constrained-efficient allocation, whereas a joint distribution is the worst scenario from the industry's viewpoint:*

$$(3.1) \quad \sum_{i=0,1} \Pi_i(0) > \sum_{i=0,1} \Pi_i(1) > 2\Pi(c).$$

To understand the ranking of aggregate profits in (3.1), we must first return to the convexity of profits stressed in Assumption 1. Granting exclusivity to either firm is an extreme allocation that thus dominates a joint distribution of rights from the point of view of the industry. This explains the second inequality in (3.1). Moreover, this effect is more important when the dominant  $F_0$  has exclusivity since the quality differential with its rival is then greater. This explains the first inequality in (3.1).

BIDDING STRATEGIES. Pay-TV firms approach  $A$  and bid for acquiring broadcasting rights. Following a methodology that was initiated by Bernheim and Whinston (1998),  $F_0$  and  $F_1$  compete by means of bidding schedules.<sup>14</sup> Those schedules are commitments that stipulate how much these firms are ready to pay  $A$  for each configuration of rights. Formally, a bidding schedule is thus a nonnegative mapping  $T_i$  on  $\mathcal{A}$ .<sup>15</sup>  $T_i(a)$  stands for the share of  $F_i$ 's profit that is left to  $A$  when an allocation of rights  $a$  is chosen by  $A$ . Hence, whether exclusivity or a joint distribution emerges is actually an equilibrium outcome.

TIMING. The overall game of bidding and downstream competition unfolds as follows:

1. *Upstream market.*  $F_0$  and  $F_1$  simultaneously offer bidding schedules  $T_0$  and  $T_1$  to  $A$ . However, since bidding schedules are nonnegative,  $A$  accepts those offers.
2. *Allocation of rights.*  $A$  chooses a distribution of rights  $a \in \mathcal{A}$  and accordingly pockets the bids  $T_i(a)$  ( $i = 0, 1$ ) from downstream firms.
3. *Downstream market.*  $F_0$  and  $F_1$  simultaneously and non-cooperatively choose their prices for the TV-offers they deliver, respectively.

The equilibrium concept is subgame-perfect equilibrium. We will sometimes rely on refinements (based on monotonicity, truthfulness or/and dominance) to pin down a unique outcome.

<sup>14</sup>Hagi and Lee (2011) adopt a similar approach in the context of competition between platforms.

<sup>15</sup>Non-negativity of the bidding schedules is without loss of generality. Indeed,  $A$  would never accept a bidding schedule from  $F_i$  with some negative payments and choose an allocation  $a$  corresponding to such a negative payment, *i.e.*,  $T_i(a) < 0$ .  $A$  could also increase its payoff by simply choosing the same allocation and excluding  $F_i$  from the auction. Beyond this, no further restrictions are made on the bidding strategies.

#### 4. MARKET STRUCTURES AND THE CONVEXITY PROPERTY

This section discusses the convexity property stated in Assumption 1 and shows that it applies under a broad range of circumstances and modes of downstream competition.

**MONOPOLY BENCHMARK.** Let us start with a simpler situation in which a monopoly firm with no costs supplies a market where viewers have a demand given by  $\alpha + D(p)$ , with  $D'(\cdot) < 0$ . The parameter  $\alpha$  is a vertical shifter of the demand function akin to a quality attribute of the firm's product. The monopoly firm chooses its price so as to maximize its profit and earns  $\Pi(\alpha) = \max_p p(\alpha + D(p))$ . As the maximum of linear and strictly increasing functions of  $\alpha$ ,  $\Pi(\alpha)$  is convex and strictly increasing. Intuitively, increasing product quality lowers the price elasticity of the demand which allows in turn to increase the price charged to viewers.

We turn next to settings with imperfect competition. The analysis now involves two effects. First, an increase in the quality of a firm has a direct effect on profit similar to that found in the monopoly benchmark. Second, such an increase also has an indirect effect since it impacts the equilibrium price charged by the rival firm. Roughly speaking, the convexity property continues to hold when the positive direct effect on profit associated to selling a product with a higher quality outweighs the possibly negative indirect effect. Details about the various market structures we discuss in this section can be found in the Appendix.

**DUOPOLY WITH VERTICAL DEMAND SHIFTERS.** Consider an allocation of rights  $a$  parametrized by the quality attributes  $(\alpha_0, \alpha_1)$ . Let the quality differential be denoted by  $\Delta\alpha \equiv \alpha_0 - \alpha_1$ . Suppose that firms compete in prices and face demands given by  $D_0(\Delta\alpha, p_0, p_1)$  and  $D_1(\Delta\alpha, p_1, p_0)$  for firms  $F_0$  and  $F_1$  respectively. Assume that  $\frac{\partial D_0}{\partial \Delta\alpha}(\Delta\alpha, p_0, p_1) \geq -\frac{\partial D_1}{\partial \Delta\alpha}(\Delta\alpha, p_1, p_0) > 0$ . These conditions mean that an allocation of rights that is favorable to  $F_0$  (or,  $\Delta\alpha \geq 0$ ) has a positive impact on  $F_0$ 's demand and a negative one on  $F_1$ 's, and the magnitude of this impact is stronger for the dominant firm than for the non-dominant one. In a nutshell, we have here an extension of our monopoly benchmark to a duopoly setting. The key point is that each firm cares about its quality relative to its rival (that is,  $\alpha_i - \alpha_{-i}$  for firm  $i$ ) rather than its absolute quality level (that is,  $\alpha_i$ ).

Profits at the Nash equilibrium in prices depend only on the quality differential and can be respectively expressed as  $\tilde{\Pi}_0(\Delta\alpha)$  for  $F_0$  and  $\tilde{\Pi}_1(\Delta\alpha)$  for  $F_1$ . Under some standard assumptions on the demand functions, it can be shown that the industry profit  $\tilde{\Pi}_0(\Delta\alpha) + \tilde{\Pi}_1(\Delta\alpha)$  increases with the quality differential  $\Delta\alpha$  for  $\Delta\alpha$  small enough. Hence, Assumption 1 holds and firms are collectively better off with an exclusive distribution of the rights compared to a joint distribution.

**DISCRETE CHOICE MODEL.** Consider a discrete choice model in which a viewer gets utility  $\alpha_i + \varepsilon_i - p_i$  when buying from  $F_i$ . The taste shocks  $\varepsilon_i$ s are i.i.d. variables. Let  $z = \varepsilon_0 - \varepsilon_1$  be the difference in taste shocks. A customer chooses  $F_0$  if  $z \geq (p_0 - p_1) - \Delta\alpha$ , and  $F_1$  otherwise. At the Nash equilibrium in prices, profits depend only on the quality differential  $\Delta\alpha$  and the distribution of  $z$ . The market is actually split at  $z^*(\Delta\alpha)$  that solves  $z^*(\Delta\alpha) + \Delta\alpha = \frac{1-2F(z^*(\Delta\alpha))}{f(z^*(\Delta\alpha))}$ . Equilibrium profits for  $F_0$  and  $F_1$  are thus given by  $\tilde{\Pi}_0(\Delta\alpha) = \frac{(1-F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$  and  $\tilde{\Pi}_1(\Delta\alpha) = \frac{(F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$ . Further assuming that  $\varepsilon_i$  follows the Gumbel distribution with location 0 and scale  $1/\lambda$  (with  $\lambda > 0$ ),  $z$  follows a logistic

distribution with cumulative distribution  $F(z) = \frac{e^{\lambda z}}{1+e^{\lambda z}}$ . It can then be shown that Assumption 1 always holds.

**COURNOT AND BERTRAND COMPETITION WITH LINEAR DEMANDS.** Assume that firms are Cournot competitors. Inverse demand functions are given by  $P_i(\alpha_i, q_i, q_{-i}) = a + \alpha_i - q_i + bq_{-i}$ , for  $i = 0, 1$  and  $-1 < b < 0$ . Equilibrium profit for  $F_i$  is given by  $\tilde{\Pi}(\alpha_i, \alpha_{-i}) = \left(\frac{a(2+b)+2\alpha_i+b\alpha_{-i}}{4-b^2}\right)^2$ . We have  $\tilde{\Pi}(\alpha + \Delta_i, 0) + \tilde{\Pi}(0, \alpha + \Delta_i) > 2\tilde{\Pi}(\alpha, \alpha)$  when  $\Delta_i$  is large enough. Hence, Assumption 1 holds true when the extra gains for firms associated to an exclusive distribution of the rights are large enough. The same result still holds if firms are instead Bertrand competitors.

**HOTELLING COMPETITION WITH SINGLE- AND MULTI-HOMING VIEWERS.** The next example introduces the possibility for some viewers to buy from both pay-TV firms rather than from only one supplier. In the case of sports broadcasting rights, an often-heard argument is that splitting rights across different suppliers somewhat forces sport fans to subscribe to several pay-TV firms. Moving beyond the realm of sport broadcasting, anecdotal evidence suggests that some viewers subscribe to more than one pay-TV offer. Checking that our insights are general enough to cover these situations is thus of particular relevance.

Assume that there is a population of viewers with unit mass. A share  $1 - \varepsilon$  of those viewers are single-homers, that is, they buy at most one unit of pay-TV services from the suppliers. The complementary share  $\varepsilon$  are multi-homers who may buy at most two units, one from each supplier. Both types of viewers are uniformly distributed on the  $[0, 1]$ -segment, and suppliers are located at the extreme points of this segment.

Consider first single-homing viewers. A viewer located in  $x \in [0, 1]$  derives a utility  $v + \alpha_i - p_i - td_i(x)$  when buying from firm  $F_i$ , where  $t$  is the per-unit of distance transportation cost,  $d_i(x)$  is the distance to  $F_i$  and  $v$  a parameter that represents the innate valuation for pay-TV services. The viewer who is indifferent between buying from either firm is located at  $\hat{x} = \frac{1}{2t}(t + \Delta\alpha - (p_0 - p_1))$ , where  $\Delta\alpha = \alpha_0 - \alpha_1$ .

Second, multi-homing viewers are modeled following Doganoglu and Wright (2006). If such a viewer buys one unit of service only from firm  $F_i$ , he obtains a utility given by  $v + \alpha_i - p_i - td_i(x)$ . Buying one unit from each supplier yields a utility  $v + \frac{1}{2}(\alpha_0 + \alpha_1) + \beta t - (p_0 + p_1) - t$ , where  $\beta > 0$  describes the viewer's gain from diversity. We can now define the marginal multi-homing viewer who is indifferent between buying two units and buying only one unit from  $F_i$  as being located respectively at  $\tilde{x}_0 = \frac{1}{t}(\Delta\alpha/2 + p_1) + 1 - \beta$  and  $\tilde{x}_1 = \frac{1}{t}(\Delta\alpha/2 - p_0) + \beta$ .

The total demand for product 0 (resp. 1) is thus given by  $D_0(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)\hat{x} + \varepsilon\tilde{x}_1$  (resp.  $D_1(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)(1 - \hat{x}) + \varepsilon(1 - \tilde{x}_0)$ ). Routine computations allow then to determine Nash equilibrium prices.<sup>16</sup> The corresponding profits can be expressed as  $\tilde{\Pi}(\Delta\alpha)$  for firm  $F_0$  and  $\tilde{\Pi}(-\Delta\alpha)$  for firm  $F_1$ , with  $\tilde{\Pi}(\cdot)$  being strictly convex. Hence, Assumption 1 holds.

<sup>16</sup>Some conditions are needed to ensure that, in equilibrium, all viewers buy at least one unit (full market coverage), and a strictly positive mass of multi-homers buy two units. These conditions are detailed in the Appendix.

## 5. EQUILIBRIA CHARACTERIZATION

### 5.1. Generalities

At equilibrium,  $F_i$  chooses its own bidding schedule  $T_i$  to induce an allocation  $\bar{a}$  that maximizes its individual profit, *i.e.*,  $\Pi_i(a) - T_i(a)$ . Given a pair of offers  $(T_0, T_1)$ ,  $A$  implements such allocation if it also maximizes its own payoff, *i.e.*,  $\sum_{i=0,1} T_i(a)$ . For future reference, we shall denote  $F_i$ 's (resp.  $A$ 's) equilibrium profit by  $\bar{\Pi}_i$  (resp.  $\bar{\Pi}_a$ ).

**DEFINITION 1.** *An equilibrium of the continuation game for stage 2 onward is a triplet  $(\bar{T}_0, \bar{T}_1, \bar{a})$  that satisfies the following conditions.*

- *Profit maximization for A. A chooses an allocation within the best-response correspondence  $\bar{\mathcal{A}}(\bar{T}_0, \bar{T}_1)$  where  $\bar{\mathcal{A}}(T_0, T_1)$  is more generally defined as follows:*

$$(5.1) \quad \bar{\mathcal{A}}(T_0, T_1) = \arg \max_{\tilde{a} \in \mathcal{A}} \sum_{i=0,1} T_i(\tilde{a}) \quad \forall (T_0, T_1).$$

- *Profit maximization for  $F_i$ ,  $i = 0, 1$ .  $\bar{T}_i$  satisfies the following:*

$$(5.2) \quad \Pi_i(\bar{a}) - \bar{T}_i(\bar{a}) = \max_{\substack{\tilde{T}_i \geq 0 \\ \tilde{a} \in \bar{\mathcal{A}}(T_i, \bar{T}_{-i})}} \Pi_i(\tilde{a}) - \tilde{T}_i(\tilde{a}),$$

where  $\bar{a} \in \bar{\mathcal{A}}(\bar{T}_0, \bar{T}_1)$ .

We can draw a few immediate consequences from conditions (5.1) and (5.2). First, at a best response to its rival's strategy  $\bar{T}_{-i}$ ,  $F_i$  always minimizes its own payment  $\bar{T}_i(\bar{a})$  to induce its most preferred allocation  $\bar{a}$ .

Second, this bidding game appears to be a *delegated common agency game*, as defined by Bernheim and Whinston (1986a). Martimort and Stole (2012) showed that those games are *aggregate games*. More specifically,  $A$ 's behavior on the equilibrium path only depends on the aggregate bidding schedule  $\sum_{i=0,1} \bar{T}_i$ . This property, together with the fact that payoffs are quasi-linear, then allows us to sum up the equilibrium conditions for each firm to obtain a more compact set of necessary conditions that must be satisfied by any equilibrium allocation. Such an allocation is actually a solution to a *self-generating* optimization problem in the vocable coined by Martimort and Stole (2012). Taking stock of these properties, the next proposition provides a set of necessary conditions that allows us to restrict the search for equilibrium allocations in a sharp way.<sup>17</sup>

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<sup>17</sup>These conditions are not necessarily sufficient because the aggregate game is not bijective in the sense defined in Martimort and Stole (2012). More specifically, because bids must remain nonnegative, not all aggregate bids can be undone by a given firm with its own offer when this offer is restricted to remain nonnegative as requested in a delegated common agency game.

PROPOSITION 1. *At any equilibrium  $(\bar{T}_0, \bar{T}_1, \bar{a})$ , the following conditions hold:*

$$(5.3) \quad \bar{a} \in \arg \max_{a \in \mathcal{A}} \sum_{i=0,1} \Pi_i(a) + \sum_{i=0,1} \bar{T}_i(a),$$

$$(5.4) \quad \bar{a} \in \arg \max_{a \in \mathcal{A}} \sum_{i=0,1} \bar{T}_i(a),$$

$$(5.5) \quad \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a) = \max_{a \in \mathcal{A}} \bar{T}_{-i}(a), \quad \forall i = 0, 1,$$

$$(5.6) \quad \bar{T}_i(a_{-i}) = 0, \quad \forall i = 0, 1,$$

where  $a_{-i}$  is any  $A$ 's best choice if  $F_i$  were to offer a null payment, i.e.,

$$(5.7) \quad a_{-i} \in \arg \max_{a \in \mathcal{A}} \bar{T}_{-i}(a).$$

The optimality condition (5.3) showcases the self-generating nature of equilibria. An equilibrium allocation maximizes the sum of the bidders' payoff  $\sum_{i=0,1} \Pi_i(a)$  plus the seller's profit  $\sum_{i=0,1} \bar{T}_i(a)$ , which is itself an equilibrium object. Condition (5.4) simply characterizes the fact that this equilibrium allocation also belongs to the seller's best-response correspondence. More interesting are conditions (5.5) and (5.6) that offer an anchor for  $A$ 's payment and  $F_i$ 's bid schedule, respectively. The first one expresses the fact that  $A$  should be indifferent between choosing the equilibrium allocation given the aggregate bid  $\bar{T}_i + \bar{T}_{-i}$  and its next best option, which is to choose an optimal allocation when dealing only with one bidder. The second condition shows that  $F_i$ 's bid is zero had any such off-path allocation been chosen by  $A$ . Formally, everything happens as if  $F_i$  had to make its own bidding schedule attractive enough to  $A$  to avoid being excluded from the market.<sup>18</sup>

We now specialize this general approach to determine what types of equilibria may emerge and discuss their constrained-efficiency properties. To this end, we first restrict the set of bidding strategies in a quite natural way, focusing on *monotonic bidding schedules* such that a bidder bids more for an action that yields a greater payoff, namely,

$$\Pi_i(a) \geq \Pi_i(a') \text{ (resp. } >) \Rightarrow T_i(a) \geq T_i(a') \text{ (resp. } >) \quad \forall (a, a') \in \mathcal{A}^2.$$

Accordingly, an equilibrium is said to be *monotonic* if it is implemented by means of *monotonic bidding schedules*. This focus has important implications that we now present.

PROPOSITION 2. *The following properties hold at any monotonic equilibrium:*

1. *The equilibrium allocation is constrained-efficient, i.e.,  $\bar{a} = (0)$ .*
2. *The set  $\Sigma$  of feasible profits  $(\bar{\Pi}_0, \bar{\Pi}_1, \bar{\Pi}_a)$  in monotonic equilibria is defined by the*

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<sup>18</sup>Readers accustomed to the literature might have recognized the similarity between Proposition 1 and Lemma 2 in Bernheim and Whinston (1986a). However, there are a couple of differences that deserve to be stressed. First, the aggregate optimality condition (5.3) replaces their Condition *iii*), which captures the fact that an equilibrium allocation maximizes the bilateral payoff of a coalition made of any one bidder and the seller. Second, conditions (5.5) and (5.6) altogether replace and supersede their Condition *iv*) but are more precise, thereby making it explicit for which allocation a bidder's bidding schedule is zero, while Condition *iv*) only provides the existence of an allocation which entails a zero bid.

following conditions:

$$(5.8) \quad \begin{aligned} \Pi_0(1) &\leq \bar{\Pi}_0 \leq \Pi_0(0) + \Pi_1(0) - \Pi_1(1), \\ \bar{\Pi}_1 &= \Pi_1(0), \\ \Pi_1(1) - \Pi_1(0) &\leq \bar{\Pi}_a \leq \Pi_0(0) - \Pi_0(1). \end{aligned}$$

The distribution of payoffs that arise at equilibrium can be readily explained. First, since  $F_0$  is a dominant firm in the market, the constrained-efficient allocation  $a^* = (0)$  is also the most preferred outcome for that firm, while at the same time it remains the worst for  $F_1$ . Overall, and even though  $A$  could choose to sell rights non-exclusively to both downstream firms, everything happens as if those firms were bidding head-to-head for exclusivity.

From condition (5.6),  $F_1$  bids zero for the equilibrium allocation  $\bar{a} = (0)$  since giving rights to  $F_0$  also minimizes  $F_1$ 's own payment. However,  $A$  can improve its bargaining position vis-à-vis  $F_0$  by threatening to sell exclusive rights to  $F_1$ .  $F_1$  could thus be ready to pay up to  $\bar{T}_1(1) = \Pi_1(1) - \Pi_1(0)$  to acquire such exclusivity. This gives the lowest bound on  $A$ 's equilibrium profit. In fact,  $F_1$  might bid even more for exclusivity, say  $\bar{T}_1(1) > \Pi_1(1) - \Pi_1(0)$ , as long as, at such an equilibrium,  $A$  still chooses to sell exclusively to  $F_0$ . Such a strategy is weakly dominated since  $F_1$ 's payoff  $\Pi_1(1) - \bar{T}_1(1)$  is less than its payoff  $\Pi_1(0)$  obtained when bidding zero. However, such a large bid also improves  $A$ 's bargaining position vis-à-vis  $F_0$  since it makes switching to  $F_1$  more attractive. To defeat such a bid,  $F_0$  should pay at least  $\bar{T}_0(0) = \bar{T}_1(1) > \Pi_1(1) - \Pi_1(0)$ , and the highest such payment that  $F_0$  is willing to make is thus  $\Pi_0(0) - \Pi_0(1)$  to gain exclusivity. With such large bids,  $A$  grasps a larger share of the overall industry's profit obtained by granting exclusivity to  $F_0$ .

Finally, we also stress that an immediate corollary of Proposition 2 is that a joint distribution of rights, *i.e.*,  $a = (c)$ , never arises at equilibrium. Proposition 2 thus showcases the need for some kind of corrective intervention aiming to implement an allocation that could be preferred from a social welfare perspective, namely, joint distribution, even though this allocation is never implemented by free competition.

### 5.2. Truthful Equilibrium

In their general investigation of menu auctions, Bernheim and Whinston (1986a) exhibited an important class of equilibria sustained with so-called *truthful strategies* which are of the following form:

$$T_i(a) = \max\{\Pi_i(a) - \Pi_i; 0\} \quad \forall a \in \mathcal{A}$$

where  $\Pi_i$  is some constant.

These strategies have attractive properties. First, those payments schedules are monotonic. From Proposition 2, truthful equilibria thus necessarily implement a constrained-efficient allocation, *i.e.*,  $\bar{a} = (0)$ . Second,  $F_i$  keeps a constant profit  $\Pi_i$  over all possible allocations for which it makes a positive bid. In other words, truthful bidding schedules align the preferences of  $F_i$  and  $A$  over possible allocations that are found attractive for  $F_i$ . Third, each downstream firm always has a truthful strategy in its best-response



correspondence.<sup>19</sup> Focusing on truthful bidding schedules thus amounts to imposing a refinement in the equilibrium correspondence. This refinement allows a sharp characterization of payoffs in truthful equilibria. In our context, this refinement even pins down a unique outcome.<sup>20</sup>

PROPOSITION 3. *There exists a unique truthful equilibrium with equilibrium profits given by:*

$$(5.9) \quad \begin{aligned} \bar{\Pi}_0^t &= \Pi_0(0) + \Pi_1(0) - \Pi_1(1) > 0, \\ \bar{\Pi}_1^t &= \Pi_1(0) > 0, \\ \bar{\Pi}_a^t &= \Pi_1(1) - \Pi_1(0) > 0. \end{aligned}$$

Together with Proposition 2, Proposition 3 shows that the constrained-efficient truthful equilibrium corresponds to an extremal point of the set of possible equilibrium profits. This equilibrium is actually sustained with strategies that are not weakly dominated. Indeed, at any other equilibrium described in Proposition 2 above,  $F_1$  would possibly pay more for acquiring rights than its incremental benefit of doing so, since for those equilibria  $\bar{T}(1) > \Pi_1(1) - \Pi_1(0)$ . This dominance criterion thus provides another argument to focus on the unique truthful allocation.

For future reference, it is worth noting that, in this unique truthful equilibrium, what  $F_0$  bids for exclusivity, namely,  $\bar{T}^t(0) = \Pi_1(1) - \Pi_1(0)$ , is the Vickrey-Clarke-Groves payment, *i.e.*, what it takes for  $F_0$  to avoid  $A$  choosing his or her next best option, namely, giving exclusivity rights to  $F_1$ .

## 6. RESALE

Resale is an important feature of the pay-TV market. From a theoretical viewpoint, resale nevertheless remains a double-edged sword. First, the renegotiation of any allocation of rights between downstream firms  $F_0$  and  $F_1$  could, *a priori*, correct any constrained inefficiency that might arise in the downstream market and improve the industry's overall profit. Given that constrained efficiency may conflict with social efficiency, the possibility of resale could go counter welfare maximization. It should thus be viewed with an eye of caution by competition authorities.

In fact, this negative stance is incomplete. When anticipated, the possibility of resale may actually change the downstream firms' bidding strategies, which may in turn lead to payoffs rather different than those found in Proposition 3. In other words, banning resale has no impact on whoever *in fine* holds the rights. The dominant firm  $F_0$  should always get those rights. However, it might impact the distribution of profits and how much the dominant firm pays for exclusivity.

To study how it can be so, consider adding a renegotiation stage in between stages 2 and 3 of the game form so far studied. Suppose that  $A$  has already chosen an arbitrary allocation  $\tilde{a}$  and assume that the corresponding bids  $T_0(\tilde{a})$  and  $T_1(\tilde{a})$  have been sunk.  $F_0$  and  $F_1$  can renegotiate away from the allocation  $\tilde{a}$  toward another allocation, say  $a$ . We model such renegotiation as a Nash bargaining game with outside options being determined by the chosen allocation  $\tilde{a}$ , and with parties having equal bargaining power.

<sup>19</sup>See Bernheim and Whinston (1986a, Theorem 1).

<sup>20</sup>Laussel and Le Breton (2001) provide general conditions for the uniqueness of a truthful equilibrium.

Renegotiation determines a final allocation  $a^*$ , as well as a compensatory payment  $z^*$ , between  $F_0$  and  $F_1$  that altogether solve the Nash-bargaining problem, *i.e.*,

$$(a^*, z^*) \in \arg \max_{(a, z)} \left( \Pi_0(a) - z - \Pi_0(\tilde{a}) \right) \left( \Pi_1(a) + z - \Pi_1(\tilde{a}) \right).$$

Renegotiation thus implements the constrained-efficient allocation  $a^* = (0)$  since it indeed maximizes the joint profit of  $F_0$  and  $F_1$  once bids are sunk. Inserting the value of the compensatory payment  $z^*(\tilde{a}) = \frac{1}{2}(\Pi_0(0) - \Pi_1(0) - \Pi_0(\tilde{a}) + \Pi_1(\tilde{a}))$  obtained into the firms' payoffs allows us to rewrite the firms' net profits once renegotiation is taken into account as  $\tilde{\Pi}_i(\tilde{a}) - T_i(\tilde{a})$ , where

$$(6.1) \quad \tilde{\Pi}_i(\tilde{a}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \left( \Pi_i(\tilde{a}) - \Pi_{-i}(\tilde{a}) \right).$$

The possibility of resale has modified the game between  $F_0$  and  $F_1$ . Remarkably, it is now a constant-sum game. By its choice of a pre-resale allocation,  $A$  determines how much should be transferred from one firm to the other through *ex post* bargaining.<sup>21</sup>

We may now apply the general methodology of Proposition 1 to the new payoff functions so-defined. It is easy to check that any *interim* allocation  $\tilde{a}$  can be part of an equilibrium of the game form extended by renegotiation. The two downstream firms could just offer the nil contracts  $T_i = 0$ , let  $A$  randomly choose an initial allocation  $\tilde{a}$  and then renegotiate away any constrained inefficiency so-obtained by trading rights to move toward the constrained-efficient allocation  $\bar{a} = (0)$ .

Among all possible equilibria that may arise with resale, there is an interesting class in which  $A$  chooses upfront a constrained-efficient allocation that is thus *not* renegotiated. Those equilibria are in a sense *resale-proof*. As soon as resale entails some (even slightly) positive transaction costs, such resale-proof equilibria may be particularly attractive.

Following a logic that is by now familiar, the constrained-efficient allocation  $\bar{a} = (0)$  can be implemented as a *truthful resale-proof* equilibrium in the bidding game modified by the possibility of resale by means of the following (monotonic) truthful schedules:

$$(6.2) \quad \tilde{T}_i(a) = \max \left\{ \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \left( \Pi_i(a) - \Pi_{-i}(a) \right) - \tilde{\Pi}_i; 0 \right\}.$$

Resale has, of course, some consequences on the distribution of equilibrium payoffs, as explained in the next proposition.

**PROPOSITION 4.** *There exists a unique truthful resale-proof equilibrium. Profits in this equilibrium are given by the following:*

$$(6.3) \quad \begin{aligned} \bar{\Pi}_0^{rp} &= \frac{1}{2} \left( \Pi_0(0) + \Pi_1(0) + \Pi_0(1) - \Pi_1(1) \right), \\ \bar{\Pi}_1^{rp} &= \Pi_1(0), \\ \bar{\Pi}_a^{rp} &= \frac{1}{2} \left( \Pi_0(0) + \Pi_1(1) - \Pi_0(1) - \Pi_1(0) \right). \end{aligned}$$

<sup>21</sup>Readers accustomed with the incomplete contracts literature may have recognized a familiar feature of implementation when renegotiation is allowed. See Maskin and Moore (1999) for instance.

*A (resp.  $F_0$ ,  $F_1$ ) always earns more (resp. less, the same) profit in the unique truthful resale-proof equilibrium than in the unique truthful equilibrium without resale.*

With resale,  $A$  can obtain a greater share of the industry profit while that of  $F_0$  decreases and that of  $F_1$  remains unchanged. Intuitively,  $A$  can now threaten each downstream firm to sell the rights to its rival as a base for future resale; this is a very unattractive outcome that forces the downstream firm that values the most those rights, *i.e.*,  $F_0$ , to pay a lot for exclusivity.

It is straightforward to check that the payoff vector  $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp}, \bar{\Pi}_a^{rp})$  belongs to  $\Sigma$ . This result comes as no surprise. The unique truthful resale-proof equilibrium implements the constrained-efficient allocation, gives to the dominated firm its reservation payoff and, by construction, satisfies all incentive constraints that must hold in equilibrium, as all equilibrium payoffs that belong to  $\Sigma$  must do.

Our results suggest that the sole role of banning resale is to change the distribution of profits between the dominant firm  $F_0$  and the producer of rights  $A$ . This distribution is of course neutral for both the allocation of rights that arises at equilibrium and consumer welfare. Hence, the only justification that can be found for such a policy is that it might have an indirect impact on investment. Suppose indeed that  $F_0$ 's promotional effort is endogenized. With resale,  $F_0$ 's profits are lower, and we may expect lower incentives to exert such effort. In turn,  $F_0$  and  $F_1$  may become more symmetric, and a non-exclusive distribution of rights may become more attractive. In other words, banning resale might have a role in preventing exclusivity; this, however, seems to be a very indirect role.

## 7. PACKAGE BIDDING FOR MULTIPLE RIGHTS

We now extend our previous approach to analyze settings in which firms in the downstream market can acquire several sets of premium contents from the upstream seller. To fix ideas, suppose that two sets of rights are available. Exclusivity can be given to either firm. Much as before,  $F_0$  is the dominant player in both markets, as it is able to improve the quality of contents more than what  $F_1$  would do if given these rights. Viewers now choose their favorite channel by comparing prices and the overall quality of the contents, taking into account how rights have been allocated to downstream firms. For simplicity, we assume that these rights have a symmetric impact on profits and surpluses across markets. Under those conditions, the model is rather similar to that presented in our previous setup.

The only difference is that we hereafter suppose that a joint distribution of either set of rights is no longer possible. There are two reasons for this assumption. First, it simplifies the analysis by limiting the number of configurations under scrutiny. However, this is only a minor restriction since the possibility remains that each downstream firm enjoys exclusively one set of rights, giving rise to a rather balanced market structure. Second, most real-world auctions for the broadcasting of premium sporting events involve exclusive distribution only. The main concern expressed by some competition authorities is to prevent the dominant firm from obtaining exclusivity over all sets of rights to maintain a long-term competitive balance in the market.

We now present the model and the structure of the equilibria with such *package bidding*.<sup>22</sup> We then analyze the consequences of a ban on package bidding on the equilibrium allocations.

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<sup>22</sup>Milgrom (2007) offers an exhaustive review of the literature on package auctions.

## 7.1. Preliminaries

Formally, we denote by  $b = (a^1, a^2)$  any arbitrary allocation of rights, where  $a^k$  (for  $k = 1, 2$ ) belongs to  $\mathcal{A}_* = \mathcal{A}/\{c\}$ . In other words, in each market,  $A$  can give exclusivity to either firm or refuse to distribute its rights. The set of possible allocations is thus the cross-product  $\mathcal{A}_*^2$ . Assume that the value-enhancing parameter that applies to firm  $F_i$ 's services can be written as  $\beta_i = \sum_{k=1,2} \alpha_i^k$ , where the superscript  $k$  indices the set of rights with, as in our baseline model,  $\alpha_i^k \in \{0, \alpha + \Delta_i\}$  depending on how rights are allocated across firms.<sup>23</sup> From there, we can derive the expressions of  $F_i$ 's profit for a given allocation of rights  $b$  and the induced value-enhancing parameters. To illustrate, if  $F_0$  acquires exclusivity over the bundle of rights, *i.e.*,  $b = (0, 0)$ , then  $\beta_0 = 2(\alpha + \Delta_0)$  and  $\beta_1 = 0$ , so that  $\Pi_0(0, 0) = \tilde{\Pi}_0(2(\alpha + \Delta_0), 0)$  and  $\Pi_1(0, 0) = \tilde{\Pi}_1(2(\alpha + \Delta_0), 0)$ . If rights are split across pay-TV firms, then  $b = (0, 1)$ ,  $\beta_0 = \alpha + \Delta_0$  and  $\beta_1 = \alpha + \Delta_1$ , so that  $\Pi_0(0, 1) = \tilde{\Pi}_0(\alpha + \Delta_0, \alpha + \Delta_1)$  and  $\Pi_1(0, 1) = \tilde{\Pi}_1(\alpha + \Delta_0, \alpha + \Delta_1)$ . Because rights are symmetric, we have  $\Pi_i(0, 1) = \Pi_i(1, 0)$  for  $i = 0, 1$ . Since  $\Delta_0 > \Delta_1$ ,  $F_0$  is dominant in both markets and we also have  $\Pi_0(0, 0) + \Pi_1(0, 0) > \Pi_0(1, 1) + \Pi_1(1, 1)$ . Of course,  $F_0$ 's and  $F_1$ 's profits can be ranked, respectively, as  $\Pi_0(0, 0) > \Pi_0(1, 0) = \Pi_0(0, 1) > \Pi_0(1, 1)$  and  $\Pi_1(1, 1) > \Pi_1(1, 0) = \Pi_1(0, 1) > \Pi_1(0, 0)$ .

As our analysis will soon unveil, with multiple rights, we need the following strengthening of Assumption 1.

ASSUMPTION 2. *Firms' profits are such that*

$$\Pi_i(i, i) + \Pi_i(-i, -i) > 2\Pi_i(i, -i), \quad \forall i \in \{0, 1\}.$$

Assumption 2 simply states that, for each firm, the incremental value of a right increases with the number of rights already obtained, or, equivalently, that firms' individual profits are strictly convex in the allocation of rights. That assumption is maintained throughout this section.

The different market structures considered in Section 4 can be easily extended to account for multiple rights. In the Appendix, we show the following results. First, for the discrete choice model, for price or quantity competition with linear demands, and for Hotelling competition with single- and multi-homers, Assumption 2 always holds. Second, for a duopoly with vertical demand shifters, we find sufficient conditions so that it also holds for  $\Delta\beta = \beta_0 - \beta_1$  small enough.

Assumption 2 and dominance, when taken in tandem, imply the following property.

LEMMA 2. *The allocation  $b^* = (0, 0)$  is the constrained-efficient allocation, or*

$$(7.1) \quad \sum_{i=0,1} \Pi_i(0, 0) = \max_{b \in \mathcal{A}_*^2} \sum_{i=0,1} \Pi_i(b).$$

An immediate consequence of Lemma 2 is that the allocation  $(0, 0)$  is also *market-by-market constrained-efficient*; *i.e.*,  $F_0$  should be given the second set of rights if it already owns the first one. In other words, rights are *strict complements* for the dominant firm.

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<sup>23</sup>Introducing some complementarity or substitutability in the valuation of multiple rights by viewers does not change our analysis as long as the ranking of profits and Assumption 2 continue to hold.

Formally, condition (7.1) indeed implies the following:

$$(7.2) \quad \sum_{i=0,1} \Pi_i(0,0) = \max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(a,0) = \max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(0,a).$$

This property is key to understanding that when  $F_0$  is a dominant firm for both sets of rights, selling those rights as a bundle or separately in different auctions has no impact on the constrained efficiency of the equilibrium allocation. Of course, whether rights are sold as a bundle or separately may have an impact on bids and payoffs. We consider those two scenarios in the following.

## 7.2. Package Bidding Equilibria

Suppose first that both sets of rights are simultaneously on sale by means of the same tender procedure. Downstream firms are unrestricted in their bidding strategies on those packages. Formally, a bidding schedule  $T_i$  is thus any arbitrary nonnegative mapping on  $\mathcal{A}_*^2$ . Such mapping determines  $F_i$ 's bid  $T_i(b)$  to  $A$  for each feasible package  $b \in \mathcal{A}_*^2$ . Still relying on our definition of monotonicity, it is straightforward to recast our analysis of Proposition 2. We again denote by  $(\bar{T}_0, \bar{T}_1, \bar{b})$  any arbitrary equilibrium pair of bidding schedules and the ensuing allocation of rights.

PROPOSITION 5.

1. All monotonic equilibria are constrained-efficient, i.e.,  $\bar{b} = (0, 0)$ .
2. The set  $\Sigma$  of profit levels  $(\bar{\Pi}_0, \bar{\Pi}_1, \bar{\Pi}_a)$  that can be achieved in any monotonic equilibrium is defined by the following conditions:

$$(7.3) \quad \begin{aligned} \Pi_0(1,1) &\leq \bar{\Pi}_0 \leq \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1), \\ \bar{\Pi}_1 &= \Pi_1(0,0), \\ \Pi_0(0,0) - \Pi_0(1,1) &\geq \bar{\Pi}_a \geq \Pi_1(1,1) - \Pi_1(0,0). \end{aligned}$$

Proposition 5 echoes, in a multimarket context, our earlier findings found in Proposition 2. There still exists a whole range of equilibrium profits corresponding to different distributions of the maximal industry profit between  $A$  and the dominant firm  $F_0$ . Indeed,  $A$  obtains a greater share of this profit when it can threaten  $F_0$  to sell exclusivity rights in both markets to  $F_1$  at price  $\bar{T}_1(1,1) \geq \Pi_1(1,1) - \Pi_1(0,0)$ . Offering such an attractive option in its bidding schedule remains, of course, a weakly dominated strategy for  $F_1$ .

To eliminate those weakly dominated strategies and focus on the distribution of profits that is the most favorable to  $F_0$ , we again consider truthful bidding schedules of the following form:

$$(7.4) \quad T_i(b) = \max\{\Pi_i(b) - \Pi_i; 0\}.$$

Those bidding schedules are of course monotonic, and any truthful equilibrium is thus constrained-efficient, thus implementing  $\bar{b} = (0, 0)$ . Echoing our earlier findings in Proposition 3, such an equilibrium determines a unique distribution of profits that again corresponds to an extremal point of the set described in Proposition 5. Then,  $F_0$  gets the highest possible profit.

PROPOSITION 6. *There exists a unique equilibrium in truthful schedules with package bidding. Equilibrium profits are given by the following expressions:*

$$(7.5) \quad \begin{aligned} \bar{\Pi}_0^b &= \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1) > 0, \\ \bar{\Pi}_1^b &= \Pi_1(0, 0) > 0, \\ \bar{\Pi}_a^b &= \Pi_1(1, 1) - \Pi_1(0, 0) > 0. \end{aligned}$$

### 7.3. Split Auctions: Pairwise-Proof Quasi-Equilibrium

Consider now a scenario where each set of rights is sold separately by means of a different auction procedure. A final allocation  $\bar{b}$  is thus determined *market by market*. To tackle the analysis of such a complex web of bilateral relationships and provide some predictive insights, we first follow an approach that has been extensively used in the literature on vertical relationships. Following Crémer and Riordan (1987), Hart and Tirole (1990), Horn and Wolinsky (1988) and O'Brien and Shaffer (1992), works in the field have often made the simplifying assumption that, in each bilateral contract (or bargain) it is part of, a given party takes as given the outcome of other bilateral contracts (or bargains) in which it participates. The benefit of this approach is of course tractability.<sup>24</sup>

There are a number of necessary requirements that any such outcome should satisfy. Mimicking our earlier analysis, downstream firms should of course play an equilibrium in each bidding market taken separately. In other words, for a given conjecture on the allocation of rights  $\bar{a}^{-k}$  that prevails on market  $-k$ , the schedules  $\bar{T}_i^k$  should comprise an equilibrium on market  $k$  and, as such, be contingent on  $a^k$  only. Definition 2 makes this statement more formal.

DEFINITION 2. *A pairwise-proof quasi-equilibrium is an array  $(\bar{T}_0^k, \bar{T}_1^k, \bar{a}^k)_{k=1,2}$  that satisfies the following necessary conditions.*

1. *Profit maximization for A in market k: A chooses an allocation in market k within the best-response correspondence  $\bar{\mathcal{A}}^k(\bar{T}_0^k, \bar{T}_1^k)$  where  $\bar{\mathcal{A}}^k(T_0^k, T_1^k)$  is more generally defined as follows:*

$$(7.6) \quad \bar{\mathcal{A}}^k(T_0^k, T_1^k) = \arg \max_{a^k \in \mathcal{A}^*} \sum_{i=0,1} T_i^k(a^k) \quad \forall (T_0^k, T_1^k) \text{ for } k = 1, 2.$$

2. *Profit maximization for  $F_i$  (for  $i = 0, 1$ ) in market  $k$  (for  $k = 1, 2$ ):  $\bar{T}_i^k$  satisfies the following:*

$$(7.7) \quad \Pi_i(\bar{a}^k, \bar{a}^{-k}) - \bar{T}_i^k(\bar{a}^k) = \max_{\substack{T_i^k \geq 0 \\ \tilde{a}^k \in \bar{\mathcal{A}}^k(T_i^k, \bar{T}_{-i}^k)}} \Pi_i(\tilde{a}^k, \bar{a}^{-k}) - T_i^k(\tilde{a}^k);$$

where  $\bar{a}^k \in \bar{\mathcal{A}}^k(\bar{T}_0^k, \bar{T}_1^k)$  for  $k = 1, 2$ .

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<sup>24</sup>To back up this approach, Hart and Tirole (1990) and McAfee and Schwartz (1994) have shown that such an outcome might be sustained as an equilibrium in some structured environments provided that out-of-equilibrium beliefs following unexpected offers are passive. A simple, often found and more practical motivation is that each firm may have delegated to a different department the responsibility to bid in a particular market, thereby avoiding a joint design of bidding schedules in both markets. The tractability of this approach can be useful for empirical purposes, as shown in Dubois and Sæthre (2020).

The pair of optimality conditions (7.7) encapsulates the requirement that a *pairwise-proof quasi-equilibrium* is computed market by market. It should already be clear that those conditions, even when taken in tandem, do not necessarily ensure that that multi-lateral deviations are not attractive. We shall come back on this issue below.

For the time being, our goal is simply to exhibit a plausible outcome of our bidding game and study its properties. To this end, we may adopt our earlier approach and focus on bidding schedules  $T_i^k$  which are monotonic. It immediately follows from Proposition 2, when applied on each market separately, together with the fact that  $\bar{b} = (0, 0)$  is the constrained-efficient allocation, that  $\bar{a}^k = (0)$  on market  $k$  inherits all the properties that were highlighted in Proposition 2 for a market where only one set of rights is sold. In particular, we already know that, by insisting on strategies that are not weakly dominated on each market taken in isolation, the outcome  $\bar{a}^k = (0)$  can be implemented with truthful schedules of the following form:

$$(7.8) \quad \bar{T}_i^k(a^k) = \max\{\Pi_i(a^k, 0) - \bar{\Pi}_i^k; 0\} \text{ for } k = 1, 2.$$

It should be stressed that because of the extra benefit of owning two sets of rights, the fees  $\bar{\Pi}_i^k$  are no longer equilibrium profits as in the single-right scenario. The next proposition describes profits in such a pairwise-proof quasi-equilibrium implemented with truthful schedules and the corresponding values of the fees  $\bar{\Pi}_i^k$ .

PROPOSITION 7.

1. *The allocation  $\bar{b} = (0, 0)$  is a pairwise-proof quasi-equilibrium allocation sustained with truthful schedules of the form (7.8) with*

$$(7.9) \quad \bar{\Pi}_0^k = \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(0, 1) \text{ and } \bar{\Pi}_1^k = \Pi_1(0, 0), \quad k = 1, 2.$$

2. *Profits in this pairwise-proof allocation are given by the following:*

$$(7.10) \quad \begin{aligned} \bar{\Pi}_0^u &= \Pi_0(0, 0) + 2(\Pi_1(0, 0) - \Pi_1(0, 1)) > \bar{\Pi}_0^b, \\ \bar{\Pi}_1^u &= \Pi_1(0, 0), \\ \bar{\Pi}_a^u &= 2(\Pi_1(0, 1) - \Pi_1(0, 0)) < \bar{\Pi}_a^b. \end{aligned}$$

As in the case of the truthful equilibrium with package bidding, the pairwise-proof allocation also gives all exclusivity rights to  $F_0$ . However, the distribution of profits is more favorable to this dominant firm. The intuition is again straightforward. Recall that  $F_0$  finds it even more attractive to acquire a second set of rights if it already owns a first set of rights. In a pairwise-proof allocation, all players anticipate that  $F_0$  already has exclusivity in market  $-k$  when bidding for exclusivity rights in market  $k$ . As a result,  $A$  can only threaten  $F_0$  to sell the rights in market  $k$  to  $F_1$  in case  $F_0$ 's bid in this market is not attractive enough. With package bidding,  $A$  can instead use the more efficient threat of giving exclusivity on both sets or rights to  $F_1$  in case  $F_0$ 's bid to obtain such joint exclusivity is not attractive enough.  $A$  thus forces  $F_0$  to pay more to obtain all exclusivity rights.<sup>25</sup> At the same time,  $F_1$  remains dominated, and the worst scenario

<sup>25</sup>At a rough level, the argument bears some similarity with the literature on repeated games in multimarket contexts (Bernheim and Whinston, 1990; Spagnolo, 1999). In the literature, it is shown that collusion may be easier to enforce under the threat of simultaneous deviations in several markets.

in each market is, for that firm, that  $F_0$  gets exclusivity for the rights on this market. From Proposition 1 (especially condition (5.6)),  $F_1$  makes a null bid for such an outcome whether it arises through package bidding or as a pairwise-proof allocation. Henceforth, splitting auctions has for the sole consequence of redistributing profit from  $A$  to  $F_0$  in comparison with package bidding, with again no impact on  $F_1$ .

**MULTILATERAL DEVIATIONS.** In the specific context of vertical manufacturer-retailer relationships or in broader contexts of contracting with externalities, McAfee and Schwartz (1994) and Rey and Vergé (2004) have criticized the concept of pairwise-proofness and showed that such a pairwise-proof allocation might unfortunately fail to be a perfect Bayesian equilibrium of the extensive form games under scrutiny. While those authors have focused on the offer-game scenario where the party (manufacturer) at the nexus of all contracts makes secret offers to his or her agents (retailers), Segal and Whinston (2003) have also studied the bidding game scenario discussed in this paper.

In our context, we might also wonder whether a pairwise-proof allocation is still immune to multilateral deviations in which one party simultaneously deviates by jointly modifying the bidding schedules in both markets. Of course, the same nonexistence problem could also *a priori* arise in our bidding environment. Before addressing this issue, we state the following definition.

**DEFINITION 3.** *A pairwise-proof quasi-equilibrium  $(\bar{T}_0^k, \bar{T}_1^k, \bar{a}^k)_{k=1,2}$  is an equilibrium if it is robust to multilateral deviations, i.e.,*

$$(7.11) \quad \Pi_i(\bar{a}^1, \bar{a}^2) - \sum_{k=1,2} \bar{T}_i^k(\bar{a}^k) = \max_{\substack{T_i^k \geq 0, k=1,2 \\ \tilde{a}^k \in \bar{\mathcal{A}}^k(T_i^k, \bar{T}_{-i}^k), k=1,2}} \Pi_i(\tilde{a}^1, \tilde{a}^2) - \sum_{k=1,2} T_i^k(\tilde{a}^k), \quad i = 0, 1.$$

Notice that the equilibrium bidding schedules found under package bidding are by definition robust to multilateral deviations. Viewed through this lens, condition (7.11) imposes a similar requirement for firm  $i$ 's additive bidding schedule  $\sum_{k=1,2} \bar{T}_i^k(\bar{a}^k)$  that emerges at a pairwise-proof quasi-equilibrium allocation. In our context, this robustness requirement is in fact quite demanding, as shown below.

**PROPOSITION 8.** *The pairwise-proof quasi-equilibrium sustained with truthful schedules characterized by means of (7.8) and (7.9) is not immune to multilateral deviations when Assumption 2 is satisfied.*

Proposition 7 showed that profits for the *pairwise-proof quasi-equilibrium* sustained with truthful schedules are tilt towards the dominant firm  $F_0$ . The inequalities in (7.10) show that while  $F_0$  earns more than in the truthful equilibrium with package bidding,  $F_1$  earns less in the quasi-equilibrium under scrutiny. From the complete characterization of profits in any equilibrium with package bidding (Proposition 5) and the fact that the truthful one lies at an extremal point of this set (Proposition 6), it immediately follows that the *pairwise-proof quasi-equilibrium* with truthful schedules characterized in Proposition 7 fails to be an equilibrium.

The logic for this finding is straightforward. In the pairwise-proof quasi-equilibrium, the dominant firm  $F_0$  ends up paying too little to obtain exclusivity in both markets relative to what she should be paying with package bidding. That firm certainly has no incentives to enter into multilateral deviations that would implement the same allocation. Instead, taking advantage of such a low bid from the dominant firm  $F_0$ , the dominated



rival  $F_1$  can bid more aggressively in both markets to obtain exclusivity over both sets of rights. It turns out that, due to Assumption 2, the pairwise-proof allocation is indeed not immune to such multilateral deviations by the dominated firm  $F_1$ .

To summarize, with the pairwise-proof quasi-equilibrium implemented with truthful schedules, the final allocation of rights remains unchanged in comparison with package bidding;  $(0,0)$  is again implemented. Consumer surplus also comes unchanged by banning package bidding. Again, such a restriction only redistributes profits from the seller to the dominant firm. As such, and by an argument that mirrors the one made in Section 6, this policy could only have a positive indirect effect in boosting investment by the dominant firm. However, such a presumption is based on the wrong premise that multilateral deviations are not attractive to dominated firms. In other words, making policy recommendations market by market based on the predictions of pairwise-proofness seems a rather flawed approach. A global perspective is necessary.

#### 7.4. Split Auctions: Additive Bidding

Because the pairwise-proof truthful schedules fail to be an equilibrium, we now return to the mere definition of schedules for package bidding and impose additional constraints on those schedules. These constraints capture the fact that auctions for each set of rights are run separately. Instead of allowing arbitrary bidding schedules contingent on global allocations, as with the scenario of package bidding, we might view the requirement of *split bidding in each market* as an additivity constraint of the following kind:

$$(7.12) \quad T_i(a^1, a^2) = \sum_{k=1,2} T_i^k(a^k) \text{ for } i = 1, 2.$$

The benefit of such additivity restrictions on bidding schedules is that multilateral deviations add no further constraints on putative equilibrium allocations. The cost of such a restriction is instead that a downstream firm's overall bidding strategy might no longer reflect the payoff complementarity that appears when this firm owns both sets of rights, such as what arises with a truthful schedule. In particular, one might expect that  $F_0$  would bid less than its incremental value to obtain exclusivity on only any single set of rights. Under such circumstances, intuition might suggest that imposing the additivity constraint (7.12) on feasible strategies could *a priori* lessen competition between downstream firms.<sup>26</sup>

Of course, the additive bidding schedule (7.12) is monotonic when each component  $T_i^k(a^k)$  is itself monotonic. From Proposition 2, any equilibrium in additive and monotonic schedules thus implements the constrained-efficient allocation  $\bar{b} = (0, 0)$ . The next proposition characterizes such equilibrium profits. Surprisingly, and in contrast with the intuition just sketched, the additivity constraint does not restrict the set of equilibrium outcomes in comparison with the scenario where package bidding is possible.

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<sup>26</sup>In the literature on package auctions, which is mainly inspired by applications to the telecom industry, this phenomenon is referred to as the risk of *demand exposure*. A bidder who cannot express his or her willingness to pay for a bundle reduces his or her bid on each component because he or she is exposed to the risk that a local competitor, who is interested in only one item, prevents him or her from acquiring both items. In such contexts, not only is competition lessened, but there is also a risk that the final allocation will be inefficient. See Kagel and Levin (2005) and Milgrom (2007).

PROPOSITION 9. *Any profit levels  $(\bar{\Pi}_0^s, \bar{\Pi}_1^s, \bar{\Pi}_a^s)$  in  $\Sigma$  that can be achieved with monotonic equilibria when package bidding is allowed can also be achieved with additive bidding schedules of the form (7.12) when Assumption 2 is satisfied.*

An immediate corollary of our findings is that the extreme payoffs obtained in (7.5) by means of truthful schedules can also be achieved when auctions are split and firms use additive bidding schedules. Let us see how this can be so. By construction, the truthful equilibrium schedules (7.4) are not additive. Indeed,  $F_0$  benefits from some complementarity in obtaining exclusivity in both markets and, in a truthful equilibrium, is ready to pay the large price  $\bar{T}_0^b(0, 0) = \Pi_1(1, 1) - \Pi_1(0, 0)$  that prevents  $A$  from selling both rights to  $F_1$ . As discussed earlier, this bid is higher than what  $F_0$  pays in a pairwise-proof quasi-equilibrium with truthful schedules, although the same allocation  $(0, 0)$  remains implemented. When restricted to additive bidding schedules,  $F_0$  can still prevent  $A$  from selling both sets of rights to  $F_1$  with a bid  $\bar{T}_0(0)$  that would replicate what the truthful payment does, namely,

$$2\bar{T}_0(0) = \bar{T}_0^b(0, 0) = \Pi_1(1, 1) - \Pi_1(0, 0).$$

This bid  $\bar{T}_0(0)$  is now much larger than in the pairwise-proof quasi-equilibrium allocation since

$$\bar{T}_0(0) = \frac{1}{2}(\Pi_1(1, 1) - \Pi_1(0, 0)) > \Pi_1(0, 1) - \Pi_1(0, 0) = \bar{T}_0^k(0),$$

where the inequality follows from the convexity of  $F_1$ 's profit as defined in Assumption 2. Multilateral deviations by  $F_1$  are thus still prevented. The flip side of the large bid  $\bar{T}_0(0)$  is that one may wonder if  $F_0$  could not simply prefer to acquire exclusivity in only one market rather than duplicating those bids in each market. This is not the case since  $F_0$ 's net benefit of acquiring the right on the second market, when already having the right on the first one, are always positive when

$$\bar{T}_0(0) \leq \Pi_0(0, 0) - \Pi_0(0, 1);$$

which is an inequality that again follows from the convexity of  $F_0$ 's profit as defined in Assumption 2.<sup>27</sup> Therefore, a restriction to split auctions does not change the distribution of profits and surplus.

## 8. CONCLUSION

Motivated by the prevalence of exclusivity in broadcasting rights in pay-TV markets, we have developed a model of competition in which firms compete both upstream for the acquisition of broadcasting rights and downstream to attract viewers. Profit functions in the downstream market exhibit a fundamental convexity property. This property implies that giving exclusive broadcasting rights to one firm, namely, a dominant player, maximizes the industry profit. We characterize all equilibria when firms can freely bid for any possible allocation of the broadcasting rights and show that all monotonic equilibria implement this profit-maximizing allocation. Light-handed regulation such as banning resale or limiting the lot size under tenders in the case of multiple rights does not change this outcome. At best, such regulation might modify the distribution of profits that is

<sup>27</sup>We show in the Appendix that, in equilibrium,  $\bar{T}_0(0)$  must be lower than  $\frac{1}{2}(\Pi_0(0, 0) - \Pi_0(1, 1))$  and, moreover, the inequality  $\frac{1}{2}(\Pi_0(0, 0) - \Pi_0(1, 1)) < \Pi_0(0, 0) - \Pi_0(0, 1)$  holds from Assumption 2.

achieved. As a result, and if anything, the dominant firm's incentives to further invest in enhancing the quality of services might be modified (and possibly diminished); however, this appears to be a very indirect way of promoting downstream competition.

Therefore, our analysis suggests that if exclusivity is to be avoided because it fails to maximize welfare, more heavy-handed tools may prove useful. As an example, the European Commission required the UK Football Association Premier League Limited to ensure that at least one package of media rights to the Premier League matches would go to an operator other than the dominant one, thereby forcing *de facto* a competitive outcome.<sup>28</sup> Perhaps unexpectedly, such a '*no single buyer rule*' is now almost always implemented by major European soccer leagues, even though competitive and regulatory landscapes differ. One may wonder, however, whether systematically forcing a non-exclusive outcome is warranted. We believe that this issue should be resolved on a case-by-case basis.

We have also remained silent on another issue that is salient in pay-TV markets and sports events broadcasting, namely, whether sports leagues that coordinate the selling of broadcasting rights on behalf of teams form a cartel and thus harm competition. Again, competition authorities differ in their assessments. Our model could be extended to take into account both the joint and individual selling of broadcasting rights and analyze how this impacts the vertical agreements that emerge between sellers of rights and pay-TV firms. This investigation is left for future research.

## APPENDIX

MARKET STRUCTURES AND ASSUMPTIONS 1 AND 2. We detail here the analysis of Assumptions 1 and 2 under the different market structures described in Section 4.

*Duopoly with Vertical Demand Shifters.* Consider a duopoly in which demands for  $F_0$  and  $F_1$  write respectively as  $D_0(\Delta\alpha, p_0, p_1)$  and  $D_1(\Delta\alpha, p_1, p_0)$  with  $\Delta\alpha = \alpha_0 - \alpha_1$  and  $\frac{\partial D_0}{\partial \Delta\alpha}(\Delta\alpha, p_0, p_1) > 0 > \frac{\partial D_1}{\partial \Delta\alpha}(\Delta\alpha, p_1, p_0)$  for all  $(p_0, p_1, \Delta\alpha)$ . Assume that products are demand substitutes, or  $\frac{\partial D_0}{\partial p_1}(\Delta\alpha, p_0, p_1) > 0$ ,  $\frac{\partial D_1}{\partial p_0}(\Delta\alpha, p_1, p_0) > 0$  for all  $(p_0, p_1, \Delta\alpha)$ . Suppose without loss of generality that  $\Delta\alpha \geq 0$ . Profits of  $F_0$  and  $F_1$  are given respectively by  $p_0 D_0(\Delta\alpha, p_0, p_1)$  and  $p_1 D_1(\Delta\alpha, p_1, p_0)$ . Assume that the Nash equilibrium prices  $(p_0^*(\Delta\alpha), p_1^*(\Delta\alpha))$  are uniquely characterized by the first-order conditions

$$(A.1) \quad \begin{aligned} D_0(\Delta\alpha, p_0, p_1) + p_0 \frac{\partial D_0}{\partial p_0}(\Delta\alpha, p_0, p_1) &= 0, \\ D_1(\Delta\alpha, p_1, p_0) + p_1 \frac{\partial D_1}{\partial p_1}(\Delta\alpha, p_1, p_0) &= 0. \end{aligned}$$

$F_0$ 's and  $F_1$ 's equilibrium profits are thus given by  $\tilde{\Pi}_0(\Delta\alpha) = p_0^*(\Delta\alpha) D_0(\Delta\alpha, p_0^*(\Delta\alpha), p_1^*(\Delta\alpha))$  and  $\tilde{\Pi}_1(\Delta\alpha) = p_1^*(\Delta\alpha) D_1(\Delta\alpha, p_1^*(\Delta\alpha), p_0^*(\Delta\alpha))$  respectively.

Let  $\varphi(\Delta\alpha) = \tilde{\Pi}_0(\Delta\alpha) + \tilde{\Pi}_1(\Delta\alpha)$ . The convexity property amounts to  $\varphi'(\Delta\alpha) > 0$ . We obtain (omitting some arguments to ease the exposition)

$$\varphi'(\Delta\alpha) = p_0^*(\Delta\alpha) \left( \frac{\partial D_0}{\partial \Delta\alpha} + \frac{\partial D_0}{\partial p_1} p_1^*(\Delta\alpha) \right) + p_1^*(\Delta\alpha) \left( \frac{\partial D_1}{\partial \Delta\alpha} + \frac{\partial D_1}{\partial p_0} p_0^*(\Delta\alpha) \right).$$

<sup>28</sup>See Commission Decision of 22/3/2006 Case COMP/38.173 – Joint selling of the media rights to the FA Premier League, available at <https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX%3A52008XC0112%2803%29>.

Suppose also that demands and that first- and second-order (direct and crossed) derivatives with respect to prices are equal at  $\Delta\alpha = 0$ . That is, when expressed at  $\Delta\alpha = 0$  and  $(p_0, p_1) = (p^*(0), p^*(0))$ , we have  $D_0(0, p^*(0), p^*(0)) = D_1(0, p^*(0), p^*(0))$ ,  $\frac{\partial D_0}{\partial p_1} = \frac{\partial D_1}{\partial p_0}$ ;  $\frac{\partial D_0}{\partial p_0} = \frac{\partial D_1}{\partial p_1}$ ;  $\frac{\partial^2 D_0}{\partial p_0^2} = \frac{\partial^2 D_1}{\partial p_1^2}$ ;  $\frac{\partial^2 D_0}{\partial p_0 \partial p_1} = \frac{\partial^2 D_1}{\partial p_0 \partial p_1}$ ; and  $\frac{\partial^2 D_0}{\partial p_1^2} = \frac{\partial^2 D_1}{\partial p_0^2}$ ,<sup>29</sup> where  $p^*(0)$  is the solution of  $D_i(0, p^*(0), p^*(0)) + p^*(0) \frac{\partial D_i}{\partial p_i}(0, p^*(0), p^*(0)) = 0$ .

Let us now differentiate the first-order conditions in (A.1) and solve the corresponding system to obtain

$$\dot{p}_0^*(\Delta\alpha) = \frac{-b_1(\Delta\alpha)a_0(\Delta\alpha) - c_0(\Delta\alpha)a_1(\Delta\alpha)}{b_0(\Delta\alpha)b_1(\Delta\alpha) - c_0(\Delta\alpha)c_1(\Delta\alpha)} \quad \text{and} \quad \dot{p}_1^*(\Delta\alpha) = \frac{b_0(\Delta\alpha)a_1(\Delta\alpha) + c_1(\Delta\alpha)a_0(\Delta\alpha)}{b_0(\Delta\alpha)b_1(\Delta\alpha) - c_0(\Delta\alpha)c_1(\Delta\alpha)},$$

where

$$\begin{aligned} - a_i(\Delta\alpha) &\equiv \frac{\partial D_i}{\partial \Delta\alpha}(\Delta\alpha, p_i(\Delta\alpha), p_{-i}(\Delta\alpha)); \\ - b_i(\Delta\alpha) &\equiv 2 \frac{\partial D_i}{\partial p_i}(\Delta\alpha, p_i(\Delta\alpha), p_{-i}(\Delta\alpha)) + p_i(\Delta\alpha) \frac{\partial^2 D_i}{\partial p_i^2}(\Delta\alpha, p_i(\Delta\alpha), p_{-i}(\Delta\alpha)); \\ - c_i(\Delta\alpha) &\equiv \frac{\partial D_i}{\partial p_{-i}}(\Delta\alpha, p_i(\Delta\alpha), p_{-i}(\Delta\alpha)) + p_i(\Delta\alpha) \frac{\partial^2 D_i}{\partial p_0 \partial p_1}(\Delta\alpha, p_i(\Delta\alpha), p_{-i}(\Delta\alpha)). \end{aligned}$$

Let  $b \equiv b_0(0) = b_1(0)$  and  $c \equiv c_0(0) = c_1(0)$ . Let us also make the standard assumption that, for  $i = 0, 1$ ,  $b_i(\Delta\alpha) < 0$  (second-order condition with respect to  $p_i$ ),  $c_i(\Delta\alpha) > 0$  ( $F_i$ 's best response is increasing in  $p_{-i}$ ), and  $-b_i(\Delta\alpha) > c_i(\Delta\alpha)$  ( $F_i$ 's best-response has a slope smaller than 1).

Under these conditions, we have  $\dot{p}_0^*(0) + \dot{p}_1^*(0) = \frac{a_0(0) - a_1(0)}{-b - c} \geq 0$ .  $\varphi'(0) > 0$  is thus equivalent to  $(a_0(0) - a_1(0)) \left(1 + \frac{1}{-b - c} \frac{\partial D_i}{\partial p_j}(0, p^*(0), p^*(0))\right) > 0$ , which always holds under our assumptions if and only if  $a_0(0) > a_1(0)$ . To summarize, Assumption 1 holds in the neighborhood of  $\Delta\alpha = 0$  when  $a_0(0) > a_1(0)$  and some standard assumptions are satisfied.

Last, let us consider a fully symmetric and ‘separable’ example in which demands are given by  $D_0(\Delta\alpha, p_0, p_1) = a\Delta\alpha + D(p_0) + d(p_1)$  and  $D_1(\Delta\alpha, p_0, p_1) = -a\Delta\alpha + D(p_1) + d(p_0)$ , with  $D'(\cdot) < 0 < d'(\cdot)$  and  $|D''(\cdot)| > |d''(\cdot)|$ . Let us further assume that  $D''(\cdot) < d''(\cdot) < 0$ . Notice first that  $\dot{p}_0^*(0) = -\dot{p}_1^*(0) = \frac{a}{-b+c} > 0$ , with  $b = 2D'(p^*(0)) + p^*(0)D''(p^*(0))$  and  $c = d'(p^*(0))$ . We have

$$\begin{aligned} \ddot{\Pi}_0(\Delta\alpha) &= \dot{p}_0^*(\Delta\alpha) (a + \dot{p}_1^*(\Delta\alpha) d'(p_1^*(\Delta\alpha))), \\ \ddot{\Pi}_1(\Delta\alpha) &= \dot{p}_1^*(\Delta\alpha) (a + \dot{p}_0^*(\Delta\alpha) d'(p_0^*(\Delta\alpha))). \end{aligned} \tag{A.2}$$

Thus, differentiating (A.2) and using the fact that  $\dot{p}_0^*(0) = -\dot{p}_1^*(0)$ , we obtain

$$\begin{aligned} \ddot{\Pi}_0(0) &= \dot{p}_0^*(0) [a - \dot{p}_0^*(0) d'(p^*(0)) + p^*(0) \dot{p}_0^*(0) d''(p^*(0))] + [p^*(0) d'(p^*(0)) \dot{p}_1^*(0)], \\ \ddot{\Pi}_1(0) &= \dot{p}_0^*(0) [a - \dot{p}_0^*(0) d'(p^*(0)) + p^*(0) \dot{p}_0^*(0) d''(p^*(0))] + [p^*(0) d'(p^*(0)) \dot{p}_0^*(0)]. \end{aligned} \tag{A.3}$$

Differentiating the first-order conditions in (A.1), we obtain

$$\begin{aligned} \dot{p}_0^*(\Delta\alpha) b_0(p_0(\Delta\alpha)) + \dot{p}_1^*(\Delta\alpha) c_0(p_1(\Delta\alpha)) + a &= 0, \\ \dot{p}_1^*(\Delta\alpha) b_1(p_1(\Delta\alpha)) + \dot{p}_0^*(\Delta\alpha) c_1(p_0(\Delta\alpha)) - a &= 0, \end{aligned} \tag{A.4}$$

<sup>29</sup>However, we also allow for some asymmetry in the impact of rights on demands:  $\frac{\partial D_0}{\partial \Delta\alpha}(\Delta\alpha, p^*(\Delta\alpha), p^*(\Delta\alpha)) \equiv a_0(\Delta\alpha) \geq -\frac{\partial D_1}{\partial \Delta\alpha}(\Delta\alpha, p^*(\Delta\alpha), p^*(\Delta\alpha)) \equiv a_1(\Delta\alpha) > 0$ .

where  $b_i(p_i) = 2D'(p_i) + p_i D''(p_i)$  and  $c_i(p_{-i}) = d'(p_{-i})$ . Differentiating equations (A.4) yields for  $\Delta\alpha = 0$

$$(A.5) \quad \begin{aligned} \ddot{p}_0^*(0)b + \ddot{p}_1^*(0)c + (\dot{p}_0^*(0))^2(\dot{b}_0(0) + \dot{c}_0(0)) &= 0, \\ \ddot{p}_1^*(0)b + \ddot{p}_0^*(0)c + (\dot{p}_0^*(0))^2(\dot{b}_1(0) + \dot{c}_1(0)) &= 0, \end{aligned}$$

where we used  $b = b_0(0) = b_1(0) = 2D'(p^*(0)) + p^*(0)D''(p^*(0))$  and  $c = c_0(0) = c_1(0) = d'(p^*(0))$  and  $\dot{p}_1^*(0) = -\dot{p}_0^*(0)$ . Simple computations show that  $\dot{b}_0(0) + \dot{c}_0(0) = \dot{p}_0^*(0)(3D''(p^*(0)) + p^*(0)D'''(p^*(0)) - d''(p^*(0))) \equiv \eta = -(\dot{b}_1(0) + \dot{c}_1(0))$ . The system (A.5) can be solved to obtain

$$\ddot{p}_0^*(0) = \frac{-\eta}{b-c} (\dot{p}_0^*(0))^2 = -\ddot{p}_1^*(0).$$

Since, by assumption,  $D''(p) < d''(p) < 0$ ,  $\eta < 0$  if we assume that  $D'''(\cdot)$  is not too strongly positive. Hence,  $\ddot{p}_0^*(0) > 0$  and  $\ddot{p}_1^*(0) < 0$ , so that  $\ddot{\Pi}_0(0) < \ddot{\Pi}_1(0)$ . Let us now focus on the condition  $\ddot{\Pi}_0(0) > 0$ , which rewrites as follows

$$(A.6) \quad \begin{aligned} (-2D'(p^*(0)) - p^*(0)D''(p^*(0)) + d'(p^*(0)))^2 &(-2D'(p^*(0)) - p^*(0)(D''(p^*(0)) - d''(p^*(0)))) \\ &> ap^*(0)d'(p^*(0))(3D''(p^*(0)) + p^*(0)D'''(p^*(0)) - d''(p^*(0))). \end{aligned}$$

The left-hand side of (A.6) is strictly positive. A sufficient condition for  $\ddot{\Pi}_0(0) > 0$  is thus that either  $d'(p^*(0))$  is small enough, or the second- and third-order derivatives of the demand functions are sufficiently small. Then, both individual profits are strictly convex in the neighborhood of  $\Delta\alpha = 0$ , and so is the industry profit.

The arguments developed in this example do not rely on whether there is only one or several rights. Hence, Assumption 2 holds under the same conditions.

*Discrete Choice Model.* Consider the following discrete choice model. A viewer gains  $\varepsilon_i + \alpha_i - p_i$  if he buys from  $F_i$ . Let  $\varepsilon_i$ ,  $i = 0, 1$ , be i.i.d. variables distributed according to cdf  $G(\cdot)$ . Let  $z = \varepsilon_0 - \varepsilon_1$  and denote by  $F(\cdot)$  its distribution obtained by convolution.  $F(\cdot)$  is symmetric, or  $F(x) + F(-x) = 1$  for all  $x$ . Last, let  $\Delta\alpha = \alpha_0 - \alpha_1$ . Assume without loss of generality that  $\Delta\alpha \geq 0$ .

Profits of  $F_0$  and  $F_1$  are respectively given by  $p_0(1 - F(p_0 - p_1 - \Delta\alpha))$  and  $p_1 F(p_0 - p_1 - \Delta\alpha)$ . Assuming that they are uniquely defined by first-order conditions, best responses are given by  $p_0 = \frac{1 - F(p_0 - p_1 - \Delta\alpha)}{f(p_0 - p_1 - \Delta\alpha)}$  and  $p_1 = \frac{F(p_0 - p_1 - \Delta\alpha)}{f(p_0 - p_1 - \Delta\alpha)}$ . Subtracting these two equations leads to  $z + \Delta\alpha = \frac{1 - 2F(z)}{f(z)}$ . Denote by  $z^*(\Delta\alpha)$  the solution of this last equation. It comes immediately that (i)  $z^*(\Delta\alpha)$  is negative, and (ii)  $z^*(\Delta\alpha)$  is non-increasing in  $\Delta\alpha$ . Equilibrium profits are thus given by  $\tilde{\Pi}_0(z^*(\Delta\alpha)) = \frac{(1 - F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$  and  $\tilde{\Pi}_1(z^*(\Delta\alpha)) = \frac{(F(z^*(\Delta\alpha)))^2}{f(z^*(\Delta\alpha))}$ .

Assume from now on that  $\varepsilon_i$ s follow the Gumbel distribution with location 0 and scale  $1/\lambda > 0$ . Then,  $z$  is distributed according to the logistic distribution with cumulative  $F(x) = \frac{e^{\lambda x}}{1 + e^{\lambda x}}$ . Consider  $\varphi(\Delta\alpha) = \tilde{\Pi}_0(z^*(\Delta\alpha)) + \tilde{\Pi}_1(z^*(\Delta\alpha))$ . We have  $\varphi(\Delta\alpha) = \frac{1}{\lambda}(e^{\lambda z^*(\Delta\alpha)} + e^{-\lambda z^*(\Delta\alpha)})$ . We obtain

$$\varphi'(\Delta\alpha) = \dot{z}^*(\Delta\alpha) \left( e^{\lambda z^*(\Delta\alpha)} - e^{-\lambda z^*(\Delta\alpha)} \right),$$

which is strictly positive since  $z^*(\Delta\alpha)$  is non-increasing and negative. Hence, Assumption 1 holds.

We also have

$$\frac{d^2}{d(\Delta\alpha)^2}\tilde{\Pi}_0(z^*(\Delta\alpha)) = \frac{d^2}{dz^2}\tilde{\Pi}_0(z)\Big|_{z=z^*(\Delta\alpha)}(\dot{z}^*(\Delta\alpha))^2 + \frac{d}{dz}\tilde{\Pi}_0(z)\Big|_{z=z^*(\Delta\alpha)}\ddot{z}^*(\Delta\alpha).$$

With a logistic distribution,  $\tilde{\Pi}_0(z^*(\Delta\alpha)) = (1/\lambda)e^{-\lambda z^*(\Delta\alpha)}$ . Moreover, we find that  $\dot{z}^*(\Delta\alpha) = -(1 + 2 \cosh(\lambda z^*(\Delta\alpha)))^{-1}$  and  $\ddot{z}^*(\Delta\alpha) = -\frac{2\lambda \sinh(\lambda z^*(\Delta\alpha))}{(1 + 2 \cosh(\lambda z^*(\Delta\alpha)))^3}$ . Finally, we obtain

$$\frac{d^2}{d(\Delta\alpha)^2}\tilde{\Pi}_0(z^*(\Delta\alpha)) = \frac{\lambda e^{2\lambda z^*(\Delta\alpha)}(1 + 2e^{\lambda z^*(\Delta\alpha)})}{(1 + e^{\lambda z^*(\Delta\alpha)} + e^{2\lambda z^*(\Delta\alpha)})^3} > 0.$$

Similar computations show that  $F_1$ 's profit is convex in  $\Delta\alpha$  as well:  $\frac{d^2}{d(\Delta\alpha)^2}\tilde{\Pi}_1(z^*(\Delta\alpha)) = \frac{\lambda(e^{\lambda z^*(\Delta\alpha)} + 2)}{(2 \cosh(\lambda z^*(\Delta\alpha)) + 1)^3} > 0$ . Therefore, individual profits are strictly convex in  $\Delta\alpha$  as well.

The arguments developed in this example do not rely on whether there is only one or several rights. Hence, Assumption 2 always holds in this example.

*Linear Cournot and Bertrand with Differentiated Products.* Consider that  $F_0$  and  $F_1$  compete in quantities and face inverse demand functions given by  $P_0(\alpha_0, q_0, q_1) = a + \alpha_0 - q_0 + bq_1$  and  $P_1(\alpha_1, q_1, q_0) = a + \alpha_1 - q_1 + bq_0$ , with  $-1 < b < 0$ . At the Nash equilibrium, profits are given by  $\tilde{\Pi}_0(\alpha_0, \alpha_1) = (\frac{a(2+b)+2\alpha_0+b\alpha_1}{4-b^2})^2$  and  $\tilde{\Pi}_1(\alpha_0, \alpha_1) = (\frac{a(2+b)+2\alpha_1+b\alpha_0}{4-b^2})^2$ . Simple manipulations shows that  $\tilde{\Pi}_0(\alpha + \Delta_0, 0) + \tilde{\Pi}_1(\alpha + \Delta_0, 0) > \tilde{\Pi}_0(0, \alpha + \Delta_1) + \tilde{\Pi}_1(0, \alpha + \Delta_1)$  for  $\Delta_0 \geq \Delta_1$ . Hence, Assumption 1 amounts to  $f(\Delta_1) \equiv \tilde{\Pi}_0(0, \alpha + \Delta_1) + \tilde{\Pi}_1(0, \alpha + \Delta_1) - (\tilde{\Pi}_0(\alpha, \alpha) + \tilde{\Pi}_1(\alpha, \alpha)) > 0$ .  $f(\cdot)$  is strictly convex in  $\Delta_1$  and admits two real roots, with the smallest one being negative and the highest  $\bar{\Delta}_1$  one being positive. Hence, Assumption 1 holds if and only if  $\Delta_1 > \bar{\Delta}_1 = \frac{(2+b)\sqrt{a^2(2+b)^2+4a\alpha(4+b^2)+2\alpha^2(4+b^2)}-a(2+b)^2-\alpha(4+b^2)}{4+b^2}$ .

Last, consider the case of multiple rights. The condition in Assumption 2 for  $F_0$  is  $\tilde{\Pi}_0(2(\alpha + \Delta_0), 0) + \tilde{\Pi}_0(0, 2(\alpha + \Delta_1)) > 2\tilde{\Pi}_0(\alpha + \Delta_0, \alpha + \Delta_1)$ . After simple computations, this condition rewrites as  $2(\frac{2\Delta_0-b\Delta_1+\alpha(2-b)}{4-b^2})^2 > 0$ , which always holds. A similar result obtains for  $F_1$ . Assumption 2 is thus always satisfied in this example.

If firms compete instead in prices and face demand functions given by  $D_i(\alpha_i, p_i, p_j) = a + \alpha_i - p_i - \frac{b}{2}(p_i - p_j)$ ,  $i = 0, 1$ , with  $0 < b$ . Computations similar to the ones performed in the case of Cournot competition show that Assumption 1 holds provided that the extra-benefits associated to exclusive distribution  $\Delta_0$  and  $\Delta_1$  are large enough. Similarly, Assumption 2 is always satisfied in this example.

*Hotelling with Single- and Multi-Homing Viewers.* There are two types of viewers: Those who single-home, *i.e.*, buy at most one unit of the product; and those who multi-home, *i.e.*, buy at most two units of the product (one unit from both firms). Both types of viewers are located on a  $[0, 1]$ -segment and there is a mass  $\varepsilon$  (resp.  $1 - \varepsilon$ ) of multi-homers (resp. single-homers).  $F_0$  and  $F_1$  compete in prices and are located at the extreme points of the  $[0, 1]$ -segment.

For single-homers, the framework is similar to a standard Hotelling model. A viewer located in  $x \in [0, 1]$  derives utility  $v + \alpha_0 - p_0 - tx$  if he buys from  $F_0$ , and  $v - \alpha_1 - p_1 - t(1 - x)$  if he buys from  $F_1$ , where  $t$  is the per-unit of distance transportation cost. Denote by  $\hat{x}$  the single-homing viewer who is indifferent between buying from either firms:  $\hat{x} = \frac{1}{2t}(t + \Delta\alpha - (p_0 - p_1))$  with  $\Delta\alpha = \alpha_0 - \alpha_1$ .

Multi-homing viewers are modeled following Doganoglu and Wright (2006). If a multi-homing viewer located in  $x \in [0, 1]$  buys only one unit from either  $F_0$  or from  $F_1$ , he obtains  $v + \alpha_0 - p_0 - tx$  or  $v + \alpha_1 - p_1 - t(1 - x)$  respectively. If he buys one unit from both firms, he obtains  $v + \frac{\alpha_0 + \alpha_1}{2} + \omega t - (p_0 + p_1) - t$ , where  $\omega \geq 1/2$  is a parameter reflecting a gain

from diversity and  $t = t(x + 1 - x)$  is the total transportation cost. Let  $\tilde{x}_0$  (resp.  $\tilde{x}_1$ ) be the viewer who is indifferent between buying two units and one unit only from  $F_0$  (resp.  $F_1$ ):  $\tilde{x}_0 = \frac{1}{t}(\Delta\alpha/2 + p_1) + 1 - \omega$  (resp.  $\tilde{x}_1 = \frac{1}{t}(\Delta\alpha/2 - p_0) + \omega$ ). We assume from now on that  $\tilde{x}_0 < \tilde{x}_1$ , or  $p_0 + p_1 < (2\omega - 1)t$ , a condition which has to be checked at equilibrium and ensures that some multi-homing viewers actually buy two units from (one from each firm) rather than one unit only from either firms. Equipped with these notations, it comes immediately that a multi-homing viewer with  $x \leq \tilde{x}_0$  (resp.  $x \geq \tilde{x}_1$ ) buys one unit only from  $F_0$  (resp.  $F_1$ ), and a multi-homing viewer with  $x \in [\tilde{x}_0, \tilde{x}_1]$  buys one unit from both firms.

The total demand for product 0 (resp. 1) is given by  $D_0(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)\hat{x} + \varepsilon\tilde{x}_1$  (resp.  $D_1(\Delta\alpha, p_0, p_1) = (1 - \varepsilon)(1 - \hat{x}) + \varepsilon(1 - \tilde{x}_0)$ ).  $F_0$ 's and  $F_1$ 's profits are then given by  $p_0D_0$  and  $p_1D_1$  respectively. Routine computations allow then to determine the Nash equilibrium prices  $p_0^*(\Delta\alpha) = \frac{\Delta\alpha}{3+\varepsilon} + \frac{t}{1+3\varepsilon}(1 + \varepsilon(2\omega - 1))$  and  $p_1^*(\Delta\alpha) = \frac{-\Delta\alpha}{3+\varepsilon} + \frac{t}{1+3\varepsilon}(1 + \varepsilon(2\omega - 1))$ . From this, we can deduce the profits at the Nash equilibrium:  $\tilde{\Pi}_0(\Delta\alpha) = \frac{1+\varepsilon}{2t} \left( \frac{\Delta\alpha(1+3\varepsilon)+t(3+\varepsilon)(1+\varepsilon(2\omega-1))}{(3+\varepsilon)(1+3\varepsilon)} \right)^2$  and  $\tilde{\Pi}_1(\Delta\alpha) = \tilde{\Pi}_0(-\Delta\alpha)$ . Let  $\varphi(\Delta\alpha) = \tilde{\Pi}_0(\Delta\alpha) + \tilde{\Pi}_0(-\Delta\alpha)$ . It comes immediately that  $\varphi(\Delta\alpha) - \varphi(0) = \frac{(\Delta\alpha)^2}{t} \frac{1+\varepsilon}{(3+\varepsilon)^2} > 0$ . Hence, Assumption 1 holds.

The condition  $p_0^*(\Delta\alpha) + p_1^*(\Delta\alpha) < (2\omega - 1)t$  amounts to  $\omega \geq \frac{3+\varepsilon}{2(1+\varepsilon)}$ , *i.e.*, the proportion of multi-homers is large enough or the gain from diversity is high enough. We must also check that the market is covered and that the utility of marginal customers is positive at the Nash equilibrium:  $v + \alpha_0 - p_0^* - t\hat{x}(p_0^*, p_1^*) \geq 0$  amounts to  $v + \frac{\alpha_0 + \alpha_1}{2} \geq \frac{t}{2} + t \frac{1+\varepsilon(2\omega-1)}{1+3\varepsilon}$ ;  $v + \alpha_0 - p_0^* - t\tilde{x}_0(p_0^*, p_1^*) \geq 0$  and  $v + \alpha_1 - p_1^* - t(1 - \tilde{x}_1(p_0^*, p_1^*)) \geq 0$  amount to  $v + \frac{\alpha_0 + \alpha_1}{2} \geq t(1 - \omega) + 2t \frac{1+\varepsilon(2\omega-1)}{1+3\varepsilon}$ . These conditions are satisfied for  $v$  sufficiently large for instance.

Last, observe that  $\tilde{\Pi}_0(\Delta\alpha)$  is strictly convex in  $\Delta\alpha$ , and so is  $\tilde{\Pi}_1(\Delta\alpha) = \tilde{\Pi}_0(-\Delta\alpha)$ . This does not rely on whether there is only one or several rights. Hence, Assumption 2 holds in this example.  $\square$

PROOF OF PROPOSITION 1. Because  $\bar{T}_i(a) \geq 0$  for all  $a \in \mathcal{A}$ , we have the following:

$$(A.7) \quad \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) = \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a) \geq \max_{a \in \mathcal{A}} \bar{T}_{-i}(a).$$

The first equality writes as (5.4).

CLAIM 1. (5.6) holds.

PROOF OF CLAIM 1. Suppose to the contrary that  $\bar{T}_i(a_{-i}) > 0$ . Consider the new bidding schedule as follows:

$$(A.8) \quad \tilde{T}_i(a) = \max\{\bar{T}_i(a) - \bar{T}_i(a_{-i}); 0\}.$$

By construction, we have the following:

$$\tilde{T}_i(a_{-i}) = 0.$$

Since  $\bar{T}_i(a) \geq 0$  for all  $a \in \mathcal{A}$ , we also have the following:

$$\tilde{T}_i(a) \leq \max\{\bar{T}_i(a); 0\} = \bar{T}_i(a).$$

Moreover, the following string of inequality holds:

$$\begin{aligned}
\max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max\{\bar{T}_i(a) - \bar{T}_i(a_{-i}) + \bar{T}_{-i}(a); \bar{T}_{-i}(a)\} \right\}, \\
&= \max \left\{ \max_{a \in \mathcal{A}} \bar{T}_i(a) - \bar{T}_i(a_{-i}) + \bar{T}_{-i}(a); \max_{a \in \mathcal{A}} \bar{T}_{-i}(a) \right\}, \\
&= \max \left\{ \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \bar{T}_i(a_{-i}); \bar{T}_{-i}(a_{-i}) \right\}, \\
&= \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \bar{T}_i(a_{-i}), \\
&\leq \tilde{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}),
\end{aligned}$$

where the last equality follows from the definition of  $\bar{a}$  and the last inequality from the definition of  $\tilde{T}_i(a)$  given in (A.8). From this, we deduce that  $\tilde{T}_i$  implements  $\bar{a}$  since

$$\bar{a} \in \arg \max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a)$$

and does so at a weakly lower cost for  $F_i$  than  $\bar{T}_i$ ; which ends the proof.  $\square$

CLAIM 2. (5.5) holds.

PROOF OF CLAIM 2. By definition, we have  $\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) = \max_{a \in \mathcal{A}} \bar{T}_i(a) + \bar{T}_{-i}(a)$  and  $\bar{T}_{-i}(a_{-i}) = \max_{a \in \mathcal{A}} \bar{T}_{-i}(a)$ . Suppose to the contrary that

$$\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) > \bar{T}_{-i}(a_{-i}).$$

There thus exists  $\varepsilon > 0$  small enough such that  $\bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) > \varepsilon + \bar{T}_{-i}(a_{-i})$ . Consider the new bidding schedule as follows:

$$\tilde{T}_i(a) = \max\{\bar{T}_i(a) - \varepsilon; 0\}.$$

Proceeding as in the Proof of Claim 1, we obtain the following:

$$\begin{aligned}
\max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max\{\bar{T}_i(a) - \varepsilon + \bar{T}_{-i}(a); \bar{T}_{-i}(a)\} \right\}, \\
&= \max \left\{ \max_{a \in \mathcal{A}} \bar{T}_i(a) - \varepsilon + \bar{T}_{-i}(a); \max_{a \in \mathcal{A}} \bar{T}_{-i}(a) \right\}, \\
&= \max \left\{ \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \varepsilon; \bar{T}_{-i}(a_{-i}) \right\}, \\
&= \bar{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}) - \varepsilon, \\
&\leq \tilde{T}_i(\bar{a}) + \bar{T}_{-i}(\bar{a}).
\end{aligned}$$

From this, we deduce that  $\tilde{T}_i$  again implements  $\bar{a}$ , or

$$\bar{a} \in \arg \max_{a \in \mathcal{A}} \tilde{T}_i(a) + \bar{T}_{-i}(a).$$

and does so at a weakly lower cost for  $F_i$  than  $\bar{T}_i$ ; which ends the proof.  $\square$

Claims 1 and 2 taken together allow to prove Proposition 1.  $\square$

PROOF OF PROPOSITION 2.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that  $a_{-0} = (1)$  (since  $\Pi_1(1) > \Pi_1(0)$ ) and  $a_{-1} = (0)$  (since  $\Pi_0(0) > \Pi_0(1)$ ). From (5.6), it immediately follows that

$$(A.9) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0.$$



From (5.5), we write the following:

$$(A.10) \quad \bar{T}_0(\bar{a}) + \bar{T}_1(\bar{a}) = \bar{T}_1(1) = \bar{T}_0(0).$$

From (5.3), any equilibrium allocation must satisfy

$$(A.11) \quad \sum_{i=0,1} \Pi_i(\bar{a}) + \sum_{i=0,1} \bar{T}_i(\bar{a}) \geq \sum_{i=0,1} \Pi_i(0) + \sum_{i=0,1} \bar{T}_i(0).$$

Inserting (A.9) and (A.10) into (A.11) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\bar{a}) \geq \sum_{i=0,1} \Pi_i(0).$$

Since (0) is constrained-efficient, we necessarily have  $\bar{a} = (0)$ , which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria. We start with the following lemma.

LEMMA A.3. *The set of equilibrium bids in monotonic equilibria is defined by the following conditions:*

$$(A.12) \quad \bar{T}_0(0) \in [\Pi_1(1) - \Pi_1(0), \Pi_0(0) - \Pi_0(1)],$$

$$(A.13) \quad \bar{T}_0(0) = \bar{T}_1(1) \geq 0,$$

$$(A.14) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0,$$

$$(A.15) \quad \bar{T}_0(c) \in [0, \bar{T}_0(0) + \Pi_1(0) - \Pi_1(c)],$$

$$(A.16) \quad \bar{T}_1(c) \in [0, \Pi_0(0) - \Pi_0(c)],$$

with

$$(A.17) \quad \bar{T}_0(c) + \bar{T}_1(c) \leq \bar{T}_0(0).$$

PROOF OF LEMMA A.3. *Necessity.* Since in any equilibrium  $\bar{a} = (0)$ ,  $F_1$  should not want to deviate to induce the alternative allocation  $a = (1)$  by offering a payment  $T_1(1) \geq \bar{T}_0(0)$ . The corresponding incentive constraint is as follows:

$$(A.18) \quad \Pi_1(0) \geq \max_{T_1(1) \text{ s.t. } T_1(1) \geq \bar{T}_0(0)} \Pi_1(1) - T_1(1) = \Pi_1(1) - \bar{T}_0(0),$$

which gives the lower bound in (A.12).

Additionally,  $F_1$  should not want to deviate from  $\bar{a} = a_{-1} = (0)$  to induce an outcome with joint distribution, *i.e.*,  $a = (c)$ , by offering a payment  $T_1(c)$  such that  $\bar{T}_0(c) + T_1(c) \geq \bar{T}_0(0)$ . The corresponding incentive constraint becomes as follows:

$$(A.19) \quad \Pi_1(0) \geq \max_{T_1(c) \text{ s.t. } \bar{T}_0(c) + T_1(c) \geq \bar{T}_0(0)} \Pi_1(c) - T_1(c) = \Pi_1(c) + \bar{T}_0(c) - \bar{T}_0(0).^{30}$$

From which, we obtain the upper bound in (A.15).

Second, turning to  $F_0$ 's incentives to abide to the equilibrium strategy rather than letting  $a = (1)$  emerges, which is simply obtained by not paying since  $a_{-0} = (1)$ , it must be that

$$(A.20) \quad \Pi_0(0) - \bar{T}_0(0) \geq \Pi_0(1).$$

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<sup>30</sup>Because  $\bar{T}_0$  is monotonic and  $\Pi_0(0) \geq \Pi_0(c)$ , we have  $\bar{T}_0(0) \geq \bar{T}_0(c)$  and the considered deviation requires  $T_1(c) \geq 0$ .

which gives the upper bound in (A.12).

Additionally,  $F_0$  should not be willing to induce  $a = (c)$  either, which requires the following:

$$(A.21) \quad \Pi_0(0) - \bar{T}_0(0) \geq \max_{T_0(c) \text{ s.t. } T_0(c) + \bar{T}_1(c) \geq \bar{T}_0(0)} \Pi_0(c) - T_0(c) = \Pi_0(c) + \bar{T}_1(c) - \bar{T}_0(0),$$

where the constraint in the maximand of the right-hand side follows from (A.9). Simplifying yields (A.16).

Putting together (A.18) and (A.20) yields (A.12).

Using (5.4) yields the following:

$$\bar{T}_0(c) + \bar{T}_1(c) \leq \bar{T}_0(0) + \bar{T}_1(0).$$

Taking into account that  $a_{-1} = (0)$  (and thus  $\bar{T}_1(0) = 0$ ) yields (A.17). The right-hand side inequality follows from bid being non-negative.

Using (5.5), the fact that  $\bar{a} = a_{-1} = (0)$  and thus  $\bar{T}_1(0) = 0$  (from (5.6)) implies (A.13).

Last, (A.14) follows from (5.6).

*Sufficiency.* Any pair of bidding schedules  $(\bar{T}_0, \bar{T}_1)$  that satisfy conditions (A.12) to (A.17) also satisfy, by construction, all the incentive constraints for  $F_0$ ,  $F_1$  and  $A$  that must hold at an equilibrium.  $\square$

Gathering everything and noting that the overall profit of the industry must be  $\Pi_0(0) + \Pi_1(0)$  yields (5.8).  $\square$

**PROOF OF PROPOSITION 3.** First, constrained efficiency follows from Proposition 2. Second, we remind that monotonicity implies  $a_{-0} = (1)$  and  $a_{-1} = (0)$ , and we note that truthful schedules of the form  $T_i(a) = \max\{\Pi_i(a) - \bar{\Pi}_i, 0\}$  satisfy those monotonicity conditions. Following Laussel and Le Breton (2001), we define the value of any arbitrary coalition  $S \subseteq N = \{F_0, F_1, A\}$  as  $W(S) = \max_{a \in \mathcal{A}} \sum_{i \in S} \Pi_i(a)$ . The cooperative game with transferable utility defined through these values is *strongly subadditive* if, for all  $S \subseteq N$ ,  $T \subseteq N$  such that  $S \cup T = N$ ,  $W(N) \leq W(T) + W(S) - W(S \cap T)$ ; this is a condition that can be readily verified since we have the following:

$$W(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W(\{0\}) = \Pi_0(0), \quad W(\{1\}) = \Pi_1(0), \quad W(\emptyset) = 0$$

and thus

$$W(\{01\}) < W(\{0\}) + W(\{1\}) \Leftrightarrow \Pi_1(0) < \Pi_1(0).$$

Following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair  $(\bar{\Pi}_0, \bar{\Pi}_1)$  must lie on the Pareto frontier of the set defined by the three following constraints:

$$(A.22) \quad \bar{\Pi}_0 \leq W(\{01\}) - W(\{1\}) = \Pi_0(0) + \Pi_1(0) - \Pi_1(0),$$

$$(A.23) \quad \bar{\Pi}_1 \leq W(\{01\}) - W(\{0\}) = \Pi_1(0),$$

$$(A.24) \quad \bar{\Pi}_0 + \bar{\Pi}_1 \leq W(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Because of strong sub-additivity, the following inequality holds:

$$W(\{01\}) - W(\{1\}) + W(\{01\}) - W(\{0\}) < W(\{01\}).$$

It implies that (A.24) necessarily holds when (A.22) and (A.23) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0, \bar{\Pi}_1) = (\Pi_0(0) + \Pi_1(0) - \Pi_1(1), \Pi_1(0)).$$

This finally gives us the expressions of profits for the whole industry in (5.9).  $\square$

PROOF OF PROPOSITION 4. We adapt the methodology developed in Proposition 3 to a scenario with renegotiation. For any arbitrary coalition  $S \subseteq N = \{F_0, F_1, A\}$ , we may define the coalitional payoff with renegotiation as  $W^{rp}(S) = \max_{a \in \mathcal{A}} \sum_{i \in S} \bar{\Pi}_i(a)$ . By Definition (6.1), we have the following:

$$W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} (\Pi_0(0) - \Pi_1(0)),$$

$$W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} (\Pi_1(1) - \Pi_0(1)), \quad W^{rp}(\emptyset) = 0.$$

The cooperative game so-defined is again *strongly subadditive* since

$$W^{rp}(\{01\}) < W^{rp}(\{0\}) + W^{rp}(\{1\}) \Leftrightarrow \Pi_1(0) - \Pi_0(0) < \Pi_1(1) - \Pi_0(1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair  $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp})$  lies on the Pareto frontier of the set defined by the three following constraints:

$$(A.25) \quad \bar{\Pi}_0^{rp} \leq W^{rp}(\{01\}) - W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} (\Pi_1(1) - \Pi_0(1)),$$

$$(A.26) \quad \bar{\Pi}_1 \leq W^{rp}(\{01\}) - W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} (\Pi_0(0) - \Pi_1(0)),$$

$$(A.27) \quad \bar{\Pi}_0^{rp} + \bar{\Pi}_1^{rp} \leq W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Strong sub-additivity implies that (A.27) necessarily holds when (A.25) and (A.26) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp}) = \left( \frac{1}{2} \sum_{i=0,1} \Pi_i(0) - \frac{1}{2} (\Pi_1(1) - \Pi_0(1)), \Pi_1(0) \right).$$

Observe that  $F_1$ 's profits with or without resale are the same (see (5.9) and (6.3)). Then  $A$ 's profit in the unique truthful resale-proof equilibrium is greater than that in the unique truthful equilibrium without resale since

$$\frac{1}{2} (\Pi_0(0) + \Pi_1(1) - \Pi_1(0) - \Pi_0(1)) > \Pi_1(1) - \Pi_1(0).$$

This concludes the proof.  $\square$

PROOF OF PROPOSITION 5.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that  $b_{-0} = (1, 1)$  and  $b_{-1} = (0, 0)$  where  $b_{-i} = \arg \max_{b \in \mathcal{A}_i^*} \bar{T}_{-i}(b)$ . From (5.6), it follows that

$$(A.28) \quad \bar{T}_0(1, 1) = \bar{T}_1(0, 0) = 0.$$

From (5.5), we write the following:

$$(A.29) \quad \bar{T}_0(\bar{b}) + \bar{T}_1(\bar{b}) = \bar{T}_1(1, 1) = \bar{T}_0(0, 0).$$

From (5.3), any equilibrium allocation must satisfy the following:

$$(A.30) \quad \sum_{i=0,1} \Pi_i(\bar{b}) + \sum_{i=0,1} \bar{T}_i(\bar{b}) \geq \sum_{i=0,1} \Pi_i(0, 0) + \sum_{i=0,1} \bar{T}_i(0, 0).$$

Inserting (A.28) and (A.29) into (A.30) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\bar{b}) \geq \sum_{i=0,1} \Pi_i(0, 0).$$

Since (0, 0) is constrained-efficient, we necessarily have  $\bar{b} = (0, 0)$ ; which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria.

LEMMA A.4. *The whole set of bids in monotonic equilibria is defined by the following inequalities:*

$$(A.31) \quad \bar{T}_0(0, 0) = \bar{T}_1(1, 1) \in [\Pi_1(1, 1) - \Pi_1(0, 0), \Pi_0(0, 0) - \Pi_0(1, 1)],$$

$$(A.32) \quad \bar{T}_0(1, 1) = \bar{T}_1(0, 0) = 0,$$

$$(A.33) \quad \begin{aligned} \bar{T}_0(0, 1) &\in [0, \bar{T}_0(0, 0) + \Pi_1(0, 0) - \Pi_1(0, 1)], \\ \bar{T}_0(1, 0) &\in [0, \bar{T}_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 0)], \end{aligned}$$

$$(A.34) \quad \begin{aligned} \bar{T}_1(0, 1) &\in [0, \Pi_0(0, 0) - \Pi_0(0, 1)], \\ \bar{T}_1(1, 0) &\in [0, \Pi_0(0, 0) - \Pi_0(1, 0)], \end{aligned}$$

$$(A.35) \quad \max\{\bar{T}_0(0, 1) + \bar{T}_1(0, 1), \bar{T}_0(1, 0) + \bar{T}_1(1, 0)\} \leq \bar{T}_0(0, 0) = \bar{T}_1(1, 1).$$

PROOF OF LEMMA A.4.

*Necessity.* Since in any equilibrium  $\bar{b} = (0, 0)$ , the first equality in (A.31) follows from (5.5) and (5.6).

Also  $F_1$  should not want to deviate to induce the alternative allocation  $a = (1, 1)$ . The corresponding incentive constraint becomes as follows:

$$(A.36) \quad \Pi_1(0, 0) \geq \max_{T_1(1,1) \text{ s.t. } T_1(1,1) \geq \bar{T}_0(0,0)} \Pi_1(1, 1) - T_1(1, 1) = \Pi_1(1, 1) - \bar{T}_0(0, 0).$$

Similarly,  $F_1$  should not induce a deviation towards  $a = (0, 1)$ , which requires the following:

$$(A.37) \quad \Pi_1(0, 0) \geq \max_{T_1(0,1) \text{ s.t. } T_0(0,1) + T_1(0,1) \geq \bar{T}_0(0,0)} \Pi_1(0, 1) - T_1(0, 1) = \Pi_1(0, 1) + T_0(0, 1) - \bar{T}_0(0, 0);$$

thus, the first condition in (A.33) follows.

$F_1$  should also not induce a deviation towards  $a = (1, 0)$ , which requires the following:

$$(A.38) \quad \Pi_1(0, 0) \geq \max_{T_1(1,0) \text{ s.t. } T_0(1,0) + T_1(1,0) \geq \bar{T}_0(0,0)} \Pi_1(0, 1) - T_1(0, 1) = \Pi_1(0, 1) + T_0(0, 1) - \bar{T}_0(0, 0);$$

thus, the second condition in (A.33) follows.

Let us now turn to  $F_0$ 's incentives to abide to the equilibrium strategy rather than to induce  $a = (1, 1)$ . Such deviation is simply obtained with a null bid since  $a_{-0} = (1, 1)$ .  $F_0$ 's incentive constraint is thus written as follows:

$$(A.39) \quad \Pi_0(0) - \bar{T}_0(0, 0) \geq \Pi_0(1, 1).$$

Putting together (A.36) and (A.39) yields the second condition in (A.31).

Also,  $F_0$  should not induce a deviation towards  $a = (0, 1)$ . Since,  $A$  is indifferent between  $\bar{a} = (0, 0)$  and  $a = (1, 1)$  (from the first equality in (A.31)), this requires avoiding that  $A$  switches to  $(1, 1)$  in case  $F_0$  no longer offers  $\bar{T}_0(0, 0)$ . We write this incentive constraint as follows:

$$(A.40) \quad \Pi_1(0, 0) - \bar{T}_0(0, 0) \geq \max_{T_0(0,1) \text{ s.t. } T_0(0,1) + \bar{T}_1(0,1) \geq \bar{T}_1(1,1)} \Pi_0(0, 1) - T_0(0, 1) = \Pi_0(0, 1) + \bar{T}_1(0, 1) - \bar{T}_1(1, 1).$$

Again taking into account the first equality in (A.31), (A.40) simplifies as follows:

$$\Pi_1(0, 0) - \bar{T}_0(0, 0) \geq \Pi_0(0, 1) + \bar{T}_1(0, 1) - \bar{T}_0(0, 0).$$

Therefore, the upper bound for  $\bar{T}_1(0, 1)$  in (A.34) immediately follows.

Finally,  $F_0$  should also not induce a deviation towards  $a = (1, 0)$ . Replicating lines above, it must be that

$$(A.41) \quad \Pi_0(0, 0) - \bar{T}_0(0, 0) \geq \max_{T_0(1,0) \text{ s.t. } T_0(1,0) + \bar{T}_1(1,0) \geq \bar{T}_1(1,1)} \Pi_0(1, 0) - T_0(1, 0) = \Pi_0(1, 0) + \bar{T}_1(1, 0) - \bar{T}_1(1, 1).$$

Again taking into account the first equality in (A.31), (A.41) simplifies as follows:

$$\Pi_0(0, 0) - \bar{T}_0(0, 0) \geq \Pi_0(1, 0) + \bar{T}_1(1, 0) - \bar{T}_0(0, 0).$$

Therefore, the upper bound for  $\bar{T}_1(1, 0)$  in (A.34) immediately follows.

*Sufficiency.* Any pair of bidding schedules  $(\bar{T}_0, \bar{T}_1)$  that satisfy conditions (A.31) also satisfy  $F_0$ ,  $F_1$  and  $A$ 's incentive constraints from the above analysis, which defines an equilibrium.  $\square$

The expression of profits immediately follows from the existing bounds on bids characterized in Lemma A.4.  $\square$

**PROOF OF PROPOSITION 6.** Truthful strategies are monotonic; thus, any putative equilibrium with truthful bidding schedules implements a constrained-efficient outcome. We again adapt the methodology already used in Propositions 3 and 4 to a scenario with multiple rights. For any arbitrary coalition  $S \subseteq N = \{F_0, F_1, A\}$ , we may define the coalitional payoff as  $W(S) = \max_{b \in \mathcal{A}_*^2} \sum_{i \in S} \tilde{\Pi}_i(a)$ . We immediately check that

$$W(\{01\}) = \Pi_0(0, 0) + \Pi_1(0, 0), \quad W(\{0\}) = \Pi_0(0, 0),$$

$$W(\{1\}) = \Pi_1(1, 1), \quad W(\emptyset) = 0.$$

The cooperative game with the so-defined coalitional payoffs is again *strongly sub-additive* since

$$W(\{01\}) < W(\{0\}) + W(\{1\}) \Leftrightarrow \Pi_1(0, 0) < \Pi_1(1, 1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair  $(\bar{\Pi}_0^{rp}, \bar{\Pi}_1^{rp})$  lies on the Pareto frontier of the set defined by the following three constraints:

$$(A.42) \quad \bar{\Pi}_0 \leq W(\{01\}) - W(\{1\}) = \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1),$$

$$(A.43) \quad \bar{\Pi}_1 \leq W(\{01\}) - W(\{0\}) = \Pi_1(0, 0),$$

$$(A.44) \quad \bar{\Pi}_0 + \bar{\Pi}_1 \leq W(\{01\}) = \Pi_0(0, 0) + \Pi_1(0, 0).$$

Strong sub-additivity implies that (A.44) necessarily holds when (A.42) and (A.43) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\bar{\Pi}_0^b, \bar{\Pi}_1^b) = (\Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1), \Pi_1(0, 0)).$$

Gathering everything, we finally obtain the expressions of equilibrium profits  $\bar{\Pi}_0^b$ ,  $\bar{\Pi}_1^b$  and  $\bar{\Pi}_a^b$  in the unique truthful equilibrium which are given in (7.5).  $\square$

#### PROOF OF PROPOSITION 7.

ITEM 1. Players conjecture that  $\bar{a}^{-k} = (0)$  in market  $-k$ . Then, (7.2) ensures that  $\bar{a}^k = (0)$  is the constrained-efficient allocation when restricted on market  $k$ . Mimicking our earlier findings from Proposition 2, any *pairwise-proof* quasi-equilibrium allocation in market  $k$  sustained with monotonic schedules implements  $\bar{a}^k = (0)$ . Focusing on truthful strategies of the form (7.8) on each market, we can then apply our earlier findings from Proposition 3. The constants  $\bar{\Pi}_0^k$  and  $\bar{\Pi}_1^k$ , for  $k = 1, 2$ , are found as

$$(A.45) \quad \begin{aligned} \bar{\Pi}_0^k &= \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 0) > 0, \\ \bar{\Pi}_1^k &= \Pi_1(0, 0) > 0. \end{aligned}$$

ITEM 2. When it offers the truthful schedules determined in (7.8)-(7.9),  $F_0$  obtains at the *pairwise-proof* quasi-equilibrium allocation a payoff worth the following:

$$(A.46) \quad \bar{\Pi}_0^u = \Pi_0(0, 0) - \bar{T}_0^1(0) - \bar{T}_0^2(0)$$

Using (7.8) and (7.9), we have

$$\bar{T}_0^1(0) = \bar{T}_0^2(0) = \max \{ \Pi_0(0, 0) - (\Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(0, 1)); 0 \} = \Pi_1(0, 1) - \Pi_1(0, 0) > 0,$$

where the latter strict inequality holds since  $\Pi_1(0, 0) < \Pi_1(0, 1)$  by assumption. Inserting into (A.46) yields the expression of  $\bar{\Pi}_0^u$  in (7.10). We check the following:

$$\bar{\Pi}_0^u > \bar{\Pi}_0^b,$$

or

$$\Pi_0(0, 0) + 2(\Pi_1(0, 0) - \Pi_1(0, 1)) > \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1).$$

This amounts to the following:

$$\Pi_1(1, 1) - \Pi_1(0, 1) > \Pi_1(0, 1) - \Pi_1(0, 0),$$

which holds thanks to Assumption 2.

We now turn to  $F_1$ .  $F_1$  obtains at the *pairwise-proof* truthful quasi-equilibrium allocation a payoff worth the following:

$$(A.47) \quad \Pi_1(0, 0) - \bar{T}_1^1(0) - \bar{T}_1^2(0).$$

Using (7.8) and (7.9), we now have

$$\bar{T}_1^1(0) = \bar{T}_1^2(0) = \max\{\Pi_1(0, 0) - \Pi_1(0, 1); 0\} = 0,$$

where the right-hand side equality follows from our assumption that  $\Pi_1(0, 0) < \Pi_1(0, 1)$ . Inserting into (A.47) yields the expression of  $\bar{\Pi}_1^u$  in (7.10).

Notice that

$$\bar{\Pi}_0^u + \bar{\Pi}_1^u + \bar{\Pi}_a^u = \bar{\Pi}_0^b + \bar{\Pi}_1^b + \bar{\Pi}_a^b = \Pi_0(0, 0) + \Pi_1(0, 0);$$

thus, we obtain the expression of  $\bar{\Pi}_a^u$  in (7.10).

Finally, since  $\bar{\Pi}_1^u = \bar{\Pi}_1^b$ , we obtain the following:  $\bar{\Pi}_a^u < \bar{\Pi}_a^b$ . □

**PROOF OF PROPOSITION 8.** First, observe that the best multilateral deviation for  $F_0$  should induce  $(0, 0)$ . In each market  $k$ , we would then have  $a_{-0}^k = (1)$ . Thus, inducing  $(0, 0)$  with a pair of additive bidding schedules  $\sum_{k=1,2} T_0^k(a^k)$  requires to at least give  $A$  a total amount  $\sum_{k=1,2} T_0^k(0)$  such that

$$(A.48) \quad \sum_{k=1,2} T_0^k(0) + \sum_{k=1,2} \bar{T}_1^k(0) = \sum_{k=1,2} \bar{T}_1^k(1) = \sum_{k=1,2} \max\{\Pi_1(0, 1) - \bar{\Pi}_1^k; 0\},$$

where the right-hand side equality follows from (7.8) when applied to  $F_1$ .

From (7.9), we know that  $\bar{\Pi}_1^k = \Pi_1(0, 0)$ . We can thus rewrite condition (A.48) as follows:

$$(A.49) \quad \sum_{k=1,2} T_0^k(0) + \sum_{k=1,2} \max\{\Pi_1(0, 0) - \Pi_1(0, 0); 0\} = \sum_{k=1,2} \max\{\Pi_1(0, 1) - \Pi_1(0, 0); 0\}.$$

Because  $\Pi_1(0, 1) > \Pi_1(0, 0)$ , we obtain the following:

$$\sum_{k=1,2} T_0^k(0) = 2(\Pi_1(0, 1) - \Pi_1(0, 0)).$$

The maximal payoff that a multilateral deviation towards  $(0, 0)$  could yield to  $F_0$  is thus

$$\Pi_0(0, 0) - 2(\Pi_1(0, 1) - \Pi_1(0, 0)) = \bar{\Pi}_0^u.$$

This payoff is also obtained at the *pairwise-proof quasi-equilibrium* implemented with truthful schedules. This shows that multilateral deviations are not attractive for  $F_0$ .

We now turn now to  $F_1$ 's benefit of a multilateral deviation. The best multilateral deviation for  $F_1$  should induce  $(1, 1)$ . In each market  $k$ , we would then have  $a_{-1}^k = (0)$ . Thus, inducing  $(1, 1)$  with a pair of additive bidding schedules  $\sum_{k=1,2} T_1^k(a^k)$  requires to give  $A$  at least an amount  $\sum_{k=1,2} T_1^k(1)$  such that:

$$(A.50) \quad \sum_{k=1,2} \bar{T}_0^k(1) + \sum_{k=1,2} T_1^k(1) = \sum_{k=1,2} \bar{T}_0^k(0) = \sum_{k=1,2} \max\{\Pi_0(0, 0) - \bar{\Pi}_0^k; 0\}$$

where the right-hand side equality follows from (7.8) when applied to  $F_0$ .

From (7.9), we know that  $\bar{\Pi}_0^k = \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,0)$ . We can thus rewrite condition (A.50) as follows:

$$(A.51) \quad \sum_{k=1,2} T_1^k(1) + \sum_{k=1,2} \max\{\Pi_0(1,0) - (\Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,0)); 0\} \\ = 2(\Pi_1(0,1) - \Pi_1(0,0)) > 0,$$

where the right-hand side inequality follows from our assumption. Because  $(0,0)$  is constrained-efficient,  $\Pi_0(0,0) + \Pi_1(0,0) > \Pi_0(1,0) + \Pi_1(1,0)$ , and we can simplify (A.51) as follows:

$$(A.52) \quad \sum_{k=1,2} T_1^k(1) = 2(\Pi_1(0,1) - \Pi_1(0,0)).$$

Therefore, the maximal gain for a multilateral deviation towards  $(1,1)$  by  $F_1$  is as follows:

$$\Pi_1(1,1) - 2(\Pi_1(0,1) - \Pi_1(0,0)).$$

$F_1$  gains from a multilateral deviation since

$$\Pi_1(1,1) - 2(\Pi_1(0,1) - \Pi_1(0,0)) > \bar{\Pi}_1^u = \Pi_1(0,0)$$

amounts to

$$\Pi_1(1,1) - \Pi_1(1,0) > \Pi_1(1,0) - \Pi_1(0,0),$$

which holds thanks to Assumption 2. This shows that a multilateral deviation is attractive for  $F_1$ .  $\square$

#### PROOF OF PROPOSITION 9.

BIDS. Inserting the additivity requirement (7.12) into constraints (A.31) to (A.35) gives a characterization of the whole set of equilibrium transfers on split markets as those satisfying the following set of linear constraints:

$$(A.53) \quad 2\bar{T}_0(0) = 2\bar{T}_1(1) \in [\Pi_1(1,1) - \Pi_1(0,0), \Pi_0(0,0) - \Pi_0(1,1)],$$

$$(A.54) \quad \bar{T}_0(1) = \bar{T}_1(0) = 0,$$

$$(A.55) \quad \bar{T}_0(0) \geq \Pi_1(0,1) - \Pi_1(0,0) \geq 0,$$

$$(A.56) \quad 0 \leq \bar{T}_1(1) \leq \Pi_0(0,0) - \Pi_0(0,1).$$

The set of possible values for  $\bar{T}_0(0) = \bar{T}_1(1)$  satisfying all those constraints is non-empty whenever

$$(A.57) \quad \max\{2(\Pi_1(0,1) - \Pi_1(0,0)); \Pi_1(1,1) - \Pi_1(0,0)\} \leq \\ \min\{2(\Pi_0(0,0) - \Pi_0(0,1)); \Pi_0(0,0) - \Pi_0(1,1)\}.$$

Using Assumption 2, the left-hand side of (A.57) is

$$(A.58) \quad \max\{2(\Pi_1(0,1) - \Pi_1(0,0)); \Pi_1(1,1) - \Pi_1(0,0)\} = \Pi_1(1,1) - \Pi_1(0,0).$$

Still using Assumption 2, the right-hand side of (A.57) is

$$(A.59) \quad \min\{2(\Pi_0(0,0) - \Pi_0(0,1)); \Pi_0(0,0) - \Pi_0(1,1)\} = \Pi_0(0,0) - \Pi_0(1,1).$$

Gathering (A.58) and (A.59) and inserting into (A.57), the set of possible values for  $\bar{T}_0(0) =$



$\bar{T}_1(1)$  is non-empty whenever

$$(A.60) \quad \Pi_0(1, 1) + \Pi_1(1, 1) \leq \Pi_0(0, 0) + \Pi_1(0, 0),$$

but again, this property follows from the fact that  $(0, 0)$  maximizes the industry's profit.

Finally, observe that conditions (A.53) to (A.56) are necessary but also sufficient for an equilibrium with additive bidding schedules. Sufficiency follows from the existence of bids satisfying those constraints.

PROFITS. From (A.53), we deduce the following characterization of  $F_0$ ,  $F_1$  and  $A$ 's profits as follows:

$$(A.61) \quad \begin{aligned} \Pi_0(1, 1) &\leq \bar{\Pi}_0^s \leq \Pi_0(0, 0) + \Pi_1(0, 0) - \Pi_1(1, 1), \\ \bar{\Pi}_1^s &= \Pi_1(0, 0), \\ \Pi_0(0, 0) + \Pi_1(0, 0) - \bar{\Pi}_0^s &= \bar{\Pi}_a^s \geq \Pi_1(1, 1) - \Pi_1(0, 0). \end{aligned}$$

This ends the proof. □

## REFERENCES

- Aghion, P. and P. Bolton, 1987, "Contracts as a Barrier to Entry," *The American Economic Review*, 77: 388-401.
- Anderson, S. and S. Coate, 2005, "Market Provision of Broadcasting: A Welfare Analysis," *The Review of Economic Studies*, 72: 947-72.
- Armstrong, M., 1999, "Competition in the Pay-TV Market," *Journal of the Japanese and International Economies*, 13: 257-280.
- Aseff, J. and H. Chade, 2008, "An Optimal Auction with Identity-Dependent Externalities," *The RAND Journal of Economics*, 39: 731-746.
- Bachelor, B., K. Lent, M. Cragg and J. Keyte, 2020, "Convergence and Divergence in Assessing the Broadcasting Practices of Professional Sports Leagues," *Business Law International*, 21(2): 101-120.
- Bedre-Defolie, Ö. and G. Biglaiser, 2022, "Competition for Exclusivity of a Superior Product and Quality Implications," Available at <https://ssrn.com/abstract=4278448> or <http://dx.doi.org/10.2139/ssrn.4278448>.
- Bernheim, D. and M. Whinston, 1986a, "Menu Auctions, Resource Allocation, and Economic Influence," *The Quarterly Journal of Economics*, 101: 1-31.
- Bernheim, D. and M. Whinston, 1986b, "Common Agency," *Econometrica*, 54: 923-942.
- Bernheim, D. and M. Whinston, 1990, "Multimarket Contact and Collusive Behavior," *The RAND Journal of Economics*, 21: 1-26.
- Bernheim, D. and M. Whinston, 1998, "Exclusive Dealing," *Journal of Political Economy*, 106: 64-103.
- Brocas, I., 2013, "Optimal Allocation Mechanisms with Type-Dependent Negative Externalities," *Theory and Decision*, 75: 359-387.
- Calzolari, G. and Vincenzo Denicolò, 2015, "Exclusive Contracts and Market Dominance," *The American Economic Review*, 105: 3321-51.
- Choné, P. and L. Linnemer, 2015, "Nonlinear Pricing and Exclusion: I. Buyer Opportunism," *The RAND Journal of Economics*, 46: 217-240.

- Collard-Wexler, A., G. Gowrisankaran and R. Lee, 2019, ““Nash-in-Nash” Bargaining: A Microfoundation for Applied Work,” *Journal of Political Economy*, 127: 163-195.
- Crémer, J. and M. Riordan, 1987, “On Governing Multilateral Transactions with Bilateral Contracts,” *The RAND Journal of Economics*, 18: 436-451.
- Doganoglu, T. and J. Wright, 2006, “Multihoming and Compatibility,” *International Journal of Industrial Organization*, 24: 45-67.
- Dubois, P. and M. Sæthre, 2020, “On the Effect of Parallel Trade on Manufacturers’ and Retailers’ Profits in the Pharmaceutical Sector,” *Econometrica*, 88: 2503-2545.
- Dukes, A. and E. Gal-Or, 2003, “Negotiations and Exclusivity Contracts for Advertising,” *Marketing Science*, 22: 222-47.
- Esö, P., V. Nocke and L. White, 2010, “Competition for Scarce Resources,” *The RAND Journal of Economics*, 41: 524-548.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2001, “Press Advertising and the Ascent of the “Pensée Unique?”,” *European Economic Review*, 45: 645-51.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2002, “Press Advertising and the Political Differentiation of Newspapers,” *Journal of Public Economic Theory*, 4: 317-34.
- Gabszewicz, J., Laussel, D. and N. Sonnac, 2004, “Programming and Advertising Competition in the Broadcasting Industry,” *Journal of Economics and Management Strategy*, 13: 657-69.
- Gal-Or, E. and A. Dukes, 2003, “Minimum Differentiation in Commercial Media Markets,” *Journal of Economics and Management Strategy*, 12: 291-325.
- Hagiu, A. and R. Lee, 2011, “Exclusivity and Control,” *Journal of Economics and Management Strategy*, 20: 679-708.
- Harbord, D. and M. Ottaviani, 2001, “Contracts and Competition in the Pay-TV Market,” University Library of Munich, Germany.
- Hart, O. and J. Tirole, 1990, “Vertical Integration and Market Foreclosure,” *Brookings Papers on Economic Activity: Microeconomics*, 205-276.
- Horn, H. and A. Wolinsky, 1988, “Bilateral Monopolies and Incentives for Merger,” *The RAND Journal of Economics*, 19: 408-419.
- Jéhiel, P. and B. Moldovanu, 2000, “Auctions with Downstream Interaction Among Viewers,” *The RAND Journal of Economics*, 31: 768-791.
- Kagel, J. and D. Levin, 2005, “Multi-Unit Demand Auctions with Synergies: Behavior in Sealed-Bid Versus Ascending-Bid Uniform-Price Auctions,” *Games and Economic Behavior*, 53: 170-207.
- Laussel, D. and M. Le Breton, 2001, “Conflict and Cooperation: The Structure of Equilibrium Payoffs in Common Agency,” *Journal of Economic Theory*, 100: 93-128.
- Martimort, D. and J. Pouyet, 2020, “Downstream Mergers In Vertically-Related Markets with Capacity Constraints,” *International Journal of Industrial Organization*, 72: 102643.
- Martimort, D. and L. Stole, 2012, “Representing Equilibrium Aggregates in Aggregate Games with Applications to Common Agency,” *Games and Economic Behavior*, 76: 753-772.

- Maskin, E. and J. Moore, 1999, "Implementation and Renegotiation," *The Review of Economic Studies*, 66: 39-56.
- McAfee, R. and M. Schwartz, 1994, "Opportunism in Multilateral Vertical Contracting: Non-Discrimination, Exclusivity, and Uniformity," *The American Economic Review*, 84: 210-230.
- McAfee, R. and M. Schwartz, 1995, "The Non-Existence of Pairwise-Proof Equilibrium," *Economics Letters*, 49: 251-259.
- Milgrom, P., 2007, "Package Auctions and Exchanges," *Econometrica*, 75: 935-965.
- Molnar, J. and G. Virag, 2008, "Revenue Maximizing Auctions with Market Interaction and Signaling," *Economics Letters*, 99: 360-363.
- O'Brien, D. and G. Shaffer, 1992, "Vertical Control with Bilateral Contracts," *The RAND Journal of Economics*, 23: 299-308.
- Peitz, M. and T. Valletti, 2008, "Content and Advertising in the Media: Pay-TV versus Free-To-Air," *International Journal of Industrial Organization*, 26: 949-65.
- Rey, P. and T. Vergé, 2004, "Bilateral Control with Vertical Contracts," *The RAND Journal of Economics*, 35: 728-746.
- Segal, I. and M. Whinston, 2003, "Robust Predictions for Bilateral Contracting with Externalities," *Econometrica*, 71: 757-791.
- Shaked, A. and J. Sutton, 1982, "Relaxing Price Competition through Product Differentiation," *The Review of Economic Studies*, 49: 3-13.
- Shaked, A. and J. Sutton, 1983, "Natural Oligopolies," *Econometrica*, 51: 1469-1483.
- Sonnac, N., 2012, "Médias Audiovisuels et Concurrence. Le Cas de la Télévision Payante," *Revue d'Economie Industrielle*, 137: 109-129.
- Spagnolo, G., 1999, "On Interdependent Supergames: Multimarket Contact, Concavity and Collusion," *Journal of Economic Theory*, 89: 127-139.
- Stennek, J., 2014, "Why Exclusive Distribution May Benefit the TV-Viewers," *Information Economics and Policy*, 26: 42-57.
- Toft, T., 2003, "Football: Joint Selling of Media Rights," *Competition Policy Newsletter*, 3: 47-52.
- Varma, G., 2002, "Standard Auctions with Identity-Dependent Externalities," *The RAND Journal of Economics*, 33: 689-708.
- Weeds, H., 2016, "TV Wars: Exclusive Content and Platform Competition in Pay TV," *Economic Journal*, 126: 1600-1633.
- Wilson, R., 1979, "Auctions of Shares," *The Quarterly Journal of Economics*, 94: 675-689.