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Résumé: Nous présentons un modèle simple d'investissements en capital humain qui peut tenir compte d'une grande hétérogénéité entre agents, et nous étudions sa compatibilité avec certains modèles de recherche d'emploi et de salaire d'équilibre qui ont été proposés dans la littérature. Nous montrons que l'équation de salaire en logarithme dérivée de la combinaison de ces modèles est additivement séparable dans le processus d'investissement en capital humain et les effets dynamiques de l'échelle des emplois sous certaines conditions parmi lesquelles figurent des constraintes de liquidité strictes et l'exogénéité de la recherche d'emploi. C'est le cas en particulier du modèle populaire proposé par Bagger, Fontaine, Postel-Vinay et Robin [2014] dans lequel l'équation de salaire prédite peut être généralisée pour tenir compte d'effets hétérogènes plus riches dus à l'accumulation endogène de capital humain.

Abstract: We review a tractable model of human capital investments that can accommodate lots of heterogeneity and we investigate its compatibility with some job search and equilibrium wage models that have been proposed in the literature. We show that the log wage equation derived from the combination of these set-ups is additively separable in the process of human capital investments and the dynamic effects of the job ladder under a few conditions among which strict liquidity constraints and exogeneity of search are prominent. This is the case in particular with the popular model proposed by Bagger, Fontaine, Postel-Vinay and Robin [2014] in which the predicted wage equation can be generalized to accommodate richer heterogeneous effects due to endogenous human capital accumulation.

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Keywords: Human capital, job search, wage inequalities, applied econometrics

Mots clés: Capital humain, recherche d'emploi, inégalités de salaires, économétrie appliquée

JEL codes: D31, I24, J24, J31, J64

"Perhaps the most widely estimated regression equation in economics is Mincer's log-earnings function that relates the log of individual earnings or wages to observed measures of schooling and potential work experience with a specification that is linear in years of schooling and quadratic in experience", Rubinstein and Weiss**,** [2006, p.3].

1 Introduction

The analysis of earnings processes over the life-cycle is motivated by different perspectives. First, a strand of the literature has taken a *policy perspective* in which it is argued that inequalities in cross-section are an imperfect snapshot of lifetime inequalities which should approximate better what inequalities of wages or labor earnings are for different individuals belonging to different cohorts (Bonhomme & Robin, 2009, Blundell, 2014). Furthermore, strong dynamic elements contribute to the formation of inequalities over the life-cycle (Bowlus & Robin, 2012), and the question of mobility that equalizes lifecycle earnings is key in this literature (Huggett, Ventura and Yaron, 2011).

Another strand of the literature has taken an *academic perspective* in which the investigation is focussed on identifying the factors that are at the root of increasing inequalities over the life-cycle as well as on the quantification of the respective role of these factors. Rubinstein & Weiss [2006] some years ago took stock of the various economic explanations of wage growth over the life-cycle using competing models of human capital, search, learning and insurance.

In this short note, we aim at updating these debates and investigate what has been learned in the last 20 years. This view will be slightly slanted towards the human capital perspective that we take with the aim of understanding how this perspective could be integrated into the search and wage equilbrium models that were published recently. We will first present a human capital investment model with lots of heterogeneity predicting that the logarithm of wages can be written as a linear factor model with simple factors (levels, growth and curvature in potential experience) in which individual-specific factor loadings have an economic interpretation.

We then turn to the integration of this model with job search and equilibrium wage models. We first interpret the time series properties of idiosyncratic shocks in the wage equation using

the elegant construction of job mobility and bargaining between firms and workers as derived by Postel-Vinay and Turon [2010]. This allows to start investigating the central property presented in this note that there are natural conditions under which there is additive separability in log wages of the effects of human capital investments and of mobility-inducing job search. These conditions include the absence of consumption smoothing and logarithmic preferences. The separability property does not always hold, and we present an example developed in Bowlus and Liu [2013] in which this property is not satisfied if search is endogenous.

Yet, we show that this property is satisfied in popular examples of equilibrium wage set-ups that we develop using two papers, the one by Burdett, Carillo-Tudela and Miles [2011] in which the negotiation game between firms and workers does not allow counter-offers, and the second one by Bagger, Fontaine, Postel-Vinay and Robin [2014] in which the bargaining game consists in sharing surplus between firms and workers. Other alternatives are described in the recent survey by Altonji, Hynsjö and Vidangos [2022].

The outline is as follows. Section 2 presents the model of human capital investments leading to a linear factor model. Next, Section 3 articulates this model with job search ones proposed in the literature. Finally, Section 4 states the formal result of *separability* of the effects of human capital investments and job search, and opens up the discussion towards more general aspects such as the existence of macro shocks, the predictions of the profile of variance of wages as well as extensions that relax the most stringent assumptions of these set-ups.

2 Human capital investments

The notion of investments in human capital encompasses somewhat different directions turning around the idea that time, or labor, can be used in the present by economic agents to learn and increase their economic productivity in the future.

We first briefly summarize what the main directions the economic literature on investments in human capital has taken in recent years with a specific slant towards adapting these models to job search in the following section. Next, we present a specific human capital investment model that leads to the justification of a linear factor representation for wages and human capital stocks (both in logarithms), and finally show how to extend this convenient model to

multiple sectors or states among which unemployment.

2.1 A brief summary of the literature

We are building upon the directions explored in the review of Rubinstein and Weiss [2006] and Das and Polachek [2019].

A first direction is taken by the theory of learning-by-doing whereby learning is acquired while working as a simple joint-product of supplying hours of work. Returns to hours of work are in consequence not only obtained in the present under the form of a hourly wage but also by future returns. This deeply modifies the interpretation of the evolution of wages and hours over the life-cycle as has been made clear by Imai and Keane [2004] and Keane and Wasi [2015]. Learning-by-doing is also the retained assumption, because of its simplicity, in most developments in job search, but not all, that we review below.

Learning is however passive, and under the indirect control of agents through the number of hours of work they provide to the market (Killingsworth, 1982). This is an implication that can be tested. Belley [2017] finds that it is rejected in the sense that given hours of work and current wages, agents overreact to anticipated future events (future hours of work, expected fertility, occupations) when looking at their wage growth so that learning is not fully passive. Agents seem to have more control on the amount of time that they invest in human capital conditionally on hours of work and wages. Blandin [2018] also documents that learning-by-doing cannot explain the estimated decrease of earnings growth rate late in the life-cycle.

A more recent direction taken in the literature is dealing with firms investing in their workers through formal training (see Fu, 2011, Lentz and Roys, 2015, Blundell, Costa-Dias, Goll and Meghir, 2020, and others). It sets questions of specifity and generality of human capital investments, and more generally questions about the bilateral relationship existing between a firm and a worker and how the surplus between them is shared. This is a dimension that could make the match between human capital and search of a different nature than the one we identify in this paper and is certainly at the top of the research agenda even if we do not explore it here. Another direction is the learning of abilities on-the-job in different occupations as exemplified in Guvenen [2007] and Papageorgiou [2014].

The final direction of research regarding human capital investments whose seminal paper is

Ben Porath [1967] is the one we set out in the following. Human capital investments are decided by workers as a function of rates of return, costs of investment and horizon of investment. These investments are the inputs of a human capital production function that might depend on existing stocks. These human capital products are accumulated into human capital stocks next period, possibly net of depreciations. Labor supply is fixed so that the terms earnings and wages are used interchangeably. A few papers developed a richer analysis in the 1970s (e.g. Blinder and Weiss, 1976, Heckman, 1976).

The decision process of agents is described in most of the literature by either: (1) maximizing expected discounted life-time earnings involving "income smoothing"; or (2) by maximizing expected discounted life-time log-earnings. The latter is justified by a model in which current-period utility is the logarithm of earnings and thereby assumes out consumption smoothing. In the continuous time set-up of Ben Porath [1967], hypothesis (1) is adopted and the accumulation process is obtained by trading-off returns at different dates. Because of decreasing returns in the production of human capital, returns appropriately discounted are equalized over time.

The first prediction of this literature is that wages are increasing over time because agents invest although wage growth decreases over time because of the horizon effect and decreasing investments (Lillard and Reville, 1999). Namely, the horizon getting shorter and shorter, returns are decreasing as well as investment levels until they become close to zero near the retirement age, or negative because of depreciations. This explains that wage or earnings profiles are increasing and concave. Moreover, Mincer [1974] proposed approximating the logarithm of wages as a quadratic function in experience. This simple proposal became extremely popular and was used in many empirical studies. Furthermore but less well-known, Mincer [1974] made also the point that the variance of wages was U-shaped over the life-cycle. This is because high-return earners are still investing time in human capital production when they start working while low-return earners are working more and investing less. This difference flips around after a few years, 5 years being the estimate derived by Mincer. This stylized U-shaped variance profile along the life-cycle is called the Mincer dip.

This rather blunt summary of the literature leads to the attempt of finding micro-foundations for a simple wage equation that would not be an approximation as in Mincer and could be used in economic models of job mobility and wages. The Ben Porath framework is not easy to deal

with in a formal model (see Bowlus and Liu, 2012, below for an instance in the job search literature in which this framework is used) and is not particularly convenient for estimation.

These micro-foundations are provided by Magnac et al. [2018] that we review now.

2.2 A convenient theoretical set up

We consider a human capital model delivering a log-wage equation which belongs to the class of factor models which are linear in potential experience and in a curvature term constructed from potential experience (Magnac, et al., 2018). This log-wage equation has parameters that are heterogeneous across individuals, interpretable economically, and belong to the set of earnings processes with lots of heterogeneity (Browning, Ejrnaes and Alvarez, 2010). In particular, an analysis of inequalities over the life-cycle remains possible even in a framework in which data is partially missing provided that some Missing at Random assumption is made as in the matrix completion literature (e.g. Bai and Ng, 2021). For the moment, it is useful to consider that the focus of interest is the observable wage process while the characteristics of firms in which agents work are not observable. In particular, we assume that human capital investments are single dimensional (see Lise and Postel-Vinay, 2020, Taber and Veljin, 2020 for alternatives) and that there are no tenure effects as in Buchinsky, Fougere, Kramarz & Tchernis [2010]. We return to these two branches of the literature among the extensions we review below.

The set-up is slightly different from Ben Porath [1967] and we detail differences when presenting the model.

An individual agent enters into the labor market at $t = 0$, and invests in human capital with a horizon date denoted R which may or may not be an exogenous retirement date. Her potential earnings are given by:

$$y_i^P(t) = \exp(\delta_i(t))H_i(t),$$

in which $\delta_i(t)$ is the (log)-price of human capital stock that is denoted $H_i(t)$. The *current-period* earnings net of investments in human capital:

$$y_i(t) = \exp(\delta_i(t))H_i(t)\exp(-\tau_i(t)) \tag{1}$$

in which $1-\exp(-\tau_i(t))$ can be interpreted as the fraction of working time devoted to investing

in human capital. Variable $\tau_i(t)$ is somewhat loosely speaking a measure of the "investment" in human capital.

The accumulation of human capital which depends on its production function is given by the dynamic equation of motion for human capital stocks :

$$H_i(t+1) = H_i(t)\exp[\rho_i\tau_i(t) - \lambda_i(t)], \tag{2}$$

which depends on ρ_i interpreted as a "rate of return" to human capital and on $\lambda_i(t)$ which is akin to a depreciation rate.

The *current-period* utility function is the logarithm of consumption, and absent consumption smoothing, the utility is given by

$$u_i(t) = \log(y_i(t)) - c_i\frac{\tau_i(t)^2}{2},$$

in which $c_i > 0$ is the parameter indexing the convex cost of investing. Using equation (1) to replace $y_i(t)$, we end up with the specification that:

$$u_i(t) = \delta_i(t) + log\ H_i(t) - \tau_i(t) - c_i\frac{\tau_i(t)^2}{2}.$$

It is interesting at this stage to note that the formulation of Ben Porath [1967] diverges with the one presented here in two ways. Ben Porath [1967] does not consider any costs of investments, and imposes the constraint $c_i = 0$. Second, equation (2) makes sure that the productivity of investments is never equal to ∞ when $\tau_i(t) = 0$ while Ben Porath's formulation does. Ben Porath has thus the prediction that human capital investments are always positive while the presented model allows for no investment at all depending on parameter values. Equation (2) could also easily be extended by introducing a non linear effect of $H_i(t)$ say $H_i(t)^\gamma$ as in the Ben Porath's production function although it introduces an additional factor without modifying the final linearity of the log-wage equation.

In this dynamic model, the discount rate is assumed homogeneous and denoted β. The last individual-specific parameter concerns the terminal value of human capital at the last period:

$$W_R(H_i(R)) = \delta^* + \kappa_i \log H_i(R).$$

For instance, if pensions are proportional to earnings, or human capital stocks, the discount rate is the same pre- and post-retirement and the horizon is infinite, $\kappa_i = \frac{1}{1-\beta}$. We assume that

in fact the terminal value is lower because of a decrease in the survival probability:

$$\kappa_i = \frac{1}{1 - \beta \Pr(\text{Survival})} \leq \frac{1}{1-\beta}.$$

This gives rise to the "curvature" of the log-wage equation.

As shown in Magnac et al. (2018), this set-up generates a condition on individual specific parameters (i.e. $\beta \rho_i \kappa_i < 1$) under which human capital investments stop before horizon R, and wages become a process determined by the evolution of log-prices and depreciation. When this condition is not fulfilled, investments remain positive and human capital investments are decaying over time and are given by the investment profile:

$$\tau_i(t) = \frac{1}{c_i} \left\{ \rho_i \left[\frac{\beta}{1-\beta} + \beta^{R-t} (\kappa_i - \frac{1}{1-\beta}) \right] - 1 \right\} > 0, \quad \forall t < R.$$

Furthermore, the logarithm of earnings in period t is written as a three factor model:

$$\log y_i(t) = \eta_{i0} + \eta_{i1} t + \eta_{i2} \beta^{-t} + \delta_i^y(t), \tag{3}$$

in which individual-specific random shocks are:

$$\delta_i^y(t) = \delta_i(t) - \sum_{l=0}^{t-1} \lambda_i(l). \tag{4}$$

Shocks are individual prices of human capital stocks (in logs) net of depreciation of human capital, and we will turn in the next section to the interpretation of these shocks using a job search framework as derived in Postel-Vinay and Turon [2010].

The linear factor representation for log-wages in equation (3) has a structural interpretation and confers an interpretation to the reduced-form dynamic models of earnings that are used (see Meghir and Pistaferri, 2010, or Blundell, 2014). In particular, the reduced form factors loadings η_{i0}, η_{i1}, η_{i2} are functions of individual-specific structural parameters ρ_i, c_i and κ_i as well as of the initial level of human capital, $H_i(0)$. In particular, this specification embeds processes that are called heterogenous income profiles (HIP) or random income profiles (RIP) for earnings equations (e.g. Guvenen, 2007).

We now briefly comment on how to deal with unemployment in order to integrate this set-up into job search models in which unemployment yields a convenient reservation value for workers.

2.3 Unemployment

The human capital model that delivers wage equation (3) can be extended to economies in which there are two sectors (see Gobillon et al., 2022). Unemployment is a candidate for being this alternative sector. Returns to investments are allowed to be different in the two sectors, and we can then allow for career interruptions for unemployment reasons as job search models do (e.g. Bagger et al., 2014). Denote $x_t^{(3)}$ the length at potential experience t of these interruptions and let $x_t^{(4)}$ be a specific weighted average of these interruptions. Real experience is defined by $t - x_t^{(3)}$, and the adapted wage equation writes:

$$\log w_i(t) = \eta_{i0} + \eta_{i1} t + \eta_{i2} \beta^{-t} + \eta_{i3} x_t^{(3)} + \eta_{i4} x_t^{(4)} + \delta_i^y(t). \tag{5}$$

in which the additional coefficients η_{i3} and η_{i4} are measures of (likely negative) individual specific effects of interruptions. In particular, these effects could include an accelerated depreciation of skills or a low rate of return to human capital investments when unemployed.

We now take (log-)wage equations (3) or (5) as given and investigate what the conditions in job search models are satisfied to accommodate such equations. Job search models are focussing not only on the univariate process of wages as human capital models do but also on the way agents change jobs, and the association of wage changes with job changes. This is by considering this association between these two processes that the marriage between the two set-ups of human capital investments and job search can be obtained under conditions that we now explore.

3 Articulating human capital investments with job search models

We first investigate whether job search models can provide an interpretation of the shocks in wage equation (3) derived in the previous section. We then turn to describing the way human capital is modelled in well known analyses of endogenous job search and equilibrum wage models. In all models we describe we adopt that labor contracts are of the piece rate type and not more elaborated ones like wage-tenure contracts e.g. as in Burdett and Coles [2003] or Balke and Lamadon [2022] in the recent literature. We return to these points in the extensions

below.

3.1 Self-enforcing wage contracts

Wage shocks are written in equation (4) above and are a compound of idiosyncratic prices of human capital stocks and depreciations. Persistence in log wages can stem from the depreciation terms, e.g. the terms $\lambda_i(s)$ that accumulate over time but not only. The second argument for having persistent shocks originates from the theory of self-enforcing contracts as proposed by Thomas and Worall [1988], and explored in Postel-Vinay and Turon [2010] in job search models. The intuition is that the surplus created by a match between a firm and a worker is shared between them in a way that satisfies two *participation* conditions at each period (1) a negative productivity shock to the match quality of the job might induce the firm to renegociate the wage down (2) another competing firm might induce the worker to renegociate the wage up either in the current firm or in the poaching one.

This is a first approach to the compatibility issue between human capital models and job search as seen from the point of view of the human capital literature. Job search and labor market mobilities are treated as market frictions, and it is the sequence of wages over the life-cycle which is the object of interest and not the joint process of wages and mobility which is the focus of job search models.

For the sake of consistency with the human capital investment model developed in the previous Section 2, we adopt a discrete-time approach. Furthermore, the environment is assumed stationary, the horizon infinite and workers risk neutral in the original version of Postel-Vinay and Turon [2010] but none of these assumptions are necessary to proceed with the argument that we state here.

Firms are collections of unrelated jobs and are risk neutral. Output in a job is the product of individual productivity, p, and a match-specific shock ε, independent across matches and which take values into a bounded interval $[\varepsilon_{\min}, \varepsilon_{\max}]$. Job shocks affecting the quality of matches are drawn each period. Wage paid is w so that firm profit is equal to $p\varepsilon - w$. Matches terminate at exogenous rate δ, and new job offers are made to employed workers at rate λ_1 and to unemployed at rate λ_0. Unemployed are also supposed to receive a fraction, $b \leq \varepsilon_{\min}$, of their productivity, p, and their surplus when they are offered a job is $w - pb$. If we denote the worker share of

output $r = w/p$ then r can take any value between b and $\varepsilon_{\max}$ and:

$$\log w = \log p + \log r,$$

in which $\log p$ is the productivity given by the deterministic terms in equations (3) or (5). The logarithm of piece rate, $\log r$, is affected by the sequence of match specific shocks ε, exogeneous termination at rate δ and rates of arrival of offers.

In this specific instance, all bargaining power is given to firms although it could be generalized (see below). For unemployed, the minimal acceptable offer is $r = b$. For employed at piece-rate r, several things can happen that will modify the bargaining position of the worker: a layoff, a productivity shock at the same firm or an offer from a poaching firm. The main drivers of the dynamic process of earnings shocks are the following: A negative productivity shock in the firm might lead to a negative profit and wage renegotiation at a lower level; A poacher firm might propose a competitive log-wage above the current level, and the worker could either quit or renegociate his wage upwards.

This delivers a first order Markov process for the random shocks in wage equation (3) that depend on productivity shocks in the current and poaching firms as well as on rates of arrival of offers. Nonetheless, this construction does not affect the human capital acccumulation in the model provided that the same assumptions as in the previous section are made, i.e. utility is logarithmic in consumption and no consumption smoothing is allowed. Both processes are compatible because of the main assumption that log wages are linear in the logarithms of human capital stocks and of piece rates that the workers are paid by firms. It is important to note that this argument applies because assumptions in the two set-ups can be made compatible.

We will show below that the same logic of additive separability of two processes - human capital accumulation and job search or ladder – may apply to other job search papers although not to the one that we present now.

3.2 Endogenous search

The model of endogenous search we consider after Bowlus & Liu [2013] does not satisfy that human capital accumulation and job search are separable. Indeed, the authors show that wage growth rates over the life-cycle are explained by a combination of human capital accumulation

(50%) and endogenous search (20%) while interactions between these two factors explain the remaining 30%.

In this paper, workers maximize expected earnings and a Ben Porath's human capital investment model (with decreasing returns to scale) is considered. The life-cycle is limited to periods 1 to R, and human capital does not depreciate. Workers face a rental rate of human capital which is exogenous and log normally distributed. Workers start as unemployed and job offer arrival rates depend on workers' effort in search using a isoelastic cost of effort. As a result, closed forms of value functions, as in the previous section, are not available although the model can be solved numerically by backward induction. Allowing for heterogeneity in human capital accumulation as well as job search, the parameters are estimated using indirect inference and US data from NLSY and SIPP.

Endogeneity of search and risk neutrality are key to the result that the value functions cannot be linear in the logarithm of productivity or human capital stocks as they are in the human capital investment model we have considered. Specifically, additional search intensity whose cost is paid in the current period has a multiplicative effect on the returns next period and this breaks the log-linearity of the effect of human capital investments.

The non separability between human capital and search leads to three interactions. First, agents invest more because the rental rates are more dispersed because of job search. Second, more effort is devoted to search since returns depend on human capital accumulation in the future. Third, unemployed are ready to lower their reservation values because of the possibility of accumulating human capital if they exit more quickly.

3.3 An equilibrium wage framework

Without endogenous search, Burdett, Castrillo-Tudela & Coles [2011] combines human capital accumulation with a relatively simple equilibrium wage set up. They argue that "[a] most useful feature of our approach is that individual wages remain consistent with a standard Mincer equation" (p.659). It uses a mechanical learning-by-doing approach but nothing prevents to adapt their approach to the endogenous accumulation process that we presented in the previous section.

Firms and workers are risk neutral and workers cannot smooth consumption over time.

Workers are heterogeneous in their productivities while firms are homogeneous. Overlapping generations of workers enter and exit the labor market with constant probability and the model is stationary. Workers accumulate human capital by learning-by-doing while working but not in unemployment where human capital stocks remain constant. Returns to experience are assumed to be log-linear. All payoffs are linear in productivity, and workers are paid using piece rates. Unemployed receive a compensation which is also linear in productivity bp. Unemployed can however accept offers which pay less than b because of gaining experience in exchange. Finally for the sake of simplicity, firms do not counter offers of poaching firms to their workers, and wage contracts are not related to tenure.

As a result of linearity of all payoffs in productivity, value functions are also linear in productivity yielding a Mincer equation :

$$\log wage = \log productivity + \log piece\_rate + \rho Experience.$$

Because of this additive separability assumption, this specification allows human capital accumulation to be optimized in a separable way as before. If in addition utility is logarithmic, we can use the human capital model presented above.

An endogenous wage equilibrium is derived in terms of a stationary distribution of experience among unemployed and among the employed given their productivity as well as the distribution of piece rates in the economy. The latter distribution satisfies the usual properties of job search models that it has no mass point, and that its support lies above the reservation wage of the unemployed.

One of the most original point of this approach is the resulting sorting of workers in firms in terms of experience at equilibrium. Namely, larger piece-rate firms have more experienced workers than those who offer low piece-rates. This generates an important fraction of wage inequality between young and inexperienced workers, and older and experienced ones.

The effect on inequalities is further evaluated in Burdett et al. [2016] which now considers that firms are heterogeneous. In this case, the separability between experience in the sense of Burdett et al. (2011) and job search effects breaks down. The logarithm of wages is indeed written as a function which is additive in experience and the specific firm (log) productivity. Moreover, the expectation of this firm effect conditional on experience is shown to be increasing

and concave in experience (note 9, page 25) because the firm that workers choose to work in is not random. It does express the lack of separability between firm and individual effects.

3.4 Equilibrium search and bargaining

Bagger, Fontaine, Postel-Vinay and Robin [2014] jointly models wages, unemployment and working spells as well as job mobility. Time is discrete, and firms and workers are heterogeneous. Workers have a logarithmic utility. When working, there is a probability of job destruction and rates of arrival of offers for employed (λ_1) and unemployed (λ_0). Unemployed receive a compensation proportional to their productivity or their human capital. Piece rates are used as in the previous paper, while a bargaining game between firms and workers is now played. Namely, the rules of this game are that firms and workers share the surplus generated given the threat points that each agent have. Finally, human capital accumulation is given by learning-by-doing and therefore related to periods of employment.

The authors show that log-wages are the sum of firm productivities, piece rates and learning-by-doing all expresssed in logs so that:

$$\log(w_{it}) = \underbrace{\alpha_i + g(t_i) + \varepsilon_{it}}_{\text{Individual productivity}} \underbrace{+p_{ij(t)} + r_{ij(t)}}_{\text{Search and matching}}, \tag{6}$$

in which $p_{ij(t)}$ is the productivity of the match with firm j and $r_{ij(t)}$ is the endogenous piece-rate obtained when working in firm j. Individual-specific productivity is composed of a level term α_i, learning-by-doing human capital $g(t_i)$ as well as an individual specific AR(1) process, ε_{it}, to allow for persistence of shocks as in Section 3.1 above. Additive separability between human capital and search again holds and implies that the two sources of wage growth are "autonomous". There is no sorting in equilibrium although Bagger et al. [2014] allows for dependence between the two blocks (human capital investments and job search) since individual-specific parameters may co-vary (e.g. offer rates λ_{i1} and λ_{i0}, and individual effects α_i can be correlated).

The empirical part of this paper is using employee-employer data (from Denmark) on wages, job and unemployment spells and firm output and is estimated using indirect inference. Results point to the decomposition between human capital formation and job search. Human capital is the most important factor for high-skilled workers, and job shopping is significant for all

education groups during the first ten years of life-cycle profiles. Wages do not grow much within firms.

Furthermore, the decomposition of the variance of earnings is possible as in Burdett et al. [2011] by using equation (6). Among the most salient results, the variance due to firm productivities is the main driver of the variance of wages during the first part of the life-cycle while the variances of individual productivity and shocks are relatively small. The importance in the variance of log wages of potential experience is also significant but at a declining rate over time. This paper has led to other analyses in which parameters of human capital formation and job ladder are correlated, and this allows for general forms of sorting (Engblom, 2021, Ozkan, Song and Karahan, 2022).

4 Summary and extensions

As a summary of results described in previous sections, job search models accommodate simple processes of human capital accumulation as the one described in Section 2 under a few conditions. The first one is that current utility is logarithmic in consumption or earnings and the second one is that endogenous search is absent. The assumption that there is no consumption smoothing in those job search models as well as in the human capital set-up of Section 2 is also a key ingredient of this result.

We can then summarize this result as:

Proposition 1 *Under the conditions of Bagger et al [2014], the log wage equation can be written as:*

$$\log(w_{it}) = \underbrace{\eta_{i0} + \eta_{i1}t + \eta_{i2}\beta^{-t} + \eta_{i3}x_t^{(3)} + \eta_{i4}x_t^{(4)} + \varepsilon_{it}}_{Human\ capital} \underbrace{+p_{ij(t)} + r_{ij(t)}}_{Search\ and\ matching}, \tag{7}$$

in which $\eta_i s$ are individual specific parameters given in Gobillon et al. [2022].

A sketch of a formal proof of this result would go as follows. As in Section 2, fix the sequence of non-negative human capital investments, possibly equal to zero over time until the retirement date. Adopt the assumptions of Bagger et al. [2014] including that utility is equal to the logarithm of earnings. Next, prove that equation (6) holds as a function of human capital

stocks at each period. We end up in the same situation as in Section 2, and we can optimize the sequence of non-negative human capital investments considering that the job-search sequence remains the same. As a result, we can additively separate the human capital and job search processes as in equation (6). To finish, we would have to adapt the proof to the case in which there are two sectors, one of which is unemployment as developed at the end of Section 2.

It is immediate to generalize sorting models by allowing correlations between the individual specific parameters affecting human capital accumulation and job search in equation (7) since they all have structural interpretations.

Possible extensions to this research line go into different directions. We here select a few of them because they play a role in Mincer equations, or because they relax strong assumptions underlying the separability property. We start with the so-called "flat spot" issue which allows aggregate macro shocks to be included in a human capital framework although much less easily in a job search model. We continue with a prediction about variances of the logarithm of wages that are U-shaped over the life-cycle, the so-called Mincer dip and that empirical job search studies seem to have overlooked. We next return briefly to the literature on wage-tenure contracts and multidimensional skills. We end up looking at models when saving and/or borrowing are allowed.

Flat spots It is well known that using panel data information on log-wages, it is impossible to identify separately time, cohort and age effects without further information. In the specific human capital set-up we considered, this also means that we cannot identify prices of human capital from stocks when analyzing wages. Heckman, Lochner and Taber [1998] proposed an ingenious solution to this identification issue. The specific framework they consider is that units of labor provided by different groups at each period (e.g. by education and cohorts) are imperfectly substitutable in production while labor units within groups are perfectly substitutable. They note that prices of human capital can then be estimated using cohorts that at a specific period have stopped investing in human capital, say the 50-55 age group with a specific education. For instance, Gobillon et al. [2022] estimate those prices of human capital for male labor in France and for cohorts entering the labor market between 1985 and 1993.

[Figure 1 about here]

Results are reported in Figure 1. Specifically, it shows that prices of human capital stocks for the less skilled group have increased over the period of observation (1985-2011] slightly more than the high-skilled workers. This places France in a strong contrast with the UK or the US where labor earnings have increased more rapidly at the top of the distribution (e.g. see the D9/D1 diagrams of D'Herbécourt, 2020]. These differences are likely to be due to the contrasting evolutions of the cost of labor over this period across countries as shown by Bozio et al. [2020].

This technique of "flat spots" is also used by Huggett, Ventura and Yaron [2011], and rigorously assessed in Bowlus and Robinson [2012]. The existence of macro shocks is not easy to reconcile with the job search models we have been considering (e.g. Moscarini and Postel-Vinay, 2018, for a survey) and taking into account macro shocks probably affects the separability between human capital investments and job search. A modelling device such as directed search might help as shown in Menzio, Telyukova and Visschers [2016].

Mincer dip Second, there is an interesting prediction of human capital models that was noted by Mincer in his 1974 book although it does not seem to have been taken up by the empirical job search literature. As investment returns are likely to differ between individuals, the high-return individuals are likely to invest more at the beginning of the life-cycle (and work and earn less) than those individuals having low-returns who invest less, and work and earn more. These differences in investments tend to disappear overtime and profiles of wages for high-return and low-return individuals cross. This implies that the variance of (log)-wages first decreases and later increases over the life-cycle, the bottom point being around 5 years after entry into the labor market after Mincer [1974]. For instance, Figure 2 displays the U-shaped profile of variances using the same data on French males described above.

[Figure 2 about here]

In the job search literature however, human capital is presumed important during the second part of the life-cycle only while job search would contribute more to the explanation of what happens in the first part. This contrasting result means that allowing for a lot of heterogeneity might lead to reject the common interpretation that initial periods in the life-cycle are devoted to job hopping without any impact of human capital investments.

Another neglected aspect is related to the correlations across time. Rubinstein and Weiss [2006] argued that we should find negative correlations between wage level and wage growth at the beginning of the career and positive correlations at the end. This is confirmed in the empirical analysis of Magnac and Roux [2021] for human capital but it does not seem to be known whether job search models are able to generate the same prediction.

Optimal wage tenure profiles The seminal paper by Burdett and Coles (2003) shows that the profile of optimal wage contracts in the absence of consumption smoothing is increasing because they lead to optimal mobility between jobs. This work is compatible with the "separability" result. In particular, their Theorem 2 can be used to show that the optimal wage-tenure contract is "proportional" to productivity.[1] The separability result between what is the effect of investments and what is the effect of job search with insurance holds in this paper as well. It means that wage-tenure profiles are shifted upwards uniformly when productivity or human capital investments increase.

Nonetheless, more recent papers have more elaborate dynamic contracts and they introduce in particular another dimension which refers to work incentives (effortwise). For instance, in Tsuyuhara (2016) or Balke and Lamadon (2022), the optimal wage-tenure contract is not only the result of controlling the mobility of workers as in Burdett and Coles (2003). It also aims at controlling moral hazard in unverifiable effort exerted by workers. To the best of my knowledge, it is very unlikely that the separability between productivity and wage tenure contracts holds in this context.

Multidimensional skills My main focus in this paper was to look at how the human capital investment literature and job search can meet in order to understand individual profiles of wages along the life cycle without using other variables like the identity of firms or of occupations between which workers can be mobile. On the other hand, the literature based on multidimensional skills use the information about occupations along the life cycle and focusses on the building up of skills that are useful in different occupations. For instance, Lise and Postel-Vinay (2020) have a matching structure embedded in a job search model in which the

[1] Provided that a typo in that Theorem is corrected in equation (21). The mass point probability should have u'() instead of u(.) in the denominator as confirmed by working out the missing mass in the previous equation.

accumulation of different skills is given as a function of current worker skills and the technology of the firm in which they work. Identification stems from observing wages as well as mobility between occupations that are described as a bundle of different skills. This richer structure allows the authors to depart from the separability assumption that is at the core of this survey paper.

Overall, any richer source of information than the limited one given by profiles of wages can allow to test and possibly reject the separability assumption. It remains that the simple combination of human capital and job search can be useful in a setting where researchers limit their focus to analyzing the growth and inequality of wages over the life cycle.

Intertemporal smoothing and other insurance devices We end this short note with a departure from the assumption of strict liquidity constraints underpinning most human capital investment and job search models that we have reviewed in this paper. This relaxation allows consumption smoothing, and therefore the possibility of savings and borrowings over the life-cycle. It first breaks the result that the wage equation (7) has a linear factor structure as was found in Section 2 because there are now two competing ways of moving resources from one period to the next, – through financial savings and through human capital investments – and these two sources interact with each other (see Magnac et al., 2018 for a derivation of the modified Euler conditions in this case). As a consequence, it also breaks the separability between job search impacts and human capital investments ones in wage equation (7). We have to turn to more general models at the risk of losing the intuition underlying the identification of the impacts in linear factor models with lots of heterogeneity.

The literature embedding consumption and savings processes into job search settings, and allowing for human capital accumulation is growing. Low, Meghir and Pistaferri [2010] and Huggett, Ventura and Yaron [2011] were among the first to propose such models as well as Altonji et al. [2013]. Those papers are reviewed in Altonji et al [2022]. Let us mention in particular Griffy [2021] who looks at inequalities generated in a model with search and human capital in which borrowing constraints affect those households which have no or few financial assets at the beginning of their working life. Even more generally, any informal or formal contracting between individuals through marriage, extended family or village economies helps

mitigating the risks related to earnings and thus affects human capital investments and search. This is a line of research which has been explored recently by Chiappori, Costa-Dias and Meghir [2018], or Gayle and Shephard [2019].

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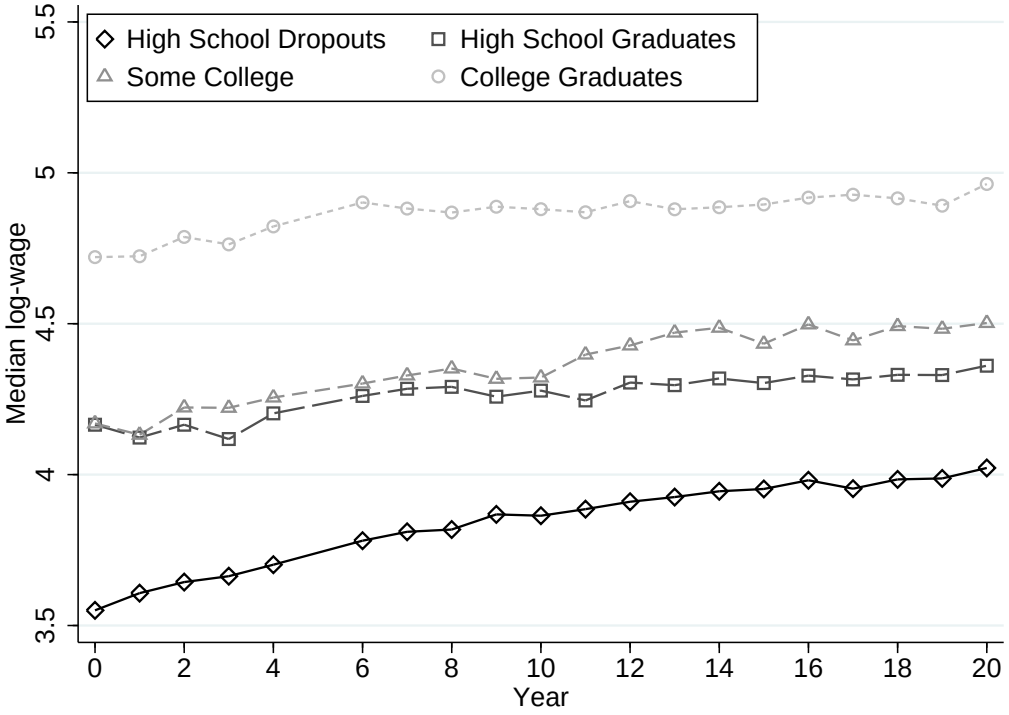
FIGURES

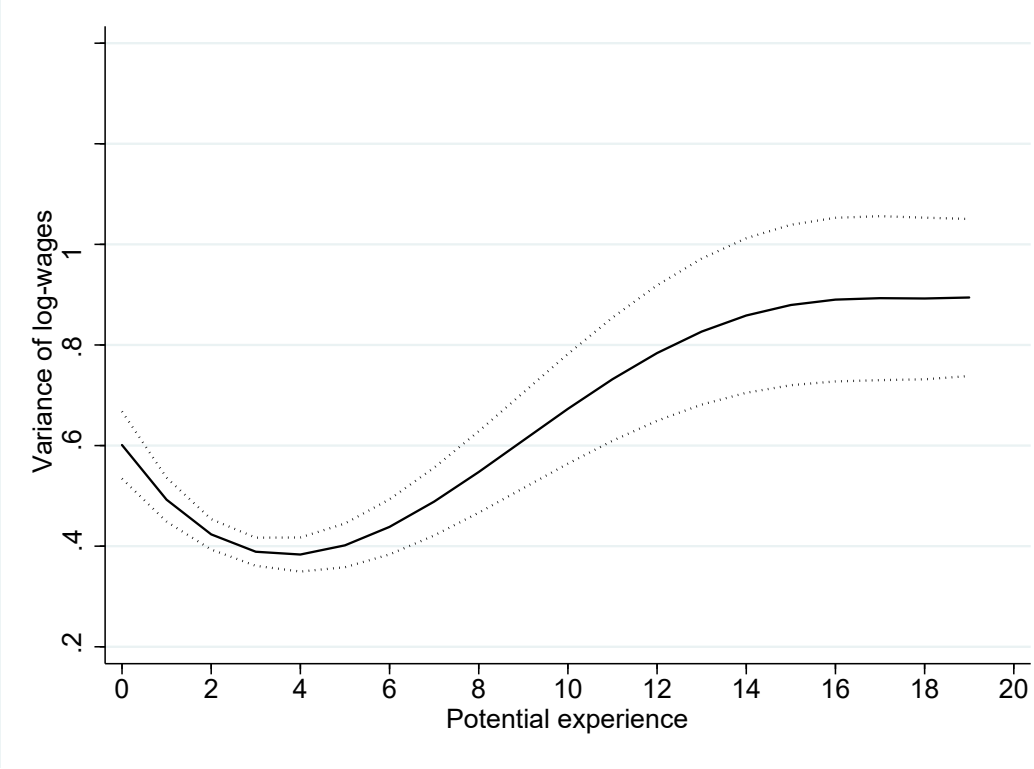
Figure 1



Source: Gobillon et al. [2022]. Male workers entering into the labor market between 1985 and 1993 and observed until 2011.

Figure 1: Estimated prices of human capital by education groups

Figure 2



Note: Bias-corrected complete model of potential wages. Source Gobillon et al. [2022]

Figure 2: Mincer dip